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Completly Revised Edition

Fast Track Objective ARITHMETIC

IAS CSAT | States' CSAT | SSC (10+2, CGL, CPO) SBI & IBPS PO/Clerk
LIC AAO | CDS | CMAT MAT & Other Management Entrances | Hotel Management
Railways | Paramilitary Forces | State Police Recruitments &
All Other Entrances, Recruitments and Aptitude Tests

Rajesh Verma

For Complete *Practice* and *Mastery* over **Arithmetic**

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Inspiring Minds.Inspiring Lives

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Fast Track
objective **Arithmetic**

Today, there is a plethora of books available in the market on Objective Arithmetic which seems to be complete in their way, but are still unable to fully satisfy the aspirants.

LET US KNOW SOME OF THE REASONS

Lack of Understanding the Basic Concepts

Mostly, students face a competitive examination on the base of their knowledge about mathematical rules, formulae and concepts. Inspite of having the knowledge, he lacks behind when he faces questions in the examination. Does he realise this inability? Yes, he does but feels confused and blocked when he is unable to solve them and is left with a sense of grudge that he could solve it. The only reason behind this problem is the understanding of basic concepts. If he would have been clear with them, he could solve any of the questions because as a matter of fact, every question is based on a particular concept which is just twisted in the examinations to judge the overall ability of a student.

Inappropriate Use of Short Tricks

This is the second biggest problem in front of the aspirants. The number of questions asked in the competitive examination is much more than the time assigned for diem. This leads die aspirants to use shortcut mediods. Although, diese mediods prove to be beneficial in some cases, but due to time management problems, he gets bound to use these methods irrationally and inappropriately. As a result, he jumbles between all the shortcuts which lead to wrong answers which could have been solved if he knew when and where to apply the shortcut methods.

Inability to Distinguish Between the Applications of Formulae

We all are aware of the amount of stress and pressure a competitive examination creates on the mindset of an aspirant. Succumbed to such pressure, an aspirant is unable to decide the appropriate formula to be applied in a particular step. During the crisis of time, such confusion adds to the problems and squeezes in more time and results to an unsatisfactory score.

Keeping in mind all kinds of problems faced by an aspirant in a competitive examination, we have developed diis book with profound interest in a step-wise method to encounter all your queries and worries. This book named 'FAST TRACK ARITHMETIC is wordiy to fulfill your expectations and will help you as loyal guide throughout.

OUTSTANDING QUALITIES OF FAST-TRACK OBJECTIVE ARITHMETIC

Use of Fundamental Formulae and Method

In this book, all the fundamental formulae and methods have been presented in such a striking yet friendly and systematic manner that just going through them once will give you an effective grasp. They have been present in such a manner that they, will never let you get confused between fast track technique and basic method.

Appropriate Short Cut Methods

An important feature of this book is its short cut methods or tricks given in the name of "fast track formulae or techniques". Each technique is given with its basic or fundamental method. So, that a student can use these tricks according to their desire and save there precious time in exams.

Division of Exercises According to the Difficulty Level

Based on the standard and level of difficulty of various questions, the exercises are divided into two parts *i.e.,* 'Base level exercise' for relatively easier questions and 'Higher skill level exercise' for difficult questions. 'Multiconcept questions' which requires a use of different concepts in a single question have also been incorporated with important chapters.

Special Emphasise on Geometry, Trigonometry and Mensuration

Now-a-days, Questions from geometry, trigonometry and mensuration are asked in large numbers in different exams. So, a large variety and number of questions are provided for tiiese chapters.

Completely Updated with Questions from Recent Exams

This book is incorporated with die questions from all the recent competitive exams, held in year 2013-14.

This book is a brain child of Mr Deepesh Jain, Director, Arihant Publications (India) Limited. Richa Agarwal, Diwakar Sharma and Shivam Mittal have given their best and sincere efforts for die completion and final presentations of die book.

The entire project has been managed and supervised by Mr Mahendra Singh Rawat and Mr Amit Verma. Aas Mohammed and Pradeep are to be complemented for very apt designing to the book cover. Amit Bansal and Mayank Saini have given their expertise in the layout of *the* book. Everyone's contribution for tiiis book is very special and is worthy of great applause. Reader's recommendations will be highly treasured.

With best compliments

Rajesh Verma

Contents

Chapter

01

Number System

A system in which we study different types of numbers, their relationship and rules govern in them is called as **number system.**

In the Hindu-Arabic system, we use the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. These symbols are called **digits.** Out of these ten digits, 0 is called an **insignificant digit** whereas the others are called **significant digits.**

Numerals

A mathematical symbol representing a number in a systematic manner is called a numeral represented by a set of digits.

How to Write a Number

To write a number, we put digits from right to left at the places designated as units, tens, hundreds, thousands, ten thousands, lakhs, ten lakhs, crores, ten crores.

Let us see how the number 308761436 is denoted

It is read as

Ten crores	Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Units
10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0
3	0	8	7	6	1	4	3	6

Thirty crore eighty seven lakh sixty one thousand four hundred and thirty six.

Face Value and Place Value of the Digits in a Number

Face Value

In a numeral, the face value of a digit is the value of the digit itself irrespective of its place in the numeral. *For example* In the numeral 486729, the face value of 8 is 8, the face value of 7 is 7, the face value of 6 is 6, the face value of 4 is 4, and so on.

Place Value (or Local Value)

In a numeral, the place value of a digit changes according to the change of its place.

Look at the following to get the idea of place value of digits in 72843016.

Crores 7 →	Place value of 7 →	$7 \times 10000000 = 70000000$
Ten Lakhs 2 →	Place value of 2 →	$2 \times 1000000 = 2000000$
Lakhs 8 →	Place value of 8 →	$8 \times 100000 = 800000$
Ten Thousands 4 →	Place value of 4 →	$4 \times 10000 = 40000$
Thousands 3 →	Place value of 3 →	$3 \times 1000 = 3000$
Hundreds 0 →	Place value of 0 →	$0 \times 100 = 0$
Tens 1 →	Place value of 1 →	$1 \times 10 = 10$
Units 6 →	Place value of 6 →	$6 \times 1 = 6$

It is clear from the above presentation that to obtain the place value of a digit in a numeral, we multiply the digit with the value of its place in the given numeral.

Types of Numbers

1. Natural Numbers

Natural numbers are counting numbers. They are denoted by N. *For example N = {1,2,3,...}.*

♦ All natural numbers are positive.

♦ Zero is not a natural number. Therefore, 1 is the smallest natural number.

2. Whole Numbers

All natural numbers and zero form the set of whole numbers. Whole numbers are

denoted by W.

For example W = {0,1,2,3,...}

♦ Zero is the smallest whole number.

Whole numbers are also called as non-negative integers.

3. Integers

Whole numbers and negative numbers form the set of integers. They are denoted by/.

For example / = {...,-4,-3,-2,-1,0,1,2,3,4,...}

Integers are of two types. (i) **Positive Integers** Natural numbers are called as positive integers. They are

denoted by *I* .

For example I^+ = {1,2,3,4,...}

(ii) **Negative Integers** Negative of natural numbers are called as negative integers. They are denoted by *I~. For example I~* ={-1,-2,-3,-4,...}

♦ '0' is neither +ve nor -ve integer.

4. Even Numbers

A counting number which is divisible by 2, is called an even number. *For example* 2, 4, 6, 8, 10, 12, ... etc.

♦ The unit's place of every even number will be 0, 2, 4, 6 or 8.

5. Odd Numbers

A counting number which is not divisible by 2, is known as an odd number.

For example 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, ... etc. ♦ The unit's place of every odd number will be 1, 3, 5, 7 or 9.

6. Prime Numbers

A counting number is called a prime number when it is exactly divisible by, 1 and itself.

For example 2, 3, 5, 7, 11, 13, ... etc.

♦ 2 is the only even number which is prime.

♦ A prime number is always greater than 1.

♦ 1 is not a prime number. Therefore, the lowest odd prime number is 3.

♦ Every prime number greater than 3 can be represented by $6n + 1$, where n is integer.

How to test a **Number is prime or not?**

If P = Given number, then

(i) Find whole number x such that $x > \sqrt{P}$.

(ii) Take all the prime numbers less than or equal to x.

(iii) If none of these divides P exactly, then P is prime otherwise P is non-prime.

For example Let $P = 193$, clearly $14 > \sqrt{(193)}$

Prime numbers upto 14 are : 2, 3, 5, 7, 11, 13.

No one of these divides 193 exactly.

Hence, 193 is a prime number.

7. Composite Numbers

Composite numbers are non-prime natural numbers. They must have atleast one factor apart from 1 and itself.

For example 4, 6, 8, 9, etc.

♦ Composite numbers can be both odd and even.

♦ 1 is neither a prime number nor composite number.

8. Coprimes

Two natural numbers are said to be coprimes, if their HCF is 1. *For example* (7, 9), (15, 16)

♦ Coprime numbers may or may not be prime.

9. Rational Numbers

A number that can be expressed as *p/q* is called a rational number, where p and *q* are inteqers and *a* ■*■ 0.

For example $\frac{3}{5}, \frac{7}{9}, \frac{8}{9}, \frac{13}{15}$ etc.

10. Irrational Numbers

The numbers that cannot be expressed in the form of *p/q* are called irrational numbers, where p and *q* are integers and *q* * 0.

For example -J2, V3, *-Jl,* VTT etc.

♦ 7C is an irrational number as 22 / 7 is not the actual value of *n* but it is its nearest value.

♦ Non-periodic infinite decimal fractions are called as irrational number.

11. Real Numbers

Real numbers include rational and irrational numbers both,

For example $\frac{7}{9}, \sqrt{2}, \sqrt{5}, \pi, \frac{8}{9}$ etc.

♦ Real numbers are denoted by *R.*

Operations on Numbers

Addition

When two or more numbers are combined together, then it is called addition. Addition is denoted by ' + ' sign. *For example* 24 + 23 + 26 = 73

Subtraction

When one or more numbers are taken out from a larger number, then it is called subtraction.

Subtraction is denoted by '-' sign.

For example 100-4-13=100-17=83

Division

When D and d are two numbers, then $\frac{D}{d}$ is called the operation of division, where

u is the **dividend** and d is the **divisor.** A number which tells how many times a

divisor (d) exists in dividend *D* is called the **quotient** Q.

If dividend *D* is not a multiple of divisor d, then *D* is not exactly divisible by *d* and

in this case **remainder** *R* is obtained.

Let us see the following operation of division

Let $D=17$ and $d=3$

Then, $\frac{D}{d}=\frac{17}{3}=5\frac{2}{3}$

Here, 5 = Quotient (Q),
3 = Divisor (d)
and 2 = Remainder (R)

We see,

3 (Divisor) × 5 (Quotient) + 2 (Remainder) = 17 (Dividend)

Hence, we can write a formula,

$$\boxed{\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}}$$

Multiolication

When *'a!* is multiplied by '£>', then *'a'* is added *'b'* times or *'b'* is added *'a'* times. It is denoted by 'x'.

Let us see the following operation on Multiplication If *a* =2 and *b* = 4, then 2x4=8

or (2+ 2+ 2+ 2) =8 Here, *'a!* is added *'b'* times or in other words 2 is added 4 times. Similarly, 4x2=8 or (4+ 4) =8 In this case, *'b'* is added *'a!* times or in other words 4 is added 2 times.

Divisibility Tests

Divisibility by 2 When the last digit of a number is either 0 or even, then the number is divisible by 2. *For example* 12, 86, 472, 520, 1000 etc., are divisible by 2.

Divisibility by 3 When the sum of the digits of a number is divisible by 3, then the number is divisible by 3. *For example* (i) **1233** 1 + 2 + 3 + 3 = 9, which is divisible by 3, so 1233

must be divisible by 3. (ii) 156 1 + 5 + 6 = 12, which is divisible by 3, so 156 must be divisible by 3.

Divisibility by 4 When the number made by last two-digits of a number is divisible by 4, then that particular number is divisible by 4. Apart from this, the number having two or more zeroes at the end, is also divisible by 4. *For example* (i) 6428 is divisible by 4 as the number made by its last two

digits *i.e.,* 28 is divisible by 4. (ii) The numbers 4300, 153000, 9530000 etc., are divisible by 4 as they have two or more zeroes at the end.

Divisibility by 5 Numbers having 0 or 5 at the end are divisible by 5.

For example 45, 4350, 135, 14850 etc., are divisible by 5 as they have 0 or 5 at the end.

Divisibility by 6 When a number is divisible by both 3 and 2, then that particular number is divisible by 6 also.

For example 18, 36, 720, 1440 etc., are divisible by 6 as they are divisible by both 3 and 2.

Divisibility by 7 A number is divisible by 7 when the difference between twice

the digit at ones place and the number formed by other digits is either zero or a multiple of 7.

For example 658 is divisible by 7 because 65 - 2 X 8 = 65 - 16 = 49. As 49 is divisible by 7, the number 658 is also divisible by 7.

Divisibility by 8 When the number made by last three digits of a number is divisible by 8, then the number is also divisible by 8. Apart from this, if the last three or more digits of a number are zeroes, then the number is divisible by 8. *For example* (i) **2256** As 256 (the last three digits of 2256) is divisible by

8, therefore 2256 is also divisible by 8.

(ii) **4362000** As 4362000 has three zeroes at the end. Therefore it will definitely divisible by 8.

Divisibility by 9 When the sum of all the digits of a number is divisible by 9, then the number is also divisible by 9. *For example* (i) 936819 9+3 + 6 + 8 + 1 + 9= 36 which is divisible by

9. Therefore, 936819 is also divisible by 9.

(ii) **4356** 4 + 3 + 5 + 6 = 18 which is divisible by 9. Therefore, 4356 is also divisible by 9.

Divisibility by II When a number ends with zero, then it is divisible by 10.

For example 20, 40, 150, 123450, 478970 etc., are divisible by 10 as these all end with zero.

Divisibility by 1' When the sums of digits at odd and even places are equal or differ by a number divisible by 11, then the number is also divisible by 11. *For example* (i) 2865423 Let us see

Sum of digits at odd places (A) = 2 + 6+4 + 3 = 15 Sum of digits at even places *(B) = 8 + 5 + 2 = 15 =>A = B* Hence, 2865423 is divisible by 11.

(ii) 217382 Let us see

Sum of digits at odd places (A) = 2 + 7 + 8 = 17

Sum of digits at even places (B) = 1 + 3 + 2 = 6

A- B = 17-6 = 11 Clearly, 217382 is divisible by 11.

Divisibility by 12 A number which is divisible by both 4 and 3 is also divisible by 12.

For example 2244 is divisible by both 3 and 4. Therefore, it is divisible by 12 also.

Divisibility by 14 A number which is divisible by both 7 and 2 is also divisible by 14.

For example 1232 is divisible by both 7 and 2. Therefore, it is divisible by 14 also.

Divisibility by 1! A number which is divisible by both 5 and 3 is divisible by 15 also.

For example 1275 is divisible by both 5 and 3. Therefore, it is divisible by 15 also.

Divisibility by II A number is divisible by 16 when the number made by its last 4-digits is divisible by 16.

For example 126304 is divisible by 16 as the number made by its last 4-digits *i.e.,* 6304 is divisible by 16.

Divisibility by 11 A number is divisible by 18 when it is even and divisible by 9. *For example* 936198 is divisible by 18 as it is even and divisible by 9.

Divisibility by 25 A number is divisible by 25 when its last 2-digits are either zero or divisible by 25.

For example 500, 1275, 13550 are divisible by 25 as last 2-digits of these numbers are either zero or divisible by 25.

Divisibility by 125 A number is divisible by 125 when the number made by its last 3-digits is divisible by 125.

For example 630125 is divisible by 125 as the number made by its last 3-digits are divisible by 125.

To Find a Number Completely Divisible by Given Number

Consider a number x_r which is when divided by d, gives a quotient q and leaves a remainder r. Then,

$$\begin{array}{l} d\,)\,x\,(\,q \\ \quad\overline{\;r\;} \end{array}$$

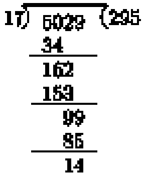
To find the number which is completely divisible by d such that remainder r is zero, follows the example given below.

Ex. 1 Find the number, which on (1) addition (2) subtraction from the number 5029 is completely divisible by 17.

Sol. Dividing 5029 by 17 we find. Remainder = 14

1. The minimum number on adding of which the given number is completely divisible by 17 = Divisor - Remainder = 17 - 14 = 3.

2. The minimum number on subtraction of which the given number is completely divisible by 17 = Remainder = 14.



Unit's Place of an Expression

Given expression can be of following two types

1. When Number is Given in the form of Product of Number

To find the units digit in the product of two or more number we take units digit of every numbers and then multiply them. Then, the unit digit of the resultant product is the units digit of the product of original numbers. *For example* 207x781x39x94

Taking units digit of every number and then multiplying them

= 7x1x9x4 = 7x36 [taking units place digit]

Again, taking units digit and then multiplying

= 7x6=42 .-. Units digit for 207 x 781x39 x 94 is 2.

2. When Number is Given in the form of Index

♦ If the unit's digit number are 0,1, 5 or 6, then the resultant unit's digit remains same.

For example, $(576)^{1151}$, its units digit is 6. $(155)^{120}$, its unit digit is 5.

$(191)^{19}$, its unit digit is 1. $(900)^{51}$, its unit digit is 0.

♦ If units place is 2, then the power of the number is first divided by 4 and there after represented in the form of 2 .

For example $(572)^{443}$

(—) i.e., $(2)^{443} = (2)^{\wedge 4}$ 'taking units place digit

$= (2^4)^{110} \times 2^3 = 2^4 \times 2^3$ [v $(2^4)^{110} = 2^4$]

= 16 x 8 taking units place digit Units place is 8

♦ In the same way, if units place digit are 4 or 8, then units digit for 4 and 8=6. e.g.,

$(124)^{372}$ taking units place digit = $(4)^{372} = (4^4)^{93} = 4^4$ =256 Unit's place digit = 6

♦ If units digit is 3 or 7, then units digit for 3 and 7 = 1.

For example $(2467)^{153}$ taking units place = $(7)^{153} \Rightarrow (7^4)^{38}$ *xf*

$\Rightarrow 7^4 \mathrm{X} 7^1 = 7^2 \times 7^2 \times 7 = 49 \times 49 \times 7$ taking unit's place digit

=9x9x7

= 81x7 taking unit's place digit

= 7 Unit's digit = 7

♦ If units place is 9 and if the power of 9 is even, then units digit will be 1 and if the power of 9 is odd, then units digit will be 9.

For example $(539)^{140}$

Since, power is even for unit's digit 9

Units digit =1$(539)^{141}$ Since, power is odd for unit's digit 9

Units digit =9

Basic Number Theory

♦ Square of every even number is an even number while square of every odd number is an odd number.

♦ A number obtained by squaring a number does not have 2, 3 ,7 or 8 at its unit place.

- Sum of first n natural numbers $= \frac{n(n+1)}{2}$
- Sum of first n odd numbers $= n^2$
- Sum of first n even numbers $= n(n+1)$
- Sum of square of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of first n natural numbers $= \left[\frac{n(n+1)}{2}\right]^2$

♦ There are 15 prime numbers between 1 and 50 and 10 prime numbers between 50 and 100.

♦ If p divides *q* and r, then p divides their sum and difference also.

e.g., 4 divides 12 and 20, then 20 + 12 = 32 and 20 -12 = 8 are also divisible by 4.

♦ For any natural number n, (n - *n)* is divisible by 6.

♦ The product of three consecutive natural numbers is always divisible by 6.

♦ (x^m - a''^1) is divisible by (x - a) for all values of *m.*

♦ *{*x^m - cr''^1) is divisible by (x + a) for even values of *m.*

♦ *{*x^m + a''^1*)* is divisible by (x + a) for odd values of *m.*

♦ Number of prime factors of a^p b^q c^r d^s is p + *q* + *I* + s, where a, *b,* c and d are prime number.

Multi Concept

QUESTIONS

1. ***If n is any odd number greater than 1, then n(n^2 -1) is***

(a) divisible by 96 always (b) divisible by 48 always

(c) divisible by 24 always (d) None of these

^ (c) Solving the question by taking two odd numbers greater than 1, *i.e.,* 3 and 5, then $n\{n^2 - 1)$ for $n = 3$ => 3(9-1) => 3x8 = 24

$n(n^2 - 1)$forn = 5 => 5(25-1) => 24x5 = 120 Using option we find that both the number are divisible by 24

2. ***$7^{6n} - 6^{6n}$, where n is a integer greater than 0, is divisible by***

(a) 13 (b) 127 (c) 559 (d) None of these

* (b) $7^{6n} - 6^{6n}$ for $n = 1$, $7^6 - 6^6$

=> $(7^3)^2 - (6^3)^2$ $\{a^2 - b^2 = (a+b)(a-b)\}$

=> $(7^3 - 6^3)(7^3 + 6^3)$ => (343 - 216) (343 + 216) => 127 x 559

∴ It is clearly divisible by 127,

3. ***Find the remainder of*** $\frac{17^{18^{19^{20\cdots\infty}}}}{8}$

(a) 2 (b) 3 (c) 4 (d) 1

➥ (d) $\frac{17^{18^{19^{20\cdots\infty}}}}{8}$ = Remainder $\left\{\frac{(8\times 2+1)^{18^{19^{20\cdots\infty}}}}{8}\right\}$

= Remainder $\left\{\frac{(1)^{18^{19^{20\cdots\infty}}}}{8}\right\}$ = Remainder $\left\{\frac{1}{8}\right\}$

∴ Remainder = 1

4. ***If the sum of first 11 terms of an arithmetic progression equal that of the first 19 terms, Then, what is the sum of first 30 terms?***

(a) 0 (b) -1 (c) 1 (d) Not unique

w (a) Let the first term be a common difference of progression be d According to the question,

$$S_{11} = S_{19}$$

$$\frac{11}{2}[2a + 10d] = \frac{19}{2}[2a + 18d]$$

$$\Rightarrow \quad 16a + 232d = 0$$

$$\Rightarrow \quad 2a + 29d = 0$$

$$\therefore \quad S_{30} = \frac{30}{2}[2a + 29d] = \frac{30}{2} \times 0 = 0$$

Fast Track Practice

Exercise© *Base Level Questions*

1. Find the place value of 4 in 46127.

(a) 4 *(b)* 400

(c) 40000 *(d)* 4000

(e) None of the above

2. Find the place value of 7 in 837218.

(a) 7000 *(b)* 7

(c) 700 *(d)* 70000

fej None of the above

What is the place value of 6 in 65489203? [SSCLDC2010]

(a) $6x10^5$ *(b)* $6x10^4$

(c) $6x10^7$ (dj $6x10^a$

(e) None of the above

4. Find the face value of 7 in 942756.

(a) 7 (b) 700

(c) 7000 *(d)* 70000

5. Find the face value of 6 in 652410.

[SSC LDC 2008]

(a) $6x10^5$ *(b)* $6x10^4$

(c) 6000 *(d)* 6

6. Find the sum of the face values of 9 and

6 in 907364. [Hotel Mgm, 2007]

(a) 15 *(b)* 20 (c) 9 *(d)* 18

7. Find the difference of the face values of

7 and 2 in 210978.

(a) 4 b) 3 *(c)* 6 r^ 5 (e) None of the above

8. Find the sum of place and face values of

8 in 43836. [Hotel Mgmt 2008]

(a) 88 *(b)* 808

(c) 880 *(d)* 888

(e) None of the above

9. Find the difference of place and face values of 4 in 324372.

(a) 3996 *(b)* 3998

(c) 3398 (dj 3396

fej None of the above

10. Find the sum of place value of 6 and face value of 9 in 927653.

(a) 608 *(b)* 508

(c) 609 *(d)* 507

(e) None of the above

11. Find the difference of place value of 4 and face value of 3 in 3784105.

(a) 3997 *(b)* 1

(c) 1000 *(d)* 3845

(e) None of the above

12. When 121012 is divided hy 12, the remainder is [CTET 2012]

(ajO *(b)2 (c)3 (d)4*

13. The sum of place values of 2 in 2424 is

[CTET 1012]

(a) 4 *(b)* 220

(c) 2002 *(d)* 2020

14. The pair of numbers which are relatively prime to each other is [CDS 2012]

(a) (68, 85) *(b)* (65, 91)

(c) (92, 85) *(d)* (102, 153)

15. Find the sum of 1st and 2nd prime numbers.

(a) 5 *(b)* 3 *(c) 7 (d)* 2 *(e)* None of the above

16. Find the product of 1st natural number and 1st prime number.

(a) 4 *(b)* 3 *(c)* 2 *(d)* 5 *(e)* None of the above

17. The product of 1st natural, 1st whole and the 1st prime numbers is equal to

(a) 5 *(b)* 0 *(c)* 9 *(d)* 7 *(e)* None of the above

18. The product of any number and the 1st whole number is equal to

(a) 0 *(b)* 2 *(c)* 1 *(d)* -1 *(e)* None of the above

19.A rational number is expressed as ... where, p and q are integers and q * 0.

(a) pq *(b)* $p+q$ *(c)* $p-q$ *(d)* $\frac{p}{q}$

(e) None of the above

20.2/3 is a rational number whereas V2/V3 is [CLAT2013]

(a) also a rational number

(b) an irrational number

(c) not a number

(d) a natural periodic number

21. Which of the following is a prime number?

(a) **35** ***(b)*** **53** ***(c)*** **88** ***(d)*** **90** *(e)* None of the above

22. The number of all prime numbers less than 40 is ...

(a) 15 *(b)* 18 *(c)* 17 *(d)* 12 *(e)* None of the above

23. Find the quotient when 445 is divided by 5.

(a) 78 *(b)* 48 *(c)* 79 fdj 89 *(e)* None of the above

24. Find the remainder when 54 is divided by 17.

(a) 5 (b) 3 (c) 2 (d) 7 (e) None of the above

25. Find the dividend when divisor is 13, quotient is 30 and remainder is 12.

(a) 402 (b) 543 (c) 436 (d) 455 (e) None of the above

26. What is the remainder in the expression

29—? 26

(a) 29 (b) 26 fc; 18 (d) 0

fej None of the above

27. Find the dividend from the expression

41±. 19

(a) 783 fbj 800 (cj 893 (dj 387

fej None of the above

28. When 1/7 of a number is subtracted from the number itself, it gives the same value as the sum of all the angles of a triangle. What is the number? [Bank PO 2010]

(a) 224 (b) 210 (c) 140 (d) 350 (e) 187

29. What least number must be added to 1057 to get a number exactly divisible by 23?

(a) 1 (b) 3 (c) 2 (d) 4 (e) None of the above

30. In a division sum, the divisor is ten times the quotient and five times the remainder. If the remainder is 46, then find the dividend. [General Insurance 2007]

(a) 5388 (b) 5343

(c) 5336 (d) 5391

(e) None of the above

31. The product of two consecutive odd numbers is 6723. What is the greater number? [Bank Clerks 2009]

(a) 89 *(b)* 85 *(c)* 91 *(d)* 83 *(e)* None of the above

32. The sum of the four consecutive even numbers is 284. What would be the smallest number? [Bank PO 2010]

(a) 72 *(b)* 74 *(c)* 68 *(d)* 66 *(e)* None of the above

33. The sum of the digits of a two-digit number is 14 and the difference between the two digits of the number is 2. What is the product of the two digits of the two-digit number? [Bank Clerks 2009]

(a) 56

(b) 48

(c) 45

(d) Couldn't be determined

(e) None of the above

34. What number should be added to 231228 to make it exactly divisible by 33?

[CDS 2012]

(a) 1 *(b)* 2 *(c)* 3 *(d)* 4

Exercise © *Higher Skill Level Questions*

1. Find the sum of first 25 natural numbers.

(a) 432 *(b)* 315 *(c)* 325 *(d)* 335 *(e)* None of the above

2. Find the sum of the squares of first 35 natural numbers.

(a) 14910 *(b)* 15510

(c) 14510 *(d)* 16510

(e) None of the above

3. Find the sum of the cubes of first 15 natural numbers.

(a) 15400 *(b)* 14400

(c) 16800 *(d)* 13300

4. Find the sum of first 37 odd numbers.

[Hotel Mgmt. 2010]

(a) 1369 *(b)* 1295 *(c)* 1388fdJ 1875 *(e)* None of the above

5. Find the sum of first 84 even numbers.

[Bank Clerks 2008] *(a)* 7140 *(b)* 7540 *(c)* 6720 *(d)* 8832 *(e)* None of the above

6. Sum of first 15 multiples of 8 is

[CUT 2013]

(a) 960 *(b)* 660

(c) 1200 *(d)* 1060

7. The product of four consecutive natural numbers plus one is [CDS 2014]

(a) a non-square

(b) always sum of two square numbers

(c) a square

(d) None of the above

Find the unit digit in the product of (268 x 539 x 826 x 102). [MBA 2009]

(a) 5 *(b)* 3 *(c)* 4 *(d)* 2

(e) None of the above

9. Find the unit digit in the product of (4326 x 5321). [Hotel Mgmt. 2010]

***(a)* 6 *(b)* 8 *(c)* 1 *(d)* 3** *(e)* None of the above

10. What is the unit digit in $(6817)^{754}$?

***(a)* 8 *(b)* 4 *(c)* 2 *(d)* 9** *(e)* None of the above

11. What is the unit digit in

$(3^{65} \text{x} 6^{59} \text{x} 7^{71})$?

***(a)* 6 *(b)* 4 *(C)* 2 *(d)* 1** *(e)* None of the above

12. Find the last two-digits of 15x37x63x51x97x17 [IBAC1O2012]

***(a)* 35 *(b)* 45 *(c)* 55 *(d)* 85**

13. How many rational numbers are there between 1 and 1000? [CDS 2012]

(a) 998 *(b)* 999

(c) 1000 *(d)* Infinite

14. The sum of 5 consecutive even numbers *A,B,C,D andE* is 130. What is the product of A and *E* ? [Bank Clerks 2009]

(a) 720 *(b)* 616 *(c)* 660 *(d)* 672 *(e)* None of the above

15. The sum of the five consecutive numbers is equal to 170. What is the product of largest and the smallest numbers?

[Bank Clerks 2011]

(a) 1512 *(b)* 1102 *(c)* 1152 *(d)* 1210 *(e)* None of the above

16. Which of the following numbers always divides the difference between the squares of two consecutive odd integers?

[Bank Clerks 2009]

***(a)* 7 *(b)* 3 *(c)* 8 *(d)* 6** *(e)* None of the above

17. A number divided by 56 gives 29 as remainder. If the same number is divided by 8, the remainder will be ...

[SSC CCL 2007]

(a) **4** ***(b)*** **5**

(c) **6** ***(d) 7***

18. On dividing a certain number by 357, the remainder is 39. On dividing the same number by 17, what will be the remainder?

(a) **5** ***(b)*** **3** ***(c) 7 (d)*** **6** *(e)* None of the above

19. A number when divided by 5, leaves 3 as remainder. What will be the remainder when the square of this number is divided by 5?

(a) **3** ***(b)*** **4** ***(c)*** **5** ***(d)*** **0** *(e)* None of the above

20. In a question on division with zero remainder, a candidate took 12 as divisor instead of 21. The quotient obtained by him was 35. Find the correct quotient.

(a) **10** ***(b)*** **12** ***(c)*** **20** ***(d)*** **15** *(e)* None of the above

21. A number when divided by a divisor leaves a remainder of 24. When twice the original number is divided by the same divisor, the remainder is 11. What is the value of the divisor? [IB ACIO 2013]

(a) **13** ***(b)*** **59**

(c) **35** ***(d)*** **37**

22. The number 58129745812974 is divisible by [CDS 2012]

(a) **11** ***(b)*9**

(c) 4 *(d)* None of these

23. How many numbers between - Hand 11 are multiples of 2 or 3? [CDS 2012]

(a) **11** ***(b)*** **14**

(c) 15 *(d)* None of these

24. Which one of the following numbers is divisible by 11? [CDS 2013]

(a) 45678940 *(b)* 54857266

(c) 87524398 *(d)* 93455120

25. When 17^{200} is divided by 18, find the remainder.

***(a)* 1 *(b)* 4 *(c)* 5 *(d)* 3** *(e)* None of the above

26. What is the remainder when 4^{1000} is divisible by 7? [CDS 2014]

***(a)* 1 *(b)* 2**

(c) 4 *(d)* None of these

27. A common factor of $(41^{43} + 43^{43})$ and $(41^{41}+43^{41})$is...

(a) (43 - 41) *(b)* $(41^{41} + 43^{41})$

(c) $(41^{43} + 43^{43})$ *(d)* (41 + 43) *(e)* None of the above

28. The remainder when 9 + 6 is divided by 8 is [SSC CGL (Main) 2012]

(a) 2 *(b)* 3

(c) 5 *(d)* 7

29. What will be the remainder when 19^{100} is divided by 20? [SSC CGL (Main) 2012]

(a) 19 *(b)* 20

(c) 3 *(d)* 1

30. It is given that $(2^{s2} + 1)$ is exactly divisible by a certain number. Which of the following is also definitely divisible by the same number?

(a) $(2^{16} + 1)$ *(b)* $(2^{16}-1)$

(c)$7 x 2^{13}$ fdj (2^{96} + 1)

(e) None of the above

31. The number *(6x + &x)* for natural number *x* is always divisible by ...

(a) 6 and 12 *(b)* 12 only

(c) 6 only *(d)* 3 only

(e) None of the above

32. $19^5 + 21^5$ is divisible by [CDS 2013]

(a) Only 10 *(b)* Only 20

(c) Both 10 and 20

(d) Neither 10 nor 20

33. If *'a'* is a natural number, then the largest number dividing (a - a) is

(a) A (b) 5

(c) 6 *(d)* 7

(e) None of the above

34. 7 - 4 is exactly divisible by which of the following number? [SSC FCI 2012]

(a) 34 *(b)* 33

(c) 36 fdj 35

35. If *N, (N* + 2) and *(N* + 4) are prime numbers, then the number of possible solutions for *N* are [CDS 2013]

(a) 1 *(b)* 2

(c) 3 *(d)* None of these

36. The smallest positive prime (say p) such that 2^P - lis not a prime is ^*DS* 2013]

raj 5 (b) 11

(c) 17 fdj 29

37. If *b* is the largest square divisor of c and *a* divides c, then which one of the following is correct? (where, *a, b* and c are integers) [CDS 2013]

(a) b divides a

(b) a does not divide *b*

(c) a divides *b*

(d) a and b are coprime

38. If re is a whole number greater than 1, then re (re - 1) is always divisible by

[CDS 2014]

(a) 12 *(b)* 24 *(c)* 48 *(d)* 60

39. What is the sum of all positive integers lying between 200 and 400 that are multiples of 7? [IB ACIO 2013]

(a) 8729 *(b)* 8700 *(c)* 8428 *(d)* 8278

40. Consider the following statements

I. To obtain prime numbers less than 121, we are to reject all the multiples of 2, 3, 5 and 7. II. Every composite number less than 121 is divisible by a prime number less than 11.

Which of the statements given above is/are correct? [CDS 2013]

(a) Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

41. Consider the following statements

I. 7710312401 is divisible by 11. II. 173 is a prime number. Which of the statements given above is/are correct? [CDS 2013]

(a) Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

42. If & is a positive integer, then every square integer is of the form [CDS 2013]

(a) Only *4k* *(b)* *Ak* or *Ak* + 3

(c) 4/c + 1 or 4/c + 3 *(d)* *Ak* or *Ak* + 1

43. Every prime number of the form $3k + 1$ can be represented in the form $6m + 1$ (where k, m are integers), when

[CDS 2013]

(a) k is odd

(b) k is even

(c) k can be both odd and even

(d) No such form is possible

Answer with **Solutions**

Exercise© *Base Level Questions*

1. (C) 4 is at the place of ten thousand. /. Required place value

= 4 x 10000 =40000

2. (a) 7 is at the thousand place. /. Required place value

= 7 X 1000 = 7000

3. (c) 6 is at the place of crore. .'. Required place value

= 6 x 10000000 =6 x 10^7

4. (a) Face value is the value of digit itself. .". Required face value of 7 = 7

5. (d) Face value is the value of digit itself. /. Required face value = 6

6. (a) The face value is the value of digit itself. So, required sum = 9 + 6 = 15

7. (d) The face value is the value of digit itself. So, required difference = 7-2 = 5

8. (6) Place value of 8 = 800

and face value of 8 = 8 ∴ Required sum = 800 + 8 = 808

9. (a) Place value of 4 = 4000

and face value of 4 = 4 ∴ Required difference = 4000 - 4 = 3996

10. (c) Place value of 6 = 600

and face value of 9 = 9 ∴ Required sum = 600 + 9 = 609

11. (a) Place value of 4 = 4000

and face value of 3 = 3 .'. Required difference = 4000 - 3 = 3997

12. (*d*)

```
12) 121012 (10084
    12
    ---
      101
       96
      ---
        52
        48
        --
         4
```

Hence, when 121012 is divided by 12, then remainder is 4.

13. (d) The sum of place values of 2 in

2424 = 2 X 1000 + 2X10 = 2000 + 20 = 2020

14. (c) 92 and 85 are coprime numbers because their HCF is 1.

15. (a) 1st prime number = 2 and 2nd prime number = 3

.'. Required sum = 2+3 = 5

16. (c) 1st natural number = 1 and 1 st prime number = 2

Required product = 1x2 = 2

17. (6) 1st natural number = 1,

1 st whole number = 0 and 1 st prime number = 2 ∴ Required product = 1x0x2 = 0

18. (a) 1st whole number = 0

Clearly, when any number is multiplied with 0 (the 1st whole number), then the result is 0.

19. (d) A proven fact.

20. (*b*) $\sqrt{2}$ is irrational and $\sqrt{3}$ is also irrational.

So, $\frac{\sqrt{2}}{\sqrt{3}}$ is also irrational.

21. (6) 53 has only two factors itself and 1. Hence, it is a prime number.

22. (d) Prime numbers less than 40 are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37

23. (d) Dividend = 445 and divisor = 5

$\therefore$ Required quotient $= \frac{445}{5} = 89$

24. (*b*) $\frac{54}{17} = 3\frac{3}{17}$

/. Remainder = 3

25. (a) Given, Divisor (d) = 13, Quotient (Q) = 30

Remainder *{R)* = 12 and Dividend *(D)* = ?

We know that,

$D=dxQ+R$ D = 13X30 + 12 = 390 + 12 = 402

26. (c) It is clear from the expression that remainder is 18.

27. (a) Given,

Quotient (Q) = 41, Divisor (d) = 19 Remainder *(.R)* = 4 and Dividend *(D)* = ? ∴ Dividend $D = dxQ + £$ = 19x41 + 4 = 779 + 4 = 783

28. (6) Let the number be x. According to the question,

$$x - \frac{x}{7} = 180$$

$$\Rightarrow \quad \frac{6x}{7} = 180$$

$$\therefore \quad x = \frac{180 \times 7}{6} = 210$$

29. (a)

$$\begin{array}{r|l} 23) & 1057 \ (45 \\ & \underline{92} \\ & 137 \\ & \underline{115} \\ & 22 \end{array}$$

∴ Number to be added = (23 - 22) = 1 30.(C) Divisor = 5 X Remainder = 5 X 46 = 230 Also, 10 x Quotient = 230

Quotient = 23 We know that

Dividend = (Divisor X Quotient) + Remainder ∴ Dividend = (230 x 23) + 46

= 5290 + 46 = 5336

(d) Let two consecutive odd number be (x + 1) and (x + 3). According to the question,

(x + 1) (x + 3) = 6723 => x^2 + 3x + x + 3 = 6723

=> x^z + 4x+3 = 6723

=> x^2+4x+3-6723 = 0 => x^z + 4x - 6720 = 0

=> x^2 + 84x - 80x - 6720 = 0

=> x (x + 84) - 80 (x + 84) = 0 => (x - 80) (x + 84) = 0

x = 80,(x*-84) Hence, the greater number = 80 + 3 = 83

32. (c) Let four consecutive even numbers are x, x + 2, x + 4 and x + 6. According to the question,

x+x + 2+x + 4 + x + 6 = 284 => 4x+ 12 = 284

=> 4x = 284-12 = 272

$\therefore \quad x = \frac{272}{4} = 68$

33. (6) Let be the ten's digit be *x* and unit's digit be *y*. The two-digit number = lOx + *y* (where, x > *y)*

According to the question,

x + *y* = 14 ...(i)

and x - *y* = 2 ... (ii)

Solving Eqs. (i) and (ii), we get

x = 8 and *y* = 6 .'. Required product = 8 x 6 = 48

34. *(c)* Given, 33)231228(7006

231 228 198 30 Now, 33 - 30 = 3

.'. On adding 3 to 231228, it is completely divisible by 33.

Exercise © *Higher Skill Level Questions*

1. (c) We know that,

Sum of first *n* natural numbers

$$= \frac{1}{2} n (n + 1)$$

Given, $n = 25$

$$\therefore \text{ Required sum} = \frac{1}{2} \times 25 \times 26$$

$$= 25 \times 13 = 325$$

2. (a) We know that,

Sum of the squares of first *n* natural

$$\text{numbers} = \frac{1}{6} n (n + 1) (2n + 1)$$

Given,

$$n = 35$$

$\therefore$ Required sum

$$= \frac{1}{6} \times 35 (35 + 1) (2 \times 35 + 1)$$

$$= \frac{1}{6} \times 35 \times 36 \times 71$$

$$= 35 \times 6 \times 71 = 14910$$

3. (6) We know that,

Sum of the cubes of first *n* natural numbers

$$= \left[\frac{n (n + 1)}{2}\right]^2$$

Given,

$$n = 15$$

$$\therefore \text{ Required sum} = \left[\frac{15 (15 + 1)}{2}\right]^2$$

$$= \left[\frac{15 \times 16}{2}\right]^2$$

$$= (15 \times 8)^2 = (120)^2 = 14400$$

4. *(a)* We know that,

2 Sum of first *n* odd numbers = *n*

Given,

n = 37

∴ Required sum = (3 *if* =37x37

= 1369

5. (a) We know that,

Sum of first *n* even numbers = *n {n* + 1)

Given, *n* = 84

∴ Required sum = 84 (84 + 1)

= 84 X 85= 7140

6. (a) first 15 multiple of 8 are

8, 16, 24.....120

So, 8(1,2,3,4.....15)

$$\text{Their sum} = 8\left[\frac{n(n+1)}{2}\right]$$
$$= 8\left[\frac{15(15+1)}{2}\right]$$
$$= 8 \times 15 \times 8 = 960$$

7. (c) Product of four consecutive numbers plus one is always a square Illustration 1 Let four consecutive numbers be 3, 4, 5 and 6.

= (3 x 4 x 5 x 6) + **1** = 361 = $(19)^z$

Illustration 2 Let four consecutive numbers be 9, 10, 11 and 12.

= (9 x 10 x 11 x 12) + 1

= 11881 = $(109)^2$

8. (c) Product of unit digits

= 8X9X6X2 = 864 /. Required digit = 4

9. (a) Product of unit digits =6x1=6 /. Required digit = 6

10. (d) Required digit = Unit digit in $(7)^{754}$

= Unit digit in $\{(7^4)^{188} \times 7^2\}$ = Unit digit in (1 x 49) = 9

11. (6) Unit digit in $3^4 = 1$ ∴ Unit digit in $(3^4)^{16} = 1$

Unit digit in $3^{85} = 3$

Unit digit in $6^{59} = 6$

Unit digit in $7^4 = 1$

∴ Unit digit in $(7^4)^{17} = 1$

771 = (74j17 x *f*

As,

Unit digit in 7^3 =3 Unit digit in 7^{71} =3 .". Required unit digit = Unit digit in

(3x6x3) = Unit digit in 54 =4

12. (a) 15 X 37 X 63 X 51 X 97 X 17

= 255X37X63X51 X 97 = 35X21 [last two digits]

= 735 = 35 [last two digits]

Hence, last two digits of the product is 35.

13. *{d)* There can be infinite number of rational numbers between 1 and 1000.

14. (c) Let five consecutive even number be A = x, .B = x + 2, C = x+4,

D = *x* + 6 and *E* = *x* + 8 According to the question, X+X+2+X+4+ x + 6+ x + 8 = 130 => 5x+20 = 130

=> 5x = 130-20 = 110

$$\therefore \quad x = \frac{110}{5} = 22$$

$$\therefore \quad A = 22$$

$$E = x + 8 = 22 + 8 = 30$$

$$\therefore \quad A \times E = 22 \times 30 = 660$$

15. (c) Let the five consecutive numbers are x, (x + 1) and (x + 2), (x + 3) and (x + 4). According to the question, x+x+l + x+2+x+3+x + 4 = 170

5x+ 10 = 170 => 5x = 160

$$\therefore \quad x = \frac{160}{5}$$

= 32 Largest number = (x + 4)

= 32 + 4 = 36 /. Required product

= 32x36 = 1152

16. (c) Let the two consecutive odd numbers be (2x + 1) and (2x + 3).

∴ Difference = $(2x + 3)^2$ - $(2x + l)^2$

= (2x + 3 + 2x + 1) (2x + 3 - 2x- 1) = (4x + 4) X2 = 8 (x+ 1),

which is exactly divisible by 8.

17. (6) Let the number be x. According to the question,

x = 56A- + 29 Then,

x = (8 X *7 k)* + (8 X 3) + 5 = 8 X (7i + 3) + 5 Therefore, when x is divided by 8, the required remainder = 5

Dired Approach

In such type of questions to get the new remainder we simply divide the first remainder by second divisor and remainder obtained is the required answer.

$$\therefore \text{Required remainder} = \frac{29}{8} = 5 \text{ as remainder.}$$

18. (a) Let the given number be (357Jr + 39).

Then,

(357* + 39) = (17 X *21k)* + (17 X 2) + 5

= 17x(21i + 2) + 5 .•. Required remainder = 5

19. (6) Let the number be x. According to the question,

x = *(5k* + 3) On squaring both sides, we get => x^2 = *(5k* + 3)^2 = (25£^{2} + 30Jc + 9)

= 5(5£^{2} + *6k* + 1) + 4 .•. On dividing x by 5, the remainder is 4.

20. (C) Number = 35 X 12 = 420

$$\therefore \text{Required correct quotient} = \frac{420}{21} = 20$$

21. (d) Let the divisor be x and quotient be *y.* Then, number = *xy* + 24

Twice the number = 2xy + 48 Now, 2xy is completely divisible by x. On dividing 48 by x remainder is x. x = 48 -11 = 37

22. *(a)* We know that, a number is divisible by 11 when the difference between the sum of its digit at even places and sum of digit at odd places is either 0 or the difference is divisible by 11.

So, number is 58129745812974 Sum of digits at odd places

= 4 + 9 + 1 + 5+7+2 + 8 = 36 Sum of digit at even places

7+2 + 8 + 4 + 9 + 1 + 5 = 36 So, the required difference = 36 - 36 = 0 /. The number is divisible by 11.

23. *(c)* Method I

Following are the numbers between - 11

and 11 which are multiples of 2 or 3?

-10,-9,-8,-6,-4,

-3,-2, 0, 2, 3, 4, 6, 8, 9, 10.

/. The numbers of multiples 2 or 3, between

- 11 and 11 are 15. Method II

Numbers between 0 and 11 which are multiples of 2 or 3

$$\frac{11}{2}+\frac{11}{3}-\frac{11}{6}=5+3-1=7$$

Numbers between 0 and - 11

$$\frac{11}{2}+\frac{11}{3}-\frac{11}{6}=7$$

/. Number be 15, including "0".

24. *(d)* we know that, if the difference between the sum of digits at even places and sum of digits at odd places is (0), then the number is divisible by 11. From options.

(a) 45678940

Sum of even places =5+7+9 + 0 = 21 Sum of odd places =4 + 6 + 8 + 4 = 22 Their difference = 22-21*0

(b) 54857266

Sum of even places =4 + 5+2+6 = 17 Sum of odd places = 5+8 + 7+6 = 26 Their difference = 26-17 = 9*0

(c) 87524398

Sum of even places =7+2+3+8= 20 Sum of odd places =8 + 5 + 4 + 9= 26 Their difference = 26-20=6*0

(d) 93455120

Sum of even places =3+5+1 + 0=9 Sum of odd places =9 + 4 + 5+2 = 20 Their difference = 20-9 = 11+11 = 1 So, it is divisible by 11. 25. *(a)* We know that, $(x^m - a^m)$ is divisible by (x + a), for even values of *m*. ..: $(17^{200} - 1^{200})$ is divisible by (17 + 1). => $(17^{200} - 1)$ is divisible by 18.

When 17^{200} is divisible by 18, then the remainder is 1.

26. *(c)* Remainder of

$$\frac{4^{1000}}{7} = \frac{(4^2)^{500}}{7} = \frac{(16)^{500}}{7}$$
$$= \frac{2^{500}}{7} = \frac{2^2 \times (2^3)^{166}}{7}$$
$$= \frac{4 \times (8)^{166}}{7} = 4$$

27. (d) We know that, when m is odd $(x^m + a^m)$ is divisible by $(x + a)$.

.•. Each one is divisible by (41 + 43). ∴ Common factor = (41 + 43)

28. *(d)* Required remainder =9 +6

$= (l)^{19} + 6=4$

[v8 = 9 - 1, so replaced by 1]

$$\text{We have} = \frac{(9)^{19} + 6}{8}$$
$$\Rightarrow \frac{(8+1)^{19} + 6}{8} \Rightarrow \frac{1^{19} + 6}{8}$$
$$\Rightarrow \frac{1+6}{8} = \frac{7}{8}$$
$$\therefore \quad \text{Remainder} = 7$$

29. *(d)* We have $= \frac{19^{100}}{20}$

$$\Rightarrow \frac{(20-1)^{100}}{20}$$
$$\Rightarrow \frac{(-1)^{100}}{20} \Rightarrow \frac{1}{20}$$
$$\therefore \quad \text{Remainder} = 1$$

Required remainder $= 19^{100}$

$$= (-1)^{100} = 1$$

$[\because 20 = 19 + 1$ so 19 replace by $(-1)]$

30. (d) Let $2^{32} = x$ and let $(2^{32} + 1) = (x + 1)$ be divisible by a number n.

Then, $(2^{96} + 1) = (x^3 + 1)$

$= (x + 1) (x^2 - x + 1)$

which is clearly divisible by n as $(x + 1)$ is divisible by n.

31. (a) $(6X^2 + 6x) = 6x (x + 1)$ which is clearly divisible by 6 and 12 as $x(x + 1)$ is even.

32. *(c)* We can check divisibility of $19^5 + 21^5$ by

10 by adding the unit digits of 9 and 1

which is equal to 9 + 1 = 10.

So, it must be divisible by 10.

Now, for divisibility by 20 we add 19 and

21 which is equal to 40. So, it is clear that

it is also divisible by 20.

So,$19^5 + 21^5$isdivisiblebybothl0and20.

33. (C) (2 - 2) = 6 is the largest natural number that divides (a - a) for every number a.

34. (6) We know that, $(x^n - y")$ is divisible by

$(x — y)$ for all n and is divisible by $(x + y)$ for even n.

$\therefore$ $(7^{12} - 4^{12})$ is divisible by(7 + 4) and(7 - 4) => (712 _ 412) is divisible by 11 and 3 ■■• $(7^{1Z} - 4^{12})$ is divisible by 33.

35. *(a)* When N is a natural number, then there is only one possible case that N, $(N + 2)$, $[N + 4)$ are prime numbers,

When $N = 3$, then N, $(N + 2)$, $\{N + 4) = 3, 5, 7$ all are primes.

36. *(b)* Taking p = 5

$2^P - 1 = 2^5 - 1 = 31$ which is prime Taking p = 11

$2^P - 1 = 2^{11} - 1 = 2047$ Since, 2047 is divisible by 23, so it is not prime. Thus, required least positive prime number is 11.

37. *(c)* Since, b is largest square divisor of c. So, c = bx

(where, x is not a whole square number)

Also, a divides c.

So, a will divide *bx*.

or a will divide *b*.

(since, it cannot divide x as it is not a whole

square)

38. *(a)* If *n* greater than 1, then $n^z(n^z - 1)$ is always divisible by 12.

Illustration 1

Put $n = 2$, then $n^z\{n^z - 1) = (2)^2(2^2 - 1) = 4 X3 = 12$ **Illustration 2** Now, put $n = 3$, then $n^2(n^z - 1) = (3)^2(3^2 - 1) = 9 X 8 = 72$

39. (a) Least number divisible by 7 and above 200 is 203.

Greatest number divisible of 7 and below 400 is 399.

Total numbers divisible by 7 between 200 to 400 are 29

Now, sum of *n* terms of AP = — (a + *1)* 2

where, a = 203, *1* = 399 and *n* = 29

$$= \frac{29}{2}(203 + 399)$$
$$= 8729$$

40. *(c)* Both the statements given are correct. As 121 is the square of 11. So, to obtain prime numbers less than 121, we reject all the multiples of prime numbers less than 11 i.e., 2, 3, 5 and 7. Similarly, every composite number less than 121 is divisible by a prime number less than 11 *i.e.,* 2, 3, 5 or 7.

41. *(c)* I. Any number in order to get completely divided by 11 must have the difference between the sum of even place digits and the sum of odd place digits equal to 0 or the multiple of 11. In 7710312401, difference between sum of even place digits and the sum of odd place digit 0.

So, it is divisible by 11.

II. To check divisibility of 173, we can

divide the number by all the prime

numbers from 2 to 13. It is not divisible by

2, 3, 5, 7, 11 and 13. So, it is a prime

number.

Hence, both Statements I and II are correct.

42. *(d)* If *k* is a positive integer, then every square integer is of the form 4i or 4£ + 1, as every square number is either a multiple of 4 or exceeds multiple of 4 by unity.

43. *(b)* Every prime number of the form $3k + 1$ can be represented in the form $6m + 1$ only, when *k* is even.

Chapter

2

Number Series

A number series is a sequence of numbers written from left to right in a certain pattern. To solve the questions on series, we have to detect/find the pattern that is followed in the series between the consecutive terms, so that the wrong/missing term can be find out.

Types of Series

There can be following types of series

1. Prime Number Series

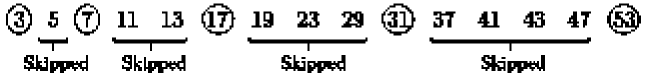
The number which is divisible by 1 and itself, is called a **prime number.** The series formed by using prime number is called prime number series.

Ex. 1 Find out the next term in the series 7,11,13,17,19,....

Sol. Given series is a consecutive prime number series. Therefore, the next term will be 23.

Ex. 2 Find out the next term in the series 3, 7,17, 31, ...

Sol. Here, every next prime number takes place skipping one, two, three and four prime numbers, respectively. Hence, the required answer will be 53. Let us see



2. Addition Series

The series in which next term is obtained by adding a specific number to the previous term, is known as addition series. Addition series are increasing order series and difference between consecutive term is equal.

Ex. 3 Find out the missing term in the series 2, 6,10,14,..., 22.

Sol. Here, every next term is obtained by adding 4 to the previous term. .•. Required term = 14 + 4 = 18

3. Difference Series

Difference series are decreasing order series in which next term is obtained by subtracting a fixed/specific number from the previous term.

Ex. 4 Find out the missing term in the series 108, 99, 90, 81,..., 63.

Sol. Here, every next number is 9 less than the previous number. So, required number = 81 - 9 = 72

4. Multiple Series

When each term of a series is obtained by multiplying a number with the previous term, then the series is called a multiplication series.

Note Number which is multiplied to consecutive terms, can be fixed or variable

Ex. 5 Find out the missing term in the series 4, 8,16, 32, 64,..., 256. Sol. Here, every next number is double the previous number. So, required number = 64 x 2 = 128

Ex. 6 Find out the missing term in the series 5, 20, 80, 320,..., 5120.

Sol. Here, every next number is 4 times the previous number. So, required number = 320 X 4 = 1280

5. Division Series

Division series are those in which the next term is obtained by dividing the previous term by a number.

Note Number which divides consecutive terms can be fixed or variable Ex. 7 Find out the missing term in the series 10080,1440, 240, ..., 12,4.

Sol. Series pattern $\frac{10080}{7} = 1440, \frac{1440}{6} = 240, \frac{240}{5} = 48, \frac{48}{4} = 12, \frac{12}{3} = 4$

Hence, missing term is 48. Ex. 8 Find out the missing term in the series 512, 216, 72,..., 12.

Sol. $\frac{512}{2} = 216, \frac{216}{3} = 72, \frac{72}{2} = 36, \frac{36}{3} = 12$

Hence, missing term is 36.

6. n^2 Series

When a number is multiplied with itself, then it is called as square of a number and the series formed by square of numbers is called *rr* series.

Ex. 9 Find out the missing term in the series 4,16, 36, 64, ..., 144. Sol. This is a series of squares of consecutive even numbers. Let us see

$2^2 = 4$, $4^2 = 16$, $6^2 = 36$, $8^2 = 64$, $10^2 = 100$, $12^2 = 144$ Hence, missing term is 100.

Ex. 10 Find out the missing term in the series 1, 4, 9,16, 25, ..., 49.

Sol. This is a series of squares of consecutive natural numbers. Let us see $1^2 = 1$,$2^2 = 4$,$3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$ Hence, missing term is 36.

7. (n^2 + 1) Series

If in a series each term is a sum of a square term and 1, then this series is called (n^2 +1) series.

Ex. 11 Find out the missing term in the series 10,17, 26, 37, ..., 65. **Sol.** Series pattern

$3^2 + 1 = 10$, $4^2 + 1 = 17$, $5^2 + 1 = 26$, $6^2 + 1 = 37$, $7^2 + 1 = 50$, $8^2 + 1 = 65$ So, required number = 50

Ex. 12 Find out the missing term in the series 122,145,170,197, ..., 257.

Sol. Series pattern

$11^2 + 1 = 122$, $12^2 + 1 = 145$, $13^2 + 1 = 170$, $14^2 + 1 = 197$, $15^2 + 1 = 226$, $16^2 + 1 = 257$ So, correct answer is 226

8. (n^2 -1) Series

In a series, if each term is obtained by subtracting 1 from square of a number, then such series is known as (n^2 -1) series.

Ex. 13 Find out the missing term in the series 0, 3, 8,15, 24,..., 48. **Sol.** Series pattern

$1^2 - 1 = 0$, $2^2 - 1 = 3$, $3^2 - 1 = 8$, $4^2 - 1 = 15$, $5^2 - 1 = 24$, $6^2 - 1 = 35$, $7^2 - 1 = 48$ So, correct answer is 35.

Ex. 14 Find out the missing term in the series 224,195,168, ..., 120. **Sol.** Series pattern 15^2 - 1,14^2 - L 13^2 - 1,12^2 - 1, 11^2 — 1 So, required number =12 - 1 = 143

9. *(n^2 + n)* Series

Those series in which each term is a sum of a number with square of that number, is called as *(ir + n)* series.

Ex. 15 Find out the missing term in the series 12, 20, 30, 42, ..., 72.

Sol. Series pattern $3^2 + 3$, $4^2 + 4$, $5^2 + 5$, $6^2 + 6$, $7^2 + 7$, $8^2 + 8$ So, required number =7 + 7 = 56

Ex. 16 Find out the missing term in the series 420 , 930,1640, ..., 3660. **Sol.** Series pattern $20^2 + 20$, $30^2 + 30$, $40^2 + 40$, $50^2 + 50$, $60^2 + 60$ So, required number = 50 + 50 = 2550

10. (n^2 - *n)* Series

Series in which each term is obtained by subtracting a number from square of that number, is known as (n^2 - *n)* series.

Ex. 17 Find out the missing term in the series 42, 30,..., 12, 6. **Sol.** Series pattern

$7^2 - 7$, $6^2 - 6$, $5^2 - 5$, $4^2 - 4$, $3^2 - 3$ So, required number =5 - 5 = 20

Ex. 18 Find out the missing term in the series 210, 240, 272, 306, ..., 380. **Sol.** Series pattern

$15^2 - 15$, $16^2 - 16$, $17^2 - 17$, $18^2 - 18$, $19^2 - 19$, $20^2 - 20$ So, required number = $19^2 - 19 = 342$

11. n^3 Series

A number when multiplied with itself twice, then the resulting number is called the cube of a number and series which consist of cube of different number following a specified sequence, is called as n^3 series.

Ex. 19 Find out the missing term in the series 1, 8, 27, ..., 125, 216. **Sol.** Series pattern 1^3, 2^3, 3^3, 4^3, 5^3, 6^3 So, required number = 4 = 64

Ex. 20 Find out the missing term in the series 1000,8000,27000,64000,125000,....

Sol. Series pattern 10^3, 20^3, 30^3, 40^3, 50^3, 60^3

So, required number = 60^3 = 216000

12. (n^3 + 1) Series

Those series in which each term is a sum of a cube of a number and 1, are known as (n^3 +1) series.

Ex. 21 Find out the missing term in the series 126, 217, 344, ..., 730. **Sol.** Series pattern 5^3 + \ 6^3 + 1, 7^3 + 1, 8^3 + 1, 9^3 + 1 So, required number = 8 + 1 = 513

Ex. 22 Find out the missing term in the series 1001, 8001, ..., 64001, 125001, 216001.

Sol. Series pattern 10^3 + 1, 20^3 + \ 30^3 + 1,40^3 + 1, 50^3 + 1, 60^3 + 1 So, required number = 30^3 + 1 = 27001

13. (n^3 -1) Series

Series in which each term is obtained by subtracting 1 from the cube of a number, is known as (n^3 -1) series.

Ex. 23 Find out the missing term in the series 0, 7, 26, 63,124, 215, ... **Sol.** Series pattern 1^3 - \ 2^3 - 1, 3^3 - 1, 4^3 - 1,5^3 - \ 6^3 - 1, 7^3 - 1 So, required number = 7^3 - 1 = 342

Ex. 24 Find out the missing term in the series......, 7999, 26999, 63999,124999.

Sol. Series pattern 10^3 - 1, 20^3 - 1, 30^3 - 1, 40^3 - \ 50^3 - 1 So, required number = 10 - 1 = 999

14. (n^3 + *n*) Series

When each term of a series is a sum of a number with its cube, then the series is known as (n^3 + *n)* series.

Ex. 25 Find out the missing term in the series 2,10, 30, ..., 130, 222. **Sol.** Series pattern $1^3 + 1$, $2^3 + 2$, + $3^3 + 3$, $4^3 + 4$, $5^3 + 5$, $6^3 + 6$ So, required number = 4 + 4 = 68

Ex. 26 Find out the missing term in the series ..., 8020, 27030, 64040.

Sol. Series pattern $10^3 + 10$, $20^3 + 20$, $30^3 + 30$,$40^3 + 40$ So, required number = $10^3 + 10 = 1010$

15. (n^3 - *n)* Series

When each term of a series is obtained by subtracting a number from its cube, then series is termed as (n - *n)* series.

Ex. 27 Find out the missing term in the series 0, 6, 24, 60,120, ... **Sol.** Series pattern

1^3 - \ $2^3 - 2$, $3^3 - 3$, $4^3 - 4$, $5^3 - 5$, $6^3 - 6$ So, required number =6 - 6 = 210

Ex. 28 Find out the missing term in the series ..., 7980, 26970, 63960,124950. **Sol.** Series pattern

$10^3 - 10$, $20^3 - 20$, $30^3 - 30$, $40^3 - 40$, $50^3 - 50$ So, required number = $10^3 - 10 = 990$

16. Alternating Series

In alternating series, successive terms increase and decrease alternately. The possibilities of alternating series are

■f It is a combination of two different series.

♦ Two different operations are performed on successive terms alternately.

Ex. 29 Find the next term in the series 15,14,19,11, 23, 8, ...

Sol. Series pattern

$$14 \xrightarrow{-3} 11 \xrightarrow{-3} 8$$

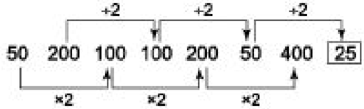
$$15 \xrightarrow{+4} 19 \xrightarrow{+4} 23 \xrightarrow{+4} \boxed{27}$$

15 14 19 11 23 8 $\boxed{27}$

So, required number = 23 + 4 = 27

Ex. 30 Find the next term in the series 50, 200,100,100,200,50,400,...

Sol.



So, required number = $\frac{50}{2} = 25$

17. Arithmetic Progression (AP)

The progression of the form *a, a+d,* a + 2d, *a + 3d,* ...is known as an arithmetic progression with first term *a* and common difference d. Then, nth term $T_n = a + (n - 1)d$

Ex. 31 In series 359, 365, 371,...., what will be the 10th term?

Sol. The given series is in the form of AP. Since, common difference *i. e., d* is same. Here, $a = 359$ and $d = 6$

10th term = $a + (n - 1)d = 359 + (10 - 1)6 = 359 + (9 \times 6) = 413$.

18. Geometric Progression (GP)

The progression of the form *a, ar, ai, ar*^{3},... is known as a GP with first term *a* and common ratio = *r*. Then, nth term of GP $T_n = ar'' \sim *$

Ex. 32 In the series 7,14, 28, • • •, what will be the 10th term?

Sol. The given series is in the form of GP. Since, common ration *i. e., r* is same. Here, $a = 7$ and $r = 2$

10th term = $ar^{n} \text{ " J} = 7(2)^{(10\ \_1)} = 7 \text{x} 2^9 = 3584$

Types of Questions Asked on Number Series

There are mainly three types of questions which are asked from this chapter which are as follows

Type O To Find the Missing Term

In this type of questions a series is given in which one of the term is missing and it is required to find the missing term by detecting the pattern of series.

Ex. 33 What should be the next term in the following series?

12, 24, 72,144, ?

Sol. Series pattern $12 \times 2 = 24$; $24 \times 3 = 72$;

$72 \times 2 = 144$; $144 \times 3 = 432$ Clearly, next term is 432.

Ex. 34 What should come in place of question mark in the series given below?

92, ?, 46, 69, 138, 345

Sol. Series pattern $92 \times \frac{1}{2} = 46$; $46 \times 1 = 46$; $46 \times 1\frac{1}{2} = 69$

$69 \times 2 = 138$; $138 \times 2\frac{1}{2} = 345$

Clearly, 46 should come in place of question mark.

Type @ To Find the Wrong Term

in this type of questions, a series is given in which one of term is required to detect the pattern of series and find that wrong term.

Ex. 35 In the following series, a wrong number is given. Find out that wrong number.

4, 5,10,18, 34, 59, 95

Sol. Series pattern

$4 + 1^2 = 5$; $5 + 2^2 = 9$ <-10; $9 + 3^2 = 18$

$18 + 4^2 = 34$; $34 + 5^2 = 59$; $59 + 6^2 = 95$ Clearly, 10 is the wrong number and will be replaced by 9.

Ex. 36 Find the wrong number in the given series.

84, 83, 79, 70, 52, 29

Sol. Series pattern $84 - 1^2 = 83$

$83 - 2^2 = 79$; $79 - 3^2 = 70$

$70 - 4^2 = 54$ <- 52 $54 - 5^2 = 29$

Clearly, wrong number is 52.

Type © To Find the Missing Series Based on a Given Series

Two number series are given in this type question. The pattern of the second series is same as the pattern of the first series and on the basis of the first series, we have to find out the unknown terms of second series.

Ex. 37. There are two number series given below. First number series is arranged in a particular way and second series is based on first series. On the basis of this, which number will come at the place of D.

49 100 153 208 265 324

16 (A) (B) (C) (D) (E)

Sol. $49 \xrightarrow{+51} 100 \xrightarrow{+53} 153 \xrightarrow{+55} 208 \xrightarrow{+57} 265 \xrightarrow{+59} 324$

Similarly,

$16 \xrightarrow{+51} \overset{A}{67} \xrightarrow{+53} \overset{B}{120} \xrightarrow{+55} \overset{C}{175} \xrightarrow{+57} \overset{D}{232} \xrightarrow{+59} \overset{E}{291}$

Hence, from the above, it is clear that the value at the place of D is 232.

Fast Track Practice

Exercise© *Base Level Questions*

Directions (Q. Nos. 1-30) *Find the missing term in each of the following series.*

1. 6, ?, 21, 33, 48

(a) 14 fbj 20 *(c)* 12 fdj 18 (e) None of the above

2. 4, 12, 36, ?, 324, 972 [SSC LDC 2009]

(a) 108 *(b)* 125 (cj 215 *(d)* 312 2, 8, 18, ? , 50, 72 [Bank Clerks 2008]

(a) 33 *(b)* 38 (c) 36 (tfj 32

4. 10000, 2000, 400, 80, 16, 3.2?

[IBPS Clerk 2011]

(a) 0.38 fbj 0.45 *(c)* 0.64 fdj 0.54 (e) None of the above

5. 2, 9, 28, 65, 126, ... [SSC (10+2) 2010]

(a) 195 fbj 199 *(C)* 208 fdj 217

6. 6, 13, 32, ?, 130, 221

(a) 75 fbj 69 (c) 100 fdj 85 *(e)* None of the above

7. ..., 9, 25, 49, 81, 121

(a) 5 r^J 1 *(c)* 4 fdj 3

(e) None of the above

8. 3, 6, 12, 24, 48, 96? [IBPS Clerk 2011]

(a) 192 *(b)* 182 *(c)* 186 fdj 198 *(e)* None of the above

9. 12, 15, 21, 30, 42, 57, ? [Bank Clerks 2011]

(a) 59 fbj 63 (c) 65 *(d)* 75 (e) None of the above

10. 3, 5, 9, 17, 33, 65, ? [Bank Clerks 2011]

(a) 129 *(b)* 128 *(c)* 126 fdj 132 *(e)* None of the above

11. In the following number series, one of the numbers is wrong. Find out the wrong number.

14, 28, 112, 672, 5374, 53760 [SBI2012]

(a) 112 *(b)* 672

(c) 5374 *(d)* 28

(e) None of the above

12. 48, 23, ?, 4.25, 1.125

(a) 10.5 fbj 10 *(c)* 2.5 fc/; 11

13. 4, 25, 130, ?, 3280 [Bank PO 2008]

(a) 685 (b) 645 *(c)* 678 fdj 655 *(e)* None of the above

14. 41, 40, 36, ?, 11

(a) 35 fbj 27 (c) 29 fdj 30 *(e)* None of the above

15. 135, 134, ?, 99, 35

(a) 126 fbj 115 (c) 85 fdj 111 *(e)* None of the above

16. 1, 9, 25, 49, 81, ?

(a) 90 fbj 135 *(c)* 121 *(d)* 125 *(e)* None of the above

17. 2, 15, 41, 80, 132, ?

(a) 197 fbj 150 (c) 178 fdj 180 *(e)* None of the above

18. Find the missing number in the following series.

4, 18..... 100, 180, 294 [IBACIO2013]

(a) 32 *(b)* 36 *(c)* 48 *(d)* 40

19. 840, ?, 420, 140, 35, 7 [Bank PO 2007]

(a) 408 fbj 840

(c) 480 fdj 804

(e) None of the above

20. 5, ?, 15, 75, 525, 4725, 51975

[IBPS Clerk 2011]

(a) 5 fbj 10 (c) 8 fdj 6

fej None of the above

21. 13, 35, 57, 79, 911, ?

(a) 1113 *(b)* 1123

(c) 1114 *(d)* 1124

(e) None of the above

22. 125, 80, 45, ?, 5

(a) 20 fbj 30 (c) 25 fdj 35 *(e)* None of the above

23. 4, 19, 49, ?, 229 [RRB 2008]

(a) 75 fbj 109 (c) 65 fdj 169

24. Complete the series 7, 26, 63, 124, 215,

342, ?, [SSC CGL (Main) 2012]

(a) 481 fbj 511

(c) 391 fdj 421

25. 2, 6, 14, 30, ... 126 [SSC (10+2) 2009]

(a) 62 fbj 63

(c) 73 *(d)* 95

26. 165, 195, 255, 285, ..., 435

(a) 340 (b) 341 (c) 345 (d) 401 (e) None of the above

27. 1220, 244, ..., 9.76, 1.952 [MBA 2009]

(a) 48.8 fbj 49.2 (c) 47 fdj 8 (e) None of the above

28. 2, 10, 42, 170, ?, 2730, 10922

[IBPS Clerk 2011]

(a) 588 fbJ 568 (c) 596 fay 682 (e) None of the above

29. 23, 42.2, 80.6, 157.4, 311? [IDB1 SO 2012]

(a) 618.2 (b) 623.6

(c) 624.2 fdj 616.6

(e) None of the above

30. 14, 33, 104, ?, 2110 [MCA 2009]

(a) 421 fbj 433 (c) 372 fdj 840 (e) None of the above

Directions (Q. Nos. 31-35) find *the wrong*

number in each of the following series. **31.2,** 11, 38, 197, 1172, 8227, 65806

(a) 11 (b) 38

(c) 197 (d) 1172

(e) None of the above 32. 16, 19, 21, 30, 46, 71, 107

(a) 19 (b) 20

(c) 30 (d) 46

fej 71

33. 7, 9, 16, 25, 41, 68, 107, 173

[Bank Clerks 2008]

(a) 107 *(b)* 16 *(c)* 41 *(d)* 68 (e) 25

34. 4, 2, 3.5, 7.5, 26.25, 118.125

[Bank Clerks 2009]

(a) 118.125 *(b)* 26.25

(cj 3.5 *(d)* 2

fej 7.5

35. 16, 4, 2, 1.5, 1.75, 1.875

(a) 1.875 foj 1.75 *(c)* 1.5 *(d)* 2 (e) 4

36. The wrong number in the series 2, 9, 28, 65, 126, 216, 344 is [SSC (10+2) 2012]

(a) 9 *(b)* 65 (c) 216 *(d)* 25

37. What should come in place of the question mark (?) in the following number series?

3, 3, 12, 108, ?, 42300 [SBI2012]

(a) 2700 *(b)* 1728 (c) 972 *(d)* 432 (e) None of the above

38. Which of the following numbers does not fit into the series?

14, 19, 29, 40, 44, 51, 59, 73

[SSC CCL 2012]

(a) 59 *(b)* 51 (c) 44 *(d)* 29

Exercise © *Higher SkiU Level Questions*

Directions (Q. Nos. 1-25) *What should come in place of the question mark in each of the following number series?*

1. 50, 60, 75, 97.5, ? 184.275, 267.19875

[Bank SO 2010]

(a) 120.50 *(b)* 130.50

(c) 131.625 fdj 124.25

(e) None of the above

2. 12, 15, 36, ?, 480, 2415, 14508

[Bank SO 2010]

(a) 115 *(b)* 109 (c) 117 *(d)* 121 (e) None of the above

3. 1, 2, 6, 21, 88, 445, ?

(a) 2230 *(b)* 2676 *(c)* 2580 *(d)* 2670 *(e)* None of the above

4. 20, 21, 25, 34, 50, ?, Ill [Bank SO 2010]

(a) 70 fbj 65

(c) 60 *(d)* 75

fej None of the above

5. 600, 125, 30, ?, 7.2, 6.44, 6.288

(a) 6 fbj 10 *(c)* 15 fa!) 11 fe) None of the above

6. 12, 24, 72, 144, 432, ?

(a) 728 fbj 852 (c) 864 fdj 1296 fe) None of the above

7. 13, 16, 22, 33, 51, ? [Bank PO 2010]

(a) 89 fbj 78 (c) 102 fd) 69 *(e)* None of the above

8. 39, 52, 78, 117, 169, ? [Bank PO 2010]

b) 246 *(b)* 182 *(c)* 234 fd) 256 fe) None of the above

9. 62, 87, 187, 412, 812, ?

b) 1012 *(b)* 1437 (c) 1337 fd) 1457 *(e)* None of the above

10. 24, 536, 487, 703, 678, ? [IDBI SO 2012]

(a) 768 *(b)* 748 fc) 764 *(d)* 742 fe) None of the above

11. 656, 432, 320, 264, 236, ? [Bank PO 2010]

(a) 222 *(b)* 229 *(c)* 232 fd) 223 fe) None of the above

12. 9, 62, ?, 1854, 7415, 22244 [IDBI SO 2010]

(a) 433 *(b)* 309 *(c)* 406 fd) 371 *(e)* None of the above

13. 4, 8, 24, 60, ?, 224 [IDBI SO 2010]

(a) 178 fbj 96

(c) 109 *(d)* 124

fej None of the above

14. 36, 154, 232, 278, 300, ? [IDBI SO 2012]

(a) 306 fbj 313

(c) 308 fay 307

fej None of the above

15. 456.5, 407, 368.5, 341, 324.5, ?

[IDBI SO 2012]

(a) 321 fbj 319

(c) 317 fdj 323

(e) None of the above

16. 7, 8, 18, ?, 232, 1165

(a) 84 *(b)* 42

(c) 57 fdj 36

fej None of the above

17. 5, 54, 90, 115, 131, 140, ? [Bank PO 2010]

(a) 149 fbj 146

(cj 144 *(d)* 152

fej None of the above

18. 7, 4, 5, 9, ?, 52.5, 160.5 [Bank PO 2010]

(a) 32 (b) 16

(c) 14 fdj 20

(e) None of the above

19. 6, 42, ?, 1260, 5040, 15120, 30240

[Bank PO 2011]

(a) 546 *(b)* 424

(c) 252 *(d)* 328

fej None of the above

20. 4, 10, 40, 190, 940, ?, 23440

[Bank Clerks 2011]

(a) 4690 fbj 2930

(c) 5140 fdj 3680

fej None of the above

21. 2, 9, 30, ?, 436, 2195, 13182

(a) 216 (b) 105

(c) 178 fdj 324

fej None of the above

22. 3, 5, 7, ?, 13, 17, 19, 23 [Bank Clerks 2009]

(a) 9 fbj 11

(c) 8 *(d)* 10

fej None of the above

23. Next number in the series 6, 16, 36, 76, 156, 316 is [SSCFCI2012]

(a) 636 fbj 638

(c) 633 *(d)* 632

24. 123, 277, 459, 669, 907, ? [IDBI SO 2012]

(a) 1179 fbj 1173

(c) 1167 fdj 1169

fej None of the above

25. 150, 152, 149, 153, 148, 154, ?

[Bank Clerks 2009]

(a) 155 fbj 152

(c) 147 *(d)* 149

(e) None of the above

Directions (Q. Nos. 26-35) find *the wrong number in each of the following series.*

26. 13, 25, 40, 57, 79, 103, 130

(a) 25 *(b)* 40

(c) 57 fdj 79

fej None of the above

27. 850, 600, 550, 500, 475, 462.5, 456.25

(a) 600 *(b)* 550

(c) 500 fdj 462.5

(e) None of the above

28. 2, 10, 18, 54, 162, 486, 1458

[Bank PO 2010]

(a) 18 fbj 54

(c) 162 fdj 10

fej None of the above

29. 8, 12, 24, 46, 72, 108, 52

(a) 12 fbj 24

(c) 46 *(d)* 72

(e) None of the above

30. 142, 119, 100, 83, 65, 59, 52

(a) 65 fbj 100

(c) 59 *(d)* 119

(e) None of the above

31. 12, 6, 7.5, 12.75, 27.5, 71.25

[Bank PO 2011]

(a) 6 fbj 7.5

(c) 27.5 *(d)* 12

fe) None of the above

32. 16, 24, 37, 54, 81, 121.5

[Hotel Mgmt. 2011]

(a) 24 fbj 54

(c) 37 fdj 121.5

fe) None of the above

33. 12, 12, 18, 48, 180, 1080

[Bank Clerks 2009]

(a) 180 fbj 12

(c) 18 *(d)* 48

fe) None of the above

34. 22, 23, 27, 36, 58, 77 [Bank Clerks 2008]

(a) 77 fbj 58

(c) 36 fdj 22

(e) None of the above

35. 16, 14, 24, 66, 250, 1270

(a) 250 *(b)* 66

(c) 16 *(d)* 1270

(e) None of the above

Directions (Q. Nos. 36-45) *There are two series given below, of which one is complete and fal-lows a certain pattern. Based on the pattern of complete series, you have to find the different answering terms of series.*

36. 1275 1307 1371 1467 1595 1755 972 A B C D E

[1BPSPO2013]

Which number will come at the place of D?

(a) 1292 *(b)* 550 *(C)* 500 *(d)* 462.5 *(e)* None of the above

37.4 23 113 449 1343 2681 7 A B C D E

[1BPSPO2013]

Which number will come at the place of E?

(a) 4793 *(b)* 4782

(c) 4841 *(d)* 4932

(e) None of the above

38.3 4 10 33 136 685

7 A B C D E [1BPSPO2013]

Which number will come at the place of E?

(a) 57 *(b)* 1165

(c) 18 *(d)* 1398

(e) None of the above

39. 6 9 18 45 135 472.5

20 A B C D E [1BPSPO2013]

Which number will come at the place of C?

(a) 30 *(b)* 1125

(c) 375 *(d)* 150

(e) None of the above

40.2 9 57 337 1681 6721

7 A B C D E [IBPSPO2013]

Which number will come from the following at place of E?

(a) 673 *(b)* 3361

(c) 13441 *(d)* 17

(e) None of the above

41.25 194 73 154 105 14 A B C D

Which number will come from the following at place of D?

(a) 183 *(b)* 62

(c) 69 *(d)* 94

42. 140 68 36 16 10 3 284 A B C D E

Which number will come from the following at place of B?

(a) 72 *(b)* 140

(c) 34 *(d)* 18

(e) None of the above

43. 3 20 118 587 2344 7027 12 A B C D E

Which number will come from the following at place of C?

(a) 2477 *(b)* 83

(c) 496 *(d)* 9904

44. 4 16 48 120 272 584 124 A B C D E

Which number will come from the following at place of B?

(a) 256 *(b)* 528

(c) 1080 *(d)* 4424

45. 8 584 143 467 242 386 43 A B C D E

Which number will come from the following at place of C?

(a) 321 *(b)* 277

(c) 519 *(d)* 502

(e) None of the above

Answer with Solutions

Exercise© *Base Level Questions*

1. (C) Series pattern

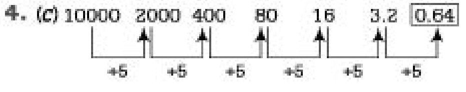
+ 6, + 9,+ 12,+ 15,... .\ Missing term = 6 + 6 = 12

2. (a) Series pattern

Every next element = 3 x Previous element ∴ Missing term = 3 X 36 = 108

3. *(d)* Series pattern

$2X1^Z,2X2^Z,2X3^2,2X4^Z, 2X5^2, 2 X 6^2$ ∴ Missing term = $2 X 4^Z = 32$



/ = U.b4

5. (d) Series pattern

$l^3 + 1.2^3 + 1,3^3 + 1,4^3 + 1,5^3 + 1,6^3 + 1$ ∴ Missing term = $6^3 + 1 = 217$

6. (h) Series pattern

$l^3 + 5,2^s + 5,3^s + 5,4^3 +5,$^+5 ∴ Missing term = $4^3 + 5 = 69$

7. *{b)* Series pattern

l^2 .& ,&."? ,9^Z ,11^2 ∴ Missing term = 1 =1

8. (a) $3 \xrightarrow{\times 2} 6 \xrightarrow{\times 2} 12 \xrightarrow{\times 2} 24 \xrightarrow{\times 2} 48 \xrightarrow{\times 2} 96 \xrightarrow{\times 2} \boxed{192}$

? = 192 .". Missing term = 192 9. (d) Series pattern

+ 3,+ 6,+ 9,+ 12,+ 15,+ 18 ∴ Missing term = 57+18 = 75

10. (a) Series pattern

(2 x Previous element) -1 ∴ Missing term = (2 X 65) - 1

= 130-1 = 129

11. (c) The given number series is based on the following pattern

$14 \xrightarrow{\times 2} 28 \xrightarrow{\times 4} 112 \xrightarrow{\times 6} 672 \xrightarrow{\times 8} 5376 \xrightarrow{\times 10} 53760$

Hence, the wrong number is 5374.

12. (a) Series pattern $= \dfrac{\text{Previous element}}{2} - 1$

∴ Missing term $= \dfrac{23}{2} - 1 = 10.5$

13. (d) Series pattern

(5 X Previous element) + 5 ∴ Missing term = (5 x 130 + 5) = 655

14. *(b)* Series pattern

$(-l)^2,(-2)^2,(-3)^2,(-4)^2$ Missing term = $36-3^2=27$

15. (a) Series pattern

$(-l)^3,(-2)^3,(-3)^3,(-4)^3$ So, missing term = 134 - 2^S =126

16. (c) Series pattern $l^z,3^2,b^z,f,9^z,ll^z$ So, missing term = 11 =121

17. (a) Series pattern

+ 13, + 26, + 39,+ 52,+65 ∴ Missing term = 132+65 = 197

18. (*c*)

$$\begin{array}{ccc} 4 & 18 & 48 \\ \downarrow & \downarrow & \downarrow \\ 2^2\times1 & 3^2\times2 & 4^2\times3 \\ 100 & 180 & 294 \\ \downarrow & \downarrow & \downarrow \\ 5^2\times4 & 6^2\times5 & 7^2\times6 \end{array}$$

19. (*b*) Series pattern +1, +2, +3, +4, +5

∴ Missing term $= \frac{840}{1} = 840$

20. (*a*)

$$5 \xrightarrow{\times1} \boxed{5} \xrightarrow{\times3} 15 \xrightarrow{\times5} 75 \xrightarrow{\times7} 525 \xrightarrow{\times9} 4725 \xrightarrow{\times11} 51975$$

∴ ? = 5

21. (a) Series pattern

The elements of the given series are the

numbers formed by joining together

consecutive odd numbers in order,

i.e., 1 and 3, 3 and 5, 5 and 7, 7 and 9,

9 and 11, ...

.'. Missing term = Number formed by

joining 11 and 13 = 1113

22. (a) Series pattern -45,-35,-25,... /. Missing term = 45 - 25 = 20

23. *(b)* Series pattern +15,+ 30,+ 60,+120 ∴ Missing term = 49 + 60 = 109

24. (*b*)

$$\begin{array}{ccccccc} 7 & 26 & 63 & 124 & 215 & 342 & \boxed{511} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2^3-1, & 3^3-1, & 4^3-1, & 5^3-1, & 6^3-1, & 7^3-1, & 8^3-1 \end{array}$$

25. (a) Series pattern

(2 X Previous number) + 2 ∴ Missing term = 2 X 30 + 2 = 62

26. (c) Series pattern

(15X11),(15X13),(15X17),

(15x19), (15 x 23), (15x29) ∴ Missing term = (15x23) =345

27. (*a*) Series pattern $\frac{\text{Previous number}}{5}$

∴ Missing term = $\frac{244}{5}$ = 48.8

28. (*d*) 2 10 42 170 $\boxed{682}$ 2730 10922

´4+2 ´4+2 ´4+2 ´4+2 ´4+2 ´4+2

∴ Missing term = 682

29. (*a*)

23 42.2 80.6 157.4 311 $\boxed{618.2}$

×2 3.8 ×2 3.8 ×2 3.8 ×2 3.8 ×2 3.8

/.Missing term = 618.2

30. (a) Series pattern

X2+5,X3+5,X4 + 5, X5+5 ∴ Missing term = 104 X4 + 5 = 416 + 5 = 421

31. *id)* Series pattern

2x3+5 = 11, 11x4-6 = 38 38x5+7 = 197, 197x6-8 = 1174 should come in place of 1172 1174x7+9 =8227 8227x8 -10=65806 Clearly, 1172 is wrong and will be replaced by 1174.

32. (a) Series pattern

$16 + 1^2$ =17

should come in place of 19

17+ 2^Z =21; 21 +3^s =30; 30 + 4^2 =46; 46 + 5^2 =71 71+ 6^2 =107

Clearly, 19 is wrong and will be replaced by 17.

33. (d) Series pattern

7+9 = 16; 9 + 16=25 16 + 25 = 41; 25 + 41 = 66 should come in place of 68 41 + 66 = 107; 66 + 107 = 173 Clearly, 68 is wrong and will be replaced by 66.

34. (c) Series pattern 4X0.5 = 2

2x1.5 = 3 should come in place of 3.5 3X2.5 = 7.5 7.5X3.5 = 26.25

26.25X4.5= 118.125 Clearly, 3.5 is wrong and is replaced by 3.

35. *{b)* Series pattern

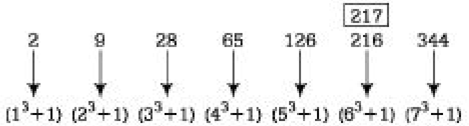
16x0.25=4; 4X 0.50 = 2 2X0.75 = 1.5; 1.5 X 1.00= 1.5

should come in place of 1.75 1.5 x 1.25 = 1.875

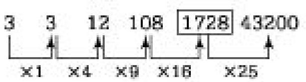
Clearly, 1.75 is wrong and will be

replaced by 1.5.

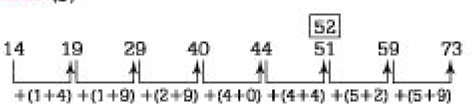
36. (c)



37. (*b*) The given number series is based on the following pattern



38. (*b*)



Exercise © *Higher SkiU Level Questions*

1. (c) Series pattern

50x1.2=60; 60x1.25 = 75 75 X 1.3 = 97.5; 97.5 X 1.35 = 131.625 should come in place of'?'

131.625 X 1.4= 184.275 184. 275 X 1.45 = 267.19875

2. (c) Series pattern

12x1 + 3x1 = 15 15x2+3x2 = 36 36X3+3X3= 117 should come in place of ?

117X4 + 3X4=480

480x5+ 3x5 = 2415

2415X6 + 3X6 = 14508

3. *{b)* Series pattern

1x1+1=2

2x2+2=6

6 X3+ 3 = 21

21x4 + 4=88

88 X5+ 5 = 445

445x6 + 6 = 2676

should come in place of'?'

4. *id)* Series pattern

20 + 1^2 =21; 21 + *2?* =25 25+ 3^Z =34; 34 + 4^Z =50 50 + 5^2 = 75 should come in place of'?' 75 + 6^2 =111

5. (*d*) Series pattern

$$\frac{600}{5} + 5 = 125$$

$$\frac{125}{5} + 5 = 30$$

$$\frac{30}{5} + 5 = 11$$

should come in place of '?'

$$\frac{11}{5} + 5 = 7.2$$

$$\frac{7.2}{5} + 5 = 6.44$$

$$\frac{6.44}{5} + 5 = 6.288$$

6. (c) Series pattern

12x2 = 24

24x3 = 72

72X2 = 144

144X3 = 432

432 x 2 = 864

should come in place of'?'

7. (*b*)

13 16 22 33 51 78

+3 +6 +11 +18 +27

+3 +5 +7 +9

8. (c) Series pattern

39 + 1x13 = 52

52 + 2x13 = 78

78 + 3X13 = 117

117 + 4 X13 = 169

169 + 5x13= 234

should come in place of ?

9. *{b)* Series pattern

$62+ 5^2 =62+ 25 = 87$

$87+ 10^2 =87+ 100 = 187$

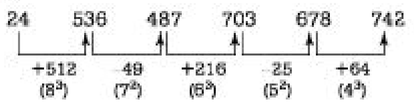
$187+ 15^2 =187+ 225 = 412$

$412+ 20^2 =412 + 400 = 812$

$812 + 25^2 = 812 + 625 = 1437$

should come in nlace of '?'

10. (*d*)



11. (a) Series pattern

656-224=432 432-112 = 320 320-56 = 264 264-28 = 236 236-14 = 222

should come in place of'?'

12. *id)* Series pattern

9x7-1=62

62x6-1 = 371

should come in place of'?'

371x5-1 = 1854

1854x4-1 = 7415

7415x3-1 = 22244

13. (d) Series pattern

$4 + 2^2$=8

$8 + 4^2$ = 24

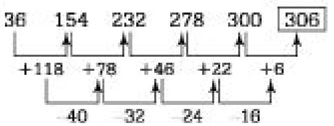
$24 + 6^2$ = 60

$60 + 8^2$ = 124

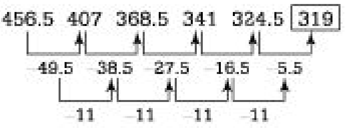
should come in place of'?'

$124 + 10^z$ =224

14. (*a*)



15. (*b*)



16. (c) Series pattern 7x1 + 1=8

8 X2+ 2 = 18

18x3+3=57

should come in place of'?'

57x4 + 4 = 232

232X5+ 5 = 1165

17. (c) Series pattern

5+ *f* =54

$54 + 6^Z = 90$

$90 + 5^2 = 115$

$115 + 4^2 = 131$

$131 + 3^2 = 140$

$140 + 2^2 = 144$

should come in place of'?'

18. (d) Series pattern 7 x 0.5 + 0.5 = 4

4x1+1=5 5X1.5+ 1.5 = 9 9 X2+ 2 = 20 should come in place of'?' 20 X 2.5+ 2.5 = 52.5 52.5X3+ 3= 160.5

19. (c) Series pattern

6X7 = 42 42X6 = 252 should come in place of'?' 252X5 = 1260 1260X4 = 5040 5040X3 = 15120 15120x2 = 30240

20. (a) Series pattern

4 + (6 x 1) = 10 10+ (6x5) =40 40 + (30X5) = 190 190 + (150 X5) =940 940 + (750 X 5) = 4690 should come in place of'?'

4690 + (3750 X 5) = 23440

21. *{b)* Series pattern

(2 + 7) X 1 = 9

(9 + 6) x 2 = 30

(30 + 5)X3= 105

should come in place of'?'

(105 + 4) X4=436

(436 + 3) X5 = 2195

(2195 + 2) x6 = 13182

22. (h) Series pattern

Prime number series 3, 5, 7, 11, 13, 17, 19,23

23. (a) 6x2 + 4 = 16

16 X2 + 4 = 36 36 X2 + 4 = 76 76x2 + 4 = 156 156x2 + 4 = 316 316x2 + 4 = 636

24. (*b*)

$$123 \quad 277 \quad 459 \quad 669 \quad 907 \quad \boxed{1173}$$

$$+154 \quad +182 \quad +210 \quad +238 \quad +266$$

$$+28 \quad +28 \quad +28 \quad +28$$

∴ Missing term = 1173

25. (*c*) Series pattern

$$-1 \qquad -1 \qquad -1$$

$$150 \quad 152 \quad 149 \quad 153 \quad 148 \quad 154 \quad 147$$

$$+1 \qquad +1$$

26. (c) Series pattern

13+ 12 = 25 25+ 15 = 40 40 + 18 = 58 should come in place of 57 58 + 21 = 79 79 + 24 = 103 103 + 27 = 130

27. (a) Series pattern

850-200 = 650 should come in place of 600 650-100 = 550 550-50 = 500 500-25 = 475 475- 12.5 = 462.5 462.5 - 6.25 = 456.25

28. (d) Series pattern

2x3 = 6

should come in place of 10

6x3 = 18

18X3 = 54

54x3 = 162

162x3 = 486

486x3 = 1458

29. (c) Series pattern

8 + 4 = 12 12 + 12 = 24 24 + 20 = 44 should come in place of 46 44 + 28 = 72 72+ 36 = 108 108 + 44 = 152

30. (a) Series pattern

142-23 = 119 119-19 = 100 100-17 = 83 83-13= 70 should come in place of 65

70-11 = 59; 59-7 = 52

31. (a) Series pattern

12x0.5+0.5 = 6.5 should come in place of 6 6.5x1 + 1 = 7.5 7.5 X 1.5 + 1.5 = 12.75 12.75x2 + 2= 27.5 27.5x2.5+ 2.5 = 71.25

32. (c) Series pattern

$$16 \times \frac{3}{2} = 8 \times 3 = 24$$

$$24 \times \frac{3}{2} = 12 \times 3 = 36$$

should come in place of 37

$$36 \times \frac{3}{2} = 18 \times 3 = 54$$

$$54 \times \frac{3}{2} = 27 \times 3 = 81$$

$$81 \times \frac{3}{2} = 40.5 \times 3 = 121.5$$

33. (d) Series pattern

12x1 = 12

12x 1.5 = 18

18 X 2.5 = 45

should come in place of 48

45X4 = 180

180x6 = 1080

34. *{b)* Series pattern

22+ 1^z =22+ 1 = 23 23+ 2^2 =23 + 4 = 27 27+ 3^2 =27+9 =36 36 + 4^z =36 + 16 = 52 should come in place of 58

52 + 5^2 = 52 + 25 = 77

35. (a) Series pattern

16x1-2 = 14 14x2-4 = 24

24x3-6=66 66x4-8 = 256 Should come in place of 250 256x5-10 = 1270

36. (a) Given number series is on the following pattern

+ 32,+64,+96,+ 128... On this pattern, starting from 972, 1292 will come at place of D.

37. (c) Given number series is on the following pattern

X6-1, X5-2,X4-3, X3-4, ...

On this pattern, starting from 7, 4841 will come at place of E.

38. (h) Given number series is on the following pattern

X 1 + 1, X 2 + 2, X 3 + 3, X 4 + 4,... On this pattern, starting from 7, 1165 will come at the place of E.

39. *id)* Given number series is on the following pattern

Xl.5,X2,X2.5,X3,X3.5,X4 ... On this pattern, starting from 20, 150 will come at the place of C.

40. *(e)* Given, number series is on the following pattern

X8-7, X7-6, X6-5, X5-4.....

On this pattern, starting from 3, 40321 will come at the place of E.

41. *{d)* Given number series is on the following pattern

+ 169,-121,+81,-49.....

On this pattern, starting from 14, 94 will come at the place of D.

42. *(a)* Given number series is on the following pattern

-5-2-2,-5-2+ 2,-5-2-2,-5-2+ 2,... On this pattern, starting from 284, 72 will come at the place of B.

43. (a) Given number series is on the following pattern

X7-1,X6 -2,X5-3,X4 -4,... On this pattern, starting from 12, 2477 will come at place of C.

{b) Given number series is on the following pattern

+ 4 X2,+ 8 X 2,+ 12X2,+ 24 X2,... On this pattern, starting from 124, 528 will come at the place of B. 45. *id)* Given number series is on the following pattern

+ 24^2,-21^2 + 18^2,-15^2... On this pattern starting from 43, 502 will come at the place of C.

Chapter

3

HCF and LCM

Factors and Multiples

If a number x divides another number y exactly (without leaving any remainder), then x is a factor of y and y is a multiple of x.

or

Factors Set of numbers which exactly divides the given

number. **Multiples** Set of numbers which are exactly divisible by

the given number. *For example* If the number is 8, then {8, 4, 2, 1} is the set

of factors, while { 8, 16, 24, 32, ...} is the set of

multiples of 8. **Note** Factors of a number are always less than or equal to the

given number 2. Multiples of a number are always more than or equal to the

given number

Common Multiple

A common multiple of two or more numbers is a number which is completely divisible (without leaving remainder) by each of them.

For example We can obtain common multiples of 3, 5 and 10 as follows

♦ Multiples of 3

= {3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33 ...}

♦ Multiples of 5 = { 5, 10, 15, 20, 25, 30, 35, ...}

♦ Multiples of 10 = { 10, 20, 30, 40, 50, 60, ...} ∴ Common multiples of 3, 5 and 10

= { 30, 60, 90, 120 ...}

Least Common Multiple (LCM)

The LCM of two or more given numbers is the least number to be exactly divisible by

each of them.

For example We can obtain LCM of 4 and 12 as follows

Multiples of 4 = 4,8,12,16,20, 24,28,32,36, ...

Multiples of 12 =12, 24,36,48,60,72..........

Common multiples of 4 and 12 =12,24,36, ...

∴ LCM of 4 and 12 =12

Methods to Calculate LCM

There are two methods to find the LCM of two or more numbers which are explained below.

1. Prime Factorisation Method

Following are the steps to obtain LCM through prime factorisation method.

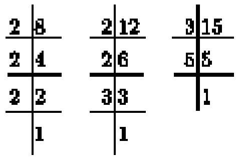
***Step* I** Resolve the given numbers into their prime factors.

***Step* II** Find the product of all the prime factors (with highest powers) that occur in the given numbers.

***Step* III** This product of all the prime factors (with highest powers) is the required LCM.

Ex. 1 Find the LCM of 8,12 and 15.

Sol.



Factors of 8 = 2x2x2 = 2^3

Factors of 12 = 2 x 2 x 3 = 2^2 x 3^1

Factors of 15 = 3 x 5 = 3^1 x 5^1

Here, the prime factors that occur in the given numbers are 2, 3 and 5 and their highest powers are 2 , 3 and 5 .

∴ Required LCM = 2^3 X 3^1 X 5^1 = 8 X 3 X 5 = 120

2. Division Method

Following are the steps to obtain LCM through division method,

***Step* I** Write down the given numbers in a line, separating them by commas.

Step II Divide them by a prime numbers which exactly divides atleast any two of the given numbers.

***Step* III** Write down the quotients and the undivided numbers in a line below the 1st.

***Step* IV** Repeat the process until you get a line of numbers which are prime to one another.

Step V The product of all divisors and the numbers in the last line will be the required LCM.

Ex. 2 What will be the LCM of 15, 24, 32 and 45?

Sol. LCM of 15, 24,32 and 45 is calculated as

2	15,	24,	32,	45
2	15,	12,	16,	45
2	15,	6,	8,	45
3	15,	3,	4,	45
5	5,	1,	4,	15
	1,	1,	4,	3

∴ Required LCM = 2x2x2x3x5x4x3= 1440 **Note** Start division with the least prime number

Common Factor

A common factor of two or more numbers is that particular number which divides

each of them exactly.

For example We can obtain common factors of 12, 48, 54 and 63 as follows

Factors of 12 = 12, 6, 4, 3, 2, 1

Factors of 48 = 48, 24, 16, 12, 8, 6, 4, 3, 2, 1

Factors of 54 = 54, 27, 18, 9, 6, 3, 2, 1

Factors of 63 = 63, 21, 9, 7, 3, 1

∴ Common factors of 12, 48, 54 and 63 = 3

Highest Common Factor

HCF of two or more numbers is the greatest number which divides each of them exactly. *For example* 6 is the HCF of 12 and 18 as there is no number greater than 6 that divides both 12 and 18. Similarly, 3 is the highest common factor of 6 and 9.

HCF is also known as 'Highest Common Divisor' (HCD) and 'Greatest Common Measure' [GCM).

Methods to Calculate HCF

There are two methods to calculate the HCF of two or more numbers which are explained below

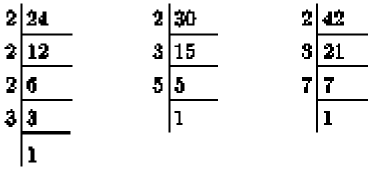
1. Prime Factorisation Method

Following are the steps for calculating HCF through prime factorisation method, ***Step*** **I** Resolve the given numbers into their prime factors. ***Step*** **II** Find the product of all the prime factors (with least power) common to all the

numbers. ***Step*** **III** The product of common prime factors (with the least powers) gives HCF.

Ex. 3 Find the HCF of 24, 30 and 42.

Sol. Resolving 24,30 and 42 into their prime factors,



∴ Factors of 24 = 2x2x2x3 = $(2^3 \times 3^1)$

Factors of 30 = 2 X 3 X 5 = $(2^1 \times 3^1 \times 5^1)$

Factors of 42 = 2 x 3 x 7 = $(2^1 \times 3^1 \times 7^1)$ /. The product of common prime factors with the least powers = 2 X 3 =6 So, HCF of 24, 30 and 42 = 6

2. Division Method

Following are the steps to obtain HCF through division method.

Step **I** Divide the larger number by the smaller one.

Step **II** Divide the divisor by the remainder.

Step **III** Repeat step II till the remainder becomes zero. The last divisor is the required HCF.

Note To calculate the HCF of more than two numbers, calculate the HCF of first two numbers then take the third number and HCF of first two numbers and calculate their HCF and so on. The resulting HCF will be the required HCF of numbers

Ex. 4 Find the HCF of 26 and 455.

Sol.

26) 455 (17
26
195
182
13) 26 (2
26
×

∴ **Required HCF = 13**

Ex. 5 What will be the HCF of 1785,1995 and 3381?

Sol. At the 1st step, use any two of three numbers.

1785) 1995 (1
1785
210) 1785 (8
1680
105) 210 (2
210
×

∴ HCF for 1785 and 1995 = 105

At the 2nd step, use obtained HCF 105 and the 3rd given number 3381.

105) 3381 (32
315
231
210
21) 105 (5
105
×

∴ Required HCF = 21

From the above examples we can notice that HCF is the factor of difference of the given numbers.

MIND IT!

To find the HCF of given numbers you can divide the numbers by their lowest possible difference. If these numbers are divisible by this difference, then this difference itself is the HCF of the given numbers otherwise any other factor of this difference will be its HCF

Ex. 6 Find the HCF of 30, 42 and 135.

Sol. We can notice that the difference between 30 and 42 is less than difference between 135 and 42 or 30.

Difference between 30 and 42 is 12, but 12 does not divide 30, 42 and 135 completely.

$\therefore$ Factors of 12 = 12, 6, 4, 3, 2, 1.

Clearly, 3 is the highest factor, which divides all the three numbers completely.

$\therefore$ 3 is the HCF of 30, 42 and 135.

Method to Calculate LCM and HCF of Fractions

The LCM and HCF can be obtained by *the following formulae*

- LCM of fractions $= \dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$
- HCF of fractions $= \dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$

Note 1. All the fractions must be in their lowest terms. If they are not in their lowest terms, then conversion in the lowest form is required before finding the HCF or LCM

2. The required HCF of two or more fractions is the highest fraction which exactly divides each of the fractions

3. The required LCM of two or more fractions is the least fraction/integer which is exactly divisible by each of them

4. The HCF of numbers of fractions is always a fraction but this is not true in case of LCM

Ex. 7 Calculate the LCM of $\frac{72}{250}, \frac{126}{75}$ and $\frac{162}{165}$.

Sol. Here, $\frac{72}{250} = \frac{36}{125}, \frac{126}{75} = \frac{42}{25}$ and $\frac{162}{165} = \frac{54}{55}$

$\therefore$ According to the formula,

$\therefore$ Required LCM $= \frac{\text{LCM of 36, 42 and 54}}{\text{HCF of 125, 25 and 55}} = \frac{756}{5} = 151\frac{1}{5}$

Ex. 8 Find the HCF of $\frac{36}{51}$ and $3\frac{9}{17}$.

Sol. Here, $\frac{36}{51} = \frac{12}{17}$ and $3\frac{9}{17} = \frac{60}{17}$

Now, we have to find the HCF of $\frac{12}{17}$ and $\frac{60}{17}$.

According to the formula,

HCF of fractions $= \frac{\text{HCF of numerators}}{\text{LCM of denominators}} = \frac{\text{HCF of 12 and 60}}{\text{LCM of 17 and 17}} = \frac{12}{17}$

Note The LCM and HCF of decimals can also be obtained from the above formulae (by converting the decimal into fraction),

Ex. 9 Find the LCM of 0.6, 9.6 and 0.12.

Sol. The given numbers are equivalent to $\frac{6}{10}, \frac{96}{10}, \frac{12}{100}$.

Now, we have to find the LCM of $\frac{6}{10}, \frac{96}{10}, \frac{12}{100}$.

According to the formula,

LCM of fractions $= \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$

$= \frac{\text{LCM of 6, 96, 12}}{\text{HCF of 10, 10, 100}}$

$= \frac{2 \times 2 \times 3 \times 8}{10} = \frac{96}{10} = 9.6$

LCM of 6, 96, 12

2	6,	96,	12
2	3,	48,	6
3	3,	24,	3
8	1,	8,	1
	1,	1,	1

Fast Track **Techniques**

to solve the **QUESTIONS**

Technique 1

Product of two numbers = HCF of the numbers x LCM of the numbers

Ex. 10 The LCM of 2 numbers is 2079 and their HCF is 27. If the 1st number is 189, then find the 2nd number. Sol. Here, LCM = 2079, HCF = 27,1st number = 189, 2nd number = ? According to the formula, Product of two numbers = HCF x LCM

$$\therefore \quad 189 \times \text{2nd number} = 2079 \times 27$$

$$\therefore \quad \text{2nd number} = \frac{2079 \times 27}{189} = 297$$

Technique 2

The greatest number which divides the numbers *x, y* and z, leaving remainders *a, b* and c respectively is given by

HCF of *(x-a), (y-b).(z-c)*

[This formula is true for any number of numbers]

Ex. 11 Find the greatest number which divides 29,60 and 103 leaving remainders 5,12 and 7, respectively. Sol. Given that,

$x = 29$, $y = 60$, $z = 103$

$a = 5$, $b = 12$ and $c = 7$ Now, according to the formula,

Required number = HCF of [(29 - 5), (60 - 12), (103 - 7)] = HCF of 24, 48, 96

Now,

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$\therefore$ Factors of 24 = 2x2x2x3 = 2^3x3^1 Factors of 48 = 2x2x2x2x3 = 2* X3^1 Factors of 96 = 2x2x2x2x2x3 = 2^5 X3^1

$\therefore$ Required HCF of 24, 48 and 96 = 2^3 x 3^1 = 8 x 3 = 24 Hence, 24 is the required number.

Technique 3

The least number which when divided by *x, y* and z leaves the remainders *a, b* and c respectively, is given by [LCM of *(x,yz)h k.*

where, *k = (x - a) =(y - b) =(z - c)*

[This formula is true for any number of numbers.]

Ex. 12 Find the least number which when divided by 24, 32 and 36 leaves the remainders 19, 27 and 31, respectively.

Sol. Given that,

x = 24, y = 32, z = 36, a = 19, 6 = 27 and c = 31 Then, 24 - 19 = 5,32 - 27 = 5,36 - 31 = 5

jfc = 5 According to the formula,

Required number = (LCM of 24, 32 and 36) - 5 Now, LCM of 24, 32 and 36 = 2 x 2 x 2 x 3 x 4 x 3 = 288 ∴ Required number = 288 - 5 = 283

2	24,	32,	36
2	12,	16,	18
2	6,	8,	9
3	3,	4,	9
	1,	4,	3

Technique 4

The least number which when divided by *x, y* and z leaves the same remainder *k* in each case, is given by [LCM of *(x, y,z) + k].*

[This formula is true for any number of numbers.]

Ex. 13 Find the least number which when divided by 24, 30 and 54 leaves 5 as remainder in each case.

Sol. Given that, $x = 24$, $y = 30$, $z = 54$ and $k = 5$ According to the formula,

Required number = [LCM of (24, 30 and 54)] + 5 Now, LCM of 24, 30, 54

$$\begin{array}{r|lll} 2 & 24, & 30, & 54 \\ \hline 3 & 12, & 15, & 27 \\ \hline & 4, & 5, & 9 \end{array}$$

∴ LCM = 2x3x4x5x9= 1080 ∴ Required number = 1080 + 5 = 1085

Technique 5

The greatest number that will divide *x, y* and z......leaving the same

remainder in each case, is given by[HCFof|x-y|,|y-z|,|z-x|......].

Ex.14 What is the greatest number that will divide 99, 123 and 183 leaving the same remainder in each case? Also, find the common remainder.

Sol. Given that, $x = 99$, $y = 123$ and $z = 183$

According to the formula,

Required number = HCF of $(|x - y|, |y - z|, |z - x|)$

Now, $|x - y| = |99 - 123| = 24$

$|y - z| = |123 - 183| = 60$

$|z - x| = |183 - 99| = 84$

Therefore,

Factors of $24 = 2^3 \times 3^1$

Factors of $60 = 2^2 \times 3^1 \times 5^1$

Factors of $84 = 2^2 \times 3^1 \times 7^1$

∴ HCF of 24, 60 and $84 = 2^2 \times 3^1 = 4 \times 3 = 12$

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

Common remainder

$$\frac{99}{12} = 8\frac{3}{12}, \frac{123}{12} = 10\frac{3}{12}, \frac{183}{12} = 15\frac{3}{12}$$

.•. Clearly, the required common remainder = 3

Technique 6

When the HCF of each pair of *n* given numbers is *a* and their LCM is *b,* then the product of these numbers is given by $(a)^{n-1}$xbor $(HCF)^{n-1}$xLCM

Ex. 15 There are five numbers. HCF of each possible pair is 4 and LCM of all the five numbers is 27720. What will be the product of all the five numbers? Sol. Given, HCF = 4, LCM = 27720 and *n* = 5 According to the formula, Required product = $(HCF)^{n}$ " ' xLCM

= $(4)^{5"1}$ x 27720= $(4)^{4}$ x 27720= 256 x 27720= 7096320

Useful Results

♦ If a is the HCF of two parts of b, then b must be divisible by a. This rule has been derived from the proven fact that, if two numbers are divisible by a certain number, then their sum is also divisible by that number.

♦ If the two numbers are prime to each other (coprimes), then their HCF should be equal to 1 Conversely, if their HCF is equal to 1, the numbers are prime to each other.

♦ LCM of numbers which are prime to each other is the product of numbers itself,

♦ If *k* is the HCF of p and *q,* then

(i) *k* is the HCF of p and p + *q* also (ii) *k* is the HCF of p and *q* - p also

♦ LCM of numbers is always divisible by the HCF of given numbers *i.e.,* if x is the LCM of a, *b,* c, d and e and y is the HCF, then x is completely divisible by y

Ex. 16 Is it possible to divide 1394 into 2 parts such that their HCF may be 34?

Sol. If 34 is the HCF of two parts of 1394, then 1394 must be divisible by 34.

Let us see $\frac{1394}{34} = 41$

Hence, it is possible to divide 1394 into 2 parts such that their HCF may be 34.

Ex.1 7 Are 6 and 11 coprimes?

Sol. HCFof6andll

Clearly, HCF of 6 and 11 is equal to 1. Therefore, 6 and 11 are coprimes or prime to each other.

```
6)11(1
  6
  5)6(1
    5
    1)5(5
      5
      x
```

Technique 7

The greatest n-digit number which when divided by *x, y* and z

(a) leaves no remainder.

Required number = n-digit greatest number - *R*

(b) leaves remainder *k.*

Required number = [n-digit greatest number - *R]* + *k*

where, *R* is the remainder obtained when n-digit greatest number is divided by

the LCM of *x, y* and z.

Ex. 18 Find the largest possible number of 5 digits which is exactly divisible by 32, 36 and 40.
Sol. Given, the numbers are 32,36 and 40. LCM of 32, 36 and 40 = 1440 Greatest 5-digit number = 99999

```
1440)99999(69
     8640
     13599
     12960
       639
```

∴ Required number = 99999-639 = 99360

Technique 8

The smallest n-digit number which when divided by *x, y* and z leaves

(a) no remainder.

Required number = [n-digit smallest number + (/_ -/?)] + *'k'*

(b) remainder *k.*

Required number = [n-digit smallest number + *{L-R)]* + *k* where, *R* is the remainder obtained when n-digit smallest number is divided by LCM of *x, y* and z. *L* is the LCM of *x, y* and z.

Ex. 19 Find the least possible 5-digit number, which when divided by 10, 12, 16 and 18 leaves remaindes 27.

Sol. First of all we will find the LCM of 10,12,16 and 18.

2	10, 12, 16, 18
2	5, 6, 8, 9
3	5, 3, 4, 9
	5, 1, 4, 3

LCM =2x2x3x5x4x3 = 720 Smallest 5-digit number = 10000 On dividing 10000 by 720,

$$
\begin{array}{r}
720)\,10000\,(13 \\
\underline{720} \\
2800 \\
\underline{2160} \\
\underline{640}
\end{array}
$$

∴ Required number = 10000 + (720 - 640) + 27= 10000 +80 + 27= 10107

Method to Solve Questions Based on Bells

To solve such questions, following steps are used.

***Step* I** Find **the LCM of given time** intervals.

***Step* II Obtained LCM is added to the initial** time **and result of this addition will be** our **answer (The next time when bells ring together).**

Note Before calculating LCM, make sure that all numbers are in the same unit. This method is also applicable to runner running round a circular track traffic light growing at different intervals.

Ex. 20 Seven bells ring at intervals of 2,3,4,6, 8,9 and 12 min, respectively. They started ringing simultaneously at 5:00 in the morning. What will be the next time when they all ring simultaneously?
Sol. LCM of 2, 3,4,6,8,9 and 12

$$\begin{array}{r|l} 2 & 2,\ 3,\ 4,\ 6,\ 8,\ 9,\ 12 \\ \hline 2 & 1,\ 3,\ 2,\ 3,\ 4,\ 9,\ \ 6 \\ \hline 3 & 1,\ 3,\ 1,\ 3,\ 2,\ 9,\ \ 3 \\ \hline & 1,\ 1,\ 1,\ 1,\ 2,\ 3,\ \ 1 \end{array}$$

LCM = 2x2x2x3x3 = 72 min = **1** h 12 min .•. Required time = (5 + 1 :12) O'clock = 6:12 O'clock in the morning.

Ex. 21 Six bells ring at intervals of 2,4,6,8,10 and 12 s, respectively. They started ringing simultaneously. How many times, will they ring together in 30 min? Sol. LCMof2,4,6.8.10andl2= 120

i.e., Bells will ring together after every 120 s or 2 min.

$\therefore$ Required number of times $= \left(\frac{30}{2} + 1\right) = 16$

Multi Concept

QUESTIONS

1. *What is the quotient, when LCM is divided by the HCF of geometric progression with first term a and common ratio r?*

(a)r"$^{+1}$ (b)r""^{2} (c)r""^{1} (d)r"$^{+2}$

***+ (c)** Given that,

First term = a and common ratio = *r*

:. Required GP = a, *ar*, *ar*^{2}, *ar*^{3}.......a/-"~^{1}

HCF of the GP = a ; **LCM** of the GP = $ar^{n \sim y}$

According to the question, $\dfrac{\text{LCM of GP}}{\text{HCF of GP}} = \dfrac{ar^{n-1}}{a} = r^{n-1}$

2. *What is the LCM of* $x^2 + 2x - 8$, $x^3 - 4x^2 + 4x$ *and* $x^2 + 4x$?

(a) $x(x+4)(x-2)^2$ (b) $x(x+4)(x-2)$ (c) $x(x+4)(x+2)^2$ (d) $x(x+4)^2(x-2)$

^ (a) $x^2 + 2x - 8 = x^2 + 4x - 2x - 8 = x(x+4) - 2(x+4) = (x-2)(x+4)$ $x^3 - 4x^2 + 4x = x^3 - 2x^2 - 2x^2 + 4x$

$= x^2(x-2) - 2x(x-2) = (x^2 - 2x)(x-2)$

$= x(x-2)(x-2)$ and $x^2 + 4x = x(x+4)$

Now, LCM of $(x^2 + 2x - 8)$, $(x^3 - 4x^2 + 4x)$ and $(x^2 + 4x)$

$= x(x-2)(x+4)(x-2) = x(x+4)(x-2)^2$

3. *What is the HCF of 36* $(3x^4 + 5x^3 - 2x^2 >$ *9* $(6x^3 +$ *4x? - 2x) and 54* $(27x^4 - x)$?

[CDS 2012]

(a) $9x(x+1)$ (b) $9x(3x-1)$ (c) $18x(3x-1)$ (d) $18x(x+1)$

w (C) Let $P(x) = 36(3x^4 + 5x^3 - 2x^2) = 36x^2(3x^2 + 5x - 2) = 36x^2\{3x^2 + 6x - x - 2\} = 36 \cdot x^2\{3x(x+2) - 1(x+2)\}$ $= 2 \times 2 \times 3 \times 3 \times x \times x \times (x+2)(3x-1)$ $Q(x) = 9(6x^3 + 4x^2 - 2x) = 9x(6x^2 + 4x - 2) = 18x(3x^2 + 2x - 1) = 18x\{3x^2 + 3x - x - 1\} = 3 \times 3 \times 2 \times x \times (3x-1)(x+1)$ $R(x) = 54(27x^4 - x) = 54x(27x^3 - 1)$

$= 2 \times 3 \times 3 \times 3 \times x \times (3x-1)(9x^2 + 3x + 1)$

So, HCFof[P(x),Q(x), fl(x)]

$= 2 \times 3 \times 3 \times x(3x-1) = 18x(3x-1)$

Fast Track Practice

Exercise© *Base Level Questions*

1. Find the LCM of 8, 15, 24 and 72.

(a) 350 *(b)* 360 *(c)* 720 *(d)* 735 *(e)* None of the above

2. If three numbers are 2a, 5a and 7a, what will be their LCM? [Bank Clerks 2011]

(a) 70a *(b)* 65a (is; 75a *(d)* $70a^3$ fe; None of the above

3. Find the LCM of (2^s x 3 x 5^2 x 7) (2^4 x 3^2 x 5 x 7^2 x 11) and (2 x 3^s x 5^4)

[RRB 2008] fa; 2^4 x 3^3 x 5^4

(b) 2 x3x7 x5x11

(c; 2^4x3^3x5^4x7^2x11

(d) 2^4 x 3^4 x 5^4 x 7

4. Find the HCF of 132, 204 and 228.

(a) 12 *(b)* 18 (c; 6 (d; 21 fe; None of the above

5. What will be the HCF of (2 x 3 x 7 x 9), *(2 x* 3 x 9 x li; and (2x3x4x5)?

[SSC CGL 2008]

fa; 2 x 3 x 7 *(b)* 2 x 3 x 9

(c) 2 x 3 *(d)* 2 x 7 x 9 x 11

6. Find the LCM of $\frac{1}{3}, \frac{2}{9}, \frac{5}{6}$ and $\frac{4}{27}$. [RRB 2007]

(a) $\frac{1}{54}$ (b) $\frac{10}{27}$ (c) $\frac{20}{3}$ (d) $\frac{3}{20}$

7. Find the LCM of $\frac{2}{3}, \frac{3}{5}, \frac{4}{7}$ and $\frac{9}{13}$.

[Delhi Police 2007]

(a) 36 (b) $\frac{1}{36}$ (c) $\frac{1}{1365}$ (d) $\frac{12}{455}$

8. Find the HCF of $\frac{4}{5}$ and $\frac{7}{15}$.

(a) $\frac{1}{13}$ (b) $\frac{1}{5}$ (c) $\frac{1}{15}$ (d) $\frac{1}{25}$

(e) None of the above

9. Find the HCF of $\frac{1}{2}, \frac{3}{4}$ and $\frac{4}{5}$.

(a) $\frac{1}{20}$ (b) $\frac{1}{40}$

(c) 20 (d) 15

(e) None of the above

10. Which of the following will be the LCM of 0.25, 0.1 and 0.125?

(a) 0.25 *(b)* 0.005

(c) 0.05 *(d)* 0.5

fe; None of the above

11. Find the LCM of 2.5, 1.2, 20 and 7.5.

(a) 60 *(b)* 65 (c; 70 fc0 50 *(e)* None of the above

12. Product of two coprime numbers is 117. Then, their LCM is [SSC CGL 2013]

(a) 9 *(b)* 13 *(c)* 39 *(d)* 117

13. The product of HCF and LCM of 18 and 15 is [CDS 2012]

(a) 120 *(b)* 150 *(c)* 175 *(d)* 270

14. The LCM of two numbers is 2376 while their HCF is 33. If one of the number is 297, then the other number is [CDS 2013]

fa; 216 *(b)* 264 *(c)* 642 *(d)* 792

15. The HCF and LCM of two numbers are 13 and 1989, respectively. If one of the numbers is 117, then determine the Other [DMRC (CRA) 2012]

fa; 121 *(b)* 131 *(c)* 221 *(d)* 231

16. The HCF of two numbers is 15 and their LCM is 225. If one of the numbers is 75, then find the another number.

[SSC CGL 2010]

fa; 105 *(b)* 90 fe; 60 *(d)* 45

17. If HCF of two numbers is 8, which of the following can never be their LCM?

[RBI Clerk 2007]

(a) 24 *(b)* 48

fe; 56 *(d)* 60

fe; None of the above

18. The difference of two numbers is 1/9 of their sum. Their sum is 45. Find the LCM. [SSC CGL 2007]

(a) 225 *(b)* 100

fe; 150 *(d)* 200

19. The ratio of two numbers is 3 : 4 and their HCF is 4. What will be their LCM?

[Hotel Mgmt. 2007]

(a) 12 *(b)* 16

fe; 24 *(d)* 48

fe; None of the above

20. The ratio of two numbers is 5 : 6 and their LCM is 480, then their HCF is

[SSC Multitasking 2013] *(a)* 20 *(b)* 16 *(c)* 6 *(d)* 5

21. The HCF of three numbers is 23. If they are in the ratio of 1: 2: 3, then find the numbers.

(a) 69, 15, 22 *(b)* 23, 46, 69

(c) 25, 31, 41 *(d)* 23, 21, 35

(e) None of the above

22. Three numbers are in the ratio of 3 : 4 : 5 and their LCM is 1200. Find the HCF of the numbers.

(a) 40 *(b)* 30 *(c)* 80 *(d)* 20 *(e)* None of the above

23. The HCF and LCM of two numbers *m* and *n* are respectively 6 and 210. If

$m + n = 72$, then $\frac{1}{m} + \frac{1}{n}$ is equal to

(a) $\frac{1}{35}$ *(b)* $\frac{3}{35}$ *(c)* $\frac{5}{37}$ *(d)* $\frac{2}{35}$

(e) None of the above

24. If a number is exactly divisible by 11 and 13, which of the following types the number must be? [Hotel Mgmt. 2008]

(a) Divisible by (11 + 13)

(b) Divisible by (13-11)

(c) Divisible by (11 x 13)

(d) Divisible by (13 + 11)

(e) None of the above

25. The LCM of two numbers is 48. The numbers are in the ratio of 2 : 3. Find the sum of the numbers. [SSC (10+2) 2011]

(a) 28 *(b)* 32 *(c)* 40 *(d)* 64

26. Four numbers are in the ratio of 10:12:15: ia If their HCF is 3, then find their LCM.

(a) 420 *(b)* 540 *(c)* 620 *(d)* 680 *(e)* None of the above

27. The product of two whole numbers is 1500 and their HCF is 10. Find the LCM.

[Bank Clerks 2008] *(a)* 15000fbJ 150 *(c)* 1500 *(d)* 15 *(e)* None of the above

28. If the HCF of a and 6 are 12 and a, 6 are positive integers and o > *b* > 12, then what will be the values of a and 6?

[RRB2012] *(a)* 12,24 *(b)* 24,12 *(c)* 24,36 *(d)* 36,24

29. The sum of HCF and LCM of two numbers is 403 and their LCM is 12 times their HCF. If one number is 93, then find the another number. [MBA 2007]

(a) 115 *(b)* 122

(c) 124 *(d)* 138

(e) None of the above

30. The LCM of two numbers is 495 and their HCF is 5. If sum of the numbers is 100, then find the difference of the numbers. [Hotel Mgmt. 2008]

(a) 10 *(b)* 46 *(c)* 70 *(d)* 90 *(e)* None of the above

31. The LCM of two numbers is 20 times of their HCF and (LCM + HCF) = 2520. If one number is 480, what will be the triple of another number?

(a) 1200 *(b)* 1500 *(c)* 2100 *(d)* 1800 *(e)* None of the above

32. The sum of two numbers is 1056 and their HCF is 66, find the number of such pairs.

(a) 6 *(b)* 2 *(c)* 4 *(d)* 8 *(e)* None of the above

33. What is the smallest possible length that can be exactly measured by the scales of lengths 3 cm, 5 cm and 10 cm?

(a) 15 cm (b) 30cm(cJ 28 cm *(d)* 40 cm *(e)* None of the above

34. What is the least number which is exactly divisible by 8, 9, 12, 15 and 18 and is also a perfect square?

(a) 3600 *(b)* 7200

(C) 5200 *(d)* 6500

(e) None of the above

35. Find the greatest number of 3-digits which when divided by 6, 9, 12 leaves 3 as remainder in each case.

[CBI 2008, BOI 2007]

(a) 975 *(b)* 996

(c) 903 *(d)* 939

(e) None of the above

36. What will be the greatest number that divides 1356, 1868 and 2764 leaving 12 as remainder in each case? [Delhi Police 2007]

(a) 64 *(b)* 124

(c) 156 *(d)* 260

37. Find the greatest number that divides 130, 305 and 245 leaving remainders 6, 9 and 17, respectively? [RBI Clerk 2008]

(a) 4 *(b)* 5 *(c)* 14 *(d)* 24 *(e)* None of the above

38. What will be the greatest number that divides 1023 and 750 leaving remainders 3 and 2, respectively?

(a) 68 *(b)* 65 *(C)* 78 *(d)* 19 *(e)* None of the above

39. In a store, there are 345 L mustard oil, 120 L sunflower oil and 225 L soyabean oil. What will be the capacity of the largest container to measure the above three types of oil?

(a) 8 L *(b)* 20 L *(c)* 23 L *(d)* 15 L *(e)* None of the above

40. The least number which should be added to 2497, so that the sum is exactly divisible by 5, 6, 4 and 3, is

(a) 3 *(b)* 13 *(c)* 23 *(d)* 33 *(e)* None of the above

41. Find the largest number which divides 1305, 4665 and 6905 leaving same remainder in each case. Also, find the common remainder. [CBI Clerk 2009]

(a) 1210, 158 *(b)* 1120, 158

(c) 1120, 185 *(d)* 1210, 185

(e) None of the above

42. The least number which when divided by 12, 16 and 18 leaves 5 as remainder in each case. Find the number.

[RBI Clerk 2009] *(a)* 139 *(b)* 144 *(c)* 149 *(d)* 154 *(e)* None of the above

43. What is the greatest number that divides 13850 and 17030 leaves a remainder 17?

[CDS 2012] *(a)* 477 *(b)* 159 *(c)* 107 *(d)* 87

44. The HCF and LCM of two natural numbers are 12 and 72, respectively. What is the difference between the two numbers, if one of the number is 24?

[CDS 2012]

(a) 12 *(b)* 18 *(c)* 21 *(d)* 24

45. What will be the least number which when divided by 12, 21 and 35 leaves 6 as remainder in each case? [UP Police 2007]

(a) 426 *(b)* 326 *(c)* 536 *(d)* 436

46. What is the least number which when diminished by 7, is divisible by each one of 21, 28, 36 and 45?

(a) 1255 *(b)* 1177 *(c)* 1265 *(d)* 1267

(e) None of the above

What is the least number which when increased by 9, is divisible by each one of 24, 32, 36 and 54?

(a) 855 *(b)* 890 *(c)* 756 *(d)* 895

(e) None of the above

48. Find the least number which when divided by 16, 18 and 20 leaves a remainder 4 in each case, but is completely divisible by 7.

(a) 2884 *(b)* 2256 *(c)* 865 *(d)* 3332 *(e)* None of the above

49. What is the greatest four digit number which when divided by 10, 15, 21 and 28 leaves remainders 4, 9, 15 and 22, respectively? [LIC ADO 2008]

(a) 9654 *(b)* 9666 *(c)* 9664 *(d)* 9864 *(e)* None of the above

50. How many numbers are there between 4000 and 6000 which are exactly divisible by 32, 40, 48 and 60?

(a) 2 *(b)* 3 *(c)* 4 *(d)* 5

51. Three pieces of timber 84 m, 98 m and 126 m long have to be divided into planks of the same length. What is the greatest possible length of each plank?

(a) 14 m *(b)* 28 m *(c)* 7 m *(d)* 21 m *(e)* None of the above

52. In a school, all the students can stand in a row, so that each row has 5, 9 or 10 students. Find the least number of students in the school.

[SSC CGL 2008] *(a)* 90 *(b)* 95 *(C)* 85 *(d)* 100

Exercise © *Higher Skill Level Questions*

1. If $(x - 6)$ is the HCF of x^L - $2x$ -24 and x — kx - 6, then what is the value of *kl*

[CDS 2012]

(a) 3 *(b)* 5 *(c)* 6 *(d)* 8

2. If a and *b* are positive integers, then what is the value of

$$\left(\frac{a}{HCF(a,b)}, \frac{b}{HCF(a,b)}\right)$$ [CDS 2014]

(a) a (b) b

(c) 1 (d) $\frac{a}{HCF(a,b)}$

3. What is the HCF of 8 (x^5 - x^3 + x) and 28(*^{6}+l)? [CDS 2014]

{a) 4 (x^4 - x^2 + 1) *(b)* x^3 - x + $4x^2$ *(c)* x^3 - x + $3x^2$ *(d)* None of these

4. The HCF of {x^4 - y^4) and (x^6 - y^6) is

[CDS 2013]

(a)x^2-/^{2} *(b)x-y*

(c) x^3 - y^3 *(d)* x^4 - y^4

5. The HCF of (x^3 - y^2 - $2x$) and (x^3 + x^2) **is [CDS 2013]**

(a) x^3 - x^2 - 2x

(b) x^2 + x

(c) x^4-x^3-$2x^2$ fc!) x-2

6. What is the HCF of a^2b^4 + $2a^2b^2$ and

(abj^7 - Aa bl [CDS 2013]

(a) ab (b) a^2b^3

(c) a^2b^2 *(d)* a^3b^2

7. For any integer n, what is HCF (22ra + 7, 33re + 10) equal to? [CDS 2014]

(a) 0 *(b)* 1

(c) 11 *(d)* None of these

8. For any integers a and *b* with HCF (a, 6) = ; what is HCF (o + *b, a - b)* equal to? [CDS 2013]

(a) It is always 1

(b) It is always 2

(c) Either 1 or 2

fc/J None of the above

9. In a fire range, 4 shooters are firing at their respective targets. The first, the second, the third and the fourth shooter hit the target once in every 5 s, 6 s, 7 s and 8 s, respectively. If all of them hit their targets at 9:00 am, when will they hit their targets together again?

[CDS 2014]

(a) 9 : 04 am

(b) 9 : 08 am

(c) 9 : 14 am

(d) None of the above

10. Three hells chime at intervals of 48, 60 and 90 min, respectively. If all the three bells chime together at 10:00 am, at what time will all the three chime again that day? [IB Grade II 2012]

(a) 1:00 pm *(b)* 2:00 pm

(c) 8:00 pm *(d)* 10:00 pm

11. Five bells begin to toll together at intervals of 9 s, 6 s, 4 s, 10 s and 8 s, respectively. How many times will they toll together in the span of 1 h (excluding the toll at the start)?

[Bank Clerks 2007]

(a) 5

(b) 8

(c) 10

(d) Couldn't be determined

(e) None of the above

12. Monica, Veronica and Rachat begin to jog around a circular stadium. They complete their revolutions in 42 s, 56 s and 63 s, respectively. After how many seconds will they be together at the starting point? [Bank Clerks 2008]

(a) 366

(b) 252

(c) 504

(d) Couldn't be determined

(e) None of the above

13. A General can draw up his soldiers in the rows of 10, 15 or 18 soldiers and he can also draw them up in the form of a solid square. Find the least number of soldiers with the General. [SSC CGL 2007]

(a) 100 *(b)* 3600 *(c)* 900 *(d)* 90

14. 6 bells commence tolling together and toll of intervals are 2, 4, 6, 8, 10 and 12 s, respectively. In 1 h, how many times, do they toll together?

(a) 16 *(b)* 32 *(c)* 21 *(d)* 35 *(e)* None of the above

15. Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm and 12 m 95 cm.

(a) 15 cm *(b)* 25 cm

(c) 35 cm *(d)* 42 cm

(e) 45 cm

16. The HCF and LCM of two numbers are 21 and 4641, respectively. If one of the numbers lies between 200 and 300, then find the two numbers.

(a) 273, 363 *(b)* 273, 359

(C) 273, 361 *(d)* 273, 357

(e) None of the above

17. Find the side of the largest possible square slabs which can be paved on the floor of a room 2 m 50 cm long and 1 m 50 cm broad. Also find the number of such slabs to pave the floor. [LIC AAO 2007]

(a) 25, 20 *(b)* 30, 15

(c) 50, 15 *(d)* 55, 10

(e) None of the above

18. When in each box 5 or 6 dozens of apples were packed, three dozens were left. Therefore, bigger boxes were taken to pack 8 or 9 dozens of apples. However, still three dozens of apples remained. What was the least number of dozens of apples to be packed?

(a) 363 dozens *(b)* 315 dozens *(c)* 345 dozens *(d)* 335 dozens *(e)* None of the above

Answer with **Solutions**

Exercise © *Base Level Questions*

1. (6) By prime factorisation method, Factors of 8 = 2x2x2 = 2^3

Factors of 15 = 3 X 5 = 3^1 X 5^1

Factors of 24 = 2 X 2 X 2 X 3 = 2^3 X 3^1

Factors of 72 = 2x2x2x3x3 = 2^3 X 3^2

Here, the prime factors that occur in the given numbers are 2, 3 and 5 and their highest powers are 2 ,3 and 5 .

∴ Required LCM = 2^3 X 3^2 X 5^1 = 360 By division method,

2	8,	15,	24,	72
2	4,	15,	12,	36
2	2,	15,	6,	18
3	1,	15,	3,	9
	1.	5.	1.	3

LCM = 2x2x2x3x5x3 = 360

2. (a) Required LCM = ax2x5x7 = 70a

3. (c) Given prime factors,

$(2^3$ X 3 X 5^Z X 7); (2^ X3^Z X 5 X *f*_ X 11); (2xf X5^]

.'. Required LCM = Product of prime factors

having highest powers

= 2^4 x3^3 X5^4 X7^Z x 11

4. (a) At the 1st step, we take the two numbers 132 and 204.

HCF of 132 and 204

132)204(1
132
72)132(1
72
60)72(1
60
12)60(5
60
x

Again. HCF of 12 and 228

12)228(19
12
108
108
x

Hence, HCF of 132, 204 and 228 = 12. 5. (c) Given factors,

2X3X7X9; 2X3X9X11; 2X3X4X5

.'. Required HCF = Product of common prime factors having least powers = 2x3

6. (c) We know that,

$$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

$$\therefore \text{ Required LCM} = \frac{\text{LCM of 1, 2, 5 and 4}}{\text{HCF 3, 9, 6 and 27}}$$

$$\therefore \qquad = \frac{20}{3}$$

7. (a) $\text{Required LCM} = \frac{\text{LCM of 2, 3, 4, 9}}{\text{HCF of 3, 5, 7, 13}}$

$$= \frac{4 \times 9}{1} = 36$$

8. (c) We know that,

$$\text{HCF of fractions} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

$$\therefore \text{ Required HCF} = \frac{\text{HCF of 4 and 7}}{\text{LCM of 5 and 15}} = \frac{1}{15}$$

9. (a) $\text{Required HCF} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$

$$= \frac{\text{HCF of 1, 3 and 4}}{\text{LCM of 2, 4 and 5}} = \frac{1}{20}$$

10. (e) Required LCM = (LCM of 250, 100 and 125) X 0.001 LCM of 250, 100 and 125

2	250,	100,	125
5	125,	50,	125
5	25,	10,	25
5	5,	2,	5
	1,	2,	1

∴ LCM = 2X5X5X5X2= 500

∴ Required LCM = 500 X 0.001 = 0.5

11. (a) Required LCM = (LCM of 25, 12, 200 and 75) X 0.1 LCM of 25, 12, 200, 75

2	25,	12,	200,	75
2	25,	6,	100,	75
3	25,	3,	50,	75
5	25,	1,	50,	25
5	5,	1,	10,	5
	1,	1,	2,	1

LCM = 2X2X3X5X5X2 = 600 ∴ Required LCM = 600 x 0.1 = 60

12. (d) LCM of two coprimes is equal to their product.

13. (d) HCF of 18 and 15

18 = 2X3 X3

15 = 3X 15 = 3 LCM of 18 and 15 = 2x3x3x5 = 90 /. Product of HCF and LCM of both numbers = 3 X 90 = 270

14. (6) Given, LCM of two numbers = 2376 HCF of two numbers = 33

One of the number = 297 ■.• (HCF of two numbers)

x (LCM of two numbers) = (First number) X (Second number)

$$\therefore \text{Second number} = \frac{33 \times 2376}{297} = 264$$

15. (c) Given,

LCM = 1989 HCF = 13 1st number =117 and 2nd number = ? According to the formula, Product of LCM and HCF

= Product of two numbers

$$\therefore \quad 1989 \times 13 = 117 \times ?$$

$$? = \frac{1989 \times 13}{117}$$

$$? = 221$$

16. (*d*) Let the other number = x

Then, $x \times 75 = 15 \times 225$

$$\therefore \quad x = \frac{15 \times 225}{75} = 45$$

17. (d) We know that, LCM of two numbers must be the multiple of their HCF. In the given options 60 is not a multiple of 8 and hence 60 cannot be the LCM of the numbers.

18. (6) Let the numbers be x and *y*. According to the question,

x + y = 45 ...(i)

Difference $= \frac{1}{9} \times$ Sum of the numbers

$\Rightarrow \qquad x - y = 5 \qquad \ldots(ii)$

By adding Eqs. (i) and (ii),

$$\begin{array}{r} x + y = 45 \\ x - y = 5 \\ \hline 2x = 50 \end{array} \quad \Rightarrow \quad x = 25$$

From Eq. (i),

$$x + y = 45$$

$\therefore \qquad y = 45 - x \quad \Rightarrow \quad y = 45 - 25 = 20$

Now, LCM of 25 and 20

$$\begin{array}{r|l} 5 & 25, \ 20 \\ \hline & 5, \ 4 \end{array}$$

∴ Required LCM = 5x5x4 = 100

19. (d) According to the question, 1 st number = 3 m 2nd number = 4 m where, m = HCF But given, m = 4 We know that,

$$\text{LCM} = \frac{\text{Product of two numbers}}{\text{HCF}}$$

$$= \frac{3\,m \times 4\,m}{m}$$

$$\text{LCM} = 12\,m = 12 \times 4 = 48$$

20. (6) Let numbers are 5x and 6x. Now, HCF of these two numbers is x. We know that,

LCM x HCF = Product of two numbers

=> 480 X x = 5x X 6x

=> 480x = $30x^2$ =>x = 16

21. (6) Let the numbers are x, 2x and 3x. Where, x = HCF

Given that, x = 23

∴ The number are 23, 46 and 69.

22. (d) Let the numbers are 3x, 4x and 5x. Where, x=HCF

Then, LCM = 60x According to the question, 60x = 1200 x = 20

23. (*d*) We have, $m \times n = 6 \times 210 = 1260$

$$\therefore \frac{1}{m} + \frac{1}{n} = \frac{m+n}{mn} = \frac{72}{1260} = \frac{4}{70} = \frac{2}{35}$$

24. (C) LCM of 11 and 13 will be (11 x 13). Hence, if a number is exactly divisible by 11 and 13, then the same number must be exactly divisible by their LCM or by (11 X 13).

25. (C) Let numbers are 2x and 3x. According to the question,

6x = 48 => x = 8 (••• LCM = 6x)

.". Required sum = (2x + 3x) = 5x = 5 X 8 = 40

26. (6) Let numbers be lOx, 12x, 15x and 18x. Then, LCM = 180x

As HCF = x

Hence, required LCM = 180 X 3 = 540

27. (6) Given that, product of two numbers

= 1500 HCF= 10

According to the formula, Product of two numbers = HCF X LCM => 1500 = 10 X LCM

$$\therefore \qquad \text{LCM} = \frac{1500}{10} = 150$$

28. (d) By Hit and Trial

From option (d), we can say that the HCF of 36 and 24 is 12 and it is also satisfies the given condition

a> *b*> 12 a = 36 and *b* = 24

29. (C) Let LCM = *m,* HCF = n, According to the question,

m = 12n, ...(i)

and *m* + *n* = 403 ...(ii)

=> 12n + *n* = 403 [from Eq. (i)]

=> 13n = 403

$$\therefore \quad n = \frac{403}{13} = 31$$

$$\therefore \quad m = 12 \times 31 = 372$$

Let the another number = k

$$\therefore \quad 93 \times k = 372 \times 31$$

(as product of two numbers = HCF × LCM)

$$\Rightarrow \quad k = \frac{372 \times 31}{93} = 124$$

30. *(a)* Given, LCM = 495, HCF = 5 Let 1st number = x, 2nd number = *y xy* = 495 X 5 [as product of two numbers = HCF x LCM] => xy = 2475

We know that,

(x - *yf* = (x + *yf* - 4xy

= $(100)^Z$ -4 X2475 = 10000 -9900 = 100

$$\therefore \quad (x - y) = \sqrt{100} = 10$$

31. (d) Let HCF = *x*

According to the question, LCM = 20x Given that. HCF + LCM = 2520

$$\Rightarrow x + 20x = 2520 \Rightarrow x = \frac{2620}{21} = 120$$

Now, LCM = 20x = 20 X 120 = 2400

We know that,

1st number X 2nd number = HCF X LCM

$$\Rightarrow \quad \text{2nd number} = \frac{\text{LCM} \times \text{HCF}}{\text{1st number}}$$

$$= \frac{120 \times 2400}{480} = 600$$

∴ Required answer = 600 X 3 = 1800

32. (c) Let the numbers be 66a and 66jb, where a and *b* are coprimes. According to the question,

66a + 66£ = 1056 => 66 (a + *b)* = 1056

$$\Rightarrow \quad (a + b) = \frac{1056}{66} = 16$$

/. Possible values of a and *b* are

(a = l,i = 15), (a = 3,6 = 13). (a = 5,i> = ll), (a = 7,£ = 9)

.". Numbers are

(66 X 1,66 X 15), (66 X 3,66 X 13), (66 X 5,66 X 11), (66 X 7,66 X 9). /. Possible number of pairs = 4

33. (6) Required length

= LCM of 3 cm, 5 cm and 10 cm = 30 cm

34. (a) Required number = Multiples of LCM Now. LCM of 8. 9. 12. 15 and 18

2	8,	9,	12,	15,	18
2	4,	9,	6,	15,	9
3	2,	9,	3,	15,	9
3	2,	3,	1,	5,	3
	2,	1,	1,	5,	1

∴ LCM =2X2X3X3X2X5 = 360 /. The factors make it clear that to make a perfect square 360 must be multiplied by (2X5).

∴ Required number = 360 X 2 X 5 = 3600 35. *(a)* Greatest number of 3-digits = 999 LCM of 6, 9, 12

2	6,	9,	12
3	3,	9,	6
	1.	3.	2

∴ Required LCM = $2 \times 3 \times 3 \times 2 = 36$

Now, $\frac{999}{36} = 27\frac{27}{36}$

∴ Remainder = 27

∴ Required number = (999 − 27 + 3) = 97!

36. (a) Given that, x = 1356, *y* = 1868, z = 2764 a = *b* = *c* = 12 According to the formula, Required number = HCF of [(1356 - 12),

(1868 -12), (2764 -12)] = HCF of (1344.1856 and 2752)

```
1344)1856(1
     1344
     ----
      512)1344(2
          1024
          ----
           320)512(1
               320
               ---
               192)320(1
                   192
                   ---
                   128)192(1
                       128
                       ---
                        64)128(2
                           128
                           ---
                            ×

64)2752(43
   256
   ---
    192
    192
    ---
     ×
```

∴ HFC of 1344, 1856 and 2752 = 64 Hence, required number = 64

37. (a) Given that,

x = 130,7 = 305, z = 245 a = 6, *b = 9,* c = 17 According to the formula, Required number = HCF of

[(130 - 6), (305 - 9), (245 - 17)] = HCF of 124, 296, 228 = 4

38. (a) Given that,

x = 1023, *y* = 750, a = 3, *b* = 2

.'. Required number

= HCF of [(1023 - 3), (750 - 2)] = HCF of 1020 and 748

```
748)1020(1
     748
     ---
     272)748(2
         544
         ---
         204)272(1
             204
             ---
              68)204(3
                 204
                 ---
                  ×
```

.'. Required number = 68

39. (d) Required capacity = HCF of 345 L, 120 L and 225 L

$$
\begin{array}{l}
120)\,225\,(1 \\
\quad\ \ \underline{120} \\
\quad\ \ 105)\,120\,(1 \\
\qquad\quad\ \ \underline{105} \\
\qquad\quad\ \ 15)\,105\,(7 \\
\qquad\qquad\quad \underline{105} \\
\qquad\qquad\quad\ \ \times
\end{array}
$$

$$
\begin{array}{l}
15)\,346\,(23 \\
\quad\ \underline{30} \\
\quad\ \ 45 \\
\quad\ \ \underline{45} \\
\quad\ \ \ \times
\end{array}
$$

/. Required capacity of container to measure the oil = 15 L

40. (c) LCM of 5, 6, 4 and 3 = 60

On dividing 2497 by 60, the remainder is 37. ∴ Number to be added = 60-37 = 23

41. (c) Given that,

x = 1305, y = 4665, z = 6905 Then,

$$
\begin{aligned}
|x - y| &= |1305 - 4665| = 3360 \\
|y - z| &= |4665 - 6905| = 2240 \\
|z - x| &= |6905 - 1305| = 5600
\end{aligned}
$$

.•. Required number

= HCF of 3360, 2240 and 5600 = 1120 On dividing 1305 by 1120, remainder is 185. On dividing 4665 by 1120, remainder is 185. On dividing 6905 by 1120, remainder is 185. .'. Common remainder =185

42. (c) Given, x = 12, y = 16, z = 18, k = 5 According to the formula,

Required number = (LCM of x, y and z) + k

= (LCM of 12, 16, 18) + 5 LCM of 12, 16, 18 is

$$
\begin{array}{r|ccc}
2 & 12, & 16, & 18 \\ \hline
2 & 6, & 8, & 9 \\ \hline
3 & 3, & 4, & 9 \\ \hline
 & 1, & 4, & 3
\end{array}
$$

∴ LCM = 2X2X3X4X3 =144 .'. Required number = 144 + 5 = 149

43. *(b)* When divide 13850 and 17030 by the number, the remainder is 17. So, find HCF of(13850-17) and (17030 -17) i.e., 13833 and 17013. Here,

$$
\begin{array}{l}
13833)\,17013\,(1 \\
\quad\quad\;\; \underline{13833} \\
\quad\quad\;\; 3180)\,13833\,(4 \\
\quad\quad\quad\quad\quad \underline{12720} \\
\quad\quad\quad\quad\quad 1113)\,3180\,(2 \\
\quad\quad\quad\quad\quad\quad\quad \underline{2226} \\
\quad\quad\quad\quad\quad\quad\quad 954)\,1113\,(1 \\
\quad\quad\quad\quad\quad\quad\quad\quad\quad \underline{954} \\
\quad\quad\quad\quad\quad\quad\quad\quad\quad 159)\,954\,(6 \\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \underline{954} \\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \times
\end{array}
$$

Hence, the number is 159.

44. (*a*) Second number $= \dfrac{\text{LCM} \times \text{HCF}}{\text{First number}}$

$= \dfrac{72 \times 12}{24} = 36$

Difference between the two numbers

$= 36 - 24 = 12$

45. (*a*) Given, $x = 12$, $y = 21$, $z = 35$, $k = 6$

∴ Required number

= (LCM of 12, 21, 35) + k

LCM of 12, 21, 35 is

3	12,	21,	35
7	4,	7,	35
	4,	1,	5

∴ LCM $= 3 \times 7 \times 4 \times 5 = 420$

∴ Required number $= 420 + 6 = 426$

46. (*d*) LCM of 21, 28, 36, 45

2	21,	28,	36,	45
2	21,	14,	18,	45
3	21,	7,	9,	45
3	7,	7,	3,	15
7	7,	7,	1,	5
	1,	1,	1,	5

LCM $= 2 \times 2 \times 3 \times 3 \times 7 \times 5 = 1260$

∴ Required number = LCM + 7

$= 1260 + 7 = 1267$

47. *(a)* LCM of 24, 32, 36, 54

2	24,	32,	36,	54
2	12,	16,	18,	27
2	6,	8,	9,	27
3	3,	4,	9,	27
3	1,	4,	3,	9
	1,	4,	1,	3

LCM = 2x2x2x3x3x3x4 = 864 .'. Required number = LCM — 9 = 864-9 = 855

48. (a) LCM of 16, 18 and 20 = 720 .'. Required number = *720k* + 4

where, *k* is a natural number to be divisible by 7, (720 *k* + 4) will be a multiple of 7. Smallest value of *k* = 4 ∴ Required number = 720 X 4 + 4 = 2884

49. (a) LCM of 10, 15, 21 and 28

2	10,	15,	21,	28
3	5,	15,	21,	14
5	5,	5,	7,	14
7	1,	1,	7,	14
	1,	1,	1,	2

LCM = 2X3X5X7X2 =420 Larqest number of 4-diqits = 9999

Now, $\frac{9999}{420} = 23\frac{339}{420}$

/. Remainder = 339

.'. Four-digit number divisible by 10, 15, 21

and 28 = 9999 - 339 = 9660

Also, *k*= 10 -4 = 15-9 = 21-15,

28-22 = 6 /. Required number = (9660 - *k)*

= (9660 - 6)

= 9654

50. (C) LCM of 32, 40, 48 and 60 = 480

The number divisible by 480 between 4000 and 6000 are 4320, 4800, 5280 and 5760. Hence, required number of numbers are 4.

51. (a) Required length = HCF of 84m, 98 m and 126 m

HCF of 84, 98 and 126

```
84)98(1
   84
   14)84(6
      84
       x

14)126(9
   126
    x
```

.". Required length = 14 m

52. (Q) Least number of students = LCM of 5, 9, 10

LCM of 5, 9, 10

```
5 | 5,  9,  10
  |------------
  | 1,  9,   2
```

LCM = 5X9X2 = 90 .'. Required number of students = 90

Exercise © *Higher SkiU Level Questions*

1. (6) Given that, (x-6) is the HCF of x^z - 2x - 24 and x^2 - *kx* - 6 i.e.,(x - 6) is a factor of both expressions. Let /(Xj) = x^2 - 2xj - 24

and /(x2) = x2 — *kx*$_z$ — 6

Now, *f(x{)* = /(x2) at (X! = x2 = 6)

=> $(6)^2$ - 2(6) - 24 = $(6)^2$ - £(6) - 6

[by condition] => 0 =30 -6£:=>6i: = 30 *k* = 5

2. (*c*) $HCF\left(\frac{a}{HCF(a,b)},\frac{b}{HCF(a,b)}\right)$ its value is always equal to 1.

Illustration 1 Let the two positive integers be $a = 24$ and $b = 36$.

$\therefore \quad HCF\left(\frac{24}{HCF(24,36)},\frac{36}{HCF(24,36)}\right)$

$\Rightarrow \quad HCF\left(\frac{24}{12},\frac{36}{12}\right)$

$\Rightarrow \quad HCF(2,3) = 1$

Illustration 2

Let the two positive integers be $a = 13$ and $b = 17$.

$\therefore \quad HCF\left(\frac{13}{HCF(13,17)},\frac{17}{HCF(13,17)}\right)$

$\Rightarrow \quad HCF\left(\frac{13}{1},\frac{17}{1}\right) = 1$

3. (a) Let $p(x) = 8(x^5 - x^3 + x)$

$= 4 X 2 X x (x^4 - x^2 + 1)$

and $q(x) = 28(x^6 + 1)$

$= 7X4 [(x^2)^3 + (l)^3]$

$= 4 X 7 X (x^2 + 1) (x^4 - x^2 + 1)$

$\therefore$HCF of p(x) and $q(x) = 4 (x^4 - x^2 + 1)$

4. (£2) Let $f(x) = (x^4 - y^4)$

$= (x^2 - y^2) (x^2 + y^2) = (x - y) (x + 7) (x^2 + y^2)$ and $gr(x) = (x^6 - y^6)$

$= (x^3)^2 - (y^3)^2$

$= (x^3 + y^3) (x^3 - y^3)$

$= (x+y)(x^2 -xy+ y^2) (x - y)$

$(x^2 + xy+y^2) = (x - y) (x + y) (x^2 - xy + y^2) (x^2 + xy + y^2)$ $\therefore$HCF of $[/(x), gr(x)] = (x - y) (x + y) = x^2-y^2$

5. (c) Let /(x) = x^3 - x^z - 2x = x (x^2 - x - 2)

= x {x^2 - 2x + x - 2)

= x {x (x - 2) + 1 (x - 2)}

= x (x + 1) (x - 2) and g(x) = x + x

= x^2 (x+ 1) = xx(x + 1) ∴ LCM of [f(x), gr(x)] = x (x + 1) • x • (x - 2)

= x^2 (x + 1) (x - 2)

= x^2 (x^2 - x-2)

= x^4-x^3-2x^2

6. (c) $a^2i>^4$ + $2a^2i^2$ = $a^2i>^2$ (i^2 + 2) ...(i) and *(ab)^7 - 4a^zb^9* = *a"*V - 4a^2 jb^9

= $a^2£^2$ (a^5jb^5-4jb^7) ...(ii) From Eqs. (i) and (ii), HCF = aV

7. (6) HCF of (22n + *7,33n* + 10) is always 1.

Illustration

For n = 1, HCF (29,43) => HCF = 1 For *n* = 2, HCF (51,76) => HCF = 1 For n = 3, HCF (73,109) => HCF = 1

8. (c) Putting arbitrary values of a and £>. **Illustration 1** Let a **= 9** and ***b = S***

HCF **(8 + 9,9-8)**

HCF **(17,1) = 1 Illustration 2** Let a = 23 and *b* = 17

HCF (17+ 23,23-17)

HCF (40,6) = 2 Hence HCF(a + *b,a- b)* can either be 1 or 2.

9. (c) Time after which they will hit the target aaain toaether = LCM (5,6,7 and 8)

$$\begin{array}{r|llll} 2 & 5, & 6, & 7, & 8 \\ \hline & 5, & 3, & 7, & 4 \end{array}$$

= 5X3X7X2X4 =840s

$$\text{They will hit target together} = \frac{840}{60} = 14 \text{ min}$$

They will hit together again at 9 : 14 am.

10. (d) We have to find out the LCM of 48, 60 and 90.

$$\begin{array}{r|l} 2 & 48,\ 60,\ 90 \\ \hline 3 & 24,\ 30,\ 45 \\ \hline 2 & 8,\ 10,\ 15 \\ \hline 5 & 4,\ 5,\ 15 \\ \hline 2 & 4,\ 1,\ 3 \\ \hline 2 & 2,\ 1,\ 3 \\ \hline 3 & 1,\ 1,\ 3 \\ \hline & 1,\ 1,\ 1 \end{array}$$

∴ LCM = 2x3x2x5x2x2x3

$$= 720 \text{ min} = \frac{720}{60} \text{ h} = 12 \text{ h}$$

/. Bell chime together again

= 10 :00 am+ 12 h= 10 :00 pm

11. (C) The bells will toll together after time in seconds equal to the LCM of 9, 6, 4, 10 and 8. LCM of 9, 6,4, 10 and 8 is

$$\begin{array}{r|l} 2 & 9,\ 6,\ 4,\ 10,\ 8 \\ \hline 2 & 9,\ 3,\ 2,\ 5,\ 4 \\ \hline 3 & 9,\ 3,\ 1,\ 5,\ 2 \\ \hline & 3,\ 1,\ 1,\ 5,\ 2 \end{array}$$

∴ LCM = 2X2X3X3X5X2 = 360 In one hour, the rings will toll together

$$\frac{3600}{360} \text{ times} = 10 \text{ times}$$

12. (c) Required time = LCM of 42, 56 and 63 s LCM of 42, 56, 63 is

2	42,	56,	63
3	21,	28,	63
7	7,	28,	21
	1,	4,	3

.'. Required time = 2x3x7x4x3= 504 s 13. (c) LCM of 10, 15 and 18 is

2	10,	15,	18
3	5,	15,	9
5	5,	5,	3
	1,	1,	3

LCM = 2X3X5X3 = 90 To make it perfect square, we multiply it with 2 x 5 = 10

.". Required number of soldiers 90 X10 =900

14. (e) LCM of 2, 4, 6, 8, 10, 12

= (2X2X3X2X5) = 120 /. After every 2 min, they toll together. .". Number of times they toll together in one hour

$$= \left(\frac{60}{2} + 1\right) \text{times} = 31 \text{ times}$$

15. (c) Required length = HCF of 7 m, 3 m 85 cm, 12 m 95 cm = HCF of 700 cm, 385 cm, 1295 cm

```
385)700(1
    385
    315)385(1
        315
         70)315(4
            280
             35)70(2
                70
                 x

   35)1295(37
      105
       245
       245
        x
```

Hence, required length is 35 cm.

16. (d) Let the numbers be 21a and 21Jb, where a and *b* are coprimes. Then, 21a X 21 *b* = (21 X 4641) => aft = 221

Two coprimes with product 221 are 13 and 17.

∴ Required number = (21x13,21x17) = (273,357) 17. (c) HCF of 250 cm and 150 cm

$$
\begin{array}{l}
150)\,250\,(1 \\
\quad\ \underline{150} \\
\quad\ 100)\,150\,(1 \\
\qquad\quad \underline{100} \\
\qquad\quad\ 50)\,100\,(2 \\
\qquad\qquad\ \ \underline{100} \\
\qquad\qquad\quad \times
\end{array}
$$

HCF = 50

$$\therefore \text{ Number of slabs} = \frac{\text{Area of floor}}{\text{Area of slab}}$$

$$= \frac{250 \times 150}{50 \times 50} = 15$$

18. (a) For such type of questions, remember the following method. Required number of apples

= (LCM of 5, 6, 8, 9) + 3 LCM of 5, 6, 8, 9 is

$$
\begin{array}{r|llll}
2 & \underline{5,} & \underline{6,} & \underline{8,} & \underline{9} \\
3 & \underline{5,} & \underline{3,} & \underline{4,} & \underline{9} \\
 & 5, & 1, & 4, & 3
\end{array}
$$

∴ LCM = 2X3X5X4X3= 360 .". Required number of apples

= 360 + 3 = 363

Chapter

4

Simple and Decimal Fractions

A digit which can be represented in *p/q* form, where *q* * 0, is called a **fraction.** Here, p is called the numerator and *q* is called the denominator.

For example 3/5 is a fraction, where 3 is called numerator and 5 is called denominator.

or When a unit is divided into any number of equal parts, then these parts are termed as a fraction of the unit. For *examnle* Tf 1 is to he divided into two ermal narts. then 1 is

divided by 2; and is represented as $\frac{1}{2}$.

Simple Fraction

The fraction which has denominator other than power of 10, is called simple fraction. *e.g.*, $\frac{3}{7}, \frac{5}{11}, \frac{7}{9}$ etc.

Note Simple fraction is also known as vulgar fraction

Types of Simple Fractions

There are following types of fractions.

1. **Proper Fraction** When the numerator of a fraction is less than its denominator, then fraction is called proper fraction.

For example $\frac{1}{2}, \frac{15}{17}, \frac{21}{43}$ etc.

2. **Improper Fraction** When the numerator of a fraction is greater than its denominator, then fraction is called improper fraction.

For example $\frac{17}{13}, \frac{18}{14}, \frac{45}{19}$ etc.

3. Compound Fraction A fraction, in which numerator or denominator or both are in fraction, then it is called compound fraction.

For example $\frac{1}{7/9}, \frac{11/9}{13}, \frac{1/4}{7/13}$ etc.

4. **Inverse Fraction** If we inverse the numerator and the denominator of a fraction, then the resultant fraction will be the inverse fraction of the original fraction.

For example Given fraction $= \frac{3}{8}$; Inverse fraction $= \frac{8}{3}$

5. **Mixed Fraction** The fraction, which is the combination of integer and fraction, is called mixed fraction.

For example $3\frac{2}{5}, 7\frac{1}{9}$ etc.

6. **Continuous Fraction** It has no certain definition but only say that a fraction contains additional fractions in its denominators, is called continuous fraction.

For example (i) $2 + \cfrac{1}{2 + \cfrac{2}{5 + \cfrac{2}{3}}}$ (ii) $6 + \cfrac{1}{1 + \cfrac{1}{\frac{1}{2} + \frac{1}{2}}}$

Note lb simplify a continuous fraction, start from bottom and work upwards

Decimal Fraction

If the fraction has denominator in the powers of 10, then fraction is called decimal fraction.

For example 10th part of unit $= \frac{1}{10} = 0.1$; 10th part of 6 $= \frac{6}{10} = 0.6$

To convert a decimal fraction into a vulgar fraction, place 1 in the denominator under the decimal point. Then, after removing the decimal point, place as many zeroes after it as the number of digits after the decimal point. Finally, reduce the fraction to its lowest terms.

For example $0.23 = \frac{23}{100}$; $0.0035 = \frac{35}{10000} = \frac{7}{2000}$

Note • Placing zeroes to the right of a decimal fraction, it does not make any change in value Hence, 0.5, 0.50, 0.500 and 0.5000 are equal. • If numerator and denominator of a fraction have same number of decimal places, then each of the decimal points be removed

Types of Decimal Fractions

1. **Recurring Decimal Fraction** The decimal fraction, in which one or more decimal digits are repeated again and again, is called recurring decimal fraction. To represent these fractions, a line is drawn on the digits which are repeated.

For example $\frac{2}{3} = 0.6666\ldots = 0.\bar{6}$; $\frac{22}{7} = 3.142857142857 = 3.\overline{142857}$

2. **Pure Recurring Decimal Fraction** When all the digits in a decimal fraction are repeated after the decimal point, then the decimal fraction is called as pure recurring decimal fraction.

For example $.\bar{5}, 0.\overline{489}$ etc.

To convert pure recurring decimal fractions into simple fractions (vulgar form), write down the repeated digits only once in numerator and place as many nines in the denominator as the number of diaits reneatincr.

For example (i) $0.\bar{3} = \frac{3}{9} = \frac{1}{3}$

Since, there is only 1 repeated digit.

Therefore, only single 9 is placed in denominator.

(ii) $0.\overline{57} = \frac{57}{99} = \frac{19}{33}$

Since, there are only 2 repeated digits. Therefore, two 9's are placed in denominator. 3. **Mixed Recurring Decimal Fraction** A decimal fraction in which some digits are repeated and some are not repeated after decimal is called as mixed recurring decimal fraction.

For example $3.22\bar{3}, 0.1\overline{9}$ etc.

To convert mixed recurring decimal fractions into simple fractions, in the numerator, take the difference between the number formed by all the digits after decimal point (repeated digits will be taken only once) and the number formed by non-repeating digits. In the denominator, place as many nines as there are repeating digits and after nine put as many zeroes as the number of non-repeating digits.

For example

$$0.3\bar{6} = \frac{(36-3)}{90} = \frac{33}{90} = \frac{11}{30}$$

$$0.42\overline{67} = \frac{(4267-42)}{9900} = \frac{4225}{9900} = \frac{169}{396}$$

Important Facts Related to Simple and Decimal Fractions

♦ If in a fraction, numerator is equal to denominator, then the value of fraction is equal to 1

♦ If the numerator of a fraction is always non-zero and denominator is zero, then the value of fraction is infinity (°°).

♦ If the numerator of a fraction is zero and denominator is not equal to zero, then the value of fraction is zero.

♦ If the numerator or denominator of any fraction is either multiplied or divided by same number, then the value of fraction remains unchanged.

♦ If the numerator and denominator have no common factor other than 1, then the fraction is said to be in its lowest form

♦ A fraction is a rational number as it can be expressed in the form of *p/q* and *q* * 0

Operations on Simple Fractions

Addition of Simple Fractions

1. **When Denominators are Same** If denominators of fractions are same, then numerators of fractions are added and their addition is divided by denominator.

For example $$\frac{1}{4}+\frac{2}{4}=(1+2)\frac{1}{4}=\frac{3}{4}$$

2. **When Denominators are Different** If denominators of fractions are not same, then make their denominators equal (by taking their LCM) and then add their numerators.

For example $$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{(1\times 6)+(1\times 4)+(1\times 3)}{12}=\frac{6+4+3}{12}=\frac{13}{12}$$

Subtraction of Simple Fractions

1. **When Denominators are Same** If denominators of fractions are same, then numerators of fractions are subtracted and their subtraction is divided by the denominator.

For example $$\frac{3}{4}-\frac{1}{4}=(3-1)\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$$

2. **When Denominators are Different** If denominators of fractions are not same, then make their denominators equal and then subtract their numerators.

For example $$\frac{2}{3}-\frac{1}{2}=\frac{(2\times 2)-(3\times 1)}{6}=\frac{4-3}{6}=\frac{1}{6}$$

Multiplication of Simple Fractions

To multiply two or more simple fractions, multiply their numerators and denominators.

For example $$\frac{1}{2}\times\frac{3}{4}=\frac{(1\times 3)}{(2\times 4)}=\frac{3}{8}$$

If fractions are given in mixed form, first convert them into improper fraction and then multiply.

For example $2\frac{4}{5} \times 1\frac{8}{3} = \frac{14}{5} \times \frac{11}{3} = \frac{154}{15}$

Division of Simple Fractions

To divide two fractions, first fraction is multiplied by the inverse of second fraction

For example $\frac{2}{3} + \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{10}{9}$

Operations on Decimal Fractions

Addition and Subtraction of Decimal Fractions

To add or subtract decimal fractions, the given numbers are written under each other such that the decimal points lie in one column and the numbers so arranged can now be added or subtracted as per the conventional method of addition and subtraction.

Ex. 1 (i) 353.5 + 2.32 + 43.23 = ? (ii) 1000 - 132.23 = ?

Sol. (i)
$$\begin{array}{r} 353.50 \\ 2.32 \\ +\ 43.23 \\ \hline 399.05 \end{array}$$

(ii)
$$\begin{array}{r} 1000.00 \\ -\ 132.23 \\ \hline 867.77 \end{array}$$

Multiplication of Two or More Decimal Fractions

Given fractions are multiplied without considering the decimal points and then in the product, decimal point is marked from the right hand side to as many places of decimal as the sum of the numbers of decimal places in the multiplier and the multiplicand together.

Ex. 2 (i) 4.3 x 0.13 = ? (ii) 1.12 x 2.3 x 4.325 = ?

Sol. (i) 43 x 13 = 559

Sum of the decimal places = (1 + 2) = 3 .•. Required product = 0.559 (ii) 112 x 23 x 4325 = 11141200 Sum of the decimal places = (2 + 1 + 3) = 6 ∴ Required product = 11.141200

Multiplication of Decimal Fraction by an Integer

Given integer is multiplied by the fraction without considering the decimal point and then in the product, decimal is marked as many places before as that in the given decimal fraction.

Ex. 3 Find the value of the following.

(i) 19.72x4 (ii) 0.0745x10 (iii) 3.52x14

Sol. (i) 19.72 x 4

Multiplying without taking decimal point into consideration

1972 x 4 = 7888

So, 19.72 x 4= 78.88

Since, in the given decimal fraction, decimal point is two places before. So, in the product, decimal point will also be put two places before.

Similarly, (ii) 0.0745 x 10 = 0.745 (iii) 3.52 x 14 = 49.28

Dividing a Decimal Fraction by an Integer

Do simple division i.e., divide the given decimal number without considering the decimal point and place the decimal point as many places of decimal as in the dividend.

Ex. 4 Divide the following.

(i) 0.81 ■*■ 9 (ii) 1.2875 ■*■ 25 (iii) 0.00049 + 7

Sol. (i) $\frac{81}{9} = 9 \Rightarrow \frac{0.81}{9} = 0.09$ [two places of decimal]

(ii) $\frac{12875}{25} = 515 \Rightarrow \frac{1.2875}{25} = 0.0515$ [four places of decimal]

(iii) $\frac{49}{7} = 7 \Rightarrow \frac{0.00049}{7} = 0.00007$ [five places of decimal]

Division of Decimal Fractions

In such divisions, dividend and divisor both are multiplied first by a suitable multiple of 10 to convert divisor into a whole number and then above mentioned rule of division is followed.

Ex. 5 Divide the following.

(i) 42 + 0.007 (ii) 0.00048 ■*■ 0.8

Sol. (i) $\dfrac{42}{0.007} = \dfrac{42}{0.007} \times \dfrac{1000}{1000} = \dfrac{42000}{7} = 6000$

(ii) $\dfrac{0.00048}{0.8} = \dfrac{0.00048}{0.8} \times \dfrac{10}{10} = \dfrac{0.0048}{8} = 0.0006$

Comparison of Simple Fractions

Following are some techniques to compare fractions.

Cross Multiplication Method

If $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are two fractions, then

(i) If $ad > bc$, then $\dfrac{a}{b} > \dfrac{c}{d}$ (ii) If $ad < bc$, then $\dfrac{a}{b} < \dfrac{c}{d}$ (iii) If $ad = bc$, then $\dfrac{a}{b} = \dfrac{c}{d}$

Ex. 6 (i) Between $\dfrac{4}{7}$ and $\dfrac{3}{8}$, which fraction is bigger?

(ii) Which one of the fraction is largest among $\dfrac{2}{3}, \dfrac{3}{4}, \dfrac{4}{3}, \dfrac{5}{4}$?

Sol. (i) Given, $\dfrac{4}{7}$ and $\dfrac{3}{8}$

$\because \quad 4 \times 8 > 7 \times 3 \qquad \therefore \quad \dfrac{4}{7} > \dfrac{3}{8}$

(ii) First find the largest among the two fractions

$\dfrac{2}{3}, \dfrac{3}{4} \qquad \dfrac{4}{3}, \dfrac{5}{4}$

$\because \quad 3 \times 3 > 4 \times 2 \qquad \because \ 4 \times 4 > 5 \times 3$

$\therefore \quad \dfrac{3}{4} > \dfrac{2}{3} \qquad \therefore \quad \dfrac{4}{3} > \dfrac{5}{4}$

Now, taking the two largest fractions, find which one is largest

$\dfrac{3}{4}, \dfrac{4}{3}$

$\because \quad 4 \times 4 > 3 \times 3 \qquad \therefore \quad \dfrac{4}{3} > \dfrac{3}{4}$

Hence, $\dfrac{4}{3}$ is the largest fraction.

By Changing Fractions in Decimal Form

To compare two or more fractions, first convert fractions into decimal form and then compare.

Ex. 7 Between $\frac{1}{7}$ and $\frac{2}{9}$, which fraction is bigger?

Sol. $\frac{1}{7} = 0.14$ and $\frac{2}{9} = 0.22$. It is clear that $0.22 > 0.14$

$\therefore$ $\frac{2}{9} > \frac{1}{7}$

Hence, bigger fraction is $\frac{2}{9}$.

By Equating Denominators of Given Fractions

For comparison of fractions, take LCM of the denominators of all fractions, so that the denominators of all fractions are same. Now, the fraction having largest numerator is the largest fraction.

Ex. 8 Arrange the following fractions in decreasing order $\frac{3}{5}, \frac{7}{9}, \frac{11}{13}$.

Sol. LCM of 5, 9 and 13 = $5 \times 9 \times 13 = 585$

$\therefore$ $\frac{3}{5} = \frac{3 \times 117}{5 \times 117} = \frac{351}{585}$; $\frac{7}{9} = \frac{7 \times 65}{9 \times 65} = \frac{455}{585}$ and $\frac{11}{13} = \frac{11 \times 45}{13 \times 45} = \frac{495}{585}$

Now, the fraction having largest numerator will be largest.

$\therefore$ Decreasing order will be $\frac{495}{585}, \frac{455}{585}, \frac{351}{585}$.

Hence, order is $\frac{11}{13}, \frac{7}{9}, \frac{3}{5}$.

By Equating Numerators of Given Fractions

For comparison of fractions, take LCM of the numerator of all fractions, so that numerators of all the fractions are same. Now, the fraction having smallest numerator will be largest.

Ex. 9 Which fraction is largest among $\frac{3}{13}, \frac{2}{15}, \frac{4}{17}$?

Sol. LCM of 2, 3 and 4 = $2 \times 2 \times 3 = 12$

$\therefore$ $\frac{3}{13} = \frac{3 \times 4}{13 \times 4} = \frac{12}{52}$; $\frac{2}{15} = \frac{2 \times 6}{15 \times 6} = \frac{12}{90}$ and $\frac{4}{17} = \frac{3 \times 4}{3 \times 17} = \frac{12}{51}$

Now, the fraction having smallest denominator will be largest.

Hence, $\frac{4}{17}$ is the biggest number.

Fast Track **Formulae**

***to solve the* QUESTIONS**

Formula 1

To represent any fraction in simplified form, divide its numerator and denominator by their HCF.

Ex. 10 Write $\frac{27}{81}$ in simplified form.

Sol. $\because 27 = 3 \times 3 \times 3$ and $81 = 3 \times 3 \times 3 \times 3$

HCF of 27 and $81 = 3 \times 3 \times 3 = 27$

$\therefore \frac{27}{81}$ can be simplified as $\frac{27/27}{81/27} = \frac{1}{3}$

Formula 2

If in the given fractions, the difference between numerator and denominator are same, then fraction having larger numerator is the largest and fraction having smaller numerator is the smallest.

Ex. 11 Arrange the given fractions in increasing order,

$$\frac{4}{5}, \frac{5}{6}, \frac{6}{7}$$

Sol. Since, all the fractions have difference in numerator and denominator are same.

$\therefore$ Increasing order is $\frac{4}{5}, \frac{5}{6}, \frac{6}{7}$.

Formula 3

if in the given fractions, the numerators are increasing by a definite value and the denominator is also increasing by a definite value but the value of denominator is greater than numerator, then the fraction having smaller numerator will be the smallest fraction and the fraction having larger numerator will be the largest fraction.

Ex. 12 Which of the fraction is largest among $\frac{2}{5}, \frac{5}{11}, \frac{8}{17}, \frac{11}{23}$?

Sol. Since, in the given fractions, numerator value is increasing by 3 and denominator value is increasing by 6 and 6 > 3. Then, the fraction having larger numerator will be the larger fraction.

$\therefore \frac{11}{23}$ is largest among the given fractions.

Formula 4

If any number is divided by a/b instead of multiplying by $\frac{a}{2}$, then the obtained value will be x greater than original value and the given number will be $\frac{abx}{b^2 - a^2}$.

Ex. 13 Arun was to find 6/7 of a fraction. Instead of multiplying, he divided the fraction by 6/7 and the result obtained was 13/70 greater than original value. Find the fraction given to Arun?

Sol. Given, $a = 6, b = 7$ and $x = \frac{13}{70}$

$\therefore$ Required fraction $= \frac{abx}{b^2 - a^2} = \frac{6 \times 7 \times \frac{13}{70}}{7^2 - 6^2} = \frac{6 \times 13}{10 \times 13} = \frac{3}{5}$

Hence, fraction given to Arun is 3/5.

Fast Track Practice

Exercise© *Base Level Questions*

1. Convert 0.34 into a vulgar fraction.

(a) $\frac{18}{50}$ (b) $\frac{16}{50}$

(c) $\frac{17}{50}$ (d) $\frac{19}{50}$

(e) None of the above

2. Find the sum $\frac{3}{10} + \frac{5}{100} + \frac{8}{1000}$ in decimal form

(a) 0.853 (b) 0.358

(c) 3.58 (d) 8.35

(e) None of the above

3. Change $\frac{75}{100}$ into decimal number.

(a) 0.075 (b) 0.75

(c) 0.0075 (d) 7.5

(e) None of the above

4. Express $0.\overline{5}$ as a vulgar fraction.

(a) $\frac{5}{10}$ (b) $\frac{5}{12}$

(c) $\frac{5}{9}$ (d) $\frac{5}{6}$

(e) None of the above

5. Convert $0.2\overline{7}$ into a vulgar fraction.

(a) $\frac{5}{18}$ (b) $\frac{5}{19}$

(c) $\frac{7}{19}$ (d) $\frac{3}{17}$

(e) None of the above

6. Find the value of $9.46\overline{7}$ in a vulgar fraction.

(a) $9\frac{421}{900}$ (b) $9\frac{422}{900}$

(c) $9\frac{435}{900}$ (d) $9\frac{437}{900}$

(e) None of the above

7. Express $0.8\overline{268}$ as a vulgar fraction.

(a) $\frac{3093}{4950}$ (b) $\frac{3043}{4850}$

(c) $\frac{4093}{4950}$ (d) $\frac{3039}{4950}$

(e) None of the above

8. Arrange $\frac{7}{12}, \frac{2}{3}$ and $\frac{3}{8}$ in the ascending order.

(a) $\frac{3}{8} < \frac{7}{12} < \frac{2}{3}$ (b) $\frac{2}{3} < \frac{7}{12} < \frac{3}{8}$

(c) $\frac{7}{12} < \frac{2}{3} < \frac{3}{8}$ (d) $\frac{3}{8} < \frac{2}{3} < \frac{7}{12}$

(e) None of the above

9. Arrange $\frac{7}{9}, \frac{5}{8}$ and $\frac{3}{7}$ in the descending order.

(a) $\frac{7}{9} > \frac{5}{8} > \frac{3}{7}$ (b) $\frac{3}{7} > \frac{5}{8} > \frac{7}{9}$

(c) $\frac{5}{8} > \frac{7}{9} > \frac{3}{7}$ (d) $\frac{3}{7} > \frac{7}{9} > \frac{5}{8}$

(e) None of the above

10. Out of the fractions $\frac{5}{7}, \frac{7}{13}, \frac{4}{7}, \frac{4}{15}$ and $\frac{9}{14}$, which is the third highest?

(a) $\frac{5}{7}$ (b) $\frac{7}{13}$ (c) $\frac{4}{7}$ (d) $\frac{4}{15}$

(e) $\frac{9}{14}$

11. Out of the fractions, $\frac{4}{7}, \frac{5}{13}, \frac{6}{11}, \frac{3}{5}$ and $\frac{2}{3}$, which is the second smallest fraction?

[SSC CGL 2010]

(a) $\frac{4}{7}$ (b) $\frac{5}{13}$ (c) $\frac{6}{11}$ (d) $\frac{3}{5}$

12.1088.88 + 1800.08 + 1880.80 = ?

[Bank PO 2010]

(a) 8790.86 *(b)* 8890.86

(c) 5588.80 *(d)* 4769.76

(e) None of the above 13.6435.9 + 7546.4 + 1203.5 = ? [Bank PO 2010]

(a) 15188.5 *(b)* 15185.8

(C) 15155.5 *(d)* 15815.8

(e) None of the above

14.726.34 + 888.12 - ? = 1001.88

[1DBISO2010]

fa; 612.58 *(b)* 602.64

fc; 654.54 *(d)* 618.78

fej None of the above

15. $\frac{16}{23} \times \frac{47}{288} \times \frac{92}{141} = ?$ **[Bank Clerks 2011]**

(a) $\frac{4}{27}$ (b) $\frac{2}{27}$ (c) $\frac{2}{29}$ (d) $\frac{3}{28}$

(e) None of the above

16. $1\frac{3}{5} + 1\frac{8}{9} + 2\frac{4}{5} = ?$ **[Bank Clerks 2011]**

(a) $6\frac{19}{45}$ (b) $6\frac{16}{45}$

(c) $6\frac{17}{45}$ (d) $6\frac{13}{45}$

(e) None of the above

17. $\frac{19999}{21111} = ?$

(a) 0.947 (b) 0.749

(c) 0.497 (d) 0.794

(e) 0.974

18. $\frac{1212}{0.5} = 6.06 \times ?$ **[SSC (10+2) 2007]**

(a) 4.04 (b) 400 (c) 0.4 (d) 0.44

19. $33 + 371 \div 7 = ?$ **[Bank Clerks 2011]**

(a) 89 (b) 85 (c) 86 (d) 84

(e) None of the above

20. $(39.3 \times 53.4) + (26.7 \times 5.9) = ?$

[Bank Clerks 2011]

(a) 2520.15 (b) 2256.15

(c) 2562.15 (d) 2652.15

(e) None of the above

21. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{10}$ of $1664 = ?$ **[Bank Clerks 2009]**

(a) 648 (b) 762

(c) 612 (d) 728

(e) None of the above

22. If the fractions $\frac{19}{21}, \frac{21}{25}, \frac{25}{29}, \frac{29}{31}$ and $\frac{31}{37}$ are arranged in ascending order of their values, then which one will be the 2nd?

[Bank Clerks 2009]

(a) $\frac{19}{21}$ (b) $\frac{21}{25}$ (c) $\frac{25}{29}$ (d) $\frac{29}{31}$

(e) None of the above

23. $783 \div 9 \div 0.75 = ?$ **[Bank Clerks 2009]**

(a) 130 (b) 124

(c) 118 (d) 116

(e) None of the above

24. $8\frac{1}{3} \div 10\frac{5}{6} = ?$

(a) $\frac{5}{6}$ (b) $\frac{2}{3}$ (c) $\frac{10}{13}$ (d) $\frac{11}{13}$

(e) None of the above

25. 0.4777 ...... is equal to

(a) $\frac{477}{100000}$ (b) $\frac{477}{100}$

(c) $\frac{437}{100}$ (d) $\frac{43}{90}$

(e) None of the above

26. $4 + 4.44 + 44.4 + 4.04 + 444 = ?$

[Bank PO 2007]

(a) 472.88 (b) 495.22

(c) 500.88 (d) 577.2

(e) None of the above

27. $(0.\overline{142857} + 0.\overline{285714}) = ?$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) 2 (d) 10

(e) None of the above

28. When 0.252525.........is converted into a fraction, then find the result. **[RRB 2009]**

(a) $\frac{25}{99}$ (b) $\frac{25}{90}$

(c) $\frac{25}{999}$ (d) $\frac{25}{9999}$

29. If $\frac{1}{1\frac{1}{4}} + \frac{1}{6\frac{2}{3}} - \frac{1}{x} + \frac{1}{10} = \frac{11}{12}$, then find the value of x.

(a) $\frac{15}{3}$ (b) $\frac{20}{15}$

(c) $\frac{15}{2}$ (d) $\frac{12}{13}$

30. The value of $0.\overline{3} + 0.\overline{6} + 0.\overline{7} + 0.\overline{8}$ in fraction will be

(a) $2\frac{3}{10}$ (b) $2\frac{2}{3}$ (c) $20.\overline{35}$ (d) $5\frac{3}{10}$

31. Find the value of

$999\frac{1}{7} + 999\frac{2}{7} + 999\frac{3}{7} + 999\frac{4}{7} + 999\frac{5}{7} + 999\frac{6}{7}$.

(a) 5997 (b) 5979 (c) 5994 (d) 2997

32. Find the value of $\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99}$.

(a) $\frac{10}{11}$ (b) $\frac{5}{11}$ (c) $\frac{9}{11}$ (d) $\frac{7}{11}$

33. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28} = ?$

(a) 2 (b) 5 (c) 4 (d) 6

(e) None of the above

34. Find the value of $27 \times 1.\overline{2} \times 5.526\overline{2} \times 0.\overline{6}$.

(a) $121.\overline{57}$ (b) $121.\overline{75}$

(c) $121.7\overline{5}$ (d) None of these

35. Out of the fractions $\frac{5}{7}, \frac{4}{9}, \frac{6}{11}, \frac{2}{5}$ and $\frac{3}{4}$, what is the difference between the largest and the smallest fractions? **[IBPS Clerk 2011]**

(a) $\frac{6}{13}$ (b) $\frac{11}{18}$

(c) $\frac{7}{18}$ (d) $\frac{11}{20}$

(e) None of the above

36. If the numerator of a fraction is increased by 200% and the denominator of the fraction is increased by 150%, the resultant fraction is 9/35. What is the original fraction? **[SBI Clerk 2011]**

(a) $\frac{3}{10}$ (b) $\frac{2}{15}$ (c) $\frac{3}{16}$ (d) $\frac{2}{7}$

(e) None of the above

37. 5/12 part of what amount will be equal to $3\frac{3}{4}$ part of ₹ 100?

(a) X 900 *(b) X* 940 *(c) X* 875 *(d) X* 975

38. A, *B, C* and *D* purchase a gift worth *X* 60. *A* pays 1/2 of what others are paying, *B* pays l/3rd of what others are paying and *C* pays l/4th of what others are paying. What is the amount paid byD? [SSCCGL2013]

(a) 14 *(b)* 15 *(c)* 16 *(d)* 13

39. l/4th of number of boys and 3/8th of number of girls participated in annual sports of the school. What fractional part of total number of students participated? [Bank PO 2007]

(a) 32%

(b) 20%

(c) 36%

(d) Data inadequate

(e) None of the above

40. The numerator of a fraction is 4 less than its denominator. If the numerator is decreased by 2 and the denominator is increased by 1, then the denominator becomes eight times the numerator, then find the fraction. [SSC CGL 2013]

(a) $\frac{3}{7}$ (b) $\frac{4}{8}$

(c) $\frac{2}{7}$ (d) $\frac{3}{8}$

41. The greatest among the numbers $\sqrt[4]{2}, \sqrt[5]{3}, \sqrt[10]{6}$ and $\sqrt[20]{15}$ is [SSC CPO 2013]

(a) $\sqrt[20]{15}$ (b) $\sqrt[4]{2}$ (c) $\sqrt[5]{3}$ (d) $\sqrt[10]{6}$

42. If the fraction *a/b* is positive, then which of the following must be true?

[SSC Multitasking 2014]

(aj a > 0 *(b) b* > 0

(c) ab>0 (d) a + *b* > 0

Exercise © *Higher SkiU Level Questions*

1. If 1.5 $x = 0.04$ y, then find the value of $\left(\frac{y-x}{y+x}\right)$. [SSC CPO 2007]

(a) $\frac{730}{77}$ (b) $\frac{73}{77}$

(c) $\frac{73}{770}$ (d) $\frac{703}{77}$

2. If 0.764 y = 1.236x, then what is the value of $\left(\frac{y-x}{y+x}\right)$? [CDS 2012]

(a) 0.764 (b) 0.236 (c) 2 (d) 0.472

3. Find the HCF of $\frac{5}{6}, \frac{15}{8}$ and $\frac{25}{32}$.

(a) 0.052 [SSC CPO 2007]

(b) 0.698

(c) 0.75

(d) Cannot be determined

4. Find the LCM of the fractions in above question.

(a) 13.5

(b) 37.5

(c) 10.5

(d) Cannot be determined

(e) None of the above

5. The pair of rational numbers that lies between $\frac{1}{4}$ and $\frac{3}{4}$ is [CDS 2014]

(a) $\frac{262}{1000}, \frac{752}{1000}$ (b) $\frac{24}{100}, \frac{74}{100}$

(c) $\frac{9}{40}, \frac{31}{40}$ (d) $\frac{252}{1000}, \frac{748}{1000}$

6. 1/8 part of a pencil is black and 1/2 part of the remaining is white. If the remaining part is blue and length of this blue part is 3% cm, then find the length of the pencil.

(a) 6 cm (b) 7 cm

(c) 8 cm (d) 9 cm

(e) None of the above

7. How many pieces of 13.2 cm can be cut from a 330 cm long rod? [Bank Clerks 2009]

(a) 25 (b) 28 (c) 21 (d) 35 (e) None of the above

8. In the year 2011, Shantanu gets ? 3832.5 as his pocket allowance. Find his pocket allowance per day.

(a) ? 9.5 (b) ? 10.5

(c) ? 12.5 Ca[)? 11.5

(e) None of the above

9. Sum of three fractions is $2\frac{11}{24}$. If the

greatest fraction is divided by the smallest fraction, the result is 7/6, which is greater than the middle fraction by 1/3. Find all the three fractions.

(a) $\frac{3}{5}, \frac{4}{7}, \frac{2}{3}$ (b) $\frac{7}{8}, \frac{5}{6}, \frac{3}{4}$

(c) $\frac{7}{9}, \frac{2}{3}, \frac{3}{5}$ (d) $\frac{7}{8}, \frac{7}{9}, \frac{7}{10}$

(e) None of the above

10. Find the value of $2 \times \left\{\frac{3.6 \times 0.48 \times 2.50}{0.12 \times 0.09 \times 0.5}\right\}$.

(a) 800 (b) 500 (c) 900 (d) 1600

(e) None of the above

11. If a fraction is multiplied by itself and then divided by the reciprocal of the same fraction, the result is $18\frac{26}{27}$. Find the fraction.

(a) $\frac{8}{27}$ (b) $1\frac{1}{3}$

(c) $2\frac{2}{3}$ (d) $3\frac{2}{3}$

(e) None of the above

12. What is the value of $0.007 + 0.00\overline{7} + 17.\overline{83} + 310.020\overline{2}$? [CDS 2012]

(a) 327.86638 (b) 328.644

(c) 327.86683 (d) $327.866\overline{8}$

13. Representation of $0.2\overline{341}$ in the form p/q, where p and q are integers, $q \neq 0$, is [CDS 2013]

(a) $\frac{781}{3330}$ (b) $\frac{1171}{4995}$

(c) $\frac{2341}{9990}$ (d) $\frac{2339}{9990}$

14. If $1\frac{2}{3} \div \frac{2}{7} \times \frac{x}{7} = 1\frac{1}{4} \times \frac{2}{3} \div \frac{1}{6}$, then find the value of x. [SSC CGL 2011]

(a) 0.006 (b) $\frac{1}{6}$

(c) 0.6 (d) 6

15. $\frac{0.04}{0.03}$ of $\frac{\left(3\frac{1}{3} - 2\frac{1}{2}\right) \div \frac{1}{2} \text{ of } 1\frac{1}{4}}{\frac{1}{3} + \frac{1}{5} \text{ of } \frac{1}{9}} = ?$ [SSC CPO 2011]

(a) 1 (b) 5

(c) $\frac{1}{5}$ (d) $\frac{1}{2}$

16. A man reads 3/8 of a book on a day and 4/5 of the remainder on the second day. If the number of pages still unread are 40, then how many pages did the book contain?

(a) 300 *(b)* 500

(c) 320 *(d)* 350

17. 4/7 of a pole is in the mud. When 1/3 of it is pulled out, 250 cm of the pole is still in the mud. Find the full length of the pole.

(a) 1000 *(b)* 1100

(c) 950 *(d)* 1050

(e) None of the above

Answer with Solutions

Exercise© *Base Level Questions*

1. (*c*) $0.34 = \frac{34}{100} = \frac{17}{50}$

2. (*b*) $\frac{3}{10} + \frac{5}{100} + \frac{8}{1000} = 0.3 + 0.05 + 0.008$
$= 0.358$

3. (*b*) $\frac{75}{100} = 0.75$

4. (*c*) $0.\overline{5} = \frac{5}{9}$

5. (*a*) $0.2\overline{7} = \frac{(27-2)}{90} = \frac{25}{90} = \frac{5}{18}$

6. (*a*) $9.46\overline{7} = 9 + \frac{(467-46)}{900}$
$= 9 + \frac{421}{900} = 9\frac{421}{900}$

7. (*c*) $0.8\overline{268} = \frac{8268-82}{9900} = \frac{8186}{9900} = \frac{4093}{4950}$

8. (*a*) $\frac{7}{12} = 0.583,\ \frac{2}{3} = 0.666,\ \frac{3}{8} = 0.375$

$\therefore \quad \frac{3}{8} < \frac{7}{12} < \frac{2}{3}$

Alternate Method

LCM of 12, 3 and 8.

2	12, 3, 8
2	6, 3, 4
3	3, 3, 2
	1, 1, 2

$\therefore \quad \text{LCM} = 2 \times 2 \times 3 \times 2$
$= 24$

$\therefore \quad \frac{7}{12} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24}; \frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24}$

and $\quad \frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$

Now, the fraction having smallest numerator will be smallest.

$\therefore$ Ascending order is

$\frac{9}{24} < \frac{14}{24} < \frac{16}{24}$

i.e., $\quad \frac{3}{8} < \frac{7}{12} < \frac{2}{3}$.

9. (*a*) $\frac{7}{9} = 0.777,\ \frac{5}{8} = 0.625,\ \frac{3}{7} = 0.428$

Given,

$0.777 > 0.625 > 0.428$

$\therefore \quad \frac{7}{9} > \frac{5}{8} > \frac{3}{7}$

10. (*c*) $\frac{5}{7} = 0.71, \frac{7}{13} = 0.54, \frac{4}{7} = 0.57,$

$\frac{4}{15} = 0.27, \frac{9}{14} = 0.64$

Clearly, $\frac{4}{7}$ is the third highest fraction.

11. (*c*) $\frac{4}{7} = 0.57, \frac{5}{13} = 0.38,$

$\frac{6}{11} = 0.54, \frac{3}{5} = 0.6, \frac{2}{3} = 0.67$

Clearly, the second smallest fraction is $\frac{6}{11}$.

12. (*d*)

$$\begin{array}{r} 1088.88 \\ 1800.08 \\ +\ 1880.80 \\ \hline 4769.76 \end{array}$$

13. (*b*)

$$\begin{array}{r} 6435.9 \\ 7546.4 \\ +\ 1203.5 \\ \hline 15185.8 \end{array}$$

14. (*a*) $726.34 + 888.12 - ? = 1001.88$

$\therefore\ ? = 726.34 + 888.12 - 1001.88$

$= 1614.46 - 1001.88 = 612.58$

15. (*b*) $\frac{16}{23} \times \frac{47}{288} \times \frac{92}{141} = \frac{4}{18 \times 3} = \frac{2}{9 \times 3} = \frac{2}{27}$

16. (*d*) $1\frac{3}{5} + 1\frac{8}{9} + 2\frac{4}{5} = (1 + 1 + 2) + \left(\frac{3}{5} + \frac{8}{9} + \frac{4}{5}\right)$

$= 4 + \frac{27 + 40 + 36}{45}$

$= 4 + \frac{103}{45} = 4 + 2\frac{13}{45} = 6\frac{13}{45}$

Alternate Method

$1\frac{3}{5} + 1\frac{8}{9} + 2\frac{4}{5}$

$= \frac{8}{5} + \frac{17}{9} + \frac{14}{5}$

LCM of 5, 9, 5 = 45

$= \frac{8 \times 9 + 17 \times 5 + 14 \times 9}{45}$

$= \frac{72 + 85 + 126}{45}$

$= \frac{283}{45} = 6\frac{13}{45}$

17. (*a*) $\dfrac{19999}{21111} = 0.947$

18. (*b*) $? = \dfrac{1212}{0.5 \times 6.06} = \dfrac{1212}{5 \times 606} \times 10 \times 100$

$$= \frac{1212}{3030} \times 1000 = \frac{1212 \times 100}{303} = 400$$

19. (*c*) $33 + 371 \div 7 = 33 + \dfrac{371}{7} = 33 + 53 = 86$

20. (*b*) $(39.3 \times 53.4) + (26.7 \times 5.9)$

$$= 2098.62 + 157.53 = 2256.15$$

21. (*d*) $? = \dfrac{3}{4} \times \dfrac{5}{6} \times \dfrac{7}{10} \times 1664 = 728$

22. (*b*) $\dfrac{19}{21} = 0.904,\ \dfrac{21}{25} = 0.84,$

$\dfrac{25}{29} = 0.86$

$\dfrac{29}{31} = 0.93,\ \dfrac{31}{37} = 0.837$

$0.837 < 0.84 < 0.86 < 0.904 < 0.93$

Clearly, $\dfrac{21}{25}$ will be on second number.

23. (*d*) $? = \dfrac{783}{9 \times 0.75} = 116$

24. (*c*) $? = \dfrac{25}{3} \div \dfrac{65}{6} = \dfrac{25}{3} \times \dfrac{6}{65} = \dfrac{10}{13}$

25. (*d*) $0.4777 \ldots\ldots = 0.4\bar{7} = \dfrac{47 - 4}{90} = \dfrac{43}{90}$

26. (*c*)

$$\begin{array}{r} 4.00 \\ 4.44 \\ 44.40 \\ 4.04 \\ +\ 444.00 \\ \hline 500.88 \\ \hline \end{array}$$

27. (*a*) $\dfrac{0.\overline{142857}}{0.\overline{285714}} = \dfrac{\dfrac{142857}{999999}}{\dfrac{285714}{999999}} = \dfrac{142857}{285714} = \dfrac{1}{2}$

28. (*a*) $0.252525 \ \ldots = 0.\overline{25} = \dfrac{25}{99}$

29. (*c*) $\dfrac{1}{1\frac{1}{4}} + \dfrac{1}{6\frac{2}{3}} - \dfrac{1}{x} + \dfrac{1}{10} = \dfrac{11}{12}$

$\Rightarrow \quad \dfrac{4}{5} + \dfrac{3}{20} - \dfrac{1}{x} + \dfrac{1}{10} = \dfrac{11}{12}$

$\Rightarrow \quad \dfrac{1}{x} = \dfrac{4}{5} + \dfrac{3}{20} + \dfrac{1}{10} - \dfrac{11}{12}$

$\Rightarrow \quad \dfrac{1}{x} = \dfrac{48 + 9 + 6 - 55}{60}$

$\Rightarrow \quad \dfrac{1}{x} = \dfrac{63 - 55}{60} = \dfrac{8}{60} = \dfrac{2}{15}$

$\Rightarrow \quad \dfrac{1}{x} = \dfrac{2}{15} \Rightarrow x = \dfrac{15}{2}$

30. (*b*) $0.\bar{3} + 0.\bar{6} + 0.\bar{7} + 0.\bar{8}$

$$= \frac{3}{9} + \frac{6}{9} + \frac{7}{9} + \frac{8}{9}$$

$$= \frac{24}{9} = \frac{8}{3} = 2\frac{2}{3}$$

31. (*a*) $999\frac{1}{7} + 999\frac{2}{7} + 999\frac{3}{7} + 999\frac{4}{7}$

$+ 999\frac{5}{7} + 999\frac{6}{7}$

$= (999 + 999 + 999 + 999 + 999 + 999)$

$$+\left(\frac{1}{7} + \frac{2}{7} + \frac{3}{7} + \frac{4}{7} + \frac{5}{7} + \frac{6}{7}\right)$$

$= 6 \times 999 + \frac{21}{7} = 6\,(1000 - 1) + \frac{21}{7}$

$= 6000 - 6 + 3 = 5997$

32. (*b*) $\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99}$

$$= \left[\frac{1}{3} + \frac{1}{15}\right] + \frac{1}{35} + \frac{1}{63} + \frac{1}{99}$$

$$= \left[\frac{6}{15} + \frac{1}{35}\right] + \frac{1}{63} + \frac{1}{99}$$

$$= \frac{45}{105} + \frac{1}{63} + \frac{1}{99} \quad = \left[\frac{3}{7} + \frac{1}{63}\right] + \frac{1}{99}$$

$$= \frac{28}{63} + \frac{1}{99} = \frac{4}{9} + \frac{1}{99}$$

$$= \frac{44 + 1}{99} = \frac{45}{99} = \frac{5}{11}$$

33. (*a*) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28}$

LCM of 2, 4, 7, 14 and 28 = 28

$$= \frac{28 + 14 + 7 + 4 + 2 + 1}{28} = \frac{56}{28} = 2$$

34. (*d*) $27 \times 1.\bar{2} \times 5\cdot 526\bar{2} \times 0.\bar{6}$

$= 27 \times \frac{12 - 1}{9} \times \frac{55262 - 5526}{9000} \times \frac{6}{9}$

$= 27 \times \frac{11}{9} \times \frac{49736}{9000} \times \frac{6}{9}$

$= \frac{1094192}{9000} = 121.576888\ldots = 121.5768\bar{8}$

35. (*e*) $\frac{5}{7} = 0.71, \frac{4}{9} = 0.44, \frac{6}{11} = 0.54, \frac{2}{5} = 0.40$

$\frac{3}{4} = 0.75$

Here, the largest fraction $= \frac{3}{4}$

and the smallest fraction $= \frac{2}{5}$

So, required difference $= \frac{3}{4} - \frac{2}{5} = \frac{15 - 8}{20}$

$$= \frac{7}{20}$$

36. (*e*) Let the original fraction be $\frac{x}{y}$.

Numerator is inereased by 200%.

$\therefore$ Numerator $= x + 200\%$ of x

$$= x + \frac{200x}{100}$$

$$= \frac{100x + 200x}{100}$$

$$= 300x/100$$

and denominator of the fraction is increased by 150%.

$$\text{Denominator} = y + \frac{150y}{100}$$

$$= \frac{100y + 150y}{100}$$

$$= \frac{250y}{100}$$

Then, according to the question,

$$\frac{300x/100}{250y/100} = \frac{9}{35}$$

$$\frac{300x}{250y} = \frac{9}{35}$$

$$\therefore \quad \frac{x}{y} = \frac{9}{35} \times \frac{250}{300} = \frac{3}{14}$$

37. (*a*) Let the amount be ₹ x.

According to the question,

$$\frac{5}{12}x = 3\frac{3}{4} \times 100$$

$$\Rightarrow \quad \frac{5}{12}x = \frac{15}{4} \times 100$$

$$\Rightarrow \quad x = \frac{12}{5} \times \frac{15}{4} \times 100$$

$$\therefore \quad x = ₹\ 900$$

38. (*d*) Let A, B, C and D pay ₹x, ₹y, ₹ z and ₹ a.

According to the question,

$$x = \frac{1}{2}(y + z + a) \quad \text{...(i)}$$

$$y = \frac{1}{3}(x + z + a) \quad \text{...(ii)}$$

$$z = \frac{1}{4}(x + y + a) \quad \text{...(iii)}$$

Also, $x + y + z + a = 60$

Now, put the value of $x + y + a = 4z$

Then, $4z + z = 60 \Rightarrow 5z = 60$

$\therefore \quad z = 12$

Similarly, on putting the value of $x + z + a = 3y$, we get

$$3y + y = 60 \Rightarrow 4y = 60$$

$\therefore \quad y = 15$

Again, on putting the value of

$(y + z + a) = 2x$, we get

$$2x + x = 60 \Rightarrow 3x = 60$$

$$\therefore \quad x = 20$$

Now, $\quad x + y + z + a = 60$

On putting the value of x, y and z, we get

$$12 + 15 + 20 + a = 60$$

$$\therefore \quad a = 60 - 47 = ₹\ 13$$

39. (*d*) Total number of students participated

$$= \left(\frac{1}{4}B + \frac{3}{8}G\right)$$

$\therefore$ Required percentage

$$= \left(\frac{\frac{1}{4}B + \frac{3}{8}G}{(B + G)} \times 100\%\right)$$

Clearly, given data is inadequate.

40. (*a*) Let denominator of fraction $= x$

Then, numerator $= x - 4$

$$\therefore \quad \text{Fraction} = \frac{x - 4}{x}$$

Now, according to the question,

$$8\,[(x - 4) - 2] = (x + 1)$$

$$(x - 4) - 2 = \frac{(x + 1)}{8}$$

$$\Rightarrow \quad x - 6 = \frac{x + 1}{8}$$

$$\Rightarrow \quad 8x - 48 = x + 1$$

$$\Rightarrow \quad 8x - x = 48 + 1$$

$$\Rightarrow \quad 7x = 49$$

$$\Rightarrow \quad x = \frac{49}{7}$$

$$\therefore \quad x = 7$$

$$\therefore \quad \text{Fraction} = \frac{7 - 4}{7} = \frac{3}{7}$$

41. (*c*) LCM of 4, 5, 10 and 20 = 20

$$\sqrt[4]{2} = (2)^{\frac{1}{4}} = (2^5)^{\frac{1}{20}} = (32)^{\frac{1}{20}}$$

$$\sqrt[5]{3} = (3)^{\frac{1}{5}} = (3^4)^{\frac{1}{20}} = (81)^{\frac{1}{20}}$$

$$\sqrt[10]{6} = (6)^{\frac{1}{10}} = (6^2)^{\frac{1}{20}} = (36)^{\frac{1}{20}}$$

$$\sqrt[20]{15} = (15)^{\frac{1}{20}}$$

Thus, greatest number is $(81)^{\frac{1}{20}}$ *i.e.*, $\sqrt[5]{3}$.

42. (*c*) If the fraction $\frac{a}{b}$ is positive, then $ab > 0$ must be true.

Exercise © *Higher SkiU Level Questions*

1. (*b*) Given, $1.5x = 0.04y$

$$\frac{y}{x} = \frac{1.5}{0.04} = \frac{1.50}{0.04} = \frac{150}{4} = \frac{75}{2}.$$

$$\left(\frac{y-x}{y+x}\right) = \left(\frac{\frac{y}{x}-1}{\frac{y}{x}+1}\right) = \frac{\left(\frac{75}{2}-1\right)}{\left(\frac{75}{2}+1\right)} = \frac{73}{77}$$

2. (*b*) Given, $\frac{y}{x} = \frac{1236}{0.764} = \frac{309}{191}$

$$\left(\frac{y-x}{y+x}\right) = \left(\frac{\frac{y}{x}-1}{\frac{y}{x}+1}\right) = \left(\frac{\frac{309}{191}-1}{\frac{309}{191}+1}\right) = \left(\frac{\frac{118}{191}}{\frac{500}{191}}\right)$$

$$= \frac{118}{500} = 0.236$$

3. (*a*) HCF of $\frac{5}{6}, \frac{15}{8}, \frac{25}{32}$

$$= \frac{\text{HCF of } 5, 15 \text{ and } 25}{\text{LCM of } 6, 8 \text{ and } 32} = \frac{5}{96} = 0.052$$

4. (*b*) LCM of $\frac{5}{6}, \frac{15}{8}, \frac{25}{32}$

$$= \frac{\text{LCM of } 5, 15 \text{ and } 25}{\text{HCF of } 6, 8 \text{ and } 32}$$

$$= \frac{75}{2} = 37.5$$

5. (*d*) $\frac{1}{4} = 0.25$ and $\frac{3}{4} = 0.75$

Only option (d) with $\frac{252}{1000} = 0.252$ and $\frac{748}{1000} = 0.748$ lies between 0.25 and 0.75.

6. (*c*) Let total length of pencil be x cm.

Then, black part $= \frac{x}{8}$

Remaining part $= x - \frac{x}{8} = \frac{7x}{8}$

White part $= \frac{1}{2}\left(\frac{7x}{8}\right) = \frac{7x}{16}$

Remaining part $= x - \left(\frac{x}{8} + \frac{7x}{16}\right) = \frac{7x}{16}$

$\therefore$ Length of blue part $= \frac{7x}{16}$

According to the question,

$$\frac{7x}{16} = 3\frac{1}{2}$$

$$\Rightarrow \quad \frac{7x}{16} = \frac{7}{2}$$

$$\therefore \quad x = 8 \text{ cm}$$

7. (*a*) Total length of the rod = 330 cm

Length of a piece = 13.2 cm.

Required number of pieces $= \dfrac{330}{13.2} = 25$

8. (*b*) Shantanu's pocket allowance = ₹ 383250

Total days in 2011 (general year)

= 365 days

Allowance per day $= \dfrac{3832.5}{365} =$ ₹ 10.5

9. (*b*) Let the greatest, middle and smallest fractions be x, y and z respectively in decreasing order.

According to the question,

$$\frac{\text{Greatest fraction}}{\text{Smallest fraction}} = \frac{7}{6}$$

$$\Rightarrow \quad \frac{x}{z} = \frac{7}{6}$$

$$\Rightarrow \quad x = \frac{7z}{6} \qquad \ldots(\text{i})$$

and

$$y = \frac{7}{6} - \frac{1}{3}$$

$$= \frac{7-2}{6} = \frac{5}{6} \qquad \ldots(\text{ii})$$

Now, $x + y + z = 2\dfrac{11}{24}$ ...(iii)

Substitute the value of x and y from Eqs. (i), (ii) and put in Eqs (iii), we get

$$\Rightarrow \quad \frac{7}{6}z + \frac{5}{6} + z = \frac{59}{24}$$

$$\Rightarrow \quad \frac{7z + 5 + 6z}{6} = \frac{59}{24}$$

$$\Rightarrow \quad \frac{13z + 5}{6} = \frac{59}{24}$$

$$\Rightarrow \quad 13z = \frac{59}{4} - 5 = \frac{39}{4}$$

$$\Rightarrow \quad z = \frac{39}{4 \times 13}$$

$$\therefore \quad z = \frac{3}{4}$$

On putting the value of z in Eq. (i), we get

$$x = \frac{7}{6}z = \frac{7}{6} \times \frac{3}{4} = \frac{7}{8}$$

10. (*d*) $2 \times \left\{\dfrac{3.6 \times 0.48 \times 2.50}{0.12 \times 0.09 \times 0.5}\right\}$

$$= 2 \times \left[\frac{36 \times 48 \times 250}{12 \times 9 \times 5}\right]$$

$$= 2 \times 4 \times 4 \times 50$$

$$= 1600$$

11. (*c*) If the required fraction be x.

According to the question,

$$\frac{x \times x}{\frac{1}{x}} = 18\frac{26}{27}$$

$$\Rightarrow \quad x^3 = \frac{512}{27}$$

$$\therefore \quad x = \frac{8}{3} = 2\frac{2}{3}$$

12. (*b*) $0.007 + \frac{7}{9} + 17\frac{83}{99} + 310\frac{(202-2)}{9000}$

$$\Rightarrow \quad 0.007 + \frac{7}{9} + \frac{1766}{99} + 310\frac{200}{9000}$$

$$\Rightarrow \quad 0.007 + \frac{7}{9} + \frac{1766}{99} + 310.022$$

$$\Rightarrow \quad 0.007 + 0.777 + 17.838 + 310.022$$

$$\Rightarrow \quad 328.644$$

13. (*d*) $0.2\overline{341} = \frac{2341-2}{9990} = \frac{2339}{9990}$

14. (*d*) Given expression

$$1\frac{2}{3} + \frac{2}{7} \times \frac{x}{7} = 1\frac{1}{4} \times \frac{2}{3} + \frac{1}{6}$$

$$\Rightarrow \quad \frac{5}{3} \times \frac{7}{2} \times \frac{x}{7} = \frac{5}{4} \times \frac{2}{3} \times 6$$

$$\Rightarrow \quad \frac{5x}{3 \times 2} = 5$$

$$\Rightarrow \quad x = \frac{5 \times 6}{5}$$

$$\therefore \quad x = 6$$

15. (*b*) Given expression

$$= \frac{0.04}{0.03} \text{ of } \frac{\left(3\frac{1}{3} - 2\frac{1}{2}\right) \div \frac{1}{2} \text{ of } 1\frac{1}{4}}{\frac{1}{3} + \frac{1}{5} \text{ of } \frac{1}{9}}$$

$$= \frac{4}{3} \times \frac{\left(\frac{10}{3} - \frac{5}{2}\right) \div \frac{1}{2} \text{ of } \frac{5}{4}}{\frac{1}{3} + \frac{1}{45}}$$

$$= \frac{4}{3} \times \frac{\left(\frac{20-15}{6}\right) \div \frac{5}{8}}{\left(\frac{15+1}{45}\right)}$$

$$= \frac{4}{3} \times \frac{\frac{5}{6} \times \frac{8}{5}}{\frac{16}{45}} = \frac{4}{3} \times \frac{5}{6} \times \frac{8}{5} \times \frac{45}{16} = 5$$

16. (*c*) Let the number of pages be x.

Number of pages read on first day $= \frac{3x}{8}$

Remaining pages $= x - \frac{3x}{8} = \frac{5}{8}x$

Number of pages read on second day

$$= \frac{4}{5} \times \frac{5}{8}x = \frac{1}{2}x$$

Now, remaining pages $= x - \left(\frac{3x}{8} + \frac{x}{2}\right)$

$$= x - \left(\frac{3x + 4x}{8}\right) = \left(x - \frac{7x}{8}\right)$$

$$= \left(\frac{8x - 7x}{8}\right) = \frac{x}{8}$$

According to the question,

$$\frac{x}{8} = 40$$

$\Rightarrow \quad x = 40 \times 8$

$\therefore \quad x = 320$

Alternate Method

Balance of the book for one day

$$= \left(1 - \frac{4}{5}\right)\left(1 - \frac{3}{8}\right)$$

$$= \left(\frac{5-4}{5}\right)\left(\frac{8-3}{8}\right) = \frac{1}{5} \times \frac{5}{8} = \frac{1}{8}$$

$\therefore$ Total pages

$$= \frac{\text{Pages unread}}{\text{Fraction related to pages unread}}$$

$$= \frac{40}{1/8} = 320$$

$\therefore \quad$ Total pages $= 320$

17. (*d*) Total length of pole

$$= \frac{\text{Length of pole in mud}}{\text{Remaining part of pole in mud}}$$

$$= \frac{250}{\frac{4}{7} - \frac{1}{3}} = 1050$$

$\therefore$ Length of pole $= 1050$

Chapter

5

Square Root and Cube Root

Square

If a number is multiplied with itself, then the result of this multiplication is called the square of that number. *For example*

(i) Square of 6 = 6 x 6 = 36

(ii) Square of 12 = 12 x 12 = 144

(iii) Square of 100 = 100 x 100 = 10000

Methods to Find Square

Different methods to calculate the square of a number are as follows

Multiplication Method

In this method, the square of any 2-digit number can be calculated by the following given steps.
Step I Square the unit's digit,

{If the square has two digits, then write ten's digit as carry.} *Step* II 2 x Ten's digit X Unit's digit + Carry *Step* III (Ten's digit)^2 + Carry from step II *Step* IV Now arrange the numbers first write step III number, then step II and at unit's place step I.

For example Find the square of 74 Sfepl $(4)^2 = 16$ {Carry = 1} *Step* II 2 x 7 x 4 + 1 = 57 {Carry = 5} Step III $(7)^2 + 5 = 49 + 5 = 54$ Step IV $(74)^2 = 5476$

Algebraic Method

To calculate square by this method, two formulae are used. (i) $(a + b)^2 = a^2 + b^2 + 2ab$ (ii) *{a-bf* $= a^2 + b^2 - 2ab$

For *example* The square of 34 is

$(34)^2 = (30 + 4)^2 = (30)^2 + (4)^2 + 2 \times 30 \times 4$

$= 900 + 16 + 240$ $(34)^2 = 1156$

Square of Decimal Numbers

To find the square of any decimal number, write the square of the number ignoring the decimal and then place the decimal twice the place of the original number starting from unit's place. *For example* The square of 3.5 is as follows $(35)^2 = 1225$

Here, the decimal is after one-digit in 3.5. Hence, the decimal will be placed twice the place of original number in the result.

$(3.5)^2 = 12.25$

Square Root

The square root of a number is that number, the square of which is equal to the given number. There are two types of square roots of a number, positive and negative. It is denoted by the sign *'yj~'.*

For example 49 has two square roots 7 and - 7, because $(7)^2 = 49$ and $(-7)^2 = 49$. Hence, we can write V49 = ± 7.

Methods to Find Square Root

Different methods to calculate the square root of a number are as follows

Prime Factorisation Method

This method has the following steps Step I Express the given number as the product of prime factors. *Step* II Arrange the factors in pairs of same prime numbers. *Step* III Take the product of these prime factors taking one out of every pair of the same primes. This product gives us the square root of the given number,

Ex. 1 Find the square root of 1089.

Sol. Prime factors of 1089 = $11 \times 11 \times 3 \times 3$

$\Rightarrow \qquad \sqrt{1089} = \sqrt{11 \times 11 \times 3 \times 3}$

Now, taking one number from each pair and multiplying them, we get

$$\sqrt{1089} = 11 \times 3 = 33$$

11	1089
11	99
3	9
3	3
	1

Ex. 2 Find the square root of 1024.

Sol. Prime factors of 1024 =2x2x2x2x2x2x2x2x2x2

$\Rightarrow \qquad \sqrt{1024} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$

Now, taking one number from each pair and multiplying them, we get

$$\sqrt{1024} = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Division Method

If it is not easy to evaluate square root using prime factorisation method, then we use division method.

The steps of this method can be easily understood with the help of following examples.

Ex. 3 Find the square root of 18769.

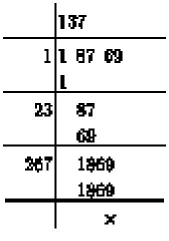
Sol. *Step* I In the given number, mark off the digits in pairs starting from the unit digit. Each pair and the remaining one-digit (if any) is called a period.

Step II Now, 1=1; On subtracting, we get 0 (zero) as remainder.

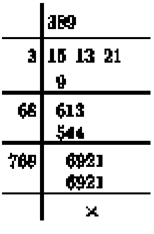
Step III Bring down the next period, *i.e.* 87. Now, the trial divisor is 1x2 = 2 and trial dividend is 87. So, we take 23 as divisor and put 3 as quotient. The remainder is 18 now.

Step IV Bring down the next period, which is 69. Now, trial divisor is 13 x 2 = 26 and trial dividend is 1869. So, we take 267 as dividend and 7 as quotient. The remainder is 0.

Step V The process (processes like III and IV) goes on till all the periods (pairs) come to an end and we get remainder as 0 (zero) now. Hence, the required square root = 137



Ex. 4 What is the square root of 151321? **Sol.** /. Required square root = 389



Properties of Squares and Square Roots

• The difference of squares of two consecutive numbers will always be equal to the sum of the number *i.e.,* $(a^2 - b^2) = (a + b)(a-b)$. Here, a>b and (a, b) being consecutive (a — b) =1.

• If a = 12 and b =11, then

$(12^2-11^2) = (12 + 11)(12-11) = 23$

• If the square of any number ends with 1, then its square root will end with 1 or 9.

• If the square of any number ends with 4, then its square root will end with 2 or 8.

• If the square of any number ends with 5, then its square root will end with 5.

• If the square of any number ends with 6, then its square root will end with 7 or 6.

• If the square of any number ends with 9, then its square root will end with 3 or 7.

• The square of any number always ends with 0,1,4, 5,6 or 9 but will never end with 2,3, 7 or 8,

• Square root of negative number is imaginary.

Important Relations

1. $\sqrt{?} = y$. The required number $= y^2$
2. $\sqrt{a^2 \times b^2} = ab$
3. $\sqrt{a^3 \times b^3} = ab\sqrt{ab}$
4. $\sqrt{a^4 \times b^4 \times c^4} = a^2 b^2 c^2$
5. $\sqrt{a^n \times b^m} = a^{n/2} \times b^{m/2}$
6. $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$
7. $\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$

Square Root of Decimal Numbers

If in a given decimal number, the number of digits after decimal are not even, then we put a 0 (zero) at the extreme right, So that these are even number of digits after the decimal point. Now, periods are marked as marked in previous explanation starting from right hand side before the decimal point and from the left hand after the decimal digit.

For example 156.694

There are odd number of digits after decimal. So, we put a zero after the digit, so that there are even

digits after the decimal 156.6940

Now, periods are marked as

Direction of marking

$\overline{1}\ \overline{56}\ .\ \overline{69}\ \overline{40}$

After the periods are marked, then previous method is used to find the square root

Ex.5 Find the square root of 147.1369.

Sol. Here, 147.1369 contains even digits after decimal, so there is no need to add zero after the the last digit, now period are marked as 147.1369

	12.13
1	1 47 . 13 69 1
22	47 44
241	313 241
2423	7269 7269
	×

∴ Required square root = 12.13

Ex.6 Find the square root of 149.597361. **Sol.**

	12.231
1	1 49 . 59 73 61 1
22	49 44
242	559 484
2443	7573 7329
24461	24461 24461
	×

∴ Required square root = 12.231

Square Root of a Fraction

To find square root of a fraction, we have to find the square roots of numerators and denominators, separately.

Ex. 7 $\sqrt{\dfrac{2704}{81}} = ?$

Sol. $\sqrt{\dfrac{2704}{81}} = \dfrac{\sqrt{2704}}{\sqrt{81}} = \dfrac{52}{9}$

	52
5	2704
	25
102	204
	204
	×

Ex. 8 Find the square root of $\dfrac{461}{8}$.

Sol. $\sqrt{\dfrac{461}{8}} = \sqrt{\dfrac{461 \times 2}{8 \times 2}} = \sqrt{\dfrac{922}{16}} = \dfrac{\sqrt{922}}{\sqrt{16}} = \dfrac{30.3644}{4} = 7.5911 \text{ (approx.)}$

Note Sometimes, numerator and denominator are not a complete square. In these types of cases, it is better to convert the given fraction into decimal fraction to find the square root.

Ex. 9 $\sqrt{\dfrac{9261}{8400}} = ?$

Sol. $\sqrt{\dfrac{9261}{8400}} = \sqrt{1.1025} = 1.05$

Fast Track **Formulae**

to solve the **QUESTIONS**

Formula **1**

If in a given number, the total number of digits are *n* and if *n* is even, then square root of that number will have *n/2* digits and if *n* is odd, then

number of digits will be $\dfrac{n+1}{2}$.

Ex. 10 How many digits are there in square root of 1838736. **Sol.** Since, the total number of digits are 7 and 7 is odd number.

$\therefore$ Number of digits $= \dfrac{7+1}{2} = 4$ digits

Formula 2

If any number has 5 in unit's place, then its square can be calculated as

$$(\underline{A}\cdot\underline{5})^2 = \underline{A\times(A+1)}\cdot\underline{25}$$

Note Student should understand the dot(-)just as a separation between numbers

Ex. 11 Find the square of 125?

Sol. $(125)^2 = 12\ (12 + 1) \bullet 25 = 15625$

Formula 3

$$\sqrt{x\sqrt{x\sqrt{x\ldots n}}} = x^{\frac{2^n-1}{2^n}}$$

Ex. 12 Find the value of $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$.

Sol. Here, value of $n = 5$

$$\therefore \quad \sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}} = 2^{\frac{2^5-1}{2^5}}$$

$$= 2^{\frac{32-1}{32}} = 2^{\frac{31}{32}}$$

Formula 4

$$\sqrt{x\sqrt{x\sqrt{x\sqrt{x\ldots\ldots\infty}}}} = x$$

Ex. 13 Find the value of $\sqrt{7\sqrt{7\sqrt{7\sqrt{\ldots\ldots\infty}}}}$.

Sol. As the expression continues to infinity, so $\sqrt{7\sqrt{7\sqrt{7\ldots\infty}}} = 7$

Formula 5

To find the value of $\sqrt{x+\sqrt{x+\sqrt{x+\ldots}}}$, find the factors of x, such that the difference between the factors is 1, then the larger factor will be result.

Ex. 14 Find the value of $\sqrt{12+\sqrt{12+\sqrt{12+\ldots}}}$.

Sol. The factors of 12 with difference of 1 are 3 and 4. Here, 4 is the larger number.

Hence, $\sqrt{12+\sqrt{12+\sqrt{12+\ldots}}} = 4$

Formula 6

The value of $\sqrt{x-\sqrt{x-\sqrt{x-}}}\ldots$, find the factors of x, such that the difference between the factors is 1, then smaller factor will be the result.

Ex. 15 Find the value of $\sqrt{30-\sqrt{30-\sqrt{30\ldots}}}$

Sol. The factors of 30 with difference of 1 are 6 and 5.

Hence, smaller number is 5.

$$\therefore \quad \sqrt{30-\sqrt{30-\sqrt{30\ldots}}} = 5$$

Fast Track **Techniques**

to solve the **QUESTIONS**

Technique 1 If any number is in $\frac{1}{\sqrt{a} \pm \sqrt{b}}$ form, then we multiply by its rationalisation factor $\sqrt{a} \mp \sqrt{b}$ in both numerator and denominator.

Ex. 16 If $\sqrt{2} = 1.414$, then find the value of $\frac{1}{\sqrt{9} - \sqrt{8}}$.

Sol. $\frac{1}{\sqrt{9} - \sqrt{8}} = \frac{1}{\sqrt{9} - \sqrt{8}} \times \frac{\sqrt{9} + \sqrt{8}}{\sqrt{9} + \sqrt{8}} = \frac{\sqrt{9} + \sqrt{8}}{(\sqrt{9})^2 - (\sqrt{8})^2} = \frac{\sqrt{9} + \sqrt{8}}{9 - 8}$

$= \frac{3 + 2\sqrt{2}}{1} = 3 + 2\,(1.414) = 3 + 2.828 = 5.828$

Technique 2

To find smallest and largest n-digit number which is a perfect square.

:. *17* Find the 5-digit number which is a perfect square. Sol. The bissest 5-digit number = 99999

	3 16
3	9 99 99
	9
61	99
	61
626	3899
	3756
	143

Clearly, the biggest 5-digit number which is a perfect square, is less by 143. ∴ Number = 99999 - 143 = 99856

Ex. 18 Find the smallest 4-digit number which is a perfect square.

Sol. Smallest 4-digit number = 1000

Clearly, the smallest 4-digit number which is a perfect square, will be greater by 24.

∴ Number = 1000 + 24 = 1024

	32
3	10 00
	9
62	100
	124
	- 24

Cube

If a number is multiplied two times with itself, then the result of this multiplication is called the cube of that number. *For example*

(i) Cube of 6=6x6x6=216 (ii) Cube of 8 =8 x8 x8=512

Methods to Find Cube

Different methods to calculate the cube of a number are as follows

Algebraic Method

To calculate cube by this method, two formulae are used.

(i) $(a + b)^3 = a^3 + 3ab\ (a+b) + b^3$ (ii) (a -*bf*= a^3 -3ab *(a-b)*- b^3

For example The cube of 16 is

$(16)^3 = (10 + 6)^3 = (10)^3 + 3 \times 10 \times 6\ (10 + 6) + (6)^3 = 1000 + 2880 + 216 = 4096$

Shortcut Method

Step I The answer consists of 4 parts each of which has to be calculated separately, *Step* II First write down the cube of ten's digit to the extreme left. Write the next two terms to the ricrht of it bv creatine; GP (Geometric Proaression) havina

common ratio which is equal to $\frac{\text{unit's digit}}{\text{ten's digit}}$ and the fourth number will be cube of unit's digit.

Step III Write the double of 2nd and 3rd number below them.

Step IV Now, add the number with numbers written below it and write the unit's

place digit in a straight line and remaining number is carried forward to

the next number.

Ex. 19 Find the cube of 35.

Sol. Here, unit's digit is 5 and ten's digit is 3.

Step I Write the cube of ten's digit at extreme left *i.e.,* $(3)^3=27$

Step II Now, the next two terms on the right will be in a GP of common ratio equals

to $\frac{\text{unit's digit}}{\text{ten's digit}}$.

So, the two terms will be $27 \times \frac{5}{3} = 45$; $45 \times \frac{5}{3} = 75$

and last term will be cube of unit's digit

i.e., $(5)^3 = 125$

So, they are arranged as 27 45 75 125

Step III Twice the second and third terms are written under it and are added.

So, carry 15 23 12

$$\begin{array}{rrrr} 27 & 45 & 75 & 125 \\ + & 90 & 150 & \\ \hline 42 & 8 & 7 & 5 \end{array}$$

$(35)^3 = 42875$

Cube Root

The cube root of a given number is the number whose cube is the given number. The cube root is denoted by the sign '$\sqrt[3]{\ }$'.

For example (i) $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$ (ii) $\sqrt[3]{512} = \sqrt[3]{8 \times 8 \times 8} = 8$

Methods to Find Cube Root

Method to calculate the Cube root of a number is as follow

Prime Factorisation Method

This method has following steps.

Step I Express the given number as the product of prime factors. *Step* II Arrange the factors in a group of three of same prime numbers. *Step* III Take the product of these prime factors picking one out of every group (group of three) of the same primes. This product gives us the cube root of given number.

Ex. 20 Find the cube root of 9261.

Sol. Prime factors of 9261 = $(3 \times 3 \times 3) \times (7 \times 7 \times 7)$

$$\Rightarrow \quad \sqrt[3]{9261} = \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7}$$

Now, taking one number from each group of three, we get

$$\sqrt[3]{9261} = 3 \times 7 = 21$$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

Properties of Cubes and Cube Roots

• If the cube of a number is of 2 or 3-digits, then its cube root will be of 1-digit

• If the cube of a number is of 4, 5 or 6 digits, then its cube root will be of 2 digits.

• If the cube of a number have 0,1, 2,3,4, 5, 6, 7,8,9 in its unit's place, then its cube root will haveO, 1, 8, 7, 4, 5, 6, 3, 2 or 9 in their unit's place, respectively.

• There are only three numbers whose cube is equal to the number

i.e., {of =0 ; (if =1 ; (-If =-1

Note
- If $\frac{p}{q}$ is a fraction, then $\sqrt[3]{\frac{p}{q}} = \frac{\sqrt[3]{p}}{\sqrt[3]{q}}$, where p and q are integers.
- If p is an integer, then $\sqrt[3]{-p} = -\sqrt[3]{p}$.

Ex. 21 Find the value of $\sqrt[3]{\frac{0.000729}{0.085184}}$.

Sol. $\sqrt[3]{\frac{0.000729}{0.085184}} = \sqrt[3]{\frac{729}{85184}} = \sqrt[3]{\frac{9 \times 9 \times 9}{44 \times 44 \times 44}} = \frac{9}{44}$

Ex. 23 Find cube root of -5832.

Sol. $\sqrt[3]{(-5832)} = -\sqrt[3]{5832} = -\sqrt[3]{18 \times 18 \times 18} = -18$

Ex. 24 Find cube root of -17576.

Sol. $\sqrt[3]{(-17576)} = -\sqrt[3]{17576} = -\sqrt[3]{26 \times 26 \times 26} = -26$

Multi Concept

QUESTIONS

1. *If $2*3=\sqrt{13}$ and $3*4=5$, then the value of $5*12$ is*

(a) 17 (b) $\sqrt{29}$ (c) 21 (d) 13

➥ (*d*) By observing, we see

$$2*3=\sqrt{(2)^2+(3)^2}=\sqrt{4+9}=\sqrt{13}$$

$$3*4=\sqrt{(3)^2+(4)^2}=\sqrt{9+16}=5$$

Hence, the value of $5*12=\sqrt{(5)^2+(12)^2}$

$$=\sqrt{25+144}=\sqrt{169}=13$$

2. *Which of the following equations are equivalent?*

1. $\left(\frac{1}{2}M+\frac{2}{3}N\right)^2$

2. $\frac{4}{9}N^2+\frac{1}{4}M^2+\frac{2}{3}MN$

3. $\left(\frac{M}{2}+\frac{2}{3}N\right)\left(\frac{1}{2}M-\frac{2}{3}N\right)$

4. $\frac{1}{4}\left(M+\frac{4}{3}N\right)^2$

(a) Only 2 and 3 (b) Only 1 and 4 (c) Only 1 and 2 (d) Only 1 and 3
(e) Only 1, 2 and 4

➥ (*e*) Simplifying all the above equations,

1. $\left(\frac{1}{2}M+\frac{2}{3}N\right)^2=\frac{1}{4}M^2+\frac{4}{9}N^2+\frac{2}{3}MN$
2. $\frac{4}{9}N^2+\frac{1}{4}M^2+\frac{2}{3}MN$
3. $\left(\frac{M}{2}+\frac{2}{3}N\right)\left(\frac{1}{2}M-\frac{2}{3}N\right)=\frac{1}{4}M^2+\frac{1}{3}MN-\frac{1}{3}MN-\frac{4}{9}N^2=\frac{1}{4}M^2-\frac{4}{9}N^2$
4. $\frac{1}{4}\left(M+\frac{4}{3}N\right)^2=\frac{1}{4}\left[M^2+\frac{16}{9}N^2+\frac{8}{3}MN\right]=\frac{1}{4}M^2+\frac{4}{9}N^2+\frac{2}{3}MN$

From the above four solutions, we find that only (1), (2) and (4) are equivalent.

3. *If $\left(\frac{x}{y}\right)=\left(\frac{z}{w}\right)$, then what is $(xy+zw)^2$ equal to?*

(a) $(x^2+z^2)(y^2+w^2)$ (b) $x^2y^2+z^2w^2$ (c) $x^2w^2+y^2z^2$ (d) $(x^2+w^2)(y^2+z^2)$

➥ (*a*) $\frac{x}{y}=\frac{z}{w}=k$ (say)

$\Rightarrow$ $x=yk;\ z=wk$

Now, $(xy+zw)^2=(y^2k+w^2k)^2$

$$=k^2(y^2+w^2)^2$$

$$=(y^2+w^2)(k^2y^2+k^2w^2)$$

$$=(y^2+w^2)(x^2+z^2)$$

Fast Track Practice

Exercise O ^{Base} *Level Questions*

1. Find the square root of 144.

(a) 15 (b) 16 (c) 12 (d) 13
(e) None of the above

2. What will be the square root of 7921? **[SSC CGL 2008]**

(a) 89 (b) 87 (c) 37 (d) 47

3. The square root of $0.\bar{4}$ is

(a) $0.\bar{6}$ (b) $0.\bar{7}$ (c) $0.\bar{8}$ (d) $0.\bar{9}$
(e) None of the above

4. Find the square root of $105\frac{4}{64}$. **[RBI 2008]**

(a) $15\frac{1}{4}$ (b) $15\frac{12}{4}$ (c) $10\frac{1}{4}$ (d) $6\frac{2}{4}$

(e) None of the above

5. $\sqrt{\frac{324}{81}} + \sqrt{\frac{324}{81}} = ?$

(a) 4 (b) 12 (c) 6 (d) 18
(e) None of the above

6. The square root of $\left(\frac{1}{4}\right) \times \left(\frac{1}{49}\right) + \left(\frac{25}{121}\right)$ is

[SSC (10+2) 2012]

(a) $\frac{11}{5}$ (b) $\frac{11}{70}$ (c) $\frac{7}{11}$ (d) $\frac{11}{7}$

7. Which of the following is false? **[Bank Clerks 2008]**

(a) $\sqrt{5184} = 72$ (b) $\sqrt{15625} = 125$
(c) $\sqrt{1444} = 38$ (d) $\sqrt{1296} = 34$
(e) None of the above

8. $\frac{\sqrt{24} + \sqrt{216}}{\sqrt{96}} = ?$ **[SSC (10+2) 2009]**

(a) $2\sqrt{6}$ (b) 2 (c) $6\sqrt{2}$ (d) $\frac{2}{\sqrt{6}}$

(e) None of the above

9. $\sqrt{1089} + \sqrt{289} = \sqrt{?}$ **[Bank Clerks 2007]**

(a) 625 (b) 50 (c) 2500 (d) 1378
(e) None of the above

10. $\sqrt{110\frac{1}{4}} = ?$ **[LIC ADO 2009]**

(a) 19.5 (b) 10.25 (c) 10.5 (d) 11.5
(e) None of the above

11. If $\sqrt{\frac{x}{169}} = \frac{18}{13}$, then x is equal to **[LIC AAO 2008]**

(a) 108 (b) 324 (c) 2916 (d) 4800
(e) None of the above

12. The value of $\sqrt[6]{0.000729}$ is **[SSC CGL 2013]**

(a) 0.027 (b) 0.3
(c) 0.03 (d) 0.09

13. Evaluate $\sqrt{16} + \sqrt{\frac{9.5 \times 0.0085 \times 18.9}{0.021 \times 0.0017 \times 1.9}}$.

(a) 154 (b) 158
(c) 160 (d) 169
(e) None of the above

14. Evaluate $\sqrt{\frac{0.289}{0.00121}} + \sqrt{\frac{64}{16}}$.

(a) $\frac{192}{11}$ (b) $\frac{170}{11}$
(c) $\frac{182}{11}$ (d) $\frac{172}{11}$
(e) None of the above

15. $\sqrt{15 \times 163 + 5 - 89} = ?$ **[Bank Clerks 2011]**

(a) 15 (b) 25 (c) 10 (d) 20
(e) None of the above

16. Find the square root of $\frac{0.204 \times 42}{0.07 \times 3.4}$.

[SSC (10+2) 2007]

(a) 5 (b) 6 (c) 3 (d) 9
(e) None of the above

17. $\sqrt{15^2 + 11 \times 3^2} = ?$ **[Bank Clerks 2007]**

(a) 18.25 (b) 19 (c) 18 (d) 19.5
(e) None of the above

18. The value of $\sqrt{400} + \sqrt{0.04} - \sqrt{0.000004}$ is **[SSC CPO 2013]**

(a) 20.22 (b) 20.198
(c) 20.188 (d) 20.022

19. $\sqrt{176 + \sqrt{2401}} = ?$ **[Hotel Mgmt. 2007]**

(a) 14 (b) 15 (c) 18 (d) 24
(e) None of the above

20. $\sqrt{?} + 136 = \frac{5}{8}$ of 320 **[Bank Clerks 2008]**

(a) 1936 (b) 4624
(c) 4196 (d) 4096
(e) None of the above

21. $\frac{2707}{\sqrt{?}} = 27.07$ [LIC AAO 2008]

(a) 10 (b) 100
(c) 1000 (d) 10000
(e) None of the above

22. $(64)^2 + \sqrt[3]{32768} = ?$ [Bank Clerks 2008]

(a) 128 (b) 132
(c) 142 (d) 104
(e) None of the above

23. $\sqrt{?} + \frac{3}{5}$ of $80 = 60 \times \frac{1}{2} \times 8$

(a) 36864 (b) 46864
(c) 56864 (d) 66864
(e) None of the above

24. $140\sqrt{?} + 315 = 1015$ [Hotel Mgmt. 2008]

(a) 16 (b) 25
(c) 36 (d) 5
(e) None of the above

25. If $\sqrt{3} = 1.732$, then find the value of $4(\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75})$ correct to 3 places of decimal.

(a) 6.928 (b) 5.928
(c) 2.732 (d) 3.732
(e) None of the above

26. If $\frac{\sqrt{1296}}{x} = \frac{x}{2.25}$, then find the value of x. [Bank Clerks 2008]

(a) 6 (b) 8
(c) 7 (d) 9
(e) None of the above

27. If $\sqrt{1 + \frac{x}{144}} = \frac{13}{12}$, then find the value of x. [UDC 2007]

(a) 1 (b) 12
(c) 13 (d) 25
(e) None of the above

28. If $x = 2 + \sqrt{2}$ and $y = 2 - \sqrt{2}$, then find the value of $(x^2 + y^2)$.

(a) 12 (b) 14
(c) 6 (d) 18
(e) None of the above

29. $(\sqrt{65025})^2 = (?)^2$ [SSC (10+2) 2009]

(a) 65.025 (b) 32512.5
(c) 255 (d) 510
(e) None of the above

30. $\sqrt{?} + 43 = \sqrt{19881}$ [Bank Clerks 2009]

(a) 9604 (b) 7744
(c) 9216 (d) 8464
(e) None of the above

31. $\sqrt{8 \times 7 - ? + 208 + 16} = 8$ [Bank Clerks 2009]

(a) 5 (b) 7
(c) 4 (d) 3
(e) None of the above

32. If $0.14 + x^2 = 14$, then x is equal to

(a) 0.01 *(b)* 10
(c) 100 *(d)* 0.1
(e) None of the above

33. If $\sqrt{18 \times 14 \times x} = 168$, then x is equal to

(a) 113 *(b)* 112
(c) 115 *(d)* 117
(e) None of the above

34. $22^2 + \sqrt{?} = 516$ **[Bank Clerks 2009]**

(a) 1028 *(b)* 1024
(c) 1124 *(d)* 1128
(e) None of the above

35. $\sqrt{?} + 28 = \sqrt{1681}$ **[Bank Clerks 2009]**

(a) 13 *(b)* 225
(c) 216 *(d)* 169
(e) None of the above

36. Evaluate $\sqrt{0.9}$ upto 3 places of decimal.

(a) 0.948 *(b)* 0.984

(c) 0.988 *(d)* 938

(e) None of the above

37. Sukhiram plants 15376 orange trees in his garden and arranges them, so that there are as many rows as there are orange trees in each row. Find the number of rows.

(a) 125 *(b)* 124

(c) 128 *(d)* 135

(e) None of the above

38. What is the least number to be multiplied with 294 to make it a perfect square? [CBI2007]

(a) 2 *(b)* 3

(c) 6 *(d)* 24

(e) None of the above

39. What least number should be subtracted from 6860, so that 19 be the cube root of the result from this subtraction?

[LIC ADO 2009]

(a) 1 *(b)* 2

(c) 3 *(d)* 5

(e) None of the above

40. What is the least number to be added to 8200 to make it a perfect square?

[Bank Clerks 2009]

(a) 81 *(b)* 100

(c) 264 *(d)* 154

(e) None of the above

41. Find the difference in 777 and its nearest perfect square number.

(a) 4 *(b)* 7 *(c)* 27 *(d)* 28 *(e)* None of the above

42. Find the value of $\sqrt{30+\sqrt{30+\ldots}}$.

(a) 5 *(b)* $3\sqrt{10}$ *(c)* 6 *(d)* 7

43. Find the value of $\sqrt{9\sqrt{9\sqrt{9 \ldots\ldots \infty}}}$.

(a) 81 (b) 3 (c) 9 (d) ∞

44. If $x = \sqrt{8+\sqrt{8+\sqrt{8 \ldots\ldots}}}$

and $y = \sqrt{8-\sqrt{8-\sqrt{8 \ldots\ldots}}}$, then

(a) $x + y = 1$ (b) $x + y + 1 = 0$

(c) $x - y = 1$ (d) $x - y + 1 = 0$

45. $\sqrt{6+\sqrt{6+\sqrt{6}}} + \ldots$ is equal to [CDS 2008]

(a) 3 (b) 2 (c) 1.5 (d) 2.5

46. Find the value of $\sqrt{6\sqrt{6\sqrt{6\sqrt{6\sqrt{6}}}}}$.

(a) 6 (b) $6^{31/32}$ (c) $6^{31/34}$ (d) 3

47. If $\sqrt{24} = 4.899$, then find the value of $\sqrt{\frac{8}{3}}$.

[SSC (10+2) 2009]

(a) 1.633 (b) 1.333 (c) 2.666 (d) 0.544

48. Find the value of $\sqrt[3]{2744} + 7$.

(a) 14 (b) 2 (c) 15 (d) 8 (e) None of the above

49. Each student of class 10 contributed some money for a picnic. The money contributed by each student was equal to the cube of the total number of students. If the total collected amount was

? 29791, then find the total number of

students. [UP Police 2008]

(a) 15 (b) 27

(c) 31 (d) 34

50. (Smallest common multiple of 12 and 16) x (Smallest common multiple of 10 and 15) is equal to [CTET 2012]

(a) 960 (b) 720

(c) 1440 (d) 480

51. Find the value of 2 x V3 upto three places of decimal.

(a) 1.732 (b) 3.464

(c) 4.464 (d) 2.732

(e) None of the above

52. If $\sqrt{2} = 1.4142$, then find the value of $2 \times \frac{\sqrt{2}}{(2 + \sqrt{2})}$.

(a) 0.8284 (b) 0.8184

(c) 0.8283 (d) 0.8483

(e) None of the above

53. If $(\sqrt{a} + \sqrt{b}) = 17$ and $(\sqrt{a} - \sqrt{b}) = 1$, then find $\sqrt{ab}$. [LIC ADO 2008]

(a) 17 (b) 18

(c) 72 (d) 21

(e) None of the above

54. If (46) is subtracted from the square of a number. The answer so obtained is 485. What is the number? [SBI 2012]

(a) 49 (b) 51

(c) 56 (d) 53

(e) None of the above

Exercise © *Higher Skill Level Questions*

$\frac{1}{4} \times \sqrt{[(12.1)^2 - (8.1)^2] + [(0.25)^2 + (0.25)(19.95)]}$

(a) 8 (b) 4 (c) 1 (d) 3
(e) None of the above

2. $\frac{1}{\sqrt{9}-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{4}} = ?$

(a) 0 (b) 1 (c) 1/3 (d) 5
(e) 1/5

3. If $3a = 4b = 6c$ and $a + b + c = 27\sqrt{29}$, then find $\sqrt{a^2 + b^2 + c^2}$.

(a) $3\sqrt{29}$ (b) 81
(c) 87 (d) 89
(e) None of the above

4. If $a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then $a + \frac{1}{a}$ is equal to

[Delhi Police 2009]

(a) 4 *(b)* 6

(c) 9 *(d)* 10

If the expression x + 809436 x 809438 be a perfect square, then the value of x is *(a)* 0 *(b)* 1

(c) 809436 *(d)* 809438

6. The sum of two numbers, when multiplied with each of the numbers separately, then the results of the above multiplications are 2418 and 3666, respectively. Find the difference between the numbers. [UP Police 2007]

(a) 16 *(b)* 21

(c) 19 *(d)* 23

7. A gardener has 1000 plants. He wants to plant them in such a way that the number of rows and the number of columns remains the same. What is the minimum number of plants that he needs more for this purpose?

(a) 14 *(b)* 24

(c) 32 *(d)* 34

8. Find the value of

$$\sqrt{\frac{(0.08)^2 + (0.21)^2 + (0.066)^2}{(0.003)^2 + (.021)^2 + (0.0066)^2}} \div 2.$$

(a) 2.8 *(b)* 5 *(c)* 10^2 *(d)* 5^2 *(e)* None of the above

9. $\sqrt[3]{1331} \times \sqrt[3]{216} + \sqrt[3]{729} + \sqrt[3]{64}$ is equal to [RBI 2009]

(a) 13.62 *(b)* 79

(c) 14.82 *(d)* 90.88

(e) None of the above

10. A toy factory manufactured a batch of electronic toys. If the toys were packed in boxes of 115 each, 13 boxes would not be filled completely. If the toys were packed in boxes of 65 each, 22 such boxes would not be enough to pack all of them. Coincidentally, in the end, the toys were packed in *n* boxes containing *n* toys each, without any remainder. The total number of toys was

(a) 1424 *(b)* 1434

(c) 1444 *(d)* 1454

11. The least number which is a perfect square and has 7936 as one of its factors is equal to [LIC ADO 2007]

(a) 12.008 *(b)* 246016

(c) 61504 *(d)* 240616

(e) None of the above

12. The positive square root of $(\sqrt{48} - \sqrt{45})$ is [SSC CGL 2011]

(a) $\frac{\sqrt[4]{3}}{\sqrt{2}}(\sqrt{5} - \sqrt{3})$

(b) $\frac{\sqrt[4]{3}}{2}(\sqrt{5} - \sqrt{3})$

(c) $\frac{\sqrt{2}}{\sqrt[4]{3}}(\sqrt{5} - \sqrt{3})$

(d) $\frac{\sqrt{2}}{\sqrt[4]{3}}(\sqrt{5} + \sqrt{3})$

13. Find the value of $\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4\ldots\ldots}}}}}}$.

(a) 2
(b) 2^2
(c) 2^3
(d) 2^5

14. What should be added to the $x(x+a)(x+2a)(x+3a)$, so that the sum be a perfect square? [CDS 2014]

(a) a^2
(b) a^4
(c) a^3
(d) None of these

Answer with **Solutions**

Exercise © *Base Level Questions*

1. (*c*) Prime factors of 144

2	144
2	72
2	36
2	18
3	9
3	3
	1

$\therefore \quad 144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$\Rightarrow \sqrt{144} = \sqrt{\overline{2 \times 2} \times \overline{2 \times 2} \times \overline{3 \times 3}}$

Now, taking one number from each pair and multiplying them, we get

$\sqrt{144} = 2 \times 2 \times 3 = 12$

2. (*a*) By division method,

	89
8	7921 64
169	1521 1521
	×

$\therefore$ Required square root = 89

3. (*a*) $\sqrt{0.\bar{4}} = \sqrt{\dfrac{4}{9}} = \dfrac{2}{3} = 0.666... = 0.\bar{6}$

4. (*c*) $\sqrt{105\dfrac{4}{64}} = \sqrt{\dfrac{6724}{64}}$

$= \dfrac{82}{8} = \dfrac{41}{4} = 10\dfrac{1}{4}$

5. (*a*) $\sqrt{\dfrac{324}{81}} + \sqrt{\dfrac{324}{81}} = 2 \times \sqrt{\dfrac{324}{81}}$

Applying prime factorisation method,

2	324
2	162
3	81
3	27
3	9
3	3
	1

$2 \times \sqrt{\dfrac{324}{81}} = 2 \times \sqrt{\dfrac{(2 \times 2) \times (3 \times 3) \times (3 \times 3)}{(3 \times 3) \times (3 \times 3)}}$

$= 2 \times \dfrac{2 \times 3 \times 3}{3 \times 3}$

$= 2 \times 2 = 4$

6. (*b*) Square root of $\frac{1}{4} \times \frac{1}{49} \div \left(\frac{25}{121}\right)$

$$= \sqrt{\frac{1}{4} \times \frac{1}{49} \times \frac{121}{25}}$$

$$= \frac{11}{2 \times 7 \times 5} = \frac{11}{70}$$

7. (*d*) (a)

$$\begin{array}{r|l} & 72 \\ \hline 7 & 5184 \\ & \;\;49 \\ \hline 142 & \;\;\;284 \\ & \;\;\;284 \\ \hline & \;\;\;\;\;\times \end{array}$$

(b)

$$\begin{array}{r|l} & 125 \\ \hline 1 & 15625 \\ & 1 \\ \hline 22 & \;56 \\ & \;44 \\ \hline 245 & \;1225 \\ & \;1225 \\ \hline & \;\;\;\;\;\times \end{array}$$

(c)

$$\begin{array}{r|l} & 38 \\ \hline 3 & 14\;44 \\ & \;\;9 \\ \hline 68 & \;\;\;544 \\ & \;\;\;544 \\ \hline & \;\;\;\;\;\times \end{array}$$

(d)

$$\begin{array}{r|l} & 36 \\ \hline 3 & 1296 \\ & \;\;9 \\ \hline 66 & \;396 \\ & \;396 \\ \hline & \;\;\;\times \end{array}$$

$\sqrt{5184} = 72$; $\sqrt{15625} = 125$

$\sqrt{1444} = 38$; $\sqrt{1296} = 36$

Clearly, $\sqrt{1296} = 36 \neq 34$

8. (*b*) $\dfrac{\sqrt{24} + \sqrt{216}}{\sqrt{96}}$

$$= \frac{\sqrt{2 \times 2 \times 2 \times 3} + \sqrt{3 \times 3 \times 3 \times 2 \times 2 \times 2}}{\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 3}}$$

$$= \frac{2\sqrt{6} + 6\sqrt{6}}{4\sqrt{6}} = \frac{8\sqrt{6}}{4\sqrt{6}} = 2$$

9. (c) By prime factorisation method,

$$\begin{array}{r|r} 3 & 1089 \\ \hline 3 & 363 \\ \hline 11 & 121 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

$$\sqrt{1089} = \sqrt{3 \times 3 \times 11 \times 11} = 3 \times 11 = 33$$

$$\sqrt{289} = \sqrt{17 \times 17} = 17$$

$\therefore \quad \sqrt{1089} + \sqrt{289} = 33 + 17 = 50$

Here, $\quad \sqrt{?} = 50$

$\therefore \quad ? = 50 \times 50 = 2500$

10. (c) $\sqrt{110\frac{1}{4}} = \sqrt{\frac{441}{4}} = \sqrt{\frac{3 \times 3 \times 7 \times 7}{2 \times 2}}$

$$= \frac{3 \times 7}{2} = \frac{21}{2} = 10.5$$

11. (b) Given, $\sqrt{\frac{x}{169}} = \frac{18}{13}$

On squaring both sides, we get

$$\frac{x}{169} = \frac{18}{13} \times \frac{18}{13}$$

$\therefore \quad x = \frac{18 \times 18 \times 169}{13 \times 13}$

$$= 18 \times 18 = 324$$

12. (b) $\sqrt[6]{0.000729} = \sqrt[6]{(0.3)^6} = 0.3$

13. (a) Given that,

$$\sqrt{16} + \sqrt{\frac{9.5 \times 0.0085 \times 18.9}{0.021 \times 0.0017 \times 1.9}}$$

$$= \sqrt{16} + \sqrt{\frac{95 \times 85 \times 18900}{21 \times 17 \times 19}}$$

$$= 4 + \sqrt{5 \times 5 \times 900}$$

$$= 4 + 5 \times 30 = 4 + 150 = 154$$

14. (a) $\sqrt{\frac{0.289}{0.00121}} + \sqrt{\frac{64}{16}} = \sqrt{\frac{0.28900}{0.00121}} + \sqrt{\frac{64}{16}}$

$$= \sqrt{\frac{17 \times 17 \times 10 \times 10}{11 \times 11}} + \sqrt{4}$$

$$= \sqrt{\frac{28900}{121}} + 2 = \frac{170}{11} + 2$$

$$= \frac{170 + 22}{11} = \frac{192}{11}$$

$$\begin{array}{r|r} & 17 \\ \hline 1 & 289 \\ & 1 \\ \hline 27 & 189 \\ & 189 \\ \hline & \times \end{array}$$

15. (d) $\sqrt{15 \times \frac{163}{5} - 89} = \sqrt{3 \times 163 - 89}$

$$= \sqrt{489 - 89} = \sqrt{400} = 20$$

16. (b) $\because \frac{0.204 \times 42}{0.07 \times 3.4} = \frac{204 \times 42}{7 \times 34} = 6 \times 6 = 36$

$\therefore \quad \sqrt{\frac{0.204 \times 42}{0.07 \times 3.4}} = \sqrt{36} = \sqrt{6 \times 6} = 6$

17. (*c*) $\sqrt{(15)^2 + 11 \times (3)^2} = \sqrt{225 + 99} = \sqrt{324}$

Applying prime factorisation method,

2	324
2	162
9	81
9	9
	1

$\sqrt{324} = \sqrt{(2 \times 2) \times (9 \times 9)} = 2 \times 9 = 18$

18. (*b*) $\sqrt{400} + \sqrt{0.04} - \sqrt{0.000004}$

$= 20 + 0.2 - 0.002$

$= 20.2 - 0.002 = 20.198$

19. (*b*) $\sqrt{2401} = \sqrt{7 \times 7 \times 7 \times 7} = 7 \times 7 = 49$

$\therefore \sqrt{176 + \sqrt{2401}} = \sqrt{176 + 49}$

$= \sqrt{225} = \sqrt{3 \times 3 \times 5 \times 5}$

$= 3 \times 5 = 15$

20. (*d*) Given, $\sqrt{?} + 136 = 320 \times \frac{5}{8}$

$\Rightarrow \sqrt{?} + 136 = 200$

$\Rightarrow \sqrt{?} = 200 - 136 = 64$

$\therefore ? = (64)^2 = 64 \times 64 = 4096$

21. (*d*) $\frac{2707}{\sqrt{?}} = 27.07$

$\Rightarrow \sqrt{?} = \frac{2707}{27.07} = \frac{270700}{2707} = 100$

$\therefore ? = (100)^2 = 100 \times 100 = 10000$

22. (*a*) We have, $(64)^2 + \sqrt[3]{32768} = ?$

$\Rightarrow ? = (64)^2 + \sqrt[3]{32 \times 32 \times 32}$

$= 64 \times 64 + 32 = \frac{64 \times 64}{32} = 64 \times 2 = 128$

23. (*a*) $\sqrt{?} + \frac{3}{5} \times 80 = 60 \times \frac{1}{2} \times 8$

$\Rightarrow \sqrt{?} + 3 \times 16 = 60 \times 4$

$\Rightarrow \sqrt{?} = 240 - 48 = 192$

$\therefore ? = 192 \times 192 = 36864$

24. (*b*) $140\sqrt{?} + 315 = 1015$

$\Rightarrow 140\sqrt{?} = 1015 - 315$

$\Rightarrow 140\sqrt{?} = 700 \Rightarrow \sqrt{?} = \frac{700}{140} = 5$

$\therefore ? = (5)^2 = 5 \times 5 = 25$

25. (*a*) $4(\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75})$

$= 4(\sqrt{64 \times 3} - \frac{1}{2}\sqrt{16 \times 3} - \sqrt{25 \times 3})$

$= 4(8\sqrt{3} - \frac{1}{2} \times 4\sqrt{3} - 5\sqrt{3})$

$= 4(3\sqrt{3} - 2\sqrt{3}) = 4 \times \sqrt{3}$

$= 4 \times 1.732 = 6.928$

26. (*d*) $\frac{\sqrt{1296}}{x} = \frac{x}{2.25}$

$\Rightarrow x^2 = 2.25 \times \sqrt{1296}$

$= 2.25 \times \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$

$= 2.25 \times 2 \times 2 \times 3 \times 3$

$= 2.25 \times 36$

$\therefore \quad x = \sqrt{2.25 \times 36}$

$= 1.5 \times 6 = 9$

27. (*d*) $\sqrt{1 + \frac{x}{144}} = \frac{13}{12}$

On squaring both sides, we get

$$\left(1 + \frac{x}{144}\right) = \left(\frac{13}{12}\right)^2 = \frac{169}{144}$$

$\Rightarrow \quad \frac{x}{144} = \frac{169}{144} - 1$

$\Rightarrow \quad \frac{x}{144} = \frac{169 - 144}{144} = \frac{25}{144}$

$\therefore \quad x = 25$

28. (*a*) $x^2 + y^2 = (2 + \sqrt{2})^2 + (2 - \sqrt{2})^2$

$= (4 + 2 + 4\sqrt{2}) + (4 + 2 - 4\sqrt{2}) = 12$

29. (*c*) $(?)^2 = (\sqrt{65025})^2$

$\therefore \quad ? = \sqrt{65025} = \sqrt{3 \times 3 \times 5 \times 5 \times 17 \times 17}$

$= 3 \times 5 \times 17 = 255$

30. (*a*) $\sqrt{?} = \sqrt{19881} - 43$

$= 141 - 43 = 98$

$\Rightarrow \quad ? = (98)^2 = 9604$

31. (*a*) $\sqrt{8 \times 7 - ? + \frac{208}{16}} = 8$

$\Rightarrow \quad 56 - ? + 13 = 64$

$\Rightarrow \quad 69 - ? = 64$

$\therefore \quad ? = 69 - 64 = 5$

32. (*d*) $\frac{0.14}{x^2} = 14 \Rightarrow x^2 = \frac{0.14}{14} = \frac{1}{100}$

$\therefore \quad x = \sqrt{\frac{1}{100}} = \frac{1}{10} = 0.1$

33. (*b*) $\sqrt{18 \times 14 \times x} = 168$

On squaring both side, we get

$$18 \times 14 \times x = 168 \times 168$$

$\therefore \quad x = \frac{168 \times 168}{18 \times 14} = 28 \times 4 = 112$

34. (*b*) $22^2 + \sqrt{?} = 516$

$\Rightarrow \quad \sqrt{?} = 516 - 484 = 32$

$\therefore \quad ? = (32)^2 = 32 \times 32 = 1024$

35. (*d*) $\sqrt{?} = \sqrt{1681} - 28$

$= 41 - 28 = 13$

$\therefore \quad ? = 13^2 = 13 \times 13 = 169$

36. (*a*)

	.948
9	0.90 00 00
	81
184	900
	736
1888	16400
	15104

$\therefore \qquad \sqrt{0.9} = 0.948$

37. (*b*) The number of rows is equal to the square root of total number of trees.

	124
1	1 53 76
	1
22	53
	44
244	976
	976
	×

$\therefore$ Required number of rows = 124

38. (*c*) Given number = 294

2	294
3	147
7	49
7	7
	1

$$394 = 2 \times 3 \times 7 \times 7$$
$$= 2^1 \times 3^1 \times 7^2$$

Hence, to make '294' a perfect square. We have to multiply with $2 \times 3 = 6$.

39. (*a*) Given number = 6860

First, we find the nearest cube values of given number.
Which are

$$(18)^3 = 5832$$
$$(19)^3 = 6859$$
$$\because \qquad (18)^3 < 6860 > (19)^3$$

Hence, required number $= 6860 - 6859$
$= 1$

40. (*a*) Given number = 8200

First, we find the nearest square values of given number.
Which are

$$(90)^2 = 8100 \text{ and } (91)^2 = 8281$$
$$\because \qquad (90)^2 < 8200 > (91)^2$$

Hence, required number $= 8281 - 8200 = 81$

41. (*b*) $28^2 = 784 > 777$

$\therefore$ Required difference $= 784 - 777 = 7$

42. (*c*) $x = \sqrt{30 + \sqrt{30 + \sqrt{30 + \ldots\ldots}}}$

$$x = \sqrt{30 + x}$$

On squaring both sides, we get

$$x^2 - x - 30 = 0$$

$\Rightarrow$ $x^2 - 6x + 5x - 30 = 0$

$\Rightarrow$ $(x - 6)(x + 5) = 0$

$\therefore$ $x = 6$

43. (*c*) Since, $\sqrt{x\sqrt{x\sqrt{x \ldots\ldots \infty}}} = x$

$\therefore$ $\sqrt{9\sqrt{9\sqrt{9 \ldots\ldots \infty}}} = 9$

44. (*c*) $x = \sqrt{8 + \sqrt{8 + \sqrt{8 + \ldots\ldots}}}$

$$x^2 = 8 + \sqrt{8 + \sqrt{8 + \sqrt{8 + \ldots\ldots}}}$$

$\therefore$ $x^2 = 8 + x$

Similarly, $y^2 = 8 - y$

$\therefore$ $x^2 - y^2 = (x + 8) - (8 - y)$

$$(x + y)(x - y) = (x + y)$$

$\Rightarrow$ $x - y = 1$

45. (*a*) Let $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}$

$\therefore$ $x = \sqrt{6 + x} \Rightarrow x^2 = (\sqrt{6 + x})^2$

$\Rightarrow$ $x^2 = 6 + x$

$\Rightarrow$ $x^2 - x - 6 = 0$

$\Rightarrow$ $(x - 3)(x + 2) = 0$

$\therefore$ $x = 3$

46. (*b*) $\sqrt{6\sqrt{6\sqrt{6\sqrt{6\sqrt{6}}}}} = 6^{\frac{2^5 - 1}{2^5}} = 6^{\left(\frac{32-1}{32}\right)}$

$\Rightarrow$ $6^{\frac{32-1}{32}} = 6^{\frac{31}{32}}$

47. (*a*) $\sqrt{\dfrac{8}{3}} = \sqrt{\dfrac{8 \times 3}{3 \times 3}} = \sqrt{\dfrac{24}{3 \times 3}}$

$$= \frac{\sqrt{24}}{3} = \frac{4.899}{3} \qquad [\because \sqrt{24} = 4.899]$$

$\therefore$ $\sqrt{\dfrac{8}{3}} = 1.633$

48. (*b*)

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$$
$$= 2^3 \times 7^3$$

$\therefore \quad \sqrt[3]{2744} = 2 \times 7 = 14$

$\therefore \quad \sqrt[3]{2744} \div 7 = 14 \div 7 = 2$

49. (*c*) Total cost amount = 29791

According to the question,

Money contributed by each student

= Cube of the total number of students

Hence, total number of students

$$= \sqrt[3]{29791} = \sqrt[3]{31 \times 31 \times 31} = 31$$

50. (*c*) (Smallest common multiple of 12 and 16) × (Smallest common multiple of 10 and 15) *i.e.*, (LCM of 12 and 16) × (LCM of 10 and 15) = 48 × 30 = 1440

51. (*b*)

	1.732
1	3.00 00 00
	1
27	200
	189
343	1100
	1029
3462	7100
	6924
	176

$\because \quad \sqrt{3} = 1.732$

$\therefore \quad 2 \times \sqrt{3} = 2 \times 1.732 = 3.464$

52. (*a*) $\dfrac{\sqrt{2}}{(2+\sqrt{2})} = \dfrac{\sqrt{2}}{(2+\sqrt{2})} \times \dfrac{(2-\sqrt{2})}{(2-\sqrt{2})}$

$$= \frac{2(\sqrt{2}-1)}{4-2} = \frac{2(\sqrt{2}-1)}{2} = (\sqrt{2}-1)$$

$$= 1.4142 - 1 = 0.4142$$

$\therefore \quad 2 \times \dfrac{\sqrt{2}}{(2+\sqrt{2})} = 2 \times 0.4142 = 0.8284$

53. (*c*) We know that, $(a+b)^2 - (a-b)^2 = 4ab$

$\therefore \quad 4\sqrt{ab} = (\sqrt{a} + \sqrt{b})^2 - (\sqrt{a} - \sqrt{b})^2$

$$= 17^2 - 1^2 = 289 - 1 = 288$$

$\therefore \quad \sqrt{ab} = \dfrac{288}{4} = 72$

54. (*b*) Let the number be x.

According to the question,

$$x^2 - (46)^2 = 485$$

$\Rightarrow \quad x^2 - 2116 = 485$

$\Rightarrow \quad x^2 = 2116 + 485 = 2601$

$\therefore \quad x = \sqrt{2601} = 51$

Exercise © *Higher Skill Level Questions*

1. (*c*) Given expression

$$= \frac{1}{4} \times \sqrt{\frac{(12.1 + 8.1)(12.1 - 8.1)}{(0.25)(0.25 + 19.95)}}$$

$$= \frac{1}{4} \times \sqrt{\frac{20.2 \times 4}{0.25 \times 20.2}}$$

$$= \frac{1}{4} \times \sqrt{\frac{4}{0.25}} = \frac{1}{4} \times \sqrt{\frac{400}{25}}$$

$$= \frac{1}{4} \times \sqrt{16} = \frac{1}{4} \times 4 = 1$$

2. (*d*) $\frac{1}{\sqrt{9} - \sqrt{8}} - \frac{1}{\sqrt{8} - \sqrt{7}} + \frac{1}{\sqrt{7} - \sqrt{6}}$

$$- \frac{1}{\sqrt{6} - \sqrt{5}} + \frac{1}{\sqrt{5} - \sqrt{4}}$$

$$\frac{1}{\sqrt{9} - \sqrt{8}} = \frac{1}{(\sqrt{9} - \sqrt{8})} \times \frac{(\sqrt{9} + \sqrt{8})}{(\sqrt{9} + \sqrt{8})}$$

[by rationalising]

$$= \frac{(\sqrt{9} + \sqrt{8})}{9 - 8} = \sqrt{9} + \sqrt{8}$$

Similarly, $\frac{1}{\sqrt{8} - \sqrt{7}} = \sqrt{8} + \sqrt{7}$

$$\frac{1}{\sqrt{7} - \sqrt{6}} = \sqrt{7} + \sqrt{6}$$

$$\frac{1}{\sqrt{6} - \sqrt{5}} = \sqrt{6} + \sqrt{5}$$

and $\frac{1}{\sqrt{5} - \sqrt{4}} = \sqrt{5} + \sqrt{4}$

$\therefore \quad (\sqrt{9} + \sqrt{8}) - (\sqrt{8} + \sqrt{7}) + (\sqrt{7} + \sqrt{6})$

$$- (\sqrt{6} + \sqrt{5}) + (\sqrt{5} + \sqrt{4})$$

$$= \sqrt{9} + \sqrt{8} - \sqrt{8} - \sqrt{7} + \sqrt{7} + \sqrt{6}$$

$$- \sqrt{6} - \sqrt{5} + \sqrt{5} + \sqrt{4}$$

$$= \sqrt{9} + \sqrt{4} = 3 + 2 = 5$$

3. (*c*) Given, $3a = 4b = 6c$

$\Rightarrow \quad a = 2c, b = \frac{3}{2} c$

Also, $\quad a + b + c = 27\sqrt{29}$

$\Rightarrow \quad 2c + \frac{3}{2} c + c = 27\sqrt{29}$

$\Rightarrow \quad \frac{9}{2} c = 27\sqrt{29}$

$\therefore \quad c = 6\sqrt{29}$

Now, $\sqrt{a^2 + b^2 + c^2} = \sqrt{4c^2 + \frac{9}{4} c^2 + c^2}$

$$= \sqrt{\frac{29}{4} c^2} = \frac{\sqrt{29}}{2} \cdot c = \frac{\sqrt{29}}{2} \cdot 6\sqrt{29} = 87$$

4. (*d*) $a + \frac{1}{a} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \frac{1}{\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}}$

$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{(\sqrt{3}-\sqrt{2})^2 + (\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$= \frac{\left[\begin{array}{r}(\sqrt{3})^2 + (\sqrt{2})^2 - 2\sqrt{3}\times\sqrt{2} + (\sqrt{3})^2 \\ + (\sqrt{2})^2 + 2\sqrt{3}\times\sqrt{2}\end{array}\right]}{(\sqrt{3})^2 - (\sqrt{2})^2}$

$= \frac{3+2+3+2}{3-2} = 10$

$$\left[\begin{array}{l}\text{as, } (a+b)^2 = a^2 + b^2 + 2ab \\ \quad (a-b)^2 = a^2 + b^2 - 2ab \\ \quad a^2 - b^2 = (a+b)(a-b)\end{array}\right]$$

5. (*b*) $x + 809436 \times 809438$

$= \text{A square number}$

$\Rightarrow x + (809437 - 1)(809437 + 1)$

$= \text{A square number}$

$\Rightarrow x + (809437)^2 - 1 = \text{A square number}$

It is possible, when $x = 1$

6. (*a*) Let the numbers be x and y.

Now, according to the question,

$x(x+y) = 2418$ ...(i)

$y(x+y) = 3666$ ...(ii)

On adding Eqs. (i) and (ii), we get

$x^2 + xy + yx + y^2 = 6084$

$\Rightarrow x^2 + 2xy + y^2 = 6084$

$\Rightarrow (x+y)^2 = 6084$

$\therefore x + y = \sqrt{6084} = 78$

On subtracting Eq. (ii) from Eq. (i), we get

$x^2 + xy - yx - y^2 = -1248$

$\Rightarrow x^2 - y^2 = -1248$

$\Rightarrow (x+y)(x-y) = -1248$

$\Rightarrow 78(x-y) = -1248$

$\Rightarrow x - y = -\frac{1248}{78} = -16$

$\therefore (y - x) = 16$

7. (6) Let the number of rows and columns be *m*.

Then, total plants should *be my. m.* Now, 1000 is not a square of any number. Let *m* = 30

Then, *m x m* = 30 x 30 = 900

Which is less than total plants.

Now, let $m = 32$

Then, $m \times m = 32 \times 32 = 1024$

Which is greater than 1000.

So, plant more than 100 plants need

= 1024 - 1000

= 24 plants

8. (*b*) Given expression

$$= \sqrt{\frac{(0.03)^2 + (0.21)^2 + (0.065)^2}{\left(\frac{0.03}{10}\right)^2 + \left(\frac{0.21}{10}\right)^2 + \left(\frac{0.065}{10}\right)^2}} + 2$$

$$= \sqrt{\frac{100\ [(0.03)^2 + (0.21)^2 + (0.065)^2]}{(0.03)^2 + (0.21)^2 + (0.065)^2}} + 2$$

$$= \sqrt{100} + 2 = 10 + 2 = 5$$

9. (*b*) $\sqrt[3]{1331} \times \sqrt[3]{216} + \sqrt[3]{729} + \sqrt[3]{64}$

$$= \sqrt[3]{11 \times 11 \times 11} \times \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} + \sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3} + \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2}$$

$$= 11 \times 6 + 9 + 4 = 66 + 9 + 4 = 79$$

10. (c) According to the question, total number of toys is a perfect square number because the toys were packed in n boxes containing n toys each, without any remainder. Hence, option (c) is the correct answer.

11. (6)

2	7936
2	3968
2	1984
2	992
2	496
2	248
2	124
2	62
	31

/. 7936 = 2x2x2x2x2x2x2x2x31

To make it a perfect square, it must be multiplied by 31.

∴ Required number = 7936 x 31 = 246016

12. (*a*) $\sqrt{48} - \sqrt{45} = \sqrt{3}\,(\sqrt{16} - \sqrt{15})$

Now, by multiplying the numerator and denominator by 2.

$$= \frac{\sqrt{3}\,(\sqrt{64} - 2\sqrt{15})}{2}$$

$$= \frac{\sqrt{3}}{2}\,(8 - 2 \times \sqrt{5} \times \sqrt{3})$$

$$= \frac{\sqrt{3}}{2}(5 + 3 - 2 \times \sqrt{5} \times \sqrt{3})$$

$$= \frac{\sqrt{3}}{2}\,((\sqrt{5})^2 + (\sqrt{3})^2 - 2 \times \sqrt{5} \times \sqrt{3})$$

$$= \frac{\sqrt{3}}{2}\,(\sqrt{5} - \sqrt{3})^2$$

$\therefore$ Required square root $= \dfrac{\sqrt[4]{3}}{\sqrt{2}}\,(\sqrt{5} - \sqrt{3})$

13. (*a*) Let $x = \sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4 \ldots}}}}$

On squaring both sides, we get

$$x^2 = 2\sqrt[3]{4x}$$

and now cubing both sides, we get

$$(x^2)^3 = 8 \times 4x$$

$\Rightarrow \quad x^5 = 32$

$\Rightarrow \quad x^5 = 2^5$

$\therefore \quad x = 2$

14. (*b*) $x\,(x + a)\,(x + 2a)\,(x + 3a)$

$= (x^2 + ax)\,(x^2 + 5ax + 6a^2)$

$= x^4 + ax^3 + 5ax^3 + 5a^2x^2 + 6a^2x^2 + 6a^3x$

$= x^4 + ax\,(x^2 + 5x^2 + 5ax + 6ax + 6a^2)$

$= x^4 + ax\,(6x^2 + 11ax + 6a^2) \quad \ldots\text{(i)}$

For terms to be perfect square,

$$(x + y)^2\,(x + y)^2$$

$= (x^2 + 2xy + y^2)\,(x^2 + y^2 + 2xy)$

$= x^4 + 2x^3y + x^2y^2 + x^2y^2 + 2xy^3 + y^4 + 2x^3y + 4x^2y^2 + 2xy^3$

$= x^4 + xy\,(4x^2 + 6xy + 4y^2) + y^4 \quad \ldots\text{(ii)}$

On comparing Eqs. (i) and (ii), as $y = a$

So, a^4 must be added to make it a perfect square.

Chapter

6

Indices and Surds

Indices

When a number *'P'* is multiplied by itself *'n'* times, then the product is called nth power of *'P'* and is written as P^n. Here, P is called the **base** and *'n'* is known as the **index** of the power.

Therefore, P^n is the exponential expression. P^n is read as *'P* raised to the power *ri* or *'P* to the power *ri*.

Rules of Indices

1. $P^m \times P^n = P^{m+n}$

2. $\dfrac{P^m}{P^n} = P^{m-n}$

3. $(P^m)^n = P^{mn}$

4. $(PQ)^n = P^n \times Q^n$

5. $\left(\dfrac{P}{Q}\right)^n = \dfrac{P^n}{Q^n}$

6. $P^0 = 1$

7. $P^{-n} = \dfrac{1}{P^n}$

Ex. 1 Simplify $(256)^{3/4}$.

Sol. Given expression

$$= (256)^{3/4} = (4^4)^{3/4} = (4)^{4 \times 3/4} = 4^3 = 64$$

Ex. 2 Simplify $(1024)^{-3/5}$.

Sol. Given expression $= (1024)^{-3/5} = (4^5)^{-3/5}$

$$= 4^{\left[5 \times \frac{(-3)}{5}\right]} = 4^{-3} = \frac{1}{4^3} = \frac{1}{64}$$

Ex. 3 Simplify $(a^4 b^{5/3})^{-3/4}$.

Sol. Given expression $= (a^4 b^{5/3})^{-3/4} = (a^4)^{-3/4} (b^{5/3})^{-3/4}$

$$= (a)^{4 \times \frac{-3}{4}} (b)^{\frac{5}{3} \times \frac{-3}{4}} = a^{-3} b^{-5/4}$$

Surds

When root of a non-negative rational number (i. e., quantities of type *tfa, a* being a rational number) does not provide an exact solution, then this root is called a surd. *For example -Jl, 4$,* 3-√8, *a* + *4b* etc,

Note 1. AH surds are irrational numbers

2. All irrational numbers are not surds

Order of Surds

Let *P* be a rational number and *m* be a positive integer such that $p^{l\,\prime m}$ = *4p* is

irrational.

Then, *4p* is called a surd of ruth order and *P* is called the **radicand.**

♦ Similarly, $7^{1/2}$ = *4l* = Surd of 2nd order

$6^{1/5}$ = ^6 = Surd of 5th order. ♦■ Surds of order 2 are known as quadratic surds

♦ Surds of order 3 are known as cubic surds

Rules of Surds

1. $\sqrt[m]{P} = P^{1/m}$

2. $\sqrt[m]{PQ} = \sqrt[m]{P} \times \sqrt[m]{Q}$

3. $\sqrt[m]{\frac{P}{Q}} = \frac{\sqrt[m]{P}}{\sqrt[m]{Q}}$

4. $(\sqrt[m]{P})^m = P$

5. $(\sqrt[m]{P})^n = (P^{1/m})^n = P^{n/m} = \sqrt[m]{P^n}$

Types of Surds

Pure Surds

Those surds which do not have factor other than 1, are known as pure surds.

For example $\sqrt{2}, \sqrt[3]{5}$

Mixed Surds

Those surds which have factor other than 1. are known as mixed surds.

For example $3\sqrt{5}, 4\sqrt{6}, 7^2\sqrt[3]{5}.$

Like and Unlike Surds

When the radicands of two surds are same, then those are known as like surds.

For example $4\sqrt{5}$ and $(6^3)\sqrt{5}$

When radicands are different, then they are called unlike surds

For example $3\sqrt{5}$ and $\sqrt{3}$

Properties of Surds

1. A quadratic surd cannot be equal to the sum and difference of a rational number and a auad-ratic surd

For example $a + b \neq \sqrt{c}$ or $\sqrt{a} - b \neq \sqrt{c}$

2. If $a + \sqrt{b} = c + \sqrt{d}$ or $a - \sqrt{b} = c - \sqrt{d}$, then $a = c$ and $b = d$.

3. If $a + \sqrt{b} = c + \sqrt{d}$, then $a - \sqrt{b} = c - \sqrt{d}$ and vice-versa.

4. If $\sqrt{a + \sqrt{b}} = \sqrt{c} + \sqrt{d}$, then $\sqrt{a - \sqrt{b}} = \sqrt{c} - \sqrt{d}$ and its vice-versa.

MIND IT!

If x is a positive real number and *a, b* and c are real numbers, then

(i) $\left(\dfrac{x^b}{x^c}\right)^a \cdot \left(\dfrac{x^c}{x^a}\right)^b \cdot \left(\dfrac{x^a}{x^b}\right)^c = 1$

(ii) $\left(\dfrac{x^a}{x^b}\right)^{a+b} \cdot \left(\dfrac{x^b}{x^c}\right)^{b+c} \cdot \left(\dfrac{x^c}{x^a}\right)^{c+a} = 1$

(iii) $\left(\dfrac{x^a}{x^b}\right)^{(a^2+b^2+ab)} \cdot \left(\dfrac{x^b}{x^c}\right)^{(b^2+c^2+bc)} \cdot \left(\dfrac{x^c}{x^a}\right)^{(c^2+a^2+ca)} = 1$

To Arrange the Surds in Increasing or Decreasing Order

Suppose given surds are *p ,q* , r

First of all, take the LCM of *a, b* and c and use it to make the denominator of the powers the same.

Then, easily we can find the required order.

Ex. 4 Arrange $\sqrt[4]{3}$, $\sqrt[3]{2}$ and $\sqrt[6]{5}$ in the decreasing order.

Sol. Terms $\sqrt[4]{3}$, $\sqrt[3]{2}$ and $\sqrt[6]{5}$ are written as $(3)^{\frac{1}{4}}$, $(2)^{\frac{1}{3}}$ and $(5)^{\frac{1}{6}}$

So, $a = 4$, $b = 3$ and $c = 6$

Now, LCM of 4, 3 and 6 = 12

Then, we will make the denominator of the power of every coefficient equal *i.e.*, 12.

$$\sqrt[4]{3} = (3)^{\frac{1}{4}} = 3^{\frac{3}{12}} = \sqrt[12]{27}$$

$$\sqrt[3]{2} = (2)^{\frac{1}{3}} = 2^{\frac{4}{12}} = \sqrt[12]{16}$$

and $$\sqrt[6]{5} = (5)^{\frac{1}{6}} = 5^{\frac{2}{12}} = \sqrt[12]{25}$$

Thus, $$\sqrt[12]{27} > \sqrt[12]{25} > \sqrt[12]{16}$$

Hence, $$\sqrt[4]{3} > \sqrt[6]{5} > \sqrt[3]{2}$$

Operations on Surds

Addition and Subtraction of Surds

Only like surds can be added or subtracted. Therefore, to add or subtract two or more surds, first simplify them and add or subtract them respectively like surds.

Note 1. $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$ 2. $\sqrt{a} - \sqrt{b} \neq \sqrt{a-b}$

3. $x\sqrt{a} + x\sqrt{b} \neq x\sqrt{a+b}$

Ex. 5 Find the value of $\sqrt{80} + 3\sqrt{245} - \sqrt{125}$.

Sol. $\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

$$3\sqrt{245} = 3\sqrt{49 \times 5} = 21\sqrt{5}$$

$$\sqrt{125} = \sqrt{25 \times 5} = 5\sqrt{5}$$

$\therefore$ $$\sqrt{80} + 3\sqrt{245} - \sqrt{125} = 4\sqrt{5} + 21\sqrt{5} - 5\sqrt{5} = 20\sqrt{5}$$

Multiplication and Division of Surds

To multiply or divide the surds, we make the denominators of the powers equal to each other. Then, multiply or divide as usual.

Ex. 6 Find the product of $\sqrt{5}, \sqrt[6]{6}$ and $\sqrt[3]{4}$.

Sol. LCM of 2, 6 and 3 = 6

$\therefore$

$$\sqrt{5} = 5^{1/2} = 5^{3/6} = (125)^{1/6}$$
$$\sqrt[6]{6} = 6^{1/6} = (6)^{1/6}$$
$$\sqrt[3]{4} = 4^{1/3} = 4^{2/6} = (16)^{1/6}$$

$\therefore$ Required product $= (125 \times 6 \times 16)^{1/6} = (12000)^{1/6}$

Ex. 7 Divide $12 \times 4^{1/3}$ by $3\sqrt{2}$.

Sol. $$\frac{12 \times 4^{1/3}}{3 \times 2^{1/2}} = \frac{4 \times 4^{2/6}}{1 \times 2^{3/6}} = \frac{4 \times (16)^{1/6}}{(8)^{1/6}} = 4\left(\frac{16}{8}\right)^{1/6} = 4(2)^{1/6} = 4\sqrt[6]{2}$$

Comparison of Surds

To compare two or more surds, first of all the denominators of the power of given surds are made equal to each other and then the radicand of the new surds are compared.

Ex. 8 Which of the following surds is greatest?

$$\sqrt[3]{2}, \sqrt{3}, \sqrt[4]{5}, \sqrt[6]{7}$$

Sol. $\sqrt[3]{2} = (2)^{1/3}$, $\sqrt{3} = (3)^{1/2}$, $\sqrt[4]{5} = (5)^{1/4}$ and $\sqrt[6]{7} = (7)^{1/6}$

LCM of 3, 2, 4 and 6 is 12.

(2)^1/^{3}=(2)^4/12=(24)l/12=(16)l/12 (3)V2 = (3)6/12 = (36)1/12 = (729)1/12 (5)^1/* = (5)^{3}A2 = (53)1/12 = (125)1/12 (7)l/6 = (7)2/12 = (72)1/12 = (49)1/12

$\therefore$ $\sqrt{3}$ is the greatest of all the given surds.

Rationalisation of Surds

The method of obtaining a rational number from a surd by multiplying it with another surd is known as rationalisation of surds. Both the surds are known as rationalising factor of each other.

For example (3)' is the rationalising factor of (3) ' and *vice-versa.*

Ex. 9 Find the fraction equivalent to $\frac{5}{2-\sqrt{3}}$, such that denominator of the fraction is not irrational.

Sol. $\frac{5}{2-\sqrt{3}} = \frac{5}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$$= \frac{5(2+\sqrt{3})}{(2)^2-(\sqrt{3})^2} = 5(2+\sqrt{3})$$

Ex. 10 Simplify $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$.

Sol. $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{5+3+2\sqrt{15}}{5-3} = \frac{8+2\sqrt{15}}{2} = 4+\sqrt{15}$$

Fast Track **Techniques**

to *solve the* QUESTIONS

Technique 1 To find the value of $\sqrt{x+\sqrt{x+\sqrt{x+\dots}}}$, factorise x. If $x = m(m+1)$, then $(m+1)$ is your answer.

Ex. 11 Find the value of $\sqrt{6+\sqrt{6+\sqrt{6+\dots}}}$.

Sol. We know that,

$$6 = 2 \times 3 = 2(2+1)$$

$$\therefore \quad \sqrt{6+\sqrt{6+\sqrt{6+\dots}}} = 3$$

Technique 2 To find the value of $\sqrt{x-\sqrt{x-\sqrt{x-\dots}}}$, factorise x. If $x = m(m+1)$, then m is your answer.

Ex. 12 Find the value of $\sqrt{12-\sqrt{12-\sqrt{12-\dots}}}$.

Sol. We know that, $12 = 3 \times 4 = 3(3+1)$

$$\therefore \quad \sqrt{12-\sqrt{12-\sqrt{12-\dots}}} = 3$$

Technique 3 $\sqrt{x\sqrt{x\sqrt{x\dots}}} = x^{\left(\frac{2^n-1}{2^n}\right)}$ where, n is the number of times x is repeated.

Ex. 13 Find the value of $\sqrt{7\sqrt{7\sqrt{7}}}$.

Sol. $\sqrt{7\sqrt{7\sqrt{7}}} = 7^{\left(\frac{2^3-1}{2^3}\right)} = 7^{7/8}$

Multi Concept

QUESTIONS

1. *If $xyz = 1$, then find the value of* $\frac{1}{(1+x+y^{-1})} + \frac{1}{(1+y+z^{-1})} + \frac{1}{(1+z+x^{-1})}$.

(a) 1 (b) 2 (c) 3 (d) 4

(a) Given that, $xyz = 1$

Then,
$$\frac{1}{(1+x+y^{-1})} + \frac{1}{(1+y+z^{-1})} + \frac{1}{(1+z+x^{-1})}$$
$$= \frac{1}{\left(1+x+\frac{1}{y}\right)} + \frac{1}{\left(1+y+\frac{1}{z}\right)} + \frac{1}{\left(1+z+\frac{1}{x}\right)} \qquad \left[\because a^{-1} = \frac{1}{a}\right]$$
$$= \frac{y}{y+xy+1} + \frac{z}{z+yz+1} + \frac{x}{x+xz+1}$$
$$= \frac{y}{y+xy+1} + \frac{z}{z+\frac{1}{x}+1} + \frac{x}{x+\frac{1}{y}+1} \qquad \left[\because yz = \frac{1}{x} \text{ and } xz = \frac{1}{y}\right]$$
$$= \frac{y}{1+xy+y} + \frac{xz}{xz+1+x} + \frac{xy}{xy+1+y}$$
$$= \frac{y}{1+xy+y} + \frac{\frac{1}{y}}{\frac{1}{y}+1+x} + \frac{xy}{1+xy+y}$$
$$= \frac{y}{1+xy+y} + \frac{1}{1+xy+y} + \frac{xy}{1+xy+y}$$
$$= \frac{1+xy+y}{1+xy+y} = 1$$

2. *If $a^x = b^y = c^z$ and $abc = 1$, then what is the value of $xy + yz + zx$?*

(a) 1 (b) 3 (c) 0 (d) 5

(c) Here, $a^x = b^y = c^z$

Let $a^x = b^y = c^z = K$

$\therefore$ $a = K^{\frac{1}{x}}, b = K^{\frac{1}{y}}$

and $c = K^{\frac{1}{z}}$

$\Rightarrow$ $abc = K^{\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)}$

$\because$ $abc = 1$ [given]

$\therefore$ $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$ or $xy + yz + zx = 0$

3. *If* $a + b + c = 0$, *then what is the value of*

$$\frac{1}{(x^a+x^{-b}+1)}+\frac{1}{(x^b+x^{-c}+1)}+\frac{1}{(x^c+x^{-a}+1)}=?$$

(a) 1 (b) 2 (c) 3 (d) 4

➥ (a) $\frac{1}{(x^a+x^{-b}+1)}+\frac{1}{(x^b+x^{-c}+1)}+\frac{1}{(x^c+x^{-a}+1)}$

$$=\frac{1}{x^a+\frac{1}{x^b}+1}+\frac{1}{x^b+\frac{1}{x^c}+1}+\frac{1}{x^c+\frac{1}{x^a}+1}$$

$$=\frac{x^b}{x^{a+b}+1+x^b}+\frac{x^c}{x^{b+c}+1+x^c}+\frac{x^a}{x^{a+c}+1+x^a}$$

$$=\frac{x^b}{x^{a+b}+x^b+1}+\frac{x^c}{x^{-a}+1+x^c}+\frac{x^a}{x^{-b}+1+x^a}$$

$[\because a+b+c=0,$
$\therefore a+c=-b$ and $b+c=-a]$

$$=\frac{x^b}{x^{a+b}+x^b+1}+\frac{x^{a+c}}{1+x^a+x^{a+c}}+\frac{x^{a+b}}{1+x^b+x^{a+b}}$$

$$=\frac{x^b}{x^{a+b}+x^b+1}+\frac{x^{-b}\cdot x^b}{1+x^b+x^{a+b}}+\frac{x^{a+b}}{1+x^b+x^{a+b}}$$

$$=\frac{x^b+1+x^{a+b}}{x^{a+b}+x^b+1}=1 \qquad [\because x^{-b}x^b=x^0=1]$$

4. *Find the value of the expression*

$$\frac{4^n+20^{m-1}\times 12^{m-n}\times 15^{m+n-2}}{16^m\times 5^{2m+n}\times 9^{m-1}}.$$

(a) $\frac{1}{200}$ (b) $\frac{1}{500}$ (c) $\frac{1}{700}$ (d) $\frac{1}{900}$

➥ (b) $\frac{4^n\times 20^{m-1}\times 12^{m-n}\times 15^{m+n-2}}{16^m\times 5^{2m+n}\times 9^{m-1}}$

$$=\frac{(2)^{2n}\times(2^2)^{m-1}\times 5^{m-1}\times(2^2)^{m-n}\times(3)^{m-n}\times(3)^{m+n-2}\times(5)^{m+n-2}}{(2^4)^m\times 5^{2m+n}\times(3^2)^{m-1}}$$

$$=\frac{(2)^{2n+2m-2+2m-2n}\times 5^{m-1+m+n-2}\times 3^{m-n+m+n-2}}{2^{4m}\times 5^{2m+n}\times 3^{2m-2}}$$

$$=\frac{2^{4m-2}\times 5^{2m+n-3}\times 3^{2m-2}}{2^{4m}\times 5^{2m+n}\times 3^{2m-2}}$$

$$=2^{4m-2-4m}\times 5^{2m+n-3-2m-n}\times 3^{2m-2-2m+2}$$

$$=2^{-2}\times 5^{-3}\times 3^0$$

$$=\frac{1}{2^2}\times\frac{1}{5^3}=\frac{1}{4}\times\frac{1}{125}$$

Fast Track Practice

Exercise © *Base Level Questions*

1. $16^{\wedge/4}$ = ?

(a) 64 *(b)* 31 *(c)* 32 *(d)* 33 *(e)* None of the above

2. $\left(\frac{32}{243}\right)^{-3/5} = ?$

(a) $\frac{27}{8}$ (b) $\frac{27}{7}$ (c) $\frac{27}{6}$ (d) $\frac{27}{2}$

(e) None of the above 3. Find the value of $(243)^{al6}$ x $(243)^{004}$

(a) 0.16 (b) $\frac{1}{3}$ (c) 3 (d) 0.04

4. $17^{3.5} \times 17^{7.3} \div 17^{4.2} = 17^{?}$ [Bank Clerks 2010]

(a) 8.4 (b) 8 (c) 6.6 (d) 6.4

(e) None of the above

5. $\sqrt{(13)^4} = ?$ [Bank Clerks 2009]

(a) 520 (b) 169

(c) 28561 (d) 14280

(e) None of the above

6. If $289 = 17^{\frac{x}{5}}$, then $x = ?$ [Bank PO 2010]

(a) 16 (b) 8 (c) 10 (d) $\frac{2}{5}$

(e) None of the above

7. $\left[\left\{\left(-\frac{1}{2}\right)^2\right\}^{-2}\right]^{-1} = ?$

(a) $\frac{1}{16}$ (b) 16 (c) $-\frac{1}{16}$ (d) -16

8. $\sqrt[L]{M}$ is a surd of order......, where M is a rational number; L is a positive integer and $\sqrt[L]{M}$ is irrational.

(a) L (b) M (c) 2 (d) 4

(e) None of the above

9. If $3^x - 3^{x-1} = 18$, then x^x is equal to

(a) 3 (b) 8 (c) 27 (d) 216

10. $16^{\frac{3}{2}} + 16^{-\frac{3}{2}} = ?$

(a) 0 (b) $\frac{4097}{64}$ (c) 1 (d) $\frac{16}{9097}$

11. If $a^{2x+2} = 1$, where a is a positive real number other than 1, then $x = ?$

[SSC CGL 2007]

(a) -2 (b) -1 (c) 0 (d) 1

12. $\dfrac{[(12)^{-2}]^2}{[(12)^2]^{-2}} = ?$ [Bank Clerks 2009]

(a) 12 (b) 4.8 (c) $\dfrac{12}{144}$ (d) 1

(e) None of the above

13. Value of ? in expression

$7^{8.9} \div (343)^{1.7} \times (49)^{4.8} = 7^{?}$ is

(a) 13.4 (b) 12.8 (c) 11.4 (d) 9.6

(e) None of the above

14. $81^{2.5} \times 9^{4.5} \div 3^{4.8} = 9^{?}$ [Bank Clerks 2009]

(a) 7.1 (b) 9.4

(c) 4.7 (d) 4.5

(e) None of the above

15. If $\{(2^4)^{1/2}\}^{?} = 256$, find the value of '?'.

[Bank PO 2007]

(a) 1 (b) 2 (c) 4 (d) 8

(e) None of the above

16. $(16)^9 \div (16)^4 \times 16^3 = (16)^{?}$

(a) 6.75 (b) 8 (c) 10 (d) 12

(e) None of the above

17. $(42 \times 229) \div (9261)^{1/3} = ?$

(a) 448 (b) 452

(c) 456 (d) 458

(e) None of the above

18. Evaluate $(0.00032)^{2/5}$.

(a) $\dfrac{1}{625}$ (b) $\dfrac{1}{225}$ (c) $\dfrac{1}{125}$ (d) $\dfrac{1}{25}$

(e) None of the above

19. If $2^{x-1} + 2^{x+1} = 2560$, find the value of x.

(a) 10 (b) 12 (c) 9 (d) 8

(e) None of the above

20. $[(11)^3 \times (6)^2] \div (4)^3 = ?$

(a) 2994.75 (b) 748.6875

(c) 272.25 (d) 4492.125

(e) None of the above

21. $\sqrt[3]{103823} = ?$ [Bank Clerks 2009]

(a) 49 (b) 51 (c) 45 (d) 47

(e) None of the above

22. Find the value of $(10)^{200} \div (10)^{196}$.

[MBA 2008]

(a) 10000 (b) 1000
(c) 100 (d) 100000
(e) None of the above

23. If $\left(\frac{1}{5}\right)^{3a} = 0.008$, find the value of $(0.25)^a$.

(a) 20.5 (b) 22.5 (c) 0.25 (d) 6.25
(e) None of the above

24. Value of $\sqrt{5+2\sqrt{6}} - \dfrac{1}{\sqrt{5-2\sqrt{6}}}$ is

(a) $2\sqrt{2}$ (b) 0 (c) $2\sqrt{3}$ (d) $\sqrt{5}-1$

25. $\dfrac{1}{1+x^{b-a}+x^{c-a}} + \dfrac{1}{1+x^{a-b}+x^{c-b}} + \dfrac{1}{1+x^{b-c}+x^{a-c}} = ?$

(a) x^{a-b-c} (b) 1
(c) 0 (d) 3

26. $[P^{(b-c)}]^{b+c} \cdot [P^{(c-a)}]^{c+a} \cdot [P^{(a-b)}]^{(a+b)} = ?$

(a) 0 (b) p^{abc} (c) 1 (d) p^{a+b+c}

27. If $\sqrt{5+\sqrt[3]{x}} = 3$, then the value of x is

(a) 64 (b) 125 (c) 9 (d) 27

28. $\left[\left(\sqrt[5]{x^{-\frac{3}{5}}}\right)^{-\frac{5}{3}}\right]^5 = ?$

(a) x^5 (b) x^{-5} (c) x (d) $\dfrac{1}{x}$

29. $\left(\sqrt{56+\sqrt{56+\sqrt{56+\ldots}}}\right) \div 2^2 = ?$

(a) 0 (b) 1 (c) 2 (d) 8
(e) None of the above

30. If $\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = a + b\sqrt{35}$, then the value of $(a-b)$ is

(a) 5 (b) 6
(c) 8 (d) None of these

31. If $2^{x-1} + 2^{x+1} = 320$, then find the value of x.

(a) 6 (b) 8 (c) 7 (d) 5

32. If $P = 124$, then $\sqrt[3]{P(P^2+3P+3)+1} = ?$

(a) 5 (b) 7 (c) 123 (d) 125

33. If $\left(\dfrac{p}{q}\right)^{n-1} = \left(\dfrac{q}{p}\right)^{n-3}$, then the value of n is

(a) $\dfrac{1}{2}$ (b) $\dfrac{7}{2}$ (c) 1 (d) 2

34. Value of ? in $\sqrt[3]{512} + \sqrt[4]{16} + \sqrt{576} = ?$ is

(a) 24 (b) 31
(c) 22 (d) 18
(e) None of the above

35. Value of $3 + \frac{1}{\sqrt{3}} + \frac{1}{3+\sqrt{3}} + \frac{1}{\sqrt{3}-3}$ is

(a) $3+\sqrt{3}$ (b) 3
(c) 1 (d) 0

36. If $a^x = b$, $b^y = c$ and $xyz = 1$, then what is the value of c^z? **[CDS 2012]**

(a) a (b) b
(c) ab (d) a / b

37. If $a = 2 + \sqrt{3}$, then what is the value of $(a^2 + a^{-2})$? **[CDS 2012]**

(a) 12 (b) 14
(c) 16 (d) 18

38. The expression $[(\sqrt{2})^{\sqrt{2}}]^{\sqrt{2}}$ gives **[CDS 2013]**

(a) a natural number
(b) a integer and not a natural number
(c) a rational number but not an integer
(d) a real number but not a rational number

39. If $16 \times 8^{n+2} = 2^m$, then m is equal to **[CDS 2013]**

(a) $n + 8$ (b) $2n + 10$
(c) $3n + 2$ (d) $3n + 10$

40. If $\sqrt{10 + \sqrt[3]{x}} = 4$, then what is the value of x? **[CDS 2012]**

(a) 150 (b) 216
(c) 316 (d) 450

41. If m and n are natural numbers, then $\sqrt[m]{n}$ is **[CDS 2013]**

(a) always irrational
(b) irrational unless n is the mth power of an integer
(c) irrational unless m is the nth power of an integer
(d) irrational unless m and n are coprime

42. Consider the following in respect of the numbers $\sqrt{2}$, $\sqrt[3]{3}$ and $\sqrt[6]{6}$

I. $\sqrt[6]{6}$ is the greatest number.
II. $\sqrt{2}$ is the smallest number.

Which of the above statements is/are correct? **[CDS 2014]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

43. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, then $x^2 + y^2$ is equal to

(a) 14 (b) 13 (c) 15 (d) 10

Exercise © *Higher Skill Level Questions*

1. Find the quotient when $(a^{-1} - 1)$ is divided by $(a - 1)$.

   (a) $\frac{2}{a}$ (b) $2a$ (c) $\frac{a}{2}$ (d) $-\frac{1}{a}$

2. Simplify $[\sqrt[3]{\sqrt[6]{2^9}}]^4 \times [\sqrt[6]{\sqrt[3]{2^9}}]^4$.

   (a) 2^4 (b) 2^9 (c) 2^3 (d) 2^{16}

3. Arrange $\sqrt[4]{3}$, $\sqrt[6]{10}$, $\sqrt[12]{25}$ in descending order.

   (a) $\sqrt[6]{10} > \sqrt[4]{3} > \sqrt[12]{25}$
   (b) $\sqrt[12]{25} > \sqrt[4]{3} > \sqrt[6]{10}$
   (c) $\sqrt[6]{10} > \sqrt[12]{25} > \sqrt[4]{3}$
   (d) $\sqrt[4]{3} > \sqrt[12]{25} > \sqrt[6]{10}$

4. If $a = \frac{\sqrt{3}}{2}$, then $\sqrt{1 + a} + \sqrt{1 - a} = ?$ **[SSC CGL 2007]**

   (a) $(2 - \sqrt{3})$ (b) $(2 + \sqrt{3})$
   (c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{3}$

5. Simplify $\frac{6a^{-2}bc^{-3}}{4ab^{-3}c^2} \div \frac{5a^{-3}b^2c^{-1}}{3ab^{-2}c^3}$.

   (a) $\frac{9}{10}ac$ (b) $\frac{9}{10}ac^{-1}$
   (c) $\frac{9}{10}ac^2$ (d) $\frac{9}{10}ac^{-3}$
   (e) None of the above

6. If $m = 7 - 4\sqrt{3}$, then $\left(\sqrt{m} + \frac{1}{\sqrt{m}}\right) = ?$

   (a) 8 (b) 3 (c) 4 (d) 9
   (e) None of the above

7. Simplify $6\sqrt{(27)^{-\frac{2}{3}} + (8)^{-\frac{2}{3}}}$ **[SSC (10+2) 2010]**

   (a) $\sqrt[6]{35}$ (b) $\frac{6}{\sqrt{13}}$ (c) $\sqrt{13}$ (d) $\sqrt[6]{6}$

8. If $2x^{1/3} + 2x^{-1/3} = 5$, then $x^{1/3}$ is equal to **[Bank Clerks 2008]**

   (a) 1 or -1 (b) 2 or $\frac{1}{2}$
   (c) 8 or $\frac{1}{8}$ (d) 3 or $\frac{1}{3}$
   (e) None of the above

9. $\left(\frac{1}{64}\right)^0 + (64)^{-\frac{1}{2}} + (32)^{\frac{4}{5}} - (32)^{-\frac{4}{5}} = ?$

   (a) $17\frac{1}{15}$ (b) $15\frac{1}{17}$ (c) $16\frac{1}{17}$ (d) $17\frac{1}{16}$

10. Which one is greatest out of $\sqrt{2}, \sqrt[6]{3}, \sqrt[3]{4}$ and $\sqrt[4]{5}$?

(a) $\sqrt[3]{4}$ (b) $\sqrt[4]{5}$ (c) $\sqrt{2}$ (d) $\sqrt[6]{3}$

(e) None of the above

11. If $2^a = 3^b = 6^c$, then $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = ?$

(a) $\frac{7}{32}$ (b) 0 (c) $\frac{7}{16}$ (d) $\frac{7}{48}$

12. Value of $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{100}+\sqrt{99}}$ is

(a) 1 (b) 9

(c) $\sqrt{99}$ (d) $\sqrt{99-1}$

13. If $\frac{2^{n+4} - 2 \cdot 2^n}{2 \cdot 2^{n+3}} + 2^{-3} = x$, then the value of x is

(a) $-2^{n+1} + \frac{1}{8}$ (b) 1

(c) 2^{n+1} (d) $\frac{n}{8} - 2^n$

14. Find the value of $m - n$, if

$$\frac{9^n \times 3^2 \times \left(3^{\frac{-n}{2}}\right)^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$$

(a) 1 (b) -2

(c) -1 (d) 2

15. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$, then the value of a and b is

(a) $a = 11, b = -6$ (b) $a = 6, b = 11$

(c) $a = -11, b = 6$ (d) $a = -11, b = -6$

16. Find the value of $\left(\frac{a^p}{a^q}\right)^{p+q-r} \times \left(\frac{a^r}{a^p}\right)^{r+p-q} \times \left(\frac{a^q}{a^r}\right)^{q+r-p}$

(a) a^{pqr} (b) a^{p+q+r}

(c) $a^{pq+qr+pr}$ (d) 1

17. If $2^P + 3^9 = 17$ and $2^{P+2} - 3^{9+1} = 5$, then, find the value of p and q. *(a)* -2, 3 *(b)* 2,-3

(cj 3, 2 (tfj 2, 3

Answer with Solutions

Exercise© *Base Level Questions*

1. (*c*) Given expression

$$= (16)^{\frac{5}{4}} = (2^4)^{\frac{5}{4}} = 2^{\left(4 \times \frac{5}{4}\right)} = 2^5 = 32$$

2. (*a*)Given expression

$$= \left(\frac{32}{243}\right)^{-\frac{3}{5}} = \left[\left\{\left(\frac{2}{3}\right)^5\right\}\right]^{-\frac{3}{5}}$$

$$= \left(\frac{2}{3}\right)^{5 \times \frac{-3}{5}} = \left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

3. (*c*) $(243)^{0.16} \times (243)^{0.04}$

$$= (243)^{0.16+0.04} = (243)^{0.2}$$

$$= (3^5)^{0.2} = 3^{5 \times 0.2} = 3$$

4. (*c*) $17^{3.5} \times 17^{7.3} \div 17^{4.2} = 17^?$

$\Rightarrow \quad 17^{3.5+7.3-4.2} = 17^?$

$\Rightarrow \quad 17^{6.6} = 17^?$

$\therefore \quad ? = 6.6$

5. (*b*) $? = \sqrt{(13)^4} = \sqrt{13 \times 13 \times 13 \times 13}$

$$= 13 \times 13 = 169$$

6. (*c*) Given that, $289 = 17^{\frac{x}{5}}$

$\Rightarrow 17^2 = 17^{\frac{x}{5}} \Rightarrow \frac{x}{5} = 2 \quad \Rightarrow x = 10$

7. (*a*) $\left[\left\{\left(-\frac{1}{2}\right)^2\right\}^{-2}\right]^{-1}$

$$= \left[\left\{\frac{1}{4}\right\}^{-2}\right]^{-1} = \left[\frac{1}{\left(\frac{1}{4}\right)^2}\right]^{-1}$$

$$= \left[\frac{1}{\frac{1}{16}}\right]^{-1} = [16]^{-1} = \frac{1}{16}$$

8. (*a*) $\sqrt[L]{M} = M^{\frac{1}{L}}$ = Surd of Lth order.

9. (*c*) $\because \; 3^x - 3^{x-1} = 18$

$\Rightarrow \quad 3^{x-1}(3-1) = 18$

$\Rightarrow \quad 3^{x-1} = 9 = 3^2$

$\Rightarrow \quad x - 1 = 2 \text{ or } x = 3$

$\therefore \quad x^x = (3)^3 = 27$

10. (*b*) $16^{\frac{3}{2}} + 16^{-\frac{3}{2}} = 16^{\frac{3}{2}} + \frac{1}{16^{\frac{3}{2}}}$

$$= \left(16^{\frac{1}{2}}\right)^3 + \frac{1}{\left(16^{\frac{1}{2}}\right)^3}$$

$$= 64 + \frac{1}{64} = \frac{4097}{64}$$

11. (*b*) Given that, $a^{2x+2} = 1$

$\Rightarrow \quad a^{2x+2} = a^0$

$\Rightarrow \quad 2x + 2 = 0$

$\Rightarrow \quad x = \frac{-2}{2} = -1$

12. (*d*) $? = \frac{[(12)^{-2}]^2}{[(12)^2]^{-2}} = \frac{(12)^{-4}}{(12)^{-4}} = 1$

13. (*a*) $7^? = 7^{8.9} \div (343)^{1.7} \times (49)^{4.8}$

$$= \frac{7^{8.9} \times (7^2)^{4.8}}{(7^3)^{1.7}}$$

$$= \frac{7^{8.9+9.6}}{7^{5.1}} = \frac{7^{18.5}}{7^{5.1}}$$

$$= 7^{18.5-5.1} = 7^{13.4}$$

$\therefore \quad ? = 13.4$

14. (*a*) $81^{2.5} \times 9^{4.5} \div 3^{4.8} = 9^?$

$\Rightarrow \quad \frac{81^{2.5} \times 9^{4.5}}{3^{4.8}} = 9^?$

$\Rightarrow \quad \frac{(3^4)^{2.5} \times (3^2)^{4.5}}{3^{4.8}} = 9^?$

$\Rightarrow \quad \frac{3^{10} \times 3^9}{3^{4.8}} = 9^?$

$\Rightarrow \quad 3^{(10+9-4.8)} = 9^?$

$\Rightarrow \quad 9^? = 3^{14.2}$

$\Rightarrow \quad (3)^{2 \times ?} = 3^{14.2}$

$\Rightarrow \quad 2 \times ? = 14.2$

$\therefore \quad ? = \frac{14.2}{2} = 7.1$

15. (*c*) Given that, $(2^{4\times\frac{1}{2}})^{?} = 256$

$\Rightarrow \quad (2^2)^? = 2^8 \Rightarrow 2 \times ? = 8$

$\therefore \quad ? = \frac{8}{2} = 4$

16. (*b*) Given that, $(16)^9 \div (16)^4 \times (16)^3 = (16)^?$

$$\Rightarrow \quad (16)^? = \frac{(16)^9 \times (16)^3}{(16)^4}$$

$$\Rightarrow \quad (16)^? = (16)^{9+3-4}$$

$$\Rightarrow \quad ? = 12 - 4 = 8$$

17. (*d*) Given that, $(42 \times 229) \div (9261)^{\frac{1}{3}} = ?$

Now, $\quad (9261)^{\frac{1}{3}} = (21 \times 21 \times 21)^{\frac{1}{3}}$

$$= 21^{3\times\frac{1}{3}} = 21$$

$$\therefore \quad ? = \frac{42 \times 229}{(9261)^{1/3}} = \frac{42 \times 229}{21} = 458$$

18. (*d*) $(0.00032)^{\frac{2}{5}} = \left(\frac{32}{100000}\right)^{\frac{2}{5}}$

$$= \left(\frac{2^5}{10^5}\right)^{\frac{2}{5}} = \left\{\left(\frac{2}{10}\right)^5\right\}^{\frac{2}{5}}$$

$$= \left(\frac{2}{10}\right)^{5\times\frac{2}{5}} = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$$

19. (*a*) $2^{x-1} + 2^{x+1} = 2560$

$$\Rightarrow \quad 2^{x-1}(1 + 2^2) = 2560$$

$$\Rightarrow \quad 2^{x-1} = \frac{2560}{5} = 512$$

$$\Rightarrow \quad 2^{x-1} = 2^9$$

$$\Rightarrow \quad x - 1 = 9$$

$$\therefore \quad x = 9 + 1 = 10$$

20. (*b*) $? = [(11)^3 \times 6^2] \times \frac{1}{4^3} = \frac{1331 \times 36}{64}$

$$= 748.6875$$

21. (*d*) $? = \sqrt[3]{103823} = \sqrt[3]{(47)^3} = [(47)^3]^{\frac{1}{3}}$

$$= 47^{3\times\frac{1}{3}} = 47$$

22. (*a*) $(10)^{200} \div (10)^{196} = (10)^{200-196}$

$$= 10^4 = 10000$$

23. (*c*) $\left(\frac{1}{5}\right)^{3a} = 0.008 = \frac{8}{1000} = \frac{1}{125} = \left(\frac{1}{5}\right)^3$

$$\Rightarrow \quad 3a = 3$$

$$\therefore \quad a = 1$$

$$\therefore \quad (0.25)^a = (0.25)^1 = 0.25$$

24. (*b*) $\sqrt{5+2\sqrt{6}} - \dfrac{1}{\sqrt{5-2\sqrt{6}}}$

$$= \frac{\sqrt{(5)^2-(2\sqrt{6})^2}-1}{\sqrt{5-2\sqrt{6}}}$$

$$= \frac{\sqrt{25-24}-1}{\sqrt{5-2\sqrt{6}}} = \frac{1-1}{\sqrt{5-2\sqrt{6}}} = 0$$

25. (*b*) $? = \dfrac{1}{1+x^{b-a}+x^{c-a}}$

$$+ \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$$

$$= \frac{1}{1+\dfrac{x^b}{x^a}+\dfrac{x^c}{x^a}} + \frac{1}{1+\dfrac{x^a}{x^b}+\dfrac{x^c}{x^b}} + \frac{1}{1+\dfrac{x^b}{x^c}+\dfrac{x^a}{x^c}}$$

$$= \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^a+x^c} + \frac{x^c}{x^c+x^b+x^a}$$

$$= \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1$$

26. (*c*) $? = [P^{(b-c)}]^{b+c} \cdot [P^{(c-a)}]^{c+a} \cdot [P^{(a-b)}]^{(a+b)}$

$$= P^{b^2-c^2} \cdot P^{c^2-a^2} \cdot P^{a^2-b^2}$$

$$= P^{b^2-c^2+c^2-a^2+a^2-b^2} = P^0 = 1$$

27. (*a*) $\sqrt{5+\sqrt[3]{x}} = 3$

$\Rightarrow \quad 5+\sqrt[3]{x} = 9$

$\Rightarrow \quad \sqrt[3]{x} = 4$

$\Rightarrow \quad x = (4)^3 = 64$

28. (*c*) $\left[\left(\sqrt[5]{x^{\frac{-3}{5}}}\right)^{-\frac{5}{3}}\right]^5 = \left[\left(x^{-\frac{3}{5}\times\frac{1}{5}}\right)^{-\frac{5}{3}}\right]^5$

$$= x^{-\frac{3}{5}\times\frac{1}{5}\times\frac{-5}{3}\times 5} = x$$

29. (*c*) Here, $56 = 7 \times 8$

[by Technique 1]

$\Rightarrow \quad \sqrt{56+\sqrt{56+\sqrt{56+\ldots}}} = 8$

$\therefore \quad ? = \dfrac{\sqrt{56+\sqrt{56+\sqrt{56+\ldots}}}}{(2)^2}$

$$= \frac{8}{4} = 2$$

30. (*d*) $\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = a + b\sqrt{35}$

Rationalising the denominator on left hand side by multiplying by $\sqrt{7}-\sqrt{5}$.

$$\therefore \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{(\sqrt{7}-\sqrt{5})\times(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})\times(\sqrt{7}-\sqrt{5})}$$

$$= \frac{(\sqrt{7})^2+(\sqrt{5})^2-2\sqrt{7\times 5}}{(\sqrt{7})^2-(\sqrt{5})^2} = \frac{7+5-2\sqrt{35}}{7-5}$$

$$= \frac{12-2\sqrt{35}}{2} = 6-\sqrt{35}$$

$\therefore \quad a = 6$ and $b = -1$

or $\quad a - b = 6-(-1) = 7$

31. (*c*) $\because 2^{x-1} + 2^{x+1} = 320$

$\Rightarrow \quad 2^x(2^{-1}+2) = 320$

$\Rightarrow \quad 2^x\left(\dfrac{1}{2}+2\right) = 320$

$\Rightarrow \quad 2^x \times \dfrac{5}{2} = 320$

$\Rightarrow \quad 2^x = 64 \times 2 = 2^7$

$\therefore \quad x = 7$

32. (*d*) $\sqrt[3]{P(P^2+3P+3)+1}$

$= \sqrt[3]{P^3+3P^2+3P+1}$

$= \sqrt[3]{(P+1)^3} = P+1$

$\because \quad P = 124 \Rightarrow P+1 = 125$

33. (*d*) $\left(\dfrac{p}{q}\right)^{n-1} = \left(\dfrac{q}{p}\right)^{n-3} \Rightarrow \left(\dfrac{p}{q}\right)^{n-1} = \left(\dfrac{p}{q}\right)^{3-n}$

$\Rightarrow \quad n-1 = 3-n$

$2n = 4$ or $n = 2$

34. (*e*) $? = \sqrt[3]{512} + \sqrt[4]{16} + \sqrt{576}$

$= 8 + 2 + 24 = 4 + 24 = 28$

35. (*b*) $3 + \dfrac{1}{\sqrt{3}} + \dfrac{1}{3+\sqrt{3}} + \dfrac{1}{\sqrt{3}-3}$

$$= 3 + \frac{1}{\sqrt{3}} + \frac{(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})} + \frac{(\sqrt{3}+3)}{(\sqrt{3}-3)(\sqrt{3}+3)}$$

$$= 3 + \frac{1}{\sqrt{3}} + \frac{3-\sqrt{3}}{9-3} + \frac{\sqrt{3}+3}{3-9}$$

$$= 3 + \frac{1}{\sqrt{3}} + \frac{3-\sqrt{3}}{6} - \frac{\sqrt{3}+3}{6}$$

$$= 3 + \frac{1}{\sqrt{3}} + \frac{3-\sqrt{3}-\sqrt{3}-3}{6}$$

$$= 3 + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} = 3$$

36. (*a*) Given, $xyz = 1$, $a^x = b$, $b^y = c$

Now, $b = a^x$

$\Rightarrow \quad b^y = a^{xy}$

$\Rightarrow \quad b^{yz} = a^{xyz}$

$\Rightarrow \quad c^z = a$

37. (*b*) Given that, $a = 2 + \sqrt{3}$

Then, $\frac{1}{a} = 2 - \sqrt{3}$ [by conjugate property]

Now, we have, $a^2 + a^{-2} = \left(a + \frac{1}{a}\right)^2 - 2$

$= (2 + \sqrt{3} + 2 - \sqrt{3})^2 - 2$

$= (4)^2 - 2 = 16 - 2 = 14$

38. (*d*) Given expression $= [(\sqrt{2})^{\sqrt{2}}]^{\sqrt{2}}$

$= (\sqrt{2})^{(2)^{\frac{\sqrt{2}}{2}}} = (\sqrt{2})^{(2)^{\frac{1}{\sqrt{2}}}}$

$= (2)^{\frac{1}{2} \times 2^{\frac{1}{\sqrt{2}}}} = 2^{\left(\frac{2^{\frac{1}{\sqrt{2}}}}{2}\right)} = (2)^{(2)^{\left(\frac{1}{\sqrt{2}} - 1\right)}}$

which denotes a real number but not a rational number.

39. (*d*) Given that, $16 \times 8^{n+2} = 2^m$

$\Rightarrow \quad (2)^4 \times 2^{3(n+2)} = 2^m$

$\Rightarrow \quad (2)^{(4 + 3n + 6)} = 2^m$

$\Rightarrow \quad 2^{(3n + 10)} = 2^m$

On comparing, we get

$3n + 10 = m$

$\Rightarrow \quad m = 3n + 10$

40. (*b*) Given, $\sqrt{10 + \sqrt[3]{x}} = 4$

On squaring both sides, we get

$10 + \sqrt[3]{x} = 16$

$\Rightarrow \quad \sqrt[3]{x} = 6$

Now, cubic on both sides, we get

$x = (6)^3 = 216$

41. (*b*) If m and n are natural numbers, then $\sqrt[m]{n}$ is irrational unless n is mth power of an integer.

42. (*d*) $\sqrt{2}, \sqrt[3]{3}, \sqrt[6]{6}$

Taking LCM of 2, 3 and 6 = 12

Now, $\sqrt{2} = (2)^{1/2} = (2)^{6/12} = \sqrt[12]{2^6} = \sqrt[12]{64}$

$\sqrt[3]{3} = (3)^{1/3} = (3)^{4/12} = \sqrt[12]{3^4} = \sqrt[12]{81}$

$\sqrt[6]{6} = (6)^{1/6} = (6)^{2/12} = \sqrt[12]{6^2} = \sqrt[12]{36}$

So, neither I nor II are correct.

43. (a) We can simply square and add both to find the answer

$$x^2 + y^2 = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2 + \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2$$

$$= \frac{3+1+2\sqrt{3}}{3+1-2\sqrt{3}} + \frac{3+1-2\sqrt{3}}{3+1+2\sqrt{3}}$$

$$= \frac{4+2\sqrt{3}}{4-2\sqrt{3}} + \frac{4-2\sqrt{3}}{4+2\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{4+3+4\sqrt{3}+4+3-4\sqrt{3}}{4-3} = 14$$

Alternate Method

Finding $x + y = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} + \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$$= \frac{4+4}{2} = 4$$

[adding both values]

and $\quad xy = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = 1$

Now, $x^2 + y^2 = (x + y)^2 - 2xy$

$$= 4^2 - 2 \times 1 = 16 - 2 = 14$$

Exercise © *Higher Skill Level Questions*

1. (*d*) $\dfrac{a^{-1}-1}{a-1} = \dfrac{\frac{1}{a}-1}{a-1} = \dfrac{(1-a)}{a} \times \dfrac{1}{(a-1)} = -\dfrac{1}{a}$

$\therefore$ Required quotient $= -\dfrac{1}{a}$.

2. (*a*) $\left[\sqrt[3]{\sqrt[6]{2^9}}\right]^4 \times \left[\sqrt[6]{\sqrt[3]{2^9}}\right]^4$

$$= 2^{9 \times \frac{1}{6} \times \frac{1}{3} \times 4} \times 2^{9 \times \frac{1}{3} \times \frac{1}{6} \times 4}$$

$$= 2^2 \times 2^2 = 2^{2+2} = 2^4$$

3. (*a*) LCM of 4, 6, 12 = 12

$$\sqrt[4]{3} = \sqrt[12]{3^3} = \sqrt[12]{27}$$

$$\sqrt[6]{10} = \sqrt[12]{10^2} = \sqrt[12]{100}$$

$$\sqrt[12]{25} = \sqrt[12]{25}$$

Clearly, $\sqrt[12]{100} > \sqrt[12]{27} > \sqrt[12]{25}$

$\Rightarrow \quad \sqrt[6]{10} > \sqrt[4]{3} > \sqrt[12]{25}$

4. (*d*) $(\sqrt{1+a} + \sqrt{1-a})^2$

$$= (1+a) + (1-a) + 2\sqrt{1-a^2}$$

$$= 2\,(1 + \sqrt{1-a^2})$$

$$= 2\left(1 + \sqrt{1 - \frac{3}{4}}\right)$$

$$= 2\left(1 + \frac{1}{2}\right) = 2 \times \frac{3}{2} = 3$$

$\therefore \quad (\sqrt{1+a} + \sqrt{1-a}) = \sqrt{3}$

5. (*b*) $\dfrac{6a^{-2}bc^{-3}}{4ab^{-3}c^2} \div \dfrac{5a^{-3}b^2c^{-1}}{3ab^{-2}c^3}$

$$= \frac{6a^{-2}bc^{-3}}{4ab^{-3}c^2} \times \frac{3ab^{-2}c^3}{5a^{-3}b^2c^{-1}}$$

$$= \frac{18a^{-2+1}b^{1-2}c^{-3+3}}{20a^{1-3}b^{-3+2}c^{2-1}}$$

$$= \frac{9}{10}\frac{a^{-1}b^{-1}c^{0}}{a^{-2}b^{-1}c^{1}} = \frac{9}{10}a^{-1+2}\,b^{-1+1}\,c^{0-1}$$

$$= \frac{9}{10}a^{1}b^{0}c^{-1} = \frac{9}{10}ac^{-1} \qquad [\because\ b^{0} = 1]$$

6. (*c*) $\frac{1}{m} = \frac{1}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{7+4\sqrt{3}}{49-48}$

$$= 7 + 4\sqrt{3}$$

$$\therefore \qquad m + \frac{1}{m} = 14$$

$$\Rightarrow \qquad m + \frac{1}{m} + 2 = 14 + 2 = 16$$

$$\Rightarrow \qquad \left(\sqrt{m} + \frac{1}{\sqrt{m}}\right)^{2} = m + \frac{1}{m} + 2$$

$$\Rightarrow \qquad \left(\sqrt{m} + \frac{1}{\sqrt{m}}\right)^{2} = 4^{2}$$

$$\Rightarrow \qquad \left(\sqrt{m} + \frac{1}{\sqrt{m}}\right) = 4$$

7. (*c*) $6\sqrt{(27)^{-\frac{2}{3}} + (8)^{-\frac{2}{3}}} = 6\sqrt{\left(\frac{1}{27}\right)^{\frac{2}{3}} + \left(\frac{1}{8}\right)^{\frac{2}{3}}}$

$$= 6\sqrt{\left(\frac{1}{3^{3}}\right)^{\frac{2}{3}} + \left(\frac{1}{2^{3}}\right)^{\frac{2}{3}}} = 6\sqrt{\frac{1}{3^{2}} + \frac{1}{2^{2}}}$$

$$= 6\sqrt{\frac{1}{9} + \frac{1}{4}} = 6\sqrt{\frac{4+9}{36}} = 6 \times \frac{\sqrt{13}}{6} = \sqrt{13}$$

8. (*b*) Given that, $2x^{1/3} + 2x^{-1/3} = 5$.

Let $x^{1/3} = m$, then $\quad 2m + \frac{2}{m} = 5$

$$\Rightarrow \qquad 2m^{2} - 5m + 2 = 0$$

$$\Rightarrow \qquad (2m-1)(m-2) = 0$$

$$\therefore \qquad m = \frac{1}{2} \quad \text{or} \quad m = 2$$

$$\Rightarrow \qquad x^{1/3} = 2 \ \text{or}\ \frac{1}{2}$$

9. (*d*) $\left(\frac{1}{64}\right)^0 + (64)^{-\frac{1}{2}} + (32)^{\frac{4}{5}} - (32)^{-\frac{4}{5}}$

$$= 1 + 8^{2\times\left(\frac{-1}{2}\right)} + 2^{5\times\frac{4}{5}} - 2^{5\times\left(\frac{-4}{5}\right)}$$
$$= 1 + 8^{-1} + 2^4 - 2^{-4}$$
$$= 1 + \frac{1}{8} + 16 - \frac{1}{16}$$
$$= 17\frac{1}{16}$$

10. (*a*) Given surds are in the form $2^{1/2}, 3^{1/6}, 4^{1/3}$ and $5^{1/4}$.

LCM of (2, 6, 3, 4) = 12

$$2^{\frac{1}{2}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$$
$$3^{\frac{1}{6}} = (3^2)^{\frac{1}{12}} = (9)^{\frac{1}{12}}$$
$$4^{\frac{1}{3}} = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$$
$$5^{\frac{1}{4}} = (5^3)^{\frac{1}{12}} = (125)^{\frac{1}{12}}$$

Clearly, greatest surd $= (256)^{\frac{1}{12}} = \sqrt[3]{4}$

11. (*b*) Let $2^a = 3^b = 6^{-c} = K$

$\therefore \quad 2 = K^{\frac{1}{a}}, 3 = K^{\frac{1}{b}}, 6 = K^{-\frac{1}{c}}$

We know that, $2 \times 3 = 6$

$\therefore \quad K^{\frac{1}{a}} \times K^{\frac{1}{b}} = K^{-\frac{1}{c}}$

$\Rightarrow \quad K^{\left(\frac{1}{a}+\frac{1}{b}\right)} = K^{\frac{-1}{c}}$

$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} = \frac{-1}{c} \Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

12. (*b*) $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{100}+\sqrt{99}}$

$$= \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{\sqrt{4}-\sqrt{3}}{4-3} + \ldots\ldots + \frac{\sqrt{100}-\sqrt{99}}{100-99}$$
$$= \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \ldots\ldots + \sqrt{100} - \sqrt{99}$$
$$= \sqrt{100} - 1 = 10 - 1 = 9$$

13. (*b*) $x = \frac{2^{n+4} - 2 2^n}{2 2^{n+3}} + 2^{-3}$

$$= \frac{2^{n+4} - 2^{n+1}}{2^{n+4}} + 2^{-3}$$
$$= \frac{2^{n+1}(2^3 - 1)}{2^{n+4}} + \frac{1}{2^3}$$
$$= \frac{8-1}{2^3} + \frac{1}{2^3} = \frac{7}{8} + \frac{1}{8} = 1$$

14. (a) $$\frac{9^n \times 3^2 \times \left(3^{\frac{-n}{2}}\right)^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \quad \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \quad \frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 8} = \frac{1}{27}$$

$$\Rightarrow \quad \frac{3^{3n} \times 8}{3^{3m} \times 8} = \frac{1}{27} \Rightarrow (3^3)^{n-m} = 3^{-3}$$

$$n - m = -1 \text{ or } \quad m - n = 1$$

15. (a) $$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{35 + 14\sqrt{3} - 20\sqrt{3} - 24}{49 - 48} = 11 - 6\sqrt{3}$$

$$\because \quad \frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$$

$$\therefore \quad a + b\sqrt{3} = 11 - 6\sqrt{3}$$

or $\quad a = 11$ and $b = -6$

16. (d) $$\left(\frac{a^p}{a^q}\right)^{(p+q-r)} \times \left(\frac{a^r}{a^p}\right)^{(r+p-q)} \times \left(\frac{a^q}{a^r}\right)^{(q+r-p)}$$

$$= (a^{p-q})^{p+q-r} \times (a^{r-p})^{r+p-q} \times (a^{q-r})^{q+r-p}$$

$$= a^{p^2 - q^2 - rp + rq} \times a^{r^2 - p^2 - qr + pq} \times a^{q^2 - r^2 - pq + pr}$$

$$= a^{p^2 - q^2 - rp + rq + r^2 - p^2 - qr + pq + q^2 - r^2 - pq + pr}$$

$$= a^0 = 1$$

17. (c) Here, $2^p + 3^q = 17$ ...(i)

$2^{p+2} - 3^{q+1} = 5$ or, $4.2^p - 3.3^q = 5$ ...(ii)

On multiplying Eq. (i) by 3 and adding it with Eq. (ii), we get

$$3.2^p + 3.3^q = 51$$
$$4.2^p - 3.3^q = 5$$

$$7.2^p = 56$$

$$\Rightarrow \quad 2^p = 8 = 2^3 \Rightarrow p = 3$$

On substituting the value of p in Eq. (i), we get

$$2^3 + 3^q = 17$$

$$\Rightarrow \quad 3^q = 9 \quad \Rightarrow q = 2$$

$$\therefore \quad p = 3 \text{ and } q = 2$$

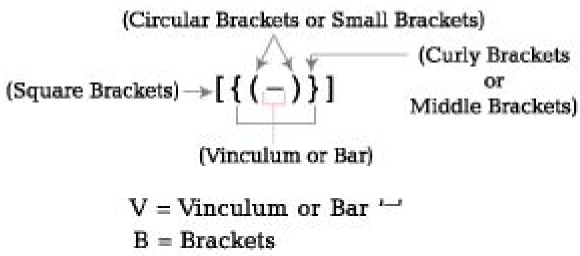
Chapter 7

Simplification

A complex arithmetical expression can be converted into a simple expression by **simplification.**

VBODMAS' Rule

To simplify arithmetic expressions, which involve various operations like brackets, multiplication, addition, etc; a particular sequence of the operations has to be followed. The operations have to be carried out in the order, in which they appear in the word **VBODMAS,** where different letters of the word stand for following operations.



Order of removing brackets

First Small brackets (Circular brackets) '( )'

Second Middle brackets (Curly brackets)'{ }' **Third** Square brackets (Big brackets) '[ ]'

O =Of

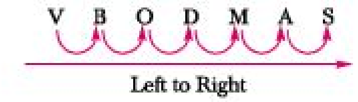
D = Division

M = Multiplication

A = Addition

S = Subtraction

Order of above mentioned operations is same as the order of letters in the 'VBODMAS' from left to right as



Clearly, the order will be as follows

First	Vinculum bracket is solved, [Remember $-5-10=-15$, but $-\overline{5-10}=-(-5)=5$]
Second	Brackets are to be solved in order given above, [first, then second, then third]
Third	Operation of 'Of' is done,
Fourth	Operation of division is performed,
Fifth	Operation of multiplication is performed,
Sixth	Operation of addition is performed and
Seventh	Operation of subtraction is performed.

MIND IT!

Absolute value of a real number

If *m* is a real number, then its absolute value is defined as

$$|m| = \begin{cases} m, \text{ if } m>0 \\ -m, \text{ if } m<0 \end{cases}$$

e.g., $|3|=3$ and $|-3|=-(-3)=3$

Ex. 1 Simplify $4 - [6 - \{12 - (10 - \overline{8 - 6})\}]$.

Sol. Given expression

$$
\begin{array}{ll}
= 4 - [6 - \{12 - (10 - 8 + 6)\}] & \text{Remove vinculum} \\
= 4 - [6 - \{12 - (10 - 2)\}] & \\
= 4 - [6 - \{12 - 8\}] & \text{Remove ()} \\
= 4 - [6 - 4] & \text{Remove \{\}} \\
= 4 - 2 = 2 & \text{Remove []}
\end{array}
$$

Ex. 2 Simplify $\left(9.6 \times 3.6 \div 7.2 + 10.8 \text{ of } \frac{1}{18} - \frac{1}{10}\right)$

Sol. Given expression

$$
\begin{array}{ll}
= 9.6 \times 3.6 \div 7.2 + 10.8 \text{ of } \frac{1}{18} - \frac{1}{10} & \\
= 9.6 \times 3.6 \div 7.2 + 0.6 - \frac{1}{10} & \text{Remove `of'} \\
= 9.6 \times \frac{3.6}{7.2} + 0.6 - 0.1 & \text{Remove `}\div\text{'} \\
= 4.8 + 0.6 - 0.1 & \text{Remove `}\times\text{'} \\
= 5.4 - 0.1 & \text{Remove `+'} \\
= 5.3 & \text{Remove `}-\text{'}
\end{array}
$$

Ex. 3 Simplify $18800 \div 940 \div 10$.

Sol. Given expression $= 18800 \div 940 \div 10$

$$= \frac{18800}{940} \div 10$$

$$= 20 \div 10 = 2$$

Ex. 4 Solve $4\frac{1}{6} + 5\frac{1}{6} + 8\frac{1}{6}$.

Sol. Given expression

$$= 4\frac{1}{6} + 5\frac{1}{6} + 8\frac{1}{6}$$

$$= (4 + 5 + 8) + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

$$= 17 + \frac{3}{6} = 17\frac{1}{2}$$

Ex. 5 Simplify $n - [n - (m + n) - \{n - (n - \overline{m - n})\} + 2m]$.

Sol. Given expression

$$= n - [n - (m + n) - \{n - (n - \overline{m - n})\} + 2m]$$
$$= n - [n - (m + n) - \{n - (n - m + n)\} + 2m]$$
$$= n - [n - (m + n) - \{n - (2n - m)\} + 2m]$$
$$= n - [n - m - n - \{n - 2n + m\} + 2m]$$
$$= n - [- m - \{- n + m\} + 2m]$$
$$= n - [- m + n - m + 2m]$$
$$= n - [n] = n - n = 0$$

Ex. 6 Simplify $\dfrac{\frac{7}{2} \div \frac{5}{2} \times \frac{3}{2}}{\frac{7}{2} \div \frac{5}{2} \text{ of } \frac{3}{2}} \div \dfrac{15}{14}$.

Sol. Given expression

$$= \frac{\frac{7}{2} \div \frac{5}{2} \times \frac{3}{2}}{\frac{7}{2} \div \frac{5}{2} \text{ of } \frac{3}{2}} \div \frac{15}{14}$$
$$= \frac{\frac{7}{2} \times \frac{2}{5} \times \frac{3}{2}}{\frac{7}{2} \times \frac{4}{15}} \div \frac{15}{14} = \frac{\frac{21}{10}}{\frac{14}{15}} \div \frac{15}{14}$$
$$= \frac{21}{10} \times \frac{15}{14} \times \frac{14}{15} = \frac{21}{10} = 2\frac{1}{10}$$

Basic Formulae

Following formulae are useful in various operations of simplification

(i) (a + *bf* = a^2 + 2ab + b^2

(ii) $(a-b)^2$ = a^2-2afa + fa^2

(iii) (a + *bf* + *{a-bf=2* $(a^2 + b^2)$

(iv) (a + *bf* - (a - *bf = Aab*

(v) a^2-b^2=(a+b){a-b)

(vi) $(a + ib)^3$ = a^3 + b^3 + *3ab* (a + b)

(vii) $(a - b)^3$ = a^3 - fa^3 - 3ab (a - b)

(viii) $a^3 + fa^3 = (a + b) \{a^2 - ab + fa^2)$

(ix) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(x) $$\frac{a^3 + b^3 + c^3 - 3abc}{(a^2 + b^2 + c^2 - ab - bc - ca)} = (a + b + c)$$

or

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
$$= (a + b + c)\frac{1}{2}(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$$
$$= \frac{(a + b + c)}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$ or $a^3 + b^3 + c^3 - 3abc = 0$

(xi) $(a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$

(xii) $$\left(a + \frac{1}{a}\right)^2 = \left(a^2 + \frac{1}{a^2}\right) + 2 = \left(a - \frac{1}{a}\right)^2 + 4$$

(xiii) $$\left(a - \frac{1}{a}\right)^2 = \left(a^2 + \frac{1}{a^2}\right) - 2 = \left(a + \frac{1}{a}\right)^2 - 4$$

(xiv) $$\left(a + \frac{1}{a}\right)^3 = \left(a^3 + \frac{1}{a^3}\right) + 3\left(a + \frac{1}{a}\right)$$

(xv) $$\left(a - \frac{1}{a}\right)^3 = \left(a^3 - \frac{1}{a^3}\right) - 3\left(a - \frac{1}{a}\right)$$

Ex. 7 Solve $\dfrac{(5.9)^3+(1.8)^3+(4.8)^3-3\times 5.9\times 1.8\times 4.8}{(5.9)^2+(1.8)^2+(4.8)^2-5.9\times 1.8-1.8\times 4.8-4.8\times 5.9}$.

Sol. We know that,

$$\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}=(a+b+c)$$

Here, $a=5.9$, $b=1.8$, $c=4.8$

$$\therefore \quad \frac{(5.9)^3+(1.8)^3+(4.8)^3-3\times 5.9\times 1.8\times 4.8}{(5.9)^2+(1.8)^2+(4.8)^2-5.9\times 1.8-1.8\times 4.8-4.8\times 5.9}$$

$$=(5.9+1.8+4.8)=12.5$$

Ex. 8 Solve $\dfrac{(9.8)^3-(6.8)^3}{9.8^2+9.8\times 6.8+6.8^2}$.

Sol. We know that,

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$\therefore \quad \frac{a^3-b^3}{a^2+ab+b^2}=(a-b)$$

Here, $a=9.8$, $b=6.8$

$$\therefore \quad \frac{(9.8)^3-(6.8)^3}{(9.8)^2+9.8\times 6.8+(6.8)^2}=(9.8-6.8)=3$$

Ex. 9 Solve $\dfrac{(835+378)^2+(835-378)^2}{835\times 835+378\times 378}$.

Sol. We know that, $\dfrac{(a+b)^2+(a-b)^2}{a^2+b^2}=\dfrac{2(a^2+b^2)}{(a^2+b^2)}=2$

$\therefore$ Given expression = 2

where, $a=835,\ b=378$

Ex. 10 If $x=3+\sqrt{8}$, then find the value of $x^2+\dfrac{1}{x^2}$.

Sol. $x=3+\sqrt{8}$

On squaring both sides, we get

$$x^2=(3+\sqrt{8})^2=9+8+6\sqrt{8} \qquad [\because (a+b)^2=a^2+b^2+2ab]$$

$$x^2=17+6\sqrt{8}=17+12\sqrt{2}$$

Now, $\dfrac{1}{x^2}=\dfrac{1}{17+12\sqrt{2}}\times\dfrac{17-12\sqrt{2}}{17-12\sqrt{2}}$ [rationalising the denominator]

$\Rightarrow \dfrac{1}{x^2}=\dfrac{17-12\sqrt{2}}{289-288}=17-12\sqrt{2}$

$\therefore x^2+\dfrac{1}{x^2}=17+12\sqrt{2}+17-12\sqrt{2}$

$$x^2+\frac{1}{x^2}=34$$

Ex. 11 If $\left(a+\dfrac{1}{a}\right)=4\sqrt{2}$, then what is the value of (a^6+a^{-6})?

Sol. $\left(a+\dfrac{1}{a}\right)=4\sqrt{2}$

On squaring both sides, we get

$$\left(a+\frac{1}{a}\right)^2=(4\sqrt{2})^2$$

$\Rightarrow (4\sqrt{2})^2=a^2+\dfrac{1}{a^2}+2 \qquad [\because (a+b)^2=a^2+b^2+2ab]$

$\Rightarrow a^2+\dfrac{1}{a^2}=32-2=30$

Now, taking cube on both sides, we get

$$\left(a^2+\frac{1}{a^2}\right)^3=(30)^3$$

$\Rightarrow a^6+\dfrac{1}{a^6}+3\cdot a^2\times\dfrac{1}{a^2}\left(a^2+\dfrac{1}{a^2}\right)=27000$

$[\because (a+b)^3=a^3+b^3+3ab(a+b)]$

$\Rightarrow a^6+\dfrac{1}{a^6}+3(30)=27000$

$\Rightarrow a^6+\dfrac{1}{a^6}=27000-90 \Rightarrow a^6+\dfrac{1}{a^6}=26910$

$\therefore a^6+a^{-6}=26910$

Multi Concept

QUESTIONS

1. *If $x + y + z = 0$, then the value of $\dfrac{x^2y^2 + y^2z^2 + z^2x^2}{x^4 + y^4 + z^4}$ is*

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

(*b*) $x + y + z = 0$ [given]

On squaring both sides, we get

$$(x + y + z)^2 = 0$$

$\Rightarrow$ $x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0 \Rightarrow x^2 + y^2 + z^2 = -2(xy + yz + zx)$

Again, squaring both sides, we get

$$(x^2 + y^2 + z^2)^2 = 4(xy + yz + zx)^2$$

$\Rightarrow$ $x^4 + y^4 + z^4 + 2(x^2y^2 + y^2z^2 + x^2z^2)$

$= 4[x^2y^2 + y^2z^2 + z^2x^2 + \underbrace{2xyz(x + y + z)}_{0}]$ [$\because$ Given $x + y + z = 0$]

$\Rightarrow$ $x^4 + y^4 + z^4 = 2(x^2y^2 + y^2z^2 + x^2z^2)$

$\therefore$ $\dfrac{x^2y^2 + y^2z^2 + x^2z^2}{x^4 + y^4 + z^4} = \dfrac{1}{2}$

2. *If $x(x + y + z) = 9$, $y(x + y + z) = 16$ and $z(x + y + z) = 144$, then value of x will be?*

(a) $\frac{9}{5}$ (b) $\frac{9}{7}$ (c) $\frac{9}{13}$ (d) $\frac{16}{13}$

(*c*) Given, $x(x + y + z) = 9$ ...(i)

$y(x + y + z) = 16$ ...(ii)

$z(x + y + z) = 144$ ...(iii)

On adding Eqs. (i), (ii) and (iii), we get

$$x(x + y + z) + y(x + y + z) + z(x + y + z) = 9 + 16 + 144$$

$\Rightarrow$ $(x + y + z)(x + y + z) = 169$

$\Rightarrow$ $(x + y + z)^2 = 169$

$\Rightarrow$ $x + y + z = 13$ ...(iv)

On putting the values of $(x + y + z)$ from Eq. (iv) in Eq. (i), we get

$$x(x + y + z) = 9$$

$\Rightarrow$ $x(13) = 9$

$\Rightarrow$ $x = \dfrac{9}{13}$

Fast Track Practice

Exercise© *Base Level Questions*

1. $7059 - 2350 + 1936 = ? \times 50$

[IBPS Clerk 2011]

(a) 123.6 (b) 132.3 (c) 132.6 (d) 123.9
(e) None of the above

2. If $\sqrt{2} = 1.4142$, then the value of $\dfrac{7}{4+\sqrt{2}}$ is **[SSC FCI 2012]**

(a) 3.5858 (b) 4.4142
(c) 1.2929 (d) 1.5858

3. What is 786 times of 964? **[SBI Clerk 2012]**

(a) 759276 (b) 749844
(c) 75416 (d) 757704
(e) None of the above

4. Solve $\dfrac{(999+588)^2 - (999-588)^2}{999 \times 588}$.

(a) 8 (b) 3 (c) 2 (d) 4
(e) None of the above

5. Solve $\dfrac{(238+131)^2 + (238-131)^2}{238 \times 238 + 131 \times 131}$.

(a) 4 (b) 2 (c) 8 (d) 9
(e) None of the above

6. $\dfrac{\dfrac{1}{5} + 999 \times \dfrac{494}{495} \times 99}{4} = ?$

(a) 25000 (b) 24225
(c) 24800 (d) 24750
(e) None of the above

7. If $x = \sqrt{3} + \sqrt{2}$, then the value of $\left(x + \dfrac{1}{x}\right)$ is **[SSC (10+2) 2012]**

(a) 2 (b) 3 (c) $2\sqrt{2}$ (d) $2\sqrt{3}$

8. Find the value of $a^3 + b^3 + c^3 - 3abc$ when $a = 225$, $b = 226$, $c = 227$

[SSC CGL 2012]

(a) 2304 (b) 2430
(c) 2034 (d) 2340

9. The value of $\dfrac{(0.96)^3 - (0.1)^3}{(0.96)^2 + (0.096) + 0.01}$ is

[SSC (10+2) 2012]

(a) 0.86 (b) 1.06 (c) 0.95 (d) 0.97

10. Value of $\dfrac{\sqrt{0.0001} \times \sqrt[3]{1000000}}{\sqrt[3]{0.125} \times \sqrt[3]{0.008}}$

[SSC Multitasking 2013]

(a) 1 *(b)* 10 *(c)* 0.1 *(d)* 0.01

11. $3.5 \times (60 \div 0.5) = ?$ **[IBPS Clerk 2011]**

(a) 62 *(b)* 96 *(c)* 74 *(d)* 88
(e) None of the above

12. Simplify $6 - [9 - \{18 - (15 - \overline{12 - 9})\}]$.

(a) 1 *(b)* 4 *(c)* 5 *(d)* 3
(e) None of the above

13. $3\frac{2}{3} + 2\frac{3}{4} + 1\frac{1}{2} = ?$ **[IBPS Clerk 2011]**

(a) $8\frac{11}{12}$ *(b)* $10\frac{12}{13}$
(c) $7\frac{11}{12}$ *(d)* $9\frac{11}{13}$
(e) None of the above

14. $\frac{5}{8} \times 2\frac{3}{5} \div \frac{4}{9} = ?$ **[IBPS Clerk 2011]**

(a) $2\frac{13}{27}$ *(b)* $1\frac{11}{27}$ *(c)* $2\frac{23}{32}$ *(d)* $3\frac{21}{32}$
(e) None of the above

15.10.8x5.5x84=? [IBPS Clerk 2011]

(a) 458.69 *(b)* 489.96

(c) 498.96 *(d)* 485.69 *(e)* None of the above

16. (4 x 4x 4x 4x 4x 4)^{B}

x (4 x 4 x 4)^{8} ■*■ (4)^{3} = (64)$^{?}$ **[IBPS Clerk 2011]**

(a) 17 *(b)* 10

(c) 16 fa; 11

(e) None of the above 17.8648 - 7652 = ? x 40 [IBPS Clerk 2011]

(a) 24.7 (b) 28.9

(c) 21A (d) 25.9

fe; None of the above

18. 68a46 - 227.39 - 34185 = ?

[IBPS Clerk 2011]

(a) 114.22 *(b)* 141.22

(c) 144.22 fa; 112.22

fe; None of the above

19. $\frac{13}{63} + \frac{104}{14} \times \frac{52}{19} = ?$ [SBI Clerk 2012]

(a) $\frac{12}{173}$ (b) $\frac{13}{171}$

(c) $\frac{17}{171}$ (d) $\frac{18}{171}$

(e) None of the above

20. $(5568 + 87)^{\frac{1}{3}} + (72 \times 2)^{\frac{1}{2}} = (?)^{\frac{1}{2}}$

[IDBI SO 2012]

(a) 256 (b) 4 (c) $\sqrt{2}$ (d) 16

(e) None of the above

21. $\sqrt{13^2 + 28 + 4 - (3)^3 + 107} = (?)^2$

[IDBI SO 2012]

(a) 2 (b) 16 (c) 256 (d) 4

(e) $(256)^2$

22. $(0.49)^4 \times (0.343)^4 + (0.2401)^4$

$= (70 + 100)^{? + 3}$ [IDBI SO 2012]

(a) 3 (b) 1 (c) 4 (d) 7

(e) None of the above

23. $$\frac{5 - \left[\frac{3}{4} + \left\{2\frac{1}{2} - \left(\frac{1}{2} + \overline{\frac{1}{6} - \frac{1}{7}}\right)\right\}\right]}{2} = ?$$

(a) $1\frac{23}{168}$ (b) $2\frac{23}{168}$ (c) $3\frac{23}{168}$ (d) $4\frac{23}{168}$

(e) None of the above

24. Simplify

$$\frac{\left[3\frac{1}{4} + \left\{1\frac{1}{4} - 0.5\left(2\frac{1}{2} - \overline{\frac{1}{4} - \frac{1}{6}}\right)\right\}\right]}{4 \times \frac{1}{12}}.$$

(a) 245 (b) 233 (c) 234 (d) 299

(e) None of the above

25. $$\frac{\left[\begin{array}{l}0.5\times 0.5\times 0.5 + 0.2\times 0.2\times 0.2 + \\ 0.3\times 0.3\times 0.3 - 3\times 0.5\times 0.3\times 0.2\end{array}\right]}{\begin{array}{l}0.5\times 0.5 + 0.2\times 0.2 + 0.3\times 0.3 - \\ 0.5\times 0.2 - 0.2\times 0.3 - 0.5\times 0.3\end{array}} = ?$$

[SSC CGL 2010]

(a) 1 (b) 0.6 (c) 0.4 (d) 0.03

26. $$\frac{\begin{array}{c}(2.247)^3 + (1.730)^3 + (1.023)^3 \\ -3\times 2.247\times 1.730\times 1.023\end{array}}{\begin{array}{l}(2.247)^2 + (1.730)^2 + (1.023)^2 - 2.247 \\ \times 1.730 - 1.730\times 1.023 - 2.247\times 1.023\end{array}} = ?$$

(a) 1.730 (b) 4

(c) 5 (d) 5.247

27. What is the value of

$$\frac{725 \times 725 \times 725 + 371 \times 371 \times 371}{725 \times 725 - 725 \times 371 + 371 \times 371} = ?$$

(a) 9610 (b) 1960
(c) 1096 (d) 1016

28. $\frac{8.73 \times 8.73 \times 8.73 + 4.27 \times 4.27 \times 4.27}{8.73 \times 8.73 - 8.73 \times 4.27 + 4.27 \times 4.27}$ is equal to **[SSC CGL 2012]**

(a) 11 (b) 13
(c) 11/7 (d) None of these

29. $\sqrt{289} - \sqrt{625} + \sqrt{25}$ is equal to **[SSC CGL 2013]**

(a) 17 (b) 15 (c) 12 (d) $-\frac{8}{5}$

30. If $a^2 + b^2 = 234$ and $ab = 108$, find the value of $\frac{a+b}{a-b}$.

(a) 10 (b) 8 (c) 5 (d) 4
(e) None of the above

31. If $a^2 + 1 = a$, then the value of $a^{12} + a^6 + 1$ is **[SSC CGL 2013]**

(a) 2 (b) 3 (c) –3 (d) 1

32. If 0.764 y = 1.236 x, then what is the value of $\left(\frac{y-x}{y+x}\right)$?

(a) 0.764 (b) 0.236 (c) 2 (d) 0.472

33. If a, b and c are non-zero, $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$, then the value of abc is **[SSC CGL 2013]**

(a) –3 (b) 1 (c) –1 (d) 3

34. If $a + b + c = 0$, then the value of $\left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right)$ $\left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right)$ is **[SSC CGL 2013]**

(a) 9 (b) 0 (c) 8 (d) – 3

35. If $a * b = a + b + \frac{a}{b}$, then the value of $12 * 4$ is **[SSC CGL 2013]**

(a) 48 (b) 19 (c) 20 (d) 21

36. If $x = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $x - y = 4\sqrt{2}$, then the value of $(x^2 + y^2)$ is **[SSC CPO 2013]**

(a) 34 (b) 38 (c) 30 (d) 32

37. If $x + \frac{a}{x} = 1$, then the value of $\frac{x^2 + x + a}{x^3 - x^2}$ is **[SSC CGL 2012]**

(a) -2 (b) $-\frac{a}{2}$ (c) $\frac{2}{a}$ (d) $-\frac{2}{a}$

38. If $\sqrt{28 - 6\sqrt{3}} = \sqrt{3}\, a + b$, (where a, b are rationals), value of $(a + b)$ is **[SSC CGL 2012]**

(a) -2 (b) 2 (c) 1 (d) -1

39. If $x + y = 1$, then the value of $(x^3 + y^3 + 3xy)$ is **[SSC (10+2) 2012]**

(a) -2 (b) 1 (c) 2 (d) 3

40. If $a = (\sqrt{2} - 1)^{1/3}$, then the value of $(a - a^{-1})^3 + 3(a - a^{-1})$ is **[SSC (10+2) 2012]**

(a) -2 (b) 2 (c) $2\sqrt{2}$ (d) $\sqrt{2}$

41. If $x = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$ and $y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$, then the value of $\frac{x^2}{y} + \frac{y^2}{x}$ is **[SSC (10+2) 2012]**

(a) 52 (b) 76 (c) 4 (d) 64

42. If $p + q = 10$ and $pq = 5$, then the numerical value of $\frac{p}{q} + \frac{q}{p}$ will be **[SSC (10+2) 2012]**

(a) 22 (b) 18 (c) 16 (d) 20

43. If $x + y = 18$ and $xy = 72$, what is the value of $x^2 + y^2$? **[SBI Clerk 2012]**

(a) 120 (b) 90 (c) 180
(d) Cannot be determined
(e) None of the above

44. $\frac{(m - n)^3 + (n - r)^3 + (r - m)^3}{6(m - n)(n - r)(r - m)} = ?$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{5}$ (d) $\frac{1}{6}$
(e) None of the above

45. If $a + b + c = 14$ and $a^2 + b^2 + c^2 = 96$, then $(ab + bc + ca) = ?$

(a) 51 (b) 55 (c) 50 (d) 65
(e) None of the above

46. $\frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \frac{1}{10 \times 13} + \frac{1}{13 \times 16} = ?$ **[SSC (10+2) 2007]**

(a) $\frac{5}{16}$ (b) $\frac{3}{16}$ (c) $\frac{7}{16}$ (d) $\frac{11}{16}$

47. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ldots\left(1-\frac{1}{m}\right)=?$

(a) $\frac{1}{m}$ (b) m (c) $m+1$ (d) $\frac{1}{(m-1)}$

(e) None of the above

48. $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\ldots\ldots\left(2-\frac{997}{999}\right)$ is equal to

[SSC CGL 2010]

(a) $\frac{1001}{999}$ (b) $\frac{999}{1001}$

(c) $\frac{1001}{3}$ (d) $\frac{5}{1001}$

49. The value of

$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots\left(1+\frac{1}{150}\right)$ is

[CG PSC 2013]

(a) 65.5 (b) 50.5 (c) 105 (d) 75.5

(e) None of the above

50. If $x=\frac{\sqrt{3}}{2}$, then the value of $\frac{1+x}{1+\sqrt{1+x}}+\frac{1-x}{1-\sqrt{1-x}}$ is equal to

[SSC CGL 2012]

(a) 0 (b) 1

(c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{3}$

51. If $a+\frac{1}{a}=\sqrt{3}$, then the value of $a^6-\frac{1}{a^6}+2$ will be

(a) $3\sqrt{3}$ (b) 5 (c) 1 (d) 2

52. If $x+\frac{1}{x}=2$, then $\frac{2x^2+2}{3x^2+5x+3}=?$

[SSC CGL 2010]

(a) $\frac{4}{11}$ (b) $\frac{1}{2}$ (c) $1\frac{3}{4}$ (d) $2\frac{1}{3}$

53. If $a+\frac{1}{b}=1$ and $b+\frac{1}{c}=1$, then $c+\frac{1}{a}=?$

[SSC CGL 2007]

(a) 4 (b) 24 (c) 3 (d) 1

54. If $x+\frac{a}{x}=b$, then the value of $\frac{x^2+bx+a}{bx^2-x^3}$ is

[SSC FCI 2012]

(a) $a+b$ (b) $\frac{2b}{a}$ (c) $\frac{b}{a}$ (d) ab

55. If $x + \frac{1}{x} = 6$, then $x^4 + \frac{1}{x^4}$ is [SSC FCI 2012]

(a) 1152 (b) 1154
(c) 1148 (d) 1150

56. If $x + \frac{1}{x} = 2$, then what is value of $x - \frac{1}{x}$?

(a) 0 (b) 1 (c) 2 (d) – 2

57. If $x + \frac{1}{x} = 3$, then $x^5 + \frac{1}{x^5}$ is equal to

[SSC CPO 2013]

(a) 123 (b) 83 (c) 92 (d) 112

58. $3x+ 1y = 12$ and $xy = 6$, then the value of $9x^2+4y^2 is$ [SSC FCI 2012]

(a) 76 (b) 80

(c) 72 (d) 74

59. If $\left(x^2 + \frac{1}{x^2}\right) = \frac{17}{4}$, then what is $\left(x^3 - \frac{1}{x^3}\right)$ equal to?

(a) $\frac{75}{16}$ (b) $\frac{63}{8}$
(c) $\frac{95}{8}$ (d) None of these

Exercise © *Higher SkiU Level Questions*

1. A 120 m long rope is cut into 3 equal parts. What is the length of each part?

[Hotel Mgmt. 2010] (a) 40 m (b) 30 m

(c) 50 m (d) 25 m

(e) None of the above

2. When a number is multiplied 5 times with itself, it gives the value 1445. Find the number. [Bank Clerks 2009]

(a) 18 (b) 15

(c) 19 (d) 17

(e) None of the above

3. A board 80 inches long is divided into

4 equal parts. What is the length of each part?

(a) 20 inches *(b)* 25 inches

(c) 15 inches *(d)* 18 inches

(e) None of the above

4. A man spends 2/5 of his income on house rent, 3/10 on food and 1/8 on conveyance. If he has ? 2800 left with him, find his expenditure on food and conveyance together.

(a) ? 7000 *(b)* ? 5500

(c) ? 6700 *(d)* ? 6800

(e) None of the above

5. Mani Ram divides *f* 17200 amongst his

5 sons, 4 daughters and 2 friends. If each daughter receives four times as much as each friend receives and each son receives five times as much as each friend receives, how much does each daughter receive? [Railways 2007]

(a) X 800 *(b)* ? 1500

(c) ? 1000 *(d) X* 1600

6. One-third of Rahul's marks in Mathematics exceeds one-half of his marks in Hindi by 30. If he got 480 marks in the two subjects together, how many marks did he get in Hindi?

[Hotel Mgmt. 2007] *(a)* 200 *(b)* 294 *(c)* 156 *(d)* 196 *(e)* None of the above

7. A container of milk was 4/5 full. When 12 bottles of milk were taken out and 8 bottles of milk were poured into it, it was 3/4 full. How many bottles of milk can the container contain?

(a) 80 *(b)* 40 *(c)* 90 *(d)* 30 *(e)* None of the above

8. The highest score in an innings was $\frac{3}{11}$ of the total score and the next highest was $\frac{3}{11}$ of the remainder. If the scores differ by 18, find the total score.

(a) 244 *(b)* 242 *(c)* 294 *(d)* 255 *(e)* None of the above

9. Mohan gets 3 marks for each correct sum and loses 2 marks for each wrong sum. He attempts 30 sums and obtains 40 marks. The number of sums solved correctly is [SSC CGL 2013]

(a) 25 *(b)* 10 *(c)* 15 *(d)* 20

10. The number obtained by interchanging the digits of a two-digit number is more than the original number by 27 and the sum of the digits is 13. What is the original number? [SSC (10+2)2011]

(a) 58 *(b)* 67

(c) 76 *(d)* 85

11. Of three positive numbers the product of the first and second is 42, that of the second and third is 56 and that of third and first is 48. The third number is

[SSCCPO2013] *(a)* 12 *(b)* 6 *(c)* 7 *(d)* 8

12. It is required to get 40% marks to qualify an exam. A candidate scored 200 marks and failed by 8 marks. What were the maximum marks of that exam?

[Bank Clerks 2011]

(a) 520

(b) 540

(c) 502

(d) Couldn't be determined

(e) None of the above

13. Mr. Arun is on tour and he has *X* 360 for his expenses. If he exceeds his tour by 4 days, he must cut down his daily expenses by *X* 3. For how many days is Mr. Arun out on tour?

(a) 40 *(b)* 20 *(c)* 60 *(d)* 15 *(e)* None of the above

14. From a group of men and women, 15 women leave from it. Then, there are 2 men for each women. After this, 45 men leave. Then, there are 5 women for each men. Find the double of the number of women in the beginning.

(a) 25 *(b)* 23 *(c)* 35 *(d)* 80 *(e)* None of the above

15. Bus fare between Raipur and Mirpur for one adult is six times the fare for one child. If an adult's bus fare is *X* 114, how much amount will be paid by 4 adults and 5 children together for travelling the same distance? [SBI PO 2012]

(a) X 505 *(b) X* 551

(c) X 572 *(d) X* 560

(e) None of the above

16. The total cost of 12 pens and 5 pencils is equal to *X* 111. Also, the cost of 1 pencil is *X* 5 less than the cost of 1 pen. What will be the cost of 8 pens and 9 pencils?

[Bank Clerks 2009] *(a) X* 89 *(b) X* 97

(c) X 91 *(d) X* 78

(e) None of the above

17. When an amount is distributed amongst 14 girls, each of them gets *X* 160 more than the amount received by each girl in that condition when the same amount is distributed equally amongst 18 girls. Find the amount.

(a) X 5040 *(b) X* 10070

(c) X 10080 *(d) X* 5000

(e) None of the above

18. A man had 170 currency notes in all, some of which were of *X* 100 denominations and some of *X* 50 denominations. The total amount of all these currency notes was *X* 10000. How much amount did he have in the denominations of *X* 50?

(a) *X* 4000 *(b)* *X* 9000

(c) *X* 7000 *(d)* *X* 6000

(e) None of the above

19. Mr. Sahni employed a worker for a certain work to be done in some days. He pays *X* 20 to the worker for each working day and the payment is reduced by *X* 3 for each non-working day of the worker. For how many days the worker does remain absent from work if at the end of 120 days, he gets *X* 560 as total remuneration?

(a) 80 *(b)* 60 *(c)* 70 *(d)* 40 *(e)* None of the above

20. A fruit seller has a crate of apples containing one bruised apple for every 40 apples in the crate. If 3 out of every 4 bruised apples are considered unsaleable and there are 9 unsaleable apples in the crate, then how many apples are there in the crate?

(a) 500 *(b)* 390

(c) 632 *(d)* 480

(e) None of the above

21. City A has a population of 136000 which is decreasing at the rate of 2400 per year. City *B* has a population of 84000 which is increasing at the rate of 1600 per year. In how many years will the population of two cities become equal?

(a) 15 *(b)* 18 *(c)* 13 *(d)* 19 *(e)* None of the above

Answer with Solutions

Exercise© *Base Level Questions*

1. (*e*) $7059 - 2350 + 1936 = ? \times 50$

$\Rightarrow \quad 8995 - 2350 = ? \times 50$

$\Rightarrow \quad ? = \dfrac{6645}{50} = 132.9$

2. (*c*) $\dfrac{7}{4+\sqrt{2}} = \dfrac{7}{4+\sqrt{2}} \times \dfrac{(4-\sqrt{2})}{(4-\sqrt{2})}$

[rationalising]

$= \dfrac{7(4-\sqrt{2})}{16-2} \quad [\because (a+b)(a-b) = a^2 - b^2]$

$= \dfrac{7(4-\sqrt{2})}{14} = \dfrac{4-\sqrt{2}}{2} = \dfrac{4-1.4142}{2}$

$= \dfrac{2.5858}{2} = 1.2929$

3. (*d*) Required number

$= 786 \times 964 = 757704$

4. (*d*) Given expression

$= \dfrac{(a+b)^2 - (a-b)^2}{ab} = \dfrac{4ab}{ab} = 4$

where, $a = 999$, $b = 588$

5. (*b*) Given expression

$= \dfrac{(a+b)^2 + (a-b)^2}{a^2+b^2} = \dfrac{2(a^2+b^2)}{(a^2+b^2)} = 2$

where, $a = 238$, $b = 131$

6. (*d*) Given expression

$= \dfrac{\frac{1}{5} + 999 \times \frac{494}{495} \times 99}{4} = ?$

$= \dfrac{\frac{1}{5} + \left(999 + \frac{494}{495}\right) \times 99}{4}$

$= \dfrac{\frac{1}{5} + 999 \times 99 + \frac{494}{495} \times 99}{4}$

$= \dfrac{\frac{1}{5} + (1000-1) \times 99 + \frac{494}{5}}{4}$

$= \dfrac{\frac{495}{5} + (99000 - 99)}{4}$

$= \dfrac{99 + 99000 - 99}{4} = \dfrac{99000}{4}$

$= 24750$

7. (*d*) Given, $x = \sqrt{3} + \sqrt{2}$

$$\therefore \quad \frac{1}{x} = \frac{1}{(\sqrt{3}+\sqrt{2})} \times \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2}} \quad \text{[rationalising]}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{\sqrt{3}-\sqrt{2}}{3-2}$$

$$\therefore \quad x + \frac{1}{x} = \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2} = 2\sqrt{3}$$

8. (*c*) We know that,

$$a^3 + b^3 + c^3 - 3abc$$

$$= (a+b+c)\frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$= (225 + 226 + 227)\frac{1}{2}[1 + 1 + 4]$$

$$= 678 \times 3 = 2034$$

9. (*a*) $\dfrac{(0.96)^3 - (0.1)^3}{(0.96)^2 + 0.096 + 0.01}$

$$(0.96)^3 - (0.1)^3 = (0.96 - 0.1)[(0.96)^2 + 0.96 \times 0.1 + (0.1)^2]$$

$$[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$$

$$= \frac{(0.96 - 0.1)[(0.96)^2 + 0.096 + 0.01]}{(0.96)^2 + 0.096 + 0.01}$$

$$= 0.96 - 0.1 = 0.86$$

10. (*b*) $\dfrac{\sqrt{0.0001} \times \sqrt[3]{1000000}}{\sqrt[3]{0.125} \times \sqrt[3]{0.008}}$

$$= \frac{\sqrt{0.01 \times 0.01} \times \sqrt[3]{100 \times 100 \times 100}}{\sqrt[3]{0.5 \times 0.5 \times 0.5} \times \sqrt[3]{0.2 \times 0.2 \times 0.2}}$$

$$= \frac{0.01 \times 100}{0.5 \times 0.2} = \frac{1}{0.1} = 10$$

11. (*e*) Given, $3.5 \times (60 \div 0.5) = ?$

$$\Rightarrow \quad ? = 3.5 \times (60 \div 2.5) = 3.5 \times \frac{60}{2.5}$$

$$= 3.5 \times 24 = 84$$

12. (*d*) Given expression

$$= 6 - [9 - \{18 - (15 - \overline{12 - 9})\}]$$

$$= 6 - [9 - \{18 - (15 - 12 + 9)\}]$$

$$= 6 - [9 - \{18 - 12\}]$$

$$= 6 - [9 - 6] = 6 - 3 = 3$$

13. (*c*) $? = 3\frac{2}{3} + 2\frac{3}{4} + 1\frac{1}{2}$

$= 3 + 2 + 1 + \left(\frac{2}{3} + \frac{3}{4} + \frac{1}{2}\right)$

$= 6 + \left(\frac{8 + 9 + 6}{12}\right) = 6 + \left(\frac{23}{12}\right)$

$= 6 + \left(1\frac{11}{12}\right) = 7\frac{11}{12}$

14. (*d*) $? = \frac{5}{8} \times 2\frac{3}{5} + \frac{4}{9}$

$= \frac{5}{8} \times \frac{13}{5} \times \frac{9}{4} = \frac{117}{32} = 3\frac{21}{32}$

15. (*c*) $? = 10.8 \times 5.5 \times 8.4 = 498.96$

16. (*a*) $(4 \times 4 \times 4 \times 4 \times 4 \times 4)^5$

$\times (4 \times 4 \times 4)^8 + (4)^3 = (64)^?$

$\Rightarrow \quad (4^6)^5 \times (4^3)^8 \times \frac{1}{(4)^3} = (4^3)^?$

$\Rightarrow \quad \frac{(4)^{30} \times (4)^{24}}{(4)^3} = (4)^{3 \times ?}$

$\Rightarrow \quad 4^{30 + 24 - 3} = 4^{3 \times ?}$

$\Rightarrow \quad 4^{51} = 4^{3 \times ?} \Rightarrow 3 \times ? = 51$

$\therefore \quad ? = \frac{51}{3} = 17$

17. (*e*) $8648 - 7652 = ? \times 40$

$996 = ? \times 40 \Rightarrow ? = \frac{996}{40} = 24.9$

18. (*a*) $? = 683.46 - 227.39 - 341.85$

$= 683.46 - 569.24 = 114.22$

19. (*b*) $\frac{13}{63} + \frac{104}{14} \times \frac{52}{19} = ?$

$\Rightarrow \quad ? = \frac{\frac{13}{63}}{\frac{104}{14}} \times \frac{52}{19} = \frac{13 \times 14}{63 \times 104} \times \frac{52}{19}$

$= \frac{13 \times 7 \times 52}{63 \times 52 \times 19}$

$= \frac{13 \times 7}{63 \times 19} = \frac{13}{9 \times 19} = \frac{13}{171}$

$\therefore \quad ? = \frac{13}{171}$

20. (*a*) $(?)^{\frac{1}{2}} = (5568 + 87)^{\frac{1}{3}} + (72 \times 2)^{\frac{1}{2}}$

$= (64)^{\frac{1}{3}} + (144)^{\frac{1}{2}}$

$\therefore \quad ? = (4 + 12)^2 = 256$

21. (*d*) $(?)^2 = \sqrt{13^2 + 28 \div 4 - (3)^3 + 107}$

$$= \sqrt{169 + 7 - 27 + 107} = \sqrt{256} = 16$$

$\therefore \quad ? = \sqrt{16} = 4$

22. (*b*) $(70 \div 100)^{? + 3} = (0.49)^4 \times (0.343)^4 \div (0.2401)^4$

$$(0.7)^{? + 3} = \left(\frac{0.49 \times 0.343}{0.2401}\right)^4$$

$$(0.7)^{? + 3} = (0.7)^4$$

On comparing the exponents both sides, we get

$$? + 3 = 4$$

$\therefore \quad ? = 4 - 3 = 1$

23. (*a*) $? = \dfrac{5 - \left[\dfrac{3}{4} + \left\{\dfrac{5}{2} - \left(\dfrac{1}{2} + \overline{\dfrac{1}{6} - \dfrac{1}{7}}\right)\right\}\right]}{2}$

$$= \frac{5 - \left[\frac{3}{4} + \left\{\frac{5}{2} - \left(\frac{1}{2} + \frac{7-6}{42}\right)\right\}\right]}{2}$$

$$= \frac{5 - \left[\frac{3}{4} + \left\{\frac{5}{2} - \left(\frac{1}{2} + \frac{1}{42}\right)\right\}\right]}{2}$$

$$= \frac{5 - \left[\frac{3}{4} + \left\{\frac{5}{2} - \left(\frac{21+1}{42}\right)\right\}\right]}{2}$$

$$= \frac{5 - \left[\frac{3}{4} + \left\{\frac{5}{2} - \frac{22}{42}\right\}\right]}{2}$$

$$= \frac{5 - \left[\frac{3}{4} + \left\{\frac{105-22}{42}\right\}\right]}{2}$$

$$= \frac{5 - \left[\frac{3}{4} + \frac{83}{42}\right]}{2} = \frac{5 - \left[\frac{63+166}{84}\right]}{2}$$

$$= \frac{5 - \frac{229}{84}}{2} = \frac{\frac{420-229}{84}}{2} = \frac{\frac{191}{84}}{2}$$

$$= \frac{191}{84 \times 2} = \frac{191}{168} = 1\frac{23}{168}$$

24. (*c*) Given expression

$$= \frac{\left[\frac{13}{4} + \left\{\frac{5}{4} - \frac{1}{2}\left(\frac{5}{2} - \frac{3-2}{12}\right)\right\}\right]}{4 \times \frac{1}{12}}$$

$$= \frac{\left[\frac{13}{4} + \left\{\frac{5}{4} - \frac{1}{2}\left(\frac{5}{2} - \frac{1}{12}\right)\right\}\right]}{\frac{1}{3}}$$

$$= \frac{\left[\frac{13}{4} + \left\{\frac{5}{4} - \frac{1}{2}\left(\frac{30-1}{12}\right)\right\}\right]}{\frac{1}{3}}$$

$$= \frac{\left[\frac{13}{4} + \left\{\frac{5}{4} - \frac{29}{24}\right\}\right]}{\frac{1}{3}}$$

$$= \frac{\left[\frac{13}{4} + \left\{\frac{30-29}{24}\right\}\right]}{\frac{1}{3}} = \frac{\left[\frac{13}{4} + \frac{1}{24}\right]}{\frac{1}{3}}$$

$$= \frac{13}{4} \times 24 \times 3 = 13 \times 18 = 234$$

25. (*a*) Given expression

$$= \frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca}$$

$$= a + b + c$$

$$= 0.5 + 0.2 + 0.3$$

where, $a = 0.5$, $b = (+0.2)$, $c = 0.3$

26. (*c*) We know that, $a^3 + b^3 + c^3 - 3abc$

$$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\Rightarrow (a + b + c) = \left(\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca}\right) \quad \ldots(\text{i})$$

Given that,

$$\frac{(2.247)^3 + (1.730)^3 + (1.023)^3 - 3 \times 2.247 \times 1.730 \times 1.023}{(2.247)^2 + (1.730)^2 + (1.023)^2 - (2.247 \times 1.730) - (1.730 \times 1.023) - (2.247 \times 1.023)}$$

$$= (2.247 + 1.730 + 1.023) \quad [\text{from Eq. (i)}]$$

$$= 5.000 = 5$$

27. (*c*) $\dfrac{725 \times 725 \times 725 + 371 \times 371 \times 371}{725 \times 725 - 725 \times 371 + 371 \times 371}$

$$= 725 + 371 = 1096$$

$$\left[\because \frac{a^3 + b^3}{a^2 - ab + b^2} = \frac{(a + b)(a^2 - ab + b^2)}{(a^2 - ab + b^2)} = a + b\right]$$

28. (*b*) Let $8.73 = a$ and $4.27 = b$

Then, given expression $= \dfrac{a^3 + b^3}{a^2 - ab + b^2}$

$$= \frac{(a + b)(a^2 - ab + b^2)}{(a^2 - ab + b^2)} = (a + b)$$

$$= 8.73 + 4.27 = 13$$

29. (*c*) $\sqrt{289} - \sqrt{625} + \sqrt{25}$

$$= \sqrt{17 \times 17} - \sqrt{25 \times 25} + \sqrt{5 \times 5}$$

$$= 17 - 25 + 5 = 17 - 5 = 12$$

30. (c) We know that, $(a+b)^2 = a^2 + b^2 + 2ab$

$$= 234 + 2 \times 108 = 450$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$= 234 - 2 \times 108 = 18$$

$$\therefore \frac{(a+b)^2}{(a-b)^2} = \frac{450}{18} = 25 \Rightarrow \left(\frac{a+b}{a-b}\right)^2 = 25$$

$$\therefore \qquad \frac{a+b}{a-b} = \sqrt{25} = 5$$

31. (b) $a^2 + 1 = a \quad \Rightarrow \quad a + \frac{1}{a} = 1$

On squaring both sides, we get

$$a^2 + \frac{1}{a^2} + 2 = 1$$

$$\Rightarrow \qquad a^2 + \frac{1}{a^2} = -1$$

On cubing both sides, we get

$$\left(a^2 + \frac{1}{a^2}\right)^3 = (-1)^3$$

$$\Rightarrow a^6 + \frac{1}{a^6} + 3a^2 \times \frac{1}{a^2}\left(a^2 + \frac{1}{a^2}\right) = -1$$

$$\Rightarrow \qquad a^6 + \frac{1}{a^6} + 3 \times (-1) = -1$$

Now, $\qquad a^6 + \frac{1}{a^6} + 1 = 3$

As, $a^{12} + a^6 + 1$ can also be written as

$$a^6 + \frac{1}{a^6} + 1$$

$$\therefore \qquad a^{12} + a^6 + 1 = 3$$

32. (b) Given, $0.764y = 1.236x$

$$\Rightarrow \qquad \frac{y}{x} = \frac{1.236}{0.764}$$

Now, $\qquad \frac{y-x}{y+x} = \frac{y/x - 1}{y/x + 1}$

$$\Rightarrow \frac{\frac{1.236}{0.764} - 1}{\frac{1.236}{0.764} + 1} = \frac{1.236 - 0.764}{1.236 + 0.764} = \frac{0.472}{2.000}$$

$$= 0.236$$

33. (c) $\because \qquad a + \frac{1}{b} = 1$

$$ab + 1 = b \qquad \ldots\text{(i)}$$

Also, $\qquad b + \frac{1}{c} = 1 \Rightarrow b = 1 - \frac{1}{c} \qquad \ldots\text{(ii)}$

From Eqs. (i) and (ii), we get

$$ab + 1 = 1 - \frac{1}{c}$$

$$\Rightarrow \qquad ab = \frac{-1}{c} \Rightarrow abc = -1$$

34. (*a*) Since, $a + b + c = 0$

Then, $a + b = -c$ ...(i)

$a + c = -b$ ...(ii)

$b + c = -a$ ...(iii)

Now, $\left[\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right]$

$\left[\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right]$

Now, putting the value of $a + b$, $b + c$ and $c + a$ from Eqs. (i), (ii) and (iii), we get

$$\left[\frac{(-c)}{c} + \frac{(-a)}{a} + \frac{(-b)}{b}\right]$$

$$\left[\frac{a}{-a} + \frac{b}{-b} + \frac{c}{-c}\right]$$

$[(-1) + (-1) + (-1)]\,[(-1) + (-1) + (-1)]$

$= (-3) \times (-3) = 9$

35. (*b*) Since, $a * b = a + b + \frac{a}{b}$

$\therefore \quad 12 * 4 = 12 + 4 + \frac{12}{4}$

$= 12 + 4 + 3 = 19$

36. (*a*) $x = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

$\Rightarrow \quad x = \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$= \frac{2 + 1 + 2\sqrt{2}}{1} = 3 + 2\sqrt{2}$

$x = 3 + 2\sqrt{2}$ ...(i)

and $\quad x - y = 4\sqrt{2}$

$\Rightarrow y = x - 4\sqrt{2} = 3 + 2\sqrt{2} - 4\sqrt{2}$

[from Eq. (i)]

$= 3 - 2\sqrt{2}$

Now, $x^2 + y^2 = (3 + 2\sqrt{2})^2 + (3 - 2\sqrt{2})^2$

$= 9 + 8 + 12\sqrt{2} + 9 + 8 - 12\sqrt{2}$

$= 34$

37. (*d*) Given, $x + \frac{a}{x} = 1$

$\Rightarrow \quad x^2 + a = x$ ...(i)

$\Rightarrow \quad x^2 - x = -a$ ...(ii)

Now, $\quad \frac{x^2 + x + a}{x^3 - x^2} = \frac{x + x}{x\,(x^2 - x)}$

[from Eq. (i)]

$= \frac{2x}{x\,(-a)}$ [from Eq. (ii)]

$= \frac{2}{-a}$

38. (*a*) $\sqrt{28 - 6\sqrt{3}} = \sqrt{3}\,a + b$

$\Rightarrow \quad \sqrt{(1)^2 + (3\sqrt{3})^2 - 6\sqrt{3}} = \sqrt{3}\,a + b$

$\Rightarrow \quad \sqrt{(1 - 3\sqrt{3})^2} = \sqrt{3}a + b$

$\Rightarrow \quad (1 - 3\sqrt{3}) = \sqrt{3}\,a + b$

On comparing, we get

$$a = -3,\ b = 1$$

$\therefore \quad a + b = -3 + 1 = -2$

39. (*b*) We know that,

$$(x + y)^3 = x^3 + y^3 + 3xy\,(x + y)$$

$$1 = x^3 + y^3 + 3xy$$

[given, $x + y = 1$]

$\Rightarrow \quad x^3 + y^3 + 3xy = 1$

40. (*a*) $a = (\sqrt{2} - 1)^{\frac{1}{3}}$

$$a^3 = \sqrt{2} - 1$$

$(a - a^{-1})^3 + 3\,(a - a^{-1})$

$$= \left(a - \frac{1}{a}\right)^3 + 3\left(a - \frac{1}{a}\right)$$

$$= a^3 - \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right) + 3\left(a - \frac{1}{a}\right)$$

$$= \sqrt{2} - 1 - \frac{1}{\sqrt{2} - 1}$$

$$= (\sqrt{2} - 1) - \frac{1}{(\sqrt{2} - 1)} \times \frac{(\sqrt{2} + 1)}{(\sqrt{2} + 1)}$$

$$= \sqrt{2} - 1 - (\sqrt{2} + 1) = -2$$

41. (*a*) Given, $x = \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$

$$y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$x + y = \frac{(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2}{(\sqrt{3} - 1)\,(\sqrt{3} + 1)}$$

$$= \frac{3 + 1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3}}{3 - 1} = \frac{8}{2}$$

$$= x + y = 4$$

and $xy = \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1} = 1$

$\therefore \quad \dfrac{x^2}{y} + \dfrac{y^2}{x} = \dfrac{x^3 + y^3}{xy}$

$$= \frac{(x + y)^3 - 3xy\,(x + y)}{xy}$$

$$= \frac{(4)^3 - 3 \times 1 \times (4)}{1}$$

$$= \frac{64 - 12}{1} = 52$$

42. (*b*) $\dfrac{p}{q}+\dfrac{q}{p}=\dfrac{p^2+q^2}{pq}$

$$=\frac{(p+q)^2-2pq}{pq}=\frac{(10)^2-2\times 5}{5}$$

$$=\frac{100-10}{5}=\frac{90}{5}=18$$

43. (*c*) $x+y=18$

$\because\ x^2+y^2=(x+y)^2-2xy$

$$=(18)^2-2\times 72=324-144=180$$

44. (*a*) Let, $(m-n)=a$, $(n-r)=b\ (r-m)=c$, we get

$$a+b+c=0$$

$\therefore\qquad a^3+b^3+c^3=3abc$

$\therefore$ Given expression

$$=\frac{a^3+b^3+c^3}{6\,abc}=\frac{3abc}{6\,abc}=\frac{1}{2}$$

45. (*c*) We know that,

$(a+b+c)^2=(a^2+b^2+c^2)+2\,(ab+bc+ca)$

$\Rightarrow\qquad 196=96+2\,(ab+bc+ca)$

$\Rightarrow\qquad 2\,(ab+bc+ca)$

$$=196-96=100$$

$\therefore\qquad (ab+bc+ca)=\dfrac{100}{2}=50$

46. (*a*) Given expression can be written as

$$\frac{1}{3}\left[\left(1-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{10}\right)+\left(\frac{1}{10}-\frac{1}{13}\right)+\left(\frac{1}{13}-\frac{1}{16}\right)\right]$$

$$=\frac{1}{3}\left[1-\frac{1}{16}\right]=\frac{1}{3}\left(\frac{16-1}{16}\right)$$

$$=\frac{1}{3}\times\frac{15}{16}=\frac{5}{16}$$

47. (*a*) Given expression

$$=\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ldots\left(1-\frac{1}{m-1}\right)\left(1-\frac{1}{m}\right)$$

$$=\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\ldots\times\frac{(m-2)}{(m-1)}\times\frac{(m-1)}{m}=\frac{1}{m}$$

48. (*c*) Given expression

$$=\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\ldots\left(2-\frac{997}{999}\right)$$

$$=\frac{5}{3}\times\frac{7}{5}\times\frac{9}{7}\times\ldots\times\frac{1001}{999}=\frac{1001}{3}$$

49. (*d*) $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\dots\left(1+\frac{1}{150}\right)$

$$=\frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\times\dots\times\frac{151}{150}$$

$$=\frac{151}{2}=75.5$$

50. (*b*) Given expression

$$\frac{1+x}{1+\sqrt{1+x}}+\frac{1-x}{1-\sqrt{1-x}}$$

On putting $x=\frac{\sqrt{3}}{2}$, we get

$$\frac{1+\frac{\sqrt{3}}{2}}{1+\sqrt{1+\frac{\sqrt{3}}{2}}}+\frac{1-\frac{\sqrt{3}}{2}}{1-\sqrt{1-\frac{\sqrt{3}}{2}}}$$

$$=\frac{2+\sqrt{3}}{\sqrt{2}\,(\sqrt{2}+\sqrt{2+\sqrt{3}})}+\frac{2-\sqrt{3}}{\sqrt{2}\,(\sqrt{2}-\sqrt{2-\sqrt{3}})}$$

$$=\frac{2+\sqrt{3}}{2+\sqrt{4+2\sqrt{3}}}+\frac{2-\sqrt{3}}{2-\sqrt{4-2\sqrt{3}}}$$

$$\Rightarrow \frac{2+\sqrt{3}}{2+\sqrt{(1)^2+(\sqrt{3})^2+2\sqrt{3}}}+\frac{2-\sqrt{3}}{2-\sqrt{(1)^2+(\sqrt{3})^2-2\sqrt{3}}}$$

$$\Rightarrow \frac{2+\sqrt{3}}{2+\sqrt{(1+\sqrt{3})^2}}+\frac{2-\sqrt{3}}{2-\sqrt{(\sqrt{3}-1)^2}}$$

$$=\frac{2+\sqrt{3}}{2+(1+\sqrt{3})}+\frac{2-\sqrt{3}}{2-(\sqrt{3}-1)}$$

$$=\frac{2+\sqrt{3}}{3+\sqrt{3}}+\frac{2-\sqrt{3}}{3-\sqrt{3}}$$

$$=\frac{(2+\sqrt{3})(3-\sqrt{3})+(2-\sqrt{3})(3+\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}=1$$

$$\Rightarrow \frac{6+3\sqrt{3}-2\sqrt{3}-3+6-3\sqrt{3}+2\sqrt{3}-3}{(3+\sqrt{3})(3-\sqrt{3})}$$

$$\Rightarrow \frac{6}{9+3\sqrt{3}-3\sqrt{3}-3}=\frac{6}{6}=1$$

51. (*d*) $a+\frac{1}{a}=\sqrt{3}$ $\quad\dots$(i)

On squaring both sides

$$\Rightarrow \quad a^2+\frac{1}{a^2}+2=3$$

$$\Rightarrow \quad a^2+\frac{1}{a^2}=1 \quad \dots\text{(ii)}$$

Now, multiplying Eqs. (i) and (ii), we get

$$\left(a+\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}\right)=\sqrt{3}$$

$\Rightarrow \quad a^3+\frac{a}{a^2}+\frac{a^2}{a}+\frac{1}{a^3}=\sqrt{3}$

$\Rightarrow \quad a^3+\frac{1}{a^3}+\left(\frac{1}{a}+a\right)=\sqrt{3}$

$\Rightarrow \quad a^3+\frac{1}{a^3}+\sqrt{3}=\sqrt{3}$ [from Eq. (i)]

$\Rightarrow \quad a^3+\frac{1}{a^3}=0$

$\Rightarrow \quad a^6=-1$

$\therefore \quad a^6-\frac{1}{a^6}+2=(-1)^6-\frac{1}{(-1)^6}+2$

$=1-1+2=2$

52. (*a*) Given expression $=\frac{2x^2+2}{3x^2+3+5x}$

On dividing numerator and denominator by x, we get

$$=\frac{2\left(x+\frac{1}{x}\right)}{3\left(x+\frac{1}{x}\right)+5}=\frac{2\times 2}{3\times 2+5}=\frac{4}{11}$$

[given $x+\frac{1}{x}=2$]

53. (*d*) Given that, $a+\frac{1}{b}=1$

$\Rightarrow \quad a=\left(1-\frac{1}{b}\right)=\frac{b-1}{b}$

$\Rightarrow \quad \frac{1}{a}=\frac{b}{(b-1)}$ [reciprocal]

and $\quad b+\frac{1}{c}=1$

$\Rightarrow \quad \frac{1}{c}=(1-b)$

$\Rightarrow \quad c=\frac{1}{(1-b)}$

$\therefore \quad c+\frac{1}{a}=\frac{1}{(1-b)}+\frac{b}{(b-1)}$

$=\frac{1}{(1-b)}-\frac{b}{(1-b)}$

$=\frac{(1-b)}{(1-b)}=1$

54. (*b*) $x+\frac{a}{x}=b \quad \Rightarrow \quad \frac{x^2+a}{x}=b$

$\Rightarrow \quad x^2+a=bx$ ...(i)

Now, $\frac{x^2+bx+a}{bx^2-x^3} \Rightarrow \frac{(x^2+a)+bx}{bx^2-x^3}$

[using Eq. (i)]

$= \dfrac{2b\ x}{bx^2 - x^3} = \dfrac{2b}{bx - x^2} = \dfrac{2b}{a}$

$\begin{bmatrix} \because x^2 + bx \\ bx - x^2 = a \end{bmatrix}$

55. (*b*) $x + \dfrac{1}{x} = 6$

On squaring both sides, we get

$$\left(x + \frac{1}{x}\right)^2 = (6)^2$$

$$\Rightarrow \qquad x^2 + \frac{1}{x^2} + 2 = 36$$

$$\Rightarrow \qquad x^2 + \frac{1}{x^2} = 34$$

On squaring both sides, we get

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (34)^2$$

$$\Rightarrow \qquad x^4 + \frac{1}{x^4} + 2 = 1156$$

$$\Rightarrow \qquad x^4 + \frac{1}{x^4} = 1154$$

56. (*a*) Given that, $x + \dfrac{1}{x} = 2$ ...(i)

On squaring both sides, we get

$$\left(x + \frac{1}{x}\right)^2 = 4$$

$$\Rightarrow \qquad x^2 + \frac{1}{x^2} + 2 = 4$$

$$\Rightarrow \qquad x^2 + \frac{1}{x^2} = 2 \qquad \text{...(ii)}$$

Now, we have

$$\left(x - \frac{1}{x}\right)^2 = \left(x^2 + \frac{1}{x^2}\right) - 2$$

$$= 2 - 2 = 0 \qquad \text{[from Eq. (ii)]}$$

$$\therefore \qquad x - \frac{1}{x} = 0$$

57. (*a*) $x + \dfrac{1}{x} = 3$ ....(i)

On squaring both sides, we get

$$\left(x + \frac{1}{x}\right)^2 = (3)^2$$

$$\Rightarrow \qquad x^2 + \frac{1}{x^2} + 2 = 9$$

$$\Rightarrow \qquad x^2 + \frac{1}{x^2} = 7 \qquad \text{...(ii)}$$

Again squaring both sides, we get

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (7)^2$$

$$\Rightarrow \quad x^4 + \frac{1}{x^4} + 2 = 49$$

$$\Rightarrow \quad x^4 + \frac{1}{x^4} = 47 \quad \text{...(iii)}$$

On cubing both sides, we get

$$\left(x + \frac{1}{x}\right)^3 = (3)^3$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} + 9 = 27 \quad \left[\because \left(x + \frac{1}{x}\right) = 3\right]$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} = 18 \quad \text{...(iv)}$$

On multiplying Eqs. (i) and (iii), we get

$$\Rightarrow \quad \left(x^4 + \frac{1}{x^4}\right)\left(x + \frac{1}{x}\right) = 47 \times 3$$

$$\Rightarrow \quad x^5 + \frac{1}{x^5} + x^3 + \frac{1}{x^3} = 141$$

$$\Rightarrow \quad x^5 + \frac{1}{x^5} + 18 = 141 \quad \text{[from Eq. (iv)]}$$

$$\Rightarrow \quad x^5 + \frac{1}{x^5} = 123$$

58. (c) Given equations are

$$3x + 2y = 12 \quad \text{...(i)}$$

$$xy = 6 \quad \text{...(ii)}$$

On squaring Eq. (i) on both sides, we get

$$(3x + 2y)^2 = (12)^2$$

$\Rightarrow \quad 9x^2 + 4y^2 + 12xy = 144$

$\Rightarrow \quad 9x^2 + 4y^2 = 144 - 12xy$

$= 144 - 12(6) = 144 - 72 = 72$

59. (*b*) $\left(x^2 + \frac{1}{x^2}\right) = \frac{17}{4}$

$\Rightarrow \quad x^2 + \frac{1}{x^2} + 2 - 2 = \frac{17}{4}$

$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 + 2 = \frac{17}{4}$

$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 = \frac{17}{4} - 2$

$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 = \frac{3}{2}$

On cubing both sides, we get

$\left(x - \frac{1}{x}\right)^3 = \left(\frac{3}{2}\right)^3$

$\Rightarrow \quad x^3 - \frac{1}{x^3} - 3 \times \frac{1}{x} \cdot x\left(x - \frac{1}{x}\right) = \frac{27}{8}$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = \frac{27}{8} + 3 \times \left(\frac{3}{2}\right)$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = \frac{27}{8} + \frac{9}{2}$

$\Rightarrow \quad \left(x^3 - \frac{1}{x^3}\right) = \frac{63}{8}$

Exercise © *Higher SkiU Level Questions*

1. (a) Required length $= \frac{120}{3} = 40$ m

2. (d) Let the number $= x$

According to the question,

$$5x^2 = 1445$$

$$\therefore \quad x^2 = \frac{1445}{5} = 289$$

$$\therefore \quad x = \sqrt{289} = 17$$

3. (a) Required length $= \frac{80}{4} = 20$ inches

4. (d) Part of income left

$$= 1 - \left(\frac{2}{5} + \frac{3}{10} + \frac{1}{8}\right)$$

$$= 1 - \left(\frac{16 + 12 + 5}{40}\right)$$

$$= 1 - \frac{33}{40} = \frac{40 - 33}{40} = \frac{7}{40}$$

Let the monthly income $= x$.

According to the question,

$$\frac{7}{40}x = 2800$$

$$\therefore \quad x = \frac{2800 \times 40}{7}$$

$$= 400 \times 40 = ₹\ 16000$$

Expenditure on food $= \frac{3}{10} \times 16000 = ₹\ 4800$

Expenditure on conveyance $= \frac{1}{8} \times 16000$

$$= ₹\ 2000$$

∴ Expenditure on food and conveyance together

$$= ₹\ (4800 + 2000) = ₹\ 6800$$

5. (d) Let share of each friend = x. Then, share of each daughter = 4x, and share of each son = $5x$. According to the question,

$$5 \times 5x + 4 \times 4x + 2 \times x = 17200$$

$\Rightarrow \quad 25x + 16x + 2x = 17200$

$\Rightarrow \quad 43x = 17200$

$\therefore \quad x = \dfrac{17200}{43} = 400$

$\therefore$ Share of each daughter

$= 4x = 4 \times 400$

$= ₹\ 1600$

6. (*c*) Let Rahul's marks in Mathematics $= x$

and in Hindi $= y$

According to the question,

$$\frac{1}{3}x - \frac{1}{2}y = 30$$

$\Rightarrow \quad 2x - 3y = 180 \quad \ldots\text{(i)}$

Also, given

$$x + y = 480 \quad \ldots\text{(ii)}$$

By solving Eqs. (i) and (ii), we get

$$x = 324$$

and $\quad y = 156$

7. (*a*) Let x bottles can fill the container completely.

Acccording to the question,

$$\frac{4}{5}x - \frac{3}{4}x = (12 - 8)$$

$\Rightarrow \quad \dfrac{16x - 15x}{20} = (12 - 8)$

$\Rightarrow \quad \dfrac{x}{20} = 4$

$\therefore \quad x = 80$

$\therefore$ Required number of bottles $= 80$

8. (*b*) Let total score $= x$.

Then, the highest score $= \dfrac{3x}{11}$.

Remainder $= \left(x - \dfrac{3x}{11}\right) = \dfrac{8x}{11}$

Next highest score $= \dfrac{3}{11}$ of $\dfrac{8x}{11} = \dfrac{24x}{121}$

According to the question,

$$\frac{3x}{11} - \frac{24x}{121} = 18$$

$\Rightarrow \quad 33x - 24x = 18 \times 121$

$\Rightarrow \quad 9x = 18 \times 121$

$\therefore \quad x = \dfrac{18 \times 121}{9} = 242$

9. (*d*) Let the questions solved correctly by Mohan $= x$

According to the question,

$$3 \times x - (30 - x) \times 2 = 40$$

$\Rightarrow \quad 3x - 60 + 2x = 40$

$\Rightarrow \quad 5x - 60 = 40$

$\Rightarrow \quad 5x = 40 + 60 \Rightarrow x = \dfrac{100}{5} = 20$

$\therefore$ Mohan attempted 20 questions correctly.

10. (*a*) Let the ten's-digit $= x$

and unit's digit $= y$

$\therefore$ The original number $= 10x + y$

After interchanging the digits

$\therefore$ New number $= 10y + x$

Now, according to the question,

$$(10y + x) - (10x + y) = 27$$

$$\Rightarrow \quad 9y - 9x = 27$$

$$\Rightarrow \quad y - x = 3 \quad \ldots(i)$$

$$\text{and} \quad y + x = 13 \quad \ldots(ii)$$

On solving Eqs. (i) and (ii), we get

$$y = 8 \quad \text{and} \quad x = 5$$

$\therefore$ Required number

$$= 10 \times 5 + 8 = 58$$

11. (*d*) Let first, second and third numbers be x, y and z, respectively.

Then, $xy = 42 \quad \ldots(i)$

$yz = 56 \quad \ldots(ii)$

$xz = 48 \quad \ldots(iii)$

Multiplying Eqs. (i), (ii) and (iii), we get

$$(xyz)^2 = 42 \times 56 \times 48$$

$$\Rightarrow \quad (xyz)^2 = 112896$$

$$\Rightarrow \quad xyz = 336 \quad \ldots(iv)$$

Dividing Eq. (iv) by Eq. (i), we get

$$\frac{xyz}{xy} = \frac{336}{42} \Rightarrow z = 8$$

12. (*a*) Let the maximum marks $= x$.

Acoding to the question,

$$\frac{40x}{100} = 200 + 8$$

$$\Rightarrow \quad x = \frac{208 \times 10}{4} = 520$$

13. (*b*) Let Mr. Arun is on tour for x days.

Then, according to the question,

Difference in expenses per day for original and extended tour $= 3$

$$\frac{360}{x} - \frac{360}{x+4} = 3$$

$$\Rightarrow \quad \frac{1}{x} - \frac{1}{x+4} = \frac{3}{360} = \frac{1}{120}$$

$$\frac{x + 4 - x}{x(x+4)} = \frac{1}{120}$$

$$\Rightarrow \quad x(x+4) = 4 \times 120 = 480$$

$$\Rightarrow \quad x^2 + 4x - 480 = 0$$

$$\Rightarrow \quad x^2 + 24x - 20x - 480 = 0$$

$\Rightarrow \quad x(x+24) - 20(x+24) = 0$

$\Rightarrow \quad (x+24)(x-20) = 0$

$\therefore \quad x = 20$

(we ignore –ve value of x, that is, –24 because days cannot be negative)

14. (*d*) Let at present, there are x men, then number of women $= 5x$

Before the men had left

number of men $= x + 45$ and

number of women $= 5x$.

$\therefore \quad x + 45 = 2 \times 5x$

$\Rightarrow \quad 9x = 45$

$$\therefore \quad x = \frac{45}{9} = 5$$

Hence, number of women in the beginning

$= 5x + 15 = 5 \times 5 + 15$

$= 25 + 15 = 40$

$\therefore$ Double of the women $= 40 \times 2 = 80$

15. (*b*) Adult fare = ₹ 114

According to the question,

$$\text{Child fare} = \frac{1}{6} \times \text{Adult fare}$$

$$= \frac{1}{6} \times 114 = ₹\ 19$$

$\therefore$ Total amount paid by 4 adults and 5 childrens are for the same distance

$= ₹\ (4 \times 114 + 5 \times 19)$

$= ₹\ (456 + 95) = ₹\ 551$

16. (*c*) Let the cost of 1 pencil $= x$

$\therefore$ Cost of 1 pen $= (x + 5)$

According to the question,

$12(x+5) + 5x = 111$

$\Rightarrow \quad 12x + 60 + 5x = 111$

$\Rightarrow \quad 17x = 111 - 60 = 51$

$$\Rightarrow \quad x = \frac{51}{17} = 3$$

$\therefore$ CP of 1 pencil = ₹ 3

CP of 1 pen = ₹ 8

$\therefore$ Total CP of 8 pens and 9 pencils

$= (8 \times 8 + 9 \times 3)$

$= ₹\ (64 + 27) = ₹\ 91$

17. (*c*) Let the amount $= x$.

According to the question,

$$\frac{x}{14} - \frac{x}{18} = 160$$

$$\Rightarrow \quad \frac{9x - 7x}{126} = 160 \Rightarrow \frac{2x}{126} = 160$$

$\Rightarrow \quad \dfrac{x}{126} = 80$

$\therefore \quad x = 126 \times 80 = ₹\ 10080$

18. (*c*) Let the number of ₹ 50 notes $= x$.

and the number of ₹ 100 notes

$= (170 - x)$

According to question,

$$50x + 100\,(170 - x) = 10000$$

$\Rightarrow \quad x + 2\,(170 - x) = 200$

$\Rightarrow \quad x + 340 - 2x = 200$

$\Rightarrow \quad -x = 200 - 340 = -140$

$\Rightarrow \quad x = 140$

$\therefore$ Required amount $= ₹50 \times 140$

$= ₹\ 7000$

19. (*a*) Let the worker remains absent for x days.

Then, his working days $= (120 - x)$ days.

According to the question,

$20 \times$ Working day $-3 \times$ Absent day $= 560$

$$20\,(120 - x) - 3x = 560$$

$\Rightarrow \quad 2400 - 23x = 560$

$\Rightarrow \quad x = \dfrac{2400 - 560}{23} = \dfrac{1840}{23}$

$= 80$ days

20. (*d*) Let total number of apples in the crate $= x$.

Then, number of bruised apples $= \dfrac{1}{40}\, x$.

Number of unsaleable apples

$$= \left(\frac{3}{4} \times \frac{1}{40}\, x\right) = \frac{3x}{160}$$

According to the question,

$$\frac{3x}{160} = 9$$

$\Rightarrow \quad x = \dfrac{9 \times 160}{3}$

$= 3 \times 160 = 480$

21. (*c*) Let population of cities A and B become equal after x yr.

According to the question,

Population of city A = Population of city B

Then,

$$136000 - 2400\,x = 84000 + 1600\,x.$$

$\Rightarrow \quad 4000x = 52000$

$\therefore \quad x = \dfrac{52}{4} = 13$ yr

Chapter

8

Approximation

In mathematical expression, which includes division and multiplication of decimal values of large number, it becomes quite complicated to solve these expressions. So, to reduce this complexity we use **approximation** method. In approximation method, we need not calculate the exact value of an expression, but we calculate the nearest **(round off)** values. When we use approximation method, then final result obtained is not equal to the exact result, but it is very close to the final result (either a little less or little more).

Basic Rules to Solve the Problems by Approximation

RuleO

To solve the complex mathematical expression, take the nearest value of numbers given in the expression. *For example,* 199.03 is approximated to 200; 94.6% is approximated to 95% etc.

Ex. 1 89% of (599.88 + 30 x 400) + 50 = ?

Sol. $\frac{90}{100} \times (600 + 30 \times 400) + 50 = ?$

$? = \frac{9}{10} \times 8000 + 50 = 7200 + 50 = 7250$

Rule®

To multiply large number, we can take the approximate value (round off) of numbers by increasing one number and decreasing the other accordingly, so that the calculation is eased

For example, 589 x 231 is approximated 590 x 230.

Ex. 2 $393 \times 197 + 5600 \times \frac{5}{4} + 8211.80 = ?$

Sol. $? = 393 \times 197 + 5600 \times \frac{5}{4} + 8211.80$

$= 390 \times 200 + 5600 \times \frac{5}{4} + 8200$

$= 78000 + 7000 + 8200 = 93200$

MIND IT!

Numbers are increased or decreased to the nearest value, so that calculation becomes easier like two numbers ending with zero will be much easier to multiply than number with digits other than ending with zero.

Rule®

When we divide large number with decimals, then we can increase or decrease both

numbers accordingly.

For example, 7987.26 ■*■ 3869 is approximated as 8000 ■*■ 40.

Ex. 3 (9615.36 +1247.18) - (2435.72 +1937.92) = ?

Sol. ? = (9615.36+ 1247.18) ■*■ (2435.72+ 1937.92)

= 10862.54 ■*■ 4373.64 = 10860 ■*■ 4370 = 2.48 = 2.5

RuleQ

To find the percentage of any number, we can use the following shortcut methods

• To calculate 10% of any number, we simply put a decimal after a digit from the right end.

• To calculate 1% of any number, we simply put a decimal after two digits from right end.

• To calculate 25% of any number, we simply divide the number by 4.

Ex. 4 24% of $3580 + 799.99 \div \frac{1000}{25} = ?$

Sol. $? = 24\%$ of $3580 + 799.99 \div \frac{1000}{25}$

$= 25\%$ of $3600 + 800 + 40 = 900 + 20 = 920$

Ex. 5 10% of $1350 + ? = 365$

Sol. 10% of $1350 + ? = 365$

$\Rightarrow \quad 135 + ? = 365$

$\Rightarrow \quad ? = 365 - 135 = 230$

Fast Track Practice

Directions (Q. Nos. 1-74) *What approximate value should come in place of the question mark(?) in the following questions? (Note that you are not expected to calculate the exact value).*

1. $(12.999)^3$ = ? [IBPS Clerk 2011]

(a) 1800 *(b)* 1650 *(c)* 2000 *(d)* 2500 *(e)* 2200

2. 50550-5-50-5-5 = ? [IBPS Clerk 2011]

***(a)* 350 *(b)* 150 *(c)* 300 *(d)* 250 *(e)* 202**

3. 49.0003 + 74.999 = ? [IBPS Clerk 2011]

***(a)* 0.05 *(b)* 0.2 *(c)* 1 *(d)* 0.7**

re; 2

4. 23.003 x 22.998 + 100.010 = ?

[IBPS Clerk 2011]

fa; 630 *(b)* 550 (c; 700 *(d)* 720 (e) 510

5. 125.009 + 69.999 + 104.989 = ?

[IBPS Clerk 2011]

***(a)* 420 *(b)* 300 fc; 285 (tfj 415 *(e)* 325**

6. 16.003x27.998-209.010 = ?

[IBPS Clerk 2011]

(a) 150 (b; 200

(c) 75 (tfj 240

fe; 110

7. 840.003-5-23.999 = ? [IBPS Clerk 2011]

fa; 47 *(b)* 8 fc; 35 *(d)* 18 fe; 23

8. 6885.009 - 419.999 - 94.989 = ?

[IBPS Clerk 2011]

(a) 6650 (b) 6830 *(c)* 6370 (c0 6200 fe; 6450

9. $(9.95)^2 x (2.01)^3 = 2x(?)^2$

[Dena Bank Clerk 2005]

(a) 12 *(b)* 15

(fc; 25 (ft) 20

fe;i6

10.503x201 = ?

(a) 101100 *(b)* 1000000

(c; 110000 (c0 100003

11. $\frac{2}{3} \times \frac{6}{8} \times \frac{2}{3} \times \frac{3}{4} = ?$ [Indian Bank PO 2003]

(a) 0.25 (b) 0.5
(c) 1.45 (d) 11.8
(e) 0.55

12. $15\frac{7}{8} \times 23\frac{1}{5} + 3\frac{4}{5} \times 5\frac{1}{8} = ?$ [IBPS Clerk 2012]

fa; 360 *(b)* 350 *(c)* 370 (c0 385 fe; None of the above

13. $2\frac{3}{5} \times \frac{15}{26} \times 283.75 = ?$
[United Bank of India PO 2005]

***(a)* 440 *(b)* 435 (cj 410 *(d)* 425 fe; 400**

14.36.98276421 x 21.00002 = ? [RBI 2003]

(a) 775 fty 785 *(c)* 800 (c0 805 fe; 750

15.14.995 x 8.001 x 20.991 = ?

[Corporation Bank 2004]

fa; 1950 *(b)* 2520 *(c)* 2200 *(d)* 1520

fe; 2760

16.26.823 x 27.923 x 4.5001 = ? [RBI 2008]

***(a)* 3500 *(b)* 3450 *(c)* 3100 (of) 3150 fe; 3400**

17.3739 + 164x27 = ?

[Canara Bank PO 2003]

***(a)* 105400 *(b)* 4000**

***(c)* 8200 (d; 690**

***(e)* 6300**

18.21 + 3.7 x 2.9 = ? [Indian Bank PO 2003]

fa; 74 *(b)* 70 *(c)* 27 *(d)* 32

fe; 44

19. (21 + 99) x (30 - 19.02) = ?

[Indian Bank PO 2003]

***(a)* 3581 *(b)* 131**

***(c)* 1290 *(d)* 1600**

fe; None of the above

20.198.001 x 25 + 112.05 x 24.998 = ?

[Indian Bank PO 2003]

fa; 7570 *(b)* 7550 *(c)* 7500 *(d)* 7750 fe; 7250

21.17.995 x 16.005 + 15.999 x 15.001 = ?

[Vijaya Bank PO 2004]

fa; 513 *(b)* 528 *(c)* 440 *(d)* 818 *(e)* 314

22.127.007 x 7.998 + 6.05 x 4.001 = ?

[SBI PO 2005]

(a) 1090 *(b)* 1200 *(c)* 1120 (c0 1040 fe; 1160

23.198.995 x 12.005 + 16.25 x 6.95 = ?

[OBC PO 2005]

fa; 2580 *(b)* 2550 *(c)* 2400 *(d)* 1450 *(e)* 2500

24. 421 x 0.9 + 130 x 101 + 10000 = ?

[IBPS Clerk 2012]

(a) 33500 *(b)* 23500

(c) 225000 *(d)* 24500

(e) None of the above 25.30.9 x 3000 - 10.1 x 1100 + 8298 - 4302 = ? [IBPS Clerk 2012]

(a) 80000 *(b)* 90000

(C) 105000 *(d)* 85000

(e) None of the above

26.56.001 x ? - 1000.999 = 231

[Vijaya Bank PO 2003]

(a) 22 *(b)* 45 *(c)* 37 fd; 16 (ej32

27.1010 -5- 36 + 187 x 20.05 = ? [SBI PO 2005]

(a) 3650 (b) 3770 *(c)* 3825 fdj 3800 *(e)* 3700

28.724 * 25 x 31.05 + 101 = ?

[United Bank of India PO 2005]

(a) 900 *(b)* 1000 *(c)* 950 fd; 1050 fe) 1010

29.2375.85 +18.01 - 4.525 x 8.05 = ?

[Corporation Bank PO 2004] *(a)* 103 *(b)* 96 (c) 88 *(d)* 90 (ej 112 30.2508 + 15.02 + ? x 11 = 200

[Corporation Bank PO 2004]

(a) 13 *(b)* 8 fc; 3 fd; 4 fej6

31.6.39 x 128.948 + 5.215 +12.189 + 25.056 = ? [IBPS PO 2011]

(a) 800 *(b)* 900 fc; 850 *(d)* 950 (e) None of the above 32.1559.999+24.001 + 11.005 x 6.999 = ?

[RBI 2003]

(a) 137 *(b)* 132 *(c)* 152 fd; 149 *(e)* 142

33.3.2x8.1 + 3185+4.95 = ?

[Corporation Bank PO 2004]

(a) 670 (b) 660 *(c)* 645 fdj 690

(e) 685 34.1205 + 2.5 = ? [SBI PO 2005]

fa; 3000 *(b)* 4800

fc; 300 *(d)* 480

fe; None of the above 35.22020+0.011 = ?

(a) 20020 *(b)* 2000000

fc; 200200 fd; 200002

36.4182.395+20.886 = ? [IBPS Clerk 2012]

fa; 300 *(b)* 200

fc; 150 *(d)* 250

fej None of the above

37.99.999 + 0.99 + 0.00991234 = ? [RBI 2003]

fa; 100 *(b)* loooofc; 1000 fd; 100000

fe; 99999 38.98743 +198 = 800 - ? [IBPS Clerk 2012]

(a) 200 *(b)* 250 fc; 300 fd; 350

(e) None of the above 39.0.0004 + 0.0001 x 36.000009 = ?

[Canara Bank PO 2003]

fa; 0.10 *(b)* 1.45 fc; 145 fd; 14.5

(e) 1450

40.4.999 +1.999 x 6.001 = ?

[Corporation Bank 2004]

(a) 24 *(b)* 20 fc; 15 fd) 10

r^ is

41.1679 + 14.95x5.02 = ?

[Syndicate Bank PO 2004]

fa; 540 *(b)* 525 fc; 545 *(d)* 565 *(e)* 520

42.447.75+28x4.99 = ?

[Syndicate Bank PO 2004]

(a) 44 *(b)* 70 fc; 32 fd; 80 fe;75

43. $\frac{137 + 17 \times 3.99}{4.02 \times 3.98} = ?$ **[Dena Bank PO 2005]**

(a) 3.2 *(b)* 3.5 fc; 2.5 *(d)* 3 ^ 2 44.425 +16.95 x ? = 225 [OBC PO 2004]

(a) 11 *(b)* 0.8 *(c)* 9 fd) 19 *(e)* 0.9

45.26.003 - 154.001 + 6.995 = ?

[Corporation Bank 2004]

(a) 4 *(b)* 18 fc; 9 fd; 10

re; 14

46.9876 + 24.96 + 215.005 - ? = 309.99

[SBI Associates PO 2002]

(a) 395 *(b)* 295 *(c)* 300 *(d)* 315 fe; 51

$47.95^{3/7}+95^{0,,89} = 95^{?}$

[Canara Bank PO 2003]

(a) 1.9 *(b)* 3 fc; 2.99 *(d)* 3.6 fe; 2.7

$48.18^{3,5} + 2T^{3,5} \times 6^{s\ 5} = 2^{?}$ [RBI 2003]

(a) 7 (b) 3.5 *(c)* 6 *(d)* 4.5 fe; None of the above

$49.3^{7}\text{-}^{5}+27^{L5}x9^{2} = 3^{?}$

[Bank of Maharashtra PO 2003]

(a) 5 *(b)* 4.5 *(c)* 7 *(d)* 6.5

fe; None of the above

50. 750.0003 + 19.999 = ? [IBPS Clerk 2011]

(a) 49 *(b)* 18 *(c)* 22 fd; 45 *(e)* 38

51. $6888.009 - 487.999 - 87.989 = ?$ **[IBPS Clerk 2011]**

(a) 6000 (b) 6570
(c) 6430 (d) 6200
(e) 6310

52. $(9.5)^2 = ?$ **[IBPS Clerk 2011]**

(a) 75 (b) 90 (c) 125 (d) 110
(e) None of the above

53. $19.003 \times 22.998 - 280.010$ **[IBPS Clerk 2011]**

(a) 220 (b) 110 (c) 160 (d) 90
(e) None of the above

54. $5454 \div 54 \div 5 = ?$ **[IBPS Clerk 2011]**

(a) 15 (b) 25 (c) 30 (d) 20
(e) None of the above

55. $685.005 \div 5 - \sqrt{?} = 16.99 \times 6.01$ **[Indian Bank PO 2002]**

(a) 625 (b) 1225 (c) 1156 (d) 841
(e) None of the above

56. $36.0001 \div 5.9998 \times \sqrt{?} = 108.0005$ **[Andhra Bank PO 2003]**

(a) 18 (b) 16 (c) 256 (d) 316
(e) 325

57. $1169.9997 \div 18.0011 + 7.0009 \times 4.9898 = \sqrt{?}$ **[Bank of Maharashtra PO 2003]**

(a) 99 (b) 9000 (c) 100 (d) 10000
(e) 9500

58. $1178.999 \times 25.0001 - \sqrt{?} \times 16.0011 = 29075$ **[Bank of Maharashtra PO 2003]**

(a) 25 (b) 15 (c) 575 (d) 225
(e) 625

59. $\sqrt{625.04} \times 16.96 + 136.001 \div 17 = ?$ **[RBI Grade 'B' 2002]**

(a) 4.18 (b) 4.41 (c) 425 (d) 433
(e) None of the above

60. $\frac{3}{5}$ of $\frac{4}{7}$ of $\frac{7}{9}$ of 425 **[Indian Bank PO 2004]**

(a) 121 (b) 118 (c) 113 (d) 110
(e) 124

61. 137% of 12345 = ? **[Canara Bank PO 2003]**

(a) 17000 (b) 15000
(c) 1500 (d) 14300
(e) 900

62. 12.49% of 839.859 = ? **[RBI 2003]**

(a) 95 (b) 100 (c) 115 (d) 105
(e) 90

63. 121% of 5103 = ? [IBPS PO 2011]

(a) 6000 (b) 8000
(c) 10000 (d) 4000
(e) None of the above

64. 124.35% of 8096 = ? [OBC PO 2004]

(a) 2000 (b) 10000
(c) 1000 (d) 12000
(e) None of the above

65. 134% of 3894 + 38.94% of 134 = ?
[Indian Bank PO 2003]

(a) 5000 (b) 5300 (c) 5500 (d) 5270
(e) 4900

66. 35% of 121 + 85% of 230.25 = ?
[Corporation Bank PO 2004]

(a) 225 (b) 230 (c) 240 (d) 245
(e) 228

67. 22.5% of 1350 + 135% of 225 = ?
[United Bank of India PO 2005]

(a) 570 (b) 610 (c) 670 (d) 630
(e) 590

68. 31% of 1508 + 26% of 2018 = ?
[IBPS PO 2011]

(a) 1500 (b) 2000 (c) 1000 (d) 1200
(e) None of the above

69. 39.8% of 400 + ?% of 350 = 230
[Indian Bank PO 2004]

(a) 15 (b) 25 (c) 18 (d) 24
(e) 20

70. 85% of 225 + 32.91×5.01 = ?
[Syndicate Bank PO 2004]

(a) 375 (b) 340 (c) 355 (d) 345
(e) 370

71. $(15.96)^2$ + 75% of 285 = ?
[Syndicate Bank PO 2004]

(a) 435 (b) 485 (c) 440 (d) 420
(e) 470

72. $0.0005 \times 76 \times 10^3$ + ? = 170% of 29.98
[United Bank of India PO 2005]

(a) 12 (b) 16 (c) 18 (d) 15
(e) 20

73. 161% of 3578 + 139.85 + $\frac{5}{8}$ of 161.56 = ?

(a) 6000 (b) 6500 (c) 6700 (d) 5500

74. ?% of 218 = $3\frac{1}{3} \times 3.3121$ [IBPS PO 2011]

(a) 2.5 (b) 5 (c) 10 (d) 15
(e) None of the above

Answer with **Solutions**

1. (e) $(12.999)^3 = ?$

Using rule 1, here 12.999 is approximated to 13 which is the nearest whole number So, $? = (13)^3$ => 7 = 2197 7=2200

2. (e) 50550 ÷ 50 ÷ 5 = ?

$$? = \frac{50550}{50} \times \frac{1}{5} \Rightarrow \frac{1011}{5}$$

$\Rightarrow$ 202.2 = 202

3. (d) 49.0003 -s- 74.999 = 7

By rule 3 for division 49.0003 is decreased to 49 and 74.999 is increased to 75 So, 49-5-75 = ?

0.653 = ? => 7 = 0.7

(CO 23.003 x 22.998 + 100.010 = ? => 7=23x23+100

=> = 529 + 100 = 629

? = 630

5. (6) 125.009 + 69.999 + 104.989 = ? Using rule 1, each value is approximated to nearest whole number

=> ? =125 + 70 + 105 => ? = 300

6. (d) ? = 16.003 X 27.998 - 209.010

? = 16 X 28 - 210 = 448 - 210 ? = 238 = 240

7. (C) 7 = 840.003+23.999

Using rule 3, here one number is increased and other is decreased to their nearest whole number

= 840 +24 = 35

8. (C) ? = 6885.009 - 419.999 - 94.989

= 6885-420 -95=6370

9. (d) $(9.95)^Z$ X $(2.01)^3$ = 2 X $(?)^2$

Here, 9.95 is approximated to 10. 2.01 is approximated to 2 because calculating the squares or cubes of whole number is much o easier than to calculate the square or cube of decimal numbers => $2x(?)^2 = (10)^z x(2)^3 = 100x8 = 800$

=> $(?)^z = 400$

7 = 20 10. (a) 503x201 = 101103 = 101100

11. (a) $\frac{2}{3}\times\frac{6}{8}\times\frac{2}{3}\times\frac{3}{4}=\frac{1}{4}=0.25$

12. (d) $? = \frac{127}{8}\times\frac{116}{5}+\frac{19}{5}\times\frac{41}{8}$

$= \frac{14732}{40}+\frac{779}{40}=\frac{15511}{40}=387.7 \approx 385$

13. (d) $? = 2\frac{3}{5}\times\frac{15}{26}\times 283.75$

$\approx \frac{13}{5}\times\frac{15}{26}\times 284 = 426 \approx 425$

14. (a) 36.98276421 X 21.00002

Using rule 2, nearest value in whole number for 36.98276421 is 37 and 21.0002 is 21 = 37X21 = 777 = 775

15. (6) ? = 14.995 X 8.001 X 20.991

= 15X8X21 = 2520

16. (e) 26.823 X 27.923 X 4.5001

= 27 X 28 X 4.5000 = 756 X 4.5000 = 3402 = 3400

17. (c) 3739 + 164 X 27 = ?

? = 3739 + 4428 = 8167 = 8200

18. (d) 21 + 3.7 X 2.9 = ?

=> 7 = 21 + 4 X3=21 + 12 = 33 = 32

19. (e) (21 + 99) x (30 - 19.02)

(21 + 99) X (30 - 19)

= 120X11 = 1320

20. (d) ? = 198.001 X 25 + 112.05 X 24.998

= 198X25 + 112X25 = 25(198 + 112) = 25X310 = 7750

21. (6) ? = 17.995 x 16.005 + 15.999

X 15.001 = 18X16+16X15=16X(18 + 15) = 16x33 = 528

22. (d) ? = 127.007 X 7.998 + 6.05 X 4.001

= 127X8 + 6X4 = 1016 + 24 = 1040

23. (e) ? = 198.995 x 12.005 + 16.25 x 6.95 = 199x12+16x7=2388 + 112 = 2500

24. (6)? = 421 x 0.9 + 130 x 101 + 10000

= 378.9 + 13130 + 10000 = 23508.9 = 23500

25. (d) ? = 30.9 x 3000 - 10.1 x 1100

+ 8298-4302 = 92700 - 11110 + 8298 - 4302 = 85586 = 85000

26. (a) 56.001 x 7 1000.999 = 231 => 56x7-1001 = 231

$$\Rightarrow \quad 56 \times 7 = 231 + 1001 = \frac{1232}{56} = 22$$

27. (6) ? = 1010 -f- 36 + 187 x 20.05

= 28.0555 + 3749.35 = 3777.40 = 3770

28. (6) 724 +25X31.05+ 101 = ? => 7 = 725+25x31 + 101

= 29 X 31 + 101 = 899 + 101 = 1000

29. (6) ? = 2375.85 +18.01 - 4.525 X 8.05

= 2376 +18 -4.5x8 = 132-36 =96

30. (C) 2508 +15.02+ ?Xll = 200

$$\Rightarrow \frac{2508}{15} + ? \times 11 = 200$$

$$\therefore ? = (200 - 167.2) \times \frac{1}{11} = 2.98 = 3$$

31. (C) ? = 6.39 X 128.948 + 5.215

$$+ 12.189 + 25.056$$

$$= 6.39 \times 128.948 + 5.215 \times \frac{1}{12.189} + 25.056$$

= 823.97772 + 0.427844778 + 25.056 = 849.4615648 = 850

32. (e) 1559.999 +24.001 + 11.005 x 6.999 = ? => ? = 1560 +24 + 11 X7=65+ 77 = 142

33. (a) ? = 3.2 X 8.1 + 3185+4.95

= 25.92 + 643.43 = 669.35 = 670

34. (d) 1205 +2.5 = 482 = 480

35. (6)22020 +0.011 = 2001818.181 = 2000000

36. (*b*) $? = \frac{4182.395}{20.886} = 200.24873 = 200$

37. (6)99.999 +0.99 +0.00991234

= 100+1 +0.01 = 10000

38. (C) 98743 + 198 = 800 - ?

$$\therefore ? = 800 - \frac{98743}{198} = 301.298 \approx 300$$

39. (C) 0.0004 +0.0001 X 36.000009 = ?

7 = 4+1x36 = 144 = 145

40. (C) 4.999 +1.999 X 6.001 = ?

$$\therefore \quad ? \approx 5 + 2 \times 6 = \frac{5}{2} \times 6 = 15$$

41. (d) ? = 1679 +14.95 X 5.02

= 1680 +15X5 = 560 =565

42. (d) ? = 447.75+28 X 4.99

$$\approx 448 + 28 \times 5 = \frac{448}{28} \times 5 = 80$$

43. (*e*) $\frac{137 + 17 \times 3.99}{4.02 \times 3.98} \approx \frac{137 + 17 \times 4}{4 \times 4} \approx 2$

44. (*c*) $425 + 16.95 \times ? = 225$

$$\Rightarrow \quad ? \approx \frac{225 \times 17}{425} = 9$$

45. (*a*) $? = 26.003 - 154.001 + 6.995$

$$= 26 - 154 + 7 = 26 - 22 = 4$$

46. (*c*) $? = \dfrac{9876}{24.96} + 215.005 - 309.99$

$\approx \dfrac{9875}{25} + 215 - 310$

$= 395 + 215 - 310 = 300$

47. (*e*) $? = 95^{3.7} \div 95^{0.9989} = 95^{?}$

$\Rightarrow \quad 95^{(3.7 - 0.9989)} = 95^{?}$

$\Rightarrow \quad 95^{2.7011} = 95^{?}$

$\therefore \quad ? = 2.7011 \approx 2.7$

48. (*a*) $18^{3.5} \div 27^{3.5} \times 6^{3.5} = 2^{?}$

$\Rightarrow \left(\dfrac{18}{27} \times 6\right)^{3.5} = 2^{?} \Rightarrow (4)^{3.5} = 2^{?}$

$\Rightarrow \quad (2^2)^{3.5} = 2^{?} \Rightarrow (2)^{7.0} = 2^{?} \Rightarrow ? = 7$

49. (*c*) $3^{7.5} \div 27^{1.5} \times 9^{2} = 3^{?}$

$\Rightarrow \quad 3^{7.5} \div (3)^{3 \times 1.5} \times 3^{2 \times 2} = 3^{?}$

$\Rightarrow \quad 3^{7.5 - 4.5 + 4} = 3^{?} \Rightarrow 3^{7} = 3^{?} \Rightarrow ? = 7$

50. (*e*) $? = 750.0003 \div 19.999$

$\Rightarrow \; ? \approx 750 \div 20 \quad \Rightarrow ? \approx 37.5 \approx 38$

51. (*e*) $? = 6888.009 - 487.999 - 87.989$

$\therefore \; ? \approx 6888 - 488 - 88$

$\Rightarrow ? \approx 6888 - 576 \Rightarrow ? \approx 6312 \approx 6310$

52. (*b*) $? = (9.5)^2 = 90.25 \approx 90$

53. (*c*) $? = 19.003 \times 22.998 - 280.010$

$\approx 19 \times 23 - 280 \approx 437 - 280$

$\approx 157 \approx 160$

54. (*d*) $? = 5454 \div 54 \div 5$

$= \dfrac{5454}{54 \times 5} = \dfrac{5454}{270} = 20.2 \approx 20$

55. (*b*) $685.005 \div 5 - \sqrt{?} = 16.99 \times 6.01$

$\therefore \sqrt{?} \approx \dfrac{685}{5} - 17 \times 6$

$\sqrt{?} = 137 - 102 = 35 \Rightarrow ? = 1225$

56. (*e*) $36.0001 \div 5.9998 \times \sqrt{?} = 108.0005$

$\Rightarrow \quad 36 \div 6 \times \sqrt{?} = 108$

$\Rightarrow \quad 6 \times \sqrt{?} = 108$

$\therefore \quad \sqrt{?} = 18 = 324 \approx 325$

57. (*d*) $1169.9997 \div 18.0011 + 7.0009 \times 4.9898 = \sqrt{?}$

$\Rightarrow \sqrt{?} \approx 1170 \div 18 + 7 \times 5 = 65 + 35 = 100$

$\therefore \quad ? = 10000$

58. (*e*) $1178.999 \times 25.0001 - \sqrt{?} \times 16.0011 = 29075$

$\Rightarrow \quad 1179 \times 25 - \sqrt{?} \times 16 = 29075$

$\Rightarrow \quad 29475 - \sqrt{?} \times 16 = 29075$

$\Rightarrow \quad \sqrt{?} \times 16 = 29475 - 29075 = 400$

$\therefore \quad \sqrt{?} = 25 \Rightarrow ? = 25 \times 25 = 625$

59. (*d*) $\sqrt{625.04} \times 16.96 + 136.001 \div 17 = ?$

$\therefore \quad ? = \sqrt{625} \times 17 + 136 \div 17$

$= 25 \times 17 + 136 \div 17 = 425 + 8 = 433$

60. (*c*) $? = \frac{3}{5}$ of $\frac{4}{7}$ of $\frac{7}{9}$ of 425

$\Rightarrow \quad ? = \frac{3}{5} \times \frac{4}{7} \times \frac{7}{9} \times 425 = 113.33 \approx 113$

61. (*a*) 137% of 12345 = ?

$\Rightarrow \quad ? = \frac{12345 \times 137}{100} = 16912.65 \approx 17000$

62. (*b*) 12.49% of 839.859 ≈ 12% of 840

$= \frac{840 \times 12}{100} = 100.80 \approx 100$

63. (*a*) $? = \frac{5103 \times 121}{100} = 6174.63 = 6000$

64. (*b*) ? = 124.35% of 8096

$= \frac{8096 \times 124.35}{100} = \frac{1006737.6}{100}$

$= 10067.376 = 10000$

65. (*d*) 134% of 3894 + 38.94% of 134 = ?

$\Rightarrow ? = \frac{3894 \times 134}{100} + \frac{38.94 \times 134}{100}$

$= 5217.96 + 52.1796$

$= 5270.1396 \approx 5270$

66. (*c*) ? = 35% of 121 + 85% of 230.25

$= \frac{121 \times 35}{100} + \frac{230 \times 85}{100}$

$= 42.35 + 195.5 = 237.85 \approx 240$

67. (*b*) ? = 22.5% of 1350 + 135% of 225

$= \frac{1350 \times 22.5}{100} + \frac{225 \times 135}{100}$

$= 303.75 + 303.75 = 607.5 \approx 610$

68. (*c*) $? = 31\%$ of $1508 + 26\%$ of 2018

$= 15.08 \times 31 + 20.18 \times 26$

$= 467.48 + 524.68$

$= 992.16 \approx 1000$

69. (*e*) 39.8% of $400 + ?\%$ of $350 = 230$

$\Rightarrow \quad 159.2 + \dfrac{350 \times ?}{100} = 230$

$\Rightarrow ? = (230 - 159.2) \times \dfrac{100}{350} = 20.23 \approx 20$

70. (*c*) $? = 85\%$ of $225 + 32.91 \times 5.01$

$\approx 85\%$ of $225 + 33 \times 5$

$= \dfrac{225 \times 85}{100} + 165$

$= 191.25 + 165 = 356.25 \approx 355$

71. (*e*) $? = (15.96)^2 + 75\%$ of 285

$\approx (16)^2 + 214 = 256 + 214 = 470$

72. (*a*) $0.0005 \times 76 \times 10^3 + ? = 170\%$ of 29.98

$\Rightarrow \quad 0.038 \times 1000 + ? = \dfrac{30 \times 170}{100}$

$\therefore \quad ? = 51 - 38 = 13 \approx 12$

73. (*a*) $? = 161\%$ of

$3578 + 139.85 + \dfrac{5}{8}$ of 161.56

$= 5760.58 + 139.85 + 100.975$

$= 6001.405 \approx 6000$

74. (*b*) $?\%$ of $218 = 3\frac{1}{3} \times 3.3121$

$\Rightarrow \dfrac{218 \times ?}{100} = \dfrac{10}{3} \times 3.3121$

$\therefore \quad ? = \dfrac{10}{3} \times 3.3121 \times \dfrac{100}{218}$

$= 5.064 \approx 5$

Chapter

9

Word Problems Based on Numbers

Numbers play an important role in our day-to-day life. Puzzles based on these numbers are known as **word problems.** To solve such problems you have to extract the information correctly and form equations based on given information. The equations formed can be single variable, multi variables, linear, quadratic etc., depending on the type of problem asked.

Types of Word Problems Based on Numbers

There are basically following types of questions that are asked on word problem on numbers.

Type O *Based on Operation with Numbers*

These types of questions includes the operations like subtraction, addition, multiplication, division of number with other number, calculation of average of consecutive numbers, calculation of parts of a number, operation on even or odd numbers, calculation of sum or difference of reciprocal of numbers etc.

Important Points

♦ Consecutive natural numbers can be assumed as

$x - 2, x - 1, x, x + 1, x + 2$......

♦ Consecutive even/odd natural numbers can be assumed

$x-3, x-1, x + 1, x + 3$.......

♦ If sum of two numbers is given as S, then take one number as x and other as (S - x).

♦ If difference of two numbers is *d,* then take one number as x and other as (x + *d)* or (x - *d).*

Ex. 1 Three-fourth of two-third of a number is 782. What is three-fifth of one-fourth of the same number? Sol. Let the number be *x*.

Then, according to the question,

$$\frac{3}{4} \text{ of } \frac{2}{3} \text{ of } x = 782$$

$$\Rightarrow \quad x \times \frac{3}{4} \times \frac{2}{3} = 782$$

$$\therefore \quad x = \frac{782 \times 4 \times 3}{2 \times 3} = 1564$$

$$\text{Now,} \quad 1564 \times \frac{3}{5} \times \frac{1}{4} = 234.6$$

Ex. 2 A number is 25 more than its two-fifth. Find the number. Sol. Let the number be *x.*

Then, according to the question,

$$x - x \times \frac{2}{5} = 25$$

$$\Rightarrow \quad \frac{3x}{5} = 25$$

$$\therefore \quad x = \frac{125}{3}$$

Ex. 3 One-fourth of a number exceeds its one-seventh by 24. What is the number? Sol. Let the number be *x.*

Then, according to the question,

$$x \times \frac{1}{4} - x \times \frac{1}{7} = 24$$

$$\Rightarrow \quad \frac{x}{4} - \frac{x}{7} = 24$$

$$\Rightarrow \quad \frac{7x - 4x}{28} = 24$$

$$\Rightarrow \quad \frac{3x}{28} = 24$$

$$\therefore \quad x = \frac{28 \times 24}{3} = 224$$

Ex. 4 The sum and product of two numbers are 12 and 35, respectively. What is the sum of their reciprocals? Sol. Let the number be a; and *y*.

Then, according to the question,

$x + y = 12$ ...(i)

and x x $y = 35$

Now, $(x - y) = (x + y) - 4xy$

$= (12)^2 - 4 \times 35 = 144 - 140 = 4$ $x - y = 41 = 2$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$\begin{aligned} x + y &= 12 \\ x - y &= 2 \\ \hline 2x &= 14 \end{aligned}$$

$$\therefore \quad x = 7$$

On putting the value of x in Eq. (i), we get

$$x + y = 12$$

$$\Rightarrow \quad 7 + y = 12$$

$$\Rightarrow \quad y = 5$$

$$\therefore \quad x = 7 \text{ and } y = 5$$

Sum of their reciprocals $= \frac{1}{x} + \frac{1}{y} = \frac{1}{7} + \frac{1}{5} = \frac{5 + 7}{35} = \frac{12}{35}$

Alternate Method

Let the two numbers be *x* and *y*. According to the question,

$x + y = 12$ and $xy = 35$

Now, sum of reciprocals of the numbers $= \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{12}{35}$

Type 0 *Based on Formation of Number with Digits*

These types of questions include formation of a number with digits and its difference with reciprocal of the same number, calculation of a number, if a number is added or subtracted to it. The digits get reversed etc.

Important Points

♦ A two-digit number with xas unit digit and y as ten's digit is formed as (10y + x) and if the digits are reversed, then number is represented as (10x + y).

♦ A three-digit number with x as unit digit, y as ten's digit and z as hundred's digit is formed as (1 OOz +10y + x).

Ex. 5 A number consists of two digits whose sum is 8. If 18 is subtracted from the number, the digits interchange their places, then what is the number? Sol. Let the unit's digit be *y* and ten's digit be a;, then the number = lCte + *y* When digits are interchanged, then the number = 10j/ + *x* According to the question,

$$x + y = 8 \quad \text{...(i)}$$

and
$$(10x + y) - 18 = 10y + x$$
$$\Rightarrow 10x + y - 10y - x = 18$$
$$\Rightarrow 9x - 9y = 18$$
$$\therefore x - y = 2 \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$\begin{array}{r} x + y = 8 \\ x - y = 2 \\ \hline 2x = 10 \end{array}$$
$$\therefore x = 5$$

On putting the value of x in Eq. (i), we get

x+*y* = *8*=>*5* + *y* = *8* => y =3

.". Required number = lOx + 2/ = 10x5 + 3 = 53

Ex. 6 The sum of the digits of a two-digit number is 10. The number formed by reversing the digits is 18 less than the original number. Find the original number. **Sol.** Suppose the two-digit number = 10s + *y*

(where' unit's digit is y and ten's digit is x)

and sum of digits = $x + y = 10$ ...(i)

After reversing the digits, the new number = lOy + x

According to the question,

(lte + y) - (lOy + x) = 18 9x - $9y$ = 18

$x - y = 2$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$\begin{aligned} x + y &= 10 \\ x - y &= 2 \\ \hline 2x &= 12 \\ \therefore \quad x &= 6 \end{aligned}$$

On putting the value of x in Eq. (i), we get

$$y = 4$$

$\therefore$ Original number = $10x + y = 10 \times 6 + 4 = 64$

Ex. 7 Find the smallest positive integer which must be subtracted from both the terms of ratio 6 : 7, so that the result gives a ratio less than 16 : 21. **Sol.** Let this positive integer be x.

Therefore, $\dfrac{6 - x}{7 - x} < \dfrac{16}{21}$

$\Rightarrow \quad 126 - 21x < 112 - 16x$

$\Rightarrow \quad 14 < 5x$

$\Rightarrow \quad x > \dfrac{14}{5}$

$\therefore \quad x > 2.8$

So, the smallest positive integer is 3.

Type © Question Regarding Calculation of Heads and Feet of Animals

If a group of animals having either two feet (like ducks, hens etc) or four feet (like horses, cows etc) is there and total number of heads in the group are Hand number of feet of these animals are L, then

Number of animals with four feet = $\frac{L-2H}{2}$

Number of animals with two feet = Total number of heads − Total number of four feeted animals

Ex. 8 In a park, there are some cows and some ducks. If total number of heads in the park are 68 and number of their legs together is 198, then find the number of ducks in the park. Sol. Let the number of cows in the park = x and number of ducks in the park = y Then, according to the question,

Total number of heads = Total number of cows and ducks

So, $x + y = 68$ ...(i)

Total number of legs = (4 x Number of cows) +(2 x Number of ducks) => $4x + 2y = 198$ [because a cow has 4 legs and a duck has 2 legs]

$2x + y = 99$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$2x + y - x - y = 99 - 68 \Rightarrow x = 31$

On putting the value of x in Eq (i), we get

$31 + y = 68$ $y = 37$ So, number of ducks = 37 **Fast Track Method**

Here, $L = 198$ and $H = 68$

So, number of cows = $\frac{L-2H}{2} = \frac{198 - 2 \times 68}{2}$

$= \frac{198-136}{2} = \frac{62}{2} = 31$

$\therefore$ Number of ducks = Number of heads − Number of cows

$= 68 - 31 = 37$

Fast Track Practice

1. A number consists of two digits. The sum of the digits is 10. On reversing the digits of the number, the number decreases by 36. What is the product of the two digits? [CDS 2012]

(a) 21 *(b)* 24 *(c)* 36 *(d)* 42

2. If a number is multiplied by three-fourth of itself, the value thus obtained is 10800. What is that number?

[Bank Clerks 2011]

(a) 210 *(b)* 180 *(c)* 120 *(d)* 160 *(e)* 140

3. The sum of five consecutive numbers is 190. What is the sum of the largest and the smallest numbers? [Bank Clerks 2011]

(a) 75 *(b)* 77 *(C)* 76 *(d)* 73 *(e)* None of the above

4. The product of two consecutive odd numbers is 19043. Which is the smaller one? [IBPS Clerk 2013]

(a) 137 *(b)* 131 *(c)* 133 *(d)* 129 *(e)* None of the above

If the three-fourth of a number is subtracted from the number, the value so obtained is 163. What is that number?

[Bank Clerks 2011]

(a) 625 *(b)* 562 *(c)* 632 *(d)* 652 *(e)* None of the above

6. A positive number, when increased by 10 equals 200 times its reciprocal. What is the number?

(a) 100 *(b)* 10 *(c)* 20 *(d)* 200

7. The sum of two numbers is 10. Their product is 20. Find the sum of the reciprocals of the two numbers.

[SSCCPO2013]

(a) 1 (b) $\frac{3}{5}$

(c) $\frac{1}{2}$ (d) $\frac{11}{6}$

Five times of a positive integer is equal to 3 less than twice the square of that number. Find the number. [SSC CPO 2013] *(a)* 3 *(b)* 13 *(C)* 23 *(d)* 33

9. A man has given one-fourth part of his property to his daughter, half part to his sons and one-fifth part given as charity. How much part of his property he has given? [RRBCGL2012]

(a) $\frac{1}{20}$ (b) $\frac{19}{20}$ (c) $\frac{1}{10}$ (d) $\frac{9}{10}$

10. A chocolate has 12 equal pieces. Manju gave l/4th of it to Anju, l/3rd of it to Sujata and l/6th of it to Fiza. The number of pieces of chocolate left with Manju is [CTET2012]

(a) 1 *(b)* 2 *(c)* 3 *(d)* 4

11. A student was asked to multiply a number by $\frac{3}{2}$ but he divided that number by $\frac{3}{2}$. His result was therefore 10 less than the correct answer. Find the number.

(a) 10 *(b)* 12 *(c)* 15 *(d)* 20

12. A number consists of two digits whose sum is 10. If the digits of the number are reversed, then the number decreased by 36. Which of the following is/are correct?

I. The number is divisible by a

composite number. II. The number is a multiple of a prime

number. [CDS 2013]

(a) Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

13. The sum of two positive numbers x and y is 2.5 times their difference. If the product of numbers is 84, then what is the sum of those two numbers?

(a) 26 *(b)* 24 *(c)* 22 *(d)* 20

14. If the numerator of a certain fraction is increased by 2 and the denominator is increased by 1, then the resulting fraction is equal to 1/2. If however, the numerator is increased by 1 and denominator is decreased by 2, then the resulting fraction

is equal to $\frac{2}{5}$. Find the original fraction.

(a) $\frac{2}{7}$ (b) $\frac{3}{5}$ (c) $\frac{1}{7}$ (d) $\frac{2}{5}$

15. The sum of five consecutive odd numbers is equal to 175. What is the sum of the second largest number and the square of the smallest number amongst them together? [IBPS PO 2013]

(a) 989 *(b)* 997

(c) 979 *(d)* 995

(e) None of the above

16. Out of three given numbers, the first number is twice the second and thrice the third. If the average of these three numbers is 154, then what is the difference between the first and the third numbers? **[Bank** Clerks 2011]

(a) 126 *(b)* 42 *(c)* 168 *(d)* 52

(e) None of the above

17.X, Yand *Z* had taken a dinner together. The cost of the meal of *Z* was 20% more than that of Y and the cost of the meal of Xwas 5/6 as much as the cost of the meal of *Z.* If Y paid *X* 100, then what was the total amount that all the three of them had paid? [CDS 2013]

(a) X 285 *(b) X* 300

(c) X 355 *(d)* None of these

18. Find the maximum number of trees which can be planted 20 m apart on the two sides of a straight road 1760 m long.

[SCCCCL2013]

(a) 174 *(b)* 176 *(c)* 180 *(d)* 178

19. The difference between two numbers is 18. If four times the second number is less than three times the first number by 18, then what is the sum of these two numbers? [SSC (10+2) 2013]

(a) 100 *(b)* 80 *(c)* 86 *(d)* 92

20. The difference between a two-digit number and the number obtained by interchanging the two digits of the number is 18. The sum of the two digits of the number is 12. What is the product of the digits of two-digit number? **[IBPS** Clerk 2012]

(a) 35 *(b)* 27 *(c)* 32

(d) Couldn't be determined

(e) None of the above

21. On Children's Day, sweets were to be equally distributed amongst 300 children. But on that particular day 50 children remained absent; hence each child got one extra sweet. How many sweets were distributed? [SSC CGL 2013]

(a) 1450 *(b)* 1700

(c) 1500 *(d)* 1650

22. There are two examination halls *P* and Q. If 10 students shifted *P* to *Q,* then the number of students will be equal in both the examination halls. If 20 students shifted from *Q* to *P,* then the students of *P* would be doubled to the students of *Q.* The numbers of students would be in *P* and *Q,* respectively are [RRB2012]

(a) 60, 40 *(b)* 70, 50

(c) 80, 60 *(d)* 100, 80

23. In a two-digit positive number, the unit digit is equal to the square of ten's digit. The difference between the original number and the number formed by interchanging the digits is 54. What is 40% of the original number?

[IBPS PO 2011] *(a)* 64 *(b)* 73 *(c)* 84

(d) Couldn't be determined

(e) None of the above

24. In a three-digit number, the digit in the unit's place is four times the digit in the hundred's place. If the digit in the unit's place and the ten's place are interchanged, the new number so formed is 18 more than the original number. If the digit in the hundred's place is one-third of the digit in the ten's place, then what is 25% of the original number?

[LICAAO2013] *(a)* 67 *(b)* 84 *(c)* 137

(d) Couldn't be determined

(e) None of the above

The sum of the squares of two numbers is 97 and the squares of their difference is 25. The product of the two numbers is

[CDS 2012]

(a) 45 *(b)* 36 *(c)* 54 *(d)* 63

There are 200 questions in a 3 h examination. Among 200 questions, 50 are from Mathematics, 100 are from GK and 50 are from Science. Ram spent twice as much time on each Mathematics question as for each other question. How many minutes did he spend on Mathematics questions? *(a)* 36 *(b)* 72 *(c)* 100 *(d)* 60

27. The taxi charges in a city contain fixed charges and additional charge per kilometre. The fixed charge is for a distance of upto 5 km and additional charge per kilometre thereafter. The charge for a distance of 10 km is *X* 350 and for 25 km is *X* 800. The charge for a distance of 30 km is

(a) *X* 800 *(b)* *X* 750 *(c)* *X* 900 *(d)* *X* 950

28. In an examination paper of five questions, 5% of the candidates answered all of them and 5% answered none. Of the rest, 25% candidates answered only one question and 20% answered 4 questions. If 396 candidates answered either 2 questions or 3 questions, the number of candidates that appeared for the examination was

(a) 800 *(b)* 1000 *(c)* 850 *(d)* 900

Answer with **Solutions**

1. (*a*) Let the unit's digit of the number be x and ten's digit be y.

$\therefore \quad \text{Number} = 10y + x$

According to the question,

$$x + y = 10 \quad \text{...(i)}$$

and $\quad 10x + y = (10y + x) - 36$

$\Rightarrow \quad 10x + y - 10y - x = -36$

$\Rightarrow \quad 9x - 9y = -36$

$\Rightarrow \quad x - y = -4 \quad \text{...(ii)}$

On adding Eqs. (i) and (ii), we get

$$\begin{array}{r} x + y = 10 \\ x - y = -4 \\ \hline 2x = 6 \end{array}$$

$\therefore \quad x = 3 \text{ and } y = 7$

$\therefore$ Required product of two digits

$$= 3 \times 7 = 21$$

2. (*c*) Let the number be x.

According to the question,

$$x \times \left(x \times \frac{3}{4} \right) = 10800$$

$\Rightarrow \quad \dfrac{3x^2}{4} = 10800$

$\Rightarrow \quad x^2 = \dfrac{10800 \times 4}{3} = 14400$

$\therefore \quad x = \sqrt{14400} = 120$

3. (*c*) Let the five consecutive numbers are $x - 2, x - 1, x, x + 1, x + 2$.

Sum of numbers $= 190$

$\therefore x - 2 + x - 1 + x + x + 1 + x + 2 = 190$

$\Rightarrow \quad 5x = 190$

$\therefore \quad x = 38$

Sum of largest and smallest numbers

$$= x + 2 + x - 2 = 2x = 2 \times 38 = 76$$

4. (*a*) Let the two consecutive odd numbers are x and $x + 2$.

According to the question,

$$x(x + 2) = 19043$$

$\Rightarrow \quad x^2 + 2x - 19043 = 0$

$\Rightarrow \quad x^2 + 139x - 137x - 19043 = 0$

$\Rightarrow \quad x(x + 139) - 137(x + 139) = 0$

$\Rightarrow \quad (x + 139)(x - 137) = 0$

$\Rightarrow \quad x = 137, -139$

$\therefore \quad x = 137 \quad$ [ignore – ve sign]

So, the smallest one is 137.

5. (*d*) Let the number be x.

According to the question,

$$x - x \times \frac{3}{4} = 163 \Rightarrow \frac{x}{4} = 163$$

$\therefore \quad x = 652$

6. (*b*) Let the positive number be x.
According to the question,

$$x + 10 = \frac{200}{x}$$

$$\Rightarrow \quad x^2 + 10x = 200$$

$$\Rightarrow \quad x^2 + 10x - 200 = 0$$

$$\Rightarrow \quad (x - 10)(x + 20) = 0$$

$$\therefore \quad x = 10 \text{ and } x = -20$$

But $x \neq -20$, since x is a positive number.
So, the required number is 10.

7. (*c*) Let the numbers be x and y. Then,

$$x + y = 10 \quad \ldots\text{(i)}$$

and $$xy = 20 \quad \ldots\text{(ii)}$$

$$\therefore \quad (x - y)^2 = (x + y)^2 - 4xy$$

$$\Rightarrow \quad (x - y)^2 = (10)^2 - 4 \times 20 = 20$$

$$\therefore \quad x - y = \sqrt{20} = 2\sqrt{5} \quad \ldots\text{(iii)}$$

On adding Eqs. (i) and (iii), we get

$$\begin{array}{r} x + y = 10 \\ x - y = 2\sqrt{5} \\ \hline 2x = 10 + 2\sqrt{5} \end{array}$$

$$\therefore \quad x = 5 + \sqrt{5}$$

On putting the value of x in Eq. (i), we get

$$x + y = 10$$

$$\Rightarrow \quad 5 + \sqrt{5} + y = 10$$

$$\therefore \quad y = 5 - \sqrt{5}$$

Sum of reciprocals of x and $y = \frac{1}{x} + \frac{1}{y}$

$$= \frac{1}{5 + \sqrt{5}} + \frac{1}{5 - \sqrt{5}} = \frac{5 - \sqrt{5} + 5 + \sqrt{5}}{(5 + \sqrt{5})(5 - \sqrt{5})}$$

$$= \frac{10}{(5)^2 - (\sqrt{5})^2} = \frac{10}{20} = \frac{1}{2}$$

Alternate Method

Here, $x + y = 10$ and $xy = 20$
According to the question,

$$\frac{1}{x} + \frac{1}{y} = \frac{x + y}{xy} = \frac{10}{20} = \frac{1}{2}$$

8. (*a*) Let the number be x.
According to the question,

$$5x = 2x^2 - 3$$

$$\Rightarrow \quad 2x^2 - 5x - 3 = 0$$

$$\Rightarrow \quad (x - 3)(2x + 1) = 0$$

$$x = 3, -1/2$$

Thus, the required number is 3.

9. *(b)* Total share given by a person

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{5} = \frac{5 + 10 + 4}{20} = \frac{19}{20}$$

10. *(c)* The number of pieces of chocolate left with Manju

$$= 1 - \left(\frac{1}{4} + \frac{1}{3} + \frac{1}{6}\right)$$

$$= 1 - \left(\frac{3 + 4 + 2}{12}\right)$$

$$= 1 - \frac{9}{12} = \frac{12 - 9}{12} = \frac{3}{12}$$

Hence, number of pieces of chocolate left with Manju is 3.

Alternate Method

Number of pieces in a chocolate = 12

Number of pieces Anju got

$$= 12 \times \frac{1}{4} = 3$$

Number of pieces Sujata got

$$= 12 \times \frac{1}{3} = 4$$

Number of pieces Fiza got

$$= 12 \times \frac{1}{6} = 2$$

∴ Number of pieces left with Manju

$$= 12 - (4 + 3 + 2) = 3$$

11. *(b)* Let the number be x.

According to the question,

$$x \times \frac{3}{2} - \frac{x}{3/2} = 10$$

$$\Rightarrow \quad \frac{3x}{2} - \frac{2x}{3} = 10$$

$$\Rightarrow \quad \frac{9x - 4x}{6} = 10$$

$$\Rightarrow \quad \frac{5x}{6} = 10$$

$$\therefore \quad x = \frac{10 \times 6}{5} = 12$$

12. *(b)* Let the two-digit number be lOx + 7. Now, according to the question,

x + y = 10 ...(i)

and (x + IO7) + 36 = (7 + lOx) => - 9y + 9x = 36

=> -*y* + x = 4 ...(ii)

On adding Eqs. (i) and (ii), we get

$2x = 14 \Rightarrow x = 7$

$x = 7$ and $7 = 3$.•. Required number is 73. So, the number is a multiple of a prime number.

13. (d) Let the numbers be x and *y*. According to the question,

$(x + y) = 2.5\ (x - 7)$

$\Rightarrow \quad x + y = 2.5x - 2.5y$

$\Rightarrow \quad 3.5y = 1.5x$

$\Rightarrow \quad \dfrac{x}{y} = \dfrac{7}{3}$ ...(i)

Now, $\quad xy = 84$

$\dfrac{7}{3}y \times y = 84$

$\Rightarrow \quad y^2 = \dfrac{84 \times 3}{7}$

$\Rightarrow \quad y^2 = 36$

$\therefore \quad y = 6$

$\therefore \quad x = \dfrac{7}{3} \times 6 = 14$

Sum of numbers $= x + y$

$= 14 + 6 = 20$

14. (*a*) Let the fraction be $\dfrac{x}{y}$.

According to the question,

$$\frac{x+2}{y+1} = \frac{1}{2}$$

$\Rightarrow \quad 2x + 4 = y + 1$

$\Rightarrow \quad 2x - y = -3$ ...(i)

Again, according to the question,

$$\frac{x+1}{y-2} = \frac{3}{5}$$

$\Rightarrow \quad 5x + 5 = 3y - 6$

$\therefore \quad 5x - 3y = -11$ ...(ii)

On multiplying Eq. (i) by 3 and then subtracting from Eq. (ii), we get

$$\begin{array}{r} 6x - 3y = -9 \\ 5x - 3y = -11 \\ - \quad + \qquad + \\ \hline x = 2 \end{array}$$

On putting the value of x in Eq. (i), we get

$2x - y = -3$

$\Rightarrow \quad 2 \times 2 - y = -3$

$\Rightarrow \quad -y = -3 - 4 = -7$

$\therefore \quad y = 7$

$\therefore \quad$ Original fraction $= \dfrac{x}{y} = \dfrac{2}{7}$

15. (*e*) Sum of five consecutive odd numbers

$= x + x + 2 + x + 4 + x + 6 + x + 8$

$= 175$

$\Rightarrow \quad 5x + 20 = 175$

$\Rightarrow \quad x = \dfrac{175 - 20}{5} = 31$

Sum of the second largest and square of smallest one

$= (31 + 6) + (31)^2$

$= 37 + 961 = 998$

16. (*c*) Let the third number $= x$

Then, the first number $= 3x$

and second number $= \frac{3x}{2}$

According to the question,

$$\frac{x + 3x + \frac{3x}{2}}{3} = 154$$

$\Rightarrow$ $$\frac{2x + 6x + 3x}{6} = 154$$

$\therefore$ $$x = \frac{154 \times 6}{11} = 84$$

$\therefore$ Required difference $= 3x - x = 2x$

$= 2 \times 84 = 168$

17. (*d*) Given that,

The cost of meal of Y = ₹ 100

Now, according to the question,

The cost of the meal of Z = 20% more than that of Y

$= \left(100 + \frac{20}{100} \times 100\right) = (100 + 20) =$ ₹ 120

and the cost of the meal of $X = \frac{5}{6}$ as much as the cost of the meal of Z

$= \frac{5}{6} \times 120 =$ ₹ 100

$\therefore$ Total amount that all the three of them has paid

$= 100 + 120 + 100 =$ ₹ 320

18. (*d*) Number of trees that can be planted on one side of road $= \frac{1760}{20} + 1$

$= 88 + 1 = 89$

$\therefore$ Trees on the both sides

$= 2 \times 89 = 178$

19. (*d*) Let first number $= x$

and second number $= y$

According to the question,

$$x - y = 18 \qquad \ldots\text{(i)}$$

and $$3x - 4y = 18 \qquad \ldots\text{(ii)}$$

On multiplying Eq. (i) by 3 and then subtracting Eq. (ii) from it, we get

$$3x - 3y = 54$$
$$3x - 4y = 18$$
$$y = 36$$

On putting the value of y in Eq. (i), we get

$$x = 18 + y = 18 + 36 \Rightarrow x = 54$$

$\therefore$ Required sum $= x + y = 54 + 36 = 90$

20. (*a*) Let the unit's digit be y and ten's digit be x.

Then, the number $= 10x + y$

When interchanging the place, the number is $10y + x$.

According to the question,

$$(10x + y) - (10y + x) = 18$$

$\Rightarrow$ $$10x + y - 10y - x = 18$$

$\Rightarrow \quad 9x - 9y = 18$

$\therefore \quad x - y = 2 \quad \ldots\text{(i)}$

and $\quad x + y = 12 \quad \ldots\text{(ii)}$

On adding Eqs. (i) and (ii), we get

$$\begin{aligned} x - y &= 2 \\ x + y &= 12 \\ \hline 2x &= 14 \\ x &= 7 \end{aligned}$$

$\therefore \quad x = 7$ and $y = 5$

Product $= xy = 7 \times 5 = 35$

21. (c) Let total number of sweets $= x$

According to the question,

$$\frac{x}{250} - \frac{x}{300} = 1 \Rightarrow \frac{6x - 5x}{1500} = 1$$

$\therefore \quad x = 1500$

$\therefore$ Required number of sweets $= 1500$

22. (d) Let number of students in examination halls P and Q is x and y, respectively.

Then, as per the first condition,

$$x - 10 = y + 10$$

$\Rightarrow \quad x - y = 20 \quad \ldots\text{(i)}$

As per the second condition,

$$x + 20 = 2(y - 20)$$

$\Rightarrow \quad x + 20 = 2y - 40$

$\Rightarrow \quad x - 2y = -60 \quad \ldots\text{(ii)}$

On subracting Eq. (ii) from Eq. (i), we get

$$-y + 2y = 20 + 60$$

$\Rightarrow \quad y = 80$

Putting the value of y in Eq. (i), we get

$$x - 80 = 20 \Rightarrow x = 100$$

Hence, number of students in examination halls P and Q is 100 and 80, respectively.

23. (e) Let ten's digit be x and unit's digit be x^2.

Original number $= 10x + x^2$

New number $= 10x^2 + x$

According to the question,

$$(10x^2 + x) - (10x + x^2) = 54$$

$\Rightarrow \quad 10x^2 + x - 10x - x^2 = 54$

$\Rightarrow \quad 9x^2 - 9x = 54$

$\Rightarrow \quad 9(x^2 - x) = 54$

$\Rightarrow \quad x^2 - x - 6 = 0$

$\Rightarrow \quad x^2 - 3x + 2x - 6 = 0$

$\Rightarrow \quad x(x - 3) + 2(x - 3) = 0$

$\Rightarrow \quad (x - 3)(x + 2) = 0$

$\therefore \quad x = 3, -2$

$\therefore$ Ten's digit $= x = 3$

Unit's digit $= x^2 = 3^2 = 9$

Original number $= 39$

$\therefore$ Required number $= 39 \times \dfrac{40}{100} = 15.6$

24. (a) Let hundred's digit = x

Then unit's digit = 4x and ten's digit = 3x Number = 100 x + 30x + 4x = 134x Again, hundred's digit = x Ten's digit = 4x and unit's digit = 3x Number = 100x+ 40x+ 3x= 143x According to the question,

143x- 134 x = 18 =>9x = 18 x = 2 Original number = 134x = 134 X 2 = 268

$$25\% \text{ of original number} = 268 \times \frac{25}{100} = 67$$

25. *(b)* Let the two numbers are x and *y*. According to the question,

Sum of squares of two numbers = 97

i.e., $x^z + y^z = 97$ ...(i)

and square of their difference = 25

i.e., $(x-y)^z = 25$ ...(ii)

=> $x - y = 5$ ...(iii)

From Eq. (ii),

$(x^z + y^2) - Zxy = 25$ => 97 - 2xy = 25 [from Eq. (i)]

=> 2xy = 72

=> xy = 36 ...(iv)

Now, we have

$(x + y)^2 = (x^2 + y^2) + 2xy = 97 + 72 = 169$ => x+7 = 13 ...(v)

Now, from Eqs. (iii) and (v), we get

2x = 18 => x = 9 and 7 = 4

.•. Product of both the numebrs = xy = 9x4 = 36

26. (b) Let Ram spends x min on each Mathematics question.

According to the question,

$$50x + 100 \times \frac{x}{2} + 50 \times \frac{x}{2} = 3 \times 60$$

$$\Rightarrow \quad x(60 + 60 + 25) = 180 \Rightarrow \quad x = \frac{180}{125}$$

$$\therefore \text{Required time} = 20 \times \frac{180}{125} = 72 \text{ min}$$

27. (d) Let the fixed charges = ? x

and the additional charges = ? *y* per km According to the question,

x+ 57 = 350 ...(i)

x+207 = 800 ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get -157 = -450 7 = 30 On putting the value of 7 in Eq. (i), we get

x = 200 .". Charge for a distance of 30 km = x + 257 = 200 + 30 X 25 = ? 950

28. (a) Let total number of candidates = x Number of candidates answered 5 questions

$$= \frac{5x}{100}$$

Number of candidates answered no question = $\frac{5x}{100}$

$\therefore$ **Remaining students**

$$= x - \left(\frac{5x}{100} + \frac{5x}{100}\right) = \frac{9x}{10}$$

Number of candidates answered only one question = $\frac{9x}{10} \times \frac{25}{100} = \frac{9x}{40}$

Number of candidates answered four questions = $\frac{9x}{10} \times \frac{20}{100} = \frac{9x}{50}$

Given, number of candidates answered either two questions or three questions

$$= 396$$

$$\Rightarrow \quad x - \left(\frac{5x}{100} + \frac{5x}{100} + \frac{9x}{40} + \frac{9x}{50}\right) = 396$$

$$\Rightarrow \quad x - \left(\frac{10 + 10 + 45 + 36}{200}\right)x = 396$$

$$\Rightarrow \quad x\left(\frac{200 - 101}{200}\right) = 396$$

$$\therefore \quad x = \frac{396 \times 200}{99} = 800$$

$$\Rightarrow \quad x = 800$$

Hence, number of candidates = 800

Chapter

10

Average

An average or an arithmetic mean of given data is the sum of the given observations divided by number of observations. *For example* If we have to find out the average of 10, 15, 25 and 30, then the required average will be equal to

$$\frac{10+15+25+30}{4}=\frac{80}{4}=20$$

Similarly, average of 30, 10 and 50

$$=\frac{30+10+50}{3}=\frac{90}{3}=30$$

Therefore, we can write the formula

$$\text{Average } (A)=\frac{\text{Sum of given observations } (S)}{\text{Number of observations } (N)}$$

$$\therefore \qquad A=\frac{S}{N}$$

Properties of Average

1. Average of a given data is less than the greatest observation and greater than the smallest observation of the given data.

For example Average of 3, 7, 9 and 13

$$=\frac{3+7+9+13}{4}=\frac{32}{4}=8$$

Clearly, 8 is less than 13 and greater than 3. 2. If the observations of given data are equal, then the average will also be the same as observations. *For example* Average of 6, 6, 6 and 6 will be 6 because

$$\frac{6+6+6+6}{4}=\frac{24}{4}=6$$

3. If 0 (zero) is one of the observation of a given data, then that 0 (zero) will also be included while calculating average. *For example* Average of 3, 6 and 0 is 3 because

$$\frac{3+6+0}{3}=\frac{9}{3}=3$$

Note • If all the numbers get increased by a, then their average must be increased by a

• If all the numbers get decreased by a, then their average must be decreased by a

• If all the numbers are multiplied by *a,* then their average must be multiplied by a

• If all the numbers are divided by *a,* then their average must be divided by a

Ex. 1 Find out the average of 308, 125, 45, 120 and 102.

Sol. Required average $= \dfrac{\text{Sum of given observations}}{\text{Number of observations}}$

$$= \frac{308+125+45+120+102}{5}$$

$$= \frac{700}{5} = 140$$

Ex. 2 If the weight of *A* is 60 kg, weight of *B* is 45 kg and weight of C is 54 kg, then what is the average weight of three persons?

Sol. Required average $= \dfrac{60+45+54}{3} = \dfrac{159}{3} = 53\text{ kg}$

Ex. 3 The average expenditure of Chandan in four days is ? 90. If his expenditures for the first three days are ? 100, ? 125 and ? 85 respectively, then what is the expenditure of Chandan for the fourth day?

Sol. Let the expenditure for the fourth day = ₹ x

Then, average expenditure = $\dfrac{\text{Sum of the expenditures of four days}}{4}$

$$\therefore \quad 90 = \frac{100 + 125 + 85 + x}{4}$$

$$\Rightarrow \quad 310 + x = 360$$

$$\Rightarrow \quad x = 360 - 310 = ₹\ 50$$

Important Formulae Related to Average of Numbers

1. Average of first n natural numbers = $\left(\dfrac{n+1}{2}\right)$
2. Average of first n even numbers = $(n + 1)$
3. Average of first n odd numbers = n
4. Average of consecutive numbers = $\dfrac{\text{First number} + \text{Last number}}{2}$
5. Average of 1 to n odd numbers = $\dfrac{\text{Last odd number} + 1}{2}$
6. Average of 1 to n even numbers = $\dfrac{\text{Last even number} + 2}{2}$
7. Average of squares of first n natural numbers = $\dfrac{(n+1)(2n+1)}{6}$
8. Average of the cubes of first n natural numbers = $\dfrac{n(n+1)^2}{4}$
9. Average of n multiples of any number = $\dfrac{\text{Number} \times (n+1)}{2}$

Ex. 4 What will be the average of numbers from 1 to 51?

Sol. According to the formula,

The average of first n natural numbers = $\left(\dfrac{n+1}{2}\right)$

Here, $n = 51$

$\therefore$ Required average = $\dfrac{51 + 1}{2} = \dfrac{52}{2} = 26$

Ex. 5 Find out the average of 2, 4, 6, 8,10,12 and 14.

Sol. As we know, average of first *n* even numbers = (n + 1) .•. Required average = (7 + 1) = 8

Ex. 6 Calculate the average of 1, 3, 5, 7, 9,11,13,15 and 17.

Sol. As we know, average of first *n* odd numbers = *n* Here, *n* = 9

.•. Required average = 9

Ex. 7 What will be the average of 1, 2, 3, 4, ..., 51, 52, 53?

Sol. As we know, average of consecutive numbers = $\frac{\text{First number} + \text{Last number}}{2}$

Here, first number = 1 and the last number = 53

$\therefore$ Required average = $\frac{1+53}{2} = \frac{54}{2} = 27$

Ex.8 Find out the average of 4, 7,10,13, ..., 28, 31. Sol. Here, the difference between any two numbers written in continuous sequence is 3. Hence, this is a series of consecutive numbers.

As we know, average of consecutive numbers = $\frac{\text{First number} + \text{Last number}}{2}$

Here, first number = 4 and last number = 31

$\therefore$ Required average = $\frac{4+31}{2} = \frac{35}{2} = 17.5$

Ex.9 Find the average of all the odd numbers and average of all the even numbers from 1 to 45.

Sol. According to the formula,

Average of 1 to n odd numbers = $\frac{\text{Last odd number} + 1}{2}$

Here, the last odd number = 45

$\therefore$ Average of 1 to 45 odd numbers = $\frac{45+1}{2} = \frac{46}{2} = 23$

Again, according to the formula,

Average of 1 to n even numbers = $\frac{\text{Last even number} + 2}{2}$

Here, last even number = 44

$\therefore$ Average of 1 to 44 even numbers = $\frac{44+2}{2} = \frac{46}{2} = 23$

Ex. 10 Calculate the average of the squares of natural numbers from 1 to 25. Sol. According to the formula,

Average of squares of first n natural numbers = $\frac{(n+1)(2n+1)}{6}$

Here, $n = 25$

$\therefore$ Required average = $\frac{(25+1)(2\times 25+1)}{6} = \frac{26\times 51}{6} = \frac{1326}{6} = 221$

Ex. 11 Calculate the average of the cubes of first 9 natural numbers. Sol. According to the formula,

Average of the cubes of first n natural numbers = ———————

4

Here, $n = 9$

D . , $9 \times (9 + 1)^2$ 9×100

Required average =-------------— =----------

4 4

= 9 X 25 = 225

Ex. 12 What will be the average of first 9 multiples of 5? Sol. According to the formula,

$$\text{Average of } n \text{ multiples of a number} = \frac{\text{Number} \times (n + 1)}{2}$$

Here, $n = 9$ and number = 5

$$\therefore \text{Required average} = \frac{5 \times (9 + 1)}{2} = \frac{50}{2} = 25$$

Fast Track **Techniques**

to solve the QUESTIONS

Technique 1

If the average of n^ observations is $a\{$, the average of n_2 observations is a_2 and so on, then

$$\text{Average of all the observations} = \frac{n_1a_1 + n_2a_2 + \ldots\ldots}{n_1 + n_2 + \ldots\ldots}$$

Ex. 13 There are 30 boys and 60 girls in a class. If the average age of boys is 12 yr and average age of girls is 10 yr, then find out the average age of the whole class. Sol. Here, nj =30,n_2 = 60,% = 12, a_2 = 10

$$\text{Average age of the whole class} = \frac{n_1a_1 + n_2a_2}{n_1 + n_2}$$

$$= \frac{30 \times 12 + 60 \times 10}{30 + 60} = \frac{360 + 600}{90} = \frac{960}{90} = 10.66 \text{ yr}$$

Ex.14 Pinky bought 20 books at the rate of ? 10 each, 45 pens at the rate of ? 5 each and 15 pencils at the rate of ? 3 each. Calculate the average price of all the stationary goods.

Sol. Here, $n_1 = 20, n_2 = 45, n_3 = 15, a_1 = 10, a_2 = 5, a_3 = 3$

$\therefore$ Average price of all the stationary goods $= \dfrac{n_1a_1 + n_2a_2 + n_3a_3}{n_1 + n_2 + n_3}$

$$= \frac{20 \times 10 + 45 \times 5 + 15 \times 3}{20 + 45 + 15}$$

$$= \frac{200 + 225 + 45}{80} = \frac{470}{80} = ₹\ 5.875$$

Technique 2

f the average of *m* observations is *a* and the average of *n* observations taken out of m is *b,* then

$$\text{Average of rest of the observations} = \frac{ma - nb}{m - n}$$

Ex.15 A man bought 20 cows in ? 200000. If the average cost of 12 cows is ? 12500, then what will be the average cost of remaining cows?

Sol. Average cost of 12 cows = ₹ 12500

Total cost of 12 cows = 12×12500 = ₹ 150000

Cost of remaining 8 cows = $200000 - 150000$ = ₹ 50000

$\therefore$ Average of remaining 8 cows $= \dfrac{50000}{8}$ = ₹ 6250

Fast Track Method

Average cost of 20 cows $= \dfrac{200000}{20}$ = ₹ 10000

Here, $m = 20, n = 12, a = 10000, b = 12500$

Average cost of remaining $(20 - 12)$ cows $= \dfrac{20 \times 10000 - 12 \times 12500}{20 - 12}$

$$= \frac{200000 - 150000}{8} = \frac{50000}{8} = ₹\ 6250$$

Technique 3

(i) if average of n observations is a but the average becomes b when one observation is eliminated, then value of eliminated observation = $n (a - b) + b$

(ii) if average of n observations is a but the average becomes b when a new observation is added, then Value of added observation =$n\{b-a) + b$

Ex. 16 In a cricket team, the average age of 11 players and the coach is 18 yr. If the age of the coach is not considered, then the average decreases by 1 yr. Find out the age of the coach.

Sol. Total age of 11 players and the coach = 12 X 18 = 216yr Total age of 11 players = 11 x 17 = 187 yr Age of the coach = 216 - 187 = 29 yr

Fast Track Method

Here,n = 11+ 1= 12; Initial average (a) = 18 yr; Last average (6) = 18- 1= 17yr Age of the coach = $n (a - b) + b$

= 12 (18 - 17) + 17 = 12 + 17 = 29yr

Ex. 17 The average run scored by a batsman in 20 innings is 32. After 21st innings, the runs average becomes 34. How much runs does the batsman score in his 21st innings? **Sol.** Runs scored in 20 innings = 20 X 32 = 640 Runs scored in 21 innings = 21 x 34 = 714 .•. Runs scored in the 21st innings = 714 - 640 = 74

Fast Track Method

Here, n = 20; Initial average, a = 32; Last average, (6) = 34 .'. Runs scored in 21st innings = $n (b - a) + b$

= 20 (34 - 32) + 34 = 20 x 2 + 34 = 74

Technique 4

We have n observations out of which some observations (a_1fa_2,a_3.....)

are replaced by some other new observations and in this way, if the average increases or decreases by b, then

Value of new observations =$a \pm nb$

where, $a = a^{\wedge}+a_2 + a_3 +$....

Note In this formula, the signs of '+' and '-' depend upon the increment or decrement in the average

Ex. 18 The average weight of 3 women is increased by 4 kg, when one of them whose weight is 100 kg, is replaced by another woman. What is the weight of the new woman? **Sol.** Total weight increased = 4x3= 12 kg

.".Weight of new woman = 100 + 12 = 112 kg

Fast Track Method

Here,n = 3, a = 100kg, 6 = 4kg

/. Weight of new woman = $a + nb$= 100 + 3 X 4 = 112 kg

[here, '+' sign has been taken as average increases in this case.]

Ex. 19 The average age of 25 boys in a class decreases by 6 months, when a new boy takes the place of a 20 yr old boy. Find out the age of new boy.

Sol. As the average age is decreased by 6 months i.e., $\frac{1}{2}$ yr.

Total age decreased $= 25 \times \frac{1}{2} = 12.5$ yr

$\therefore$ Age of new boy = (Age of boy replaced − Total age decreased) = 20 − 12.5 = 7.5 yr

Fast Track Method

Here, $n = 25, a = 20, b = 6$ months $= \frac{6}{12} = 0.5$ yr

$\therefore$ Age of the new boy $= a - nb = 20 - 25 \times 0.5 = 20 - 12.5 = 7.5$ yr

[here, '−' sign has been taken as average decreases in this case.]

Technique 5

If the average of n students in a class is a, where average of passed students is x and average of failed students is y, then

Number of students passed

$$= \frac{\text{Total students (Total average} - \text{Average of failed students)}}{\text{Average of passed students} - \text{Average of failed students}} = \frac{n(a-y)}{(x-y)}$$

Ex.20 In a class, there are 75 students and their average marks in the annual examination is 35. If the average marks of passed students is 55 and average marks of failed students is 30, then find out the number of students who failed. Sol. Let number of failed students = *x*

Then, number of passed students = 75 - *x* According to the question,

75 x 35 = 30z + 55 (75 - i) => 15 x 35 = 6i + 11 (75 - *x)*

=> 5a; = 300

x = 60

Fast Track Method

Here, $n = 75, a = 35, x = 55, y = 30$

$\therefore$ **Number of students who passed** $= \dfrac{n(a-y)}{x-y}$

$$= \frac{75\,(35-30)}{55-30} = \frac{75 \times 5}{25} = 15$$

$\therefore$ **Number of students who failed** $= 75 - 15 = 60$

Technique 6

if the average of total components in a group is a, where average of *n* components (1st part) is *b* and average of remaining components (2nd part) is c, then

$$\text{Number of remaining components (2nd part)} = \frac{n(a-b)}{(c-a)}$$

Ex.21 The average salary of the entire staff in an office is ? 200 per day. The average salary of officers is ? 550 and that of non-officers is ? 120. If the number of officers is 16, then find the numbers of non-officers in the office. Sol. Let number of non-officers = *x*

Then, 120x + 550 x 16 = 200 (16 + *x)*

=> 12x + 55 x 16 = 20 (16 + *x)*

=> 3z + 55 x 4 = 5 (16 + *x)*

=» *3x* + 220 = 80 + 5x

=> $5x - 3x = 220 - 80$

$$\Rightarrow \quad 2x = 140$$
$$\therefore \quad x = \frac{140}{2} = 70$$

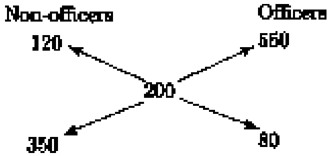
Fast Track Method

Here, $n = 16, a = 200, b = 550, c = 120$

According to the formula,

$$\text{Number of non-officers} = \frac{n\,(a-b)}{c-a}$$
$$= \frac{16\,(200-550)}{120-200}$$
$$= \frac{16 \times (-350)}{-80} = \frac{16 \times 35}{8}$$
$$= 2 \times 35 = 70$$

Alternate Method



Ratio = 35 : 8

Number of officers (8 ratio) $\rightarrow$ 16

$\Rightarrow$ 1 ratio $\rightarrow$ 2

Hence, number of non-officers (35)

(35 ratio) $\rightarrow 35 \times 2 = 70$

Note Here, we have taken the office as a mixture of officers and non-officers

Average Speed

Average speed is defined as total distance travelled divided by total time taken.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

• Case I

If a person covers a certain distance at a speed of A km/h and again covers the same

distance at a speed of B km/h, then the average speed during the whole journey will be $\frac{2AB}{A+B}$.

Ex. 22 A person covers a certain distance by car at a speed of 25 km/h and comes back at a speed of 40 km/h. What is his average speed during his travel? **Sol.** Here, A = 25 km/h and B = 40 km/h

$$\therefore \quad \text{Required average speed} = \frac{2AB}{A+B} = \frac{2 \times 25 \times 40}{25 + 40} = \frac{2000}{65} = 30.76 \text{ km/h}$$

* ***Case* II**

If a person covers three equal distances at the speed of A km/h, B km/h and C km/h respectively, then the average speed during the whole journey will be $\frac{3ABC}{AB+BC+CA}$.

X.23 If a person covers three equal distances at the speed of 30 km/h, 15 km/h and 10 km/h respectively, then find out his average speed during the whole journey.

Sol. Here, A = 30 km/h, B = 15 km/h and C = 10 km/h

$$\therefore \text{ Required average speed} = \frac{3ABC}{AB+BC+CA} = \frac{3 \times 30 \times 15 \times 10}{30 \times 15 + 15 \times 10 + 10 \times 30}$$
$$= \frac{3 \times 30 \times 15 \times 10}{450 + 150 + 300} = \frac{3 \times 30 \times 15 \times 10}{900} = 15 \text{ km/h}$$

* ***Case* III**

If distance P is covered with speed x, distance Q is covered with speed y and distance R is covered with speed z, then for the whole journey,

$$\text{Average speed} = \frac{P+Q+R+\ldots\ldots}{\frac{P}{x}+\frac{Q}{y}+\frac{R}{z}+\ldots\ldots}$$

Ex. 24 A person covers 20 km distance with a speed of 5 km/h, then he covers the next 15 km with a speed of 3 km/h and the last 10 km is covered by him with a speed of 2 km/h. Find out his average speed for the whole journey. Sol. Here.P = 20km,Q = 15 km, R = 10 km and $x = 5$ km/h, $y = 3$ km/h, $z = 2$ km/h

$$\therefore \text{ Required average speed } = \frac{P+Q+R}{\frac{P}{x}+\frac{Q}{y}+\frac{R}{z}} = \frac{20+15+10}{\frac{20}{5}+\frac{15}{3}+\frac{10}{2}}$$

$$= \frac{45}{4+5+5} = \frac{45}{14} = 3\frac{3}{14} \text{ km/h}$$

***■ *Case* IV**

If a person covers *P* part of his total distance with speed of x, Q part of total distance with speed of y and *R* part of total distance with speed of z, then

$$\text{Average speed} = \frac{1}{\frac{P}{x}+\frac{Q}{y}+\frac{R}{z}+\ldots\ldots}$$

Ex.25 Mr. Sharma travels by car and covers 25% of his journey with a speed of 10 km/h, 45% of his journey with a speed of 5 km/h and remaining 30% of his journey with a speed of 15 km/h. What will be the average speed of Mr. Sharma for the whole journey?

Sol. Here, $P = 25\% = \frac{1}{4};\ x = 10 \text{ km/h}$

$$Q = 45\% = \frac{45}{100} = \frac{9}{20};\ y = 5 \text{ km/h}$$

$$R = 30\% = \frac{30}{100} = \frac{3}{10};\ z = 15 \text{ km/h}$$

$$\therefore \text{ Required average speed } = \frac{1}{\frac{P}{x}+\frac{Q}{y}+\frac{R}{z}}$$

$$= \frac{1}{\frac{1}{4\times 10}+\frac{9}{20\times 5}+\frac{3}{10\times 15}} = \frac{1}{\frac{1}{40}+\frac{9}{100}+\frac{1}{50}} = \frac{1}{\frac{5+18+4}{200}}$$

$$= \frac{200}{27} = 7.40 \text{ km/h}$$

Fast Track Practice

Exercise© *Base Level Questions*

1. Find the average of the following set of scores 253, 124, 255, 534, 836, 375, 101, 443,760 [IBPS Clerk 2011]

(a) 427 (b) 413 (c) 141 (d) 490 (e) None of the above

2. Find the average of the following set of scores. 124, 856, 331, 227, 963, 338, 259 and 662.

(a) 570 (b) 660 (cJ141 (d) 350 (e) None of the above

3. What will be the average of even numbers between 11 to 63? [RRB 2008]

(a) 37.5 (b) 47

(c) 42 (d) 37

4. Find average of all prime numbers between 60 and 90. [SSC (10+2) 2012]

(a) 72 (b) 74.7 (C) 74 (d) 73.6

5. Calculate the average of the cubes of first five natural numbers. [LIC ADO 2007]

(a) 55 (b) 65 (c) 45 (d) 35 (e) None of the above

6. The average of 8 numbers is 14. The average of 6 of these numbers is 16. What is the average of the remaining two numbers? [DMRC (CRA) 2012]

(a) 12 (b) 6 (c) 8 (d) 10

7. Average age of 5 boy is 16 yr, of which that of 4 boys is 16 yr 3 months. The age of the 5th boy is

(a) 15 yr (b) 15 yr 6 months

(c) 15 yr 4 months (d) 15 yr 2 months

8. If the algebraic sum of deviations of 20 observations measured from 23 is 70, mean of these observations would be

[SNAP 2012]

(a) 24 (b) 25 (c) 26 (d) 27 (e) None of the above

9. Eight consecutive numbers are given. If the average of the two numbers that appear in the middle is 6, then the sum of the eight given numbers is

[SSC (10+2) 2012] *(a)* 36 *(b)* 48 *(c)* 54 *(d)* 64

10. The average of 4 consecutive even numbers is 27. What is the greatest number? [SSC SAS 2010]

(a) 28 *(b)* 26

(c) 32 *(d)* 30

11. A businessman purchased 30 TV sets in ? 300000. If the average cost of 15 TV sets is ? 9000, then find out the average cost of remaining TV sets.

(a) ? 20000 *(b)* ? 12000

(c) ? 11000 *(d)* ? 10000

(e) None of the above

12. The average age of 30 girls is 13 yr. The average of first 18 girls is 15 yr. Find out the average age of remaining 12 girls.

[Hotel Mgmt. 2009]

(a) 12 yr *(b)* 10 yr

(c) 16 yr *(d)* 10.5 yr

(e) None of the above

13. The average of four consecutive numbers A, *B, C* and *D* is 49.5. What is the product of *B* and *IP*.

(a) 2499 *(b)* 2352

(C) 2450 *(d)* 2550

(e) None of the above

14. The average height of the basketball team A is 5 ft 11 inches and that of *B* is 6 ft 2 inches. There are 20 players in team A and 18 players in team *B*. The overall average height is

(a) 72.42 inches *(b)* 72 inches *(c)* 70.22 inches *(d)* 70 inches

15. The average of 13 results is 60. If the average of first 7 results is 59 and that of last 7 results is 61, what will be the seventh result?

(a) 90 *(b)* 50 *(c)* 75 *(d)* 60 *(e)* None of the above

16. The average of 50 numbers is 38. If the two numbers 45 and 55 are not considered, what will be the average of remaining numbers? [UP Police 2009]

(a) 36.5 *(b)* 37

(c) 37.5 *(d)* 37.52

17. Average weight of 19 men is 74 kg and the average weight of 38 women is 63 kg. What is the average weight (rounded off to the nearest integer) of all the men and the women together? [SBI Clerk 2012]

(a) 59 kg *(b)* 65 kg *(c)* 69 kg *(d)* 67 kg *(e)* 71 kg

18. The average marks of 30 students are 45 but after checking, there are two mistakes found. After adjustment, if a student got 45 more marks and other student got 15 less marks, then what will be the adjusted average? [SSC CGL 2012]

(a) 45 *(b)* 44 *(c)* 47 *(d)* 46

The average of nine numbers is 50. The average of the first five numbers is 54 and that of the last three numbers is 52. Then, the sixth number is *(a)* 34 *(b)* 24 *(c)* 44 *(d)* 30

20. The mean marks obtained by seven students in a group is 226. If the marks obtained by six of them are 340, 180, 260, 56, 275 and 307 respectively, find the marks obtained by the seventh student.

(a) 164

(b) 226

(c) 340

(d) Cannot be determined

21. A cricketer has a mean score of 60 runs in 10 innings. Find out how many runs are to be scored in the eleventh innings to raise the mean score to 62?

(a) 80 *(b)* 81 *(c)* 83 *(d)* 82

22. The mean weight of 34 students of a school is 42 kg. If the weight of the teacher be included, the mean rises by 400 g. Find the weight (in kg) of the teacher.

(a) 66 *(b)* 56 *(c)* 55 *(d)* 57

23. The average weight of 21 boys was recorded as 64 kg. If the weight of the 'teacher was added, the average increased by one kg. What was the teacher's weight? [Bank Clerks 2011]

(a) 86 kg *(b)* 64 kg *(c)* 72 kg *(d)* 98 kg

(e) None of the above

24. The average age of 14 girls and their teacher's age is 15 yr. If teacher's age is excluded, then the average reduced by 1. What is the teacher's age? [SSC CGL 2013]

(a) 29 yr *(b)* 35 yr

(c) 32 yr *(d)* 30 yr

25. The average of six observations is 12. The average decreases by 1 when a new observation is included. What is the seventh observation? [Delhi Police 2009]

(a) 1 *(b)* 3 *(c)* 5 *(d)* 6

26. The average age of a class is 35 yr. 6 new students with an average age of 33 yr joined in that class, there by decreasing the average by half year. The original strength of the class was [SSC (10+2) 2012]

(a) 14 *(b)* 18 *(c)* 16 *(d)* 20

27. The average age of 5 members of a family is 25 yr. If the servant of the family is included, the average age increases by 40%. What is the age of servant?

(a) 85 yr *(b)* 90 yr *(c)* 73 yr *(d)* 55 yr

28. The average age of 4 members of a family is 25 yr. If head of the family is included in this group, then average age increases by 20%. Find out the age of the head. [Delhi Police 2008]

(a) 45 yr *(b)* 50 yr *(c)* 55 yr *(d)* 60 yr

29. The average weight of 10 women is increased by 1 kg if 2 women of 42 kg and 38 kg are replaced by 2 men. Find the average weight of both the men.

(a) 46 kg *(b)* 45 kg

(c) 42 kg *(d)* 44 kg

(e) None of the above

30. The average age of a committee of 11 persons increases by 2 yr when 3 men of 32 yr, 34 yr and 33 yr are replaced by 3 women. What will be the average age of those 3 women? [RBI 2008]

(a) 40 yr *(b)* $41\frac{1}{3}$ yr

(c) 41 yr *(d)* $40\frac{1}{3}$ yr

(e) None of the above

31. The average age of 30 women decreases by 3 months if a new person Priyanka is included in place of a 25 yr old woman. Calculate the age of Priyanka.

(a) 17.5 yr *(b)* 20 yr

(c) 30 yr *(d)* 22 yr

(e) None of the above

32. Average of ten numbers is 7. If every number is multiplied by 12, what will be the average of new numbers?

[SSC (10+2) 2009]

(a) 7 (b) 9

(c) 82 *(d)* 84

33. The average temperature of Delhi for four days in a particular month is 48°C. If the average temperature of second and third days is 34° C and the ratio of the temperatures of first and fourth days is 9 : 11, then find out the temperatures of first and fourth days.

(a) 45.5°C, 49.9°C *(b)* 32.4°C, 39.8°C *(c)* 43.4"C, 68.2°C *(d)* 52.4°C, 46.8°C *(e)* None of the above

34. The mean temperature of Monday to Wednesday was 37°C and of Tuesday to Thursday was 34°C. If the temperature on Thursday was 4/5 that of Monday, the temperature of Thursday was

[SSCFCI2012]

(a) 35.5°C *(b)* 34°C

(c) 36.5-C *(d)* 36°C

35. The average of 11 numbers is 36. Out of these 11 numbers, the average of 9 numbers is 34. If the ratio of remaining two numbers is 2 : 3, find out the smaller number. [Hotel Mgmt. 2009]

(a) 18 *(b)* 36 *(c)* 54 *(d)* 48 *(e)* None of the above

36. Roshan bought 5 pants at ? 25 each, 10 shirts at ? 50 each and 15 ties at ? 35 each. Find the average price of all the articles.

(a) ? 38.33 *(b)* ? 45

(c) ? 60 *(d)* ? 45.33

(e) None of the above

37. The average weight of a class of 15 boys and 10 girls is 38.4 kg. If the average weight of the boys is 40 kg, then what is the average weight of the girls? [CDS 2013]

(a) 36.5 kg *(b)* 35 kg

(c) 36 kg *(d)* 35.6 kg

38. The mean weight of 150 students in a class is 60 kg. The mean weight of boys is 70 kg and that of girls is 55 kg. What is the number of boys in the class?

[CDS 2012]

(a) 50 *(b)* 60

(c) 75 *(d)* 100

39. The average monthly income of 4 earning members of a family is ? 7350. One member passes away and the average monthly income becomes ? 6500. What was the monthly income of the person, who is no more? [SSC CGL 2007]

(a) ? 6928 *(b)* ? 8200

(c) ? 9900 *(d) X* 13850

40. The average of 5 consecutive odd positive integers is 9. The least one among them is

[SSC Multitasking 2014]

(a) 5 *(b)* 3

(c) 1 *(d) 7*

41. The average of the test scores of a class of W students is 70 and that of *'ri* students is 91. When the scores of both the classes are combined, the average is 80. What is re/m?

(a) 11/10 *(b)* 13/10

(c) 10/13 *(d)* 10/11

42. If 47a + 476 = 5452, what is the average of a and 6? [SNAP 2012]

(a) 116 *(b)* 23.5

(c) 96 *(d)* 58

(e) None of the above

43. The average salary per head of all workers of an institution is ? 60. The average salary per head of 12 officers is ? 400. The average salary per head of the rest is ? 56. Then, the total number of workers in the institution is

(a) 1030 *(b)* 1032

(c) 1062 *(d)* 1060

44. A car travels a certain distance from town *A* to town *B* at the speed of 42 km/h and from town *B* to town A at a speed of 48 km/h. What is the average speed of the car? [Bank Clerks 2008]

(a) 45 km/h

(b) 46 km/h

(c) 44 km/h

(d) Cannot be determined

(e) None of the above

45. A man divides his total route of journey into three equal parts and decides to travel the three parts with speed of 15 km/h, 10 km/h and 5 km/h, respectively. Find his average speed of during journey.

(a) 2.28 km/h *(b)* 9 km/h

(c) 14 km/h *(d)* 8.18 km/h

(e) None of the above

46. Mr. Bundda travels in his car and covers 1/4 part of his journey with 8 km/h, 3/5 part with 6 km/h and remaining 3/20 part with a speed of 10 km/h. Find out his average speed during the whole journey.

(a) 6.83 km/h *(b)* 9 km/h

(c) 4 km/h *(d)* 8.5 km/h

(e) None of the above

47. A man spends an average of ? 1694. 70 per month for the first 7 month and ? 1810.50 per month for the next 5 months. His monthly salary if he saves ? 3084.60 during the whole year is

(a) ? 2400 *(b)* ? 3000

(c)?1000 (dj? 2000

48. The average marks of 120 students are 35. If the average of passed students was 39 and failed students was 15, then find the number of students who have passed.

[SSCCGL2012] *(a)* 80 *(b)* 100

(c) 120 *(d)* 140

49. A person covers 9 km with a speed of 3 km/h, 25 km with a speed of 5 km/h and 30 km with a speed of 10 km/h. Find out the average speed of person.

[LIC ADO 2007]

(a) $5\frac{9}{11}$ km/h *(b)* $11\frac{6}{9}$ km/h

(c) $8\frac{5}{11}$ km/h *(d)* $5\frac{5}{11}$ km/h

(e) None of the above

50. In the afternoon, a student read 100 pages at the rate of 60 pages/h. In the evening, when she was tired, she read 100 more pages at the rate of 40 pages/h. What was her average rate of reading, in pages per hour?

(a) 48 *(b)* 50 *(c)* 60 *(d)* 70

51. Five years ago, the average age of *P* and *Q* was 15 yr. Now, average age of *P, Q* and *R* is 20 yr. What whould be the age *ofR* after 10 yr?

(a) 35 yr *(b)* 40 yr *(c)* 30 yr *(d)* 50 yr

52. The average of four positive integers is 73.5. The highest integer is 108 and the lowest integer is 29. The difference between the remaining two integers is 15. Which of the following is the smaller of the remaining two integers?

[Bank Clerks 2009] *(a)* 80 *(b)* 86 *(c)* 73

(d) Cannot be determined

(e) None of the above

53. The daily minimum temperature of a place in a certain week, beginning from Monday was 20°, 18.6°, 17°, 21.4°, 19° and 19.8°. It was known that the average of the whole week was 19.5°. The minimum temperature on Sunday was

(a) 16.9° *(b)* 11.6°

(c) 20.7° *(d)* 12.8°

54. The average age of two boys and their father is greater than the average age of those two boys and their mother by 3 yr. The average age of the four is 19 yr, If the average age of the two boys be 5%yr, then find the age of the father and mother. [SSC Multitasking 2014]

(a) 37 yr and 28 yr

(b) 47 yr and 38 yr

(c) 50 yr and 41 yr

(d) 35 yr and 32 yr

55. The average salary of the entire staff in an office is ? 500 per day. The average salary of officers is ? 750 and that of non-officers is ? 250. If the number of officers is 15, then find the number of non-officers in the office.

(a) 25 *(b)* 10 *(c)* 15 *(d)* 20

Exercise © *Higher Skill Level Questions*

1. *A* has double the money of *B* and *B* has 50% more money than C. If average money of all the three persons is ? 12000, how much money does *A* have?

(a) ₹ $\frac{211000}{11}$ (b) ₹ $\frac{315000}{11}$

(c) ₹ $\frac{216000}{11}$ (d) ₹ $\frac{316000}{11}$

(e) None of the above

2. A cricketer scored some runs in his continuous 9 innings. He scored 100 runs in his 10th innings and increases his average by 8 runs. What

was the average of his runs at the end of 10th innings? [LIC ADO 2008]]

(a) 20 *(b)* 24

(c) 28 *(d)* 32

(e) None of the above 3. Three year ago, the average age of a family of five members was 27 yr. The present average age is also 27 yr after the inclusion of a child into the family. Find the age of the child. [UP Police 2008]

(a) 16 yr *(b)* 12 yr

(c) 24 yr *(d)* 20 yr

4. Average weight of three boys P, T and R is $54\frac{1}{3}$ kg, while the average weight of three boys T, F and G is 53 kg. What is the average weight of P, T, R, F and G?

[Bank Clerks 2008]

(a) 53.8 kg

(b) 52.4 kg

(c) 53.2 kg

(d) Cannot be determined

(e) None of the above

5. The average runs scored by a cricketer in 42 innings, is 30. The difference between his maximum and minimum scores in an innings is 100. If these two innings are not taken into consideration, then the average score of remaining 40 innings is 28. Calculate the maximum runs scored by him in an innings? [SSC SAS 2010]

(a) 125 *(b)* 120 *(c)* 110 *(d)* 100

6. The average age of 5 sisters is 20 yr. If the age of the youngest sister be 4 yr, what was the average age of the group of sisters at the birth of the youngest sister?

(a) 25 yr *(b)* 15 yr *(c)* 18 yr *(d)* 20 yr *(e)* None of the above

7. The average marks obtained by 120 students in an examination is 30. If the average marks of passed students is 40 and that of the failed students is 10, what is the 25% of the number of students who passed the examination?

(a) 30 (b) 20 (c) 40 (d) 15 (e) None of the above

8. A cricketer played 80 innings and scored an average of 99 runs. His score in the last inning is zero run. To have an average of 100 at the end, his score in the last innings should have been

(a) 10 runs (b) 1 run

(c) 60 runs (d) 80 runs

If $x + y + z - 13$, then the maximum value of $(x - 2)(y + 1)(z - 3)$ is

[SSC FCI 2012]

(a) 25 (b) 30 (c) 54 (d) 27 10. A class is divided into two sections *A* and *B*. Passing average of 20 students of section *A* is 80% and passing average of 30 students of section *B* is 70%. What is the passing average of both the sections? [SSC CGL 2009]

(a) 72% (b) 74%

(c) 75% (d) 77%

11. The average age of 4 members of a family is 20 yr. If youngest member is 4 yr old, what was the average age of family at the time of the birth of youngest member?

(a) 27 yr (b) $23\frac{1}{3}$ yr (c) $21\frac{1}{3}$ yr (d) 22 yr

(e) None of the above

12. The arithmetic mean of the scores of a group of students in a test was 52. The brightest 20% of them secured a mean score of 80 and the dullest 25%, a mean score of 31. The mean score of remaining 55% is

(a) 54.6% (b) 45%

(c) 50% (d) 51.4%

13. A car reached Raipur from Somgarh in 35 min with an average speed of 69 km/h. If the average speed is increased by 36 km/h, how much time will it take to cover the same distance?

[Bank Clerks 2009] (a) 24 min (b) 27 min

(c) 23 min *(d)* 29 min

(e) None of the above

14. The average weight of 4 men *A, B, C* and *D,* is 67 kg. The 5th man *E* is included and the average weight decreases by 2 kg. *A* is replaced by *F.* The weight of *F* is 4 kg more than *E.* Average weight decreases because of the replacement of *A* and now the average weight is 64 kg. Find the weight of A.

(a) 78 kg *(b)* 66 kg *(c)* 75 kg *(d)* 58 kg *(e)* None of the above

15. 3yr ago, the average age of a family of 5 members was 17 yr. Even after the birth of a child, the average is same today. Find out the age of child.

[LIC AAO 2008]

(a) 1 yr *(b)* 3 yr *(c)* $2\frac{1}{2}$ yr *(d)* 2 yr
(e) None of the above

16. The average age of three boys is 15 yr. If the ratio of their ages is 3 : 5 : 7, what is the age of the oldest boy? [LIC AAO 2008]

(a) 7 yr *(b)* 14 yr *(c)* 20 yr *(d)* 21 yr

(e) None of the above

17. 5 yr ago, the average age of Rashmi and Surbhi was 20 yr. Now, the average age of Rashmi, Surbhi and Meeta is 30 yr. What will be the age of Meeta after 15 yr?

(a) 35 yr *(b)* 55 yr *(c)* 59 yr *(d)* 67 yr *(e)* None of the above

18. The average weight of 60 students in a class was calculated to be 40 kg. Later, it was found out that the weight of one of the students was calculated as 36 kg, whereas his actual weight was 33 kg. What is the actual average weight of the students in the class? [Bank Clerks 2009]

(a) 39.90 kg *(b)* 39.95 kg

(c) 39 kg *(d)* 38 kg

(e) None of the above

19. A person bought some oranges worth *X* 36 from each of the five markets at *X* 1, *X* 1.50, *X* 1.80, *X* 2 and *X* 2.25 per orange, respectively. What is the average price of an orange? [SSC FCI 2012]

(a) X 1.91 *(b) X* 2.00

(c) X 1.58 *(d) X* 1.80

20. Average of *a, b* and c is 11; average of c, *d* and e is 17; average of e and/is 22 and average of *e* and c is 17. Find out the average of *a, b, c, d, e* and *f.* [CBI 2009]

(a) $15\frac{2}{3}$ *(b)* $18\frac{1}{2}$ *(c)* $21\frac{1}{3}$ *(d)* $16\frac{1}{2}$

21. The average age of 54 girls in a class was calculated as 14 yr. It was later realized that the actual age of one of the girls in the class was 10.5 yr but it was calculated as 13 yr. What is the actual average age (round off) of the girls in the class? *(Find the approximate answer.)*

(a) 10.50 yr *(b)* 12.50 yr

(c) 12 yr *(d)* 14 yr

(e) None of the above

22. The average age of a man and his two twin sons born on the same day is 30 yr. The ratio of the ages of father and one son is 5 : 2, what is the father's age?

[Bank Clerks 2009] *(a)* 50 yr *(b)* 30 yr *(c)* 45 yr *(d)* 20 yr *(e)* None of the above

23. The ratio of pens and pencils in a stationary shop is 3 : 2, respectively. The average number of pens and pencils is 180. What is the 30% of number of pencils in the shop?

(a) 150 *(b)* 144

(c) 245 *(d)* 115

(e) None of the above

24. If the average of the ages of Rakesh and Mohan is 15, average of the ages of Mohan and Ramesh is 12 and the average of the ages of Rakesh and Ramesh is 13, then the age of Mohan, is

[CGPSC2013]

(a) 16 *(b)* 13 *(c)* 14 *(d)* 12 *(e)* None of the above

25. A cricketer has a certain average for 10 innings. In the eleventh innings, he scored 216 runs, there by increasing his average by 12 runs. Find out his new average. [Bank Clerks 2011]

(a) 96 *(b)* 84 *(c)* 97 *(d)* 87 *(e)* None of the above

26. A cricketer whose bowling average is 12.4 runs per wicket, takes 5 wickets for 26 runs and there by decreases his average by 0.4. Find out the number of wickets taken by him till the last match.

(a) 75 *(b)* 81 *(c)* 85 *(d)* 79 *(e)* None of the above

27. Nine friends have a dinner in a hotel. Eight of them spent *X* 12 each on their meals and the ninth spent *X* 16 more than the average expenditure of all the nine. Find out the total money spent by them. [Hotel Mgmt. 2010]

(a) X 126 *(b) X* 135 *(c) X* 111 *(d) X* 141 *(e)* None of the above

28. The average weight of the students in four sections *A, B, C* and *D* is 60 kg. The average weight of the students of A, *B, C* and *D* individually are 45 kg, 50 kg, 72 kg and 80 kg, respectively. If the average weight of the students of section *A* and *B* together is 48 kg and that of *B* and C together is 60 kg, what is the ratio of the number of students in sections *A* andD? [SSCCPO2013]

(a) 12 : 7 *(b)* 4 : 3

(c) 3 : 2 *(d)* 8 : 5

29. The average age of students of a class is 15.8 yr. The average age of boys in the class is 16.4 yr and that of the girls is 15.4 yr. Find out the ratio of the number of boys to the number of girls in the class. [SSC CGL 2011]

(a) 3 : 1 *(b)* 5 : 2 *(c)* 2 : 3 *(d)* 3 : 7

30. In a certain examination, the average marks of an examinee is 64 per paper. If he had obtained 18 more marks for his Mathematics paper and 4 more marks for his English paper, his average per paper would have been 66. How many papers were there in the examination?

[Bank PO 2010]

(a) 11 *(b)* 13

(c) 9 *(d)* ^

(e) None of the above

Answer with Solutions

Exercise © *Base Level Questions*

1. (*e*) Required average $= \dfrac{\text{Sum of all scores}}{\text{Number of scores}}$

$$= \frac{253 + 124 + 255 + 534 + 836 + 375 + 101 + 443 + 760}{9}$$

$$= \frac{3681}{9} = 409$$

2. (*e*) Required average

$$= \frac{124 + 856 + 331 + 227 + 963 + 338 + 259 + 662}{8}$$

$$= \frac{3760}{8} = 470$$

3. (*d*) Even numbers between 11 to 63 = 12, 14, 16, 18, 20, ..., 62. Clearly, this is a series of consecutive even numbers.

According to the formula,

Average of consecutive even numbers

$$= \frac{\text{First number} + \text{Last number}}{2}$$

$$= \frac{12 + 62}{2} = \frac{74}{2} = 37$$

4. (*b*) Average of all prime numbers between 60 and 90

$$= \frac{61 + 67 + 71 + 73 + 79 + 83 + 89}{7}$$

$$= \frac{523}{7} = 74.7$$

5. (*c*) As per the formula,

Average of the cubes of 1st 'n' natural numbers $= \dfrac{n(n+1)^2}{4}$.

where, $n = 5$.

$\therefore$ Required average $= \dfrac{5(5+1)^2}{4}$

$$= \frac{5 \times 36}{4} = 5 \times 9 = 45$$

6. (*c*) Average of 8 numbers = 14

Sum of 8 numbers $= 14 \times 8 = 112$

Similarly, average of 6 numbers = 16

Sum of 6 numbers $= 16 \times 6 = 96$

$\therefore$ Sum of remaining two numbers

= Sum of 8 numbers − Sum of 6 numbers

$= 112 - 96 = 16$

Average $= \dfrac{\text{Sum of two number}}{2} = \dfrac{16}{2} = 8$

7. (*a*) Average age of 5 boys = 16 yr

Sum of age of 5 boys = $16 \times 5 = 80$ yr

$\therefore$ Average age of 4 boys = 16- yr 3 months

$$= 16 \text{ yr} + \frac{3}{12} \text{ yr} = 16 \text{ yr} + \frac{1}{4} \text{ yr}$$

Sum of ages of 4 boys = $4 \times \left(16 + \frac{1}{4}\right)$ yr

$$= 64 + 1 = 65 \text{ yr}$$

$\therefore$ Age of 5th boy = $(80 - 65)$ yr = 15 yr

8. (*e*) Mean = $\dfrac{\text{Sum of observation}}{\text{Number of observation}}$

Sum of observation is given as = 70

Number of observation = 20

$\therefore \quad \text{Mean} = \dfrac{70}{20} = 3.5$

But, here observation starts from 23.

$\therefore$ Mean = 23 + 3.5 = 26.5

9. (*b*) Let's eight sequencially number are $x, x+1, x+2, x+3, x+4, x+5, x+6$ and $x+7$.

According to the question,

$$\frac{x+3+x+4}{2} = 6$$

$\Rightarrow \quad \dfrac{2x+7}{2} = 6 \quad \Rightarrow \quad 2x + 7 = 12$

$\Rightarrow \quad 2x = 5 \quad \Rightarrow \quad x = \dfrac{5}{2}$

$\therefore$ Sum of all eight numbers

$= x + x + 1 + x + 2 + x + 3 + x + 4$
$\quad + x + 5 + x + 6 + x + 7 = 8x + 28$

$= 8 \times \dfrac{5}{2} + 28 = 20 + 28 = 48$

10. (*d*) Let the four consecutive even numbers are $x, (x+2), (x+4), (x+6)$.

Clearly, this is a series of consecutive even numbers.

According to the formula,

Average of consecutive numbers

$$= \frac{\text{First number} + \text{Last number}}{2}$$

$\Rightarrow \quad 27 = \dfrac{x + (x+6)}{2}$

$\Rightarrow \quad x + 3 = 27$

$\therefore \quad x = 24$

$\therefore$ Greatest number = 4th consecutive even number = $(x+6) = 24 + 6 = 30$

11. (C) Total cost of 30 TV sets = ? 300000 Total cost of 15 TV sets

= 9000 x 15 = 135000 Total cost of remaining 15 TV sets

= 300000 - 135000 = 165000 .'. Average cost of remaining TV sets

$$= \frac{165000}{15} = ₹\ 11000$$

Fast Track Method

Here, $m = 30$, $n = 15$

$a = 10000$, $b = 9000$

Average of remaining TV sets $= \frac{ma - nb}{m - n}$

[by Technique 2]

Average cost of remaining (30 − 15) TV sets

$$= \frac{30 \times 10000 - 15 \times 9000}{30 - 15}$$

$$= \frac{300000 - 135000}{15}$$

$$= \frac{165000}{15} = ₹\ 11000$$

12. (*b*) Total age of 30 girls = 30 × 13 = 390 yr

Total age of 18 girls = 18 × 15 = 270 yr

∴ Age of remaining 12 girls

= 390 − 270 = 120 yr

∴ Required average $= \frac{120}{12} = 10$ yr

Fast Track Method

Here, $n_1 = 18$, $n_2 = 12$

$a_1 = 15$ yr, $a_2 = ?$

∴ Average $= \frac{n_1a_1 + n_2a_2}{n_1 + n_2}$ [by Technique 1]

$$\Rightarrow \quad 13 = \frac{18 \times 15 + 12a_2}{18 + 12}$$

$$\Rightarrow \quad 13 \times 30 = 270 + 12a_2$$

$$\Rightarrow \quad 12a_2 = 390 - 270$$

$$\therefore \quad a_2 = \frac{120}{12} = 10 \text{ yr}$$

13. (*a*) Let the numbers A, B, C and D are x, $x + 1$, $x + 2$ and $x + 3$, respectively.

According to the question,

$$\frac{x + (x + 1) + (x + 2) + (x + 3)}{4} = 49.5$$

$$\Rightarrow \quad 4x + 6 = 4 \times 49.5$$

$$\Rightarrow \quad 4x = 198 - 6$$

$$\Rightarrow \quad x = \frac{192}{4} \Rightarrow x = 48$$

The required product = $B \times D$

$= (x + 1) \times (x + 3)$

$= (48 + 1)(48 + 3) = 49 \times 51 = 2499$

14. (*a*) Overall average height $= \frac{n_1a_1 + n_2a_2}{n_1 + n_2}$

$n_1 = 20$, $a_1 = 5$ ft 11 inches

$= 5 \times 12 + 11 = 71$ inches

$n_2 = 18, a_2 = 6$ ft 2 inches

$= 6 \times 12 + 2 = 74$ inches

$\therefore$ Overall height $= \dfrac{(20)(71) + (18)(74)}{20 + 18}$

$= \dfrac{1420 + 1332}{38} = 72.42$ inches

15. (*d*) According to the fundamental formula,

$$A = \frac{S}{N}$$

[A = Average, S = Sum, N = Number]

From the question, $60 = \dfrac{S}{13}$

$\therefore \quad S = 60 \times 13 = 780$

Sum of first seven results $= 59 \times 7 = 413$

Sum of last seven results $= 61 \times 7 = 427$

$\therefore$ 7th result = Sum of first seven results + Sum of last seven results − Sum of all the results

$= (413 + 427 - 780)$

$= (840 - 780) = 60$

Note Sum of first seven results when added to the sum of last seven results, then repetition of 7th result takes place.

16. (*c*) According to the basic formula of average, $A = \dfrac{S}{N}$

[A = Average, S = Sum, N = Number]

$\therefore \quad 38 = \dfrac{S}{50}$

$\Rightarrow \quad S = 38 \times 50 = 1900$

Sum of remaining numbers after eliminating 45 and 55

$= 1900 - (45 + 55) = 1800$

Average of remaining numbers

$= \dfrac{\text{Sum of remaining numbers}}{\text{Remaining number}}$

$= \dfrac{1800}{(50 - 2)} = \dfrac{1800}{48} = 37.5$

17. (*d*) Here, $n_1 = 19$, $a_1 = 74$, $n_2 = 38$, $a_2 = 63$

Total average weight $= \dfrac{n_1 a_1 + n_2 a_2}{n_1 + n_2}$

$= \dfrac{(19)(74) + (38)(63)}{19 + 38} = \dfrac{3800}{57} \approx 67$ kg

18. (*b*) Total number of all students of the class

$= 30 \times 45 = 1350$

$\therefore$ New average after adjustment

$= \dfrac{1350 - 45 + 15}{30} = \dfrac{1320}{30} = 44$

19. (*b*) Required number

$= 50 \times 9 - (54 \times 5 + 3 \times 52)$

$= 450 - (270 + 156) = 450 - 426 = 24$

20. (a) Sum of marks of 7 student

= Mean X Number of student

= 226 X 7 = 1582 Now, total marks of 6 students

= (340 + 180 + 260 + 56 + 275 + 307)

= 1418 .•. Marks obtained by seventh student

= Total marks of seven students

- Total marks of six students

= 1582- 1418 = 164

21. (d)Here, $n = 10$. $a = 60$, $b = 62$

Number of runs scored in 11th inning = $n (b - a) + b$ [by Technique 3] = 10 (62 - 60) + 62 = 20 + 62 = 82

22. (6) Here, $n = 34$,a = 42 and $b = 42.4$

Weight of the teacher.

= $n\{b - a) + b$ = 34(42.4 - 42) + 42.4 = 13.6 + 42.4 = 56.0 kg

23. (a) Here, $n = 21$, a = 64 , $b = 65$

Weight of teacher = $n(b - a) + b$

= 21(65 - 64) + 65 = 21 + 65 = 86 kg

24. (a) Sum of 14 girl's and teacher's age

= 15x 15 = 225 Without teacer's age, sum of 14 girl's age

= 14 X14 = 196 ∴ Teacher's age = 225 - 196 = 29 year

25. (C) Total of 6 observations = 6 X 12 = 72

Total of 7 observations (including the 7th) = 7 x (12-1) = 77 ∴ 7th observation = 77 - 72 = 5 **Fast Track Method** 7th observation = *n (b* - a) + *b*

[by Technique 3] where, initial average, a = 12 Last average, i> = (12-l) = ll

n = *6* Now, 7th observation = 6 (11-12) + 11 = -6 + 11 = 5

26. (6) Let the number of students in the class = *x*

According to the question.

$$\frac{35x + 6 \times 33}{x + 6} = 35 - \frac{1}{2}$$

$$\Rightarrow \quad 35x + 198 = 34.5(x + 6)$$

$$\Rightarrow \quad 35x + 198 = 34.5x + 207$$

$$\Rightarrow \quad 0.5x = 207 - 198 \; x = \frac{9}{0.5} = 18$$

$$\therefore \quad x = 18 \text{ days}$$

27. (a) Total age of 5 members

= 25x 5 = 125yr Total age of 5 members and the servant

$$= \left(25 + 25 \times \frac{40}{100}\right) \times 6$$

= 35 X 6 = 210 yr ∴ Age of the servant = 210 - 125 = 85 yr

Fast Track Method

Here, $n = 5$

Initial average $a = 25$

Last average, $b = 25 + 25 \times \frac{40}{100}$

$= 25 + 10 = 35$

$\therefore$ Age of the servant $= n(b - a) + b$

$= 5(35 - 25) + 35$

$= 50 + 35 = 85$ yr

28. (*b*) Total age of 4 members $= 25 \times 4 = 100$ yr

Total age of 5 members including the head

$= 5 \times \left(25 + 25 \times \frac{20}{100}\right)$

$= 5 \times 30 = 150$ yr

$\therefore$ Age of the head $= 150 - 100 = 50$ yr

Fast Track Method

Here, $n = 4$ Initial average, $a = 25$

Last average, $b = 25 + 20\% \times 25$

$= 25 + \frac{20 \times 25}{100} = 30$

$\therefore$ Age of the head $= n(b - a) + b$

[by Technique 3]

$= 4(30 - 25) + 30 = 4 \times 5 + 30 = 50$ yr

29. (*b*) Total weight of 2 men

$= 1 \times 10 + (42 + 38)$

$= 10 + 80 = 90$ kg

$\therefore$ Average weight of 2 men

$= \frac{90}{2} = 45$ kg

Fast Track Method

Here, $n = 10, a = 42 + 38 = 80$ kg, $b = 1$ kg

Total weight of both men $= a + nb$

[by Technique 4]

$= 80 + 10 \times 1 = 90$ kg

$\therefore$ Average weight of both men $= \frac{90}{2} = 45$ kg

30. (*d*) Total age of 3 women

$= 2 \times 11 + (32 + 33 + 34)$

$= 22 + 99 = 121$ yr

$\therefore$ Average age of 3 women

$= \frac{121}{3} = 40\frac{1}{3}$ yr

Fast Track Method

Here, $n = 11$,

$a = 32 + 33 + 34 = 99$ yr

$b = 2$ yr

$\therefore$ Age of 3 women $= a + nb$

$= 99 + 11 \times 2 = 99 + 22 = 121$ yr

$\therefore$ Average age of 3 women

$$= \frac{121}{3} = 40\frac{1}{3} \text{ yr}$$

[Average increases, hence '+' sign $(a + nb)$ has been taken]

31. (*a*) Here, average decreases by 3 months or $\frac{3}{12}$ yr.

Sum of ages decreased

$$= 30 \times \frac{3}{12} = \frac{15}{2} = 7.5 \text{ yr}$$

$\therefore$ Age of Priyanka $= (25 - 7.5) = 17.5$ yr

Fast Track Method

Here, $n = 30$, $a = 25$ yr, $b = \frac{3}{12}$ yr

$\therefore$ Age of Priyanka $= a - nb$ [by Technique 4]

$$= 25 - 30 \times \frac{3}{12}$$

$$= 25 - \frac{15}{2}$$

$$= 25 - 7.5 = 17.5 \text{ yr}$$

[Here, '–' sign $(a - nb)$ has been taken as average decreases in this case.]

32. (*d*) Here, initial average = 7

As we know that, if all the numbers are multiplied by a certain number, then their average must be a multiple of that number.

$\therefore$ New average $= 7 \times 12 = 84$

33. (*e*) Total temperature of first and fourth day

$$= (4 \times 48 - 2 \times 34)^\circ\text{C}$$

$$= (192 - 68)^\circ\text{C} = 124^\circ\text{C}$$

Now, according to the question,

Temperature of the first day

$$= \frac{9}{20} \times 124 = 55.8^\circ\text{C}$$

Temperature of fourth day

$$= \frac{11}{20} \times 124 = 68.2^\circ\text{C}$$

34. (*d*) Temperature of (Mon + Tue + Wed)

$$= 37 \times 3 = 111^\circ\text{C}$$

Temperature of (Tue + Wed + Thu)

$$= 34 \times 3 = 102^\circ\text{C}$$

Temperature of (Mon – Thu)

$$= 111^\circ\text{C} - 102^\circ\text{C} = 9^\circ\text{C}$$

Temperature of [Mon $- \frac{4}{5}$ (Mon)] = 9

Temperature of Monday $= 9 \times 5 = 45^\circ$C

$\therefore$ Temperature of Thursday

$$= 45 \times \frac{4}{5} = 36^\circ\text{C}$$

35. (*b*) Sum of 11 numbers $= 36 \times 11 = 396$

Sum of 9 numbers $= 34 \times 9 = 306$

Sum of remaining two numbers

$$= 396 - 306 = 90$$

$\therefore$ The smaller number $= \dfrac{2 \times 90}{2+3} = \dfrac{180}{5} = 36$

36. (*a*) Here, $n_1 = 5$, $n_2 = 10$, $n_3 = 15$, $a_1 = 25$, $a_2 = 50$, $a_3 = 35$

According to the formula,

Average price $= \dfrac{n_1a_1 + n_2a_2 + n_3a_3}{n_1 + n_2 + n_3}$

[by Technique 1]

$$= \frac{5 \times 25 + 10 \times 50 + 15 \times 35}{5 + 10 + 15}$$

$$= \frac{25 + 100 + 105}{1 + 2 + 3} = \frac{230}{6} = ₹\, 38.33$$

37. (*c*) Given that, number of boys $(n_1) = 15$

Number of girls $(n_2) = 10$

Average weight of boys and girls = 38.4 kg

Average of boys = 40 kg

Let average of girls be a_2

By formula, average $= \dfrac{na_1 + n_2a_2}{n_1 + n_2}$

$$\Rightarrow \quad 38.4 = \frac{15\,(40) + 10 \times a_2}{15 + 10}$$

$$\Rightarrow \quad 38.4 \times 25 = 600 + 10a_2$$

$$\Rightarrow \quad a_2 = \frac{960 - 600}{10} = \frac{360}{10} = 36\text{kg}$$

38. (*a*) Given that, total number of students $= 150 = (n_1 + n_2)$ and mean weight of 150 students = 60 kg. The mean weight of boys $a_1 = 70$ kg and the mean weight of girls $a_2 = 55$ kg

Let n_1 and n_2 be the number of boys and girl, respectively.

$\therefore\ n_1 + n_2 = 150$ ...(i)

$11n_1 + 11n_2 = 1650$ ...(ii)

Average weight $= \dfrac{n_1a_1 + n_2a_2}{n_1 + n_2}$

[by Technique 1]

$$\Rightarrow \quad 60 = \frac{70n_1 + 55n_2}{150}$$

$\Rightarrow \quad 70n_1 + 55n_2 = 9000$

$\Rightarrow \quad 14n_1 + 11n_2 = 1800$ ...(iii)

On subtracting Eq. (ii) from Eq. (iii), we get

$$3n_1 = 150 \Rightarrow n_1 = 50$$

Hence, required number of boys is 50.

39. (*c*) Monthly income of 4 persons

$= 7350 \times 4 = ₹\, 29400$

Monthly income of 3 persons (excluding the dead person) $= 6500 \times 3 = ₹\, 19500$

$\therefore$ Monthly income of dead person

$= 29400 - 19500 = ₹\, 9900$

Fast Track Method

Required income $= n\,(a - b) + b$

where, $\quad n = 4$

Initial average, $a = ₹\ 7350$
Last average, $b = ₹\ 6500$
Monthly income of the person, who is no more $= 4\ (7350 - 6500) + 6500$
$$= 4 \times 850 + 6500$$
$$= 3400 + 6500 = ₹\ 9900$$

40. (*a*) Let the 5 consecutive odd positive integers be $x + 1, x + 3, x + 5, x + 7, x + 9$

$$\text{Average} = \frac{\text{Sum of all terms}}{\text{Number of terms}}$$

$$\Rightarrow \quad 9 = \frac{x + 1 + x + 3 + x + 5 + x + 7 + x + 9}{5}$$

$$\Rightarrow \quad 9 = \frac{5x + 25}{5}$$

$$\Rightarrow \quad 45 = 5x + 25$$
$$\Rightarrow \quad 5x = 45 - 25$$
$$\therefore \quad 5x = 20 \Rightarrow x = 4$$

The least one is $x + 1 = 4 + 1 = 5$

41. (*d*) According to the question,

$$70m + 91n = 80\ (m + n)$$
$$\Rightarrow \quad 70m + 91n = 80m + 80n$$
$$\Rightarrow \quad 10m = 11n \Rightarrow \frac{n}{m} = \frac{10}{11}$$

42. (*d*) $47(a + b) = 5452$

$$\Rightarrow \quad a + b = \frac{5452}{47} = 116$$

$\therefore$ Average value

$$= \frac{a + b}{2} = \frac{116}{2} = 58$$

43. (*b*) Let the total number of workers $= x$

According to the question,

$$60x = 12 \times 400 + 56(x - 12)$$
$$\Rightarrow 60x - 56x = 4800 - 672$$
$$\Rightarrow \quad 4x = 4128 \Rightarrow x = 1032$$

44. (*e*) If two equal distances are covered at different speeds at A km/h and B km/h respectively, then

Average speed during the whole journey

$$= \frac{2AB}{A + B} \text{ km/h}$$

$\therefore$ Average speed of the car

$$= \frac{2 \times 42 \times 48}{42 + 48} = \frac{2 \times 42 \times 48}{90}$$
$$= 44.8 \text{ km/h}$$

45. (*d*) We know that, if three equal distances are covered at different speeds of A, B and C km/h respectively, then average speed during whole journey

$$= \frac{3ABC}{AB + BC + CA} \text{ km/h}$$

$\therefore$ Required average speed

$$= \frac{3 \times 15 \times 10 \times 5}{15 \times 10 + 10 \times 5 + 15 \times 5}$$

$$= \frac{3 \times 15 \times 10 \times 5}{150 + 50 + 75} = \frac{3 \times 15 \times 10 \times 5}{275}$$

$= 8.18$ km/h

46. (*a*) According to the average speed formula,

$$P = \frac{1}{4}, Q = \frac{3}{5}, R = \frac{3}{20}$$

$x = 8$ km/h, $y = 6$ km/h, $z = 10$ km/h

$\therefore$ Required average speed $= \dfrac{1}{\dfrac{P}{x} + \dfrac{Q}{y} + \dfrac{R}{z}}$

$$= \frac{1}{\frac{1}{4 \times 8} + \frac{3}{5 \times 6} + \frac{3}{20 \times 10}}$$

$$= \frac{1}{\frac{1}{32} + \frac{1}{10} + \frac{3}{200}}$$

$$= \frac{1}{\frac{25 + 80 + 12}{800}} = \frac{800}{117} = 6.83 \text{ km/h}$$

47. (*d*) Let the monthly salary of a man = ₹ x

$\therefore$ The annual salary = ₹ $12x$

According to the question,

Annually spends of a man

$= 7 \times 1694.70 + 5 \times 1810.50$

= ₹ 20915.40

$\therefore$ His monthly spends

$= \dfrac{20915.40}{12} =$ ₹ 1742.95

and monthly saving $= \dfrac{3084.60}{12} =$ ₹ 257.05

$\therefore$ His monthly salary

$= 1742.95 + 257.05 =$ ₹ 2000

48. (*b*) Here, $n = 120, a = 35, x = 39, y = 15$

Number of students passed $= \dfrac{n(a - y)}{(x - y)}$

$$= \frac{120(35 - 15)}{(39 - 15)} = \frac{120 \times 20}{24}$$

[by Technique 5]

= 100 students passed

49. (*a*) Here, $P = 9$ km, $Q = 25$ km, $R = 30$ km

$x = 3$ km/h, $y = 5$ km/h and $z = 10$ km/h

$\therefore$ Required average speed

$$= \frac{P + Q + R}{\frac{P}{x} + \frac{Q}{y} + \frac{R}{z}} = \frac{9 + 25 + 30}{\frac{9}{3} + \frac{25}{5} + \frac{30}{10}}$$

$$= \frac{64}{3 + 5 + 3} = \frac{64}{11}$$

$$= 5\frac{9}{11} \text{ km/h}$$

50. (*a*) Required average rate of reading

$$= \frac{2AB}{A + B} = \frac{2 \times 60 \times 40}{60 + 40}$$

$$= \frac{2 \times 60 \times 40}{100} = 48 \text{ pages/h}$$

51. (*c*) According to the question,

$$\left(\frac{P + Q}{2}\right) - 5 = 15$$

$$\Rightarrow \quad \frac{P + Q}{2} = 20$$

$$\Rightarrow \quad P + Q = 40 \quad \text{...(i)}$$

and $\quad \dfrac{P + Q + R}{3} = 20$

$$\Rightarrow \quad P + Q + R = 60 \quad \text{...(ii)}$$

On solving Eqs. (i) and (iii), we get

$$R = 20 \text{ yr}$$

$\therefore$ Age of R after 10 yr from now

$$= 20 + 10 = 30 \text{ yr}$$

52. (*e*) Let one integer be x.

$\therefore$ Other integer $= x + 15$

According to the question,

$$29 + x + x + 15 + 108 = 4 \times 73.5$$

$$\Rightarrow \quad 2x + 152 = 294$$

$$\Rightarrow \quad 2x = 294 - 152 = 142$$

$$\Rightarrow \quad x = \frac{142}{2} = 71$$

53. (*c*) Let temperature on Sunday be x°C.

Then, $\dfrac{20° + 18.6° + 17° + 21.4° + 19° + 19.8 + x}{7} = 19.5°$

$$\Rightarrow \quad 115.8 + x = 19.5 \times 7$$

$$\Rightarrow \quad x = 136.5 - 115.8$$

$$\Rightarrow \quad x = 20.7°\text{C}$$

54. (*a*) Let the ages of mother, father and boys be M, F, B_1 and B_2, respectively.

The total age of four members

$$= 19 \times 4 = 76 \text{ yr}$$

Given, $\dfrac{B_1 + B_1}{2} = \dfrac{11}{2}$

$\Rightarrow \quad B_1 + B_2 = 11$

$M + F + B_1 + B_2 = 76$

$\Rightarrow \quad M + F = 76 - 11$

$\Rightarrow \quad M + F = 65 \quad \ldots(\text{i})$

According to the question,

$$\frac{B_1 + B_2 + F}{3} = \frac{B_1 + B_2 + M}{3} + 3$$

$\Rightarrow \quad B_1 + B_2 + F = B_1 + B_2 + M + 9$

$\Rightarrow \quad F = M + 9$

$\Rightarrow \quad F - M = 9 \quad \ldots(\text{ii})$

From Eqs. (i) and (ii), we get

$F = 37$ yr and $M = 28$ yr

55. (*c*) Let number of non-officers = x

Then, $250x + 750 \times 15 = 500\,(15 + x)$

$\Rightarrow \quad 250x + 112\,50 = 7500 + 500x$

$\Rightarrow \quad 11250 - 7500 = 500x - 250x$

$\Rightarrow \quad 3750 = 250x$

$\Rightarrow \quad x = \dfrac{3750}{250}$

$\therefore \quad x = 15$

Fast Track Method

Here, $n = 15$, $a = 500$, $b = 750$, $c = 250$

$\therefore$ Number of non-officers $= \dfrac{n\,(a - b)}{c - a}$

$$= \frac{15\,(500 - 750)}{250 - 500}$$

$$= \frac{15 \times (-250)}{-250} = 15$$

Exercise © *Higher Skill Level Questions*

1. (*c*) Let money of C be ₹ x.

According to the question,

Total money of $B = x + x \times 50\%$

$$= x + \frac{50x}{100} = \frac{3x}{2}$$

Total money of $A = 2 \times \frac{3x}{2} = 3x$

Average money of three persons = 12000

$\therefore$ Total money of three 12000×3

$$\therefore \quad 3x + \frac{3x}{2} + x = 12000 \times 3$$

$$\Rightarrow \quad \frac{6x + 3x + 2x}{2} = 36000$$

$$\therefore \quad x = \frac{36000 \times 2}{11} = \frac{72000}{11}$$

Now, money of $A = 3x = 3 \times \frac{72000}{11}$

$$= ₹ \frac{216000}{11}$$

2. (*c*) Let average of a inning = x

Total runs = $9x$

According to the question.

$$\frac{9x + 100}{10} = x + 8$$

$$\Rightarrow \quad 9x + 100 = 10x + 80$$

$$x = 20$$

Average after 10 innings = 20 + 8 = 28

3. (6) Total age of 5 members (3 yr ago) = 27X5 = 135 yr

Total age of 5 members (at present) = 135+ 5x3 = 150 yr

Let the present age of child = x

$\therefore \quad 27 = \dfrac{150 + x}{6}$

$\Rightarrow \quad x = 162 - 150 = 12$ yr

4. (*d*) According to the question,

$$\frac{P + T + R}{3} = 54\frac{1}{3}$$

$\Rightarrow \quad P + T + R = \dfrac{163}{3} \times 3$

$\Rightarrow \quad P + T + R = 163 \quad \ldots(i)$

Also, $\quad \dfrac{T + F + G}{3} = 53$

$\Rightarrow \quad T + G + F = 159 \quad \ldots(ii)$

Clearly, from the two Eqs. (i) and (ii), the problem cannot be solved.

5. (*b*) Let the minimum score = x.

$\therefore$ Maximum score = $x + 100$

$\therefore \quad x + (x + 100) = 30 \times 42 - 40 \times 28$

$\Rightarrow \quad 2x + 100 = 1260 - 1120 = 140$

$\Rightarrow \quad 2x = 140 - 100 = 40$

$\therefore \quad x = 20$

Hence, the maximum score = $x + 100$

$= 20 + 100 = 120$

6. (*d*) Total age of 5 sisters = $20 \times 5 = 100$ yr

4 yr ago, total sum of ages

$= 100 - (5 \times 4) = 100 - 20 = 80$ yr

But at that time (4 yr ago), there were 4 sisters in the group.

$\therefore$ Average age at that time (4 yr ago)

$= \dfrac{80}{4} = 20$ yr

7. (*b*) Let number of passed students = x

Total marks = 120×30

According to the question,

$40x + (120 - x) \times 10 = 120 \times 30$

$\Rightarrow \quad 3600 = 40x + 1200 - 10x$

$\Rightarrow \quad 30x = 2400$

$\therefore \quad x = \dfrac{2400}{30} = 80$

$\therefore$ 25% of 80 = $\dfrac{80}{4} = 20$

Fast Track Method

Here, $n = 120$, $a = 30$, $x = 40$, $y = 10$

$\therefore$ Number of passed students

$$= \frac{n(a - y)}{x - y} = \frac{120\,(30 - 10)}{40 - 10}$$

$= 4\,(30 - 10) = 80$

$\therefore$ 25% of 80 = $\dfrac{80}{4} = 20$

8. (*d*) Let the required runs be x.

According to the question,

$$\frac{80 \times 99 + x}{80} = 100$$

$\Rightarrow \quad 7920 + x = 8000 \Rightarrow x = 80$

9. (*d*) We know that, AM ≥ GM

$\therefore \quad \frac{(x-2)+(y+1)+(z-3)}{3} \geq \sqrt[3]{(x-2)(y+1)(z-3)}$

$\Rightarrow \quad \frac{13-4}{3} \geq \sqrt[3]{(x-2)(y+1)(z-3)}$

$\Rightarrow \quad 3 \geq \sqrt[3]{(x-2)(y+1)(z-3)}$

$\Rightarrow \quad 27 \geq (x-2)(y+1)(z-3)$

Thus, maximum value of

$(x-2)(y+1)(z-3)$ is 27.

10. (*b*) Here, $n_1 = 20$, $n_2 = 30$, $a_1 = 80\%$, $a_2 = 70\%$

$\therefore$ Total average $= \frac{n_1 a_1 + n_2 a_2}{n_1 + n_2}$

[by Technique 4]

$= \frac{20 \times 80 + 30 \times 70}{20 + 30}$

$= \frac{1600 + 2100}{50} = 74\%$

11. (*c*) Total age of all members of the family (at present) $= 20 \times 4 = 80$ yr

4 yr ago, total age of all members

$= 80 - (4 \times 4) = 80 - 16 = 64$ yr

As 4 yr ago, there were only 3 members in the family.

$\therefore$ Average age of the family at the time of the birth of the youngest member

$= \frac{64}{3} = 21\frac{1}{3}$ yr

12. (*d*) Let the total number of students be 100.
Also, let the mean score $= x$
According to the question,

$20 \times 80 + 25 \times 31 + (100 - 20 - 25) \times x = 100 \times 52$

$\Rightarrow \quad 1600 + 775 + (55) \times x = 5200$

$\Rightarrow \quad 55x = 5200 - 1600 - 775$

$\therefore \quad x = \frac{2825}{55} = 51.36 = 51.4\%$

So, mean score of remaining 55% is 51.4%

13. (*c*) Distance between Raipur and Somgarh
= Average speed × Time

$= \frac{69 \times 35}{60} = \frac{161}{4}$ km

New speed = (69 + 36) km/h = 105 km/h

$\therefore$ Time $= \frac{\text{Distance}}{\text{Speed}} = \frac{161}{4 \times 105}$ h

$= \frac{161 \times 60}{4 \times 105}$ min $= 23$ min

14. (6) Sum of the weights of *A, B, C* and *D*

= 67 X 4 = 268 kg and average weight of *A, B, C, D* and *E*

= 67 - 2 = 65 kg /. Sum of the weights of *A, B, C, D* and *E*

= 65X 5 = 325 kg ∴ Weight of *E* = 325 - 268 = 57 kg ∴ Weight of .F = 57 + 4 = 61 kg Now, average weight of *F, B, C, D* and *E*

= 64 kg ∴ Sum of the weights of *F.B.C.D* and *E*

= 64 x 5 = 320 kg /. Sum of the weights of *B, C* and *D*

= 320 -57-61 = 202 kg ∴ Weight of *A* = 268 - 202 = 66 kg

15. (d) 3 yr ago, total age of 5 members of the family = 17x5 = 85 yr

Present age of all the members of the family = 85 + 3 X 5 = 100 yr Let the age of the child = *x* yr

$$\text{Present average age of family} = \frac{100 + x}{6}$$

[At present total number of members $= 5 + 1 = 6$]

$$17 = \frac{100 + x}{6} \Rightarrow 102 = 100 + x$$

x = 2 yr 16. (d) Total age of all the three boys = 3 X 15 = 45yr Ratio of ages = 3:5:7 /. Age of the oldest boy

$$= \frac{7}{3+5+7} \times 45 = \frac{7}{15} \times 45 = 21 \text{ yr}$$

17. (6) Total age of Rashmi and Surbhi (5 yr ago) = 20 x 2 = 40 yr.

Total age of Rashmi and Surbhi (at present)

= 40 + 5 + 5 = 50 yr.

Total age of Rashmi, Surbhi and Meeta (at

present) = 30 x 3 = 90 yr

.". Present age of Meeta = 90 — 50 = 40 yr

∴ AgeofMeetaafterl5yr=40 + 15 = 55yr

18. (6) Total correct weight

= 40 X 60 - 36 + 33 = 2400 - 3 = 2397 kg .". Required average weight

$$= \frac{2397}{60} = 39.95 \text{ kg}$$

19. (c) Number of oranges bought by the person

$$= \frac{36}{1} + \frac{36}{1.50} + \frac{36}{1.80} + \frac{36}{2} + \frac{36}{2.25}$$

= 36 + 24 + 20 + 18 + 16 = 114 Total expenditure = 36 X 5 = ? 180 Thus, average price of each orange

$$= ₹\frac{180}{114} = ₹\ 1.58$$

20. (a) According to the question,

a+ *b*+ *c* = 11X3 = 33 ...(i)

c+d+e = 17x3 = 51 ...(ii)

e+f = 22x2 = 44 ...(iii)

e + c = 17x2 = 34 ...(iv)

From Eqs. (ii) and (iv), we get

34 + d = 51ord = 17 ...(v)

Now, by adding Eqs. (i), (iii) and (v), we get

a+£ + c+d+e + / = 33 + 44 + 17=94

$\therefore$ Average of a, b, c, d, e and $f = \frac{94}{6}$

$$= \frac{47}{3} = 15\frac{2}{3}$$

21. (*d*) Total actual age of 54 girls

$$= (54 \times 14 - 13 + 10.5) = 753.5 \text{ yr}$$

$\therefore$ Required average age $= \frac{753.5}{54}$

= 13.95yr = 14 yr (approx).

22. (a) According to the question, Average age of man and his twin sons = 30 .'. Total age = 30 x 3 = 90 yr According to the question,

Ratio of father and one son = 5:2 /. Ratio of father and both the sons = 5:2:2 (As sons are born on the same day.) ∴ 5x + 2x + *2x* = 90 or 9x = 90 .\x = 10 ∴ Age of father = 5 X 10 = 50 yr

23. (e) The average number of pens and pencils

= 180 .•. Total number of pens and pencils

= 180 X2 = 360 /. Total number of pencils in the shop

$$= \frac{360 \times 2}{5} = 144$$

$$\therefore 30\% \text{ of } 144 = \frac{30}{100} \times 144 = 43.2$$

= 43 (approx).

24. (c) Let ages of Rakesh, Mohan and Ramesh be *R, M* and r, respectively.

Then, *R* + *M* = 15X2 = 30 ...(i)

M+r = 12x2 = 24 ...(ii)

r+ i? = 13x2 = 26 ...(iii)

Adding Eqs. (i), (ii) and (iii), we get 2(i? + *M* + r) = 30 + 24 + 26 = 80 => i? + M+r = 40

Subtracting Eq. (iii) from Eq. (iv), we get *M* = 40 -26 = 14

25. (G) Let average after 10th innings = *x*. According to the question,

10x+ 216 = ll(x+ 12) => x = 216-132 = 84

∴ New average = 84 + 12 = 96

Note This problem is similar to Q. 3 (Exercise 2) and therefore can also be solved according to the solution given for that (Q. 3, Exercise 2).

26. (c) Let x be the number of wickets taken till the last match. According to the question,

(12.4 x+ 26) = 12(x+ 5) => 12.4 x + 26 = 12x + 60

=> 0.4 x = 34

$$\therefore \quad x = \frac{34}{0.4} = 85$$

27. (*a*) Let average expenditure of 9 persons = x.

According to the question,

$$12 \times 8 + (x + 16) = 9x$$

$$\Rightarrow \quad 8x = 112 \Rightarrow x = \frac{112}{8} = 14$$

∴ Total money spent = $9x = 9 \times 14 =$ ₹ 126

28. (6) Let number of students in the sections *A, B, C* and *D* be a, *b, c* and *d,* respectively. Then, total weight of students of section A = 45a

Total weight of students of section *B* = 50 £ Total weight of students of section *C* = 72c Total weight of students of section *D* = 80d According to the question, Average weight of students of sections *A*

and *B* = 48 kg

$$\therefore \quad \frac{45a + 50b}{a + b} = 48$$

=> 45a + *50b* = 48a + 48*

=> *3a* = *2b*

=> 15a = 10i

And average weight of students of sections

Band C=60kg

=> *50b* + *72c* = 60(i> + c)

=> 10i> = 12c

Now, average weight of students of *A, B, C* and *D* = 60 kg

45a + 506 + 72c + 80d = 60(a + *b* + *c* + *d)* => 15a + 10jb-12c-20d = 0 => 15a = 20d => a:d = 4 :3

29. (*c*)

Girls		Boys
15.4		16.4
	15.8	
(16.4 – 15.8)		(15.8 – 15.4)
0.6		0.4

Clearly, Boys : Girls = 0.4 : 0.6 = 2 : 3

30. (*a*) Let the number of papers = x.

According to the question,

$$64x + 18 + 4 = 66x$$

$$\Rightarrow \quad 2x = 22$$

$$\therefore \quad x = \frac{22}{2} = 11$$

Chapter

11

Percentage

The term per cent means *'for every hundred'*. It can be defined as follows

"A per cent is a fraction whose denominator is 100 and the numerator of the fraction is called the **rate per cent."** Per cent is denoted by the sign '%'.

Formula to Calculate Per Cent

If we have to find y% of x, then

$$y\% \text{ of } x = x \times \frac{y}{100}$$

Some Quick Results

5% of a number $= \frac{\text{Number}}{20}$, 10% of a number $= \frac{\text{Number}}{10}$

$12\frac{1}{2}\%$ of a number $= \frac{\text{Number}}{8}$, 20% of a number $= \frac{\text{Number}}{5}$

25% of a number $= \frac{\text{Number}}{4}$, 60% of a number $= \frac{\text{Number}}{2}$

Ex. 1 20% of 300 = ?

Sol. According to the formula,

$$20\% \text{ of } 300 = 300 \times \frac{20}{100} = 3 \times 20 = 60$$

Alternate Method

Here, number = 300

$$20\% \text{ of number} = \frac{\text{Number}}{5} = \frac{300}{5} = 60$$

Ex. 2 If 30% of a = 60, then find a.

Sol. According to the formula,

$$30\% \text{ of } a = 60$$

$$\Rightarrow \quad \frac{30a}{100} = 60$$

$$\Rightarrow \quad a = \frac{60 \times 100}{30} = 200$$

Conversion of Per Cent into Fraction

Expressing per cent (x%) into fraction.

$$\textbf{Required fraction} = \frac{x}{100}$$

Ex. 3 Express 25% in fraction.

Sol. $25\% = \frac{25}{100} = \frac{1}{4}$

Ex. 4 Express 84% in fraction.

Sol. $84\% = \frac{84}{100} = \frac{21}{25}$

Conversion of Fraction into Percentage

Expressing a fraction $\left(\frac{x}{y}\right)$ in per cent.

Required percentage $= \left(\frac{x}{y} \times 100\right)\%$

Ex. 5 Convert $\frac{3}{8}$ into per cent.

Sol. Required percentage $= \left(\frac{3}{8} \times 100\right)\% = 37.5\%$

Ex. 6 Express $2\frac{1}{4}$ in per cent.

Sol. Required percentage $= \left(2\frac{1}{4} \times 100\right)\% = \left(\frac{9}{4} \times 100\right)\% = 225\%$

Expressing One Quantity as a Per Cent with Respect to Other

To express a quantity as a per cent with respect to other quantity following formula is used.

$$\frac{\text{The quantity to be expressed in per cent}}{\text{2nd quantity (in respect of which the per cent has to be obtained)}} \times 100\%$$

Note To apply this formula, both the quantities must be in same metric unit,

Ex. 7 60 kg is what per cent of 240 kg? **Sol.** According to the formula, Reauired Dercentaee

$$= \frac{\text{The quantity to be expressed in per cent}}{\text{2nd quantity (in respect of which the per cent has to be obtained)}} \times 100\%$$

$$= \frac{60}{240} \times 100\% = \frac{100}{4}\% = 25\%$$

Ex. 8 10 g is what per cent of 1 kg?

Sol. Here, units of both the quantities are different. Hence, first, we will make both the units same.

1 quantity = 10 g ; 2 quantity = 1kg = 1000 g

Now, both the quantities are in same unit.

According to the formula,

Required per cent $= \frac{10}{1000} \times 100\% = 1\%$

Conversion of Per Cent into Decimal

Expressing per cent ($x\%$) in decimal.

$$\text{Required decimal} = \frac{x}{100} = 0.0x$$

Ex. 9 Express 9% in decimal.

Sol. Required decimal $= \frac{9}{100} = 0.09$

Ex. 10 Express 18% in decimal.

Sol. Required decimal $= \frac{18}{100} = 0.18$

Fast Track **Techniques**

to solve the QUESTIONS

Technique 1

if $x\%$ of A is equal to $y\%$ of B, then

$$z\% \text{ of } A = \left(\frac{yz}{x}\right)\% \text{ of } B$$

Ex. 11 If 10% of *A* is equal to 12% of *B*, then 15% of *A* is equal to what per cent of B?

Sol. Given that, $x = 10$, $y = 12$ and $z = 15$

$\therefore \quad 15\% \text{ of } A = \left(\frac{12 \times 15}{10}\right)\% \text{ of } B = 18\% \text{ of } B$

Technique 2

when a number x is increased or decreased by $y\%$, then the new

number will be $\frac{100 \pm y}{100} \times x$.

Note 1. '+' sign is used in case of increase 2. '-' sign is used in case of decrease

Ex. 12 The monthly income of a person is ? 8000. If his income is increased by 20%, then what will be his new monthly income?

Sol. Monthly income of a person = ₹ 8000

Increment in income = $8000 \times \frac{20}{100} = 1600$

New income = 8000 + 1600 = ₹ 9600

Fast Track Method

Here, x = ₹ 8000 and $y = 20\%$

According to the formula,

New income $= \frac{100 + 20}{100} \times 8000$ ['+' sign is used for increase in income]

$= \frac{120}{100} \times 8000 =$ ₹ 9600

Ex. 13 The price of a computer is ? 20000. What will be the price of computer after reduction of 25%?

Sol. Here, x = ₹ 20000 and $y = 25\%$

According to the formula,

New price $= \frac{100 - y}{100} \times x = \frac{100 - 25}{100} \times 20000$

$= \frac{75}{100} \times 20000 = 75 \times 200 =$ ₹ 15000

Technique 3

(i) If x is $a\%$ more than y, then y is $\left(\frac{a}{100 + a} \times 100\right)\%$ less than x.

(ii) If x is $a\%$ less than y, then y is $\left(\frac{a}{100 - a} \times 100\right)\%$ more than x.

Ex. 14 If income of Ravi is 20% more than that of Ram, then income of Ram is how much per cent less than that of Ravi?

Sol. Let Ram's income be 100.

Then, Ravi's Income = 120

$\therefore$ Required percentage $= \dfrac{120 - 100}{120} \times 100\% = \dfrac{20}{120} \times 100\% = 16\dfrac{2}{3}\%$

Fast Track Method

Here, $a = 20\%$

According to the formula,

$$\text{Required percentage} = \left(\frac{a}{100 + a} \times 100\right)\%$$
$$= \left(\frac{20}{100 + 20} \times 100\right)\% = \frac{50}{3}\% = 16\frac{2}{3}\%$$

Alternate Method

Ravi's income = 20% more than that of Ram $= \dfrac{1}{5}$ more than that of Ram

i.e., 1 out of 5 part is more that Ravi have, hence we can say Ravi's income is 6, if Ram's Income is 5 and 1 part is less in the salary of Ram as compared to Ravi.

$$\text{Lesser\%} = \frac{\text{Part which is less in salary of Ram}}{\text{Ravi's salary}} \times 100$$
$$= \frac{1}{6} \times 100 = 16\frac{2}{3}\%$$

Ex. 15 If in an examination, the marks obtained by Preeti is 20% less than that of Vandana, then marks obtained by Vandana is how much per cent more than marks obtained by Preeti? **Sol. Here, $a = 20\%$**

According to the formula,

$$\text{Required percentage} = \left(\frac{a}{100 - a} \times 100\right)\% = \left(\frac{20}{100 - 20} \times 100\right)\%$$
$$= \left(\frac{20}{80} \times 100\right)\% = 25\%$$

Technique 4

If the value of a number is first increased by a% and later decreased by *a%,* then the net effect is always a decrease which is equal to a% of

a and is written as $\dfrac{a^2}{100}\%$ or $\left(\dfrac{a}{10}\right)^2\%$.

Ex. 16 The salary of a worker is first increased by 5% and then it is decreased by 5%. What is the change in his salary?

Sol. Let the initial salary of the worker be ₹ 100.

Firstly, the salary of worker is increased by 5%.

So, increased salary = 105% of 100 = $\frac{105 \times 100}{100}$ = ₹ 105

Now, the salary is reduced by 5% after the increase.

$\therefore$ Reduced salary = 95% of 105 = $\frac{95 \times 105}{100} = 99.75$

$\therefore$ Required change is a decrease.

i.e., $100 - 99.75 = 0.25$

So, required percentage decrease in salary $= \frac{0.25 \times 100}{100}\% = 0.25\%$

Fast Track Method

Here, $a = 5\%$

We know that, change in the salary of worker is a decrease.

According to the formula,

Decreased percentage = $\frac{a^2}{100}\% = \frac{5^2}{100}\% = \frac{25}{100}\% = 0.25\%$

Technique 5

When the value of an object is first changed (increased or decreased) by *a%* and then changed (increased or decreased) by *b%,* then

$$\text{Net effect} = \left[\pm a \pm b + \frac{(\pm a)(\pm b)}{100}\right]\%$$

Net effect is a increase or a decrease according to the +ve or -ve sign, respectively of the final result.

MIND IT!

1. Signs of *a* and *b* depends on increment and decrement of the quantity. '+' sign is used for increment and '-' sign for decrement.

2. The above formula can also be used to find net change in the product of two numbers, if they are increased or decreased by a% and *b%.*

Ex. 17 The price of an article is first increased by 20% and later on the price were decreased by 25% due to reduction in sales. Find the net percentage change in final price of Article.

Sol. Let the original price of article be ₹ 100

Then after the first change *i.e.*, increase of 20%

the price of article = $\frac{100+20}{100} \times 100 = ₹\ 120$

Again, the price were decreased by 25%, so the reduced price

$$= \frac{100-25}{100} \times 120 = \frac{75 \times 120}{100} = ₹\ 90$$

$\therefore$ Required decrease = $100 - 90 = ₹\ 10$ [$\because$ final price < initial price]

Fast Track Method

Here, $a = 20\%$, $b = 25\%$

So required percentage = $\frac{10}{100} \times 100 = 10\%$

Required change = $\left[(\pm a) + (\pm b) + \frac{(\pm a)(\pm b)}{100}\right]\%$

$$= \left[20 - 25 + \frac{20 \times (-25)}{100}\right]\% \qquad \left[\begin{array}{l} +\text{ ve sign for increase} \\ -\text{ ve sign for decrease} \end{array}\right]$$

$= [-5 - 5]\%$

$= -10\%$

/.Net percentage change is a decrease of 10% because final result is negative.

Technique 6

If the price of a commodity increases or decreases by *a%,* then the decrease or increase in consumption, so as not to increase or decrease

the expenditure is equal to $\left(\frac{a}{100 \pm a}\right) \times 100\%$.

Ex. 18 If the price of a commodity be raised by 40%, by how much per cent must a householder reduce his consumption of that commodity, so as not to increase his expenditure?

Sol. Let initial price be ₹ 100.

Price after increase $= 140\%$ of $100 = \frac{140 \times 100}{100} = ₹\ 140$

$\therefore$ Required reduction $= \frac{140 - 100}{140} \times 100\%$

$= \frac{40}{140} \times 100\% = 28\frac{4}{7}\%$

Fast Track Method

Here, $a = 40\%$

According to the formula,

Reduction in consumption $= \left(\frac{40}{100 + 40} \times 100\right)\%$

$= \left(\frac{40}{140} \times 100\right)\%$

$= \frac{200}{7}\% = 28\frac{4}{7}\%$

Ex. 19 If the price of milk falls down by 20%, by how much per cent must a householder increase its consumption, so as not to decrease his expenditure on this item?

Sol. Here, $a = 20\%$

According to the formula,

Increase in consumption $= \left(\frac{20}{100 - 20} \times 100\right)\% = \left(\frac{20}{80} \times 100\right)\% = 25\%$

Technique 7

The passing marks in an examination is P%. If a candidate scores *R* marks and fails bv *F* marks, then

$$\text{Maximum marks, } M = \frac{100(R + F)}{P}$$

Ex. 20 A student has to score 30% marks to get through. If he gets 30 marks and fails by 30 marks, then find the maximum marks set for the examination.

Sol. Let the maximum marks be x.

According to the question,

$$30\% \text{ of } x = 30 + 30$$

$$\Rightarrow \quad \frac{30x}{100} = 30 + 30$$

$$\Rightarrow \quad \frac{30x}{100} = 60 \Rightarrow x = 200$$

Fast Track Method

Here, $P = 30$, $R = 30$ and $F = 30$

According to the formula,

$$\text{Maximum marks} = \frac{100\,(R + F)}{P}$$

$$= \frac{100\,(30 + 30)}{30} = \frac{100 \times 60}{30} = 200$$

Technique 8

A candidate scores $x\%$ marks in an examination and fails by a marks, while an another candidate who scores $y\%$ marks, gets b marks more than the minimum required passing marks. The maximum marks for the examination is given as

$$M = \frac{100\,(a + b)}{y - x} = \frac{\text{Sum of scores}}{\text{Difference in \% marks}} \times 100$$

Ex. 21 A candidate scores 25% and fails by 60 marks, while an another candidate who scores 50% marks, gets 40 marks more than the minimum required marks to pass the examination. Find the maximum marks for the examination.

Sol. Let maximum marks be x.

Marks scored bv first candidate + 60 = Marks scored bv second candidate - 40

$$\Rightarrow \quad 25\% \text{ of } x + 60 = 50\% \text{ of } x - 40$$

$$\Rightarrow \quad \frac{25x}{100} + 60 = \frac{50x}{100} - 40$$

$$\Rightarrow \quad \frac{25x}{100} = 100$$

$$\Rightarrow \quad x = 400$$

Fast Track Method

Here, $x = 25$, $y = 50$, $a = 60$ and $b = 40$

According to the formula,

$$M = \frac{100\,(a + b)}{y - x} = \frac{100\,(60 + 40)}{50 - 25} = \frac{100 \times 100}{25} = 400$$

Technique 9

Suppose in an examination, *x%* of total number of students failed in subject *A* and y% of total number of students failed in subject *B* and *z%* failed in both the subjects. Then,

(i) Percentage of students who passed in both the subjects = [100 - *(x + y - z)* % (ii) Percentage of students who failed in either subject = *(x + y -z)%*

Ex.22 In an examination, 20% of total number of students failed in History, 15% of total number of students failed in Hindi and 5% of total number of students failed in both. Find the percentage of students who passed in both the subjects.

Sol. Let total number of students be 100.

Then, students failed in History only = 20 - 5 = 15 and students failed in Hindi only = 15 - 5 = 10 .'. Total number of failed students = 15 + 10 + 5 = 30 .•. Number of students passed in both subjects = 100 - 30 = 70 .•. Required percentage = 70% **Fast Track Method** Here, *x* = 20, *y* = 15 and *z* = 5 According to the formula, Required percentage = [100- (x + *y - z)]%*

= [100 - (20 + 15 - 5)]%

= (100 - 30)% = 70%

(Technique^

if due to *r%* decrease in the price of an item, a person can buy *A* kg more in ? *x.* then

$$\text{Actual price of that item} = ₹\,\frac{r\,x}{(100 - r)A} \text{ per kg}$$

Ex. 23 If due to 10% decrease in the price of sugar, Ram can buy 5 kg more sugar in ? 100, then find the find actual price of sugar.

Sol. Let the actual price of sugar be ₹ y.

Amount of suger bought in ₹ 100 $= \frac{100}{y}$ ...(i)

Price of suger after 10% decrease = 90% of $y = \frac{9}{10}y$

Now, amount of sugar bought in ₹ 100 $= \frac{1000}{9y}$ ...(ii)

According to the question,

$$\frac{1000}{9y} - \frac{100}{y} = 5$$

$$\Rightarrow \quad \frac{1000-900}{9y} = 5$$

$$\Rightarrow \quad y = \frac{100}{9 \times 5}$$

$$\therefore \quad y = \frac{20}{9} = ₹\, 2\frac{2}{9}$$

Fast Track Method

Here, $r = 10\%$, $x =$ ₹ 100 and $A = 5$ kg

$\therefore$ Actual price of sugar $= \frac{r \times x}{(100-r)A}$

$$= \frac{10 \times 100}{(100-10) \times 5} = \frac{1000}{450}$$

$$= ₹\, 2\frac{2}{9}$$

Technique 11

If two candidates contested in an election and one candidate got x% of total votes casted and still lose by y votes, then

$$\text{Total number of votes casted} = \frac{100 \times y}{100 - 2x}$$

Ex. 24 In an. election contested by two candidates, one candidate got 30% of total votes and still lost by 500 votes, then find the total number of votes casted.

Sol. Let total number of votes casted be x.

Number of votes got by first candidate $= \frac{30}{100}x$.

Number of votes got by second candidate $= \frac{70}{100}x$

According to the question,

$$\text{Difference in votes} = 500$$

$$\Rightarrow \quad \frac{70}{100}x - \frac{30}{100}x = 500$$

$$\Rightarrow \quad \frac{40}{100}x = 500$$

$$\therefore \quad x = \frac{500 \times 100}{40} = 1250$$

Fast Track Method

Here, $x = 30$ and $y = 500$

$$\therefore \text{Total number of votes} = \frac{100 \times y}{100 - 2x}$$

$$= \frac{500 \times 100}{100 - 2 \times 30}$$

$$= \frac{500 \times 100}{40} = 1260$$

Technique 12

If the population of a town is P and it increases (or decreases) at the rate of /?% per annum, then

(i) Population after n yr $= P\left(1 \pm \frac{R}{100}\right)^n$

(ii) Population, n yr ago $= \dfrac{P}{\left(1 \pm \frac{R}{100}\right)^n}$

Note Use '+' sign for increment and '-' sign for decrement

Ex.25 The population of a town is 352800. If it increases at the rate of 5% per annum, then what will be its population 2 yr hence. Also, find the population 2 yr ago.

Sol. Given that, $P = 352800$, $R = 5\%$ and $n = 2$

According to the formula,

$$\text{Population after 2 yr} = P\left(1+\frac{R}{100}\right)^n = 352800 \times \left(1+\frac{5}{100}\right)^2$$

$$= 352800 \times \left(\frac{100+5}{100}\right)^2 = 352800 \times \left(\frac{21}{20} \times \frac{21}{20}\right) = 388962$$

$$\text{Population 2 yr ago} = \frac{P}{\left(1+\frac{R}{100}\right)^n} = \frac{352800}{\left(1+\frac{5}{100}\right)^2} = 352800 \times \frac{20}{21} \times \frac{20}{21} = 320000$$

Students may also solve problems through simple increasing-decreasing method of percentage because 5% increase for 2 yr means population after 2 yr, which is equal

to $352800 \times \frac{105}{100} \times \frac{105}{100}$. In case, when population decrease at the rate of 5%, then population after 2 yr is equal to $352800 \times \frac{95}{100} \times \frac{95}{100}$.

Technique 13

If the present population of a city is *P* and there is a increment or decrement of /?.,%, $R_2\%$ and $R_3\%$ in first, second and third year respectively, then

$$\text{Population of city after 3 yr} = P\left(1 \pm \frac{R_1}{100}\right)\left(1 \pm \frac{R_2}{100}\right)\left(1 \pm \frac{R_3}{100}\right)$$

Ex. 26 Population of a city in 2004 was 1000000. If in 2005 there is an increment of 15%, in 2006 there is a decrement of 35% and in 2007 there is an increment of 45%, then find the population of city at the end of year 2007.

Sol. Given that, $P = 1000000$, $R_x = 15\%$, R^ $= 35\%$ (decrease) and $R_3 = 45\%$ Population of city at the end of year 2007

$$= P\left(1+\frac{R_1}{100}\right)\left(1-\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

$$= 1000000\left(1+\frac{15}{100}\right)\left(1-\frac{35}{100}\right)\left(1+\frac{45}{100}\right)$$

$$= 1000000 \times \frac{115}{100} \times \frac{65}{100} \times \frac{145}{100} = 1083875$$

Fast Track Practice

Exercise© *Base Level Questions*

1. What is 5% of 50% of 500? [CDS 2012]

(a) 12.5 *(b)* 25 *(c)* 1.25 *(d)* 6.25

2. If 20% of* = 90, find*.

(a) 350 *(b)* 450 *(c)* 250 (tfj 550 (e) None of the above 89% of ? + 365 = 1075.22 [Bank Clerks 2010] *(a)* 798 *(b)* 897 *(c)* 898 *(d)* 752 *(e)* None of the above 4. 50 kg is what per cent of 250 kg? *(a)* 26% *(b)* 15%

(c) 20% *(d)* 18%

fej None of the above

5. What is 32% of $\frac{3}{8}$ th of 5000? **[Bank Clerks 2011]**

(a) 606 (b) 610 (c) 580 (d) 600
(e) None of the above

6. 12 of 26% of $\frac{5}{78}$ of 38% of $\frac{7}{152}$ of 10000 = ? **[Hotel Mgmt. 2010]**

(a) 38 (b) 35 (c) 41 (d) 52
(e) None of the above

7. If 15% of A is equal to 20% of B, then 25% of A is equal to what per cent of B?

(a) 30% (b) $33\frac{1}{3}$%
(c) 35% (d) 25%

8. A number increased by $137\frac{1}{2}$% and the increment is 33. The number is **[SSC CGL 2013]**

(a) 27 (b) 22
(c) 24 (d) 25

9. A person's salary has increased from ₹ 7200 to ₹ 8100. What is the percentage increase in his salary? **[CDS 2013]**

(a) 25% (b) 18%
(c) $16\frac{2}{3}$% (d) $12\frac{1}{2}$%

10. A's salary is 20% less than B's salary. Then B's salary is more than A's salary by **[SSC CGL 2013]**

(a) $33\frac{1}{2}$% (b) $16\frac{2}{3}$% (c) 20% (d) 25%

11. If the price of petrol is increased by 20%, by what percentage should the consumption be decreased by the consumer, if the expenditure on petrol remains unchanged? [CG PSC 2013]

(a) $16\frac{2}{3}$% (b) $6\frac{2}{3}$% (c) 8% (d) 15%
(e) None of the above

12. If A's income is $\frac{5}{6}$th of B's income, then B's income is more than A's income by **[SSC (10+2) 2012]**

(a) $\frac{25}{2}$% (b) 15% (c) 20% (d) 25%

13. 25% of what amount of money is equal to $12\frac{1}{2}$% of ₹ 180? **[SSC FCI 2012]**

(a) ? 120 *(b)* ? 75 *(c)* ? 80 *(d)* ? 90

14. 30% of a 3-digit number is 190.8. What will be 125% of that number?

[Bank Clerks 2011]

(a) 759 *(b)* 785 *(c)* 795 *(d)* 779 *(e)* None of the above

15. The price of 10% increased of an article of ? 100, then again 10% increased the price. How much is increased in total price?

(a) 20 *(b)* 21 *(c)* 110 *(d)* 121

16. In a company, there are 724 employees. 25% employees left the job. Find the number of rest of employees.

(a) 543 *(b)* 550 *(c)* 578 *(d)* 565 *(e)* None of the above

17. An agent sells goods of value of ? 15000. The commission which he receives at the

ratio of $12\frac{1}{2}\%$, is

fa; ?1875 (b)?2125

(c) X 2000 *(d)* ? 2700

(e) None of the above

18. There are 1240 employees in an

organisation. Out of which, 25% are

promoted. How many such employees are

there who got promotion? [Bank Clerks 2009]

(a) 398 *(b)* 345 *(c)* 310 *(d)* 372

(e) None of the above

19. The difference between 78% of a number and 59% of the same number is 323. What is 62% of that number?

[Bank Clerks 2009]

(a) 1054 *(b)* 1178 *(c)* 1037 *(d)* 1159 *(e)* None of the above

20. The monthly income of a person is *X* 5000. If his income is increased by 30%, then what is his monthly income now?

(a) *X* 7000 *(b)* *X* 5500

(c) *X* 4500 *(d)* *X* 6500

(e) None of the above

21. The price of a certain article is *X* 15000. But due to slump in the market, its price decreases by 8%. Find the new price of the article.

(a) *X* 14000 *(b)* *X* 13800

(c) *X* 16500 *(d)* *X* 12600

(e) None of the above

22. A water pipe is cut into two pieces. The longer piece is 70% of the length of the pipe. By how much percentage is the longer piece longer than the shorter piece? [CDS 2014]

(a) 140% *(b)* $\frac{400}{3}$%

(c) 40% *(d)* None of these

23. When 35 is subtracted from a number, it reduces to its 80%. What is $\frac{4}{5}$th of that number?

(a) 70 *(b)* 90 *(c)* 120 *(d)* 140 *(e)* None of the above

24. A man losses 20% of his money. After spending 25% of the remainder, he has *X* 480 left. What is the amount of money he originally had? [CDS 2012]

(a) *X* 600 *(b)* *X* 720

(c) X 800 (d) X 840

25. If income of Vandana is 25% more than that of Aarti, then income of Aarti is how much per cent less than that of Vandana?

(a) 18% (b) 21% (c) 20% (d) 24%

(e) None of the above

26. In an examination, the marks obtained by Shantanu is 40% less than the marks obtained by Kamal, then marks obtained by Kamal is how much per cent more than the marks obtained by Shantanu?

[Bank Clerks 2009]

(a) $55\frac{2}{3}\%$ (b) $44\frac{3}{5}\%$ (c) $33\frac{1}{3}\%$ (d) $66\frac{2}{3}\%$

(e) None of the above

27. Because of scarcity of rainfall, the price of a land decreases by 12% and its production also decreases by 4%. What is the total effect on revenue?

[LIC ADO 2008] (a) Loss of 16% (b) Gain of 15% (c) Loss of 15.48% (d) Gain of 15.48% (e) Loss of 15.52%

28.Mangulal, a shopkeeper, marks the prices of his goods at 20% higher than the original price. After that, he allows a discount of 5%. What profit or loss did he get? (a) 14% (profit) (b) 16% (loss) (c) 14% (loss) (d) 16% (profit)

(e) None of the above

29. The marked price of brand A watches is 15% higher than its original price. Due to increase in demand, the price is further increased by 10%. How much profit will be obtained in selling the watches?

(a) 25% (b) 35% (c) 26.5% (d) 27% (e) None of the above

30. The price of sugar is increased by 25%. If a family wants to keep its expenses on sugar unaltered, then the family will have to reduce the consumption of sugar by [SSCCGL2010]

(a) 20% (b) 21% (c) 22% (d) 25%

31. If the price of tea falls down by 6%, by how much per cent must a householder increase its consumption, so as not to decrease expenditure?

(a) $5\frac{16}{47}\%$ (b) $4\frac{18}{67}\%$
(c) $6\frac{18}{47}\%$ (d) $6\frac{17}{47}\%$
(e) None of the above

32. A student has to score 40% marks to get through. If he gets 40 marks and fails by 40 marks, then find the maximum marks set for the examination.

(a) 200 *(b)* 250 *(c)* 300 *(d)* 150 *(e)* None of the above

33. A candidate scores 20% and fails by 50 marks, while another candidate who scores 40% marks, gets 30 marks more than the minimum required marks to pass the examination. Find the maximum marks for the examination.

(a) 500 *(b)* 450

(c) 300 *(d)* 400

(e) None of the above

34. The population of a town is 126800. It increase by 15% in the 1st year and decrease by 20% in the 2nd year. What is the population of the town at the end of 2 yr? [Bank Clerks 2008]

(a) 174984 *(b)* 135996

(c) 116656 *(d)* 145820

(e) None of the above

35. The population of a town is 705600. If it increases at the rate of 5% per annum, then what will be its population 2 yr hence?

(a) 777924 *(b)* 777881

(c) 778781 *(d)* 797724

(e) None of the above

36. The population of a town is 1058400. If it increases at the rate of 5% per annum, then find the population of the town 2 yr ago.

(a) 949000 *(b)* 930000

(c) 960000 *(d)* 950000

(e) None of the above

37. In an examination, 25% students failed in Mathematics, 20% failed in Economics and 5% failed in both. Find the percentage of students who passed in both the subjects.

(a) 60% *(b)* 45% *(c)* 39% *(d)* 62% *(e)* None of the above

38. The population of a city is 250000. It is increasing at the rate of 2% every year. The growth in the population after 2 yr is?

[CLAT2013]

(a) 2500 *(b)* 10000

(c) 252000 *(d)* 10100

39. In a school, 10% of boys are equal to the one-fourth of the girls. What is the ratio of boys and girls in that school? [RRB 2012]

(a) 3 :2 *(b)* 5 :2 (cj 2 :1 *(d)* 4 : 3

40. Income of Suman is first increased by 7% and then it is decreased by 7%. What is the change in her income?

[Hotel Mgmt. 2008]

(a) 0.49% (increase)

(b) 0.39% (decrease)

(c) 0.39% (increase)

(d) 0.49% (decrease)

(e) None of the above

41. 48% of the 1st number is 60% of the 2nd number. What is the ratio of the 1st number to the 2nd number?

[Bank Clerks 2008] *(a)* 4 : 7 *(b)* 3 : 4 *(c)* 5 : 4

(d) Couldn't be determined

(e) None of the above

42. If the numerator of a fraction is increased by 200% and the denominator of the fraction is increased by 150%, the

resultant fraction is $\frac{9}{35}$. What is the original fraction?

(a) $\frac{3}{10}$ *(b)* $\frac{2}{15}$

(c) $\frac{3}{14}$ *(d)* $\frac{2}{7}$

(e) None of the above

43. If length and breadth of a rectangle became half and double respectively, then what will be the resultant area?

(a) 25% *(b)* 55%

(c) 75% *(d)* 80%

(e) None of the above

44. Two numbers are respectively 20% and 50% more than a 3rd number. What is the percentage of 2nd with respect to 1st? [SSCCCL2011]

(a) 125% *(b)* 90%

(c) 80% *(d)* 75%

45. The sum of 15% of a positive number and 20% of the same number is 126. What is one-third of that number? [IBPS Clerk 2011]

(a) 360 *(b)* 1080

(c) 120 *(d)* 40

(e) None of the above

46. In a test, *A* scored 10% more than *B* and *B* scored 5% more then C. If C scored 300 marks out of 400, then *A's* marks are

[SSCCGL2012]

(a) 310 *(b)* 325

(c) 350 *(d)* 360

47. In order to pass in an exam, a student is required to get 780 marks out of the aggregate marks. Sonu got 728 marks and was declared failed by 5%. What are the maximum aggregate marks a student can get in the examination?

(a) 1040 *(b)* 1100 *(c)* 1000

(d) Cannot be determined

(e) None of the above

48.Mathew scored 42 marks in Biology, 51 marks in Chemistry, 58 marks in Mathematics, 35 marks in Physics and 48 marks in English. The maximum marks, a student can score in each subject, are 60. How much overall percentage did Mathew get in this exam? [SBI Clerk 2012]

(a) 76% *(b)* 82%

(c) 68% *(d)* 78%

(e) None of the above

49.Reena got 76 marks in Hindi, 48 marks in Science, 84 marks in Mathematics, 38 marks in Social Science and 72 marks in English. The maximum marks of each subject was 100. How much over all percentage of marks she got?

[Bank Clerks 2011]

(a) 63.8% *(b)* 62.6%

(c) 63.6% *(d)* 64.8%

(e) None of the above 50. Sahil got 45 marks in Hindi, 47 marks in Science, 49 marks in Mathematics, 45 marks in Social Science and 32 marks in English. The maximum marks of each subject is 50. How much overall percentage of marks did he get?

(a) 75% *(b)* 56.4%*(c)* 78% *(d)* 87.2%

(e) None of the above

51-A student was asked to measure the length and breadth of a rectangle. By mistake, he measured the length 20% less and the breadth 10% more. If its original area is 200 sq cm, then find the area after this measurement? *(a)* 176 sq cm *(b)* 206 sq cm

(c) 226 sq cm *(d)* 316 sq cm

(e) None of the above

52. A box has 100 blue balls, 50 red balls and 50 black balls. 25% of blue balls and 50% of red balls are taken away. Then, percentage of black balls at present is

[SSCCGL2013]

(a) 25% (b) $33\frac{1}{3}$% (c) 40% (d) 50

53. A jogger desires to run a certain course in $\frac{1}{4}$ less time than he usually takes. By what per cent must be increase his average running speed to accomplish the goal? [SSC CPO 2013]

(a) 50% (b) 20% (c) 25% (d) $33\frac{1}{3}$%

54. The prices of two articles are as 3:4. If the price of the first article is increased by 10% and that of the second by *X* 4, one original ratio remains the same. The original price of the second article is

[SSCCPO2013]

(a) ? 40 *(b)* ? 10

(c) *X* 30 *(d)* *X* 35

55. A saves 20% of his monthly salary. If his monthly expenditure is *X* 6000, then his monthly savings is [SSC (10+2) 2012]

(a) ? 1200 *(b)* ? 4800

(c)X 1500 (by ? 1800

56. From 2008 to 2009, the sales of a book decreased by 80%. If the sales in 2010 was the same as in 2008, by what per cent did it increase from 2009 to 2010?

[SSC (10+2) 2012] *{a)* 80% *(b)* 100% *(c)* 120% *(d)* 400%

57. In a particular constituency, 75% of voters cast their votes, out of which 2% were rejected. The winning candidate received 75% of the valid votes and bagged a total of 9261 votes. The total number of voters in the constituency is

[SSC (10+2) 2012]

(a) 14500 *(b)* 18900

(c) 16800 *(d)* 24000

58. If increasing 20 by *P* percentage gives the same result as decreasing 60 by *P* percentage, what is *P* percentage of 70?

[IB Assistant Central 2013]

(a) 50 *(b)* 140

(c) 14 *(d)* 35

59. Last year, there were 610 boys in a school. The number decreased by 20% this year. How many girls are there in the school, if the number of girls is 175% of the total number of boys in the school this year? [SBI Clerk 2012]

(a) 854 *(b)* 848

(c) 798 *(d)* 782

(e) None of the above

60. Aryan got 350 marks and Vidya scored 76% marks in the same test. If Vidya scored 296 marks more than Aryan, then what were the maximum marks of the test?

(a) 650 *(b)* 900

(C) 850 *(d)* 950

(e) None of the above

61. A student was awarded certain marks in an examination. However, after re-evaluation, his marks were reduced by 40% of the marks that were originally awarded to him, so that the new score now became 96. How many marks did the student lose after re-evaluation?

(a) 58 *(b)* 68

(c) 63 *(d)* 56

(e) 64

62. What should come in place of the question mark (?), so that it satisfies equality of the given equation?

32% of 750 = ? *(a)* 23% of 600 *(b)* 46% of 207 *(c)* 98% of 250 *(d)* 75% of 320

(e) None of the above

Exercise © *Higher SkiU Level Questions*

1. An alloy of gold and silver weights 50g. It contains 80% gold. How much gold should he added to the alloy, so that percentage of gold is increased to 90?

(a) 50g *(b)* 60g

(c) 30g *(d)* 40g

(e) None of the above

2. Veena spends 25% of her monthly income on household expenses. Her annual income is *X* 4.32 lakh. What is the total amount that Veena spends on household expenses in 8 months together? [Bank Clerks 2011]

(a) X 74000 *(b) X* 71000

(C) X 73000 *(d) X* 72000

(e) None of the above

3. The salary of an employee of a company increases every month by 4%. If his salary in August was X 6300, then what would be his approximate salary in month of October of the same year?

[Bank Clerks 2010] (a) X 6552 (b) X 6967

(c) X 6814 (d) X 6627

(e) None of the above

4. 1 L of water is added to 5 L of alcohol and water solution containing 40% alcohol strength. The strength of alcohol in the new solution will be

[SSC CGL 2007]

(a) 30% (b) $33\frac{1}{3}$% (c) $33\frac{2}{3}$% (d) 33%

5. In an examination, 49% students failed in English, 36% students failed in Hindi, while 15% failed in both. If total number of passed students is 450, then how many students did appear in the examination?

(a) 1800 (b) 2000 (C) 1100 (d) 1500 (e) None of the above

6. In a test, Rajesh got 112 marks which was 32 more than the passing marks. Sonal got 75% marks which was 70 more than the passing marks. What is the minimum passing percentage of the test?

(a) 35% (b) 45% (c) 40% (d) 48% (e) None of the above

7. Bina's monthly income is 90% of Anita's monthly income. The total of both their monthly incomes is Mr. Sen's monthly income. Mr. Sen's annual income is X 775200. What is Bina's monthly income?

(a) X 34000 (b) X 36000

(c) X 30600 (d) X 30000

(e) None of the above

8. In an examination, it is required to get 296 marks out of aggregate marks to pass. A student gets 222 marks and is declared failed by 10% marks. What are the maximum aggregate marks a student can get? [Bank Clerks 2011]

(a) 830 (b) 810 (c) 780 (d) 740 (e) None of the above

9. Sohan spends 23% of an amount of money on an insurance policy, 33% on food, 19% on children's education and 16% on recreation. He deposits the remaining amount of X 504 in bank. How much total amount does he spend on food and insurance policy together?

[Bank Clerks 2010] (a) X 3200 (b) X 3126

(C) X 3136 (d) X 3048

(e) None of the above

10. Sonali invests 15% of her monthly salary in insurance policies. She spends 55% of her monthly salary in shopping and on household expenses. She saves the remaining amount at X 12750. What is Sonali is monthly income? [SNAP 2012]

(a) X 42500 (b) X 38800

(C) X 40000 (d) X 35500

(e) None of the above

11. In an office, 40% of the staff is female. 40% of the female and 60% of the male voted for me. The percentage of votes I got was [SSC CPO 2011]

(a) 24% (b) 42% (c) 50% (d) 52%

12. Ram sells his goods 25% cheaper than Shyam and 25% dearer than Hari. How much percentage is Hari's goods cheaper than Shyam? [SSC CPO 2011]

(a) 25 (b) $33\frac{1}{3}$

(c) 40 (d) 50

13. The ratio of the number of boys and girls in a school is 3 : 2. If 20% of the boys and 25% of the girls are scholarship holders, then the percentage of the students who do not get the scholarship, is

[SSC CGL 2010] (a) 78% (b) 75%

(c) 60% (d) 55%

14. A reduction of 20% in the price of wheat enables a person to buy 3.5 kg more wheat for *X* 770. The original price of rice is

(a) *X* 55 per kg *(b)* *X* 45 per kg *(c)* *X* 65 per kg *(d)* *X* 37 per kg *(e)* None of the above

15. Fresh grapes contain 80% water, while dry grapes contain 10% water. If the weight of dry grapes is 500 kg, then what is its total weight when it is fresh?

(a) 2350 kg *(b)* 2085 kg

(c) 2255 kg *(d)* 2250 kg

(e) None of the above

16. Due to a 25% increase in the price of rice per kilogram, a person is able to purchase 20 kg less for *X* 400. What is the increased price of rice per kilogram?

[IB AC1O2012]

(a) X5 *(b) X* 6

(c) X 10 *(d) X 4*

17. The price of rice is reduced by 5%. How many kilogrames of rice can now be bought for the money, which was sufficient to buy 50 kg of rice earlier?

(a) 52.63 kg *(b)* 50.40 kg

(c) 42.30 kg *(d)* 60.50 kg

18. The price of rice decreases by 6.25% and because of this reduction, Vandana is able to buy 1 kg more for *X* 120. Find the reduced rate of rice.

(a) *X* 7.50 per kg *(b)* *X* 9 per kg *(c)* *X* 5.50 per kg *(d)* ? 19 per kg

19. Due to an increase of 30% in the price of eggs, 6 eggs less are available for *X* 7.80. The present rate of eggs per dozen is

(a) *X* 5.50(b) *X* 4.68 *(c)* *X* 6.49 *(d)* *X* 3.58 *(e)* None of the above

20. Wheat is now being sold at X 27 per kg. During last month, its cost was X 24 per kg. Find by how much per cent a family reduces its consumption, so as to keep the expenditure fixed.

(a) 10.2% *(b)* 12.1%

fc; 12.3% (dj 11.1%

21. In a class of 40 students and 5 teachers, each student got sweets that are 15% of the total number of students and each teacher got sweets that are 20% of the total number of students. How many sweets were there? [Bank Clerks 2008]

(a) 280 *(b)* 240

(c) 320 *(d)* 360

(e) None of the above

22. The price of ghee is increased by 32%. Therefore, a family reduces its consumption, so that the increment in price of ghee is only 10%. If consumption of ghee is 10 kg before the increment, then what is the consumption now?

(a) $8\frac{1}{3}$ kg *(b)* $8\frac{3}{4}$ kg

(c) $8\frac{1}{2}$ kg *(d)* 9 kg

23. The expenses on wheat, meat and vegetables of a family are in the ratio 12 : 17 : 3. The prices of these articles are increased by 20%, 30% and 50%, respectively. The total expenses of the familv nn these articles are increased bv

(a) $23\frac{1}{3}$% *(b)* $28\frac{1}{8}$%

(c) $27\frac{1}{8}$% *(d)* $25\frac{1}{7}$%

(e) None of the above

24. In 1998, ratio of the numbers of students taking examinations in *x* and *z* states are respectively 3:5:6. Next year, the numbers of students are increased by 20%, 10% and 20% respectively. If ratio of the numbers of students in states *x* and *z* is 1 : 2, then find the number of students who sit to take examination in 1998.

{a) 5000 *(b)* 6000 *(c)* 75000

(d) Data is insufficient

(e) None of the above

25. The tank-full of petrol in Aran's motor-cycle lasts for 10 days. If he starts using 25% more everyday, how many days will the tank-full of petrol last? [CSAT 2013]

(a) 5 *(b)* 6 *(c)* 7 *(d)* 8

26. In an examination out of 480 students, 85% of the girls and 70% of the boys passed. How many boys appeared in the examination, if total pass percentage was 75%? [SNAP 2012]

(a) 370 *(b)* 340 *(c)* 320 *(d)* 360 *(e)* None of the above

27. In a class-X of 30 students, 24 passed in first class; in another class *Y* of 35 students, 28 passed in first class. In which class was the percentage of students first class more?

(a) Class X has more percentage of students getting first class.

(b) Class Y has more percentage of students getting first class

(c) Both classes have equal percentage of students getting first class

(d) None of the above

Answer with Solutions

Exercise *Q* *Base Level Questions*

1. (*a*) 5% of 50% of 500

$$= \frac{5}{100} \times \frac{50}{100} \times 500 = 12.5$$

2. (*b*) 20% of $x = 90$

$$\Rightarrow \quad x \times \frac{20}{100} = 90 \Rightarrow x = 90 \times 5 = 450$$

3. (*a*) 89% of ? + 365 = 1075.22

$$\Rightarrow \quad \frac{89}{100} \times ? = 1075.22 - 365$$

$$\therefore \quad ? = \frac{71022}{89} = 798$$

4. (*c*) Required percentage

$$= \frac{50}{250} \times 100\% = 20\%$$

5. (*d*) 32% of $\frac{3}{8}$ of $5000 = \frac{32}{100} \times \frac{3}{8} \times 5000$

$$= 4 \times 3 \times 50 = 600$$

6. (*b*) $12 \times \frac{26}{100} \times \frac{5}{78} \times \frac{38}{100} \times \frac{7}{152} \times 10000$

$$= 12 \times \frac{5}{3} \times \frac{7}{4} = 35$$

7. (*b*) Given, $x = 15$, $y = 20$ and $z = 25$

According to the formula,

$z\%$ of $A = \left(\frac{yz}{x}\right)\%$ of B [by Technique 5]

$$25\% \text{ of } A = \frac{20 \times 25}{15}\% \text{ of } B$$

$$= \frac{100}{3}\% \text{ of } B = 33\frac{1}{3}\% \text{ of } B$$

8. (*c*) Let the number be x.

Then, 137.5% of $x = 33$

$$\therefore \quad x = \frac{33 \times 100}{137.5} = 24$$

9. (*d*) Percentage increase in salary

$$= \frac{8100 - 7200}{7200} \times 100$$

$$= \frac{900}{7200} \times 100 = 12.5\% = 12\frac{1}{2}\%$$

10. (*d*) Given, $a = 20\%$

Required percentage $= \frac{a}{100 - a} \times 100\%$

[by Technique 7]

$$= \frac{20}{100 - 20} \times 100\% = \frac{20}{80} \times 100\% = 25\%$$

11. (*a*) Here, $a = 20\%$

Required percentage loss $= \dfrac{a}{100 + a} \times 100\%$

$$= \frac{20}{100 + 20} \times 100\%$$

$$= \frac{20}{120} \times 100\% = 16\frac{2}{3}\%$$

12. (*c*) Given, $A = \dfrac{5}{6} B$

Let income of $B = x$

$\therefore \quad A = \dfrac{5}{6} x$

Required percentage $= \dfrac{x - \dfrac{5x}{6}}{\dfrac{5x}{6}} \times 100$

$$= \frac{6x - 5x}{5x} \times 100$$

$$= \frac{1}{5} \times 100 = 20\%$$

13. (*d*) Let required amount of money be x.

Then, $x \times 25\% = 180 \times 12\dfrac{1}{2}\%$

$\therefore \quad x = \dfrac{180 \times 12.5}{25} =$ ₹ 90

14. (*c*) Let x be the number.

Then, 30% of $x = 190.8$

$\Rightarrow \quad x = \dfrac{190.8 \times 100}{30} = 636$

$\therefore$ 125% of 636 $= \dfrac{125}{100} \times 636 = 795$

15. (*b*) 10% of 100 $= \dfrac{100 \times 10}{100}$

New price = 110

Again, 10% of 110

$$= \frac{110 \times 10}{100} = 11$$

New price = 110 + 11 = 121

$\therefore$ Required increment = 121 − 100 = 21

Fast Track Method

Here, $a = 10$ and $b = 10$

Increased price $= \left[a + b + \dfrac{ab}{100}\right]\%$

$$= \left[10 + 10 + \frac{10 \times 10}{100}\right]\% = 21\%$$

$\therefore$ Required Increment

$$= \frac{21}{100} \times 100 = ₹\,21$$

16. (*a*) Number of employees leaving job

$$= 25\% \text{ of } 724 = \frac{1}{4} \times 724 = 181$$

$\therefore$ Number of rest of the employees

$$= 724 - 181 = 543$$

17. (*a*) Agent receives commission of the rate

$$= 12\frac{1}{2}\%$$

Total value of goods sold = ₹ 15000

$\therefore$ Agent's commission $= 12\frac{1}{2}\%$ of 15000

$$= \frac{25}{2} \times \frac{15000}{100} = ₹\,1875$$

18. (*c*) Required number $= \frac{1240 \times 25}{100} = 310$

19. (*a*) Let the number be x.

According to the question,

$$(78 - 59)\% \text{ of } x = 323$$

$$\Rightarrow \quad \frac{19 \times x}{100} = 323$$

$$\therefore \quad x = \frac{323 \times 100}{19} = 1700$$

$$\therefore \quad 62\% \text{ of } 1700 = \frac{62 \times 1700}{100} = 1054$$

20. (*d*) Monthly income of a person = ₹ 5000

Increment in income $= \frac{30}{100} \times 5000 = 1500$

New income = 5000 + 1500 = ₹ 6500

Fast Track Method

Here, x = ₹ 5000 and $y = 30\%$

According to the formula,

Required new income $= \frac{100 + y}{100} \times x$

$$= \frac{100 + 30}{100} \times 5000$$

$$= \frac{130}{100} \times 5000 = ₹\,6500$$

21. (*b*) Given that, x = ₹ 15000, and $y = 8$

According to the formula,

Required new price $= \frac{100 - y}{100} \times x$

$$= \frac{100 - 8}{100} \times 15000$$

$$= \frac{92}{100} \times 15000 = ₹\,13800$$

Direct Approach

New price = 92% of 15000

$$= \frac{92}{100} \times 15000 = ₹\,13800$$

22. (*b*) Increase in percentage of longer pipe compared to shorter pipe

$$= \frac{70 - 30}{30} \times 100\%$$

$$= \frac{40}{30} \times 100\% = \frac{400}{3}\%$$

23. (*d*) Let the number be x.

Then, $x - 35 = \frac{80}{100} x$

$\Rightarrow \quad x - \frac{4}{5} x = 35$

$\Rightarrow \quad \frac{x}{5} = 35 \Rightarrow x = 175$

$\therefore$ Required number $= 175 \times \frac{4}{5}$

$= 35 \times 4 = 140$

24. (*c*) Let a man have ₹ x.

Remainder money, after losses 20% of his money

$= x - 20\% \text{ of } x = x - \frac{x \times 20}{100} = \frac{5x - x}{5}$

$= \frac{4x}{5}$

Remainder money, after spending 25% of his remainder money $= \frac{4x}{5} - 25\% \text{ of } \left(\frac{4x}{5}\right)$

$= \frac{4x}{5} - \left(\frac{4x}{5} \times \frac{25}{100}\right)$

$= \frac{4x}{5} - \frac{x}{5} = \frac{4x - x}{5} = \frac{3x}{5}$

According to the question,

$\frac{3x}{5} = 480$

$\Rightarrow \quad x = \frac{480 \times 5}{3} = ₹\ 800$

25. (*c*) Let Aarti's income be ₹ 100.

$\therefore$ Vandana's income = ₹ 125

$\therefore$ Aarti's income is ₹ 25 less than 125.

$\therefore$ Required percentage

$= \frac{25}{125} \times 100\% = 20\%$

Fast Track Method

Here, $a = 25\%$

According to the formula,

Required percentage

$$= \left(\frac{a}{100 + a} \times 100\right)\%$$

$$= \left(\frac{25}{125} \times 100\right)\% = 20\%$$

26. (*d*) Let Kamal's marks be 100.

$\therefore$ Shantanu's marks = 60

Kamal has 40 marks more than Shantanu.

∴ Required percentage

$$= \frac{40}{60} \times 100\% = 66\frac{2}{3}\%$$

Fast Track Method

Here, $a = 40\%$

According to the formula,

Required percentage

$$= \left(\frac{a}{100 - a} \times 100\right)\%$$

$$= \left(\frac{40}{100 - 40} \times 100\right)\%$$

$$= \left(\frac{40}{60} \times 100\right)\% = \frac{400}{6}\% = 66\frac{2}{3}\%$$

27. (*e*) Net effect $= \left[-12 - 4 + \frac{(-12)(-4)}{100}\right]\%$

[by Technique 9]

$$= (-16 + 0.48)\% = -15.52\%$$

28. (*a*) Given that, $a = 20\%$ and $b = 5\%$

According to the formula,

Required percentage

$$= \left(20 - 5 - \frac{20 \times 5}{100}\right)\% = 14\%$$

Result is +ve, hence required answer is profit.

29. (*c*) Given that, $a = 15\%$ and $b = 10\%$

According to the formula,

Required profit $= \left(a + b + \frac{ab}{100}\right)\%$

$$= \left(15 + 10 + \frac{15 \times 10}{100}\right)\% = (25 + 1.5)\%$$

$$= 26.5\%$$

30. (*a*) Let original price be ₹ 100.

Then, increased price = ₹ 125

∴ Reduction in consumption

$$= \left(\frac{125 - 100}{125} \times 100\right)\%$$

$$= \left(\frac{25}{125} \times 100\right)\% = 20\%$$

Fast Track Method

Here, $a = 25\%$

According to the formula,

Reduction in consumption

$$= \left(\frac{a}{100 + a} \times 100\right)\%$$

$$= \left(\frac{25}{125} \times 100\right)\% = 20\%$$

31. (*c*) Let original price be ₹ 100.

Then, reduced price = ₹ 94

∴ Increase in consumption

$$= \left(\frac{100 - 94}{94} \times 100\right)\%$$

$$= \left(\frac{6}{94} \times 100\right)\% = 6\frac{18}{47}\%$$

Fast Track Method

Here, $a = 6\%$

According to the formula,

Increase in consumption

$$= \left(\frac{a}{100 - a} \times 100\right)\% = \left(\frac{6}{94} \times 100\right)\%$$

$$= \frac{600}{94}\% = 6\frac{18}{47}\%$$

32. (*a*) Let maximum marks be x.

According to the question,

$$\frac{40x}{100} = 40 + 40 \Rightarrow \frac{40x}{100} = 80$$

$\therefore \quad x = 200$

Fast Track Method

Here, $P = 40\%$, $R = 40$ and $F = 40$

According to the formula,

$$\text{Maximum marks} = \frac{100\,(R + F)}{P}$$

$$= \frac{100\,(40 + 40)}{40} = \frac{100 \times 80}{40} = 200$$

33. (*d*) Let maximum marks be x.

According to the question,

$$\frac{20x}{100} + 50 = \frac{40x}{100} - 30 \Rightarrow \frac{20x}{100} = 80$$

$$\therefore \quad x = \frac{80 \times 100}{20} = 400$$

Fast Track Method

Here, $x = 20\%$, $y = 40\%$, $a = 50$ and $b = 30$

According to the formula,

$$\text{Maximum marks} = \frac{100\,(a + b)}{y - x}$$

$$= \frac{100\,(50 + 30)}{40 - 20} = \frac{100 \times 80}{20} = 400$$

34. (*c*) Given, $R = 15\%$ and $R_2 = 20\%$

Required population

$$= P\left(1 + \frac{R_1}{100}\right)\left(1 - \frac{R_2}{100}\right) \text{[by Technique 17]}$$

$$= 126800\left(1 + \frac{15}{100}\right)\left(1 - \frac{20}{100}\right)$$

$$= 126800\left(1 + \frac{3}{20}\right)\left(1 - \frac{1}{5}\right)$$

$$= 126800 \times \frac{23}{20} \times \frac{4}{5} = 116656$$

35. (*a*) Given that,

$P = 705600$, $R = 5\%$ and $n = 2$

According to the formula,

$$\text{Population after } n \text{ yr} = P\left(1 + \frac{R}{100}\right)^n$$

[by Technique 13]

$\therefore$ Population after 2 yr

$$= 705600 \times \left(1 + \frac{5}{100}\right)^2$$

$$= 705600 \times \left(\frac{105}{100}\right)^2$$

$$= 705600 \times \left(\frac{21}{20} \times \frac{21}{20}\right) = 777924$$

36. (*c*) Given that,

$$P = 1058400, R = 5\% \text{ and } n = 2$$

According to the formula,

$$\text{Population } n \text{ yr ago} = \frac{P}{\left(1 + \frac{R}{100}\right)^n}$$

[by Technique 13]

$$\therefore \text{Population 2 yr ago} = \frac{1058400}{\left(1 + \frac{5}{100}\right)^2}$$

$$= 1058400 \times \frac{20}{21} \times \frac{20}{21} = 960000$$

37. (*a*) Let total number of students be 100.
Then, students failed in Mathematics only

$$= 25 - 5 = 20$$

and students failed in Economics only

$$= 20 - 5 = 15$$

$\therefore$ Total number of failed students

$$= 20 + 15 + 5 = 40$$

$\therefore$ Number of passed students in both the subjects $= 100 - 40 = 60$

$\therefore$ Required percentage $= 60\%$

Fast Track Method

Here, $x = 25, y = 20$ and $z = 5$

According to the formula,

Required percentage

$= [100 - (25 + 20 - 5)]\%$

$= [100 - (45 - 5)]\% = (100 - 40)\% = 60\%$

38. (*d*) Population after 2 yr

$$= P\left(1 + \frac{R}{100}\right)^2 = 250000\left(1 + \frac{2}{100}\right)^2$$

$$\Rightarrow 250000 \times \frac{51}{50} \times \frac{51}{50} = 260100$$

$\therefore$ Growth $= 260100 - 250000 = 10100$

39. (*b*) Let number of boys $= B$

and number of girls $= G$

Then, 10% of $B = \frac{1}{4}$ of G

$$\Rightarrow \frac{B}{10} = \frac{G}{4} \Rightarrow \frac{B}{G} = \frac{10}{4} = \frac{5}{2} \Rightarrow B : G = 5 : 2$$

40. (*d*) In such case, there is always decrease.

Given that, $a =$ (common increase or decrease)

$= 7\%$

According to the formula,

Decreased percentage

$= \frac{a^2}{100}\%$ [by Technique 8]

$= \frac{7^2}{100}\% = 0.49\%$

41. (*c*) Let 1st number be x and 2nd number be y.

According to the question,

48% of $x = 60\% y$

$\Rightarrow \quad x \times \frac{48}{100} = y \times \frac{60}{100}$

$\Rightarrow \quad \frac{x}{y} = \frac{60}{100} \times \frac{100}{48} = \frac{5}{4}$

$\therefore \quad x : y = 5 : 4$

42. (*c*) Let the fraction be $\frac{x}{y}$.

Then, $\frac{x + 2x}{y + \frac{3}{2}y} = \frac{9}{35} \Rightarrow \frac{3x}{5y} \times 2 = \frac{9}{35}$

$\therefore \quad \frac{x}{y} = \frac{9}{35} \times \frac{5}{6} = \frac{3}{14}$

43. (*e*) According to the question,

Change in length of rectangle $= -50\% = a$

Change in breadth of rectangle $= 100\% = b$

Net effect $= \left(a + b + \frac{ab}{100}\right)\%$

[by Technique 9]

$= \left[-50 + 100 + \frac{(-50)(100)}{100}\right]\% = 0\%$

44. (*a*) III II I

100 150 120

Required percentage

$= \frac{150}{120} \times 100\% = 125\%$

45. (*c*) Let the positive number be x.

According to the question,

$x \times (15 + 20)\% = 126$

$\Rightarrow \quad x \times 35 = 126 \times 100$

$\therefore \quad x = \frac{12600}{35} = 360$

So, one-third of the number $= \frac{360}{3} = 120$

46. (*d*) B's marks $= C$'s marks $+ 5\%$ of 400

$= 300 + 20 = 320$

Now, A's marks $= B$'s marks $+ 10\%$ of 400

$= 320 + 40 = 360$

47. (*a*) Let maximum marks be x.

According to the question,

Then, $780 - 728 = 5\%$ of $x \Rightarrow 52 = 5\%$ of x

$\therefore \quad x = \frac{5200}{5} = 1040$

48. (*d*) Total marks scored by Mathew in all subjects $= 42 + 51 + 58 + 35 + 48 = 234$

$\because$ Maximum marks is 60 in any subject.

$\therefore$ Maximum marks $= 60 \times 5 = 300$

$\therefore$ Percentage of Mathew's marks

$$= \frac{\text{Marks obtained}}{\text{Maximum marks}} \times 100$$

$$= \frac{234}{300} \times 100 = \frac{234}{3} = 78\%$$

49. (*c*) Total marks obtained

$$= (76 + 48 + 84 + 38 + 72) = 318$$

Maximum marks $= 5 \times 100 = 500$

$\therefore$ Required percentage

$$= \frac{318}{500} \times 100\% = 63.6\%$$

50. (*d*) Total marks obtained

$$= [45 + 47 + 49 + 45 + 32] = 218$$

Maximum marks $= 5 \times 50 = 250$

$\therefore$ Overall percentage

$$= \frac{218}{250} \times 100\% = 87.2\%$$

51. (*a*) Net effect on area

$$= \left[-20 + 10 + \frac{(-20)(10)}{100}\right]\%$$

[by Technique 9]

$$= (-10 - 2)\% = -12\%$$

Now, after this mistake new area

$$= (100 - 12)\% \text{ of } 200 = \frac{88}{100} \times 200$$

$$= 176 \text{ sq cm}$$

52. (*b*) After removing 25% of blue balls, total blue balls left = 75% of 100

$$= \frac{75 \times 100}{100} = 75$$

After removing 50% of red balls, total red balls left $= 50\%$ of $50 = \frac{50 \times 50}{100} = 25$

$\therefore$ Required percentage

$$= \frac{50}{(75 + 25 + 50)} \times 100$$

$$= \frac{50}{150} \times 100 = 33\frac{1}{3}\%$$

53. (*d*) Let usual speed and usual time taken by the jogger are x and t, respectively. Let his new speed be x. Then,

$$xt = x' \ \frac{3}{4}t \Rightarrow x = \frac{3}{4}x' \Rightarrow x' = \frac{4}{3}x$$

Thus, he has to increase his speed by

$$\frac{\frac{4}{3}x - x}{x} \times 100\% \quad i.e., 33\frac{1}{3}\%$$

54. (*a*) Let cost prices of two articles be $3x$ and $4x$, respectively. Then,

$$\frac{110\% \text{ of } 3\,x}{4x + 4} = \frac{3}{4} \Rightarrow \frac{1.1x}{x + 1} = 1$$

$$\Rightarrow \qquad 1.1x = x + 1$$

$\Rightarrow \quad 0.1x = 1$

$\Rightarrow \quad x = 10$

Thus, cost price of the second article is

$$4 \times 10 = ₹\,40$$

55. (*c*) Let's *A*'s monthly income be ₹ x.

According to the question,

Monthly expenditure of A = ₹ 6000

$$x \times (100 - 20)\% = 6000$$

$$\Rightarrow \quad x \times \frac{80}{100} = 6000$$

$$\Rightarrow \quad x = \frac{6000 \times 100}{80} = 7500$$

$\therefore \quad x = ₹\,7500$

$\therefore$ Monthly savings of A = 20% of 7500

$$= \frac{7500 \times 20}{100} = ₹\,1500$$

56. (*d*) Let sale in 2008 = 100

Sale in 2009 = 20

Sale in 2010 = 100

$$\therefore \text{Required increase} = \frac{100 - 20}{20} \times 100$$

$$= \frac{80}{20} \times 100 = 400\%$$

57. (*c*) Let the total number of voters = x

According to the question,

75% of 98% of 75% of x = 9261

[$\because$ 2% votes were rejected]

$$\Rightarrow \quad \frac{x \times 75 \times 98 \times 75}{100 \times 100 \times 100} = 9261$$

$$\therefore x = \frac{9261 \times 100 \times 100 \times 100}{75 \times 75 \times 98} = 16800$$

58. (*d*) According to the question,

$$\frac{(100 + P) \times 20}{100} = \frac{(100 - P) \times 60}{100}$$

$\Rightarrow \quad (100 + P) = (100 - P) \times 3$

$\Rightarrow \quad P + 3P = 300 - 100$

$$\Rightarrow \quad P = \frac{200}{4} = 50\%$$

$$\text{Then, } 50\% \text{ of } 70 = \frac{50 \times 70}{100} = 35$$

59. (*a*) Last year number of boys in school

= 610

$\therefore$ Number of boys in school this year

$$= 610 - 610 \times \frac{20}{100}$$

$$= 610 - 122 = 488$$

$\therefore$ Number of girls in school this year

= Number of boys in school this year × 175%

$$= 488 \times \frac{175}{100} = 488 \times \frac{7}{4}$$

$$= 122 \times 7 = 854$$

Hence, number of girls in school this year in 854.

60. (c) Let maximum marks of the exam is x.
According to the question,
Marks obtained by Vidya = Marks obtained by Aryan + 296

$$\Rightarrow x \times \frac{76}{100} = 350 + 296 \Rightarrow x \times \frac{76}{100} = 646$$

$$\Rightarrow x = \frac{646 \times 100}{76} \Rightarrow x = 8.5 \times 100 = 850$$

$\therefore\ x = 850$

61. (e) Let a student was awarded in exam $= x$ marks
According to the question,
After re-evaluation his score

$$\Rightarrow x - x \times \frac{40}{100} = 96 \Rightarrow x - \frac{2x}{5} = 96$$

$$\Rightarrow \frac{5x - 2x}{5} = 96 \Rightarrow \frac{3x}{5} = 96$$

$$\Rightarrow x = 5 \times \frac{96}{3} = 5 \times 32 \Rightarrow x = 160$$

So, a student was awarded by 160 marks in examination.

.•. Reduced marks after re-evaluation = 160 -96 = 64

62. (d) 32% of 750 $= 750 \times \frac{32}{100} = 240$

From option (a), 23% of 600

$$= 600 \times \frac{23}{100} = 138$$

From option (b), 46% of 207

$$= 207 \times \frac{46}{100} = 95.22$$

From option (c), 98% of 250

$$= 250 \times \frac{98}{100} = 245$$

From option (d), 75% of 320

$$= 320 \times \frac{75}{100} = 240$$

Hence, from option (d), satisfies equality of the equation.

Exercise © *Higher SkiU Level Questions*

1. (*a*) Gold in 50g of alloy

$$= 80 \times \frac{50}{100} = 40 \text{ g}$$

Let x g gold must be added.

Now, according to the question,

$$\frac{40 + x}{50 + x} = \frac{90}{100}$$

$\Rightarrow$ $100\,(40 + x) = 90\,(50 + x)$

$\Rightarrow$ $10\,(40 + x) = 9\,(50 + x)$

$\Rightarrow$ $400 + 10x = 450 + 9x$

$\Rightarrow$ $x = 450 - 400 \Rightarrow x = 50$ g

Thus, 50 g of gold must be added to make it 90%.

2. (*d*) Annual household expenses

$$= ₹\,(25\% \text{ of } 4.32 \times 10^5)$$

$$= ₹\left(\frac{25}{100} \times 4.32 \times 10^5\right)$$

$$= ₹\,(25 \times 4.32 \times 10^3)$$

$\therefore$ Household expenses for 8 months

$$= ₹\left(\frac{25 \times 4.32}{12} \times 8 \times 10^3\right) = ₹\ 72000$$

3. (*c*) Required salary in October

$$= \left(6300 \times \frac{104}{100} \times \frac{104}{100}\right)$$

$$= (63 \times 104 \times 1.04)$$

$$= ₹\ 6814.08 \approx ₹\ 6814$$

4. (*b*) Quantity of alcohol in 5 L of solution

$$= \frac{40}{100} \times 5 = 2 \text{ L}$$

Quantity of alcohol in 6 L of solution = 2 L

$\therefore$ Strength of alcohol in new solution

$$= \left(\frac{2}{6} \times 100\right)\% = 33\frac{1}{3}\%$$

5. (*d*) According to the formula,

Percentage of students passed in both the subjects

$$= [100 - (x + y - z)]\%$$
$$= [100 - (49 + 36 - 15)]\% = 30\%$$

Let total number of students = x

According to the question,

$$x \times \frac{30}{100} = 450$$

$$\Rightarrow \quad x = \frac{450}{30} \times 100$$
$$= 1500$$

$\therefore$ Total number of students = 1500

6. (*c*) Let maximum marks be x.

According to the question,

$$\frac{75x}{100} - 70 = 112 - 32$$

$$\Rightarrow \quad \frac{75x}{100} = 80 + 70 \Rightarrow \frac{75x}{100} = 150$$

$$\therefore \quad x = 200$$

Passing marks = 112 − 32 = 80

(as Rajesh got 112 marks which is 32 more than the passing marks.)

$\therefore$ Minimum passing marks percentage

$$= \frac{80}{200} \times 100\% = 40\%$$

7. (*c*) Mr. Sen's monthly income

$= \text{(Bina + Anita)'s monthly income}$

$= \dfrac{775200}{12} = ₹\ 64600$

According to the question,

Bina's monthly income = 90% of Anita's monthly income

$\Rightarrow \quad \dfrac{\text{Bina}}{\text{Anita}} = \dfrac{90}{100} = \dfrac{9}{10}$

$\Rightarrow \quad \text{Bina : Anita} = 9 : 10$

$\therefore$ Bina's monthly salary

$= \dfrac{9}{19} \times 64600$

$= ₹\ 30600$

8. (*d*) Let the maximum aggregate marks be x.

According to the question,

$10\% \text{ of } x = 296 - 222$

$\Rightarrow \quad \dfrac{x}{10} = 74$

$\therefore \quad x = 74 \times 10 = 740$

9. (*c*) Let total amount be ₹ x.

Total expenditure $= (23 + 33 + 19 + 16)\%$ of $x = 91\%$ of x

Remaining money

$= (100 - 91)\% \text{ of } x = 9\% \text{ of } x$

According to the question,

$9\% \text{ of } x = 504$

$\therefore \quad x = \dfrac{504}{9} \times 100$

$= ₹\ 5600$

Now, total money (food + insurance)%

$= (23 + 33)\% \text{ of } x = 56\% \text{ of } x = 56\% \text{ of } 5600$

$= \dfrac{56}{100} \times 5600 = ₹\ 3136$

10. (*a*) Total salary spent on insurance, shopping and household expenses $= 15 + 55 = 70\%$

$\therefore$ Saving $= 100 - 70 = 30\ \%$

Let the total salary be x.

$\therefore 30\% \text{ of } x = ₹\ 12750$

$\Rightarrow \quad \dfrac{30}{100} \times x = ₹\ 12750$

$\therefore \quad x = \dfrac{12750 \times 100}{30}$

$= ₹\ 42500$

11. (*d*) Let total number of staff be 100.

$\therefore \quad \text{Female staff} = 40$

Male staff $= (100 - 40) = 60$

Votes cast by females $= \dfrac{40}{100} \times 40 = 16$

Votes cast by males $= \dfrac{60}{100} \times 60 = 36$

Votes cast by both (males + females)

$= 16 + 36 = 52$

$\therefore$ Percentage votes obtained = 52%

12. (*c*) Let selling price of goods by Shyam be ₹ 100.

$\therefore$ Selling price of goods by Ram = ₹ 75

Now, according to the question,

125% of selling price of goods by Hari = 75

$\Rightarrow$ Selling price of goods by Hari

$$= \frac{100}{125} \times 75 = ₹\ 60$$

So, Hari's goods are cheaper than Shyam's goods by

$$\frac{40}{100} \times 100\% = 40\%$$

13. (*a*) Let number of boys be 300.

Number of girls = 200

Boys holding scholarship

$$= \frac{20}{100} \times 300 = 60$$

Girls holding scholarship

$$= \frac{25}{100} \times 200 = 50$$

Total students holding scholarship

$$= 60 + 50 = 110$$

$\therefore$ Percentage of students not holding scholarship

$$= \frac{500 - 110}{500} \times 100\%$$

$$= \frac{390}{500} \times 100\% = 78\%$$

14. (*a*) Let original price be x.

New price per kg $= \left(\frac{80}{100} \times x\right) = ₹\ \frac{4x}{5}$

According to the question,

$$\frac{770}{\frac{4x}{5}} - \frac{770}{x} = 3.5$$

$$\Rightarrow \quad \frac{3850}{4x} - \frac{770}{x} = \frac{7}{2}$$

$$\Rightarrow \quad (3850 - 3080) = 14x$$

$$\Rightarrow \quad 14x = 770$$

$$\therefore \quad x = \frac{770}{14} = ₹\ 55$$

$\therefore$ Original price of the wheat is ₹ 55 per kg.

Fast Track Method

Here, $r = 20\%$, $A = 3.5$ kg and $x = ₹\ 770$

According to the formula,

Actual price of rice $= \frac{rx}{(100 - r)\ A}$

$$= \frac{20 \times 770}{(100 - 20) \times 3.5} = \frac{20 \times 770}{80 \times 3.5} = ₹\ 55$$

15. (*d*) Let the weight of fresh grapes be x.

Quantity of water in it $= \frac{80}{100} \times x = \frac{4x}{5}$

Quantity of pulp in it $= \left(x - \frac{4x}{5}\right) = \frac{x}{5}$

Quantity of water in 500 kg dry grapes

$$= \frac{10}{100} \times 500 = 50 \text{ kg}$$

$\therefore$ Quantity of pulp in it

$$= (500 - 50) = 450 \text{ kg}$$

$$\frac{x}{5} = 450$$

$\therefore$ $x = 2250$ kg

16. (*a*) Increase in the price of rice = 25%

$\therefore$ Increase in ₹ 400 = 25% of ₹ 400

$$= 400 \times \frac{25}{100} = ₹\ 100$$

$\because$ Increase in rate of rice in 25%, there are 20 kg less rice in ₹ 400 *i.e.*, rate of 20 kg rice

$$= ₹\ 100$$

$\therefore$ Rate of 1kg rice $= \frac{100}{20} = ₹\ 5$

$\therefore$ Increased price of rice = ₹ 5 per kg

17. (*a*) Let original price be ₹ 100 per kg.

Money required to buy 50 kg of rice

$$= ₹\ (100 \times 50) = ₹\ 5000$$

New price = (100 − 5) = ₹ 95 per kg

$\therefore$ Quantity of rice bought $= \left(\frac{5000}{95}\right)$ kg

$$= 52.63 \text{ kg}$$

18. (*a*) Let original rate of rice be ₹ x per kg.

Reduced rate $= ₹\left[(100 - 6.25) \times \frac{1}{100} \times x\right]$

$$= ₹\ \frac{15x}{16} \text{ per kg}$$

According to the question,

$$\frac{120}{\frac{15x}{16}} - \frac{120}{x} = 1$$

$$\Rightarrow \quad \frac{120 \times 16}{15x} - \frac{120}{x} = 1$$

$$\Rightarrow \quad 120 \times 16 - 1800 = 15x$$

$$\Rightarrow \quad 128 - 120 = x$$

$\therefore$ $x = ₹\ 8$ per kg

$\therefore$ Reduced rate $= ₹\left(\frac{15}{16} \times 8\right)$ per kg

$$= ₹\ 7.50 \text{ per kg}$$

19. (*b*) Let the original price per egg be x.

Then, increased price $= ₹\left(\frac{130}{100}x\right)$

According to the question,

$$\frac{7.80}{x} - \frac{7.80}{\frac{130x}{100}} = 6$$

$\Rightarrow \quad \frac{7.80}{x} - \frac{780}{130x} = 6$

$\Rightarrow \quad 1014 - 780 = 6 \times 130x$

$\Rightarrow \quad 780x = 234$

$\therefore \quad x = \frac{234}{780} = 0.3$

$\therefore$ Present price per dozen

$= ₹\left(12 \times \frac{130}{100} \times 0.3\right) = ₹\ 4.68$

20. (*d*) Old price of rice = ₹ 24/kg

New price of rice = ₹ 27 per/kg

$\therefore$ Increase in price of rice = New price − Old price = 27 − 24 = ₹ 3/kg

$\therefore$ Increased percentage in price

$= \frac{3}{24} \times 100 = 12\frac{1}{2}\%$

Now, reduction in consumption, so as to keep the expenditure fixed.

According to the formula,

$$\left(\frac{a}{100 + a} \times 100\right)\%$$

$$\left(\frac{12.5}{100 + 12.5} \times 100\right)\% = 11.1\%$$

21. (*a*) Number of sweets received by every student = 15% of 40 $= \frac{40 \times 15}{100} = 6$

$\therefore$ Number of sweets received by 40 students

$= 40 \times 6 = 240$

Number of sweets received by each teacher

$= 20\%$ of $40 = \frac{40 \times 20}{100} = 8$

$\therefore$ Number of sweets received by 5 teachers

$= 8 \times 5 = 40$

$\therefore$ Total number of sweets

$= 240 + 40 = 280$

22. (*a*) Let price of ghee before increment = ₹ x

Consumption = 10 kg

Then, expenditure on ghee = ₹ $10x$

After increment,

Expenditure on ghee = 110% of $10x = 11x$

Price of ghee = 132% of $x = x \times \frac{132}{100}$

$= \frac{33x}{25}$ per kg

$\therefore$ New consumption

$= \frac{11x \times 25}{33x}$ kg

$= 8\frac{1}{3}$ kg

23. (*b*) Let expenses on wheat be $12x$.

Expenses on meat = $17x$

Expenses on vegetables = $3x$

$\therefore$ Total expenses = $32x$

Increased expenses = ₹ (120% of $12x$)
+ (130% of $17x$) + (150% of $3x$)]

$$= ₹\left[\left(\frac{120}{100}\times 12x\right)+\left(\frac{130}{100}\times 17x\right)+\left(\frac{150}{100}\times 3x\right)\right]$$

$$= ₹\left[\frac{72x}{5}+\frac{221x}{10}+\frac{9x}{2}\right]$$

$$= ₹\left(\frac{144x+221x+45x}{10}\right)$$

$$= ₹\left(\frac{410x}{10}\right) = ₹\,41x$$

$\therefore$ Total increase percentage

$$=\left(\frac{9x}{32x}\times 100\right)\% = \frac{225}{8}\% = 28\frac{1}{8}\%$$

24. (*d*) In 1998,

Let number of students in $x = 3k$

Number of students in $y = 5k$ and

Number of students in $z = 6k$

Next year, number of students in

$$x = 3k + 20\% \text{ of } 3k = \frac{18k}{5}$$

Number of students in $y = 5k + 10\%$ of $5k$

$$= \frac{11k}{2}$$

Number of students in $z = 6k + 20\%$ of $6k$

$$= \frac{36k}{5}$$

According to the question,

$$\frac{\frac{18k}{5}}{\frac{36k}{5}} = \frac{1}{2}$$

Thus, data is insufficient.

25. (d) Let us assume that Arun uses x units of petrol everyday, so, the amount of petrol in the tank when it is full will be lOx. If he starts using 25% more petrol everyday, then the units of petrol he now use everyday will be

$$x\left(1+\frac{25}{100}\right)=125x$$

So, the number of days his petrol will now last will be equal to (amount of petrol in tank/number of units used everyday)

$$=\frac{10x}{1.25x}=\frac{10}{125}=8 \text{ days}$$

26. (*c*) Total number of students = 480

Percentage of total students passed

$= 75\%$ of total student

$$=\frac{75\times 480}{100}=360 \text{ students}$$

Now, using the condition from the question.

Let the number of boys be x.

Then, 70 % of x + 85% of $(480 - x) = 360$

$$\Rightarrow \quad \frac{70\times x}{100}+\frac{85\times(480-x)}{100}=360$$

$$\Rightarrow \quad 70x - 85x + 40800 = 36000$$

$$\Rightarrow \quad 40800 - 36000 = 85x - 70x$$

$$\Rightarrow \quad 4800 = 15x$$

$$\Rightarrow \quad x=\frac{4800}{15}=320$$

$\therefore$ There are 320 boys who appeared for the examination,

27. (*c*) For class X, let the student passed in first class = a%

Then, by condition given in question,

$a\%$ of $30 = 24$

$$\Rightarrow \quad \frac{a\times 30}{100}=24$$

$$\therefore \quad a = 80\%$$

Now, for class y let the student passed in first class = $b\%$

Then, according to the question,

b % of $35 = 28$

$$\Rightarrow \quad \frac{b\times 35}{100}=28$$

$$\therefore \quad b=28\times\frac{100}{35}=80\ \%$$

Hence, both classes have equal percentage of students getting first class.

Chapter

12

Profit and Loss

Profit and loss are the terms related to monetary transactions in trade and business. Whenever a purchased article is sold, then either profit is earned or loss is incurred. **Cost Price** (CP) This is the price at which an article is purchased or manufactured.

Selling Price (SP) This is the price at which an article is sold.

Overhead Charges Such charges are the extra expenditures on purchased goods apart from actual cost price. Such charges include freight charges, rent, salary of employees, repairing cost on purchased articles etc.

Note If overhead charges are not specified in the question, then they are

not considered

Profit (SP>CP) When an article is sold at a price more than its cost price, then profit is earned,

Loss (CP>SP) When an article is sold at a price lower than its cost price, then loss is incurred.

Ex. 1 A man buys an article for ? 300 and sells it for ? 900. Find profit/loss.

Sol. Here, SP > CP

.". Profit is earned. According to the formula, Profit = SP - CP = 900 - 300 = ? 600

Ex. 2 Raman purchased a car for ? 5 lakh and sold it for ? 4 lakh. Find profit/loss in this transaction.

Sol. Here, SP < CP

.". Loss is incurred in this case. According to the formula, Loss= CP- SP= (5-4) lakh=?l lakh

Basic Formulae Related to Profit and Loss

1. Profit = SP – CP

2. Loss = CP – SP

3. $\text{Profit \%} = \frac{\text{Profit}}{\text{Cost price}} \times 100\% = \frac{P}{CP} \times 100\ \%$

4. $\text{Loss \%} = \frac{\text{Loss}}{\text{Cost price}} \times 100\% = \frac{L}{CP} \times 100\ \%$

5. $SP = \left(\frac{100 + \text{Gain \%}}{100}\right) \times CP$

6. $SP = \left(\frac{100 - \text{Loss \%}}{100}\right) \times CP$

7. $CP = \left(\frac{100}{100 + \text{Gain \%}}\right) \times SP$

8. $CP = \left(\frac{100}{100 - \text{Loss \%}}\right) \times SP$

MIND IT!

1. Profit and loss are always calculated on cost price unless otherwise stated in the question.

2. If an article is sold at a certain gain (say 45%), then SP = 145% of CP

3. If an article is sold at a certain loss (say 25%), then SP =75% of CP

Ex. 3 A person buys a toy for ? 50 and sells it for ? 75. What will be his gain per cent?

Sol. Given,CP=?50andSP=?75

Profit = SP -CP = 75 - 50 = ? 25 According to the formula,

$$\text{Gain \%} = \frac{\text{Profit}}{CP} \times 100\% = \frac{25}{50} \times 100\% = 50\%$$

Ex. 4 A person buys a cycle for ? 450 but because of certain urgency, he sells it for ? 350. Find his loss per cent.

Sol. Given, CP = ₹ 450 and SP = ₹ 350

$$\text{Loss} = \text{CP} - \text{SP} = 450 - 350 = ₹\ 100$$

According to the formula,

$$\text{Loss }\% = \frac{\text{Loss}}{\text{CP}} \times 100\% = \frac{100}{450} \times 100\% = \frac{200}{9}\ \% = 22\frac{2}{9}\ \%$$

Ex. 5 Find the SP, when CP is ? 80 and gain is 20%.

Sol. Given, CP = ? 80 and gain = 20%

$$\text{SP} = \left(\frac{100 + \text{Gain}\%}{100}\right) \times \text{CP}$$

$$\text{SP} = \left(\frac{100 + 20}{100}\right) \times 80 = \frac{120}{100} \times 80 = 12 \times 8 = ₹\ 96$$

Ex. 6 Find the SP when CP is ? 80 and loss is 20%.

Sol. Given, CP = ? 80 and loss =20%

$$\text{SP} = \left(\frac{100 - \text{Loss}\%}{100}\right) \times \text{CP}$$

$$\text{SP} = \left(\frac{100 - 20}{100}\right) \times 80 = 8 \times 8 = ₹\ 64$$

Ex. i Find the CP when SP is ? 40 and gain is 15%.

Sol. Given, SP = ₹ 40 and gain = 15%

$$\text{CP} = \left(\frac{100}{100 + \text{Gain}\%}\right) \times \text{SP}$$

$$\text{CP} = \left(\frac{100}{100 + 15}\right) \times 40 \text{ of SP} = \frac{100}{115} \times 40 = ₹\ 34.78$$

Ex. 8 Find the CP when SP is ? 200 and loss is 35%.

Sol. Given, SP = ₹ 200 and loss = 35%

$$\text{CP} = \left(\frac{100}{100 - \text{Loss}\%}\right) \times \text{SP}$$

$$\text{CP} = \left(\frac{100}{100 - 45}\right) \times 200 \text{ of SP} = \frac{100}{65} \times 200 = 307.6$$

Ex. 9 A toy is bought for ? 150 and sold at a gain of 8%. Find its selling price.

Sol. Given, CP = ₹ 150 and gain = 8%

$$\therefore \quad SP = \left(\frac{100 + \text{Gain}\%}{100}\right) \times CP = \frac{108}{100} \times 150 = 54 \times 3 = ₹\ 162$$

Ex. 10 A table is bought for ?1500 and sold at a loss of 6%. Find its selling price.

Sol. Given, CP = ₹ 1500 and loss = 6%

$$\therefore \quad SP = \left(\frac{100 - \text{Loss}\%}{100}\right) \times CP = \frac{94}{100} \times 1500 = ₹\ 1410$$

Ex. 11 By selling a watch for ? 1440, a man losses 10%. At what price should he sell it to gain 10%?

Sol. Given, SP = ₹ 1440 and loss = 10%

$$\therefore \quad CP = \left(\frac{100}{100 - \text{Loss}\%}\right) \times SP = \frac{100}{90} \times 1440 = ₹\ 1600$$

Now, CP = ₹ 1600 and gain = 10%

$$\therefore \text{ Required } SP = 1600 \times \frac{110}{100} = ₹\ 1760$$

Ex. 12 Ravish lost 20% by selling a radio set for ? 3072. What per cent will he gain by selling it for ? 4080?

Sol. Given, SP = ? 3072 and loss = 20%

$$\therefore \quad CP = \frac{100}{80} \times 3072 = ₹\ 3840$$

Now, CP = ₹ 3840 and SP = ₹ 4080

Gain = 4080 − 3840 = ₹ 240 [∵ gain = SP − CP]

$$\therefore \quad \text{Gain }\% = \frac{240}{3840} \times 100\% = 6.25\% \quad \left[\because \text{gain }\% = \frac{\text{gain}}{CP} \times 100\%\right]$$

Ex. 13 A vendor sells apples at 10 for ? 1 gaining 40%. How many apples did he buy for ? 1?

Sol. Given, SP of 10 apples = ₹ 1 and gain = 40%

$$\text{CP of 10 apples} = 1 \times \frac{100}{140} = ₹\ \frac{5}{7}$$

$\because$ ₹ $\frac{5}{7}$ yield 10 apples.

$\therefore$ ₹ 1 will yield $10 \times \frac{7}{5} = 14$ apples.

Ex. 14 A grocer buys 160 kg of rice at ? 27 per kg and mixes it with 240 kg of rice available at ? 32 per kg. At what rate per kg should he sell the mixture to gain 20% on the whole?

Sol. CP of 400 kg of rice $= 160 \times 27 + 240 \times 32 = 4320 + 7680 = ₹\,12000$

Now, $\quad CP = ₹\,12000$ and gain $= 20\%$

$\therefore \quad SP = \frac{120}{100} \times 12000 = ₹\,14400$

$\therefore \quad$ SP per kg $= \frac{14400}{400} = ₹\,36$

Ex. 15 A man purchases a certain number of apples at 3 per rupee and the same number of apples at 4 per rupee. He mixes them together and sells them at 3 per rupee. What is his gain or loss per cent?

Sol. CP of 1 apple for 1st rate $= ₹\,\frac{1}{3}$

CP of 1 apple for 2nd rate $= ₹\,\frac{1}{4}$

CP of 2 apples after mixing $= \frac{1}{3} + \frac{1}{4} = ₹\,\frac{7}{12}$

CP of 1 apple after mixing $= \frac{7}{12 \times 2} = ₹\,\frac{7}{24}$

SP of 1 apple after mixing $= ₹\,\frac{1}{3}$

$\therefore \quad$ Gain $= \frac{1}{3} - \frac{7}{24} = \frac{8-7}{24} = ₹\,\frac{1}{24}$

$\therefore \quad$ Gain % $= \left(\frac{1}{24} \times \frac{24}{7} \times 100\right)\% = \frac{100}{7}\% = 14\frac{2}{7}\%$

Fast Track **Techniques**

to solve the **QUESTIONS**

Technique 1

If a person sells two similar articles, one at a gain of *a%* and another at a loss of a%, then the seller always incurrs a loss which is given by

$$\text{Loss }\% = \left(\frac{a}{10}\right)^2 \%$$

Note In this case, SP is immaterial

Ex. 16 A man sold two radios for ? 2000 each. On one he gains 16% and on the other he losses 16%. Find his gain or loss per cent in the whole transaction.

Sol. Here, $a = 16\%$

According to the formula,

$$\text{Loss }\% = \left(\frac{a}{10}\right)^2\% = \left(\frac{16}{10}\right)^2\% = \frac{256}{100}\%$$

$$= 2.56\%$$

Technique 2

If 'a'th part of some items is sold at x% loss, then required gain per cent in selling rest of the items in order that there is neither gain nor loss in

whole transaction, is $\frac{ax}{1-a}\%$.

Ex. 17 A medical store owner purchased medicines worth ? 6000 from a company. He sold 1/3 part of the medicine at 30% loss. On which gain he should sell his rest of the medicines, so that he has neither gain nor loss?

Sol. Given, $a = \frac{1}{3}$ and $x = 30\%$

According to the formula,

$$\text{Required gain }\% = \frac{ax}{1-a}\% = \frac{\frac{1}{3} \times 30}{1 - \frac{1}{3}}\% = \frac{10 \times 3}{2}\% = 15\%$$

Technique 3

A businessman sells his items at a profit/Loss of a%. If he had sold it for ? *R* more, he would have gained/lost &%. Then,

$$\text{CP of items} = \frac{R}{b \pm a} \times 100$$

'-' = when both are either profit or loss '+' = When one is profit and other is loss

Ex.18 A person sold a table at a profit of $6\frac{1}{2}\%$. If he had sold it for ₹ 1250 more, he would have gained 19%. Find the CP of the table.

Sol. In this problem it is clearly stated that for ₹ 1250 more the gain will rise to 10% hence.

$$6\frac{1}{2}\% \text{ profit} + ₹\ 1250 = 19\% \text{ profit}$$

$$\Rightarrow \quad ₹\ 1250 = \left(19 - \frac{13}{2}\right)\% \text{ CP}$$

[profit% is calculated on CP, hence we can write it % of CP]

$$\Rightarrow \quad ₹\ 1250 = \left(\frac{38 - 13}{2}\right)\% \text{ CP}$$

$$\Rightarrow \quad ₹\ 1250 = \frac{25}{2}\% \left(\text{or } \frac{1}{8}\right) \text{of CP}$$

$\therefore$ CP = ₹ 1250 × 8 = ₹ 10000

Fast Track Method

Here, $a = 6\frac{1}{2}\% = \frac{13}{2}\%$

$b = 19\%$ and $R = ₹\ 1250$

According to the formula,

$$\text{CP of table} = \frac{R}{b - a} \times 100$$

$$= \frac{1250}{19 - \frac{13}{2}} \times 100 = \frac{1250 \times 2}{25} \times 100$$

$$= ₹\ 10000$$

Technique 4

If cost price of *'a'* articles is equal to the selling price of *'b'* articles, then

$$\text{Profit percentage} = \frac{a - b}{b} \times 100\%$$

Ex. 19 If the cost price of 20 articles is equal to the selling price of 18 articles, then find the profit per cent.

Sol. Let CP of 1 article = ₹ x

CP of 20 articles = ₹ $20x$ and SP of 18 articles = $20x$

$\therefore$ SP of 1 article = $\frac{20}{18}x = \frac{10x}{9}$

Profit = $\frac{10x}{9} - x = \frac{x}{9}$

$\therefore$ Profit % = $\frac{\frac{x}{9}}{x} \times 100 = \frac{100}{9} = 11\frac{1}{9}$ %

Direct Approach

2 articles are gained on selling 18 articles.

$\therefore$ Profit % = $\left(\frac{2}{18} \times 100\right)\% = 11\frac{1}{9}\%$

Fast Track Method

Here, $a = 20$ and $b = 18$

According to the formula,

$$\text{Profit } \% = \left(\frac{a-b}{b} \times 100\right)\% = \left(\frac{20-18}{18} \times 100\right)\%$$

$$= \frac{100}{9}\% = 11\frac{1}{9}\%$$

Technique 5

If a man purchases m items for ₹ x and sells n items for ₹ y, then

Profit or loss per cent is given by $\frac{my - nx}{nx} \times 100\%$

[Positive result means profit and negative result means loss.]

Ex. 20 If Karan purchases 10 oranges for ? 25 and sells 9 oranges for ? 25, then find the gain percentage.

Sol. CP of 10 oranges = ₹ 25

Then, CP of 1 orange = ₹ $\frac{25}{10}$

SP of 9 oranges = ₹ 25

SP of 1 orange = ₹ $\frac{25}{9}$

$\therefore$ Gain per cent $= \frac{SP - CP}{CP} \times 100\%$

$$= \frac{\frac{25}{9} - \frac{25}{10}}{\frac{25}{10}} \times 100\%$$

$$= \frac{25 \times 10 - 25 \times 9}{9 \times 10} \times \frac{10}{25} \times 100\%$$

$$= \frac{25}{9 \times 25} \times 100\% = \frac{100}{9}\% = 11\frac{1}{9}\%$$

Direct Approach

Let the total number of oranges be 90 (LCM of 10 and 9).

Then, SP of 90 oranges = 25 × 10 = ₹ 250

CP of 90 oranges = 25 × 9 = 225

$\therefore$ Profit per cent $= \frac{SP - CP}{CP} \times 100\%$

$$= \frac{250 - 225}{225} \times 100\%$$

$$= \frac{25}{225} \times 100\% = \frac{100}{9}\% = 11\frac{1}{9}\%$$

Fast Track Method

Here, $m = 10$, $x = 25$, $n = 9$ and $y = 25$

$\therefore$ Profit per cent $= \frac{my - nx}{nx} \times 100\%$

$$= \frac{25 \times 10 - 9 \times 25}{9 \times 25} \times 100\%$$

$$= \frac{250 - 225}{225} \times 100\%$$

$$= \frac{25}{225} \times 100\% = \frac{100}{9}\% = 11\frac{1}{9}\%$$

Ex. 21 Shakshi bought pens at a rate of 10 pens for ? 11 and sold them at a rate of 11 pens for ? 10. Then, find the profit or loss per cent.

Sol. CP of 10 pens = ₹ 11

Then, CP of 1 pen = ₹ $\frac{11}{10}$

Similarly, SP of 11 pens = ₹ 10

$\therefore$ SP of 1 pens = $\frac{10}{11}$

Here, SP < CP

$$\therefore \quad \text{Loss per cent} = \frac{\text{CP} - \text{SP}}{\text{CP}} \times 100\% = \frac{\frac{11}{10} - \frac{10}{11}}{\frac{11}{10}} \times 100\%$$

$$= \frac{121 - 100}{10 \times 11} \times \frac{10}{11} \times 100\%$$

$$= \frac{21}{121} \times 100\%$$

$$= \frac{2100}{121}\% = 17\frac{43}{121}\%$$

Fast Track Method

Here, $m = 10$, $x = 11$, $n = 11$ and $y = 10$

Then,

$$\text{Profit or loss per cent} = \frac{my - nx}{nx} \times 100\%$$

$$= \frac{10 \times 10 - 11 \times 11}{11 \times 11} \times 100\%$$

$$= \frac{-21}{121} \times 100\% = -\frac{2100}{121}\%$$

$$= -17\frac{43}{121}\%$$ [negative sign indicates a loss]

Technique 6

If *A* sold an article to e at a profit (loss) of r_1% and *B* sold this article to C at a profit (loss) of r_2%, then cost price of article for *C* is given by cost

$$\text{price for } A \times \left(1 \pm \frac{r_1}{100}\right)\left(1 \pm \frac{r_2}{100}\right)$$

[Positive for profit and negative for loss is used.]

Ex.22 Nikunj sold a machine to Sonia at a profit of 30%. Sonia sold this machine to Anu at a loss of 20%. If Nikunj paid ? 5000 for this machine, then find the cost price of machine for Anu.

Sol. Nikunj sold the machine for a profit of 30%.

$\therefore$ CP of machine for Sonia = 130% of CP of machine for Nikunj

$$= \frac{130 \times 5000}{100} = ₹\ 6500$$

Now, Sonia sold the machine to Anu at a loss of 20%.

$\therefore$ CP of machine for Anu = 80% of CP of machine for Sonia

$$= \frac{80 \times 6500}{100} = ₹\ 5200$$

Hence, CP of machine for Anu is ₹ 5200.

Fast Track Method

Here, $r_1 = 30\%$ and $r_2 = 20\%$

CP of a machine for Nikunj = ₹ 5000

$\therefore$ CP of machine for Anu = CP of machine for Nikunj $\left(1 + \frac{r_1}{100}\right)\left(1 - \frac{r_2}{100}\right)$

$$= 5000\left(1 + \frac{30}{100}\right)\left(1 - \frac{20}{100}\right) = 5000 \times \frac{130}{100} \times \frac{80}{100} = ₹\ 5200$$

Technique 7

If a dishonest trader professes to sell his items at CP but uses false weight, then

$$\text{Gain \%} = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100\ \%$$

$$\text{Gain \%} = \frac{\text{True weight} - \text{False weight}}{\text{False weight}} \times 100\ \%$$

Here, while calculating gain or profit per cent, we have taken false weight as a base. Because CP is what is paid when an item is purchased or manufactured. Here, in this case dishonest trader is telling false weight to be the CP and he is gaining only when sells at false weight.

Ex.23 A dishonest dealer professes to sell his goods at cost price but he uses a weight of 930 g for 1 kg weight. Find his gain per cent.

Sol. 70 g is gained on 930 g.

$\therefore$ $$\text{Gain \%} = \left(\frac{70}{930} \times 100\right)\% = 7\frac{49}{93}\ \%$$

Fast Track Method

According to the formula,

$$\text{Gain \%} = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100\%$$

$$= \frac{70}{1000 - 70} \times 100\% = \frac{70}{930} \times 100\% = \frac{700}{93}\ \% = 7\frac{49}{93}\ \%$$

Technique 8

If a shopkeeper sells his goods at *a%* loss on cost price but uses *b* g instead of c g, then his percentage profit or loss is

$$\left[(100-a)\frac{c}{b}-100\right]\%$$ as sign positive or negative.

Ex. 24 A dealer sells goods at 6% loss on cost price but uses 14 g instead of 16 g. What is his percentage profit or loss?

Sol. Given, $a = 6\%$, $b = 14\,\text{g}$ and $c = 16\,\text{g}$

According to the formula,

$$\text{Required answer} = \left[(100-6)\frac{16}{14}-100\right]\% = \left(94 \times \frac{8}{7} - 100\right)\%$$

$$= \left(\frac{752-700}{7}\right)\% = \frac{52}{7}\%$$

$$= 7\frac{3}{7}\% \text{ gain}$$ [+ve sign shows that there is a gain]

Technique 9

If a dealer sells his goods at *a%* profit or loss on cost price and uses *b%* less weight, then his percentage profit or loss will be $\frac{(b \pm a)}{100-b} \times 100\%$ according to +ve and −ve signs.

Ex. 25 A dealer sells his goods at 20% loss on cost price but uses 40% less weight. What is his percentage profit or loss?

Sol. Given, $a = 20\%$ and $b = 40\%$

According to the formula,

$$\text{Required answer} = \frac{(b \pm a)}{100-b} \times 100\% = \frac{(40-20)}{60} \times 100\% = \frac{2}{6} \times 100\%$$

$$= \frac{100}{3}\% = 33\frac{1}{3}\% \text{ profit}$$

Technique 10

If *'a'* part of an article is sold at *x%* profit/loss, *'b'* part at y% profit/loss and c part atz% profit/loss and finally there is a profit/loss of?/?, then

$$\text{Cost price of entire article} = ₹\,\frac{R \times 100}{ax + by + cz}.$$

Ex. 26 If $\frac{2}{3}$ part of an article is sold at 30% profit, $\frac{1}{4}$ part at 16% profit and remaining part at 12% profit and finally, there is a profit of ₹ 75, then find the cost price of the article.

Sol. Here, $a = \frac{2}{3}, x = 30\%, b = \frac{1}{4}, y = 16\%, z = 12\%$ and $R =$ ₹ 75

Then, remaining part $= c = 1 - \left(\frac{2}{3} + \frac{1}{4}\right) = \frac{1}{12}$

$$\therefore \text{Cost price of entire article} = \frac{R \times 100}{ax + by + cz}$$

$$= \frac{75 \times 100}{\frac{2}{3} \times 30 + \frac{1}{4} \times 16 + \frac{1}{12} \times 12}$$

$$= \frac{75 \times 100}{20 + 4 + 1} = \frac{7500}{25} = ₹\,300$$

Fast Track Practice

Exercise© *Base Level Questions*

1. A person buys a book for *X* 200 and sells it for *X* 225. What will be his gain per cent? [SSCLDC2011]

(a) 13% *(b)* 14%

(c) 18.4% *(d)* 12.5%

A person buys a watch for *X* 500 and sells it for *X* 300. Find his loss per cent.

(a) 30% *(b)* 40% *(c)* 35% *(d)* 45%

(e) None of the above

3. A salesman expects a gain of 13% on his cost price. If in a month, his sale was *X* 791000, what was his profit?

[SSC (10+2) 2012]

(a)X 91000 (b)X 97786

(c) X 85659 (d) X 88300

4. A gold bracelet is sold for X 14500 at a loss of 20%. What is the cost price of the gold bracelet? [SBI PO 2012]

(a) ? 18125 (b) X 17400

(c) X 15225 (d) X 16800

(e) None of the above

5. Find the SP when CP is X 40 and gain is 25%.

(a) X 50 (b) X 45

(c) X 60 (d) X 49

(e) None of the above

6. Anita purchased a bicycle at a cost of X 3200. She sold it at a loss of X 240. At what price did she sell the bicycle?

[Bank Clerks 2011] (a) X 2960 (b) X 2690

(C) X 3440 (d) X 3360

(e) None of the above

Find the CP when SP is X 400 and loss is

70%.

(a) ₹ $\frac{4000}{3}$ (b) ₹ $\frac{5000}{3}$

(c) ₹ $\frac{2000}{3}$ (d) ₹ $\frac{7000}{3}$

(e) None of the above

8. By selling a cycle for X 2345, a student loses 19%. His cost price is nearly

[CLAT2013]

(a) X 4000 *(b)* X 5000

(c) X 3000 *(d)* X 3500

(e) None of the above

9. A man loses 10% by selling an article for X 180. At what price should he sell it to gain 10%? [SSC FCI 2012]

(a)?220 *(b)*X 217.80

(c) X 200 *(d)* X 216

10. A calculator is bought for X 350 and sold at a gain of 15%. What will be the selling price of calculator (in ?)? [Hotel Mgmt. 2010]

(a) 385 *(b)* 375 *(c)* 472 *(d)* 402.50

(e) None of the above By selling a cellphone for X 2400, a shopkeeper make a profit of 25%. Then, his profit percentage, if he had sold it for X 2040, is [SSC (10+2) 2012]

(a) 10% *(b)* 6.25% *(C)* 6.5% *(d)* 15% 12. If the cost price is 95% of the selling price, what is the profit per cent?

[SSC CPO2011]

(a) 4 *(b)* 4.75 *(c)* 5 *(d)* 5.26

The owner of a cell phone shop charges his customer 28% more than the cost price. If the customer paid X 8960 for the cell phone, what was the cost price of the cell phone? [Bank Clerks 2009]

(a) X 7800 *(b)* X 7000

(c) X 6900 *(d)* X 6850

(e) None of the above

14. Selling price of an article is X 2220 and the per cent profit earned is 20%. What is the cost price of the article?

[Bank Clerks 2009] *(a)* X 1750 *(b)* X 1876

(C) X 1776 *(d)* X 1850

(e) None of the above

15. By selling an article for *X* 720, a man loses 10%. At what price should he sell it to gain 5%?

(a) X 840 *(b)* X 890 *(c)* X 1000 *(d)* X 995 *(e)* None of the above

16. Rajdeep loses 20% by selling a radio for *X* 768. What per cent will he gain by selling it for *X* 1020?

(a) 7.25% *(b)* 5.25%

(c) 6.25% *(d)* 8.25%

(e) None of the above

17. If a watch is sold for *X* 120, there is a loss of 15%. For a profit of 2%, the watch is to be sold for

(a) X 144 *(b)* X 175

(c) X 185 *(d)* X 165

(e) None of the above

18. Rajan sold an article for *X* 6000 at a loss of 25%. Find the cost price. [Bank Clerks 2010]

(a) X 7500 *(b)* X 7200

(c) X 8000 *(d)* X 8500

(e) None of the above

19. A trader buys some goods for *X* 150. If the overhead expenses be 12% of the cost price, at what price should it be sold to earn 10%? [Railways 2007]

(a) X 184.80 *(b) X* 185.80

(c) X 187.8 *(d) X* 188.80

20. A person sold an article for X 3600 and got a profit of 20%. Had he sold the article for X 3150, how much profit would he have got? [CDS 2013]

(a) 4% *(b)* 5% *(c)* 6% *(d)* 10%

21. Meera purchased 23 bracelets at the rate of X 160 per bracelet. At what rate per bracelet should she sell the bracelets so that profit earned is 15%? [SBI Clerk 2012]

(a) X 184 *(b)* X 186 *(c)* X 192 *(d)* X 198 *(e)* None of the above

22. Neeta got profit of 10% on selling an article in X 220. To get the profit of 30%, she should sell the article in how many rupees? [SSCCGL2012]

(a) X 220 *(b)* X 230 *(c)* X 260 *(d)* X 280

23. Niraj incurred a loss of 55% on selling an article for X 9549. What was the cost price of the article? [SSC CCL (Main) 2012]

(a) X 27700 *(b)* X 25600

(c) X 21220 *(d)* X 29000

24. Find the percentage loss when the cost price and selling price of an article are in the ratio of 5 : 3.

(a) 40% *(b)* 35% *(c)* 45% *(d)* 26% *(e)* None of the above

25. A shopkeeper sells eggs at 20 for a rupee gaining 30%. How many eggs did he buy for a rupee?

(a) 40 *(b)* 30 *(c)* 35 *(d)* 26 *(e)* None of the above

26. A man bought toffees at 6 for a rupee. How many toffees for a rupee must he sell to gain 50%?

(a) 5 *(b)* 2 *(c)* 4 *(d)* 6 *(e)* None of the above

27. An article is sold for X 300 at a profit of 20%. Had it been sold for X 235, the loss percentage would have been

[SSC CGL2013]

(a) 3% *(b)* 5% *(c)* 6% *(d)* 16%

28. The profit earned after selling a pair of shoes for *X* 2033 is the same as the loss incurred after selling the same pair of shoes for *X* 1063. What is the cost price of the shoes? [Bank Clerks 2009]

(a) X 1650 *(b) X* 1548 *(c) X* 1532

(d) Cannot be determined

(e) None of the above

29. A man sells an article at a profit of 40%. If he had bought it at 40% less and sold for *X* 5 less, he would have gained 50%. Find the cost price of the article. [MBA 2010]

(a) X 10 *(b) X* 15 *(c) X* 20 *(d) X* 30 *(e)* None of the above

30. Charu purchased a dinner set at 3/10 th of its selling price and sold it at 10% more than its CP. Find the gain per cent.

[Hotel Mgmt. 2008] *(a)* 15% *(b)* 5% *(c)* 9% *(d)* 10% *(e)* None of the above

31. The owner of a furniture shop charges his customer 18% more than the CP. If a customer paid *X* 10207 for a dining table, then find its original price. [LIC ADO 2010]

(a) X 9240 *(b) X* 8650

(c) X 9840 *(d) X* 7670

(e) None of the above Meena purchased two fans each at *X* 1200. She sold one fan at the loss of 5% and other at the gain of 10%. Find total gain or loss per cent. [Bank Clerks 2009]

(a) 1.2% loss *(b)* 1.2% profit

(c) 2.5% profit *(d)* 2.5% loss

(e) None of the above

33. A woman bought eggs at *X* 30 per dozen. The selling price per hundred so as to gain 12% will be (in *X)*

(a) 280 *(b)* 250 *(c)* 300 *(d)* 360 *(e)* None of the above

34. A man purchases a certain number of oranges at 4 a rupee and the same number of oranges at 5 a rupee. He mixes them together and sells them at 4 a rupee. What is his gain or loss per cent?

(a) $11\frac{2}{9}\%$ gain (b) $11\frac{2}{9}\%$ loss

(c) $11\frac{1}{9}\%$ gain (d) $11\frac{1}{9}\%$ loss

35. A man gains 10% by selling an article for a certain price. If he sells it at double the price, then the profit made is

[SSCCGL2013]

(a) 120% *(b)* 20%

(c) 40% *(d)* 100%

36. A man sold an article for *X* 322, gaining l/6th of his outlay. Find the cost price of the article.

(a) X 300 *(b) X* 376

(c) X 175 *(d) X* 276

(e) None of the above

37. The cost price of an item is two-third of its selling price. What is the gain or loss per cent on that item? [Bank Clerks 2010]

(a) 45 *(b)* 50

(c) 35 *(d)* 54

(e) None of the above

38. The difference between the CP and SP of an article is *X* 240. If the profit is 20%, the selling price is [SSC (10+2)2011]

(a) X 1440 *(b) X* 1400

(c) X 1240 *(d) X* 1200

39. Pankaj purchased an item for X 7500 and sold it at the gain of 24%. From that amount he purchased another item and sold it at the loss of 20%. What is his over all gain/loss? [Bank Clerks 2011]

(a) Loss of X 140

(b) Gain of X 60

(c) Loss of X 60

(d) Neither gain nor loss

(e) None of the above

40. Sumit purchased an item for X 4000 and sold it at the gain of 35%. From that amount, he purchased another item and sold it at the loss of 20%. What is his over all gain/loss? [Bank Clerks 2011]

(a) Loss of X 340

(b) Gain of X 320

(c) Loss of X 360

(d) Neither gain nor loss

(e) None of the above

41. A man purchases a certain number of toffees at 6 per rupee and the same number of toffees at 7 per rupee. He mixes the toffees and sells them at 6 per rupee. What is his gain or loss per cent?

(a) $6\frac{9}{13}$% loss *(b)* $7\frac{9}{13}$% gain

(c) $7\frac{6}{13}$% loss *(d)* $7\frac{5}{13}$% gain

(e) None of the above

42. Raj sold an item for X 6384 and incurred a loss of 30%. At what price should he have sold the item to have gained a profit of 30% [SBI Clerk 2011]

(a) X 14656

(b) X 11856

(c) X 13544

(d) Cannot be determined

(e) None of the above

43. The price of a land passing through three hands, rises on the whole by 65%. If the first and second sellers earned 20% and 25% profit, respectively. Find the profit earned by the third seller. [SSC (10+2) 2007]

(a) 20% (b) 55% (c) 10% (d) 25%

44. If the difference between the selling prices of an article at profit of 6% and 4% is X 3, then the cost price of the article should be [SSC (10+2)2010]

(a) X 100 (b) X 150

(c) X 175 (d) ?200

45. A man sells calculator at the rate of X 250 each which includes a profit of 14%. What amount of profit will he earn in 19 days, if he sells seven calculators per day? [IDBISO2012]

(a) X 4665 (b) X 4565

(c) X 4545 (d) X 4655

(e) None of the above

46. A fruit seller buys 700 oranges at the rate of X 500 for 100 oranges and another variety of 500 oranges at the rate of X 700 for 100 oranges and sells them at X 84 per dozen. The profit per cent is

[SSC Multitasking 2014]

(a) 20% (b) 40%

(c) 30% (d) 10%

47. A person sold his watch for X 75 and got a percentage profit equal to the cost price. The cost price of the watch is

[SSC Multitasking 2014]

(a) X 40 *(b)* X 45 *(c)* X 50 *(d)* X 55

48. Two lots of onions with equal quantity, one costing X 10 per kg and the other costing X 15 per kg, are mixed together and whole lot is sold at X 15 per kg. What is the profit or loss? [CDS 2013]

(a) 10% loss *(b)* 10% profit

(c) 20% profit *(d)* 20% loss

49. A fruit-seller buys lemons at 2 for a rupee and sells them at 5 for three rupees What is his gain per cent? [CDS 2011]

(a) 10% *(b)* 15% *(c)* 20% *(d)* 25%

50. A vender sells lemons at the rate of 5 for ? 14, gaining thereby 40%. For how much did he buy a dozen lemon? [SSC CGL 2010]

(a) ? 20 *(b)* ? 21 *(c)* ? 24 *(d)* ? 28

51. By selling 32 oranges for ? 30 a man loses 25%. How many oranges should be sold for ? 24 so as to gain 20% in the transaction?

(a) 16 *(b)* 24 *(c)* 32 *(d)* 28

(e) None of the above 52.Kamlesh purchased 120 reams of paper at ? 100 per ream and the expenditure on transport was ? 480. He had to pay an octroi duty of 50 paise per ream and the coolie charges were ? 60. What should he charge per ream to gain 40%?

(a) X 155 *(b)* X 147 *(c)* X 138(d) X 165

(e) None of the above

53. A producer of tea blends two varieties of tea from tea gardens one costing ? 18 per kg and another f 20 per kg in the ratio 5 : 3. If he sells the blended variety at X 21 per kg, then his gain percentage is

[SSCFCI 2012] *(a)* 18 *(b)* 8 *(c)* 10 *(d)* 12

54. A dealer sold three-fourth of his articles at a gain of 24% and the remaining at the cost price. Percentage of gain in the whole transaction is [SSC CPO 2011]

(a) 15 *(b)* 18 *(C)* 24 *(d)* 32

55. A merchant buys *p* apples for *X q* and sells *q* apples for ? *p.* If $p < q$, then in the whole outlay, he makes [SSC CGL 2007]

(a) $10\left(\frac{p^2+q^2}{q^2}\right)$% gain

(b) $100\left(\frac{q^2-p^2}{q^2}\right)$% loss

(c) $100\left(\frac{p^2+q^2}{p^2}\right)$% loss

(d) $100\left(\frac{q^2-p^2}{p^2}\right)$% gain

56. Fens are bought at 5 for ? 4 and sold at 4 for *X* 5. Find the gain per cent.

(a) 55% *(b)* 45.26%

(c) 56.25% *(d)* 60%

(e) None of the above

57. A person purchased 11 articles for ? 10 and at 10 articles for ? 11. Find the gain percentage. [SSC CGL (Main) 2012]

(a) 22 *(b)* 20 *(c)* 1 *(d)* 21

58. The selling price of 20 articles is equal to the cost price of 22 articles. The gain percentage is [SSC CGL 2013]

(a) 12% *(b)* 9% *(c)* 10% (dj 11%

59. By selling 100 pens, a shopkeeper gains the selling price of 40 pens. Find his gain per cent.

(a) 4.47%*(b)* 6.67% (cj 8.8% *(d)* 5.59% *(e)* None of the above

60. If the cost price of 16 tables be equal to the selling price of 12 tables, the gain per cent is [SSC CPO 2013]

(a) $33\frac{1}{3}$% (b) 20% (c) 30% (d) 15%

61. If the cost price of 23 toys is equal to selling price of 20 toys, then the gain or loss percentage is [SSC (10+2) 2012]

(a) 12 (b) 14 (c) 15 (d) $12\frac{1}{2}$

62. The cost price of 24 apples is same as the selling price of 18 apples. The percentage of gain is [SSCFCI 2012]

(a) $12\frac{1}{2}$% (b) $14\frac{2}{3}$% (c) $16\frac{2}{3}$% (d) $33\frac{1}{3}$%

63. If the cost price of 16 pens is equal to the selling price of 12 pens, then the gain or loss per cent is [SSC Multitasking 2014]

(a) $33\frac{1}{3}$% gain (b) 25% gain

(c) 25% loss (d) $33\frac{1}{3}$% loss

64. If the cost price and selling price of an article are in the ratio of 10 : 11, then the percentage profit is [SSC CGL 2010]

(a) 10% *(b)* 9% *(c)* 3% *(d)* 1%

65. The cost price and selling price of an article are in the ratio of 3 : 7. If the selling price is ? 700, then find the cost price

(a) ? 500 *(b)* ? 400 *(c)* ? 300 *(d)* ? 800

66. A dishonest dealer sells his goods at 10% loss on cost price but uses 20% less weight. What is his profit or loss per cent?

(a) 12% loss *(b)* 22.5% gain

(c) 13.9% loss *(d)* 12.5% gain *(e)* None of the above

67. A man sells rice at 10% profit and uses weight 30% less than the actual measure. His gain per cent is

(a) $57\frac{2}{8}\%$ (b) $57\frac{1}{7}\%$

(c) $57\frac{2}{5}\%$ (d) $57\frac{3}{7}\%$

(e) None of the above

68. A seller uses 920 g in place of one kg to sell his goods. When he sells his article at 15% gain on cost price, the actual percentage of profit is

[SSCCGL (Main) 2012]

(a) 20% *(b)* 15% *(c)* 25% *(d)* 30%

69. A trader sells wheat at 20% profit and uses weight 20% less than the actual measure. His gain per cent is

(a) 35% *(b)* 38% *(c)* 48% *(d)* 50% *(e)* None of the above

70. A dishonest dealer sells his goods at 10% loss on cost price and uses 30% less weight. What is his profit or loss per cent?

(a) $28\frac{4}{7}\%$ loss (b) $28\frac{3}{7}\%$ gain

(c) $28\frac{3}{7}\%$ loss (d) $28\frac{4}{7}\%$ gain

(e) None of the above

71. A dishonest dealer professes to sell his goods at cost price but he uses a weight of 920 g for 1 kg weight. Find his gain per cent.

(a) $7\frac{16}{23}\%$ (b) $8\frac{16}{23}\%$

(c) $5\frac{16}{23}\%$ (d) $3\frac{16}{23}\%$

(e) None of the above

72. A dealer sells goods at 4% loss on cost price but he uses 28 g instead of 32 g. What is his per cent profit or loss?

(a) $9\frac{5}{7}\%$ gain (b) $14\frac{3}{7}\%$ loss

(c) $16\frac{3}{7}\%$ gain (d) $16\frac{3}{7}\%$ loss

(e) None of the above

73. If the cost price of 20 articles is equal to the selling price of 15 articles, find the profit per cent. [Bank PO 2008]

(a) $33\frac{1}{2}\%$ (b) $33\frac{1}{3}\%$

(c) $33\frac{1}{5}\%$ (d) $33\frac{1}{7}\%$

(e) None of the above

74. A furniture seller sells two tables at ? 1500 each. He earned a profit of 20% on one table and suffered a loss of 20% on the another table. Net profit or loss in this deal is [CC PSC 2013]

(a) 4% loss

(b) 4% profit

(c) Neither profit nor loss

(d) 10% loss

(e) 10% profit

75. A dishonest dealer sells articles at 10% loss on cost price but uses the weight of 16 g instead of 18 g. What is his profit or loss per cent?

(a) $1\frac{1}{4}\%$ gain (b) $1\frac{1}{4}\%$ loss

(c) $3\frac{1}{4}\%$ loss (d) $6\frac{1}{4}\%$ gain

(e) None of the above

76. A man sold two watches, each for *X* 495. If he gained 10% on one watch and suffered a loss of 10% on the other, then what is the loss or gain percentage in the transaction? [CDS 2011]

(a) 1% gain

(b) 1% loss

(c) 100/99% loss

(d) Neither gain nor loss

77. If an article is sold at a gain of 6% instead of at a loss of 6%, then the seller gets ? 6 more. The cost price of the article is [SSC CCL 2013]

(a) ? 106 *(b)* ? 50 *(c)* *f* 94 *(d)* *X* 100

78. Some apples are bought at 5 for ? 10 and sold at 6 for ? 15. What is the gain per cent?

(a) 35% *(b)* 45% *(c)* 20% *(d)* 25%

(e) None of the above

79. A man sold two houses for ? 96000 each. In the sale of the first house, he incurred 20% profit and in the sale of the second, he incurred 20% loss, what is the gain or loss percentage in total? [SSC CCL (Main) 2012]

(a) 6% gain *(b)* 6% loss

(c) 4% gain *(d)* 4% loss

80. The profit earned after selling an article for ? 625 is same as the loss incurred after selling the article for *X* 435. The cost price of the article is [SSC (10+2)2012]

(a) ? 520 *(b)* ? 530 *(c)* *X* 540 *(d)* ? 550

81. A shopkeeper purchased some books from a publication worth *X* 750. Because of some reasons, he had to sell two-fifth part of the book at a loss of 15%. On which gain he should sell his rest of the books, so that he gets neither gain nor loss?

{a) 10% *(b)* 9% *(c)* 12% *(d)* 15% *(e)* 18%

82. A sold a watch to *B* at 40% gain and *B* sold it to *C* at a loss of 20%. If C bought the watch for ? 432, at what price did *A* purchase it?

(a) *X* 385.71 *(b)* *X* 216

(c) ? 250 *(d)* ? 550

83. A man sold an article at a loss of 20%. If he sells the article for *X* 12 more, he would have gained 10%. The cost price of the article is

(a) X 60 *(b)* X 40

(c) X 30 *(d)* X 22

(e) None of the above

84. A person sold a watch at a profit of 10%. If he had sold it for *X* 2000 more, he would have gained 20%. Find the CP of watch.

(a) X 15000 *(b) X* 10000

(c) X 20000 *(d) X* 25000

85. A person sold an article for *X* 136 and got

15% loss. Had he sold it for *X x,* he would

have got a profit of 15%. Which one of

the following is correct? [CDS 2012]

(a) $190<x<200$

(b) $180< x<190$

(c) $170<x<180$

(d) $160< x<170$

Exercise © *Higher Skill Level Questions*

1. An article passing through two hands is sold at a profit of 40% at the original cost price. If the 1st dealer makes a profit of 20%, then the profit per cent made by the second is

(a) $15\frac{2}{3}\%$ *(b)* $16\frac{2}{3}\%$

(c) $13\frac{2}{3}\%$ *(d)* $11\frac{2}{3}\%$

(e) None of the above

2. A merchant fixed the selling price of his articles at *X* 700 after adding 40% profit to the cost price. As the sale was very low at this price level, he decided to fix the selling price at 10% profit. Find the new selling price [SSC (10+2) 2012]

(a) X 450 *(b) X* 490 *(C) X* 500 *(d) X* 550 A trader purchases a watch and a wall clock for *X* 390. He sells them making a profit of 10% on the watch and 15% on the wall clock. He earns a profit of *X* 51.50. The difference between the original prices of the wall clock and the watch is equal to [SSC (10 + 2) 2012]

(a) X 110 *(b) X* 100 *(c) X* 80 *(d) X* 120

4. A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is

[SSC CGL2010]

(a) 500 kg *(b)* 600 kg

(c) 400 kg *(d)* 640 kg

5. A dealer buys an article marked at *X* 25000 with 20% and 5% off. He spends *X* 2000 on its repair and sells it for *X* 25000. What is his gain or loss percent? [SSCFCI 2012]

(a) 21% loss *(b)* 10.50% loss

(c) 19.05% gain *(d)* 25% gain

6. By selling an umbrella for *X* 30, a shopkeeper gains 20%. During a clearance sale, the shopkeeper allows a discount of 10% of the marked price. His gain percentage during the sale season is [SSC CGL (Main) 2012]

(a) 7 *(b)* 7.5 *(C)* 8 *(d)* 9

7. On selling an article at *X* 530, the gain is 20% more than the loss incurred on selling it at *X* 475. In order to gain 20%, the selling price will be

(a) X 900 *(b) X* 600

(c) X 700 *(d) X* 500

(e) None of the above

8. A person bought two bicycles for X 1600 and sold the first at 10% profit and the second at 20% profit. If he sold the first at 20% profit and the second at 10% profit, he would get X 5 more. The difference in the cost price of the two bicycles was [SSC CGL 2013]

(a) X 25 *(b)* X 75

(c) X 50 *(d)* X 40

9. A person sold a table at a gain of 15%. Had he bought it for 25% less and sold it for X 60 less, he would have made a profit of 32%. The cost price of table was

[SSC (10+2) 2013] *(a)* X 300 *(b)* X 350

(c) X 375 *(d)* X 400

10. Cost of a packet of coffee powder and a litre of milk are X 20 and X 30, respectively. 10 cups of coffee is made with one packet coffee powder and for each cup 200 mL of milk is used. If coffee is sold at 25% profit, the selling price of each cup of coffee is [SSC CPO 2013]

fa; r 12.50 *(b)*X 6.25

*(c) X8 (d)*X 10

11. Assuming that profit of a shopkeeper in a particular commodity is a linear expression of transportation charge *(t)* and the quantity of commodity *(q)*. He earns a profit of X 10000 by selling 20 units at the transport charge of X 400. He also earns a profit of X 12000 by selling 25 units at the transport charge of X 600. What is the linear expression in tandg? [CDS 2012]

(a) 600q - 5f *(b)* 500q - *4t*

(c) e00q - 4f *(d)* 500q - 5f

12. A person bought 8 quintal of rice for certain rupees. After a week, he sold 3 quintal of rice at 10% profit, 3 quintal of rice with neither profit nor loss and 2 quintal at 5% loss. In this transaction, what is the profit? [CDS 2012]

(a) 10% *(b)* 20%

(c) 25% *(d)* None of these

13. A bookseller sells a book at a gain of 10%. If he had bought it at 4% less and sold it for X 6 more, he would have

gained 118-%. The CP of the book is

4 [SSC CGL 2007]

(a) X 130 *(b) X* 140 *(c) X* 150fdJ *X* 160

14. A dealer bought 80 cricket bats for *X* 50 each. He sells 20 of them at a gain of 5%. What must be the gain percentage of the remaining bats, so as to get 10% gain on the whole? [SSC (10+2) 2012]

(a) $3\frac{2}{11}\%$ (b) $12\frac{1}{2}\%$

(c) $11\frac{2}{3}\%$ (d) ₹ 3350

15. A merchant earns a profit of 20% by selling a basket containing 80 apples whose cost is *X* 240 but he gives one-fourth of it to his friend at cost price and sells the remaining apples. In order to earn the same profit, at what price must he sell each apple? [CDS 2012]

(a) X 3.00 *(b) X* 3.60

(c) X 3.80 *(d) X* 4.80

16. A shopkeeper sells a transistors at 15% above its cost price. If he had bought it at 5% more than what he paid for it and sold it for *X* 6 more, he would have gained 10%. The cost price of the transistor is [SSC CGL (Main) 2012]

(a)X 800 (b)? 1000

(c) X 1200 *(d) X* 1400

Answer with Solutions

Exercise © *Base Level Questions*

1. (*d*) Given that, CP = ₹ 200 and SP = ₹ 225

Profit = SP − CP = 225 − 200 = ₹ 25

$\therefore \quad \text{Gain }\% = \dfrac{\text{Profit}}{\text{CP}} \times 100\%$

$= \dfrac{25}{200} \times 100\% = 12.5\%$

2. (*b*) Given that, CP = ₹ 500 and SP = ₹ 300

Loss = CP − SP = 500 − 300 = ₹ 200

$\therefore \quad \text{Loss }\% = \dfrac{\text{Loss}}{\text{CP}} \times 100\%$

$= \dfrac{200}{500} \times 100\% = 40\%$

3. (*a*) Given, total sale = ₹ 791000

$\therefore$ Cost price $= \dfrac{791000 \times 100}{113} =$ ₹ 700000

$\therefore$ Required profit

= 791000 − 700000 = ₹ 91000

4. (*a*) CP of bracelet $= \left(\dfrac{100}{100 - \text{Loss}\%}\right) \times \text{SP}$

$= \dfrac{100}{80} \times 14500 =$ ₹ 18125

5. (*a*) SP = 125% of CP

$\left[\because \text{SP} = \left(\dfrac{100 + \text{Gain }\%}{100}\right) \times \text{CP}\right]$

$= \dfrac{125}{100} \times 40 = \dfrac{5}{4} \times 40 = 5 \times 10 =$ ₹ 50

6. (*a*) We know that,

Loss = CP − SP

240 = 3200 − SP

$\therefore$ SP = 3200 − 240 = ₹ 2960

7. (*a*) $\text{CP} = \dfrac{100}{30} \times 400 = ₹\ \dfrac{4000}{3}$

$\left[\because \text{CP} = \left(\dfrac{100}{100 - \text{Loss}\%}\right) \times \text{SP}\right]$

8. (*c*) Let cost price be ₹ x.

Selling price = 2345

Loss = 19%

Cost price, $x = \dfrac{\text{SP} \times 100}{(100 - \text{Loss}\%)}$

$= \dfrac{2345 \times 100}{(100 - 19\%)}$

$\Rightarrow \quad x = \dfrac{234500}{81} = ₹\ 2895 \approx ₹\ 3000$

9. (*a*) CP of the article $= 180 \times \frac{100}{90} = ₹\ 200$

To gain 10%, SP of article

$= 110\%$ of $200 = ₹\ 220$

10. (*d*) Given, CP = ₹ 350 and gain = 15%

$\therefore \quad SP = \left(\frac{100 + \text{Gain }\%}{100}\right) \times CP$

$= \frac{115}{100} \times 350 = \frac{115 \times 7}{2} = ₹\ 402.50$

11. (*b*) SP of cellphone = ₹ 2400

$\therefore$ CP of cellphone $= 2400 \times \frac{100}{100 + 25}$

$= ₹\ 1920$

$\therefore$ Required percentage profit

$= \frac{2040 - 1920}{1920} \times 100 = 6.25\%$

12. (*d*) Let SP = ₹ 100

$\therefore \quad CP = ₹\ 95$

Profit $= 100 - 95 = ₹\ 5$

$\therefore$ Profit $\% = \frac{\text{Profit}}{CP} \times 100\%$

$= \frac{5}{95} \times 100\% = \frac{100}{19}\% = 5.26\%$

13. (*b*) $CP = \left(\frac{100}{100 + \text{Gain}\%}\right) \times SP$

$= 8960 \times \frac{100}{128} = 70 \times 100 = ₹\ 7000$

14. (*d*) $CP = \frac{100}{120} \times 2220 = ₹\ 1850$

15. (*a*) SP = ₹ 720 and loss = 10%

$\therefore \quad CP = \frac{100}{90} \times 720 = ₹\ 800$

Now, CP = ₹ 800 and gain = 5%

$\therefore \quad SP = \frac{105}{100} \times 800 = ₹\ 840$

16. (*c*) SP = ₹ 768 and loss = 20%

$\therefore \quad CP = \frac{100}{80} \times 768 = \frac{5}{4} \times 768 = ₹\ 960$

Now, CP = ₹ 960 and SP = ₹ 1020

Gain $= 1020 - 960 = ₹\ 60$

$\therefore$ Gain% $= \frac{60}{960} \times 100\% = 6.25\%$

17. (*a*) SP = ₹ 120 and loss = 15%

$\therefore \quad CP = \frac{100}{85} \times 120 = \frac{2400}{17},$

Gain = 2%

$\therefore$ Required SP $= \frac{102}{100} \times \frac{2400}{17} =$ ₹ 144

18. (*c*) $CP = \left(\frac{100}{100 - \text{loss\%}}\right) \times SP$

$CP = 6000 \times \frac{100}{75} =$ ₹ 8000

19. (*a*) Total CP = (CP + Overhead expenses)

= 150 + 12% of 150

$= 150 + \frac{12}{100} \times 150 =$ ₹ 168

Given that, gain = 10%

$\therefore \quad SP = \frac{110}{100} \times 168 =$ ₹ 184.80

20. (*b*) Let the cost price of the article be ₹ x.

Now, $\quad x + \frac{20x}{100} = 3600$

$\Rightarrow \quad \frac{120x}{100} = 3600$

$\therefore \quad x = 3000$

Now, profit percentage when the article is sold for ₹ 3150 $= \frac{3150 - 3000}{3000} \times 100$

$= \frac{150}{3000} \times 100 = 5\%$

21. (*a*) Cost price of one bracelet = ₹ 160

Profit earned = 15%

$\therefore$ Selling price of one bracelet

= Cost price + Profit earned

$= 160 + 160 \times \frac{15}{100}$

$= 160 + \frac{16 \times 15}{10} = 160 + \frac{240}{10}$

= 160 + 24 = 184 = ₹ 184

Hence, Meera should sale her bracelet ₹ 184 per piece.

22. (*c*) Cost price of an article

$= 220 \times \frac{100}{110} =$ ₹ 200

For getting the profit of 30%, selling price of an article = 30% of ₹ 200

$= \frac{130}{100} \times 200 =$ ₹ 260

23. (*c*) Let cost price of article is CP.

According to the formula,

$CP = \frac{SP}{100 - \text{Loss\%}} \times 100$

$CP = \frac{100 \times 9549}{100 - 55} =$ ₹ 21220

24. (*a*) Let CP = $5x$ and SP = $3x$

$$\text{Loss} = (\text{CP} - \text{SP}) = (5x - 3x) = 2x$$

$$\therefore \quad \text{Loss \%} = \frac{2x}{5x} \times 100\% = 40\%$$

25. (*d*) SP of 20 eggs = ₹ 1 and gain = 30%

$$\text{CP of 20 eggs} = \left(1 \times \frac{100}{130}\right) = ₹\,\frac{10}{13}$$

Now, ₹ $\frac{10}{13}$ yields 20 eggs.

$$\therefore ₹\,1 \text{ will yield } \left(20 \times \frac{13}{10}\right) = 26 \text{ eggs}$$

Clearly, 26 eggs were bought for ₹ 1.

26. (*c*) CP of 6 toffees = ₹ 1,

$$\text{SP of 6 toffees} = 150\% \text{ of } 1 = \frac{150}{100} = \frac{3}{2}$$

$\because$ For ₹ $\frac{3}{2}$, the number of toffees sold = 6

$\therefore$ For ₹ 1, the number of toffees sold

$$= \left(6 \times \frac{2}{3}\right) = 4$$

27. (*c*) Let the cost price of article = ₹ x

Then, 120% of $x = 300$

$$\frac{120 \times x}{100} = 300$$

$$\therefore \quad x = \frac{300 \times 100}{120} = ₹\,250$$

Now, SP = 235

Then, loss percentage

$$= \frac{250 - 235}{250} \times 100 = 6\%$$

28. (*b*) Let the cost price of the shoes = ₹ x

According to the question,

$$2033 - x = x - 1063$$

$$\Rightarrow \quad 2x = 2033 + 1063 = 3096$$

$$\therefore \quad x = \frac{3096}{2} = ₹\,1548$$

29. (*a*) Let CP = x

$$\text{Then,} \quad \text{SP} = \frac{140x}{100} = ₹\,\frac{7x}{5}$$

$$\text{New} \quad \text{CP} = 60\% \text{ of } x = \frac{60x}{100} = \frac{3x}{5},$$

Gain = 50%

$$\text{New} \quad \text{SP} = 150\% \text{ of } \frac{3x}{5} = \frac{150}{100} \times \frac{3x}{5} = \frac{9x}{10}$$

According to the question,

$$\frac{7x}{5} - \frac{9x}{10} = 5 \Rightarrow \frac{14x - 9x}{10} = 5$$

$$\Rightarrow \quad \frac{5x}{10} = 5$$

$\Rightarrow \quad \frac{1}{2}x = 5$

$\therefore \quad x = ₹\ 10$

Alternate Method

Let CP = ₹ 100, then SP = 140

New CP = ₹ 60 and new SP = $60 \times \frac{150}{100}$

$= ₹\ 90$

According to the question,

₹ (140 − 90) = ₹ 50 is equivalent to ₹ 5.

∴ ₹ 100 is equivalent to ₹ 10.

∴ CP = ₹ 10

30. (*d*) Let SP = ₹ 100

$\therefore \quad CP = \frac{3}{10} \times 100 = ₹\ 30$

New $SP = 30 \times \frac{110}{100} = ₹\ 33$

Gain = 33 − 30 = ₹ 3

$\therefore \quad \text{Gain \%} = \frac{3}{30} \times 100\% = \frac{300}{30}\% = 10\%$

31. (*b*) Original $CP = \left(\frac{100}{100 + \text{Gain\%}}\right) \times SP$

$= \frac{100}{118} \times 10207 = ₹\ 8650$

32. (*c*) Total CP = 2 × 1200 = ₹ 2400

SP at 5% loss $= \frac{95}{100} \times 1200 = ₹\ 1140$

SP at 10% gain $= \frac{110}{100} \times 1200$

$= ₹\ 1320$

Total SP = 1140 + 1320 = ₹ 2460

∴ Gain = 2460 − 2400 = ₹ 60

$\therefore \quad \text{Gain \%} = \frac{60}{2400} \times 100\% = 2.5\%$

33. (*a*) 12 eggs cost = ₹ 30

Then, cost price of 1 egg $= \frac{30}{12} = ₹\ 2.5$

∴ Cost price of 100 eggs

= 2.5 × 100 = ₹ 250

Now, let the SP of 100 eggs be ₹ x.

Then, $\frac{SP - CP}{CP} \times 100 = \text{Profit \%}$

$\frac{x - 250}{250} \times 100 = 12$

$\Rightarrow \quad \frac{(x - 250)}{5} \times 2 = 12$

$\Rightarrow \quad x - 250 = \frac{12 \times 5}{2} = 30$

$\therefore \quad x = 250 + 30 = ₹\ 280$

34. (*c*) CP of 1 orange for 1st rate = ₹ $\frac{1}{4}$

CP of 1 orange for 2nd rate = ₹ $\frac{1}{5}$

CP of 2 oranges after mixing

$$= \frac{1}{4} + \frac{1}{5} = ₹\ \frac{9}{20}$$

CP of 1 orange after mixing

$$= \frac{9}{20 \times 2} = ₹\ \frac{9}{40}$$

SP of 1 orange after mixing = ₹ $\frac{1}{4}$

$\therefore$ Gain $= \frac{1}{4} - \frac{9}{40} = ₹\ \frac{1}{40}$

$\therefore$ Gain % $= \left(\frac{1}{40} \times \frac{40}{9} \times 100\right)\%$

$$= \frac{100}{9}\% = 11\frac{1}{9}\%$$

35. (*a*) Let CP of the article be ₹ x.

Then, SP = 110% of $x = 1.1x$

If SP be double *i.e.*, $2.2x$, then

Profit per cent $= \frac{2.2x - x}{x} \times 100\% = 120\%$

36. (*d*) Let CP = x, then gain $= \frac{x}{6}$

According to the question,

$$x + \frac{x}{6} = 322$$

$\Rightarrow$ $\frac{7x}{6} = 322$

$\therefore$ $x = \frac{322 \times 6}{7} = ₹\ 276$

37. (*b*) Let SP = x, then CP $= \frac{2x}{3}$

Gain $= \left(x - \frac{2x}{3}\right) = \frac{x}{3}$

$\therefore$ Gain % $= \dfrac{\frac{x}{3}}{\frac{2x}{3}} \times 100\%$

$$= \frac{x}{3} \times \frac{3}{2x} \times 100\% = 50\%$$

38. (*a*) Given, gain% = 20

Difference between SP and CP = 240

Now, gain% $= \frac{SP - CP}{CP} \times 100$

$\Rightarrow$ $20 = \frac{240}{CP} \times 100$

$\Rightarrow$ CP = 240 × 5

$\Rightarrow$ CP = 1200

$\therefore$ SP = 1200 + 240 [$\because$ SP − CP = 240]

= 1440

39. (c) $CP_1 = ₹\ 7500$

$\therefore SP_1 = 7500 \times \frac{124}{100} = ₹\ 9300 = CP_2$

$SP_2 = 9300 \times \frac{80}{100} = ₹\ 7440 \Rightarrow CP_1 > SP_2$

Hence, loss is incurred in this transaction.

$\therefore$ Required loss $= (CP_1 - SP_2)$

$= 7500 - 7440 = ₹\ 60$

40. (b) $CP_1 = ₹\ 4000$

$SP_1 = 4000 \times \frac{135}{100} = ₹\ 5400 = CP_2$

$SP_2 = 5400 \times \frac{80}{100} = ₹\ 4320$

$CP_1 < SP_2$

Hence, profit is earned here.

$\therefore$ Required gain $= (SP_2 - CP_1)$

$= 4320 - 4000 = ₹\ 320$

41. (b) CP of 1 toffee for 1st rate $= \frac{1}{6}$

CP of 1 toffee for 2nd rate $= \frac{1}{7}$

CP of 2 toffees after mixing $= \frac{1}{6} + \frac{1}{7} = \frac{13}{42}$

CP of 1 toffee after mixing $= \frac{13}{84}$

SP of 1 toffee after mixing $= \frac{1}{6}$

$$\text{Gain} = \frac{1}{6} - \frac{13}{84} = \frac{1}{84}$$

$$\text{Gain\%} = \frac{\frac{1}{84}}{\frac{13}{84}} \times 100\% = \frac{100}{13}\% = 7\frac{9}{13}\%$$

42. (b) $CP = \frac{100}{70} \times 6384 = ₹\ 9120$

To gain 30%, SP = 130% of 9120

$$SP = \frac{130}{100} \times 9120 = ₹\ 11856$$

43. (c) Required profit

$$= (100 + 65) \times \frac{100}{120} \times \frac{100}{125} - 100$$

$$= 110 - 100 = 10\%$$

44. (b) Let $CP = x$

According to the question,

$$\frac{106x}{100} - \frac{104x}{100} = 3 \Rightarrow \frac{2x}{100} = 3$$

$\therefore \quad x = \frac{300}{2} = ₹\ 150$

Direct Approach

According to the question,

$$(6\% - 4\%) = 3 \Rightarrow 2\% = 3$$

$\Rightarrow \quad (2 \times 50)\% = 3 \times 50$

$\therefore \quad 100\% = ₹\ 150$

45. (*e*) Profit on one calculator

$$= 250\left(1 - \frac{100}{114}\right) = ₹\ 30.70$$

$\therefore$ Total amount of the profit

$$= 19 \times 7 \times 30.70$$

$$= ₹\ 4083.10$$

46. (*a*) CP of 700 oranges

$$= \frac{500}{100} \times 700 = ₹\ 3500$$

CP of 500 oranges of another variety

$$= \frac{700}{100} \times 500 = ₹\ 3500$$

Total CP = 3500 + 3500 = ₹ 7000

and total number of oranges purchased

$$= 700 + 500 = 1200$$

$$\text{Total SP} = \frac{84}{12} \times 1200 = ₹\ 8400$$

$$\therefore \text{Profit per cent} = \frac{8400 - 7000}{7000} \times 100\%$$

$$= 20\%$$

47. (*c*) Let CP of the watch be ₹ x. Then,

According to the question,

$$\frac{75 - x}{x} \times 100 = x$$

$$\Rightarrow \quad (75 - x) \times 100 = x^2$$

$$\Rightarrow \quad x^2 + 100x - 7500 = 0$$

$$\Rightarrow \quad x^2 + 150x - 50x - 7500 = 0$$

$$\Rightarrow \quad x(x + 150) - 50(x + 150) = 0$$

$$\Rightarrow \quad (x + 150)(x - 50) = 0$$

$$\Rightarrow \quad x = ₹\ 50$$

48. (*c*) Let each lot of onion-contains x kg onion, then total cost price of these two lots together

$$= 10x + 15x = 25x$$

Selling price of whole lot

$$= 15 \times (x + x) = 15 \times 2x = 30x$$

Profit percentage

$$= \frac{30x - 25x}{25x} \times 100\%$$

$$= \frac{5x}{25x} \times 100\% = 20\%$$

49. (*c*) Since, CP of 2 lemons is ₹ 1.

$$\therefore \text{CP of 1 lemon is} = \frac{1}{2} = ₹\ 0.5$$

SP of 5 lemons is ₹ 3.

$$\therefore \text{SP of 1 lemon is} = \frac{3}{5} = ₹\ 0.6$$

$$\therefore \text{Gain per cent} = \frac{0.6 - 0.5}{0.5} \times 100$$

$$= \frac{0.1 \times 100}{0.5}$$

$$= 20\%$$

50. (*c*) SP of 1 lemon = ₹ $\frac{14}{5}$

Let CP of 1 lemon = x

According to the question,

$$x \times \frac{140}{100} = \frac{14}{5}$$

$\therefore$ $x = ₹\ 2$

$\therefore$ CP for 12 lemons = 12 × 2 = ₹ 24

51. (*a*) Let the cost price be ₹ x.

SP of 1 orange = ₹ $\frac{30}{32}$ = ₹ $\frac{15}{16}$

According to the question,

$$\frac{75x}{100} = \frac{15}{16}$$

$\therefore$ $x = \frac{15 \times 100}{75 \times 16} = ₹\ \frac{5}{4}$

SP of 1 orange with 20% profit

$$= ₹\left(\frac{5}{4} \times \frac{120}{100}\right) = ₹\ \frac{3}{2}$$

$\therefore$ In ₹ $\frac{3}{2}$, the number of oranges sold = 1

$\therefore$ In ₹ 24, the number of oranges sold

$$= \frac{2}{3} \times 24 = 16$$

52. (*b*) Total cost price = ₹ (120 × 100) = ₹ 12000

Total expenditure

$$= 480 + \frac{1}{2} \times 120 + 60 = ₹\ 600$$

Total cost price = 12000 + 600 = ₹ 12600

Gain = 40%

SP of 120 reams = 12600 × $\frac{140}{100}$ = ₹ 17640

$\therefore$ SP per ream = $\frac{17640}{120}$ = ₹ 147

53. (*d*) Let quantities of two types of tea costing ₹ 18 per kg and ₹ 20 per kg be 5x kg and 3x kg, respectively.

Then, CP of tea = $18 \times 5x + 20 \times 3x$

$$= 90x + 60x = 150x$$

SP of tea = $21 \times 8x = 168x$

$$\text{Profit per cent} = \frac{168x - 150x}{150\ x} \times 100\%$$

$$= \frac{18}{150} \times 100\% = 12\%$$

54. (*b*) Let CP = ₹ 100

$\therefore$ $SP = 75 \times \frac{124}{100} + 25$

$$= 93 + 25 = ₹\ 118$$

Clearly, gain % = 18%

55. (*b*) Given,

$$\text{CP of } p \text{ apples} = q$$

$\therefore$ CP of 1 apple $= \dfrac{q}{p}$

and SP of q apples $= p$

$\therefore$ SP of 1 apple $= \dfrac{p}{q}$

Given, $p < q$ [CP > SP]

$\therefore$ Loss $= \dfrac{q}{p} - \dfrac{p}{q} = \dfrac{q^2 - p^2}{pq}$

and loss% $= \dfrac{\text{loss}}{\text{CP}} \times 100$

$$= \frac{\dfrac{q^2 - p^2}{pq}}{\dfrac{q}{p}} \times 100 = \frac{(q^2 - p^2)}{q^2} \times 100$$

Therefore, when $p < q$, then the person had a loss which is given by

$$100 \times \left(\frac{q^2 - p^2}{q^2}\right)\% \text{ loss.}$$

56. (*c*) Let the number of pens bought

$$= 5 \times 4 = 20$$

Then, CP $= \dfrac{4}{5} \times 20 = ₹\ 16$

and SP $= \dfrac{5}{4} \times 20 = ₹\ 25$

$\therefore$ Gain $= 25 - 16 = ₹\ 9$

Gain % $= \dfrac{9}{16} \times 100\% = 56.25\%$

Fast Track Method

Here, $m = 5$, $x = 4$, $n = 4$, $y = 5$

Profit% $= \dfrac{my - nx}{nx} \times 100$ [by Technique 5]

$$= \frac{5 \times 5 - 4 \times 4}{4 \times 4} \times 100$$

$$= \frac{9}{16} \times 100 = 56.25\%$$

57. (*d*) Cost price of 11 Articles = 10

Cost price of 1 Article $= \dfrac{10}{11}$

and SP of 10 Articles = 11

$\therefore$ SP of 1 Article $= \dfrac{11}{10}$

Now, gain $= \dfrac{11}{10} - \dfrac{10}{11}$

$$= \frac{121 - 100}{110} = \frac{21}{110}$$

and gain% $= \dfrac{\dfrac{21}{110}}{\dfrac{10}{11}} \times 100$

$$= \frac{21 \times 11}{11} = 21$$

58. (*c*) Here, $a = 22$ and $b = 20$, then

$\therefore$ Gain% $= \frac{a-b}{b} \times 100\%$ **[by Technique 4]**

$= \frac{22-20}{20} \times 100\% = 10\%$

59. (*e*) (SP of 100 pens) – (CP of 100 pens)

= SP of 40 pens

$\Rightarrow$ SP of 60 pens = CP of 100 pens

Let CP of each pen = ₹ 1

Then, CP of 60 pens = ₹ 60

SP of 60 pens = ₹ 100

Gain = 100 – 60 = ₹ 40

$\therefore$ Gain % $= \frac{40}{60} \times 100\% = 66.67\%$

60. (*a*) Here, $a = 16$, $b = 12$

Required gain per cent $= \left(\frac{a-b}{b}\right) \times 100$

[by Technique 4]

$= \frac{16-12}{12} \times 100\%$

$= \frac{1}{3} \times 100\% = 33\frac{1}{3}\%$

61. (*c*) Here, $a = 23$, $b = 20$

$\therefore$ Required percentage profit

$= \left(\frac{a-b}{b}\right) \times 100\%$ **[by Technique 4]**

$= \frac{23-20}{20} \times 100\% = 15\%$

62. (*d*) Given that, $a = 24$ and $b = 18$

According to the formula,

Gain % $= \left(\frac{a-b}{b} \times 100\right)\%$

[by Technique 4]

$= \left(\frac{24-18}{18} \times 100\right)\% = \left(\frac{6}{18} \times 100\right)\%$

$= \frac{100}{3}\% = 33\frac{1}{3}\%$

Direct Approach

Here, 6 apples are gained over 18 apples.

$\therefore$ Gain % $= \left(\frac{6}{18} \times 100\right)\% = 33\frac{1}{3}\%$

63. (*a*) Here, $a = 16$, $b = 12$

According to the formula,

Required per cent $= \frac{a-b}{b} \times 100$

[by Technique 4]

$= \frac{16-12}{12} \times 100$

$= \frac{4}{12} \times 100 = 33\frac{1}{3}\%$ (gain)

64. (*a*) Let CP = $10x$ and SP = $11x$

$$\text{Gain }\% = \frac{11x - 10x}{10x} \times 100\%$$

$$= \frac{1}{10} \times 100\% = 10\%$$

65. (*c*) Let CP = $3x$ and SP = $7x$

According to the question,

$$7x = 700$$

$$\therefore \quad x = 100$$

$$\therefore \quad \text{CP} = 3x = 3 \times 100 = ₹\,300$$

66. (*d*) Here, $a = 10\%$ and $b = 20\%$

According to the formula,

$$\text{Required answer} = \left[\frac{b \pm a}{100 - b} \times 100\right]\%$$

[by Technique 9]

$$= \frac{(20 - 10)}{(100 - 20)} \times 100\%$$

$$= \frac{10}{80} \times 100\% = 12.5\% \text{ gain}$$

+ve sign shows that profit is earned here.

67. (*b*) Let the marked weight = 1 kg = 1000 g

Real weight = 70% of 1000 = 700 g

Let CP of 1 g = ₹ 1

$\therefore$ CP of 700 g = ₹ 700

CP of 1000 g = ₹ 1000

$$\text{SP} = 110\% \text{ of } 1000$$

$$= \frac{110}{100} \times 1000 = ₹\,1100$$

$$\text{Gain} = 1100 - 700 = ₹\,400$$

$$\therefore \quad \text{Gain }\% = \frac{400}{700} \times 100\% = 57\frac{1}{7}\%$$

68. (*c*) Let cost price = ₹ 1000

$\therefore$ Actual cost price = ₹ 920

$$\text{Selling price} = \frac{1000 \times 115}{100} = ₹\,1150$$

$\therefore$ Actual profit percentage

$$= \frac{1150 - 920}{920} \times 100 = 25\%$$

69. (*d*) Let the marked weight = 1 kg

= 1000 g

Real weight = 80% of 1000 = 800 g

Let the CP of 1 g = ₹ 1

$\therefore$ CP of 800 g = ₹ 800,

CP of 1000 g = ₹ 1000

$$\text{SP} = 120\% \text{ of } 1000 = \frac{120}{100} \times 1000 = ₹\,1200$$

$$\text{Gain} = 1200 - 800 = ₹\,400$$

$$\therefore \text{ Gain }\% = \frac{400}{800} \times 100\% = 50\%$$

70. (*d*) Given that, $a = 10\%$ and $b = 30\%$

According to the formula,

Required per cent

$$= \frac{b-a}{100-b} \times 100 \quad \text{[by Technique 9]}$$

$$= \frac{30-10}{100-30} \times 100$$

$$= \frac{20}{70} \times 100 = \frac{200}{7}\% = 28\frac{4}{7}\% \text{ profit}$$

71. (*b*) According to the formula,

$$\text{Gain\%} = \left(\frac{\text{Error}}{\text{True value} - \text{Error}}\right) \times 100\%$$

[by Technique 1]

$$= \frac{80}{1000-80} \times 100\%$$

$$= \frac{80}{920} \times 100\% = 8\frac{16}{23}\%$$

Direct Approach

80 g is gained on 920 g.

$\therefore$ Gain % $= \frac{80}{920} \times 100\ \% = 8\ \frac{16}{23}\%$

72. (*a*) Given that, $a = 4\%$, $b = 28$ g and $c = 32$ g

Required profit [by Technique 8]

$$= \left[(100-a)\ \frac{c}{b} - 100\right]\%$$

$$= \left[(100-4)\ \frac{32}{28} - 100\right]\%$$

$$= \left[96 \times \frac{32}{28} - 100\right]\%$$

$$= \left[96 \times \frac{8}{7} - 100\right]\% = \left[\frac{768}{7} - 100\right]\%$$

$$= \left[\frac{768-700}{7}\right]\%$$

$$= \frac{68}{7}\% = 9\frac{5}{7}\% \text{ gain}$$

+ve sign shows that profit is earned here.

73. (*b*) Given that, $a = 20$ and $b = 15$

According to the formula,

$$\text{Profit \%} = \left(\frac{a-b}{b} \times 100\right)\%$$

[by Technique 4]

$$= \frac{5}{15} \times 100\% = \frac{100}{3}\%$$

$$= 33\frac{1}{3}\%$$

Direct Approach

5 articles are gained on selling 15 articles.

$$\therefore \text{Gain \%} = \left(\frac{5}{15} \times 100\right)\%$$

$$= 33\ \frac{1}{3}\%$$

74. (*a*) Required loss $= \left(\frac{a}{10}\right)^2 \%$

[by Technique 1]

$$= \left(\frac{20}{10}\right)^2 \% = 4\%$$

Thus, seller gets 4% loss in the deal.

75. (*a*) Given that, $a = 10\%$, $b = 16g$ and $c = 18g$

$= \left[(100 - a)\frac{c}{b} - 100\right]\%$ [by Techniques 8]

$= \left[(100 - 10)\frac{18}{16} - 100\right]\%$

$= \left[90 \times \frac{18}{16} - 100\right]\% = \left[90 \times \frac{9}{8} - 100\right]\%$

$= \left[\frac{810}{8} - 100\right]\% = \left[\frac{810 - 800}{8}\right]\%$

$= \frac{10}{8}\% = \frac{5}{4}\% = 1\frac{1}{4}\%$ gain

76. (*b*) Loss% $= \left(\frac{a}{10}\right)^2 \%$ [by Technique 1]

$$= \left(\frac{10}{10}\right)^2 \% = 1\%$$

77. (*b*) Let CP of the article be ₹ x, then 106% of $x - 94\%$ of $x = 6 \Rightarrow 12\%$ of $x = 6$

$\therefore \quad x = \frac{6 \times 100}{12} =$ ₹ 50

78. (*d*) Let number of apples bought $= 5 \times 6 = 30$ [LCM of 5 and 6]

$$\text{CP} = \frac{10}{5} \times 30 = ₹\ 60$$

$$\text{SP} = \frac{15}{6} \times 30 = ₹\ 75$$

$\therefore \quad$ Gain = SP − CP = 75 − 60 = ₹ 15

$\therefore \quad$ Gain % $= \left(\frac{15}{60} \times 100\right)\% = 25\%$

Fast Track Method

Here, $m = 5$, $x = 10$, $n = 6$, $y = 15$

Profit% $= \frac{my - nx}{nx} \times 100\%$

[by Technique 5]

$= \frac{(5 \times 15 - 6 \times 10)}{6 \times 10} \times 100\%$

$= \frac{75 - 60}{60} \times 100\% = \frac{15}{60} \times 100\% = 25\%$

79. (*d*) Here, $x = 20\%$

$\therefore$ Total loss percentage

$= \frac{a^2}{100}\%$ [by Technique 1]

$= \frac{20 \times 20}{100}$

$= 4\%$

80. (*b*) Let CP = x

According to the question,

$$625 - x = x - 435$$

$\Rightarrow \quad 2x = 1060$

$\Rightarrow \quad x = \frac{1060}{2} = ₹\, 530$

81. (*a*) Here, $a = \frac{2}{5}$, $x = 15\%$

According to the formula,

Gain % $= \frac{ax}{1-a}\,\%$ **[by Technique 2]**

$$= \frac{\frac{2}{5} \times 15}{1 - \frac{2}{5}}\,\% = \frac{6 \times 5}{3}\,\% = 10\%$$

82. (*a*) Let A purchased the watch for ₹ x.

$A \times \left(1 + \frac{r_1}{100}\right)\left(1 - \frac{r_2}{100}\right)$

= CP of article given by C

[by Technique 6]

$\Rightarrow \quad x \times \frac{140}{100} \times \frac{80}{100} = 432$

$\therefore \quad x = \frac{432 \times 100}{14 \times 8} = ₹\, 385.71$

83. (*b*) Let the CP = x

According to the question,

$$\frac{110x}{100} - \frac{80x}{100} = 12$$

$$\Rightarrow \quad \frac{11x}{10} - \frac{4x}{5} = 12$$

$$\Rightarrow \quad 11x - 8x = 120$$

$$\therefore \quad x = \frac{120}{3} = 40$$

$$\therefore \quad CP = ₹\ 40$$

84. (*c*) Here, $a = 10\%$, $b = 20\%$, $R = 2000$

According to the formula,

$$\text{CP of watch} = \frac{R}{b - a} \times 100$$

[by Technique 3]

$$= \frac{2000}{20 - 10} \times 100$$

$$= \frac{2000 \times 100}{10}$$

$$= ₹\ 20000$$

85. (*b*) $$\text{Cost price} = \frac{\text{Selling price} \times 100}{(100 - \text{loss\%})}$$

$$= \frac{136 \times 100}{100 - 15}$$

$$= \frac{136 \times 100}{85} = ₹\ 160$$

$$\text{Selling price } (x) = \frac{160 \times (100 + 15)}{100}$$

$$= \frac{160 \times 115}{100} = ₹\ 184$$

$\therefore$ Option (b) is correct because

$[180 < x < 190]$.

Exercise © *Higher Skill Level Questions*

1. (*b*) Let CP = ₹ 100

Then, SP = ₹ 140

Let the profit made by the 2nd dealer = x%

Then, $(100 + x)$% of 120% of ₹ 100 = ₹ 140

$$\Rightarrow \frac{100 + x}{100} \times \frac{120}{100} \times 100 = 140$$

$$\Rightarrow \quad 6(100 + x) = 700$$

$$\Rightarrow \quad 600 + 6x = 700$$

$$\Rightarrow \quad 6x = 100$$

$$\therefore \quad x = \frac{100}{6}\% = \frac{50}{3}\% = 16\frac{2}{3}\%$$

2. (*d*) Let cost price = ₹ x

According to the question,

$$x \times \frac{(100 + 40)}{100} = 700,$$

$$\Rightarrow \quad x = \frac{700 \times 100}{140} = 500$$

$$\therefore \quad \text{New selling price} = \frac{500 \times (100 + 10)}{100}$$

$$= 5 \times 110 = 550 = ₹\ 550$$

3. (*a*) Let cost price of a wall clock = ₹ x

∴ Cost price of a watch = ₹ $(390 - x)$

According to the question,

$$\frac{x \times 10}{100} + \frac{(390 - x) \times 15}{100} = 5150$$

$$\Rightarrow \quad 390 \times 15 - 5x = 5150 \times 100$$

$$\Rightarrow \quad 5x = 5850 - 5150 = 700$$

$$x = \frac{700}{5} = 140$$

$$\Rightarrow \quad x = 140$$

$$\therefore \text{ Required difference} = (390 - x) - x$$

$$= 390 - 2 \times 140$$

$$= 390 - 280 = 110$$

$$= ₹\ 110$$

4. (*b*) Let the sugar sold at 8% gain = x

∴ Sugar sold at 18% gain = $(1000 - x)$

Let CP of sugar = ₹ y per kg

Total CP = ₹ $1000y$

$\therefore \left(\frac{108}{100} \times xy\right) + \frac{118}{100}(1000 - x)y$

$$= \frac{114}{100} \times 1000y$$

$\Rightarrow 108xy + 118000y - 118xy = 114000y$

$\Rightarrow \quad 10x = 4000$

$\therefore \quad x = 400$

$\therefore$ Quantity sold at 18% profit

$= 1000 - 400 = 600$ kg

5. (*c*) Total CP = (95% of 80% of ₹ 25000) + (2000)

$$= \left(\frac{95}{100} \times \frac{80}{100} \times 25000\right) + 2000$$

$= 19000 + 2000 =$ ₹ 21000

$\therefore$ CP = ₹ 21000 and SP = ₹ 25000

$\therefore$ Gain = 25000 − 21000 = ₹ 4000

$\therefore$ Gain % $= \frac{4000}{21000} \times 100\% = \frac{400}{21}\%$

$= 19.05\%$ (approx.)

6. (*c*) Given, selling price of an umbrella = ₹ 30

Profit percentage = 20%

$\therefore$ Cost price of an umbrella

$= \frac{30 \times 100}{120} =$ ₹25

During the clearance sale, selling price of an umbrella

$= \frac{30 \times 90}{100} =$ ₹ 27

$\therefore$ Required profit percentage

$= \frac{27 - 25}{25} \times 100 = 8\%$

7. (*b*) Let CP = x

Then, according to the question,

$$\frac{120}{100}(x - 475) = (530 - x)$$

$\Rightarrow \quad \frac{6}{5}(x - 475) = (530 - x)$

$\Rightarrow \quad (6x - 2850) = (2650 - 5x)$

$\Rightarrow \quad 11x = 5500$

$\therefore \quad x =$ ₹ 500

$\therefore$ Required SP $= 500 \times \frac{120}{100} =$ ₹ 600

8. (*c*) Let the cost price of first bicycle be ₹ x.

Then, the cost price of second bicycle

= ₹ $(1600 - x)$

According to the given condition,

[20% of x + 10% of $(1600 - x)$] −

[10% of x + 20% of $(1600 - x)$] = 5

$\Rightarrow \quad \left[\frac{20 \times x}{100} + \frac{10 \times (1600 - x)}{100}\right]$

$$-\left[\frac{10 \times x}{100}+\frac{20 \times (1600-x)}{100}\right]=5$$

$$\Rightarrow \left(\frac{x}{5}+\frac{1600-x}{10}\right)-\left(\frac{x}{10}+\frac{1600-x}{5}\right)=5$$

$$\Rightarrow \quad \frac{x}{5}-\frac{x}{10}+\frac{(1600-x)}{10}-\frac{(1600-x)}{5}=5$$

$$\Rightarrow \quad \frac{2x-x}{10}+\frac{(1600-x)-2(1600-x)}{10}=5$$

$$\Rightarrow \quad \frac{x+1600-x-3200+2x}{10}=5$$

$$\Rightarrow \quad \frac{-1600+2x}{10}=5$$

$$\Rightarrow \quad 2x=1600+50$$

$$\therefore \quad x=\frac{1650}{2}=825$$

$\therefore$ Cost of second bicycle = (1600 − 825)

= ₹ 775

$\therefore$ Required difference = 825 − 775 = ₹ 50

9. (c) Let the cost price of table = ₹ x.

Then, selling price with 15% gain

$$=\frac{(100+\text{Gain \%}) \times \text{CP}}{100}$$

$$=\frac{(100+15)\% \times \text{CP}}{100}$$

$$=\frac{115 \times x}{100}=₹\,\frac{115x}{100}$$

$$\text{New CP}=\frac{(100-25\%) \times \text{CP}}{100}=₹\,\frac{75x}{100}$$

$$\text{New SP}=₹\left(\frac{115x}{100}-60\right)$$

Now, according to the question,

$$\frac{\left(\frac{115x}{100}-60\right)-\frac{75x}{100}}{\frac{75x}{100}} \times 100=32$$

$$\Rightarrow \quad \frac{\frac{115x-6000-75x}{100}}{\frac{75x}{100}} \times 100=32$$

$$\Rightarrow \quad \frac{40x-6000}{75x} \times 100=32$$

$$\Rightarrow \quad \frac{40x-6000}{3x} \times 4=32$$

$$\Rightarrow \quad 160x-24000=96x$$

$$\Rightarrow \quad 160x-96x=24000$$

$$\Rightarrow \quad 64x=24000$$

$$\Rightarrow \quad x=\frac{24000}{64}$$

$$\Rightarrow \quad x=₹\,375$$

$\therefore$ The cost price of table is ₹ 375.

10. (*d*) Cost of coffee powder used in one cup

$$= \frac{20}{10} = ₹\ 2$$

Cost of milk used in one cup

$$= \frac{30}{1000} \times 200 = ₹\ 6$$

$\therefore$ Cost of each cup of coffee

$$= 2 + 6 = ₹\ 8$$

To gain 25% profit, sale price of each cup of coffee $= 125\%$ of $8 = ₹\ 10$

11. (*a*) By hit and trial,

Case I $t = ₹\ 400$ and $q = 20$ unit

Then, from option (a),

Total profit $= 600q - 5t$

$= 600 \times 20 - 5 \times 400$

$= 12000 - 2000 = ₹\ 10000$

Case II $t = ₹\ 600$ and $q = 25$ units

Total profit $= 600q - 5t$

$= 600 \times 25 - 5 \times 600$

$= 600\ (25 - 5)$

$= 600 \times 20$

$= ₹\ 12000$

12. (*d*) Let CP of 8 quintal $= ₹\ x$

$\therefore$ CP of 1 quintal $= ₹\ \dfrac{x}{8}$

$\therefore$ SP of rice 10% profit $= \dfrac{3x}{8} + \dfrac{3x}{8} \times \dfrac{10}{100}$

$$= \frac{3x}{8} + \frac{3x}{80} = \frac{33x}{80}$$

SP of rice without profit or loss $= ₹\ \dfrac{3x}{8}$

SP of rice at 5% loss $= \dfrac{2x}{8} - \dfrac{2x}{8} \times \dfrac{5}{100}$

$$= \frac{x}{4} - \frac{x}{4 \times 20} = \frac{19x}{4 \times 20} = \frac{19x}{80}$$

$\therefore$ Total selling price $= \dfrac{33x}{80} + \dfrac{3x}{8} + \dfrac{19x}{80}$

$$= \frac{33x + 30x + 19x}{80} = \frac{82x}{80}$$

$\therefore$ Profit $= \dfrac{\text{SP} - \text{CP}}{\text{CP}} \times 100 = \dfrac{\dfrac{82x}{80} - x}{x} \times 100$

$$= \frac{(82 - 80)x}{80x} \times 100 = \frac{2}{80} \times 100$$

$= 2.5\%$

13. (*c*) Let CP $= x$

Then, SP $= \dfrac{110x}{100} = ₹\ \dfrac{11x}{10}$

Now, CP $= 96\%$ of $x = \dfrac{96x}{100} = ₹\ \dfrac{24x}{25}$

Now, according to the question,

$$SP = ₹\left(\frac{11x}{10} + 6\right)$$

$\therefore \quad \left(\frac{11x}{10} + 6\right) = 118\frac{3}{4}\%$ of $\frac{24x}{25}$

$\Rightarrow \quad \frac{11x + 60}{10} = \frac{475}{400} \times \frac{24x}{25} = \frac{57x}{50}$

$\Rightarrow \quad 550x + 3000 = 570x$

$\Rightarrow \quad 20x = 3000$

$\therefore \quad x = \frac{3000}{20} = 150$

$\therefore \quad CP = ₹\ 150$

14. (*c*) Let required percentage profit = $x\%$

According to the question,

10% of (80 × 50)

= 5% of (20 × 50) + x% of (60 × 50)

$\Rightarrow \quad \frac{80 \times 50 \times 10}{100}$

$= \frac{20 \times 50 \times 5}{100} + \frac{60 \times 50 \times x}{100}$

$\Rightarrow \quad 80 = 10 + 6x$

$\Rightarrow \quad x = \frac{70}{6} = 11\frac{2}{3}\%$

15. (*c*) $\because$ CP of 80 apples = ₹ 240

$\therefore$ CP of 1 apple = ₹ 3

$\therefore$ CP of 20 apples = ₹ 60

To earn a profit of 20%,

SP = 120% of 240 = ₹ 288

But he sales $\frac{1}{4}$ of his apples *i.e.*, 20 apples for ₹ 60.

$\therefore$ SP of remaining 60 apples

= 288 − 60 = ₹ 228

$\therefore$ SP of 1 apple = $\frac{228}{60}$ = ₹ 3.80

16. (*c*) Let cost price of transistor = ₹ x

According to the question,

CP of transistor = $x \times \frac{105}{100}$

and SP of transistor = $\left(\frac{115x}{100} + 6\right)$

$\therefore$ Profit percentage = $\frac{SP - CP}{CP} \times 100$

$$\Rightarrow \quad 10 = \frac{\frac{115x}{100} + 6 - \frac{105x}{100}}{\frac{105x}{100}} \times 100$$

$$\Rightarrow \quad 10 = \frac{(10x + 600)\,100}{105x}$$

$\Rightarrow \quad 105x = 100x + 6000$

$\Rightarrow \quad 5x = 6000$

$\Rightarrow \quad x = ₹\ 1200$

Chapter

13

Discount

Discount is defined as the amount of rebate given on a fixed price (called as marked price) of an article. It is given by merchants/ shopkeepers to increase their sales by attracting customers.

Discount = Marked Price - Selling Price

Marked Price (List Price)

The price on the label of an article/product is called the marked price or list price. This is the price at which product is intended to be sold. However, there can be some discount given on this price and actual selling price of the product may be less than the marked price. It is generally denoted by MP.

Basic Formulae Related to Discount

- $\text{Discount \%} = \dfrac{\text{Marked price} - \text{Selling price}}{\text{Marked price}} \times 100$
- $\text{Discount \%} = \dfrac{\text{Discount}}{\text{Marked price}} \times 100$
- $\text{Selling price} = \text{Marked price} \times \left(1 - \dfrac{r}{100}\right)$

♦ Selling price = Marked price - Discount where, $r\%$ is the rate of discount allowed

Note Discount is always calculated with respect to marked price of an article

Successive Discount

When a series of discounts (one after the other) are allowed on marked price of an article, then these discounts are called successive discounts.

Let $r_1\%, r_2\%, r_3\%$......be the series of discounts on an article

with marked price of ? *P,* then the selling price of the article after all the discounts is given as

$$P \times \left(1-\frac{r_1}{100}\right)\times\left(1-\frac{r_2}{100}\right)\times\left(1-\frac{r_3}{100}\right)\times......$$

Ex. 1 An item is sold for ? 680 by allowing a discount of 15% on its marked price. The marked price of item is

Sol. Selling price of an item = ? 680 and rate of discount, r = 15%

.•. Selling price = Marked price 1-------

I 100 J

=> 680 = MP {1 - —1

I 100 J

680 = MP f100-15! =* MP=680X100= ? 800 L 100 *J* 85

Ex. 2 A shopkeeper on the eve of Diwali allowed a series of discount on television sets. Find the selling price of a television set, if the marked price of television is ? 1000 and successive discounts are 10% and 5%.

Sol. Selling price of a television = Marked price $\times \left(1-\frac{r_1}{100}\right)\left(1-\frac{r_2}{100}\right)$

Here, marked price = ₹ 1000, R_1 = 10% and R_2 = 5%

$\therefore$ Selling price = $1000\left(1-\frac{10}{100}\right)\times\left(1-\frac{5}{100}\right)$

$$= 1000 \times \left(\frac{100-10}{100}\right)\times\left(\frac{100-5}{100}\right) = 1000 \times \frac{90}{100} \times \frac{95}{100} = ₹\ 855$$

Fast Track **Techniques**

to solve the QUESTIONS

(Technique^

Single discount equivalent to two successive discounts r-,% and r_2%

$$= \left(r_1 + r_2 - \frac{r_1 \times r_2}{100}\right)\%$$

Ex. 3 What will be a single equivalent discount for successive discounts of 10% and 5% on marked price of an article?

Sol. Given, r_1 = 10% and r_2 = 5%

Single equivalent discount $= \left(r_1 + r_2 - \frac{r_1 \times r_2}{100}\right)\%$

$$= \left(10 + 5 - \frac{10 \times 5}{100}\right)\%$$

$$= \left(15 - \frac{50}{100}\right)\%$$

$$= (15 - 0.5)\%$$

$$= 14.5\%$$

$\therefore$ Single equivalent discount = 14.5%

Technique 2

Single discount equivalent to three successive discounts r.,%, r_2% and r_3%

$$= \left[1-\left(1-\frac{r_1}{100}\right)\left(1-\frac{r_2}{100}\right)\left(1-\frac{r_3}{100}\right)\right] \times 100\%$$

Ex. 4 What is the single equivalent discount for successive discounts of 10%, 20% and 15% on marked price for motor bike?

Sol. Given, $r_1 = 10\%$, $r_2 = 20\%$ and $r_3 = 15\%$

Single equivalent discount for three successive discounts

$$= \left[1 - \left(1 - \frac{r_1}{100}\right)\left(1 - \frac{r_2}{100}\right)\left(1 - \frac{r_3}{100}\right)\right] \times 100\%$$

$$= \left[1 - \left(1 - \frac{10}{100}\right)\left(1 - \frac{20}{100}\right)\left(1 - \frac{15}{100}\right)\right] \times 100\%$$

$$= \left[1 - \left(\frac{90}{100}\right)\left(\frac{80}{100}\right)\left(\frac{85}{100}\right)\right] \times 100\% = \left[1 - \frac{9}{10} \times \frac{4}{5} \times \frac{17}{20}\right] \times 100\%$$

$$= \left[\frac{1000 - 612}{1000}\right] \times 100\% = \frac{388}{1000} \times 100 = 38.8\%$$

Technique 3

If a shopkeeper wants a profit of $R\%$ after allowing a discount of

$r\%$, then Marked Price (MP) of the item $= \text{CP}\left(\frac{100 + R}{100 - r}\right)$

or

Cost Price (CP) of the item $= \text{MP}\left(\frac{100 - r}{100 + R}\right)$

Ex. 5 The marked price of a bicycle is ? 1100. A shopkeeper allows a discount of 10% and get a profit of 10%. Find the cost price of the bicycle.

Sol. Let CP of the bicycle = ₹ x

Then, SP of the bicycle $= \dfrac{110x}{100} = ₹\ \dfrac{11x}{10}$

Now, MP of the bicycle = ₹ 1100

$\therefore$ SP of the bicycle $= \dfrac{1100 \times 90}{100} = ₹\ 990$

According to the question, $\dfrac{11x}{10} = 990$

$\Rightarrow$ $x = \dfrac{990 \times 10}{11}$

$\Rightarrow$ $x =$ ₹ 900

Fast Track Method

Here, MP = ₹ 1100, r = 10% and R = 10%

$\therefore$ $CP = MP\left(\dfrac{100 - r}{100 + R}\right)$

$= 1100 \times \dfrac{(100 - 10)}{(100 + 10)}$

$= \dfrac{1100 \times 90}{110} =$ ₹ 900

Technique 4

A merchant fixes the marked price of an article in such a way that after allowing a discount of *r%,* he earns a profit of/?%. Then, marked price of

the article is $\left(\dfrac{r + R}{100 - r} \times 100\right)$% more than its cost price.

Ex. 6 A shopkeepper allows a discount of 10% on the marked price of calculator. then by what per cent higher than cost price, should marked the price, so as to gain 20% on selling it at the discount?

Sol. Let the cost price of article be ₹ 100.

Then, selling price should be 120% of 100 $\quad$ [∵ profit per cent is 20%]

$$= \frac{120 \times 100}{100} = ₹\ 120$$

Now, selling price of the article should be 10% lower than marked price because of the discount of 10%.

$$\therefore \quad \text{Marked price} = \frac{100 \times SP}{100 - 10} = \frac{100 \times 120}{90} = \frac{400}{3}$$

∴ Required percentage at which article is marked higher than cost price

$$= \frac{MP - CP}{CP} \times 100\%$$

$$= \frac{\frac{400}{3} - 100}{100} \times 100 = \frac{(400 - 300)}{3 \times 100} \times 100 = \frac{100}{3} = 33\frac{1}{3}\%$$

Fast Track Method

Here, $r = 10\%$ and $R = 20\%$

$$\therefore \text{Required percentage} = \left(\frac{r + R}{100 - r} \times 100\right)\% = \frac{10 + 20}{100 - 10} \times 100$$

$$= \frac{30 \times 100}{90} = \frac{100}{3} = 33\frac{1}{3}\%$$

Technique 5

If a shopkeeper allows a discount of r-,% on an article and marked price of the article is r% more than the cost price, then profit or loss per cent in this transaction

$$= \frac{r \times (100 - r_1)}{100} - r_1$$

Note Positive value shows a profit, while negative value shows loss

Ex. 7 A shopkeeper marked the price 10% more than its cost price. If he allows a discount of 20%, then find his loss per cent.

Sol. Here, $r_1 = 20\%$ and $r = 10\%$

$$\therefore \quad \text{Loss or profit per cent} = \frac{r \times (100 - r_1)}{100} - r_1 = \frac{10 \times (100 - 20)}{100} - 20$$

$$= \frac{10 \times 80}{100} - 20 = -12\%$$

∴ Loss per cent = 12%

Fast Track Practice

1. On a 20% discount sale, an article costs 596. What was the original price of the article? [CDS 2013]

(a) X 720 *(b)* X 735 *(c)* X 745 *(d)* X 775

2. If an electricity bill is paid before due date, one gets a reduction of 4% on the amount of the bill. By paying the bill before due date, a person got a reduction of X 13. The amount of his electricity bill was [SSCCGL2013]

(a) ?125 *(b)* ?225

(c) X 325 *(d)* X 425

3. If on a marked price, the difference of selling prices with a discount of 30% and two successive discounts of 20% and 10% is X 72, then the marked price (in X) is

[SSCCGL2010] *(a)* X 3600 *(b)* X 3000

(c) X 2500 *(d)* X 2400

4. Two successive discounts of 20% and 20% are equivalent to a single discount of [SSCCGL2013]

(a) 42% *(b)* 40%

(c) 36% *(d)* 34%

5. Two successive discounts of 20% and 5% are equivalent to a single discount of

[SSCSAS2010]

(a) 25% *(b)* 24%

(c) 18% *(d)* 15%

6. A shopkeeper earns a profit of 12% on selling a book at 10% discount on the printed price. The ratio of the cost price and the printed price of the book is

[SSCCPO2013] *(a)* 45 : 56 *(b)* 8 :11

(c) 47 : 56 (d) 3 : 4

7. A manufacturer marked an article at X 50 and sold it allowing 20% discount. If his profit was 25%, then the cost price of the article was [SSC CGL 2012]

(a) ?40 (b) ?35

(c) X 32 (d) X 30

8. A seller marks his goods 30% above their cost price but allows 15% discount for cash payment. His percentage of profit when sold in cash, is [SSC CGL 2010]

(a) 10.5% (b) 15%

(c) 9% (d) 8.5%

9. Successive discounts of 10%, 20% and 30% is equivalent to single discount of

[SSC CGL 2010]

(a) 60% (b) 49.6%(c) 40.5% (d) 36%

10. By selling an article at 3/4th of the marked price, there is a gain of 25%. The ratio of the marked price and the cost price is [SSC CGL 2012]

(a) 5 : 3 (b) 3 : 5 (c) 3 : 4 (d) 4 : 3

11. A retailer offers the following discount schemes for buyers on an article.

I. Two successive discounts of 10%. II. A discount of 12% followed by a discount of 8%.

III. Successive discounts of 15% and 5%.

IV. A discount of 20%.

The selling price will be minimum under the scheme [SSC CGL 2012]

(a) I (b) II

(c) III (d) IV

12. While selling, a businessman allows 40% discount on the marked price and there is a loss of 30%. If it is sold at the marked price, profit per cent will be

[SSC CGL 2012]

(a) 10% *(b)* 20%

(c) 16-% *(d)* 16 -%

3 3

13. A man bought an article listed at X 1500 with a discount of 20% offered on the list price. What additional discount must be offered to the man to bring the net price to? 1104? [SSC CGL 2011]

(a) 8% *(b)* 10%

(c) 12% *(d)* 15%

14. The cost price of an article is X 800. After allowing a discount of 10%, a gain of 12.5% was made. Then, the marked price of the article is [SSC CGL 2011]

(a) ?1000 *(b)* ?1100

(c) ?1200 *(d)* ?1300

15. The marked price of a radio is X 480. The shopkeeper allows a discount of 10% and gains 8%. If no discount is allowed, his gain per cent would be [SSC CGL 2011]

(a) 18% *(b)* 18.5%

(c) 20.5% *(d)* 20%

16. The marked price of a clock is X 3200. It is to be sold at X 2448 at two successive discounts. If the first discount is 10%, then the second discount is [SSC CCL 2010]

(a) 5% *(b)* 10%

(c) 15% *(d)* 20%

17. The marked price of a TV is *X* 16000. After two successive discounts, it is sold for *X* 11400. If the first discount is 5%, then the rate of second discount is

[SSCCPO2012] *(a)* 15% *(b)* 20% *(c)* 30% *(d)* 25%

18. A dealer marks his goods 30% above his cost price and then allows 15% discount on it. What is the cost price of an article on which he gains ? 84 ? [SSC CPO 2010]

(a) X 800 *(b) X* 560

(c) X 373.33 *(d) X* 280

19. A shopkeeper has announced 14% rebate on marked price of an article. If the selling price of the article is *X* 645, then the marked price of the article will be

[SSC CPO 2013] *(a) X* 800 *(b) X* 810 *(c) X* 750 *(d) X* 775

20. A merchant has announced 25% rebate on prices of ready made garments at the time of sale. If a purchaser needs to have a rebate of *X* 400, then how many shirts, each costing *X* 320, should he purchase?

[SSC CCL 2010] *(a)* 10 *(b) 7 (c)* 6 *(d)* 5

21. A shopkeeper sells notebooks at the rate of *X* 457 each and earns a commission of 4%. He also sells pencil boxes at the rate of *X* 80 each and earns a commission of 20%. How much amount of commission will he earn in two weeks, if he sells 10 notebooks and 6 pencil boxes a day?

[CBIPO2010] *(a)* ?1956 *(b)* ?1586

(c) ?1496 *(d)* ?1596

(e) None of the above

22. The difference between a discount of 40% on *X* 500 and two successive discounts of 36% and 4% on the same amount is

(a) X0 (b) X2

(c) ?1.93 *(d)* ?7.20

If the price of an item is increased by 30% and then allows two successive discounts of 10% and 10%. In last the price of an item is [SSC CCL 2011]

(a) increased by 10%

(b) increased by 5.3%

(c) decreased by 3%

(d) decreased by 5.3%

24. The difference between a discount of 30% on *X* 2000 and two successive discounts of 25% and 5% on the same amount is

[SSC FCI 2012]

(a) ?30 *(b)* ?35

(c) *X* 25 *(d)* *X* 40

25. The cost price of an article is 64% of the marked price. The gain percentage after allowing a discount of 12% on the marked price is [SSC CCL 2013]

(a) 37.5% *(b)* 48%

(c) 50.5% *(d)* 52%

26. A merchant marked the price on his goods 20% more than its cost price and allows a discount of 15%. His profit per cent is [SSC CCL 2011]

(a) 1% *(b) 2%*

(c) 10% *(d)* 15%

27. A trader lists his articles 20% above their cost prices and allows a discount of 10% at the time of sale. His gain per cent is [SSC CPO 2010]

(a) 5% *(b)* 6%

(c) 8% *(d)* 10%

While selling a shirt, a shopkeeper gives a discount of 7%. If he had given a discount of 9%, he would have got *X* 15 less as profit. The marked price of the shirt is [SSC SAS 2010]

(a) ?750 *(b) X720*

(c) X 712.50 *(d)* ?600

29. While selling a watch, a shopkeeper gives a discount of 5%. If he gives a discount of 6%, he earns *X* 15 less as profit. What is the marked price of the watch? [SSC CCL 2011]

(a) ?1250 *(b)* ?1400

(c) ?1500 *(d)* ?750

30. After allowing a discount of 16%, there was still a gain of 5%. Then, the percentage of marked price over the cost price is [SSC CPO 2011]

(a) 15% *(b)* 18% *(c)* 21% *(d)* 25%

31. A shopkeeper marks an article at *X* 60 and sells at a discount of 15%. He also gives a gift worth *X* 3. If the still makes 20% profit, the cost price is [SSC CPO 2011]

(a) ?22 *(b)* ?32 *(c) XA0 (d) XA2*

32. A shopkeeper allows 23% commission on his advertised price and still makes a profit of 10%. If he gains *X* 56 on one item, his advertised price of the item (in?) is [SSCCCL2013]

(a) 820 *(b)* 780

(c) 790 *(d)* 800

33. A shopkeeper allows a discount of 10% to his customers and still gains 20%, the marked price of the article which costs ? 450, is [SSCCCL2013]

(a) X 600 *(b) X* 540

(c) X 660 *(d) X* 580

34. A dozen pair of socks quoted at *X* 80 are available at a discount of 10%. How many pair of socks can be bought for *X* 24? [SSC Multitasking 2013]

(a) 4 (b) 5 (c) 3 (d) 6

35. A tradesman allows a discount of 15% on the marked price. How much above the cost price must he mark his goods as to gain 19%? [SSC CPO 2008]

(a) 34% (b) 40% (c) 25% (d) 30%

36. If a commission of 10% is given on the written price of an article, the gain is 20%. The gain per cent, when the commission is increased to 20%, will be

[SSSCFCI2012]

(a) $6\frac{2}{3}\%$ (b) 5% (c) 8% (d) $5\frac{1}{3}\%$

37. Rita bought a television set with 20% discount on the labelled price. She made a profit of X 800 by selling it for X 16800. The labelled price of the set was

[SSC CGL 2010] (a) ? 10000 (b) ?20000

(c) ? 20800 (d) ? 24000

38. The marked price of an article is X 500. It is sold on two successive discounts of 20% and 10%. The selling price of that article is [SSC CCL 2002]

(a) ?350 (b) ?375

(c) X 360 (d) X 400

39. A shopkeeper allows a discount of 4% on his article. If cost price of the article is X 100 and he wants to get a profit of 20%. Then, its marked price will be

[SSC DEO 2008] (a) X 96 (b) X 120 (c) X 125 (d) X 130

40. If marked price of an article is 30% more than its cost price and 10% discount is given, then profit per cent is

[RRB ASM 2004]

(a) $18\frac{1}{2}\%$ (b) 20%

(c) $1\frac{1}{2}\%$ (d) 17%

41. If a shopkeeper sold a book with 20% profit after giving a discount of 10% on marked price. The ratio of cost price and marked price of the book is

[SSC Constable 2012] *(a)* 6 : 5 *(b)* 5 : 6

(c) 3 : 4 *(d)* 2 : 3

42. A shopkeeper marked 50% more price than cost price of the article. If he allows 30% discount to his customers, then his profit per cent is [Delhi Police 2010]

(a) 5% *(b)* 10%

(c) 12% *(d)* 15%

43. A man purchased a shirt and pant with a discount of 25% on its marked price. He sold them at a price 40% more than the price at which he bought them. How much per cent the new selling price to its marked price? [Delhi Police 2007]

(a) 5% *(b)* 7.5%

(c) 9% *(d)* 12.5%

44. By selling an article at 80% of its marked price, a trader makes a loss of 10%, what will be a profit percentage, if he sells it at 95% of its marked price? [SSC CCL 2012]

(a) 6.9% *(b)* 5%

(c) 5.9% *(d)* 12.5%

45. What is the maximum percentage discount (approximately) that a merchant can offer on his marked price, so that he ends up selling at no profit or loss, if he initially marked his goods up by 40%? [SSC CGL 2013]

(a) 60% *(b)* 28.5%

(c) 33.5% *(d)* No discount

46. A merchant purchases a wrist watch for *X* 450 and fixes its list price in such a way that after allowing a discount of 10%, he earns a profit of 20%. Then, the list price of the watch is

[SSC Multitasking 2013; SSC CPO 2010] *(a)* *X* 600 *(b)* *X* 650

(c) *X* 700 *(d)* *X* 550

47. In order that there may be a profit of 20% after allowing a discount of 10% on the marked price, the cost price of an article has to be increased by

[SSC CGL 2012]

(a) 30% (b) 33%

(c) $33\frac{1}{3}\%$ (d) $33\frac{2}{3}\%$

Answer with Solutions

1. (*c*) Let the original price of be *x*.

$\therefore$ At discount of 20% article costs ₹ 596

$$\therefore \quad 596 = \frac{80}{100} \times x$$

$$\Rightarrow \quad x = \frac{596 \times 100}{80}$$

$$\therefore \quad x = 745$$

$\therefore$ Original price = ₹ 745

2. (*c*) Let the amount of electricity bill = ₹ x

$$\therefore \quad 4\% \text{ of } x = 13 \Rightarrow \frac{4 \times x}{100} = 13$$

$\Rightarrow \quad x = 13 \times 25 =$ ₹ 325

3. (*a*) Let the marked price = ₹ x

The discount equivalent to successive discounts 20% and 10% $= \left(r_1 + r_2 - \frac{r_1 r_2}{100}\right)\%$

where, $r_1 = 20$ and $r_2 = 10$

Then, $30 - 2 = 28\%$

According to the question,

$$\frac{(100-28)x}{100} - \frac{(100-30)x}{100} = 72$$

$$\Rightarrow \quad \frac{72x - 70x}{100} = 72$$

$$\therefore \quad x = \frac{72 \times 100}{2} = ₹\ 3600$$

4. (*c*) Given, $r_1 = 20\%$ and $r_2 = 20\%$

$\therefore$ Single discount equal to r_1 and r_2

$= \left(r_1 + r_2 - \frac{r_1 \times r_2}{100}\right)\%$ [by Technique 1]

$$= 20 + 20 - \frac{20 \times 20}{100} = 40 - 4 = 36\%$$

5. (*b*) Here, $r_1 = 20\%$ and $r_2 = 5\%$

$\therefore$ Required percentage

$$= \left(r_1 + r_2 - \frac{r_1 \times r_2}{100}\right)\%$$

[by Technique 1]

$$= 20 + 5 - \frac{20 \times 5}{100} = 25 - 1 = 24\%$$

6. (*a*) Let the CP of book = ₹ x

Then, SP of book $= \frac{(100+12) \times x}{100} = \frac{112x}{100}$

Now, the printed price = ₹ y

Then, after discount, the SP

$$= \frac{(100-10) \times y}{100} = \frac{90y}{100}$$

Since, both SP are same.

Then, $\frac{112x}{100} = \frac{90y}{100} \Rightarrow \frac{x}{y} = \frac{45}{56} \Rightarrow 45 : 56$

7. (*c*) $\because$ Marked price of an article = ₹ 50

$\therefore$ SP of an article $= \dfrac{50 \times (100 - 20)}{100}$

$= \dfrac{50 \times 80}{100} = ₹\ 40$

Hence, cost price of an article $= \dfrac{40 \times 100}{(100 + 25)}$

$= \dfrac{40 \times 100}{125} = ₹\ 32$

8. (*a*) Let CP of the goods = ₹ x

$\therefore$ Marked price of the goods

$= \dfrac{x \times (100 + 30)}{100}$

$= ₹\ \dfrac{13x}{10}$

Now, SP of the goods $= \dfrac{13x}{10} \times \left(\dfrac{100 - 15}{100}\right)$

$= \dfrac{13x}{10} \times \dfrac{85}{100} = ₹\ \dfrac{221}{200}x$

$\Rightarrow$ Profit $= \left(\dfrac{221}{200}x - x\right) = ₹\ \dfrac{21}{200}x$

Hence, profit per cent $= \dfrac{\frac{21}{200}x}{x} \times 100\%$

$= \dfrac{2100}{200} = 10.5\%$

9. (*b*) Here, $r_1 = 10\%$, $r_2 = 20\%$ and $r_3 = 30\%$

$\therefore$ Required discount

$= \left[1 - \left(1 - \dfrac{r_1}{100}\right)\left(1 - \dfrac{r_2}{100}\right)\left(1 - \dfrac{r_3}{100}\right)\right] \times 100\%$

[by Technique 2]

$= \left[1 - \left(1 - \dfrac{10}{100}\right)\left(1 - \dfrac{20}{100}\right)\left(1 - \dfrac{30}{100}\right)\right] \times 100\%$

$= \left(1 - \dfrac{9}{10} \times \dfrac{4}{5} \times \dfrac{7}{10}\right) \times 100\%$

$= (1 - 0.504) \times 100\% = 49.6\%$

10. (*a*) Let MP of an article = ₹ x

$\therefore$ SP of an article $= ₹\ \dfrac{3}{4}x$

and CP of an article $= \dfrac{3x}{4} \times \dfrac{100}{100 + 25}$

$= \dfrac{3x}{4} \times \dfrac{100}{125} = ₹\ \dfrac{3x}{5}$

Required ratio $= x : \dfrac{3x}{5} = 5 : 3$

11. (*d*) I. Equivalent single discount to 10% and 10%

$$= \left(10 + 10 - \frac{10 \times 10}{100}\right)\% = 19\%$$

II. Equivalent single discount to 12% and 8%

$$= 12 + 8 - \frac{12 \times 8}{100}$$

$$= 20 - 0.96 = 19.04\%$$

III. Equivalent single discount to 15% and 5%

$$= 15 + 5 - \frac{15 \times 5}{100}$$

$$= 20 - 0.75 = 19.25\%$$

IV. Equivalent single discount to 20% = 20%

So, the selling price will be minimum under the scheme IV as in this scheme, the discount is maximum.

12. (*c*) Let MP of an article = ₹ x

$$\therefore \text{ SP of an article} = \frac{x \times (100 - 40)}{100} = ₹ \frac{3x}{5}$$

$$\text{CP of an article} = \frac{3x}{5} \times \frac{100}{100 - 30}$$

$$= \frac{3x}{5} \times \frac{100}{70} = ₹ \frac{6x}{7}$$

$$\therefore \text{ Profit when sold at MP} = \left(x - \frac{6x}{7}\right) = ₹ \frac{x}{7}$$

Hence, profit per cent

$$= \frac{\frac{x}{7}}{\frac{6x}{7}} \times 100\% = \frac{50}{3}\%$$

$$= 16\frac{2}{3}\%$$

13. (*a*) $\because$ Listed price of an article = ₹ 1500

$\therefore$ Price after first discount

$$= 1500 \times \left(1 - \frac{20}{100}\right) = 1500 \times \frac{4}{5} = ₹ 1200$$

Now, second discount = 1200 − 1104 = ₹ 96

Hence, required percentage

$$= \frac{96}{1200} \times 100\% = 8\%$$

Alternate Method

Let rate of second discount = r_2%

$\therefore \quad x = 1500, r_1 = 20\%, r_2 = ?$ and $y = 1104$

$$\Rightarrow y = x\left(1 - \frac{r_1}{100}\right)\left(1 - \frac{r_2}{100}\right)$$

$$\Rightarrow 1104 = 1500 \times \left(1 - \frac{20}{100}\right)\left(1 - \frac{r_2}{100}\right)$$

$$\Rightarrow \frac{1104}{1500} \times \frac{5}{4} \times 100 = 100 - r_2$$

$$\Rightarrow 92 = 100 - r_2$$

$$\therefore r_2 = 100 - 92 = 8\%$$

14. (*a*) Let marked price of the article = ₹ x

$\therefore$ SP of the article after a discount of 10%

$$= x \times \left(\frac{100 - 10}{100}\right) = ₹\ \frac{9x}{10}$$

$\Rightarrow$ CP of the article with a profit of 12.5%

$$= \frac{9x}{10} \times \frac{100}{100 + 12.5} = ₹\ \frac{9x}{10} \times \frac{100}{112.5}$$

But CP of the article = ₹ 800

$$\frac{9x}{10} \times \frac{100}{112.5} = 800$$

$$\therefore \quad x = \frac{800 \times 112.5 \times 10}{9 \times 100} = ₹\ 1000$$

Fast Track Method

Here, CP = ₹ 800, r = 10% and R = 12.5%

$$\therefore \text{ Marked price (MP)} = \frac{\text{CP} \times (100 + R)}{(100 - r)}$$

[by Technique 3]

$$= \frac{800 \times (100 + 12.5)}{(100 - 10)}$$

$$= \frac{800 \times 112.5}{90} = ₹\ 1000$$

15. (*d*) $\because$ Marked price of a radio = ₹ 480

$$\therefore \quad \text{SP of a radio} = 480 \times \left(\frac{100 - 10}{100}\right)$$

$$= \frac{480 \times 90}{100} = ₹\ 432$$

$$\Rightarrow \quad \text{CP of a radio} = \frac{432 \times 100}{100 + 8}$$

$$= \frac{432 \times 100}{108} = ₹\ 400$$

Profit when article is sold at MP

$$= 480 - 400 = ₹\ 80$$

Hence, profit per cent

$$= \frac{80}{400} \times 100\% = 20\%$$

Fast Track Method

Here, MP = ₹ 480, r = 10% and R = 8%

$$\therefore \text{ CP} = \frac{\text{MP} \times (100 - r)}{100 + R} \quad \textbf{[by Technique 3]}$$

$$= \frac{480 \times (100 - 10)}{100 + 8} = \frac{480 \times 90}{108} = ₹\ 400$$

$\therefore$ Profit = 480 − 400 = ₹ 80

$$\text{Hence, profit per cent} = \frac{80}{400} \times 100\% = 20\%$$

16. (*c*) Let rate of second discount = r%

Now, marked price of the clock = ₹ 3200

$\therefore$ SP of the clock after first discount

$$= \frac{3200 \times (100 - 10)}{100} = \frac{3200 \times 90}{100}$$

$$= ₹\ 2880$$

SP of the clock after second discount

$$= \frac{2880 \times (100 - r)}{100}$$

But SP of the clock after second discount

$$= 2448$$

$$\therefore \quad 2448 = \frac{2880 \times (100 - r)}{100}$$

$$\Rightarrow \quad 100 - r = \frac{2448 \times 100}{2880} = 85$$

$$\therefore \quad r = 100 - 85 = 15\%$$

Alternate Method

Let rate of second discount = r %

$x = ₹\,3200, y = ₹\,2448,$

$r_1 = 10\%$ and $r_2 = r\%$

$$\therefore \quad y = x \times \left(\frac{100 - r_1}{100}\right)\left(\frac{100 - r_2}{100}\right)$$

$$\Rightarrow 2448 = \frac{3200 \times (100 - 10) \times (100 - r)}{100 \times 100}$$

$$\Rightarrow \quad \frac{2448 \times 100 \times 100}{3200 \times 90} = 100 - r$$

$$\therefore \quad r = 100 - 85 = 15\%$$

17. (*d*) Let the rate of second discount = r%

$$\therefore \quad 11400 = 16000\left(1 - \frac{5}{100}\right)\left(1 - \frac{r}{100}\right)$$

$$\Rightarrow \quad \frac{11400}{16000} \times \frac{20}{19} = 1 - \frac{r}{100}$$

$$\therefore \quad r = 100 \times (1 - 0.75) = 25\%$$

18. (*a*) Let cost price of an article = ₹x

$\therefore$ Marked price of an article

$$= \frac{x \times (100 + 30)}{100} = ₹\,\frac{13x}{10}$$

$$\Rightarrow \quad \text{SP of the article} = \frac{13x}{10} \times \frac{(100 - 15)}{100}$$

$$= \frac{13x}{10} \times \frac{85}{100} = ₹\,\frac{221x}{200}$$

$$\therefore \quad \text{Profit} = \left(\frac{221}{200}x - x\right)$$

$$\Rightarrow \quad 84 = \frac{221x - 200x}{200}$$

$$\therefore \quad x = \frac{200 \times 84}{21} = ₹\,800$$

19. (*c*) Let the marked price of item = ₹ x

$$\therefore \quad \left(\frac{100 - 14}{100}\right) \text{of } x = 645$$

$$\Rightarrow \quad x = \frac{645 \times 100}{86} = ₹\,750$$

Alternate Method

$\because$ $r = 14\%$ and SP = ₹ 645

$$\therefore \quad \text{Marked price of an item} = \frac{\text{SP} \times 100}{100 - r}$$

$$= \frac{645 \times 100}{100 - 14} = \frac{64500}{86} = ₹\,750$$

20. (*d*) $\because$ Marked price of a shirt = ₹ 320

and discount on a shirt $= \dfrac{320 \times 25}{100} = ₹\, 80$

$\therefore$ Number of shirts has to be purchased to get a rebate of ₹ 400 $= \dfrac{400}{80} = 5$

21. (*e*) $\because$ SP of the notebook = ₹ 457

$\therefore$ Commission on one notebook $= ₹\, \dfrac{4 \times 457}{100}$

and commission on 10 notebooks

$$= \frac{10 \times 4 \times 457}{100} = ₹\, 182.80$$

Now, SP of the pencil box = ₹ 80

$\therefore$ Commission on 1 pencil box $= ₹\, \dfrac{80 \times 20}{100}$

Commission on 6 pencil boxes

$$= \frac{80 \times 20 \times 6}{100} = ₹\, 96$$

Hence, total commission of 1 day

$$= (182.80 + 96) = ₹\, 278.80$$

Thus, total commission of 2 weeks

$$= 278.80 \times 14 = ₹\, 3903.20$$

22. (*d*) $\because$ Discount of 40% on ₹ 500

$$= \frac{500 \times 40}{100} = ₹\, 200$$

and single discount per cent equivalent to 36% and 4% $= \left(r_1 + r_2 - \dfrac{r_1 r_2}{100}\right)\%$

$$= 36 + 4 - \frac{36 \times 4}{100} = 40 - 1.44 = 38.56\%$$

$\therefore$ Discount by 38.56% on 500

$$= \frac{38.56 \times 500}{100} = ₹\, 192.80$$

Hence, required difference = (200 − 192.80)

$$= ₹\, 7.20$$

Alternate Method

$\because$ Single discount equivalent to 36% and 4%

$$= 36 + 4 - \frac{36 \times 4}{100} = 38.56$$

$\therefore$ Required difference

$$= \frac{(40 - 38.56) \times 500}{100} = 1.44 \times 5 = ₹\, 7.20$$

23. (*b*) Let the cost price of item = ₹ x

$\therefore$ Marked price of item

$$= \frac{x \times (100 + 30)}{100} = ₹\, \frac{13x}{10}$$

Selling price of the item after two successive discount

$$= \frac{13x}{10} \times \left(\frac{100 - 10}{100}\right) \times \left(\frac{100 - 10}{100}\right)$$

$$= \frac{13x}{10} \times \frac{90}{100} \times \frac{90}{100} = ₹\, \frac{1053}{1000}\, x$$

Increment in the price of item

$$= \frac{\frac{1053x}{1000} - x}{x} \times 100\% = \frac{53}{10}\% = 5.3\%$$

24. (*c*) $\because$ Single equivalent discount per cent to 25% and 5%

$$= \left(r_1 + r_2 - \frac{r_1 \cdot r_2}{100}\right)\% = 25 + 5 - \frac{25 \times 5}{100}$$

$= 30 - 1.25 = 28.75\%$

$\therefore$ Required difference

$$= \frac{(30 - 28.75) \times 2000}{100}$$

$$= \frac{1.25 \times 2000}{100} = ₹\, 25$$

25. (*a*) Let MP of the article = ₹ x

$\therefore$ CP of the article $= \frac{x \times 64}{100} = ₹\, \frac{16x}{25}$

and SP of the article $= \frac{x \times (100 - 12)}{100}$

$$= ₹\, \frac{x \times 22}{25}$$

$\Rightarrow$ Profit $= \left(\frac{22x}{25} - \frac{16x}{25}\right) = ₹\, \frac{6x}{25}$

Hence, profit per cent $= \frac{\frac{6x}{25}}{\frac{16x}{25}} \times 100\%$

$$= \frac{6 \times 100}{16} = 37.5\%$$

26. (*b*) Let the cost price of item = ₹ x

$\because$ Marked price of item $= \frac{x \times (100 + 20)}{100}$

$$= ₹\, \frac{6x}{5}$$

Then, selling price of item

$$= \frac{6x}{5} \times \left(\frac{100 - 15}{100}\right) = \frac{6x \times 17}{5 \times 20} = ₹\, \frac{102}{100}x$$

$\Rightarrow$ Profit $= \left(\frac{102x}{100} - x\right) = ₹\, \frac{2}{100}x$

$\therefore$ Gain per cent $= \frac{\frac{2x}{100} \times 100\%}{x} = 2\%$

Fast Track Method

Here, $r = 20\%$ and $r_1 = 15\%$

Required profit per cent $= \frac{r\,(100 - r_1)}{100} - r_1$

[by Technique 5]

$$= \frac{20\,(100 - 15)}{100} - 15 = 17 - 15 = 2\%$$

27. (*c*) Let the CP = x

$\therefore$ Marked price of item $= x \times \frac{120}{100} = \frac{6x}{5}$

Then, SP of item $= \frac{6x}{5} \times \left(\frac{100-10}{100}\right) = \frac{27x}{25}$

Profit $= \frac{27x}{25} - x = \frac{2x}{25}$

Profit% $= \frac{\frac{2x}{25}}{x} \times 100\% = 8\%$

Fast Track Method

Here, $r_1 = 10\%$ and $r = 20\%$

$\therefore$ Gain per cent $= \frac{r \times (100 - r_1)}{100} - r_1$

[by Technique 5]

$= \frac{20 \times (100-10)}{100} - 10 = \frac{20 \times 90}{100} - 10$

$= 18 - 10 = 8\%$

28. (*a*) Let MP of the shirt = ₹ x

According to the question,

$$\frac{(9-7)x}{100} = 15$$

$\Rightarrow$ $x = \frac{15 \times 100}{2} =$ ₹ 750

29. (*c*) Given, $r_1 = 5\%$, $r_2 = 6\%$ and $x =$ ₹ 15

Let marked price of the watch be ₹ x.

Then, $(6-5)\%$ of $x = 15$

$\Rightarrow$ $x = 15 \times 100$

$\therefore$ $x =$ ₹ 1500

30. (*d*) Here, $r = 16\%$ and $R = 5\%$

$\therefore$ Required percentage

$= \left(\frac{r+R}{100-r} \times 100\right)\%$ **[by Technique 4]**

$= \frac{16+5}{100-16} \times 100\% = \frac{21 \times 100}{84}\% = 25\%$

31. (*c*) $\because$ MP of the article = ₹ 60

and SP of the article $= \frac{60 \times (100-15)}{100}$

$= \frac{60 \times 85}{100} =$ ₹ 51

Thus, actual SP of the article

$= (51 - 3) =$ ₹ 48

Hence, CP of the article $= \frac{48 \times 100}{100 + 20}$

$= \frac{48 \times 100}{120} =$ ₹ 40

32. (*d*) Advertised price of the item = ₹x

$\therefore$ Commission on advertised price of the article

$=$ ₹ $\frac{x \times 23}{100}$

Price after commission $= x - \frac{23x}{100} =$ ₹ $\frac{77x}{100}$

$\therefore$ CP of the item $= \frac{77x}{100} \times \frac{100}{100+10}$

$= \frac{77x}{110} = ₹\, \frac{7x}{10}$

Profit $= \frac{77x}{100} - \frac{7x}{10} = ₹\, \frac{7x}{100}$

$\Rightarrow \quad 56 = \frac{7x}{100}$

$\therefore \quad x = \frac{56 \times 100}{7} = ₹\, 800$

33. (*a*) Here, $r = 10\%$ and $R = 20\%$

$\therefore$ Required per cent $= \left(\frac{r+R}{100-r} \times 100\right)\%$

[by Technique 4]

$= \frac{10+20}{100-10} \times 100\% = \frac{30 \times 100}{90}\% = \frac{100}{3}\%$

This shows that the marked price of the item is $\frac{100}{3}\%$ more than its cost price.

Now, cost price of the article = ₹ 450

$\therefore$ Marked price of the article

$= \frac{450 \times 400}{300} = ₹\, 600$

34. (*a*) $\because$ MP of one dozen of pairs of socks = ₹ 80

$\therefore$ SP of one dozen of pairs of socks

$= \frac{80 \times (100-10)}{100} = \frac{80 \times 90}{100} = ₹\, 72$

Hence, required number of pairs of socks purchased for ₹ 24 $= \frac{12 \times 24}{72} = 4$

35. (*b*) Here, $r = 15\%$ and $R = 19\%$

$\therefore$ Required percentage $= \frac{r+R}{100-r} \times 100\%$

[by Technique 4]

$= \frac{15+19}{100-15} \times 100\% = \frac{34 \times 100}{85}\% = 40\%$

36. (*a*) Let MP of the article = ₹x

$\therefore$ SP of the article $= \frac{x \times (100-10)}{100} = ₹\, \frac{9x}{10}$

$\Rightarrow$ CP of the article $= \frac{9x}{10} \times \frac{100}{100+20}$

$= \frac{9x \times 10}{120} = ₹\, \frac{3x}{4}$

Now, new SP of the article

$= \frac{x \times (100-20)}{100} = ₹\, \frac{4x}{5}$

New profit $= \left(\frac{4x}{5} - \frac{3x}{4}\right) = \frac{16x - 15x}{20}$

$= ₹\, \frac{x}{20}$

Hence, profit per cent $= \dfrac{\frac{x}{20}}{\frac{3x}{4}} \times 100\%$

$$= \frac{4 \times 100}{3 \times 20}\% = 6\frac{2}{3}\%$$

37. (*b*) Let labelled price of TV= ₹ x

$\therefore$ SP of the TV $= \dfrac{x \times (100 - 20)}{100}$

$$= ₹\,\frac{4x}{5}$$

But $16800 - 800 = \dfrac{4x}{5}$

$\therefore\ x = \dfrac{16000 \times 5}{4} = ₹\,20000$

38. (*c*) $\because$ Marked price of an article = ₹ 500

The selling price of article after discount 20% and 10%

$$= 500\left(\frac{100-20}{100}\right) \times \left(\frac{100-10}{100}\right)$$

$$= 500 \times \frac{80}{100} \times \frac{90}{100} = ₹\,360$$

Alternate Method

Single discount equivalent to

$$= \left(r_1 + r_2 - \frac{r_1 \cdot r_2}{100}\right)\%$$

Here, $r_1 = 20\%$ and $r_2 = 10\%$

$$= \left(20 + 10 - \frac{20 \times 10}{100}\right) = 28\%$$

$\therefore$ Selling price of article

$$= \frac{500 \times (100 - 28)}{100} = \frac{500 \times 72}{100}$$

$= ₹\,360$

39. (*c*) $\because$ Cost price of article = ₹ 100

$\therefore$ Selling price of the article for 20% profit

$$= \frac{100 \times (100 + 20)}{100} = ₹\,120$$

Hence, marked price of the article for 4% discount

$$= \frac{120 \times 100}{100 - 4} = \frac{120 \times 100}{96} = ₹\,125$$

40. (*d*) Here, $r = 30\%$ and $r_1 = 10\%$

$\therefore$ Profit per cent $= \dfrac{r \times (100 - r_1)}{100} - r_1$

[by Technique 5]

$$= \frac{30 \times (100 - 10)}{100} - 10$$

$$= \frac{30 \times 90}{100} - 10 = 27 - 10 = 17\%$$

41. (*c*) Let the marked price of book = ₹x

Selling price after 10% discount

$$= \frac{90x}{100} = ₹\ \frac{9}{10}x$$

Profit = 20%

$$\therefore \text{Cost price of book} = \frac{9}{10} \times \frac{100x}{120} = ₹\ \frac{3x}{4}$$

Hence, required ratio $= \frac{3x}{4} : x = 3:4$

42. (*a*) Here, $r = 50\%$ and $r_1 = 30\%$

$$\therefore \text{Gain per cent} = \frac{r(100 - r_1)}{100} - r_1$$

[by Technique 5]

$$= \frac{50 \times (100 - 30)}{100} - 30$$

$$= 35 - 30 = 5\%$$

43. (*a*) Let the original price of pant and shirt be = ₹x.

$\therefore$ Cost price of point and shirt

$$= \frac{x \times (100 - 25)}{100} = ₹\ \frac{3x}{4}$$

and selling price of shirt and pant

$$= \frac{3x}{4} \times \left(\frac{100 + 40}{100}\right) = \frac{3x}{4} \times \frac{140}{100} = ₹\ \frac{21}{20}x$$

Hence, required percentage

$$= \frac{\frac{21}{20}x - x}{x} \times 100\% = \frac{100}{20}\% = 5\%$$

44. (*a*) Let marked price = ₹ 100

and selling price = ₹ 80

In condition of 10% loss the cost price of article

$$= \frac{80 \times 100}{90} = ₹\ \frac{800}{9}$$

According to the question,

When SP = 95, then

$\therefore$ Required profit percentage

$$= \frac{95 - \frac{800}{9}}{\frac{800}{9}} \times 100 = \frac{55}{8}$$

$$= 6.9\% \text{ (approx)}$$

45. (*b*) Let cost price = ₹ 100

and marked price = 100 + 40 = ₹ 140

Let required discount be x%.

According to the question,

$$140 \times \left(\frac{100 - x}{100}\right) = 100$$

$$\Rightarrow \qquad 100 - x = \frac{100 \times 100}{140}$$

$$\therefore \qquad x = 100 - \frac{100 \times 100}{140} = \frac{40 \times 100}{140}$$

$$= 28.5\ \%\ \text{(approx)}$$

46. (*a*) CP of watch = ₹ 450

$$\text{SP of watch} = 450\left(\frac{100 + 20}{100}\right) = ₹\ 540$$

∴ List price of watch

$$= 540 \times \frac{100}{(100 - 10)} = ₹\ 600$$

Alternate Method

We know that to get a profit of R% after allowing a discount of r%, the marked price should be marked $\left(\frac{r + R}{100 - r} \times 100\right)$% more than its cost price.

Here, $r = 10\%$ and $R = 20\%$

∴ Required per cent

$$= \left(\frac{10 + 20}{100 - 10} \times 100\right)\% = \frac{100}{3}\%$$

and CP of the watch = ₹ 450

Hence, MP of the watch

$$= 450 \times \left(\frac{100 + \frac{100}{3}}{100}\right)$$

$$= \frac{450 \times 400}{300} = ₹\ 600$$

47. (*c*) Let MP = ₹ x

$$\therefore \quad \text{SP} = x \times \left(\frac{100 - 10}{100}\right) = ₹\ \frac{9}{10}x$$

$$\therefore \quad \text{CP} = \frac{9}{10}x\left(\frac{100}{100 + 20}\right) = ₹\ \frac{3}{4}x$$

Thus, CP has to be increased by

$$= \frac{x - \frac{3x}{4}}{\frac{3x}{4}} \times 100\% = \frac{\frac{x}{4}}{\frac{3x}{4}} \times 100\%$$

$$= 33\frac{1}{3}\%$$

Chapter

14

Simple Interest

When a person borrows some amount of money from another person or organisation (bank), then the person borrowing money (borrower) pays some extra money during repayment, that extra money during repayment is called **interest.** *For example* If A takes ? 50 from *B* and after using ? 50, A returns ? 55 to *B,* then *A* pays (55 - 50) *i.e.,* ? 5 as interest.

Let us consider following definitions before proceeding chapter

Principal *(P)* Principal is the money borrowed or deposited for a certain time.

Amount *(A)* The sum of principal and interest is called amount.

Amount = Principal + Simple Interest

Rate of Interest *(R)* It is the rate at which the interest is charged on principal. It is always specified in percentage terms.

Time (T) The period, for which the money is borrowed or deposited, is called time.

Simple Interest (Si)

If the interest is calculated on the original principal for any length of time, then it is called simple interest.

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

or $$\text{SI} = \frac{P \times R \times T}{100}$$

Basic Formulae Related to Simple Interest

- $P = \frac{100 \times A}{100 + RT}$
- $\text{SI} = \frac{ART}{100 + RT}$

where, SI = Simple Interest, P = Principal, R = Rate of interest, T = Time, A = Amount Ex. 1 Find the simple interest on ? 200 for 5 yr at 6% per annum.

Sol. Here, $P = ₹\, 200, \ T = 5 \text{ yr}, R = 6\%$

$\therefore \quad \text{SI} = \frac{P \times R \times T}{100} = \frac{200 \times 5 \times 6}{100} = ₹\, 60$

Ex. 2 In what time, X 1200 will become ? 1450 when annual rate of interest is 20%?

Sol. Here, $P = ₹\, 1200, A = ₹\, 1450, R = 20\%$

We know that, $A = P + \text{SI}$

$\Rightarrow \quad 1450 = 1200 + \text{SI}$

$\Rightarrow \quad \text{SI} = 1450 - 1200 = ₹\, 250$

Again, $\text{SI} = \frac{P \times R \times T}{100} \Rightarrow 250 = \frac{1200 \times 20 \times T}{100} = 240\,T$

$\therefore \quad T = \frac{25}{24} = 1\frac{1}{24} \text{ yr}$

Ex. 3 A sum at simple interest of 4% per annum amounts to ? 3120 in 5 yr. Find the sum.

Sol. Here, $T = 5\text{yr}, R = 4\%, A = ₹3120$

We know that, $P = \dfrac{100 \times A}{100 + RT} = \dfrac{100 \times 3120}{100 + 4 \times 5} = \dfrac{100 \times 3120}{120} = ₹2600$

Alternate Method

Let sum be P.

Then, $\text{SI} = \dfrac{P \times R \times T}{100} = \dfrac{P \times 4 \times 5}{100} = \dfrac{P}{5}$

$\therefore$ Amount $= P + \text{SI} = P + \dfrac{P}{5} = \dfrac{6P}{5}$

According to the question,

$$\text{Amount} = 3120 \Rightarrow \frac{6P}{5} = 3120$$

$\therefore$ $P = \dfrac{3120 \times 5}{6} = ₹2600$

Ex. 4 *A* takes ? 3000 from *B* for 2 yr at the rate of 10% half-yearly interest. What amount will be paid by *A* to *B* after the end of 2 yr?

Sol. Here, $P = ₹3000, T = 2\text{yr} = 4\text{ half-yr}, R = 10\% = \dfrac{10}{2}\%$

$$\text{SI} = \frac{P \times R \times T}{100} = \frac{3000 \times 4 \times 10}{100 \times 2} = ₹600$$

$\therefore$ Amount paid to $B = P + \text{SI} = 3000 + 600 = ₹3600$

MIND IT!

1. If rate of interest is half-yearly, then rate $= \left(\dfrac{R}{2}\right)\%$ and time $= 2T$

2. If rate of Interest is quarterly, then rate $= \left(\dfrac{R}{4}\right)\%$ and time $= 4T$

3. If rate of interest is monthly, then rate $= \left(\dfrac{R}{12}\right)\%$ and time $= 12T$

4. To calculate interest, the day on which amount is deposited, is not counted but the day on which amount is withdrawn, is counted.

Ex. 5 Amit takes some loan from Akash for 2 yr at the rate of 5% per annum and after 2 yr he gave back ? 6600 to Akash and completed the payment of his loan. Find the interest paid by Amit.

Sol. Here, $T = 2$ yr, $R = 5\%$, $A =$ ₹ 6600

We know that,

$$SI = \frac{ART}{100 + RT} = \frac{6600 \times 5 \times 2}{100 + 5 \times 2}$$

$$= \frac{6600 \times 10}{110} = ₹\ 600$$

Alternate Method

$$SI = \frac{P \times R \times T}{100} = \frac{P \times 5 \times 2}{100} = \frac{P}{10}$$

We know that, $A = P + SI$

$$P + \frac{P}{10} = 6600$$

$\Rightarrow$ $$\frac{11P}{10} = 6600$$

$\Rightarrow$ $$11P = 66000$$

$\therefore$ $$P = 6000$$

$\therefore$ Interest paid = Amount − Principal = 6600 − 6000 = ₹ 600

Instalments

When a borrower paid the total money in some equal parts (i.e., not in a single amount), then we say that he/she is paying in instalments

For example A borrowed ? 100 from *B* and he pays back it to *B* in several parts i.e., ? 20 in 5 times or ? 50 in 2 times etc. The important point is that borrower has to also pay the interest for using the borrowed sum or purchased article.

In general, the value of each instalment is kept constant even when the interest charged on each instalment vary for each instalment for *n* equal instalments we only calculate upto *(n* -1) term.

For simple interest

$$A = \left[x + \left(x + \frac{x \times R \times 1}{100}\right) + \left(x + \frac{x \times R \times 2}{100}\right) + \left(x + \frac{x \times R \times 3}{100}\right) + \ldots\right]$$

where, A = Total amount paid

x = Value of each instalment

Also, $$A = P + \frac{P \times n \times R}{100}$$

where, *P* is the principal

n is the number of instalments *R* is the rate of interest

Ex. 6 A scooty is sold by an automobile agency for ? 19200 cash or for ? 4800 cash down payment together with five equal monthly instalments. If the rate of interest charged by the company is 12% per annum, then find each instalment.

Sol. Balance of the price to be paid through instalments

$$P = 19200 - 4800 = 14400$$

Now, according to the formula,

$$A = \left[x + \left(\frac{x \times R \times 1}{100}\right) + \left(x + \frac{x \times R \times 2}{100}\right) + \dots + \left(x + \frac{x \times R \times 4}{100}\right)\right]$$

where, $$A = P + \frac{P \times n \times R}{100}$$

$$\Rightarrow \left(14400 + \frac{14400 \times 12 \times 5}{100 \times 12}\right) = \left[x + \left(x + \frac{12x}{12 \times 100}\right) + \left(x + \frac{12x \times 2}{12 \times 100}\right) + \dots + \left(x + \frac{12x \times 4}{12 \times 100}\right)\right]$$

$$\Rightarrow 15120 = 5x + \frac{x}{10}$$

$$\Rightarrow x = \frac{151200}{51}$$

$$\therefore x = ₹\ 2964.70$$

Note In the left hand side and right hand side, given amounts are equal. Each amount is equal to the total amount payable after 5 months.

Fast Track **Techniques**

to solve the **QUESTIONS**

Technique 1

If a sum of money becomes *n* times in *T* yr at simple interest, then formula for calculating rate of interest will be given as

$$R = \frac{100(n-1)}{T}\%$$

Ex. 7 A sum of money becomes four times in 20 yr at SI. Find the rate of interest. Sol. Given, $T = 20$ yr, Let sum = P Then, the sum after 20 yr = 4P

SI = 4P - $P = 3P$

Now, $\quad 3P = \frac{PRT}{100} = \frac{P \times R \times 20}{100}$

$\Rightarrow \quad 3 = \frac{20R}{100} = \frac{R}{5}$

$\therefore \quad R = 15\%$

Fast Track Method

Here, $T = 20$ yr, $n = 4$

$\therefore \quad R = \frac{100(n-1)}{T} = \frac{100(4-1)}{20} = \frac{100 \times 3}{20} = 15\%$

Technique 2

(i) if a sum of money at a certain rate of interest becomes *n* times in ^ yr and *m* times in T_2 yr, then formula for T_2 will be given as

$$T_2 = \left(\frac{m-1}{n-1}\right) \times T_1$$

(ii) If a sum of money in a certain time becomes *n* times at /?., rate of interest and *m* times at R_2 rate of interest, then formula for R_2 will be given as

$$R_2 = \left(\frac{m-1}{n-1}\right) \times R_1$$

Ex. 8 A sum becomes two times in 5 yr at a certain rate of interest. Find the time in which the same amount will be 8 times at the same rate of interest.

Sol. Let sum $= P$

Then, for 5 yr,

$$SI = 2P - P = P$$

$\because$ $$SI = \frac{P \times R \times T}{100}$$

$\therefore$ $$P = \frac{P \times R \times 5}{100} = \frac{PR}{20}$$

$\Rightarrow$ $$R = 20\%$$

Again, for another time (T),

$$SI = 8P - P = 7P$$

$\therefore$ $$7P = \frac{P \times 20 \times T}{100} = \frac{20\,TP}{100} = \frac{TP}{5}$$

$\therefore$ $$T = 7 \times 5 = 35 \text{ yr}$$

Fast Track Method

Here, $n = 2, m = 8, T_1 = 5, T_2 = ?$

$\therefore$ $$T_2 = \left(\frac{m-1}{n-1}\right) \times T_1 = \left(\frac{8-1}{2-1}\right) \times 5 = 35 \text{ yr}$$

Ex. 9 In a certain time, a sum becomes 3 times at the rate of 5% per annum. At what rate of interest, the same sum becomes 6 times in same duration?

Sol. Let sum = P

Then, for 5% rate of interest,

SI = 3P - $P = 2P$

$\Rightarrow$ $$2P = \frac{P \times 5 \times T}{100} = \frac{PT}{20}$$

$\therefore$ $$T = 40 \text{ yr}$$

Again, for another rate (R),

$$SI = 6P - P = 5P$$

$\Rightarrow$ $$5P = \frac{P \times R \times 40}{100} = \frac{2PR}{5}$$

$\therefore$ $$R = \frac{25}{2} = 12.5\%$$

Fast Track Method

Here, $n = 3, m = 6, R_1 = 5\%, R_2 = ?$

$\therefore$ $$R_2 = \left(\frac{m-1}{n-1}\right) \times R_1 = \left(\frac{6-1}{3-1}\right) \times 5 = \frac{5}{2} \times 5 = \frac{25}{2} = 12.5\%$$

Technique 3

If a certain sum P in a certain time amounts to ? A, at the rate of /?.,% and the same sum amounts to ? A_2 at the rate of R_2%, then

$$P = \left(\frac{A_2R_1 - A_1R_2}{R_1 - R_2}\right) \text{ and } T = \left(\frac{A_1 - A_2}{A_2R_1 - A_1R_2}\right) \times 100$$

Note If the rate of interest is uniform in above mentioned condition and time is variable, then

$$P = \left(\frac{A_2T_1 - A_1T_2}{T_1 - T_2}\right) \text{ and } R = \left(\frac{A_1 - A_2}{A_2T_1 - A_1T_2}\right) \times 100$$

where, 7} and T_2 are the time for first and second conditions, respectively

Ex. 10 A certain sum in certain time becomes *X* 500 at the rate of 8% per annum SI and the same sum amounts to *X* 200 at the rate of 2% per annum SI in the same duration. Find the sum and time.

Sol. According to the question,

$$A_1 - A_2 = 500 - 200$$

$$\left(P + \frac{P \times 8 \times T}{100}\right) - \left(P + \frac{P \times 2 \times T}{100}\right) = 500 - 200$$

$$\Rightarrow \quad \frac{6PT}{100} = 300$$

$$\Rightarrow \quad PT = \frac{300 \times 100}{6} = 5000$$

Again, for 8% rate, $SI = \frac{PTR}{100} = \frac{5000 \times 8}{100}$ [as $PT = 5000$]

$$\Rightarrow \quad SI = ₹\,400$$

$\therefore$ Sum $(P) = 500 - 400 = ₹\,100$ [as P = amount $-$ SI]

We have, $PT = 5000$

$$\therefore \quad T = \frac{5000}{P} = \frac{5000}{100} = 50 \text{ yr}$$

Fast Track Method

Here, $R_1 = 8\%$, $R_2 = 2\%$, $A_1 = ₹\,500$, $A_2 = ₹\,200$

Now, according to the formula,

$$P = \frac{A_2R_1 - A_1R_2}{R_1 - R_2} = \frac{200 \times 8 - 500 \times 2}{8 - 2} = \frac{1600 - 1000}{6} = \frac{600}{6} = ₹\,100$$

and

$$\text{Time, } T = \frac{A_1 - A_2}{A_2R_1 - A_1R_2} \times 100$$

$$= \frac{500 - 200}{200 \times 8 - 500 \times 2} \times 100 = \frac{300}{600} \times 100 = 50 \text{ yr}$$

Ex. 11 A certain sum at a certain rate of SI amounts to ? 1125 in 4 yr and ? 1200 in 7 yr. Find the sum and rate of interest.

Sol. According to the question,

$$\left(P + \frac{P \times R \times 7}{100}\right) - \left(P + \frac{P \times R \times 4}{100}\right) = 1200 - 1125$$

$$\Rightarrow \quad \frac{7PR}{100} - \frac{4PR}{100} = 75 \Rightarrow \frac{3PR}{100} = 75 \Rightarrow PR = 2500$$

For 4 yr, $\quad SI = \frac{PRT}{100} = \frac{2500 \times 4}{100} = ₹\,100$

$\therefore \quad$ Sum $= 1125 - 100 = ₹\,1025$

Again, we have $PR = 2500$

$$\therefore \quad R = \frac{2500}{1025} = 2.43\%$$

Fast Track Method

Here, $A_1 = ₹\,1125$, $A_2 = ₹\,1200$, $T_1 = 4$ yr, $T_2 = 7$ yr

According to the formula,

$$P = \frac{A_2T_1 - A_1T_2}{T_1 - T_2} = \frac{1200 \times 4 - 1125 \times 7}{4 - 7}$$

$$= \frac{4800 - 7875}{-3} = \frac{-3075}{-3} = ₹\,1025$$

We know that, $\quad R = \frac{A_1 - A_2}{A_2T_1 - A_1T_2} \times 100 = \frac{1125 - 1200}{1200 \times 4 - 1125 \times 7} \times 100$

$$= \frac{-7500}{4800 - 7875} = \frac{-7500}{-3075}$$

$\therefore \quad R = 2.43\%$

Technique 4

if Si for a certain sum P, for time ^ and rate of interest /?., is A, and Si for another sum P2 for time r2 and rate of interest *R*2 is /2, then Difference

$$\text{of SI} = I_2 - I_1 = \frac{P_2R_2T_2 - P_1R_1T_1}{100}$$

1. In above mentioned condition, if all the parameters are constant but time is variable, then

$$I_2 - I_1 = \frac{PR\,(T_2 - T_1)}{100}$$

2. When only rate of interest is variable, then $I_2 - I_1 = \frac{PT\,(R_2 - R_1)}{100}$

3. When only sum is variable, then $I_2 - I_1 = \frac{RT\,(P_2 - P_1)}{100}$

4. When only one parameter remains constant and the remaining are variables, then

(a) $I_2 - I_1 = \frac{P(R_2T_2 - R_1T_1)}{100}$ (b) $I_2 - I_1 = \frac{R(P_2T_2 - P_1T_1)}{100}$

(c) $I_2 - I_1 = \frac{T(P_2R_2 - P_1R_1)}{100}$

Ex. 12 Simple interest for the sum of ? 1500 is ? 50 in 4 yr and ? 80 in 8 yr. Find the rate of SI.

Sol. According to the question,

$$\frac{1500 \times R \times 8}{100} - \frac{1500 \times R \times 4}{100} = 80 - 50$$

$$\Rightarrow \quad \frac{6000\,R}{100} = 30$$

$$\Rightarrow \quad 60\,R = 30$$

$$\therefore \quad R = \frac{30}{60} = \frac{1}{2} = 0.5\%$$

Fast Track Method

Here, $I_1 = 50, I_2 = 80, T_1 = 4$ yr, $T_2 = 8$ yr, $P =$ ₹ 1500

According to the formula,

$$I_2 - I_1 = \frac{P \times R \times (T_2 - T_1)}{100}$$

$$\Rightarrow \quad 80 - 50 = \frac{1500 \times R\,(8 - 4)}{100}$$

$$\Rightarrow \quad 30 = 15 \times R \times 4$$

$$\Rightarrow \quad 1 = 2R \Rightarrow R = \frac{1}{2}$$

$$\therefore \quad R = 0.5\,\%$$

X. 13 Simple interest for a sum of ? 1550 for 2 yr is ? 20 more than the simple interest for *X* 1450 for the same duration. Find the rate of interest.

Sol. Given that, $P_1 = ₹1550, T_1 = 2, P_2 = ₹1450, T_2 = 2$

According to the question,

$$\Rightarrow \quad \frac{P_1 \times R \times T_1}{100} - \frac{P_2 \times R \times T_2}{100} = 20$$

$$\frac{1550 \times 2 \times R}{100} - \frac{1450 \times 2 \times R}{100} = 20$$

$$\Rightarrow \quad \frac{200\,R}{100} = 20$$

$$\therefore \quad R = 10\%$$

Fast Track Method

Here, $I_1 - I_2 = ₹\,20, P_1 = ₹\,1550, P_2 = ₹\,1450, T = 2\,\text{yr}$

According to the formula,

$$I_2 - I_1 = \frac{RT\,(P_2 - P_1)}{100}$$

$$\Rightarrow \quad -20 = \frac{R \times 2\,(1450 - 1550)}{100} \qquad \left[\begin{array}{l} \because I_1 - I_2 = 20 \\ \therefore I_2 - I_1 = 20 \end{array}\right]$$

$$\Rightarrow \quad -10 = \frac{-100\,R}{100}$$

$$\therefore \quad R = 10\%$$

Ex. 14 For a certain sum, the simple interest in 2 yr at 8% per annum is ? 90 more

than the simple interest in 1.5 yr at the rate of 10% per annum for the same sum. Find the sum.

Sol. According to the question,

$$\frac{P \times 8 \times 2}{100} - \frac{P \times 10 \times 1.5}{100} = 90 \Rightarrow \frac{P}{100} = 90$$

$$\therefore \quad P = ₹\,9000$$

Fast Track Method

Here, $I_1 - I_2 = ₹\,90, T_1 = 2\,\text{yr}, R_1 = 8\%, R_2 = 10\%, T_2 = 1.5\,\text{yr}$

According to the formula, $\quad I_2 - I_1 = \dfrac{P\,(R_2T_2 - R_1T_1)}{100}$

$$\Rightarrow \quad -90 = \frac{P}{100}(10 \times 1.5 - 8 \times 2)$$

$$\Rightarrow \quad P = \frac{-90 \times 100}{15 - 16} = ₹\,9000$$

Technique 5 If $\frac{1}{x}$ part of a certain sum P is lent out at $R_1\%$ SI, $\frac{1}{y}$ part is lent out at $R_2\%$ SI and the remaining $\frac{1}{z}$ part at $R_3\%$ SI and this way the interest received be I, then $P = \dfrac{I \times 100}{\dfrac{R_1}{x} + \dfrac{R_2}{y} + \dfrac{R_3}{z}}$

Ex. 15 Alok lent out a certain sum. He lent 1/3 part of his sum at 7% SI, 1/4 part at 8% SI and remaining part at 10% SI. If ? 510 is his total interest, then find the money lent out.

Sol. Let entire sum = P

Now, according to the question,

$$\frac{1}{3}P \times 7\% + \frac{1}{4}P \times 8\% + \left[1 - \left(\frac{1}{3} + \frac{1}{4}\right)\right] \times P \times 10\% = 510$$

$$\Rightarrow \frac{\frac{1}{3}P \times 7}{100} + \frac{\frac{1}{4}P \times 8}{100} + \frac{\frac{5}{12}P \times 10}{100} = 510$$

$$\Rightarrow \frac{7P}{3} + 2P + \frac{25P}{6} = 510 \times 100$$

$$\therefore P = \frac{510 \times 100 \times 6}{51} = ₹\,6000$$

Fast Track Method Here, $R_1 = 7\%, R_2 = 8\%, R_3 = 10\%$

and $\frac{1}{x} = \frac{1}{3}, \frac{1}{y} = \frac{1}{4}, I = ₹\,510$

$$\therefore \frac{1}{z} = \left[1 - \left(\frac{1}{3} + \frac{1}{4}\right)\right] = \frac{5}{12}$$

According to the formula,

$$P = \frac{I \times 100}{\frac{R_1}{x} + \frac{R_2}{y} + \frac{R_3}{z}} = \frac{510 \times 100}{\frac{7}{3} + \frac{8}{4} + \frac{50}{12}} = \frac{51000}{\frac{7}{3} + 2 + \frac{25}{6}} = \frac{51000}{51} \times 6 = ₹\,6000$$

Technique 6

A sum of ? P is lent out in n parts in such a way that the interest on first part at /?., % for 7"., yr, the interest on second part at R_2 % for T_2 yr and the interest on third part at R_3 % for T_3 yr and so on, are equal, then the ratio in which the sum was divided in n parts is given by

$$\frac{1}{R_1T_1} : \frac{1}{R_2T_2} : \frac{1}{R_3T_3} : \dots\dots : \frac{1}{R_nT_n}$$

Ex. 16 A sum of ? 7700 is lent out in two parts in such a way that the interest on one part at 20% for 5 yr is equal to that on another part at 9% for 6 yr. Find the two sums.

Sol. Let the first sum be ₹ x. Then, second sum = ₹ $(7700 - x)$

Now, according to the question,

$$\frac{x \times 20 \times 5}{100} = \frac{(7700 - x) \times 9 \times 6}{100}$$

$$\Rightarrow \quad 50x = (7700 - x) \times 27$$

$$\Rightarrow \quad 50x = 7700 \times 27 - 27x$$

$$\Rightarrow \quad 77x = 7700 \times 27$$

$$\Rightarrow \quad x = ₹\ 2700$$

$\therefore$ The second part $= (7700 - x) = 7700 - 2700 =$ ₹ 5000

Fast Track Method

Here, sum = 7700, $R_1 = 20\%$, $T_1 = 5$ yr, $R_2 = 9\%$, $T_2 = 6$ yr

Ratio of two sums $= \frac{1}{R_1T_1} : \frac{1}{R_2T_2} = \frac{1}{20 \times 5} : \frac{1}{9 \times 6}$

$$= \frac{1}{10 \times 5} : \frac{1}{9 \times 3} = \frac{1}{50} : \frac{1}{27} = 27 : 50$$

$\therefore$ First part $= \frac{27}{27 + 50} \times 7700 = \frac{27}{77} \times 7700 =$ ₹ 2700

Second part $= \frac{50}{27 + 50} \times 7700 = \frac{50}{77} \times 7700 =$ ₹ 5000

Technique 7

The annual payment that will discharge a debt of ? P due in T yr at the rate of interest $R\%$ per annum is given by

$$\frac{100P}{100T + \frac{RT(T-1)}{2}}$$

Ex. 17 What annual payment will discharge a debt of ? 848 in 8 yr at 8% per annum?

Sol. Here, $P =$ ₹ 848, $T = 8$ yr, $R = 8\%$

According to the formula,

Annual payment $= \dfrac{100\,P}{100\,T + \dfrac{RT\,(T-1)}{2}}$

$$= \frac{100 \times 848}{100 \times 8 + \frac{8 \times 8\,(8-1)}{2}}$$

$$= \frac{848 \times 100}{800 + 32 \times 7} = \frac{84800}{1024} = ₹\ 82.8125$$

Multi Concept

QUESTIONS

1. *A person invested some amount at the rate of 12% simple interest and the remaining at 10%. He received yearly an interest ofX 130. Had he interchanged the amount invested, he would have received an interest ofX 134. How much money did he invest at different rates?*

(a) ? 500 @ 10%, ? 800 @ 12% (b) ? 700 @ 10%, f 600 @ 12%

(c) ? 800 @ 10%, ? 400 @ 12% (d) ? 700 @ 10%, ? 500 @ 12%

➥ (*d*) Let ₹ x be invested at 12% per annum and ₹ y be invested at 10% per annum.

According to the question,

$$12\% \text{ of } x + 10\% \text{ of } y = 130$$

$$\Rightarrow \quad 12x + 10y = 13000 \quad \ldots \text{(i)}$$

and after interchanged the amount

$$10\% \text{ of } x + 12\% \text{ of } y = 134$$

$$10x + 12y = 13400 \quad \ldots \text{(ii)}$$

On solving Eqs. (i) and (ii) and evaluating x and y, we get

$$x = 500 \text{ and } y = 700$$

∴ ₹ 500 are invested at 12% per annum and ₹ 700 are invested at ₹ 10% per annum.

2. A private *finance company A claims to be lending money at simple interest. But the company includes the interest every 6 months for calculating principal. If company A is charging an interest of 10%, the effective rate of interest after 1 yr becomes*

(a) 10.25% (b) 12.50% (c) 11.25% (d) 10.75%

➥ (*a*) Let the sum be ₹ 100

$$\text{SI for first 6 months} = \frac{100 \times 10 \times \frac{1}{2}}{100} = ₹\ 5$$

Now, principal becomes 100 + 5 = 105

$$\therefore \quad \text{SI for last 6 months} = \frac{105 \times 10 \times 1}{100 \times 2} = ₹\ 5.25$$

Hence, amount at the end of 1 yr = 105 + 5.25 = ₹ 110.25

∴ Effective SI = 110.25 − 100 = ₹ 10.25%

$$\text{Effective rate } (R) = \frac{100 \times \text{SI}}{P \times T} = \frac{100 \times 10.25}{100 \times 1} = ₹\ 10.25\%$$

3. *The rates of simple interest in two banks x and y are in the ratio of 10:8. Rajni wants to deposit his total savings in two banks in such a way that she receive equal half-yearly interest from both. She should deposit the savings in banks x and y in the ratio of*

(u) 4:5 *(b)* 3:5 (c) 5:4 *(d)* 2:1

(a) Here, $R_1 = 10x$, $R_2 = 8x$ and $T_1 = T_2 = \frac{1}{2}$ yr

Let the savings be P and Q and rates of simple interest be 10x and 8x, respectively.

Then, $$\frac{P \times R_1 \times T_1}{100} = \frac{Q \times R_2 \times T_2}{100}$$

$$P \times 10x \times \frac{1}{2} \times \frac{1}{100} = Q \times 8x \times \frac{1}{2} \times \frac{1}{100} \Rightarrow 10P = 8Q$$

$$\Rightarrow \frac{P}{Q} = \frac{8}{10} = \frac{4}{5}$$

$$\therefore P : Q = 4 : 5$$

Fast Track Practice

Exercise© *Base Level Questions*

1. What would be the simple interest accrued in 4 yr on a principle of *X* 18440 at the rate 15% per annum?

[IBPS Clerk 2011]

(a) ? 11075 *(b) X* 12250

(C) ? 11500 *(d) X* 12985

(e) None of the above

2. What will be the simple interest on *X* 4000 at 12V£% per annum for the period from 4th February, 2005 to 18th April, 2005?

(a) X 215 *(b) X* 120

(c) X 200 *(d) X* 100

(e) None of the above

Rakesh lent out *X* 8750 at 7% annual interest. Find the simple interest in 3 yr. [SSC LDC 2007] *(a) X* 1870 *(b) X* 1837.50

(c) X 1560 *(d) X* 2200

What will be simple interest for 1 yr and

4 months on a sum of *X* 25800 at the rate of 14% per annum? [Delhi Police 2007]

(a) X 48-\6 (b) ?2580

(c) X 4816.75 *(d) X* 4815

5. A sum at simple interest of 13V£% per annum amounts to *X* 3080 in 4 yr. Find the sum.

(a) X 1550 *(b) X* 1680

(c) X 2000 *(d) X* 1850

(e) None of the above

6. The sum which amounts to *X* 364.80 in 8 yr at 3.5% simple interest per annum is [CDS 2011]

(a) X 285 *(b) X* 280

(c) X 275 *(d) X* 270

7. A sum of *X* 2668 amounts to *X* 4669 in

5 yr at the rate of simple interest. Find the rate per cent. [SSC CGL 2008]

(a) 15.2% *(b)* 14.9%

(c) 16% *(d)* 15%

"• Find the difference in amount and principal for *X* 4000 at the rate of 5% annual interest in 4 yr. [GIC 2007]

(a) X 865.50 *(b) X* 865

(C) X 400 *(d) X* 800

(e) None of the aobve

9. At the rate of 8H% per annum simple interest, a sum of *X* 5800 will earn how much interest in 2 yr 3 months?

(a) X 1300 *(b) X* 1109

(c) X 1509 *(d) X* 1725

(e) None of the above

10. Kriya deposits an amount of *X* 65800 to obtain a simple interest at the rate of 14% per annum for 4 yr. What total amount will Kriya get at the end of 4 yr?

[Bank Clerks 2009] *(a) X* 102648 *(b) X* 115246

(C) X 125578 *(d) X* 110324

(e) None of the above

11. A sum becomes its double in 10 yr. Find the annual rate of simple interest.

[Hotel Mgmt. 2007]

(a) 1% *(b)* 5% *(c)* 10% *(d)* 20% *(e)* None of the above

12. A certain sum becomes 3 fold at 4% annual rate of interest. At what rate, it will become 6 fold? [CBI 2008]

(a) 10% *(b)* 12%

(c) 8% *(d)* 9%

(e) None of the above

13. In a certain time, a sum becomes 4 times at the rate of 5% per annum. At what rate of simple interest, the same sum becomes 8 times in the same duration?

(a) $12\frac{2}{3}\%$ *(b)* $11\frac{3}{5}\%$

(c) $11\frac{2}{3}\%$ *(d)* $12\frac{3}{5}\%$

(e) None ol the above

14. In what time, does a sum of money become four fold at the simple interest rate of 10% per annum?

(a) 40 yr *(b)* 30 yr

(c) 15 yr *(d)* 25 yr

(e) None of the above

15. How long will a sum of money invested at 5% per annum SI take to increase its value by 50%?

(a) 10 yr *(b)* 12 yr

(c) 15 yr *(d)* 7 yr

(e) None of the above

16. A sum becomes 6 fold at 5% per annum. At what rate, the sum becomes 12 fold?

[LIC AAO 2009]

(a) 10% *(b)* 12% *(c)* 9% *(d)* 11% *(e)* None of the above

17. A sum becomes two fold in 6 yr at a certain rate of interest. Find the time, in which the same amount will be 10 fold at the same rate of interest.

(a) 35 yr *(b)* 49 yr *(c)* 59 yr *(d)* 54 yr *(e)* None of the above

18. At simple interest, a sum becomes 3 times in 20 yr. Find the time, in which the sum will be double at the same rate of interest. [RRB 2007]

(a) 8 yr *(b)* 10 yr *(c)* 12 yr *(d)* 14 yr

19. Ajay takes some loan from Rashmi at the rate of 5% per annum and after 2 yr, Ajay gave back *X* 8800 to Rashmi and this way paid his whole loan. Find the interest paid by Ajay.

(a) X 825 *(b) X* 975 *(c) X* 800 *(d) X* 850 *(e)* None of the above

20. A certain sum becomes *X* 600 in a certain time at the rate of 6% simple interest. The same sum amounts to *X* 200 at the rate of 1% simple interest in the same duration. Find the sum and time.

(a) ₹ 120 and $66\frac{2}{3}$ yr

(b) ₹ 150 and $66\frac{2}{3}$ yr

(c) ₹ 130 and $66\frac{2}{3}$ yr

(d) ₹ 160 and $66\frac{2}{3}$ yr

(e) None of the above

21. At a simple interest, a sum amounts to ₹ 1012 in $2\frac{1}{2}$ yr and becomes ₹ 1067.20 in 4 yr. What is the rate of interest? [SSC (10+2) 2009]

(a) 2.5% (b) 3% (c) 4% (d) 5

22. Simple interest for the sum of X 1500 is X 30 in 4 yr and ? 60 in 8 yr. Find the rate of simple interest.

(a) 2.5% (b) 1.5%

(c) 0.5% (d) 0.25%

(e) None of the above

23. Simple interest for the sum of X 1230 for 2 yr is ? 10 more than the simple interest for X 1130 for the same duration. Find the rate of interest. [SSC Multitasking 2012]

(a) 5% (b) 6%

(c) 8% (d) 2%

24. A certain sum becomes 8 fold in 15 yr at simple interest. What will be the rate of interest? [SSC (10+2)2012]

(a) $46\frac{5}{3}$% (b) $46\frac{2}{3}$%

(c) $46\frac{5}{8}$% (d) $46\frac{11}{12}$%

25. For a certain sum, the simple interest in 2 yr at 8% per annum is ? 110 more than the simple interest in 1 yr at the rate of 5% per annum for the same sum. Find the sum. [SSC Multitasking 2013]

(a) X 5000 (b) X 1000

(c) X 1050 (d) X 950

""•A sum was lent out for a certain time. The sum amounts to X 400 at 10% annual interest rate. When the sum was lent out at 4% annual interest rate, it amounts to X 200. Find the sum.

[SSC (10+2) 2008]

(a) ₹ $\frac{200}{3}$ (b) ₹ 100 (c) ₹ $\frac{400}{3}$ (d) ₹ $\frac{500}{3}$

27. A sum was invested for 4 yr at a certain rate of simple interest. If it had been invested at 2% more annual rate of interest, then X 56 more would have been obtained. What is the sum? [GIC 2007]

(a) X 680 *(b)* X 700

(c) X 720 *(d)* X 820

(e) None of the above

28. The simple interest on a certain sum of money for 2V4 yr at 12% per annum is X 20 less than the simple interest on the same sum for 3% yr at 10% per annum. Find the sum.

(a) X 800 *(b)* X 750 *(c) X 625(d)* X 400 *(e)* None of the above

29. At what rate per annum will the simple interest on a certain sum of money be 1/5 of the amount in 10 yr?

(a) 5% *(b) 2% (c)* 1% *(d)* 8% *(e)* None of the above

30. Mr. Deepak invested an amount of X 21250 for 6 yr. At what rate of simple interest, will he obtain the total amount of X 26350 at the end of 6 yr?

[SBI Clerk 2012]

(a) 6% pa *(b)* 5% pa

(c) 8% pa *(d)* 12% pa

(e) None of the above

31. The difference of simple interest from two banks for X 1000 in 2 yr is X 20. Find the difference in rates of interest.

[UP Police 2008]

(a) 2% *(b)* 1.5% *(c)* 1% *(d)* 2.5%

32. Raju lent *X* 400 to Ajay for 2 yr and *X* 100 to Manoj for 4 yr and received from both *X* 60 as collective interest. Find the rate of interest, simple interest being calculated. [SSC Assistant 2007]

(a) 5% *(b)* 6% *(c)* 8% *(d)* 9%

33.2/3 part of my sum is lent out at 3%, 1/6 part is lent out at 6% and remaining part is lent out at 12%. All the three parts are lent out at simple interest. If the annual income is *X* 25, what is the sum? [Bank Clerks 2009]

[a) *X* 500 *(b)* *X* 650 *(c)* *X* 600 *(d)* *X* 450

(e) None of the above

34. A sum of *X* 1521 is lent out in two parts in such a way that the interest on one part at 10% for 5 yr is equal to that of another part at 8% for 10 yr. What will be the two parts of sum? [Hotel Mgmt. 2007]

(a) *X* 926 and *X* 595

(b) *X* 906 and ? 615

(c) ? 916 and *X* 605

(d) *X* 936 and *X* 585

(e) None of the above

35.Rashmi lent *X* 600 to Geeta for 2 yr and ? 150 to Seeta for 4 yr and received altogether *X* 80 as simple interest from both. Find the rate of interest.

(a) $3\frac{4}{9}\%$ *(b)* $2\frac{4}{9}\%$ *(c)* $5\frac{4}{9}\%$ *(d)* $4\frac{4}{9}\%$

(e) None of the above

36. Harsha makes a fixed deposit of *X* 20000 in Bank of India for a period of 3 yr. If the rate of interest be 13% SI per annum charged half-yearly, what amount will he get after 42 months? [SSC CPO 2007]

(a) *X* 27800 *(b)* *X* 28100

(c) *X* 29100 *(d)* *X* 30000

37. The simple interest on a sum of money is 1/16 of the principal and the number of years is equal to the rate per cent annum. Find the rate per cent.

(a) $2\frac{1}{2}\%$ (b) $3\frac{1}{2}\%$ (c) $4\frac{1}{2}\%$ (d) $8\frac{1}{2}\%$

(e) None of the above

38. The simple interest on *X* 4000 in 3 yr at the rate of *x%* per annum equals to the simple interest on *X* 5000 at the rate of 12% per annum in 2 yr. The value of *x* is

[SSC CCL 2013] *(a)* 6% *(b)* 8%

(c) 9% *(d)* 10%

39. The simple interest on a sum of money is 1/144 of the principal and the number of years is equal to the rate per cent per annum. What will be the rate per cent per annum?

(a) $\frac{3}{5}\%$ (b) $\frac{5}{6}\%$ (c) $\frac{7}{6}\%$ (d) $\frac{1}{6}\%$

(e) None of the above

40. Suresh borrowed *X* 800 at 6% and Naresh borrowed *X* 600 at 10%. After how much time, will they both have equal debts?

[SSC CCL 2008]

(a) $15\frac{1}{3}$ yr (b) $14\frac{1}{2}$ yr

(c) $18\frac{1}{3}$ yr (d) $16\frac{2}{3}$ yr

41. What annual payment will discharge a debt of *X* 1696 in 4 yr at 4% per annum?

(a) *X* 525 *(b)* *X* 425

(c) *X* 325 *(d)* *X* 400

(e) None of the above

42. What annual payment will discharge a debt of *X* 1092 due in 2 yr at 12% simple interest?

(a) X 725 *(b) X* 325 *(c) X tt(d) X* 900 *(e) X* 400

43. The annual payment of *X* 160 in 5 yr at 5% per annum simple interest will discharge a debt of

(a) X 980 *(b) X* 880 *(c) X 440(d) X* 220 *(e)* None of the above

Exercise © *Higher Skill Level Questions*

1. A sum of money amounts to *X* 2240 at 4% per annum simple interest in 3 yr. The interest on the same sum for 6 months at 3.5% per annum is

[SSC CCL 2013] *(a) X* 30 *(b) X* 50 *(c) X* 35 *(d) X* 150

2. The simple interest on a sum of money at 9% per annum for 5 yr is half the sum. What is the sum?

(a) X 5800

(b) X 5000

(c) X 7000

(d) Data is inadequate

3. A certain sum at simple interest amounts to *X* 1350 in 5 yr and to *X* 1620 in 8 yr. What is the sum? [CDS 2011]

(a) X 700 *(b) X* 800

(c) X 900 *(d) X* 1000

4. The principal on which a simple interest of ₹ 55 will be obtained after 9 months at the rate of $3\frac{2}{3}$% per annum is [CDS 2013]

(a) 1000 *(b)* 1500 *(c)* 2000 *(d)* 2500

5. The simple interest on a sum of money will be *X* 200 after 5 yr. In the next 5 yr, principal is tripled. What will be the total interest at the end of the 10th yr?

(a) X 650 *(b) X* 850 *(c) X* 800 *(d) X* 750

6. Ramesh invested an amount that is 10% of *X* 10000 at simple interest. After 3 yr, the amount becomes *X* 2500. Find out the 4 times of actual interest rate.

(a) 5000% *(b)* 250%

(c) 200% *(d)* 600%

(e) None of the above

7. The simple interest on a sum of money at 8% per annum for 6 yr is half the sum.What is the sum? [RBI 2008]

(a) X 4800 *(b) X* 6000

(c) X 8000

(d) Data is inadequate

(e) None of the above

8. In 4 yr, ? 6000 amounts to *X* 8000. In what time at the same rate, will *X* 525 amount to *X* 700? [SNAP 2012]

(a) 2 yr *(b)* 3 yr *(c)* 4 yr fdj 5 yr *(e)* None of the above

9. A principal amounts to *X* 944 in 3 yr and to *X* 1040 in 5 yr, each sum being invested at the same simple interest. The principal was [SSC CCL 2013]

(a) X 800 *(b) X* 991 *(c) X 750(d) X* 900

10. What will be the ratio of simple interest earned by certain amount at the same rate of interest for 12 yr and for 18 yr?

(a) 2 : 5 *(b)* 1 : 3 *(c)* 2 : 3 *(d)* 3 : 1 *(e)* None of the above

11. A sum of *X* 1550 was lent partly at 5% and partly at 8% per annum simple interest. The total interest received after 4 yr was *X* 400. The ratio of the money lent at 5% to that lent at 8% is

[SSCCPO2013] *(a)* 16 : 15 *(b)* 17 : 15

(c) 16 : 13 *(d)* 16 : 19

12. Neeta borrowed some money at the rate of 6% per annum for the first 3 yr, at the rate of 9% per annum for the next 5 yr and at the rate of 13% per annum for the period beyond 8 yr. If she pays a total interest of ? 8160 at the end of 11 yr, how much money did she borrow?

[Bank PO 2008]

(a) *X* 12000

(b) *X* 10000

(c) *X* 8000

(d) Data is inadequate

(e) None of the above

13. Reena had *X* 10000 with her. Out of this money, she lent some money to Akshay for 2 yr at 15% simple interest. She lent remaining money to Brijesh for an equal number of years at the rate of 18%. After 2 yr, Reena found that Akshay had given her *X* 360 more as interest as compared to Brijesh. The amount of money which Reena had lent to Brijesh must be

(a) *X* 4000 *(b)* *X* 2500

(c) *X* 3500 *(d)* *X* 4200

(e) None of the above

14. Mr. Pawan invests an amount of *X* 24200 at the rate of 4% per annum for 6 yr to obtain a simple interest, later he invests the principal amount as well as the amount obtained as simple interest for another 4 yr at the same rate of interest. What amount of simple interest will be obtained at the end of the last 4 yr?

[Bank Clerks 2009] *(a)* *X* 4800 *(b)* *X* 4850.32

(c) *X* 4801.28 *(d)* *X* 4700

(e) None of the above

15. A person invests *X* 12000 as fixed deposit at a bank at the rate of 10% per annum simple interest. But due to some pressing needs, he has to withdraw the entire money after 3 yr, for which the bank allowed him a lower rate of interest. If he gets *X* 3320 less than, what he would have got at the end of 5 yr, the rate of interest allowed by **bank is [SSC (10+2)2012]**

(a) $7\frac{8}{9}\%$ (b) $8\frac{7}{9}\%$

(c) $7\frac{8}{9}\%$ (d) $7\frac{7}{9}\%$

16. Rajnish invested certain sum in three different schemes *P, Q* and *R* with the rates of interest 10% per annum, 12% per annum and 15% per annum, respectively. If the total interest accrued in 1 yr was *X* 3200 and the amount invested in scheme *R* was 150% of the amount invested in scheme *Q,* what was the amount invested in scheme Q? *(a)* *X* 8000 *(b)* *X* 9000

(c) *X* 5000 *(d)* *X* 3050

(e) None of the above

Answer with Solutions

Exercise© *Base Level Questions*

1. (*c*) Given, $P = 18440$, $R = 15\%$, $T = 4$

$\therefore$ Simple interest (SI) $= \dfrac{T \times P \times R}{100}$

$$= \frac{4 \times 18440 \times 15}{100} = ₹\ 11064$$

2. (*d*) Here, $P = ₹\ 4000$, $R = 12\frac{1}{2}\% = \frac{25}{2}\%$

$$T = (24 + 31 + 18) \text{ days} = 73 \text{ days}$$

$$\Rightarrow \quad = \frac{73}{365} \text{ yr} = \frac{1}{5} \text{ yr}$$

$$\therefore \text{ SI} = \frac{P \times R \times T}{100} = 4000 \times \frac{25}{2} \times \frac{1}{5} \times \frac{1}{100}$$

$$= ₹\ 100$$

3. (*b*) Given, $P = ₹\ 8750$, $R = 7\%$, $T = 3$ yr

According to the formula,

$$\text{SI} = \frac{P \times R \times T}{100} = \frac{8750 \times 7 \times 3}{100}$$

$$= ₹\ 1837.50$$

4. (*a*) Here, $P = ₹\ 25800$, $R = 14\%$

$$T = 1 \text{ yr } 4 \text{ months}$$

$$= \left(1 + \frac{4}{12}\right) = \left(1 + \frac{1}{3}\right) = \frac{4}{3} \text{ yr}$$

According to the formula,

$$\text{SI} = \frac{P \times R \times T}{100} = \frac{25800 \times 14 \times \frac{4}{3}}{100}$$

$$= \frac{258 \times 14 \times 4}{3} = ₹\ 4816$$

5. (*c*) Let sum $= P$

Then, $\text{SI} = \dfrac{P \times R \times T}{100}$

$$= \frac{P \times \frac{27}{2} \times 4}{100}$$

$$= \frac{P \times 54}{100} = \frac{27P}{50}$$

$$\therefore \text{ Amount} = P + \frac{27P}{50} = \frac{77P}{50}$$

According to the question,

$$\frac{77P}{50} = 3080$$

$$\therefore \quad P = \frac{3080 \times 50}{77} = ₹\ 2000$$

Alternate Method

Here, $T = 4$ yr, $R = 13\frac{1}{2}\% = \frac{27}{2}\%$

$A =$ ₹ 3080, $P = ?$

According to the formula,

$$P = \frac{100 \times A}{100 + RT}$$

$$= \frac{100 \times 3080}{100 + \frac{27}{2} \times 4}$$

$$= \frac{100 \times 3080}{154}$$

$$= 100 \times 20 = ₹\ 2000$$

6. (*a*) Given, $t = 8$ yr, $r = 3.5\%$, $A =$ ₹ 364.80

Let amount = ₹ P

Since, $A = P\left(1 + \frac{RT}{100}\right)$

$\therefore \quad 364.80 = P\left(1 + 3.5 \times \frac{8}{100}\right)$

$\Rightarrow \quad 364.80 = P\left(1 + \frac{35 \times 8}{1000}\right)$

$\Rightarrow \quad \frac{3648}{10} = P\left(\frac{128}{100}\right)$

$\Rightarrow \quad P = \frac{3648}{128} = ₹\ 285$

7. (*d*) Here, $P =$ ₹ 2668, $T = 5$ yr, $A =$ ₹ 4669

We know that,

Amount (A) = Principal (P) + (Simple Interest) SI

$4669 = 2668 + \text{SI}$

$\Rightarrow \quad \text{SI} = 4669 - 2668 = ₹\ 2001$

Again, $\text{SI} = \frac{P \times R \times T}{100}$

$\therefore \quad 2001 = \frac{2668 \times R \times 5}{100}$

$\Rightarrow R = \frac{2001 \times 100}{2668 \times 5} = \frac{2001 \times 5}{667} = 15\%$

Alternate Method

Here, $A = 4669$, $P = 2668$, $T = 5$, $R = ?$

According to the formula,

$$P = \frac{100 \times A}{100 + RT}$$

$$\Rightarrow \quad \frac{2668}{1} = \frac{100 \times 4669}{100 + 5R}$$

$$\Rightarrow \quad 266800 + 13340R = 466900$$

$$\Rightarrow \quad 13340R = 466900 - 266800$$

$$\Rightarrow \quad R = \frac{200100}{13340} \Rightarrow R = 15\%$$

8. (*d*) The required difference in amount and principal is SI $= A - P$.

Here, $P =$ ₹ 4000, $R = 5\%$, $T = 4$ yr

According to the formula,

$$\text{SI} = \frac{P \times R \times T}{100}$$

$$= \frac{4000 \times 5 \times 4}{100} = ₹\ 800$$

9. (*b*) Time = 2 yr 3 months $= 2\frac{1}{4}$ yr $= \frac{9}{4}$ yr

$$\therefore \quad \text{SI} = 5800 \times \frac{17}{2} \times \frac{9}{4} \times \frac{1}{100}$$

$$= \frac{8874}{8} = ₹\ 1109.25 \approx ₹\ 1109$$

10. (*a*) Here, $P =$ ₹ 65800, $R = 14\%$, $T = 4$ yr

Hence, $\quad \text{SI} = \frac{65800 \times 14 \times 4}{100} = ₹\ 36848$

$\therefore$ Required amount $= P +$ SI

$= 65800 + 36848 =$ ₹ 102648

Alternate Method

Here, $P = 65800$, $R = 14\%$, $T = 4$ yr

$$\Rightarrow \quad P = \frac{100 \times A}{100 + RT}$$

$$\Rightarrow \quad 65800 = \frac{100 \times A}{100 + 14 \times 4}$$

$$\Rightarrow \quad 100A = 65800 \times 156$$

$$\Rightarrow \quad A = 658 \times 156$$

$$\Rightarrow \quad A = 102648$$

11. (*c*) Let the sum be P.

Then, after 10 yr

Sum $= 2P$

$$\therefore \quad \text{SI} = 2P - P = P$$

Now, SI $= \frac{P \times R \times T}{100} \Rightarrow P = \frac{P \times R \times 10}{100}$

$$\therefore \quad R = 10\%$$

Fast Track Method

Here, $n = 2$, $T = 10$ yr

$\therefore \quad R = \frac{100\,(n-1)}{T}$ **[by Technique 1]**

$$= \frac{100\,(2-1)}{10} = \frac{100}{10} = 10\%$$

12. (*a*) Let the sum be P.

Then, for 4%

$$\text{SI} = (3P - P) = 2P$$

$\therefore \quad 2P = \dfrac{P \times 4 \times T}{100} \Rightarrow 1 = \dfrac{2T}{100} = \dfrac{T}{50}$

$\therefore \quad T = 50$ yr

Again, for another rate (R),

$$SI = (6P - P) = 5P$$

$\therefore \quad 5P = \dfrac{P \times R \times 50}{100} = \dfrac{PR}{2}$

$\therefore \quad R = 10\%$

Fast Track Method

Here, $n = 3$, $m = 6$, $R_1 = 4\%$

$\therefore \quad R_2 = \dfrac{m-1}{n-1} \times R_1$ **[by Technique 2]**

$$= \frac{6-1}{3-1} \times 4 = \frac{5}{2} \times 4 = 10\%$$

13. (*c*) Let the sum be P.

Then, for $R = 5\%$

$$SI = (4P - P) = 3P$$

$\therefore \quad 3P = \dfrac{P \times 5 \times T}{100} = \dfrac{PT}{20}$

$\Rightarrow \quad T = 60$ yr

Again, for another rate (R),

$$SI = (8P - P) = 7P$$

$\therefore \quad 7P = \dfrac{P \times R \times 60}{100}$

$\Rightarrow \quad R = \dfrac{7 \times 100}{60} = \dfrac{35}{3}\% = 11\dfrac{2}{3}\%$

Fast Track Method

Here, $n = 4$, $m = 8$, $R_1 = 5\%$, $R_2 = ?$

$\therefore \quad R_2 = \dfrac{m-1}{n-1} \times R_1$ **[by Technique 2]**

$$= \frac{8-1}{4-1} \times 5$$

$$= \frac{7 \times 5}{3} = \frac{35}{3} = 11\frac{2}{3}\%$$

14. (*b*) As P becomes $4P$ in time T.

$\therefore \quad SI = (4P - P) = 3P$

Now, $\quad 3P = \dfrac{P \times R \times T}{100}$

$\Rightarrow \quad 3 = \dfrac{10 \times T}{100} \Rightarrow T = 30$ yr

Fast Track Method

Here, $n = 4$, $R = 10$, $T = ?$

According to the formula,

$R = \dfrac{100\,(n-1)}{T}$ **[by Technique 1]**

$\Rightarrow \quad 10 = \dfrac{100\,(4-1)}{T} = \dfrac{100 \times 3}{T}$

$\therefore \quad T = 30$ yr

15. (*a*) Let sum be P.

$\therefore$ 50% of $P = \dfrac{P}{2} = \text{SI}$

Now, $\dfrac{P}{2} = \dfrac{P \times 5 \times T}{100}$ [as time = 10 yr]

$\Rightarrow \quad \dfrac{P}{2} = \dfrac{5PT}{100} \Rightarrow \dfrac{1}{2} = \dfrac{T}{20}$

$\therefore \quad T = 10$ yr

16. (*d*) SI at 5% $= 6P - P = 5P$

$\therefore \quad 5P = \dfrac{P \times 5 \times T}{100}$

$\Rightarrow \quad T = 100$ yr

Now, for new rate (R),

$$11P = \frac{P \times R \times 100}{100}$$

$\therefore \quad R = 11\%$

Fast Track Method

Hence, $R_1 = 5\%$, $n = 6$, $m = 12$

According to the formula,

$R_2 = \dfrac{m-1}{n-1} \times R_1$ **[by Technique 2]**

$= \dfrac{12-1}{6-1} \times 5 = \dfrac{11}{5} \times 5 = 11\%$

17. (*d*) Let sum $= P$

Then, for 6 yr

$$\text{SI} = (2P - P) = P$$

$\therefore \quad P = \dfrac{P \times R \times 6}{100} = \dfrac{3PR}{50}$

$\therefore \quad R = \dfrac{50}{3}\%$

Again, for another time (T),

$$\text{SI} = 10P - P = 9P$$

$\therefore \quad 9P = \dfrac{P \times \dfrac{50}{3} \times T}{100}$

$\Rightarrow \quad 9P = \dfrac{P \times 50 \times T}{300}$

$\Rightarrow \quad 9 = \dfrac{T}{6}$

$\therefore \quad T = 54$ yr

Fast Track Method

Here, $n = 2$, $m = 10$, $T_1 = 6$ yr, $T_2 = ?$

$\therefore \quad T_2 = \dfrac{m-1}{n-1} \times T_1$ **[by Technique 2]**

$= \dfrac{10-1}{2-1} \times 6 = 54$ yr

18. (*b*) Let sum $= P$, then for 20 yr,

$$\text{SI} = (3P - P) = 2P$$

$\therefore \quad 2P = \dfrac{P \times R \times 20}{100}$

$\Rightarrow \qquad 2 = \frac{R}{5}$

$\Rightarrow \qquad R = 10\%$

Again, for another time (T),

$$SI = (2P - P) = P$$

$\therefore \qquad P = \frac{P \times 10 \times T}{100}$

$\therefore \qquad T = 10$ yr

Fast Track Method

Here, $n = 3$, $m = 2$, $T_1 = 20$ yr

$\therefore \qquad T_2 = \frac{m-1}{n-1} \times T_1$ **[by Technique 2]**

$$= \frac{2-1}{3-1} \times 20 = 10 \text{ yr}$$

19. (*c*) $SI = \frac{P \times R \times T}{100} = \frac{P \times 5 \times 2}{100} = \frac{P}{10}$

According to the question,

$$P + \frac{P}{10} = 8800$$

$\Rightarrow \qquad \frac{11P}{10} = 8800$

$\Rightarrow \qquad P = \frac{88000}{11}$

$\therefore \qquad P = ₹\ 8000$

$\therefore$ Interest paid $= 8800 - 8000 = ₹\ 800$

Alternate method

Here, $A = ₹\ 8800$, $T = 2$ yr, $R = 5\%$

We know,

$$SI = \frac{ART}{100 + RT} = \frac{8800 \times 5 \times 2}{100 + 5 \times 2}$$

$$= \frac{8800 \times 10}{110} = ₹\ 800$$

20. (*a*) According to the question,

$$\left(P + \frac{P \times 6 \times T}{100}\right) - \left(P + \frac{P \times 1 \times T}{100}\right)$$

$$= 600 - 200$$

$\Rightarrow \qquad \frac{5PT}{100} = 400$

$\Rightarrow \qquad PT = 8000$

Again, for 6% rate,

$$SI = \frac{PTR}{100} = \frac{8000 \times 6}{100} = ₹\ 480$$

$\therefore$ Sum $= 600 - 480 = ₹\ 120$

As we have, $PT = 8000$

$\therefore \qquad T = \frac{8000}{120} = \frac{200}{3} = 66\frac{2}{3}$ yr

Fast Track Method

Here, $R_1 = 6\%$, $R_2 = 1\%$, $A_1 = ₹\ 600$,

$A_2 = ₹\ 200$

$$P = \frac{A_2R_1 - A_1R_2}{R_1 - R_2} \text{ [by Technique 3]}$$
$$= \frac{200 \times 6 - 600 \times 1}{6 - 1}$$
$$= \frac{1200 - 600}{5} = \frac{600}{5} = ₹\,120$$
$$\text{Time } T = \frac{A_1 - A_2}{A_2R_1 - A_1R_2} \times 100$$
$$= \frac{600 - 200}{1200 - 600} \times 100$$
$$= \frac{400 \times 100}{600} = \frac{200}{3} = 66\frac{2}{3} \text{ yr}$$

21. (c) Given, $T_1 = 2\frac{1}{2}$ yr, $T_2 = 4$ yr

According to the question,
$$\left(P + \frac{P \times R \times 4}{100}\right) - \left(P + \frac{P \times R \times 2.5}{100}\right)$$
$$= 1067.20 - 1012 = 55.2$$
$$\Rightarrow \quad \frac{1.5\,PR}{100} = 55.2$$
$$\Rightarrow \quad PR = \frac{552 \times 100}{15} = 3680$$
For 4 yr,
$$\text{SI} = \frac{PRT}{100} = \frac{3680 \times 4}{100} = ₹\,147.2$$
$\therefore$ Sum $(P) = 1067.2 - 147.2 = ₹\,920$

We have, $PR = 3680$
$$\therefore \quad R = \frac{3680}{P} = \frac{3680}{920} = 4\%$$

Fast Track Method

Here, $T_1 = 2\frac{1}{2}$ yr $= 2.5$ yr, $T_2 = 4$ yr ,

$A_2 = ₹\,1067.20$, $A_1 = 1012$
$$\therefore \quad R = \frac{A_1 - A_2}{A_2T_1 - A_1T_2} \times 100$$
[by Technique 3]
$$= \frac{1012 - 106720}{(10672 \times 2.5) - (1012 \times 4)} \times 100$$
$$= \frac{-5520}{2668 - 4048} \times 100 = \frac{-5520}{-1380}$$
$$\therefore \quad R = 4\%$$

22. (c) Given, $\text{SI}_2 = 60$, $\text{SI}_1 = 30$, $T_1 = 4$ yr, $T_2 = 8$ yr

According to the question,
$$\frac{1500 \times R \times 8}{100} - \frac{1500 \times R \times 4}{100} = 60 - 30$$
$$\Rightarrow \quad \frac{6000\,R}{100} = 30$$
$$\therefore \quad R = \frac{30}{60} = \frac{1}{2} = 0.5\%$$

Fast Track Method

Here, $I_1 = ₹\ 30$, $I_2 = ₹\ 60$, $T_1 = 4$ yr,
$T_2 = 8$ yr, $P = ₹\ 1500$

$$I_2 - I_1 = \frac{P \times R\,(T_2 - T_1)}{100}$$

[by Technique 4]

$$\Rightarrow \quad 60 - 30 = \frac{1500 \times R\,(8-4)}{100}$$

$$\Rightarrow \quad 30 = 15 \times R \times 4$$

$$\therefore \quad R = \frac{30}{4 \times 15} = \frac{1}{2} = 0.5\%$$

23. (*a*) According to the question,

$$\frac{1230 \times 2 \times R}{100} - \frac{1130 \times 2 \times R}{100} = 10$$

$$\Rightarrow \quad \frac{200}{100} R = 10$$

$$\therefore \quad R = 5\%$$

Fast Track Method

Here, $I_1 - I_2 = ₹\ 10$, $P_1 = ₹\ 1230$,
$P_2 = ₹\ 1130$, $T = 2$ yr

According to the formula,

$$I_2 - I_1 = \frac{RT\,(P_2 - P_1)}{100}$$ [by Technique 4]

$$\Rightarrow \quad -10 = \frac{R \times 2\,(1130 - 1230)}{100}$$

$$\Rightarrow \quad -10 = \frac{-200\,R}{100}$$

$$\therefore \quad R = 5\%$$

24. (*b*) Let sum $= P$

Then, after 15 yr,

$$\text{Sum} = 8P$$

$$\therefore \quad \text{SI} = 8P - P = 7P$$

Now, $$7P = \frac{P \times R \times 15}{100}$$

$$\Rightarrow \quad 7 = \frac{15R}{100} = \frac{3R}{20}$$

$$\therefore \quad R = \frac{20 \times 7}{3}$$

$$= \frac{140}{3} = 46\frac{2}{3}\%$$

Fast Track Method

Here, $n = 8$, $T = 15$ yr

$$R = \frac{100\,(n-1)}{T}$$ [by Technique 1]

$$\Rightarrow \quad R = \frac{100\,(8-1)}{15}$$

$$= \frac{100 \times 7}{15}$$

$$= \frac{20 \times 7}{3} = \frac{140}{3} = 46\frac{2}{3}\%$$

25. (*b*) Given, $SI_1 - SI_2 = 110$, $T_1 = 2$ yr,

$T_2 = 1$ yr, $R_1 = 8\%$, $R_2 = 5\%$

According to the question,

$$\frac{P \times 8 \times 2}{100} - \frac{P \times 5 \times 1}{100} = 110$$

$$\Rightarrow \quad \frac{11P}{100} = 110$$

$$\therefore \quad P = ₹\,1000$$

Alternate Method

Given, $I_1 - I_2 = ₹\,110$, $T_1 = 2$ yr

$R_1 = 8\%$, $R_2 = 5\%$, $T_2 = 1$ yr

According to the formula,

$$I_2 - I_1 = \frac{P\,(R_2T_2 - R_1T_1)}{100}$$

$$\Rightarrow \quad -110 = \frac{P\,(5 \times 1 - 8 \times 2)}{100}$$

$$\Rightarrow \quad -110 = \frac{P\,(5 - 16)}{100} = \frac{-11P}{100}$$

$$\therefore \quad P = ₹\,1000$$

26. (*a*) According to the question,

$$\left(P + \frac{P \times 10 \times T}{100}\right) - \left(P + \frac{P \times 4 \times T}{100}\right) = 400 - 200$$

$$\Rightarrow \quad \frac{6PT}{100} = 200$$

$$\Rightarrow \quad PT = \frac{200 \times 100}{6} = \frac{10000}{3}$$

Again, for 10% rate,

$$SI = \frac{P \times 10 \times T}{100} = \frac{\frac{10000}{3} \times 10}{100} = \frac{1000}{3}$$

$$\therefore \text{ Sum } (P) = 400 - \frac{1000}{3} = \frac{1200 - 1000}{3}$$

$$= ₹\,\frac{200}{3}$$

Fast Track Method

Given, $R_1 = 10\%$, $R_2 = 4\%$

$A_1 = ₹\,400$, $A_2 = ₹\,200$

According to the formula,

$$P = \frac{A_2R_1 - A_1R_2}{R_1 - R_2} \quad \text{[by Technique 3]}$$

$$= \frac{200 \times 10 - 400 \times 4}{10 - 4}$$

$$= \frac{2000 - 1600}{6} = \frac{400}{6} = ₹\,\frac{200}{3}$$

27. (*b*) According to the question,

$$\frac{P \times (R + 2) \times 4}{100} - \frac{P \times R \times 4}{100} = 56$$

$\Rightarrow \quad \dfrac{4PR + 8P - 4PR}{100} = 56$

$\Rightarrow \quad \dfrac{8P}{100} = 56$

$\therefore \quad P = \dfrac{56 \times 100}{8} = ₹\ 700$

Fast Track Method

$I_2 - I_1 = ₹\ 56$, $R_2 - R_1 = 2\%$, $T = 4$ yr

According to the formula,

$I_2 - I_1 = \dfrac{PT\,(R_2 - R_1)}{100}$ [by Technique 4]

$\Rightarrow \quad 56 = \dfrac{P \times 4 \times 2}{100}$

$\therefore \quad P = \dfrac{56 \times 100}{8} = ₹\ 700$

28. (*d*) Given, $T_1 = \dfrac{5}{2}$, $R_1 = 12\%$, $T_2 = \dfrac{7}{2}$ yr

and $R_2 = 10\%$

Let the sum be P.

Then, $\dfrac{P \times 10 \times 7}{100 \times 2} - \dfrac{P \times 12 \times 5}{100 \times 2} = 20$

$\Rightarrow \quad \dfrac{7P}{20} - \dfrac{3P}{10} = 20$

$\therefore \quad P = 20 \times 20 = ₹\ 400$

29. (*b*) Let sum $= P$, then SI $= \dfrac{P}{5}$, time $= 10$ yr

$\therefore$ Rate $= \dfrac{100 \times P}{P \times 5 \times 10} = 2\%$

30. (*e*) SI $= 26350 - 21250 = ₹\ 5100$

$\therefore \quad \text{Rate} = \dfrac{\text{SI} \times 100}{\text{Principal} \times \text{Time}}$

$= \dfrac{5100 \times 100}{21250 \times 6} = 4\%$

Alternate Method

Given, $P = 21250$, $A = 26350$, $T = 6$ yr

$R = ?$

$\Rightarrow \quad P = \dfrac{A \times 100}{100 + RT}$

$\Rightarrow \quad 21250 = \dfrac{26350 \times 100}{100 + R \times 6}$

$\Rightarrow \quad 2125000 + 127500R = 2635000$

$\Rightarrow \quad 127500R = 2635000 - 2125000$

$\Rightarrow \quad R = \dfrac{510000}{127500}$

$\Rightarrow \quad R = 4\%$

31. (*c*) Let the two rates be R_1 and R_2.

According to the question,

$\dfrac{1000 \times 2 \times R_1}{100} - \dfrac{1000 \times 2 \times R_2}{100} = 20$

$$\Rightarrow \quad \frac{2000\,(R_1 - R_2)}{100} = 20$$

$$\therefore \quad (R_1 - R_2) = \frac{20}{20} = 1\%$$

Fast Track Method

Here, $P = ₹\ 1000$, $T = 2$ yr, $I_2 - I_1 = 20$

According to the formula,

$$I_2 - I_1 = \frac{PT\,(R_2 - R_1)}{100} \quad \text{[by Technique 4]}$$

$$\Rightarrow \quad 20 = \frac{1000 \times 2\,(R_2 - R_1)}{100}$$

$$\Rightarrow \quad (R_2 - R_1) = \frac{20}{20} = 1\%$$

32. (*a*) According to the question,

$$\frac{R \times 400 \times 2}{100} + \frac{R \times 100 \times 4}{100} = 60$$

$$\Rightarrow \quad 12R = 60$$

$$\therefore \quad R = \frac{60}{12} = 5\%$$

33. (*a*) Let entire sum $= P$

According to the question,

$$\frac{2}{3}P \times 3\% + \frac{1}{6}P \times 6\% + \left[1 - \left(\frac{2}{3} + \frac{1}{6}\right)\right] P \times 12\%$$

$$\Rightarrow \frac{2P}{3} \times \frac{3}{100} + \frac{P}{6} \times \frac{6}{100} + \left[1 - \frac{4+1}{6}\right]\frac{12P}{100} = 25$$

$$\Rightarrow \quad \frac{2P}{100} + \frac{P}{100} + \frac{2P}{100} = 25$$

$$\Rightarrow \quad 5P = 2500, \quad P = 500$$

Fast Track Method

Given, $R_1 = 3\%$, $R_2 = 6\%$, $R_3 = 12\%$

$$\frac{1}{x} = \frac{2}{3}, \; \frac{1}{y} = \frac{1}{6},$$

$$\frac{1}{z} = 1 - \left(\frac{2}{3} + \frac{1}{6}\right)$$

$$= \frac{6 - (4+1)}{6} = \frac{6-5}{6} = \frac{1}{6}$$

$$I = ₹\ 25$$

Now, according to the formula,

$$P = \frac{I \times 100}{\frac{R_1}{x} + \frac{R_2}{y} + \frac{R_3}{z}} \quad \text{[by Technique 5]}$$

$$= \frac{25 \times 100}{3 \times \frac{2}{3} + 6 \times \frac{1}{6} + 12 \times \frac{1}{6}}$$

$$= \frac{2500}{2 + 1 + 2} = ₹\ 500$$

34. (*d*) Given, $T_1 = 5$ yr, $R_1 = 10\%$ and $T_2 = 10$ yr and $R_2 = 8\%$

Let the first part $= x$

Then, second part $= (1521 - x)$

Now, according to the question,

$$\frac{x \times 5 \times 10}{100} = \frac{(1521 - x) \times 10 \times 8}{100}$$

$\Rightarrow \quad 5x = 12168 - 8x$

$\Rightarrow \quad 13x = 12168 \Rightarrow x = ₹\,936$

and second parts $= 1521 - 936 = ₹\,585$

Fast Track Method

Given, $T_1 = 5$ yr, $T_2 = 10$ yr, $R_1 = 10\%$, $R_2 = 8\%$

According to the formula,

Ratio of two parts $= \frac{1}{R_1 T_1} : \frac{1}{R_2 T_2}$

[by Technique 6]

$= \frac{1}{10 \times 5} : \frac{1}{8 \times 10} = \frac{1}{50} : \frac{1}{80} = \frac{1}{5} : \frac{1}{8} = 8 : 5$

First part $= \frac{8}{8 + 5} \times 1521 = \frac{8}{13} \times 1521$

$= 8 \times 117 = ₹\,936$

$\therefore$ Second part $= 1521 - 936 = ₹\,585$

35. (*d*) Given, $T_1 = 2$ yr and $T_2 = 4$ yr, $P_1 = 600$, $P_2 = 150$ and $SI_1 + SI_2 = 80$

According to the question,

$$\frac{600 \times R \times 2}{100} + \frac{150 \times R \times 4}{100} = 80$$

$\Rightarrow \quad 120R + 60R = 800$

$\Rightarrow \quad 180R = 800$

$\therefore \quad R = \frac{800}{180} = \frac{80}{18} = \frac{40}{9} = 4\frac{4}{9}\%$

36. (*c*) Given, time $= 42$ months

$\Rightarrow \quad \frac{42}{12}$ yr $= 3\frac{1}{2}$ yr

$\Rightarrow \frac{7}{2} \times 2 = 7$ half-yr, rate $= \frac{13}{2}\%$ half-yearly

$$SI = \frac{20000 \times 13 \times 7}{100 \times 2} = ₹\,9100$$

$\therefore$ Amount $(A) = 20000 + 9100 = ₹\,29100$

37. (*a*) Let principal $= P$, time $= T$

and rate $= T$

According to the question,

$$\frac{P \times T \times T}{100} = \frac{P}{16}$$

[$\because$ time and rate are equal]

$\Rightarrow \quad T^2 = \frac{100}{16}$ or $T = \frac{10}{4} = \frac{5}{2} = 2\frac{1}{2}\%$

38. (*d*) Since, the two simple interests are equal.

Then, $\frac{4000 \times 3 \times x}{100} = \frac{5000 \times 12 \times 2}{100}$

$\therefore \quad x = 10\%$

39. (*b*) Let the principal be *P*.

Then, according to the question,

$$\frac{P \times T \times T}{100} = \frac{P}{144}$$

[$\because$ time and rate are equal]

$$\Rightarrow \quad T^2 = \frac{100}{144}$$

$$\therefore \quad T = \frac{10}{12} = \frac{5}{6}\%$$

40. (*d*) Given, $R_1 = 6\%$, $R_2 = 10\%$

According to the question,

$$800 + \frac{800 \times 6 \times T}{100} = 600 + \frac{600 \times 10 \times T}{100}$$

$$800 + 48T = 600 + 60T$$

$$\Rightarrow \quad 12T = 200$$

$$\Rightarrow \quad 3T = 50$$

$$\therefore \quad T = \frac{50}{3} = 16\frac{2}{3} \text{ yr}$$

41. (*d*) Given, debt $(P) = ₹\ 1696$,

$R = 4\%$, $T = 4$ yr

According to the formula,

Annual payment

$$= \frac{100\,P}{100 \times T + \frac{RT\,(T-1)}{2}}$$ **[by Technique 7]**

$$= \frac{1696 \times 100}{4 \times 100 + \frac{4 \times 3 \times 4}{2}} = \frac{1696 \times 100}{400 + 24}$$

$$= \frac{1696 \times 100}{424} = ₹\ 400$$

42. (*c*) According to the formula,

$$\text{Annual payment} = \frac{100\,P}{100\,T + \frac{R\,T\,(T-1)}{2}}$$

[by Technique 7]

$$= \frac{1092 \times 100}{100 \times 2 + \frac{24\,(2-1)}{2}}$$

$$= \frac{1092 \times 100}{212}$$

$$= ₹\ 515.09 \approx ₹\ 515$$

43. (*b*) Given, annual payment $= ₹\ 160$

$R = 5\%$, $T = 5$ yr, debt $(P) = ?$

According to the formula,

$$\text{Annual payment} = \frac{100\,P}{100 \times T + \frac{RT\,(T-1)}{2}}$$

[by Technique 7]

$$\Rightarrow\ 160 = \frac{100\,P}{5 \times 100 + \frac{5 \times 4 \times 5}{2}}$$

$$\Rightarrow\ 160 = \frac{100\,P}{550}$$

$$\therefore\quad P = \frac{550 \times 160}{100} = 55 \times 16$$

$$= ₹\ 880$$

Exercise © *Higher Skill Level Questions*

1. (*c*) If the sum be ₹ *P*, then

$$2240 - P = \frac{P \times 4 \times 3}{100}$$

$$\Rightarrow \quad 2240 = \frac{12P}{100} + P \Rightarrow 2240 = \frac{112}{100}P$$

$$\therefore \quad P = \frac{2240 \times 100}{112} = ₹\,2000$$

Now, required interest,

$$\text{SI} = \frac{PRT}{100} = 2000 \times \frac{7}{2} \times \frac{1}{2} \times \frac{1}{100} = ₹\,35$$

2. (*d*) Let the sum be *P*.

$$\because \quad \text{SI} = \frac{P}{2}$$

$$\therefore \quad \frac{P}{2} = \frac{P \times 9 \times 5}{100}$$

Clearly, data is inadequate.

3. (*c*) Given, $A_1 = ₹\,1350$, $A_2 = ₹\,1620$

$T_1 = 5$ yr and $T_2 = 8$ yr

Let principal amount be ₹ *P*.

$\therefore$ In time $= 8 - 5 = 3$ yr

Simple interest will be

$$1620 - 1350 = ₹\,270$$

$$\therefore \quad R = \frac{(A_2 - A_1) \times 100}{A_1T_2 - A_2T_1}$$

$$= \frac{(1620 - 1350) \times 100}{(1350 \times 8 - 1620 \times 5)}$$

$$= \frac{270 \times 100}{10800 - 8100} = \frac{27000}{2700}$$

$$\Rightarrow \quad R = 10\%$$

$$\therefore \quad P = \frac{\text{SI} \times 100}{R \times T} = \frac{270 \times 100}{10 \times 3} = ₹\,900$$

4. (*c*) Let, principal $= P$

Given, SI $= 55$, Time $t = 9$ months $= \frac{9}{12}$ yr,

Rate $\quad r = 3\frac{2}{3}\% = \frac{11}{3}\%$

$\therefore \quad \text{SI} = \frac{P \times r \times t}{100}$

$\Rightarrow \quad P = \frac{100 \times \text{SI}}{R \times T}$

$\Rightarrow \quad \frac{55 \times 100}{11 \times 9} \times 3 \times 12 = 2000$

$\therefore$ Principal (P) = ₹ 2000

5. (*c*) According to the question,

SI for 5 yr = ₹ 200

$\therefore$ Total SI for 10 yr $= 200 + 600$

= ₹ 800

When principal is trebled, then SI for 5 yr will also be treble and hence SI for next 5 yr will be ₹ (200 × 3) *i.e.*, ₹ 600.

6. (*c*) Investment of Ramesh

= 10% of 10000 = ₹ 1000

After 3 yr = 2500 − 1000 = ₹ 1500

We know,

$\text{SI} = \frac{P \times R \times T}{100}$

$\Rightarrow \quad 1500 = \frac{1000 \times R \times 3}{100}$

$\therefore \quad R = \frac{1500 \times 100}{1000 \times 3} = 50\%$

$\therefore$ 4 times of 50% = 200%

7. (*d*) Let sum $= P$

Then, according to the question,

$\text{SI} = \frac{P}{2}$

$\therefore \quad \frac{P}{2} = \frac{P \times 8 \times 6}{100}$

$\therefore$ It is clear that data is inadequate.

8. (*c*) Amount = ₹ 8000

Time (T) = 4 yr; Principal (P) = ₹ 6000

Simple interest (SI) $= (A - P)$

= Amount − Principal

= 8000 − 6000 = ₹ 2000

Rate (R) = ?

According to the formula,

$\text{SI} = \frac{P \times R \times T}{100}$

$\Rightarrow \quad 2000 = \frac{6000 \times R \times 4}{100}$

$\Rightarrow \quad R = \frac{2000 \times 100}{6000 \times 4} = \frac{25}{3}\%$

Now, again

Amount $(A) = ₹\ 700$

Principle $(P) = ₹\ 525$, Time $(T) = ?$

Rate $(R) = \frac{25}{3}\%$

Simple interest $= A - P$

$\Rightarrow \quad 700 - 525 = ₹\ 175$

Using formula, $SI = \frac{P \times R \times T}{100}$

$$\Rightarrow \quad 175 = \frac{525 \times \frac{25}{3} \times T}{100}$$

$$\Rightarrow \quad T = \frac{175 \times 100 \times 3}{525 \times 25} = 4 \text{ yr}$$

9. (a) Let the principle be ₹ x.

Rate of interest $= R\%$

Case I

$P = ₹\ x$, $T = 3$ yr

$R = R\%$, $SI = ₹\ (944 - x)$

$$SI = \frac{P \times R \times T}{100}$$

$$\Rightarrow \quad 944 - x = \frac{x \times R \times 3}{100}$$

$$\Rightarrow \quad \frac{100\,(944 - x)}{3x} = R \qquad \ldots\text{(i)}$$

Case II

$P = ₹\ x$, $\quad T = 5$ yr,

$R = R\%$, $\quad SI = ₹\ (1040 - x)$

$$SI = \frac{P \times R \times T}{100}$$

$$\Rightarrow \quad 1040 - x = \frac{x \times R \times 5}{100}$$

$$\Rightarrow \quad \frac{100\,(1040 - x)}{5x} = R \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{100\,(944 - x)}{3x} = \frac{100\,(1040 - x)}{5x}$$

$$\Rightarrow \quad \frac{944 - x}{3} = \frac{1040 - x}{5}$$

$\Rightarrow \quad (944 - x) \times 5 = 3 \times (1040 - x)$

$\Rightarrow \quad 4720 - 5x = 3120 - 3x$

$\Rightarrow \quad 4720 - 3120 = 5x - 3x$

$\Rightarrow \quad 1600 = 2x$

$$x = \frac{1600}{2} = ₹\ 800$$

Fast Track Method

By formula, $P = \frac{A_2T_1 - A_1T_2}{T_1 - T_2}$

[by Technique 3]

Here, $A_1 = ₹\,944$, $T_1 = 3$

$A_2 = ₹\,1040$, $T_2 = 5$

$$\therefore \quad P = \frac{(1040)(3) - (944)(5)}{3-5}$$

$$= \frac{3120 - 4720}{-2}$$

$$= \frac{-1600}{-2} = ₹\,800$$

$\therefore$ Principal $= ₹\,800$

10. (c) If the principal $= P$ and interest $= R\%$

$$\text{Then, required ratio} = \frac{\frac{P \times R \times 12}{100}}{\frac{P \times R \times 18}{100}}$$

$$= \frac{12}{18} = \frac{2}{3} = 2:3$$

11. (a) Let the sum lent at 5% $= P$

$\therefore$ Sum lent at 8% $= (1550 - P)$

$$\text{Then, } \frac{P \times 5 \times 4}{100} + \frac{(1550 - P) \times 8 \times 4}{100} = 400$$

$$\Rightarrow \quad 20P - 32P + 1550 \times 32 = 40000$$

$$\Rightarrow \quad -12P + 49600 = 40000$$

$$\Rightarrow \quad -12P = -9600$$

$$\therefore \quad P = ₹\,800$$

Sum lent at 8% $= 1550 - 800 = ₹\,750$

$\therefore$ Required ratio $= 800 : 750 = 16 : 15$

12. (c) Let the sum borrowed $= P$

Then, according to the question,

$$\frac{P \times 6 \times 3}{100} + \frac{P \times 9 \times 5}{100} + \frac{P \times 13 \times 3}{100} = 8160$$

$$\Rightarrow \quad \frac{18P + 45P + 39P}{100} = 8160$$

$$\Rightarrow \quad \frac{102P}{100} = 8160 \Rightarrow P = \frac{8160 \times 100}{102}$$

$\therefore \quad P = ₹\,8000$

13. (a) Let the money lent to Akshay $= ₹\,x$

Then, money lent to Brijesh

$= ₹\,(10000 - x)$

[as total amount $= ₹\,10000$]

$$\text{SI for Akshay} = \frac{x \times 15 \times 2}{100} = \frac{3x}{10}$$

$$\text{SI for Brijesh} = \frac{(10000 - x) \times 18 \times 2}{100}$$

$$= \frac{9}{25}(10000 - x)$$

According to the given condition,

$$\frac{3x}{10} - \frac{9}{25}(10000 - x) = 360$$

[as SI (Akshay) – SI (Brijesh) = 360]

$$\Rightarrow \quad \frac{3x}{10} - 3600 + \frac{9x}{25} = 360$$

$$\Rightarrow \quad \frac{3x}{10} + \frac{9x}{25} = 360 + 3600 = 3960$$

$$\Rightarrow \quad \frac{33x}{50} = 3960$$

$$\Rightarrow \quad x = \frac{3960 \times 50}{33}$$

$$\Rightarrow \quad x = 6000$$

$\therefore$ The amount of money lent to Brijesh

$$= 10000 - 6000 = ₹\ 4000$$

14. (*c*) In the case I,

$$SI = \frac{P \times R \times T}{100} = \frac{24200 \times 4 \times 6}{100}$$

$$= ₹\ 5808$$

$\therefore$ Amount = Principal + SI

$$SI = 24200 + 5808 = 30008$$

In the case II,

$$SI = \frac{30008 \times 4 \times 4}{100} = ₹\ 4801.28$$

15. (*d*) Let the rate of interest allowed by bank be r%.

According to the question,

$$\frac{12000 \times 5 \times 10}{100} - \frac{12000 \times 3 \times r}{100} = 3320$$

$$\Rightarrow \quad 6000 - 360r = 3320$$

$$\Rightarrow \quad 360r = 6000 - 3320 = 2680$$

$$\Rightarrow \quad r = \frac{2680}{360} = 7\frac{4}{9}\%$$

16. (*c*) Let x, y and z be the amounts invested in schemes P, Q and R, respectively.

Then, according to the question,

$$\frac{x \times 10 \times 1}{100} + \frac{y \times 12 \times 1}{100} + \frac{z \times 15 \times 1}{100} = 3200$$

$$\Rightarrow \quad 10x + 12y + 15z = 320000 \quad \ldots\text{(i)}$$

Now, $\quad z = 240\% \text{ of } y = \frac{12}{5}y \quad \ldots\text{(ii)}$

and $\quad z = 150\% \text{ of } x = \frac{3}{2}x$

$$\Rightarrow \quad x = \frac{2}{3} \times z = \left(\frac{2}{3} \times \frac{12}{5}\right)y = \frac{8}{5}y \quad \ldots\text{(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$16y + 12y + 36y = 320000$$

$$\Rightarrow \quad 64y = 320000$$

$$\therefore \quad y = 5000$$

$\therefore$ Sum invested in scheme Q = ₹ 5000

Chapter

15

Compound Interest

As we know that when we borrow some money from bank or any person, then we have to pay some extra money at the time of repaying. This extra money is known as **interest.** If interest accrued on principal, it is known as simple interest.

Sometimes it happens that we repay the borrow money some late. After the completion of specific period, interest accrued on principal as well as interest due of the principal. Then, it is known as **compound interest.**

Compound interest = Amount - Principal

Basic Formulae Related to Compound Interest

Let principal = *P,* rate = *R%* pa and time = *n* yr 1. If interest is compounded annually, then

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

Compound interest = Amount - Principal

$$\text{Compound Interest} = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$$

2. If interest is compounded half-yearly, then $R = \frac{R}{2}$ and $n = 2n$

$$\text{Amount} = P\left(1+\frac{R}{2\times 100}\right)^{2n}$$

3. If interest is compounded quarterly, then $R = \frac{R}{4}$ and $n = 4n$

$$\text{Amount} = P\left(1+\frac{R}{4\times 100}\right)^{4n}$$

4. If interest is compounded annually but time is in fraction (suppose time $= n\frac{a}{b}$ yr), then

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n \times \left(1+\frac{(a/b)R}{100}\right)$$

5. If rates of interest are $R_1\%$, $R_2\%$ and $R_3\%$ for 1st, 2nd and 3rd yr respectively, then

$$\text{Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

Ex. 1 Find the compound interest on? 8000 at 4% pa for 2 yr, compounded annually.

Sol. Given that, $P =$ ₹ 8000, $R = 4\%$ and time = 2 yr

Now, according to the formula,

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

$$= 8000\left(1+\frac{4}{100}\right)^2 = 8000 \times \frac{26}{25} \times \frac{26}{25} = ₹\ 8652.80$$

$\therefore$ CI = Amount − Principal = ₹ 8652.80 − ₹ 8000 = ₹ 652.80

Ex. 2 Ruchi invested ? 1600 at the rate of compound interest for 2 yr. She got ? 1764 after the specified period. Find the rate of interest.

Sol. Given that, $P = ₹\,1600$, $n = 2$ yr and $A = ₹\,1764$

Now, according to the formula,

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^n \Rightarrow 1764 = 1600\left(1 + \frac{R}{100}\right)^2$$

$$\Rightarrow \quad \frac{1764}{1600} = \left(\frac{100 + R}{100}\right)^2 \Rightarrow \frac{441}{400} = \left(\frac{100 + R}{100}\right)^2$$

$$\Rightarrow \quad \left(\frac{21}{20}\right)^2 = \left(\frac{100 + R}{100}\right)^2 \Rightarrow \frac{100 + R}{100} = \frac{21}{20} \Rightarrow 100 + R = \frac{21}{20} \times 100$$

$$\Rightarrow \quad 100 + R = 105$$

$$\therefore \quad R = 105 - 100 = 5\%$$

Ex. 3 Find the compound interest on ? 5000 in 2 yr at 4% pa, if the interest being compounded half-yearly.

Sol. Given that, principal $(P) = ₹\,5000$, rate $(R) = 4\%$ pa and time $(n) = 2$ yr

Now, according to the formula,

$$\text{Amount} = P\left(1 + \frac{R}{2 \times 100}\right)^{2n} = 5000\left(1 + \frac{4}{200}\right)^4$$

$$= 5000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} = \frac{51 \times 51 \times 51 \times 51}{1250} = ₹\,5412.16$$

$\therefore$ Compound interest $=$ Amount $-$ Principal$=$ $5412.16 - 5000 = ₹\,412.16$

Ex. 4 Find the compound interest on ? 8000 at 20% pa for 9 months, compounded quarterly.

Sol. Given that, $P = ₹\,8000$, $n = 9$ months $= 3/4$ yr and $R = 20\%$ pa

According to the formula,

$$\text{Amount} = P\left(1 + \frac{R}{4 \times 100}\right)^{4n} = 8000\left(1 + \frac{20}{400}\right)^{3/4 \times 4} = 8000\left(1 + \frac{5}{100}\right)^3$$

$$= 8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = ₹\,9261$$

$$\therefore \quad CI = 9261 - 8000 = ₹\,1261$$

Ex. 5 Find the compound interest on ? 2000 at 15% pa for 2 yr 4 months, compounded annually.

Sol. Given that, $P = ₹\,2000, n = 2$ and $\frac{a}{b} = \frac{4}{12} = \frac{1}{3}$, $R = 15\%$

Now, according to the formula,

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n \times \left(1+\frac{\frac{a}{b}R}{100}\right) = \left[2000\left(1+\frac{15}{100}\right)^2 \times \left(1+\frac{\frac{1}{3}\times 15}{100}\right)\right]$$

$$= 2000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{21}{20} = \frac{11109}{4} = 277725$$

$\therefore$ $\quad CI = 2777.25 - 2000 = ₹\,777.25$

Ex. 6 What sum of money at compound interest will amount to ? 4499.04 in 3 yr, if the rate of interest is 3% for the 1st yr, 4% for the 2nd yr and 5% for the 3rdyr?

Sol. Given that, $A = ₹\,4499.04$, $R_1 = 3\%$, $R_2 = 4\%$, $R_3 = 5\%$ and $P = ?$

Now, according to the formula,

$$\text{Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

$$4499.04 = P\left(1+\frac{3}{100}\right)\left(1+\frac{4}{100}\right)\left(1+\frac{5}{100}\right)$$

$\Rightarrow$ $\quad 4499.04 = P\,(103)\,(104)\,(105)$

$\therefore$ $\quad P = \frac{4499.04}{103 \times 104 \times 105} = \frac{4499.04}{112476} = ₹\,4000$

Instalments

When a borrower pays the sum in parts, then we say that he/she is paying in instalments.

$$\therefore \quad P = \left[\frac{x}{\left(1+\frac{R}{100}\right)} + \frac{x}{\left(1+\frac{R}{100}\right)^2} + \frac{x}{\left(1+\frac{R}{100}\right)^3} + \ldots\ldots + \frac{x}{\left(1+\frac{R}{100}\right)^n}\right]$$

$x =$ Value of each instalment

Total amount paid in instalments, $A = P\left(1+\frac{R}{100}\right)^n$

$n =$ Number of instalments

Ex. 7 Sapna borrowed some money on compound interest and returned it in 3 yr in equal annual instalments. If rate of interest is 15% per annum and annual instalment is ? 486680, then find the sum borrowed.

Sol. Given that, rate of interest, *R* = 15% pa Annual instalment, *x* = ? 486680 Total number of instalments, n = 3

$$\therefore \quad P = \left[\frac{x}{\left(1+\frac{R}{100}\right)} + \frac{x}{\left(1+\frac{R}{100}\right)^2} + \frac{x}{\left(1+\frac{R}{100}\right)^3}\right]$$

$$\Rightarrow \quad P = x\left[\frac{100}{(100+R)} + \frac{(100)^2}{(100+R)^2} + \frac{(100)^3}{(100+R)^3}\right]$$

$$= 486680\left[\left(\frac{100}{100+15}\right) + \left(\frac{100}{100+15}\right)^2 + \left(\frac{100}{100+15}\right)^3\right]$$

$$= 486680 \times \left[\frac{20}{23} + \left(\frac{20}{23}\right)^2 + \left(\frac{20}{23}\right)^3\right]$$

$$= 486680 \times \frac{20}{23}\left(1 + \frac{20}{23} + \frac{400}{529}\right) = 1111200$$

$\therefore$ **Principal borrowed = ₹ 1111200**

Fast Track **Techniques**

to *solve the* QUESTIONS

Technique 1 Difference between (Compound Interest) CI and (Simple Interest) SI

(a) Difference between CI and SI for 2 yr $(D) = P\left(\frac{R}{100}\right)^2 = \frac{SI \times R}{200}$

(b) Difference between CI and SI for 3 yr $(D) = P\left(\frac{R}{100}\right)^2\left(\frac{R}{100}+3\right)$

Note SI and CI for one year on the same sum and at same rate are equal

Ex. 8 The difference between compound interest and simple interest for 2 yr at rate of 5% per annum is ? 5, then find the sum.

Sol. Given that, difference (D) = ₹ 5, rate $(R) = 5\%$

Then, according to the formula,

$$D = P\left(\frac{R}{100}\right)^2 \Rightarrow 5 = P\left(\frac{5}{100}\right)^2 \Rightarrow 5 = \frac{P \times 5 \times 5}{100 \times 100} \Rightarrow P = ₹\ 2000$$

Ex. 9 The difference between CI and SI for 3 yr at the rate of 20% pa is ? 152. What is the principal lent?

Sol. Difference between CI and SI for 3 yr = ₹ 152

$$\therefore \quad P\left(\frac{R}{100}\right)^2\left(\frac{R}{100}+3\right)=152$$

$$\Rightarrow \quad P\left(\frac{20}{100}\right)^2\left(\frac{20}{100}+3\right)=152$$

$$\Rightarrow \quad P\left(\frac{1}{25}\right)\left(\frac{16}{5}\right)=152 \Rightarrow P=\frac{152\times 25\times 5}{16}$$

$$\Rightarrow \quad P=9.5\times 25\times 5 \Rightarrow P=₹\ 1187.5$$

Technique 2

If simple interest for a certain sum for 2 yr at the annual rate of interest *R%* is SI. then

$$\text{Compound Interest (CI)} = \text{SI}\left(1+\frac{R}{200}\right)$$

Note This formula is applicable only for 2 yr,

Ex. 10 If the simple interest for a certain sum for 2 yr at 5% pa is ? 200, then what will be the compound interest for same sum for same period and at the same rate of interest?

Sol. Given that, SI = ₹ 200 and R = 5%

∴ According to the formula,

$$\text{CI} = \text{SI}\left(1+\frac{R}{200}\right)=200\left(1+\frac{5}{200}\right)=200\times\frac{205}{200}=₹\ 205$$

Technique 3

If a certain sum at compound interest becomes x times in n^ yr and y

times in n_2 yr, then $x^{\frac{1}{n_1}} = y^{\frac{1}{n_2}}$

Ex. 11 If a certain sum at compound interest becomes double in 5 yr, then in how many years, it will be 16 times at the same rate of interest?

Sol. If sum = x, then

x becomes $2x$ in 5 yr.
$2x$ becomes $4x$ in 10 yr.
$4x$ becomes $8x$ in 15 yr.
$8x$ becomes $16x$ in 20 yr.

Fast Track Method

Here, $n_1 = 5$ yr, $x = 2$, $y = 16$ and $n_2 = ?$

According to the formula,

$$x^{\frac{1}{n_1}} = y^{\frac{1}{n_2}} \Rightarrow 2^{\frac{1}{5}} = 16^{\frac{1}{n_2}}$$

$$\Rightarrow \quad 2^{\frac{1}{5}} = (2)^{4 \times \frac{1}{n_2}}$$

$$\Rightarrow \quad \frac{1}{5} = \frac{4}{n_2} \quad \text{[on comparing both sides]}$$

$$\therefore \quad n_2 = 5 \times 4 = 20 \text{ yr}$$

Technique 4

If a certain sum at compound interest becomes A_1 in n yr and A_2 in $(n + 1)$ yr, then

(i) Rate of compound interest $= \dfrac{(A_2 - A_1)}{A_1} \times 100\%$

(ii) Sum $= A_1 \left(\dfrac{A_1}{A_2}\right)^n$

Ex. 12 A sum of money invested at compound interest amounts to ? 800 in 2 yr and ? 840 in 3 yr. Find the rate of interest per annum and the sum.

Sol. SI on ₹ 800 for 1 yr = 840 − 800 = ₹ 40

According to the formula,

$$SI = \frac{P \times R \times T}{100}$$

$$\Rightarrow \quad 40 = \frac{800 \times R \times 1}{100}$$

$$\Rightarrow \quad R = 5\%$$

Now, let the sum = x, then

$$\text{Amount} = \text{Sum}\left(1 + \frac{R}{100}\right)^T$$

$$x\left(1 + \frac{5}{100}\right)^2 = 800$$

$$\Rightarrow \quad \frac{105}{100} \times \frac{105}{100} \times x = 800 \Rightarrow \frac{21}{20} \times \frac{21}{20} \times x = 800$$

$$\therefore \quad x = \frac{800 \times 20 \times 20}{21 \times 21} = \frac{320000}{441} = ₹\ 725.62$$

Fast Track Method

Here, A_1 = ₹ 800 and A_2 = ₹ 840

According to the formula,

$$\text{Rate of compound interest} = \frac{A_2 - A_1}{A_1} \times 100\% = \frac{840 - 800}{800} \times 100\% = \frac{40}{8}\% = 5\%$$

$$\text{and sum} = A_1\left(\frac{A_1}{A_2}\right)^n = 800 \times \left(\frac{800}{840}\right)^2 = 800 \times \frac{800}{840} \times \frac{800}{840} = \frac{320000}{441} = ₹\ 725.62$$

Technique 5

If the population of a city is P and it increases with the rate of $R\%$ per annum, then

(i) Population after n yr $= P\left(1 + \frac{R}{100}\right)^n$ (ii) Population n yr ago $= \dfrac{P}{\left(1 + \frac{R}{100}\right)^n}$

Note • If population decreases with the rate of $R\%$, then (-) sign will be used in place of (+) in the above mentioned formula.

- If the rate of growth per year is $R_1\%$, $R_2\%$, $R_3\%$, ..., $R_n\%$, then
  Population after n yr $= P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right)\ldots\ldots\left(1 + \frac{R_n}{100}\right)$

(This formula can also be used, if there is increase\decrease in the price of an article.)

Ex. 13 The population of a city increases at the rate of 15% pa. If its population was 4000 at the end of year 2013, then what will be its population at the end of year 2015?

Sol. Given that, $R = 15\%, n = 2$ yr and $P = 4000$

According to the formula,

$$\text{Population after } n \text{ yr} = P\left(1 + \frac{R}{100}\right)^n$$

$$\therefore \text{Population after 2 yr} = 4000\left(1 + \frac{15}{100}\right)^2$$

$$= 4000 \times \frac{23}{20} \times \frac{23}{20} = 10 \times 529 = 5290$$

Ex. 14 The population of a city increases at the rate of 5% pa. If the present population of the city is 370440, then what was its population 3 yr ago?

Sol. Given that, $P = 370440$, $R = 5\%$ and $n = 3$ yr

According to the formula,

$$\text{Population } n \text{ yr ago} = \frac{P}{\left(1 + \frac{R}{100}\right)^n}$$

$$\therefore \text{Population 3 yr ago} = \frac{370440}{\left(1 + \frac{5}{100}\right)^3} = \frac{370440}{\left(\frac{21}{20}\right)^3} = \frac{370440 \times 20 \times 20 \times 20}{21 \times 21 \times 21}$$

$$= \frac{370440 \times 8000}{9261} = 40 \times 8000 = 320000$$

Ex. 15 The population of a particular area A of a city is 5000. It increases by 10% in 1st yr. It decreases by 20% in the 2nd yr because of some reason. In the 3rd yr, the population increases by 30%. What will be the population of area *A* at the end of 3 yr?

Sol. Given that, $P = 5000$, $R_1 = 10\%$, $R_2 = -20\%$ (decrease) and $R_3 = 30\%$

$\therefore$ Population at the end of 3rd year

$$= P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right)$$

$$= 5000\left(1 + \frac{10}{100}\right)\left(1 - \frac{20}{100}\right)\left(1 + \frac{30}{100}\right)$$

$$= 5000 \times \frac{11}{10} \times \frac{4}{5} \times \frac{13}{10} = 10 \times 11 \times 4 \times 13 = 5720$$

Ex. 16 The cost price of a car is ? 400000. If its price decreases 10% every year, then what will be the cost of car after 3 yr?

Sol. Given that, $P = ₹\,400000$, $R = -10\%$ and $n = 3$

According to the formula,

Price of car after n yr $= P\left(1 + \frac{R}{100}\right)^n$

$\therefore$ Price of car after 3 yr $= 400000\left(1 - \frac{10}{100}\right)^3 = 400000 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$

$= 400 \times 9 \times 9 \times 9 = ₹\,291600$

['–' sign has been taken because of the decrease in price]

Multi Concept

QUESTIONS

1. *Suneeta borrowed certain sum from Reena for 2 yr at simple interest. Suneeta lent this sum to Venu at the same rate for 2 yr at compound interest. At the end of 2 yr, she received ? 110 as compound interest but paid ? 100 as simple interest. Find the sum and the rate of interest.*

(a) ? 250, race 10 % pa (b) ? 250, rate 20 % pa (c) ? 250, rate 25 % pa (d) ? 250, rate 30 % pa

➥ (b) Let the sum borrowed be ₹ x and rate of Interest be R%.

Given, Compound interest (CI) = ₹ 110, Simple interest (SI) = ₹ 100 and time (t) = 2 yr

By using the formula,

$$CI - SI = \frac{SI \times R}{200} \Rightarrow 110 - 100 = \frac{100 \times R}{200} \Rightarrow R = \frac{10 \times 200}{100} = 20\%$$

Again, using the same formula,

$$CI - SI = \frac{PR^2}{100^2} \Rightarrow 110 - 100 = \frac{x \times 20 \times 20}{100 \times 100} \Rightarrow x = \frac{10 \times 100 \times 100}{20 \times 20} \Rightarrow x = 250$$

$\therefore$ Sum borrowed = ₹ 250 and rate of interest = 20%

2. *Find the least number of complete year in which a sum of money put out at 20% compound interest will be more than double.*

(a) 3yr (b) 4 yr (c) 5 yr (d) 8 yr

➥ (b) Let the sum = P

According to the question, $P\left(1 + \frac{20}{100}\right)^n > 2P$ or $\left(\frac{6}{5}\right)^n > 2$; So, $n > 4$

By Hit and Trial This is true for $n = 4$; As, $\left(\frac{6}{5} \times \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5}\right) > 2$; Here, $n = 4$ yr

3. *The halflife ofUranium -233 is 160000 yr i.e., Uranium -233 decays at a constant rate in such a way that it reduces to 50% in 160000 yr. In how many years, will it reduce to 25%?*

(a) 200000 yr (b) 150000 yr (c) 280000 yr (d) 320000 yr

(*d*) Let the Initial amount of Uranium – 233 be 1 unit and the rate of decay be R per unit per year.

$\therefore$ Amount of Uranium – 233 after 160000 yr $= 1 \times \left(1 - \frac{R}{100}\right)^{160000}$

$$= \frac{1}{2} = 1 \times \left(1 - \frac{R}{100}\right)^{160000} \qquad \left[\because 50\% = \frac{1}{2}\right] \quad \ldots \text{(i)}$$

Assume that Uranium – 233 reduces to 25% in t yr, then

$$\frac{25}{100} = 1 \times \left(1 - \frac{R}{100}\right)^{n} \Rightarrow \left(\frac{1}{2}\right)^{2} = \left(1 - \frac{R}{100}\right)^{n} \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\left[\left(1 - \frac{R}{100}\right)^{160000}\right]^{2} = \left(1 - \frac{R}{100}\right)^{n} \Rightarrow \left(1 - \frac{R}{100}\right)^{320000} = \left(1 - \frac{R}{100}\right)^{n} \Rightarrow n = 320000 \text{ yr}$$

Thus, Uranium – 233 will reduce to 25% in 320000 yr.

Fast Track Practice

Exercise© *Base Level Questions*

1. What will be the compound interest on a sum of ? 50000 after 3 yr at the rate of 12% pa?

(a) X 80000 *(b)* X 70246.40

(c) X 20246.40 *(d)* X 70000

(e) None of the above

2. Vandana invested an amount of *X* 8000 in a fixed deposit scheme for 2 yr at the rate of 5% pa compound interest. How much amount will Vandana get on maturity of the fixed deposit?

(a) X 8000 *(b)* X 8890

(c) X 8888 *(d)* X 8820

(e) None of the above

3. A sum amounts to ? 1352 in 2 yr at 4% compound interest. The sum is

[SSCFCI2012]

(a) X 1250 *(b)* X 1200

(c) X 1300 *(d)* X 1260

4. Raja invested X 15000 at the rate of 10% pa for 1 yr. If the interest is compounded half-yearly, then find the amount received by Raja at the end of the year.

(a) X 16537.50 *(b)* X 18000 *(c)* X 19000.50 *(d)* X 20000

(e) None of the above

5. What will be the present worth of X 169 due in 2 yr at 4% pa compound interest?

[SSC LDC 2008]

(a) X 156.25 *(b)* X 160

(c) X 150.50 *(d)* X 154.75

6. Find the compound interest on X 31250 at 16% pa compounded quarterly for 9 months.

(a) X 4000 *(b)* X 3902

(c) X 3500 *(d)* X 4200

(e) None of the above

What amount will be received on a sum of X 1750 in 2V6 yr, if the interest is compounded at the rate of 8% pa?

[SBI PO 2009]

(a) X 2125 *(b)* X 2122.85

(C) X 2100 *(d)* X 2200

(e) X 2300

8. What amount will be received on a sum of? 15000 in VA yr at 12% pa, if interest is compounded quarterly? [SSC LDC 2008]

(a) X 16596.88 (b) X 16789.08

(c) ? 17630.77 (d) ? 17389.12

9. The compound interest on ? 30000 at 7% pa for a certain time is X 4347. The time is

(a) 2 yr (b) 2.5 yr (c) 3 yr (d) 4 yr

10. The compound interest on a sum of ?4000 becomes ? 630.50 in 9 months. Find the rate of interest, if interest is compounded quarterly. [SSC CGL 2008]

(a) 20% (b) 23% (c) 19% (d) 21%

11. The population of a city increases at the rate of 5% pa. If the present population of the city is 185220, then what was its population 3 yr ago? [LIC ADO 2008]

(a) 181500 (b) 183433

(C) 160000 (d) 127783

(e) None of the above

12. The population of a country is 10 crore and it is the possibility that the population will become 13.31 crore in 3 yr. What will be the the annual rate per cent on this growth? [SSC DEO 2009]

(a) 8% (b) 12.7%(cJ 10% (d) 15%

13. A sum of ? 400 amounts to ? 441 in 2 yr. What will be its amount, if the rate of interest is increased by 5%? [SSC CGL 2008]

(a) ? 484 (b) ? 560 (c) ? 512 (d) ? 600

14. The cost price of an LCD TV set is ? 100000. If its price value depreciates at the rate of 10% pa, then what will be the price at the end of 3 yr?

(a) X 80000 *(b)* f 85000

(C) ? 90000 *(d)* ? 72900

(e) None of the above

15. Income of Shantanu was ? 4000. In the first 2 yr, his income decreased by 10% and 5% respectively but in the third year, the income increased by 15%. What was his income at the end of third year?

(a) ? 3933 *(b)* ? 4000

(c) X 3500 *(d)* X 3540

(e) None of the above

16. A sum, at the compound rate of interest, becomes $2^{l}A$ times in 6 yr. The same sum becomes what times in 18 yr?

[Hotel Mgmt. 2007]

(a) $\frac{5}{2}$ *(b)* $\frac{25}{4}$ *(c)* $\frac{125}{8}$ *(d)* $\frac{625}{16}$

(e) None of the above

What is the difference between compound interest and simple interest for 2 yr on the sum of ? 1250 at 4% pa?

[RBI 2008]

(a) ? 3 *(b)* ? 4 *(c)* ? 2 *(d)* ? 8 *(e)* None of the above

18. The difference between compound and simple rates of interest on ? 10000 for 3 yr at 5% per annum is [SSC CGL (Main) 2012]

(a) ? 76.25 *(b)* ? 76.75

(c) ? 76.50 *(d)* ? 76

19. What is the difference between the compound interest and simple interest calculated on an amount of ? 16200 at the end of 3 yr at 25% pa? (Rounded off to two digits after decimal)

[IBPS Clerk 2011] fa;? 3213.44 *(b)* ? 3302.42

(c) ? 3495.28 (d) ? 3290.63

(e) None of the above

20. If the difference between the compound interest and the simple interest on a certain sum for 2 yr at 8% per annum is ? 32, then the sum is [SSCfl 0+2) 2012]

fa; ? 5000 (b) ? 5500

(c) ? 6000 (d) ? 5250

21. If the difference between the compound interest and simple interest on a sum of money at 5% pa for 2 yr is ? 16, then find the simple interest.

fa; ? 840 (b) ? 932 (c) ? 520 (d) ? 640 (e) None of the above

22. The simple interest for certain sum in 2 yr at 4% pa is ? 80. What will be the compound interest for the same sum, if conditions of rate and time period are same? [SSC DEO 2009]

(a) ? 91.60 (b) ? 81.60

(c) ? 71.60 (d) ? 80

23. The compound interest is ? 6.40 more than the simple interest, if a sum is lent for 2 yr at 8% compound interest. Find the sum. [UP Police 2007]

(a) ? 1800 (b) ? 10000

(c) ?800 (d) ? 1000

24. A certain sum at compound interest amounts to ? 12960 in 2 yr and ? 13176 in 3 yr. Find the rate per cent per annum. [Bank Clerks 2009]

(a) $1\frac{1}{3}\%$ (b) $2\frac{1}{3}\%$ (c) $1\frac{2}{3}\%$ (d) $2\frac{2}{3}\%$
(e) None of the above

25. A sum amounts to ? 2916 in 2 yr and ? 3149.28 in 3 yr at compound interest. The sum is [SSC (10+2)2012]

fa;? 1500 (b;?2500

(c) ? 2000 *(d)* ? 3000

26. At what per cent annual compound interest rate, a certain sum amounts to its 27 times in 3 yr? [RRB 2007]

(a) 100% *(b)* 150% *(c)* 75% *(d)* 200%

27. What will be the principal, if the rate of 5% per annum can yield ? 410 as compound interest in 2 yr? [RRB 2012]

(a) ? 4000 *(b)* ? 42000

fc; ? 8000 *(d)* ? 21000

28. Find the compound interest for a sum of ? 9375 in 2 yr, if the rate of interest for the first year is *2%* and for the second year is 4%. [RRB 2007]

(a) ? 570 *(b)* ? 670 *(c)* ? 770 *(d)* ? 760

29. A sum invested for 3 yr compounded at 5%, 10% and 20% respectively. In 3 yr, if the sum amounts to ? 16632, then find the sum. [SSC Multitasking 2014]

(a) ? 11000 *(b)* ? 12000

fc; ?9000 *(d)* ? 15000

The simple interest on a certain sum for 2 yr is ? 120 and compound interest is ? 129. Find the rate of interest.

[LIC ADO 2009]

(a) 14% *(b)* 15% *(c)* 12% *(d)* $12\frac{1}{2}\%$
(e) None of the above

31. The height of a tree increases every year 1/8 times. If the present height of the tree is 64 cm, then what will be its height after 2 yr? [Bank Clerks 2007]

(a) 76 cm *(b)* 80 cm

fc; 81 cm *(d)* 84 cm

(e) None of the above

32. An amount at the rate of 5% pa becomes ? 10 more at compound interest than that of simple interest after 2 yr. Calculate the principal.

(a) ? 4000 *(b)* ? 4050

(c) ? 5000 *(d)* ? 5500

(e) None of the above

33. The principal amount which yields a compound interest of ? 208 in the second year at 4% is [SSC CGL 2012]

(a) ? 5000 *(b)* ? 10000

fc; ? 130000 *(d)* ? 6500

Exercise © *Higher SkiU Level Questions*

1. The population of a town is 126800. It increases by 15% in the first year and decreases by 20% in the second year. What is the population of the town at the end of 2 yr? [Bank Clerks 2007]

(a) 174984 *(b)* 135996

(c) 116656 *(d)* 145820

(e) None of the above

2. A man borrows *X* 5100 to be paid back with compound interest at the rate of 4% pa by the end of 2 yr in two equal yearly instalments. How much will each instalment be?

(a) X 2704 *(b) X* 2800

(c) X 3000 *(d) X* 2500

(e) None of the above Divide *X* 2602 between *X* and Y, so that the amount of *X* after 7 yr is equal to the amount of *Y* after 9 yr, the interest being compounded at 4% pa. *(a) X* 1352, *X* 1250 *(b) X* 1400, *X* 1350 *(c) X* 1415, *X* 1300 *(d) X* 1500, *X* 1450 *(e)* None of the above

4. A borrowed sum was paid in the two annual instalments of ? 121 each. If the rate of compound interest is 10% pa, what sum was borrowed?

(a) X 217.80 (b) X 210

(c) X 220 (d) X 200

5. If X 3000 amounts to X 4320 at compound interest in a certain time, then X 3000 amounts to what in half of the time?

(a) X 3400 (b) X 3600

(C) X 38000 (d) X 3520

6. SBI lent X 1331 lakh to the TATA group at compound interest and got X 1728 lakh after 3 yr. What is the rate of interest charged, if compounded annually? [SSC CGL 2007]

(a) 11% (b) 9.09% (c) 12% (d) 8.33%

7. A sum of money lent at compound interest for 2 yr at 20% pa would fetch X 964 more, if the interest was payable half-yearly than if it was payable annually. What is the sum?

(a) X 40000 (b) X 60000

(c) X 90000 (d) X 500000

(e) None of the above An amount is invested in a bank at compound rate of interest. The total amount, including interest, after first and third year is X 1200 and X 1587, respectively. What is the rate of interest ? [SSC CGL (Main) 2012]

(a) 10% (b) 3.9% (c) 12% (d) 15%

9. A sum of ? 11000 was taken as loan. This is to be paid in two equal annual instalments. If the rate of interest be 20% compounded annually, then the value of each instalment is

[SSCCPO2013] (a) ?7500 (b) ?7000

(c) ?7100 (d) ?7200

10. A sum of X 8448 is to be divided between A and B who are respectively 18 and 19 yr old, in such a way that if their shares be invested at 6.25% per annum at compound interest, they will receive equal amounts on attaining the age of 21 yr. The present share of A is

(a) ? 4225 *(b)* ? 4352

(c) *X* 4096 *(d)* *X* 4000

11. The simple interest on a certain sum of money for 3 yr at 8% pa is half the compound interest on *X* 8000 for 2 yr at 10% pa. Find the sum on which simple interest is calculated

(a) *X* 3500 *(b)* *X* 3800

(c) *X* 4000 *(d)* *X* 3600

(e) None of the above

12. During the first year, the population of a village is increased by 5% and the second year, it is diminished by 5%. At the end of the second year, its population was 47880. What was the population at the beginning of the first year? [SSC Multitasking 2014]

(a) 45500 *(b)* 48000

(c) 43500 *(d)* 53000

Answer with **Solutions**

Exercise© *Base Level Questions*

1. (*c*) Given, $P = ₹\ 50000$, $R = 12\%$

and $n = 3$ yr

According to the formula,

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

$$= 50000\left(1+\frac{12}{100}\right)^3$$

$$= 50000 \times \frac{28}{25} \times \frac{28}{25} \times \frac{28}{25}$$

$$= \frac{16 \times 28 \times 28 \times 28}{5} = \frac{351232}{5}$$

$$= ₹\ 70246.40$$

$\therefore$ CI $= 70246.40 - 50000 = ₹\ 20246.40$

2. (*d*) Given, $P = ₹\ 8000$, $R = 5\%$ and $n = 2$ yr

According to the formula,

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

$$= 8000\left(1+\frac{5}{100}\right)^2$$

$$= 8000 \times \left(\frac{105}{100}\right)^2$$

$$= 8000 \times \frac{21}{20} \times \frac{21}{20}$$

$$= 20 \times 21 \times 21 = ₹\ 8820$$

3. (*a*) Using the formula, $A = P\left(1+\frac{R}{100}\right)^n$

$\Rightarrow \quad 1352 = P\left(1+\frac{4}{100}\right)^2$

$\Rightarrow \quad 1352 = P\,(1.04)^2$

$\therefore \quad P = \frac{1352}{(1.04)^2} = ₹\ 1250$

4. (*a*) Given, $P = ₹\ 15000$, $R = 10\%$

and $n = 1$ yr

According to the formula,

$$\text{Amount} = P\left(1+\frac{R}{2 \times 100}\right)^{2n}$$

$$= 15000\left(1+\frac{5}{100}\right)^2$$

$$= 15000 \times \left(\frac{105}{100}\right)^2$$

$$= 15000 \times \frac{21}{20} \times \frac{21}{20}$$
$$= ₹\ 16537.50$$

5. (a) Given, $R = 4\%$, $n = 2$ yr and $A = ₹\ 169$, $P = ?$

According to the formula,

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^n$$

$$\Rightarrow \quad 169 = P\left(1 + \frac{4}{100}\right)^2$$

$$\Rightarrow \quad 169 = P\left(\frac{26}{25}\right)^2$$

$$\Rightarrow \quad P = \frac{169 \times 25 \times 25}{26 \times 26}$$

$$\therefore \quad P = \frac{105625}{676} = ₹\ 156.25$$

6. (b) Given, $P = ₹\ 31250$, $n = 9$ months $= 3$ quarters and $R = 16\%$ pa $= 4\%$ per quarter

According to the formula,

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^n$$
$$= 31250\left(1 + \frac{4}{100}\right)^3$$
$$= 31250 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25}$$
$$= ₹\ 35152$$

$$\therefore \quad CI = 35152 - 31250 = ₹ 3902$$

7. (b) Given, $P = ₹\ 1750, R = 8\%$,

$$n = 2 \text{ and } \frac{a}{b} = \frac{1}{2}$$

According to the formula,

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^n \times \left(1 + \frac{\frac{a}{b} \times R}{100}\right)$$

$$= 1750\left(1 + \frac{8}{100}\right)^2\left(1 + \frac{\frac{1}{2} \times 8}{100}\right)$$

$$= 1750\left(\frac{27}{25}\right)^2 \times \frac{26}{25} = 1750 \times \frac{27}{25} \times \frac{27}{25} \times \frac{26}{25}$$

$$= ₹\ 2122.848 = ₹\ 2122.85$$

8. (*d*) Given, $P = ₹\ 15000$, $R = 12\%$

and $n = 1\frac{1}{4} = \frac{5}{4}$ yr

According to the formula,

$$\text{Amount} = P\left(1 + \frac{R}{100 \times 4}\right)^{4n}$$

$$= 15000\left(1 + \frac{12}{100 \times 4}\right)^{4 \times \frac{5}{4}}$$

$$= 15000\left(\frac{412}{400}\right)^5 = 15000\left(\frac{103}{100}\right)^5$$

$$= 15000 \times \frac{103}{100} \times \frac{103}{100} \times \frac{103}{100} \times \frac{103}{100} \times \frac{103}{100}$$

$$= \frac{15 \times 103 \times 103 \times 103 \times 103 \times 103}{10000000}$$

$= ₹\ 17389.111 = ₹\ 17389.12$ (approx)

9. (*a*) Given, $\text{CI} = ₹\ 4347$, $P = ₹\ 30000$

and $R = 7\%$

By formula, $\text{CI} = P\left[\left(1 + \frac{R}{100}\right)^n - 1\right]$

$$\Rightarrow 4347 = 30000\left[\left(1 + \frac{7}{100}\right)^n - 1\right]$$

$$\Rightarrow \left(\frac{107}{100}\right)^n = \frac{4347}{30000} + 1$$

$$\Rightarrow \left(\frac{107}{100}\right)^n = \frac{34347}{30000} = \frac{11449}{10000}$$

$$\Rightarrow \left(\frac{107}{100}\right)^n = \left(\frac{107}{100}\right)^2$$

$\therefore \quad n = 2$

10. (*a*) Given, $P = ₹\ 4000$,

$n = 9$ months $= \frac{3}{4}$ yr and $\text{CI} = ₹\ 630.50$

$\text{Amount} = P + \text{CI} = 4000 + 630.50$
$= ₹\ 4630.50$

According to the formula,

$$\text{Amount} = P\left(1 + \frac{R}{100 \times 4}\right)^{4n}$$

$$\Rightarrow \quad 4630.50 = 4000\left(1 + \frac{R}{400}\right)^{4 \times \frac{3}{4}}$$

$$\Rightarrow \quad 4630.50 = 4000\left(\frac{400 + R}{400}\right)^3$$

$$\Rightarrow \quad \frac{4630.50}{4000} = \left(\frac{400 + R}{400}\right)^3$$

$$\Rightarrow \quad \frac{9261}{8000} = \left(\frac{400 + R}{400}\right)^3$$

$$\Rightarrow \quad \left(\frac{21}{20}\right)^3 = \left(\frac{400+R}{400}\right)^3$$

$$\Rightarrow \quad \frac{400+R}{400} = \frac{21}{20}$$

$$\Rightarrow \quad 400 + R = 21 \times 20 = 420$$

$$\therefore \quad R = 420 - 400 = 20\%$$

11. (c) Given, $P = 185220$, $R = 5\%$ (increases) and $n = 3$ yr

According to the formula,

$$\text{Population } n \text{ yr ago} = \frac{P}{\left(1+\frac{R}{100}\right)^n}$$

[by Technique 5]

$$= \frac{185220}{\left(1+\frac{5}{100}\right)^3} = \frac{185220}{\frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}}$$

$$= \frac{185220 \times 20 \times 20 \times 20}{21 \times 21 \times 21}$$

$$= 20 \times 20 \times 20 \times 20 = 160000$$

12. (c) Given, $P = 10$ crore

and population after 3 yr = 13.31 crore

According to the formula,

$$\text{Population after } n \text{ yr} = P\left(1+\frac{R}{100}\right)^n$$

[by Technique 5]

$$\Rightarrow \quad 13.31 = 10\left(1+\frac{R}{100}\right)^3$$

$$\Rightarrow \quad \frac{1331}{1000} = \left(1+\frac{R}{100}\right)^3$$

$$\Rightarrow \quad \left(\frac{11}{10}\right)^3 = \left(1+\frac{R}{100}\right)^3$$

$$\Rightarrow \quad 1 + \frac{R}{100} = \frac{11}{10}$$

$$\Rightarrow \quad \frac{R}{100} = \frac{11}{10} - 1 = \frac{1}{10}$$

$$\therefore \quad R = 10\%$$

13. (a) According to the given condition,

$$441 = 400\left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \quad \frac{441}{400} = \left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \quad \left(\frac{21}{20}\right)^2 = \left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \quad \frac{21}{20} = 1 + \frac{R}{100}$$

$$\Rightarrow \quad \frac{21}{20} - 1 = \frac{R}{100} \quad \Rightarrow \quad \frac{R}{100} = \frac{1}{20}$$

$$\therefore \quad R = 5\%$$

$\therefore$ New rate $= 5 + 5 = 10\%$

$\therefore$ Amount $= 400\left(1+\frac{10}{100}\right)^2$

$= 400 \times \frac{11}{10} \times \frac{11}{10} = ₹\ 484$

14. (*d*) Given, $P = ₹\ 100000$, $R = 10\%$, $n = 3$ yr

According to the formula,

Price of LCD TV set after 3 yr

$= P\left(1-\frac{R}{100}\right)^n = 100000\left(1-\frac{10}{100}\right)^3$

$= 100000\left(\frac{90}{100}\right)^3$

$= 100000 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} = ₹\ 72900$

15. (*a*) Given, $P = ₹\ 4000$,

$R_1 = 10\%$ (decreases), $R_2 = 5\%$ (decreases) and $R_3 = 15\%$ (growth)

$\therefore$ According to the formula,

Income

$= P\left(1-\frac{R_1}{100}\right)\left(1-\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

[by Technique 5]

$= 4000\left(1-\frac{10}{100}\right)\left(1-\frac{5}{100}\right)\left(1+\frac{15}{100}\right)$

$= 4000 \times \frac{9}{10} \times \frac{19}{20} \times \frac{23}{20}$

$= 9 \times 19 \times 23 = ₹\ 3933$

16. (*c*) If sum is x, then x becomes $\frac{5}{2}x$ in 6 yr.

$\frac{5}{2}x$ becomes $\frac{25}{4}x$ in 12 yr.

$\frac{25}{4}x$ becomes $\frac{125}{8}x$ in 18 yr.

The sum becomes $\frac{125}{8}$ times in 18 yr.

Fast Track Method

Here, $n_1 = 6$ yr, $x = 2\frac{1}{2} = \frac{5}{2}$, $n_2 = 18$ yr

According to the formula,

$x^{\frac{1}{n_1}} = y^{\frac{1}{n_2}}$ [by Technique 5]

$\Rightarrow \left(\frac{5}{2}\right)^{\frac{1}{6}} = y^{\frac{1}{18}} \Rightarrow y = \left(\frac{5}{2}\right)^{18/6}$

$\Rightarrow y = \left(\frac{5}{2}\right)^3 = \frac{5 \times 5 \times 5}{2 \times 2 \times 2}$

$\therefore y = 125/8$

17. (*c*) Given, $P = ₹\ 1250$, $n = 2$ yr and $R = 4\%$

According to the formula,

Difference between compound interest and simple interest $= \frac{PR^2}{100^2}$ [by Technique 1]

$\therefore$ Required difference $= \dfrac{1250 \times 4 \times 4}{100 \times 100} = ₹\, 2$

18. (*a*) Required difference

$$= P\left(\frac{R}{100}\right)^2\left(\frac{300+R}{100}\right)$$

[by Technique 1]

$$= 10000\left(\frac{5}{100}\right)^2\left(\frac{305}{100}\right) = 76.25$$

19. (*d*) Let required difference be ₹ D.

By formula,

$$D = P \times \left(\frac{R}{100}\right)^2 \times \left(3 + \frac{R}{100}\right)$$

[by Technique 1]

$$= 16200 \times \left(\frac{25}{100}\right)^2 \times \left(3 + \frac{25}{100}\right)$$

$$= 16200 \times \frac{625}{10000} \times \frac{13}{4}$$

$$= \frac{162 \times 625 \times 13}{4 \times 100} = ₹\, 3290.63$$

20. (*a*) Given, $R = 8\%$, $D = ₹\, 32$ and $P = ?$

By formula, $D = P\left(\dfrac{R}{100}\right)^2$ **[by Technique 1]**

$$\Rightarrow \quad 32 = P\left(\frac{8}{100}\right)^2$$

$$\therefore \quad P = \frac{32 \times 100 \times 100}{8 \times 8} = ₹\, 5000$$

21. (*d*) Given, CI $-$ SI $= 16$, $R = 5\%$, $n = 2$ yr and SI $= ?$

According to the formula,

$$\text{CI} - \text{SI} = \frac{\text{SI} \times R}{200}$$ **[by Technique 2]**

$$\Rightarrow \quad 16 = \frac{\text{SI} \times 5}{200}$$

$$\Rightarrow \quad 16 = \frac{\text{SI}}{40}$$

$$\therefore \quad \text{SI} = ₹\, 640$$

22. (*b*) Given, $n = 2$ yr, $R = 4\%$ and SI $= 80$

According to the formula,

$$\text{CI} = \text{SI}\left(1 + \frac{R}{100}\right)$$ **[by Technique 2]**

$$= 80\left(1 + \frac{4}{200}\right) = \frac{80 \times 51}{50} = ₹\, 81.60$$

23. (*d*) Given, CI $-$ SI $= 6.40$

$T = 2$ yr and $R = 8\%$

$$\Rightarrow P\left[\left(1 + \frac{R}{100}\right)^T - 1\right] - \frac{P \times R \times T}{100} = 6.40$$

$$\Rightarrow P\left[\left(1 + \frac{8}{100}\right)^2 - 1\right] - \frac{P \times 8 \times 2}{100} = 6.40$$

$$\Rightarrow P\left[\left(\frac{27}{25}\right)^2 - 1\right] - \frac{16P}{100} = 6.40$$

$$\Rightarrow P\left[\frac{27^2 - 25^2}{625}\right] - \frac{16P}{100} = 6.40$$

$$\Rightarrow \frac{104\ P}{625} - \frac{16\ P}{100} = 6.40$$

$$\Rightarrow \frac{416\ P - 400\ P}{2500} = 6.40$$

$$\Rightarrow 16\ P = 6.40 \times 2500$$

$$\therefore P = \frac{6.40 \times 2500}{16} = ₹\,1000$$

Fast Track *Method*

Here, *D* = f 6.40, i? = 8% and *n* = *2* yr Then, according to the formula, Difference between compound interest and simole interest

$$D = \frac{PR^2}{100^2} \qquad \textbf{[by Technique 1]}$$

$$\Rightarrow \quad 6.40 = \frac{P \times 64}{10000}$$

$$\therefore \quad P = ₹\,1000$$

24. (*c*) SI for on ₹ 12960 for 1 yr

$$= 13176 - 12960 = ₹\,216$$

By formula, $\text{SI} = \dfrac{P \times R \times T}{100}$

$$\Rightarrow \quad 216 = \frac{12960 \times R \times 1}{100}$$

$$\therefore \quad R = \frac{5}{3} = 1\frac{2}{3}\%$$

Fast Track Method

Here, $A_1 = ₹\,12960$ and $A_2 = ₹\,13176$

Required rate per cent

$$= \frac{A_2 - A_1}{A_1} \times 100 \quad \textbf{[by Technique 4]}$$

$$= \frac{13176 - 12960}{12960} \times 100$$

$$= \frac{216 \times 100}{12960} = \frac{2160}{1296} = \frac{5}{3} = 1\frac{2}{3}\%$$

25. (*b*) Let required amount be ₹ P.

According to the question,

$$2916 = P\left(1 + \frac{R}{100}\right)^2 \qquad \text{... (i)}$$

and $$3149.28 = P\left(1 + \frac{R}{100}\right)^3 \qquad \text{...(ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$1 + \frac{R}{100} = \frac{3149.28}{2916} \Rightarrow \frac{R}{100} = \frac{3149.28}{2916} - 1$$

$$\Rightarrow \quad R = \frac{233.28}{2916} \times 100 = 8\%$$

From Eq. (i),

$$P = \frac{2916 \times 100 \times 100}{108 \times 108} = ₹\,2500$$

26. (*d*) Let sum $= P$

and given that $n = 3$ yr

According to the question,

Amount $= 27P$

According to the formula,

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^n$$

$$\Rightarrow \quad 27P = P\left(1 + \frac{R}{100}\right)^3$$

$$\Rightarrow \quad 27 = \left(1 + \frac{R}{100}\right)^3$$

$$\Rightarrow \quad \left(1 + \frac{R}{100}\right)^3 = (3)^3$$

$$\Rightarrow \quad 1 + \frac{R}{100} = 3$$

$$\Rightarrow \quad \frac{R}{100} = 3 - 1 = 2$$

$$\therefore \quad R = 200\%$$

27. (*a*) Let the amount be ₹ P

$R = 5\%$, $n = 2$ yr and CI ₹ 410

By formula, $\text{CI} = P\left[\left(1 + \frac{R}{100}\right)^n - 1\right]$

$$\Rightarrow \quad 410 = P\left[\left(1 + \frac{5}{100}\right)^2 - 1\right]$$

$$\Rightarrow \quad 410 = P\left[\left(1 + \frac{1}{20}\right)^2 - 1\right]$$

$$\Rightarrow \quad 410 = P\left[\left(\frac{21}{20}\right)^2 - 1\right]$$

$$\Rightarrow \quad 410 = P\left[\frac{441}{400} - 1\right]$$

$$\Rightarrow \quad 410 = P\left(\frac{441 - 400}{400}\right) = P \times \frac{41}{400}$$

$$\Rightarrow \quad P = \frac{410 \times 400}{41}$$

$$\therefore \quad P = ₹\,4000$$

28. (*a*) Given, $P = ₹\,9375$, $R_1 = 2\%$

and $R_2 = 4\%$

According to the formula,

$$A = P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)$$

$$= 9375\left(1 + \frac{2}{100}\right)\left(1 + \frac{4}{100}\right)$$

$$= 9375 \times \frac{51}{50} \times \frac{26}{25}$$

$$= 7.5 \times 51 \times 26 = ₹\,9945$$

Now, $\text{CI} = A - P = 9945 - 9375$

$= ₹\,570$

29. (*b*) Let the required sum be P. Then,

$$P\left(1+\frac{5}{100}\right)\left(1+\frac{10}{100}\right)\left(1+\frac{20}{100}\right)=16632$$

$$\Rightarrow \quad P\times\frac{105}{100}\times\frac{110}{100}\times\frac{120}{100}=16632$$

$$\Rightarrow P=\frac{16632\times100\times100\times100}{105\times110\times120}=₹\,12000$$

30. (*b*) Given, SI = ₹ 120, $n = 2$ yr

and CI = ₹ 129

By formula,

$$\text{SI}=\frac{P\times R\times T}{100}$$

$$\Rightarrow \quad 120=\frac{P\times R\times 2}{100}$$

$$\Rightarrow \quad PR = 6000$$

$$\text{CI}=P\left[\left(1+\frac{R}{100}\right)^n-1\right]$$

$$\Rightarrow \quad 129=P\left[\left(1+\frac{R}{100}\right)^2-1\right]$$

$$\Rightarrow \quad 1290000=P\,[(100+R)^2-100^2]$$

$$\Rightarrow \quad 1290000=P\,[100^2+R^2+2R\times100-100^2]$$

$$\Rightarrow \quad 1290000=P\,(R^2+R\times200)$$

$$\Rightarrow \quad 1290000=\frac{6000}{R}(R^2+200R) \qquad \left[\because P=\frac{6000}{R}\right]$$

$$\Rightarrow \quad 1290000=6000\,(R+200)$$

$$\Rightarrow \quad 1290000=6000\,R+1200000$$

$$\Rightarrow \quad 6000\,R=90000$$

$$\therefore \quad R=\frac{90000}{6000}=15\%$$

Fast Track Method

Here, $n = 2$ yr, SI = ₹ 120 and CI = ₹ 129

By formula,

$$\text{CI}=\text{SI}\left(1+\frac{R}{200}\right) \quad \textbf{[by Technique 2]}$$

$$\Rightarrow \quad 129=120\left(1+\frac{R}{200}\right)$$

$$\Rightarrow \quad \frac{129}{120}=1+\frac{R}{200}$$

$$\Rightarrow \quad \frac{129}{120}-1=\frac{R}{200}$$

$\Rightarrow \quad \dfrac{9}{120} = \dfrac{R}{200}$

$\therefore \quad R = \dfrac{9 \times 200}{120} = 15\%$

31. (*c*) Percentage growth

$$= \left(\frac{1}{8} \times 100\right)\% = 12.5\%$$

Height after 2 yr $= 64 \times \left(1 + \dfrac{12.5}{100}\right)^2$

$$= 64 \times \frac{9}{8} \times \frac{9}{8} = 81 \text{ cm}$$

32. (*a*) Given, CI − SI = 10, $R = 5\%$

and $n = T = 2$ yr

$$P\left[\left(1 + \frac{R}{100}\right)^n - 1\right] - \frac{P \times R \times T}{100} = 10$$

$$\Rightarrow P\left[\left(1 + \frac{5}{100}\right)^2 - 1\right] - \frac{P \times 5 \times 2}{100} = 10$$

$$\Rightarrow P\left[\left(\frac{21}{20}\right)^2 - 1\right] - \frac{P}{10} = 10$$

$$\Rightarrow P\left[\frac{21^2 - 20^2}{400}\right] - \frac{P}{10} = 10$$

$$\Rightarrow \frac{41}{400}P - \frac{P}{10} = 10 \Rightarrow \frac{41P - 40P}{400} = 10$$

$\therefore \quad P = 400 \times 10 = ₹\,4000$

Fast Track Method

Here, CI − SI = 10, $R = 5\%$, $n = 2$ yr and $P = ?$

According to the formula,

$$\text{CI} - \text{SI} = \frac{PR^2}{100^2} \quad \text{[by Technique 1]}$$

$$\Rightarrow \quad 10 = \frac{P \times 25}{10000}$$

$\therefore \quad P = 400 \times 10 = ₹\,4000$

33. (*a*) Let the principal be ₹ x.

Amount after first year $= \dfrac{x \times 104}{100}$

and amount after second year

$$\frac{x \times 104}{100} \times \frac{4}{100}$$

According to the question,

$$= \frac{x \times 104 \times 4}{100 \times 100} = 208$$

$\therefore \quad x = ₹\,5000$

Exercise © *Higher Skill Level Questions*

1. (*c*) Given, $P = 126800$, $R_1 = 15\%$ (increases) and $R_2 = 20\%$ (decreases)

According to the formula,

Population after 2 yr

$= P\left(1+\frac{R_1}{100}\right)\left(1-\frac{R_2}{100}\right)$ **[by Technique 5]**

$= 126800\left(1+\frac{15}{100}\right)\left(1-\frac{20}{100}\right)$

$= 126800 \times \frac{115}{100} \times \frac{80}{100} = 116656$

2. (*a*) Let each instalment be ₹ x.

Then, according to the question,

$$\frac{x}{\left(1+\frac{4}{100}\right)} + \frac{x}{\left(1+\frac{4}{100}\right)^2} = 5100$$

[by Technique 6]

$\Rightarrow \quad \frac{25x}{26} + \frac{625x}{676} = 5100$

$\Rightarrow \quad 1275x = 5100 \times 676$

$\therefore \quad x = \frac{5100 \times 676}{1275} =$ ₹ 2704

3. (*a*) Let the first part be ₹ x.

Then, second part $= (2602 - x)$

According to the question,

$$x\left(1+\frac{4}{100}\right)^7 = (2602 - x)\left(1+\frac{4}{100}\right)^9$$

$\Rightarrow \quad \frac{x}{(2602 - x)} = \frac{\left(1+\frac{4}{100}\right)^9}{\left(1+\frac{4}{100}\right)^7}$

$\Rightarrow \quad \frac{x}{(2602 - x)} = \left(1+\frac{4}{100}\right)^2$

$= \frac{26}{25} \times \frac{26}{25} = \frac{676}{625}$

$\Rightarrow \quad 625x = 676 \times 2602 - 676x$

$\Rightarrow \quad 1301x = 676 \times 2602$

$\therefore \quad x = \frac{676 \times 2602}{1301} = 676 \times 2 = 1352$

$\therefore$ The two parts are ₹ 1352 and ₹ $(2602 - 1352) =$ ₹ 1250

4. (*b*) According to the question,

Borrowed sum $= \frac{121}{\left(1+\frac{10}{100}\right)} + \frac{121}{\left(1+\frac{10}{100}\right)^2}$

$= \frac{121}{\frac{11}{10}} + \frac{121}{\frac{11}{10} \times \frac{11}{10}}$

$= 110 + 100 =$ ₹ 210

5. (*b*) Let rate = R% and time = n yr

Then, $$4320 = 3000\left(1+\frac{R}{100}\right)^n$$

$\Rightarrow$ $$\left(1+\frac{R}{100}\right)^n = \frac{4320}{3000} = 1.44$$

$\therefore$ $$\left(1+\frac{R}{100}\right)^{n/2} = \sqrt{1.44} = 1.2$$

$\therefore$ Required amount for $\frac{n}{2}$ yr

$$= 3000\left(1+\frac{R}{100}\right)^{n/2}$$

$$= 3000 \times 1.2 = ₹\ 3600$$

6. (*b*) According to the question,

$$1728 = 1331\left(1+\frac{R}{100}\right)^3$$

$\Rightarrow$ $$\frac{1728}{1331} = \left(1+\frac{R}{100}\right)^3$$

$\Rightarrow$ $$\left(\frac{12}{11}\right)^3 = \left(1+\frac{R}{100}\right)^3$$

$\Rightarrow$ $$1+\frac{R}{100} = \frac{12}{11} \Rightarrow \frac{R}{100} = \frac{12}{11} - 1 = \frac{1}{11}$$

$\therefore$ $$R = \frac{100}{11} = 9.09\%$$

7. (*a*) Let the sum be ₹ x.

Then, CI when compounded half-yearly

$$= \left[x \times \left(1+\frac{10}{100}\right)^4 - x\right] = \frac{4641}{10000}x$$

CI when compounded annually

$$= \left[x \times \left(1+\frac{20}{100}\right)^2 - x\right] = \frac{11}{25}x$$

According to the question,

$$\frac{4641}{10000}x - \frac{11}{25}x = 964$$

$\Rightarrow$ $$x\left(\frac{4641-4400}{10000}\right) = 964$$

$\therefore$ $$x = \frac{964 \times 10000}{241} = ₹\ 40000$$

8. (*d*) Let amount be ₹ x and rate of interest be r% annually.

According to the question,

Amount after 1st yr = ₹ 1200

$\Rightarrow$ $$x\left(1+\frac{R}{100}\right) = 1200 \quad \text{...(i)}$$

Amount after 3rd yr = 1587

$$\Rightarrow \quad x\left(1+\frac{R}{100}\right)^3 = 1587 \quad \dots \text{(ii)}$$

On dividing Eq. (ii) from Eq. (i), we get

$$\left(1+\frac{R}{100}\right)^2 = \frac{1587}{1200} = \frac{529}{400}$$

$$\Rightarrow \quad 1+\frac{R}{100} = \frac{23}{20} \Rightarrow \frac{R}{100} = \frac{3}{20}$$

$$\therefore \quad R = 15\%$$

9. (*d*) Let value of each instalment be ₹ x.

Then, $\dfrac{x}{\left(1+\dfrac{20}{100}\right)} + \dfrac{x}{\left(1+\dfrac{20}{100}\right)^2} = 11000$

$$\Rightarrow \quad x\left(\frac{5}{6}+\frac{25}{36}\right) = 11000$$

$$\Rightarrow \quad x\left(\frac{55}{36}\right) = 11000$$

$$\therefore \quad x = ₹\ 7200$$

10. (*c*) Let shares of A and B be ₹ x and ₹ $(8448 - x)$, respectively.

Amount got by A after 3 yr

= Amount got by B after 2 yr

$$x\left(1+\frac{6.25}{100}\right)^3 = (8448 - x) \times \left(1+\frac{6.25}{100}\right)^2$$

$$\Rightarrow \quad 1+\frac{6.25}{100} = \frac{8448 - x}{x}$$

$\Rightarrow \quad 1 + \frac{1}{16} = \frac{8448 - x}{x}$

$\Rightarrow \quad \frac{17}{16} = \frac{8448 - x}{x}$

$\Rightarrow \quad 17x = 135168 - 16x$

$\Rightarrow \quad x = 4096$

11. (*a*) $\text{CI} = 8000 \times \left(1 + \frac{10}{100}\right)^2 - 8000$

$= 8000 \times \frac{11}{10} \times \frac{11}{10} - 8000$

$= 9680 - 8000 = ₹\ 1680$

$\therefore \text{Sum} = \left(\frac{840 \times 100}{3 \times 8}\right) = ₹\ 3500$

$\left[\begin{array}{l} \because \text{SI is half of CI} \\ \therefore \text{SI} = \frac{1680}{2} = 840 \end{array}\right]$

12. (*b*) Let the population at the beginning of the first year be x.

Then, according to the question,

$x\left[1 + \frac{5}{100}\right] \times \left[1 - \frac{5}{100}\right] = 47880$

[by Technique 5]

$\Rightarrow \quad x \times \frac{105}{100} \times \frac{95}{100} = 47880$

$\therefore \quad x = 47880 \times \frac{100}{105} \times \frac{100}{95} = 48000$

Chapter

16

True Discount and Banker's Discount

True Discount

If a person borrows certain money from another person for a certain period and the borrower wants to clear off the debt right now, then for paying back the debt, the borrower gets certain discount which is called True Discount (TD),

For example If a person has to pay *X* 160 after 4 yr and the rate of interest is 15% per annum. It is clear that ? 100 at 15% will amount to ? 160 in 4 yr. Therefore, the payment of ? 100 now will clear off the debt of ? 160 due 4 yr hence at 15% per annum.

Thus, following points are the outcomes

1. Sum due is equal to ? 160 due 4 yr hence,

2. Present Worth (PW) is ? 100.

3. True Discount (TD) = 160-100 = ? 60

Now, it is easy for us to define terms like Present Worth (PW), Amount (A) and True Discount (TD).

Present Worth The money to be paid back is called the Present Worth (PW).

Amount Sum due is called Amount (A). *Amount (A) = PW + TD*

True Discount It is the difference between the Amount (A) and the Present Worth (PW).

Discount (TD) =A-PW

MIND IT!

1. True discount is the interest on Present Worth (PW).

2. Interest is reckoned on PW and TD is reckoned on amount. According to the definition, we have TD = *A* - PW

Fast Track **Formulae**

to *solve the* QUESTIONS

Formula 1

If rate of interest is R% per annum, time is T yr and present worth is PW, then True Discount (TD) $= \dfrac{PW \times R \times T}{100}$

Ex. 1 What will be the true discount for the present worth of ? 6000 for a period of 9 months at 12% per annum?

Sol. Given that, $T = 9 \text{ months} = \dfrac{9}{12}$ yr and $R = 12\%$ PW = ₹ 6000;

$$TD = \frac{PW \times R \times T}{100} = \frac{6000 \times 12 \times \frac{9}{12}}{100} = 60 \times 9 = ₹\,540$$

Formula 2

If rate of interest is $R\%$ and monev due for amount) is A after T vr. then

$$\text{Present Worth (PW)} = \frac{100 \times A}{100 + R \times T}$$

:X. 2 Find the present worth of ? 1860 due 3 yr hence at 8% per annum. Also, find the true discount.

Sol. Let PW be x.

Then, according to the formula,

$$\therefore \quad 1860 - x = \frac{x \times 8 \times 3}{100} \Rightarrow 1860 - x = \frac{6x}{25} \Rightarrow 31x = 1860 \times 25$$

$$\therefore \quad x = \frac{1860 \times 25}{31} = ₹\,1500$$

$$\therefore \quad TD = A - PW$$

$$= TD = 1860 - 1500 = ₹\,360$$

Alternate Method

Given that, $A = ₹\,1860$, $R = 8\%$ and $T = 3$ yr

According to the formula,

$$PW = \frac{100 \times A}{100 + R \times T} = \left[\frac{100 \times 1860}{100 + (8 \times 3)}\right]$$

$$= \frac{100 \times 1860}{124} = ₹\,1500$$

$$TD = A - PW$$

$$= 1860 - 1500 = ₹\,360$$

Formula 3

If money due A, rate of interest $R\%$ and time T are given, then

$$\text{True Discount (TD)} = \frac{A \times T \times R}{100 + R \times T}$$

Ex. 3 Find the sum due, if the true discount on a sum due 2 yr hence at 14% rate of interest be ? 336.

Sol. Let PW be x.

Then, according to the formula,

$$336 = \frac{x \times 14 \times 2}{100} = \frac{7x}{25} \Rightarrow x = \frac{336 \times 25}{7} = ₹\ 1200$$

$\therefore$ Amount $= 1200 + 336 = ₹\ 1536$

Alternate Method

Given that, $R = 14\%$, $T = 2$ yr, TD $= ₹\ 336$ and Sum due $(A) = ?$

According to the formula,

$$\text{TD} = \frac{A \times R \times T}{100 + R \times T} \Rightarrow 336 = \frac{A \times 14 \times 2}{100 + 14 \times 2} = \frac{28A}{128}$$

$$\therefore \quad A = \frac{336 \times 128}{28} = ₹\ 1536$$

Formula 4

If the true discount on a certain sum of money due certain year hence and the simple interest on the same sum for the same time and at the same rate is given, then

$$\text{Money due } (A) = \frac{\text{SI} \times \text{TD}}{\text{SI} - \text{TD}}$$

(SI = Simple interest and TD = True discount)

Ex. 4 The true discount on a certain sum of money due 10 yr hence is ? 68 and the simple interest on the same sum for the same time and at the same rate of interest is ? 102. Find the sum due.

Sol. Given that, TD $= ₹\ 68$ and SI $= ₹\ 102$

$$\therefore \quad \text{Sum due } (A) = \frac{\text{SI} \times \text{TD}}{\text{SI} - \text{TD}} = \frac{102 \times 68}{102 - 68} = \frac{102 \times 68}{34} = ₹\ 204$$

Formula 5

If the true discount on a certain sum of money due *T* yr hence and the simple interest on the same sum for the same time and at the same rate of interest *R*% per annum are given, then

$$\text{SI} - \text{TD} = \frac{\text{TD} \times R \times T}{100}$$

Ex. 5 The true discount on a certain sum of money due 16 yr hence is ? 300 and the simple interest on the same sum for the same time and at the same rate of interest is ? 900, find the rate per cent.

Sol. Given that, SI = ₹ 900, TD = ₹ 300, T = 16 yr and R = ?

According to the formula, $SI - TD = \frac{TD \times R \times T}{100}$

$\Rightarrow \quad 900 - 300 = \frac{300 \times R \times 16}{100}$

$\Rightarrow \quad 600 = \frac{300 \times 4 \times R}{25}$

$\Rightarrow \quad 50 = 4R$

$\therefore \quad R = \frac{50}{4} = \frac{25}{2} = 12.5\%$

Alternate Method

According to the formula,

Sum due $= \frac{SI \times TD}{SI - TD} = \frac{900 \times 300}{600} = ₹\ 450$

TD is SI on amount.

$\therefore$ ₹ 900 is SI on ₹ 450 for 16 yr.

$\therefore \quad 900 = \frac{450 \times R \times 16}{100}$

$\Rightarrow \quad R = \frac{900 \times 100}{450 \times 16} = 12.5\%$

Formula 6

When the sum is put at compound interest, then $PW = \frac{A}{\left(1 + \frac{R}{100}\right)^T}$

Ex. 6 What will be the present worth of ? 4840 due 2 yr hence, when the interest is compounded at 10% per annum? Also, find true discount.

Sol. Given that, A = ₹ 4840, T = 2 yr, R = 10%, PW = ? and TD = ?

According to the formula,

$$PW = \frac{A}{\left(1 + \frac{R}{100}\right)^T} = \frac{4840}{\left(1 + \frac{10}{100}\right)^2} = \frac{4840}{\left(\frac{11}{10}\right)^2} = \frac{4840 \times 10 \times 10}{11 \times 11}$$

$= 40 \times 10 \times 10 = ₹\ 4000$

$TD = A - PW = 4840 - 4000 = ₹\ 840$

Ex. 7 If the present worth of a certain sum due 2 yr hence at 10% per annum compound interest is ? 2000, find the sum.

Sol. Given that, PW = ₹ 2000, T = 2 yr, R = 10% and A = ?

According to the formula,

$$PW = \frac{A}{\left(1 + \frac{R}{100}\right)^T}$$

$$\Rightarrow \quad 2000 = \frac{A}{\left(1 + \frac{10}{100}\right)^2} = \frac{A \times 10 \times 10}{11 \times 11}$$

$$\therefore \quad A = \frac{2000 \times 11 \times 11}{10 \times 10} = 20 \times 121$$

$$= ₹\ 2420$$

Banker's Discount

The difference between the amount shown on a bond that is bought by a bank from a customer and the amount that the customer actually receives from the bank is called Banker's Discount.

For example If a person X buys goods having value of ? 5000 from another person Y at a credit of 6 months, then X makes payment of this purchase through 'Bill of Exchange' (Hundi).

The *'Bill of Exchange'* is prepared by Y and X signing this bill (Bill of Exchange) to allow Y to withdraw the amount from his (X's) bank account on a date that falls exactly after 6 months. This is the date which is known as *'Nominally Due Date'.*

If we add some more days (Grace days/Period), we get *'Legally Due Date'. Now, following outcomes can be written as* Banker's Discount (BD)

$$= \text{Interest on bill for remaining time (unexpired time)}$$
$$= \frac{\text{Value of bill} \times \text{Rate} \times \text{Remaining time}}{100}$$

Banker's Gain (BG) =BD-TD (unexpired time)

Note • 'Banker's Discount' (BD) is the 'Simple Interest' (SI) on the face value for the period from that particular date on which the bill was discounted and the legally due date

• Banker's Discount (BD) is little bit more than ''True Discount' (TD)

• When the date of bill is not given, the grace days are not added

Formula 7

If the face value *A*, remaining time *T* and rate /?% are given, then

$$\text{Banker's Discount (BD)} = \frac{A \times R \times T}{100}$$

Ex. 8 Find the banker's discount at a bill of ? 25500 due 2 months hence, when rate of interest is 3% per annum.

Sol. Given that, $A =$ ₹ 25500, $R = 3\%, T = 2 \text{ months} = \frac{2}{12} = \frac{1}{6}$ yr and BD = ?

According to the formula,

$$\text{BD} = \frac{A \times R \times T}{100} = \frac{25500 \times 3 \times \frac{1}{6}}{100} = ₹\ 127.5$$

Formula 8

If the face value of a bill due *T* yr hence is *A* and rate is /?%, then

$$\text{Banker's Gain (BG)} = \frac{A\,(R \times T)^2}{100\,(100 + R \times T)} \text{ or Banker's Gain (BG)} = \frac{(\text{TD})^2}{\text{PW}}$$

Ex.9 The face value of a bill due 3 yr hence is ? 13800. If the rate of simple interest is 5% per annum, what will be the banker's gain at the bill?

Sol. Given that, $A =$ ₹ 13800, $R = 5\%, T = 3$ yr and BG = ?

According to the formula,

$$\text{BG} = \frac{A\,(R \times T)^2}{100\,(100 + R \times T)} = \frac{13800\,(5 \times 3)^2}{100\,(100 + 5 \times 3)} = \frac{13800 \times 225}{100 \times 115} = ₹\ 270$$

Ex. 10 The true discount at a bill of ? 3720 due 8 months hence is *X* 120. Find the banker's gain.

Sol. Present worth of bill = 3720 − 120 = ₹ 3600

$$\therefore \quad \text{Banker's Gain (BG)} = \frac{(\text{TD})^2}{\text{PW}} = \frac{120 \times 120}{3600} = \frac{144}{36} = ₹\ 4$$

Formula 9

If the true discount at a bill due *T* yr hence is TD and annual rate of interest is $R\%$, then Banker's Gain (BG) $= \dfrac{\text{TD} \times R \times T}{100}$

Ex.11 If the banker's gain is ? 180 at a bill due 6 yr hence at 10% per annum, find the banker's discount.

Sol. Given that, BG = ₹ 180, R = 10% and T = 6 yr

According to the formula,

$$\text{BG} = \frac{\text{TD} \times R \times T}{100}$$

$$\Rightarrow \quad 180 = \frac{\text{TD} \times 10 \times 6}{100}$$

$$\therefore \quad \text{TD} = \frac{100 \times 180}{10 \times 6} = ₹\ 300$$

As we know that BG = BD − TD

$$\Rightarrow \quad \text{BD} = \text{BG} + \text{TD} = 180 + 300 = ₹\ 480$$

Formula 10

If the face value on a bill is A and true discount is TD

then, $\text{BD} = \dfrac{A \times \text{TD}}{A - \text{TD}}$

Ex. 12 The true discount on a bill of ? 900 is ? 100. Find the banker's discount.

Sol. Given that, A = ₹ 900 and TD = ₹ 100

According to the formula,

$$\text{Banker's Discount (BD)} = \frac{A \times \text{TD}}{A - \text{TD}} = \frac{900 \times 100}{900 - 100} = \frac{900 \times 100}{800} = ₹\ 112.50$$

Alternate Method

PW = ₹ (900 − 100) = ₹ 800; SI on ₹ 800 = ₹ 100,

$$\text{SI on ₹ 900} = ₹\ \frac{100}{800} \times 900 = ₹\ \frac{900}{8} = ₹\ 112.50$$

Formula 11

If the banker's gain on a bill due T time hence at $R\%$ rate of simnle interest

is BG, then Present Worth (PW) $= \text{BG}\left(\dfrac{100}{R \times T}\right)^2$

Ex. 13 The banker's gain of a sum due 4 yr, hence at 12% per annum is ? 72. Find the present worth of that bill.

Sol. Given that, BG = ? 72, R = 12%, T = 4yr and PW = ?

According to the formula,

$$\text{PW} = \text{BG}\left(\frac{100}{R \times T}\right)^2 = 72\left(\frac{100}{12 \times 4}\right)^2 = 72 \times \frac{100}{48} \times \frac{100}{48}$$

$$= 72 \times \frac{25}{12} \times \frac{25}{12} = \frac{625}{2}$$

$$= ₹\ 312.50$$

Formula 12

If on a bill, banker's gain is BG and present worth is PW, then

True Discount, $\text{TD} = \sqrt{(\text{PW}) \times (\text{BG})}$.

Ex. 14 The present worth of a sum due sometime hence is ? 400 and the true discount is ? 80. Find the banker's gain.

Sol. $\because$ SI on ₹ 400 = ₹ 80

$\therefore$ SI on ₹ 480 $= \frac{80}{400} \times 480 =$ ₹ 96

$\therefore$ TD = ₹ 80 and BD = ₹ 96

$\therefore$ BG = BD − TD = 96 − 80 = ₹ 16

Alternate Method

Given that, PW = ₹ 400, TD = ₹ 80 and BG = ?

According to the formula, $\text{TD} = \sqrt{(\text{PW}) \times (\text{BG})}$

$\Rightarrow$ $80 = \sqrt{400 \times \text{BG}} = 20\sqrt{\text{BG}}$

$\therefore$ BG = 4 × 4 = ₹ 16

Formula 13

If BD and TD have been given on a bill, then

$$\text{Amount of bill } (A) = \frac{\text{BD} \times \text{TD}}{\text{BD} - \text{TD}} = \frac{\text{BD} \times \text{TD}}{\text{BG}}$$

Ex. 15 If BD and TD on a sum due certain time hence at a certain rate are ? 72 and ? 60 respect-ively, find the sum.

Sol. Given that, BD = ? 72, TD = ? 60 and *A = 1*

According to the formula,

$$\text{Required sum }(A) = \frac{BD \times TD}{BD - TD}$$

$$= \frac{72 \times 60}{72 - 60} = \frac{72 \times 60}{12}$$

$$= ₹\ 360$$

Fast Track Practice

True Discount

1. What will be the true discount for the present worth of *X* 600 for a period of 4 yr at 4% per annum rate of interest?

(a) *X* 100 *(b)* *X* 32

(c) *X* 86 *(d)* *X* 96

(e) None of the above

2. If the true discount for a sum of *X* 50000 for a period of 4 yr at a certain rate of interest per annum is *X* 2000, find the rate of interest.

(a) 1% *(b)* 1.5%

(c) 2% *(d)* 5%

(e) None of the above

3. What will be the present worth of *X* 3720 due 3 yr hence at 8% per annum? Also, find the true discount.

(a) PW = ? 3000 and TD = ? 720

(b) PW = X 15000 and TD = X 720

(c) PW = X 3000 and TD = X 320

(d) PW = X 1000 and TD = X 420

(e) None of the above

4. Find the sum due, if the true discount on a sum due 2 yr hence at 7% be X 672.

(a) X 5500 (b) X 5425

(c) X 5472 (d) X 5300

(e) None of the above

5. The true discount on a certain sum of money due 4 yr hence is X 75 and the simple interest on the same sum for the same time and at the same rate of interest is X 225. Find the rate per cent.

(a) 25% (b) 50%

(c) 31% (d) 45%

(e) None of the above

6. Kailash wants to sell his television. There are two offers, one at X 10000 cash and the other at a credit of X 6440 to be paid after 8 months, money being at 18% per annum. Which one is better offer?

(a) X 6440 at credit

(b) X 6440 in cash

(c) X 6440 at credit and X 10000 in cash both

(d) X 10000 in cash

(e) None of the above

7. Vandana bought a watch for X 600 and sold it the same day for X 688.50 at a credit of 9 months and this way she gained 2%. Find the rate of interest per annum.

(a) $16\frac{2}{3}\%$ (b) $15\frac{2}{3}\%$

(c) $11\frac{2}{3}\%$ (d) $5\frac{2}{3}\%$

(e) None of the above

8. Jagatram, a trader, owes a merchant Maganlal *X* 5014 due 1 yr hence. Jagatram wants to settle the account after 3 months. If the rate of interest is 12% per annum, how much cash should Jagatram pay?

(a) *X* 9200 (b) *X* 5600

(c) *X* 4600 (d) *X* 6600

(e) None of the above

9. Aarti has to pay *X* 440 to Babita after

1 yr. Babita asks Aarti to pay *X* 220 in cash and defer the payment of *X* 220 for

2 yr. Aarti agrees to it. If the rate of interest be 10% per annum, find Aarti's gain or loss.

(a) Aarti gains *X* 3.33

(b) Aarti gains *X* 8

(c) Aarti losses *X* 9

(d) Aarti losses *X* 3.33

(e) None of the above

10. X owes *Y X* 3146 payable *VA* yr hence. Also, *Y* owes *X, X* 2889 payable 6 months hence. If they want to settle the account forthwith, keeping 14% the rate of interest, who should pay and how much?

(a) /should pay? 100

(b) *Y* should pay *X* 50

(c) X should pay *X* 100

(d) *X* should pay *X* 50

(e) None of the above

Banker's Discount

11. Find the banker's discount at a bill of *X* 12750 due four months hence when rate of interest is 6% per annum.

(a) X 250 *(b) X* 120

(c) X 255 *(d) X* 300

(e) None of the above

12. The face value of a bill due 5 yr hence is *X* 13800. If the rate of simple interest is 5% per annum, what will be the banker's gain at the bill?

(a) X 690 *(b) X* 600

(c) X 590 *(d) X* 625

(e) None of the above

13. The true discount at a bill of *X* 7440 due 16 months, hence is *X* 240. Find the banker's gain.

(a) X 10 *(b) X* 6

(c) X 4 (d) X8

(e) None of the above

14. The true discount on a bill 24 months, hence at 24% per annum, is *X* 144. What will be the banker's discount?

(a) ? 213.12 *(b)* ? 415

(c) X 515.12 *(d) X* 616

(e) None of the above

15. The banker's gain of a sum due 8 yr, hence at 24% per annum, is *X* 144. Find the present worth of that bill.

(a) *X* 39.06 *(b)* *X* 45

(c) *X* 38.06 *(d)* *X* 50

(e) None of the above

16. If the banker's discount and banker's gain on a certain bill, are *X* 196 and *X* 28, respectively. Find the amount of bill.

(a) *X* 1200 *(b)* *X* 1376

(c) *X* 1176 *(d)* *X* 1400

(e) None of the above

17. The present worth of a sum due sometime, hence is *X* 576 and the banker's gain is *X* 9. Find the true discount.

(a) *X8* *(b)* *X* 70

(c) *X* 95 *(d)* *X* 72

(e) None of the above

18. The present worth of a bill due sometime, hence is *X* 2200 and true discount on the bill is *X* 220. Find the banker's discount and banker's gain.

(a) BG = *X* 22 and BD = *X* 242

(b) BG = *X* 242 and BD = *X* 22

(c) BG = ? 11 andBD = ? 121

(d) BG = *X* 31 and BD = *X* 343

(e) None of the above

19. If rate of interest and time on a certain bill are numerically equal and true discount is 81 times of banker's gain, find the rate of interest.

(a) $2\frac{8}{13}\%$ (b) $1\frac{2}{9}\%$

(c) $1\frac{7}{9}\%$ (d) $1\frac{1}{9}\%$

(e) None of the above

20. A bill for *X* 10200 is drawn on July 14 at 5 months. It is discounted on 5th October at 10%. Find the banker's discount, true discount, banker's gain and the money that the holder of the bill receives.

(a) X 204, *X* 200, *X* 4 and *X* 9996

(b) X 100, *X* 200, *X* 2 and *X* 4989

(c) X 121, *X* 172, *X* 132 and *X* 4046

(d) X 136, *X* 132, *X* 138 and *X* 3649

(e) None of the above

21. The true discount on a certain sum due 1 yr hence at 30% per annum, is *X* 240. What is the banker' discount on the same sum for the same time and at the same rate?

(a) X 400 *(b) X* 212

(c) X 312 *(d) X* 445

(e) None of the above

22. The banker's discount on a certain sum

due 4 yr, hence is — of the true discount. 10

Find the rate per cent per annum.

(a) 2.5% *(b)* 5%

(c) 5.5% *(d)* 1.5%

(e) None of the above

23. What rate per cent does Shantanu get for his money when in discounting a bill due 30 months hence he deducts 30% of the amount of the bill?

(a) $17\frac{2}{7}\%$ (b) $17\frac{3}{7}\%$

(c) $17\frac{5}{7}\%$ (d) $17\frac{1}{7}\%$

(e) None of the above

Answer with Solutions

1. (*d*) Given that, $T = 4$ yr, $R = 4\%$,

PW = ₹ 600 and TD = ?

According to the formula, (1)

$$\text{TD} = \frac{\text{PW} \times R \times T}{100}$$

$$\text{TD} = \frac{600 \times 4 \times 4}{100} = 6 \times 4 \times 4 = ₹\ 96$$

2. (*a*) Given that, PW = ₹ 50000,

TD = ₹ 2000, $T = 4$ yr and $R = ?$

According to the formula, (1)

$$\text{TD} = \frac{\text{PW} \times R \times T}{100}$$

$$\Rightarrow \quad 2000 = \frac{50000 \times R \times 4}{100}$$

$$\therefore \quad R = \frac{2000 \times 100}{50000 \times 4} = \frac{20}{20} = 1\%$$

3. (*a*) Let PW be x.

$$\therefore \quad 3720 - x = \frac{x \times 8 \times 3}{100} = \frac{6x}{25}$$

$$\Rightarrow \quad 31x = 3720 \times 25$$

$$\therefore \quad x = \frac{3720 \times 25}{31} = ₹\ 3000$$

$$\text{TD} = \text{Amount} - \text{PW}$$
$$= 3720 - 3000 = ₹\ 720$$

Alternate Method

Here, $A =$ ₹ 3720, $R = 8\%$, $T = 3$ yr, PW = ?
and TD = ?

According to the formula, (2)

$$\text{PW} = \frac{100 \times A}{100 + R \times T}$$

$$\Rightarrow \quad \text{PW} = \frac{100 \times 3720}{100 + 8 \times 3}$$

$$= \frac{100 \times 3720}{100 + 24} = \frac{100 \times 3720}{124}$$

$$= ₹\ 3000$$

and TD = Amount − PW

$$= 3720 - 3000 = ₹\ 720$$

4. (*c*) Given that, $R = 7\%$, $T = 2$ yr,

TD = ₹ 672 and Sum due (A) = ?

According to the formula, (3)

$$\text{TD} = \frac{A \times R \times T}{100 + R \times T}$$

$$\Rightarrow \quad 672 = \frac{A \times 7 \times 2}{100 + 7 \times 2} = \frac{14A}{114}$$

$$\therefore \quad A = \frac{114 \times 672}{14} = ₹\ 5472$$

Alternate Method

Let PW be x.

$$672 = \frac{x \times 7 \times 2}{100} = \frac{14x}{100}$$

$$\therefore \quad x = \frac{672 \times 100}{14} = ₹\, 4800$$

$\therefore$ Amount $(A) = 4800 + 672 = ₹\, 5472$

5. (*b*) Given that, SI = ₹ 225, TD = ₹ 75,

$T = 4$ yr and $R = ?$

According to the formula, (5)

$$\text{SI} - \text{TD} = \frac{\text{TD} \times R \times T}{100}$$

$$\Rightarrow \quad 225 - 75 = \frac{75 \times R \times 4}{100}$$

$$\Rightarrow \quad 150 = \frac{75 \times R \times 4}{100} \Rightarrow 3R = 150$$

$$\therefore \quad R = 50\%$$

Alternate Method

Given, SI = ₹ 225, TD = ₹ 75

According to the formula, (6)

$$\text{Sum due } (A) = \frac{\text{SI} \times \text{TD}}{\text{SI} - \text{TD}} = \frac{225 \times 75}{225 - 75}$$

$$= \frac{225 \times 75}{150} = ₹\, 112.5$$

TD is SI on amount.

$\therefore$ ₹ 225 is SI on ₹ 112.5 for 4 yr.

$$225 = \frac{112.5 \times R \times 4}{100}$$

$$\therefore \quad R = \frac{225 \times 100}{112.5 \times 4} = 50\%$$

6. (*d*) PW of ₹ 6440 due 8 months hence

$$= \frac{6440 \times 100}{100 + 18 \times \frac{8}{12}} = \frac{6440 \times 100}{112} = ₹\, 5750$$

Clearly, ₹ 10000 in cash is better offer.

7. (*a*) SP = 102% of ₹ 600 $= \frac{102}{100} \times 600$

$$= ₹\, 612$$

Now, PW = ₹ 612 and sum = ₹ 688.50

$\therefore$ TD = 688.50 − 612 = ₹ 76.50

Thus, SI on ₹ 612 for 9 months is ₹ 76.50.

$$\therefore \quad \text{Rate} = \frac{100 \times 76.50}{612 \times \frac{3}{4}}$$

$$= \frac{100 \times 76.50}{153 \times 3}$$

$$= 16\frac{2}{3}\%$$

8. (*c*) Required cash = PW of ₹ 5014 due 9 months hence

$$= \frac{5014 \times 100}{100 + \left(12 \times \frac{9}{12}\right)} = \frac{5014 \times 100}{100 + 9}$$

$$= \frac{5014 \times 100}{109} = ₹\ 4600$$

9. (*d*) Money to be paid by Aarti

$$= \text{PW of } ₹\ 440 \text{ due 1 yr hence}$$

$$= \frac{440 \times 100}{100 + (10 \times 1)} = \frac{440 \times 100}{110}$$

$$= 4 \times 100 = ₹\ 400$$

Aarti actually pays ₹ 220 + PW of ₹ 220 due 2 yr hence

i.e., $220 + \frac{220 \times 100}{100 + (10 \times 2)}$

$$= \left[220 + \frac{220 \times 100}{120}\right]$$

$$= [220 + 183.33] = ₹\ 403.33$$

$\therefore$ Loss of Aarti = 403.33 − 400

$$= ₹\ 3.33$$

10. (*a*) X owes PW of ₹ 3146 due $\frac{3}{2}$ yr hence

i.e., $\frac{3146 \times 100}{100 + \left(14 \times \frac{3}{2}\right)} = \frac{3146 \times 100}{100 + 21}$

$$= \frac{314600}{121} = ₹\ 2600$$

Y owes PW of ₹ 2889 due 6 months hence

i.e., $\frac{2889 \times 100}{100 + \left(14 \times \frac{1}{2}\right)} = \frac{2889 \times 100}{100 + 7}$

$$= \frac{2889 \times 100}{107} = ₹\ 2700$$

$\therefore$ Y must pay ₹ (2700 − 2600) *i.e.*, ₹ 100 to X.

11. (*c*) Given that, $A = ₹\ 12750$, $R = 6\%$

and $T = 4$ months $= \frac{4}{12} = \frac{1}{3}$ yr, BD = ?

According to the formula, (7)

$$\text{BD} = \frac{A \times R \times T}{100} = \frac{12750 \times 6 \times \frac{1}{3}}{100}$$

$$= \frac{12750 \times 2}{100} = ₹\ 255$$

12. (*a*) Given that, $A = ₹\ 13800$, $R = 5\%$,

$T = 5$ yr and BG = ?

According to the formula, (8)

$$\text{BG} = \frac{A\,(R \times T)^2}{100\,(100 + R \times T)}$$

$$= \frac{13800\,(5 \times 5)^2}{100\,(100 + 5 \times 5)} = \frac{13800 \times 625}{100 \times 125} = ₹\ 690$$

13. (*d*) Present worth of bill = ₹ (7440 − 240)

$= ₹\ 7200$

$\therefore$ Banker's Gain (BG) $= \dfrac{(\text{TD})^2}{\text{PW}}$

$$= \frac{240 \times 240}{7200} = \frac{576}{72} = ₹\ 8$$

14. (*a*) Given that, $R = 24\%$, $T = 24$ months

$= 2$ yr and TD = ₹ 144

BD = TD + BG

According to the formula, (9)

$$\text{BG} = \frac{\text{TD} \times R \times T}{100} = \frac{144 \times 24 \times 2}{100}$$

$$= ₹\ 69.12$$

$$\therefore \quad \text{BD} = 144 + 69.12$$

$$= ₹\ 213.12$$

15. (*a*) Given that, BG = ₹ 144, $R = 24\%$, $T = 8$ yr and PW = ?

According to the formula, (11)

$$\text{PW} = \text{BG}\left(\frac{100}{R \times T}\right)^2 = 144 \times \left(\frac{100}{24 \times 8}\right)^2$$

$$= 144 \times \frac{100}{192} \times \frac{100}{192} = ₹\ 39.06$$

Alternate Method

$$\text{TD} = \frac{\text{BG} \times 100}{R \times T} = \frac{144 \times 100}{24 \times 8}$$

$$= 6 \times 12.5 = ₹\ 75$$

Again, we know that

$$\text{TD} = \frac{\text{PW} \times R \times T}{100}$$

$$\Rightarrow \quad 75 = \frac{\text{PW} \times 24 \times 8}{100}$$

$$\therefore \quad \text{PW} = \frac{75 \times 100}{24 \times 8} = 39.06$$

16. (*c*) Given that, BD = ₹ 196 and BG = ₹ 28

$\therefore \quad$ TD = BD − BG = 196 − 28 = ₹ 168

According to the formula, (13)

$$A = \frac{\text{BD} \times \text{TD}}{\text{BG}} = \frac{196 \times 168}{28}$$

$$= 196 \times 6 = ₹\ 1176$$

17. (*d*) Given that, PW = ₹ 576, BG = ₹ 9 and TD = ?

According to the formula, (12)

$$\text{TD} = \sqrt{\text{PW} \times \text{BG}}$$

$$= \sqrt{576 \times 9}$$

$$= 24 \times 3 = ₹\ 72$$

18. (*a*) Given that, PW = ₹ 2200, TD = ₹ 220

According to the formula, (8)

$$BG = \frac{(TD)^2}{PW} = \frac{220 \times 220}{2200} = ₹\ 22$$

$$BD = (TD + BG) = ₹\ (220 + 22) = ₹\ 242$$

19. (*d*) Given that, $n = 81$

According to the formula,

$$R = 10\sqrt{\frac{1}{n}}\% = 10\sqrt{\frac{1}{81}}\%$$

$$= 10 \times \frac{1}{9}\% = 1\frac{1}{9}\%$$

20. (*a*) Face value of the bill = ₹ 10200

Date on which the bill was drawn

= July 14 at 5 months

Nominally due date = Dec. 14

Legally due date = Dec. 17

Date on which the bill was discounted

= Oct. 05

Unexpired time

Oct.	Nov.	Dec.	
26	30	17	= 73 days

$$= \frac{73}{365}\text{ yr} = \frac{1}{5}\text{ yr}$$

$\therefore$ BD = SI on ₹ 10200 for $\frac{1}{5}$ yr

$$= \left(10200 \times \frac{10}{100} \times \frac{1}{5}\right) = ₹\ 204$$

$$TD = \frac{\left(10200 \times \frac{1}{5} \times 10\right)}{100 + \left(10 \times \frac{1}{5}\right)}$$

$$= \frac{10200 \times 2}{102} = ₹\ 200$$

$$BG = (BD) - (TD) = (204 - 200) = ₹\ 4$$

Money received by the holder of the bill

$$= ₹\ (10200 - 204) = ₹\ 9996$$

21. (*c*) We know that

$$BG = SI \text{ on } TD = \left(240 \times 30 \times 1 \times \frac{1}{100}\right)$$

$$= ₹\ 72$$

$$BG = BD - TD \Rightarrow BD = BG + TD$$

$$= 72 + 240 = ₹\ 312$$

22. (*a*) Let TD = x, then BD = $\frac{11x}{10}$

According to the formula, (13)

$$\text{Sum} = \frac{BD \times TD}{BD - TD} = \frac{\frac{11x}{10} \times x}{\frac{11x}{10} - x}$$

$$= \frac{\frac{11x^2}{10}}{\frac{x}{10}} = 11x$$

SI on ₹ $11x$ for 4 yr is ₹ $\frac{11x}{10}$.

$$\therefore \text{ Rate} = \left(\frac{100 \times \frac{11x}{10}}{11x \times 4}\right)\% \text{ per annum}$$

$$= 2.5\% \text{ per annum}$$

23. (*d*) Let the amount of the bill = ₹ 100.

Money deducted = ₹ 30

Money received by the holder of the bill

$$= ₹\ (100 - 30) = ₹\ 70$$

SI on ₹ 70 for 30 months = ₹ 30

$$\therefore \text{ Rate} = \left(\frac{100 \times 30}{70 \times \frac{30}{12}}\right)\% \text{ per annum}$$

$$= \frac{10 \times 3 \times 12}{21}\%$$

$$= \frac{10 \times 12}{7}\% = \frac{120}{7}\% = 17\frac{1}{7}\%$$

Chapter

17

Ratio and Proportion

Ratio

When two or more similar quantities are compared, then to represent this comparison, ratios are used.

or Ratio of two quantities is the number of times one quantity contains another quantity of same kind.

The ratio between x and y can be represented as x : y, where x is called **antecedent** and y is called **consequent.**

$$\frac{x}{y} \text{ or } x : y$$

For example There can be a ratio between ? 100 and ? 500, but there cannot be the ratio between ? 100 and 500 apples. Hence, the units of quantity for the comparison of ratio should be same.

Types of Ratio

The different types of ratio are explained as under

1. **Duplicate Ratio** If two numbers are in ratio, then the ratio of their squares is called duplicate ratio. If x and y are two numbers, then the duplicate ratio of x and y would be xr : jr.

For example Duplicate ratio of 3 :4 = 3 :4 =9:16

2. **Sub-duplicate Ratio** If two numbers are in ratio, then the ratio of their square roots is called sub-duplicate ratio. If x and y are two numbers, then the sub-duplicate ratio of x and y would be Vx : *Jy.*

3. **Triplicate Ratio** If two numbers are in ratio, then the ratio of their cubes is called triplicate ratio. If x and y are two numbers, then the triplicate ratio of x and y would be $x\text{^}y^3$.

For example Triplicate ratio of 2 :3 = $2^3 : 3^3$ = 8 :27

4. **Sub-triplicate Ratio** If two numbers are in ratio, then the ratio of their cube roots is called sub-triplicate ratio. If x and y are two numbers, then the sub-triplicate ratio of x and y would be 3/x : ^/y.

For example Sub-triplicate ratio of 1:125 = 3/1 : 3/125 =1:5

5. Inverse **Ratio** If two numbers are in ratio, then their antecedent and consequent are interchanged and the ratio obtained is called inverse ratio, If x and y are two numbers and their ratio is x : y, then its inverse ratio will be y:x.

For example Inverse ratio of 4 :5 is 5 :4.

6. **Compound Ratio** If two or more ratios are given, then the antecedent of one is multiplied with antecedent of other and respective consequents are also multiplied. If *a* : *b, c*: d and e : / are three ratios, then their compound ratio will be *ace*: *bdf.*

For example The compount ratio of 2 : 5, 6 : 7 and 9 : 13 $= \frac{2\times6\times9}{5\times7\times13} = \frac{108}{455}$

Note 1. If the **antecedent** is gieatei than the **consequent,** then the ratio is known as the ratio of greater inequality, such as 7 : 5

2. If the **antecedent** is less than the **consequent,** then the ratio is called the ratio of less inequality, such as 5 : 7

Comparison of Ratios

Rules used to compare different ratios are as follows

RuleO

if the given ratios are a: *b* and *c*: *d,* then

(i) a *:b> c: d,* if *ad > be* (ii) a *:b< c: d,* if *ad < be* (iii) *a:b = c:d,\f ad = bc*

Ex. 1 Which is greater $\frac{5}{8}$ or $\frac{9}{14}$?

Sol. Let $\frac{a}{b} = \frac{5}{8}$ and $\frac{c}{d} = \frac{9}{14}$; $ad = 5 \times 14 = 70$ and $bc = 8 \times 9 = 72$

$\because \quad ad < bc$

$\therefore \quad \frac{a}{b} < \frac{c}{d} = \frac{5}{8} < \frac{9}{14}$

Rule®

if two ratios are given for comparison, convert each ratio in such a way that both ratios have same denominator, then compare their numerators, the fraction with greater numerator will be greater.

Ex. 2 Find the greater ratio between 2 : 3 and 4 : 5.

Sol. $\frac{2 \times 5}{3 \times 5} = \frac{10}{15}$ and $\frac{4 \times 3}{5 \times 3} = \frac{12}{15}$ $\quad$ [$\because$ LCM of $\frac{2}{3}$ and $\frac{4}{5}$ i.e., 3 and 5 is 15]

$\Rightarrow \quad \frac{12}{15} > \frac{10}{15}$

$\therefore \quad 4:5 > 2:3$

Rule®

If two ratios are given for comparison, convert each ratio in such a way that both ratios have same numerator, then compare their denominators, the fraction with lesser denominator will be greater.

Ex. 3 Find the least fraction between $\frac{6}{7}$ and $\frac{7}{9}$.

Sol. $\frac{6 \times 7}{7 \times 7} = \frac{42}{49}$ and $\frac{7 \times 6}{9 \times 6} = \frac{42}{54}$ $\quad$ [$\because$ LCM of $\frac{6}{7}$ and $\frac{7}{9}$ i.e., 6 and 7 is 42]

$\Rightarrow \quad \frac{42}{49} > \frac{42}{54} \quad \therefore \quad \frac{6}{7} > \frac{7}{9}$

Proportion

An equality of two ratios is called the proportion. If $\frac{a}{b} = \frac{c}{d}$ or $a : b = c : d$, then we can say that a, b, c and d are in proportion and can be written as $a : b :: c : d$, where symbol'::' represents proportion and it is read as 'a is to b' as 'c is to d.'

Here, a and d are called **'Extremes'** and b and c are called as **'Means'.**

Basic Rules of Proportion

RuleO

if a:b:\b\c, then c is called third proportional to a and b, which are in continued proportion, c will be calculated as

$$a:b::b:c \Rightarrow a:b=b:c \Rightarrow a\times c=b\times b \Rightarrow b^2=ac \Rightarrow c=\frac{b^2}{a}$$

Ex. 4 Calculate the 3rd proportional to 16 and 32. Sol. Let 3rd proportional be x. Then, 16 : 32 : : 32 : x

$$\Rightarrow \quad \frac{16}{32} = \frac{32}{x}$$

$$\therefore \quad x = \frac{32 \times 32}{16} = 64$$

Rule®

if a: b:: c: d, then d is called the 4th proportional to a, b and c, d will be calculated as

$$a:b::c:d \Rightarrow a:b=c:d \Rightarrow a\times d=c\times b \Rightarrow d=\frac{bc}{a}$$

Ex. 5 Find the 4th proportional to 3,7 and 9.

Sol. Let 4th proportional be x.

Then, $3 : 7 :: 9 : x \Rightarrow \frac{3}{7} = \frac{9}{x}$

$\Rightarrow 3x = 9 \times 7 \Rightarrow x = \frac{9 \times 7}{3} = 21$

Rule®

Mean proportional between a and b is $\sqrt{ab}$. If mean proportional is x, then

$$a : x :: x : b$$

$$\Rightarrow a \times b = x \times x$$

$$\Rightarrow x^2 = ab$$

$$\Rightarrow x = \sqrt{ab}$$

Ex. 6 What will be the mean proportional between 4 and 25?

Sol. Let mean proportional be x.

Then, $4 : x :: x : 25$

$$\Rightarrow 4 : x = x : 25$$

$$\Rightarrow 4 \times 25 = x \times x$$

$$\Rightarrow x = \sqrt{4 \times 25} = 10$$

Invertendo, Alternendo, Componendo and Dividendo

If $\frac{a}{b} = \frac{c}{d}$, then

1. **Invertendo** $\rightarrow \frac{b}{a} = \frac{d}{c}$
2. **Alternendo** $\rightarrow \frac{a}{c} = \frac{b}{d}$
3. **Componendo** $\rightarrow \frac{a+b}{b} = \frac{c+d}{d}$
4. **Dividendo** $\rightarrow \frac{a-b}{b} = \frac{c-d}{d}$
5. **Componendo and dividendo** $\rightarrow \left(\frac{a+b}{a-b}\right) = \left(\frac{c+d}{c-d}\right)$
6. **If** $\frac{a}{b} = \frac{c}{d} = \frac{i}{j} = \ldots = k$, **then** $\frac{a+c+i+\ldots}{b+d+j+\ldots} = k$

Ex. 7 Find the value of $\frac{a+b}{a-b}$, if $\frac{a}{b}=\frac{5}{3}$.

Sol. Given, $\frac{a}{b}=\frac{5}{3}$

By componendo and dividendo,

$$\frac{a+b}{a-b}=\frac{5+3}{5-3}=\frac{8}{2}=\frac{4}{1}$$

Ex. 8 If $\frac{3}{2}=\frac{18}{12}=\frac{24}{16}=\alpha$, then find the value of α.

Sol. $\alpha=\frac{\text{Sum of numerators}}{\text{Sum of denominators}}$

$$=\frac{3+18+24}{2+12+16}=\frac{45}{30}=\frac{3}{2}$$

Fast Track **Techniques**

to solve the QUESTIONS

Technique 1

(i) if *A: B = a: b* and *B: C = m*: *n,* then *A: B: C = am: mb: nb* and

A:C = am:bn (ii) If *A: B = a: b, B: C = c:* of and *C*: D = e: *f,* then

A:B:C:D = ace:bce: bde: bdf

Ex. 9 *If a:b =5:14 andb: c = 7 : 3, then find a:b:c.*

Sol. *a* : 6 : c= (5 x 7) : (7 x 14): (14 x 3) [dividing the ratio by 7]

= 5: 14: (3 x2)= 5: 14: 6

Ex.10 The ratio of *A:B* = \ : 3, *B:C* = 2:5 andC :D = 2: 3. Find the value of *A:B:C:D.* Sol. *A: B= 1:3, B:C =2:5, C :D =2:3*

∴ *A : B : C : D* = (1 x 2 x 2): (3 x 2 x 2): (3 x 5 x 2) : (3 x 5 x 3) = 4 : 12 : 30 : 45

Technique 2

(i) If x is divided in $a:b$, then 1st part $= \dfrac{ax}{a+b}$; 2nd part $= \dfrac{bx}{a+b}$

(ii) If x is divided in $a:b:c$, then

$$\text{1st part} = \frac{ax}{a+b+c};\ \text{2nd part} = \frac{bx}{a+b+c};\ \text{3rd part} = \frac{cx}{a+b+c}$$

Ex. 11 Divide 1111 in the ratio of 8 : 3.

Sol. Let 1st part be $8x$ and 2nd be $3x$.

According to the question,

$$8x + 3x = 1111 \Rightarrow 11x = 1111$$

$$\therefore \quad x = \frac{1111}{11} = 101$$

Now, 1st part $= 8x = 8 \times 101 = 808$

2nd part $= 3x = 3 \times 101 = 303$

Fast Track Method

$$\text{1st part} = \frac{8}{8+3} \times 1111 = \frac{8}{11} \times 1111 = 8 \times 101 = 808$$

$$\text{2nd part} = \frac{3}{11} \times 1111 = 3 \times 101 = 303$$

Ex. 12 Divide 2324 in the ratio of 35 : 28 : 20.

Sol. 1st part $= \dfrac{35}{35+28+20} \times 2324 = \dfrac{35}{83} \times 2324 = 35 \times 28 = 980$

2nd part $= \dfrac{28}{83} \times 2324 = 28 \times 28 = 784$; 3rd part $= \dfrac{20}{83} \times 2324 = 20 \times 28 = 560$

Ex. 13 The sum of three numbers is 315. If the ratio between 1st and 2nd is 2 : 3 and the ratio between 2nd and 3rd is 4 : 5, then find the 2nd number.

Sol. 1st number : 2nd number $= 2:3 = (2 \times 4):(3 \times 4) = 8:12$

2nd number : 3rd number $= 4:5 = (4 \times 3):(5 \times 3) = 12:15$

$\therefore$ 1st number : 2nd number : 3rd number $= 8:12:15$

2nd number $= \dfrac{12}{8+12+15} \times 315 = \dfrac{12}{35} \times 315 = 108$

Technique 3

The incomes of two persons are in ratio of *a: b* and their expenditures are in the ratio of c: cf. If each of them saves ? *X,* then their incomes are given by $\frac{X(d-c)}{ad-bc} \times a$ and $\frac{X(d-c)}{ad-bc} \times b$ respectively and their expenditures are given by $\frac{X(b-a)}{ad-bc} \times c$ and $\frac{X(b-a)}{ad-bc} \times d$, respectively.

Ex.14 The ratio of incomes of Raman and Gagan is 4 : 3 and ratio of their expenditures is 3 : 2. If each person saves ? 2500, then find their incomes and expenditures.

Sol. Let the income of Raman be ₹ $4x$ and that of Gagan be ₹ $3x$.

Expenditure of Raman = ₹ $(4x - 2500)$; Expenditure of Gagan = ₹ $(3x - 2500)$

According to the question,

$$\frac{4x - 2500}{3x - 2500} = \frac{3}{2}$$

$\Rightarrow \quad 8x - 5000 = 9x - 7500 \Rightarrow x = 7500 - 5000 = 2500$

Income of Raman = $4x = 4 \times 2500 =$ ₹ 10000

Income of Gagan = $3x = 3 \times 2500 =$ ₹ 7500

Expenditure of Raman = $4x - 2500 = 10000 - 2500 =$ ₹ 7500

Expenditure of Gagan = $3x - 2500 = 7500 - 2500 =$ ₹ 5000

Fast Track Method

Here, $a = 4, b = 3, c = 3, d = 2$ and $X =$ ₹ 2500

$$\text{Income of Raman} = \frac{X(d-c)}{ad-bc} \times a$$

$$= \frac{2500(2-3)}{8-9} \times 4 = ₹\ 10000$$

$$\text{Income of Gagan} = \frac{X(d-c)}{ad-bc} \times b$$

$$= \frac{2500(2-3)}{8-9} \times 3 = ₹\ 7500$$

$$\text{Expenditure of Raman} = \frac{X(b-a)}{ad-bc} \times c$$

$$= \frac{2500(3-4)}{8-9} \times 3 = ₹\ 7500$$

$$\text{Expenditure of Gagan} = \frac{X(b-a)}{ab-bc} \times d$$

$$= \frac{2500(3-4)}{8-9} \times 2 = ₹\ 5000$$

Technique 4

If two numbers are in ratio *a: b* and *x* is added to the numbers, then the ratio becomes $c : d$. The two numbers will be $\frac{xa(c-d)}{ad-bc}$ and $\frac{xb(c-d)}{ad-bc}$, respectively.

Ex. 15 Two numbers are in the ratio of 2 : 3. If 15 is added to both the numbers, then the ratio between two numbers becomes $\frac{11}{14}$. Find the greater number.

Sol. Let the numbers be $2x$ and $3x$.

According to the question,

$$\frac{2x+15}{3x+15}=\frac{11}{14}$$

$$\Rightarrow \quad 14(2x+15)=11(3x+15)$$

$$\Rightarrow \quad 28x+210=33x+165$$

$$\Rightarrow \quad 33x-28x=210-165$$

$$\Rightarrow \quad 5x=45 \Rightarrow \frac{45}{5}=9$$

$\therefore$ Greater number $=3x=3\times 9=27$

Fast Track Method

Here, $a=2, b=3, c=11, d=14$ and $x=15$

Then, two numbers are

$$\frac{xa(c-d)}{ad-bc} \text{ and } \frac{xb(c-d)}{ad-bc}$$

$$\Rightarrow \quad \frac{15\times 2(11-14)}{2\times 14-3\times 11} \text{ and } \frac{15\times 3(11-14)}{2\times 14-3\times 11}$$

$$\Rightarrow \quad \frac{30\times(-3)}{28-33} \text{ and } \frac{45\times(-3)}{28-33}$$

$$\Rightarrow \quad \frac{30\times(-3)}{-5} \text{ and } \frac{45\times(-3)}{-5}$$

$$18 \text{ and } 27$$

$\therefore$ Greater number is 27.

Technique 5

Two numbers are in ratio $a:b$ and x is subtracted from the numbers, then the ratio becomes $c:d$. The two numbers will be $\frac{xa(d-c)}{ad-bc}$ and $\frac{xb(d-c)}{ad-bc}$, respectively.

Ex. 16 Two numbers are in the ratio of 3 : 5. If 9 is subtracted from each, the ratio becomes 12 : 23. Find the greater number.

Sol. Here, $a=3, b=5, c=12, d=23$ and $x=9$

Then, 1st number $=\frac{xa(d-c)}{ad-bc}=\frac{9\times 3(23-12)}{3\times 23-5\times 12}=\frac{27\times 11}{69-60}=\frac{297}{9}=33$

2nd number $=\frac{xb(d-c)}{ad-bc}=\frac{9\times 5(23-12)}{3\times 23-5\times 12}=\frac{45\times 11}{69-60}=\frac{45\times 11}{9}=55$

Multi Concept

QUESTIONS

1. *A sum* o/? *430 has been distributed among 45 people consisting of men, women and children. The total amounts given to men, women and children are in the ratio 12 :15 :16. But the amounts received by each man, woman and child are in the ratio 6:5: 4. Find what each man, woman and child receives (in %)?* (a) 12, 10, 8 (b) 18,15, 12 (c) 120, 150,160 (d) 60, 75, 80

➥ (*a*) Total amount = 430

Total people = 45

Ratio of personal shares = 6 : 5 : 4

Ratio of the amounts = 12 : 15 : 16

$$\text{Ratio of men, women and children} = \frac{12}{6} : \frac{15}{5} : \frac{16}{4} = 2 : 3 : 4$$

Sum of these ratios = 2 + 3 + 4 = 9

$$\text{Number of men} = \left(\frac{45 \times 2}{9}\right) = 10$$

$$\text{Number of women} = \left(\frac{45 \times 3}{9}\right) = 15$$

Number of children = 45 − (10 + 15) = 20

Now, divide ₹ 430 in the ratio 12 : 15 : 16.

$$\text{Total amount of men's share} = \frac{430 \times 12}{43} = ₹\,120$$

$$\text{Total amount of women's share} = \frac{430 \times 15}{43} = 150$$

Total amount of children's share = 430 − (120 + 150) = ₹ 160

$$\therefore \quad \text{Each man's share} = \frac{120}{10} = ₹\,12$$

$$\text{Each woman's share} = \frac{150}{15} = ₹\,10$$

$$\text{Each child's share} = \frac{160}{20} = ₹\,8$$

2. *The ratio between the number of passengers travelling by 1st and 2nd class between the two railway stations is 1 : 50, whereas the ratio of 1st and 2nd class fares between the same stations is 3 :1. If on a particular day, ? 1325 were collected from the passengers travelling between these stations, then what was the amount collected from the 2nd class passengers?* (a)? 750 (b)?850 (c)?1000 (d)?1250

→ (*d*) Let number of passengers in 1st class be x and number of passengers in 2nd class be $50x$.

Then, total amount of 1st class = $3x$ and total amount of 2nd class = $50x$

Ratio of the amounts collected from the 1st class and the 2nd class passengers = 3 : 50

$\therefore$ Amount collected from the 2nd class passengers $= \frac{b}{a+b} \times x$

where, x = total amount $a = 3$, $b = 50$

$\Rightarrow \qquad = \frac{50}{53} \times 1325 =$ ₹ 1250

Fast Track Practice

Exercise© *Base Level Questions*

1. If $P:Q$ = 8:15 and $Q:R = 3:2$, then find $P:Q:R$.

fa; 8:15:7 *(b)* 7 : 15 : 8

(c) 8 : 15 : 10 fdj 10 : 15 : 8

(e) None of the above

2. If P : Q = 8:15, Q : i? = 5: 8 and R : S = 4: 5, then P : S is equal to

(a) 4:15 fbj 2 : 15

(c) 3 : 19 fdj 7 : 15

fej None of the above

3. If A : B = 2: 3, B : C = 5: 7 and C : D = 3:10, then what is A : D equal **to? [CDS 2014]**

(a) 1 : 7 *(b)* 2 : 7

(c) 1 : 5 (t/> 5 : 1

4. Find the 4th proportional to 4, 16 and 7.

(a) 28 fbj 29

(c) 22 fdj 25

(e) None of the above

5. Calculate the 3rd proportional to 15 and 30.

(a) 55 *(b)* 15

(c) 65 *(d)* 60

fej None of the above

6. Find the mean proportional between 9 and 64.

(a) 25 fbj 24

(c) 27 *(d)* 35

(e) None of the above

7. What is the mean proportional between (15 + V200) and (27 - V648)? [CDS 2012]

(a) 4 *(b)* 14V7

(c) 3V5 *(d)* 3V5

What will be the duplicate ratio of 2 : 7? *(a)* 4 : 49 fbj 49 : 4

(c) 4 : 14 fdj 8 : 343

(e) None of the above 9. Find the sub-duplicate ratio of 81 : 64. *(a)* 8 : 9 fbj 4 : 9

(c) 9 : 8 *(d) 7 : 8*

(e) None of the above

10. Find the triplicate ratio of 7 : 5. *(a)* 125 : 343 fbj 343 : 125

(c) 344 : 125 fdj 343 : 126

fej None of the above

Calculate the sub-triplicate ratio of 512 : 729.

(a) 9 : 8 fbj 4 : 9 (c) 7 : 8 fdj 8 : 9

fej None of the above

12. What will be the inverse ratio of 17 : 19?

(a) 19 : 17 *(b)* 18 : 17

(c) 17 : 18 *(d)* 19 : 5

(e) None of the above

13. Find the compound ratio of 2 : 7, 5 : 3 and 4 : 7.

(a) 147 : 40 *(b)* 40 : 147

(c) 147 : 30 fdj 30 : 147

(e) None of the above

14. If *A : B* = 3: 4 and *B* : C = 8: 9, then find the value of *A : B : C.* [SSC CCL 2010]

(a) 3 : 4 : 5 fbj 1 : 2 : 3

(c) 7 : 12 : 17 fdj 6 : 8 : 9

15. If *a : b = b: c,* then ratio *a : b* is equal **to [SSC (10+2) 2010]**

(a) $ac:b^2$ *(b)* $a^2:c^2$

(c) $c^2:a^2$ fdj $b^2:ac$

16. If a : *b* = 3: 5 and *b: c* = 4: 7, then a : c is equal to

(a) 11:35 fbj 35:11

(c) 35:12 fdj 12:35

fej None of the above

17. If 4 a = 56 and 76 = 9c, then *a: b: c* is equal to

(a) 45 : 36 : 28 *(b)* 44 : 33 : 28 *(c)* 28 : 36 : 45 *(d)* 36 : 28 : 45 (e) None of the above

18. If $\frac{a}{7} = \frac{b}{9} = \frac{c}{11}$, then find $a : b : c$. **[Hotel Mgmt. 2007]**

(a) 11 : 9 : 7 *(b)* 9 : 7 : 11
(c) 7 : 9 : 11 *(d)* 11 : 7 : 9
(e) None of the above

19. If 30% of *A* = 20% of *B,* then find the value of A : *B.*

(a) 1 : 3 *(b)* 3 : 2

(93:1 *(d)* 2 : 3

(e) None of the above

20. If $P : Q : R = 2 : 3 : 4$, then find $\frac{P}{Q} : \frac{Q}{R} : \frac{R}{P}$.

(a) 8 : 9 : 24 *(b)* 9 : 8 : 24
(c) 24 : 8 : 9 *(d)* 8 : 24 : 9
(e) None of the above

21. If $A = \frac{1}{4} B$ and $B = \frac{1}{2} C$, then find the value of $A : B : C$. **[SSC (10+2) 2010]**

(a) 8 : 4 : 1 *(b)* 4 : 2 : 1
(c) 1 : 4 : 8 *(d)* 1 : 2 : 4

22. If 0.8 x A = 0.09 x *B,* then find *A : B.* *(a)* 9 : 80 *(b)* 3 : 80

fc; 80 : 9 fdj 7 : 80

(e) None of the above

23. If $\frac{1}{x}:\frac{1}{y}:\frac{1}{z} = 2:3:5$, then determine $x:y:z$.

(a) 6 : 15 : 10 (b) 3 : 15 : 10
(c) 15 : 3 : 10 (d) 15 : 10 : 6
(e) None of the above

24. If $\frac{a}{3} = \frac{b}{8}$, then $(a+3):(b+8)$ is equal to

(a) 3 : 8 (b) 8 : 3
(c) 5 : 8 (d) 3 : 5
(e) None of the above

25. If $(a+b):(a-b) = 5:3$, then find $(a^2+b^2):(a^2-b^2)$. [SSC CPO 2011]

(a) 17 : 15 (b) 25 : 9
(c) 4 : 1 (d) 16 : 1

26. If $\frac{x}{2y} = \frac{6}{7}$, then find the value of $\frac{x-y}{x+y} + \frac{14}{19}$.

(a) 5 (b) 1
(c) 4 (d) 3
(e) None of the above

27. If $\frac{1}{2}$ of $A = \frac{2}{5}$ of $B = \frac{1}{3}$ of C, then $A:B:C$ is equal to [SSC (10+2) 2013]

(a) 4 : 5 : 6 (b) 6 : 4 : 5
(c) 5 : 4 : 6 (d) 4 : 6 : 5

28. If $P^2 + 4Q^2 = 4PQ$, then determine P : *Q*. *(a)* 1 : 3 (b) 3 : 1

(cj 2 : 1 (c[M : 2

(ej None of the above

29. If *xy* = 36, then which of the following is correct?

(a) x : 9 = 4 : *y* *(b)* 9 : x = 4 : y (cj x: 17 = y:7 (c[) x:6 = y:6 (e; None of the above

30. If 10% of (A + *B)* = 50% of (A - B), then findA:£.

(a) 1 : 2 *(b) 5 :2 (c)* 2 : 3 (tfj 3 : 2 (ej None of the above

31.If a:6=c:d = e:/ = I:2, then find (3a + 5c + 7e): (36 + 5d + *If). (a)* 1 : 2 fbj 2 : 1

(cj 3 : 1 (dj 1 : 3

(ej None of the above

32. If $x: y$ = 7: 5, then what is the value of (5* - $2y)$: $(Sx + 2y)l$ [CDS 2012]

(a) 5/4 (b) 6/5 (c) 25/31 *(d)* 31/42

33. If *a,b,c, d* and e are in continued proportion, then *ale* is equal to

[CDS 2013]

(a) a^3/b^3 *(b)* a^4/b*

(c) b^3/a^3 *(d)* b^4/a^4

34. If 2A = 3B = 4C, then find A : *B* : C.

[SSC (10+2) 2010]

(aj 2 : 3 : 4 fbj 4 : 3 : 2

(cj 6 : 4 : 3 fdj 3 : 4 : 6

35. In a certain school, the ratio of boys to girls is 7 : 5. If there are 2400 students in the school, then how many girls are there in the school? [CDS 2012]

(a) 500 *(b)* 700 *(c)* 800 *(d)* 1000

36. If a+6:6+c:c+a = 6:7:8 and $a + b + c$ = 14, then find c.

(a) 6 *(b)* 7 *(c)* 8 *(d)* 10

re; 14

37. The quantity that must be added to each term of o : *b,* so as to make it c: *d,* is

[SSC Multitasking 2013]

(a) $\dfrac{ab - cd}{a - b}$ (b) $\dfrac{ac + bd}{c + a}$

(c) $\dfrac{ad + bc}{c + d}$ (d) $\dfrac{ad - bc}{c - d}$

38. One-half of a certain number is equal to 65% of the 2nd number. Find the ratio of 1st to 2nd number.

[Bank Clerks 2010]

(a) 10 : 13 *(b)* 8 : 13

(c) 13 : 8 *(d)* 13 : 10

(e) None of the above

39. The ratio between two numbers is 3 : 4. If each number is increased by 3, the ratio becomes 4:5. Find the difference between the numbers.

(a) 3 *(b)* 9 *(c)* 2 *(d)* 7 *(e)* None of the above

40. From each of two given numbers, half the smaller number is subtracted. After such subtraction, the larger number is

4 times as large as the smaller number. What is the ratio of the numbers ?

[SSCCGL (Main) 2012] *(a)* 5 : 2 *(b)* 1 : 4

(c) 4 : 1 *(d)* 4 : 5

41. Two numbers are in the ratio of 4 : 7. If

5 is subtracted from each, the ratio becomes 1 : 2. Find the greater number.

(a) 15 *(b)* 40 *(c)* 20 *(d)* 35

42. Two numbers are in the ratio 2 : 3. If 9 is added to each number, they will be in the ratio 3 : 4. What is the product of the two numbers? [CDS 2012]

(a) 360 *(b)* 480

(c) 486 *(d)* 512

43. The total number of students in a school is 2140. If the number of girls in the school is 1200, then what is the ratio of the total number of boys to the total number of girls in the school?

[Bank Clerks 2009]

(a) 26 : 25 *(b)* 47 : 60

(c) 18 : 13 *(d)* 31 : 79

(e) None of the above

44. The total weight of Sanjay and Suresh is 120 kg. If Sanjay weighs 30 kg more than Suresh, then what is the ratio of the weight of Suresh to that of Sanjay?

[Bank PO 2008] *(a)* 0.4 *(b)* 0.6

(c) 0.3 *(d)* 0.25

(e) None of the above

45. In a school, the ratio of boys and girls is 4 : 5. When 100 girls leave the school, the ratio becomes 6 : 7. How many boys are there in the school?

(a) 1800 *(b)* 1200

(c) 1000 *(d)* 1500

(e) None of the above

46. In a class, the number of boys and girls is in the ratio of 4 : 5. If 10 more boys join the class, the ratio of numbers of boys and girls becomes 6 : 5. How many girls are there in the class?

[Bank Clerks 2009]

(a) 20

(b) 30

(c) 25

(d) Couldn't be determined

(e) None of the above

47. The total number of students in a school is 8670. If the number of boys in the school is 4545, then what will be the ratio of the total number of boys to the total number of girls in the school?

[Hotel Mgmt. 2008]

(a) 303 : 275

(b) 275 : 303

(c) 11 : 12

(d) 12 : 11

(e) None of the above

48. A truck covers a distance of 640 km in 10 h. A car covers the same distance in 8 h. What is the respective ratio of the speed of the truck to that of car?

[IBPS Clerk 2011] *(a)* 3 : 4 *(b)* 1 : 2

(c) 5 : 6 *(d)* 6 : 7

(e) None of the above

49. A certain distance is covered at a certain speed . If half of this distance is covered in double the time, the ratio of the two speeds is [SSC CCL 2013]

(a) 4 : 1 *(b)* 1 : 4

(c) 2 : 1 *(d)* 1 : 2

50. The speeds of three cars are in the ratio of 2:3:4 Find the ratio between the time taken by these cars to cover the same distance. [Bank PO 2008]

(a) 2 : 3 : 4 *(b)* 4 : 3 : 2

(c) 4 : 3 : 6 *(d)* 6 : 4 : 3

(e) None of the above

51. A person distributes his pens among four friends A, B, C and D in the ratio $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}:\frac{1}{6}$. What is the minimum number of pens that the person should have ? [SSC CGL 2013]

(a) 75 *(b)* 45 *(c)* 57 *(d)* 65

52. Divide ? 990 into 3 parts in such a way that half of the first part, one-third of the second part and one-fifth of the third part are equal [SSC FCI 2013]

(a) 198, 494, 298

(b) 198, 297, 495

(c) 200, 300, 490

(d) 196, 298, 496

53. A bag contains ?1, 50 paise and 25 paise coins in the ratio of 8 : 9 : 11. If the total money in the bag is *X* 366, then find the number of 25 paise coins.

(a) 264 *(b)* 364 *(c)* 241 *(d)* 245 *(e)* None of the above

54. Weekly incomes of two persons are in the ratio of 7 : 3 and their weekly expenses are in the ratio of 5 : 2. If each of them saves *X* 300 per week, then the weekly income of the first person is [SNAP 2012]

(a) X 7500 *(b) X* 4500

(c) X 6300 *(d) X* 5400

(e) None of the above

55.Amit and Sudesh have invested in the ratio of 4:7. If both invested a total amount of *X* 49500, then find the investment of Sudesh. [Bank Clerks 2010]

(a) X 31500 *(b) X* 1800

(c) X 31000 *(d) X* 18500

(*e*) None of the above

56. If a sum of *X* 1664 is divided between *P* and Q in the ratio of 1/3 : 1/5, then find fs share.

(*a*) 1085 (*b*) 1015 (*c*) 1090 (*d*) 1040 (*e*) None of the above

57. The marks of 3 students *A, B* and C are in the ratio 10:12:15. If the maximum marks of the paper are 100, then the marks of *B* cannot be in the range of

[SSCCGL (Main) 2013] (*a*) 20-30 (*b*) 40-50

(*c*) 70-80 (*d*) 80-90

58. A sum of *X* 7000 is divided among *A,B* and *C* in such a way that the shares of *A* and *B* are in the ratio 2 : 3 and those of *B* and C are in the ratio 4: 5. The share of *B* is [SSC (10+2)2012]

(*a*) ?1600 (*b*) ?2000

(*c*) *X* 2400 (*d*) *X* 3000

59. A sum of *X* 300 is divided among *P, Q* and *R* in such a way that *Q* gets *X* 30 more than *P* and *R* gets *X* 60 more than *Q*. Then, ratio of their shares is

[SSCCGL 2013] (*a*) 2 : 3 : 5 (*b*) 3 : 2 : 5

(*c*) 2 : 5 : 3 (*d*) 5 : 3 : 2

60. A sum of money is divided amongst *A, B, C* and *D* in the ratio of 3 : 7 : 9 : 13. If the share of *B* is *X* 4872, then what will be the total amount of money of *A* and C together? [Bank Clerks 2008]

(*a*) *X* 8352 (*b*) *X* 6998

(*c*) *X* 9784 (*d*) *X* 7456

(*e*) None of the above

What sum of money is to be divided among 3 persons in the ratio 3 : 4 : 7, so that the second person receives *X* 12 only? [SSC (10+2) 2013]

(*a*) *X* 21 (*b*) *X* 32

(c) *XQ* *(d)* *X* 42

62. The prices of a scooter and a television are in the ratio of 8 : 7. If a scooter costs *X* 2000 more than a television, then find out the price of television.

(a) *X* 14000 *(b)* *X* 28000

(c) *X* 10000 *(d)* *X* 18000

(e) None of the above

63. The ratio between the ages of A and *B* is 3: 5 and the sum of their ages is 56 yr. The ratio between their ages 7 yr ago was [SSC FCI 2013]

(a) 3 : 4 *(b)* 3 : 5

(c) 1 : 2 *(d)* 2 : 3

64. The ratio of the ages of a father to that of his son is 5 : 2. If the product of their ages (in years) is 1000, then find the father's age after 10 yr. [SSC CCL 2010]

(a) 50 yr *(b)* 60 yr

(c) 80 yr *(d)* 100 yr

65. A certain number is divided into two parts such that 5 times the first part added to 11 times the second part makes 7 times the whole. The ratio of the first part to the second part is

[SSC (10+2) 2013] *(a)* 2 : 1 fb; 5:11

(c) 1 : 2 *(d)* 2 : 3

35% of a number is two times 75% of another number. What is the ratio between the first and the second number, respectively? [SBI Clerk 2012]

(a) 35 : 6 *(b)* 31 : 7

(c) 23 : 7 *(d)* 32 : 9

(e) None of the above

67. Brothers *A* and *B* had some savings in the ratio 4 : 5. They decided to buy a gift for their sister, sharing the cost in the ratio 3 : 4. After they bought, *A* spent two-third of his amount, while *B* is left with *X* 145. Then, the value of the gift is [SSC CCL (Main) 2013] *(a)* ?70 *(b)* ?105

(c) ?140 *(d)* ? 175

Exercise © *Higher SkiU Level Questions*

1. In a town, 80% of the population are adults of which the men and women are in the ratio of 9 : 7, respectively. If the number of adult women is 4.2 lakh, what is the total population of the town?

[Bank Clerks 2009]

(a) 12 lakh *(b)* 9.6 lakh

(c) 9.8 lakh *(d)* 11.6 lakh

(e) None of the above

2. Mr. Shrimant inherits 2505 gold coins and divides them among his three sons; Bharat, Parat and Marat; in a certain ratio. Out of the total coins received by each of them, Bharat sells 30 coins, Parat donates his 30 coins and Marat losses 25 coins. Now, the ratio of gold coins with them is 46:41 :34, respectively. How many coins did Parat receive from his father? [Bank Clerks 2009]

(a) 705 *(b)* 950 *(c)* 800 *(d)* 850 *(e)* None of the above

3. *X* 600 are divided among *A, B* and C such that *X* 40 more than 2/5th of *A's* share, *X* 20 more than 2/7 th of *ffs* share and *X* 10 more than 9/17 th of Cs share, all are equal. What is *B's* share?

(a) X 150 *(b) X* 185

(c) X 280 *(d) X* 285

(e) None of the above

4. Salaries of Akash, Bablu and Chintu are in the ratio of 2 : 3 : 5. If their salaries were increased by 15%, 10% and 20% respectively, then what will be the new ratio of their salaries? [SSC FCI 2013]

(a) 3 : 3 : 10 *(b)* 23 : 33 : 60

(c) 20 : 22 : 40 *(d)* None of these

5. Salary of Mr. *X* is 80% of the salary of Mr. *Y* and the salary of Mr. *Z* is 120% of the salary of Mr. *X*. What is the ratio between the salaries of *X, Y* and *Z,* respectively? [Bank Clerks 2009]

(a) 4 : 6 : 5

(b) 4 : 5 : 6

(c) 16 : 24 : 25

(d) 16 : 25 : 24

(e) None of the above

6. If the positions of the digits of a two-digit number are interchanged, the number newly formed is smaller than the original number by 45. Also, the ratio of the new number to the original number is 3 : 8. What is the original number?

(a) 61 *(b)* 72 *(c)* 94

(d) Couldn't be determined

(e) None of the above

7. A cat takes 5 leaps for every 4 leaps of a dog but 3 leaps of the dog are equal to 4 leaps of the cat. What is the ratio of the speeds of the cat to that of the dog?

[Delhi Police 2007] *(a)* 11 : 15 *(b)* 15 : 11

(c) 16 : 15 *(d)* 15 : 16

In the month of January, Aran's income and expenses were *X* 15000 and *X* 9000, respectively. His monthly expenses vary directly as the square of his monthly income. What is his income when it just equals his expenses? *(a) X* 25000 *(b) X* 2000

(c) X 35000 *(d) X* 15000

(e) None of the above

9. The respective ratio of Sita's, Riya's and Kunal's monthly incomes is 84 : 76 : 89. If Riya's annual income is *X* 456000, then what is the sum of Sita's and Kunal's annual incomes? (In some cases monthly income and in some cases annual income is used.) [IBPS Clerk 2011]

(a) X 195000 *(b) X* 983500

(C) X 1130000 *(d) X* 1038000

(e) None of the above

10. *X* 5625 are divided among *A, B* and C, so that *A* receives 1/2 as much as *B* and C together receive and *B* receives 1/4 as much as *A* and C together receive. Find the sum of shares of A and *B.*

(a) X 5000 *(b) X* 3000

(C) X 15000 *(d) X* 9000

(e) None of the above

11.? 710 were divided among *A, B* and *C* in such a way that *A* had *X* 40 more than *B* and *C* had *X* 30 more than *A*. How much was Cs share?

(a) X 270 *(b) X* 300 *(c) X* 135 *(d) X* 235

(e) None of the above

12. The ratio of 1st and 2nd classes train fairs between two stations is 3 : 1 and that of the number of passengers travelling between these stations by 1st and 2nd classes is 1 : 50. If on a particular day, *X* 2650 be collected from the passengers travelling between these stations, then find the amount collected from 2nd class passengers.

(a) X 3000 *(b) X* 3500

(c) X 2800 *(d) X* 2500

(e) None of the above

13. Out of two sections *A* and *B,* 10 students of section *B* shift to *A,* as a result strength of *A* becomes 3 times the strength of *B*. But, if 10 students shift over from *A* to *B,* both *A* and *B* become equal in strength. Ratio of the number of students in section *A* that of section *B* is

[SSCCPO2013] *(a)* 2 : 1 *(b)* 5 : 3

(93:1 (dj 9 : 4

14. A sum of money is to be divided equally among *P, Q* and *R* in the respective ratio of 5 : 6 : 7 and another sum of money is to be divided between *S* and *T* equally. If *S* got *X* 2100 less than *P,* then how much amount did *Q* receive? [Bank Clerks 2008]

(a) X 2500

(b) X 2000

(c) X 1500

(d) Couldn't be determined

(e) None of the above

15. Nandita scores 80% marks in five subjects together, *viz.,* Hindi, Science, Mathematics, English and Sanskrit, where in the maximum marks of each subject were 105. How many marks did Nandita score in Science, if she scored 89 marks in Hindi, 92 marks in Sanskrit, 98 marks in Mathematics and 81 marks in English? [IBPS Clerk 2011]

(a) 60 *(b)* 75

(c) 65 *(d)* 70

(e) None of the above

16. *X* 2186 are distributed among *A,B* and C. If money given to them is decreased by *X* 26, *X* 28 and *X* 32 respectively, then they have money in the ratio of 9 : 13 : 8. What is the amount given to A?

(a) X 575 *(b) X* 640

(c) X 656 *(d) X* 672

(e) None of the above

17. A person gave 2/5 part of his income to his elder son and 30% part to his younger son. He saved his remaining money in

three trusts *A*, *B* and *C* in the ratio of 3 : 5 : 2. If difference between the amount got by his both sons is *X* 2000, how much amount he saved in trust C? *(a)* ?1000 *(b)* ? 1140

(c) X 1200 *(d) X* 1256

(e) X 1300

18. In a factory, the ratio of the numbers of employees of three types *A, B* and C is 9: 13 :18 and their wages are in the ratio of 10 : 7 : 4. If number of employees of type C is 54 and wages of every employee of type *B* is *X* 1400, then find the total wages of all the employees of type A.

(a) X 51000 *(b) X* 54000

(c) X 56000 *(d) X* 57000

(e) X 59000

19. Number of employees in a factory decreases in the ratio of 8 : 7 and salary of employees increases in the ratio of 5:6. Find wheather the total salary given to the empolyees is increased or decreased and in what ratio?

(a) 19 : 20, increased

(b) 20 : 19, decreased

(c) 19 : 21, increased

(d) 21 : 20, decreased

(e) None of the above

20. Out of 120 applications for a post, 70 are males and 80 have a driver's license. What is the ratio between the minimum to maximum number of males having driver's license ? [CSAT 2013]

(a) 1 : 2 *(b)* 2 : 3

(c) 3 : 7 *(d) 5:7*

(e) None of the above

21. In a certain examination, the number of those who passed was 4 times the number of those who failed. If there had been 35 fewer candidates and 9 more had failed, the ratio of passed and failed candidates would have been 2:1, then the total number of candidates was

[SSC (10+2) 2013]

(a) 135 *(b)* 155

(c) 145 *(d)* 150

Answer with **Solutions**

Exercise © *Base Level Questions*

1. (*c*) Given that, $P : Q = 8 : 15$, $Q : R = 3 : 2$

$$P : Q : R = (8 \times 3) : (15 \times 3) : (15 \times 2)$$
$$= 24 : 45 : 30$$

$\therefore \quad P : Q : R = 8 : 15 : 10$

Here, consequent of first ratio should be equal to the antecedent of second ratio.

2. (*a*) Given, $\frac{P}{Q} = \frac{8}{15}$, $\frac{Q}{R} = \frac{5}{8}$, $\frac{R}{S} = \frac{4}{5}$

$$\therefore \frac{P}{S} = \frac{P}{Q} \times \frac{Q}{R} \times \frac{R}{S} = \frac{8}{15} \times \frac{5}{8} \times \frac{4}{5} = \frac{4}{15}$$

$\therefore \quad P : S = 4 : 15$

3. (*a*) Here, $A : B = 2 : 3$, $B : C = 5 : 7$ and $C : D = 3 : 10$

$$\therefore \quad \frac{A}{D} = \frac{A}{B} \times \frac{B}{C} \times \frac{C}{D}$$
$$= \frac{2}{3} \times \frac{5}{7} \times \frac{3}{10}$$
$$= \frac{1}{7}$$

$\therefore \quad A : D = 1 : 7$

4. (*a*) Let the 4th proportional $= x$

Then, $4 : 16 :: 7 : x$ or $\frac{4}{16} = \frac{7}{x}$

$\Rightarrow \quad 4x = 7 \times 16$

$$\therefore \quad x = \frac{7 \times 16}{4} = 7 \times 4 = 28$$

5. (*d*) Let the 3rd proportional $= x$

Then, $15 : 30 :: 30 : x$

$$\Rightarrow \quad \frac{15}{30} = \frac{30}{x}$$

$\Rightarrow \quad 15x = 30 \times 30$

$$\therefore \quad x = \frac{30 \times 30}{15} = 60$$

6. (*b*) Required mean proportional

$$= \sqrt{9 \times 64} = 3 \times 8 = 24$$

7. (*c*) Here, $a = 15 + \sqrt{200}$, $b = 27 - \sqrt{648}$

Mean proportional between two numbers $= \sqrt{ab}$

Mean proportional between $(15 + \sqrt{200})$ and $(27 - \sqrt{648})$

$$= \sqrt{(15 + \sqrt{200})\,(27 - \sqrt{648})}$$

$$= \sqrt{\begin{matrix}(15 \times 27) - 15 \times \sqrt{648} + 27\sqrt{200} \\ - (\sqrt{200} \times \sqrt{648})\end{matrix}}$$

$$= \sqrt{\begin{matrix}405 - (15 \times 18\sqrt{2}) + (27 \times 10\sqrt{2}) \\ - (10\sqrt{2} \times 18\sqrt{2})\end{matrix}}$$

$$= \sqrt{405 - 270\sqrt{2} + 270\sqrt{2} - 180 \times 2}$$

$$= \sqrt{405 - 180 \times 2}$$

$$= \sqrt{405 - 360} = \sqrt{45} = 3\sqrt{5}$$

8. (*a*) Required duplicate ratio of

$$2 : 7 = 2^2 : 7^2 = 4 : 49$$

9. (*c*) Required sub-duplicate ratio of

$$81 : 64 = \sqrt{81} : \sqrt{64} = 9 : 8$$

10. (*b*) Required triplicate ratio of

$$7 : 5 = 7^3 : 5^3 = 343 : 125$$

11. (*d*) Required sub-triplicate ratio of 512 : 729

$$= \sqrt[3]{512} : \sqrt[3]{729} = 8 : 9$$

12. (*a*) Required inverse ratio of 17 : 19

$$= \frac{1}{17} : \frac{1}{19} = 19 : 17$$

13. (*b*) Required compound ratio

$$= \frac{2 \times 5 \times 4}{7 \times 3 \times 7} = \frac{40}{147} \Rightarrow \quad 40 : 147$$

14. (*d*) Given that,

$A : B = 3 : 4 = (3 \times 2) : (4 \times 2) = 6 : 8$

$B : C = 8 : 9$

$\therefore \quad A : B : C = 6 : 8 : 9$

As consequent of the first ratio is equal to the antecedent of second ratio.

15. (*b*) Given that, $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$

$$\therefore \quad a^4 : b^4 = a^4 : a^2c^2 = a^2 : c^2$$

16. (*d*) Given that, $\frac{a}{b} = \frac{3}{5}, \frac{b}{c} = \frac{4}{7}$

$$\therefore \quad \frac{a}{c} = \frac{a}{b} \times \frac{b}{c} = \frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$$

$$\therefore \quad a : c = 12 : 35$$

17. (*a*) Given that, $4a = 5b$

$$\therefore \quad \frac{a}{b} = \frac{5}{4}$$

Also, $\quad 7b = 9c$

$$\therefore \quad \frac{b}{c} = \frac{9}{7}$$

$\therefore\ a : b = 5 : 4 = (5 \times 9) : (4 \times 9) = 45 : 36$

$b : c = 9 : 7 = (9 \times 4) : (7 \times 4) = 36 : 28$

$\therefore\quad a : b : c = 45 : 36 : 28$

18. (*c*) Let $\dfrac{a}{7} = \dfrac{b}{9} = \dfrac{c}{11} = K$

Then, $a = 7K,\ b = 9K,\ c = 11K$

$\therefore\quad a : b : c = 7K : 9K : 11K = 7 : 9 : 11$

19. (*d*) Given that, 30% of A = 20 % of B

$$\Rightarrow\quad \frac{A}{B} = \frac{20}{30} = \frac{2}{3} \Rightarrow A : B = 2 : 3$$

20. (*a*) Given that, $P : Q : R = 2 : 3 : 4$

Let $P = 2K,\ Q = 3K,\ R = 4K$

$$\therefore\quad \frac{P}{Q} = \frac{2K}{3K} = \frac{2}{3},\ \frac{Q}{R} = \frac{3K}{4K} = \frac{3}{4},$$

$$\frac{R}{P} = \frac{4K}{2K} = \frac{2}{1}$$

$$\therefore\quad \frac{P}{Q} : \frac{Q}{R} : \frac{R}{P} = \frac{2}{3} : \frac{3}{4} : \frac{2}{1}$$

$$= \frac{2 \times 12}{3} : \frac{3 \times 12}{4} : \frac{2 \times 12}{1}$$

$$= 2 \times 4 : 3 \times 3 : 24 = 8 : 9 : 24$$

21. (*c*) Given that, $A = \dfrac{1}{4} B$

$$\therefore\quad \frac{A}{B} = \frac{1}{4} \quad \text{or} \quad A : B = 1 : 4$$

Also, $\quad B = \dfrac{1}{2} C \quad$ or $\quad \dfrac{B}{C} = \dfrac{1}{2}$

or $\quad B : C = 1 : 2$

Now, we have

$A : B = 1 : 4; \quad B : C = 1 : 2$

$A : B : C = (1 \times 1) = (1 \times 4) : (2 \times 4)$

$= 1 : 4 : 8$

22. (*a*) Given that, $0.8 \times A = 0.09 \times B$

$$\Rightarrow\quad \frac{A}{B} = \frac{0.09}{0.8} = \frac{9}{80}$$

$\therefore\quad A : B = 9 : 80$

23. (*d*) Let $\dfrac{1}{x} = 2K,\ \dfrac{1}{y} = 3K$ and $\dfrac{1}{z} = 5K$

Then, $\quad x = \dfrac{1}{2K},\ y = \dfrac{1}{3K}$ and $z = \dfrac{1}{5K}$

$$\therefore\ x : y : z = \frac{1}{2K} : \frac{1}{3K} : \frac{1}{5K}$$

$$= \frac{1}{2} : \frac{1}{3} : \frac{1}{5} = 15 : 10 : 6$$

24. (*a*) Let $\dfrac{a}{3} = \dfrac{b}{8} = K$

Then, $\quad a = 3K,\ b = 8K$

$$\therefore\quad \frac{(a+3)}{(b+8)} = \frac{3K+3}{8K+8} = \frac{3(K+1)}{8(K+1)} = \frac{3}{8}$$

$\therefore\quad (a + 3) : (b + 8) = 3 : 8$

25. (*a*) $\dfrac{a+b}{a-b}=\dfrac{5}{3} \Rightarrow\ 3a+3b=5a-5b$

$\Rightarrow\quad 2a=8b\ \Rightarrow\ a=4b,\ \Rightarrow \dfrac{a}{b}=\dfrac{4}{1}$

Now, $\quad \dfrac{(a^2+b^2)}{(a^2-b^2)}=\dfrac{\dfrac{a^2}{b^2}+1}{\dfrac{a^2}{b^2}-1}$

$$=\frac{\left(\dfrac{a}{b}\right)^2+1}{\left(\dfrac{a}{b}\right)^2-1}=\frac{\left(\dfrac{4}{1}\right)^2+1}{\left(\dfrac{4}{1}\right)^2-1}=\frac{16+1}{16-1}=\frac{17}{15}$$

$\therefore\quad (a^2+b^2):(a^2-b^2)=17:15$

26. (*b*) $\dfrac{x}{2y}=\dfrac{6}{7} \Rightarrow \dfrac{x}{y}=\dfrac{12}{7}$

By componendo and dividendo,

$$\frac{x-y}{x+y}=\frac{12-7}{12+7}=\frac{5}{19}$$

$\therefore\quad \dfrac{x-y}{x+y}+\dfrac{14}{19}=\dfrac{5}{19}+\dfrac{14}{19}=\dfrac{19}{19}=1$

27. (*a*) $\dfrac{1}{2}$ of $A=\dfrac{2}{5}$ of $B=\dfrac{1}{3}$ of C

$$\frac{1}{2}\times A=\frac{2}{5}\times B=\frac{1}{3}\times C$$

Let $\quad \dfrac{A}{2}=\dfrac{2B}{5}=\dfrac{C}{3}=K$

Then, $A=2K, B=\dfrac{5K}{2}, C=3K$

$\therefore\quad A:B:C=2K:\dfrac{5K}{2}:3K \Rightarrow 4:5:6$

28. (*c*) $P^2+4Q^2=4PQ$

$\Rightarrow\ P^2+4Q^2-4PQ=0$

$\Rightarrow\ (P-2Q)^2=0 \Rightarrow P-2Q=0 \Rightarrow P=2Q$

$\therefore\quad \dfrac{P}{Q}=\dfrac{2}{1}$

$\therefore\quad P:\ Q=2:1$

29. (*a*) $xy=36$

$\therefore\ xy=4\times 9, \dfrac{x}{9}=\dfrac{4}{y};\ x:9=4:y$

30. (*d*) Given that, 10% of $(A+B)=50\%$ of $(A-B)$

$\Rightarrow\ \dfrac{A+B}{A-B}=\dfrac{50}{10}=\dfrac{5}{1} \Rightarrow A+B=5A-5B$

$\Rightarrow\quad 5A-A=B+5B$

$\Rightarrow\quad 4A=6B$

$\Rightarrow\quad \dfrac{A}{B}=\dfrac{6}{4}=\dfrac{3}{2}$

$\therefore\quad A:B=3:2$

31. (*a*) Given that, $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{1}{2}$

$$\Rightarrow \quad a=\frac{b}{2},\, c=\frac{d}{2},\, e=\frac{f}{2}$$

$$\left(\frac{3a+5c+7e}{3b+5d+7f}\right)=\frac{\left(\frac{3b}{2}+\frac{5d}{2}+\frac{7f}{2}\right)}{(3b+5d+7f)}$$

$$=\frac{\frac{1}{2}(3b+5d+7f)}{(3b+5d+7f)}=\frac{1}{2}$$

$\therefore$ Required ratio $=1:2$

32. (*c*) Given, $\frac{x}{y}=\frac{7}{5}$

$$\therefore \frac{5x-2y}{3x+2y}=\frac{(5\times 7-2\times 5)}{(3\times 7+2\times 5)}=\frac{35-10}{21+10}=\frac{25}{31}$$

33. (*b*) Since, a, b, c, d and e are in continued proportion.

$$\therefore \quad \frac{a}{b}=\frac{b}{c}=\frac{c}{d}=\frac{d}{e}\Rightarrow\frac{e}{d}=\frac{d}{c}=\frac{c}{b}=\frac{b}{a}$$

$$\therefore \quad c=\frac{b^2}{a} \qquad \left[\because \frac{c}{b}=\frac{b}{a}\right]$$

$$\therefore \quad d=\frac{c^2}{b}=\frac{b^4}{a^2}\cdot\frac{1}{b}=\frac{b^3}{a^2}$$

$$\therefore \quad e=\frac{d^2}{c}=\frac{b^6}{a^4}\cdot\frac{a}{b^2}=\frac{b^4}{a^3}$$

$$\therefore \quad \frac{a}{e}=\frac{a}{(b^4/a^3)}=\frac{a^4}{b^4}$$

34. (*c*) Given, $\quad 2A=3B=4C$

Now, $\quad 2A=3B \Rightarrow \frac{A}{B}=\frac{3}{2}$

or $\quad A:B=3:2$

$\quad =(3\times 2):(2\times 2)=6:4$

Again, $\quad 3B=4C \Rightarrow \frac{B}{C}=\frac{4}{3}$

or $\quad B:C=4:3$

$\therefore \quad A:B:C=6:4:3$

35. (*d*) Let the number of boys and girls are $7x$ and $5x$, respectively.

Given, total number of students $=2400$

$\Rightarrow \quad 7x+5x=2400$

$\Rightarrow \quad 12x=2400$

$\Rightarrow \quad x=200$

$\therefore$ Required number of girls

$\quad =5x=5\times 200=1000$

Fast Track Method

Here, $\quad a=7,\, b=5,\, x=2400$

$$\text{Number of girls}=\frac{bx}{a+b}=\frac{5\times 2400}{7+5}$$

$$=\frac{5}{12}\times 2400=1000$$

36. (*a*) Let $a + b = 6K$

$$b + c = 7K \quad \ldots(i)$$

$$c + a = 8K$$

and $a + b + c = 14 \quad \ldots(ii)$

From Eq. (i),

$$a + b + b + c + c + a = 6K + 7K + 8K$$

$\Rightarrow \quad 2(a + b + c) = 21K$

$\Rightarrow \quad 2 \times 14 = 21K \Rightarrow K = \frac{28}{21} = \frac{4}{3}$

$\therefore \quad a + b = 6K$

$\Rightarrow \quad a + b = 6 \times \frac{4}{3} = 8 \quad \ldots(iii)$

Subtracting Eq. (iii) from Eq. (ii), we get

$$(a + b + c) - (a + b) = 14 - 8 = 6$$

37. (*d*) Let the quantity be x.

Then, $\frac{a + x}{b + x} = \frac{c}{d}$

$\Rightarrow \quad (a + x)d = (b + x)c$

$\Rightarrow \quad ad + dx = bc + cx$

$\Rightarrow \quad ad - bc = cx - dx$

$\Rightarrow \quad ad - bc = x(c - d)$

$\therefore \quad x = \frac{ad - bc}{c - d}$

38. (*d*) Let 1st number $= x$ and 2nd number $= y$

According to the question,

$$\frac{1}{2} \text{ of } x = 65\% \text{ of } y \Rightarrow \frac{x}{2} = \frac{65y}{100}$$

$\Rightarrow \quad \frac{x}{y} = \frac{130}{100} = \frac{13}{10}$

$\therefore \quad x : y = 13 : 10$

39. (*a*) Let the numbers be $3x$ and $4x$.

According to the question,

$$\frac{3x + 3}{4x + 3} = \frac{4}{5}$$

$\Rightarrow \quad 5(3x + 3) = 4(4x + 3)$

$\Rightarrow \quad 15x + 15 = 16x + 12$

$\therefore \quad x = 15 - 12 = 3$

$\therefore$ Difference between the numbers

$$= (4x - 3x) = x = 3$$

Fast Track Method

Here, $a = 3$, $b = 4$, $c = 4$, $d = 5$ and $x = 3$

First number $= \frac{xa(c - d)}{ad - bc}$ [by Technique 4]

$$= \frac{3 \times 3 \times (4 - 5)}{3 \times 5 - 4 \times 4} = \frac{9 \times (-1)}{15 - 16} = \frac{9 \times (-1)}{(-1)} = 9$$

Second number

$$= \frac{x\,b(c - d)}{ad - bc} = \frac{3 \times 4 \times (4 - 5)}{3 \times 5 - 4 \times 4}$$

$$= \frac{12 \times (-1)}{15 - 16} = \frac{12 \times (-1)}{(-1)} = 12$$

Difference between two numbers

$= 12 - 9 = 3$

40. (*a*) Let the smaller number $= x$ and the greater number $= y$.

According to the question,

$$\left(y - \frac{x}{2}\right) = 4\left(x - \frac{x}{2}\right)$$

$\Rightarrow \quad y - \frac{x}{2} = 4\,\frac{x}{2} \Rightarrow y = 2x + \frac{x}{2} \Rightarrow y = \frac{5x}{2}$

$\therefore \quad y : x = 5 : 2$

41. (*d*) Let 1st number be $4x$ and 2nd number be $7x$.

According to the question,

$$\frac{4x - 5}{7x - 5} = \frac{1}{2}$$

$\Rightarrow \quad 8x - 10 = 7x - 5$

$\Rightarrow \quad 8x - 7x = 10 - 5$

$\therefore \quad x = 5$

$\therefore$ Greater number $= 7x = 7 \times 5 = 35$

Fast Track Method

Here, $a = 4$, $b = 7$, $c = 1$, $d = 2$ and $x = 5$

Then, 1st number

$$= \frac{xa(d - c)}{ad - bc} \qquad \text{[by Technique 5]}$$

$$= \frac{5 \times 4\,(2 - 1)}{4 \times 2 - 7 \times 1} = \frac{20 \times 1}{8 - 7} = 20$$

2nd number $= \dfrac{xb\,(d - c)}{ad - bc}$

$$= \frac{5 \times 7\,(2 - 1)}{4 \times 2 - 7 \times 1} = \frac{35 \times 1}{8 - 7} = 35$$

42. (*c*) Let the numbers be x and y.

According to the question,

$$\frac{x}{y} = \frac{2}{3}$$

$\Rightarrow \quad 3x = 2y$

$\Rightarrow \quad 3x - 2y = 0 \qquad \ldots\text{(i)}$

and $\quad \dfrac{x + 9}{y + 9} = \dfrac{3}{4}$

$\Rightarrow \quad 4x + 36 = 3y + 27$

$\Rightarrow \quad 4x - 3y = -9 \qquad \ldots\text{(ii)}$

Multiplying Eq. (i) by 4 and Eq. (ii) by 3 and substracting, we get

$$\begin{array}{r} 12x - 8y = 0 \\ 12x - 9y = -27 \\ - \quad + \quad\quad + \\ \hline y = 27 \end{array}$$

Now, putting $y = 27$ in Eq. (i), we get

$3x = 54$

$\Rightarrow \quad x = 18$

$\therefore$ Product of the given numbers

$= 27 \times 18 = 486$

43. (*b*) Number of boys $= (2140 - 1200) = 940$

$\therefore \quad$ Boys : Girls $= 940 : 1200 = 47 : 60$

44. (*b*) Given that,

Sanjay + Suresh = 120 ...(i)

Sanjay – Suresh = 30 ...(ii)

By adding Eqs. (i) and (ii), we get

$$2 \times \text{Sanjay} = 150$$

$$\therefore \quad \text{Sanjay} = \frac{150}{2} = 75 \text{ kg}$$

$\therefore$ Weight of Suresh = 75 – 30 = 45 kg

$\therefore$ Suresh : Sanjay = 45 : 75 = 3 : 5

$$= \frac{3}{5} = 0.6$$

45. (*b*) Let number of boys = $4x$ and number of girls = $5x$

According to the question,

$$\frac{4x}{5x - 100} = \frac{6}{7}$$

$$\Rightarrow \quad 28x = 6(5x - 100)$$

$$\Rightarrow \quad 28x = 30x - 600$$

$$\Rightarrow \quad 2x = 600$$

$$\therefore \quad x = \frac{600}{2} = 300$$

$\therefore$ Number of boys = $4x = 4 \times 300 = 1200$

46. (*c*) Let original number of boys = $4x$ and number of girls = $5x$

According to the question,

$$\frac{4x + 10}{5x} = \frac{6}{5}$$

$$\Rightarrow \quad 30x = 20x + 50$$

$$\Rightarrow \quad 10x = 50$$

$$\therefore \quad x = \frac{50}{10} = 5$$

$\therefore$ Number of girls = $5x = 5 \times 5 = 25$

47. (*a*) Total number of students = 8670

Total number of boys = 4545

$\therefore$ Total number of girls

= 8670 – 4545 = 4125

$\therefore$ Required ratio = 4545 : 4125 = 303 : 275

48. (*e*) Speed of truck = $\frac{640}{10}$ = 64 km/h

Speed of car = $\frac{640}{8}$ = 80 km/h

So, the required ratio = $\frac{64}{80} = \frac{4}{5}$ = 4 : 5

49. (*b*) *Case* I Speed = $\frac{\text{Distance}}{\text{Time}}$

Case II $\quad \text{Speed} = \dfrac{\frac{\text{Distance}}{2}}{2 \times \text{Time}} = \dfrac{\text{Distance}}{4 \times \text{Time}}$

$\therefore$ Required ratio = $\frac{1}{4}$ = 1 : 4

50. (*d*) Ratio of speed = 2 : 3 : 4

Ratio of time taken = $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ = 6 : 4 : 3

51. (*c*) The ratio among A, B, C and D

$$= \frac{1}{3} : \frac{1}{4} : \frac{1}{5} : \frac{1}{6}$$

On rearranging the ratio

$$\frac{60}{3} : \frac{60}{4} : \frac{60}{5} : \frac{60}{6} = 20 : 15 : 12 : 10$$

So, minimum number of pens can be when the common ratio is 1.

So, minimum number of pen

$$= 20 + 15 + 12 + 10 = 57$$

52. (*b*) If three parts be x, y and z, then

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{5} \Rightarrow x : y : z = 2 : 3 : 5$$

$\therefore \quad x = \frac{2}{10} \times 990 = 198$

$y = \frac{3}{10} \times 990 = 297$

and $\quad z = \frac{5}{10} \times 990 = 495$

53. (*a*) Let number of ₹ 1 coins $= 8x$

Number of 50 paise coins $= 9x$

Number of 25 paise coins $= 11x$

According to the question,

$$8x + \frac{9x}{2} + \frac{11x}{4} = 366$$

$\Rightarrow \quad 32x + 18x + 11x = 1464$

$\Rightarrow \quad 61x = 1464$

$\therefore \quad x = \frac{1464}{61} = 24$

$\therefore$ Number of 25 paise coins

$$= 11x = 11 \times 24 = 264$$

54. (*c*) Let the incomes of two persons are $7x$ and $3x$.

Expenditure of first person $= 7x - 300$

Expenditure of second person $= 3x - 300$

According to the question,

$$\frac{7x - 300}{3x - 300} = \frac{5}{2}$$

$\Rightarrow \quad 14x - 600 = 15x - 1500$

$\therefore \quad x = 900$

Income of first person

$$= 7x = 7 \times 900 = ₹\ 6300$$

Fast Track Method

Ratio of incomes $= 7 : 3$

Ratio of expenses $= 5 : 2$

So, $\quad a = 7, b = 3, c = 5, d = 2\ x = ₹\ 300$

$\therefore$ Income of 1st person

$$= \frac{xa\,(d - c)}{(ad - bc)} \quad \text{[by Technique 3]}$$

$$= \frac{300 \times 7(2 - 5)}{14 - 15}$$

$$= \frac{300 \times (-3) \times 7}{-1} = ₹\ 6300$$

55. (*a*) Given, total amount = ₹ 49500

Let part of Amit's investment = $4x$

and part of sudesh's investment = $7x$

According to the question,

$$4x + 7x = 49500 \Rightarrow 11x = 49500$$

$$\Rightarrow \quad x = \frac{49500}{11}$$

$$\therefore \quad x = 4500$$

Hence, investment of sudesh = $7x$

$= 7 \times 4500 = 31500$

Fast Track Method

Here, $x = 49500$, $a = 4$ and $b = 7$

$\therefore$ Investment of sudesh $= \dfrac{bx}{a + b}$

[by Technique 2]

$$= \frac{7 \times 49500}{7 + 4} = 7 \times 4500 = 31500$$

56. (*d*) Given that, P's share : Q's share

$$= \frac{1}{3} : \frac{1}{5} = 5 : 3$$

Here, $a = 5$, $b = 3$, $x = 1664$

P's share $= \dfrac{ax}{a + b}$ [by Technique 2]

$$\therefore \quad = \frac{5 \times 1664}{5 + 3} = \frac{5}{8} \times 1664 = ₹\ 1040$$

57. (*d*) Let the marks of A, B and C are $10x, 12x$ and $15x$, respectively.

Let $x = 6$

$\therefore$ Maximum marks of C can be

$= 15 \times 6 = 90$

So, maximum marks of B can be

$= 12 \times 6 = 72$

As the marks are fixed and they cannot exceed the maximum marks.

So, the marks of B cannot be in the range of (80-90) *i.e.*, B cannot score above 80.

58. (*c*) Given, $A : B = 2 : 3$

and $B : C = 4 : 5$

Then, $A : B : C = 8 : 12 : 15$

$\therefore$ Share of $B = \dfrac{12}{8 + 12 + 15} \times 7000$

$$= \frac{12 \times 7000}{35} = 2400 = ₹\ 2400$$

59. (*a*) Let the share of $P = x$

Then, Q's share = $x + 30$

and R's share = $(x + 30) + 60 = x + 90$

Sum of money with P, Q and R = 300

$$\therefore \quad x + x + 30 + x + 90 = 300$$

$$\Rightarrow \quad 3x + 120 = 300$$

$$\therefore \quad x = \frac{300 - 120}{3} = 60$$

$\therefore$ Required ratio = 60 : (60 + 30) : (60 + 90)

$= 2 : 3 : 5$

60. (*a*) Let A's share $= 3x$

B's share $= 7x$

C's share $= 9x$

and D's share $= 13x$

According to the question,

$$7x = 4872$$

$$\therefore \quad x = \frac{4872}{7} = 696$$

Share of A and $C = 3x + 9x = 12x$

$= 12 \times 696 =$ ₹ 8352

61. (*d*) Let required money be ₹ x.

Then,

$$\frac{4}{14}x = 12 \Rightarrow x = \frac{12 \times 14}{4} = ₹\,42$$

62. (*a*) Let the price of scooter $= 8x$ and price of television $= 7x$

According to the question,

$$8x - 7x = 2000$$

$$\therefore \quad x = 2000$$

$\therefore$ Price of television $= 7x = 7 \times 2000$

$=$ ₹ 14000

63. (*c*) Age of $A = \frac{3}{8} \times 56 = 21$ yr

Age of $B = \frac{5}{8} \times 56 = 35$ yr

Before 7 yr, ratio of the ages of A and B

$= (21 - 7) : (35 - 7) = 14 : 28 = 1 : 2$

64. (6) Let the age of father = 5x and the age of son = 2x According to the question,

$5x \times 2x = 1000 \Rightarrow 10x^2 = 1000$

$\Rightarrow \quad x^2 = 100$

$\therefore \quad x = \sqrt{100} = 10$

$\therefore$ Father's age after 10 yr $= (5x + 10)$

$= 5 \times 10 + 10 = 60$ yr

65. (*a*) Let first and second part of the number be x and y, respectively. Then,

$$5x + 11y = 7(x + y)$$

$$\Rightarrow \quad 11y - 7y = 7x - 5x$$

$$\Rightarrow \quad 4y = 2x$$

$$\therefore \quad x : y = 2 : 1$$

66. (*e*) Let first number is x and second number is y.

Then, according to the question,

$$x \times 35\% = 2 \times y \times 75\%$$

$$\Rightarrow \quad x \times \frac{35}{100} = 2y \times \frac{75}{100}$$

$$\Rightarrow \quad \frac{x}{y} = \frac{2 \times 75}{35} = \frac{2 \times 15}{7} = \frac{30}{7}$$

$$\therefore \quad x : y = 30 : 7$$

67. (*c*) Let the savings of A and B are $4x$, $5x$ and the share in cost of gift are $3y$, $4y$ respectively.

According to the question,

For A, $4x - 3y = \frac{2}{3} \times 4x$

$$\Rightarrow \quad x = \frac{9y}{4} \quad \ldots \text{(i)}$$

For B, $5x - 4y = 145$

$\Rightarrow 5 \times \frac{9y}{4} - 4y = 145$ [by Eq. (i)]

$$\therefore \quad y = 20$$

$\therefore$ Cost of gift $= 3y + 4y = 7 \times 20$

$= ₹\ 140$

Exercise © *Higher SkiU Level Questions*

1. (*a*) Let the number of adult men be x.

Then, ratio of the numbers of adult men and adult women

$$9 : 7 = x : 4.2$$

$$\Rightarrow \quad x = \frac{9}{7} \times 4.2 = 5.4 \text{ lakh}$$

Total adult population

$$= 4.2 + 5.4 = 9.6 \text{ lakh}$$

If the population of the town be y lakh, then

$$\frac{80y}{100} = 9.6$$

$$\Rightarrow \quad y = \frac{9.6 \times 100}{80} = 12 \text{ lakh}$$

2. (*d*) According to the question,

$$46x + 30 + 41x + 30 + 34x + 25 = 2505$$

$$\Rightarrow \quad 121x = 2505 - 85 = 2420$$

$$\therefore \quad x = \frac{2420}{121} = 20$$

$\therefore$ Number of coins received by Parat

$$= 41x + 30$$

$$= 41 \times 20 + 30 = 850$$

3. (*c*) $\frac{2}{5}A + 40 = \frac{2}{7}B + 20 = \frac{9}{17}C + 10 = K$

Then, $\quad A = \frac{5}{2}(K - 40),\ B = \frac{7}{2}(K - 20),$

$$C = \frac{17}{9}(K - 10)$$

According to the question,

$$A + B + C = 600$$

$$\Rightarrow \quad \frac{5}{2}(K - 40) + \frac{7}{2}(K - 20) + \frac{17}{9}(K - 10) = 600$$

$\Rightarrow \quad 45(K-40)+63(K-20)$
$+34(K-10)=10800$
$\Rightarrow \quad 45K-1800+63K-1260+34K$
$-340=10800$
$\Rightarrow \quad 142K-3400=10800$
$\Rightarrow \quad 142K=10800+3400$
$\Rightarrow \quad 142K=14200$
$\therefore \quad K=\frac{14200}{142}=100$
$\therefore \quad B=\frac{7}{2}(K-20)=\frac{7}{2}(100-20)$
$=\frac{7\times 80}{2}=7\times 40=₹\,280$

4. (*b*) Ratio of salaries
Akash : Babloo : Chintu = 2 : 3 : 5
Let the common ratio be x.
Then, salaries of Akash, Babloo and Chintu will be $2x, 3x$ and $5x$, respectively.
Now, 15% increase in Akash's salary
$=15\%\text{ of }2x=\frac{15\times 2x}{100}=0.3x$
$\therefore$ New salary $=2x+0.3x=2.3x$
Also, 10% increase in Babloo's salary
$=10\%\text{ of }3x=\frac{10\times 3x}{100}=0.3x$
$\therefore$ New salary $=3x+0.3x=3.3x$
Again, 20% increase in Chintu's salary
$=20\%\text{ of }5x=\frac{20\times 5x}{100}=1x$
$\therefore$ New salary $=5x+x=6x$
$\therefore$ New ratio = Ratio of new salaries
$=2.3x:3.3x:6x$
On multiplying with 10 and dividing by x ratio will be 23 : 33 : 60.

5. (*e*) Let Y's salary = 100
$\therefore \quad X$'s salary = 80
and $\quad Z$'s salary $=\frac{80\times 120}{100}=96$
$\therefore$ Required ratio
$=80:100:96=20:25:24$

6. (*b*) According to the question,
$8x-3x=45 \Rightarrow 5x=45$
$\therefore \quad x=\frac{45}{5}=9$
$\therefore \quad$ Original number $=8x=8\times 9=72$

7. (*d*) 4 leaps of cat = 3 leaps of dog
$\Rightarrow \quad 1\text{ leap of cat}=\frac{3}{4}\text{ leap of dog}$
Cat takes 5 leaps for every 4 leaps of dog
$\therefore$ Required ratio
$=(5\times\text{Cat's leap}):(4\times\text{Dog's leap})$
$=\left(5\times\frac{3}{4}\text{ dog's leap}\right):(4\times\text{Dog's leap})$

$= 15 : 16$

Direct Approach

$$\begin{array}{ccc} \text{Cat} & & \text{Dog} \\ 5 & & 4 \\ 4 & \times & 3 \\ 15 & : & 16 \end{array}$$

8. (*a*) According to the question,

$$\text{Expenses} \propto (\text{Income})^2$$

$\Rightarrow \quad 9000 = K\,(15000)^2$

$\therefore \quad K = \dfrac{9000}{(15000)^2} = \dfrac{1}{25000}$

Again, $E = K \times I^2$

$\Rightarrow \quad I = K \times I^2 \quad [\because \text{expenses} = \text{income}]$

$\therefore \quad I = \dfrac{1}{K} = 25000$

$\therefore$ Required answer = ₹ 25000

9. (*d*) Let monthly income of Sita, Riya and Kunal be $84x, 76x$ and $89x$, respectively.

Given, annual income of Riya = 456000

$\therefore$ Monthly income of Riya

$$= \frac{456000}{12} = 38000$$

$\therefore \quad 76x = 38000 \Rightarrow x = 500$

So, the monthly income of Sita and Kunal

$= 84x + 89x = 173x$

$= 173 \times 500 = 86500$

Therefore, annual income $= 86500 \times 12$

= ₹ 1038000

10. (*b*) According to the question,

$A = \dfrac{1}{2}(B + C)$

$\Rightarrow \quad B + C = 2A \Rightarrow A + B + C = 3A$

[adding A on both sides]

$\therefore \quad 3A = 5625$

$\therefore \quad A = \dfrac{5625}{3} =$ ₹ 1875

and $\quad B = \dfrac{1}{4}(A + C)$

$\Rightarrow \quad A + C = 4B$

$\Rightarrow \quad A + B + C = 5B$

[adding B on both sides]

$\Rightarrow \quad 5B = 5625$

$\therefore \quad B = \dfrac{5625}{5} =$ ₹ 1125

$\therefore \quad A + B = 1875 + 1125 =$ ₹ 3000

11. (*a*) Let B gets x.

Then, A gets $(x + 40)$ and C gets $(x + 70)$.

According to the question,

$x + 40 + x + x + 70 = 710$

$\Rightarrow \quad 3x = 710 - 110 = 600$

$\therefore \quad x = \dfrac{600}{3} = 200$

$\therefore$ C's share $= 200 + 70 =$ ₹ 270

12. (*d*) Ratio of the amounts collected from 1st and 2nd classes fairs

$$= (3 \times 1) : (1 \times 50) = 3 : 50$$

$\therefore$ Amount collected from 2nd class passengers $= 2650 \times \dfrac{50}{53} = ₹\ 2500$

13. (*a*) $A + 10 = 3(B - 10)$

$\Rightarrow \quad A - 3B = -20 \quad \ldots(\text{i})$

and $\quad (A - 10) = (B + 10)$

$\Rightarrow \quad A - B = 20 \quad \ldots(\text{ii})$

On subtracting Eq. (ii) from Eq. (i), we get

$A - 3B - A + B = -20 - 20$

$\Rightarrow \quad -2B = -40$

$\therefore \quad B = 20$

Now, by using Eq. (ii),

$$A = 20 + 20 = 40.$$

$\therefore$ Ratio of the numbers of students of A and $B = 40 : 20 = 2 : 1$

14. (*d*) Couldn't be determined, since the total amount of money is not given in either of the case.

15. (*a*) Total of maximum marks of all subjects $= 105 \times 5 = 525$

$$80\% \text{ of } 525 = \frac{525 \times 80}{100} = 420$$

Obtained marks of four subjects (Hindi, Sanskrit, Mathematics and English)
$= 89 + 92 + 98 + 81 = 360$

So, the obtained marks in Science

$$= 420 - 360 = 60$$

16. (*c*) Amount given to $A = 9K + 26$

Amount given to $B = 13K + 28$

and amount given to $C = 8K + 32$

According to the question,

$(9K + 26) + (13K + 28) + (8K + 32)$
$= 2186$

$\Rightarrow \quad 30K + 86 = 2186$

$\Rightarrow \quad 30K = 2100$

$\therefore \quad K = 70$

Hence, amount given to $A = 9K + 26$
$= 9 \times 70 + 26$
$= 630 + 26 = ₹\ 656$

17. (*c*) Let the person have ₹ x.

Then, share of elder son $= ₹\ \dfrac{2x}{5}$

and share of younger son $= ₹\ \dfrac{3x}{10}$

According to the question,

$$\frac{2x}{5} - \frac{3x}{10} = 2000$$

$\Rightarrow \quad \dfrac{x}{10} = 2000$

$\Rightarrow \quad x = 20000$

Remaining amount saved in trusts

$$= 1 - \left(\frac{2x}{5} + \frac{3x}{10}\right) = 1 - \left(\frac{4x + 3x}{10}\right)$$

$$= 1 - \frac{7x}{10} = \frac{3x}{10} = \frac{3}{10} \times 20000 = 6000$$

Given that, $A : B : C = 3 : 5 : 2$

$\therefore$ Share of $C = \frac{2}{3 + 5 + 2} \times 6000$

$$= \frac{2}{10} \times 6000 = ₹1200$$

18. (*b*) Given ratio of employees = 9 : 13 : 18

Let number of A's employees = $9x$

Number of B's employees = $13x$

and number of C's employees = $18x$

According to the question,

$$18x = 54$$

$\therefore \quad x = 3$

$\therefore$ Number of employees of type A

$$= 9 \times 3 = 27$$

Similarly, Wages of every employee of type A

$$= \frac{10}{7} \times 1400 = ₹2000$$

Required wages = 27 × 2000 = ₹54000

19. (e) Let total number of employees in beginning = 8 *K* and total number of employees in present = *IK*

Let salary of an employee in beginning = 5J and salary of an employee in present = *61 .:* Total salary in beginning

= *8K X 51* = 40*Kl* and total salary in present

= *7K* X *61* = *42.K1* Required ratio = 40 *Kl*: 42 *Kl* = 20 : 21 Clearly, salary is increased.

20. (c) Since, there are 70 males out of 120 applicants, there must be 50 females. For the minimum number of males to have a driver's license all 50 females must have a driver's license. Thus, the number of males having a driver's license, will be 80 - 50 = 30. The maximum possible number of males having a driver's license is 70. The ratio between the minimum and the maximum is 30 : 70 or 3 : 7.

21. (6) Let the number of failed and passed candidates be x and 4x, respectively. Therefore, total number of candidates was 5x.

According to the question,

If total number of students had been 5x - 35, then $\dfrac{4x - 35 - 9}{x + 9} = \dfrac{2}{1}$

$\Rightarrow \quad 4x - 44 = 2(x + 9)$

$\Rightarrow \quad 4x - 2x = 18 + 44$

$\Rightarrow \quad 2x = 62$

$\therefore \quad x = 31$

Thus, total number of candidates was 31×5 *i.e.*, 155.

Chapter

18

Mixture or Alligation

Mixture

The new product obtained by mixing two or more ingredients in a certain ratio is called a mixture.

or Combination of two or more quantities is known as mixture,

Mean Price

The cost price of a unit quantity of the mixture is called the mean price. It will always be higher than cost price of cheaper quantity and lower than cost price of dearer quantity.

Rule of Mixture or Alligation

It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.

According to this rule, $\frac{n_1}{n_2} = \frac{A_2 - A_w}{A_w - A_1}$

Where, *ri\lni* is the ratio, in which two quantities should be mixed, while \, A_2 and A_w are the cheaper price, dearer price and mean price, respectively. The above rule can be represented pictorially as shown below.

Cheaper price A_1 | Dearer price A_2

Mean price A_W

$A_2 - A_W$ | $A_W - A_1$

[Remember, $A_1 < A_w < A_2$]

$$\text{Amount of cheaper : Amount of dearer } (n_1 : n_2) = \frac{A_2 - A_w}{A_w - A_1}$$

MIND IT!

The rule is also applicable for solving questions based on average *i.e.,* speed, percentage, price, ratio etc., and not for absolute values. In other words, we can use this method whenever per cent, per hour, per kg etc., are being compared.

Ex. 1 In what proportion, must wheat at ? 6.20 per kg be mixed with wheat at ? 7.20 per kg, so that the mixture be worth ? 6.50 per kg?

Sol. Given, cost price of cheaper quantity = ₹ 6.20 per kg

Cost price of dearer quantity = ₹ 7.20 per kg

Mean price = ₹ 6.50 per kg

According to the rule of alligation,

CP of cheaper (620 paise) | CP of dearer (720 paise)

Mean price (650 paise)

(720 – 650) = 70 paise | (650 – 620) = 30 paise

∴ Required ratio = 70 : 30 = 7 : 3

Ex. 2 A mixture of a certain quantity of milk with 16 L of water is worth ? 0.75 per litre. If pure milk be worth ? 2.25 per litre, then how much milk is there in the mixture?

Sol. Water is available free of cost, so its cost price = ₹ 0

Now, according to the rule of alligation,

Water (₹ 0)		Milk (₹ 2.25)
	Mean price (0.75)	
(2.25 − 0.75) = ₹ 1.5		(0.75 − 0) = ₹ 0.75

∴ Water : Milk = 1.5 : 0.75 = 2 : 1

Clearly, quantity of milk = $\frac{1}{2}$ of water = $\frac{1}{2} \times 16 = 8$ L

Ex. 3 If 50 L of milk solution has 40% milk in it, then how much milk should be added to make it 60% in the solution?

Sol. Total quantity = 50 L

Quantity of milk = $\frac{50 \times 40}{100} = 20$ L

Let x L milk should be added.

Then, $(20 + x) = \frac{(50 + x) \times 60}{100}$

or $\frac{20 + x}{50 + x} = \frac{6}{10}$

$\Rightarrow 200 + 10x = 300 + 6x$

$\Rightarrow 4x = 100$

$\therefore x = 25$ L

Fast Track **Techniques**

to solve the **QUESTIONS**

Technique 1

If a container initially contains *a* unit of liquid and *b* unit of liquid is taken out and it is filled with *b* unit of another liquid, then after *n* operations, the final quantity of the original liquid in the container is given as

$\left[a\left(1 - \frac{b}{a}\right)^n\right]$ units.

Ex. 4 A container contains 40 L of milk. From this container, 4 L of milk was taken out and replaced by water. This process was further repeated two times. How much milk is now there in the container?

Sol. Original quantity of milk = 40 L

Since, 4 L of milk was taken out

$\therefore$ Quantity of milk in the new mixture = 40 − 4 = 36 L

Now, when 4 L of this mixture taken out

Quantity of milk taken out $= 4 \times \frac{36}{40} = 3.6$ L

$\therefore$ Quantity of milk left = 36 − 3.6 = 32.4 L

Similarly, quantity of milk taken out in third step $= \frac{4 \times 32.4}{40} = 3.24$ L

$\therefore$ Quantity of milk left = 32.4 − 3.24 = 29.16 L

Fast Track Method Here, $a = 40$ L, $b = 4$ L, $n = 3$

According to the formula,

After n operations, quantity of milk $= \left[a\left(1 - \frac{b}{a}\right)^n\right]$

$$\therefore \text{Quantity of milk left} = \left[40\left(1 - \frac{4}{40}\right)^3\right] = \left(40 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}\right) = \frac{40 \times 729}{10 \times 10 \times 10}$$

$= 29.16$ L

Technique 2

In a container, milk and water are present in the ratio $a : b$. If xL of water is added to this mixture, the ratio becomes a: c. Then,

Quantity of milk in original mixture $= \frac{ax}{c-b}$ L

and quantity of water in original mixture $= \frac{bx}{c-b}$ L

Ex. 5 In a container, milk and water are present in the ratio 7 : 5. If 15 L water is added to this mixture, the ratio of milk and water becomes 7: 8. Find the quantity of water in the new mixture.

Sol. Let the quantity of milk and water in initial mixture be $7x$ and $5x$ L. Then, according to the question,

$$\frac{7x}{5x + 15} = \frac{7}{8} \Rightarrow 7x \times 8 = 7\,(5x + 15)$$

$\Rightarrow$ $$56x = 35x + 105 \Rightarrow 56x - 35x = 105$$

$\Rightarrow$ $$21x = 105 \Rightarrow x = \frac{105}{21} = 5$$

$\therefore$ Quantity of water in initial mixture $= 5 \times 5 = 25$ L

and quantity of water in new mixture $= 25 + 15 = 40$ L

Fast Track Method

Here, $a = 7$, $b = 5$, $c = 8$ and $x = 15$ L

According to the formula,

Quantity of water in original mixture $= \frac{bx}{c - b} = \frac{5 \times 15}{8 - 5} = 25$ L

$\therefore$ Quantity of water in new mixture $= 25 + 15 = 40$ L

Technique 3

A container has milk and water in the ratio *a : b,* a second container has milk and water in the ratio c : *d.* if both the mixtures are emptied into a third container, then the ratio of milk to water in third container is given by

$$\left(\frac{a}{a+b} + \frac{c}{c+d}\right) : \left(\frac{b}{a+b} + \frac{d}{c+d}\right).$$

Ex. 6 2 containers have milk and water in the ratio 2 :1 and 3 :1, respectively. If both containers are emptied into a bigger container, then find the ratio of milk to water in bigger container?

Sol. Given, ratio of milk and water in 1st container = 2 : 1

$\therefore$ Quantity of milk in 1st container = $\frac{2}{3}$

and quantity of water in 1st container = $\frac{1}{3}$

Similarly, ratio of milk and water in 2nd container = 3 : 1

$\therefore$ Quantity of milk in 2nd container = $\frac{3}{4}$

and quantity of water in second container = $\frac{1}{4}$

Now, after pouring both mixture in one container

Quantity of milk = $\frac{2}{3} + \frac{3}{4} = \frac{17}{12}$

and quantity of water = $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$

Hence, required ratio = 17 : 7

Fast Track Method

Here, $a = 2, b = 1, c = 3, d = 1$

Ratio of milk to water in bigger container

$$= \left(\frac{a}{a+b} + \frac{c}{c+d}\right) : \left(\frac{b}{a+b} + \frac{d}{c+d}\right)$$

$$= \left(\frac{2}{2+1} + \frac{3}{3+1}\right) : \left(\frac{1}{2+1} + \frac{1}{3+1}\right) = \frac{17}{12} : \frac{7}{12} = 17 : 7$$

Multi Concept

QUESTIONS

1. *Jagatram, a milk seller has certain quantity of milk to sell. In what ratio, he should mix water to gain 5% by selling the mixture at the cost price?*

(a) 1 :10 *(b)* 1 :5 *(c)* 1 : 20 *(d)* 1 :15

➥ (c) Let the cost price of milk be ₹ 1 per litre.

∴ SP of 1 L of mixture = ₹ 1; Gain = 5%

∴ CP of 1 L of mixture $= \frac{100}{105} \times 1 = ₹\ \frac{20}{21}$

According to the rule of alligation,

Water (₹ 0)   Milk (₹ 1)

Mean price (₹ 20/21)

[1 − (20/21)] = ₹ 1/21   [(20/21) − 0)] = ₹ 20/21

∴ Required ratio $= \frac{1}{21} : \frac{20}{21} = 1:20$

2. *If the price of three types of rice are ? 480, ? 576 and ? 696 per quintal, then find the ratio in which these types of rice should be mixed, so that the resultant mixture cost ? 564 per quintal?*

(a) 22:11:9 (b) 28:14:7 (c)14:7:21 (d) 11:77:7

➥ (d) Let amount of 3 types of rice be x, y and z, respectively.

Using rule of alligation for x and y

x ₹ 480 y ₹ 576

₹ 564

(576 − 564) = ₹ 12   (564 − 480) = ₹ 84

∴ $x : y = 12 : 84 = 1 : 7$

Similarly, using the rule of alligation for x and z

x ₹ 480 z ₹ 696

₹ 564

(696 − 564) = ₹ 132   (564 − 480) = ₹ 84

∴ $x : z = 132 : 84 = 11 : 7 = 1 : \frac{7}{11}$

On combining the two ratios, we get

$x : y : z = 1 : 7 : \frac{7}{11} = 11 : 77 : 7$

Fast Track Practice

Exercise© *Base Level Questions*

1. In what ratio must a grocer mix two types of rice costing ? 7.50 per kg and ? 10 per kg, respectively, so as to get a mixture worth ? 8.25 per kg?

(a) 4 : 3 *(b)* 7 : 3 *(c)* 8 : 3 *(d)* 2 : 5

2. In what proportion must a grocer mix wheat at ? 2.04 per kg and *X* 2.88 per kg so as to make a mixture of worth ? 2.52 per kg? [Bank Clerks 2008]

(a) 2 : 3 *(b)* 3 : 2 *(c)* 5 : 3 *(d)* 3 : 4 (ej None of the above

3. A milkman bought 15 L of milk and mixed 3 L of water in it. If the price per kg of the mixture becomes ? 22, what is cost price of the milk per litre? [CDS 2012]

(a) ? 28.00 *(b)* ? 26.40

(c) ? 24.00 *(d)* ? 22.60

4. A mixture of certain quantity of milk with 8 L of water is worth 45 paise per litre. If pure milk be worth 54 paise per litre, how much milk is there in the mixture? [Hotel Mgmt. 2010]

(a) 40 L *(b)* 35 L *(c)* 25 L *(d)* 45 L *(e)* None of the above

5. The ratio of milk and water mixture of four containers are 5 : 3, 2 : 1, 3 : 2 and 7 : 4, respectively. In which container, is the quantity of milk relative to water minimum?

(a) First *(b)* Second

(c) Third *(d)* Fourth

A merchant has 2000 kg of rice, one part of which he sells at 36% profit and the rest at 16% profit. He gains 28% on the whole. Find the quantity sold at 16%.

[Bank Clerks 2011]

(a) 400 kg *(b)* 300 kg

(c) 900 kg *(d)* 800 kg

(e) None of the above 7. A trader has 50 kg of pulses, part of which he sells at 8% profit and rest at 18% profit. He gains 14% on the whole. What is the quantity sold at 18% profit? [SSC CGL 2008]

(a) 30 kg *(b)* 35 kg

(c) 40 kg *(d)* 60 kg

8. **A person had ₹ 8400. He lent a part of it at 4% and the remaining at $3\frac{1}{3}$ % simple interest. His total annual income was ₹ 294. Find the sum he lent at 4%.**

(a) ₹ 2310 *(b)* ₹ 2110
(c) ₹ 2500 *(d)* ₹ 2100
(e) None of the above

9. A merchant had 50 kg of pulse. He sells one part at a profit of 10% and other at 5% loss. Overall he had a gain of 7%. Find the quantity of pulses, which he sold at 10% profit and 5% loss.

(a) 40 kg, 10 kg *(b)* 40 kg, 15 kg *(c)* 40 kg, 12 kg *(d)* 40 kg, 9 kg

10. A goldsmith has two qualities of gold, one of 24 carats and another of 32 carats purity. In what proportion should he mix both to make an ornament of 30 carats purity?

(a) 1 : 3 *(b)* 2 : 3

(c) 3 : 2 *(d)* 1 : 5

11. 300 g of salt solution has 40% salt in it. How much salt should be added to make it 50% in the solution?

(a) 40 g *(b)* 60 g *(c)* 70 g *(d)* 80 g

(e) None of the above 12.600g of sugar solution has 40% sugar in it. How much sugar should be added to make it 50% in the solution?

(a) 160 g *(b)* 120 g *(c)* 130 g*(d)* 140 g

(e) None of the above

13. A milk seller has a milk of ? 100 per litre. In what ratio should water be mixed in that milk, so that after selling the mixture at ? 80 per litre, he may get a profit of 50%?

(a) 7 : 8 *(b)* 7 : 9 *(c)* 9 : 7 *(d)* 7 : 5 *(e)* None of the above

14. How many kilograms of tea worth ? 25 per kg must be blended with 30 kg of tea worth ? 30 per kg, so that by selling the blended variety at ? 30 per kg, there should be a gain of 10%?

(a) 36 kg *(b)* 40 kg *(c)* 32 kgfdj 42 kg *(e)* None of the above

15. In two types of stainless steel, the ratio of chromium and steel are 2 : 11 and 5 : 21, respectively. In what proportion should the two types be mixed, so that the ratio of chromium to steel in the mixed type become 7 : 32?

[SSC (10+2) 2012]

(a) 1 : 2 *(b)* 1 : 3

(c) 2 : 3 *(d)* 3 : 4

16. A vessel is filled with milk and water. 70% of milk and 30% of water is taken out of the vessel. It is found that the vessel is vacated by 55% and has 160 L mixture. Find the quantity of milk and water in this mixture.

(a) Milk =100 L; Water = 60 L

(b) Milk = 50 L; Water = 110 L

(c) Milk = 70 L; Water = 90 L

(d) Milk = 60 L; Water = 100 L

(e) None of the above

Exercise © *Higher Skill Level Questions*

1. A butler stole wine from a butt of sherry which contained 80% of spirit and he replaced it by wine containing only 32% spirit. Then, the butt was of 48% strength only. How much of the butt did he steal? [UP Police 2007]

(a) $\frac{1}{4}$ (b) $\frac{3}{5}$

(c) $\frac{2}{5}$ (d) $\frac{2}{3}$

2. In a mixture of 60 L the ratio of acid and water is 2 : 1. If the ratio of acid and water is to be 1 : 2, then the amount of water (in litres) to be added to the mixture is [SSC CPO 2013]

(a) 55 *(b)* 60

(c) 50 *(d)* 45

3. Tea worth ? 126 per kg and ? 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth ? 153 per kg, the price of the third variety per kg will be [SSC (10+2) 2012]

(a) X 169.5 *(b) X* 170.0

(c) ? 175.5 *(d)* ? 180.0

4. A butler stole wine from a butt of sherry which contains 15% of spirit and he replaced what he had stolen by wine containing 6% of spirit. The butt was then 9% strong only. How much of the **hutt did he steal?**

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{2}{6}$ (d) $\frac{3}{5}$

(e) None of the above

5. 4 L are drawn from a container full of milk and then is filled with water. This operation is performed three more

times. The ratio of the quantity of milk left in the container and that of water is 16:65. How much milk did the container hold initially?

(a) 24 L *(b)* 12 L

(c) 15 L *(d)* 25 L

(e) None of the above 6. A container is filled with liquid, 6 part of which are water and 10 part milk. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half milk? [Hotel Mgmt. 2007]

(a) $\frac{1}{3}$ (b) $\frac{1}{7}$

(c) $\frac{1}{5}$ (d) $\frac{1}{8}$

(e) None of the above

7. A container contains a mixture of two liquids *A* and *B* in the ratio of 7 : 5. When 9 L of mixture is drawn off and the container is filled with *B,* the ratio of *A* and *B* becomes 7 : 9. How many litres of liquid *A* was contained by the container initially?

(a) 10 *(b)* 20

(c) 21 *(d)* 25

(e) None of the above

8. 60 kg of a certain variety of rice at ? 32 per kg is mixed with 48 kg of another variety of rice and the mixture is sold at the average price of ? 28 per kg. If there be no profit or loss due to the new sale price, then the price of the second variety of rice is [SSC Multitasking 2013]

(a) ? 25.60 per kg *(b)* ? 25 per kg *(c)* ? 23 per kg *(d)* *X* 30 per kg

Answer with Solutions

Exercise© *Base Level Questions*

1. (*b*) According to the rule of alligation,

Cheaper price (₹ 7.50)   Dearer price (₹ 10)

Mean price (₹ 8.25)

$(10 - 8.25) = 1.75$ $(8.25 - 7.50) = 0.75$

$\therefore$ Required ratio $= 1.75 : 0.75 = 7 : 3$

2. (*d*) According to the rule of alligation,

Cheaper price (₹ 2.04)   Dearer price (₹ 2.88)

Mean price (₹ 2.52)

$(2.88 - 2.52) =$ ₹ 0.36 $(2.52 - 2.04) =$ ₹ 0.48

$\therefore$ Required ratio $= 0.36 : 0.48 = 3 : 4$

3. (*b*) Let CP of milk be ₹ x per kg.

Then, according to the rule of alligation,

Price of milk ₹ x Price of water ₹ 0

Mean price ₹ 22

₹ 22   ₹ $(x - 22)$

$\therefore \quad 22 : (x - 22) = 15 : 3$

$\Rightarrow \quad \dfrac{22}{x - 22} = \dfrac{15}{3}$

$\Rightarrow \quad \dfrac{22}{x - 22} = 5$

$\Rightarrow \quad 22 = 5x - 110$

$\Rightarrow \quad 5x = 132$

$\therefore \quad x =$ ₹ 26.40

4. (*a*) As water is available free of cost, so its cost price = ₹ 0

According to the rule of alligation,

Water (0)   Milk (54 paise)

Mean price (45 paise)

$(54 - 45) = 9$ $(45 - 0) = 45$

Water : Milk $= 9 : 45 = 1 : 5$

$\therefore$ Quantity of milk $= 5 \times 8 = 40$ L

5. (*c*) Milk in first container $= \frac{5}{8} = 0.625$

Milk in second container $= \frac{2}{3} = 0.66$

Milk in third container $= \frac{3}{5} = 0.6$

Milk in fourth container $= \frac{7}{11} = 0.636$

So, it is clear that quantity of milk relative to water is minimum in third container.

6. (*d*) According to the rule of alligation,

Part I (16%) | Part II (36%)

Mean value 28%

$(36 - 28) = 8$ | $(28 - 16) = 12$

Part I (16%) : Part II (36%) = 8 : 12 = 2 : 3

$\therefore$ Quantity sold at 16% profit

$$= \frac{a}{a+b} \times \text{Total quantity}$$

$$= \frac{2}{5} \times 2000 = 800 \text{ kg}$$

7. (*a*) According to the rule of alligation,

Part I (8%) | Mean value 14% | Part II (18%)

$(18 - 14) = 4$ | $(14 - 8) = 6$

Part I (8%) : Part II (18%) = 4 : 6 = 2 : 3

$\therefore$ Quantity sold at 18% $= \frac{3}{5} \times 50 = 30$ kg

8. (*d*) SI on ₹ 8400 for 1 yr = ₹ 294

$\therefore$ Rate of interest $= \frac{100 \times 294}{8400 \times 1} = \frac{7}{2} = 3\frac{1}{2}\%$

According to the rule of alligation,

Rate I (4%) | Rate II (10/3%)

Mean price (7/2%)

$[7/2 - (10/3)] = 1/6$ | $(4 - 7/2) = 1/2$

Rate I (4%) : Rate II $\left(\frac{10}{3}\%\right) = \frac{1}{6} : \frac{1}{2} = 1 : 3$

$\therefore$ Money lent at 4% $= \frac{1}{4} \times 8400 = ₹\ 2100$

9. (*a*) According to the rule of alligation,

[− ve sign indicates loss]

Pulse sold at profit (10) | Mean value 7 | Pulse sold at loss (− 5)

$7 - (-5) = 12$ | $(10 - 7) = 3$

$\therefore$ Ratio of pulses sold at 10% profit and 5% loss = 12 : 3 = 4 : 1

$\therefore$ Quantity of pulse sold at 10% profit

$$= \frac{4}{4+1} \times 50$$

$$= \frac{4}{5} \times 50 = 40 \text{ kg}$$

and quantity of pulse sold at 5% loss

$$= \frac{1}{4+1} \times 50 = \frac{1}{5} \times 50 = 10 \text{ kg}$$

10. (*a*) According to the rule of alligation,

Part I (24) | 30 | Part II (32)

$(32 - 30) = 2$ | $(30 - 24) = 6$

$\therefore$ Required ratio = Part I : Part II

$= 2 : 6 = 1 : 3$

11. (*b*) 40% is salt in 300 g of salt solution.

Then, quantity of salt $= \frac{40 \times 300}{100} = 120$ g

Now, by the condition in the question,

$$\frac{120 + x}{300 + x} = \frac{50}{100}$$

$$\Rightarrow \quad \frac{120 + x}{300 + x} = \frac{1}{2}$$

$$\Rightarrow \quad 240 + 2x = 300 + x$$

$$\therefore \quad x = 60 \text{ g}$$

12. (*b*) The existing solution contains 40% sugar and here sugar has to be mixed. Therefore, the other solution has 100% sugar.

According to the rule of alligation,

Mixture I (40%) | 50% | Mixture II (100%)

$(100 - 50) = 50$ | $(50 - 40) = 10$

Required ratio = 50 : 10 = 5 : 1

The two mixtures should be added in the ratio of 5 : 1.

$\therefore$ Required sugar $= \dfrac{600}{5} \times 1 = 120$ g

Alternate Method

40% sugar is in 600 g of sugar solution.

$\therefore$ Quantity of sugar $= \dfrac{600 \times 40}{100} = 240$ g

Let x g sugar should be added.

According to the question,

$$240 + x = \frac{(600 + x) \times 50}{100}$$

$$\Rightarrow \quad \frac{240 + x}{600 + x} = \frac{1}{2}$$

$$\Rightarrow \quad 600 + x = 480 + 2x$$

$$\therefore \quad x = 120 \text{ g}$$

13. (*a*) Here, SP of milk = ₹ 80

and gain = 50%

$\therefore$ CP of milk $= \dfrac{100}{150} \times 80$

$$\left[\because \text{CP} = \frac{100}{(100 + \text{gain\%})} \times \text{SP}\right]$$

$= ₹ \dfrac{160}{3}$ per litre

According to the rule of alligation,

Water (₹ 0) | Milk (₹ 100)

Mean price (₹ 160/3)

[100 – (160/3)] = ₹ 140/3 | [(160/3) – 0)] = ₹ 160/3

$\therefore$ Water : Milk $= \dfrac{140}{3} : \dfrac{160}{3}$

$= 14 : 16 = 7 : 8$

14. (*a*) Let the quantity of tea worth ₹ 25 be x kg.

According to the question,

$$(25x + 30 \times 30) \times \frac{110}{100} = 30\,(30 + x)$$

$$\Rightarrow \quad (275x + 9900) = (9000 + 300x)$$

$$\Rightarrow \quad 300x - 275x = 900 \Rightarrow x = \frac{900}{25}$$

$$\therefore \quad x = 36 \text{ kg}$$

15. (*a*) Quantity of chromium in first kind of steel $= \dfrac{2}{13}$

Quantity of chromium in second kind of steel

$= \dfrac{5}{26}$

Quantity of chromium in both kind mixtures

$= \dfrac{7}{39}$

According to the rule of alligation,

Type I $\quad\quad\quad\quad$ Type II

$\frac{2}{13} \quad\quad \frac{5}{26}$

$\frac{7}{39}$

$\frac{5}{26} - \frac{7}{39} = \frac{1}{78} \quad\quad \frac{7}{39} - \frac{2}{13} = \frac{1}{39}$

$\therefore$ Requied ratio $= \frac{1}{78} : \frac{1}{39} = 1 : 2$

16. (a) Here, the percentage value of water and milk that is taken from the vessel should be taken into consideration.

Percentage of milk (30) $\quad\quad$ Percentage of water (70)

45

$(70 - 45) = 25 \quad\quad (45 - 30) = 15$

Milk : Water = 25 : 15 = 5 : 3

$\therefore$ Quantity of milk in the remaining mixture

$= \frac{5}{8} \times 160$

$= 100$ L

and quantity of water $= \frac{3}{8} \times 160$

$= 60$ L

Exercise © *Higher SkiU Level Questions*

1. (*d*) According to the rule of alligation,

32%   80%

48%

$(80 - 48) = 32$ $(48 - 32) = 16$

$\therefore$ Required ratio $= 32 : 16 = 2 : 1$

Clearly, $\frac{1}{3}$ of the butt of sherry was left and the butler stole $\frac{2}{3}$ of the butt.

2. (*b*) Quantity of acid in the mixture

$$= \frac{2}{3} \times 60 = 40 \text{ L}$$

Quantity of water in the mixture

$$= \frac{1}{3} \times 60 = 20 \text{ L}$$

Let required quantity of water be x L.

According to the question,

$$\frac{40}{20 + x} = \frac{1}{2} \Rightarrow 80 = 20 + x$$

$$\Rightarrow \qquad x = 60 \text{ L}$$

3. (*c*) Let the cost of third variety tea will be ₹ x per kg.

According to the question,

$$\frac{126 + 135 + 2x}{4} = 153$$

$$\Rightarrow \qquad 261 + 2x = 4 \times 153$$

$$\Rightarrow \qquad 2x = 612 - 261 = 351$$

$$\therefore \qquad x = \frac{351}{2} = ₹\ 175.50$$

$$\Rightarrow \qquad x = ₹\ 175.5$$

Hence, the cost of third variety tea is ₹ 175.50 per kg.

4. (*a*) According to the rule of alligation,

$$\begin{array}{ccc} 15\% & & 6\% \\ & 9\% & \\ (9-6)=3 & & (15-9)=6 \end{array}$$

$\therefore$ Required ratio $= 3 : 6 = 1 : 2$

Clearly, $\frac{1}{3}$ of the butt of sherry was left. It means the butler stole $\frac{2}{3}$ of the butt.

5. (*b*) According to the formula,

After n operations, quantity of pure liquid

$$= \left[a\left(1-\frac{b}{a}\right)^n\right] \text{ units} \qquad \text{[by formula 1]}$$

Here, $\quad b = 4,\ n = 4,\ a = ?$

Quantity of milk left in the container after four operations $= \left[a\left(1-\frac{4}{a}\right)^4\right]$ L

Then, according to the question,

$$\frac{a\left(1-\frac{4}{a}\right)^4}{a-a\left(1-\frac{4}{a}\right)^4} = \frac{16}{65} \Rightarrow \frac{\left(1-\frac{4}{a}\right)^4}{1-\left(1-\frac{4}{a}\right)^4} = \frac{16}{65}$$

$$\Rightarrow \qquad \left(1-\frac{4}{a}\right)^4 = \frac{16}{65} - \frac{16}{65}\left(1-\frac{4}{a}\right)^4$$

$$\Rightarrow \quad \frac{81}{65}\left(1-\frac{4}{a}\right)^4 = \frac{16}{65} \Rightarrow \left(1-\frac{4}{a}\right)^4 = \frac{16}{81}$$

$$\Rightarrow \qquad 1-\frac{4}{a} = \frac{2}{3} \Rightarrow \frac{4}{a} = \frac{1}{3}$$

$$\therefore \qquad a = 12 \text{ L}$$

6. (*c*) Let the container initially contains 16 L of liquid.

Let a L of liquid be compressing water.

Quantity of water in the new mixture

$$= \left(6 - \frac{6a}{16} + a\right) \text{L}$$

Quantity of milk in the new mixture

$$= \left(10 - \frac{10a}{16}\right) \text{L}$$

According to the question,

$$6 - \frac{6a}{16} + a = 10 - \frac{10a}{16}$$

$$\Rightarrow \quad 96 - 6a + 16a = 160 - 10a$$

$$\Rightarrow \quad 96 + 10a = 160 - 10a$$

$$\Rightarrow \quad 20a = 64$$

$$\Rightarrow \quad a = \frac{64}{20} = \frac{16}{5}$$

$\therefore$ Part of mixture replaced $= \frac{1}{16} \times \frac{16}{5} = \frac{1}{5}$

7. (*c*) Suppose the container initially contains $7x$ and $5x$ L of mixtures A and B, respectively.

Quantity of A in mixture left

$$= \left(7x - \frac{7}{12} \times 9\right) = \left(7x - \frac{21}{4}\right) \text{L}$$

Quantity of B in mixture left

$$= \left(5x - \frac{5}{12} \times 9\right)$$

$$= \left(5x - \frac{15}{4}\right) \text{L}$$

According to the question,

$$\frac{7x - \frac{21}{4}}{\left(5x - \frac{15}{4}\right) + 9} = \frac{7}{9}$$

$$\Rightarrow \quad \frac{28x - 21}{20x + 21} = \frac{7}{9}$$

$$\Rightarrow \quad 252x - 189 = 140x + 147$$

$$\Rightarrow \quad 112x = 336$$

$$\Rightarrow \quad x = 3$$

$\therefore$ Container contained $= 7 \times 3 = 21$ L of A

8. (*c*) Let price of the second variety of rice be ₹ x per kg.

Then, total cost of first variety of rice

$$= 60 \times 32 = ₹\,1920$$

and total cost of second variety of rice

$$= 48 \times x = ₹\,48x$$

$\therefore$ Total SP of both varieties of rice

$$= (60 + 48) \times 28$$

$$= 108 \times 28$$

$$= ₹\,3024$$

Since, there is neither profit nor loss.

Therefore,

$$1920 + 48x = 3024$$

$$48x = 3024 - 1920$$

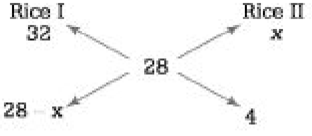
$$\Rightarrow \quad 48x = 1104$$

$$\therefore \quad x = 23$$

Thus, price of second variety of rice is ₹ 23 per kg.

Alternate Method

Let price of second variety of rice be ₹ x per kg.



According to the question,

$$\frac{28 - x}{4} = \frac{60}{48}$$

$$\Rightarrow \quad x = ₹\,23$$

Chapter

19

Partnership

When two or more persons make an association and invest money for running a certain business and after certain time receive profit in the ratio of their invested money and time period of investment, then such an association is called **partnership** and the persons involved in the partnership are called **partners.**

Partnership is of Two Types

Simple Partnership

If all partners invest their different capitals (money) for the same time period or same capital for different time period then their profit or loss is in the ratio of their investments or time period of investment then such a partnership is called simple partnership.

Compound Partnership

If all partners invest their different capitals (money) for different time period, then their profit not only depends on their investments but also on the time period of their investment, then such a partnership is called compound partnership.

Partners are of Two Types

Active or Working Partner

A partner who not only invests money, but also take part in the business activities for which he draws a defined salary or gets some share from profit before its division is called an active partner.

Sleeping Partner

A partner who only invests money and does not take part in business activities is called sleeping partner.

***Case* I Ratio of Profit/Loss in Case of Simple Partnership**

♦ When the investments made by all the partners are for the same time period, then gain or loss is distributed amongst them in the ratio of their investments,

♦ When the amount of capital invested by different partners is same (say ? x) for different time periods, tj, t_2, (3,..., then

Ratio of profit/loss = Ratio of time period for which the capital is invested $P_1 : P_2 : P_3$ '▪▪▪▪ =k'-h -h '▪▪▪▪

Ex. 1 *A* and *B* start a business by investing ? 4000 and ? 12000, respectively. Find the ratio of their profits after 1 yr.

Sol. The gain will be distributed amongst *A* and *B* in the ratio of their investments

$$\frac{\text{Investment of } A}{\text{Investment of } B} = \frac{\text{Profit of } A}{\text{Profit of } B}$$

Hence, ratio of their profits $= \dfrac{4000}{12000} = \dfrac{1}{3} \Rightarrow 1:3$

Ex. 2 *A* and *B* jointly start a business. The investment of *A* is equal to three times the investment of *B*. Find the share of *A* in the annual profit of ? 52000.

Sol. Let the share of B = ₹ x

Then, the share of A = ₹ $3x$

$$\frac{\text{Profit of } A}{\text{Profit of } B} = \frac{\text{Investment of } A}{\text{Investment of } B}$$

Hence, ratio of their profit $= \dfrac{3x}{x} = \dfrac{3}{1} = 3:1$

According to the question, $3x + x = 52000 \Rightarrow 4x = 52000$

$\therefore \quad x = 13000$

Clearly, A's share $= 3x = 3 \times 13000 =$ ₹ 39000

Ex. 3 A and B invested ? 24000 and ? 8000 for a period of 2 yr. After 2 yr, they earned ? 48000. What will be the shares of A and B out of this earning?

Sol. A's share : B's share = A's investment : B's investment = 24000 : 8000 = 3 : 1

Now, let A's share = $3x$ and B's share = x

According to the question, $3x + x = 48000 \Rightarrow 4x = 48000$

$\therefore$ $x = 12000$

Clearly, share of $B = x$ = ₹ 12000 and share of $A = 3x = 3 \times 12000$ = ₹ 36000

Direct Approach

A's share : B's share = A's investment : B's investment = 24000 : 8000 = 3 : 1

Now, A's share $= \frac{3}{3+1} \times 48000 = 3 \times 12000 =$ ₹ 36000

and B's share $= \frac{1}{3+1} \times 48000 = 1 \times 12000 =$ ₹ 12000

Ex. 4 A,B and C start a business with investment of ? 50000 each. A remains in partnership for 9 months, B for 6 months and C for 12 months. Then, find the ratio of their profits.

Sol. Ratio of profit of A, B and C will be in the ratio of time period of investment So, *As* profit : B's profit: C's profit =9:6:12=3:2:4

***Case* II Ratio of Profit/Loss in Case of Compound Partnership**

When capital invested by the partners is given as XlrX2,X3,... for different time period tj, f2 <*3--■- in a business, then

Ratio of their profits *P^ : fy* : P3 :... = Amount of capital invested x Time period for which the capital is invested = X^ : X2*2 : X3f3 :...

Note To find the amount of rent paid by the person for using the piece of land or any other property for different time periods. When the total amount of rent is given, then the concept of compound partnership is used

Ex. 5 A starts a business with ? 2000 and B joins him after 3 months with ? 8000. Find the ratio of their profits at the end of the year.

Sol. Here, A invested for 1 yr (12 months) and B invested 3 months later. It does mean B invested for (12-3) months.

Ratio of profit A and B = Capital invested x Time period of investment

So, *A's* share : *B's* share = 2000 x 12 : 8000 x (12 - 3)

= 2x12 : 8x9=1:3

Ex. 6 *A* starts a business with ? 4000 and *B* joins the business 4 months later with an investment of ? 5000. After 1 yr, they earn a profit of ? 22000. Find the share of *A* and *B*.

Sol. *A*'s share : *B*'s share

$= 4000 \times 12 : 5000 \times (12 - 4) = 4 \times 12 : 5 \times 8 = 6 : 5$

Now, let the share of $A = 6x$, and the share of $B = 5x$

According to the question, $6x + 5x = 22000 \Rightarrow 11x = 22000$

$\therefore \quad x = ₹\ 2000$

$\therefore$ Share of $A = 6x = 6 \times 2000 = ₹\ 12000$,

and share of $B = 5x = 5 \times 2000 = ₹\ 10000$

Direct Approach

A's share : *B*'s share $= 4000 \times 12 : 5000 \times 8 = 6 : 5$

Now, *A*'s share $= \dfrac{6}{6+5} \times 22000 = ₹\ 12000$

and *B*'s share $= \dfrac{5}{6+5} \times 22000 = ₹\ 10000.$

Ex. 7 *A* starts a business with ? 2000. *B* joins him. after 3 months with *X* 4000. C

invests a sum of ? 10000 in the business for 2 months only. At the end of year, the business gave a profit of ? 112000. How should the profit be divided amongst *A, B* and *C?*

Sol. Ratio of profit of *A* and *B* = Capital invested x Time period of investment

A's share : *B's* share : Cs share = 2000 X 12 : 4000 X (12 - 3) : 10000 X 2

= 2x12:4x9:10x2=6:9:5 Now, let *A's* share = *6x, B's* share = 9s; Cs share = *5x* According to the question,

6x + *9x* + *5x* = 112000 =>20x = 112000 *x* = *X* 5600 Now, *A's* share = *6x* = 6 x 5600 = ? 33600;

B's share = ftc = 9 x 5600 = ? 50400; *Cs* share = *5x* = 5 x 5600 = ? 28000

Fast Track **Techniques**

to solve the **QUESTIONS**

Technique 1

If $x_1 : x_2 : x_3 : \ldots$ is the ratio of investments and $P_1 : P_2 : P_3 : \ldots$ is the ratio of profits, then ratio of time periods of investment is given by $\frac{P_1}{x_1} : \frac{P_2}{x_2} : \frac{P_3}{x_3} : \ldots$

Ex. 8 *A, B* and C invested capitals in the ratio of 2 : 3 : 5. At the end of the business terms, they received the profit in the ratio of 5 : 3 : 12. Find the ratio of time for which they contributed their capitals,

Sol. Let required ratio of time be \ *: t% '■ %■*

Then, ratio of investments = Ratio of profits Ratio of product of time and investment = Ratio of profit 2*l : 3i2 : 5i3 = 5 : 3 : 12

Taking first two terms of the ratio,

$$\frac{2t_1}{3t_2} = \frac{5}{3} \Rightarrow \frac{t_1}{t_2} = \frac{5}{2} = \frac{25}{10} \Rightarrow t_1 : t_2 = 25 : 10 \quad \ldots(i)$$

Taking last two terms of the ratio,

$$\frac{3t_2}{5t_3} = \frac{3}{12} \Rightarrow \frac{t_2}{t_3} = \frac{5}{12} = \frac{10}{24}$$

$$\Rightarrow \quad t_2 : t_3 = 10 : 24 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$t_1 : t_2 : t_3 = 25 : 10 : 24$$

which is the required ratio.

Fast Track Method

Here, $P_1 : P_2 : P_3 = 5 : 3 : 12$ and $x_1 : x_2 : x_3 = 2 : 3 : 5$

$$\text{Required ratio} = \frac{P_1}{x_1} : \frac{P_2}{x_2} : \frac{P_3}{x_3} = \frac{5}{2} : \frac{3}{3} : \frac{12}{5} = \frac{5}{2} : 1 : \frac{12}{5}$$

$$= \frac{5 \times 10}{2} : 1 \times 10 : \frac{12 \times 10}{5} = 25 : 10 : 24$$

Technique 2

If $P_1 : P_2 : P_3 : \ldots$ is the ratio of profits and $t_1 : t_2 : t_3 : \ldots$ is the ratio of time periods, then ratio of investments is given by $\frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3} : \ldots$

Ex.9 *A, B* and C each does certain investments for time periods in the ratio of 5 : 6 : 8. At the end of the business terms, they received the profits in the ratio of 5 : 3 :12. Find the ratio of investments of *A, B* and C.

Sol. Here, $t_1 : t_2 : t_3 = 5 : 6 : 8$ and $P_1 : P_2 : P_3 = 5 : 3 : 12$

$$\text{Required ratio} = \frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3} = \frac{5}{5} : \frac{3}{6} : \frac{12}{8} = 1 : \frac{1}{2} : \frac{3}{2} = 2 : 1 : 3$$

Multi Concept

QUESTIONS

1. *A started a business with ? 52000 and after 4 months B joined him with ? 39000. At the end of the year, out of the total profit B recieved total ? 20000 including 25% of the profits as commission for managing the business. What amount did A receive?*

(a) 20500 (b) 21000 (c) 20000 (d) 30000

➥ (c) Ratio of profit of *A* and *B* = Amount of capital invested × Time period of investment

$= 52000 \times 12 : 39000 \times 8 = 2 : 1$

Let the total profit = ₹ x

B receives 25% as commission for managing business. Then, the remaining 75% of the total profit *x* is shared between *A* and *B* in the ratio 2 : 1.

Hence, *B* will get $\frac{1}{3}$rd part of 75% in addition to his commission.

Hence, *B*'s total earning = 25% of total profit + $\frac{1}{3}$ × 75% of total profit

$= 0.25x + \frac{1}{3} \times (075x)$

$\Rightarrow \quad 20000 = 0\,25x + 0.25x$

$\Rightarrow \quad 0\,5x = 20000$

$\therefore \quad x = 40000$

So, the remaining amount of profit goes to *A*.

Hence, *A*'s total earning = Total profit − *B*'s total earning = 40000 − 20000 = ₹ 20000

$\therefore$ *A*'s profit = ₹ 20000

2. *A, B and C started a business with their investments in the ratio 1 : 2 : 4, after 6 months, A invested the half amount more as before and B invested twice the amount more as before, while C withdraws $\frac{1}{4}$th of their investments. Find the ratio of their profits at the end of the year.*

(a) 5 : 10 : 15 (b) 15 : 20 : 25 (c) 5 : 16 : 20 (d) 5 : 16 : 14

(*d*) Let us assume their initial investments be x, $2x$ and $4x$, respectively.

According to the question,

$\therefore$ Investment of *A* for 1yr = x for 6 months + $\left(x + \frac{x}{2}\right)$ for 6 months

$$= 6x + \frac{3x}{2} \times 6 = 6x + 9x \Rightarrow 15x$$

Investment of *B* for 1yr = $2x$ for 6 months + $(2x + 4x)$ for 6 months

$$= 12x + 36x = 48x$$

Investment of *C* for 1yr = $4x$ for 6 months + $\left(4x - \frac{4x}{4}\right)$ for 6 months

$$= 24x + (3x \times 6)$$
$$= 24x + 18x = 42x$$

$\therefore$ $A : B : C = 15x : 48x : 42x = 15 : 48 : 42$

$$= 5 : 16 : 14$$

$\therefore$ Ratio of their profits = 5 : 16 : 14

Fast Track Practice

Exercise© *Base Level Questions*

1. Ravi and Kavi start a business by investing *X* 8000 and *X* 72000, respectively. Find the ratio of their profits at the end of year.

(a) 2 : 9 *(b)* 5 : 9

(c) 7 : 9 *(d)* 1 : 9

(e) None of the above

2. *P* and *Q* entered a partnership for 3 yr. At the start of the business, they invested *X* 13000 and *X* 25000, respectively. At the end of 3 yr their total profit was *X* 76000. What will be share of *Q* out of this profit?

(a) X 50000 *(b) X* 26000

(c) ?55000 *(d)* ?21000

(e) None of the above

3. Srikant and Vividh started a business investing amounts of X 185000 and X 225000, respectively. If Vividh's share in the profit earned by them is X 9000, what is the total profit earned by them together? [Bank Clerks 2009]

(a) ? 17400 (b) ? 16400

(C) ? 16800 (d) ?17800

(e) None of the above

4. Raj an and Sajan started a business initially with ? 14200 and ? 15600, respectively. If total profits at the end of year is X 74500, what is the Rajan's share in the profit? [Bank Clerks 2009]

(a) X 39000 (b) X 39600

(c) X 35000 (d) X 35500

(e) None of the above

5. *A* starts a business with X 4000 and *B* joins him after 3 months with X 16000. Find the ratio of their profits at the end of year.

(a) 1 : 3 (b) 2 : 3 (cj 1 : 9 (c0 1 : 7 (e) None of the above

6. *A* starts a business with X 9000 and *B* joins him after 6 months with an investment of X 45000. What will be the ratio of the profits of *A* and *B* at the end of year? [Hotel Mgmt. 2008]

(a) 1 : 5 (b) 5:2

(c) 2 : 5 (d) 5 : 1

(e) None of the above

7. *X* starts a business with X 300000 and *Y* joins him after 3 months with X 1200000. Find the ratio of their profits at the end of the year. [Bank Clerks 2011]

(a) 2 : 5 (b) 3 : 5 (c) 5 : 1 (d) 1 : 3 (e) None of the above

8. Ajay started a business investing X 25000. After 3 months, Vijay joined him with a capital of X 30000. At the end of the year, they made a profit of X 38000. What will be the Ajay's share in the profit?

(a) ? 10000 *(b)* ? 18000

(c) ? 15000 *(d)* ? 20000

9. Ram, Shyam and Kamal together start a business in partnership. The ratio of their capitals is 3 : 4 : 7. If their annual profit be ? 21000, what will be Kamal's share in this profit? [RRB 2007]

(a) ? 12500 *(b)* ? 10500

(c) ? 15000 *(d)* ? 10000

lO.Rakesh, Dinesh and Mahesh start a business by investing X 5000, X 8000 and X 12000, respectively. At the end of year, they got the profit of X 12500. What is the share of Dinesh in profit? [RRB CCL 2012] *(a)* X 4000 *(b)* X 4500

(C) X 6000 *(d)* X 7500

11. A starts a business with X 4000. *B* joins him after 3 months with X 8000. C puts a sum of X 20000 in the business for 2 months only. At the end of year, the business gives a profit of X 16800. How should the profit be divided amongst them?

(a) *A* = ? 5040 S = ? 7560, C = ? 4200

(b) $A = X$ 7560, $B =$? 5040, C = ? 4200 *(c)*$A = X$ 4200, fi = ? 7560, $C = X$ 5040 fd; A = ? 4200, e = X 5040, C = X 7560

12.A and *B* invest in the ratio of 3:5, respectively. After 6 months, C enters the business with the investment of the capital equal to that of *B*. What will be the ratio of the profits of *A, B* and C at the end of year?

[SSC LDC 2008] *(a)* 6 : 10 : 5 *(b)* 3 : 5 : 5

(c) 3 : 5 : 2 *(d)* 6 : 2 : 3

13.Reena, Reema and Meera take a car on rent and they use that car for 14 h, 16 h and 22 h, respectively. What amount will be paid by Meera, if total rent of the car is X 2080? [RRB 2006]

(a) ?880 *(b)* ?1280

(c) ?1040 (d) ?960

14. Pinku, Rinku and Tinku divide an amount of X 4200 amongst themselves in the ratio of 7 : 8 : 6, respectively. If an amount of X 200 is added to each of their shares, what will be the new respective ratio of their shares of amount? [RRB 2006]

(a) 8 : 9 : 5 (b) 7 : 9 : 5

(c) 7 : 8 : 6 (d) 8 : 9 : 7

15. *P, Q* and *R* start a business jointly. Twice the capital of *P* is equal to thrice the capital of *Q* and the capital of *Q* is four times the capital of *R*. Find the share of *Q* in an annual profit of *X* 148500.

(a) X 54000 (b) X 64000

(C) ? 56000 (d) X 55000

(e) None of the above

16. *P, Q* and *R* are three partners in a business. If *Ps* capital is equal to twice the capital of *Q* and *Qs* capital is three times the capital of *R,* find the ratio of the capitals of *P, Q* and *R.*

(a) 6 : 3 : 1 (b) 1 : 3 : 6

(c) 2 : 3 : 6 (d) 3 : 6 : 1

(e) None of the above

17. *P, Q* and *R* start a business. *P* invests

3 times as much as *Q* invests and

2 0 invests —rd as much as *R* invests. Find

3 the ratio of capitals of *P, Q* and *R. (a)* 3 : 2 : 6 *(b)* 2 : 6 : 3

(c) 6 : 2 : 3 *(d)* 5 : 2 : 3

18.Ramesh and Priya started a business initially with *X* 5100 and *X* 6600, respectively. Investments done by both the persons are for different time periods. If the total profit is *X* 5460, what is the profit of Ramesh? [Bank PO 2010] *(a) X* 1530 *(b) X* 1600

(c) X 1400 *(d)* Data inadequate

(e) None of the above 19. *M, N* and *P* invest *X* 50000 for a business. *M* invests *X* 4000 more than *N* and *N* invests *X* 5000 more than *P*. Out of the total profit of *X* 70000, what is the share received by *MP*. *(a) X* 29400 *(b) X* 30000

(c) X 35000 *(d) X* 40000

(e) None of the above

20. *P, Q* and *R* hire a meadow for *X* 2920. Pputs 10 cows for 20 days, *Q* puts 30 cows for 8 days and *R* putsl6 cows for 9 days. Find the rent paid by *R*.

(a) X 735 *(b) X* 820

(c) X 575 *(d) X* 720

(e) None of the above

21. Four milkmen rented a pasture. A grazed 18 cows for 4 months, *B* grazed 25 cows for 2 months, *C* grazed 28 cows for

5 months and *D* grazed 21 cows for 3 months. If A's share of rent is *X* 720, what will be the total rent of the pasture?

(a) X 1650 *(b) X* 3250

(C) X 3500 *(d) X* 3000

22. A, B and C enter into a partnership. *A* invests some amount at the beginning, *B* invests double the amount of *A* after

6 months and C invests thrice the amount of *A* after 8 months. If the annual profit be *X* 54000, find the Cs share.

(a) X 3000 *(b) X* 18000

(c) X 15000 (d) X 21000

(e) None of the above

23. A, B and C together start a business. B invests 1/6 of the total capital while investments of A and C are equal. If the annual profit on this investment is X 33600, find the difference between the profits of B and C. [SSC CGL 2007]

(a) X 8400 (b) X 7200

(c) X 6000 (d) X 9600

24. A and B invest in a business in the ratio of 3 : 2. If 5% of the total profit goes to a charity and A's share is X 4275, then what will be the total profit?

(a) X 8000 (b) X 7700

(c) X 7500 (d) X 6500

25. A and B together start a business by investing in the ratio of 4 : 3. If 9% of the total profit goes to charity and A's share is X 1196, find the total profit.

[LIC ADO 2008] (a) X 2300 (b) X 4435

(c) X 2093 (d) X 2700

(e) None of the above

26. A, B and C invested capitals in the ratio of 4 : 6 : 9. At the end of the business term, they received the profit in the ratio of 2 : 3 : 5. Find the ratio of their time for which they contributed their capitals.

[Delhi Police (SI) 2008]

(a) 1 : 1 : 9

(b) 2 : 2 : 9

(c) 10 : 10 : 9

(d) 9 : 9 : 10

27. A, *B* and C do certain investments for time periods in the ratio of 2:1: 8. At the end of the business term, they received the profits in the ratio of 3 : 4 : 2. Find the ratio of investments of *A, B* and C.

(a) 6 : 16 : 1 *(b)* 2 : 5 : 1

fc; 6:17:1 *(d)* 6 : 19 : 3

(e) None of the above

28. A, *B* and C invested their capitals in the ratio of 5:6:8. At the end of the business, they received the profits in the ratio of 5 : 3 : 1. Find the ratio of time for which they contributed their capitals.

[Bank Clerks 2007]

(a) 12 : 9 : 7

(b) 25 : 18 : 8

(c) 5 : 6 : 8

(d) 8 : 4 : 1

(e) None of the above

29.Sonu invested 10% more than the investment of Mona and Mona invested 10% less than the investment of Raghu. If the total investment of all the three persons is *X* 5780, find the investment of Raghu. [Bank PO 2010]

(a) X 2010 *(b) X* 2000

(c) X 2100 *(d) X* 2210

(e) None of the above

Exercise © *Higher SkiU Level Questions*

1. *A* and *B* started a business by investing *X* 20000 and *X* 25000, respectively. After

4 months *B* left and C joined by investing *X* 15000. At the end of the year, there was a profit of *X* 23000. What is Cs share?

(a) X 8000 (b) X 9000

(c) X 6000 (d) X 12000

(e) None of the above

2. A, B and C entered into partnershp in a business. A got 3/5 of the profit. B and C distributed the remaining profit equally. If C got X 400 less than A, the total profit was [SSCCPO2013]

(a) X 1600 (b) X 1200

(c) X 1000 (d) X 800

3. A and B started a business by investing X 35000 and X 20000, respectively. After

5 months B left the business and C joined the business with a sum of X 15000. The profit earned at the end of year is X 84125. What is the share of B in profit? [Bank Clerks 2011]

(a) X 14133 (b) X 15000

(c) X 13460

(d) Couldn't be determined

(e) None of the above

4. X and Y make a partnership. X invests X 8000 for 8 months and Y remains in the business for 4 months. Out of the total profit, Y claims 2/7 of the profit. How much money was contributed by Y?

(a) X 5000 (b) X 5400

(c) X 7400 (d) X 6400

(e) None of the above

5. A and B invest X 3000 and X 4000, respectively in a business. A receives X 10 per month out of the profit as a remuneration for running the business

and the rest of the profit is divided in proportion to the investments. If in a year A totally receives X 390, what does B receive? [SSC CPO 2008]

(a) X 630 *(b)* X 360

(c) X 480 *(d)* X 380

6. Avinash, Manoj and Arun started a business in partnership investing in the ratio of 3 : 2 : 5 respectively. At the end of the year, they earned a profit of X 45000 which is 15% of their total investment. How much did Manoj invest?

(a) X 60000 *(b)* X 180000

(c) X 30000 *(d)* X 90000

(e) None of the above

7. *A* began a business with X 2250 and was joined afterwards by *B* with X 2700. If the profits at the end of the year were divided in the ratio of 2: X after how much time *B* joined the business?

(a) 5 months *(b)* 6 months

(c) 3 months *(d)* 7 months

(e) None of the above

8. Amit and Brijesh started a business with initial investments in the ratio of 12 : 11 and their annual profits were in the ratio of 4 : 1. If Amit invested the money for 11 months, then for what time Brijesh invested the money?

(a) 9 months *(b)* 3 months

(c) 5 months *(d)* 10 months

(e) None of the above

9. Aarti, Vinita and Kamla became partners in a business by investing money in the ratio of 5: 7: 6. Next year, they increased their investments by 26%, 20% and 15%, respectively. In what ratio should profit earned during 2nd year be distributed?

(a) 21 : 28 : 23 *(b)* 23 : 28 : 21 (cj 28 : 23 : 21 *(d)* 35 : 41 : 7 (e) None of the above 10. Anil is an active and Vimal is a sleeping partner in a business. Anil invests X 12000 and Vimal invests X 20000. Anil receives

10% profit for managing, the rest being divided in proportion to their capitals. Out of the total profit of *X* 9000, the money received by Anil is

[Hotel Mgmt. 2010] *(a)* *X* 4500 *(b)* *X* 4800

(c) *X* 4600 *(d)* *X* 3937.5

(e) None of the above Aayush and Babloo are partners in a business. Aayush contributes 1/4 of the capital for 15 months and Babloo received 2/3 of the profit. How long Babloo's money was used? *(a)* 10 months *(b)* 9 months *(c)* 11 months *(d)* 7 months *(e)* None of the above

12. Amitabh, Brijesh and Kamlesh enter a partnership with shares in the ratio of

7 4 6

— :—:—. After 4 months, Amitabh

2 3 5

increases his share by 50%. If the total

profit at the end of the year be *X* 21600,

what will be the share of Brijesh in the

profit? [SSC (10+2)2013]

(a) *X* 8000 *(b)* *X* 12000

(c) *X* 4000 *(d)* *X* 7000

13. *A* and *B* started a joint business. *A's* investment was thrice the investment of *B* and the period of his investment was twice the period of investment of *B*. If *B*

got *X* 6000 as profit, then what will be the 20% of total profit? [Hotel Mgmt. 2007] *(a)* *X* 5000 *(b)* *X* 8400

(c) *X* 3500 *(d)* *X* 4500

(e) None of the above

14. *A* and *B* started a business with *X* 20000 and *X* 35000 respectively. They agreed to share the profit in the ratio of their capital. C joins the partnership with the condition that *A*,*B* and *C* will share profit equally and pays *X* 220000 as premium for this, to be shared between *A* and *B*. This is to be divided between *A* and *B* in the ratio of [SSC CCL 2013]

(a) 10 : 1 *(b)* 1 : 10

(c) 9 : 10 *(d)* 10 : 9

15. Two partners invest *X*125000 and *X* 85000, respectively in a business and agree that 60% of the profit should be divided equally between them and the remaining profit is to be treated as interest on capital. If one partner gets *X* 600 more than the other, find the total profit made in the business.

(a) *X* 8800 *(b)* *X* 8885

(c) *X* 8995 *(d)* *X* 7875

(e) None of the above

16. *A* and *B* entered a partnership investing *X* 16000 and *X* 12000, respectively. After 3 months *A* takes out *X* 5000, while *B* puts in *X* 5000 more. After 3 months more, C joins the business with a capital of *X* 21000. After a year, they earned a profit of *X* 13200. By what value does the share of *B* exceeds the share of C?

(a) *X* 1600 *(b)* *X* 1800

(c) *X* 2100 *(d)* *X* 2300

Answer with Solutions

Exercise© *Base Level Questions*

1. (*d*) Ratio of profits = Ratio of investments

$= 8000 : 72000 = 1 : 9$

2. (*a*) P's share : Q's share

= Ratio of their investments

$= 13000 : 25000 = 13 : 25$

Now, let P's share $= 13x$

Q's share $= 25x$

According to the question,

$$13x + 25x = 76000$$

$$\Rightarrow \quad x = \frac{76000}{38} = 2000$$

$\therefore$ Q's share $= 25x = 25 \times 2000 =$ ₹ 50000

Direct Approach

P's share : Q's share

= Ratio of their investments

$= 13000 : 25000 = 13 : 25$

$$\therefore Q\text{'s share} = \frac{25}{13+25} \times 76000$$

$$= \frac{25}{38} \times 76000 = ₹\ 50000$$

3. (*b*) As we know,

Ratio of profits = Ratio of investments

$\therefore$ Srikant's share : Vividh's share

$= 185000 : 225000 = 37 : 45$

Let Srikant's share $= 37x$

Vividh's share $= 45x$

According to the question,

$$45x = 9000$$

$$\therefore \quad x = \frac{9000}{45} = 200$$

$\therefore$ Total profit $= 37x + 45x = 82x$

$= 82 \times 200 =$ ₹ 16400

4. (*d*) Ratio of profits = Ratio of investments

$\therefore$ Rajan's share : Sajan's share

$= 14200 : 15600 = 142 : 156 = 71 : 78$

Let Rajan's share $= 71x$

and Sajan's share $= 78x$

According to the question,

$$71x + 78x = 74500$$

$$\Rightarrow \quad 149x = 74500$$

$$\therefore \quad x = \frac{74500}{149} = 500$$

$\therefore$ Rajan's share $= 71x = 71 \times 500 =$ ₹ 35500

Direct Approach

Ratio of profits of Rajan and Sajan

= Ratio of their investments

$= 14200 : 15600$
$= 142 : 156 = 71 : 78$

$\therefore$ Rajan's share $= \dfrac{71}{71 + 78} \times 74500$

$= \dfrac{71}{149} \times 74500 = ₹\ 35500$

5. (*a*) *A*'s share : *B*'s share

= Ratio of product of investment and time period of investment

$= 4000 \times 12 : 16000 \times (12 - 3)$
$= 4 \times 12 : 16 \times 9 = 1 : 3$

Note Here, the partnership is compound, in which *A* invested for 1 yr or 12 month and *B* invested 3 months later. It does mean *B* invested for (12 − 3) months.

6. (*c*) *A*'s share : *B*'s share

$= 9000 \times 12 : 45000 \times (12 - 6)$
$= 12 : 5 \times 6 = 2 : 5$

7. (*d*) *X*'s share : *Y*'s share

$= (300000 \times 12) : (1200000 \times 9)$
$= 36 : 108 = 1 : 3$

8. (*d*) Ajay's share : Vijay's share

$= 25000 \times 12 : 30000 \times (12 - 3)$
$= 25 \times 12 : 30 \times 9 = 10 : 9$

Ajay's share $= \dfrac{10}{10 + 9} \times 38000$

$= \dfrac{10}{19} \times 38000 = 10 \times 2000 = ₹\ 20000$

9. (*b*) Let Ram's share $= 3x$

Shyam's share $= 4x$
Kamal's share $= 7x$

According to the question,

$3x + 4x + 7x = 21000$

$\Rightarrow \quad 14x = 21000$

$\therefore \quad x = \dfrac{21000}{14} = 1500$

$\therefore$ Kamal's share $= 7x = 7 \times 1500$
$= ₹\ 10500$

Direct Approach

Given ratio of profits of Ram, Shyam and Kamal = Ratio of their capitals

$= 3 : 4 : 7$

Kamal's share $= \dfrac{7}{3 + 4 + 7} \times 21000$

$= 7 \times 1500 = ₹\ 10500$

10. (*a*) Share of Rakesh : Share of Dinesh : Share of Mahesh

$= 5000 : 8000 : 12000 = 5 : 8 : 12$

Total earned profit = ₹ 12500

$\therefore$ Share of Dinesh in profit

$$= \frac{8}{5+8+12} \times 12500 = \frac{8}{25} \times 12500$$

$= 8 \times 500 = ₹\ 4000$

11. (*a*) *A*'s share : *B*'s share : *C*'s share

= Ratio of product of investment and time period of investment

$= 4000 \times 12 : 8000 \times (12 - 3) : 20000 \times 2$

$= 4 \times 12 : 8 \times 9 : 20 \times 2 = 6 : 9 : 5$

Let A's share $= 6x$

B's share $= 9x$

C's share $= 5x$

According to the question,

$6x + 9x + 5x = 16800$

$\Rightarrow \quad 20x = 16800$

$\therefore \quad x = \frac{16800}{20} = 840$

$\therefore \quad A$'s share $= 6x = 6 \times 840$

$= ₹\ 5040$

B's share $= 9x = 9 \times 840 = ₹\ 7560$

C's share $= 5x = 5 \times 840 = ₹\ 4200$

12. (*a*) Let investment of $A = 3x$

$\therefore$ Investment of $B = 5x$

Investment of $C = 5x$

As we know,

Ratio of profits = Ratio of investments

$\therefore$ *A*'s share : *B*'s share : *C*'s share

$= 3x \times 12 : 5x \times 12 : 5x \times (12 - 6)$

$= 3x \times 12 : 5x \times 12 : 5x \times 6 = 6 : 10 : 5$

13. (*a*) Reena's share : Reema's share : Meera's share

Ratio of time period for which the car was used

$= 14 : 16 : 22 = 7 : 8 : 11$

Amount paid by Meera

$$= \frac{11}{7+8+11} \times 2080 = \frac{11}{26} \times 2080$$

$= 11 \times 80 = ₹\ 880$

14. (*d*) Let Pinku's share $= 7x$

Rinku's share $= 8x$

Tinku's share $= 6x$

According to the question,

$7x + 8x + 6x = 4200$

$\Rightarrow \quad 21x = 4200$

$\therefore \quad x = \frac{4200}{21} = 200$

Hence, amount of Pinku

$= 7x = 7 \times 200 = ₹\ 1400$

Amount of Rinku

$= 8x = 8 \times 200 = ₹\ 1600$

Amount of Tinku

$= 6x = 6 \times 200 = ₹\ 1200$

Now, ₹ 200 is added to each share

Then,

New ratio of shares

$= (1400 + 200) : (1600 + 200) : (1200 + 200)$

$= 1600 : 1800 : 1400$

$= 16 : 18 : 14 = 8 : 9 : 7$

15. (*a*) Let *R*'s capital = 1

Then, Q's capital = 4

2 (P's capital) = 3 (Q's capital)

$= 3 \times 4 = 12$

$\therefore$ P's capital $= \frac{12}{2} = 6$

$\therefore$ P's share : Q's share : R's share

$= 6 : 4 : 1$

Thus, Q's share profit

$= \frac{4}{6 + 4 + 1} \times 148500$

$= \frac{4}{11} \times 148500$

$= 4 \times 13500 = ₹\ 54000$

16. (*a*) Let the capital of $R = x$

Then, capital of $Q' = 3x$,

and capital of $P = 6x$

$\therefore$ P's capital : Q's capital : R's capital

$= 6x : 3x : x = 6 : 3 : 1$

17. (*c*) Let investment of $R = x$

Then, investment of $Q = \frac{2x}{3}$

and investment of $P = 2x$

$\therefore$ Ratio of capitals of P, Q and R

$= 2x : \frac{2x}{3} : x = 6x : 2x : 3x = 6 : 2 : 3$

18. (*d*) Time is not given in the question. So, we cannot find the profit of Ramesh. Hence, the data is inadequate.

19. (*a*) Let investment of $P = ₹\ x$

Then, investment of $N = ₹\ (x + 5000)$

and investment of $M = (x + 5000) + 4000$

$= ₹\ (x + 9000)$

According to the question,

$x + (x + 5000) + (x + 9000) = 50000$

$\Rightarrow$ $3x + 14000 = 50000$

$\Rightarrow$ $3x = 50000 - 14000 = 36000$

$\therefore$ $x = \frac{36000}{3} = 12000$

Clearly, investment of $P = ₹\ 12000$

Investment of $N = (x + 5000)$

$= 12000 + 5000 = ₹\ 17000$

Investment of $M = (x + 9000)$

$= 12000 + 9000 = ₹\, 21000$

M's share : N's share : P's share

$= 21000 : 17000 : 12000$

$= 21 : 17 : 12$

Hence, M's share

$$= \frac{21}{21 + 17 + 12} \times 70000$$

$$= \frac{21}{50} \times 70000 = ₹\, 29400$$

20. (*d*) Ratio of rents to be paid by P, Q and R

Ratio of monthly equivalent

$= (10 \times 20) : (30 \times 8) : (16 \times 9)$

$= 200 : 240 : 144 = 25 : 30 : 18$

Hence, R's share

$$= \frac{18}{25 + 30 + 18} \times 2920$$

$$= \frac{18}{73} \times 2920 = 18 \times 40 = ₹\, 720$$

21. (*b*) Ratio of rents of A, B, C and D

$= (18 \times 4) : (25 \times 2) : (28 \times 5) : (21 \times 3)$

$= 72 : 50 : 140 : 63$

Let the total rent be x.

Then, A's share

$$= \frac{72x}{(72 + 50 + 140 + 63)} = \frac{72x}{325}$$

According to the question,

$$\frac{72x}{325} = 720$$

$$\therefore \quad x = \frac{720 \times 325}{72} = 3250$$

$\therefore$ Total rent $= ₹\, 3250$

22. (*b*) Let investment of $A = x$,

Investment of $B = 2x$,

Investment of $C = 3x$

$\therefore$ A's share : B's share : C's share

$= (x \times 12) : (2x \times 6) : (3x \times 4)$

$= 12x : 12x : 12x = 1 : 1 : 1$

$$\therefore \ C\text{'s share} = \frac{1}{1 + 1 + 1} \times 54000$$

$$= \frac{1}{3} \times 54000 = ₹\, 18000$$

23. (*a*) Given,

Investment of $B = \frac{1}{6}$ of total capital

$\therefore$ Investments of A and C each

$$= \frac{1}{2}\left(1 - \frac{1}{6}\right) \text{ of total capital}$$

$$= \frac{1}{2} \times \frac{5}{6} \text{ of total capital}$$

$$= \frac{5}{12} \text{ of total capital}$$

Now, A's share : B's share : C's share

$$= \frac{5}{12} : \frac{1}{6} : \frac{5}{12} = 5 : 2 : 5$$

Let A's share $= 5x$

B's share $= 2x$

C's share $= 5x$

According to the question,

$$5x + 2x + 5x = 33600$$

$\therefore$ $12x = 33600$

$\therefore$ $x = \frac{33600}{12} = 2800$

$\therefore$ Difference in the profits of B and C

$= 5x - 2x = 3x = 3 \times 2800 =$ ₹ 8400

24. (*c*) Let total profit $= x$

Paid to charity $= 5\%$ of $x = \frac{5}{100} \times x = \frac{x}{20}$

$\therefore$ Balance profit $= x - \frac{x}{20} = \frac{19x}{20}$

$\therefore$ A's share $= \frac{19x}{20} \times \frac{3}{5} = \frac{57x}{100}$

According to the question,

$$\frac{57x}{100} = 4275$$

$\therefore$ $x = \frac{4275 \times 100}{57} = 7500$

Hence, total profit = ₹ 7500

25. (*a*) Let total profit $= x$

Paid to charity $= 9\%$ of $x = \frac{9x}{100}$

$\therefore$ Balance profit $= x - \frac{9x}{100} = \frac{91x}{100}$

$\therefore$ A's share $= \frac{4}{(4+3)} \times \frac{91x}{100} = \frac{4}{7} \times \frac{91x}{100}$

According to the question,

$$\frac{4}{7} \times \frac{91x}{100} = 1196$$

$\therefore$ $x = \frac{1196 \times 7 \times 100}{4 \times 91} =$ ₹ 2300

Hence, total profit = ₹ 2300

26. (*d*) Let required ratio of time be $t_1 : t_2 : t_3$.

Then using,

Ratio of investments = Ratio of profits

$$4t_1 : 6t_2 : 9t_3 = 2 : 3 : 5$$

Taking first two terms of the ratio,

$$\frac{4t_1}{6 + t_2} = \frac{2}{3} \Rightarrow \frac{t_1}{t_2} = \frac{1}{1} = \frac{9}{9}$$

$\Rightarrow$ $t_1 : t_2 = 9 : 9$

Taking last two terms of the ratio,

$$\frac{6t_2}{9t_3} = \frac{3}{5} \Rightarrow \frac{t_2}{t_3} = \frac{9}{10}$$

$\Rightarrow$ $t_2 : t_3 = 9 : 10$

$\therefore$ $t_1 : t_2 : t_3 = 9 : 9 : 10$

Fast Track Method

Here, $P_1 : P_2 : P_3 = 2 : 3 : 5$ [profit's ratio]

and $x_1 : x_2 : x_3 = 4 : 6 : 9$

[investment's ratio]

According to the rule,

Required ratio

$$= \frac{P_1}{x_1} : \frac{P_2}{x_2} : \frac{P_3}{x_3} \quad \text{[by Technique 1]}$$

$$= \frac{2}{4} : \frac{3}{6} : \frac{5}{9}$$

$$= \frac{1}{2} : \frac{1}{2} : \frac{5}{9} = 9 : 9 : 10$$

27. (*a*) Let required ratio of investments be $I_1 : I_2 : I_3$.

Ratio of investments = Ratio of profits

$$2I_1 : I_2 : 8I_3 = 3 : 4 : 2$$

Taking first two terms of the ratio

$$\frac{2I_1}{I_2} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{3}{8} = \frac{6}{16}$$

$$\Rightarrow \quad I_1 : I_2 = 6 : 16$$

Taking last two terms of the ratio,

$$\frac{I_2}{8I_3} = \frac{4}{2}$$

$$\Rightarrow \quad \frac{I_2}{I_3} = \frac{16}{1}$$

$$\Rightarrow \quad I_2 : I_3 = 16 : 1$$

$$\therefore \quad I_1 : I_2 : I_3 = 6 : 16 : 1$$

Fast Track Method

Here, $t_1 : t_2 : t_3 = 2 : 1 : 8$

and $P_1 : P_2 : P_3 = 3 : 4 : 2$

According to the rule,

Required ratio

$$= \frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3} \quad \text{[by Technique 2]}$$

$$= \frac{3}{2} : \frac{4}{1} : \frac{2}{8}$$

$$= \frac{3 \times 8}{2} : \frac{4 \times 8}{1} : \frac{2 \times 8}{8}$$

$$= 12 : 32 : 2 = 6 : 16 : 1$$

28. (*d*) Here, $P_1 : P_2 : P_3 = 5 : 3 : 1$

and $x_1 : x_2 : x_3 = 5 : 6 : 8$

According to the formula,

Required ratio

$$= \frac{P_1}{x_1} : \frac{P_2}{x_2} : \frac{P_3}{x_3} \quad \text{[by Technique 1]}$$

$$= \frac{5}{5} : \frac{3}{6} : \frac{1}{8}$$

$$= 1 : \frac{1}{2} : \frac{1}{8} = 8 : \frac{8}{2} : \frac{8}{8} = 8 : 4 : 1$$

29. (*b*) Let share of Raghu be 100. Then share of Mona = 90 and share of Sonu = 99

Sonu : Mona : Raghu = 99 : 90 : 100

∴ Investment of Raghu

$$= \frac{5780}{99 + 90 + 100} \times 100$$

$$= \frac{5780}{289} \times 100 = ₹\ 2000$$

Exercise © *Higher SkiU Level Questions*

1. (c) A's share : B's share : C's share

$= (20000 \times 12) : (25000 \times 4) : (15000 \times 8)$

$= 240000 : 100000 : 120000$

$= 24 : 10 : 12 = 12 : 5 : 6$

Let $\quad A$'s share $= 12x$

B's share $= 5x$

C's share $= 6x$

According to the question,

$$12x + 5x + 6x = 23000$$

$\Rightarrow \quad 23x = 23000$

$\therefore \quad x = \dfrac{23000}{23} = 1000$

$\therefore$ C's share $= 6x = 6 \times 1000 =$ ₹ 6000

2. (c) Let the total proft be ₹ x.

Then, A's share in profit = ₹ $\dfrac{3}{5}x$

Remaining profit $= x - \dfrac{3}{5}x = \dfrac{5x - 3x}{5} = \dfrac{2x}{5}$

$\therefore$ B's share in profit = ₹ $\dfrac{x}{5}$

C's share in profit = ₹ $\dfrac{x}{5}$

According to the question,

$$\left(\frac{3x}{5} - \frac{x}{5}\right) = 400$$

$$\frac{2x}{5} = 400; \quad x = \frac{400 \times 5}{2} = ₹\ 1000$$

3. (c) Ratio of equivalent capitals of A, B and C

$= (35000 \times 12) : (20000 \times 5) : (15000 \times 7)$

$= 35 \times 12 : 20 \times 5 : 15 \times 7$

$= 84 : 20 : 21$

Let $\quad A$'s share $= 84x$

B's share $= 20x$

C's share $= 21x$

According to the question,

$$84x + 20x + 21x = 84125$$

$\Rightarrow \quad 125x = 84125$

$\Rightarrow \quad x = \dfrac{84125}{125} = 673$

$\therefore$ B's share $= 20x = 20 \times 673 =$ ₹ 13460

4. *(d)* Y gets $\dfrac{2}{7}$ of the profit.

$\therefore \quad X$ gets $\left(1 - \dfrac{2}{7}\right) = \dfrac{5}{7}$ of the profit

$\therefore \quad X : Y = \dfrac{5}{7} : \dfrac{2}{7} = 5 : 2$

Let the contribution of Y be a.

Then, $8000 \times 8 : a \times 4 = 5 : 2$

$\Rightarrow \quad \dfrac{64000}{4a} = \dfrac{5}{2}$

$\Rightarrow \quad 20a = 128000$

$\therefore \quad a = \dfrac{128000}{20} =$ ₹ 6400

5. *(b)* Let the annual profit be *X* *x*

Then, ?(x-120) will be distributed between *A* and *B* as their shares of profit. Ratio of profits = Ratio of investments So, *A* : *B* = 3000 : 4000 = 3:4

$\therefore \quad A\text{'s share} = 120 + (x - 120) \times \frac{3}{7}$

$\Rightarrow \quad 120 + (x - 120) \times \frac{3}{7} = 390$

$\Rightarrow \quad (x - 120) \times \frac{3}{7} = 390 - 120 = 270$

$\Rightarrow \quad x - 120 = 270 \times \frac{7}{3} = 630$

$\therefore \quad B\text{'s share} = \frac{4}{7} \times (x - 120)$

$= \frac{4}{7} \times 630 = ₹\ 360$

6. (*a*) Total investment $= \frac{100}{15} \times 45000$

$= ₹\ 300000$

$\because$ Investment of Avinash : Investment of Manoj : Investment of Arun = 3 : 2 : 5

$\therefore$ Investment of Manoj

$= \frac{2}{10} \times 300000 = ₹\ 60000$

7. (*d*) Let B remained in the business for x months. Then,

$A : B = (2250 \times 12) : (2700 \times x)$

$\Rightarrow \quad 2 : 1 = (27000 : 2700x) = (10 : x)$

$\therefore \quad \frac{10}{x} = \frac{2}{1}$ [$\because$ Ratio of profit is 2 : 1]

$\Rightarrow \quad 2x = 10$

$\therefore \quad x = \frac{10}{2} = 5$

Clearly, B joined after $(12 - 5) = 7$ months.

8. (*b*) Let investment of Amit = $12x$

and investment of Brijesh = $11x$

Let Brijesh invested the money for y months. Then,

Amit's investment : Brijesh's investment

$$= (12x \times 11) : (11x \times y) = 132x : 11xy$$
$$= 12 : y$$

$\therefore$ $\frac{12}{y} = \frac{4}{1}$ [$\because$ ratio of profit is 4 : 1]

$\Rightarrow$ $4y = 12$

$\therefore$ $y = \frac{12}{4} = 3$ months

Clearly, Brijesh invested money for 3 months.

9. (*a*) Let investment of Aarti during first year $= 5x$

Investment of Vinita during first year = $7x$

Investment of Kamla during first year = $6x$

Then, their investments during second year are

(126% of $5x$) : (120% of $7x$) : (115% of $6x$)

$$\Rightarrow \left(\frac{126}{100} \times 5x\right) : \left(\frac{120}{100} \times 7x\right) : \left(\frac{115}{100} \times 6x\right)$$

$$\Rightarrow \quad 630 : 840 : 690 = 21 : 28 : 23$$

10. (*d*) For management, money received by Anil = 10% of 9000 = ₹ 900

Balance = ₹ (9000 − 900) = ₹ 8100

Now, ratio of investments

$$= 12000 : 20000 = 3 : 5$$

$\therefore$ Anil's share $= ₹\, 8100 \times \frac{3}{3+5}$

$$= ₹\, 8100 \times \frac{3}{8} = ₹\, 3037.50$$

$\therefore$ Amount received by Anil

$$= ₹\, 900 + 3037.50 = ₹\, 3937.50$$

11. (*a*) Let the total capital be ₹ x and Babloo's money is used for y months.

Capital of Aayush $= \frac{x}{4}$

$\therefore$ Capital of Babloo $= \left(x - \frac{x}{4}\right) = \frac{3x}{4}$

Ratio of investments of Aayush and Babloo

$$= \left(\frac{x}{4} \times 15\right) : \left(\frac{3x}{4} \times y\right) = \frac{15}{4} : \frac{3y}{4} = 5 : y$$

Ratio of profits of Aayush and Babloo

$$= \frac{1}{3} : \frac{2}{3} = 1 : 2$$

$\therefore$ $\frac{5}{y} = \frac{1}{2}$

$\therefore \quad y = 10$ months

Hence, Babloo's money was used for 10 months.

12. (c) Given, ratio $= \frac{7}{2} : \frac{4}{3} : \frac{6}{5}$

$$= \frac{7}{2} \times 30 : \frac{4}{3} \times 30 : \frac{6}{5} \times 30$$

$$= 105 : 40 : 36$$

Let investment of Amitabh $= 105x$

Investment of Brijesh $= 40x$

Investment of Kamlesh $= 36x$

Ratio of their investments

$$= [105x \times 4 + 150\% \times 8 \text{ of } 105x] : (40x \times 12) : (36x \times 12)$$

$$= \left(420x + \frac{150}{100} \times 105x \times 8\right) : (480x) : (432x)$$

$$= (1680x) : (480x) : (432x)$$

$$= 35 : 10 : 9$$

$\therefore$ Brijesh's share

$$= \left(21600 \times \frac{10}{35 + 10 + 9}\right)$$

$$= \left(21600 \times \frac{10}{54}\right) = ₹\, 4000$$

13. (b) Let investment of B be x for y months.

Then, A's investment $= 3x$ for $2y$ months

$\therefore \quad A : B = (3x \times 2y) : (x \times y)$

$$= 6xy : xy = 6 : 1$$

Let the total profit $= m$.

Then, $\quad m \times \frac{1}{7} = 6000$

$\therefore \quad m = 6000 \times 7 = ₹\, 42000$

$\therefore$ 20% of 42000 = ₹ 8400

14. (a) Ratio of total capital of A and B

$$= 20000 \times 12 : 35000 \times 12$$

$$= 240000 : 420000$$

Now, C gives ₹ 220000 to both to make the capital equal.

$\therefore A$'s capital : B's capital

$$\begin{array}{r} = 240000 : 420000 \\ +200000 : 20000 \\ \hline 440000 : 440000 \\ \hline \end{array}$$

If A takes ₹ 200000 and B takes ₹ 20000 from C, then both have the equal capital.

∴ Required ratio of divided amount

$$= 200000 : 20000$$
$$= 20 : 2 = 10 : 1$$

Hence, A and B should divide the amount in the ratio of 10 : 1.

15. (*d*) The difference counts only due to 40% of the profit which was distributed according to their investments.

Let total profit $= x$.

40% of x is distributed in the ratio,

$$125000 : 85000 = 25 : 17$$

Share of 1st partner

$$= 40\% \text{ of } x\left(\frac{25}{25+17}\right)$$

$$= 40\% \text{ of } \frac{25x}{42} = \frac{40}{100} \times \frac{25x}{42} = \frac{5x}{21}$$

Share of 2nd partner

$$= 40\% \text{ of } \frac{17x}{42} = \frac{40}{100} \times \frac{17x}{42} = \frac{17x}{105}$$

Now, according to the question

$$\frac{5x}{21} - \frac{17x}{105} = 600$$

$$\Rightarrow \quad \frac{x(25-17)}{105} = 600$$

$$\Rightarrow \quad x = \frac{600 \times 105}{8} = ₹\ 7875$$

16. (*b*) A's share : B's share : C's share

$$= [16000 \times 3 + (16000 - 5000) \times 9] : [12000 \times 3 + (12000 + 5000) \times 9] : (21000 \times 6)$$

$$= (16 \times 3 + 11 \times 9) : (12 \times 3 + 17 \times 9) : (21 \times 6)$$

$$= 147 : 189 : 126 = 7 : 9 : 6$$

Hence, B's share exceeds C's share by

$$\frac{13200}{7+9+6}(9-6) = \frac{13200 \times 3}{22} = ₹\ 1800$$

Chapter

20

Unitary Method

Unitary method is a fundamental tool to solve arithmetic problems based on variation in quantities. The method endorses a simple technique to find the amount related to unit quantity.

This method can be applied in questions based on time and work, speed and distance, work and wages etc.

Direct Proportion

Two quantities are said to be in direct proportion to each other, if on increasing (decreasing) a quantity, the other quantity also increases (decreases) to the same extent

i.e., (Quantity 1) < (Quantity 2)*

For example

Number of men <* Volume of workdone (time constant)

i.e., if number of men increases, then the volume of workdone

also increases. Similarly, if volume of work increases, then

number of men required to finish the work also increases.

Ex. 1 If the price of 8 bananas is ? 40, then find out the price of 12 bananas.

Sol. $\because$ Price of 8 bananas = ₹ 40

$\therefore$ Price of 1 banana = ₹ $\frac{40}{8}$

$\therefore$ Price of 12 bananas = $\frac{40}{8} \times 12 = 5 \times 12 =$ ₹ 60

Alternate Method

Let the price of 12 bananas be ₹ x.

More bananas, More cost (Direct proportion)

$$= 8 : 12 :: 40 : x$$

$$8 \times x = 12 \times 40$$

$$x = \frac{12 \times 40}{8} = ₹\ 60$$

Ex. 2 Ramesh walks 160 m everyday, how many kilometres will he walk in 4 weeks.

Sol. Given,

1 day walk = **160** m

•/Total days = 4 x 7 = 28 days

Hence, total walking distance = 28 **x 160** = 4480 m

= 4.480 km (v 1 km = 1000 m)

Alternate Method

Let the walking distance **in** 4 week **or** 28 days = *x* m More days : More walking distance (Direct Proportion) Hence, 1:28 :: **160** : *x*

1 X *x* = 28 X 160 Total distance *(x)* = 28 x 160 = 4480 m or = 4.48 km

Indirect Proportion

Two quantities are said to be in indirect proportion to each other, if on increasing (or decreasing) a quantity, the other quantity decreases (or increases) to the same extent

i.e., $$(\text{Quantity } 1) \propto \frac{1}{(\text{Quantity } 2)}$$

For example **The time taken by a vehicle in covering a certain distance is inversely proportional to the speed of the vehicle.**

i.e., $$\text{Speed} \propto \frac{1}{\text{Time}}$$

Ex. 3 If Karan travels at a speed of 60 km/h and covers a distance in 9 h, then how much time will he take to travel the same distance at a speed of 90 km/h?

Sol. $\because$ $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

or $\qquad \text{Distance} = \text{Speed} \times \text{Time}$

Given, speed = 60 km/h, time = 9 h

$\therefore \qquad \text{Distance} = 60 \times 9 = 540 \text{ km}$

Now to cover the same distance at speed of 90 km/h

$$\text{Time taken} = \frac{\text{Distance}}{\text{Speed}} = \frac{540}{90} = 6 \text{ h}$$

Alternate Method

Let the time taken to cover the distance = x

More speed, Less time

$$60 : 90 :: x : 9 \qquad \text{[Indirect proportion]}$$

$$\Rightarrow \qquad \frac{60}{90} = \frac{x}{9}$$

$$\Rightarrow \qquad \frac{60 \times 9}{90} = x \Rightarrow x = 6 \text{ h}$$

Ex. 4 If 15 chains cost *X 12A,* then how much do 70 chains cost?

Sol. Let cost of 70 chains be ₹ x.

More chains, More cost (Direct proportion)

$$15 : 70 :: 234 : x \Rightarrow x = \frac{234}{15} \times 70 = 78 \times 14 = ₹\ 1092$$

Ex. 5 If the wages of 12 men for 30 days be ? 4200, then find out the wages of 18 men for 24 days.

Sol. Let the required wages be ₹ x.

More men, More wages (Direct proportion)

Less days, Less wages (Direct proportion)

$$\left.\begin{array}{lccc} \text{Men} & 12 & : & 18 \\ \text{Days} & 30 & : & 24 \end{array}\right\} :: 4200 : x$$

$\Rightarrow \quad 12 \times 30 \times x = 18 \times 24 \times 4200 \quad \Rightarrow x = \dfrac{18 \times 24 \times 4200}{12 \times 30} = ₹\,5040$

$\therefore$ Required wages = ₹ 5040

MIND IT!

If *My* persons can do \Ny work in *Dy* days and M_2 persons can do I/I/2 work in D_2 days, then we have a general formula, *MfyW*$_2$ = $M_2D_2W_V$

If we include working hours (says *T* and T_2) for the two groups, then the relationship is

Ex. 6 If 30 men working 18 h per day can reap a field in 32 days, in how many days can 36 men reap the field working 16 h per day?

Sol. Let the required number of days be x.

More men, Less days (Indirect proportion)

Less hours, More days (Indirect proportion)

$$\left.\begin{array}{lccc} \text{Men} & 36 & : & 30 \\ \text{Hours per day} & 16 & : & 18 \end{array}\right\} :: 32 : x$$

$\Rightarrow \quad 36 \times 16 \times x = 30 \times 18 \times 32$

$\Rightarrow \quad x = \dfrac{30 \times 18 \times 32}{36 \times 16} = 30$

$\therefore$ Required number of days = 30

Fast Track Method

Here, $M_1 = 30$, $D_1 = 32$, $T_1 = 18\text{h}$, $M_2 = 36$, $T_2 = 16\text{ h}$

$\because \quad M_1D_1T_1 = M_2D_2T_2$

$\therefore \quad 30 \times 32 \times 18 = 36 \times 16 \times D_2$

$\Rightarrow \quad D_2 = \dfrac{30 \times 32 \times 18}{36 \times 16} = 30$ days

Fast Track Practice

Exercise Q *Base Level Questions*

1. If cost of 24 oranges is ? 72, then find out the cost of 120 oranges. [Bank Clerks 2009]

(a) ? 180 *(b)* ? 360

(c) X 172 *(d)* ?500

(e) None of the above

2. If cost of 12 pens is ? 84, then what is the cost of 10 such pens?

(a) ? 90 *(b)* ? 72 *(c)* ? 65 fdj ? 70

(ej None of the above If cost of 15 eggs is ? 75, then find out the cost of 4 dozens eggs. [Bank Clerks 2010]

(a) ? 240 *(b)* ? 300 *(c)* ? 150(d) ? 185

(e) None of the above If 16 dozens bananas cost ? 360, then how many bananas can be bought in f 60?

(a) 16 bananas *(b)* 48 bananas

(c) 32 bananas *(d)* 50 bananas

(e) None of the above

5. A worker makes a toy in every 2 h. If he works for 80 h, then how many toys will he make? [Hotel Mgmt. 2008]

(a) 40 *(b)* 54 *(c)* 45 *(d)* 39 *(e)* None of the above

6. If price of *m* articles is *X n,* then what is the price of 5 articles?

(a) ₹ $\frac{5n}{m}$ *(b)* ₹ $\frac{mn}{5}$ *(c)* ₹ $\frac{m}{n}$ *(d)* ₹ $\frac{5m}{n}$

(e) None of the above

7. 12 men can do a piece of work in 24 days. How many days are needed to complete the work, if 8 men are engaged in the same work? [IDBI Officers 2010]

(a) 28 *(b)* 36 *(c)* 48 *(d)* 52 *(e)* None of the above

8. If 45 m of a uniform rod weighs 171 kg, then what will be the weight of 12 m of the same rod? [MBA 2008]

(a) 49 kg *(b)* 42.5 kg

(c) 55 kg *(d)* 45.6 kg

(e) None of the above

9. Maganlal, a worker, makes an article in every $\frac{2}{3}$ h. If he works for $7\frac{1}{2}$ h, then how many articles will he make?

(a) $11\frac{1}{4}$ *(b)* $11\frac{1}{3}$ *(c)* $11\frac{1}{6}$ *(d)* $11\frac{2}{5}$

(e) None of the above

10. Shantanu completes 5/8 of a job in 20 days. At this rate, how many more days will he take to finish the job?

(a) 6 days *(b)* 18 days

(c) 5 days *(d)* 12 days

(e) None of the above 11.20 men can build 56 m long wall in 6 days. What length of a similar wall can be built by 70 men in 3 days?

(a) 100 m *(b)* 98 m

(c) 48 m *(d)* 85 m

(e) None of the above

12. 22 men can complete a job in 16 days. In how many days, will 32 men complete that job? [Bank Clerks 2009]

(a) 14 *(b)* 12 *(c)* 16 *(d)* 9 *(e)* None of the above

13. If 30 men working 9 h per day can reap a field in 16 days, in how many days, will 36 men reap the field working 8 h per day?

(a) 15 *(b)* 25

(c) 18 *(d)* 10

(e) None of the above

14. In a race, Ravi covers 5 km in 20 min. How much distance will he cover in 100 min?

(a) 40 km *(b)* 35 km b) 26 km fdj 25 km *(e)* None of the above

15. A garrison of 1000 men had provisions for 48 days. However, a reinforcement of 600 men arrived. How long will now food last for?

(a) 35 days *(b)* 30 days

(c) 25 days *(d)* 45 days

(e) None of the above

16. A tree is 12 m tall and casts an 8 m long shadow. At the same time, a flag pole casts a 100 m long shadow. How long is the flag pole?

(a) 150 m *(b)* 200 m

(c) 125 m *(d)* 115 m

(e) None of the above

17. If 12 persons working 16 h per day earn ? 33600 per week, then how much will 18 persons earn working 12 h per day?

(a) *X* 40000 *(b)* *X* 35000

(c) ? 28800 *(d)* ? 37800

(e) None of the above

18. If 10 spiders can catch 10 flies in 10 min, then how many flies can 200 spiders catch in 200 min? [MBA 2010]

(a) 2000 *(b)* 5000 *(c)* 4000 *(d)* 3000 *(e)* None of the above

19. 2000 soldiers in a fort had enough food for 20 days. But some soldiers were transferred to another fort and the food lasted for 25 days. How many soldiers were transferred? [SSC (10+2) 2013]

(a) 400 *(b)* 450 *(c)* 525 *(d)* 500

20. If in a hostel, food is available for 45 days for 50 students. For how many days will this food be sufficient for 75 students? [CGPSC2013]

(a) 25 days *(b)* 28 days

(c) 30 days *(d)* 40 days

(e) None of the above

21. In a garrison, there was food for 1000 soldiers for one month. After 10 days, 1000 more soldiers joined the garrison. How long would the soldiers be able to carry on with the remaining food?

[CSAT2013]

(a) 25 days

(b) 20 days

(c) 15 days

(d) 10 days

(e) None of the above

22. A garrison is provided with ration for 72 soldiers to last for 54 days. Find how long would the same amount of food last for 90 soldiers, if the individual ration is reduced by 10%? [SSC CGL 2013]

(a) 48 days *(b)* 72 days

(c) 54 days *(d)* 126 days

Exercise © *Higher Skill Level Questions*

1. 3 men can do a piece of work in 18 days. 6 boys can also do the same work in 18 days. In how many days, 4 men and 4 boys together will finish the work?

[Bank Clerks 2010]

(a) 10 days *(b)* 6 days

(c) 12 days *(d)* 9 days

(e) None of the above A man and a boy working together can complete a work in 24 days. If for the last 6 days, the man alone does the work, then it is completed in 26 days. How long will the boy take to complete the work alone? *(a)* 72 days *(b)* 73 days

(c) 49 days *(d)* 62 days

(e) None of the above 3. 25 men can reap a field in 20 days. When should 15 men leave the work, if the whole field is to be reaped in 37V& days after they leave the work? *(a)* After 5 days *(b)* After 10 days *(c)* After 9 days *(d)* After 7 days *(e)* None of the above 10 men and 8 women can together complete a work in 5 days. Work done by a woman is equal to the half of the work done by a man. In how many days, will 4 men and 6 women complete that work? [Bank Clerks 2010]

(a) 12 days *(b)* 10 days

(c) 8 | days *(d)* 9 | days

(e) None of the above 5. If 12 engines consume 30 metric tonnes of coal when each is running 18 h per day, how much coal will be required for

16 engines, each running 24 h per day, it being given that 6 engines of former type consume as much as 8 engines of latter type? *(a)* 10 tonnes *(b)* 5 tonnes

(c) 25 tonnes *(d)* 40 tonnes

(e) None of the above

6. If 12 men or 18 women can do a piece of work in 14 days, then how long will 8 men and 16 women take to finish the work? [RRB 2007]

(a) 9 days *(b)* 10 days

(c) 12 days *(d)* 14 days

(e) None of the above

7. 8 men can complete a work in 12 days, 4 women can complete it in 48 days and 10 children can complete the same work in 24 days. In how many days can 10 men, 4 women and 10 children complete the same work?

(a) 10 days *(b)* 5 days

(c) *7* days *(d)* 6 days

(e) None of the above

8. 20 men complete one-third of a work in 20 days. How many more men should be employed to finish the rest of the work in 25 more days?

(a) 10 *(b)* 12 *(c)* 15 *(d)* 20 *(e)* None of the above

9. 40 men complete one-third of a work in 40 days. How many more men should be employed to finish the rest of the work in 50 more days?

(a) 12 *(b)* 20 *(c)* 18 *(d)* 24 *(e)* None of the above

Answer with **Solutions**

Exercise© *Base Level Questions*

1. (*b*) $\because$ Cost of 24 oranges = ₹ 72

$\therefore$ Cost of 1 orange = ₹ $\frac{72}{24}$

$\therefore$ Cost of 120 oranges = $\frac{72}{24} \times 120$

$= 3 \times 120 =$ ₹ 360

Alternate Method

Let the required cost be ₹ x.

More oranges, More cost

(Direct proportion)

$24 : 120 :: 72 : x$

$\therefore \quad x = \frac{120 \times 72}{24} =$ ₹ 360

2. (*d*) $\because$ Cost of 12 pens = ₹ 84

$\therefore$ Cost of 1 pen = ₹ $\frac{84}{12}$

$\therefore$ Cost of 10 pens = $\frac{84}{12} \times 10$

$= 7 \times 10 =$ ₹ 70

3. (*a*) $\because$ Cost of 15 eggs = ₹ 75

$\therefore$ Cost of 1 egg = ₹ $\frac{75}{15}$

$\therefore$ Cost of 4 dozens $(4 \times 12 = 48)$ eggs

$= \frac{75}{15} \times 48 =$ ₹ 240

4. (*c*) Let the required number of bananas be x.

16 dozens bananas

$= 16 \times 12 = 192$ bananas

Less bananas, Less cost

(Direct proportion)

$360 : 60 :: 192 : x$

$\therefore \quad x = \frac{60 \times 192}{360} = 32$ bananas

5. (*a*) Let number of toys be x.

More hours, More toys (Direct proportion)

$2 : 80 :: 1 : x \Rightarrow x = \frac{80}{2} = 40$ toys

6. (*a*) $\because$ Price of m articles = ₹ n

$\therefore$ Price of 1 article = ₹ $\frac{n}{m}$

$\therefore$ Price of 5 articles = ₹ $\frac{5n}{m}$

7. (*b*) $\because$ 12 men can do the work in 24 days.

$\therefore$ 1 man will do it in 24×12 days.

$\therefore$ 8 men will do it in $\frac{24 \times 12}{8}$ days.

$\therefore$ Required number of days $= \frac{24 \times 12}{8}$

$= 3 \times 12 = 36$ days

Alternate Method

Let the required number of days be x.

Less men, More days (Indirect proportion)

$8 : 12 :: 24 : x$

$\therefore \quad x = \frac{12 \times 24}{8} = 36$ days

8. (*d*) $\because$ Weight of 45 m rod = 171 kg

$\therefore$ Weight of 1 m rod $= \frac{171}{45}$ kg

$\therefore$ Weight of 12 m rod

$= \frac{171}{45} \times 12 = 45.6$ kg

Alternate Method

Let the required weight be x kg.

Less length, Less weight

(Direct proportion)

$45 : 12 :: 171 : x$

$\therefore \quad x = \frac{12 \times 171}{45} = 45.6$ kg

9. (*a*) $\because$ In $\frac{2}{3}$ h, 1 article is made.

$\therefore$ In 1h, $\frac{3}{2}$ articles are made.

$\therefore$ In $7\frac{1}{2} = \frac{15}{2}$ h, $\frac{3}{2} \times \frac{15}{2} = \frac{45}{4}$

articles are made.

$\therefore$ Required articles $= \frac{45}{4} = 11\frac{1}{4}$

10. (*d*) Let the required number of days be x.

Remaining work $= 1 - \frac{5}{8} = \frac{3}{8}$

Less work, Less days (Direct proportion)

$\frac{5}{8} : \frac{3}{8} :: 20 : x$

$\Rightarrow \quad x = \frac{3}{8} \times 20 \times \frac{8}{5}$

$\therefore \quad x = 12$ days

Alternate Method

$\because \frac{5}{8}$ work is done in 20 days.

$\therefore$ 1 work will be done in $\frac{20 \times 8}{5} = 32$ days.

$\therefore$ Required number of days to complete the work

$$= (30 - 20) = 12 \text{ days}$$

11. (*b*) Let the required length be x m.

More men, More length

(Direct proportion)

Less days, Less length

(Direct proportion)

$$\left.\begin{array}{ll} \text{Men} & 20:70 \\ \text{Days} & 6:3 \end{array}\right\} :: 56 : x$$

$\therefore (20 \times 6 \times x) = (70 \times 3 \times 56)$

$$\therefore \quad x = \frac{70 \times 3 \times 56}{20 \times 6} = 98 \text{ m}$$

12. (*e*) $\because$ 22 men do the work in 16 days.

$\therefore$ 1 man will do the work in 16×22 days.

$\therefore$ 32 men will do the job in

$$= \frac{16 \times 22}{32} \text{ days}$$

$\therefore$ Required number of days

$$= \frac{16 \times 22}{32} = 11 \text{ days}$$

13. (*a*) Let the required number of days be x.

More men, Less days (Indirect proportion)

Less hours, More days

(Indirect proportion)

$$\left.\begin{array}{ll} \text{Men} & 36:30 \\ \text{Hours / day} & 8:9 \end{array}\right\} :: 16 : x$$

$\therefore \quad (36 \times 8 \times x) = (30 \times 9 \times 16)$

$$\therefore \quad x = \frac{30 \times 9 \times 16}{36 \times 8} = 15 \text{ days}$$

Alternate Method

Here, $\quad M_1 = 30, D_1 = 16,$

$T_1 = 9, M_2 = 36$ and $T_2 = 8$

By formula, $\quad M_1 D_1 T_1 = M_2 D_2 T_2$

$$30 \times 16 \times 9 = 36 \times D_2 \times 8$$

$$\therefore \quad D_2 = \frac{30 \times 16 \times 9}{36 \times 8} = 15 \text{ days}$$

14. (*d*) $\because$ Distance covered in 20 min = 5 km

$\therefore$ Distance covered in 1 min $= \frac{5}{20}$ km

$\therefore$ Distance covered in 100 min

$$= \frac{5}{20} \times 100 = 5 \times 5 = 25 \text{ km}$$

15. (*b*) $\because$ For 1000 men, provision lasts for 48 days.

$\therefore$ For 1 man, provision lasts for (48×1000) days.

$\therefore$ For $(1000 + 600)$ men, provision will last for $\frac{48 \times 1000}{1600}$ days.

$\therefore$ Required number of days

$$= \frac{48 \times 1000}{1600} = 3 \times 10 = 30$$

16. (*a*) $\because$ 8 m shadow means original height

$$= 12 \text{ m}$$

$\therefore$ 1 m shadow means original height

$$= \frac{12}{8} \text{ m}$$

$\therefore$ 100 m shadow means original height

$$= \frac{12}{8} \times 100 \text{ m} = \frac{6}{4} \times 100$$

$$= 6 \times 25 = 150 \text{ m}$$

17. (*d*) Let the required earnings be ₹ x.

More persons, More earnings

(Direct proportion)

Less hours per day, Less earnings

(Direct proportion)

$$\left.\begin{array}{ll} \text{Persons} & 12 : 18 \\ \text{Hours per day} & 16 : 12 \end{array}\right\} :: 33600 : x$$

$\therefore (12 \times 16 \times x) = (18 \times 12 \times 33600)$

$$\therefore \quad x = \frac{18 \times 12 \times 33600}{12 \times 16}$$

$$= 18 \times 2100 = ₹\ 37800$$

Alternate Method

$M_1 = 12, H = 16, W_1 = 33600$

$M_2 = 18, H = 12, W_2 = ?$

By formula, $\frac{M_1 H_1}{W_1} = \frac{M_2 H_2}{W_2}$

$$\frac{12 \times 16}{33600} = \frac{18 \times 12}{W_2}$$

$$\Rightarrow \quad W_2 = \frac{33600 \times 18}{16}$$

$$\Rightarrow \quad W_2 = 2100 \times 8$$

$$\therefore \quad W_2 = 37800$$

18. (*c*) Let the required number of flies be x.

More spiders, More flies

(Direct proportion)

More time, More flies (Direct proportion)

$$\left.\begin{array}{ll} \text{Spiders} & 10 : 200 \\ \text{Time} & 10 : 200 \end{array}\right\} :: 10 : x$$

$$\therefore \quad (10 \times 10 \times x) = (200 \times 200 \times 10)$$

$$\therefore \quad x = \frac{200 \times 200 \times 10}{10 \times 10} = 4000 \text{ flies}$$

19. (*a*) Let the number of soldiers transferred be x

Now, the food would last for 25 days for $(2000 - x)$ soldiers.

Less men, More days

(Indirect proportion)

$$25 : 20 :: 2000 : 2000 - x$$

$$\Rightarrow \quad 2000 - x = \frac{2000 \times 20}{25}$$

$$\Rightarrow \quad 2000 - x = 1600$$

$$\therefore \quad x = 2000 - 1600 = 400$$

Fast Track Method

Here, $M_1 = 2000$, $D_1 = 20$

and let the soldiers leaving the fort be x.

Then, $M_2 = 2000 - x$, $D_2 = 25$

Using formula, $M_1 D_1 = M_2 D_2$

$$2000 \times 20 = (2000 - x)\,25$$

$$\Rightarrow \quad \frac{2000 \times 20}{25} = 2000 - x$$

$$\Rightarrow \quad 2000 - x = 1600$$

$$\therefore \quad x = 400$$

20. (*c*) $\because$ For 50 students, food is sufficient for 45 days.

$\therefore$ For 1 student, food is sufficient for 45×50 days.

$\therefore$ For 75 students, food is sufficient for $\frac{45 \times 50}{75}$ days *i.e.*, for 30 days.

21. (*d*) Let us assume that each soldier eats one unit of food per day. Thus, total units of food at the begining will be

$$1000 \times 30 = 30000.$$

After 10 days 1000 soldiers would have eaten $1000 \times 10 = 10000$ units of food. Thus, food left after 10 days equals 20000 units. Now, there are total of 2000 soldiers who eat one unit of food every day. So, the number of days that 20000 units of food will serve 2000 soldiers is $\frac{20000}{2000} = 10$.

Alternate Method

$\because$ For 1000 soldiers, remaining food is sufficient for

$$30 - 10 = 20 \text{ days}$$

$\therefore$ For $1000 + 1000 = 2000$ soldiers, food will be sufficient for

$$\frac{1000}{2000} \times 20 = 10 \text{ days}$$

22. (*a*) Let the required number of days be x.
More men, Less days

(Indirect proportion)

$$\left.\begin{array}{lll} \text{Soldiers} & 90 & 72 \\ \text{Ratio} & \frac{9}{10} & 1 \end{array}\right\} :: 54 : x$$

$$x = \frac{72 \times 54 \times 10}{90 \times 9} = 48 \text{ days}$$

Exercise © *Higher Skill Level Questions*

1. (*d*) 3 men = 6 boys

$\Rightarrow$ 1 man = 2 boys

$\therefore$ 4 men + 4 boys = 4 men + 2 men

= 6 men

$\because$ 3 men can do a work in 18 days.

$\therefore$ 1 man can do a work in 18×3 days.

$\therefore$ 6 men can do the work in $\frac{18 \times 3}{6}$ days.

$\therefore$ Required number of days

$$= \frac{18 \times 3}{6} = 9 \text{ days}$$

Alternate Method

3 men = 6 boys

$\Rightarrow$ 1 man = 2 boys

$\therefore$ 4 men + 4 boys = 4 men + 2 men

= 6 men

More men, Less days (Indirect proportion)

$6 : 3 :: 18 : x$

$$\Rightarrow \quad x = \frac{3 \times 18}{6}$$

$$\therefore \quad x = 3 \times 3 = 9 \text{ days}$$

2. (*a*) Let man's 1 day's work $= \frac{1}{m}$

and boy's 1 day's work $= \frac{1}{n}$

1 day's work man and boy $= \frac{1}{24}$

Man's 6 day's work $= \frac{6}{m}$

Now, for 20 days, both man and boy do the work and for last 6 days, only man does the work.

According to the question,

$$\frac{1}{m} + \frac{1}{n} = \frac{1}{24}$$

$$\Rightarrow \quad 20\left(\frac{1}{m} + \frac{1}{n}\right) + \frac{6}{m} = 1$$

$$\Rightarrow \quad \left(20 \times \frac{1}{24}\right) + \frac{6}{m} = 1$$

$$\Rightarrow \quad \frac{6}{m} = \left(1 - \frac{20}{24}\right) = \frac{4}{24} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{m} = \frac{1}{36}$$

Now from eq. (i)

$$\frac{1}{m} + \frac{1}{n} = \frac{1}{24}$$

$$\frac{1}{36} + \frac{1}{n} = \frac{1}{24}$$

$$\Rightarrow \quad \frac{1}{n} = \left(\frac{1}{24} - \frac{1}{36}\right) = \frac{1}{72}$$

Hence, the boy alone can do the work in 72 days.

3. (*a*) Let 15 men work for m days.

$$\text{Work done in 1 days} = \frac{m}{20}$$

$$\text{Remaining work} = \left(1 - \frac{m}{20}\right)$$

$$\text{25 men's 1 days work} = \frac{1}{20}$$

$$\text{1 man's 1 days work} = \frac{1}{20} \times \frac{1}{25} = \frac{1}{500}$$

$$\text{10 men's 1 days work} = \frac{1}{500} \times 10 = \frac{1}{50}$$

10 men's $\frac{75}{2}$ days work

$$= \frac{1}{50} \times \frac{75}{2} = \frac{75}{100} = \frac{3}{4}$$

$$\therefore \quad \left(1 - \frac{m}{20}\right) = \frac{3}{4} \Rightarrow \frac{m}{20} = \frac{1}{4}$$

$$\Rightarrow \quad m = \frac{1}{4} \times 20 = 5$$

Clearly, 15 men leave after 5 days.

4. (*b*) 1 man $\equiv$ 2 women

10 men + 8 women $\equiv$ 20 women + 8 women = 28 women

4 men + 6 women $\equiv$ 8 women + 6 women = 14 women

$\because$ 28 women do the work in 5 days.

$\therefore$ 1 woman can do the same work in (28×5) days.

$\therefore$ 14 women will do the same work in $\left(\frac{28 \times 5}{14}\right)$ days.

$\therefore$ Required number of days

$$= \frac{28 \times 5}{14} = 2 \times 5 = 10$$

Alternate Method

1 women $\equiv \frac{1}{2}$ man

10 men + 8 women $\equiv$ 10 men + 4 men

$= 14$ men

4 men + 6 women $\equiv$ 4 men + 3 men

$= 7$ men

More men, Less days (Indirect proportion)

$7 : 14 :: 5 : x$

$\therefore \quad x = \frac{14 \times 5}{7} = 10$ days

5. (*d*) Let the required quantity of coal consumed be x.

More engines, More coal consumption (Direct proportion)

More hours, More coal consumption (Direct proportion)

Less rate of consumption, Less coal consumption (Direct proportion)

$$\left.\begin{array}{ll} \text{Engines} & 12 : 16 \\ \text{Working hours} & 18 : 24 \\ \text{Rate of consumption} & \frac{1}{6} : \frac{1}{8} \end{array}\right\} :: 30 : x$$

$\therefore 12 \times 18 \times \frac{1}{6} \times x = 16 \times 24 \times \frac{1}{8} \times 30$

$\Rightarrow \quad 36x = 1440$

$\therefore \quad x = \frac{1440}{36} = 40$

Hence, quantity of coal consumed will be 40 tonnes.

6. (*a*) 12 men $\equiv$ 18 women

$\Rightarrow \quad 1$ man $\equiv \frac{18}{12}$ women $\equiv \frac{3}{2}$ women

$\therefore \quad 8$ men $\equiv \frac{3}{2} \times 8 = 12$ women

$\therefore$ 8 men + 16 women

$=$ 12 women + 16 women

$=$ 28 women

$\because$ 18 women can do the work in 14 days.

$\therefore$ 1 woman can do the same work in (14×18) days.

$\therefore$ 28 women will do the same work in $\left(\frac{14 \times 18}{28}\right)$ days.

$\therefore$ Required number of days

$= \frac{14 \times 18}{28} = 9$ days

7. (*d*) 1 man can finish the work in (8×12)

$= 96$ days

1 woman can finish the work in (4×48)

$= 192$ days

1 child can finish the work in (10×24)

$= 240$ days

$$1 \text{ man's } 1 \text{ day's work} = \frac{1}{96}$$

$$1 \text{ woman's } 1 \text{ day's work} = \frac{1}{192}$$

$$1 \text{ child's } 1 \text{ day's work} = \frac{1}{240}$$

(10 men + 4 women + 10 children)'s 1 day's work

$$= \left(\frac{10}{96} + \frac{4}{192} + \frac{10}{240}\right)$$

$$= \left(\frac{5}{48} + \frac{1}{48} + \frac{1}{24}\right)$$

$$= \left(\frac{5 + 1 + 2}{48}\right)$$

$$= \frac{8}{48} = \frac{1}{6}$$

Hence, they will finish the work in 6 days.

8. (*b*) Work done $= \frac{1}{3}$

Remaining work $= \left(1 - \frac{1}{3}\right) = \left(\frac{3-1}{3}\right) = \frac{2}{3}$

Let the number of more men to be employed be x.

More work, More men (Direct proportion)

More days, Less men (Indirect proportion)

$$\left.\begin{array}{lc} \text{Work} & \frac{1}{3} : \frac{2}{3} \\ \text{Days} & 25 : 20 \end{array}\right\} : : 20 : (20 + x)$$

$$\therefore \quad \frac{1}{3} \times 25 \times (20 + x) = \frac{2}{3} \times 20 \times 20$$

$$\Rightarrow \quad (20 + x) = \frac{800}{25} = 32$$

$$\therefore \quad x = 32 - 20 = 12$$

9. (*d*) Work done $= \frac{1}{3}$

Remaining work $= \left(1 - \frac{1}{3}\right) = \left(\frac{3-1}{3}\right) = \frac{2}{3}$

Let the number of additional men be x.

More work, More men

(Direct proportion)

More days, Less men

(Indirect proportion)

$$\left.\begin{array}{lc} \text{Work} & \frac{1}{3} : \frac{2}{3} \\ \text{Days} & 50 : 40 \end{array}\right\} : : 40 : (40 + x)$$

$$\therefore \quad \frac{1}{3} \times 50 \times (40 + x) = \frac{2}{3} \times 40 \times 40$$

$$\Rightarrow \quad 5 \times (40 + x) = 2 \times 40 \times 4$$

$$\Rightarrow \quad 200 + 5x = 320$$

$$\Rightarrow \quad 5x = 320 - 200 = 120$$

$$\therefore \quad x = \frac{120}{5} = 24$$

$\therefore$ Required number of men = 24

Chapter

21

Problem Based on Ages

Age is defined as a period of time that a person has lived or a thing has existed. Age is measured in **months, years, decades** and so on.

Problem based on ages generally consists of information of ages of two or more persons and a relation between their ages in present/future/past.

Using the information, it is asked to calculate the ages of one 01 more persons in present/future/ past.

Important Rules for Problem Based on Ages

RuleO

If ratio of present ages of *A* and *B* is $x : y$ and n yr ago, the ratio of

their ages was $p : q$, then $\frac{x-n}{y-n} = \frac{p}{q}$

Ex. 1 A father is twice as old as his son. 20 yr back, he was twelve times as old as his son. What is the present age of father?

Sol. Let present age of father be $2k$ yr and that of son is k yr.

Then, $x = 2k$ and $y = k$

Also, 20 yr before the ratio of their ages was 12 : 1.

So, $\dfrac{p}{q} = \dfrac{12}{1}$

Then, $\dfrac{x-n}{y-n} = \dfrac{p}{q} \Rightarrow \dfrac{2k-20}{k-20} = \dfrac{12}{1}$

$\Rightarrow 2k - 20 = 12k - 240 \Rightarrow 240 - 20 = 12k - 2k$

$\Rightarrow 10k = 220$

$\therefore k = \dfrac{220}{10} = 22$

$\therefore$ Present age of father $= 22 \times 2 = 44$ yr

Rule®

If ratio of present ages of *A* and *B* is *x: y* and after *n* yr, the ratio of their ages will be

$p : q$, then $\dfrac{x+n}{y+n} = \dfrac{p}{q}$

Ex. 2 I am three times as old as my son. 15 yr hence, I will be twice as old as my son. What is the sum of our present ages.

Sol. Let the present ages of man and his son be $3k$ and k yr, respectively.

So, $x = 3k$ and $y = k$

Also, $\dfrac{p}{q} = \dfrac{2}{1}$ [$\because$ 15 yr hence their ratio is 2 : 1]

Then, $\dfrac{x+n}{y+n} = \dfrac{p}{q}$

$\Rightarrow \dfrac{3k+15}{k+15} = \dfrac{2}{1}$

$\Rightarrow 3k + 15 = 2(k + 15)$

$\Rightarrow 3k + 15 = 2k + 30$

$\Rightarrow 3k - 2k = 30 - 15$

$\therefore k = 15$

$\therefore$ Sum of their ages $= 3k + k = 4k$

$= 4 \times 15 = 60$ yr

MIND IT!

Mostly questions on ages can be solved with the use of linear equations. So, the method to solve linear equations is important for this chapter which are discussed in chapter equations.

Fast Track **Techniques**

to solve the QUESTIONS

Technique 1

If t yr after age of one person is n times of that of another person and at present the age of first person is m times of that of another person, then

$$\text{Age of first person} = tm\left(\frac{n-1}{m-n}\right)\text{ yr}$$

$$\text{and age of second person} = t\left(\frac{n-1}{m-n}\right)\text{ yr}$$

Ex. 3 Present age of Karan is 5 times the age of Shivam. After 10 yr, Karan will be 3 times as old as Shivam. What are the present ages of Karan and Shivam? Sol. Let the present age of Shivam be a; yr. Then, present age of Karan = $5x$ yr

After 10 yr, the ratio of ages will be 3 : 1 According to the question,

$$\frac{5x+10}{x+10}=\frac{3}{1}$$

$\Rightarrow \quad 5x + 10 = 3(x + 10)$

$\Rightarrow \quad 5x + 10 = 3x + 30$

$\Rightarrow \quad 5x - 3x = 30 - 10$

$\Rightarrow \quad 2x = 20 \Rightarrow x = \frac{20}{2} = 10$

$\therefore$ Karan's present age = $5 \times 10 = 50$ yr and Shivam's present age = 10 yr

Fast Track Method

Here, $t = 10$, $m = 5$ and $n = 3$

Karan's present age $= tm\left(\frac{n-1}{m-n}\right)$

$$= 10 \times 5\left(\frac{3-1}{5-3}\right) = 50 \times \frac{2}{2} = 50 \text{ yr}$$

Shivam's present age $= t\left(\frac{n-1}{m-n}\right)$

$$= 10\left(\frac{3-1}{5-3}\right)$$

$$= 10 \times \frac{2}{2} = 10 \text{ yr}$$

Technique 2

if ^ yr before, age of a person was m times of age of another person. After t_2 yr, age of a person will be n times of age of second person, then

$$\text{Age of first person} = \frac{t_2 m(n-1) + t_1 n(m-1)}{m-n} \text{ yr}$$

$$\text{and age of second person} = \frac{t_2(n-1) + t_1(m-1)}{m-n} \text{ yr}$$

Ex. 4 5 yr ago, age of Sachin was 4 times the age of Vineet and after 10 yr, Sachin will be twice as old as Vineet. Find the present ages of Sachin and Vineet.

Sol. Let the present ages of Sachin and Vineet be x yr and y yr, respectively.

Case I 5 yr ago,

$$\frac{x-5}{y-5}=\frac{4}{1}$$

$$\Rightarrow \quad x-5=4(y-5)$$

$$\Rightarrow \quad x-4y=-20+5$$

$$\Rightarrow \quad x-4y=-15 \quad \text{...(i)}$$

Case II After 10 yr,

$$\frac{x+10}{y+10}=\frac{2}{1}$$

$$\Rightarrow \quad x+10=2(y+10)$$

$$\Rightarrow \quad x+10=2y+20$$

$$\Rightarrow \quad x-2y=20-10$$

$$\Rightarrow \quad x-2y=10 \quad \text{...(ii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$\begin{array}{r} x-4y=-15 \\ x-2y=10 \\ -\quad+\quad\quad- \\ \hline -2y=-25 \end{array}$$

$$\Rightarrow \quad y=\frac{25}{2}=12\frac{1}{2}$$

On putting value of y in Eq. (i), we get

$$x-4\times\frac{25}{2}=-15 \quad \Rightarrow \quad x-50=-15$$

$$x=-15+50=35$$

Hence, present age of Sachin is 35 yr and present age of Vineet is $12\frac{1}{2}$ yr.

Fast Track Method

Here, $t_1=5$, $t_2=10$, $m=4$ and $n=2$

Then, Sachin's present age $=\dfrac{t_1 n(m-1)+t_2 m(n-1)}{m-n}$

$$=\frac{5\times2(4-1)+10\times4(2-1)}{4-2}$$

$$=\frac{30+40}{2}=\frac{70}{2}=35 \text{ yr}$$

Vineet's present age $=\dfrac{t_1(m-1)+t_2(n-1)}{m-n}$

$$=\frac{5(4-1)+10(2-1)}{4-2}=\frac{5\times3+10\times1}{2}$$

$$=\frac{15+10}{2}=\frac{25}{2}=12\frac{1}{2} \text{ yr}$$

Technique 3

If M is as elder to N as he is younger to P and sum of ages of N and P is t yr, then $N<M<P$

$$M\text{'s age} = \frac{\text{Sum of ages of } N \text{ and } P}{2} = \frac{S}{2}, \text{ where } S = \text{Sum}$$

Ex. 5 If Akshay is as much elder than Vinay as he is younger to Kartik and sum of ages of Vinay and Kartik is 48 yr, then find the age of Akshay.

Sol. Let present age of Akshay be x yr and he is younger to Kartik by y yr.

Then, Kartik's age $= (x + y)$ yr and Vinay's age $= (x - y)$ yr

Now, according to the question,

Sum of ages of Kartik and Vinay $= 48$

$\Rightarrow$ $(x + y) + (x - y) = 48$

$\Rightarrow$ $2x = 48$

$\therefore$ $x = \frac{48}{2} = 24$ yr

Hence, present age of Akshay is 24 yr.

Fast Track Method

Present age of Akshay $= \frac{\text{Sum of ages}}{2} = \frac{48}{2} = 24$ yr

Fast Track Practice

1. The present ages of Reena and Usha are 24 and 36 yr, respectively. What was the ratio between the ages of Usha and Reena respectively 8 yr ago?

[Allahabad Bank Clerk 2010]

(a) 7 :4 *(b)* 4:7 *(c)* 11:8 *(d)* 8:11 *(e)* None of the above

2. Raju decided to marry 3 yr after he gets a job. He was 17 yr old when he passed class 12th. After passing class 12th, he had completed his graduation course in 3 yr and PG course in 2 yr. He got the job exactly 1 yr after completing his PG course. At what age will he get married?

[Allahabad Bank Clerk 2010]

(a) 27 yr *(b)* 26 yr

(c) 28 yr *(d)* 23 yr

(e) None of the above

3. The average age of 8 children of a family is 12 yr. If the age of 7 children are 12, 8, 14, 11, 9, 13 and 15 yr, then the age of 8th child will be [RRB 2008]

(a) 12 yr *(b)* 14 yr

(c) 13 yr *(d)* 15 yr

4. The ratio between the present ages of A and B is 4: 5. If the ratio between their ages 4 yr hence becomes 14:17, then what is B's age at present? [RRB 2009]

(a) 30 yr *(b)* 28 yr

(c) 34 yr *(d)* Data inadequate

5. In a family, the average age of father and mother is 35 yr. The average age of the father, mother and their only son is 27 yr. What is the age of the son?

[SSCCGL2010]

(a) 12 yr *(b)* 11 yr

(c) 10.5 yr *(d)* 10 yr

6. The sum of the present ages of Varun and Kapil is 42 yr. The ratio of their ages after 5 yr will be 15:11 What is the present age of Kapil? [RRB 2010]

(a) 17 yr *(b)* 24 yr *(c)* 25 yr *(d)* 22 yr

7. The average age of 9 students and their teacher is 16 yr. The average age of the first four students is 19 yr and that of the last five is 10 yr. The teacher's age is

[SSC (10+2) 2010]

(a) 36 yr *(b)* 34 yr

(c) 30 yr *(d)* 28 yr

8. At present, Anil is 1.5 times of Purvi's age. 8 yr hence, the respective ratio between Anil and Purvi's ages will be 25: 1& What is Purvi's present age?

[IBPS Clerk 2011]

(a) 50 yr *(b)* 28 yr *(c)* 42 yr *(d)* 36 yr *(e)* None of the above

9. The average of the present ages of Sachin and Saurabh is 36 yr. If Sachin is 8 yr older than Saurabh, what is the Saurabh's present age?

[Allahabad Bank Clerk 2010]

(a) 30 yr *(b)* 34 yr *(c)* 32 yr *(d)* 40 yr *(e)* None of the above

10. At present Meena is eight times her daughter's age. 8 yr from now, the ratio of the ages of Meena and her daughter will be 10:3, respectively. What is Meena's present age? [IDBI PO 2009]

(a) 32 yr *(b)* 40 yr *(c)* 36 yr

(d) Cannot be determined

(e) None of the above

Raman's present age is three times his daughter's and nine-thirteenth of his mother's present age. The sum of the present ages of all three of them is 125 yr. What is the difference between the present ages of Raman's daughter and Raman's mother? [Allahabad Bank PO 2009] *(a)* 45 yr *(b)* 40 yr *(c)* 50 yr

(d) Cannot be determined

(e) None of the above

12. The ages of Harsh and Sumit are 40 yr and 60 yr, respectively. How many years before the ratio of their ages was 3:5?

(a) 15 yr *(b)* 20 yr *(c)* 37 yr *(d)* 10 yr

13. The average age of a man and his son is 27 yr. The ratio of their ages is 8:1, respectively. What will be the son's age after 6 yr? [UBI Clerk 2011]

(a) 6 yr *(b)* 14 yr *(c)* 12 yr *(d)* 8 yr *(e)* None of the above

14. The difference between the ages of Meena and Seema is 3 yr and the ratio between their ages is 7 : 8. What is the sum of their ages? [UCO Bank Clerk 2011]

(a) 43 yr *(b)* 41 yr *(c)* 45 yr *(d)* 48 yr *(e)* 47 yr

15. At present, the ratio of the ages of Maya and Chhaya is 6: 5 and fifteen years from now, the ratio will get changed to 9: 8. Maya's present age is [SSC CGL 2011]

(a) 21 yr *(b)* 24 yr *(c)* 30 yr *(d)* 40 yr

16. The ratio of the present age of Manoj to that of Wasim is 3:11 Wasim is 12 yr younger than Rehana. Rehana's age after 7 yr will be 85 yr. What is the present age of Manoj's father, who is 25 yr older than Manoj? [IOBPO2011]

(a) 43 yr *(b)* 67 yr *(c)* 45 yr *(d)* 69 yr *(e)* None of the above

17. Before 7 yr, the ratio of ages of A and *B* was 3: After 9 yr, ratio of their ages will be 7: 8. The present age of *B* will be

[SBIPO2012]

(a) 16 yr *(b)* 19 yr *(c)* 28 yr *(d)* 23 yr *(e)* None of the above

18. The ratio between the present ages of Indira and Lizzy is 3:8, respectively. After 8 yr, Indira's age will be 20 yr. What was Lizzy's age 5 yr ago?

[BOI Clerk 2010]

(a) 37 yr *(b)* 27 yr *(c)* 28 yr *(d)* 38 yr *(e)* None of the above

19. The ratio between the present ages of Tarun and Varun is 3:7, respectively. After 4 yr, Varun's age will be 39 yr. What was Tarun's age 4 yr ago?

[CBI Clerk 2010]

(a) 12 yr *(b)* 13 yr *(c)* 19 yr *(d)* 18 yr *(e)* None of the above 20.7 yr ago, the ages of A and *B* were in the ratio 4: 5 and 7 yr hence, they will be in the ratio 5: 6. The present age of *B* is

[SSC CGL 2010] *(a)* 56 yr *(b)* 63 yr *(c)* 70 yr *(d)* 77 yr

21. At present, Kavita is twice Sarita's age. 8 yr hence, the respective ratio between Kavita's and Sarita's ages will be 22:13 What is Kavita's present age?

[IBPS Clerk 2011]

(a) 26 yr *(b)* 18 yr

(c) 42 yr *(d)* 36 yr

(e) None of the above

22. The average age of a family of five members is 24. If the present age of the youngest member is 8 yr, what was the average age of the family at the time of the birth of the youngest member?

[Corporation Bank PO 2011]

(a) 20 yr *(b)* 16 yr

(c) 12 yr *(d)* 18 yr

(e) 21 yr

23. Four years ago, Shyam's age was 3/4 times of that of Ram. Four year hence, Shyam's age will be 5/6 times that of Ram. What is the present age of Shyam?

[Corporation Bank PO 2011]

(a) 15 yr *(b)* 20 yr *(c)* 16 yr *(d)* 24 yr *(e)* 8 yr

24. The present ages of two persons are 36 and 50 yr, respectively. If after n yr the ratio of their ages will be 3 : 4, then the value of n is [SSC Multitasking 2013]

(a) 3 *(b)* 4 *(c) 7 (d)* 6

25. The ratio of the ages of Ram and Rahim 10 yr ago was 1: 3 The ratio of their ages five years hence will be 2: *3.* Then, the ratio of their present ages is [SSC CGL 2011]

(a) 1:2 *(b) 3:5 (c)* 3 : 4 *(d)* 2 : 5

26. At present, Palash is three times Arnav's age. After 7 yr, Palash will be twice Arnav's age, then how many times will Palash's age be in another 14 yr time with respect to Arnav's age then?

[IBPS Clerk 2011]

(a) 1 *(b)* 3 *(c)* 2 *(d)* 1.5

(e) None of the above

27.10 yr before, the ratio of ages of A and *B*

was 13:17. After 17 yr from now, the

ratio of their ages will be 10:11 The

present age of *B* is [SBI PO 2012]

(a) 23 yr *(b)* 40 yr *(c)* 27 yr *(d)* 44 yr

(e) None of the above

28. 3 yr before, the average age of a family of

5 members was 17 yr. Due to birth of a

new child, the average age is same today.

The present age of child is [SSC FCI 2012]

(a) 1 yr *(b)* 3 yr *(c)* $2\frac{1}{2}$ yr *(d)* 2 yr

29. The age of a man after 15 yr is 4 times the age of that man 15 yr before. His present age is [RRB 2012]

(a) 10 yr *(b)* 15 yr *(c)* 20 yr *(d)* 25 yr

30. The present age of Ravi's father is four times Ravi's present age. Five years back, Ravi's father was seven times as old as Ravi was at that time. What is the present age of Ravi's father? [CDS 2013]

(a) 84 yr *(b)* 70 yr *(c)* 40 yr *(d)* 35 yr

31. Ravi's brother is 3 yr elder to him. His father was 28 yr of age when his sister was born, while his mother was 26 yr of age when he was born. If his sister was 4 yr of age when his brother was born, the ages of Ravi's father and mother respectively when his brother was born were [CDS 2013]

(a) 32 yr and 23 yr *(b)* 32 yr and 29 yr

(c) 35 yr and 29 yr *(d)* 35 yr and 33 yr

Answer with Solutions

1. (*a*) Required ratio $= \dfrac{36-8}{24-8} = \dfrac{28}{16} = 7:4$

2. (*b*) $\because$ Age of Raju when he got the job

$$= 17 + 3 + 2 + 1 = 23 \text{ yr}$$

$\therefore$ Age of Raju at the time of marriage

$$= 23 + 3 = 26 \text{ yr}$$

3. (*b*) $\because$ Total age of 8 children

$$= 8 \times 12 = 96 \text{ yr}$$

and total age of 7 children

$= 12 + 8 + 14 + 11 + 9 + 13 + 15 = 82$

$\therefore$ The age of 8th child $= 96 - 82 = 14$ yr

4. (*a*) Let the ages of A and B are $4x$ and $5x$ yr, respectively.

According to the question,

$$\therefore \quad \frac{4x+4}{5x+4} = \frac{14}{17}$$

$$\Rightarrow \quad 68x + 68 = 70x + 56$$

$$\Rightarrow \quad 2x = 12 \Rightarrow x = 6$$

Hence, present age of $B = 5 \times 6 = 30$ yr

5. (*b*) $\because$ Total age of father and mother

$$= 35 \times 2 = 70 \text{ yr}$$

and total age of father, mother and son

$$= 27 \times 3 = 81 \text{ yr}$$

$\therefore \quad$ Age of son $= 81 - 70 = 11$ yr

6. (*a*) Let the present age of Varun be x and present age of Kapil be y.

According to the questions,

$$x + y = 42 \quad \text{...(i)}$$

Again, according to the question,

After 5 yr, $\dfrac{x+5}{y+5} = \dfrac{15}{11}$

$$\Rightarrow \quad 11x + 55 = 15y + 75$$

$$\Rightarrow \quad 11x - 15y = 20 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii),

$$11x + 11y = 462$$

[multiply by 11 in Eq. (i)]

$$\begin{array}{r} 11x - 15y = 20 \\ - \quad + \quad\quad - \\ \hline 26y = 442 \end{array}$$

$$\therefore \quad y = \frac{442}{26} = 17 \text{ yr}$$

Hence, present age of Kapil = 17 yr

7. (*b*) $\because$ Total age of 9 students and a teacher

$$= 10 \times 16 = 160 \text{ yr}$$

Total age of first 4 students $= 4 \times 19 = 76$ yr

and total age of last 5 students

$$= 5 \times 10 = 50 \text{ yr}$$

$\therefore \quad$ Age of teacher $= 160 - 76 - 50 = 34$ yr

8. (*b*) Let age of Purvi = x yr

Then, age of Anil = $1.5x$ yr

According to the question,

$$\frac{1.5x + 8}{x + 8} = \frac{25}{18}$$

$\Rightarrow \quad 27x + 144 = 25x + 200$

$\therefore \quad x = \frac{56}{2} = 28$

Hence, present age of Purvi = 28 yr

9. (*c*) $\because$ Total age of Sachin and Saurabh

$= 36 \times 2 = 72$ yr

Let age of Saurabh = x yr

Then, age of Sachin = $(x + 8)$ yr

According to the question,

$x + x + 8 = 72$

$\Rightarrow \quad 2x = 64$

$\therefore \quad x = 32$

Hence, Saurabh's present age = 32 yr

10. (*a*) Let age of daughter = x yr

$\therefore$ Age of Meena = $8x$ yr

According to the question,

$$\frac{8x + 8}{x + 8} = \frac{10}{3}$$

$\Rightarrow \quad 24x + 24 = 10x + 80$

$\Rightarrow \quad 14x = 56$

$\therefore \quad x = 4$

Hence, Meena's present age = $4 \times 8 = 32$ yr

11. (*c*) Let age of Raman's daughter = x yr

Then, age of Raman = $3x$ yr

and age of Raman's mother

$$= \frac{13}{9} \times 3x = \frac{13}{3} x \text{ yr}$$

According to the question,

$$x + 3x + \frac{13}{3} x = 125$$

$\Rightarrow 3x + 9x + 13x = 125 \times 3$

$\therefore \quad x = \frac{125 \times 3}{25} = 15$

Hence, required difference

$$= \frac{13}{3} x - x = \frac{10x}{3} = \frac{10 \times 15}{3} = 50 \text{ yr}$$

12. (*d*) Let x yr before, the ratio of their ages was 3 : 5.

According to the question,

$$\frac{40 - x}{60 - x} = \frac{3}{5}$$

$\Rightarrow \quad 200 - 5x = 180 - 3x \Rightarrow 2x = 20$

$\therefore \quad x = 10$

13. (*c*) Let ages of man and his son are $8x$ yr and x yr, respectively.
According to the question,
$$\frac{8x + x}{2} = 27 \Rightarrow x = \frac{27 \times 2}{9} = 6$$
Hence, age of son after 6 yr $= 6 + 6 = 12$ yr

14. (*c*) Let ages of Meena and Seema are $7x$ yr and $8x$ yr, respectively.
$\therefore \quad 8x - 7x = 3$
$\therefore \quad x = 3$
Hence, required sum $= 8x + 7x$
$= 15 \times 3 = 45$ yr

15. (*c*) Let the present ages of Maya and Chhaya are $6x$ and $5x$ yr, respectively.
According to the question,
After 15 yr,
$$\frac{6x + 15}{5x + 15} = \frac{9}{8}$$
$\Rightarrow \quad 48x + 120 = 45x + 135$
$\Rightarrow \quad 3x = 15$
$\therefore \quad x = 5$
Hence, present age of Maya $= 5 \times 6 = 30$ yr

16. (*a*) Let the present ages of Manoj and Wasim are $3x$ yr and $11x$ yr, respectively.
According to the question,
$11x = 85 - 7 - 12$
$\Rightarrow \quad 11x = 66$
$\therefore \quad x = 6$
$\therefore$ Present age of Manoj $= 3 \times 6 = 18$ yr
Hence, present age of Manoj's father
$= 18 + 25 = 43$ yr

17. (*d*) Let the ages of A and B before 7 yr were $3x$ yr and $4x$ yr, respectively.
$\therefore$ Present age of $A = 3x + 7$
and present age of $B = 4x + 7$
Now, according to the question,
$$\therefore \quad \frac{3x + 7 + 9}{4x + 7 + 9} = \frac{7}{8}$$
$\Rightarrow \quad 24x + 128 = 28x + 112$
$\Rightarrow \quad 4x = 16$
$\therefore \quad x = 4$
Hence, present age of $B = 4 \times 4 + 7$
$= 16 + 7 = 23$ yr

18. (*b*) Let present ages of Indira and Lizzy are $3x$ yr and $8x$ yr, respectively.
$\therefore \quad 3x + 8 = 20 \Rightarrow 3x = 12$
$\therefore \quad x = 4$
Hence, Lizzy's required age $= 8x - 5$
$= 32 - 5 = 27$ yr

19. (*e*) Let Tarun and Varun's ages are $3x$ yr and $7x$ yr, respectively.
$\therefore \quad 7x + 4 = 39 \Rightarrow x = 5$
Hence, Tarun's age 4 yr ago
$= 3 \times 5 - 4 = 11$ yr

20. (*d*) Let 7 yr ago, ages of A and B are $4x$ yr and $5x$ yr, respectively.
Then, present age of $A = 4x + 7$
and present age of $B = 5x + 7$
Now, according to the question,

$$\therefore \quad \frac{4x + 7 + 7}{5x + 7 + 7} = \frac{5}{6}$$

$$\Rightarrow \quad 24x + 84 = 25x + 70$$

$$\Rightarrow \quad x = 14$$

Hence, B's present age $= 5 \times 14 + 7 = 77$ yr

21. (*d*) Let Sarita's present age $= x$ yr
Then, Kavita's present age $= 2x$ yr
According to the question,

$$\frac{2x + 8}{x + 8} = \frac{22}{13}$$

$$\Rightarrow \quad 26x + 104 = 22x + 176$$

$$\Rightarrow \quad 4x = 72$$

$$\therefore \quad x = 18$$

Hence, Kavita's age $= 2 \times 18 = 36$ yr

22. (*a*) $\because$ Total age of five members of a family

$$= 24 \times 5 = 120$$

$\therefore$ Total age of four members at the time of birth of youngest $= 120 - 8 \times 5$

$$= 120 - 40 = 80 \text{ yr}$$

Hence, required average age $= \frac{80}{4} = 20$ yr

23. (*c*) 4 yr ago, let Ram's age $= x$ yr and Shyam's age $= \frac{3}{4}x$ yr
Now, Ram's present age $= (x + 4)$ yr
and Shyam's present age $= \left(\frac{3}{4}x + 4\right)$ yr
According to the question,

$$\frac{5}{6}(x + 4 + 4) = \left(\frac{3}{4}x + 4 + 4\right)$$

$$\Rightarrow \quad 4(5x + 40) = 6(3x + 32)$$

$$\Rightarrow \quad 20x + 160 = 18x + 192$$

$$\Rightarrow \quad 2x = 32$$

$$\therefore \quad x = 16$$

Hence, present age of Shyam

$$= \frac{3}{4} \times 16 + 4 = 16 \text{ yr}$$

Fast Track Method

Here $t_1 = 4, t_2 = 4, m = \frac{3}{4}$ and $n = \frac{5}{6}$

Then, Shyam's present age

$$= \frac{t_1 n(m - 1) + t_2\, m(n - 1)}{m - n}$$

$$= \frac{4 \times \frac{5}{6}\left(\frac{3}{4} - 1\right) + 4 \times \frac{3}{4}\left(\frac{5}{6} - 1\right)}{\frac{3}{4} - \frac{5}{6}}$$

$$= \frac{\frac{10}{3}\left(\frac{-1}{4}\right) + 3\left(\frac{-1}{6}\right)}{\frac{9 - 10}{12}}$$

$$= \frac{\frac{-10}{12} - \frac{3}{6}}{\frac{9-10}{12}} = \frac{\frac{-10-6}{12}}{\frac{-1}{12}} = 16 \text{ yr}$$

24. (*d*) According to the question,

$$\frac{36+n}{50+n} = \frac{3}{4}$$

$\Rightarrow \quad 144 + 4n = 150 + 3n$

$\therefore \quad n = 6$

25. (*b*) Let 10 yr ago, ages of Ram and Rahim were x yr and $3x$ yr, respectively.

Then, present age of Ram $= (x + 10)$

and present age of Rahim $= (3x + 10)$

According to the question,

$$\frac{x + 10 + 5}{3x + 10 + 5} = \frac{2}{3}$$

$\Rightarrow \quad 3x + 45 = 6x + 30$

$\Rightarrow \quad 3x = 15$

$\therefore \quad x = 5$

Hence, required ratio $= \dfrac{5+10}{3 \times 5 + 10}$

$$= \frac{15}{25} = 3:5$$

26. (*a*) Let Arnav's age $= x$ yr

Then, Palash's age $= 3x$ yr

According to the question,

$3x + 7 = 2(x + 7)$

$\Rightarrow \quad 3x + 7 = 2x + 14$

$x = 7$

$\therefore$ Age of Arnav after 14 yr $= 7 + 14 = 21$ yr

and Palash's present age $= 21$ yr

Hence,

Palash's age is one time of Arnav's age.

27. (*c*) Let the ages of A and B 10 yr before were $13x$ yr and $17x$ yr, respectively.

Then, present age of $A = 13x + 10$

and present age of $B = 17x + 10$

According to the question,

$\because \quad \dfrac{13x + 10 + 17}{17x + 10 + 17} = \dfrac{10}{11}$

$\Rightarrow \quad \dfrac{13x + 27}{17x + 27} = \dfrac{10}{11}$

$\Rightarrow \quad 143x + 297 = 170x + 270$

$\Rightarrow \quad 27x = 27$

$\therefore \quad x = 1$

Hence, present age of $B = 17 \times 1 + 10$

$= 27$ yr

28. (*d*) 3 yr before,

Total age of 5 members

$= 17 \times 5 = 85$ yr

$\therefore$ The present age of all members of family

$= 85 + 3 \times 5 = 100$ yr

Let the age of child be x yr.

$\therefore$ Present average age of family including the child

$$= \frac{100 + x}{6}$$

$$\Rightarrow \quad 17 = \frac{100 + x}{6}$$

$\therefore \quad x = 102 - 100 = 2$ yr

29. *{d)* Let present age of man be x yr. According to the question,

x + 15 = 4(x- 15) => x+ 15 = 4x-60 => 3x = 75

x = 25 Hence, the present age of man is 25 yr.

30. (c) Let present age of Ravi be *x*. Then, present age of Ravi's father = 4x Now, 5 yr ago,

Ravi's father's age = 7 X Ravi's age => 4x - 5 = 7(x - 5)

=> 4x - 5 = 7x - 35

=> 3x = 30

x = 10 /. Ravi's present age = x = 10 yr .•. Ravi's father's present age

4x = 4 X 10 = 40 yr

31. (a) When Ravi was born, his mother's age was 26 yr and his elder brother was 3 yr elder to him.

.". Mother's age when brother was born = 26 -3 = 23yr

Ravi's father was 28 yr of age when his sister

was born and his sister was 4 yr of age when his brother was born.

.'. Age of father when brother was born = 28 + 4 = 32 yr

Chapter

22

Work and Time

In this chapter, we will study techniques to solve problems based on work and its completion time as well as number of persons required to finish the given work in stipulated time.

Suppose that you are a contractor and you got a contract to construct a flyover in a certain time. For this, you need to calculate the number of men required to finish the work according to their work efficiency.

Important Relations

1. Work and Person Directly proportional (more work, more men and conversely more men, more work).

2. Time and Person Inversely proportional (more men, less time and conversely more time, less men).

3. Work and Time Directly proportional (more work, more time and conversely more time, more work).

Note While solving these types of problems, the work done is always supposed to be equal to 1

Basic Rules Related to Work and Time

RuleQ

If a person can do a piece of work in n days (hours), then that person's 1 day's (hour's) work $= \frac{1}{n}$.

Ex. 1 Vandana completes a work in 35 days. What work will she do in 1 day?

Sol. We know that, if a person can do a piece of work in n days, then

Person's 1 day's work $= \frac{1}{n}$

Here, $n = 35$

$\therefore$ Required work done $= \frac{1}{35}$

Rule®

If a person's 1 day's (hour's) work $= \frac{1}{n}$, then the person will complete the work in n days (hours).

Ex. 2 Kavi does $\frac{1}{13}$ part of a certain work in 1 day. In how many days, will he complete the whole work?

Sol. We know that, if a person's 1 day's work $= \frac{1}{n}$, then the person will complete the whole work in n days.

Here, $\frac{1}{n} = \frac{1}{13}$

$\therefore$ Required number of days $= 13$

Rule©

If a person is n times efficient than the second person, then work done by

First person: Second person = n : 1 and time taken to complete a work by

First person : Second person = 1 : n

Ex. 3 P can do a work 3 times faster than Q and therefore takes 40 days less than Q. Find the time in which P and Q can complete the work individually.

Sol. We know that, if a person is n times efficient than the second person, then time taken to complete a work by

First person : Second person = 1 : n

∴ Time taken to complete the work by $P: Q = K: 3K$

According to the question,

Time taken by Q — Time taken by P = 40

=> $3K - K = 40$ => $2K = 40$

$K = 20$

Number of days required by P = 20 and number of days required by Q = 60

RuleQ

If ratio of numbers of men required to complete a work is $m : n$, then the ratio of time taken by them will be $n : m$.

Ex. 4 If 12 men can finish a work in 20 days, then find the number of days required to complete the same work by 15 men.

Sol. We know that, if ratio of numbers of men required to complete a work is $m : n$, then ratio of time taken by them will be n: m. According to the question,

Ratio of numbers of men = 12 : 15 = 4 : 5 Ratio of times taken =5:4 Let us suppose 15 men can finish a work in x days. Then, 20 : x = 5 : 4

x = 16 Required number of days =16

Fast Track **Techniques**

to solve the **QUESTIONS**

Technique 1

If M, persons can do I/K, work in D, days working T-, h in a day and M_2 persons can do W_2 work in D_2 days working T_2 h in a day, then the relationship between them is

M^D^W2 = M2D2T2W^

Ex. 5 10 men can make 20 toys in 12 days working 12 h a day. Then, in how many days can 24 persons make 32 toys working 16 h a day?

Sol. Given that, $M_1 = 10, M_2 = 24, D_1 = 12, D_2 = ?,$
$T_1 = 12, T_2 = 16, W_1 = 20$ and $W_2 = 32$

According to the formula,

$$M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1$$

$$\Rightarrow \quad 10 \times 12 \times 12 \times 32 = 24 \times D_2 \times 16 \times 20$$

$$\therefore \quad D_2 = \frac{10 \times 12 \times 12 \times 32}{24 \times 16 \times 20} = \frac{12}{2} = 6 \text{ days}$$

Technique 2

(i) If A can do a piece of work in x days and B can do the same work in y days, then $(A + B)$'s 1 day's work $= \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$

Time taken by $(A + B)$ to complete the work $= \frac{xy}{x+y}$ days

(ii) If A can do a piece of work in x days, B can do the same work in y day and C can do the same work in z days, then

Time taken by $(A + B + C)$ to complete the work $= \frac{xyz}{xy + yz + zx}$ days

For n persons, their 1 day's work $= \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}$

where, x-j,x2,x3,...,xn represents numbers of days taken by them to complete a work.

Ex. 6 *A* can do a piece of work in 10 days and *B* can do the same work in 12 days. How long will they take to finish the work, if both work together?

Sol. A's 1 day's work $= \frac{1}{10}$ and B's 1 day's work $= \frac{1}{12}$

$(A + B)$'s 1 day's work $= \frac{1}{10} + \frac{1}{12} = \frac{6+5}{60} = \frac{11}{60}$ day

$\therefore$ $(A + B)$ complete the whole work in $\frac{60}{11}$ days $= 5\frac{5}{11}$ days.

Fast Track Method

Here, $x = 10$ and $y = 12$

$\therefore$ Number of days taken by A and $B = \frac{xy}{x+y} = \frac{12 \times 10}{12 + 10} = 5\frac{5}{11}$ days

Ex. 7 If A can do a piece of work in 4 days, B can do the same work in 8 days and C can do the same work in 12 days, then working together, how many days will they take to complete the work?

Sol. A's 1 day's work $= \frac{1}{4}$; B's 1 day's work $= \frac{1}{8}$; C's 1 day's work $= \frac{1}{12}$

According to the question,

$(A + B + C)$'s 1 day's work $= \frac{1}{4} + \frac{1}{8} + \frac{1}{12} = \frac{6+3+2}{24} = \frac{11}{24}$

$\therefore$ $(A + B + C)$ complete the whole work in $\frac{24}{11}$ days $= 2\frac{2}{11}$ days.

Technique 3

If A and B can complete a work in x days and A alone can finish that work in y days, then

Number of days required to complete the work by $B = \frac{xy}{y-x}$ days

Ex. 8 A and B together can do a piece of work in 12 days and A alone can do it in 18 days. In how many days can B alone do it?

Sol. $(A + B)$'s 1 day's work $= \frac{1}{12}$ and A's 1 day's work $= \frac{1}{18}$

$\therefore$ B's 1 day's work $= (A + B)$'s 1 day's work $-$ A's 1 day's work $= \frac{1}{12} - \frac{1}{18}$

$= \frac{3-2}{36} = \frac{1}{36}$

$\therefore$ Time taken by B to complete the work alone $= 36$ days

Fast Track Method Here, $x = 12$ and $y = 18$

$\therefore$ Time taken by $B = \frac{xy}{y-x} = \frac{12 \times 18}{18 - 12} = 36$ days

(Technique;/

If *A* and *B* can do a piece of work in *x* days, e and C can do the same work in y days and *A* and C can do it in *z* days, then working together

$$A, B \text{ and } C \text{ can do that work in } \frac{2xyz}{xy + yz + zx} \text{ days.}$$

Ex. 9 *A* and *B* can do a piece of work in 3 days. *B* and C can do the same work in 9 days, while C and *A* can do it in 12 days. Find the time in which *A, B* and C can finish the work, working together.

Sol. $(A + B)$'s 1 day's work $= \frac{1}{3}$ ...(i)

$(B + C)$'s 1 day's work $= \frac{1}{9}$ ...(ii)

$(C + A)$'s 1 day's work $= \frac{1}{12}$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$2\,(A + B + C)\text{'s 1 day's work} = \frac{1}{3} + \frac{1}{9} + \frac{1}{12} = \frac{12 + 4 + 3}{36} = \frac{19}{36}$$

$$\therefore \quad (A + B + C)\text{'s 1 day's work} = \frac{19}{36 \times 2} = \frac{19}{72}$$

$$\therefore \quad (A + B + C) \text{ complete the work in } \frac{72}{19} \text{ days or in } 3\frac{15}{19} \text{ days.}$$

Fast Track Method

Here, $x = 3$, $y = 9$ and $z = 12$

According to the formula,

$$\text{Required time taken by } A, B \text{ and } C = \frac{2xyz}{xy + yz + zx}$$

$$= \frac{2 \times 3 \times 9 \times 12}{3 \times 9 + 9 \times 12 + 3 \times 12} = \frac{2 \times 3 \times 9 \times 12}{27 + 108 + 36} = \frac{2 \times 3 \times 9 \times 12}{171}$$

$$= \frac{6 \times 12}{19} = \frac{72}{19} = 3\frac{15}{19} \text{ days}$$

Technique 5

If a, men or *b,* women can finish a work in *D* days, then time taken by

a_2 men and b_2 women to complete the work $= \dfrac{D(a_1b_1)}{(a_2b_1 + a_1b_2)}$ days

Ex. 10 If 6 men or 8 women can reap a field in 86 days, how long will 14 men and 10 women take to rear? it?

Sol.

$$6 \text{ men} = 8 \text{ women}$$

$$\therefore \quad 1 \text{ man} = \frac{8}{6} = \frac{4}{3} \text{ women}$$

$$\therefore \quad 14 \text{ men} = \frac{56}{3} \text{ women}$$

$$14 \text{ men} + 10 \text{ women} = \frac{56}{3} + 10 = \frac{86}{3} \text{ women}$$

Now, $M_1 = 8, M_2 = \dfrac{86}{3}$, $\quad W_1 = W_2 = 1, \quad D_1 = 86 \quad$ and $\quad D_2 = ?$

According to the formula,

$$M_1 D_1 W_2 = M_2 D_2 W_1$$

$$\Rightarrow \quad 8 \times 86 \times 1 = \frac{86}{3} \times D_2 \times 1$$

$$\therefore \quad D_2 = \frac{8 \times 86 \times 3}{86} = 24 \text{ days}$$

Fast Track Method

Here, $a_1 = 6$, $b_1 = 8$ $a_2 = 14$, $b_2 = 10$ and $D = 86$

$$\therefore \quad \text{Number of days} = \frac{D(a_1 b_1)}{(a_2b_1 + a_1b_2)} = \frac{86 \times 6 \times 8}{14 \times 8 + 10 \times 6} = \frac{86 \times 6 \times 8}{172} = 24 \text{ days}$$

Technique 6

If A can do a work in x days and B can do $y\%$ fast than A, then B will complete the work in $\dfrac{100x}{100+y}$ days.

Ex. 11 Kamal can do a work in 15 days and Vimal is 50% more expert than Kamal to complete the same work, then find total time taken to complete the work by Vimal.

Sol. Here, $x = 15$ days and $y = 50\%$

Now, time taken by Vimal $= \dfrac{100x}{100+y}$

$$\Rightarrow \quad = \frac{100 \times 15}{100 + 50} = \frac{1500}{150} = 10 \text{ days}$$

Technique 7

If *a* men can do a piece of work in *x* days and *b* boys can do the same work in *y* days, then time taken to complete the same work by c men

and *d* boys will be $\dfrac{1}{\dfrac{c}{ax}+\dfrac{d}{by}}$ days.

Ex. 12 If 5 men can do a work in 2 days and 3 boys can do the same work in 5 days, then find the time taken to complete same work by 10 men and 3 boys.

Sol. Given, $a = 5, b = 3, x = 2, y = 5, c = 10$ and $d = 3$

Time taken by 10 men and 3 boys $= \dfrac{1}{\dfrac{c}{ax}+\dfrac{d}{by}} = \dfrac{1}{\dfrac{10}{5\times 2}+\dfrac{3}{3\times 5}} = \dfrac{1}{1+\dfrac{1}{5}} = \dfrac{5}{6}$ days

if a1 men and ^ boys can complete a work in x days, while a2 men and *by* boys can complete the same work in *y* days, then

$$\frac{\text{One day work of 1 man}}{\text{One day work of 1 boy}} = \frac{(yb_2 - xb_1)}{(xa_1 - ya_2)}$$

Ex.13 If 12 men and 16 boys can finish a work in 5 days, while 13 men and 24 boys can finish the same work in 4 days. Compare the one day work of 1 man and 1 boy.

Sol. Here, $a_1 = 12, b_1 = 16, x = 5, a_2 = 13, b_2 = 24$ and $y = 4$

$$\therefore \quad \frac{\text{One day work of 1 man}}{\text{One day work of 1 boy}} = \frac{(yb_2 - xb_1)}{(xa_1 - ya_2)}$$

$$= \frac{4 \times 24 - 5 \times 16}{5 \times 12 - 4 \times 13}$$

$$= \frac{96 - 80}{60 - 52} = \frac{16}{8} = \frac{2}{1}$$

Technique 9

If *x* takes a days more to complete a work than the time taken by (x + y) to do same work and y takes *b* days more than the time taken

by *[x + y)* to do the same work, then (x + y) do the work in *Jab* days.

Ex. 14 When *A* alone does a piece of work, he takes 16 days more than the time taken by *(A + B)* to complete the work, while *B* alone takes 9 days more than the time taken by *(A + B)* to finish the work. What time, *A* and *B* together will take to finish this work?

Sol. Here, $a = 16$ and $b = 9$

According to the formula,

Required time $= \sqrt{ab}$

$\therefore$ Required time $= \sqrt{16 \times 9}$

$= 4 \times 3 = 12$ days

Technique 10

A and *B,* each alone can do a piece of work in *a* and *b* days, respectively Both begin together and if

(i) *A* leaves the work *x* days before its completion, then total time taken for completion of work will be given as $T = \dfrac{(a + x)b}{(a + b)}$ days.

(ii) *B* leaves the work *x* days before its completion, then total time taken for completion of work will be given as $T = \dfrac{(b + x)a}{(a + b)}$ days.

Ex. 15 *A* can do a piece of work in 10 days while *B* can do it in 15 days. They begin together but 5 days before the completion of the work, *B* leaves off. Find the total number of days for the work to be completed.

Sol. *A*'s 5 days' work $= \dfrac{5}{10} = \dfrac{1}{2}$, Remaining work $= 1 - \dfrac{1}{2} = \dfrac{1}{2}$

$(A + B)$'s 1 day's work $= \dfrac{1}{10} + \dfrac{1}{15} = \dfrac{3 + 2}{30} = \dfrac{5}{30} = \dfrac{1}{6}$ days

$(A + B)$ finish $\dfrac{1}{6}$ work in 1 day.

$(A + B)$ will finish $\dfrac{1}{2}$ work in $\dfrac{1}{2} \times 6$ days or in 3 days.

$\therefore$ Required time $= 5 + 3 = 8$ days

Fast Track Method Here, $a = 10$ days, $b = 15$ days, $x = 5$ and $T = ?$

According to the formula,

Required time $= \dfrac{(b + x)\,a}{(a + b)} = \dfrac{(15 + 5)\,10}{10 + 15} = \dfrac{20 \times 10}{25} = 4 \times 2 = 8$ days

Technique 11

A and B do a piece of work in a and b days, respectively. Both begin together but after some days, A leaves off and the remaining work is completed by e in x days. Then, the time after which A left, is given by

$$T = \frac{(b - x)\,a}{a + b}$$

Ex. 16 A and B can do a piece of work in 40 days and 50 days, respectively. Both begin together but after a certain time, A leaves off. In this case B finishes the remaining work in 20 days. After how many days did A leave?

Sol. B's 20 day's work $= \frac{20}{50} = \frac{2}{5}$; Remaining work $= 1 - \frac{2}{5} = \frac{3}{5}$

$(A + B)$'s 1 day's work $= \frac{1}{40} + \frac{1}{50} = \frac{5 + 4}{200} = \frac{9}{200}$

$(A + B)$ do $\frac{9}{200}$ work in 1 day.

$\therefore$ $(A + B)$ will do $\frac{3}{5}$ work in $= \frac{3}{5} \times \frac{200}{9} = \frac{40}{3} = 13\frac{1}{3}$ days.

Fast Track Method

Here, $a = 40$ days, $b = 50$ days, $x = 20$ and $T = ?$

$\therefore$ Required time $= \frac{(b - x)\,a}{a + b} = \frac{(50 - 20) \times 40}{(40 + 50)} = \frac{30 \times 40}{90} = \frac{40}{3} = 13\frac{1}{3}$ days

Fast Track Practice

Exercise© *Base Level Questions*

1. A can complete a piece of work in 18 days and B can complete the same work in half time taken by A. Then, working together, what part of the same work they can complete in a day?

(a) $\frac{1}{6}$ (b) $\frac{1}{9}$ (c) $\frac{2}{5}$ (d) $\frac{2}{7}$
(e) $\frac{2}{9}$

2. A and B together can do a piece of work in 12 days, while B alone can finish it in 30 days. A alone can finish the work in

[SSC (10+2) 2012]

(a) 15 days *(b)* 18 days

(c) 20 days *(d)* 25 days

3. Aarti can do a piece of work in 6 days. In how many days will she complete the three time of work of same type?

(a) 18 days *(b)* 21 days

(c) 3 days *(d)* 6 days

(e) None of the above

4. A and B can do a piece of work in 6 and 12 days, respectively. They (both) will complete the work in how many days?

[SSC CCL 2012]

{a) 9 days *(b)* 18 days

(c) 6 days *(d)* 4 days

5. A alone can complete a work in 12 days and B alone can complete the same work in 24 days. In how many days can A and B together complete the same work?

[Bank Clerks 2009]

(a) 6 days *(b)* 4 days

(c) 10 days *(d)* 8 days

(e) None of the above

6. X can complete a job in 12 days. If X and Y work together, they can complete the job in 6%f days. Y alone can complete the job in

(a) 10 days *(b)* 12 days

(c) 15 days *(d)* 18 days

7. If 3 men and 4 boys can do a piece of work in 8 day, then 6 men and 8 boys can do the same work in

(a) 2 days *(b)* 4 days

(c) 6 days *(d)* 16 days

8. *A'* can do a piece of work in *x* days and *B* can do the same work *3x* days. To finish the work together they take 12 days. What is the value of *x*>. [CDS 2012]

(a) 8 *(b)* 10 *(c)* 12 *(d)* 16

9. *A* can do $\frac{1}{2}$ of the work in 5 days and *B* can do $\frac{2}{5}$ of the work in 10 days. In how many days both *A* and *B* together can do the work?

(a) $7\frac{2}{7}$ days *(b)* $7\frac{1}{7}$ days

(c) $7\frac{3}{7}$ days *(d)* $7\frac{5}{7}$ days

(e) None of the above

10. *A* can do a piece of work in 4 days and *B* can complete the same work in 12 days. What is the number of days required to do the same work together? [CDS 2013]

(a) 2 days *(b)* 3 days

(c) 4 days *(d)* 5 days

11. A can do a piece of work in 8 days, *B* can do it in 10 days and C can do it in 20 days. In how many days can *A, B* and *C* together complete the work?

(a) $3\frac{7}{11}$ days *(b)* $3\frac{5}{11}$ days

(c) $3\frac{2}{11}$ days *(d)* $3\frac{9}{11}$ days

(e) None of the above

12. A, B and *C* can complete a work in 2 h. If *A* does the job alone in 6 h and *B* in 5 h, how long will it take for C to finish the job alone? [SSC Multitasking 2014]

(a) $6\frac{1}{2}$ h *(b)* $7\frac{1}{2}$ h *(c)* 9 h *(d)* $4\frac{1}{2}$ h

13. *A* and *B* can do a piece of work in 18 days. *B* and *C* in 24 days, *C* and *A* in 36 days. Find the time in which *A, B* and C working together can finish the work.

(a) 8 *(b)* 16 *(c)* 24 *(d)* 36

14. A and *B* can do a piece of work in 72 days. *B* and *C* can do it in 120 days. *A* and C can do it in 90 days. In what time can *A* alone do it? [SSC CCL 2011]

(a) 80 days *(b)* 100 days

(c) 120 days (d) 150 days

15. A and *B* can do a piece of work in 10 h. *B* and C can do it in 15 h, while *A* and C take 12 h to complete the work. *B* independently can complete the work in [CDS 2012]

(a) 12 h *(b)* 16 h *(c)* 20 h *(d)* 24 h

16. A can do a piece of work in 10 days and *B* in 20 days. They begin together but *A* leaves 2 days before the completion of the work. The whole work will be done in

[SSC (10+2) 2012]

(a) 8 days *(b)* $7\frac{5}{3}$ days

(c) 7 days *(d)* 6 days

17. A and *B* together can complete a work in

3 days. They started together but after 2 days, *B* left the work. If the work is completed after 2 more days, *B* alone could do the work in how many days?

[SSC CCL 2007] *(a)* 5 *(b)* 6 *(c)* 7 fa; 10

18. A can do a piece of work in 10 days, while *B* can do it in 6 days. *B* worked at it for

4 days. How long will *A* take to finish the remaining work?

(a) $3\frac{1}{3}$ days (b) $3\frac{2}{3}$ days

(c) $3\frac{2}{5}$ days (d) $3\frac{5}{7}$

(e) None of the above

19. A and *B* can complete a job in 24 days working together. *A* alone can complete it in 32 days. Both of them worked together for 8 days and then *A* left. The number of days *B* will take to complete the remaining job is [SSC CCL 2012]

(a) 16 *(b)* 32 *(c)* 64 fa; 128

20. A contractor undertook to do a certain piece of work in 18 days. He employed certain number of men but 12 of them being absent from the very 1st day, the rest could finish the work in 30 days. Find the number of men originally employed.

(a) 40 *(b)* 15 *(c)* 45 *(d)* 30

(e) None of the above

21. P and *Q* can finish a work in 30 days. They worked at it for 10 days and then *Q* left. The remaining work is done by *P* alone in 20 more days. How long will *P* take to finish the work alone?

fa; 30 days *(b)* 20 days

(c) 60 days *(d)* 50 days

(e) None of the above

22. A and *B* can do a piece of work in 60 days and 75 days, respectively. Both begin together but after a certain time *A* leaves off. In such case *B* finishes the remaining work in 30 days. After how many days did *A* leave?

(a) 25 days *(b)* 21 days

(c) 20 days *(d)* 24 days

23. Ajay can do a piece of work in 25 days and Sanjay can finish it in 20 days. They work together for 5 days and then Ajay goes away. In how many days will Sanjay finish the remaining work? [DMRC CRA 2012]

(a) 11 days *(b)* 12 days

fc; 14 days fa; None of these

24. A and *B* can complete a work in 8 days, working together. *B* alone can do it in 12 days. After working for 4 days, *B* left the work. How many days will *A* take to complete the remaining work?

fa; 16 days *(b)* 18 days

fc; 20 days *(d)* 22 days

(e) 24 days 25.10 men can make a wall in 8 days. How many men required to complete the same work in half day ? [SSC CCL 2012]

(a) 80 *(b)* 100 *(c)* 120 *(d)* 160

26.6 boys can complete a piece of work in 16 h. In how many hours will 8 boys complete the same work? [Bank Clerks 2011]

(a) 10 *(b)* 8 *(c)* 12 *(d)* 14

fe; None of the above

27. In a hostel, there are 120 students and food stock is for 45 days. If 30 new students join the hostel, in how many days will the complete stock be exhausted? [SSC FCI 2012]

fa; 38 *(b)* 40 *(c)* 32 *(d)* 36

28. If 5 boys take 7 h to pack 35 toys, how many boys can pack 65 toys in 3 h?

[Bank Clerks 2007]

(a) 26 *(b)* 39 *(c)* 45 *(d)* 65 *(e)* None of the above

29.20 women can complete a piece of work in 7 days. If 8 more women are put on the job. In how many days will they complete the work? [DMRC CRA 2012] *(a)* 4.5 days *(b)* 5 days

(c) 5.5 days fa; 4.5 days

30.12 men can do a piece of work in 24 days. How many days are needed to complete the work, if 8 men do this work?

[IDBI Officers 2010] *(a)* 28 *(b)* 36 *(c)* 48 fa; 52 fe; None of the above

20 men can cut 30 trees in 4 h. If 4 men leave the job, how many trees will be cut in 12 h? [UP Police 2007]

(a) 72 *(b)* 80 *(c)* 68 *(d)* 79

32.40 men can build a wall 200 m long in 12 days, working 8 h a day. What will be the number of days that 30 men will take to build a similar wall 300 m long, working 6 h per day [SSC FCI 2012]

(a) 32 *(b)* 18 *(c)* 36 *(d)* 9

33. If *m* men working *m* h per day, can do *m* units of work in *m* days, then n men working *n* h per day would be able to complete how many units of work in *n* days?

(a) $\frac{n^3}{m^2}$ (b) $\frac{m^3}{n^2}$ (c) $\frac{m^4}{n^2}$ (d) $\frac{n^4}{m^3}$

34.20 men complete one-third of a work in 20 days. How many more men should be employed to finish the rest of work in 25 more days? [CBO 2007]

(a) 15 *(b)* 12

(c) 18 *(d)* 25

(e) None of the above

35.15 men complete a work in 16 days. If 24 men are employed, then the time required to complete that work will be

[CDS 2014]

(a) 7 days *(b)* 8 days

(c) 10 days *(d)* 12 days

36.20 workers working for 5 h per day complete a work in 10 days. If 25 workers are employed to work 10 h per day, what is the time required to complete the work? [CDS 2013]

(a) 4 days *(b)* 5 days

(c) 6 days *(d)* 8 days

37. A certain number of men can do a piece of work in 80 days. If there were 10 men less, It could be finished in 20 days more. How many men are there in the starting?

(a) 45 *(b)* 50

(c) 40 *(d)* 60

38. A stock of food is enough for 240 men for 48 days. How long will the same stock last for 160 men? [CDS 2012]

(a) 54 days *(b)* 60 days

(c) 64 days *(d)* 72 days

39.45 people take 18 days to dig a pond. If the pond would have to be dug in 15 days, then the number of people to be employed will be [CDS 2012]

(a) 50 *(b)* 54

(c) 60 *(d)* 72

40. If 3 men or 4 women can build a wall in 43 days, in how many days can 7 men and 5 women build this wall ?

[SSC (10+2) 2012]

(a) 16 days *(b)* 25 days

(c) 21 days *(d)* 12 days

41. If 12 men or 18 women can do a piece of work in 14 days. How long will 8 men and 16 women take to finish the work?

[RRB 2007]

(a) 9 days *(b)* 12 days

(c) 13 days *(d)* 10 days

42. In a school, Mid-Day Meal food is sufficient for 250 students for 33 days, if each student is given 125 g meals. 80 more students joined the school. If same amount of meal is given to each student, then the food will last fbkAT 2013]

(a) 20 days *(b)* 40 days

(c) 30 days *(d)* 25 days

43.90 men are engaged to do a piece of work in 40 days but it is found that in 25 days, 2/3 work is completed. How many men should be allowed to go off, so that the work may be finished in time?

[SSC (10+2) 2008]

(a) 10 *(b)* 15 *(c)* 20 *(d)* 25

44.36 workmen are employed to finish a certain work in 48 days. But it is found that in 24 days only 2/5 work is done. How many more men must be employed to finish the work in time? [LIC ADO 2007]

(a) 16 *(b)* 18 *(c)* 20 *(d)* 22

(e) None of the above

45. It is given that 16 men working 18 h a day can build a wall 36 m long, 4 m broad and 24 m high in 20 days. How many men will be required to build a wall 64 m long, 6 m broad and 18 m high working 12 h a day in 16 days? *(a)* 60 *(b)* 20 *(c)* 30 *(d)* 35

46.3 men can do a piece of work in 6 days.

5 women can do the same work in 18 days. If 4 men and 10 women work together, then how long will it take to finish the work? [Bank PO 2010]

(a) 3 days (b) 5 days

(c) 2 days (d) 4 days

(e) None of the above

47.3 men can do a piece of work in 18 days.

6 children can also do that work in 18 days. 4 men and 4 children together will finish the work in how many days?

[Bank Clerks 2010] (a) 10 (b) 6 (c) 12 (d) 9 (e) None of the above

48. A man can do a piece of work in 5 days. With the help of his friend, he can do the same work in 3 days. In how many days can his friend complete the work?

(a) 8 days (b) $6\frac{1}{2}$ days

(c) 7 days (d) $7\frac{1}{2}$ days

(e) None of the above

49. *P* can do a piece of work in 12 days, while *Q* alone can finish it in 8 days. With the help of *R*, they can finish the work in 4 days. How long will *R* take to finish the work alone?

(a) 25 days (b) 34 days

(c) 14 days (d) 24 days

(e) None of the above

50. A alone can do a certain job in 15 days, while *B* alone can do it in 10 days. A started the work and was joined by *B* after 5 days. The work lasted for how many days?

(a) 4 (b) 8 (c) 5 (d) 9 (e) None of the above

51. A mason can build a tank in 12 h. After working for 6 h, he took the help of a boy and finished the work in another 5 h. The time that the boy will take alone to complete the work is [CDS 2013]

(a) 30 h (b) 45 h (c) 60 h (d) 64 h

52. A and *B* can do a job together in 12 days. A is 2 times as efficient as *B*. In how many days can *B* alone complete the work? [SSC (10+2) 2012]

(a) 36 *(b)* 12 *(c)* 18 *(d)* 9

53.16 children and 24 men complete a certain work in 18 days. If each child takes twice the time taken by a man to finish the work, in how many days will 24 men finish the same work? *(a)* 12 days *(b)* 24 days

(c) 36 days *(d)* 48 days

(e) None of the above

54. A does 20% less work than *B*. If A can complete a piece of work in 7V£h, then *B* can do it in [SSC CGL 2013]

(a) 4 h *(b)* 6 h *(c)* 8 h *(d)* 10 h

55. A takes twice as much time as *B* and C takes thrice as much time as *B* to finish a work. Working together, they can finish the work in 12 days. Find the number of days needed for A to do the work alone. [SSC CGL 2011]

(a) 20 *(b)* 22 *(c)* 33 *(d)* 44

56. A is thrice as good a workman as *B* and therefore is able to finish a job in 30 days less than *B*. How many days will they take to finish the job working together?

[RRB 2007]

(a) $10\frac{1}{4}$ *(b)* $11\frac{3}{4}$ *(c)* $11\frac{1}{4}$ *(d)* $11\frac{1}{3}$

57.X can do a work in 16 days. In how many

days will the work be completed by *Y,* if

the efficiency of *Y* is 60% more than that

of-X? [CDS 2013]

(a) 10 days *(b)* 12 days

(c) 25 days *(d)* 30 days

58.10 men and 8 women together can complete a work in 5 days. Work done by one woman in a day is equal to half the work done by a man in 1 day. How many days will it take for 4 men and 6 women to complete that work? [Bank Clerks 2009]

(a) 12 days *(b)* 10 days

2 3

(c) 8- days *(d)* 4- days

(e) None of the above

59. A takes twice the time taken by *B* and thrice the time taken by *C* to do a particular piece of work. Working together, they can complete the work in 2 days. Find the number of days taken by A, *B* and C respectively to complete the work alone. [LIC ADO 2007]

(a) 12, 6 and 4 *(b)* 18, 9 and 6 *(c)* 24, 12 and 8 *(d)* 6, 3 and 2 *(e)* None of the above

60. *X* can do 20% of a work in a day, *Y* can do 25% of the same work in a day and *X, Y* and *Z* together can do 50% of the same work in a day. How many per cent of work can be done by *Z* in a day?

(a) 5% *(b)* 10% *(c)* 15% *(d)* 20% *(e)* 25%

61. If one man or two women or three boys can finish a work in 88 days, then how many days will one man, one woman and one boy together take to finish the same work? [Bank Clerks 2009]

(a) 46 days *(b)* 54 days

(c) 48 days *(d)* 44 days

(e) 60 days

62. When A alone does a piece of work, he takes 25 days more than the time taken by (A + *B)* to complete that particular work, while *B* alone takes 49 days more than the time taken by (A + *B)* to finish the same work. A and *B* together will take what time to finish this work?

(a) 35 days *(b)* 25 days

(c) 15 days *(d)* 45 days

(e) None of the above 63.X can do 1/4 of a work in 10 days, *Y* can do 40% of the same work in 40 days and *Z* can do 1/ 3 of the work in 13 days. Who will complete the work first?

(a) X *(b)* Y

(c) Z *(d)* Both X and Z

(e) None of the above 64.5 men start working to complete a work in 15 days. After 5 days, 10 women are accompanied by them to complete the work in next 5 days. If the work is to be done by women only, when could the work be over, if 10 women have started it? [Bank Clerks 2007]

(a) 10 days *(b)* 18 days

(c) 15 days *(d)* 12 days

(e) None of the above

65.4 men or 6 women or 10 children can

paint a house in 5 days. The painting is

given to a couple and their 5 sons. They

finish the job in [SSC Multitasking 2014]

(a) $\frac{11}{60}$ days *(b)* $5\frac{5}{11}$ days

(c) $5\frac{6}{11}$ days *(d)* $11\frac{1}{5}$ days

66.4 goats or 6 sheeps can graze a field in 50 days. 2 goats and 9 sheeps can graze the field in [CDS 2013]

(a) 100 days *(b)* 75 days

(c) 50 days *(d)* 25 days

67. If 14 men and 12 boys can finish work in 4 days, while 8 men and 16 boys can finish the same work in 5 days. Compare the 1 day work of 1 man and 1 boy.

(a) 2 (b) $1\frac{1}{2}$

(c) $\frac{1}{2}$ (d) 3

68. If the work done by 8 men and 4 boys in 1 day is 7 times the work done by 1 man and 1 boy, then compare the work done by 1 man and 1 boy in 1 day?

[CDS 2013]

(a) 1 (b) 2 (c) 3 (d) $\frac{1}{2}$

Exercise © *Higher Skill Level Questions*

1. *A* and *B* can complete a work in 12 days and 18 days, respectively. *A* begins to do the work and they work alternately one at a time for 1 day each. The whole work will be completed in how many days?

[SSC CGL 2007]

(a) $14\frac{2}{3}$ (b) $14\frac{1}{9}$ (c) $14\frac{3}{5}$ (d) $14\frac{5}{7}$

2. In a factory, there are equal number of women and children. Women work for 6 h a day and children for 4 h a day. During festival time, the work load goes up by 50%. The government rule does not allow children to work for more than 6 h a day. If they are equally efficient and the extra work is done by women, then extra hours of work put in by women everyday are [SSC CGL 2012]

(a) 5 *(b)* 3 *(c)* 4 *(d)* 9

3. 3 women and 18 children together take

2 days to complete a piece of work. How many days will 9 children alone take to complete the piece of work, if 6 women alone can complete the piece of work in

3 days? [IBPS Clerk 2011] *(a)* 9 *(b) 7 (c)* 5 *(d)* 6

(e) None of the above

4. A contractor undertook to finish a certain work in 124 days and employed 120 men. After 64 days, he found that he had already done 2/3 of the work. How many men can be discharged now, so that the work may finish in time ?

[SSC CGL 2013] *(a)* 40 *(b)* 50 *(c)* 48 *(d)* 56

5. A contract is to be completed in 92 days and 234 men were set to work, each working 16 h a day. After 66 days, 4/7 of the work is completed. How many additional men may be employed, so that the work may be completed in time, each man now working 18 h a day?

(a) 162 *(b)* 234 *(c)* 262 *(d)* 81 *(e)* None of the above

6. *A, B* and C can do a piece of work in 16, 32 and 48 days, respectively. They all begin together. *A* work continuously till it is finished, C leaves the work 2 days before its completion and *B* leaves the work 1 day before its completion. In what time is the work finished?

(a) 10 days *(b)* 20 days

(c) 15 days *(d)* 30 days

(e) None of the above

7. Rashmi can do a piece of work in 16 days. Ravina can do the same work in $12\frac{4}{5}$ days, while Gitika can do it in 32 days. All of them started to work together but Rashmi leaves after 4 days. Ravina leaves the job 3 days before the completion of the work. How long would the work last?

(a) 9 days *(b)* 6 days

(c) 18 days *(d)* 5 days

8. A contract is to be completed in 50 days and 105 men were set to work, each working 8 h a day. After 25 days, 2/5th of the work is finished. How many additional men be employed, so that the work may be completed on time, each man now working 9 h a day ? [SNAP 2012]

(a) 34 *(b)* 36 *(c)* 35 *(d)* 37 *(e)* None of the above

9. *A* can build up a structure in 8 days and *B* can break it in 3 days. *A* has worked for 4 days and then *B* joined to work with *A* for another 2 days only. In how many days will *A* alone build up the remaining part of the structure ? [SNAP 2012]

(a) 10 days *(b)* 9 days

(c) 12 days *(d)* 13 days *(e)* None of these

10. *A, B* and C can do a piece of work in 18, 24 and 36 days, respectively. They work alternately, first day C, second day *B* and third day *A,* fourth day C and so on. How many days will be needed to complete the work like this way?

(a) 18 *(b)* 20 *(c)* 24 *(d)* 30

(e) 36 11.6 men can do a piece of work in 12 days while 8 women can do the same work in 18 days. The same work can be done by 18 children in 10 days. 4 men, 12 women and 20 children work together for 2 days. If only men have to complete remaining work in 1 day, then find the required number of men. [Bank PO 2010]

(a) 36 *(b)* 24 *(c)* 18

(d) Couldn't be determined

(e) None of the above

12. *A* can complete a work in 20 days and *B* in 30 days. *A* worked alone for 4 days and then *B* completed the remaining work along with C in 18 days. In how many days can *C* working alone complete the work ? [SSC CGL 2012]

(a) 12 *(b)* 68

(c) 72 *(d)* 90

13. *A* and *B* each working alone can do a work in 15 days and 25 days, respectively. They started the work together but *B* left after some time and *A* finished the remaining work in 7 days. After how many days from the start did *B* leave? [SSC CGL 2012]

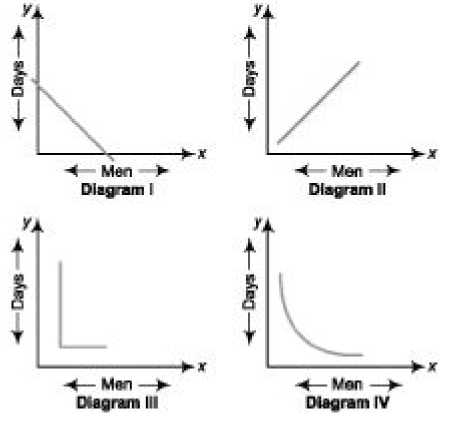
(a) 3 *(b)* 5 *(c)* 7 *(d)* 9

14. *A* and *B* can do a piece of work in 9 days and 18 days, respectively. As they were ill, they could do 90% and 72% of their efficiency, respectively. How many days will they take to complete the work together?

(a) $7\frac{1}{7}$ (b) 8 (c) 5 (d) 10

(e) $12\frac{1}{2}$

15. Consider the following diagrams *x* men, working at constant speed, do a certain job in *y* days. Which one of these diagrams shows the relation between xand;y? [CSAT2013]



(a) Diagram I *(b)* Diagram II

(c) Diagram III *(d)* Diagram IV

16. *A, B* and C can do a piece of work individually in 8, 12 and 15 days, respectively. *A* and *B* start working but *A* quits after working for 2 days. After this, C joins *B* till the completion of work. In how many days will the work be completed? [CDS 2014]

(a) $5\frac{8}{9}$ days (b) $4\frac{6}{7}$ days

(c) $6\frac{7}{13}$ days (d) $3\frac{3}{4}$ days

Answer with Solutions

Exercise© *Base Level Questions*

1. (*a*) A's 1 day's work $= \frac{1}{18}$

B's 1 day's work $= \frac{1}{9}$

$(A + B)$'s 1 day's work $= \frac{1}{9} + \frac{1}{18} = \frac{3}{18} = \frac{1}{6}$

2. (*c*) $(A + B)$'s 1 day's work $= \frac{1}{12}$

B's 1 day's work $= \frac{1}{30}$

$\therefore A$'s 1 day's work $= \frac{1}{12} - \frac{1}{30}$

$$= \frac{5-2}{60} = \frac{1}{20}$$

Fast Track Method

Here, $x = 12$ and $y = 30$

$\therefore$ Required number of days

$$= \frac{xy}{y-x} = \frac{12 \times 30}{30 - 12} \text{ [by Technique 3]}$$

$$= \frac{12 \times 30}{18} = 20 \text{ days}$$

$\therefore A$ alone can finish the work in 20 days.

3. (*a*) We have the important relation,

More work, More time (days)

$\because$ A piece of work can be done in 6 days.

$\therefore$ Three times of work of same type can be done in $6 \times 3 = 18$ days.

4. (*d*) A's 1 day's work $= \frac{1}{6}$

B's 1 day's work $= \frac{1}{12}$

$(A + B)$'s 1 day's work $= \frac{1}{6} + \frac{1}{12}$

$$= \frac{2+1}{12} = \frac{3}{12} = \frac{1}{4}$$

Hence, both will complete the work in 4 days.

Fast Track Method

Here, $x = 6$, and $y = 12$

$\therefore$ Time taken by $(A + B)$

[by Technique 2]

$$= \frac{xy}{x+y} = \frac{6 \times 12}{12 + 6} = 4 \text{ days}$$

5. (*d*) A's 1 day's work $= \dfrac{1}{12}$

B's 1 day's work $= \dfrac{1}{24}$

$(A + B)$'s 1 day's work

$$= \frac{1}{12} + \frac{1}{24}$$

$$= \frac{2+1}{24} = \frac{3}{24} = \frac{1}{8}$$

$\therefore$ $(A + B)$ will finish the work in 8 days.

6. (*c*) X's 1 day's work $= \dfrac{1}{12}$

$(X + Y)$'s 1 day's work $= \dfrac{3}{20}$

$\therefore Y$'s 1 day's work $= \dfrac{3}{20} - \dfrac{1}{12} = \dfrac{4}{60} = \dfrac{1}{15}$

$\therefore$ Number of day's taken by Y to complete the work = 15 days

7. (*b*) Let the men do the work in a days and the boys in b days.

$$\therefore \quad \frac{3}{a} + \frac{4}{b} = \frac{1}{8} \quad \ldots(i)$$

Now, $\dfrac{6}{a} + \dfrac{8}{b} = 2\left(\dfrac{3}{a} + \dfrac{4}{b}\right) = 2 \times \dfrac{1}{8} = \dfrac{1}{4}$

[from Eq. (i)]

So, 6 men and 8 boys can do the same work in 4 days.

8. (*d*) A's 1 day's work $A = \dfrac{1}{x}$

B's 1 day's work $B = \dfrac{1}{3x}$

$(A + B)$'s 1 day's work

$$= \frac{1}{x} + \frac{1}{3x} = \frac{4}{3x}$$

and given one day work of both A

and $\quad B = \dfrac{1}{12}$

$$\Rightarrow \quad \frac{4}{3x} = \frac{1}{12} \Rightarrow 3x = 48 \Rightarrow x = 16$$

9. (*b*) A completes the work in

$$5 \times 2 = 10 \text{ days}$$

B completes the work in

$$10 \times \frac{5}{2} = 25 \text{ days}$$

$\therefore$ A's 1 day's work $= \frac{1}{10}$

B's 1 day's work $= \frac{1}{25}$

$(A + B)$'s 1 day's work $= \frac{1}{10} + \frac{1}{25}$

$= \frac{5+2}{50} = \frac{7}{50}$

$\therefore$ $(A + B)$ will complete the whole work in $\frac{50}{7}$ days.

$\therefore$ Required time $= \frac{50}{7} = 7\frac{1}{7}$ days

10. (*b*) A's 1 day's work $= \frac{1}{4}$

B's 1 day's work $= \frac{1}{12}$

$(A + B)$'s 1 day's work $= \frac{1}{4} + \frac{1}{12}$

$= \frac{3+1}{12} = \frac{4}{12} = \frac{1}{3}$

Days required by A and B together to do the work

$$= \frac{1}{(A + B)\text{'s 1 day's work}} = 3 \text{ days}$$

11. (*a*) A's 1 day's work $= \frac{1}{8}$

B's 1 day's work $= \frac{1}{10}$

C's 1 day's work $= \frac{1}{20}$

$(A + B + C)$'s 1 day's work

$= \frac{1}{8} + \frac{1}{10} + \frac{1}{20} = \frac{5+4+2}{40} = \frac{11}{40}$

$\therefore$ $(A + B + C)$ can finish the work in

$\frac{40}{11}$ days or $3\frac{7}{11}$ days.

Fast Track Method

Here, $x = 8$, $y = 10$, $z = 20$

According to the formula,

Time taken by A, B and C to complete

the work $= \frac{xyz}{xy + yz + zx}$ [by Technique 2]

$$= \frac{8 \times 10 \times 20}{8 \times 10 + 10 \times 20 + 20 \times 8}$$

$$= \frac{1600}{80 + 200 + 160}$$

$$= \frac{1600}{440}$$

$$= 3\frac{7}{11} \text{ days}$$

12. (*b*) Let C alone can finish the job in x h.
According to the question,

Work done by A, B and C in 1 h $= \frac{1}{2}$

$$\Rightarrow \quad \frac{1}{6} + \frac{1}{5} + \frac{1}{x} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{2} - \frac{1}{6} - \frac{1}{5} = \frac{15 - 5 - 6}{30}$$

$$= \frac{4}{30} = \frac{2}{15}$$

$$\therefore \quad x = 7\frac{1}{2} \text{ h}$$

13. (*b*) $(A + B)$'s 1 day's work $= \frac{1}{18}$

$(B + C)$'s 1 day's work $= \frac{1}{24}$

$(C + A)$'s 1 day's work $= \frac{1}{36}$

From Eqs. (i), (ii) and (iii), we get

2$(A + B + C)$'s 1 day's work

$$= \frac{1}{18} + \frac{1}{24} + \frac{1}{36}$$

$(A + B + C)$'s 1 day's work $= \frac{4 + 3 + 2}{72 \times 2}$

$$= \frac{9}{72 \times 2} = \frac{1}{16}$$

$\therefore (A + B + C)$ can complete the work in 16 days

Fast Track Method

Here, $x = 18$, $y = 24$ and $z = 36$

According to the formula,

Required time taken by A, B and C

$$= \frac{2xyz}{xy + yz + zx} \quad \text{[by Technique 4]}$$

$$= \frac{2 \times 18 \times 24 \times 36}{18 \times 24 + 24 \times 36 + 36 \times 18}$$

$$= \frac{2 \times 18 \times 24 \times 36}{1944} = 16 \text{ days}$$

14. (*c*) $(A + B)$'s 1 day's work $= \frac{1}{72}$

$(B + C)$'s 1 day's work $= \frac{1}{120}$

$(A + C)$'s 1 day's work $= \frac{1}{90}$

2$(A + B + C)$'s 1 day's work

$$= \frac{1}{72} + \frac{1}{120} + \frac{1}{90}$$

$\therefore$ $(A + B + C)$'s 1 day's work

$$= \frac{5 + 3 + 4}{360 \times 2} = \frac{12}{360 \times 2} = \frac{1}{60}$$

$\therefore$ A's 1 day's work = $(A + B + C)$'s 1 day's work − $(B + C)$'s 1 day's work

$$= \frac{1}{60} - \frac{1}{120} = \frac{2-1}{120} = \frac{1}{120}$$

$\therefore$ A alone can finish the work in 120 days.

15. (*d*) A's and B's 1 h work $= \frac{1}{10}$

B's and C's 1 h work $= \frac{1}{15}$

and A's and C's 1 h work $= \frac{1}{12}$

$\therefore$ A's, B's and C's 1 h work

$$= \frac{1}{2}\left(\frac{1}{10} + \frac{1}{15} + \frac{1}{12}\right) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

Hence, B's work in 1 h $= \frac{1}{8} - \frac{1}{12} = \frac{1}{24}$

$\therefore$ B independently can complete the work in 24 h.

16. (*a*) Let the required days be x.

A works for $(x - 2)$ days, while B works for x days.

According to the question,

$$\frac{x-2}{10} + \frac{x}{20} = 1$$

$$\Rightarrow \quad 2x - 4 + x = 20$$

$$\Rightarrow \quad 3x = 24$$

$$\therefore \quad x = 8 \text{ days}$$

Fast Track Method

Here, $a = 10$, $b = 20$ and $x = 2$

$\therefore$ Required time

$$= \frac{(a + x) \times b}{(a + b)} \qquad \text{[by Technique 12]}$$

$$= \frac{(10 + 2) \times 20}{30} = 8 \text{ days}$$

17. (*b*) $(A + B)$'s 2 day's work $= 2 \times \frac{1}{3} = \frac{2}{3}$

Remaining work $= 1 - \frac{2}{3} = \frac{1}{3}$

A will complete $\frac{1}{3}$ work in 2

A will complete 1 work in 6

A's 1 day's work $= \frac{1}{6}$

B's 1 day's work $= \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

$\therefore$ B will take 6 days to complete the work alone.

18. (*a*) B's 4 day's work $= \frac{1}{6} \times 4 = \frac{2}{3}$

$\therefore$ Remaining work $= 1 - \frac{2}{3} = \frac{1}{3}$

A's 1 day's work $= \frac{1}{10}$

$\therefore$ $\frac{1}{3}$ work is finished by A in

$$\left(10 \times \frac{1}{3}\right) = 3\frac{1}{3} \text{ days}$$

19. (*c*) Let B will take x days to complete the remaining job.

According to the question,

$$\frac{1}{A} + \frac{1}{B} = \frac{1}{24} \text{ and } \frac{1}{A} = \frac{1}{32}$$

$$\therefore \quad \frac{1}{B} = \frac{1}{24} - \frac{1}{32} = \frac{1}{96} \Rightarrow B = 96 \text{ days}$$

According to the question,

$$8\left(\frac{1}{A} + \frac{1}{B}\right) + x \times \frac{1}{B} = 1$$

$$\Rightarrow \quad 8 \times \frac{1}{24} + \frac{x}{96} = 1$$

$$\Rightarrow \quad \frac{1}{3} + \frac{x}{96} = 1$$

$$\Rightarrow \quad \frac{x}{96} = 1 - \frac{1}{3}$$

$$\therefore \quad x = \frac{2 \times 96}{3} = 64$$

Hence, B complete the remaining job in 64 days.

20. (*d*) Let the number of men at the beginning $= x$

Given, $M_1 = x$, $M_2 = x - 12$, $D_1 = 18$ and $D_2 = 30$

According to the formula,

$$M_1 D_1 = M_2 D_2$$

$$\Rightarrow \quad x \times 18 = (x - 12) \times 30$$

$$\Rightarrow \quad 3x = 5x - 60$$

$$\Rightarrow \quad 2x = 60$$

$$\therefore \quad x = 30$$

21. (*a*) $(P + Q)$'s 10 day's work $= \frac{1}{30} \times 10 = \frac{1}{3}$

Remaining work $= \left(1 - \frac{1}{3}\right) = \frac{2}{3}$

$\frac{2}{3}$ work is done by P in 20

$\therefore$ Whole work is done by P in

$$20 \times \frac{3}{2} = 30 \text{ days}$$

22. (*c*) B's 30 day's work $= \frac{30}{75} = \frac{2}{5}$

$$\text{Remaining work} = 1 - \frac{2}{5} = \frac{3}{5}$$

$(A + B)$'s 1 day's work $= \frac{1}{60} + \frac{1}{75}$

$$= \frac{5+4}{300} = \frac{9}{300} = \frac{3}{100}$$

$(A + B)$ do $\frac{3}{100}$ work in 1 day.

$\therefore$ $(A + B)$ will do $\frac{3}{5}$ work in $\frac{3}{5} \times \frac{100}{3}$

or in 20 days.

Fast Track Method

Here, $a = 60$ days, $b = 75$ days,
$x = 30$, $T = ?$

$\therefore$ Required time $= \frac{(b - x)\, a}{a + b}$

[by Technique 13]

$$= \frac{(75 - 30) \times 60}{60 + 75} = \frac{45 \times 60}{135} = 20 \text{ days}$$

$\therefore$ Required time $= 20$ days

23. (*a*) Ajay's 1 day's work $= \frac{1}{25}$

Similarly, Sanjay's 1 day's work $= \frac{1}{20}$

Ajay's and Sanjay's together 1 day's work

$$= \frac{1}{25} + \frac{1}{20} = \frac{4+5}{100} = \frac{9}{100}$$

Their 5 day's work together

$$= 5 \times 1 \text{ day's work}$$

$$= 5 \times \frac{9}{100} = \frac{45}{100}$$

Remaining work $= 1 - 5$ day's work

$$= 1 - \frac{45}{100} = \frac{55}{100}$$

Now, this remaining work is done by Sanjay.
Let Sanjay takes x days to complete it.

Then, $\quad \frac{1}{20} \times x = \frac{55}{100}$

$\Rightarrow \quad x = \frac{55 \times 20}{100}$

$\therefore \quad x = 11$ days

So, remaining work is done in 11 days by Sanjay.

Fast Track Method

Here, $a = 25$ days, $b = 20$ days,
$x = ?$ and $T = 5$

According to the formula,

$$T = \frac{(b - x) \times a}{a + b} \quad \text{[by Technique 13]}$$

$\Rightarrow \quad 5 = \frac{(20 - x) \times 25}{45}$

$\Rightarrow \quad 45 = 100 - 5x$

$\Rightarrow \quad 5x = 55$

$\therefore \quad x = 11$ days

24. (*a*) $(A + B)$'s 1 day's work $= \frac{1}{8}$

B's 1 day's work $= \frac{1}{12}$

$\therefore$ A's 1 day's work

$$= \frac{1}{8} - \frac{1}{12} = \frac{3-2}{24} = \frac{1}{24}$$

$\therefore$ A can complete the work in 24 days.

Now, B's 4 day's work $= \frac{4}{12} = \frac{1}{3}$

Remaining work $= 1 - \frac{1}{3} = \frac{2}{3}$

As, time taken by A to complete the work is 24 days.

$\therefore$ Time taken by A to do $\frac{2}{3}$ of the work

$$= \frac{2}{3} \times 24 = 16 \text{ days}$$

25. (*d*) Given, $M_1 = 10$

$$D_1 = 8, M_2 = ? \text{ and } D_2 = \frac{1}{2}$$

From $M_1 D_1 = M_2 D_2$ **[by Technique 1]**

$\Rightarrow$ $10 \times 8 = M_2 \times \frac{1}{2}$

$\Rightarrow$ $M_2 = 10 \times 8 \times 2$

$\therefore$ $M_2 = 160$

26. (*c*) Given, $M_1 = 6$, $M_2 = 8$, $T_1 = 16$ h,

and $T_2 = ?$, $W_1 = W_2 = 1$

According to the formula,

$$M_1 T_1 W_2 = M_2 T_2 W_1$$

$\therefore$ $6 \times 16 \times 1 = 8 \times T_2 \times 1$

$\therefore$ $T_2 = \frac{16 \times 6}{8} = 2 \times 6 = 12$ h

27. (*d*) Let food stock will be exhausted in x days.

Given, $M_1 = 120, D_1 = 45$

$M_2 = 120 + 30 = 150$

and $D_2 = x$

Then, using $M_1 D_1 = M_2 D_2$

$\Rightarrow$ $120 \times 45 = 150 \times x$

$\therefore$ $x = \frac{120 \times 45}{150} = 36$

28. (*e*) Given, $M_1 = 5$, $M_2 = ?$, $T_1 = 7$, $T_2 = 3$,

$W_1 = 35$ and $W_2 = 65$

According to the formula,

$M_1 T_1 W_2 = M_2 T_2 W_1$ **[by Technique 1]**

$\Rightarrow$ $5 \times 7 \times 65 = M_2 \times 3 \times 35$

$\therefore$ $M_2 = \frac{5 \times 7 \times 65}{35 \times 3} = \frac{65}{3}$ boys

29. (*b*) Given, $M_1 = 20$

$D_1 = 7$

$M_2 = 20 + 8 = 28$

and $D_2 = ?$

Using formula, $M_1D_1 = M_2D_2$

[by Technique 1]

$\Rightarrow \quad 20 \times 7 = 28 \times D_2$

$\therefore \quad D_2 = \dfrac{20 \times 7}{28} = 5$ days

30. (*b*) Given, $M_1 = 12$, $M_2 = 8$, $D_1 = 24$, $W_1 = 1$

$W_2 = 1$

According to the formula,

$M_1D_1W_2 = M_2D_2W_1$

[by Technique 1]

$\Rightarrow \quad 12 \times 24 \times 1 = 8 \times D_2 \times 1$

$\therefore \quad D_2 = \dfrac{12 \times 24}{8}$

$= 12 \times 3 = 36$ days

31. (*a*) Given, $M_1 = 20$, $T_1 = 4$, $W_1 = 30$,

$M_2 = 20 - 4 = 16$

$T_2 = 12$ and $W_2 = ?$

According to the formula,

$M_1T_1W_2 = M_2T_2W_1$ **[by Technique 1]**

$\Rightarrow \quad 20 \times 4 \times W_2 = 16 \times 12 \times 30$

$\therefore \quad W_2 = \dfrac{16 \times 12 \times 30}{20 \times 4}$

$= 12 \times 6 = 72$ trees

32. (*a*) Given, $M_1 = 40, W_1 = 200, T_1 = 8,$

$D_1 = 12, \; M_2 = 30, W_2 = 300,$

$T_2 = 6$ and $D_2 = ?$

Then, using

$M_1T_1D_1W_2 = M_2T_2D_2W_1$ **[by Technique 1]**

$\Rightarrow 40 \times 8 \times 12 \times 300 = 30 \times 6 \times D_2 \times 200$

$\therefore \quad D_2 = \dfrac{40 \times 8 \times 12 \times 300}{30 \times 6 \times 200} = 32$ days.

33. (*a*) Let the required number of units of work $= x$

According to the formula,

$W_1 = m$ and $W_2 = x$

$M_1T_1D_1W_2 = M_2T_2D_2W_1$

$\Rightarrow m \times m \times m \times x = n \times n \times n \times m$

$\therefore \quad x = \dfrac{m \times n^3}{m^3} = \dfrac{n^3}{m^2}$

34. (*b*) Given, $M_1 = 20$,

$M_2 = 20 + x$

$W_1 = \dfrac{1}{3}, W_2 = 1 - \dfrac{1}{3} = \dfrac{2}{3}$

$D_1 = 20$ and $D_2 = 25$

According to the formula,

$$M_1 D_1 W_2 = M_2 D_2 W_1$$

$\Rightarrow 20 \times 20 \times \left(\frac{2}{3}\right) = (20 + x) \times 25 \times \frac{1}{3}$

$\Rightarrow \quad 20 \times 20 \times \frac{2}{3} = (20 + x) \times \frac{25}{3}$

$\Rightarrow \quad 16 \times 2 = 20 + x$

$\therefore \quad x = 32 - 20 = 12$

35. (*c*) Let the work done be 1 work.

Here, $M_1 = 15$, $D_1 = 16$, $W_1 = W_2 = 1$

$M_2 = 24$, $D_2 = ?$

$\therefore$ According to the formula,

$M_1 D_1 W_2 = M_2 D_2 W_1$ **[by Technique 1]**

$\Rightarrow \quad 15 \times 16 \times 1 = 24 \times D_2 \times 1$

$\therefore \quad D_1 = \frac{15 \times 16}{24} = 10$

Therefore, 10 days are required to complete the work.

36. (*a*) Given, $M_1 = 20$, $M_2 = 25$, $T_1 = 5$, $T_2 = 10$

$D_1 = 10$ and $D_2 = ?$

According to the formula,

$M_1 T_1 D_1 = M_2 T_2 D_2$ **[by Technique 1]**

$\Rightarrow \quad 20 \times 5 \times 10 = 25 \times 10 \times D_2$

$\therefore \quad D_2 = \frac{20 \times 5 \times 10}{25 \times 10} = 4$ days

37. (*b*) Let original number of men $= x$

Time taken by $x = 80$ days

Now, $(x - 10)$ men can finish the work in $(80 + 20) = 100$ days

Here, $M_1 = x$, $M_2 = (x - 10)$, $D_1 = 80$

and $\quad D_2 = 100$

According to the formula,

$M_1 D_1 = M_2 D_2$ **[by Technique 1]**

$\Rightarrow \quad x \times 80 = 100 \times (x - 10)$

$\Rightarrow \quad 8x = 10x - 100$

$\Rightarrow \quad 10x - 8x = 100 \Rightarrow x = 50$

Fast Track Method

Here, $a = 10$, $D = 80$ and $d = 20$

$\therefore$ Number of men in starting

$= \frac{a(D + d)}{d}$ **[by Technique 6]**

$= \frac{10 \times (80 + 20)}{20} = \frac{10 \times 100}{20} = 50$

38. (*d*) Given, $M_1 = 240$, $D_1 = 48$, $M_2 = 160$

and $D_2 = ?$

According to the formula,

$M_1 D_1 = M_2 D_2$ **[by Technique 1]**

$\Rightarrow 240 \times 48 = 160 \times D_2$

$\therefore \quad D_2 = \frac{240 \times 48}{160} = 72$

39. (*b*) Given, $M_1 = 45$, $D_1 = 18$

$$M_2 = ?, D_2 = 15$$

By using the formula,

$$M_1 D_1 = M_2 D_2 \quad \text{[by Technique 1]}$$

$$\therefore \quad M_2 = \frac{M_1 D_1}{D_2}$$

$$\Rightarrow \quad M_2 = \frac{45 \times 18}{15} = 3 \times 18$$

$$\Rightarrow \quad M_2 = 54$$

40. (*d*) 3 men = 4 women

$$\Rightarrow \quad 1 \text{ man} = \frac{4}{3} \text{ women}$$

$\therefore$ 7 men + 5 women

$$= \left(7 \times \frac{4}{3} + 5\right) = \frac{43}{3} \text{ women}$$

$$\therefore M_1 = 4, D_1 = 43, M_2 = \frac{43}{3}, D_2 = ?$$

and $W_1 = W_2 = 1$

According to the formula,

$$M_1 D_1 W_2 = M_2 D_2 W_1$$

$$\Rightarrow \quad 4 \times 43 \times 1 = \frac{43}{3} \times D_2 \times 1$$

$$\therefore \quad D_2 = 3 \times 4 = 12 \text{ days}$$

41. (*a*) 12 men = 18 women

$$\Rightarrow \quad 1 \text{ man} = \frac{18}{12} \text{ women}$$

$$\Rightarrow \quad 8 \text{ men} = \frac{18}{12} \times 8 = 12 \text{ women}$$

Given, $M_1 = 18$, $M_2 = 12 + 16 = 28$,
$D_1 = 14$, $D_2 = ?$ and $W_1 = W_2 = 1$

According to the formula,

$$M_1 D_1 W_2 = M_2 D_2 W_1$$

$$\Rightarrow \quad 18 \times 14 \times 1 = 28 \times D_2 \times 1$$

$$\therefore \quad D_2 = \frac{18 \times 14}{28} = 9 \text{ days}$$

Fast Track Method

Here, $a_1 = 12$, $b_1 = 18$,

$$a_2 = 8, b_2 = 16 \text{ and } D = 14$$

Number of days

$$= \frac{D a_1 b_1}{a_2 b_1 + a_1 b_2} \quad \text{[by Technique 5]}$$

$$= \frac{14 \times 12 \times 18}{8 \times 18 + 12 \times 16}$$

$$= \frac{14 \times 12 \times 18}{336} = 9 \text{ days}$$

42. (*d*) $M_1 = 250$, $D_1 = 33$ day,

per day meal $W_1 = W_2 = 125$ g

$$M_2 = (250 + 80) = 330 \text{ and } D_2 = ?$$

According to the formula,

$$M_1 D_1 W_2 = M_2 D_2 W_1$$

$\Rightarrow \quad 250 \times 33 \times 125 = 330 \times D_2 \times 125$

$\Rightarrow \quad D_2 = \dfrac{250 \times 33}{330}$

$\therefore \quad D_2 = 25$ days

43. (*b*) Let x men are allowed to go off,

$M_1 = 90,\ D_1 = 25,\ D_2 = 15$

$W_1 = \dfrac{2}{3},\ W_2 = 1 - \dfrac{2}{3} = \dfrac{1}{3}$

$M_2 = 90 - x$

According to the formula,

$M_1 D_1 W_2 = M_2 D_2 W_1$ [by Technique 1]

$\Rightarrow (90 \times 25)\left(\dfrac{1}{3}\right) = (90 - x) \times 15 \times \dfrac{2}{3}$

$\Rightarrow 90 \times 25 \times \dfrac{1}{3} = 10\,(90 - x) \Rightarrow 75 = 90 - x$

$\therefore \quad x = 90 - 75 = 15$

44. (*b*) Let x men are employed.

Given, $M_1 = 36,\ M_2 = 36 + x,\ D_1 = 24,$

$D_2 = 24,\ W_1 = \dfrac{2}{5},\ W_2 = 1 - \dfrac{2}{5} = \dfrac{3}{5}$

According to the formula,

$M_1 D_1 W_2 = M_2 D_2 W_1$ [by Technique 1]

$\Rightarrow 36 \times 24 \times \dfrac{3}{5} = (36 + x) \times 24 \times \dfrac{2}{5}$

$\Rightarrow \quad 36 \times 3 = 2\,(36 + x) \Rightarrow 108 = 72 + 2x$

$\therefore \quad x = \dfrac{36}{2} = 18$ men

45. (*a*) Given, $M_1 = 16,\ T_1 = 18,$

$W_1 = 36 \times 4 \times 24,\ D_1 = 20$ and $M_2 = ?,$

$T_2 = 12,\ W_2 = 64 \times 6 \times 18,\ D_2 = 16$

According to the formula,

$\dfrac{M_1 D_1 T_1}{W_1} = \dfrac{M_2 D_2 T_2}{W_2}$ [by Technique 1]

$\Rightarrow \quad \dfrac{16 \times 20 \times 18}{36 \times 4 \times 24} = \dfrac{M_2 \times 12 \times 16}{64 \times 6 \times 18}$

$\Rightarrow \quad \dfrac{15}{36 \times 24} = \dfrac{M_2 \times 2}{64 \times 6 \times 18}$

$\therefore \quad M_2 = 60$ men

46. (*a*) (3×6) men $= (5 \times 18)$ women

18 men = 90 women

$\therefore$ 1 man = 5 women

$\therefore$ 4 men + 10 women

$= 4 \times 5 + 10 = 30$ women

Given, $M_1 = 5,\ M_2 = 20,\ D_1 = 18,$

$W_1 = W_2 = 1$ and $D_2 = ?$

According to the formula,

$M_1 D_1 W_2 = M_2 D_2 W_1$

$\Rightarrow \quad 5 \times 18 \times 1 = 30 \times D_2 \times 1$

$\therefore \quad D_2 = \dfrac{5 \times 18}{30} = 3$ days

47. (*d*) 3 men = 6 children

$\Rightarrow \quad$ 1 man = 2 children

$\therefore$ 4 men + 4 children

$$= 4 \text{ men} + \frac{4}{2} \text{ men} = 6 \text{ men}$$

Given, $M_1 = 3$, $M_2 = 6$, $D_1 = 18$,

$W_1 = W_2 = 1$ and $D_2 = ?$

According to the formula,

$$M_1 D_1 W_2 = M_2 D_2 W_1$$

$$\Rightarrow \quad 3 \times 18 \times 1 = 6 \times D_2 \times 1$$

$$\therefore \quad D_2 = \frac{3 \times 18}{6} = 9 \text{ days}$$

48. (*d*) (Man and his friend)'s 1 day's work $= \frac{1}{3}$

Man's 1 day's work $= \frac{1}{5}$

His friend's 1 day's work $= \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

Hence, his friend can complete the work in $\frac{15}{2} = 7\frac{1}{2}$ days.

49. (*d*) P's 1 day's work $= \frac{1}{12}$

Q's 1 day's work $= \frac{1}{8}$

$(P + Q)$'s 1 day's work

$$= \frac{1}{12} + \frac{1}{8} = \frac{2+3}{24}$$

$$= \frac{5}{24}$$

$(P + Q + R)$'s 1 day's work $= \frac{1}{4}$

$\therefore \quad R$'s 1 day's work

$$= \frac{1}{4} - \frac{5}{24} = \frac{6-5}{24} = \frac{1}{24}$$

$\therefore$ R will finish the work alone in 24 days.

$\therefore$ Required time = 24 days

50. (*d*) A's 5 days' work $= \frac{1}{15} \times 5 = \frac{1}{3}$

Remaining work $= 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore$ $(A + B)$'s 1 day's work

$$= \frac{1}{15} + \frac{1}{10} = \frac{2+3}{30}$$

$$= \frac{5}{30} = \frac{1}{6}$$

$\therefore$ $\frac{2}{3}$ work will be finished by $(A + B)$ in $\frac{2}{3} \times 6$ days or in 4 days.

$\therefore$ Total time taken = (5 + 4) days = 9 days

51. (c) Mason's 1 h work $= \frac{1}{12}$

Mason's 6 h work $= \frac{6}{12} = \frac{1}{2}$

Remaining work $= 1 - \frac{1}{2} = \frac{1}{2}$

Remaining work can be finished in 5 h total work can be finished in $2 \times 5 = 10$ h.

$$\frac{1}{12} + \frac{1}{B} = \frac{1}{10} \Rightarrow \frac{1}{B} = \frac{1}{10} - \frac{1}{12} = \frac{1}{60}$$

Boy can complete the work in 60 h.

52. (a) Time taken by A to complete a work $= x$ days

$\therefore$ Time taken by B to complete the same work $= 2x$ days

According to the question,

$$\frac{1}{x} + \frac{1}{2x} = \frac{1}{12}$$

$$\Rightarrow \quad \frac{3}{2x} = \frac{1}{12}$$

$$\therefore \quad x = \frac{3 \times 12}{2} = 18$$

Time taken by B to complete the same work $= 2x = 2 \times 18 = 36$ days

53. (b) According to the question,

2 children = 1 man

$\therefore$ (16 children + 24 men) = (8 + 24) men = 32 men

Given, $M_1 = 32$, $M_2 = 24$, $D_1 = 18$, $D_2 = ?$ and $W_1 = W_2 = 1$

According to the formula,

$$M_1 D_1 W_2 = M_2 D_2 W_1$$

$$\Rightarrow \quad 32 \times 18 \times 1 = 24 \times D_2 \times 1$$

$$\therefore \quad D_2 = \frac{32 \times 18}{24} = 8 \times 3 = 24 \text{ days}$$

54. (b) Let time taken by $B = x$

$$\text{Efficiency} \propto \frac{1}{\text{Time taken}}$$

So, if B is 100% efficient, then A is 80% efficient.

$$\text{So,} \quad \frac{80}{100} = \frac{x}{\frac{15}{2}}$$

$$\therefore \quad x = 6 \text{ h}$$

55. (d) Let time taken by $B = x$

Then, time taken by $A = 2x$

and time taken by $C = 3x$

According to the question,

$$\frac{1}{x} + \frac{1}{2x} + \frac{1}{3x} = \frac{1}{12}$$

$\Rightarrow \quad 1 + \frac{1}{2} + \frac{1}{3} = \frac{x}{12}$

$\Rightarrow \quad \frac{6+3+2}{6} = \frac{x}{12}$

$\Rightarrow \quad 11 = \frac{x}{2}$

$\therefore \quad x = 22$

$\therefore$ Required number of days

$= 2x = 2 \times 22 = 44$ days

56. (c) Let A takes x and B takes $3x$ days to finish the job.

According to the question,

$3x - x = 30$

$\therefore \quad x = \frac{30}{2} = 15$ days

$\therefore$ B's time to complete the work

$= 3x = 3 \times 15 = 45$ days

$\therefore$ $(A + B)$'s 1 day's work

$= \frac{1}{15} + \frac{1}{45} = \frac{3+1}{45} = \frac{4}{45}$

$\therefore$ $(A + B)$ will finish the work in $\frac{45}{4}$ days.

$\therefore$ Required time $= \frac{45}{4} = 11\frac{1}{4}$ days.

57. (a) Ratio of times taken by X

and $Y = 160 : 100 = 1.6 : 1$

Suppose Y takes y days to do the work.

Then, $1.6 : 1 :: 16 : y$

$\therefore \quad y = \frac{16 \times 1}{1.6} = 10$ days

58. (b) According to the question,

1 man = 2 women

$\therefore$ 10 men + 8 women

= (20 + 8) women = 28 women

4 men + 6 women = (8 + 6) women

= 14 women

Given, $M_1 = 28$, $M_2 = 14$, $D_1 = 5$, $D_2 = ?$

and $W_1 = W_2 = 1$

According to the formula,

$M_1 D_1 W_2 = M_2 D_2 W_1$

$\Rightarrow \quad 28 \times 5 \times 1 = 14 \times D_2 \times 1$

$\therefore \quad D_2 = \frac{28 \times 5}{14} = 10$ days

59. (a) Let A, B and C take $x, \frac{x}{2}$ and $\frac{x}{3}$ days respectively to complete the work.

$(A + B + C)$'s 1 day's work

$= \frac{1}{x} + \frac{2}{x} + \frac{3}{x} = \frac{6}{x}$

According to the question, $\frac{6}{x} = \frac{1}{2}$

$$\left[\text{as, } (A+B+C)\text{'s 1 day's work} = \frac{1}{2}\right]$$

$\Rightarrow \quad x = 12$

$\therefore$ Time taken by A to complete the work

$= 12$ days

Time taken by B to complete the work

$$= \frac{12}{2} = 6 \text{ days}$$

Time taken by C to complete the work

$$= \frac{12}{3} = 4 \text{ days}$$

60. (*a*) Work done by $(x + y + z)$ in 1 day

$$= 50\% = \frac{1}{2}$$

Work done by x in 1 day $= 20\% = \frac{1}{5}$

Work done by y in 1 day $= 25\% = \frac{1}{4}$

$\therefore$ Work done by z in 1 day

$$= \frac{1}{2} - \frac{1}{5} - \frac{1}{4} = \frac{10-4-5}{20}$$

$$= \frac{1}{20} = 5\%$$

61. (*c*) 1 man = 2 women = 3 boys

$\therefore \quad$ 1 man + 1 woman + 1 boy

$$= \left(3 + \frac{3}{2} + 1\right) \text{boys} = \frac{11}{2} \text{ boys}$$

Here, $M_1 = 3$, $M_2 = \frac{11}{2}$, $D_1 = 88$ and $D_2 = ?$

According to the formula,

$$M_1 D_1 = M_2 D_2$$

$$\therefore \quad D_2 = \frac{M_1 D_1}{M_2}$$

$\therefore$ Required number of days

$$= \frac{3 \times 2 \times 88}{11} = 3 \times 2 \times 8 = 48 \text{ days}$$

62. (*a*) According to the formula,

Required time $= \sqrt{ab}$ **[by Technique 9]**

where, $a = 25$ and $b = 49$

$\therefore$ Required time $= \sqrt{25 \times 49} = 5 \times 7$

$= 35$ days

63. (*c*) Time taken to complete the work by X

$= 10 \times 4 = 40$ days

Time taken to complete the work by

$$Y = 40 \times \frac{100}{40} = 100 \text{ days}$$

Time taken to complete the work by Z

$= 13 \times 3 = 39$ days

Hence, Z will complete the work first.

64. (c) According to the question,

$$5 \text{ men's 1 day's work} = \frac{1}{15} \quad \ldots\text{(i)}$$

$$\therefore \; 5 \text{ men's 5 day's work} = \frac{1}{15} \times 5 = \frac{1}{3}$$

$$\therefore \quad \text{Remaining work} = 1 - \frac{1}{3} = \frac{2}{3}$$

5 men + 10 women can do $\frac{2}{3}$ of the work in 5 days.

$\therefore$ 5 men + 10 women can do the whole of the work in $\frac{15}{2}$ days.

$\therefore$ (5 men + 10 women's) 1 day's work

$$= \frac{2}{15} \quad \ldots\text{(ii)}$$

10 women's 1 day's work

$= \frac{2}{15} -$ 5 men's 1 day's work

$$= \frac{2}{15} - \frac{1}{15} = \frac{1}{15}$$

$\therefore$ 10 women can finish the work in 15 days.

65. (b) 4 men = 6 women = 10 children

$\Rightarrow$ 1 man $= \frac{5}{2}$ children

1 woman $= \frac{5}{3}$ children

Now, 1 couple + 5 children

= 1 man + 1 woman + 5 children

$$= \left(\frac{5}{2} + \frac{5}{3} + 5\right) = \frac{55}{6} \text{ children}$$

According to the formula,

$$M_1 D_1 = M_2 D_2$$

$$\Rightarrow \quad 10 \times 5 = \frac{55}{6} \times D_2$$

$\therefore \quad D_2 = \dfrac{60}{11} = 5\dfrac{5}{11}$ days

66. (*d*) Part of field grazed by 4 goats in 1 day $= \dfrac{1}{50}$

Part of field grazed by 1 goat in 1 day

$$= \frac{1}{50 \times 4} = \frac{1}{200}$$

$\Rightarrow$ 4 g = 6 s [here, g = goats, and s = sheeps]

$\Rightarrow \quad 1\text{ s} = \dfrac{4}{6}\text{ g} = \dfrac{2}{3}\text{ g}$

Now, $2\text{ g} + 9\text{ s} = 2\text{ g} + 9 \times \dfrac{2}{3}\text{ g}$

$$= 2\text{ g} + 6\text{ g} = 8\text{ g}$$

$\therefore$ 8 goats can graze the field in

$$\frac{1}{\frac{8}{200}} = 25 \text{ days.}$$

67. (*a*) Here, $a_1 = 14$, $b_1 = 12$, $x = 4$, $a_2 = 8$, $b_2 = 16$ and $y = 5$

$$\frac{\text{One day work of 1 man}}{\text{One day work of 1 boy}} = \frac{(yb_2 - xb_1)}{(xa_1 - ya_2)}$$

[by Technique 9]

$$= \frac{5 \times 16 - 4 \times 12}{4 \times 14 - 5 \times 8}$$

$$= \frac{80 - 48}{56 - 40} = \frac{32}{16} = \frac{2}{1}$$

68. (*c*) Here, $a_1 = 8$, $b_1 = 4$, $a_2 = 1$, $b_2 = 1$

and $\quad n = 7$

$$\therefore \quad \frac{\text{Work done by 1 man}}{\text{Work done by 1 boy}} = \frac{(nb_2 - b_1)}{(a_1 - na_2)}$$

[by Technique 10]

$$= \frac{7 \times 1 - 4}{8 - 7 \times 1} = \frac{3}{1}$$

Exercise © *Higher Skill Level Questions*

1. (*b*) $(A + B)$'s 2 day's work $= \dfrac{1}{12} + \dfrac{1}{18} = \dfrac{5}{36}$

$(A + B)$'s 14 day's work $= \dfrac{5}{36} \times 7 = \dfrac{35}{36}$

Remaining work $= 1 - \dfrac{35}{36} = \dfrac{1}{36}$

Now, it is the turn of A.

A's 1 day's work $= \dfrac{1}{12}$

$\therefore \quad \dfrac{1}{36}$ work is done by A in

$$\left(12 \times \frac{1}{36}\right) \text{days} = \frac{1}{3} \text{ day}$$

$\therefore$ Total time taken $= 14\frac{1}{3}$ days.

2. (*b*) Let extra hours a day are x.

According to the formula,

$$\frac{M_1 D_1 T_1}{W_1} = \frac{M_2 D_2 T_2}{W_2} \quad \textbf{[by Technique 1]}$$

$$\Rightarrow \quad \frac{1 \times 1 \times (6 + 4)}{1} = \frac{1 \times 1 \times (6 + 6 + x)}{1\frac{1}{2}}$$

$\Rightarrow \quad \frac{3}{2} \times 10 = 12 + x$

$\Rightarrow \quad 15 = 12 + x$

$\therefore \quad x = 15 - 12 = 3$

Extra hours of work everyday are 3.

3. (*d*) Since, 3 women + 18 children complete work in 2 days. Therefore, (3×2) women + (18×2) children complete work in 1 day *i.e.*, 6 women + 36 children complete work in 1 day.

Work of 36 children for 1 day $= 1 - \frac{1}{3} = \frac{2}{3}$

$$\left[\because \text{ work of 6 women for 1 day} = \frac{1}{3}\right]$$

$\therefore$ 36 children do $\frac{2}{3}$ part of the work in 1 day.

36 children can do the work in $\frac{3}{2}$ days.

9 children can do the work in

$$\left(\frac{3}{2} \times 4\right) = 6 \text{ days}$$

4. (*d*) Given, $M_1 = 120$, $M_2 = 120 - x$

$D_1 = 64$, $D_2 = 60$

$W_1 = \frac{2}{3}$ and $W_2 = \frac{1}{3}$

According to the formula,

$\frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2}$ [by Technique 1]

$\Rightarrow \quad \frac{120 \times 64}{2/3} = \frac{(120 - x) \times 60}{1/3}$

$\Rightarrow \quad \frac{120 \times 64}{2 \times 60} = (120 - x)$

$\Rightarrow \quad (120 - x) = 64$

$\therefore \quad x = 120 - 64 = 56$

Now, 56 men can be discharged to finish the work in time.

5. (*a*) Remaining work $= \left(1 - \frac{4}{7}\right) = \frac{3}{7}$

Remaining period $= (92 - 66) = 26$ days

Let the number of additional men $= x$

Given, $M_1 = 234$, $D_1 = 66$, $T_1 = 16$,

$W_1 = \frac{4}{7}$, $M_2 = (234 + x)$, $D_2 = 26$,

$T_2 = 18$, $W_2 = \frac{3}{7}$

According to the question,

$M_1 W_2 T_1 D_1 = M_2 W_1 T_2 D_2$ [by Technique 1]

$\Rightarrow \quad 234 \times \frac{3}{7} \times 16 \times 66$

$= (234 \times x) \times \frac{4}{7} \times 18 \times 26$

$\Rightarrow \quad 234 + x = \dfrac{3 \times 66 \times 16 \times 234}{4 \times 26 \times 18}$

$\Rightarrow \quad 234 + x = 36 \times 11 = 396$

$\therefore \quad x = 396 - 234 = 162$

Additional men to be employed = 162

6. (*e*) Let the work be finished in x days.

Then, A's x day's work + B's $(x - 1)$ days work + C's $(x - 2)$ day's work = 1

$\Rightarrow \quad \dfrac{x}{16} + \dfrac{x-1}{32} + \dfrac{x-2}{48} = 1$

$\Rightarrow \quad \dfrac{x}{1} + \dfrac{x-1}{2} + \dfrac{x-2}{3} = 16$

$\Rightarrow \quad \dfrac{6x + 3x - 3 + 2x - 4}{6} = 16$

$\Rightarrow \quad 11x - 7 = 96$

$\Rightarrow \quad 11x = 103$

$\therefore \quad x = \dfrac{103}{11} = 9\dfrac{4}{11}$ days

7. (*a*) Let the work lasted for x days.

Then, Rashmi's 4 day's work + Ravina's $(x - 3)$ day's + Gitika's x day's work = 1

$\Rightarrow \quad \dfrac{4}{16} + \dfrac{x-3}{12\frac{4}{5}} + \dfrac{x}{32} = 1$

$\Rightarrow \quad \dfrac{5(x-3)}{64} + \dfrac{x}{32} = 1 - \dfrac{1}{4}$

$\Rightarrow \quad \dfrac{5(x-3) + 2x}{64} = \dfrac{3}{4}$

$\Rightarrow \quad 7x - 15 = 48$

$\therefore \quad x = \dfrac{48 + 15}{7} = \dfrac{63}{7} = 9$ days

8. (*c*) According to the formula,

$$\frac{M_1 D_1 T_1}{W_1} = \frac{M_2 D_2 T_2}{W_2} \quad \text{[by Technique 1]}$$

Given, $M_1 = 105, D_1 = 25, T_1 = 8, W_1 = \dfrac{2}{5}$

Now, let the additional men be x.

Then, $\quad M_2 = 105 + x, T_2 = 9$

$\quad D_2 = 25$

and $\quad W_2 = 1 - \dfrac{2}{5} = \dfrac{3}{5}$

On putting these values in the above formula,

$$\frac{105 \times 25 \times 8}{2/5} = \frac{(105 + x) \times 25 \times 9}{3/5}$$

$\Rightarrow \quad \dfrac{105 \times 8}{2} = \dfrac{(105 + x) \times 9}{3}$

$\Rightarrow \quad 105 \times 4 = (105 + x) \times 3$

$\Rightarrow \quad 105 \times 4 = 105 \times 3 + 3x$

$\Rightarrow \quad 3x = 105$

$\therefore \quad x = 35$ men

9. (*e*) A's 1 day's work $= \frac{1}{8}$

B's 1 day's work in breaking the building

$$= \frac{1}{3}$$

Now, according to the question,

A's 4 day's work $= 4 \times \frac{1}{8} = \frac{1}{2}$

Now, A's and B's 2 day's work

$$= 2\left(\frac{1}{8} - \frac{1}{3}\right) = 2 \times \frac{-5}{24} = \frac{-10}{24}$$

Total work done in 6 days

$$= \frac{1}{2} + \left(\frac{-10}{24}\right) = \frac{12-10}{24} = \frac{2}{24} = \frac{1}{12}$$

Remaining work $= 1 - \frac{1}{12} = \frac{11}{12}$

Now, A has to complete the work, so A can complete in x days.

$$\frac{1}{8} \times x = \frac{11}{12} \Rightarrow \quad x = \frac{11 \times 8}{12}$$

$$\therefore \qquad x = \frac{11 \times 2}{3} = 7\frac{1}{3} \text{ days}$$

10. (*c*) Work done in first 3 days

$$= \frac{1}{18} + \frac{1}{24} + \frac{1}{36} = \frac{4+3+2}{72} = \frac{1}{8}$$

$\because$ Time taken to complete $\frac{1}{8}$ part of work

$$= 3 \text{ days}$$

$\therefore$ Time taken to complete the whole work

$$= 3 \times 8 = 24 \text{ days}$$

11. (*a*) 6×12 men $= 8 \times 18$ women

$= 18 \times 10$ children

$\Rightarrow$ 12 men = 24 women = 30 children

$\Rightarrow$ 2 men = 4 women = 5 children

Now, 4 men + 12 women + 20 children

= 4 men + 6 men + 8 men = 18 men

Time to do remaining work for 1 man

$= (6 \times 12 - 18 \times 2) = 36$ days

$\therefore$ Required number of men to finish the work in 1 day = 36

12. (*d*) Work done by A in 4 days $= \frac{4}{20} = \frac{1}{5}$

$\therefore$ Remaining work $= 1 - \frac{1}{5} = \frac{4}{5}$

Let C can working alone complete the work in x days.

According to the question,

$$\frac{18}{30} + \frac{18}{x} = \frac{4}{5}$$

$\Rightarrow \qquad 18(x + 30) \times 5 = 4 \times 30x$

$\Rightarrow \qquad 90x + 2700 = 120x$

=> 30x = 2700

x = 90 days

13. (6) Let *B* left the work after x days from the

start.

According to the question,

x x + 7 ,, — + -------= 1

25 15

3x + 5x + 35 ,,

=> ------------------= 1

75

40 => 8x = 75-35=>x = —

8

x = 5

(a) Because of illness,

1 1 A's 1 day's work = 90% of - = — 9 10

B's 1 day's work = 72% of— = — 18 25

/. *(A + B)'s 1* day's work

_ 1 1_5+2_7

10 25 50 50

Hence, time taken by them to complete the work = — days = 7 - days. 7 7

15. (d) As the number of men increase the number of days taken to do the work must decrease which means that diagrams II and III are ruled out. Moreover, the men work at constant speed which means that in no case will the work be done in zero days no matter, how many men are put to work? Thus, diagram I is ruled out because, here the graph touches the zero line on both the axes. Thus, the right answer is diagram IV

16. (*a*) Work done by A and B in 1 day

$$= \frac{1}{8} + \frac{1}{12} = \frac{5}{24}$$

2 day's work of A and $B = \frac{10}{24}$

After 2 day's A left the work

$\therefore$ Remaining work $= 1 - \frac{10}{24} = \frac{14}{24}$

One day work of B and C together

$$= \frac{1}{12} + \frac{1}{15} = \frac{9}{60}$$

So, the number of days required by B and C to finish work

$$= \frac{14/24}{9/60} = \frac{14}{24} \times \frac{60}{9} = \frac{35}{9}$$

$\therefore$ Total days to complete the work

$$= 2 + \frac{35}{9} = \frac{53}{9} = 5\frac{8}{9} \text{ days}$$

Chapter

23

Work and Wages

Activity involving physical efforts, done in order to achieve a result is known as **work.**

Money received by a person for a certain work is called the **wages** of the person for that particular work,

In other words, we can find the entire wages of any person by *the following formula*

$$\boxed{\text{Entire wages} = \text{Total number of days} \times \text{Wages of 1 day of any person}}$$

For example If Arjun's monthly wages ? 4200 and he worked for all 30 days, then his daily wages will be calculated as Total wages = Number of days x Daily wages 4200 = 30 x Daily wages

$$\text{Daily wages} = \frac{4200}{30} = ₹140$$

Important Points

Wages is directly proportional to the work done. It means, more money will be received for more work and less money will be received for less work.

♦ Wages is indirectly proportional to the time taken by the individual.

♦ Wages is directly proportional to 1 day work of each individual,

For example If Karan can do a piece of work in 10 days and Arun can do the same piece of work in 15 days. Then, ratio of Karan and Arun's wages will be 15 :10 *i.e.,* 3 :2.

♦ If *X, Y* and Z can do a piece of work in *d\, a* and d3 days respectively, then ratio of their shares is

d2 *d3 -.djdi-.* dj d2

Fast Track **Formulae**

to solve the **QUESTIONS**

Formula **1**

If *A* and e can do a piece of work in *x* and *y* days respectively, the ratio of their wages will be *y*: *x*. Then, the wages earned by *A* and e will be

$$A\text{'s wages} = \frac{\text{Total wages}}{x+y} \times y; \quad B\text{'s wages} = \frac{\text{Total wages}}{x+y} \times x$$

Ex. 1 Akansha can do a piece of work in 6 days, while Vasudha can do the same work in 5 days. If the total amount to be given for this work is ? 660, then what will be the share of Vasudha, if both work together.

Sol. Time taken by Akansha = 6 days

$\therefore$ 1 day's work = $\frac{1}{6}$ and time taken by Vasudha = 5 days

$\therefore$ 1 day's work = $\frac{1}{5}$

Total amount earned = ₹ 660

$\therefore$ Ratio of their incomes = $\frac{1}{6} : \frac{1}{5} = 5 : 6$

$\therefore$ Vasudha's share = $\frac{660}{5+6} \times 6 =$ ₹ 360

Formula **2**

if *A, B* and *C* can do a piece of work in *x, y* and z days respectively, the ratio of their wages will be *yz: xz: xy.* Then, wages earned by *A, B* and C respectively will be

$$A\text{'s wages} = \frac{\text{Total wages}}{(yz + xz + xy)} \times yz\ ; \quad B\text{'s wages} = \frac{\text{Total wages}}{(yz + xz + xy)} \times xz;$$

$$C\text{'s wages} = \frac{\text{Total wages}}{(yz + xz + xy)} \times xy$$

Ex. 2 *A, B* and C take ? 535 for doing a piece of work together. If working alone, each takes 5 days, 6 days and 7 days respectively, then find the share of each.

Sol. Total wages = ₹535

A can do a work in 5 days; *B* can do a work in 6 days; *C* can do a work in 7 days.

$$\therefore \quad A\text{'s share} = \frac{535}{(7\times 6)+(7\times 5)+(6\times 5)} \times (7\times 6)$$

$$= \frac{535}{42+35+30} \times 42 = \frac{535}{107} \times 42 = 42 \times 5 = ₹\,210$$

$$B\text{'s share} = \frac{535}{107} \times 35 = 5 \times 35 = ₹\,175;$$

$$C\text{'s share} = \frac{535}{107} \times 30 = 5 \times 30 = ₹\,150$$

: *Formula* 3

Total wages earned by certain persons in doing certain work. Total wages = (1 person's 1 day's wages) x (Number of persons)

x (Number of days)

∴ Required number of persons

$$= \frac{\text{Total wages}}{\text{Number of days} \times 1 \text{ person's } 1 \text{ day's wages}}$$

Ex. 3 Wages of 45 women for 48 days amount to ? 31050. How many men must work for 16 days to receive ? 11500, if the daily wages of a man being double those of a woman?

Sol. 1 day wages of a woman $= \dfrac{31050}{45 \times 48} = ₹\dfrac{115}{8}$

1 day wages of a man $= \dfrac{115}{8} \times 2 = ₹\dfrac{115}{4}$

$\therefore$ Required number of men $= \dfrac{11500}{16 \times \dfrac{115}{4}} = 25$ men

Formula 4

A can do a piece of work in *x* days. With the help of *B, A* can do the same work in y days. If they get ? a for that work, then

$$\text{Share of } A = ₹\left(\frac{ay}{x}\right)$$

$$\text{Share of } B = ₹\left(\frac{a(x-y)}{x}\right)$$

Ex. 4 Suresh can do a work in 20 days. Suresh and Surendra together do the same work in 15 days. If they got ? 400 for that work, then find the share of Suresh and Surendra?

Sol. Here, $x = 20, y = 15, a = 400$

Now, according to the formula,

Share of Suresh $= \dfrac{a \times y}{x} = \dfrac{400 \times 15}{20} = ₹\, 300$

Share of Surendra $= \dfrac{a(x-y)}{x} = \dfrac{400 \times (20-15)}{20} = \dfrac{400 \times 5}{20} = ₹\, 100$

Formula 5

X, Y and Z undertake to do a work for ? *R.* If together they do only *m/n* of the work and rest is done by C alone, then the share of *C* is given by

$$R\left(1 - \frac{m}{n}\right).$$

Ex. 5 *A, B* and C undertake to do a work for ? 480. *A* and *B* together do 1/4 of the work and rest is done by the C alone. How much should C get?

Sol. Here, $R = ₹\, 480, m = 1$ and $n = 4$

Now, according to the formula,

Share of $C = R\left(1 - \dfrac{m}{n}\right) = 480\left(1 - \dfrac{1}{4}\right) = 480 \times \dfrac{3}{4} = ₹\, 360$

Ex. 6 X and Y contracted a piece of work for ? 1600. X alone can do it in 6 days, while Y alone can do that work in 8 days. They completed the work in 3 days taking help of *A*. Find the share of *A*.

Sol. Time taken by $X = 6$ days

$\therefore$ $$X\text{'s 1 day's work} = \frac{1}{6}$$

and time taken by $Y = 8$ days

$\therefore$ $$Y\text{'s 1 day's work} = \frac{1}{8}$$

If A can do a work in n days, then A's 1 day's work $= \frac{1}{n}$

According to the question,

$(X + Y + A)$ complete the work in 3 days.

$\therefore$ X's 3 days work + Y's 3 days work + A's 3 days work = 1

$$\Rightarrow \frac{3}{6} + \frac{3}{8} + \frac{3}{n} = 1 \Rightarrow \frac{3}{n} = \left(1 - \frac{3}{6} - \frac{3}{8}\right) = 1 - 3\left(\frac{4+3}{24}\right)$$

$$\Rightarrow \frac{3}{n} = 1 - \frac{7}{8} = \frac{1}{8} \Rightarrow n = 3 \times 8 \Rightarrow n = 24$$

$\therefore$ X's share : Y's share : A's share $= \frac{1}{6} : \frac{1}{8} : \frac{1}{24} = \frac{12}{72} : \frac{9}{72} : \frac{3}{72} = 12 : 9 : 3 = 4 : 3 : 1$

Now, A's share $= \frac{1}{4 + 3 + 1} \times 1600 = \frac{1}{8} \times 1600 =$ ₹ 200

Ex. 7 *A* can do a piece of work in 10 days, while *B* alone can do it in 15 days. They work together for 5 days and rest of the work is done by M in 2 days. If they get ? 9000 for the whole work, then how should they divide the money?

Sol. Time taken by A = 10 days

$\therefore$ A's 1 day's work $= \frac{1}{10}$ and time taken by B = 15 days

$\therefore$ B's 1 day's work $= \frac{1}{15}$

Now, $(A + B)$'s 5 days work $= 5\left(\frac{1}{10} + \frac{1}{15}\right)$

$$= 5\left(\frac{3+2}{30}\right) = 5 \times \frac{5}{30} = \left(5 \times \frac{1}{6}\right) = \frac{5}{6}$$

Remaining work $= 1 - \frac{5}{6} = \frac{6-5}{6} = \frac{1}{6}$

$\therefore$ M's 2 days work $= \frac{1}{6}$

Now, (A's 5 days work) : (B's 5 days work) : (M's 2 days work)

$$= \frac{5}{10} : \frac{5}{15} : \frac{1}{6} = \frac{1}{2} : \frac{1}{3} : \frac{1}{6} = \frac{3}{6} : \frac{2}{6} : \frac{1}{6} = 3 : 2 : 1$$

Let A's share $= 3x$, B's share $= 2x$ and M's share $= x$

According to the question,

$$3x + 2x + x = 9000 \Rightarrow 6x = 9000 \Rightarrow x = \frac{9000}{6} = 1500$$

$\therefore$ A's share $= 3x = 3 \times 1500 =$ ₹ 4500

B's share $= 2x = 2 \times 1500 =$ ₹ 3000

and M's share $= x =$ ₹ 1500

Fast Track Practice

Exercise © *Base Level Questions*

1. Shantanu can do a piece of work in 12 days and Manu can do the same work in 10 days. If they work together, in what ratio Shantanu and Manu will receive their wages?

(a) 5 : 6 *(b)* 3 : 2 *(c)* 1 : 6 *(d)* 5 : 7 *(e)* None of the above

2. *A* can do a piece of work in 9 days and *B* can do the same work in 15 days. If they work together, in what ratio *A* and *B* will receive their wages? [Bank Clerks 2008]

(a) 3 : 5 *(b)* 5 : 3

(c) 2 : 5 fdj 5 : 2

(ej None of the above

3. A man and a hoy received X 1400 as wages for 10 days for the work they did together. The man's efficiency in the work was six times that of the boy. What is the daily wages of the boy?

(a) no (b) X 15 (c) ?20 (d) X 30

4. A alone can do a piece of work in 8 days while B alone can do it in 10 days. If they together complete this work and get X 900 as their remuneration, then find the shares of both the persons.

(a) A = ?800ande=? 100

(b) A = X 500 and B = X 400

(c) A = X 600 and B = X 300

(d) A = X 300 and S = X 600

(e) None of the above

5. A can do a piece of work in 3 days while B can do the same work in 4 days. If they work together for a total wages of X 2800, how much does A get? [Bank Clerks 2009]

(a) X 1200 (b) X 1900

(c) X 1300 (d) X 1600

(e) None of the above

6. A alone can do a piece of work in 3 days, while B alone can do the same work in 2 days. If they work together to complete the work, their total wages is fixed X 225. Find the share of A. [RRB 2008]

(a) X 90 (b) X 75

(c) X 95 (d) X 65

(e) None of the above

7. A can finish a work in 15 days, B in 20 days and C in 25 days. All these three worked together and earned X 4700. The share of C is [CDS 2012]

(a) ?1200 (b) ?1500

(c) ?1800 (d) ?2000

8. A person can do a piece of work in 26 days and an another person can do the same work in 39 days. If they work together, then by what per cent the wages of 1st person is more than that of 2nd person? [UP Police 2007]

(a) 25% (b) 35% (c) 15% (d) 50%

9. *A* alone can finish a work in 2 days, while *B* alone can finish it in 3 days. If they work together to finish it, then out of total wages of *X* 6000, what will be the 20% of *A's* share? [Hotel Mgmt. 2009]

(a) *X* 720 (b) *X* 350

(c) *X* 820 (d) *X* 420

(e) None of the above

10. Vandana can do a piece of work in 10 days. With the help of Ruchi, she can do the same work in 6 days. If they get *X* 200 for that work, then what will be the share of Ruchi in the received remuneration?

(a) *X* 120 (b) *X* 100 (c) *X* 60 (d) *X* 80 (e) None of the above

11. A sum of money is sufficient to pay A's wages for 21 days and B's wages for 28 days. The same money is sufficient to pay the wages of both for [SSC CCL 2013]

(a) $24\frac{1}{2}$ days (b) 12 days

(c) $12\frac{1}{4}$ days (d) 14 days

12. Amit, Vipul and Kapil can do a piece of work in 12, 16 and 24 days, respectively. If they complete the work together and get an amount of *X* 2700, then what is the share of Vipul in that amount?

(a) *X* 450 (b) *X* 1000

(c) *X* 225 (d) *X* 900

(e) None of the above

13. *P, Q* and *R* enter into a contract for a piece of work for *X* 1100. *P* and *Q* together are supposed to do 7/22 of the work. How much does *R* get?

(a) X 750 *(b)* X 350

(c) X 4751 *(d)* X 900

(e) None of the above

14. Ramesh and Suresh undertaken to do a piece of work for *X* 800. Ramesh alone can do it in 12 days, while Suresh alone can do it in 16 days. With the assistance of a boy, they finish it in 6 days. How should the money be divided?

(a) Ramesh = *X* 100, Suresh = *X* 300 and boy = *X* 400

(b) Ramesh = *X* 200, Suresh = *X* 200 and boy = *X* 400

(c) Ramesh = *X* 400, Suresh = *X* 300 and boy = *X* 100

(d) Ramesh = *X* 300, Suresh = *X* 400 and boy = *X* 100

(e) None of the above

15. *A, B* and C completed a work costing *X* 1800. *A* worked for 6 days, *B* worked for 4 days and C worked for 9 days. If their daily wages are in the ratio of 5:6:4, how much amount will be received by *A*"> [SSC CPO 2007]

(a) X 600 *(b) X* 500 *(c) X 900(d) X* 450

16. A and *B* undertaken to do a piece of work for ? 1200. *A* alone can do it in 8 days, while *B* can do it in 6 days. With the help of C, they complete it in 3 days. Find Cs share. [SSC CGL (Main) 2012]

(a) ? 450 *(b) X* 300 *(c) X* 150 *(d) X* 100

17. *A, B* and *C* can do a piece of work in 20, 24 and 30 days, respectively. They undertook to do the piece of work for *X* 5400. They begin the work together but *B* left 2 days before the completion of work and C left 5 days before the completion of work. The share of *A* from the assured money is [SSC CPO 2013]

(a) ? 2700 *(b) X* 540 *(c) X* 1800 *(d) X* 600

Exercise © *Higher Skill Level Questions*

1. *A, B* and C get *X* 5400 for doing a work in 36 days. *A* and C get *X* 1880 for doing the same work in 20 days, while *B* and C get *X* 3040 for doing the same work in 40 days. Find the amount received by C per day.

(a) X 50 *(b) X* 25 *(c) X* 30 *(d) X* 20 *(e)* None of the above

2. Total wages of 3 men, 2 women and 4 boys is *X* 26. If the wages of 3 men is equal to that of 4 women and the wages of 2 women is equal to that of 3 boys, then find out the total wages of 4 men, 3 women and 2 boys.

(a) X 29 *(b) X* 35 *(c) X* 65 *(d) X* 20 *(e)* None of the above

3. A person was appointed for a 50 days job on a condition that he will be paid *X* 12 for every working day but he will be fined *X* 6 for every day he remains absent. After the completion of the work, he got *X* 420. For how many days, he did not work?

(a) 15 days *(b)* 5 days

(c) 10 days *(d)* 20 days

(e) None of the above

4. Total wages for a work is *X* 1280. *A* alone can do a piece of work in 8 days, while *B*

alone can do it in 12 days. If *A* and *B* work on alternate days, then find the share of A.

(a) X 800 *(b) X* 500 *(c) X 600(d) X* 700 *(e)* None of the above

5. 4 men and 6 women get *X* 1600 by doing a piece of work in 5 days. 3 men and 7 women get *X* 1740 by doing the same work in 6 days. In how many days, 7 men and 6 women can complete the same work getting *X* 3760? [SSC CGL 2009]

(a) 6 days *(b)* 8 days

(c) 10 days *(d)* 12 days

6. Men, women and children are employed to do a work in the proportion of 3 : 2 : 1 and their wages as 5:3:2. When 90 men are employed, total daily wages of all amounts to ? 10350. Find the daily wages of a man.

[SSC CGL (Main) 2012]

(a) ? 45 *(b)* ? 57:50 *(c)* ? 115 *(d)* ? 75

7. 2 men and 1 woman can do a piece of work in 14 days, while 4 women and 2 men can do the same work in 8 days. If a man gets X 90 per day, what should be the wages per day of a women? [CDS 2013]

(a)? 48 (b)? 60 *(c)* ? 72 fdj?135

Answer with Solutions

Exercise © *Base Level Questions*

1. (*a*) Shantanu's 1 day's work $= \frac{1}{12}$

Manu's 1 day's work $= \frac{1}{10}$

Shantanu's share : Manu's share

$$= \frac{1}{12} : \frac{1}{10} = \frac{5}{60} : \frac{6}{60} = 5 : 6$$

2. (*b*) A's 1 day's work $= \frac{1}{9}$

B's 1 day's work $= \frac{1}{15}$

A's share : B's share

$$= \frac{1}{9} : \frac{1}{15} = \frac{5}{45} : \frac{3}{45} = 5 : 3$$

3. (*c*) The ratio of efficiency of man to boy $= 6 : 1$

Efficiency $\propto$ Wages

$\therefore$ Boy's share $= \frac{1}{1+6} \times 1400$

$$= \frac{1}{7} \times 1400 = ₹\, 200$$

Now, they worked for 10 days.

$\therefore$ Daily wages of a boy $= \frac{200}{10} = ₹\, 20$

4. (*b*) A's 1 day's work $= \frac{1}{8}$

B's 1 day's work $= \frac{1}{10}$

$\therefore$ A's share : B's share

$$= \frac{1}{8} : \frac{1}{10} = \frac{5}{40} : \frac{4}{40} = 5 : 4$$

Let A's share $= 5x$

and B's share $= 4x$.

According to the question,

$$5x + 4x = 900 \Rightarrow 9x = 900$$

$\therefore$ $x = 100$

$\therefore$ A's share $= 5x = 5 \times 100 = ₹\, 500$

and B's share $= 4x = 4 \times 100 = ₹\, 400$.

Fast Track Method

Ratio of wages of A and $B = 10 : 8$

A's wages $= \frac{A}{A+B} \times$ Total wages

[by Formula 1]

$$= \frac{10}{10+8} \times 900 = ₹\, 500$$

Similarly,

$$B\text{'s wages} = \frac{8}{10+8} \times 900 = ₹\,400$$

5. (*d*) A's 1 day's work $= \dfrac{1}{3}$

B's 1 day's work $= \dfrac{1}{4}$

A's share : B's share

$$= \frac{1}{3} : \frac{1}{4} = \frac{4}{12} : \frac{3}{12}$$

$$= 4 : 3$$

$$A\text{'s share} = \frac{4}{7} \times 2800$$

$$= 4 \times 400$$

$$= ₹\,1600$$

6. (*a*) A's 1 day's work $= \dfrac{1}{3}$

B's 1 day's work $= \dfrac{1}{2}$

A's share : B's share $= \dfrac{1}{3} : \dfrac{1}{2} = \dfrac{2}{6} : \dfrac{3}{6}$

$$= 2 : 3$$

Share of $A = \dfrac{2}{5} \times 225 = ₹\,90$

7. (*a*) A's 1 day's work $= \dfrac{1}{15}$

B's 1 day's work $= \dfrac{1}{20}$

C's 1 day's work $= \dfrac{1}{25}$

A, B and C worked together.

$\therefore$ $(A + B + C)$'s 1 day's work

$$= \frac{1}{15} + \frac{1}{20} + \frac{1}{25}$$

$$= \frac{20 + 15 + 12}{300}$$

$$= \frac{47}{300}$$

Days taken to complete work by A, B and C working together $= \dfrac{300}{47}$

$\therefore$ Share of $C = \dfrac{1}{25} \times \dfrac{300}{47} \times 4700$

$$= ₹\,1200$$

8. (*d*) Let 1st person be x and 2nd person be y.

Then, x's 1 day's work $= \frac{1}{26}$

y's 1 day's work $= \frac{1}{39}$

x's share : y's share

$$= \frac{1}{26} : \frac{1}{39} = \frac{3}{78} : \frac{2}{78} = 3:2$$

Difference of ratio $= 3 - 2 = 1$

$\therefore$ Required percentage $= \frac{1}{2} \times 100\% = 50\%$

$\therefore$ 1st person's wages is 50% more than the 2nd person's wages.

9. (*a*) A's 1 day's work $= \frac{1}{2}$

B's 1 day's work $= \frac{1}{3}$

A's share : B's share $= \frac{1}{2} : \frac{1}{3} = \frac{3}{6} : \frac{2}{6} = 3:2$

A's share $= \frac{3}{5} \times 6000 = 3 \times 1200 =$ ₹ 3600

$\therefore$ 20% of A's share $= 3600 \times \frac{20}{100} =$ ₹ 720

10. (*d*) Vandana's 1 day's work $= \frac{1}{10}$

(Vandana + Ruchi)'s 1 day's work $= \frac{1}{6}$

$\therefore$ Ruchi's 1 day's work

$$= \frac{1}{6} - \frac{1}{10} = \frac{10-6}{6 \times 10} = \frac{4}{60} = \frac{1}{15}$$

$\therefore$ Vandana's share : Ruchi's share

$$= \frac{1}{10} : \frac{1}{15} = \frac{3}{30} : \frac{2}{30} = 3:2$$

Ruchi's share $= \frac{2}{5} \times 200 =$ ₹ 80

Fast Track Method

Ratio of wages of Vandana and Ruchi

$$= 15:10$$

Ruchi's wages $= \frac{A}{A+B} \times$ Total wages

[by Formula 1]

$$= \frac{10}{25} \times 200 = ₹\ 80$$

11. (*b*) A's 1 day's work $= \frac{1}{21}$

B's 1 day's work $= \frac{1}{28}$

$\therefore$ Same money is sufficient to pay the wages of both for

$$= \frac{1}{\frac{1}{21} + \frac{1}{28}} = \frac{21 \times 28}{21 + 28} = \frac{21 \times 28}{49}$$
$$= 3 \times 4 = 12 \text{ days}$$

12. (*d*) 1 day's work of Amit $= \frac{1}{12}$

1 day's work of Vipul $= \frac{1}{16}$

1 day's work of Kapil $= \frac{1}{24}$

Amit's share : Vipul's share : Kapil's share

$$= \frac{1}{12} : \frac{1}{16} : \frac{1}{24} = \frac{4}{48} : \frac{3}{48} : \frac{2}{48} = 4 : 3 : 2$$

Now, Vipul's share $= \frac{3}{9} \times 2700$

$$= 3 \times 300 = ₹\ 900$$

13. (*a*) Work done by $(P + Q) = \frac{7}{22}$

Work done by $R = \left(1 - \frac{7}{22}\right) = \frac{22 - 7}{22} = \frac{15}{22}$

$\therefore$ $(P + Q)$'s share : R's share

$$= \frac{7}{22} : \frac{15}{22} = 7 : 15$$

R's share $= \frac{15}{22} \times 1100 = 15 \times 50 = ₹\ 750$

14. (*c*) Ramesh's 1 day's work $= \frac{1}{12}$

Suresh's 1 day's work $= \frac{1}{16}$

(Ramesh + Suresh)'s 1 day's work

$$= \frac{1}{12} + \frac{1}{16} = \frac{16 + 12}{12 \times 16} = \frac{28}{192} = \frac{7}{48}$$

Given that, (Ramesh + Suresh + Boy)'s 1 day's work = 1/6

$\therefore$ Boy's 1 day's work $= \frac{1}{6} - \frac{7}{48} = \frac{8 - 7}{48} = \frac{1}{48}$

$\therefore$ Ramesh's share : Suresh's share : Boy's share

$$= \frac{1}{12} : \frac{1}{16} : \frac{1}{48} = \frac{8}{96} : \frac{6}{96} : \frac{2}{96}$$
$$= 8 : 6 : 2 = 4 : 3 : 1$$

Let Ramesh's share be $4x$, Suresh's share be $3x$ and boy's share be x.

According to the question,

$$4x + 3x + x = 800$$

$\Rightarrow$ $8x = 800$

$\therefore$ $x = \frac{800}{8} = 100$

$\therefore$ Ramesh's share $= 4x = 4 \times 100 = ₹\ 400$

Suresh's share $= 3x = 3 \times 100 = ₹\ 300$

Boy's share $= x = ₹\ 100$

15. (*a*) Ratio of the wages of A, B and C $= 5:6:4$

A's share : B's share : C's share

$$= (6 \times 5) : (4 \times 6) : (9 \times 4)$$

$$= 30 : 24 : 36 \; = 5 : 4 : 6$$

$$\therefore \quad A\text{'s share} = \frac{5}{15} \times 1800 = ₹\, 600$$

16. (*c*) According to the question,

$$\frac{1}{A} + \frac{1}{B} + \frac{1}{C} = \frac{1}{3} \Rightarrow \frac{1}{8} + \frac{1}{6} + \frac{1}{C} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{3} - \left(\frac{1}{8} + \frac{1}{6}\right) = \frac{1}{24}$$

$\therefore$ Ratio in shares of A, B and C

$$= \frac{1}{8} : \frac{1}{6} : \frac{1}{24} = \frac{3}{24} : \frac{4}{24} : \frac{1}{24}$$

$$= 3 : 4 : 1$$

$$\therefore \; C\text{'s share} = \frac{1}{3+4+1} \times 1200 = ₹\, 150$$

17. (*a*) Let the number of days to complete the work be x.

According to the question,

$$\frac{x}{20} + \frac{x-2}{24} + \frac{x-5}{30} = 1$$

$$\Rightarrow \quad \frac{6x + 5(x-2) + 4(x-5)}{120} = 1$$

$$\Rightarrow \quad 6x + 5x + 4x = 120 + 10 + 20$$

$$\Rightarrow \quad 15x = 150$$

$$\therefore \quad x = 10$$

$$\therefore \quad \text{Work done by } A = \frac{10}{20}$$

$$= \frac{1}{2}$$

$\therefore$ Share of A from the assured money

$$= \frac{1}{2} \times 5400$$

$$= ₹\, 2700$$

Exercise © *Higher Skill Level Questions*

1. (*d*) Amount received by $(A+B+C)$ per day

$$= \frac{5400}{36} = ₹\,150$$

$\therefore \quad A+B+C = ₹\,150 \quad \ldots(\text{i})$

Similarly, amount received by $(A+C)$ per day

$$= \frac{1880}{20} = ₹\,94$$

$\therefore \quad A+C = ₹\,94 \quad \ldots(\text{ii})$

Amount received by $(B+C)$ per day

$$= \frac{3040}{40} = ₹\,76$$

$\therefore \quad B+C = ₹\,76 \quad \ldots(\text{iii})$

From Eqs. (i) and (iii), we get

$A + 76 = ₹\,150$

$A = 150 - 76 = ₹\,74$

By putting the value of A in Eq. (ii), we get

$74 + C = ₹\,94$

$\therefore \quad C = 94 - 74 = ₹\,20$

Hence, amount received by C per day

$= ₹\,20$

2. (a) Let the wages of 1 man, 1 woman and 1 boy are ? x, ? *y* and ? z, respectively. According to the question,

3x+2y+4z = 26 ...(i)

3x = 4v ...(ii)

and 2y = 3z ... (iii)

From Eqs. (i) and (ii), we get

4y + *2y* + 4z = 26 => 6/+4z = 26 ...(iv)

From Eqs. (iii) and (iv), we get

9z + 4z = 26 => 13z = 26

z = 2 From Eqs. (ii) and (iii), we get

y = *3* and x = 4 .•. Wages of 4 men, 3 women and 2 boys =4X4+3X3+2X2 = 16 + 9 + 4=?29 3. (c) Let the person did not work for x days. It means that he worked for (50 — x) days. .'. Fine for being absent = ? 6x Wages for working days = 12 (50 - x) According to the question, Received wages = 12(50-x)-6x = 420 => 600-12x-6x = 420

=> 18x = 600-420 = 180

$\therefore \quad x = \frac{180}{18} = 10$ days

4. (a) Work for 1st 2 days $= \frac{1}{8} + \frac{1}{12} = \frac{5}{24}$

$\therefore$ Work for 8 days $= \frac{5}{24} \times \frac{8}{2} = \frac{5}{6}$

$\therefore$ Remaining work $= 1 - \frac{5}{6} = \frac{6-5}{6} = \frac{1}{6}$

On 9th day, A's work $= \frac{1}{8}$

Remaining work after 9 days

$$= \frac{1}{6} - \frac{1}{8} = \frac{4-3}{24} = \frac{1}{24}$$

B will finish this work in $12 \times \frac{1}{24} = \frac{1}{2}$ day

Clearly, A worked for 5 days from starting.

$\therefore$ Work done by A in 5 days

$$= \frac{1}{8} \times 5 = \frac{5}{8}$$

$\therefore$ A's share $= \frac{5}{8} \times 1280 = 5 \times 160 =$ ₹ 800

5. (*b*) $\because$ In 5 days, (4 men + 6 women) get $=$ ₹ 1600

$\therefore$ In 1 day, (4 men + 6 women) get

$$= \frac{1600}{5} = ₹\ 320 \quad \ldots(\text{i})$$

In 1 day, number of persons to get ₹ 1

$$= \frac{320}{4 \text{ men} + 6 \text{ women}} \quad \ldots(\text{ii})$$

Similarly, in second condition,

In 1 day, number of persons to get ₹ 1

$$= \frac{1740}{6\,(3 \text{ men} + 7 \text{ women})}$$

$$= \frac{290}{3 \text{ men} + 7 \text{ women}} \quad \ldots(\text{iii})$$

From Eqs. (ii) and (iii), we get

$$\frac{320}{4 \text{ men} + 6 \text{ women}} = \frac{290}{3 \text{ men} + 7 \text{ women}}$$

96 men + 224 women = 116 men + 174 women

$\Rightarrow$ 20 men = 50 women

$$\Rightarrow \quad \frac{\text{Man}}{\text{Woman}} = \frac{5}{2}$$

$\therefore$ 1 woman $= \frac{2}{5}$ men

From Eq. (i), in 1 day,

$$(4 \text{ men} + 6 \text{ women}) = \left(4 \text{ men} + 6 \times \frac{2}{5} \text{ men}\right)$$

$$= \frac{32}{5} \text{ men get ₹ } 320$$

$\therefore$ In 1 day, 1 man get $= \frac{320 \times 5}{32} =$ ₹ 50

$\therefore$ In 1 day, 1 woman get

$$= 50 \times \frac{2}{5} = ₹\ 20$$

$\therefore$ In 1 day, (7 men + 6 women) get

$$7 \times 50 + 6 \times 20 = ₹\ 470$$

$\therefore$ Required number of days

$$= \frac{3760}{470} = 8 \text{ days}$$

6. (d) Let the numbers of men, women and children are *3y, 2y* and *y,* respectively. Given, 3y = 90 => *y* = 30 Number of women = 60 and number of children = 30 Let the men's, women's and children's wages be ? 5x, ? 3xand ? 2x, respectively. According to the question,

Total daily wages = ? 10350 => 90 x5x + 60 x3x+ 30 x2x= 10350 => x(450 + 180 + 60) = 10350

$$\therefore \qquad x = \frac{10350}{690} = 15$$

$\therefore$ Daily wages of a man = $15 \times 5 = ₹\ 75$

7. (*b*) Let man be represent by *m* and woman be represented by *w*.

$$\because \qquad 2m + 1w = \frac{1}{14}$$

$$\Rightarrow \qquad 14(2m + 1w) = 1 \qquad \text{...(i)}$$

$$\text{and} \qquad 4w + 2m = \frac{1}{8}$$

$$8(4w + 2m) = 1 \qquad \text{...(ii)}$$

On equating Eqs. (i) and (ii), we get

$$14(2m + 1w) = 8(4w + 2m)$$

$$\Rightarrow \qquad 28m + 14w = 32w + 16m$$

$$\Rightarrow \qquad 28m - 16m = 32w - 14w$$

$$\Rightarrow \qquad 12m = 18w$$

$$\therefore \qquad \frac{m}{w} = \frac{18}{12} = \frac{3}{2}$$

So, efficiency of 1 man and 1 woman is 3 : 2.

So, their wages must be in the same ratio

$$\frac{90}{x} = \frac{3}{2}$$

[here, x = wages of a woman]

$$\therefore \qquad x = \frac{90 \times 2}{3} = ₹\ 60$$

Chapter

24

Pipes and Cisterns

Problems on Pipes and Cisterns are based on the basic concept of time and work. Pipes are connected to a tank or cistern and are used to fill or empty the tank or cistern. In pipe and cistern, the work is done in form of filling or emptying a cistern/tank.

Inlet pipe It fills a tank/cistern/reservoir. **Outlet pipe** It empties a tank/cistern/reservoir.

Important Points

- If a pipe can fill/empty a tank in 'm' h, then the part of tank filled/emptied in 1 h $= \frac{1}{m}$.

  For example A pipe can fill the tank in 7 h, then the volume of tank filled in 1 h $= \frac{1}{7}$
- If a pipe can fill/empty '$1/m$' part of a tank in 1 h, then it can fill/empty the whole tank in 'm' h.

  For example If a pipe can fill 1/5 part of a tank in 1h, then it can fill the whole tank in 5 h.
- Generally, time taken to fill a tank is taken positive (+ve) and time taken to empty a tank is taken negative (–ve).
- If a pipe fills a tank in m h and an another pipe fills in n h. Then, part filled by both pipes in 1 h $= \frac{1}{m} + \frac{1}{n}$.

Ex. 1 An outlet pipe can empty a cistern in 5 h. In what time will the pipe empty $\frac{2}{5}$ part of the cistern?

Sol. $\because$ Time taken to empty full cistern = 5 h

$\therefore$ Time taken to empty $\frac{2}{5}$ part of the cistern $= \frac{2}{5} \times 5 = 2$ h

Ex. 2 If a pipe can fill a tank in 2 h and another pipe can fill the same tank in 6 h, then what part of a tank will be filled by both the pipes in 1 h, if they are opened simultaneously?

Sol. In 1 h, part filled by 1st pipe $= \frac{1}{m} = \frac{1}{2}$;

In 1 h, part filled by 2nd pipe $= \frac{1}{n} = \frac{1}{6}$;

$\therefore$ In 1 h, part filled by both the pipes together

$$= \left(\frac{1}{m} + \frac{1}{n}\right)$$

$$= \left(\frac{1}{2} + \frac{1}{6}\right)$$

$$= \frac{3+1}{6} = \frac{4}{6} = \frac{2}{3} \text{ part}$$

Ex. 3 If a pipe can fill a tank in 5 h and an another pipe can empty the tank in 10 h, then part fill by both pipes in 1 h, if both pipes are open simultaneonsly.

Sol. In 1 h, part filled by 1st pipe $= \frac{1}{m} = \frac{1}{5}$

In 1 h, part emptied by 2nd pipe $= \frac{1}{n} = \frac{1}{10}$

$\therefore$ In 1 h, part filled by both pipes when open simultaneously

$$= \frac{1}{m} - \frac{1}{n} \quad \text{[– ve sign is used, as 2nd pipe empties the tank]}$$

$$= \frac{1}{5} - \frac{1}{10}$$

$$= \frac{2-1}{10} = \frac{1}{10} \text{ part}$$

Fast Track **Techniques**

to solve the QUESTIONS

Technique 1

If a pipe can fill/empty a tank in *'m'* h and an another pipe can fill/empty the same tank in *'n'* h, then

(i) If both pipes either fills or empties the tank, then the time taken to fill or emotv the

tank when both pipes are opened is $t = \frac{mn}{m+n}$

(ii) If first pipe fills the tank and second pipe **empties** the tank, then the time taken to

fill the tank when both pipes are opened is $t = \frac{mn}{m-n}$; $m > n$

(iii) if first pipe fills the tank and second pipe **empties** the tank, then the time taken to

empty the tank when both pipes are opened is $t = \frac{mn}{n-m}$; $n > m$

Ex. 4 Two pipes *A* and *B* can fill a tank in 18 h and 12 h, respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Sol. Part filled by A alone in 1 h = 1/18

Part filled by B alone in 1 h = 1/12

Part filled by $(A + B)$ in 1 h $= \frac{1}{18} + \frac{1}{12} = \frac{2+3}{36} = \frac{5}{36}$

Hence, both the pipes together will fill the tank $= \frac{36}{5}$ h $= 7\frac{1}{5}$ h

Fast Track Method

Time taken by both pipes to fill the tank $= \frac{mn}{m+n}$ where, m and n are the time taken to fill the tank by individual pipes.

Here, $m = 18$, $n = 12$

Time taken to fill the tank $= \frac{m \times n}{m+n} = \frac{18 \times 12}{18 + 12}$

$$= \frac{18 \times 12}{30} = \frac{3 \times 12}{5} = \frac{36}{5} = 7\frac{1}{5} \text{ h}$$

Ex. 5 A pipe can fill a tank in 5 h, while another pipe can empty it in 6 h. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Sol. Part filled by 1st pipe in 1 h $= \frac{1}{5}$

Part filled by 2nd pipe in 1 h $= \frac{1}{6}$

$\therefore$ Part filled in 1 h by both pipes $= \frac{1}{5} - \frac{1}{6} = \frac{6-5}{30} = \frac{1}{30}$

Hence, the tank will be filled completely in 30 h.

Fast Track Method

Here, $m = 5$ h and $n = 6$ h

Time taken to fill the tank $= \frac{m \times n}{n - m}$

$$= \frac{5 \times 6}{6 - 5} = \frac{30}{1} = 30 \text{ h}$$

Ex. 6 A pipe can fill a tank in 10 h. Due to a leak in the bottom, it fills the tank in 20 h. If the tank is full, how much time will the leak take to empty it?

Sol. Let the leak empties full tank in x h, then part emptied in 1 h by leak $= \frac{1}{x}$

Also, part filled by inlet pipe in 1 h $= \frac{1}{10}$

According to the question,

$$\frac{1}{10} + \frac{1}{x} = \frac{1}{20}$$

$\therefore$ $\frac{1}{x} = \frac{1}{20} - \frac{1}{10} = \frac{1-2}{20} = -\frac{1}{20}$ [–ve sign means leak empties the tank]

$\therefore$ Leak will empty the full tank in 20 h.

Direct Approach

Work done by the leak in 1 h $= \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$

$\therefore$ Leak will empty the full tank in 20 h.

Fast Track Method

Here, $m = 10$ and $n = 20$

According to the formula,

Required time taken to empty the tank $= \frac{m \times n}{n - m} = \frac{10 \times 20}{20 - 10} = \frac{200}{10} = 20$ h

Technique 2

If three pipes can fill a tank separately in m, n and p h, respectively, then part of tank filled in 1 h by all the three pipes is given by $\left(\frac{1}{m} + \frac{1}{n} + \frac{1}{p}\right)$ and total time taken to fill the tank is given by $\frac{mnp}{np + mp + mn}$ h.

MIND IT!

If any one of the three pipes is used to empty the tank, then time taken by that particular pipe will be negative (-ve). Suppose, 3rd pipe is used to empty the tank. Then, the above formulae takes the form as

$$\left(\frac{1}{m} + \frac{1}{n} - \frac{1}{p}\right) \text{ and } \frac{mnp}{np + mp - mn} \text{ h.}$$

Ex. 7 Three pipes *m, n* and *p* can fill a tank separately in 4, 5 and 10 h, respectively. Find the time taken by all the three pipes to fill the tank when the pipes are opened together.

Sol. Part filled by pipe m in 1 h $= \frac{1}{4}$

Part filled by pipe n in 1 h $= \frac{1}{5}$

Part filled by pipe p in 1 h $= \frac{1}{10}$

$\therefore$ Part filled by $(m + n + p)$ pipes in 1 h $= \frac{1}{4} + \frac{1}{5} + \frac{1}{10}$

$$= \frac{5 + 4 + 2}{20} = \frac{11}{20}$$

$\therefore$ Required time to fill the tank $= \frac{20}{11}$ h $= 1\frac{9}{11}$ h

Ex. 8 Pipe A can fill a tank in 20 h while pipe B alone can fill it in 10 h and pipe C can empty the full tank in 30 h. If all the pipes are opened together, how much time will be needed to make the tank full?

Sol. Part filled by pipe A alone in 1 h = —

20

Part rilled by pipe B alone in 1 h = —

10

Part emptied by pipe C alone in 1 h = —

30

Net part filled by $(A + B + C)$ in 1 h = f — + — - — | V ^ l20 10 30j

$$= \left(\frac{3 + 6 - 2}{60}\right)$$

$$= \frac{7}{60} \text{ h}$$

$\therefore$ Required time to fill the tank $= \frac{60}{7} \text{ h} = 8\frac{4}{7}$ h.

Fast Track Method

Here, $m = 20$, $n = 10$ and $p = 30$

$\therefore$ Required time to fill the tank $= \frac{mnp}{np + mp - mn}$

$$= \frac{20 \times 10 \times 30}{10 \times 30 + 20 \times 30 - 20 \times 10}$$

$$= \frac{6000}{300 + 600 - 200} = \frac{6000}{700} = \frac{60}{7} = 8\frac{4}{7} \text{ h.}$$

Technique 3

Two pipes A and B together can fill a tank in time t. If time taken by/* alone is more than t by a and time taken by B alone is more than t by b, then t = $4ab$.

Ex. 9 Two pipes A and B are opened together to fill a tank. Both the pipes fill the tank in time t. If A separately takes 4 min more time than t to fill the tank and B takes 64 min more time than t to fill the tank, find the value of t.

Sol. We know that, time taken by both pipes to fill the tank $(t) = \sqrt{ab}$

[here, $a = 4$ and $b = 64$]

$$\therefore \quad t = \sqrt{4 \times 64} = 2 \times 8 = 16 \text{ min}$$

Technique 4

A full tank get emptied in $'a'$ h due to presence of a leak in it. If a tap which fills it at a rate of $'b'$ L/h, is opened, then it get emptied in $'c'$ h.

Therefore, volume of tank $= \dfrac{abc}{c-a}$

Ex. 10 A full tank get emptied in 6 min due to presence of an orifice in it. On opening a tap which can fill the tank at the rate of 8 L/min, the tank get emptied in 10 min. find the capacity of tank.

Sol. Here, $a = 6$, $b = 8$ and $c = 10$

Capacity of tank $= \dfrac{abc}{c-a} = \dfrac{6 \times 8 \times 10}{10 - 6} = 120$ L.

Technique 5

If two taps A and B, which can fill a tank, such that efficiency of A is n times of B and takes t min less/more than B to fill the tank, then

(i) time taken to fill the tank by both pipes together $= \dfrac{nt}{n^2 - 1}$ min

(ii) time taken to fill the tank by faster tap $= \dfrac{t}{n-1}$ min

(iii) time taken to fill the tank by slower tap $= \dfrac{nt}{n-1}$ min

Ex. 11 If tap A can fill a tank 3 times faster than tap B and takes 28 min less than tap B to fill the tank. If both the taps are opened simultaneously, then find the time taken to fill the tank.

Sol. Let the time taken by pipe A to fill the tank be x min. Then, time taken by pipe B to fill the tank be $3x$ min.

Then, according to the question,

$3x - x = 28;\ 2x = 28$

$\Rightarrow \quad x = \frac{28}{2} = 14$ min

$\therefore$ Time taken by pipe A = 14 min.

Time taken to fill the tank by pipe A in 1 min = $\frac{1}{14}$

Time taken by pipe $B = 3 \times 14 = 42$ min

Time taken to fill the tank by pipe B in 1 min = $\frac{1}{42}$

Time taken to fill the tank by pipes in 1 min

$$= \frac{1}{14} + \frac{1}{42} = \frac{3+1}{42}$$

$$= \frac{4}{42} = \frac{2}{21}$$

So, time taken by both pipes to fill the tank working together = $\frac{21}{2}$ min

Fast Track Method

Here, $n = 3$ and $t = 28$

According to the formula,

So, time taken to fill the tank by both pipes together

$$= \frac{nt}{n^2 - 1} = \frac{28 \times 3}{(3)^2 - 1} = \frac{28 \times 3}{8} = \frac{21}{2} \text{ min}$$

Technique 6

Two pipes A and B can fill a tank in x min and y min, respectively. If both the nines are onened simultaneously, then the time after which nine S

should be closed so that the tank is full in t min, is $\left[y\left(1 - \frac{t}{x}\right)\right]$ min.

Ex. 12 Two pipes A and B can fill a tank in 12 and 16 min, respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 9 min?

Sol. Here, $x = 12$, $y = 16$ and $t = 9$

Required time after which B should be closed

$$= y\left(1 - \frac{t}{x}\right) = 16\left(1 - \frac{9}{12}\right)$$

$$= 16 \times \frac{3}{12} = 4 \text{ min}$$

Fast Track Practice

Exercise© *Base Level Questions*

1. If a pipe fills a tank in 6 h, then what part of the tank will the pipe fill in 1 h?

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$
(e) None of the above

2. An outlet pipe can empty a cistern in 30 min, then what part of the cistern will it empty in lmin?

(a) $\frac{1}{20}$ (b) $\frac{1}{15}$ (c) $\frac{1}{30}$ (d) $\frac{1}{18}$
(e) None of the above

3. An inlet pipe fills 1/8 part of a tank in 1 h. How much time will the pipe take to fill the empty tank?

(a) 4 h *(b)* 2 h *(c)* 6 h *(d)* 8 h *(e)* None of the above

4. An outlet pipe can empty a cistern in 3 h. In what time will the pipe empty two-third part of the cistern?

(a) 4 h *(b)* 2 h *(c)* 3 h *(d)* 5 h *(e)* None of the above

5. Two pipes *A* and *B* can fill a tank in 18 and 6 h, respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

[Bank Clerks 2008]

(a) $4\frac{1}{2}$ h (b) 7 h (c) 8 h (d) 10 h
(e) None of the above

6. There are two tanks *A* and *B* to fill up a water tank. The tank can be filled in 40 min, if both taps are on. The same tank can be filled in 60 min, if tap *A* alone is on. How much time will tap *B* alone take, to fill up the same tank?

[CDS 2012] *(a)* 64 min *(b)* 80 min

(c) 96 min *(d)* 120 min

7. A cistern can be filled up in 4 h by an inlet *A*. An outlet *B* can empty the cistern in 8 h. If both *A* and *B* are opened simultaneously, then after how much time will the cistern get filled?

[Bank Clerks 2009]

(a) 5 h *(b)* 7 h *(c)* 8 h *(d)* 6 h *(e)* None of the above

8. A pipe can fill a tank in 20 h. Due to a leak in the bottom, it is filled in 40 h. If the tank is full, how much time will the leak take to empty it? [CDS 2013]

(a) 40 h *(b)* 30 h *(c)* 50 h *(d)* 30 h

9. A pipe can fill a tank in 10 h, while an another pipe can empty it in 6 h. Find the time taken to empty the tank, when both the pipes are opened up simultaneously. [Bank Clerks 2010]

(a) 11 h *(b)* 15 h *(c)* 18 h *(d)* 16 h *(e)* None of the above

10. Three pipes *A, B* and C can fill a tank separately in 8 h, 10 h and 20h, respectively. Find the time taken by all the three pipes to fill the tank when the pipes are opened together.

(a) $5\frac{7}{11}$ h *(b)* $4\frac{7}{11}$ h *(c)* $8\frac{7}{11}$ h *(d)* $3\frac{7}{11}$ h
(e) None of the above

11. Three taps are fitted in a cistern. The empty cistern is filled by the first and the second taps in 3 and 4 h, respectively. The full cistern is emptied by the third tap in 5 h. If all three taps are opened simultaneously, the empty cistern will be filled up in [SSC CCL 2013]

(a) $1\frac{14}{23}$ h *(b)* $2\frac{14}{23}$ h
(c) 2 h 40 min *(d)* 1 h 58 min

12. Pipe *A* can fill a tank in 30 min, while pipe *B* can fill the same tank in 10 min and pipe C can empty the full tank in 40 min. If all the pipes are opened together, how much time will be needed to make the tank full? [Hotel Mgmt. 2010]

(a) $9\frac{3}{13}$ h *(b)* $9\frac{4}{13}$ h *(c)* $8\frac{7}{13}$ h *(d)* $9\frac{9}{13}$ h
(e) None of the above

13. Pipes A and B can fill a tank in 5 and 6 h, respectively. Pipe C can fill it in 30 h. If all the three pipes are opened together, then in how much time the tank will be filled un? fBank PO 2007]

(a) $3\frac{3}{14}$ h (b) $2\frac{1}{2}$ h (c) $3\frac{9}{14}$ h (d) $2\frac{1}{14}$ h

(e) None of the above

14. Through an inlet, a tank takes 8 h to get filled up. Due to a leak in the bottom, it takes 2 h more to get it filled completely. If the tank is full, how much time will the leak take to empty it? [SSC CGL 2010]

(a) 16 h *(b)* 20 h *(c)* 32 h *(d)* 40 h

15. A tap can fill an empty tank in 12 h and a leakage can empty the tank in 20 h. If tap and leakage both work together, then how long will it take to fill the tank?

[Bank Clerks 2010] *(a)* 25 h *(b)* 40 h *(c)* 30 h *(d)* 35 h *(e)* None of the above

16. Three taps A, B and C together can fill an empty cistern in 10 min. The tap A alone can fill it in 30 min and the tap B alone can fill it in 40 min. How long will the tap C alone take to fill it?

[SSC CPO 2010]

(a) 16 min *(b)* 24 min

(c) 32 min *(d)* 40 min

17. Two pipes A and B can fill a tank in 1 h and 75 min, respectively. There is also an outlet C. If all the three pipes are opened together. The tank is full in 50 min. How much time will be taken by C to empty the full tank?

(a) 100 min *(b)* 150 min

(c) 200 min *(d)* 125 min

(e) None of the above

18. A tank has a leak which would empty it in 8 h. A tap is turned on which admits 3 L a min into the tank and it is now emptied in 12 h. How many litres does the tank hold?

(a) 4320 L *(b)* 4000 L

(c) 2250 L *(d)* 4120 L

19. *A, B* and *C* are three pipes connected to a tank. *A* and *B* together fill the tank in 6 h, *B* and C together fill the tank in 10 h and *A* and *C* together fill the tank in 12 h. In how much time *A, B* and *C* fill up the tank together?

(a) 9 h *(b)* $5\frac{3}{7}$ h *(c)* $5\frac{2}{7}$ h *(d)* $5\frac{5}{7}$ h

(e) None of the above

20. Two pipes *P* and *Q* can fill a cistern in 12 and 15 min, respectively. If both are opened together and at the end of 3 min, the first is closed. How much longer will the cistern take to fill? [SSC CGL 2013]

(a) $8\frac{1}{4}$ min *(b)* $8\frac{3}{4}$ min

(c) 5 min *(d)* $8\frac{1}{2}$ min

21. Two pipes *A* and *B* are opened together to fill a tank. Both pipes fill the tank in a certain time. If *A* separately takes 16 min more than the time taken by *(A + B)* and *B* takes 9 min more than the time taken by *(A + B)*. Find the time taken by *A* and *B* to fill the tank when both the pipes are opened together.

(a) 10 min *(b)* 12 min

(c) 15 min *(d)* 8 min

(e) None of the above

22. There are three pipes connected with a tank. The first pipe can fill 1/2 part of the tank in 1 h, second pipe can fill 1/3 part of the tank in 1 h. Third pipe is connected to empty the tank. After opening all the three pipes, 7/12 part of the tank can be filled in 1 h, then how long will third pipe take to empty the full tank? [SSC CGL 2007]

(a) 3 h *(b)* 4 h *(c)* 5 h *(d)* 6 h

23. Two pipes can fill a tank in 20 and 24 min, respectively and a waste pipe can empty 6 gallon per min. All the three pipes working together can fill the tank in 15 min. Find the capacity of the tank.

(a) 210 gallon *(b)* 50 gallon

(c) 150 gallon *(d)* 240 gallon

(e) None of the above

24. Inlet *A* is four times faster than inlet *B* to fill a tank. If *A* alone can fill it in 15 min, how long will it take if both the pipes are opened together?

(a) 10 min *(b)* 12 min

(c) 15 min *(d)* 14 min

(e) None of the above

25. There are two inlets *A* and *B* connected to a tank. *A* and *B* can fill the tank in 16 h and 10 h, respectively. If both the pipes are opened alternately for 1 h, starting from *A,* then how much time will the tank take to be filled?

(a) $13\frac{1}{4}$ h *(b)* $11\frac{6}{8}$ h *(c)* $12\frac{2}{5}$ h *(d)* $12\frac{1}{4}$ h

(e) None of the above

26. Two pipes *X* and *Y* can fill a cistern in 6 and 7 min, respectively. Starting with pipe *X,* both the pipes are opened alternately, each for 1 min. In what time will they fill the cistern?

(a) $6\frac{2}{7}$ min *(b)* $6\frac{3}{7}$ min

(c) $6\frac{5}{7}$ min *(d)* $6\frac{1}{7}$ min

(e) None of the above

Exercise © *Higher Skill Level Questions*

1. A tap having diameter *'d'* can empty a tank in 40 min. How long another tap having diameter *'2d'* take to empty the same tank?

(a) 5 min *(b)* 20 min

(c) 10 min *(d)* 40 min

(e) 80 min

2. Two pipes can fill a cistern in 14 and 16 h, respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom, it took 92 min more to fill the cistern. When the cistern is full, in what time will the leak empty it?

(a) $43\frac{19}{23}$ h (b) $43\frac{17}{23}$ h

(c) $43\frac{13}{23}$ h (d) $43\frac{19}{23}$ h

(e) None of the above

3. Two pipes *A* and *B* can fill a tank in 24 and 32 min, respectively. If both the pipes are opened together, after how much time pipe *B* should be closed so that the tank is full in 9 min?

(a) 40 min *(b)* 30 min

(c) 10 min *(d)* 20 min

(e) None of the above

4. Two pipes *A* and *B* can fill a cistern in 15 and 20 min, respectively. Both the pipes are opened together, but after 2 min, pipe *A* is turned off. What is the total time required to fill the tank?

(a) $\frac{46}{3}$ min (b) $\frac{52}{3}$ min

(c) $\frac{43}{3}$ min (d) $\frac{41}{3}$ min

(e) None of the above

5. A pipe can fill a cistern in 12 min and another pipe can fill it in 15 min, but a third pipe can empty it in 6 min. The first two pipes are kept open for 5 min in the beginning and then the third pipe is also opened. Time taken to empty the cistern is [SSC CCL (Mains) 2013]

(a) 38 min *(b)* 22 min

(c) 42 min *(d)* 45 min

6. A tank can be filled by a tap in 20 min and by another tap in 60 min. Both the taps are kept open for 5 min and then the 1st tap is shut off. After this, how much time the tank will be completely filled?

(a) 20 min *(b)* 30 min

(c) 45 min *(d)* 40 min

(e) None of the above

7. A cistern has three pipes *A, B* and C. Pipes *A* and *B* can fill it in 3 and 4 h, respectively, while pipe C can empty the completely filled cistern in 1 h. If the pipes are opened in order at 3:00 pm, 4:00 pm and 5:00 pm, respectively, at what time will the cistern be empty?

[SSC (10+2) ***2007]***

(a) 6:15 pm *(b)* 7:12 pm

(c) 8:12 pm *(d)* 8:35 pm

8. If two pipes function together, the tank will be filled in 12 h. One pipe fills the tank in 10 h faster than the other. How many hours does the faster pipe take to fill up the tank?

(a) 20 h *(b)* 60 h *(c)* 15 h *(d)* 25 h *(e)* None of the above

9. A pipe *P* can fill a tank in 12 min and another pipe *R* can fill it in 15 min. But, the 3rd pipe *M* can empty it in 6 min. The 1st two pipes *P* and *R* are kept open for double the 2.5 min in the beginning and then the 3rd pipe is also opened. In what time is the tank emptied?

(a) 30 min *(b)* 25 min

(c) 45 min *(d)* 35 min

(e) None of the above

10. Three pipes *A, B* and C can fill a tank in 30 min, 20 min and 10 min, respectively. When the tank is empty, all the three pipes are opened. If *A, B* and C discharge chemical solutions *P, Q* and *R* respectively, then the part of solution *R* in the liquid in the tank after 3 min is

[SSC (10+2) 2013]

(a) $\frac{8}{11}$ *(b)* $\frac{5}{11}$ *(c)* $\frac{6}{11}$ *(d)* $\frac{7}{11}$

11. There are 7 pipes attached with a tank out of which some are inlets and some are outlets. Every inlet can fill the tank in 10 h and every outlet can empty the tank in 15 h. When all the pipes are opened simultaneously, the tank is filled

up in $2\frac{8}{11}$ h. Find the numbers of inlets and outlets.

(a) 5, 2 *(b)* 6, 1 *(c)* 4, 3 *(d)* 3, 4 *(e)* None of the above

12. Capacity of tap *B* is 80% more than that of *A*. If both the taps are opened simultaneously, they take 45 h to fill the tank. How long will *B* take to fill the tank alone?

{a) 72 h *(b)* 48 h

(c) 66 h *(d)* 70 h

(e) None of the above

13. Three taps *A, B* and C fill a tank in 20 min, 15 min and 12 min, respectively. If all the taps are opened simultaneously, how long will they take to fill 40% of the tank?

(a) 1 min *(b)* 2 min

(c) 3 min *(d)* 4 min

(e) None of the above

14. Taps *A, B* and *C* are attached with a tank and velocity of water coming through them are 42 L/h, 56 L/h and 48 L/h, respectively. *A* and *B* are inlets and *C* is outlet. If all the taps are opened simultaneously, tank is filled in 16 h. What is the capacity of the tank?

(a) 2346 L *(b)* 1600 L

(c) 800 L *(d)* 960 L

(e) None of the above

15. Two taps *A* and *B* can fill a tank in 25 min and 20 min, respectively. But taps are not opened properly, so the taps

A and *B* allow $\frac{5}{6}$th and $\frac{2}{3}$rd part of water, respectively. How long will they take to fill the tank?

(a) 12 min *(b)* 13 min

(c) 14 min *(d)* 15 min

(e) 16 min 16. Two taps *A* and *B* can fill a tank in 20 min and 30 min, respectively. An outlet pipe *C* can empty the full tank in 15 min. *A, B* and C are opened alternatively, each for 1 min. How long will the tank take to be filled? *(a)* 105 min *(b)* 120 min

(c) 167 min *(d)* 185 min

(e) 198 min

17. **There are three taps of diameters 1 cm, $\frac{4}{3}$ cm and 2 cm, respectively. The ratio of the water flowing through them is equal to the ratio of the square of their diameters. The biggest tap can fill the tank alone in 61 min. If all the taps are opened simultaneously, how long will the tank take to be filled?**

(a) 44 min *(b)* 45 min

(c) $44\frac{1}{4}$ min *(d)* 46 min

(e) None of the above

Answer with Solutions

Exercise© *Base Level Questions*

1. (*b*) We know that, when a pipe fills a tank in m h, then the part of tank filled in 1 h

$$= \frac{1}{m}$$

Here, $m = 6$

$\therefore$ Required part of the tank to be filled in 1 h $= \frac{1}{6}$ part

2. (*c*) We know that, when a pipe empties a cistern in n min, then the part emptied by the pipe in 1 min $= \frac{1}{n}$.

Here, $n = 30$

$\therefore$ Required part of the tank emptied in 1 min $= \frac{1}{30}$ part

3. (*d*) We know that, when a pipe fills $\frac{1}{m}$ part of a tank in 1 h, then the pipe takes m h to fill the tank.

Here, $\frac{1}{m} = \frac{1}{8} \Rightarrow m = 8$

$\therefore$ Equired time to fill the tank = 8 h

4. (*b*) Time taken by pipe to empty the cistern = 3 h

Then, time taken by the pipe to empty the $\frac{2}{3}$ part $= 3 \times \frac{2}{3} = 2$ h

5. (*a*) Part filled by A in 1 h $= \frac{1}{18}$

Part filled by B in 1 h $= \frac{1}{6}$

Part filled by $(A + B)$

$$= \frac{1}{18} + \frac{1}{6} = \frac{1+3}{18} = \frac{4}{18} = \frac{2}{9}$$

Hence, both the pipes together will fill the tank in $\frac{9}{2}$ h or $4\frac{1}{2}$ h.

Fast Track Method

Here, $m = 18$ and $n = 6$

Required time $= \frac{m \times n}{m + n}$

[by Technique 1(i)]

$$= \frac{18 \times 6}{18 + 6} = \frac{18 \times 6}{24} = \frac{18}{4} = \frac{9}{2} = 4\frac{1}{2} \text{ h}$$

6. (*d*) Part filled by tap A in 1 min $= \frac{1}{60}$

Let tap B fills the tank in x min

Then, part filled by tap, B in 1 min $= \frac{1}{x}$

According to the question,

$$= \frac{1}{60} + \frac{1}{x} = \frac{1}{40}$$

$$= \frac{1}{x} = \frac{1}{40} - \frac{1}{60}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{3-2}{120}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{120}$$

$\therefore$ Tap B can fill the tank in 120 min.

7. (*c*) Part filled by A in 1 h $= \frac{1}{4}$

Part emptied by B in 1 h $= \frac{1}{8}$

Part filled by $(A + B)$ in 1 h

$$= \frac{1}{4} + \left(-\frac{1}{8}\right) = \frac{1}{4} - \frac{1}{8} = \frac{2-1}{8} = \frac{1}{8}$$

$\therefore$ Required time to fill the cistern = 8 h

Note $\frac{-1}{8}$ has been taken because it empties the tank.

Fast Track Method

Here, $n = 8$ and $n = 4$.

Required time $= \frac{m \times n}{m - n}$ **[by Technique 1 (ii)]**

$$= \frac{8 \times 4}{8 - 4} = 8 \text{ h}$$

8. (*a*) Let the leak empties the full tank in x h, then

Part emptied in 1 h by leak $= \frac{1}{x}$

Part filled by inlet in 1 h $= \frac{1}{20}$

According to the question,

$$\frac{1}{20} + \frac{1}{x} = \frac{1}{40}$$

$$\therefore \quad \frac{1}{x} = \frac{1}{40} - \frac{1}{20} = \frac{1-2}{40} = -\frac{1}{40}$$

[-ve sign indicates emptying.]

Clearly, leak will empty the full tank in 40 h.

Direct Approach

Work done by the leak in 1 h

$$= \frac{1}{20} - \frac{1}{40} = \frac{2-1}{40} = \frac{1}{40}$$

$\therefore$ Leak will empty the full tank in 40 h.

Fast Track Method

Here, $m = 20$ and $n = 40$

Required time

$$= \frac{m \times n}{n - m} \quad \text{[by Technique 1(iii)]}$$

$$= \frac{20 \times 40}{40 - 20} = 40 \text{ h}$$

9. (*b*) Part filled by 1st pipe in 1 h $= \frac{1}{10}$

Part emptied by 2nd pipe in 1 h $= \frac{1}{6}$

Part emptied when both the pipes are opened up

$$= \frac{1}{6} - \frac{1}{10} = \frac{5-3}{30} = \frac{2}{30} = \frac{1}{15}$$

Hence, time taken to empty the full tank in 15 h.

Fast Track Method

Here, $n = 10$ and $m = 6$

Time taken to empty the tank

$$\frac{m \times n}{n - m} \quad \text{[by Technique 1 (iii)]}$$

$$= \frac{10 \times 6}{10 - 6} = \frac{60}{4} = 15 \text{ h}$$

10. (*d*) Part filled by A in 1 h $= \frac{1}{8}$

Part filled by B in 1 h $= \frac{1}{10}$

Part filled by C in 1 h $= \frac{1}{20}$

Part filled by $(A + B + C)$ in 1 h

$$= \frac{1}{8} + \frac{1}{10} + \frac{1}{20} = \frac{5 + 4 + 2}{40} = \frac{11}{40}$$

$\therefore$ Required time to fill the tank

$$= \frac{40}{11} \text{ h} = 3\frac{7}{11} \text{ h}$$

Fast Track Method

Here, $m = 8$, $n = 10$ and $p = 20$

Required time to fill the tank

$$= \frac{mnp}{mn + np + mp} \quad \text{[by Technique 2]}$$

$$= \frac{8 \times 10 \times 20}{8 \times 10 + 10 \times 20 + 8 \times 20}$$

$$= \frac{1600}{80 + 200 + 160}$$

$$= \frac{1600}{440} = \frac{40}{11} = 3\frac{7}{11} \text{ h}$$

11. (*b*) Part of tank filled by first tap in 1 h = $\frac{1}{3}$

Part of tank filled by second tap in 1 h = $\frac{1}{4}$

Part of tank emptied by third tap in 1 h = $\frac{1}{5}$

Part of the tank filled by all pipes opened simultaneously in 1 h

$$= \frac{1}{3} + \frac{1}{4} - \frac{1}{5} = \frac{20 + 15 - 12}{60} = \frac{23}{60}$$

Time taken by all the taps to fill the tank when it is empty

$$= \frac{60}{23} \text{ h} = 2\frac{14}{23} \text{ h}$$

Fast Track Method

Here, $m = 3$, $n = 4$ and $p = 5$

$\therefore$ Required time $= \dfrac{mnp}{np + mp - mn}$

[by Technique 2]

$$= \frac{3 \times 4 \times 5}{4 \times 5 + 3 \times 5 - 3 \times 4}$$

$$= \frac{60}{20 + 15 - 12} = \frac{60}{23} = 2\frac{14}{23} \text{ h}$$

12. (*a*) Part filled by A in 1 min = $\frac{1}{30}$

Part filled by B in 1 min = $\frac{1}{10}$

Part emptied by C in 1 min = $-\frac{1}{40}$

Net part filled in 1 h by $(A + B + C)$

$$= \left(\frac{1}{30} + \frac{1}{10} - \frac{1}{40}\right)$$

$$= \frac{4 + 12 - 3}{120} = \frac{13}{120}$$

$\therefore$ Required time to fill the tank

$$= \frac{120}{13} = 9\frac{3}{13} \text{ h}$$

13. (*b*) Part filled by A in 1 h = $\frac{1}{5}$

Part filled by B in 1 h = $\frac{1}{6}$

Part filled by C in 1 h = $\frac{1}{30}$

Net part filled by $(A + B + C)$ in 1 h

$$= \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{30}\right) = \frac{6 + 5 + 1}{30} = \frac{12}{30} = \frac{2}{5}$$

$\therefore$ Required time to fill the tank $= \frac{5}{2} = 2\frac{1}{2}$ h

14. (*d*) Let the leak takes x h to empty the tank.

Now, part filled by inlet in 1 h = $\frac{1}{8}$

Part filled in 1 h when both tap and leak works together

$$= \frac{1}{8+2} = \frac{1}{10}$$

According to the question,

$$\frac{1}{x} = \frac{1}{8} - \frac{1}{10} = \frac{5-4}{40} = \frac{1}{40}$$

$\therefore \quad x = 40$ h

15. (c) Part filled by tap in 1 h $= \frac{1}{12}$

Part emptied by leak in 1 h $= \frac{1}{20}$

Net part filled in 1 h when both (tap and leakage) work

$$= \frac{1}{12} - \frac{1}{20} = \frac{5-3}{60} = \frac{2}{60} = \frac{1}{30}$$

$\therefore$ Required time to fill the tank = 30 h

Fast Track Method

Here, $m=20$ and $n=12$

$\therefore \quad t = \dfrac{m \times n}{m-n}$ [by Technique 1(iii)]

$$= \frac{12 \times 20}{8} = 30\text{h}$$

16. (b) Part filled by $(A + B + C)$ in 1 min

$$= \frac{1}{10}$$

Part filled by A in 1 min $= \frac{1}{30}$

Part filled by B in 1 min $= \frac{1}{40}$

Part filled by $(A + B)$ in 1 min

$$= \frac{1}{30} + \frac{1}{40} = \frac{4+3}{120} = \frac{7}{120}$$

$\therefore$ Part filled by C in 1 min

$$= \frac{1}{10} - \frac{7}{120} = \frac{12-7}{120} = \frac{5}{120} = \frac{1}{24}$$

$\therefore$ Tap C will fill the cistern in 24 min.

17. (a) Work done by C in 1 min

$$= \left(\frac{1}{60} + \frac{1}{75} - \frac{1}{50}\right)$$

$$= \frac{5+4-6}{300} = \frac{3}{300} = \frac{1}{100}$$

Hence, C can empty the full tank in 100 min.

18. (a) Work done by the inlet in 1 h

$$= \left(\frac{1}{8} - \frac{1}{12}\right) = \frac{1}{24}$$

Work done by the inlet in 1 min

$$= \frac{1}{24} \times \frac{1}{60} = \frac{1}{1440}$$

$\therefore \quad$ Volume of $\frac{1}{1440}$ part = 3 L

$\therefore \quad$ Volume of the whole

$$= 3 \times 1440 = 4320 \text{ L}$$

Fast Track Method

Here, $a=8$, $b=3$ L / min

$= 3 \times 60 = 180$ and $c=12$

$\therefore$ Capacity of tank

$$= \frac{abc}{c-a} \quad \text{[by Technique 3]}$$

$$= \frac{180 \times 8 \times 12}{4} = 4320 \text{ L}$$

19. (*d*) Part filled by $(A + B)$ in 1 h $= \frac{1}{6}$

Part filled by $(B + C)$ in 1 h $= \frac{1}{10}$

Part filled by $(A + C)$ in 1 h $= \frac{1}{12}$

$\therefore$ Part filled by 2 $(A + B + C)$ in 1 h

$$= \frac{1}{6} + \frac{1}{10} + \frac{1}{12}$$

$$= \frac{10 + 6 + 5}{60} = \frac{21}{60} = \frac{7}{20}$$

$\therefore$ Part filled by $(A + B + C)$ in 1 h

$$= \frac{7}{2 \times 20} = \frac{7}{40}$$

$\therefore$ Required time $= \frac{40}{7} = 5\frac{5}{7}$ h

20. (*a*) Part filled by pipe P in 1 min $= \frac{1}{12}$

Part filled by pipe Q in 1 min $= \frac{1}{15}$

Part filled by both pipes in 1 min

$$= \frac{1}{12} + \frac{1}{15} = \frac{5 + 4}{60} = \frac{9}{60}$$

Now, part filled by both pipes in 3 min

$$= \frac{3 \times 9}{60} = \frac{27}{60} = \frac{9}{20}$$

$\therefore$ Remaining part $= 1 - \frac{9}{20} = \frac{11}{20}$

Let the remaining part is filled by pipe Q in x min.

Then, $x \times \frac{1}{15} = \frac{11}{20}$

$$x = \frac{15 \times 11}{20} = \frac{3 \times 11}{4} = \frac{33}{4} = 8\frac{1}{4} \text{ min}$$

21. (*b*) Here, $a = 16$ and $b = 9$

Required time

$$= \sqrt{ab} \quad \text{[by technique 3]}$$

$$= \sqrt{16 \times 9} = 4 \times 3 = 12 \text{ min}$$

22. (*b*) 1st pipe takes 1 h to fill $\frac{1}{2}$ part of the tank.

So, time taken to fill the whole tank $(m) = 2$ h

2nd pipe takes 1 h to $\frac{1}{3}$ part of the tank

So, time taken to fill the whole tank $(n) = 3$ h

Let 3rd pipe takes P h to empty the tank

$= x$

$\therefore \frac{1}{m} + \frac{1}{n} - \frac{1}{x} = \frac{7}{12} \Rightarrow \frac{1}{2} + \frac{1}{3} - \frac{1}{x} = \frac{7}{12}$

$\Rightarrow \quad \frac{1}{x} = \frac{6+4-7}{12} = \frac{3}{12} = \frac{1}{4}$

$\therefore \quad x = 4$ h

23. (*d*) Part filled by Ist pipe in 1 min $= \frac{1}{20}$

Part filled by 2nd pipe in 1 min $= \frac{1}{24}$

Part filled by all the pipes in 1 min $= \frac{1}{15}$

Work done by the waste pipe in 1 min

$= \frac{1}{15} - \left(\frac{1}{20} + \frac{1}{24}\right) = \frac{1}{15} - \left(\frac{6+5}{120}\right)$

$= \frac{1}{15} - \frac{11}{120}$

$= \frac{8-11}{120} = \left(-\frac{3}{120}\right) = \left(-\frac{1}{40}\right)$

[– ve sign indicates emptying]

Now, volume of $\frac{1}{40}$ part = 6 gallon

$\therefore$ Volume of whole tank

$= 40 \times 6 = 240$ gallon

24. (*b*) Time taken by A to fill the tank, m

$= 15$ min

$\therefore$ Time taken by B to fill the tank,

$n = 15 \times 4 = 60$ min

$\therefore$ Required time taken

$\frac{m \times n}{m + n}$ [by Technique 1(i)]

$= \frac{15 \times 60}{15 + 60} = \frac{15 \times 60}{75} = 12$ min

25. (*c*) Part filled by A in 1 h $= \frac{1}{16}$

Part filled by B in 1 h $= \frac{1}{10}$

Part filled by $(A + B)$ in 2 h

$$= \frac{1}{16} + \frac{1}{10} = \frac{13}{80}$$

$\therefore$ Part filled by $(A + B)$ in 12 h

$$= \frac{6 \times 13}{80} = \frac{78}{80}$$

$\therefore$ Remaining part $= 1 - \frac{78}{80} = \frac{2}{80} = \frac{1}{40}$

Now, it is the turn of A.

Time taken by A to fill $\frac{1}{40}$ part of the tank

$$= \frac{1}{40} \times 16 = \frac{2}{5} \text{ h}$$

$\therefore$ Total time taken

$$= \left(12 + \frac{2}{5}\right) \text{h} = 12\frac{2}{5} \text{ h}$$

26. (*b*) Part filled by X in 1st min and Y in the 2nd min $= \left(\frac{1}{6} + \frac{1}{7}\right) = \frac{13}{42}$

Part filled by $(X + Y)$ working alternately in 6 min $= \frac{1}{2} \times \frac{13}{42} \times 6 = \frac{13}{14}$

$\therefore$ Remaining part $= \left(1 - \frac{13}{14}\right) = \frac{1}{14}$

Now, it is the turn of x, one-sixth part is filled in 1 min. One-fourteenth part is filled in $\left(6 \times \frac{1}{14}\right)$ min $= \frac{3}{7}$ min

$\therefore$ Required time $= \left(6 + \frac{3}{7}\right) = 6\frac{3}{7}$ min

Exercise © *Higher Skill Level Questions*

1. (*c*) Area of tap $\propto$ Work done by pipe.

When diameter is doubled, area will be four times. So, it will work four times faster.

Hence, required time taken to empty the tank $= 40 \times \frac{1}{4} = 10$ min.

2. (*a*) Part filled by 1st pipe in 1 h $= \frac{1}{14}$

Part filled by 2nd pipe in 1 h $= \frac{1}{16}$

Part filled by the two pipes in 1 h

$$= \left(\frac{1}{14} + \frac{1}{16}\right) = \frac{8+7}{112} = \frac{15}{112}$$

$\therefore$ Time taken by these two pipes to fill the cistern $= \frac{112}{15}$ h $= 7$ h 28 min

Due to leakage, the time taken

7 h 28 min + 92 min = 9 h

$\therefore$ Work done by (two pipes + leak) in 1 h = 1 / 9

Work done by the leak in 1 h

$$= \frac{1}{9} - \frac{15}{112} = \frac{112-135}{1008}$$

$$= -\frac{23}{1008}$$

$\therefore$ Time taken by leak to empty the full cistern

$$= \frac{1008}{23} = 43\frac{19}{23} \text{ h}$$

3. (*d*) Part filled by A in 1 min $= \frac{1}{24}$

Part filled by B in 1 min $= \frac{1}{32}$

Let B is closed after x min. Then, [Part filled by $(A + B)$ in x min] + [Part filled by A in $(9 - x)$ min] = 1

$$\therefore \quad x\left(\frac{1}{24} + \frac{1}{32}\right) + (9 - x) \times \frac{1}{24} = 1$$

$$\Rightarrow \quad x\left(\frac{4 + 3}{96}\right) + \frac{(9 - x)}{24} = 1$$

$$\Rightarrow \quad \frac{7x}{96} + \frac{(9 - x)}{24} = 1$$

$$\Rightarrow \quad \frac{7x + 4(9 - x)}{96} = 1$$

$$\Rightarrow 7x + 4(9 - x) = 96$$

$$\Rightarrow 7x + 36 - 4x = 96$$

$$\Rightarrow \quad 7x - 4x = 96 - 36$$

$$\Rightarrow \quad 3x = 60 \Rightarrow x = \frac{60}{3} = 20$$

Hence, B must be closed after 20 min.

Fast Track Method

Here, $x = 24$ min, $y = 32$ min, $t = 9$ min

$\therefore$ Required time

$$= y\left(1 - \frac{t}{x}\right) \quad \text{[by Technique 6]}$$

$$= 32\left(1 - \frac{9}{24}\right)$$

$$= 32 \times \frac{15}{24} = 20 \text{ min.}$$

4. (*b*) Part filled by both in 2 min

$$= 2 \times \left(\frac{1}{15} + \frac{1}{20}\right) = 2 \times \left(\frac{4 + 3}{60}\right)$$

$$= 2 \times \frac{7}{60} = \frac{7}{30}$$

Part unfilled $= 1 - \frac{7}{30} = \frac{30 - 7}{30} = \frac{23}{30}$

Now, B fills $\frac{1}{20}$ part in 1 min.

$\therefore \quad \frac{23}{30}$ part will be filled by B in

$$\left(20 \times \frac{23}{30}\right) \text{ min or in } \frac{46}{3} \text{ min.}$$

$\therefore$ Required time taken to fill the tank

$$= \left(2 + \frac{46}{3}\right) = \frac{52}{3} \text{ min.}$$

5. (*d*) Let the number of minutes taken to empty the cistern be x min.

According to the question,

$$\frac{x}{6} - \frac{x + 5}{12} - \frac{x + 5}{15} = 0$$

$\Rightarrow \quad \frac{x}{6} - \frac{x}{12} - \frac{5}{12} - \frac{x}{15} - \frac{5}{15} = 0$

$\Rightarrow \quad \frac{x}{6} - \frac{x}{12} - \frac{x}{15} = \frac{5}{12} + \frac{5}{15}$

$\Rightarrow \quad \frac{10x - 5x - 4x}{60} = \frac{25 + 20}{60}$

$\Rightarrow \quad \frac{x}{60} = \frac{45}{60} \Rightarrow x = 45$ min.

6. (*d*) Part of the tank filled by both taps in 5 min

$= 5 \times \left(\frac{1}{20} + \frac{1}{60}\right) = \frac{5 \times (6 + 2)}{120} = \frac{8}{24} = \frac{1}{3}$

$\therefore$ Remaining part $= \left(1 - \frac{1}{3}\right) = \frac{2}{3}$

$\because \frac{1}{60}$ part is now filled in 1 min.

$\therefore \frac{2}{3}$ part is now filled in $60 \times \frac{2}{3} = 40$ min.

7. (*b*) Let the cistern gets emptied in m h after 3:00 pm.

Work done by A in m h, by B in $(m - 1)$ h and by C in $(m - 2)$ h $= 0$

$\Rightarrow \quad \frac{m}{3} + \frac{m-1}{4} - (m - 2) = 0$

$\Rightarrow \quad 4m + 3(m - 1) - 12(m - 2) = 0$

$\Rightarrow \quad 5m = 21$

$\Rightarrow \quad m = \frac{21}{5} = 4.2$

$\therefore \; m = 4$ h 12 min

$\therefore$ Required time $= 7 : 12$ pm.

8. (*a*) Let one pipe takes m h to fill the tank. Then, the other pipe takes $(m - 10)$ h.

According to the question.

$\therefore \quad \frac{1}{m} + \frac{1}{(m - 10)} = \frac{1}{12}$

$\Rightarrow \quad \frac{m - 10 + m}{m(m - 10)} = \frac{1}{12}$

$\Rightarrow \quad 12(m - 10 + m) = m(m - 10)$

$\Rightarrow \quad m^2 - 34m + 120 = 0$

$\Rightarrow \quad m^2 - 30m - 4m + 120 = 0$

$\Rightarrow \quad (m - 30)(m - 4) = 0$

$\therefore \quad m = 30$ or 4

$\therefore$ Faster pipe will take $(30 - 10)$h=20 h to fill the tank.

9. (*c*) According to the question,

Double the 2.5 min = 5 min

Now, part filled in 5 min

$= 5 \times \left(\frac{1}{12} + \frac{1}{15}\right) = 5 \times \left(\frac{5 + 4}{60}\right)$

$= 5 \times \frac{9}{60} = \frac{3}{4}$

Part emptied in 1 min when P, R and M, all are opened.

$= \frac{1}{6} - \left(\frac{1}{12} + \frac{1}{15}\right) = \frac{1}{6} - \left(\frac{5+4}{60}\right)$

$= \left(\frac{1}{6} - \frac{3}{20}\right) = \frac{1}{60}$

One-sixtieth part is emptied in 1 min.

$\therefore$ Three-fourth part will be emptied in

$$60 \times \frac{3}{4} = 15 \times 3 = 45 \text{ min.}$$

10. (*c*) Total quantity of solutions P, Q and R from A, B and C respectively, after 3 min

$$= \frac{3}{30} + \frac{3}{20} + \frac{3}{10} = 3\left(\frac{2+3+6}{60}\right)$$

$$= \frac{3 \times 11}{60} = \frac{11}{20}$$

Quantity of solution R in liquid in 3 min

$$= \frac{3}{10}$$

$\therefore$ Part of solution $R = \dfrac{\frac{3}{10}}{\frac{11}{20}} = \dfrac{3 \times 20}{10 \times 11} = \dfrac{6}{11}$

11. (*a*) Let number of outlets be x

$\therefore$ Number of inlets $= (7 - x)$

Time taken to fill the tank when all the pipes are opened $= \frac{30}{11}$ h

Part of tank filled in 1 h when all the pipes are opened $= \frac{11}{30}$ h

According to the question,

$$\frac{7-x}{10} - \frac{x}{15} = \frac{11}{30}$$

$$\Rightarrow \quad \frac{3(7-x) - 2x}{30} = \frac{11}{30}$$

$$\Rightarrow \quad 21 - 3x - 2x = 11$$

$$\Rightarrow \quad 5x = 10$$

$$\therefore \quad x = 2$$

Hence, number of outlets = 2

and number of inlets = 7 − 2 = 5

12. (*d*) Let time taken by B to fill the tank,

$a = x$ h.

$\therefore$ Time taken by A to fill the tank,

$$b = x + \frac{x \times 80}{100} = \frac{9x}{5} \text{ h}$$

According to the formula,

Time taken by both the taps to fill the tank

$$t = \frac{ab}{a+b} \qquad \text{[by Technique 1(i)]}$$

$$\Rightarrow \quad 45 = \frac{x \times \frac{9x}{5}}{x + \frac{9x}{5}}$$

$$\Rightarrow \quad 45 \times \frac{14x}{5} = \frac{9x^2}{5}$$

$$\therefore \quad x = \frac{45 \times 14}{9} = 70 \text{ h}$$

13. (*b*) Part of the tank filled in 1 min by A, B and C

$$= \frac{1}{20} + \frac{1}{15} + \frac{1}{12}$$

$$= \frac{3+4+5}{60} = \frac{12}{60} = \frac{1}{5}$$

$\therefore$ Time taken by A, B and C to fill the tank
$= 5$ min

$\therefore$ Time taken by A, B and C to fill 40% of the tank $= 40\%$ of $5 = \frac{40}{100} \times 5 = 2$ min

14. (*c*) Quantity of water admitted by tap 1 in 1 h
$= 42$ L

Quantity of water admitted by tap 2 in 1 h
$= 56$ L

Quantity of water removed by tap 3 in 1 h
$= 48$ L

So, quantity of water filled in the tank in 1 h
$= (42 + 56 - 48)$ L $= 50$ L

$\therefore$ Quantity of water filled in 16 h
$= 16 \times 50 = 800$ L

Hence, capacity of tank $= 800$ L

15. (*d*) Part of the tank filled with A and B in

$$1 \text{ min} = \frac{1}{25} \times \frac{5}{6} + \frac{1}{20} \times \frac{2}{3} = \frac{1}{30} + \frac{1}{30}$$

$$= \frac{2}{30} = \frac{1}{15}$$

Hence, time taken to fill the tank
$= 15$ min.

16. (*c*) Part of the tank filled by the taps A, B and C in 3 min

$$= \frac{1}{20} + \frac{1}{30} - \frac{1}{15} = \frac{3+2-4}{60} = \frac{1}{60}$$

$\therefore$ Time taken to fill $\left[1 - \left(\frac{1}{20} + \frac{1}{30}\right)\right]$ or

$\frac{55}{60}$th part of the tank $= 3 \times 55 = 165$ min

Remaining part of the tank

$$= 1 - \frac{55}{60} = \frac{5}{60} = \frac{1}{12}$$

Tap A fills $\frac{1}{20}$ part in 1 min, then

Remaining part $= \frac{1}{12} - \frac{1}{20} = \frac{5-3}{60} = \frac{2}{60}$

$= \frac{1}{30}$

i.e., $\frac{1}{30}$th part is filled by *B* in 1 min

Hence, required time to fill the whole tank = (165 + 1 + 1) min = 167 min

17. (*e*) Time taken to fill the tank by the tap having 2 cm diameter = 61 min

∴ Time taken to fill the tank by the tap having 1 cm diameter

$$= 61 \times \left(\frac{2}{1}\right)^2 = 244 \text{ min}$$

Similarly, time taken to fill the tank by the tap having $\frac{4}{3}$ cm diameter

$$= 61 \times \left(\frac{2}{4/3}\right)^2 = 61 \times \frac{9}{4} = \frac{549}{4} \text{ min}$$

∴ Part of the tank filled by all the three pipes in 1 min

$$= \frac{1}{61} + \frac{1}{244} + \frac{1}{549/4}$$

$$= \frac{36 + 9 + 16}{2196} = \frac{61}{2196} = \frac{1}{36}$$

Hence, required time taken = 36 min

Chapter

25

Speed, Time and Distance

Speed

The rate at which a body or an object travels to cover a certain distance is called speed of that body.

Time

The duration in hours, minutes or seconds spent to cover a certain distance is called the time.

Distance

The length of the path travelled by any object or a person between two places is known as distance.

Relation between Speed, Time and Distance

Speed is the distance covered by an object in unit time. It is calculated by dividing the distance travelled by the time taken.

- $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$
- $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$
- $\text{Distance} = \text{Speed} \times \text{Time}$

Note Remember, units of speed, time and distance should be in the same metric system

Ex. 1 A car covers 125 km in 5 h, then find **the** speed of the car.

Sol. We know that, $\text{speed} = \dfrac{\text{Distance}}{\text{Time}}$

$\therefore$ Required speed $= \dfrac{125}{5} = 25$ km/h

Ex. 2 A train covers a distance of 200 km with a speed of 10 km/h. What time is taken by the train to cover this distance?

Sol. Given,

Speed = 10 km/h and distance = 200 km

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \Rightarrow \text{Time} = \frac{200}{10}$$

$\therefore$ Required time = 20 h

Ex. 3 A bike crosses a bridge with a speed of 108 km/h. What will be the length of the bridge, if the bike takes 8 h to cross the bridge?

Sol. Here, length of the bridge = Distance travelled by bike in 8 h

= Speed x Time Given that, speed = 108 km/h

Time = 8 h $\therefore$ Length of the bridge = 108 X 8 = 864 km

Basic Formulae Related to Speed, Time and Distance

***Formula* 1**

$$\text{Conversion of units } a \text{ km/h} = \frac{a \times 1000 \text{ m}}{3600 \text{ s}} = \frac{5a}{18} \text{ m/s}$$

$$a \text{ m/s} = \frac{a \times 1/1000 \text{ km}}{1/3600 \text{ h}} = \frac{18a}{5} \text{ km/h}$$

Ex. 4 Convert 72 km/h into m/s.

Sol. We know that,

$$a \text{ km/h} = \left(a \times \frac{5}{18}\right) \text{m/s}$$

$$\therefore \quad 72 \text{ km/h} = \left(72 \times \frac{5}{18}\right) \text{m/s} = 4 \times 5 = 20 \text{ m/s}$$

Ex. 5 Convert 25 m/s to km/h.

Sol. We know that,

$$a \text{ m/s} = \left(a \times \frac{18}{5}\right) \text{km/h}$$

$$\therefore \quad 25 \text{ m/s} = \left(25 \times \frac{18}{5}\right) = 5 \times 18 = 90 \text{ km/h}$$

Formula 2

If speed is kept constant, then the distance covered by an object is proportional to time.

$$i.e., \quad \text{Distance} \propto \text{Time (speed constant) or } \frac{D_1}{T_1} = \frac{D_2}{T_2}$$

Ex. 6 A person covers $20\frac{2}{5}$ km in 3 h. What distance will he cover in 5 h?

Sol. Here, speed is kept constant. Therefore, according to the formula,

$$\frac{D_1}{T_1} = \frac{D_2}{T_2}$$

Given that $D_1 = 20\frac{2}{5} = \frac{102}{5}$ km

$T_1 = 3$ h, $T_2 = 5$ h and $D_2 = ?$

$$\therefore \quad \frac{102/5}{3} = \frac{D_2}{5} \Rightarrow D_2 = \frac{102 \times 5}{5 \times 3} = 34 \text{ km}$$

$\therefore$ Distance covered by the object in 5 h = 34 km

Formula 3

If time is kept constant, then the distance covered by an object is proportional to speed *i.e.*, Distance $\propto$ Speed (time constant) or $\frac{D_1}{S_1} = \frac{D_2}{S_2}$

Ex. 7 A person covers a distance of 12 km, while walking at a speed of 4 km/h. How much distance he would cover in same time, if he walks at a speed of 6 km/h?

Sol. Given that, $D_1 = 12\,\text{km}, S_1 = 4\,\text{km/h}, D_2 = ?$ and $S_2 = 6\,\text{km/h}$

Since, the time is kept constant. Therefore, according to the formula, $\frac{D_1}{S_1} = \frac{D_2}{S_2}$

$$\Rightarrow \quad \frac{12}{4} = \frac{D_2}{6} \Rightarrow D_2 = 18\,\text{km}$$

Therefore, the person will cover 18 km.

Formula 4

If distance is kept constant, then the speed of a body is inversely proportional to time. *i.e.*, Speed $\propto \frac{1}{\text{Time}}$ (distance constant)

or $\quad S_1 T_1 = S_2 T_2 = S_3 T_3 = \ldots$

Note If the ratio of speeds of two objects is $x : y$, then to cover same distance, the ratio of time taken will be $y : x$

Ex. 8 A person covers a certain distance with a speed of 18 km/h in 8 min. If he wants to cover the same distance in 6 min, what should be his speed?

Sol. We know that, Speed $= \frac{\text{Distance}}{\text{Time}} \Rightarrow 18 = \frac{\text{Distance} \times 60}{8}$ $\quad \left[\because 8\text{ min} = \frac{8}{60}\text{ h}\right]$

$\therefore \quad$ Distance $= \frac{18 \times 8}{60} = \frac{12}{5}$ km

$\therefore$ Speed to cover $\frac{12}{5}$ km in 6 min $= \frac{\text{Distance}}{\text{Time}} = \frac{\frac{12}{5}}{\frac{1}{10}}$ $\quad \left[\because 6\text{ min} = \frac{1}{10}\text{ h}\right]$

$$= \frac{12}{5} \times 10 = 12 \times 2 = 24\,\text{km/h}$$

Alternate Method We know that, if distance is same, then speed is inversely proportional to time.

Given that, $S_1 = 18$ km/h, $S_2 = ?$, $T_1 = \frac{8}{60}$ h and $T_2 = \frac{6}{60}$ h

According to the formula,

$$S_1T_1 = S_2T_2$$

where, S_1 and S_2 are speeds and T_1 and T_2 are times.

$$18 \times \frac{8}{60} = S_2 \times \frac{6}{60}$$

$$\therefore \quad S_2 = \frac{18 \times 8}{6} = 3 \times 8 = 24 \text{ km/h}$$

Formula 5

When two bodies *A* and *B* are moving with speed a km/h and *b* km/h respectively, then the relative speed of two bodies is (i) (a + *b)* km/h (if they are moving in opposite direction) (ii) (a - *b)* km/h (if they are moving in same direction)

Ex. 9 Two persons are moving in the directions opposite to each other. The speeds of the both persons are 5 km/h and 3 km/h, respectively. Find the relative speed of the two persons in respect of each other.

Sol. We know that, the two speeds will be added, if the motions of two objects are in opposite directions.

.•. Required relative speed =5 + 3 = 8 km/h

Ex. 10 Two trains are running in the same direction. The speeds of two trains are 5 km/h and 15 km/h, respectively. What will be the relative speed of second train with respect to first?

Sol. We know that, if two trains are running in same direction, then difference in speeds is the required relative speed.

Required relative speed = 15 - 5 = 10 km/h

Formula 6

When a body travels with different speeds for different durations, then average speed of that body for the complete Journey is defined as the total distance covered by the body divided by the total time taken to cover the distance.

$$\text{Average speed} = \frac{\text{Total distance covered by a body}}{\text{Total time taken by the body}}$$

Ex. 11 A person covers a distance of 20 km by bus in 35 min. After deboarding the bus, he took rest for 20 min and covers another 10 km by a taxi in 20 min. Find his average speed for the whole journey.

Sol. Total distance covered = (20 + 10) km = 30 km

Total time taken = (35 + 20 + 20) min = 75 min = $\frac{5}{4}$ h

∴ According to the formula,

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{30}{\frac{5}{4}} = 24 \text{ km/h}$$

So, the average speed of the person for the whole journey is 24 km/h.

Ex. 12 If a person covers 40 km at a speed of 10 km/h by a cycle, 25 km at 5 km/h on foot and another 100 km at 50 km/h by bus. Then, find his average speed for the whole journey?

Sol. Here, total distance covered by the person = (40 + 25 + 100) km = 165 km

Time taken to cover 40 km = $\frac{40}{10}$ = 4 h

Time taken to cover 25 km = $\frac{25}{5}$ = 5 h

Time taken to cover 100 km = $\frac{100}{50}$ = 2 h

∴ Total time taken for whole journey = (4 + 5 + 2) h = 11 h

According to the formula,

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{165}{11} = 15 \text{ km/h.}$$

MIND IT!

If a body covers a distance *D*^ at ^ km/h, D2 at ^ km/h, D3 at S3 km/h and so on upto D_n at S_n, then

$$\text{Average speed} = \frac{D_1 + D_2 + D_3 + D_4 + \ldots D_n}{\frac{D_1}{S_1} + \frac{D_2}{S_1} + \frac{D_3}{S_3} + \frac{D_4}{S_4} + \ldots \frac{D_n}{S_n}}$$

$$\text{Average speed} \neq \frac{S_1 + S_2 + S_3 + S_4 \ldots S_n}{n}$$

Fast Track **Techniques**

to *solve the* QUESTIONS

Technique 1

When a certain distance is covered at speed A and the same distance is covered at speed B, then the average speed during the whole journey is

given by $\frac{2AB}{A+B}$

Ex.13 Shantanu covers a certain distance by car driving at 35 km/h and he returns back to the starting point riding on a scooter with a speed of 25 km/h. Find the average speed for the whole journey.

Sol. Let us assume that the distance covered by Shantanu in one direction = D km

$\therefore$ Time taken in first case $= \frac{D}{35}$ h.

Time taken in second case $= \frac{D}{25}$ h

$$\therefore \quad \text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{D+D}{\frac{D}{35}+\frac{D}{25}} = \frac{2D}{D\left(\frac{1}{35}+\frac{1}{25}\right)}$$

$$= \frac{2 \times 35 \times 25}{60} = 29.16 \text{ km/h.}$$

Fast Track Method

Here, $A = 35$ km/h and $B = 25$ km/h

According to the formula,

$$\text{Average speed} = \frac{2AB}{A+B} = \frac{2 \times 35 \times 25}{60} = \frac{175}{6} = 29.16 \text{ km/h}$$

Technique 2

If a body covers $\frac{1}{a}$ part of journey at A km/h, $\frac{1}{b}$ part at B km/h, $\frac{1}{c}$ part at C km/h and so on, then the average speed of body for the whole journey is $\frac{1}{\frac{1}{aA}+\frac{1}{bB}+\frac{1}{cC}+\frac{1}{dD}\cdots}$

Ex. 14 A person covers half of his journey at 30 km/h, one-third at 40 km/h and rest of his iour-ney at 20 km/h. Find his average speed for the whole iourney.

Sol. Here, the person cover $\frac{1}{2}$ of his journey at 30 km/h, $\frac{1}{3}$ at 40 km/h and rest $\left(1-\frac{1}{2}-\frac{1}{3}=\frac{1}{6}\right)$ at 20 km/h

$\therefore$ Average speed for whole journey $= \dfrac{1}{\dfrac{1}{2\times 30}+\dfrac{1}{3\times 40}+\dfrac{1}{6\times 20}}$

$$= \frac{1}{\dfrac{1}{60}+\dfrac{1}{120}+\dfrac{1}{120}} = \frac{1}{\dfrac{2+1+1}{120}} = \frac{120}{2+1+1} = 30 \text{ km/h}$$

Technique 3

When a person covers a certain distance between two certain places with speed 'a', he gets his destination late by time t, but when he covers the same distance with speed *'tf,* he reaches his destination t_2 time earlier. In this case, the distance between two places is given by

$$D = \frac{ab(t_1+t_2)}{b-a}$$

Ex. 15 Aashutosh covers a certain distance between his home and college by cycle. Having an average speed of 30 km/h, he is late by 20 min. However, with a speed of 40 km/h, he reaches his college 10 min earlier. Find the distance between his house and college.

Sol. Let the distance $= x$

Difference between the time taken $= 20 + 10 = 30$ min $= \frac{30}{60}$ h

According to the question,

$$\frac{x}{30}-\frac{x}{40}=\frac{30}{60} \Rightarrow \frac{x}{30}-\frac{x}{40}=\frac{1}{2}$$

$\Rightarrow$ $4x - 3x = 60$

$\therefore$ $x = 60$ km

Fast Track Method Here, $a = 30$, $b = 40$

$$t_1 = \frac{20}{60} \text{ and } t_2 = \frac{10}{60}$$

According to the formula,

$$D = \frac{ab(t_1+t_2)}{b-a}$$

$\therefore$ Required distance $= \dfrac{30\times 40}{40-30}\times\left(\dfrac{20+10}{60}\right) = \dfrac{30\times 40}{10}\times\dfrac{30}{60} = (30\times 4)\times\dfrac{1}{2} = 60$ km

Note t_t time late and t_2 time earlier make a difference of ($t_j + t_2$)

Technique 4

When a person reaches a certain distance with speed *'a',* he gets late by f., time and when he increases his speed by '£>' to cover the same

distance, then he still gets late by *u* time, in this case, the distance is

calculated by $D = (t_1 - t_2)(a + b)\frac{a}{b}$

Ex. 16 A boy walking at a speed of 20 km/h reaches his school 30 min late. Next time he increases his speed by 4 km/h but still he is late by 10 min. Find the distance of the school from his home.

Sol. Let the distance $= x$

Here, the difference in time $= 30 - 10 = 20 \text{ min} = \frac{20}{60} = \frac{1}{3}$ h

Speed during next journey $= (20 + 4) = 24$ km/h

According to the question, $\frac{x}{20} - \frac{x}{24} = \frac{1}{3} \Rightarrow \frac{6x - 5x}{120} = \frac{1}{3} \Rightarrow x = 40$ km

Fast Track Method

Here, $a = 20$ km/h, $b = 4$ km/h, $t_1 = 30$ min and $t_2 = 10$ min

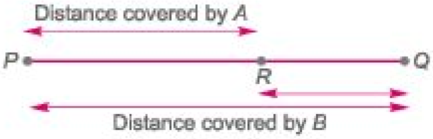
According to the formula,

Required distance $= (t_1 - t_2)(a + b)\frac{a}{b}$

$$= \frac{(30 - 10)}{60}(20 + 4)\frac{20}{4} = \frac{20}{60} \times 24 \times \frac{20}{4} = 5 \times 8 = 40 \text{ km}$$

Technique 5

When two persons *A* and e travel from points *P* to *Q,* a distance of D with speeds *'a'* and '£>', respectively and B reaches Q first, returns immediately and meets *A* at *R,* then



Distance travelled by A (from points P to R) $= 2 \times D\left(\dfrac{a}{a+b}\right)$

Distance travelled by B $(PQ + QR) = 2 \times D\left(\dfrac{b}{a+b}\right)$

Ex. 17 Sonu and Monu travel from point P to Q, a distance of 42 km, at 6 km/h and 8 km/h, respectively. Monu reaches *Q* first and returns immediately and meets Sonu at *R*. Find the distance from points *P* to *R*.

Sol. Given that, *D* = 42 km, a = 6 km/h and 6 = 8 km/h

According to the formula,

Distance travelled by Sonu = *PR* = *2D* x —— = 2 x 42 x ——

a + *b* 6+8

= 2 x 42 x — = 2x3x6 = 36 km 14

(technique^

A policeman sees a thief at a distance of d. He starts chasing the thief who is running at a speed of 'a' and policeman is chasing with a speed of 'ft' *(b* > a). In this case, the distance covered by the thief when he is

caught by the policeman, is given by $d\left(\dfrac{a}{b-a}\right)$

Ex. 18 A policeman sees a chain snatcher at a distance of 50 m. He starts chasing the chain snatcher who is running with a speed of 2 m/s while the policeman chasing him with a speed of 4 m/s. Find the distance covered by the chain snatcher when he is caught by the policeman.

Sol. Here, $d = 50$ m, $a = 2$ m/s and $b = 4$ m/s

According to the formula,

Required distance $= d\left(\dfrac{a}{b-a}\right) = 50 \times \dfrac{2}{4-2} = 50$ m

Technique 7

Two persons *A* and *B* start running at the same time in opposite directions from two points and after passing each other they complete their journeys in *Y* and '/ h, respectively. Then, *As* speed : e's speed = *Jy*: Vx

Ex. 19 A man sets out to cycle from points *P* to *Q* and at the same time another man starts to cycle from points Q to *P.* After passing each other, they complete their journeys in 9 h and 4 h, respectively. Find the ratio of speeds of 1st man to that of 2nd man.

Sol. Given that, *x* = 9 handy = 4h According to the formula, 1st man's speed : 2nd man's speed = *Jy : 4x* = V4 : V9 = 2:3

Technique 8

If a man changes his speed to $\left(\frac{x}{y}\right)$ of his usual speed and gets late by t min or reaches early by t min, then the usual time taken by him

$$= \frac{t \times x}{(y - x)} \text{ if } (y > x) \text{ and } \frac{t \times x}{(x - y)} \text{ if } (x > y)$$

Ex.20 A man increases his speed to 7/5 times of his original speed and reaches his office 20 min before to fixed time, then find the usual time taken by him?

Sol. Given that, $\frac{x}{y} = \frac{7}{5}, x = 7, y = 5$ and $t = 20$ min

Now, Required time $= \frac{t \times x}{(x - y)}$ $[\because x > y]$

$$= \frac{20 \times 7}{(7 - 5)} = \frac{20 \times 7}{2} = 70 \text{ min.}$$

Multi Concept

QUESTIONS

1. *Distance between A and B is 72 km. Two men started walking from A and B at the same time towards each other. The person who started from A travelled uniformly with average speed 4 km/h. While the other man travelled with varying speed as follows In first hour, his speed was 2 km/h, in the second hour, it was 2.5 km/h, in the third hour, it was 3 km/h, and so on. When will they meet each other?* (a) 7 h (b) 10 h

(c) 35 km from A (d) Midway between A and 8

(*d*) Both of them are moving in opposite direction hence relative speed will be sum of their speeds. It means that in 1st hour they will travel 6 km, in second hour they will travel 6.5 km, in the third hour they will travel 7 km and so on.

Now, relative speeds of both of them in consecutive hours forms an arithmetic progression *i.e.*, 6, 6.5, 7, 7.5, ...so on

where, $a = 6$ and $d = 6.5 - 6 = 0.5$, n = number of required hours to travel 72 km

Now, according to the formula of AP,

Total distance covered $= \frac{n}{2}[2a + (n-1)d]$

$$72 = \frac{n}{2}[2 \times 6 + (n-1)(0.5)]$$

$$\Rightarrow \quad 144 = 12n + 0.5n^2 - 0.5n$$

$$\Rightarrow \quad 288 = 24n + n^2 - n$$

$$\Rightarrow \quad n^2 + 23n - 288 = 0$$

$$(n-9)(n+32) = 0$$

$$n = 9, n \neq -32 \quad \text{[time cannot be negative]}$$

Hence, a distance of 72 km will be covered by them in 9 h

Now, the distance travelled by *A* in 9 h = 9 x 4 = 36 km. Hence, both of them would meet

midway between *A* and 8.

2. *Two cars start together in the same direction from the same place. The first goes with a uniform speed of 10 km/h. The second goes at a speed of 8 km/h in the first hour and increase the speed by $\frac{1}{2}$ km each succeeding hour. After how many hours will the second car overtake the first, if both go non-stop?*

(a) 9 h (b) 5 h (c) 7 h (d) 8 h

(*a*) Let the second car over takes the first car after n h.

$\therefore$ Distance covered by first car = Distance covered by second car

$\Rightarrow \quad 10n = 8 + \left(8 + \frac{1}{2}\right) + \left(8 + \frac{2}{2}\right) + \ldots + \left(8 + \frac{n-1}{2}\right)$

$\Rightarrow \quad 10n = 8n + \frac{1}{2}[1 + 2 + \ldots + (n-1)]$

$\Rightarrow \quad 10n = 8n + \frac{1}{2}\frac{n(n-1)}{2}$

$\Rightarrow \quad 2n = \frac{1}{4}(n^2 - n) \Rightarrow 8n = n^2 - n \Rightarrow 8 = n - 1 \Rightarrow n = 9$

So, the second car over takes the first car after 9 h.

Fast Track Practice

Exercise© *Base Level Questions*

1. If speed of $3\frac{1}{3}$ m/s is converted to km/h, then it would be [SSC CGL 2012]

(a) 8 km/h *(b)* 9 km/h

(c) 10 km/h *(d)* 12 km/h

2. A car covers 300 km in 15 h. Find the speed of the car.

(a) 20 km/h *(b)* 25 km/h

(c) 15 km/h (d) 24 km/h

fej None of the above

3. A car covers a distance of 690 km in 30 h. What is the average speed of the car?

[Bank Clerks 2009] *(a)* 25 km/h *(b)* 23 km/h *(c)* 20 km/h *(d)* 18 km/h

fe) None of the above

4. A bus is running with a uniform speed of 37 km/h. What distance will be covered by bus in 8 h? [Bank Clerks 2009]

(a) 246 km *(b)* 289 km

(c) 296 km *(d)* 276 km

(e) None of the above

5. A bus covers a distance of 400 km with a speed of 20 km/h. What time is taken by the bus to cover this distance?

(a) 25 h *(b)* 5 h *(c)* 21 h *(d)* 20 h

(e) None of the above The speed of a bus is 72 km/h. The distance covered by the bus in 5 s is

[SSC (10+2) 2012]

(a) 50 m *(b)* 74.5 m

(c) 100 m *(d)* 60 m

7. A person riding a bike crosses a bridge with a speed of 54 km/h. What is the length of the bridge, if he takes 4 min to cross the bridge? [Hotel Mgmt. 2010]

(a) 3600 m *(b)* 2800 m

(c) 3500 m *(d)* 4500 m

(e) None of the above

8. Ram and Shyam are moving in the directions opposite to each other. The speeds of both persons are 10 km/h and 6 km/h, respectively. Find the speeds of Ram with respect of Shyam.

(a) 6 km/h *(b)* 16 km/h

(c) 4 km/h *(d)* 8 km/h

(e) None of the above

9. Two cars are moving in the same direction with their respective speeds of 9 km/h and 5 km/h. Find the relative speed of 1st car with respect of 2nd.

(a) 2 km/h *(b)* 4 km/h

(c) 8 km/h *(d)* 6 km/h

fej None of the above

10. A man covered a distance of 12 km in 90 min by cycle. How much distance will he cover in 3 h, if he rides the cycle at a uniform speed? [Bank Clerks 2010]

(a) 36 km *(b)* 24 km

(c) 30 km *(d)* 27 km

(e) None of the above

11. Two men start together to walk a certain distance, one at 4 km/h and another at

3 km/h. The former arrives half an hour before the latter. Find the distance.

[SSC (10+2) 2012]

(a) 6 km *(b)* 9 km *(c)* 8 km *(d)* 7 km

12. Two trains *A* and *B* travel from points *X* to *Y* and the ratio of the speeds of *A* to that of *B* is 2 : 7. Find the ratio of time taken by *A* and *B* to reach from *X* to *Y*.

(a) 2 : 5 *(b)* 3 : 5 *(c)* 3 : 8 *(d)* 7 : 2 *(e)* None of the above 13.Aashutosh can cover a certain distance in 84 min by covering 2/3rd of distance at

4 km/h and the rest at 5 km/h. Find the total distance.

(a) 6 km *(b)* 8 km

(c) 9 km *(d)* 15 km

(e) None of the above 14. A man completes 30 km of a journey at 6 km/h and the remaining 40 km of the journey in 5 h. Find the average speed for the whole iournev. TSSC CGL 20071

(a) $6\frac{4}{11}$ km/h *(b)* 7 km/h

(c) $7\frac{1}{2}$ km/h *(d)* 8 km/h

15. A certain distance is covered at a certain speed. If half of the distance is covered in double time, the ratio of the two speeds is [Bank Clerks 2011]

(a) 4 : 1 *(b)* 1 : 4 *(c)* 1 : 2 *(d)* 2 : 1 *(e)* 1 : 1

16. A bullock cart has to cover a distance of 80 km in 10 h. If it covers half of the journey in $\frac{3}{5}$th time, what should be its speed to cover the remaining distance in the left time?

(a) 5 km/h *(b)* 10 km/h

(c) 15 km/h (d) 18 km/h

(ej 20 km/h

17. Moving 6/7 of its usual speed a train is 10 min late. Find its usual time to cover the journey. [Hotel Mgmt. 2008]

(a) 25 min *(b)* 15 min

(c) 35 min *(d)* 60 min

(e) None of the above

18. Rani covers a certain distance by car driving at 5 km/h and returns the starting point riding on a scooter at 2 km/h. Find her average speed for the whole journey.

(a) $3\frac{6}{7}$ km/h *(b)* $2\frac{6}{7}$ km/h

(c) $5\frac{6}{7}$ km/h *(d)* $7\frac{6}{7}$ km/h

(e) None of the above

19. A car covers a distance from town *A* to town *B* at the speed of 58 km/h and covers the distance from town *B* to town *A* at the speed of 52 km/h. What is the approximate average speed of the car?

[Bank Clerks 2009] *(a)* 55 km/h *(b)* 52 km/h

(c) 48 km/h (d) 60 km/h

fej None of the above

20. A car reached Raipur from Sonagarh in 35 min with an average speed of 69 km/h. If the average speed is increased by 36 km/h, how long will it take to cover the same distance?

[Bank Clerks 2009] *(a)* 24 min *(b)* 27 min

(c) 23 min *(d)* 29 min

(e) None of the above

21. A student walks from his house at $2\frac{1}{2}$ km/h and reaches his school late by 6 min. Next day, he increases his speed by 1 km/h and reaches 6 min before school time. How far is the school from his house? [SSC CGL 2007]

(a) $\frac{5}{4}$ km *(b)* $\frac{7}{4}$ km

(c) $\frac{9}{4}$ km *(d)* $\frac{11}{4}$ km

22. A man riding a bicycle from his house at 10 km/h and reaches his office late by 6 min. He increases his speed by 2 km/h and reaches 6 min before. How far is the office from his house? [SSC CGL 2012]

(a) 6 km *(b)* 7 km *(c)* 12 km *(d)* 16 km

23. A thief is noticed by a policeman from a distance of 200 m. The thief starts running and the policeman, chases him. The thief and the policeman run at the rate of 10 km/h and 11 km/h, respectively. The distance between them after 6 min will be [SSC CGL 2013]

(a) 100 m *(b)* 180 m

(c) 150 m *(d)* 125 m

24. A person can walk a certain distance and drive back in 6 h. He can also walk both ways in 10 h. How much time will he take to drive both ways? [CSM 2013]

(a) 2 h *(b)* $2\frac{1}{2}$ h *(c)* $5\frac{1}{2}$ h *(d)* 4 h

Two men *A* and *B* travel from point *P* to Q, a distance of 84 km at 12 km/h and 16 km/h, respectively. *B* reaches *Q* and returns immediately and meets *A* at *R*. Find the distance from *P* to *R*. *(a)* 72 km *(b)* 76 km

(c) 78 km *(d)* 68 km

(e) None of the above

26. A thief is spotted by a policeman from a distance of 200 m. When the policeman starts chasing, the thief also starts running. If the speed of the thief be 16 km/h and that of the policeman be 20 km/h, how far the thief will have run before he is overtaken?

(a) 800 m *(b)* 850 m

(c) 700 m *(d)* 650 m

(e) None of the above

27. A person sets out to cycle from *A* to *B* and at the same time another person starts from *B* to *A*. After passing each other, they complete their journeys in 16 h and 25 h, respectively. Find the ratio of speeds of the 1st man to that of the 2nd man.

(a) 5 : 4 *(b)* 5 : 3 *(c)* 4 : 5 *(d)* 3 : 5 *(e)* None of the above

28. John started from *A* to *B* and Vinod from *B* to *A*. If the distance between *A* and *B* is 125 km and they meet at 75 km from *A*, what is the ratio of John's speed to that of Vinod's speed? [SSC CGL 2008]

(a) 2 : 3 *(b)* 3 : 2 *(c)* 4 : 3 *(d)* 5 : 4

29. A certain distance is covered at a certain speed. If half of this distance is covered in 4 times of the time, then find the ratio of the two speeds.

(a) 1 : 8 *(b)* 1 : 4 *(c)* 4 : 1 *(d)* 8 : 1 *(e)* None of the above

30. A man covers half of his journey at 6 km/h and the remaining half at 3 km/h. Find his average speed. [SSC (10 +2) 2007]

(a) 3 km/h *(b)* 4 km/h

(c) 4.5 km/h *(d)* 9 km/h

31. A is twice as fast as *B* and *B* is thrice as fast as *C*. The journey covered by *C* in 56 min will be covered by *A* in

[Bank PO 2010]

(a) $5\frac{1}{3}$ min *(b)* $2\frac{1}{3}$ min

(c) $7\frac{1}{3}$ min *(d)* $9\frac{1}{3}$ min

(e) None of the above

32. Shantanu drives a motorcycle and covers a distance of 715 km at a constant speed. If the speed of the motorcycle had been 10 km/h more, he would have taken 2 h less to cover the same distance. What is the original speed of the motorcycle?

(a) 65 km/h *(b)* 55 km/h

(c) 60 km/h *(d)* 36 km/h

fej None of the above The ratio of speeds of a train and a car is 16:15, respectively and a bus covered a distance of 480 km in 8 h. The speed of the bus is 3/4th of the speed of train. What distance will be covered by car in 6h? [Bank PO 2010]

(a) 450 km *(b)* 480 km

(c) 360 km

(d) Couldn't be determined

(e) None of the above

34. The ratio of the speeds of *A* and *B* is 3 : 4. *A* takes 20 min more than the time taken by *B* to reach a particular place. Find the time taken by *A* and *B,* respectively to reach that place. [LIC ADO 2010]

(a) 40 min and 30 min

(b) 80 min and 60 min

(c) 90 min and 45 min

(d) 90 min and 50 min

(e) None of the above

35. The ratio between the speeds of two cars is 7 : 8. If the 2nd car runs 200 km in 5 h, then find the speed of the 1st car.

(a) 25 km/h *(b)* 28 km/h

(c) 40 km/h *(d)* 35 km/h

(e) None of the above

36. The ratio between the speeds of two buses is 5 : 3. If the 1st bus runs 400 km in 8 h, then find the speed of the 2nd bus. [SSC (10+2) 2010]

(a) 30 km/h *(b)* 15 km/h

(c) 27 km/h *(d)* 37 km/h

37. The speeds of three cars are in the ratio of 2 : 3 : 5 Find the ratio of the time taken by the above cars to travel the same distance.

(a) 15 : 10 : 6 *(b)* 6 : 10 : 15

(c) 10 : 15 : 6 *(d)* 10 : 6 : 15

(e) None of the above Nilu covers a distance by walking for 6 h. While returning, his speed decreases by 2 km/h and he takes 9 h to cover the same distance. What was her speed while returning? *(a)* 2 km/h *(b)* 5 km/h

(c) 4 km/h *(d)* 7 km/h

(e) None of the above

39. A takes 4 h more than the time taken by *B* to walk *D* km. If *A* doubles his speed, he can make it in 2 h less than that of *B*. How much time does *B* require for walking *D* km?

(a) 8 h (b) 4 h (c) 6 h (d) 9 h (e) None of the above

40. Walking with 3/4 of his usual speed, Sachin covers a certain distance in 2 h more than the time he takes to cover the distance at his usual speed. Find the time taken by him to cover this distance with usual speed.

(a) 5 h (b) 6 h (c) 9 h (d) 4 h (e) None of the above

41. If Sohail walks from his home to office at 16 km/h, he is late by 5 min. If he walks at 20 km/h, he reaches 10 min before the office time. Find the distance of his office from his house. [SSC CGL 2010]

(a) 22 km (b) 20 km

(c) 18 km (d) 16 km

42. A car covers a distance of 200 km in 2 h 40 min whereas a jeep covers the same distance in 2 h. What is the ratio of their speeds? [Bank Clerks 2007]

(a) 3 : 4 (b) 4 : 3 (c) 4 : 5 (d) 5 : 4 (e) None of the above

43. Amit walks at a uniform speed of 4 km/h and 4 h after his start, Brijesh cycles after him at the uniform rate of 20 km/h. How far from the starting point will Brijesh catch Amit?

(a) 15km (b) 18km(cJ 13km(dJ 20km (e) None of the above

44. A walks at a uniform rate of 2 km/h and 2 h after his start, *B* cycles after him at the uniform rate of 5 km/h. How far from the starting point will *B* catch *A1*

(a) $6\frac{2}{3}$ km (b) $6\frac{1}{3}$ km

(c) $6\frac{5}{7}$ km (d) $6\frac{3}{7}$ km

(e) None of the above

45. A person goes from one point to another point with a speed of 5 km/h and comes back to starting point with a speed of 3 km/h. Find the average speed for the whole journey. [SSC CCL 2010]

(a) 4.5 km/h (b) 4 km/h

(c) 4.25 km/h (d) 3.75 km/h

46. A train travels at the rate of 50 km/h without stoppages and it travels at 40 km/h with stoppages. How many minutes does the train stop on an average per hour?

(a) 6 min *(b)* 12 min

(c) 18 min *(d)* 14 min

(e) None of the above

47. A bus travels at the rate of 54 km/h without stoppages and it travels at 45 km/h with stoppages. How many minutes does the bus stop on an average per hour? [SSC CCL 2010]

(a) 8 min *(b)* 10 min

(c) 12 min *(d)* 4 min

48. A bus covers a certain distance in 16 h. It covers half the distance at 40 km/h and the rest at 60 km/h. Find the length of the journey.

(a) 520 km *(b)* 448 km

(c) 384 km *(d)* 768 km

(e) None of the above

49. The ratio between the rates of travelling of A and B is 2 : 3 and therefore A takes 20 min more than time taken by B to reach a destination. If A had walked at double the speed, how long would he have taken to cover the distance?

(a) 30 min *(b)* 35 min

(c) 20 min *(d)* 45 min

(e) None of the above

50. A train covers a distance in 1 h 40 min, if it runs at a speed of 96 km/h on an average. Find the speed at which the train must run to reduce the time of journey to 1 h 20 min.

(a) 120 km/h *(b)* 90 km/h

(c) W.A km/h (d) 150 km/h

(e) None of the above

51. A car travels a distance of 75 km at the speed of 25 km/h. It covers the next 25 km of its journey at the speed of 5 km/h and the last 50 km of its journey at the speed of 25 km/h. What is the average speed of the car? [Bank Clerks 2008]

(a) 40 km/h (b) 25 km/h

(c) 15 km/h (d) 12.5 km/h

(e) None of the above

52. A car runs at the speed of 50 km/h when not serviced and runs at 60 km/h, when serviced. After servicing, the car covers a certain distance in 6 h. How much time will the car take to cover the same distance when not serviced?

[Bank Clerks 2009]

(a) 8.2 h (b) 6.5 h (c) 8 h (d) 7.2 h (e) None of the above

53. A train runs for 3 h at the speed of 40 km/h and then for $4\frac{1}{2}$ h at the speed of 60 km/h. Thus, it covers 3/5th part of the whole distance. It wants to cover the remaining distance in 4 h, find the average speed of the train for remaining distance. [Bank Clerks 2007]

(a) 45 km/h (b) 35 km/h

(c) 80 km/h (d) 65 km/h

(e) None of the above

54. If you travel 39 km at a speed of 26 km/h, another 39 km at a speed of 39 km/h and again 39 km at a speed of 52 km/h, what is your average speed for the entire journey? [SSC CCL 2008]

(a) 39 km/h (b) 37 km/h

(c) 33.33 km/h *(d)* 36 km/h

55. Dalbir Singh, a policeman, is 114 m behind a thief. Dalbir Singh runs 21 m and the thief 15 m in a minute. In what time will Dalbir Singh catch the thief?

(a) 19 min *(b)* 16 min

(c) 21 min *(d)* 23 min

(e) None of the above

56. A person covers half of his journey at 40km/h, one-third at 60 km/h and rest of journey at 30 km/h. Find his average speed for the whole journey.

(a) 45 km/h *(b)* 42.35 km/h

(c) 50 km/h *(d)* 40 km/h

57. A boy walking at a speed of 15 km/h reaches his school 20 min late. Next time he increases his speed by 5 km/h but still he late by 5 min. Find the distance of the school from his home.

(a) 5 km *(b)* 10 km

(c) 15 km *(d)* 20 km

58. Two cars *A* and *B* start simultaneously from a certain place at the speed of 30 km/h and 45 km/h, respectively. The car *B* reaches the destination 2 h earlier than *A*. What is the distance between the starting point and destination? [CDS 2013] *(a)* 90 km *(b)* 180 km

(c) 270 km *(d)* 360 km

59. A person travels a certain distance at 3 km/h and reaches 15 min late. If he travels at 4 km/h, he reaches 15 min earlier. The distance he has to travel is

[CDS 2013] *(a)* 4.5 km *(b)* 6 km

(c) 7.2 km b) 12 km

Exercise © *Higher Skill Level Questions*

1. A truck covers a distance of 368 km at a certain speed in 8 h. How much time would a car take at an average speed which is 18 km/h more than that of the speed of the truck to cover a distance which is 16 km more than that travelled by the truck? [SBI Clerk 2012]

(a) 7 h *(b)* 5 h

(c) 6 h *(d)* 8 h

(e) None of the above

2. Sumit drove at the speed of 45 km/h from home to a resort. Returning over the same route, he got stuck in traffic and took an hour longer. Also, he could drive only at the speed of 40 km/h. How many kilometres did he drive each way?

[DMRCCRA2012]

(a) 250 km *(b)* 360 km

(c) 375 km (a) None of these

3. A car runs at the speed of 40 km/h when not serviced and runs at 65 km/h when serviced. After servicing, the car covers a certain distance in 5 h. How much approximate time will the car take to cover the same distance when not serviced? [Bank Clerks 2009]

(a) 10 h *(b)* 7 h

(c) 12 h *(d)* 8 h

(e) 6 h

4. An express train travelled at an average speed of 100 km/h, stopping for 3 min after every 75 km. A local train travelled at a speed of 50 km/h, stopping for 1 min after every 25 km. If the trains began travelling at the same time, how many kilometres did the local train travel in the time in which the express train travel 600 km?

(a) 400 km

(b) 405 km

(c) 307.25 km

(d) 415.5 km

(e) None of the above

5. A man started 20 min late and travelling at a speed of $1\frac{1}{2}$ times of his usual speed reaches his office in time. The time taken by the man to reach his office at his usual speed is [SSC CGL (Main) 2012]

(a) 40 min (b) 1 h 20 min

(c) 1 h (d) 30 min

6. A man decides to travel 80 km in 8 h partly by foot and partly on a bicycle. If his speed on foot is 8 km/h and on bicycle 16 km/h, what distance would he travel on foot? [IB ACIO 2012]

(a) 20 km (b) 30 km

(c) 48 km b) 60 km

7. A man covers a certain distance on scooter. Had he moved 3 km/ faster, he would have taken 40 min less. If he had moved 2 km/h slower, he would have taken 40 min more. The distance (in km) is [SSC Multitasking 2013]

(a) 42.5 (b) 36

(c) 37.5 (d) 40

8. *A* and *B* are two stations 1000 km apart. A train starts from *A* and moves towards *B* at 40 km/h. Another train starts from *B* at the same time and moves towards *A* at 60 km/h. How far from *A* will they cross each other?

(a) 350 km (b) 400 km

(c) 525 km (d) 300 km

(e) None of the above

9. X and Y start walking towards each other at 8 : 00 am at the speeds of 3 km/h and 4 km/h, respectively. They were initially 17.5 km apart. At what time do they meet?

(a) 10 : 30 am

(b) 10: 30 pm

(c) 11 : 30 am

(d) 11 : 30 pm

(e) None of the above

10. A train leaves Manipur at 6:00 am and reaches Dispur at 10:00 am. Another train leaves Dispur at 8:00 am and reaches Manipur at 11 : 30 am. At what time do the two trains cross each other?

(a) 7 : 56 am

(b) 7 : 56 pm

(c) 8 : 56 am

(d) 8 : 56 pm

(e) None of the above

11. The average speed of a car is 75 km/h. The driver first decreases its average speed hy 40% and then increases it by 50%. What is the new average speed now? [Hotel Mgmt. 2010]

(a) 67.5 km

(b) 60 km

(c) 90 km

(d) 60.5 km

(e) None of the above

12. Shantanu started cycling along the boundaries of a square field *ABCD* from corner point *A*. After 1/2 h, he reached the corner point C, diagonally opposite to point *A*. If his speed was 16 km/h, find the area of the field.

(a) 8 sq km

(b) 9 sq km

(c) 32 sq km

(d) 19 sq km

(e) None of the above

13. A car driver covers a distance between two cities at a speed of 60 km/h and on the return his speed is 40 km/h. He goes again from the 1st to the 2nd city at twice the original speed and returns at half the original return speed. Find his average speed for the entire journey.

[LIC AAO2007]

(a) 55 km/h

(b) 50 km/h

(c) 48 km/h

(d) 40 km/h

(e) None of the above

14. The distance between two stations '*X* and '*Y* is 450 km. A train *L* starts at 6:00 pm from *X* and moves towards *Y* at an average speed of 60 km/h. Another train *M* starts from *Y* at 5 : 20 pm and moves towards *X* at an average speed of 80 km/h. How far from *x* will the two trains meet and at what time?

(a) 170 km, 8 : 50 pm

(b) 150 km, 7 : 50 pm

(c) 170 km, 6 : 50 pm

(d) 150 km, 9 : 50 pm

(e) None of the above

15. Two cars *A* and *B* are running towards each other from two different places 88 km apart. If the ratio of the speeds of the cars *A* and *B* is 5 : 6 and the speed of the car *B* is 90 km/h, after what time will they meet each other?

(a) 38 min

(b) 39 min

(c) 45 min

(d) 32 min

(e) None of the above

16. A car starts running with the initial speed of 40 km/h with its speed increasing every hour by 5 km/h. How many hours will it take to cover a distance of 385 km? [SSC Multitasking 2014]

(a) $8\frac{1}{2}$ h
(b) $9\frac{1}{2}$ h
(c) 8 h
(d) 7 h

Answer with Solutions

Exercise © *Base Level Questions*

1. (*d*) $\because$ $1 \text{ m/s} = \frac{18}{5}$ km/h

$$\therefore\ 3\frac{1}{3} \text{ m/s} = \frac{10}{3} \text{ m/s} = \frac{10}{3} \times \frac{18}{5} \text{ km/h}$$
$$= 12 \text{ km/h}$$

2. (*a*) We know that, speed $= \frac{\text{Distance}}{\text{Time}}$

$\therefore$ Required speed $= \frac{300}{15} = 20$ km / h

3. (*b*) Required speed

$$= \frac{\text{Distance}}{\text{Time}} = \frac{690}{30} = 23 \text{ km/h}$$

4. (*c*) We know that, speed $= \frac{\text{Distance}}{\text{Time}}$

$\therefore$ Required distance = Speed $\times$ Time

$$= 37 \times 8 = 296 \text{ km}$$

5. (*d*) We know that, speed $= \frac{\text{Distance}}{\text{Time}}$

$\therefore$ Required time $= \frac{\text{Distance}}{\text{Speed}} = \frac{400}{20} = 20$ h

6. (*c*) Given, speed of bus = 72 km/h

$$= 72 \times \frac{5}{18} = 20 \text{ m/s}$$

$\therefore$ Distance travelled in 5 s $= 5 \times 20 = 100$ m

7. (*a*) Length of the bridge = Distance travelled by the person in 4 min

$$= \text{Speed} \times \text{Time}$$

Speed $= 54$ km/h $= 54 \times \frac{5}{18} = 3 \times 5 = 15$ m/s

Time $= 4$ min $= 4 \times 60 = 240$ s

$\therefore$ Required length $= 15 \times 240 = 3600$ m

8. (*b*) We know that for relative speed, the two speeds will be added, if the motions of the two objects are in opposite directions.

$\therefore$ Required speed $= 10 + 6 = 16$ km/h

9. (*b*) We know that for relative speed, the subtraction of speeds takes place, if the motions of the two objects are in the same direction.

$\therefore$ Required relative speed $= 9 - 5 = 4$ km/h

10. (*b*) We know that, speed $= \frac{\text{Distance}}{\text{Time}}$

$$= \frac{12}{\frac{90}{60}} = \frac{12 \times 60}{90} = 8 \text{ km/h}$$

$\therefore$ Distance covered in 3 h

$= \text{Speed} \times \text{Time}$

$= 8 \times 3 = 24$ km

11. (*a*) Let total distance $= x$ km

According to the question,

$$\frac{x}{3} - \frac{x}{4} = \frac{1}{2}; \quad \frac{x}{12} = \frac{1}{2} \Rightarrow x = 6 \text{ km}$$

12. (*d*) We know that speed is inversely proportional to time.

Given that,

(Speed of *A*) : (Speed of *B*) = 2 : 7

$\therefore$ (Time taken by *A*) : (Time taken by *B*)

$$= \frac{1}{2} : \frac{1}{7} = 7 : 2$$

13. (*a*) Let the total distance $= x$

Then, according to the question,

$$\frac{\frac{2}{3}x}{4} + \frac{\left(1 - \frac{2}{3}\right)x}{5} = \frac{84}{60}$$

$$\Rightarrow \quad \frac{\frac{2}{3}x}{4} + \frac{\frac{1}{3}x}{5} = \frac{84}{60}$$

$$\Rightarrow \frac{x}{6} + \frac{x}{15} = \frac{84}{60} \Rightarrow 5x + 2x = 42$$

$$\Rightarrow \quad 7x = 42$$

$$\therefore \quad x = \frac{42}{7} = 6 \text{ km}$$

14. (*b*) Total distance $= 30 + 40 = 70$ km

Total time taken $= \frac{30}{6} + 5 = \frac{60}{6} = 10$ h

$\therefore$ Required average speed $= \frac{70}{10} = 7$ km/h

15. (*a*) Let x km distance be covered in y h.

So, speed in first case $= \frac{x}{y}$ km/h

and speed in second case

$$= \frac{x/2}{2y} = \frac{x}{4y} \text{ km/h}$$

$\therefore$ Required ratio $= \frac{x}{y} : \frac{x}{4y} = 1 : \frac{1}{4} = 4 : 1$

16. (*b*) Total distance to covered in

$10 \text{ h} = 80$ km

But it covers 40 km in $\frac{3}{5}$th of time, *i.e.*, 40 km in 6 h.

$\therefore$ Required time $= 10 - 6 = 4$ h

and remaining distance $= 40$ km

Thus, required speed $= \dfrac{40}{4} = 10$ km/h

17. (*d*) New speed $= \dfrac{6}{7}$ of usual speed

Now, time taken $= \dfrac{7}{6}$ of usual time

$$\left(\frac{7}{6} \text{ of the usual time}\right)$$

$$- \text{(usual time)} = 10 \text{ min}$$

$\Rightarrow$ $\dfrac{1}{6}$ of the usual time $= 10$ min

$\therefore$ Usual time $= 60$ min

18. (*b*) According to the formula,

Average speed $= \dfrac{2AB}{A+B}$ [by Technique 1]

Here, $A = 5$ km/h, and $B = 2$ km/h

$\therefore$ Required average speed

$$= \frac{2 \times 5 \times 2}{7} = \frac{20}{7} = 2\frac{6}{7} \text{ km/h}$$

19. (*a*) We know that, if two equal distances are covered at two different speeds A and B, then

Average speed

$$= \frac{2AB}{A+B} = \frac{2 \times 58 \times 52}{58 + 52} \quad \text{[by Technique 1]}$$

$= 54.8$ km/h $= 55$ km/h (approx.)

20. (*c*) Distance between Sonagarh and Raipur

$$= \text{Average speed} \times \text{Time}$$

$$= \frac{69 \times 35}{60} \text{ km} = \frac{161}{4} \text{ km}$$

New speed $= (69 + 36)$ km/h $= 105$ km/h

$\therefore$ Required time $= \dfrac{\text{Distance}}{\text{Speed}}$

$$= \frac{161}{4 \times 105} \text{ h}$$

$$= \frac{161 \times 60}{4 \times 105} \text{ min} = 23 \text{ min}$$

21. (*b*) Let the required distance $= x$

According to the question,

$$\frac{x}{5/2} - \frac{x}{7/2} = \frac{12}{60}$$

(difference between two times

$= 6 + 6 = 12$ min)

$$\Rightarrow \quad \frac{2x}{5} - \frac{2x}{7} = \frac{1}{5}$$

$$\Rightarrow \quad 14x - 10x = 7$$

$$\Rightarrow \quad 4x = 7$$

$$\Rightarrow \quad x = \frac{7}{4} \text{ km}$$

Fast Track Method

Here,

$a = 2\frac{1}{2} = \frac{5}{2}$, $b = \frac{7}{2}$, $t_1 = 6$ and $t_2 = 6$

According to the formula,

Required distance $= \frac{ab(t_1 + t_2)}{b - a}$

[by Technique 3]

$\therefore$ Required distance

$$= \frac{\frac{5}{2} \times \frac{7}{2}\left(\frac{6+6}{60}\right)}{\frac{7}{2} - \frac{5}{2}} = \frac{35}{4} \times \frac{12}{60} = \frac{7}{4} \text{ km}$$

22. (c) Time $= \frac{\text{Distance}}{\text{Speed}}$

As per first condition,

$$\frac{d}{10} = t + \frac{6}{60}$$

$\Rightarrow \quad d = 10t + 1 \quad \ldots$(i)

As per second condition,

$$\frac{d}{12} = t - \frac{6}{60} \Rightarrow d = 12t - \frac{6}{5} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$10t + 1 = 12t - \frac{6}{5}$

$\Rightarrow \quad 2t = 1 + \frac{6}{5} = \frac{11}{5} \Rightarrow t = \frac{11}{10}$

$\therefore$ From Eq (i), $d = 10 \times \frac{11}{10} + 1$

$d = 11 + 1 = 12$

$\therefore \quad d = 12$ km

Fast Track Method

Here, $a = 10$ km/h,

$b = (10 + 2) = 12$ km/h,

$t_1 = 6 \text{ min} = \frac{1}{10}$ h

and $\quad t_2 = \frac{1}{10}$ h.

$\therefore$ Required distance $= \frac{ab(t_1 + t_2)}{b - a}$

[by Technique 3]

$$= \frac{10 \times 12 \times \left(\frac{1}{10} + \frac{1}{10}\right)}{12 - 10} = \frac{10 \times 12 \times 1}{5 \times 2}$$

$= 12$ km

23. (a) Relative speed of policeman with respect to thief $= (11 - 10) = 1$ km/h.

Now, relative distance covered by policeman in 6 min

$= \text{Speed} \times \text{Time} = 1 \times \frac{6}{60}$

$= \frac{1}{10}$ km $= 100$ m

= The distance between the policeman and thief after 6 min = (200 - 100) = 100 m

24. (a) Since, he takes 10 h to walk both ways, the number of hours to walk one way for him will be 5. If he walks one way and drives back the same way it takes him 6 h which means that the number of h taken to drive one way is 6 - 5 = 1 h. Thus, number of hours to drive both ways will be 2.

Alternate Method

Given that, W + *D* = 6 ...(i)

[W = Time taken while walking and *D* = Time taken while driving] Also, *ZW = 10 =>W* = 5 From Eq. (i)

5+ *D* = *6=>D = 1*

2D = 2X1 =2 /. He will take 2 h to drive both ways.

25. (*a*) Given, $D = 84$ km, $a = 12$ km/h

and $b = 16$ km/h

According to the formula,

Distance travelled by

$$A = PR = 2D \times \frac{a}{a+b}$$

[by Technique 5]

$$= 2 \times 84 \times \frac{12}{12+16} = 2 \times 84 \times \frac{12}{28}$$

$$= 2 \times 6 \times 6 = 72 \text{ km}$$

26. (*a*) Relative speed of policeman

$$= (20 - 16) \times \frac{5}{18} = \frac{10}{9} \text{ m/s}$$

To catch the thief, the policeman has to gain 200 m.

Time taken to gain 200 m

$$= \frac{200 \times 9}{10} = 180 \text{ s}$$

Actual distance covered by policeman in 180 s $= 180 \times \frac{50}{9} = 1000$ m

$\therefore$ Distance covered by the thief

$$= 1000 - 200 = 800 \text{ m}$$

Fast Track Method

Here, $d = 200$ m, $a = 16$ km/h

$$= 16 \times \frac{5}{18} = \frac{40}{9} \text{ m/s}$$

and $b = 20 \text{ km/h} = 20 \times \frac{5}{18} = \frac{50}{9}$ m/s

According to the formula,

Required distance $= d \times \left(\frac{a}{b-a}\right)$

[by Technique 6]

$$= 200 \times \frac{\frac{40}{9}}{\frac{50}{9} - \frac{40}{9}}$$

$$= 200 \times \frac{40}{10} = 800 \text{ m}$$

27. (*a*) Given, $x = 16$ and $y = 25$

According to the formula,

1st man's speed : 2nd man's speed

[by Technique 7]

$$= \sqrt{y} : \sqrt{x} = \sqrt{25} : \sqrt{16} = 5 : 4$$

28. (*b*) John's speed : Vinod's speed

$$= 75 : (125 - 75)$$
$$= 75 : 50 = 3 : 2$$

29. (*d*) Let x km is covered in y h. Then, first speed $= \frac{x}{y}$ km/h

Again, $\frac{x}{2}$ km is covered in $4y$ h.

$\therefore$ New speed $= \left(\frac{x}{2} \times \frac{1}{4y}\right) = \left(\frac{x}{8y}\right)$ km/h

Ratio of speeds $= \frac{x}{y} : \frac{x}{8y} = 1 : \frac{1}{8} = 8 : 1$

30. (*b*) Given, $A = 6$ km/h, $B = 3$ km/h

According to the formula,

Average speed $= \frac{2AB}{A + B}$ **[by Technique 1]**

$\therefore$ Required average speed

$$= \frac{2 \times 6 \times 3}{6 + 3} = \frac{36}{9} = 4 \text{ km/h}$$

31. (*d*) Let time taken by $A = y$

Let speed of $C = x$

Then, speed of $B = 3x$

$\therefore$ Speed of $A = 6x$

Now, ratio of speeds of A and C

= Ratio of time taken by C and A

$$6x : x = 56 : y \Rightarrow \frac{6x}{x} = \frac{56}{y}$$

$$\therefore \quad y = \frac{56}{6} = 9\frac{2}{6} = 9\frac{1}{3} \text{ min}$$

32. (*b*) Let the constant speed $= x$

According to the question,

$$\frac{715}{x} - \frac{715}{x + 10} = 2$$

$$\Rightarrow \quad \frac{1}{x} - \frac{1}{x + 10} = \frac{2}{715}$$

$$\Rightarrow \quad \frac{(x + 10) - x}{x(x + 10)} = \frac{2}{715}$$

$\Rightarrow \quad x^2 + 10x = \dfrac{10 \times 715}{2}$

$\Rightarrow \quad x^2 + 10x - 3575 = 0$

$\Rightarrow \quad x^2 + 65x - 55x - 3575 = 0$

$\Rightarrow \quad x(x + 65) - 55(x + 65) = 0$

$\Rightarrow \quad (x - 55)(x + 65) = 0$

$\therefore \quad x = 55 \text{ or } x = -65$

Ignoring the negative value, we get

$x = 55$

$\therefore$ Original speed of the motorcycle

$= 55$ km/h

33. (*a*) Let speed of train $= 16x$

Speed of car $= 15x$

Speed of bus $= \dfrac{480}{8} = 60$ km/h

According to the question,

$$16x = 60 \times \frac{4}{3} = 80$$

$\Rightarrow \quad x = 5$

$\therefore$ Speed of car $= 15x = 15 \times 5 = 75$ km/h

$\therefore$ Required distance

$=$ Speed $\times$ Time

$= 75 \times 6 = 450$ km

34. (*b*) Ratio of speeds of A and $B = 3:4$

$\therefore$ Ratio of time taken $= 4:3$

Let time taken by A and B be $4x$ and $3x$, respectively.

Then, according to the question,

$$4x - 3x = 20 \Rightarrow x = 20$$

Hence, time taken by A

$= 4 \times 20 = 80$ min

and time taken by $B = 3 \times 20 = 60$ min

35. (*d*) Let speed of 1st car $= 7x$

and speed of the 2nd car $= 8x$

According to the question,

$$8x = \frac{200}{5} = 40$$

$$\left[\text{using, Speed} = \frac{\text{Distance}}{\text{Time}}\right]$$

$\therefore \quad x = \dfrac{40}{8} = 5$

$\therefore$ Speed of the 1st car

$= 7x = 7 \times 5 = 35$ km/h

36. (*a*) Let speed of 1st bus $= 5x$

and speed of 2nd bus $= 3x$

According to the question,

$$5x = \frac{400}{8} = 50$$

$\therefore \quad x = \dfrac{50}{5} = 10$

$\therefore$ Speed of 2nd bus $= 3x = 3 \times 10$

$= 30$ km/h

37. (*a*) We know that speed and time are inversely proportional.

$\therefore$ Ratio of time taken

$$= \frac{1}{2} : \frac{1}{3} : \frac{1}{5} = \frac{30}{2} : \frac{30}{3} : \frac{30}{5}$$

[LCM of 2, 3, 5 = 30] = 15 : 10 : 6

38. (*c*) Let the speed in return journey = x

According to the question,

$$6(x + 2) = 9x$$

$$\Rightarrow \quad 6x + 12 = 9x$$

$$\Rightarrow \quad 9x - 6x = 12$$

$$\Rightarrow \quad 3x = 12$$

$$\therefore \quad x = \frac{12}{3} = 4 \text{ km/h}$$

39. (*a*) Let B takes x h to walk D km.

Then, A takes $(x + 4)$ h to walk D km.

With double of the speed,

A will take $\frac{1}{2}(x + 4)$ h.

According to the question,

$$x - \frac{1}{2}(x + 4) = 2$$

$$\Rightarrow \quad 2x - (x + 4) = 4$$

$$\Rightarrow \quad 2x - x - 4 = 4$$

$$\therefore \quad x = 4 + 4 = 8 \text{ h}$$

40. (*b*) Let the distance = x

and usual speed = y

According to the question,

$$\frac{x}{\left(\frac{3}{4}y\right)} - \frac{x}{y} = 2$$

$$\Rightarrow \quad \frac{x}{y} \times \left(\frac{4}{3} - 1\right) = 2 \Rightarrow \frac{x}{y} \times \frac{1}{3} = 2$$

$$\therefore \quad \frac{x}{y} = 3 \times 2 = 6$$

Time taken to cover the distance with usual speed = 6 h

41. (*b*) Let required distance = x

According to the question,

$$\frac{x}{16} - \frac{x}{20} = \frac{15}{60} \Rightarrow \frac{5x - 4x}{80} = \frac{1}{4}$$

$$\therefore \quad x = \frac{1}{4} \times 80 = 20 \text{ km}$$

Fast Track Method

Here, $a = 16$ km/h, $b = 20$ km/h, $t_1 = \frac{5}{60}$,

$t_2 = \frac{10}{60}$ and $D = ?$

$\therefore$ Required distance

$$D = \frac{ab(t_1 + t_2)}{b - a} \quad \text{[by Technique 3]}$$

$$= \frac{16 \times 20 \left(\frac{5 + 10}{60}\right)}{20 - 16}$$

$$= \frac{16 \times 20 \times \frac{15}{60}}{4} = 4 \times 5 = 20 \text{ km}$$

42. (*a*) Speed of car $= \dfrac{\text{Distance}}{\text{Time}} = \dfrac{200}{2\frac{40}{60}}$

$$= \frac{600}{8} = 75 \text{ km/h}$$

Speed of jeep $= \dfrac{200}{2} = 100$ km/h

$\therefore$ Required ratio $= \dfrac{75}{100} = \dfrac{3}{4} = 3:4$

Alternate Method

Speed of car : speed of jeep

= Time taken by jeep : Time taken by car

= 120 min : 160 min = 3 : 4

43. (*d*) Let Brijesh catches Amit after x h.

Then, distance travelled by Amit in $(x + 4)$ h

= Distance travelled by Brijesh in x h.

According to the question,

$$4(x + 4) = 20x$$

$\Rightarrow \quad 4x + 16 = 20x$

$\Rightarrow \quad 16x = 16$

$\therefore \quad x = \dfrac{16}{16} = 1$

Distance travelled by Brijesh in 1 h

$= (1 \times 20)$ km $= 20$ km

44. (*a*) Let B catches A after x h.

Then, distance travelled by A in $(x + 2)$ h

= Distance travelled by B in x h

According to the question,

$$2(x + 2) = 5x$$

$\Rightarrow \quad 2x + 4 = 5x \quad \Rightarrow \quad 3x = 4$

$\therefore \quad x = \dfrac{4}{3}$

Distance travelled by B in $\dfrac{4}{3}$ h

$$= \frac{4}{3} \times 5 = \frac{20}{3} = 6\frac{2}{3} \text{ km}$$

45. (*d*) Average speed

$$= \frac{2AB}{A + B} = \frac{2 \times 5 \times 3}{5 + 3}$$

[by Technique 1]

$$= \frac{30}{8} = 3.75 \text{ km/h}$$

46. (*b*) Due to stoppages, it covers 10 km less per hour.

Time taken to cover 10 km

$$= \left(\frac{10}{50} \times 60\right) = 12 \text{ min}$$

Hence, the train stops on an average 12 min per hour.

47. (*b*) Due to stoppages, bus covers 9 km less per hour.

Time taken to cover 9 km

$$= \left(\frac{9}{54} \times 60\right) = 10 \text{ min}$$

Hence, the train stops on an average 10 min per hour.

48. (*d*) Let the total distance covered $= x$

$= \text{Length of journey}$

According to the question,

$$\left(\frac{x}{2} \times \frac{1}{40}\right) + \left(\frac{x}{2} \times \frac{1}{60}\right) = 16$$

$$\Rightarrow \frac{x}{80} + \frac{x}{120} = 16 \Rightarrow \frac{3x + 2x}{240} = 16$$

$$\Rightarrow \quad 5x = 16 \times 240$$

$$\therefore \quad x = \frac{16 \times 240}{5} = 16 \times 48 = 768 \text{ km}$$

49. (*a*) We know that speed is inversely proportional to time. Therefore,

Time taken by A : Time taken by $B = \frac{1}{2} : \frac{1}{3}$

Let B takes x min. Then, A takes $(x + 20)$ min.

$$\therefore \quad (x + 20) : x = \frac{1}{2} : \frac{1}{3} = 3 : 2$$

$$\Rightarrow \quad \frac{x + 20}{x} = \frac{3}{2} \Rightarrow 2x + 40 = 3x$$

$$\therefore \quad x = 40$$

$\therefore$ A takes $(x + 20)$

or $(40 + 20) = 60$ min

$\therefore$ At double the speed, A would have covered it in 30 min.

50. (*a*) Distance = Speed $\times$ Time

$$= 96 \times \frac{100}{60} = 160 \text{ km}$$

New time $= \frac{80}{60}$ h $= \frac{4}{3}$ h

New speed $= 160 \times \frac{3}{4} = 40 \times 3$

$= 120$ km/h

51. (*c*) Time taken to cover first 75 km

$$= \frac{75}{25} = 3 \text{ h}$$

Time taken to cover next 25 km

$$= \frac{25}{5} = 5 \text{ h}$$

Time taken to cover last 50 km of its journey $= \frac{50}{25} = 2$ h

Total distance $= 75 + 25 + 50 = 150$ km

Total time taken $= 3 + 5 + 2 = 10$ h

$\therefore$ Required average speed

$$= \frac{\text{Total distance}}{\text{Total time taken}} = \frac{150}{10} = 15 \text{ km/h}$$

52. (*d*) After servicing speed of car

$= 60$ km/h.

$\therefore$ Distance covered in 6 h

$= (60 \times 6)$ km $= 360$ km.

When not serviced, then time taken to cover

360 km $= \dfrac{360}{50} = 7.2$ h

53. (*d*) Let total distance $= x$

Now, according to the question,

$$40 \times 3 + 60 \times 4.5 = \frac{3x}{5}$$

$$\Rightarrow \quad 120 + 270 = \frac{3x}{5}$$

$$\therefore \quad x = \frac{390 \times 5}{3} = 650 \text{ km}$$

$\therefore$ Remaining distance

$$= \left(650 - \frac{3 \times 650}{5}\right)$$

$$= 650 - 390 = 260 \text{ km}$$

Required average speed $= \dfrac{260}{4} = 65$ km/h.

54. (*d*) Required average speed

$$= \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{3 \times 39}{\frac{39}{26} + \frac{39}{39} + \frac{39}{52}} = \frac{3 \times 13}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}$$

$$= \frac{3 \times 13 \times 12}{13} = 36 \text{ km/h}$$

55. (*a*) In this case, $(21 - 15)$ m or 6 m is covered by Dalbir singh in 1 min.

$\therefore$ 114 m will be covered in

$$\left(\frac{1}{6} \times 114\right) \text{min} = 19 \text{ min}$$

56. (*b*) Here, the person covers $\frac{1}{2}$ of his journey at 40 km/h, $\frac{1}{3}$ at 60 km/h and rest $\left(1 - \frac{1}{2} - \frac{1}{3} = \frac{1}{6}\right)$ at 30 km/h.

$\therefore$ Average speed for whole journey

[by Technique 2]

$$= \frac{1}{\frac{1}{2 \times 40} + \frac{1}{3 \times 60} + \frac{1}{6 \times 30}}$$

$$= \frac{1}{\frac{1}{80} + \frac{1}{180} + \frac{1}{180}}$$

$$= \frac{720}{9 + 4 + 4} = \frac{720}{17} = 42.35 \text{ km / h}$$

57. (*c*) Let the distance = x

Here, the difference in time

$$= 20 - 5 = 15 \text{ min}$$
$$= \frac{15}{60} = \frac{1}{4} \text{ h}$$

Speed during next journey

$$= 15 + 5 = 20 \text{ km / h}$$

According to the question,

$$\frac{x}{15} - \frac{x}{20} = \frac{1}{4}$$
$$\Rightarrow \quad \frac{4x - 3x}{60} = \frac{1}{4}$$
$$\Rightarrow \quad x = \frac{60}{4} = 15$$
$$\therefore \quad x = 15 \text{ km}$$

Fast Track Method

Here, $a = 15$ km/h, $b = 5$ km / h,

$t_1 = 20$min and $t_2 = 5$min

Required distance $= (t_1 - t_2)\,(a + b)\dfrac{a}{b}$

[by Technique 4]

$$= \frac{(20 - 5)}{60}(15 + 5)\,\frac{15}{5}$$
$$= \frac{15 \times 20 \times 3}{60} = 15 \text{ km}$$

58. (*b*) Let the time taken by car B to reach destination is x h.

So, the time taken by car A to reach destination is $(x + 2)$ h.

Now, $\quad S_1T_1 = S_2T_2$

$$\Rightarrow \quad 30 \times (x + 2) = 45 \times x$$
$$\Rightarrow \quad 30x + 60 = 45x$$
$$\Rightarrow \quad 15x = 60$$
$$\therefore \quad x = 4 \text{ h}$$

Now, distance between starting point and destination

$$= S_2T_2 = 45 \times 4 = 180 \text{ km}$$

59. (*b*) Let the certain distance be d and time t.

Now, by given condition,

$$\frac{d}{3} = (t + 15) \text{ min} = \frac{(t + 15)}{60} \text{ h}$$
$$\Rightarrow \quad 20\,d = t + 15$$
$$\Rightarrow \quad t = 20\,d - 15 \qquad \text{...(i)}$$

and $\quad \dfrac{d}{4} = (t - 15)$ min

$$= \frac{(t - 15)}{60} \text{ h}$$
$$\Rightarrow \quad 15d = t - 15$$
$$\Rightarrow \quad t = 15\,d + 15 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$20\,d - 15 = 15\,d + 15$$
$$\Rightarrow \quad 5d = 30$$
$$\therefore \quad d = 6 \text{ km.}$$

Exercise © *Higher SkiU Level Questions*

1. (*c*) Speed of truck

$$= \frac{\text{Distance}}{\text{Time}} = \frac{368}{8} = 46 \text{ km / h}$$

Now, speed of car =

(speed of truck + 18) km/h

$$= (46 + 18) = 64 \text{ km/h}$$

and distance travelled by car

$$= 368 + 16 = 384 \text{ km}$$

$\therefore$ Time taken by car

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{384}{64} \text{ h} = 6\text{h}$$

2. (*b*) Let distance he drove be x.

$$\text{Using Time} = \frac{\text{Distance}}{\text{Speed}}$$

From given condition,

$$\frac{x}{40} - \frac{x}{45} = 1$$

$$\Rightarrow \quad \frac{45x - 40x}{40 \times 45} = 1$$

$$\Rightarrow \quad 5x = 40 \times 45 \quad \Rightarrow \quad x = 360 \text{ km}$$

3. (*d*) When serviced, the distance covered by the car $= 65 \times 5 = 325$ km.

When not serviced, the time taken by the car to cover 325 km.

$$= \frac{325}{40} = 8.125 = 8 \text{ h (approx)}$$

4. (*c*) Time taken by express train to cover 75 km including stoppage.

$$= \left(\frac{60}{100} \times 75\right) \text{min} + 3 \text{ min} = 48 \text{ min}$$

Time taken by express train to cover 600 km.

= Time taken by it to cover 525 km

+ Time taken by it to cover 75 km

$$= \left(\frac{48}{75} \times 525\right) \text{min} + \left(\frac{60}{100} \times 75\right) \text{min}$$

$$= (336 + 45) \text{ min} = 381 \text{ min}$$

Time taken by local train to cover 25 km including stoppage

$$= \left(\frac{60}{50} \times 25\right) \text{min} + 1 \text{ min} = 31 \text{ min}$$

In 31 min, distance covered = 25 km

In (31×12) min, distance covered

$$= \left(\frac{25}{31} \times 31 \times 12\right) = 300 \text{ km}$$

In last 9 min, distance covered

$$= \left(\frac{25}{31} \times 9\right) = 7.25 \text{ km}$$

$\therefore$ Required total distance

$$= 300 + 7.25 = 307.25 \text{ km}$$

5. (*c*) Let usual speed of a man is V and time is t h.

By the formula,

$$V_1t_1 = V_2t_2$$

$$Vt = 1\frac{1}{2}V\left(t - \frac{20}{60}\right)$$

$$\Rightarrow \quad t = \frac{3}{2}\left(t - \frac{1}{3}\right)$$

$$\Rightarrow \quad \frac{3}{2}t - t = \frac{1}{2}$$

$$\therefore \quad t = 1\text{h}$$

Hence, the time taken by the man to reach his office at his usual speed is 1 h.

6. (*c*) Here, $S_1 = 8$ km / h

and $\quad S_2 = 16$ km/h

$\therefore \quad d_1 = S_1 \times t_1 = 8t_1 \qquad \ldots$(i)

and $\quad d_2 = S_2 \times t_2 = 16t_2 \qquad \ldots$(ii)

We know that,

$t_1 + t_2 = 8 \qquad \ldots$(iii)

and $\quad d_1 + d_2 = 80 \quad$ (given) $\qquad \ldots$(iv)

From Eqs. (i) and (ii) put the value of d_1 and d_2 in Eq. (iv), we get

$$d_1 + d_2 = 80$$

$$8t_1 + 16t_2 = 80$$

$$\Rightarrow \quad 8t_1 + 8t_2 + 8t_2 = 80$$

$$8(t_1 + t_2) + 8t_2 = 80$$

$$8 \times 8 + 8t_2 = 80$$

From Eq (iii),

$$8t_2 = 80 - 64 = 16$$

$$\Rightarrow \quad t_2 = \frac{16}{8} = 2\text{ h}$$

$$\therefore \quad t_1 = 8 - 2 = 6\text{ h}$$

$\therefore$ Distance travelled by foot.

$$= d_1 = 8 \times 6 = 48\text{ km.}$$

7. (*d*) Let distance and original speed of the man be d km and s km/h. Then,

$$\frac{d}{s} - \frac{d}{s+3} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{d(s+3-s)}{s(s+3)} = \frac{2}{3}$$

$\Rightarrow \quad 9d = 2s(s+3) \qquad \ldots$(i)

and $\quad \dfrac{d}{s-2} - \dfrac{d}{s} = \dfrac{2}{3}$

$$\Rightarrow \quad \frac{d(s-s+2)}{s(s-2)} = \frac{2}{3}$$

$\Rightarrow \quad 3d = s(s-2) \qquad \ldots$(ii)

From Eqs. (i) and (ii), we get

$$3s(s-2) = 2s(s+3)$$

$$\Rightarrow \quad 3s^2 - 6s = 2s^2 + 6s$$

$$\Rightarrow \quad s^2 = 12s \Rightarrow s = 12$$

From Eq. (ii), we get

$$3d = 12\,(12 - 2)$$

$$\therefore \quad d = 40 \text{ km}$$

8. (*b*) Let they meet x km from A.

Then, according to the question,

$$\frac{x}{40} = \frac{(1000 - x)}{60}$$

$$\Rightarrow \quad 60x = 40\,(1000 - x)$$

$$\Rightarrow \quad 3x = 2000 - 2x$$

$$\Rightarrow \quad 3x + 2x = 2000$$

$$\Rightarrow \quad 5x = 2000 \text{ km}$$

$$\therefore \quad x = \frac{2000}{5} = 400 \text{ km}$$

9. (*a*) Let they meet after x h.

Then, according to the question,

$$3x + 4x = 17.5$$

$$\Rightarrow \quad 7x = 17.5$$

$$\therefore \quad x = \frac{17.5}{7} = 2.5 \text{ h}$$

So, they meet 2.5 h after 8 : 00 am.

It means they meet at 10 : 30 am.

10. (*c*) Let distance between Manipur and Dispur = x km

Average speed of train from Manipur

$$= \frac{x}{4} \text{ km/h}$$

Average speed of train from Dispur

$$= \frac{2x}{7} \text{ km/h}$$

Let they meet y h after 6 : 00 am.

Then, according to the question,

$$\left(\frac{x}{4} \times y\right) + \frac{2x}{7} \times (y - 2) = x$$

$$\Rightarrow \quad \frac{y}{4} + \frac{2\,(y - 2)}{7} = 1$$

$$\Rightarrow \quad 7y + 8\,(y - 2) = 28$$

$$\Rightarrow \quad 15y = 44$$

$$\therefore \quad y = \frac{44}{15} \text{ h} = 2 \text{ h } 56 \text{ min}$$

Clearly, trains meet 2 h 56 min after 6 : 00 am.

It means the trains meet at 8 : 56 am.

11. (*a*) Required average speed

$$= 75 \times \frac{60}{100} \times \frac{150}{100} = 75 \times \frac{3}{5} \times \frac{3}{2}$$

$$= 67.5 \text{ km/h}$$

12. (*e*) Distance = $(AB + BC) = 2x$

$$= 16 \times \frac{1}{2} = 8 \text{ km}$$

$$\therefore \quad x = \frac{8}{2} = 4 \text{ km}$$

$\therefore$ Area of the field = $4 \times 4 = 16$ sq km.

13. (*d*) Required average speed

$$= \frac{4}{\frac{1}{60} + \frac{1}{40} + \frac{1}{120} + \frac{1}{20}}$$

$$= \frac{4 \times 120}{12} = 40 \text{ km/h}$$

14. (*a*) Let two trains meet at *a* km from '*X*'.

[Time taken by *M* to cover (450 − *a*) km]

− [Time taken by *L* to cover a km]

$$= \frac{40}{60}$$

$$\Rightarrow \quad \frac{(450 - a)}{80} - \frac{a}{60} = \frac{40}{60}$$

$$\Rightarrow \quad \frac{(450 - a)}{80} = \frac{40}{60} + \frac{a}{60}$$

$$\Rightarrow \quad \frac{(450 - a)}{8} - \frac{(a + 40)}{6} = 0$$

$$\Rightarrow \quad 3(450 - a) - 4(a + 40) = 0$$

$$\Rightarrow \quad 7a = 1190$$

$$\therefore \quad a = \frac{1190}{7} = 170$$

Time taken by *L* to cover 170 km

$$= \frac{170}{60} \text{ h} = 2 \text{ h } 50 \text{ min}$$

So, the two trains will meet 2 h 50 min after 6 : 00 pm. It means that the two trains will meet at 8 : 50 pm.

15. (*d*) Let speed of $A = 5x$

Then, speed of $B = 6x$

Given that, speed of $B = 6x = 90$

$$\therefore \quad x = \frac{90}{6} = 15$$

$\therefore$ Speed of $A = 5x = 5 \times 15 = 75$ km/h

Let *A* and *B* meet after *t* h. Then,

$$75 \times t + 90 \times t = 88$$

$$\therefore \quad t = \frac{88}{165} \text{ h} = \frac{88 \times 60}{165} = 32 \text{ min}$$

16. (*d*) Required number of hours is the number of terms of the series 40 + 45 + 50 + ... as speed increases every hour.

Given, sum of the series is 385

$\therefore \quad a = 40, d = 5, S = 385$ and $n = ?$

Using $\quad S = \frac{n}{2}[2a + (n - 1)d]$

$$\Rightarrow \quad 385 = \frac{n}{2}[80 + 5n - 5]$$

$$\Rightarrow \quad 770 = 5n^2 + 75n$$

$$\Rightarrow \quad n^2 + 15n - 154 = 0$$

$$\Rightarrow \quad n^2 + 22n - 7n - 154 = 0$$

$$\Rightarrow \quad n(n + 22) - 7(n + 22) = 0$$

$$\Rightarrow \quad (n + 22)(n - 7) = 0$$

$$\therefore \quad n = 7$$

Chapter

26

Problems Based on Trains

Problems based on trains are same as the problems related to 'Speed, Time and Distance' and some concepts of 'Speed, Time and Distance' are also applicable to these problems. The only difference is that the length of the moving object (train) is taken into consideration in these types of problems.

Problems Based on Trains

RuleO

Speed of train $(S) = \dfrac{\text{Distance covered } (d)}{\text{Time taken } (t)}$ or $S = \dfrac{d}{t}$.

Here, unit of speed is m/s or km/h.

(a) a km/h $= \left(a \times \dfrac{5}{18}\right)$ m/s (b) a m/s $= \left(a \times \dfrac{18}{5}\right)$ km/h

Ex. 1 Convert 360 km/h into m/s.

Sol. $360 \text{ km/h} = 360 \times \frac{5}{18} \text{ m/s} = 100 \text{ m/s}$

Ex. 2 Convert 150 m/s into km/h.

Sol. $150 \text{ m/s} = 150 \times \frac{18}{5} \text{ km/h} = 30 \times 18 \text{ km/h} = 540 \text{ km/h}$

Rule®

The distance covered by train in passing a pole or a standing man or a signal post or any other object (of negligible length) is equal to the length of the train.

Ex. 3 A train covers 85 m in passing a signal post. What is the length of the train?

Sol. We know that, the distance covered by a train in passing a pole or a standing man or a signal post or any other object (of negligible length) is equal to the length of the train. So, in this case, train covers 85 m to pass a signal post.

/. Length of the train = 85 m

Rule®

If a train passes a stationary object (bridge, platform etc;) having some length, then the distance covered by train is equal to the sum of the lengths of train and that particular stationary object which it is passing.

Ex. 4 A 29 m long train passes a platform which is 100 m long. Find the distance covered by the train in passing the platform.

Sol. We know that, when a train passes a stationary object having some length, then the distance covered by train is equal to the sum of the lengths of train and that particular stationary object. In this case, station-ary object is 100 m long platform.

.•. Required distance = Length of train + Length of platform = 29 + 100 = 129 m

RuleQ

If two trains are moving in opposite directions, then their relative speed is equal to the sum of the speeds of both the trains.

Ex. 5 Two trains are moving in opposite directions with speeds of 4 m/s and 8 m/s, respectively. Find their relative speed.

Sol. When two trains are moving in opposite directions, then their relative speed

= Sum of the speeds of both the trains

Required relative speed = 4 m/s + 8 m/s = 12 m/s

Rule®

if two trains are moving in the same direction, then the relative speed is the difference of speeds of both trains.

Ex. 6 Two trains are moving in the same direction with speeds of 19 km/h and 25 km/h, respectively. What will be the relative speed of the train running at 25 km/h in respect of the train running at 19 km/h?

Sol. We know that, when two trains are running in the same direction, then the relative speed is equal to the difference of speeds of both the trains.

.•. Required relative speed = 25 km/h - 19 km/h = 6 km/h

Rule®

If two trains of lengths *x* and y are moving in opposite directions with speeds of *u* and *v*

respectively, then time taken by the trains to cross each other is equal to $\frac{(x+y)}{(u+v)}$.

Ex. 7 Two trains of lengths 80 m and 90 m are moving in opposite directions at 10 m/s and 7 m/s, respectively. Find the time taken by the trains to cross each other.

Sol. According to the formula,

Required time = $\frac{x+y}{u+v}$

where, $x = 80$ m, $y = 90$ m, $u = 10$ m/s and $v = 7$ m/s

$\therefore$ Required time = $\frac{80+90}{10+7} = \frac{170}{17} = 10$ s

RuleO

If two trains of lengths x and y are moving in the same direction with speeds of u and v respectively, then time taken by the faster train to cross the slower train is equal to

$$\left(\frac{x+y}{u-v}\right). \qquad [\text{here, } u > v]$$

Ex. 8 Two trains of lengths 75 m and 95 m are moving in the same direction at 9 m/s and 8 m/s, respectively. Find the time taken by the faster train to cross the slower train.

Sol. According to the formula,

Required time $= \dfrac{x+y}{u-v}$

where, $x = 75$ m, $y = 95$ m, $u = 9$ m/s and $v = 8$ m/s

$\therefore$ Required time $= \dfrac{75+95}{9-8} = 170$ s

Rule©

If two trains start at the same time from points P and Q towards each other and after crossing each other, they take f., and t_2 time in reaching points Q and P respectively, then (P's speed): (Q's speed) = *Jt^: ^*

Ex. 9 Two trains start at the same time from points *A* and *B* towards each other and after crossing each other, they take 25 h and 9 h in reaching points B and *A,* respectively. Find the ratio of speeds of 1st train to that of 2nd train. Sol. Given that, \ = 25hand£2 = 9h According to the formula, (1st train's speed) : (2nd train's speed) = *Ji^ : J* = (V9 : *-J25)* = 3:5

Ex. 10 A train passes a standing man in 6 s and a 210 m long platform in 16 s. Find the length and the speed of the train.

Sol. Let length of the train be L.

As the speed while crossing a standing man and while crossing the platform remains same. Then,

According to the question, $\dfrac{L}{6} = \dfrac{L+210}{16}$ $\qquad \left[\text{Speed} = \dfrac{\text{Length}}{\text{Time}}\right]$

$16L = 6L + 1260$

$\Rightarrow$ $10L = 1260 \Rightarrow L = \dfrac{1260}{10} = 126$ m

$\therefore$ Speed of the train $= \dfrac{\text{Length}}{\text{Time}} = \dfrac{126}{6} = 21$ m/s

Ex. 11 A 250 m long train is running at 100 km/h. In what time, will it pass a man running at 10 km/h in the same direction in which the train is going?

Sol. Speed of the train relative to man = 100 − 10 = 90 km/h

$$= 90 \times \frac{5}{18} \text{ m/s} = 5 \times 5 = 25 \text{ m/s}$$

Distance covered in passing the man = 250 m

$$\therefore \quad \text{Time taken} = \frac{250}{25} = 10 \text{ s}$$

Ex. 12 A 220 m long train is running at 120 km/h. In what time, will it pass a man running in the direction opposite to that of the train at 12 km/h?

Sol. Speed of the train relative to man = (120 + 12) km/h

$$= 132 \text{ km/h} = \left(132 \times \frac{5}{18}\right) \text{m/s} = \frac{110}{3} \text{ m/s}$$

Distance covered in passing the man = 220 m

$$\therefore \quad \text{Time taken} = \frac{220}{110} \times 3 = 2 \times 3 = 6 \text{ s}$$

Fast Track **Techniques**

to solve the **QUESTIONS**

(Technique^

If a train of length *L* m passes a platform of *x* m in *t^s,* then time taken *t2s* by the same train to pass a platform of length y m is given as

$$t_2 = \left(\frac{L+y}{L+x}\right) t_1$$

Ex. 13 A train of length 250 m, passes a platform of 350 m length in 50 s. what time will this train take to pass the platform of 230 m length.

Sol. Length of train = 250 m; Length of platform = 350 m

Time taken to cover the distance = 50 s

So, to cross the platform, the train has to cover a distance

$$= \text{Length of train} + \text{Length of platform} = 250 + 350 = 600 \text{ m}$$

$$\therefore \quad \text{Speed of train} = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{600}{50} = 12 \text{ m/s}$$

Now, the time taken to cross the platform of length 230 m

$$= \frac{\text{Total distance covered}}{\text{Speed of train}} = \frac{\text{Length of train} + \text{Length of platform}}{\text{Speed of train}}$$

$$= \frac{250 + 230}{12} = \frac{480}{12} = 40 \text{ s}$$

Fast Track Method

Here, $L = 250$ m, $x = 350$ m, $t_1 = 50$ s,

$y = 230$ m and $t_2 = ?$

$$\therefore \quad t_2 = \left(\frac{L + y}{L + x}\right) t_1 = \left(\frac{250 + 230}{250 + 350}\right) \times 50 = \frac{480}{600} \times 50 = 40 \text{s}$$

Technique 2

From stations *P* and *Q*, two trains start moving towards each other with the speeds of a and *b*, respectively. When they meet each other, it is found that one train covers distance *d* more than that of another train. In such cases, distance between stations *P* and Q is given

as $\left(\frac{a+b}{a-b}\right) \times d$.

Ex. 14 From stations *A* and *B*, two trains start moving towards each other with the speeds of 150 km/h and 130 km/h, respectively. When the two trains meet each other, it is found that one train covers 20 km more than that of another train. Find the distance between stations *A* and *B*.

Sol. Let the trains meet after time *t* at a distance *x* from station *B*. Then, another train coming from *A* covers a distance of *(x* + 20).

For station *A* , distance covered by first train (x + 20) = 150*1*

=> *x* = 150i - 20 ...(i)

For station B,distance covered by second train *x* = 130t ...(ii)

From Eqs. (i) and (ii), we get

130i = 150i - 20 => 150t - 130i = 20 => 20t = 20 => *t* = lh

Distance between stations *A* and *B* - 150t + 130£

= 280< = 280 x 1= 280 km

Fast Track Method

Here, a = 150 km/h, b = 130 km/h and d = 20 km

According to the formula,

$$\text{Distance between stations } A \text{ and } B = \left(\frac{a+b}{a-b}\right) \times d$$

$$= \left(\frac{150+130}{150-130}\right) \times 20 = \frac{280}{20} \times 20 = 280 \text{ km}$$

Technique 3

If two trains leave P for *Q* at time t, and t_2 and travel with speeds a and *b* respectively, then the distance *d* from *P,* where the two trains meet, is given as

$$d = \text{Difference in time} \times \frac{\text{Product of speeds}}{\text{Difference in speeds}} = (t_2 - t_1) \times \frac{a \times b}{b - a}$$

where, $t_2 > t_1$ and $b > a$

Ex. 15 Two trains leave Patna for Delhi at 10:00 am and 10:30 am respectively and travel at 120 km/h and 150 km/h, respectively. How many kilometres from Patna, will the two trains meet?

Sol. Speed of 1st train = 120 km/h and speed of 2nd train = 150 km/h

Relative speed of 2nd train = 150 − 120 = 30 km/h

As 1st train leaves $\frac{1}{2}$ h before, hence in $\frac{1}{2}$ h, it will cover 60 km as it moves 120 km in 1 h.

Time taken by 2nd train to gain 60 km = $\frac{60}{30}$ = 2 h

∴ Actual distance covered by 2nd train in 2 h = 2 × 150 = 300 km

Fast Track Method

Here, $a = 120$ km/h, $b = 150$ km/h, $t_1 = 10{:}00$ am and $t_2 = 10{:}30$ am

According to the formula,

$$\text{Required distance} = (t_2 - t_1) \times \frac{a \times b}{b - a}$$

$$= (10{:}30 - 10{:}00) \times \frac{(120 \times 150)}{(150 - 120)}$$

$$= \frac{1}{2} \times \frac{120 \times 150}{30}$$

$$= \frac{120 \times 150}{60} = 2 \times 150 = 300 \text{ km}$$

Technique 4

If a train overtakes two persons who are walking with speeds of *a* and *b* respectively, in the same direction and passes the two persons completely in t, and t_2 time, respectively, then

$$\text{Length of the train} = \frac{\text{Difference in speeds} \times t_1 \times t_2}{(t_2 - t_1)}, \text{ where } t_2 > t_1$$

Note In case tj > £2. *(t\ - h*) is taken in place of *{t2 -1{)* in the denominator

Ex. 16 A train overtakes two persons who are walking at the rate of 4 km/h and 8 km/h in the same direction and passes them completely in 18 and 20 s, respectively. Find the length of the train.

Sol. Let speed of the train be x.

Then, relative speeds are $\left(x - \frac{10}{9}\right)$ m/s and $\left(x - \frac{20}{9}\right)$ m/s. $\left[\because 4 \text{ km/h} = \frac{10}{9} \text{ m/s} \quad 8 \text{ km/h} = \frac{20}{9} \text{ m/s}\right]$

For 1st man,

Length of the train $= \left(x - \frac{10}{9}\right) \times 18$ m ...(i)

For 2nd man,

Length of the train $= \left(x - \frac{20}{9}\right) \times 20$ m ...(ii)

From Eqs. (i) and (ii), we get

$$\left(x - \frac{10}{9}\right) \times 18 = \left(x - \frac{20}{9}\right) \times 20$$

$$\Rightarrow \left(\frac{9x - 10}{9}\right) 18 = \left(\frac{9x - 20}{9}\right) 20$$

$$\Rightarrow 9(9x - 10) = 10(9x - 20)$$

$$\Rightarrow 81x - 90 = 90x - 200$$

$$\Rightarrow 9x = 110$$

$$\Rightarrow x = \frac{110}{9}$$

From Eq. (i),

$$\text{Length of the train} = \left(\frac{110}{9} - \frac{10}{9}\right) \times 18 = \frac{100}{9} \times 18 = 200 \text{ m}$$

Fast Track Method

Here, $t_1 = 18$ s, $t_2 = 20$ s

$a = 4$ km/h $= 4 \times \frac{5}{18}$ m/s $= \frac{10}{9}$ m/s and $b = 8$ km/h $= 8 \times \frac{5}{18}$ m/s $= \frac{20}{9}$ m/s

According to the formula,

$$\text{Length of the train} = \frac{\text{Difference in speeds} \times t_1 \times t_2}{t_2 - t_1}$$

$$= \frac{\left(\frac{20}{9} - \frac{10}{9}\right) \times 18 \times 20}{20 - 18}$$

$$= \frac{\frac{10}{9} \times 18 \times 20}{2} = 10 \times 20 = 200 \text{ m}$$

(Technique^

if two trains A and B start from stations/points P and Q towards Q and P respectively and after passing each other, they take t, and t_2 time to reach Q and P respectively and speed of train A is given as a, then

Speed of train $B = a\sqrt{\frac{t_1}{t_2}}$

Ex. 17 Two trains x and y start from Mumbai and Delhi towards Delhi and Mumbai, respectively. After passing each other, they take 12 h 30 min and 8 h to reach Delhi and Mumbai, respectively. If the train from Mumbai is moving at 60 km/h, then find the speed of the other train.

Sol. Here, $a = 60$ km/h, $t_1 = 12$ h and 30 min $= 12 + \frac{30}{60} = \frac{25}{2}$ h, $t_2 = 8$ h

According to the formula,

Speed of $y = a\sqrt{\frac{t_1}{t_2}} = 60 \times \sqrt{\frac{25}{2 \times 8}} = 60 \times \frac{5}{4}$ km/h $= 75$ km/h

Technique 6

The distance between p and Q is d km. A train with a km/h starts from station P towards Q and after a difference of t h another train with b km/h starts from Q towards station P, then both the trains will meet at a certain point after time T. Then,

$$T = \left(\frac{d \pm tb}{a + b}\right)$$

if second train starts after the first train, then t is taken as positive. If second train starts before the first train, then t is taken as negative.

Ex. 18 The distance between two stations P and Q is 110 km. A train with speed of 20 km/h leaves station P at 7:00 am towards station Q. Another train with speed of 25 km/h leaves station Q at 8:00 am towards station P. Then, at what time both trains meet? Sol. First train leaves at 7:00 am and second at 8:00 am. So, first train *i.e.* from P to Q has travelled lh. Distance covered by first train = 20 km So, distance left between the station = 110 - 20 = 90 km

Now, trains are travelling in opposite directions.

So, relative speed = 20 + 25 = 45 km/h

Time taken to cover 90 km = $\frac{90}{45} = 2$ h

$\therefore$ The time, at which they will meet, is 2 h after second train left

i.e. 8:00 am + 2h = 10:00 am

Fast Track Method

Here, $d = 110$ km, $t = 8:00 - 7:00 = 1$ h

$$a = 20 \text{ km/h and } b = 25 \text{ km/h}$$

Time taken by trains to meet, $T = \left(\frac{d + tb}{a + b}\right)$

$$\Rightarrow \quad T = \frac{110 + (1)(25)}{20 + 25} = \frac{135}{45}$$

$$\Rightarrow \quad t = 3 \text{ h}$$

$\therefore$ They will meet at = 7 : 00 am + 3 h = 10:00 am.

Technique 7

The distance between two stations P and Q is *d* km. A train starts from P towards Q and another train starts from Q towards P at the same time and they meet at a certain point after *t* h. If train starting from *P* travels with a speed of *x* km/h slower or faster than another train, then

(i) Speed of faster train = $\left(\frac{d + tx}{2t}\right)$ km/h

(ii) Speed of slower train = $\left(\frac{d - tx}{2t}\right)$ km/h

Ex. 19 The distance between two stations *A* and B is 138 km. A train starts from *A* towards *B* and another from *B* to *A* at the same time and they meet after 6 h. The train travelling from *A* to *B* is slower by 7 km/h compared to other train from *B* to *A*, then find the speed of the slower train?

Sol. Let the speed of slower train be x km/h.

Then, speed of faster train $= (x + 7)$ km/h

As the trains are moving in opposite directions.

So, the relative speed $= x + (x + 7) = (2x + 7)$ km/h

$$\text{Time taken} = \frac{\text{Distance travelled}}{\text{Relative speed}}$$

$$\Rightarrow \quad 6 = \frac{138}{2x + 7} \Rightarrow 2x + 7 = \frac{138}{6} \Rightarrow 2x + 7 = 23 \Rightarrow 2x = 23 - 7$$

$$\therefore \quad x = \frac{16}{2} = 8 \text{ km/h}$$

$\therefore$ Speed of slower train = 8 km/h

Fast Track Method

Here, $d = 138$ km, $t = 6$ h and $x = 7$ km/h

$$\therefore \text{ Speed of slower train} = \frac{d - tx}{2t} = \frac{138 - (6)(7)}{2(6)}$$

$$= \frac{138 - 42}{12} = \frac{96}{12} = 8 \text{ km/h}$$

Technique 8

A train covers distance d between two stations P and Q in t_1 h. If the speed of train is reduced by a km/h, then the same distance will be covered in t_2 h. (i) Distance between P and Q is

$$d = a\left(\frac{t_1 t_2}{t_2 - t_1}\right) \text{ km}$$

(ii) Speed of the train $= \left(\frac{a t_2}{t_2 - t_1}\right)$ km/h

Ex. 20 A train covers distance between two stations A and B in 2 h. If the speed of train is reduced by 6 km/h, then it travels the same distance in 3 h. Calculate the distance between two stations and speed of the train.

Sol. Let the initial speed of train be x km/h.

Distance between stations A and $B = d$ km

Case I With initial speed,

$$\text{Time taken} = \frac{\text{Distance}}{\text{Speed}} \Rightarrow 2 = \frac{d}{x} \quad \text{...(i)}$$

Case II With decreased speed,

$$\text{Time taken} = \frac{\text{Distance}}{\text{Speed}} \Rightarrow 3 = \frac{d}{(x-6)} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$2x = 3(x-6) \Rightarrow 2x = 3x - 18 \Rightarrow 3x - 2x = 18 \Rightarrow x = 18$$

On putting the value of x in Eq. (i) we get

$$2 = \frac{d}{18} \Rightarrow d = 2 \times 18 = 36 \text{ km}$$

$\therefore$ Initial speed of train = 18 km/h and distance between two stations = 36 km

Fast Track Method

Here, $t_1 = 2$ h, $t_2 = 3$ h, $a = 6$ km/h and $d = ?$

(i) Distance between A and B is

$$d = a\left(\frac{t_1 t_2}{t_2 - t_1}\right) \text{km}$$

$$\Rightarrow \quad d = 6\left(\frac{2 \times 3}{3-2}\right) \Rightarrow d = 36 \text{ km}$$

(ii) Speed of the train $= \dfrac{a\,t_2}{t_2 - t_1} = \dfrac{6 \times 3}{3-2} = 18$ km/h

Technique 9

Without stoppage, a train travels at an average speed of *a* and with stoppage, it covers the same distance at an average speed of *b*, then

$$\text{Time of rest per hour} = \frac{\text{Difference in average speeds}}{\text{Speed without stoppage}} = \frac{a-b}{a}$$

where, $a > b$.

Ex. 21 Without stoppage, the speed of a train is 54 km/h and with stoppage, it is 45 km/h. For how many minutes, does the train stop per hour?

Sol. Decrease in speed due to stoppage = 54 - 45 = 9 km/h

Because of stoppage, train covers 9 km less per hour.

$\therefore$ Time taken to cover 9 km $= \frac{9}{54} = \frac{1}{6}$ h $= \frac{1}{6} \times 60 = 10$ min

Fast Track Method

Here, $a = 54$ km/h and $b = 45$ km/h

According to the formula,

$$\text{Required rest time} = \frac{a-b}{a} = \frac{54-45}{54}$$

$$= \frac{9}{54} = \frac{1}{6}\text{ h} = \frac{1}{6} \times 60 = 10\text{ min}$$

Technique 10

If two trains of equal lengths and different speeds take t, and t_2 time to cross a pole, then time taken by them to cross each other is

$$T = \frac{2t_1t_2}{t_2 \pm t_1}.$$

We use'+' sign, if trains are moving in opposite directions and'-' sign, if they are moving in same direction.

Ex.22 Two trains of equal lengths take 5 s and 6 s respectively to cross a pole. If these trains are moving in the same direction, then how long will they take to cross each other?

Sol. Let the length of train be x m.

Then, speed of first train $= \frac{\text{Distance covered}}{\text{Time taken}} = \frac{x}{5}$ m/s

Similarly, speed of second train $= \frac{x}{6}$ m/s

Now, time taken to cross each other when moving in same direction

$$= \frac{\text{Sum of lengths of two trains}}{\text{Relative speed of two trains}} = \frac{x + x}{\frac{x}{5} - \frac{x}{6}}$$

$$= \frac{2x}{\frac{6x - 5x}{30}}$$

$$= \frac{2x \times 30}{x} = 60 \text{ s}$$

Fast Track Method

Here, $t_1 = 5$ s and $t_2 = 6$ s

According to the formula,

$$\text{Required time} = \frac{2t_1 t_2}{t_2 - t_1}$$

$$= \frac{2 \times 5 \times 6}{6 - 5} = 60 \text{ s}$$

Multi Concept

QUESTIONS

1. *After travelling 80 km, a train meets with an accident and then proceeds at 3/4 of its former speed and arrives at its destination 35 min late. Had the accident occurred 24 km further, it would have reached the destination only 25 min late. Find the speed of the train.*

(a) 50km/h (b) 30 km/h (c) 48 km/h (d) 55 km/h

^ (c) When we analyse the question minutely, we see that the speeds of the train upto 80 km are the same in both the cases. Further, the speeds after (80 + 24) = 104 km are same in both the cases. Therefore, the difference in time (35 min - 25 min) = 10 min is only because of the difference in speeds for the 24 km journey. Now, let speed of the train be x km/h Then, according to the question, Difference in time when 24 km are travelled with $\frac{3}{4}$th of speed and with usual speed = 10 min

$$\Rightarrow \quad \frac{24}{\frac{3x}{4}} - \frac{24}{x} = \frac{10}{60}$$

$$\Rightarrow \quad \frac{24 \times 4}{3x} - \frac{24}{x} = \frac{10}{60}$$

$$\Rightarrow \quad \frac{32}{x} - \frac{24}{x} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{8}{x} = \frac{1}{6} \Rightarrow x = 48 \text{ km/h}$$

2. *A goods train and a passenger train are running on the parallel tracks in the same direction. The driver of the goods train observes that the passenger train coming from behind, overtakes and crosses his train completely in 1 min whereas a passenger on the passenger train marks that he crosses the goods train in 2/3 min. If the speeds of the trains is in the ratio of 1* : *2, then find the ratio of their lengths.*

(a) 4 :1 (b) 3 :1 (c) 1 : 4 (d) 2:1

w (d) Let the speeds of the two trains be xand 2x and length be *L*-\ and L2, respectively

***Case* I** When driver of goods train observes that passenger train crosses his train

$$\frac{L_1 + L_2}{2x - x} = 1 \text{ min} = 60 \text{ s} \quad \ldots(\text{i})$$

Case II When a passenger on passenger train observes that he crosses the goods train

$$\frac{L_1}{2x - x} = \frac{2}{3} \text{ min} = \frac{2}{3} \times 60 = 40 \text{ s} \quad \ldots(\text{ii})$$

On dividing Eq. (i) by Eq. (ii), we have

$$\frac{L_1 + L_2}{L_1} = \frac{60}{40}$$

$$\Rightarrow \quad \frac{L_2}{L_1} + 1 = \frac{3}{2} \Rightarrow \frac{L_2}{L_1} = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\therefore \quad L_1 : L_2 = 2 : 1$$

Fast Track Practice

Exercise© *Base Level Questions*

1. A train covers 90 m in passing a standing man. Find the length of the train. *(a)* 20 m *(b)* 87 m

(c) 71 m *(d)* 90 m

(e) None of the above

A 220 m long train passes a signal post in 12 s. Find the speed of the train.

[Bank Clerks 2010]

(a) 72 km/h *(b)* 60 km/h

(c) 66 km/h *(d)* 69 km/h

(e) None of the above

3. A train takes 9 s to cross a pole. If the speed of the train is 48 km/h, then length of the train is [CDS 2014]

(a) 150 m *(b)* 120 m

(c) 90 m *(d)* 80 m

4. A train running at the speed of 72 km/h goes past a pole in 15 s. What is the length of the train? [CDS 2013]

(a) 150 m *(b)* 200 m

(c) 300 m *(d)* 350 m

5. 150 m long train running with the speed of 90 km/h to cross a bridge in 26s. What is the length of the bridge?

[SSCCGL2012] *(a)* 500 m *(b)* 600 m

(c) 659 m *(d)* 550 m

6. A train travelling at a speed of 30 m/s crosses a 600 m long platform in 30 s. Find the length of the train. [SSC CCL 2007]

(a) 120 m *(b)* 150 m

(c) 200 m *(d)* 300 m

7. A 240 m long train crosses a platform of equal length in 27 s. What is the speed of the train in km/h? [Bank Clerks 2009]

(a) 48 *(b)* 60

(c) 56 *(d)* 64

(e) None of the above

8. A train crossed a platform in 43 s. The length of the train is 170 m . What is the speed of the train? [IBPS Clerk 2011]

(a) 233 km/h

(b) 243 km/h

(c) 265 km/h

(d) Cannot be determined

(e) None of the above

9. A train crosses a platform in 30 s travelling with a speed of 60 km/h. If the length of the train be 200 m, then the length of the platform is [SSC CPO 2013]

(a) 420 m *(b)* 500 m

(c) 300 m *(d)* 250 m

10. A train crosses a bridge of length 150 m in 15 s and a man standing on it in 9 s. The train is travelling at a uniform speed. Length of the train is

[SSC CGL (Mains) 2012] *(a)* 225 m *(b)* 200 m

(c) 135 m *(d)* 90 m

11. A train moving with uniform speed crosses a pole in 2 s and a 250 m long bridge in 7 s. Find the length of the train. [SSC CCL 2010]

(a) 150 m *(b)* 120 m

(c) 100 m *(d)* 80 m

12. The lengths of a train and a platform are equal. If a train running at a speed of 90 km/h, crossed the platform in 1 min, then find the length of the train.

(a) 500 m *(b)* 550 m *(c)* 600 m *(d)* 700 m *(e)* 750 m

13. A train travelling with uniform speed crosses two bridges of lengths 300 m and 240 m in 21 s and 18 s, respectively. Find the speed of the train. [SSC CPO 2011]

(a) 72 km/h *(b)* 68 km/h

(c) 65 km/h *(d)* 60 km/h

14. Two trains are moving in opposite directions with speeds of 6 m/s and 12 m/s, respectively. Find their relative speed.

(a) 18 m/s *(b)* 15 m/s

(c) 20 m/s *(d)* 6 m/s

(e) None of the above

15. Two trains are moving in the same direction with speeds of 15 km/h and 21 km/h, respectively. What is the speed of trains in respect of each other?

(a) 26 km/h

(b) 15 km/h

(c) 6 km/h

(d) 24 km/h

(e) None of the above

16. The relative speed of a train in respect of a car is 90 km/h when train and car are moving opposite to each other. Find the actual speed of train, if car is moving with a speed of 15 km/h. [Bank PO 2011]

(a) 80 km/h *(b)* 105 km/h

(c) 75 km/h *(d)* 100 km/h

(e) None of the above

17. Two trains of lengths 70 m and 90 m are moving in opposite directions at 10 m/s and 6 m/s, respectively. Find the time taken by trains to cross each other.

(a) 10 s *(b)* 8 s *(c)* 12 s *(d)* 16 s

18. A 440 m long train is running at 240 km/h. In what time will it pass a man running in the direction opposite to that of the train at 24 km/h?

[Hotel Mgmt. 2009]

(a) 9 s *(b)* 6 s *(c)* 12 s *(d)* 4 s *(e)* None of the above

19. A 400 m long train takes 36 s to cross a man walking at 20 km/h in the direction opposite to that of the train. What is the speed of the train? [IBPS Clerks 2011]

(a) 20 km/h *(b)* 30 km/h

(c) 15 km/h *(d)* 11 km/h

(e) None of the above

20. A 280 m long train crosses a platform which is three times of its length, in 6 min 40 s. What is the speed of the train? [IBPS Clerk 2011]

(a) 3.2 m/s

(b) 1.4 m/s

(c) 2.8 m/s

(d) Cannot be determined

(e) None of the above

21. Two trains of length 512 m and 528 m are running towards each other on parallel lines at 84 km/h and 60 km/h, respectively. In what time, will they be clear of each other from the moment they meet?

(a) 26 s *(b)* 25 s *(c)* 15 s *(d)* 27 s *(e)* None of the above

22. Two trains of lengths 50 m and 65 m are moving in the same direction at 18 m/s and 17 m/s, respectively. Find the time taken by the faster train to cross the slower train.

(a) 100 s *(b)* 114 s *(c)* 95 s *(d)* 115 s *(e)* None of the above

23. Two trains of lengths 105 m and 90 m, respectively run at the speeds of 45 km/h and 72 km/h, respectively in opposite directions on parallel tracks. Find the time which they take to cross each other.

[SSC CGL 2007]

(a) 5 s *(b)* 6 s *(c)* 7 s *(d)* 8 s

24. A train passes two persons who are walking in the direction opposite to the direction of train at the rate of 10 m/s and 20 m/s respectively in 12 s and 10 s, respectively. Find the length of the train.

[SSC CGL 2013] *(a)* 500 m *(b)* 900 m

(c) 400 m *(d)* 600 m

25. A train *A* is 180 m long, while another train *B* is 240 m long. *A* has a speed of 30 km/h and *B's* speed is 40 km/h. If the trains move in opposite directions, find when will *A* pass *B* completely?

[SSC (10+2) 2012]

(a) 21 s *(b)* 21.6 s

(c) 26.1 s *(d)* 26 s

26. A train *A* of length 180 m, running by 72 km/h to cross the another train *B* which is running in the opposite direction to speed of 108 km/h and length is 120 m, in how much time?[SSC CGL 2012]

(a) 23 s *(b)* 12 s *(c)* 6 s *(d)* 30 s

27. A 110 m long train is running at a speed of 60 km/h. How many seconds, does it take to cross an another train of length 170 m, which is standing on parallel track? [SSC CGL 2011]

(a) 15.6 s*(b)* 16.8 s*(c)* 17.2 s *(d)* 18 s

28. Two trains of equal length are running on parallel lines in the same direction at 46 km/h and 36 km/h. The faster train passes the slower train in 36 s. The length of each train is [SSC (10+2)2012]

(a) 82 m *(b)* 50 m *(c)* 80 m *(d)* 72 m

29. Two trains, of same length are running in parallel tracks in opposite directions with speeds 65 km/h and 85 km/h respectively. They cross each other in 6 s. The length of each train is

[SSC (10+2) 2013]

(a) 100 m *(b)* 115 m

(c) 125 m *(d)* 150 m

30. Two trains start at the same time from points *x* and *y* towards each other and after crossing, they take 9 h and 4 h in reaching points *y* and *x*, respectively. Find the ratio of speeds of the 1st train to that of the 2nd train.

(a) 2 : 3 *(b)* 3 : 2 *(c)* 2 : 5 *(d)* 5 : 3 *(e)* None of the above

31. The ratio between the speeds of two train is 8 : 9. Second train covers 360 km in 4 h. Distance covered by first train in **3 h (in km) is [SSC (10+2)2012]**

(a) 240 *(b)* 480 *(c)* 120 *(d)* 60

32. The average speed of a bus is three-fourth the average speed of a train. The train covers 240 km in 12 h. How much distance will the bus cover in 7 h?

[IBPS Clerk 2011]

(a) 110 km *(b)* 115 km

(c) 105 km *(d)* 100 km

(e) None of the above

33. From stations M and N, two trains start moving towards each other at speed 125 km/h and 75 km/h, respectively. When the two trains meet each other, it is found that one train covers 50 km more than another. Find the distance between M and N.

(a) 190 km *(b)* 200 km

(c) 145 km *(d)* 225 km

34. Two trains A and B start from Delhi and Patna towards Patna and Delhi, respectively. After passing each other, they take 16 h and 9 h to reach Patna and Delhi, respectively. If the train from Delhi is moving at 90 km/h, then find the speed of the other train (in km/h).

(a) 120 *(b)* 190

(c) 125 *(d)* 145

35. Excluding stoppages, the speed of a train is 108 km/h and including stoppages, it is 90 km/h. For how many minutes does the train stop per hour? [SSC CPO 2012]

(a) 5 *(b)* 9

(c) 10 *(d)* 6

36. Without stoppages, the speed of a train is 150 km/h and with stoppages, it is 100 km/h. How many minutes, does the train stop? [MBA 2008]

(a) 20 *(b)* 15

(c) 25 *(d)* 45

(e) None of the above

37. Two trains running in opposite directions cross a man standing on the platform in 54s and 34s respectively and they cross each other in 46 s. Find the ratio of their speeds.

(a) 3 : 2 *(b)* 2 : 3

(c) 5 : 3 *(d)* 3 : 5

(e) None of the above

38. Two trains are running 40km/h and 20 km/h respectively, in the same direction. The fast train completely passes a man sitting in the slow train in 5 s. The length of the fast a train is

rssr *cm ?m* 31

(a) $23\frac{2}{9}$ m (b) 27 m

(c) $27\frac{7}{9}$ m (d) 23 m

39. Two trains of same length take 6 s and 9 s, respectively to cross a pole. If both the trains are running in the same direction, then how long will they take to cross each other?

(a) 30 s *(b)* 36 s *(c)* 40 s *(d)* 42 s *(e)* None of the above

40. The distance between two stations *P* and Q is 145 km. A train with speed of 25 km/h leaves station at 8:00 am towards station *Q*. Another train with speed of 35 km/h leaves station *Q* at 9:00 am towards station *P*. Then, at what time both trains meet?

[SSC Multitasking 2013] *(a)* 10 : 00 am *(b)* 11 : 00 am

(c) 12 : 00 am *(d)* 11 : 30 am

4LP and *Q* are 27 km away. Two trains will having speeds of 24 km/h and 18 km/h respectively start simultaneously from *P* and *Q* and travel in the same direction. They meet at a point *R* beyond *Q*. Distance *QR* is [SSC CCL 2012]

(a) 126 km *(b)* 81 km

(c) 48 km *(d)* 36 km

42. Two trains *A* and *B* start from Howrah and Patna towards Patna and Howrah respectively at the same time. After passing each other, they take 4, h, 48 min and 3 h, 20 min to reach Patna and Howrah, respectively. If the train from Howrah is moving at 45 km/h, then the speed of the other train is

[SSC Multitasking 2014] *(a)* 60 km/h *(b)* 45 km/h

(c) 35 km/h *(d)* 54 km/h

Exercise © *Higher Skill Level Questions*

1. Two stations A and B, 100 km far away to each other. Two trains starts at the same time from station A and station B. The train starts from station A is running with the speed of 50 km/h to station B. The train starts from station

B is running with the speed of 75 km/h to station A. At what distance both the trains meet with each other from station A? [SSC CCL 2012]

(a) 40 km *(b)* 20 km

(c) 30 km *(d)* None of these

2. Two stations P and Q are at a distance of 160 km. Two trains start moving from P and Q to Q and P respectively and meet each other after 4 h. If speed of the train strating from P is more than that of other train by 6 km/h, then find the speeds of both the trains, respectively.

(a) 19 km/h, 13 km/h

(b) 13 km/h, 9 km/h

(c) 17 km/h, 23 km/h

(d) 16 km/h, 10 km/h

(e) None of the above

3. A train travelling at 48 km/h completely crosses an another train having half length of first train and travelling in opposite directions at 42 km/h in 12 s. It also passes a railway platform in 45 s. The length of the platform is

(a) 400 m *(b)* 450 m

(c) 560 m *(d)* 600 m

(e) None of the above

4. A train leaves Mumbai for Goa at 3 : 45 pm and goes at the rate of 50 km/h. Another train leaves Goa for Mumbai at 2:35 pm and goes at the rate of 60 km/h. If the distance between Mumbai and Goa is 510 km, at what distance from Mumbai will the two trains meet?

(a) 200 km *(b)* 150 km

(c) 145 km *(d)* 300 km

(e) None of the above

5. A train P starts from Mokama at 5:00 pm and reaches Hazipur at 6:00 pm. An another train Q starts from Hazipur at 5:00 pm and reaches Mokama at 6:30 pm. At what time, two trains will cross each other?

(a) 5:36 pm *(b)* 4:36 pm

(c) 6:00 pm *(d)* 7:00 pm

(e) None of the above

6. Train A crosses a pole in 25 s and train B crosses the pole in 1 min 15 s. Length of train A is half the length of train B. What is the ratio between the speeds of A and B, respectively?

(a) 3 : 2

(b) 3 : 4

(c) 4 : 3

(d) Couldn't be determined

(e) None of the above

7. A 110 m long train crosses a tree in 3 s. How long will it take to cross 165 m long platform and 135 m long bridge, if platform and bridge are at a distance of 30 m?

(a) 12 s *(b)* 15 s *(c)* 16 s *(d)* 18 s *(e)* 20 s

8. The distance travelled by a train is 1830 km. The speed of the train is one more than twice the time taken to travel the distance. What will be the respective ratio of the speed of the train and time taken to travel? [Bank Clerks 2010]

(a) 30 : 61 *(b)* 61 : 30

(c) 25 : 51 *(d)* 51 : 25

(e) None of the above

9. The average speed of a train in the onward journey is 25% more than that in the return journey. The train halts for 2 h on reaching the destination. The total time taken to complete to and for journey is 32 h, covering a distance of 1600 km. Find the speed of the train in the onward journey.

(a) 56.25 km/h *(b)* 60 km/h *(c)* 66.50 km/h *(d)* 67 km/h *(e)* None of the above

10. A train overtakes two persons walking along a railway track. The first one walks at 4.5 km/h and the other one walks at 5.4 km/h. The train needs 8.4 s and 8.5 s respectively, to overtake them. What is the speed of the train, if both the persons are walking in the same direction as the train?

(a) 66 km/h *(b)* 72 km/h

(c) 78 km/h *(d)* 81 km/h

(e) None of the above

11. A 125 m long train take 25 s to cross a person who is going in the same direction with the speed of 4 km/h. After crossing that person, the train can reach next station in 30 min. How long that person takes to reach that station after being crossed by them?

(a)3±h fbJ2^h

4 4

(c) 3 - h *(d)2-h*

4 4

(e) None of the above

12. A train covers certain distance between two places at a uniform speed. If the train moved 10 km/h faster, it would take 2 h less, and if the train were slower by 10 km/h, it would take 3 h more than the scheduled time. Find the distance covered by the train.

(a) 300 km *(b)* 600 km

(c) 800 km *(d)* 1200 km

(e) None of the above

Answer with Solutions

Exercise© *Base Level Questions*

1. *id)* We know that, the distance covered by a train in passing a pole or a standing man or a signal post or any other object (of negligible length) is equal to the length of the train. In this case, train covers 90 m to cross a standing man. .'. Length of the train = 90 m

2. (*c*) We know that,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Speed of the train} = \frac{220}{12} \times \frac{18}{5} = 66 \text{ km/h}$$

3. (*b*) Let the length of the train be x m.

Now, speed $= 48$ km/h $= 48 \times \frac{5}{18}$ m/s

Train takes 9 s to cross a pole.

$\therefore$ Length of train, x = Speed $\times$ Time

$$= 48 \times \frac{5}{18} \times 9 = 120 \text{ m}$$

4. (*c*) Speed of train

$$= \frac{\text{Length of train}}{\text{Time taken to cross the stationary object}}$$

$\therefore$ Length of train = Speed of train $\times$ Time taken to cross the stationary object

$$= \frac{72 \times 5 \times 15}{18} = 300 \text{ m}$$

5. (*a*) Let length of bridge be x m.

We know, Speed $= \frac{\text{Distance}}{\text{Time}}$

According to the question,

$$90 \times \frac{5}{18} = \frac{150 + x}{26}$$

$$\Rightarrow \quad 25 \times 26 = 150 + x$$

$$\Rightarrow \quad 650 = 150 + x$$

$$\therefore \quad x = 500 \text{ m}$$

6. (*d*) Let length of the train be x m.

Relative length of the train $= (x + 600)$ m

According to the question,

$$\frac{x + 600}{30} = 30 \Rightarrow x + 600 = 900$$

$$\therefore \quad x = 900 - 600 = 300 \text{ m}$$

7. (*d*) By the formula, speed $= \frac{\text{Distance}}{\text{Time}}$

$$= \frac{(240 + 240)}{27} \times \frac{18}{5}$$

$$= \frac{480}{27} \times \frac{18}{5} = 64 \text{ km/h}$$

8. *{d)* Length of platform is not given. So, it cannot be determined.

9. (*c*) Speed of train = 60 km/h

$$= 60 \times \frac{5}{18} \text{ m/s} = \frac{50}{3} \text{ m/s}$$

Let length of the platform be x m.

According to the question,

$$200 + x = \frac{50}{3} \times 30$$

$$\Rightarrow \quad 200 + x = 500$$

$$\therefore \quad x = 300 \text{ m}$$

10. (*a*) Let the length of the train be x m.

According to the question,

$$\frac{x}{9} = \text{Speed} \quad \ldots(\text{i})$$

and $$\frac{x+150}{15} = \text{Speed} \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\frac{x}{9} = \frac{x+150}{15}$$

$$\Rightarrow \quad \frac{x}{3} = \frac{x+150}{5}$$

$$\Rightarrow \quad 5x = 3x + 450$$

$$\Rightarrow \quad x = 225 \text{ m}$$

11. (*c*) Let length of the train be L m.

According to the question,

$$\frac{L}{2} = \frac{L+250}{7}$$

$$\Rightarrow \quad 7L = 2L + 500$$

$$\Rightarrow \quad 7L - 2L = 500$$

$$\Rightarrow \quad 5L = 500$$

$$\therefore \quad L = \frac{500}{5} = 100 \text{ m}$$

12. (*e*) Let length of both train and platform be x.

Distance covered by the train to cross the platform = $x + x = 2x$

Time = 1 min = 60 s

and speed = 90 km/h

$$= 90 \times \frac{5}{18} = 25 \text{ m/s}$$

$\therefore$ Distance = Speed × Time

$$\Rightarrow \quad 2x = 25 \times 60$$

$$\Rightarrow \quad x = 750 \text{ m}$$

13. (a) Let length of the train = L

According to the question,

$$\frac{L+300}{21}=\frac{L+240}{18}$$

$$\Rightarrow \quad \frac{L+300}{7}=\frac{L+240}{6}$$

$$\Rightarrow \quad 6L+1800=7L+1680$$

$$\therefore \quad L=120\text{ m}$$

Taking the length of the 2nd bridge into consideration,

$$\text{Speed of train}=\frac{L+240}{18}=\frac{120+240}{18}\text{ m/s}$$

$$=\frac{360}{18}\times\frac{18}{5}\text{ km/h}$$

$$=72\text{ km/h}$$

(GO When two trains are moving in opposite directions, then their relative speed is equal to the sum of the speeds of both the trains. /. Required relative speed

= 6+ 12= 18m/s

15. (c) We know that, if two trains are running in the same direction, then relative speed is equal to the difference of the speeds of both the trains.

.'. Required relative speed =21-15=6 km/h

16. (c) Relative speed of train

= Speed of train + Speed of car => 90 = Speed of train +15

Speed of train = 90-15 = 75 km/h

17. (*a*) According to the formula,

Required time $= \frac{x + y}{u + v}$

Here, $x = 70$ m, $y = 90$ m, $u = 10$ m/s and $v = 6$ m/s

$\therefore$ Required time $= \frac{70 + 90}{10 + 6} = \frac{160}{16} = 10$ s

18. (*b*) Speed of the train relative to man

$= (240 + 24)$ km/h $= 264$ km/h

$= 264 \times \frac{5}{18}$ m/s $= \frac{220}{3}$ m/s

Distance covered in passing the man

$= 440$ m

$\therefore$ Time taken $= \frac{440}{220} \times 3 = 6$ s

19. (*a*) Relative speed of train

$= \frac{400}{36}$ m/s $= \frac{400}{36} \times \frac{18}{5} = 40$ km/h

Relative speed of train

= Speed of train + Speed of man

$\Rightarrow$ 40 = Speed of train + 20

$\therefore$ Speed of train $= 40 - 20$

$= 20$ km/h

20. (*c*) Length of the train = 280 m

Length of the platform = 280 × 3 = 840 m

Taken time = 6 min 40 s

$= (360 + 40)\text{ s} = 400\text{ s}$

$\therefore$ Required speed $= \dfrac{\text{Distance}}{\text{Time}}$

$= \dfrac{840 + 280}{400} = \dfrac{1120}{400} = 2.8\text{ m/s}$

21. (*a*) Relative speed = (84 + 60) km/h

$= 144\text{ km/h} = 144 \times \dfrac{5}{18}\text{ m/s}$

$= 8 \times 5 = 40\text{ m/s}$

Distance covered in passing each other

$= 512 + 528 = 1040\text{ m}$

$\therefore$ Required time $= \dfrac{1040}{40} = 26\text{ s}$

22. (*d*) According to the formula,

Required time $= \dfrac{x + y}{u - v}$

Here, $x = 50$ m, $y = 65$ m, $u = 18$ m/s and $v = 17$ m/s

$\therefore$ Required time $= \dfrac{50 + 65}{18 - 17} = \dfrac{115}{1} = 115\text{ s}$

23. (*b*) Total length of the train

$= 105 + 90 = 195\text{ m}$

Relative speed = 72 + 45 km/h

$= 117\text{ km/h} = \left(117 \times \dfrac{5}{18}\right) = \dfrac{585}{18}\text{ m/s}$

$\therefore$ Required time $= \left(195 \times \dfrac{18}{585}\right)\text{s} = 6\text{ s}$

24. (*d*) Let the speed of the train be x.

According to the question,

$(x + 10) \times 12 = (x + 20) \times 10$

$\Rightarrow \quad 6x + 60 = 5x + 100$

$\Rightarrow \quad x = 100 - 60 = 40\text{ m/s}$

$\therefore$ Length of the train $= (x + 10) \times 12$

$= (40 + 10) \times 12 = 600\text{ m}$

Fast Track Method

Here, $t_1 = 12$ s, $t_2 = 10$ s, $a = 10$ m/s and $b = 20$ m/s

According to the formula,

Length of the train

$= \dfrac{\text{Difference in speeds} \times t_1 \times t_2}{t_1 - t_2}$

[by Technique 4]

$= \dfrac{(20 - 10) \times 10 \times 12}{12 - 10}$

$= \dfrac{10 \times 10 \times 12}{2}$

$= \dfrac{1200}{2} = 600\text{ m}$

25. (*b*) Total distance $= x + y$

$$= 180 + 240 = 420 \text{ m}$$

Active speed $= u + v = (30 + 40) \times \frac{5}{18}$

$$= \frac{70 \times 5}{18} \text{ m/s}$$

$\therefore$ Required time $= \left(\frac{x + y}{u + v}\right) = \frac{420 \times 18}{70 \times 5}$

$$= 21.6 \text{ s}$$

26. (*c*) Required time

$$= \frac{\text{Distance covered by train}}{\text{Relative speed of train}}$$

$$= \frac{180 + 120}{(72 + 108) \times \frac{5}{18}} = \frac{300 \times 18}{180 \times 5} = 6 \text{ s}$$

27. (*b*) Speed $= 60 \text{ km/h} = 60 \times \frac{5}{18} \text{ m/s} = \frac{50}{3} \text{ m/s}$

Required time $= \frac{110 + 170}{\frac{50}{3}}$

$$= 280 \times \frac{3}{50} = 16.8 \text{ s}$$

28. (*b*) Let the length of each train be x m.

Relative speed $= 46 - 36 = 10$ km/h

$$= \frac{10 \times 5}{18} \text{ m/s} = \frac{25}{9} \text{ m/s}$$

$$\frac{\text{Sum of length of train}}{\text{Relative speed of train}} = \text{Time taken}$$

$\therefore \quad \frac{2x}{\frac{25}{9}} = 36 \Rightarrow 2x = \frac{36 \times 25}{9} = 100$

$\therefore \quad x = 50$ m

29. (*c*) Let length of each train be x m.

Then, $(65 + 85) \times \frac{5}{18} = \frac{x + x}{6}$

$\Rightarrow \quad \frac{150 \times 5 \times 6}{18} = 2x$

$\Rightarrow \quad 2x = 250$

$\therefore \quad x = 125$ m

30. (*a*) Given that, $T_1 = 9$ h and $T_2 = 4$ h

According to the formula,

(1st train's speed) : (2nd train's speed)

$$= \sqrt{4} : \sqrt{9} = 2 : 3$$

31. (*a*) Let speed of 1st train be 8 x and speed of 2nd train be 9 x.

According to the question,

Speed of second train

$$= \frac{360}{4} = 90 \text{ km/h}$$

$$9x = 90 \Rightarrow x = 10$$

So, speed of 1st train $= 8 \times 10 = 80$ km/h

$\therefore$ Required distance $= 80 \times 3 = 240$ km

32. (*c*) Average speed of train

$$= \frac{240}{12} = 20 \text{ km/h}$$

Average speed of bus

$$= \frac{3}{4} \times 20 = 3 \times 5 = 15 \text{ km/h}$$

Required distance $= 15 \times 7 = 105$ km

33. (*b*) Let the trains meet after time t at a distance x from station N, then another train coming from station M covers a distance of $(x + 50)$.

For station M, $(x + 50) = 125t$

$\Rightarrow \quad x = 125t - 50 \quad \text{...(i)}$

For station N, $x = 75t \quad \text{...(ii)}$

From Eqs. (i) and (ii), we get

$$75t = 125\,t - 50 \quad \Rightarrow \quad t = 1 \text{ h}$$

Distance between stations M and N

$$= 125 + 75\,t = 200 \times 1 = 200 \text{ km}$$

Fast Track Method

Here, $a = 125$ km/h, $b = 75$ km/h and $d = 50$ km

According to the formula,

Distance between the stations

M and $N = \left(\frac{a+b}{a-b}\right) \times d$ [by Technique 2]

$$= \left(\frac{125+75}{125-75}\right) \times 50$$

$$= \frac{200}{50} \times 50 = 200 \text{ km}$$

34. (*a*) Given, $t_1 = 16$ h, $t_2 = 9$ h and $a = 90$ km/h

According to the question,

$\therefore$ Speed of $B = a\sqrt{\frac{t_1}{t_2}}$ [by Technique 5]

$$= 90 \times \sqrt{\frac{16}{9}} = 90 \times \frac{4}{3}$$

$$= 30 \times 4 = 120 \text{ km/h}$$

35. (*c*) Because of stoppages, train covers 18 km less per hour.

$\therefore$ Time taken to cover 18 km

$$= \frac{18}{108} = \frac{1}{6} \text{ h} = \frac{1}{6} \times 60 = 10 \text{ min}$$

Fast Track Method

Here, $a = 108$ km/h and $b = 90$ km/h

According to the formula,

Required rest time $= \frac{a-b}{a}$

[by Technique 9]

$$= \frac{108-90}{108}$$

$$= \frac{18}{108} = \frac{1}{6} \text{ h} = \frac{1}{6} \times 60 \text{ min}$$

$$= 10 \text{ min}$$

36. (*a*) Because of stoppages, train covers 50 km less per hour.

$\therefore$ Time taken to cover 50 km

$$= \frac{50}{150} \text{ h} = \frac{50}{150} \times 60 = 20 \text{ min}$$

Fast Track Method

Here, $a = 150$ km/h and $b = 100$ km/h

According to the formula,

Required rest time $= \frac{a-b}{a}$ [by technique 9]

$$= \frac{150-100}{150}$$

$$= \frac{50}{150} = \frac{1}{3} \text{ h} = \frac{1}{3} \times 60 = 20 \text{ min}$$

37. (*a*) Let the speeds of two trains be x and y, respectively.

$\therefore$ Length of 1st train $= 54x$

Length of the 2nd train $= 34y$

According to the question,

$$\frac{54x + 34y}{x + y} = 46$$

$$\Rightarrow \quad 54x + 34y = 46x + 46y$$

$$\Rightarrow \quad 27x + 17y = 23x + 23y$$

$$\Rightarrow \quad 4x = 6y \Rightarrow 2x = 3y \Rightarrow \frac{x}{y} = \frac{3}{2}$$

$$\therefore \quad x : y = 3 : 2$$

38. (*c*) The length of the fast train

$=$ Relative speed $\times$ Time

$$= (40 - 20) \times \frac{5}{18} \times 5 = 27\frac{7}{9} \text{ m}$$

39. (*b*) Given that, $t_1 = 6$ s and $t_2 = 9$ s

Then, time taken by the trains to cross each other

$$= \frac{2t_1 t_2}{t_2 - t_1} \quad \text{[by Technique 10]}$$

$$= \frac{2 \times 6 \times 9}{9 - 6} = 36 \text{ s}$$

40. (6) First train leaves at 8:00 am and second at 9:00 am.

So, first train i.e., from *P* to *Q* has covered 25 km distance in 1 h. So, distance left between the station =145-25 = 120 km

Now, trains are travelling in opposite directions.

So, relative speed = 25 + 35 = 60 km/h

Time taken to cover 120 km $= \frac{120}{60} = 2$ h

$\therefore$ The time, at which both the trains will meet, is 2 h after second train left *i.e.*, 9:00 am + 2h = 11: 00 am

Fast Track Method

Here, $d = 145$ km, $t = 9:00 - 8:00 = 1$ h

$a = 25$ km/h and $b = 35$ km/h

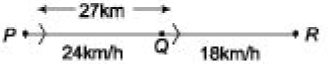
Time taken by trains to meet

$$T = \left(\frac{d + tb}{a + b}\right) \quad \text{[by Technique 6]}$$

$$= \frac{145 + 1 \times 35}{25 + 35} = \frac{180}{60} = 3 \text{ h}$$

$\therefore$ They will meet at = 8 : 00 am + 3 h
= 11 : 00 am

41. (*b*) If the trains meet after t h.



Relative speed of train = (24 − 18)
= 6 km/h

$\Rightarrow$ Distance = 27

$\therefore$ $t = \frac{27}{6} = \frac{9}{2}$ h

$\therefore$ QR distance travel by train which is travelling at a speed of 18 km/h

$$= 18t = 18 \times \frac{9}{2} = 81 \text{ km}$$

42. (*d*) Given, $a = 45$ km/h, $y = ?$, $t_1 = 4$ h 48 min and $t_2 = 3$ h 20 min

Using $y = a\sqrt{\frac{t_1}{t_2}} = 45\sqrt{\frac{4 \text{ h and } 48 \text{ min}}{3 \text{ h and } 20 \text{ min}}}$

$$= 45\sqrt{\frac{\frac{24}{5}\text{h}}{\frac{10}{3}\text{h}}} = 45\sqrt{\frac{24 \times 3}{5 \times 10}}$$

$$= 45 \times \sqrt{1.44} = 45 \times 1.2$$

$$= 54 \text{ km/h}$$

Exercise © *Higher Skill Level Questions*

1. (a) Let both the trains meet to each other x km from station A. Then,

$$\frac{x}{50} = \frac{100 - x}{75}$$

=> 3x = 200 - 2x

=> *5x* = 200

x = 40 km.

2. (c) Let the speed of both trains be x km/h and (x + 6) km/h, respectively. Then, according to the question,

160 = xX4 + (x+ 6) X4 => 160=4x + 4x+24

$\Rightarrow \quad 40 = x + x + 6$

$\Rightarrow \quad 2x + 6 = 40$

$\Rightarrow \quad 2x = 34$

$\therefore \quad x = 17$

Hence, speeds of both the trains are 17 km/h and (17 + 6) km/h *i.e.*, 17 km/h and 23 km/h.

Fast Track Method

Here, $d = 160$ km, $t = 4$ h and $x = 6$ km/h.

$\therefore$ Speed of faster train $= \left(\frac{d + tx}{2t}\right)$ km/h

[by Technique 7]

$$= \frac{160 + 24}{8} = 23 \text{ km/h}$$

Speed of slower train $= \left(\frac{d - tx}{2t}\right)$

$$= \frac{160 - 24}{8} = 17 \text{ km/h}$$

3. (*a*) Let the length of the first train be x m.

Then, the length of second train is $\left(\frac{x}{2}\right)$ m.

$\therefore$ Relative speed = (48 + 42) km/h

$$= \left(90 \times \frac{5}{18}\right) \text{ m/s} = 25 \text{ m/s}$$

According to the question,

$$\frac{\left(x + \frac{x}{2}\right)}{25} = 12$$

$\Rightarrow \quad \frac{3x}{2} = 300 \quad \Rightarrow \quad x = 200$ m

$\therefore$ Length of first train = 200 m

Let the length of platform be y m.

Speed of the first train

$$= \left(48 \times \frac{5}{18}\right) \text{ m/s} = \frac{40}{3} \text{ m/s}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$\therefore \quad (200 + y) \times \frac{3}{40} = 45$

$\Rightarrow \quad 600 + 3y = 1800$

$\therefore \quad y = 400$ m

4. (*a*) Let the two trains meet at x km from Mumbai. Then,

$$3{:}45 \text{ pm} + \frac{x}{50} = 2{:}35 \text{ pm} + \frac{510 - x}{60}$$

$\Rightarrow (3{:}45 \text{ pm} - 2{:}35 \text{ pm}) + \left(\frac{x}{50} + \frac{x}{60}\right) = \frac{510}{60}$

$\Rightarrow \quad \left(1\frac{1}{6} \text{ h}\right) + x\left(\frac{50 + 60}{50 \times 60}\right) = \frac{17}{2}$

$\Rightarrow \quad x\left(\frac{110}{3000}\right) = \frac{17}{2} - \frac{7}{6}$

$\Rightarrow \quad x\left(\frac{110}{3000}\right) = \frac{51-7}{6} = \frac{44}{6}$

$\therefore \quad x = \frac{44}{6} \times \frac{3000}{110} = 200$ km

Alternate Method

The 2nd train leaving Goa starts its journey earlier and it travels = 60 × (3:45 pm − 2:35) $= 60 \times 1\frac{1}{6}$ h = 70 km, when the 1st train (that leaves Mumbai) starts its journey.

Now, both the trains cover (510 − 70) km *i.e.*, 440 km with relative speed (50 + 60) km/h = 110 km/h.

Therefore, the trains meet after $\frac{440}{110} = 4$ h after the 1st train starts at 3:45 pm.

Now, the 1st train covers 4 × 50 km = 200 km to meet the 2nd train.

5. (*a*) Let the distance between Mokama and Hazipur be x km.

Time taken by train P to cover x km = 1 h

Time taken by Q to cover x km = $\frac{3}{2}$ h

$\therefore$ Speed of train P = x km/h

Speed of train $Q = \left(\frac{2x}{3}\right)$ km/h.

Let they cross each other y h after 5:00 pm.

Then, $xy + \frac{2xy}{3} = x$

$\Rightarrow \quad y + \frac{2y}{3} = 1 \Rightarrow y\left(1 + \frac{2}{3}\right) = 1$

$\Rightarrow \quad y = \frac{3}{5}\text{ h} = \frac{3}{5} \times 60 = 3 \times 12 = 36$ min

Hence, the two trains meet 36 min after 5:00 pm.

$\therefore$ Meeting time = 5:36 pm

6. (*a*) Let the lengths of the trains A and B be a and $2a$, respectively.

When a train crosses a pole, it covers the distance equal to its length.

$\therefore$ Required ratio of speeds $= \frac{a}{25} : \frac{2a}{75} = 3:2$

7. (*a*) The distance covered by the train to cross the platform and bridge

= Length of the platform + Length of bridge + Distance between platform and bridge + Length of the train

= 165 + 135 + 30 + 110 = 440 m

Speed of train $= \frac{110}{3}$ m/s

$\therefore$ Required time taken $= \frac{440}{110/3} = 12$ s

8. (*b*) Let time taken to cover the distance = t

$\therefore$ Speed = $(2t + 1)$

$\Rightarrow$ $t(2t + 1) = 1830 \Rightarrow 2t^2 + t = 1830$

$\Rightarrow$ $2t^2 + t - 1830 = 0$

$$\therefore \quad t = \frac{-1 \pm \sqrt{(1)^2 - 4 \times 2 \times (-1830)}}{2 \times 2}$$

$$= \frac{-1 \pm \sqrt{1 + 14640}}{4} = \frac{-1 \pm 121}{4}$$

Taking '+' sign, $t = \frac{-1 + 121}{4} = \frac{120}{4} = 30$

$\therefore$ Required ratio $= (2 \times 30 + 1) : 30 = 61 : 30$

9. (*b*) Let speed in the return journey = x

$\therefore$ Speed in onward journey

$$= \frac{125}{100} x = \left(\frac{5}{4} x\right) \text{km/h}$$

Average speed $= \dfrac{2 \times \frac{5}{4} x \times x}{\frac{5}{4} x + x} = \dfrac{10x}{9}$ km/h

$\therefore$ $1600 \times \frac{9}{10x} = 30$ [$\because$ Train halts for 2 h]

$$\therefore \quad x = \frac{1600 \times 9}{30 \times 10} = 48 \text{ km/h}$$

$\therefore$ Speed in onward journey $= \frac{5}{4} x = \frac{5}{4} \times 48$

$= 60$ km/h

10. (*d*) Speed of Ist person

$$= 4.5 \text{ km/h} = \left(4.5 \times \frac{5}{18}\right) \text{m/s}$$

$$= \frac{5}{4} \text{ m/s} = 1.25 \text{ m/s}$$

= Speed of IInd person = 5.4 km/h

$$= \left(5.4 \times \frac{5}{18}\right) \text{m/s} = \frac{3}{2} \text{ m/s} = 1.5 \text{ m/s}$$

Let the speed of the train be x m/s.

Then, $(x - 1.25) \times 8.4 = (x - 1.5) \times 8.5$

$\Rightarrow$ $8.4x - 10.5 = 8.5x - 12.75$

$\Rightarrow$ $0.1x = 2.25 \Rightarrow x = 22.5$

$\therefore$ Speed of the train $= \left(22.5 \times \frac{18}{5}\right) = 81$ km/h.

11. (*b*) Speed of the person = 4 km/h

$$= 4 \times \frac{5}{18} \text{ m/s} = \frac{10}{9} \text{ m/s}$$

Let speed of train be x m/s.

Then, relative speed of train $= \left(x - \frac{10}{9}\right)$ m/s

As, train takes 25 s to cross the person

$$\therefore \quad 25 = \frac{125}{\left(x - \frac{10}{9}\right)} \Rightarrow x - \frac{10}{9} = 5$$

$\therefore \qquad x = \frac{55}{9}$ m/s

Now, distance covered by the train in 30 min

$$= \frac{55}{9} \times 30 \times 60 = 11000 \text{ m} = 11 \text{ km}$$

Thus, time taken by the person to cover the distance of 11 km $= \frac{11}{4}$ h $= 2\frac{3}{4}$ h.

12. (*b*) Let the distance covered be x km and speed of train be y km/h.

Now, according to the question,

$$\frac{x}{y} - \frac{x}{y+10} = 2 \qquad \ldots(\text{i})$$

$$\frac{x}{y-10} - \frac{x}{y} = 3 \qquad \ldots(\text{ii})$$

On adding Eqs. (i) and (ii), we get

$$\frac{x}{y} - \frac{x}{y+10} = 2$$

$$\frac{x}{y-10} - \frac{x}{y} = 3$$

$$\overline{\frac{x}{y-10} - \frac{x}{y+10} = 5}$$

$$\Rightarrow \quad x\left(\frac{y+10-y+10}{(y-10)(y+10)}\right) = 5$$

$$\Rightarrow \quad x\left(\frac{20}{y^2-100}\right) = 5$$

$$\therefore \ x = 5\left(\frac{y^2-100}{20}\right) = \frac{y^2-100}{4} \qquad \ldots(\text{iii})$$

On substituting the value of x from Eq. (iii) in Eq. (ii), we get

$$\frac{y^2-100}{4(y-10)} - \frac{y^2-100}{4(y)} = 3$$

$$\Rightarrow \quad \frac{y+10}{4} - \frac{y^2-100}{4y} = 3$$

$$\Rightarrow \quad \frac{y(y+10)-(y^2-100)}{4y} = 3$$

$$\Rightarrow \quad \frac{y^2+10y-y^2+100}{4y} = 3$$

$$\Rightarrow \quad 10y - 12y = -100$$

$$\Rightarrow \quad -2y = -100$$

$$\therefore \quad y = 50$$

On putting the value of y in Eq. (iii), we get

$$x = \frac{(50^2)-100}{4} = \frac{2500-100}{4}$$

$$\therefore \quad x = \frac{2400}{4} = 600$$

Hence, distance covered by train is 600 km.

Chapter

27

Boats and Streams

Boats and streams is an application of concepts of speed, time and distance. Speed of river flowing either aides a swimmer (boat), while travelling with the direction of river or it opposes when travelling against the direction of river.

Here, we shall explain different concepts like

Still Water If the speed of water of a river is zero, then water

is considered to be still water.

Stream Water If the water of a river is moving at a certain speed, then it is called as stream water.

Speed of Boat Speed of boat means speed of boat (swimmer) in still water. In other words, if the speed of a boat (swimmer) is given, then that particular speed is the speed in still water.

Downstream Motion If the motion of a boat (swimmer) is along the direction of stream, then such motion is called downstream motion,

Upstream Motion If the motion of a boat (swimmer) is against the direction of stream, then such motion is called upstream motion.

Basic Formulae Related to Boats and Streams

If the speed of a boat in still water is x km/h and speed of the stream is y km/h, then

1. Speed downstream = $(x + y)$ km/h
2. Speed upstream = $(x - y)$ km/h
3. Speed of a boat in still water $(x) = \frac{1}{2}$ (Speed downstream + Speed upstream)
4. Speed of stream $(y) = \frac{1}{2}$ (Speed downstream − Speed upstream)

Ex. 1 A man can row with a speed of 6 km/h in still water. What will be his speed with the stream, if the speed of stream is 2 km/h?

Sol. Given, speed of man in still water = x = 6 km/h and speed of stream = y = 2 km/h

.'. Speed downstream = $x + y = 6 + 2 = 8$ km/h

Ex. 2 If the speed of a boat in still water is 8 km/h and the rate of stream is 4 km/h, then find upstream speed of the boat.

Sol. Given, speed of a boat = x - 8 km/h Speed of stream = y = 4 km/h .•. Speed upstream = x - y= 8 - 4 = 4 km/h

Ex. 3 Shantanu can row upstream at 10 km/h and downstream at 18 km/h. Find the man's rate in still water and the rate of the current. **Sol.** Speed upstream = 10 km/h and speed downstream = 18 km/h According to the formula,

$$\text{Man's rate in still water} = \frac{1}{2}(\text{Speed downstream} + \text{Speed upstream})$$
$$= \frac{1}{2}(18 + 10) = \frac{28}{2}$$
$$= 14 \text{ km/h}$$
$$\text{Speed of current} = \frac{1}{2}(\text{Speed downstream} - \text{Speed upstream})$$
$$= \frac{1}{2}(18 - 10) = \frac{8}{2}$$
$$= 4 \text{ km/h}$$

Ex. 4 What time will be taken by a boat to cover a distance of 64 km along the stream, if speed of boat in still water is 12 km/h and speed of stream is 4 km/h?

Sol. Given that, distance = 64 km, speed of boat in still water = x = 12 km/h and speed of stream = y = 4 km/h

$\therefore$ Downstream speed of boat = $x + y = 12 + 4 = 16$ km/h

$$\therefore \text{ Required time} = \frac{\text{Distance}}{\text{Speed (downstream)}} = \frac{64}{16} = 4\text{ h}$$

Ex. 5 A boat takes 8 h to row 48 km downstream and 12 h to row the same distance upstream. Find the boat's rate in still water and rate of current.

Sol. Speed downstream $= \frac{\text{Distance}}{\text{Time}} = \frac{48}{8} = 6$ km/h

Speed upstream $= \frac{48}{12} = 4$ km/h

Now, rate of boat in still water $= \frac{(\text{Speed downstream} + \text{Speed upstream})}{2}$

$$= \frac{6+4}{2} = 5\text{ km/h}$$

and rate of current $= \frac{(\text{Speed downstream} - \text{Speed upstream})}{2}$

$$= \frac{6-4}{2} = 1\text{ km/h}$$

Fast Track **Techniques**

to solve the **QUESTIONS**

Technique 1

if speed of stream is *a* and a boat (swimmer) takes *n* times as long to row up as to row down the river, then

$$\text{Speed of boat (swimmer) in still water} = \frac{a(n+1)}{(n-1)}$$

Note This formula is applicable for equal distances

Ex. 6 Rajnish can row 12 km/h in still water. It takes him twice as long to row up as to row down the river. Find the rate of stream. Sol. Let rate of stream = a km/h and distance travelled = j/km Given, rate of Rajnish in still water = 12 km/h Rate of downstream = (12 + a) km/h Rate of upstream = (12 - a) km/h According to the question, Time taken to travel downstream = 2 x Time taken to travel upstream

$$\Rightarrow \quad \frac{12+a}{y} = \frac{2(12-a)}{y}$$

$$\Rightarrow \quad 12 + a = 2(12 - a)$$

$$\Rightarrow \quad 12 + a = 24 - 2a$$

$$\Rightarrow \quad a + 2a = 24 - 12$$

$$\Rightarrow \quad 3a = 12$$

$$\Rightarrow \quad a = \frac{12}{3} = 4 \text{ km/h}$$

Fast Track Method

Here, speed of Rajnish in still water = 12 km/h

$n = 2$; Speed of stream $(a) = ?$

According to the formula,

$$\text{Speed in still water} = \frac{a(n+1)}{(n-1)}$$

$$\Rightarrow \quad 12 = \frac{a(2+1)}{(2-1)}$$

$$\Rightarrow \quad 3a = 12$$

$$\therefore \quad a = \frac{12}{3} = 4 \text{ km/h}$$

Technique 2

A person can row at a speed of *x* in still water. If stream is flowing at a speed of y, it takes time *T* to row to a place and back, then

$$\text{Distance between two places} = \frac{T(x^2 - y^2)}{2x}$$

Ex. 7 A man can row 12 km/h in still water. When the river is running at 2.4 km/h, it takes him 1 h to row to a place and to come back. How far is the place?

Sol. Man's rate downstream = 12 + 2.4 = 14.4 km/h

Man's rate upstream = 12 − 2.4 = 9.6 km/h

Let the required distance be x km.

According to the question,

Total time taken to travel x km upstream and downstream = 1 h

$$\Rightarrow \quad \frac{x}{14.4} + \frac{x}{9.6} = 1$$

$$9.6x + 14.4x = 14.4 \times 9.6$$

$$\Rightarrow \quad 24x = 138.24$$

$$\therefore \quad x = \frac{138.24}{24} = 5.76 \text{ km}$$

Fast Track Method Here, speed of man in still water = x = 12 km/h

Speed of river = y = 2.4 km/h; T = 1 h

According to the formula,

$$\text{Required distance} = \frac{T(x^2 - y^2)}{2x} = \frac{1 \times [(12)^2 - (2.4)^2]}{2 \times 12}$$

$$= \frac{138.24}{24} = 5.76 \text{ km}$$

Technique 3

A man rows a certain distance downstream in x h and returns the same distance in y h. When the stream flows at the rate of a km/h. then

$$\text{Speed of the man in still water} = \frac{a(x+y)}{(y-x)}$$

Ex. 8 Kamal can row a certain distance downstream in 12 h and can return the same distance in 18 h. If the stream flows at the rate of 6 km/h, then find the speed of Kamal in still water. Sol. Let the speed of Kamal in still water = x

Then, Kamal's speed downstream = $(x + 6)$ Kamal's speed upstream = $(x - 6)$ According to the question,

Distance travelled downstream = Distance traveled upstream => $12(x + 6) = 18(x - 6)$

=> $2x + 12 = 3x - 18$

=> $3x - 2x = 18 + 12$

$x = 30$ km/h

Fast Track Method Here, x = 12 h; y = 18 h Rate of stream (a) = 6 km/h According to the formula,

$$\text{Speed of Kamal in still water} = \frac{a(x+y)}{(y-x)}$$

$$= \frac{6 \times (12 + 18)}{(18 - 12)} = \frac{6 \times 30}{6} = 30 \text{ km/h}$$

Note If in case of technique 3, man's speed in still water is *b* km/h and we are asked to find the speed of stream, then technique 3 takes the form as

$$\text{Speed of stream} = \frac{b(y-x)}{(x+y)}$$

Ex. 9 If in the above example, the speed of Kamal in still water is 12 km/h, then

find the speed of the stream.

Sol. Let the speed of stream = *x* km/h

Then, Kamal's speed downstream = (12 + x)km/h

Kamal's speed upstream = (12 - x)km/h

According to the question,

Distance travelled downstream = Distance travelled upstream

=> 12 (12 + *x)* = 18 (12 - *x)*

=> 2(12 + *x)* = 3(12 -*x)*

=> 24 + *2x* = 36 - 3x

=> 3x + *2x* = 36 - 24

=> 5z = 12

x = — = 2.4 km/h 5

Fast Track Method Here, $b = 12$, $y = 18$ and $x = 12$

According to the formula,

Speed of stream $= \dfrac{b(y-x)}{(x+y)} = \dfrac{12(18-12)}{18+12} = \dfrac{12 \times 6}{30} = \dfrac{12}{5} = 2.4$ km/h

Technique 4

If boat's (swimmer's) speed in still water is a km/h and river is flowing with a speed of *b* km/h, then average speed in going to a certain place

and coming back to starting point is given by $\dfrac{(a+b)(a-b)}{a}$ km/h.

Ex. 10 Ramesh rows in still water with a speed of 4.5 km/h to go to a certain place and to come back. Find his average speed for the whole journey, if the river is flowinc with a speed of 1.5 km/h.

Sol. Ramesh's spood upstream $= 4.5 - 1.5 = 3$ km/h

Ramesh's speed downstream $= 4.5 + 1.5 = 6$ km/h

Let the distance in one direction be x.

Then, time taken in upstream $= \dfrac{x}{3}$ and time taken in downstream $= \dfrac{x}{6}$

$\therefore$ Average speed $= \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{2x}{\dfrac{x}{3} + \dfrac{x}{6}} = \dfrac{2x \times 18}{6x + 3x} = 4$ km/h

Fast Track Method Here, $a = 4.5$ km/h, $b = 1.5$ km/h

Average speed $= \dfrac{(a+b)(a-b)}{a}$

$= \dfrac{(4.5+1.5)(4.5-1.5)}{4.5} = \dfrac{6 \times 3}{4.5} = \dfrac{18}{4.5} = 4$ km/h

Technique 5

When boat's speed (swimmer's speed) in still water is *a* km/h and river is flowing with a speed of *b* km/h and time taken to cover a certain distance upstream is *T* more than the time taken to cover the same

distance downstream, then Distance $= \dfrac{(a^2 - b^2)T}{2b}$

Ex. 11 A boat's speed in still water is 10 km/h, while river is flowing with a speed of 2 km/h and time taken to cover a certain distance upstream is 4 h more than time taken to rover the same distance downstream. Find the distance.

Sol. Let distance be x.

Then, boat's rate downstream = 10 + 2 = 12 km/h

and boat's rate upstream = 10 − 2 = 8 km/h

According to the question,

Difference between the time = Time taken by boat to travel upstream − Time taken by boat to travel downstream

$$\Rightarrow \quad \frac{x}{8} - \frac{x}{12} = 4 \Rightarrow \frac{3x - 2x}{24} = 4$$

$$\therefore \quad x = 96 \text{ km}$$

Fast Track Method Here, $a = 10$ km/h, $b = 2$ km/h and $T = 4$ h

According to the formula,

$$\therefore \text{Required distance} = \frac{(a^2 - b^2)}{2b} \times T = \frac{(10^2 - 2^2)}{2 \times 2} \times 4$$

$$= \frac{100 - 4}{4} \times 4 = 100 - 4 = 96 \text{ km}$$

Technique 6

If a man covers / km distance in ^ h along the direction of river and he covers same distance in t_9 h against the direction of river, then

$$\text{Speed of man} = \frac{1}{2}\left(\frac{l}{t_1} + \frac{l}{t_2}\right) = \frac{l}{2}\left(\frac{1}{t_1} + \frac{1}{t_2}\right)$$

$$\text{Speed of the stream} = \frac{1}{2}\left(\frac{l}{t_1} - \frac{l}{t_2}\right) = \frac{l}{2}\left(\frac{1}{t_1} - \frac{1}{t_2}\right)$$

Ex. 12 A boat covers 20 km in an hour with downstream and covers the same distance in 2 h with upstream. Then, find the speed of boat in still water and speed of stream.

Sol. Here, $l = 20$ km, $t_1 = 1$ h, $t_2 = 2$ h

$$\text{Speed of boat in still water} = \frac{l}{2}\left(\frac{1}{t_1} + \frac{1}{t_2}\right) = \frac{20}{2}\left(\frac{1}{1} + \frac{1}{2}\right) = 10 \times \frac{3}{2} = 15 \text{ km/h}$$

$$\text{Speed of stream} = \frac{l}{2}\left(\frac{1}{t_1} - \frac{1}{t_2}\right) = \frac{20}{2}\left(\frac{1}{1} - \frac{1}{2}\right) = 10 \times \frac{1}{2} = 5 \text{ km/h}$$

Fast Track Practice

Exercise© *Base Level Questions*

1. When the speed of a boat in still water is 4 km/h and the rate of stream is 2 km/h, find upstream speed of the boat.

(a) 6 km/h *(b)* 5 km/h

(c) 2 km/h (tfj 7 km/h

fej None of the above

2. If the speed of a swimmer in still water is 9 km/h. Find the downstream speed of the swimmer, when the river is flowing with the speed of 6 km/h. [Bank Clerks 2010]

(a) 15 km/h *(b)* 18 km/h

(c) 3 km/h fd) 12 km/h

(ej None of the above

3. Ravi can row downstream at 9 km/h and upstream at 5 km/h. Find the speed of Ravi in still water and speed of current.

(a) 7 km/h and 2 km/h

(b) 8 km/h and 6 km/h

(c) 7 km/h and 3 km/h (tfj 8 km/h and 2 km/h *(e)* None of the above

4. A swimmer's speed downstream is 11 km/h and speed of the stream is 1.5 km/h. Find the upstream speed of swimmer. [RRB 2008]

(a) 8 km/h *(b)* 9.5 km/h

(c) 9 km/h fd) 6.25 km/h

5. A boatman rows 1 km in 5 min along the stream and 6 km in 1 h against the stream. The speed of the stream is

[SSCCGL2010]

(a) 3 km/h *(b)* 6 km/h

(c) 10 km/h fd) 12 km/h

6. Keshav can row 60 km downstream and 36 km upstream, taking 10 h each time. What is the velocity of the current?

(a) 3 km/h *(b)* 2.2 km/h

(c) 4 km/h fd) 1.2 km/h

(ej None of the above

7. What time will be taken by a boat to cover a distance of 128 km along the stream, if speed of a boat in still water is 24 km/h and speed of stream is 8 km/h?

(a) 8 km/h *(b)* 4 km/h

(c) 7 km/h fay 6 km/h

fej None of the above

8. A motorboat can travel at 10 km/h in still water. It travelled 91km downstream in a river and then returned to the same place, taking altogether 20 h. The rate of flow of river is

[SSCCGL2011]

(a) 3 km/h *(b)* 4 km/h

(c) 2 km/h *(d)* 5 km/h

9. A man can row against the current three-fourth of a kilometre in 15 min and returns same distance in 10 min, then ratio of his speed to that of current is

[SSC CGL 2010] *(a)* 3 : 5 *(b)* 5 : 3 *(c)* 1: 5 *(d)* 5 :1

10. The speed of the current is 5 km/h. A motorboat goes 10 km upstream and back again to the starting point in 50 min. The speed, (in km/h) of the motorboat in still water is [SSC CPO 2011]

(a) 20 km/h *(b)* 26 km/h

fc) 25 km/h fd) 28 km/h

11. A man can row 6 km/h in still water. If the speed of the current is 2 km/h, it takes 3 h more in upstream than in the downstream for the same distance. The distance is [SSC CCL 2012]

(a) 30 kmfbj 24 km(c) 20 km fdj 32 km

12. A boat goes 48 km downstream in 20 h. It takes 4 h more to cover the same distance against the stream. What is the speed of the boat in still water?

[SSC CPO 2013]

(a) 2.2 km/h *(b)* 2 km/h

(c) 4 km/h fd) 4.2 km/h

13. Pa wan can row 24 km/h in still water. When the river is running at 4.8 km/h, it takes him 1 h to row to a place and to come back. How far is the place?

(a) 11.52 km *(b)* 14 km

(c) 12.52 km fd) 15 km

(e) None of the above

14. Sameer can row a certain distance downstream in 24 h and can come back covering the same distance in 36 h. If the stream flows at the rate of 12 km/h, find the speed of Sameer in still water.

[Hotel Mgmt. 2009]

(a) 30km/h *(b)* 15 km/h

(c) 40 km/h *(d)* 60 km/h

(e) None of the above

15. A sailor sails a distance of 48 km along the flow of a river in 8 h. If it takes 12 h to return the same distance, then the speed of the flow of the river is

[CDS 2013]

(a) 0.5 km/h *(b)* 1 km/h

(c) 1.5 km/h *(d)* 2 km/h

16. Speed of motorboat in still water is 45 km/h. If the motorboat travels 80 km along the stream in 1 h 20 min, then the time taken by it to cover the same distance against the stream will be

[SSC CPO 2008]

(a) 4 h 20 min *(b)* 3 h 40 min

(c) 2 h 40 min *(d)* 2 h 55 min

17. A boat has to travel upstream 20 km distance from point *X* of a river to point *Y*. The total time taken by boat in travelling from point *X* to *Y* and *Y* to *X* is 41 min 40 s. What is the speed of the boat?

(a) 66 km/h *(b)* 72 km/h

(c) 48 km/h

(d) Cannot be determined

A boat covers a distance of 30 km downstream in 2 h while it takes 6 h to cover the same distance upstream. What is the speed of the boat (in km/h)?

[SNAP 2012]

(a) 5 *(b)* 7.5 *(c)* 13 *(d)* 10

(e) None of the above

19. A boat's speed in still water is 5 km/h. While river is flowing with a speed of 2 km/h and time taken to cover a certain distance upstream is 2 h more than time taken to cover the same distance downstream. Find the distance.

[Bank Clerks 2007]

(a) 10.5 km *(b)* 11 km

(c) 10.9 km *(d)* 15 km

(e) None of the above

20. A boat takes 9 h to travel a distance upstream and takes 3 h to travel the same distance downstream. If the speed of the boat in still water is 4 km/h, then what is the velocity of the stream?

[SSC CPO 2013]

(a) 4 km/h *(b)* 3 km/h

(c) 6 km/h *(d)* 2 km/h

21. A boat running upstream covers a distance of 10 km in 30 min and while running downstream, it covers the same distance in 25 min. What is the speed of the river current (in km/h)? [Bank Clerks 2009]

(a) 20 *(b)* 2.2 *(c)* 2

(d) Couldn't be determined

(e) None of the above

22. A man can row at 10 km/h in still water. If he takes total 5 h to go to a place 24 km away and return, then the speed of the water current is

[SSC CGL (Main) 2012]

(a) 2 km/h *(b)* 3 km/h

(c) $\frac{1}{2}$ km/h *(d)* 1 km/h

23. A steamer goes downstream from one port to another in 4 h. It covers the same distance upstream in 5 h. If the speed of the stream is 2 km/h, then find the distance between the two ports.

[SSC CGL 2007] *(a)* 50 km *(b)* 60 km

(c) 70 km *(d)* 80 km

24. A man can row 754 km/h in still water. If in a river running at 1.5 km/h, it takes him 50 min to row to a place and back, how far off is the place?

(a) 3 km *(b)* 4 km *(c)* 1 km *(d)* 2 km

25. Ashutosh can row 24 km/h in still water. It takes him twice as long to row up as to row down the river. Find the rate of stream.

(a) 4 km/h *(b)* 18 km/h

(c) 8 km/h *(d)* 15 km/h

(e) None of the above

26. A boat goes 12 km in 1 h in still water. It takes thrice time in covering the same distance against the current. Find the speed of the current.

(a) 8 km/h *(b)* 12 km/h

(c) 6 km/h *(d)* 7 km/h

(e) None of the above

27. A boatman takes twice as long to row a distance against the stream as to row the same distance with the stream. Find the ratio of speeds of the boat in still water and the stream.

(a) 2 : 1 *(b)* 3 : 1

(c) 1 : 2 *(d)* 1 : 3

(e) None of the above

Exercise © *Higher SkiU Level Questions*

1. A boat running upstream takes 528 min to cover a certain distance, while it takes 240 min to cover the same distance running downstream. What is the ratio between the speed of the boat and speed of the water current, respectively?

(a) 2 : 1 *(b)* 3 : 2

(c) 8 : 3

(d) Couldn't be determined

(e) None of the above

2. A river is flowing with a steady speed of

4 km/h. One rows his boat downstream in the river and then returns by rowing upstream in the same river. When he returns to the starting point, the total distance covered by him is 42 km. If the return journey takes 2 h more than his outward journey, then the speed of his rowing in still water must be

(a) 12 km/h *(b)* 10 km/h

(c) 9 km/h *(d)* 8 km/h

3. The ratio of speeds of a motorboat to that of the current of water is 36 : 5. The motorboat goes along with the current in

5 h 10 min. Find the time to come back of motorboat. [SSC (10+2) 2007]

(a) 5 h 50 min *(b)* 6 h

(c) 6 h 50 min *(d)* 12 h 10 min

4. Ishwar is rowing a boat. He takes half time in moving a certain distance downstream than upstream. What is the ratio of the rate of boat in still water to the rate of current? [Hotel Mgmt. 2007]

(a) 2 : 1 *(b)* 5 : 1 *(c)* 7 : 1 *(d)* 3 : 1 *(e)* None of the above

5. In a river, the ratio of the speed of stream and speed of a boat in still water is 2:5. Again, ratio of the speed of stream and speed of an another boat in still water is 3: 4 What is the ratio of the speeds of the first boat to the second boat in still water?

(a) 10:7 *(b)* 15:8 *(c)* 4:3 *(d)* 5: 4 *(e)* None of the above

6. A motorboat travelling at the some speed, can cover 25 km upstream and 39 km downstream in 8 h. At the same speed, it can travel 35 km upstream and 52 km downstream in 11 h. The speed of the stream is [SSC CCL 2011]

(a) 2 km/h (b) 3 km/h

(c) 4 km/h (d) 5 km/h

7. A boat covers 24 km upstream and 36 km downstream in 6 h, while it covers 36 km upstream and 24 km downstream in $6\frac{1}{2}$ h. The speed of the current is [SSC CPO 2010]

(a) 1 km/h (b) 2 km/h

(c) 1.5 km/h (d) 2.5 km/h

8. *A, B* and C are situated at the bank of river which is flowing at a constant rate. *B* is at an equal distance with *A* and *C.* A swimmer Avinash takes 10 h to swim from *A* to *B* and *B* to *A*. Also, he takes 4 h to swim from *A* to C. What is the ratio of speed of Avinash in still water and speed of stream?

(a) 5 : 3 (b) 3 : 5 (c) 2 : 5 (d) 1:2 (e) None of the above

9. A river is flowing at a speed of 5 km/h in a particular direction. A man, who can swim at a speed of 20 km/h in still water, starts swimming along the direction of flow of the river from point *A* and reaches another point *B* which is at a distance of 30 km from the starting point *A*. On reaching point *B,* the man turns back and starts swimming against the direction of flow of the river and stops after reaching point *A*. The total time taken by the man to complete his journey is

(a) 2 h 30 min (b) 3 h 12 min

(c) 3 h 30 min (d) 3 h 45 min

Answer with **Solutions**

Exercise© *Base Level Questions*

1. (c) Given,

Boat's speed in still water x = 4 km/h Rate of stream y = Z km/h .'. Upstream speed of boat = x - y = 4-2 = 2 km/h

2. (a) Given,

Swimmer's speed in still water

$= x = 9$ km/h Rate of stream = 7 = 6 km/h .'. Speed downstream $= x + y = 9 + 6 = 15$ km/h

3. (a) Given, downstream speed = 9 km/h
and upstream speed = 5 km/h
According to the formula,
Ravi's speed in still water

$$= \frac{1}{2}(\text{Speed downstream} + \text{Speed upstream})$$
$$= \frac{1}{2}(9+5) = \frac{14}{2}$$
$$= 7 \text{ km/h}$$

and speed of current

$$= \frac{1}{2}(\text{Speed downstream} - \text{Speed upstream})$$
$$= \frac{1}{2}(9-5) = \frac{4}{2} = 2 \text{ km/h}$$

4. (a) Given, rate of stream = 1.5 km/h
We know that,

$$\text{Rate of stream} = \frac{1}{2}(\text{Speed downstream} - \text{Speed upstream})$$
$$\Rightarrow \quad 1.5 = \frac{1}{2}(11 - \text{Speed upstream})$$
$$\Rightarrow \quad 11 - \text{Speed upstream} = 3$$
$$\therefore \text{Speed upstream} = 11 - 3 = 8 \text{ km/h}$$

5. (a) Let the speed of boat and stream be x and y km/h.
$\therefore$ Speed of boat along stream
$= (x + y)$ km/h
and speed of boat against stream
$= (x - y)$ km/h
According to the question,

$$x + y = \frac{1}{5/60} = \frac{60}{5}$$
$$\Rightarrow \quad x + y = 12 \quad \text{... (i)}$$
$$\text{and} \quad x - y = 6 \quad \text{... (ii)}$$

On adding Eqs. (i) and (ii), we get

$$2x = 18$$

$$x = \frac{18}{2} = 9$$

On putting the value of x in Eq. (i), we get

$$9 + y = 12$$

$$y = 12 - 9 = 3$$

Hence, speed of boat = 9 km/h

and speed of stream = 3 km/h

6. (*d*) Speed downstream $= \frac{\text{Distance}}{\text{Time}}$

$$= \frac{60}{10} = 6 \text{ km/h}$$

Speed upstream $= \frac{\text{Distance}}{\text{Time}} = \frac{36}{10}$

$= 3.6$ km/h

$\therefore$ Velocity of the current

$= \frac{1}{2}$ (Speed downstream – Speed upstream)

$= \frac{1}{2}(6 - 3.6) = \frac{2.4}{2} = 1.2$ km/h

7. (*b*) Given,

Speed of boat in still water = x = 24 km/h

Speed of stream = y = 8 km/h

$\therefore$ Downstream speed of boat

$= x + y = 24 + 8 = 32$ km/h

$\therefore$ Required time $= \frac{\text{Distance}}{\text{Speed (downstream)}}$

$$= \frac{128}{32} = 4 \text{ km/h}$$

8. (*a*) Given, speed of boat = 10 km/h

Let speed of flow of river = x km/h

$\therefore$ Upstream speed of boat = $(10 - x)$ km/h

and downstream speed of boat

According to question,

$= (10 + x)$ km/h

$$\frac{91}{10 - x} + \frac{91}{10 + x} = 20$$

$$\Rightarrow \quad \frac{91(10 + x + 10 - x)}{(10 - x)(10 + x)} = 20$$

$$\Rightarrow \quad \frac{91(20)}{100 - x^2} = 20$$

$$\Rightarrow \quad 91 = 100 - x^2 \Rightarrow x^2 = 9$$

$$\therefore \quad x = 3$$

9. (d) Let the speed of man and current be x and y km/h, respectively. Speed upstream = $(x - y)$ km/h Speed down stream = $(x + y)$ km/h

According to the question,

$$\frac{3 \times 60}{4 \times 15} = x - y$$

$$\Rightarrow \quad x - y = 3 \quad \text{... (i)}$$

and $$\frac{3}{4} \times \frac{60}{10} = x + y$$

$$\Rightarrow \quad x + y = \frac{9}{2} \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2x = 3 + \frac{9}{2}$$

$$2x = \frac{6 + 9}{2}$$

$$\Rightarrow \quad x = \frac{15}{4}$$

On putting the value of x in Eq. (ii), we get

$$\frac{15}{4} + y = \frac{9}{2}$$

$$y = \frac{9}{2} - \frac{15}{4} = \frac{18 - 15}{4}$$

$$\Rightarrow \quad y = \frac{3}{4}$$

Hence, speed of man $x = \frac{15}{4}$ and

Speed of current $y = \frac{3}{4}$

Hence, required ratio $= \frac{15}{4} : \frac{3}{4} = 5 : 1$

10. (*c*) Let speed of boat be x km/h.

Given speed of current = 5 km/h

$\therefore$ Upstream speed of boat = $(x - 5)$ km/h

Downstream speed of boat = $(x + 5)$ km/h

According to the question,

$$\frac{10}{x - 5} + \frac{10}{x + 5} = \frac{50}{60}$$

$$\Rightarrow \quad 10\left(\frac{x + 5 + x - 5}{x^2 - 25}\right) = \frac{5}{6}$$

$$\Rightarrow \quad 12 \times 2x = x^2 - 25$$

$$\Rightarrow \quad x^2 - 24x - 25 = 0$$

$$\Rightarrow \quad x^2 - 25x + x - 25 = 0$$

$$\Rightarrow \quad (x - 25)(x + 1) = 0$$

$$\therefore \quad x = 25 \quad [\because x \neq -1]$$

11. (*b*) Let total distance be x km.

According to the question,

$$\frac{x}{6 + 2} + 3 = \frac{x}{6 - 2}$$

$$\Rightarrow \quad \frac{x}{8} + 3 = \frac{x}{4} \Rightarrow \frac{x}{4} - \frac{x}{8} = 3$$

$$\Rightarrow \quad \frac{2x - x}{8} = 3$$

$$\therefore \quad x = 8 \times 3 = 24 \text{ km}$$

12. (*a*) Speed downstream $= \frac{48}{20} = 2.4$ km/h

Speed upstream $= \frac{48}{24} = 2$ km/h

$\therefore$ Speed of boat in still water

$$= \frac{1}{2}(2.4 + 2) = \frac{4.4}{2} = 2.2 \text{ km/h}$$

13. (*a*) Pawan's speed downstream

$= 24 + 4.8 = 28.8$ km/h

Pawan's speed upstream

$= 24 - 4.8 = 19.2$ km/h

Let the required distance be x.

According to the question,

$$\frac{x}{28.8} + \frac{x}{19.2} = 1$$

$$\Rightarrow \quad \frac{19.2\,x + 28.8\,x}{552.96} = 1$$

$$\Rightarrow \quad 19.2x + 28.8x = 552.96$$

$$\Rightarrow \quad 48x = 552.96$$

$$\therefore \quad x = \frac{552.96}{48} = 11.52 \text{ km}$$

Fast Track Method

x = Speed of Pawan in still water

$= 24$ km/h

y = Speed of river $= 4.8$ km/h

$T = 1$ h

$\therefore$ According to the formula,

$\therefore$ Required distance $= \frac{T(x^2 - y^2)}{2x}$

[by Technique 2]

$$= \frac{1 \times [(24)^2 - (4.8)^2]}{2 \times 24}$$

$$= \frac{576 - 23.04}{2 \times 24} = \frac{552.96}{48} = 11.52 \text{ km}$$

14. (*d*) Let the speed of Sameer in still water be x km/h.

Sameer's speed downstream $= (x + 12)$ km/h

Sameer's speed upstream $= (x - 12)$ km/h

According to the question,

$$24(x + 12) = 36(x - 12)$$

$$\Rightarrow \quad 2x + 24 = 3x - 36$$

$$\Rightarrow \quad x = 36 + 24 = 60 \text{ km/h}$$

Fast Track Method

Here, $x = 24$ h, $y = 36$ h

Rate of stream $= 12$ km/h

According to the formula,

Speed of Sameer in still water

$$= \frac{a(x + y)}{(y - x)} \qquad \text{[by Technique 3]}$$

$$= \frac{12 \times (24 + 36)}{(36 - 24)} = \frac{60 \times 12}{12} = 60 \text{ km/h}$$

15. (*b*) Let speed of the flow of water be x km/h and rate of sailing of sailer be y km/h.

Downstream speed $(x + y) = \frac{48}{8}$

$\Rightarrow \quad x + y = 6 \quad \ldots(i)$

and upstream speed $x - y = \frac{48}{12}$

$\Rightarrow \quad x - y = 4 \quad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$y = 1$ km/h

16. (*c*) Let speed of stream be x km/h.

Given speed of motorboat in still water = 45 km/h

$\therefore$ Speed of boat along stream

$= (45 + x)$ km/h

According to the question,

$$45 + x = \frac{80}{1\frac{1}{3}}$$

$$\Rightarrow \quad 45 + x = \frac{80 \times 3}{4}$$

$$\Rightarrow \quad x = 60 - 45 = 15$$

$\Rightarrow$ Speed of boat against stream

$= 45 - 15 = 30$ km/h

Hence, required time $= \frac{\text{Distance}}{\text{Speed}}$

$$= \frac{80}{30} = \frac{8}{3} \times 60 = 160 \text{ min} = 2 \text{ h } 40 \text{ min}$$

17. (*d*) Let x be the speed of the boat and y be the speed of the current.

Speed upstream $= (x - y)$ km/h

Speed downstream $= (x + y)$ km/h

According to the question,

$$\frac{20}{x - y} + \frac{20}{x + y} = \frac{25}{36}$$

In this equation, there are two variables but only one equation. So, the value of x cannot be determined.

18. (*d*) Speed of boat downstream

$$= \frac{\text{Distance covered}}{\text{Time taken}}$$

$$= \frac{30}{2} = 15 \text{ km/h}$$

Now, speed of boat upstream

$$= \frac{\text{Distance covered}}{\text{Time taken}} = \frac{30}{6} = 5 \text{ km/h}$$

Now, speed of boat

$$= \frac{\text{Speed downstream} + \text{Speed upstream}}{2}$$

$$= \frac{15 + 5}{2} = \frac{20}{2} = 10 \text{ km/h}$$

19. (*a*) Let the distance be x km.

Speed downstream $= (5 + 2) = 7$ km/h

and speed upstream $= (5 - 2) = 3$ km/h

According to the question,

$$\frac{x}{3}-\frac{x}{7}=2 \Rightarrow 7x-3x=21\times 2$$

$$\therefore \quad x=\frac{21\times 2}{4}=10.5 \text{ km}$$

Fast Track Method

Here, $a=5$ km/h, $b=2$ km/h, $T=2$ h

According to the formula,

[by Technique 5]

$$\text{Required distance}=\left(\frac{a^2-b^2}{2b}\right)\times T$$

$$=\frac{5^2-2^2}{2\times 2}\times 2=\frac{25-4}{2}=\frac{21}{2}=10.5 \text{ km}$$

20. (*d*) Let velocity of the stream be x km/h.

$\therefore$ Velocity of the boat downstream

$$=(4+x) \text{ km/h}$$

and velocity of the boat upstream

$$=(4-x) \text{ km/h}$$

According to the question,

$$3(4+x)=9(4-x)$$

$$\Rightarrow \quad 12+3x=36-9x$$

$$\Rightarrow \quad 12x=24$$

$$\therefore \quad x=2 \text{ km/h}$$

21. (*c*) Speed upstream $=\dfrac{10}{\frac{30}{60}}=\dfrac{10\times 60}{30}$

$$=20 \text{ km/h}$$

Speed downstream $=\dfrac{10}{\frac{25}{60}}=\dfrac{10\times 60}{25}$

$$=24 \text{ km/h}$$

$\therefore$ Speed of the river's current

$$=\frac{1}{2}(24-20)=\frac{4}{2}=2 \text{ km/h}$$

22. (*a*) Let speed of water current be x km/h.

$\therefore$ Speed downstream $=(10+x)$ km/h,

water $=(10-x)$ km/h

According to the question,

$$\frac{24}{10+x}+\frac{24}{10-x}=5$$

$$\Rightarrow \quad 24[10-x+10+x]=5(10^2-x^2)$$

$$\Rightarrow \quad 100-x^2=\frac{24\times 20}{5}$$

$$\Rightarrow \quad 100-x^2=96$$

$$\Rightarrow \quad x^2=4$$

$$\Rightarrow \quad x=\sqrt{4}=2 \text{ km/h}$$

23. (*d*) Let the distance between the two ports be x km.

Then, speed downstream $=\dfrac{x}{4}$

And speed upstream $=\dfrac{x}{5}$

$\therefore$ Speed of the stream

$= \frac{1}{2}$ [speed downs tream + speed upstream]

$= \frac{1}{2}\left(\frac{x}{4} - \frac{x}{5}\right)$

$\Rightarrow \quad \frac{1}{2}\left(\frac{5x - 4x}{20}\right) = 2$

$\Rightarrow \quad \frac{x}{40} = 2 \Rightarrow x = 80$ km

24. (*a*) Let the distance be x km.

$\because$ Speed of the man in still water

$= 7\frac{1}{2}$ km/h

and speed of the river = 1.5 km/h

$\therefore$ Speed of the man downsteam

$= 7.5 + 1.5 = 9$

Speed of the man upstream

$= 7.5 - 1.5 = 6$ km/h

According to the question,

$$\frac{x}{9} + \frac{x}{6} = \frac{50}{60}$$

$$\Rightarrow \quad \frac{4x + 6x}{36} = \frac{50}{60}$$

$$\Rightarrow \quad \frac{10x}{36} = \frac{50}{60}$$

$$\Rightarrow \quad x = \frac{50 \times 36}{10 \times 60} \Rightarrow x = 3 \text{ km}$$

25. (c) Let rate of stream be a km/h. According to the question,

24 + a = 2(24 - a) => 24 + a = 48 - 2a

=> 3a = 48 - 24 = 24

24 a = — = 8 km/h 3

Fast Track Method

Here, speed of boat in still water

= 24 km/h and n = 2 Speed of stream = (a) = ? According to the formula,

.'. Speed of boat in still water =-----------

(rj-1)

[by Technique 1]

24 = f^±iU3a = 24 (2-1)

24 => a = — = 8 km/h

3

26. (a) Speed of a boat in still water

Distance 12

=-----------= — = 12 km/h

Time 1

12 Speed against the current = — = 4 km/h 3

Let the speed of the current = x km/h

According to the question,

12- x = 4 => x = 8km/h

27. (6) Let boatman's speed upstream x And his speed downstream = 2x

.'. Ratio = (Speed in still water) : (Speed of stream)

= 3£:£ = 3:1 2 2

Exercise © *Higher Skill Level Questions*

1. (c) Let boat's rate upstream be x and Boat's rate downstream = *y* According to the question, Distance covered in 528 min = Distance covered in 240 min => Distance covered in 8 h 48 min

= Distance covered in 4 h

$$\Rightarrow \quad x \times 8\frac{4}{5} = (y \times 4)$$

$$\Rightarrow \quad \frac{44}{5}x = 4y \Rightarrow y = \frac{11}{5}x$$

$$\therefore \text{ Required ratio} = \left(\frac{y+x}{2}\right) : \left(\frac{y-x}{2}\right)$$

$$\Rightarrow \quad = \left(\frac{\frac{11}{5}x + x}{2}\right) : \left(\frac{\frac{11x}{5} - x}{2}\right)$$

$$= \left(\frac{16x}{5} \times \frac{1}{2}\right) : \left(\frac{6x}{5} \times \frac{1}{2}\right)$$

$$= \frac{8x}{5} : \frac{3x}{5} = 8 : 3$$

2. (6) Let the speed of rowing in still water be *u* km/h.

Distance in downstream motion = 21 km and speed downstream = (u + 4) km/h

$$\therefore \quad \text{Time taken} = \frac{21}{u+4}$$

Distance upstream motion = 21 km and speed upstream

$$= (u - 4) \text{ km}$$

$$\therefore \quad \text{Time taken} = \frac{21}{u-4}$$

According to the question,

$$\frac{21}{u+4} + 2 = \frac{21}{u-4}$$

$\Rightarrow \quad \dfrac{21 + 2u + 8}{u + 4} = \dfrac{21}{u - 4}$

$\Rightarrow \quad \dfrac{2u + 29}{u + 4} = \dfrac{21}{u - 4}$

$\Rightarrow \quad (u - 4)(2u + 29) = 21(u + 4)$

$\Rightarrow \quad 2u^2 - 8u + 29u - 116 = 21u + 84$

$\Rightarrow \quad 2u^2 + 21u - 116 = 21u + 84$

$\Rightarrow \quad 2u^2 = 84 + 116$

$\Rightarrow \quad u^2 = \dfrac{200}{2}$

$\Rightarrow \quad u^2 = 100$

$\Rightarrow \quad u = 10$ km/h

3. (*c*) Let speed of a motorboat be

$= 36x$ km/h

and speed of the current $= 5x$ km/h

$\therefore$ Speed downstream

$= (36 + 5)\,x = 41x$ km/h

and speed upstream

$= (36 - 5)\,x = 31x$ km/h

Let distance be a km.

According to the question,

When boat goes along with the current,

Distance = Time × Speed

$\Rightarrow \quad a = \left(5 + \dfrac{10}{60}\right) \times 41x$

$\Rightarrow \quad a = \dfrac{31}{6} \times 41x \quad \ldots(\text{i})$

Again, when boat come back

a = Time × Speed

From Eq. (i),

$\dfrac{31}{6} \times 41x = \text{Time} \times 31x$

$\Rightarrow \quad \text{Time} = \dfrac{41}{6}$

Time = 6 h 50 min.

4. (*d*) Let speed of Ishwar's boat in still water be x km/h

and speed of current $= y$ km/h

Rate downstream $= (x + y)$ km/h

Rate upstream $= (x - y)$ km/h

Let the distance covered in each case be a.

According to the question,

$$\frac{2a}{x + y} = \frac{a}{x - y}$$

$\Rightarrow \quad 2(x - y) = x + y$

$2x - 2y = x + y$

$\Rightarrow \quad x = 3y$

$\therefore \quad \dfrac{x}{y} = \dfrac{3}{1}$

$\Rightarrow \quad x : y = 3 : 1$

5. (*b*) For the first boat,

Speed of stream : Speed of boat = 2 : 5

Let speed of stream be $2x$ km/h
and speed of boat = $5x$ km/h
Similarly, for the second boat
Speed of stream : Speed of boat = 3 : 4
Let speed of stream be $3y$ Km/h
and speed of boat = $4y$
In both of the conditions, river is same.

$$\therefore \quad 2x = 3y$$

$$\Rightarrow \quad x = \frac{3}{2}y$$

Thus, required ratio in speeds of boats in still water

$$= 5x : 4y = 5 \times \frac{3}{2}y : 4y = 15 : 8$$

6. (*c*) Let the speed of a boat and stream be x and y km/h.

$\therefore$ Speed of boat along stream = $(x + y)$ km/h
and speed of boat against stream = $(x - y)$ km/h
According to the question,

$$\frac{25}{x - y} + \frac{39}{x + y} = 8 \quad \ldots(\text{i})$$

and
$$\frac{35}{x - y} + \frac{52}{x + y} = 11 \quad \ldots(\text{ii})$$

On multiplying Eq. (i) by 4 and Eq. (ii) by 3, then subtract Eq. (ii) from Eq. (i), we get

$$\frac{100}{x - y} + \frac{156}{x + y} = 32$$

$$\frac{105}{x - y} + \frac{156}{x + y} = 33$$

$$- \qquad - \qquad -$$

$$\frac{100}{x - y} - \frac{105}{x - y} = -1$$

$$\Rightarrow \quad \frac{5}{x - y} = 1$$

$$\Rightarrow \quad x - y = 5 \quad \ldots(\text{iii})$$

On substituting the value of $(x - y = 5)$ in Eq. (i), we get

$$\frac{25}{5} + \frac{39}{x + y} = 8$$

$$\Rightarrow \quad \frac{39}{x + y} = 8 - 5$$

$$\Rightarrow \quad x + y = \frac{39}{3}$$

$$\Rightarrow \quad x + y = 13 \quad \ldots(\text{iv})$$

On solving Eqs. (iii) and (iv), we get

$$x = 9$$

and
$$y = 4$$

Hence, speed of stream = 4 km/h.

7. (*b*) Let speed of boat and current be x and y km/h.

$\therefore$ Speed of boat Upstream = $(x - y)$ km/h
And speed of boat downstream

$= (x + y)$ km/h

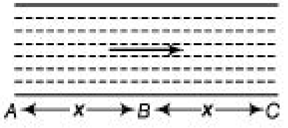
$\therefore \quad \dfrac{24}{x-y} + \dfrac{36}{x+y} = 6 \quad \ldots(i)$

and $\quad \dfrac{36}{x-y} + \dfrac{24}{x+y} = \dfrac{13}{2} \quad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$x = 10$ and $y = 2$

8. (*a*) Let speed of Avinash in still water be a km/h and speed of stream be b km/h.



Let $AB = BC = x$ km

From first condition,

$$\frac{x}{a+b} + \frac{x}{a-b} = 10 \quad \ldots(i)$$

From second condition,

$$\frac{2x}{a+b} = 4$$

$\Rightarrow \quad \dfrac{x}{a+b} = 2 \quad \ldots(ii)$

Putting the value in Eq. (i), we get

$$2 + \frac{x}{a-b} = 10$$

$\Rightarrow$ $$\frac{x}{a-b} = 8 \qquad \text{...(iii)}$$

On dividing Eqs. (iii) by Eq. (ii), we get

$$\frac{a+b}{a-b} = \frac{8}{2} = 4$$

$\Rightarrow$ $a + b = 4a - 4b$

$\Rightarrow$ $3a = 5b$

$\therefore$ $a : b = 5 : 3$

9. (*b*) Given,

Speed of the stream = 5 km/h

and speed of the man in still water

= 20 km/h

$\therefore$ Speed of the man downstream

= 20 + 5 = 25 km/h

and speed of the man upstream

= 20 − 5 = 15 km/h

$\therefore$ Total time taken to complete the whole journey

$$= \frac{30}{25} + \frac{30}{15}$$

$$= 30\left(\frac{3+5}{75}\right) = \frac{30 \times 8}{75}$$

= 3 h 12 min

Chapter

28

Races and Games of Skill

A race or a games of skill includes the contestants in a contest and their skill in the concerned contest/game.

Important Terms

Race A race is a contest of speed in running, driving, riding, sailing or rowing.

Race Course The ground/path on which a contest is organised in a systematic way, is called a race course. **Starting Point** The exact point/place from where a race begins, is called starting point.

Start If two persons A and B are contesting a race and before the start of the race, A is at the starting point and B is ahead of A by 20 m, then it is said that A gives B a start of 20 m. *For example If* A and B are the contestants for a 100 m race and A has to cover 100 m, while B has to cover (100 - 20) = 80 m.

Finishing Point The exact point/place where a race ends, is known as finishing point.

Winning Point/Goal A person who reaches the finishing point first, is called the **winner.**

Note For a winner, finishing point is as same as the winning point/goal

Dead Heat Race A race is said to be a dead heat race, if all

the contestants reach the finishing point exactly at the same

time.

Game A game of 100 means that the contestant who scores

100 points first, is declared the winner.

For example If in a 100 points game, *A* scores 100 points,

while *B* scores 85 points,

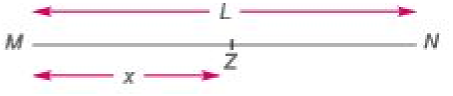
Then, it is said that *A* can give (100 - 85) = 15 points to *B*.

Some Facts about Race

For Two Contestants *A* and *B*

1. If *A* beats Bbyxm, then

Distance covered by *A* (winner) = L m Distance covered by *B* (loser) = (L - x) m 2. If *B* starts from x m ahead of *A* (or *A* gives *B* a start of x m), then



A starts from M and *B* starts from Z. Distance covered by *B* = *(L* - x) m

3. If *A* beats B by *T* s, then

A and *B* both start from point M.

Time taken by *A* (winner) = Time taken by *B* (loser) - *T* It means that *A* completes the race in T s less time than that of *B*. 4. If *B* starts the race *T* s before the time *A* starts (or if *A* gives *B* a start of T s), then

In such case, we say that *A* starts *T* s after the time *B* starts. 5. If both of the contestants get at the finishing point at the same time, then Difference in time of defeat = 0; Difference in distance of defeat = 0

Ex. 1 In a race of 100 m, *A* gives *B* a start of 10 m. What distance will be covered byB? Sol. Required distance = (100 - 10) m = 90 m

Ex. 2 In a race, *x* gives *y* a start of 30 m making length of race for *y* a distance of 170 m. Find the total length of race. Sol. Required length = (170 + 30) = 200 m

Ex. 3 In a 300 m race, M gives some start to *N* and this makes the length of race for *N* 225 m. What start does *N* get from M? Sol. Start given by *M*to *N*= (300 - 225) = 75 m

Ex. 4 In a 100 m race, Ajay runs at the speed of 4 km/h. Ajay gives Brijesh a start of 4 m and still beats him bv 15 s. Find the speed of Briiesh.

Sol. Time taken by Ajay to cover 100 m $= \left(\frac{60 \times 60}{4000} \times 100\right) \text{s} = 90 \text{ s}$

$\therefore$ Brijesh covers $(100 - 4) = 96$ m in $(90 + 15) = 105$ s.

$$\therefore \text{Brijesh's speed} = \frac{96}{105} \text{ m/s} = \frac{96}{105} \times \frac{18}{5} = 3.29 \text{ km/h}$$

Ex. 5 *P* covers 1 km in 4 min 40 s, while *Q* covers the same distance in 5 mirt. By what distance does *P* defeat O?

Sol. Clearly, P beats Q by 20 s.

Distance covered by Q in 20 s $= \frac{1000}{300} \times 20 = 66\frac{2}{3}$ m

Ex. 6 *A* can run 1 km in 5 min and B can run the same distance in 6 min. How many metres start, can *A* give to *B* in 1 km race, so that the race may end in a dead heat? **Sol.** Time taken by *A* to run 1 km = 300 s; Time taken by Btorunlkm = 360s. *A* can give *B* a start of (360 - 300) = 60 s

$\because$ In 360 s, B runs 1000 m.

$\therefore$ In 60 s, B runs $\frac{1000}{360} \times 60$ m $= \frac{1000}{6}$ m $= \frac{500}{3}$ m $= 166\frac{2}{3}$ m

Hence, A can give a start of $166\frac{2}{3}$ m.

Ex. 7 In a game of 100 points, *A* scores 100 points, while *B* scores only 75 points. In this game, how many points can *A* give to B? **Sol.** Score of *A* = 100 points; Score of *B* = 75 points ∴ *A* can give (100 - 75) = 25 points to *B*.

Ex. 8 In 100 m race, *A* runs at 8 km/h. If *A* gives *B* a start of 4 m and still beats him by 15 s, then what is the speed of B?

Sol. Time taken by A to cover 8 km = 1h

$\Rightarrow$ Time taken by A to cover 8000 m $= 60 \times 60$ s

Time taken by A to cover 100 m $= \frac{60 \times 60}{8000} \times 100 = 45$ s

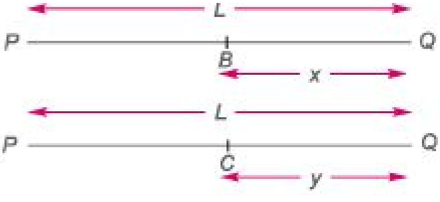
Now, B covers $(100 - 4) = 96$ m in $(45 + 15) = 60$ s

$\therefore$ B's speed $= \frac{\text{Distance covered}}{\text{Time taken}} = \frac{96}{60}$ m/s $= \frac{96 \times 60 \times 60}{60 \times 1000}$ km/h $= 5.76$ km/h

For Three Contestants *A, B* and C

Let *A, B* and C participate in a race of L m.

Let *A* comes 1st in the race by beating *B* by x m and C by y m, respectively,



Here, the values of x and y will decide 2nd and 3rd positions. If x <y, then *B* will beat C, *i.e., B* will get the 2nd position. If x > y, then C will beat *B, i. e., C* will get the 2nd position,

Ex. 9 *A, B* and C are three contestants in 1 km race. If *A* can give *B* a start of 40 m and *A* can give C a start of 64 m, how many metres start can *B* give C?

Sol. While A covers 1000 m, B covers $(1000 - 40)$

$$= 960 \text{ m and } C \text{ will cover } (1000 - 64) = 936 \text{ m.}$$

So, when B covers 1000 m, C will cover $\left(\frac{936}{960} \times 1000\right) = 975 \text{ m}$

$\therefore$ B can give C a start of $(1000 - 975)$ m *i.e.*, 25 m.

Fast Track **Techniques**

to solve the QUESTIONS

Technique 1

For 2 contestants, following relation will be valid.

$$\frac{\text{Time taken by the winner}}{\text{Distance covered by the loser}} = \frac{\text{Time taken by the loser}}{\text{Distance covered by the winner}}$$

$$= \frac{\text{Difference of winning time} + \text{Initial time}}{\text{Difference of winning distance} + \text{Initial distance}}$$

Ex. 10 In 1 km race, *A* beats *B* by 36 m or 18 s. Find the *A's* time over the course.

Sol. Clearly, B covers 36 m in 18 s.

$\therefore$ B's time over the course $= \frac{18}{36} \times 1000 = 500$ s

$\therefore$ A's time over the course $= (500 - 18) = 482$ s

Alternate Method

Initial time = 0, initial distance = 0, difference of winning time = 18 s and difference of winning distance = 36 m

According to the formula,

$$\frac{\text{Time taken by the winner}}{\text{Distance covered by the loser}} = \frac{\text{Difference of winning time + Initial time}}{\text{Difference of winning distance + Initial distance}}$$

$$\Rightarrow \quad \frac{\text{Time taken by } A}{1000 - 36} = \frac{18}{36}$$

$\therefore$ Time taken by $A = \frac{18}{36} \times 964 = \frac{1}{2} \times 964 = 482$ s

Technique 2

If in a race of L m, 1st contestant beats 2nd contestant and 3rd contestant by distances of a_{12} and a_{13} respectively and the 2nd contestant beats the 3rd contestant by a distance of a_{23}, then we get the following relation

$$(L - a_{12})\, a_{23} = L\,(a_{13} - a_{12})$$

Ex. 11 P, Q and R are three contestants in a 2 km race. If P can give Q a start of 100 m and P can give R a start of 138 m, then how many metres start can Q give to R? Sol. When P covers 2000 m, then Q covers (2000 - 100) = 1900 m

and R covers $(2000 - 138) = 1862$ m

When Q covers 1900 m, then R covers 1862 m.

When Q covers 2000 m, then R covers $\left(\frac{1862}{1900} \times 2000\right) = 1960$ m

$\therefore$ Q can give R a start of $(2000 - 1960) = 40$ m

Fast Track Method Here, $a_{12} = 100$ m, $a_{13} = 138$ m, $a_{23} = ?$ and $L = 2000$ m

According to the formula,

$$(L - a_{12})\, a_{23} = L\,(a_{13} - a_{12})$$

$$\Rightarrow \quad (2000 - 100)\, a_{23} = 2000\,(138 - 100)$$

$$\Rightarrow \quad 1900\, a_{23} = 2000 \times 38$$

$$\therefore \quad a_{23} = \frac{2000 \times 38}{1900} = \frac{760}{19} = 40 \text{ m}$$

Hence, Q can give R a start of 40 m.

Technique 3

if in a race of L, m, 1st contestant beats the 2nd contestant by a distance of a_{12}, in a race of L_2 m 2nd contestant beats the 3rd contestant by a distance of a_{23} and in a race of L_3 m 1st contestant beats the 3rd contestant by a distance of a_{13}, then for a race of L m

$$A_{12} = \frac{a_{12}}{L_1} \times L; \quad A_{23} = \frac{a_{23}}{L_2} \times L; \quad A_{13} = \frac{a_{13}}{L_3} \times L$$

Now, we get the following relation

$$(L - A_{12}) A_{23} = L (A_{13} - A_{12})$$

Ex. 12 In a race of 1200 m, *A* can beat *B* by 120 m and in a race of 500 m, *B* can beat C by 100 m. By how many metres will *A* beat C in a race of 800 m?

Sol. If *A* runs 1200 m, then *B* runs 1080 m.

If *A* runs 800 m, then *B* runs $\left(\frac{1080}{1200} \times 800\right) = 720$ m

When *B* runs 500 m, then *C* runs 400 m.

When *B* runs 720 m, then *C* runs $\left(\frac{400}{500} \times 720\right) = 576$ m

$\therefore$ *A* beats *C* by $(800 - 576)$ m $= 224$ m

Alternate Method According to the question, the length of each race is different.

$$a_{12} = 120 \text{ m (for 1200 m)}$$

$$\therefore \quad A_{12} \text{ (for 800 m)} = \frac{120}{1200} \times 800 = 80 \text{ m}$$

Similarly $\quad a_{23} = 100$ m (for 500 m)

$$A_{23} \text{ (for 800 m)} = \frac{100}{500} \times 800 = 160 \text{ m and } A_{13} = ?$$

According to the formula, $(L - A_{12}) A_{23} = L (A_{13} - A_{12})$

$\Rightarrow \quad (800 - 80)\, 160 = 800 (A_{13} - 80)$

$\Rightarrow \quad 720 \times 160 = 800 A_{13} - 800 \times 80$

$\Rightarrow \quad 720 \times 16 = 80 A_{13} - 6400$

$\Rightarrow \quad 80 A_{13} = 17920$

$\therefore \quad A_{13} = \frac{17920}{80} = 224$ m

Hence, *A* will beat *C* by a distance of 224 m.

Multi Concept

QUESTIONS

1. *In 1 km race, the ratio of the speeds of two contestants L and M is 3 :4. L has a start of 280 m. Then, L wins by how many metres?*

(a) 40 m (b) 60 m (c) 45 m (d) 65 m

➥ (*a*) To reach the winning post, L will have to cover a distance of $(1000 - 280) = 720$ m

When L covers 3 m, then M covers 4 m.

When L covers 720 m, then M covers $\left(\frac{4}{3} \times 720\right) = 960$ m

Thus, when *L* reaches the winning post, *M* covers 960 m and therefore, remains 40 m

behind.

.·. *L* wins by 40 m

2. *In a game of 80 points, A can give 10 points to B and 20 points to C. Then, how many points can B give C in a game of 70 points?*

(a) 20 (b) 30 (c) 40 (d) 10

➥ (*d*) $A : B = 80 : 70$ and $A : C = 80 : 60$

$$\frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{70}{80} \times \frac{80}{60}\right) = \frac{70}{60} = 70 : 60$$

Hence, in a game of 70 points, S can give C 10 points

3. ***In a 500 m race, A gives B a start of 6 s and beat him by 20 m. In another race of 500 m, A beats B by $8\frac{1}{7}$ s. Find their speeds.***

(a) A's speed = 11 m/s, B's speed = 10 m/s (b) A's speed = 11 m/s, B's speed = 9.33 m/s
(c) A's speed = 12 m/s, B's speed = 9.33 m/s (d) A's speed = 9.33 m/s, B's speed = 11 m/s

➡ (*b*) In a 500 m race, *B* takes $8\frac{1}{7}$ s more time than *A*.

In another 500 m race, *B* takes 6 s more time and run 20 m less distance than *A*.

∴ *B* can run in $\left(8\frac{1}{7} - 6\right)$ s a distance of 20 m; *B* can run in 1 s a distance of $\frac{20}{2\frac{1}{7}}$.

∴ Speed of *B* is $\frac{28}{3}$ m/s ≈ 9.33 m/s

Let the speed of *A* be V_A m/s.

Then, according to question,

$$\frac{500}{\text{Speed of } B} - \frac{500}{\text{Speed of } A} = 8\frac{1}{7} \Rightarrow \frac{500}{28/3} - \frac{500}{V_A} = \frac{57}{7}$$

$$\Rightarrow \quad \frac{1500}{28} - \frac{57}{7} = \frac{500}{V_A}$$

$$\therefore \quad V_A \approx 11 \text{ m/s}$$

Hence, speed of *A* is 11 m/s and speed of *B* is 9.33 m/s.

Fast Track Practice

1. In a race of 150 m, *A* gives B a start of 20 m. What distance will be covered by B?

(*a*) 100 m (*b*) 130 m

(*c*) 170 m (*d*) 160 m

2. In a race, *P* gives *Q* a start of 25 m making length of race for *Q* a distance of 175 m. Find the total length of race.

(*a*) 250 m (*b*) 200 m

(*c*) 225 m (*d*) 235 m

(*e*) None of the above

3. In a 800 m race, *A* gives some start to B and this makes the length of race for *B* 725 m. What start does *B* get from A?

(*a*) 22 m (*b*) 35 m

(c) 45 m *(d)* 75 m

4. In a game of 100 points, *A* scores 100 points while *B* scores only 65 points. In this game, how much points can *A* give to B?

(a) 45 *(b)* 35

(c) 25 *(d)* 55

(e) None of the above

5. In 1 km race, *A* beats B by 18 m or 9 s. Find the *A's* time over the course.

(a) 391 s *(b)* 591 s

(c) 491 s *(d)* 500 s

6. In a 1000 m race, *X* beats *Y* by 140 m or 14 s. What will be the *X's* time over the course?

{*a)* 86 s *(b)* 90 s

(c) 95 s *(d)* 76 s

7. In 1 km race, *P* beats *Q* by 72 m in 12 s. Find the *Ps* time over the course.

(a) 155-s *(b)* 151 s

(c) 154-s (a; 160 s

8. In a 400 m race, *A* runs at a speed of 16 m/s. If A gives *B* a start of 16 m and still beats him by 40 s, what will be the speed of B?

(a) 6 m/s

(b) 8 m/s (c) 15 m/s (C0 5.9 m/s

9. In a 200 m race, *A* runs at a speed of 2 m/s. If A gives B a start of 10 m and still beats him by 5 s, what will be the SDeed of B?

(a) $1\frac{17}{21}$ m/s (b) $3\frac{17}{21}$ m/s

(c) 21 m/s (d) 2 m/s

10. *X* covers 1 km in 8 min 40 s while *Y* covers the same distance in 10 min. By what distance does *X* defeat Y?

(a) $13\frac{1}{3}$ m (b) $133\frac{2}{3}$ m

(c) $133\frac{2}{5}$ m (d) $133\frac{1}{3}$ m

11. *A* can run 45 m while B runs 50 m. In a km race B beats *A* by how many metres?

(a) 150 m *(b)* 200 m

(c) 100 m *(d)* 125 m

(e) None of the above

12. A can run 40 m while B runs 50 m. In a km race, B beats *A* by which of the following distances?

(a) 175 m *(b)* 225 m

(c) 335 m *(d)* 200 m

(e) None of the above

13. Raman covers 1 km in 8 min while Suman covers the same distance in 10 min. By what distance does Raman beat Suman?

(a) 150 m *(b)* 65 m

(c) 190 m *(d)* 200 m

14. *L* can run 1 km in 10 min while *M* can run it in 12 min. How many metres start can *L* give *M* in a km race, so that the race may end in a dead heat?

(a) $166\frac{2}{3}$ m (b) $166\frac{1}{3}$ m

(c) $166\frac{3}{5}$ m (d) 200 m

(e) None of the above

15.Yogesh can run 1 km in 6 min 20 s and Vijay can cover the same distance in 6 min 40 s. By what distance can Yogesh beat Vijay?

(a) 50 m *(b)* 90 m

(c) 45 m *(d)* 30 m

16. In a game of 200 points, *A* can give 40 points to *B* and 56 points to C How many points can *B* give to C?

(a) 20 points *(b)* 15 points

(c) 10 points *(d)* 5 points

(e) None of the above

17. In a game of 160 points, A can give 10 points to *B* and 30 points to C. How many points *B* can give C in a game of 60?

(a) 10 *(b)* 15

(c) 6 *(d)* 8

(e) None of the above

18. In a 200 m race, *A* can beat *B* by 50 m and *B* can beat *C* by 8 m. In the same race, A can beat C by what distance?

(a) 60 m (b) 72 m

(c) 56 m *(d)* 66 m

19. A, B and C are three contestants in a 500 m race. If *A* can give B a start of 20 m and *A* can give C a start of 32 m, how many metres start can *B* give to C?

(a) 12 m *(b)* 14 m

(c) 12.5 m fdj 13.5 m

(e) None of the above

20. In a race of 600 m, *A* can beat *B* by 30 m and in a race of 500 m, *B* can beat C by 25 m. By how many metres will *A* beat C in a race of 400 m?

(a) 39 m *(b)* 49 m

(c) 55 m *(d)* 25 m

21. In a game of billiards, *A* can give £ 20 points in the game of 120 points and he can give C 30 points in the game of 120 points. How many points can *B* give C in a game of 90?

(a) 9 points *(b)* 18 points *(c)* 6 points *(d)* 3 points

22. Arun and Bhaskar start from place *P* at 6:00 am and 7 : 30 am, respectively and run in the same direction. Arun and Bhaskar run at 8 km/h and 12 km/h, respectively. Bhaskar overtakes Arun at

[SSCCGL (Main) 2012]

(a) 10 : 30 am *(b)* 9:00 am *(c)* 11 : 30 am *(d)* 1:00 am

23. A runs *1%* times as fast as *B*. If A gives *B* a start of 30 m, how far must the winning post be, so that A and *B* reach it at the same time?

(a) 52 m *(b)* 75 m

(c) 69 m *(d)* 70 m

(e) None of the above

24. A 10 km race is organised at 800 m circular race course. *P* and *Q* are the contestants of this race. If the ratio of the speeds of *P* and *Q* is 5 : 4, how many times will the winner overtake the loser?

(a) 4 times *(b)* 1 time

(c) 2 times *(d)* 3 times

(e) None of the above

25. A runs $1\frac{2}{3}$ times as fast as *B*. If *A* gives *B* a start of 40 m, how far must the winning post be, so that *A* and *B* might reach it at the same time?

(a) 75 m *(b)* 200 m

(c) 100 m *(d)* 125 m

(e) None of the above

26. In a 250 m race, the ratio of speeds of A and *B* is 1 : 2. If A has a start of 140 m, then A wins by how many metres?

(a) 45 m *(b)* 15 m

(c) 25 m *(d)* 30 m

(e) None of the above

27. A, *B* and *C* walk 1 km in 5 min, 8 min and 10 min, respectively. C starts walking from a point at a certain time, *B* starts from the same point 1 min later and A starts from the same point 2 min later than C Then, A meet *B* and C at times [SSCCGL 2013]

(a) 2 min, 3 min
(b) $\frac{4}{3}$ min, 3 min
(c) 2 min, $\frac{5}{3}$ min
(d) 1 min, 2 min

Answer with Solutions

1. (*b*) Required distance $= (150 - 20)$ m

$= 130$ m

2. (*b*) Required length $= (175 + 25)$ m

$= 200$ m

3. (*d*) Start given by A to B

$= (800 - 725)$ m $= 75$ m

4. (*b*) Score of $A = 100$ points

Score of $B = 65$ points

$\therefore$ A can give $(100 - 65) = 35$ points to B.

5. (*c*) Clearly, B covers 18 m in 9 s.

$\therefore$ B's time over the course

$$= \frac{9}{18} \times 1000 = 500 \text{ s}$$

$\therefore$ A's time over the course

$= (500 - 9) = 491$ s

Alternate Method

According to the formula,

$$\frac{\text{Time taken by the winner}}{\text{Distance covered by the loser}} = \frac{\text{Difference of winning time} + \text{Initial time}}{\begin{bmatrix}\text{Difference of winning distance}\\ + \text{ Initial distance}\end{bmatrix}}$$

[by Technique 1]

$$\therefore \quad \frac{\text{Time taken by } A}{1000 - 18} = \frac{9 + 0}{18 + 0}$$

$$\Rightarrow \text{Time taken by } A = \frac{9}{18} \times 982$$

$$= \frac{1}{2} \times 982 = 491 \text{ s}$$

6. (*a*) Clearly, Y covers 140 m in 14 s

$\therefore$ Y's time over the course

$$= \frac{14}{140} \times 1000 = 100 \text{ s}$$

$\therefore$ X's time over the course

$= 100 - 14 = 86$ s

Alternate Method

According to the formula,

$$\frac{\text{Time taken by the winner}}{\text{Distance covered by the loser}} = \frac{\text{Difference of winning time}}{\text{Difference of winning distance}}$$

[by Technique 1]

[$\because$ initial time $= 0$ and initial distance $= 0$]

$$\Rightarrow \quad \frac{\text{Time taken by } X}{1000 - 140} = \frac{14}{140}$$

$\therefore$ Time taken by $X = \frac{1}{10} \times 860 = 86$ s

7. (*c*) According to the formula,

$$\frac{\text{Time taken by the winner}}{\text{Distance covered by the loser}} = \frac{\text{Difference of winning time}}{\text{Difference of winning distance}}$$

[by Technique 1]

$$\Rightarrow \quad \frac{\text{Time taken by } P}{1000 - 72} = \frac{12}{72}$$

$\therefore$ Time taken by $P = \frac{12}{72} \times 928$

$$= \frac{464}{3} = 154\frac{2}{3} \text{ s}$$

8. (*d*) Time taken by A to cover 400 m

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{400}{16} \text{ s} = 25 \text{ s}$$

$\therefore$ B covers $(400 - 16)$

$= 384$ m in $(25 + 40) = 65$ s

$\therefore$ B's speed $= \frac{384}{65} = 5.9$ m/s

9. (*a*) Time taken by A to cover 200 m

$$= \frac{200}{2} = 100 \text{ s}$$

$\therefore$ B covers $(200 - 10) = 190$ m in

$(100 + 5) = 105$ s

$\therefore$ B's speed $= \frac{190}{105} = \frac{38}{21} = 1\frac{17}{21}$ m/s

10. (*d*) Clearly, X beats Y by 80 s.

Distance covered by y in 600 s = 1000 m

($\because$ 10 min = 600 s)

Distance covered by Y in 80 s

$$= \frac{1000}{600} \times 80 = \frac{400}{3} \text{ m}$$

$$= 133\frac{1}{3} \text{ m}$$

Alternate Method

According to the formula,

$$\frac{\text{Time taken by the loser}}{\text{Distance covered by the winner}}$$

[by Technique 1]

$$= \frac{\text{Difference of winning time} + \text{Initial time}}{\left[\begin{array}{r}\text{Difference of winning distance} \\ + \text{Initial distance}\end{array}\right]}$$

$$\Rightarrow \quad \frac{600}{1000} = \frac{80+0}{x+0} \Rightarrow \quad \frac{6}{10} = \frac{80}{x}$$

$$\Rightarrow \quad 3x = 400 \quad \Rightarrow \quad x = \frac{400}{3} = 133\frac{1}{3} \text{ m}$$

11. (*c*) When B runs 50 m, then A runs 45 m.
When B runs 1000 m, then A runs

$$\left(45 \times \frac{1}{50} \times 1000\right) \text{m} = 900 \text{ m}$$

$\therefore$ B beats A by 100 m.

12. (*d*) In a 50 m race, B beats A by 10 m.
In 1 km race, B beats A by

$$\left(\frac{10}{50} \times 1000\right) \text{m} = 200 \text{ m}$$

13. (*d*) Raman covers 1 km in 8 min
and Suman cover 1 km in 10 min.
If they starts together, then distance covered by Suman in 8 min

$$= \frac{1000}{10} \times 8 = 800 \text{ m}$$

$\therefore$ Raman will beat Suman by

$$(1000 - 800) \text{ m} = 200 \text{ m}$$

14. (*a*) Time taken by L to run 1 km = 600 s
Time taken by M to run 1 km = 720 s
$\therefore L$ can give M a start of

$$(720 - 600) = 120 \text{ s}$$

$\because$ In 720 s, M runs 1000 m.
In 120 s M runs

$$= \frac{1000}{720} \times 120 = \frac{500}{3} \text{ m} = 166\frac{2}{3} \text{ m}$$

Hence, L can give a start of $166\frac{2}{3}$ m.

15. (*a*) Clearly, Yogesh beats Vijay by 20 s.
Distance covered by Vijay in 400 s

$$= 1000 \text{ m}$$

[$\because$ 6 min 40 s = 400 s]

Distance covered by Vijay in 20 s

$$= \left(\frac{1000}{400} \times 20\right) \text{m} = 50 \text{ m}$$

$\therefore$ Yogesh beats Vijay by 50 m.

16. (*a*) $A : B = 200 : 160$

$$A : C = 200 : 144$$

$$\therefore \quad \frac{B}{C} = \frac{B}{A} \times \frac{A}{C} = \frac{160}{200} \times \frac{200}{144}$$

$$= \frac{160}{144} = \frac{10}{9} = \frac{200}{180}$$

Hence, B can give C 20 points.

17. (*d*) $A : B = 160 : 150$, $A : C = 160 : 130$

$$\frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{150}{160} \times \frac{160}{130}\right)$$

$$= \frac{15}{13} = \frac{60}{52} = 60 : 52$$

$\therefore$ In a game of 60, B can give C

$= (60 - 52) = 8$ points

18. (*c*) According to the question,

$A : B = 200 : 150$

and $B : C = 200 : 192$

$$\therefore A : C = \left(\frac{A}{B} \times \frac{B}{C}\right) = \left(\frac{200}{150} \times \frac{200}{192}\right) = \frac{200}{144}$$

So, A beats C by $(200 - 144) = 56$ m

19. (*c*) Given that, $a_{12} = 20$ m, $a_{13} = 32$ m and $a_{23} = ?$

According to the formula,

$$(L - a_{12})\, a_{23} = L\,(a_{13} - a_{12})$$

[by Technique 3]

$\Rightarrow \quad (500 - 20)\, a_{23} = 500\,(32 - 20)$

$\Rightarrow \quad 480\, a_{23} = 500 \times 12$

$\Rightarrow \quad a_{23} = \dfrac{500 \times 12}{480}$

$\therefore \quad a_{23} = \dfrac{50}{4} = 12.5$ m

Hence, A can give C a start of 12.5 m.

20. (*a*) If A runs 600 m, then B runs 570 m.

If A runs 400 m, B then runs

$$\left(\frac{570}{600} \times 400\right) \text{m} = 380 \text{ m}$$

When B runs 500 m, then C runs 475 m

When B runs 380 m, then C runs

$$\frac{475}{500} \times 380 = 361 \text{ m}$$

$\therefore$ A beats C by $(400 - 361) = 39$ m

21. (*a*) If A scores 120 points, then B scores 100 points and C scores 90 points.

When B scores 100 points, then C scores 90 points.

When B scores 90 points, then C scores $\left(\dfrac{90}{100} \times 90\right)$ points $= 81$ points

$\therefore$ B can give C, 9 points in a game of 90.

22. (*a*) Distance between Arun and Bhaskar at 7 : 30 am $= 8 \times 1\dfrac{1}{2} = 12$ km

Time taken by Bhaskar in covering a distance of 12 km $= \dfrac{12}{(12 - 8)} = 3$ h

$\therefore$ Required time = 10 : 30 am

23. (*d*) A is $1\frac{3}{4}$ faster than B.

Then, ratio of the rates of A and $B = 7:4$

3 m are gained in a race of 7 m.

30 m are gained in a race of

$$\frac{7}{3} \times 30 = 70 \text{ m}$$

24. (*c*) Speed of P : Speed of $Q = 5:4$

Time taken by P to cover 5 rounds

= Time taken by Q to cover 4 rounds

Distance covered by P in 5 rounds

$$= 5 \times \frac{800}{1000} = 4 \text{ km}$$

Distance covered by Q in 4 rounds

$$= 4 \times \frac{800}{1000} = \frac{16}{5} \text{ km}$$

In 5 rounds, P will overtake Q every time.

It means that after covering 4 km, P will overtake Q one time.

$\therefore$ After covering 10 km P will overtake Q, $\frac{1}{4} \times 10 = 2\frac{1}{2}$ times $\approx$ 2 times

25. (*c*) Ratio of the speeds of A and B

$$= 5 : 3$$

Thus, in a race of 5m, A gains 2 m over B.

2 m are gained by A in a race of 5 m.

40 m will be gained by A in a race of $\left(\frac{5}{2} \times 40\right)$ m = 100 m

$\therefore$ Winning post is 100 m away from the starting point.

26. (*d*) To reach the winning post, A has to cover (250 − 140) m = 110 m

From the ratio of 1 : 2,

When A covers 1m, then B covers 2 m.

When A covers 110 m, then B covers

$$\left(\frac{2}{1}\times 110\right)\text{m} = 220\text{ m}$$

$\therefore$ A wins by $(250 - 220) = 30$ m

27. (*c*) A walks 1 km in 5 min.

Then, distance covered by A in 1 min

$$= \frac{1000}{5} = 200\text{ m}$$

B walks 1 km in 8 min.

Then, distance covered by B in 1 min

$$= \frac{1000}{8} = 125\text{ m}$$

C walks 1 km in 10 min.

Then, distance covered by C in 1 min

$$= \frac{1000}{10} = 100\text{ m}$$

Let A meets B and C in X and Y min, respectively.

Then, according to the question,

Distance covered by C in $(x + 2)$ min

= Distance covered by A in x min

$$\Rightarrow \quad 100\,(x + 2) = 200\,x$$

$$\Rightarrow \quad 100\,x + 200 = 200\,x$$

$$\Rightarrow \quad 200 = 100\,x$$

$$\therefore \quad x = \frac{200}{100} = 2\text{ min}$$

Now, for A and B,

Distance covered by B in $(y + 1)$ min

= Distance covered by A in y min

$$\Rightarrow \quad 125\,(y + 1) = 200 \times y$$

$$\Rightarrow \quad 125y + 125 = 200y$$

$$\Rightarrow \quad 125 = 200y - 125y$$

$$\Rightarrow \quad 125 = 75y$$

$$\therefore \quad y = \frac{125}{75} = \frac{5}{3}\text{ min}$$

Chapter

29

Clock and Calendar

Clock

A clock is an instrument which displays time divided into

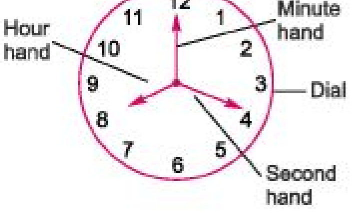
hours, minutes and seconds.

A clock mainly consists of four components.

Dial

A clock is a circular dial. The periphery of the dial is numbered 1 through 12 indicating the hours in a 12 h cycle. The circumference of a dial is divided into 60 equal spaces

cauea minute spaces or *iz* equal spaces called hour spaces. Every clock has mainly two hands, one is smaller and other is bigger. The smaller hand is slower the and the bigger hand is faster.



Hour Hand

The smaller or slower hand of a clock is called the hour hand. It makes two revolutions in a day.

Minute Hand

The bigger or faster hand of a clock is called the minute hand. It makes one revolution in every hour.

Second Hand

Second hand indicates seconds on a circular dial. It makes one revolution per minute.

Note In 1 h minute hand covers 60 min spaces whereas the hour hand covers 5 min spaces Therefore, minute hand gains (60 - 5) = 55 min in 1 h

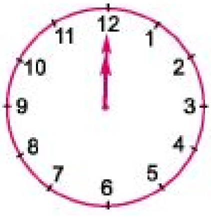
Important Points Related to Clock

1. In 1 h, both hands coincide once (i.e., 0° apart) *For example*

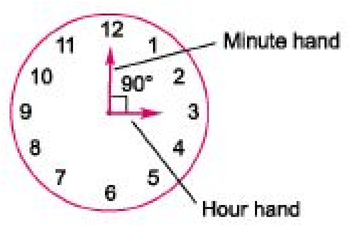
Between 3 and 4'o clock, hands are together as shown in adjacent figure.

2. In 12 h, both hands coincide 11 times (between 11 and l'o clock they coincide once) and in a day both hands coincide 22 times. *For example*

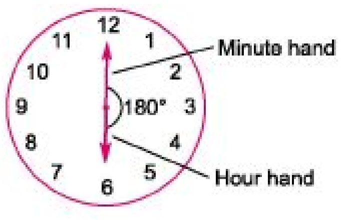
Between 11 and l'o clock, hands are together as shown in adjacent figure.



3. If two hands are at 90° they are 15 min spaces apart. This happens twice in 1 h. In a period of 12 h, the hands are at right angle 22 times (2 common positions) and in a day both hands are at right angle 44 times.



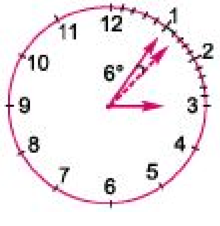
4. If two hands are in opposite direction. (i.e., 180°apart), then they are 30 min spaces apart. This happens once in 1 h. In a period of 12 h both hands are in opposite direction 11 times and in a day both hands are in opposite direction 22 times.



5. Angle covered by minute hand in 1 min

$$= \frac{\text{Total angle}}{\text{Number of spaces}}$$

$$= \frac{360°}{60} = 6° \text{ in 1 min.}$$



6. Angle covered by hour hand in 1 min.

As hour hand covers $360°$ in 12 h.

Hence, hour hand covers $\left(\frac{360°}{12}\right) = 30°$ in 1 h.

Hence, hour hand covers $\left(\frac{30°}{60}\right) = \frac{1°}{2}$ in 1 min.

7. From point 5 and 6, we can say that the minute hand goes ahead by $5\frac{1°}{2}$ in comparison to hour hand.

Concept of Slow or Fast Clocks

If a watch/clock indicates 9 : 15, when the correct time is 9, then it is said to be 15 min too fast. On the other hand, if the watch/clock indicates 6 : 45, when the correct time is 7, then it is said to be 15 min too slow.

Ex. 1 What will be angle between the two hands of a clock at 9:50?

Sol. $\because$ Angle traced by the hour hand in 12 h = $360°$

$\therefore$ Angle traced by the hour hand in 1 h = $\frac{360°}{12}$

and angle traced by the hour hand in 9 h, 50 min

i.e., $\frac{59}{6}$ h $= \frac{360°}{12} \times \frac{59}{6} = 295°$

Similarly,

$\because$ Angle traced by the minute hand in 60 min = $360°$

$\therefore$ Angle traced by minute hand in 50 min = $\frac{360°}{60} \times 50 = 300°$

$\therefore$ Required angle = $300° - 295° = 5°$

Ex. 2 A clock gains 10 s in every 3 h. If the clock was set right at 4:00 am on Monday morning, then the time it will indicate on Tuesday evening at 7:00 pm.

Sol. Difference of time between 4:00 am on Monday to 7:00 pm Tuesday

$$24 + 12 + 3 = 39 \text{ h}$$

Now, time gained by clock in 3 h = 10 s

Time gained by clock in 1 h $= \frac{10}{3}$ s

$\therefore$ Time gained by clock in 39 h $= \frac{10 \times 39}{3} = 130$ s

$\because$ Correct time is 7 : 02 : 10 pm.

Fast Track **Techniques**

to solve the QUESTIONS

Technique 1

Between *n* o'clock and (*n* +1) o'clock, the two hands of a clock will

coincide at $\left(\frac{60n}{11}\right)$ min past n.

Ex. 3 At what time between 4 o'clock and 5 o'clock, will the hands of a clock be

together?

Sol. At 4 o'clock, the hour hand is at 4 and the minute hand is at 12. It means that they are 20 min spaces apart.

To be together, the minute hand must gain 20 min over the hour hand.

As we know,

55 min is gained by minute hand in 60 min.

$\therefore$ 20 min will be gained in $\left(\frac{60}{55} \times 20\right)$ min $= \frac{60 \times 4}{11} = \frac{240}{11}$ min $= 21\frac{9}{11}$ min

Hence, the hands will coincide at $21\frac{9}{11}$ min past 4.

Fast Track Method

Here, $n = 4$ and $(n + 1) = 5$

According to the formula,

The two hands will coincide at $\frac{60n}{11}$ min past n.

or $\frac{60 \times 4}{11}$ min past 4 or $21\frac{9}{11}$ min past 4.

Ex. 4 At what time between 1 o'clock and 2 o'clock, will the hands of a clock be together?

Sol. Given, $n = 1$ and $(n + 1) = 2$

$\therefore$ The two hands will coincide at $\left(\frac{60n}{11}\right)$ min past 1.

or $\frac{60 \times 1}{11}$ min past 1 or $5\frac{5}{11}$ min past 1.

Technique 2

Between n o'clock and $(n + 1)$ o'clock, the two hands of a clock will

mutually make right angle at $(5n \pm 15) \times \frac{12}{11}$ min past n.

Ex. 5 At what time between 7 o'clock and 8 o'clock in the morning, will the both

hands of a clock be at right angle?

Sol. At 7 o'clock the minute hand will be 35 min spaces behind the hour hand. Now, when the two hands are at right angle, they are 15 min spaces apart. So, they are at right angles in the following cases.

Case I When minute hand is 15 min spaces behind the hour hand.

In this case, minute hand will have to gain $(35 - 15) = 20$ min spaces.

55 min spaces are gained by it in 60 min.

20 min spaces will be gained by it in $\left(\frac{60}{55} \times 20\right)$ min $= \frac{240}{11}$ min $= 21\frac{9}{11}$ min

Hence, they are at right angle at $21\frac{9}{11}$ min past 7.

Case II When the minute hand is 15 min spaces ahead of the hour hand.

To be in this position, the minute hand will have to gain $(35 + 15) = 50$ min spaces.

55 min spaces are gained in 60 min.

50 min spaces are gained in $\left(\frac{60}{55} \times 50\right)$ min $= \left(\frac{60}{11} \times 10\right)$ min $= 54\frac{6}{11}$ min

Hence, they are at right angle at $54\frac{6}{11}$ min past 7.

Fast Track Method

Here, $n = 7$ and $(n + 1) = 8$

According to the formula,

The hands will make right angle at

$(5n \pm 15) \times \frac{12}{11}$ min past 7 $\Rightarrow (5 \times 7 \pm 15) \times \frac{12}{11}$ min past 7.

$\Rightarrow$ $(35 + 15) \times \frac{12}{11}$ min past 7 and $(35 - 15) \times \frac{12}{11}$ min past 7.

$\Rightarrow$ $\frac{50 \times 12}{11}$ min past 7 and $\frac{20 \times 12}{11}$ min past 7.

$\Rightarrow$ $\frac{600}{11}$ min past 7 and $\frac{240}{11}$ min past 7.

Clearly, the two hands will make right angle at $54\frac{6}{11}$ min past 7 and $21\frac{9}{11}$ min past 7.

Technique 3

Between n o'clock and $(n + 1)$ o'clock, the hands of a clock will be in the same straight line (without being together) at

(i) $(5n - 30) \times \frac{12}{11}$ min past n, when $n > 6$

(ii) $(5n + 30) \times \frac{12}{11}$ min past n, when $n < 6$

Ex. 6 At what time between 7 o'clock and 8 o'clock, will the hands of a clock be in the same straight line but not together?

Sol. At 7 o'clock, the hour hand is at 7 and the minute hand is at 12. It means that the two hands are 25 min spaces apart.

To be in the same straight line (but not together), they will be 30 min spaces apart.

∴ The minute hand will have to gain (30 - 25) = 5 min spaces over the hour hand. As we know,

55 min spaces are gained in 60 min.

$\therefore$ 5 min will be gained in $\left(\frac{60}{55} \times 5\right)$ min $= 5\frac{5}{11}$ min

Hence, the hands will be in the same straight line but not together at $5\frac{5}{11}$ min past 7.

Fast Track Method

Here, $n = 7$ and $(n + 1) = 8$, also $n > 6$

According to the formula,

The hands will be in the same straight line at $(5n - 30) \times \frac{12}{11}$ min past n.

$\Rightarrow$ at $(5 \times 7 - 30) \times \frac{12}{11}$ min past 7 or at $(35 - 30) \times \frac{12}{11}$ min past 7.

$\Rightarrow$ at $\frac{12 \times 5}{11}$ min past 7 $\Rightarrow$ at $5\frac{5}{11}$ min past 7.

Ex. 7 At what time between 3 o'clock and 4 o'clock, will the hands of a clock be in opposite directions?

Sol. At 3 o'clock, the hour hand is at 3 and the minute hand is at 12. It means that the two hands are 15 min spaces apart. But to be in opposite directions, the hands must be 30 min spaces apart. Therefore, the minute hand will have to gain (30 + 15) = 45 min spaces over the hour hand.

$\because$ 55 min spaces are gained in 60 min.

$\therefore$ 45 min spaces are gained in $\left(\frac{60}{55} \times 45\right)$ min $= \frac{60 \times 9}{11}$ min $= \frac{540}{11} = 49\frac{1}{11}$ min

Hence, required time = $49\frac{1}{11}$ past 3.

Fast Track Method

Here, $n = 3$ and $(n + 1) = 4$, also $(n < 6)$

According to the formula,

The hands will be at the same straight line at $(5n + 30) \times \frac{12}{11}$ min past n.

or at $(5 \times 3 + 30) \times \frac{12}{11}$ min past 3 or at $(15 + 30) \times \frac{12}{11}$ min past 3.

or at $\frac{45 \times 12}{11}$ min past 3 or at $49\frac{1}{11}$ min past 3.

Technique 4

Between n o'clock and $(n+1)$ o'clock, the hands of a clock are x min apart at $(5n \pm x) \times \frac{12}{11}$ min past n. If minute hand is ahead, then '+' sign is used and if hour hand is ahead, then '–' sign is used.

Ex. 8 At what time between 3 o'clock and 4 o'clock, will the hands of a clock be 4 min apart?

Sol. At 3 o'clock, the minute hand is 15 min spaces behind the hour hand.

***Case* I** When the minute hand is 4 min spaces behind the hour hand. In such case, the minute hand has to gain (15 - 4)= llmin spaces

$\because$ 55 min are gained in 60 min.

$\therefore$ 11 min are gained in $\left(\frac{60 \times 11}{55}\right)$ min $= \frac{12 \times 11}{11} = 12$ min

The hands will be 4 min apart at 12 min past 3.

***Case* II** When the minute hand is 4 min spaces ahead of the hour hand.

In such case, the minute hand has to gain (15 + 4) min spaces = 19 min spaces

$\because$ 55 min are gained in 60 min.

19 min are gained in $\left(\frac{60}{55} \times 19\right)$ min

$= \left(\frac{12 \times 19}{11}\right)$ min $= 20\frac{8}{11}$ min

Hence, the hands will be 4 min apart at $20\frac{8}{11}$ min past 3.

Fast Track Method

Here, $n = 3$, $(n+1) = 4$ and $x = 4$

According to the formula,

$(5n \pm x) \times \frac{12}{11}$ min past n $= (5 \times 3 \pm 4) \times \frac{12}{11}$ min past 3

$= (15 \pm 4) \times \frac{12}{11}$ min past 3

$= (15 + 4) \times \frac{12}{11}$ min past 3 and $(15 - 4) \times \frac{12}{11}$ min past 3

$= \frac{19 \times 12}{11}$ min past 3 and $11 \times \frac{12}{11}$ min past 3

$= 20\frac{8}{11}$ min past 3 and 12 min past 3

Technique 5

If the minute hand of a clock overtakes the hour hand at intervals of xmin of the correct time, then the clock losses or gains by

$$\left(\frac{720}{11} - x\right)\left(\frac{60 \times 24}{x}\right) \text{min.}$$

Ex. 9 The minute hand of a clock overtakes the hour hand at intervals of 63 min of the correct time. How much does a clock eain or loss in a dav?

Sol. As we know that in a correct clock, the min hand gains 55 min spaces over the hour hand in 60 min.

To be together again, the minute hand must gain 60 min over the hour hand.

$\because$ 55 min are gained in $\left(\frac{60}{55} \times 60\right)$ min $= 65\frac{5}{11}$ min

But they are together after 63 min.

$\therefore$ Gain in 63 min $= \left(65\frac{5}{11} - 63\right) = 2\frac{5}{11}$ min $= \frac{27}{11}$ min

$\therefore$ Gain in 24 h (one day) $= \left(\frac{27}{11} \times \frac{60 \times 24}{63}\right)$ min $= \frac{4320}{77}$ min $= 56\frac{8}{77}$ min

As the result is positive, therefore the clock gains $56\frac{8}{77}$ min.

Fast Track Method

Here, $x = 63$ min

According to the formula,

The required result $= \left(\frac{720}{11} - x\right)\left(\frac{60 \times 24}{x}\right)$ min $= \left(\frac{720}{11} - 63\right)\left(\frac{60 \times 24}{63}\right)$ min

$= \frac{27}{11} \times \frac{60 \times 8}{21} = 56\frac{8}{77}$ min

As result is positive, therefore the clock gains $56\frac{8}{77}$ min.

Calendar

A calendar is chart or series of pages showing the days, weeks and months of a particular year. A calendar consist of 365 or 366 days divided into 12 months.

Ordinary Year

A year having 365 days is called an ordinary year

(52 complete weeks + 1 extra day = 365 days)

Leap Year

A leap year has 366 days (the extra day is 29th of Februrary) (52 complete weeks + 2 extra days =366 days.)

A leap year is divisible by 4 except for a century. For a century to be a leap year it must be divisible by 400. e.g.,

♦ Years like 1988, 2008 are leap year (divisible by 4).

♦ Centuries like 2000, 2400 are leap year (divisible by 400).

♦ Years like 1999, 2003 are not leap year (not divisible by 4).

■f Centuries like 1700, 1800 are not leap year (not divisible by 400). ■f In a century, there is 76 ordinary year and 24 leap years.

Odd Days

Extra days, apart from the complete weeks in a given period are called odd days. An ordinary year has 1 odd day while a leap year has 2 odd days. ***To find the numbers of odd days***

Months	Odd days
January	3
February	0/1 (ordinary/leap)
March	3
April	2
May	3
June	2
July	3
August	3
September	2
October	3
November	2
December	3

*■ Number of days in an ordinary year

= 365 = (52 x 7) +1 = 52 weeks + 1 odd day

.•. An ordinary year has 1 odd day. ♦ Number of days in a leap year = 366 = (52 x 7) + 2

= 52 weeks + 2 days

.•. A leap year has 2 odd days. ■f Number of days in a century (100 yr)

= 76 ordinary years + 24 leap years

(76x1+ 24x2) =124 =17x7+5 =17 week+ 5 odd days

.•. 100 yr *has* 5 *odd days.*

+ Number of odd days in 200 yr = 5x2=10 days = 1 week + 3 days = 3 odd days ■f Number of odd days in 300 yr = 5 x 3 = 15 days = 2 weeks + 1 day = 1 odd day ♦ Number of odd days in 400 yr = (5 x 4 +1) days =21 days = 3 weeks = 0 odd days

Note As 400th is a leap year, therefore 1 more day has been taken

Similarly, each one of 800 yr, 1200 yr, 1600 yr, 2000 yr, 2400 yr etc., has no odd days. Remember the adjacent table for the number of odd days in different months of an year.

• In an ordinary year, February has no odd days, but in a leap year, February has one odd day.

• The 1 st day of a century must be Tuesday, Thursday or Saturday.

• The last day of a century cannot be Tuesday, Thursday or Saturday.

Day Gain/Loss

Ordinary Year (± 1 day)

■f When we proceed forward by 1 yr, then 1 day is gained.

For example 9th August 2013 is Friday, then 9th August 2014 has to be Friday +1 = Saturday.

■f When we move backward by 1 yr, then 1 day is lost.

For example 24th December 2013 is Tuesday, then 24th December 2012 has to be Tuesday -1 = Monday.

Leap Year (+ 2 days)

♦ When we proceed forward by 1 leap year, then 2 days are gained.

For example If it is Wednesday on 25th December 2011, then it would be Friday on 25th December 2012 [Wednesday + 2] because 2012 is a leap year.

♦ When we move backward by 1 leap year, then 2 days are lost.

For example If its is Wednesday on 18th December 2012, then it would be Monday on 18th December 2011. [Wednesday -2] because 2012 is a leapyear.

Exception

• The day must have crossed 29th February for adding 2 days otherwise 1 day.

For *example* If 26th January 2011 is Wednesday, 26th January 2012 would be Wednesday + 1 = Thursday (even if 2012 is leap year, we have added + 1 day because 29 February is not crossed). If 23rd March 2011 is Wednesday, then 23rd March 2012 would be Wednesday + 2 = Friday (+ 2 days 29th February of leap year is crossed).

To Find a Particular Day on the Basis of Given Day and Date

Following steps are taken into consideration to solve such questions

***Step* I Firstly, you have to find the number of odd days between the given date and**

the date for which the day is to be determined. *Step* II The day (for a particular date) to be determined, will be that day of the week which is equal to the total number of odd days and this number is counted forward from the given day, in case the given day comes before the day to be determined. But, if the given day comes after the day to be determined, then the same counting is done backward from the given day.

Ex. 10 If 5th January, 1991 was Saturday, what day of the week was it on 4th March, 1992? Sol. Number of days between 5th January, 1991 and 4th March, 1992 = (365 - 5) days of year 1991 + 31 days of January 1992

+ 29 days of February 1992 + 4 days of March 1992

[as 1992 is completely divisible by 4, hence it is a leap year and that's why February has 29 days]

= 360 + 31+29+4= 424= 60 weeks + 4 days

Number of odd days = 4

.•. 4th March, 1992 will be 4th day beyond Saturday.

/. The required day will be Wednesday.

Ex. 11 What day of the week was it on 5th November, 1987, if it was Monday on 4th April, 1988?

Sol. Number of days between 5th November, 1987 and 4th April, 1988 = (30 - 5) days of November 1987 + 31 days of December 1987 + 31 days of January 1988 + 29 days of February 1988 + 31 days of March 1988 + 4 days **of April 1988.** [1988 is a leap year. So, February has 29 days.]

= 25 + 31+31+29 + 31+4= 151days = 21 weeks + 4 days

Number of odd days = 4

.•. 5th November 1987 will be 4 days before Monday.

Required day is Thursday.

To Find a Particular Day without Given Date and Day

Following steps are taken into consideration to solve such questions

***Step* I Firstly, you have to find the number of odd days upto the date for which the**

day is to be determined. *Step* II Your required day will be according to the following conditions

(a) If the number of odd days = 0, then required day is Sunday.

(b) If the number of odd days = 1, then required day is Monday.

(c) If the number of odd days = 2, then required day is Tuesday.

(d) If the number of odd days = 3, then required day is Wednesday.

(e) If the number of odd days = 4, then required day is Thursday.

(f) If the number of odd days = 5, then required day is Friday. (g) If the number of odd days = 6, then required day is Saturday.

Ex. 12 Find the day of the week on 26th January 1950. Sol. Number of odd days upto 26th January, 1950.

= Odd days for 1600 yr + Odd days for 300 yr + Odd days for 49 yr

+ Odd days of 26 days of January 1950 = 0+1+ (12 X2 +37)+5=0+1+ 61+5 = 67 days = 9 weeks + 4 days = 4 odd days .•. It was Thursday on 26th January 1950.

Note 49 yr has 12 leap year and 37 ordinary year

Ex. 13 Mahatma Gandhi was born on 2 October 1869. What was the day of the week?

Sol. Odd days till the year 1868.

$= 1600 + 200 + 68 = 0 + 3 + (17 \text{ leap} + 51 \text{ ordinary years})$

$= 0 + 3 + (17 \times 2 + 51 \times 1) = 3 + 85 = 88 \Rightarrow 12 \text{ weeks} + 4 \text{ days}.$

Now, total number of odd days 1869 till October 2, 1869 are.

January = 3	February = 0	March = 3
April = 2	May = 3	June = 2
July = 3	August = 3	September = 2
October = 2		

23 days = 3 weeks + 2 days.

$\therefore$ Total number of odd days = 4 + 2 = 6 days

$\therefore$ 2 October 1869 was Saturday.

Ex. 14 How many days are there in *x* weeks *x* days? Sol. Number of days in x weeks = 7 x *x* - *7x*

∴ Total number of days is *x* week *x* days = *7x* + *x* = *8x* days

Ex. 15 The calendar for the year 2007 on will be same for the year.

Sol. Count the number of odd days from year 2007 and onwards to get the sum equal to odd days.

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Odd days	1	2	1	1	1	2	1	1	1	2	1

Sum = 14 odd days = 0 odd days.

.•. Calendar for the year 2018 will be same as for the year 2007.

Fast Track Practice

Clock

1. Number of hours and minutes from 6 : 14 am to 8 : 02 pm on the same days is

[CTET2012]

(a) 2 h and 12 min

(b) 12 h and 16 min

(c) 13 h and 48 min

(d) 14 h and 16 min

(e) None of the above

2. Find the angle between the hour hand and the minute hand of a clock when the time is 4 : 10.

(a) 65° *(b)* 35° *(c)* 125° *(d)* 60° *(e)* None of the above

3. At what time between 3 o'clock and 4 o'clock, will the hands of a clock be together?

(a) $18\frac{3}{11}$ min past 3

(b) $14\frac{3}{11}$ min past 3

(c) $13\frac{2}{11}$ min past 3

(d) $16\frac{4}{11}$ min past 3

(e) None of the above

4. At what point of time after 3 O' clock, hour hand and the minute hand are at right angles for the first time? [CDS 2014]

(a) 8 O' clock (b) 4 h $37\frac{1}{8}$ min

(c) 3 h $30\frac{8}{11}$ min (d) 3 h $32\frac{8}{11}$ min

5. At what time between 9 o'clock and 10 o'clock, will the hands of a clock be in the same straight line but not together?

(a) $16\frac{2}{11}$ min past 9

(b) $16\frac{2}{11}$ min past 10

(c) $16\frac{4}{11}$ min past 9

(d) $16\frac{4}{11}$ min past 10

(e) None of the above

6. At what time between 2 o'clock and 3 o'clock, will the hands of a clock point in opposite directions?

(a) $\frac{480}{11}$ min past 2

(b) $\frac{485}{11}$ min past 2

(c) $\frac{580}{11}$ min past 2

(d) $\frac{520}{11}$ min past 2

(e) None of the above

7. At what time between 5 oclock and 6 o'clock in the morning, will the both hands of a clock be at right angle?

(a) $43\frac{7}{11}$ min past 5 and $1\frac{10}{11}$ min past 6

(b) $43\frac{7}{11}$ min past 5 and $10\frac{10}{11}$ min past 5

(c) $42\frac{7}{11}$ min past 5 and $11\frac{10}{11}$ min past 5

(d) $47\frac{7}{11}$ min past 5 and $1\frac{10}{11}$ min past 6

(e) None of the above

8. A watch which gains uniformly, is 5 min slow at 7 o'clock in the morning on Sunday and it is 5 min 48 s fast at 7 pm on following Sunday. Which day was it correct?

(a) Friday (b) Saturday

(c) Tuesday (d) Wednesday

(e) None of the above

9. A clock is set right at 6 am. The clock losses 16 min in 24 h. What will be true time, if the clock indicates 11 pm on 4th day?

(a) 12 o'clock (b) 11 pm

(c) 9 pm (d) 2 pm

(e) None of the above

10. A clock is set right at 9 am. The clock gains 10 min in 24 h. What will be true time, when the clock indicates 2 pm on the following day?

(a) 48 min past 12 (b) 48 min past 11 (c) 48 min past 1 (d) 48 min past 2 (e) None of the above

The minute hand of a clock overtakes the hour hand at intervals of 70 min of the correct time. How much in a day does the clock gain or loss?

(a) $\frac{7200}{77}$ min gain (b) $\frac{7200}{77}$ min loss

(c) $\frac{7300}{77}$ min loss (d) $\frac{7300}{78}$ min gain

12. How much does a watch gain per day, if its hands coincide every 64 min?

[SSCCGL2013]

(a) 37 min (b) $32\frac{8}{11}$

(c) 31 min (d) None of these

Calendar

13. By which of the following, a leap year must be divisible?

(a) 9 *(b)* 6 *(c)* 5 *(d)* 4 *(e)* None of the above

14. Which of the following is a leap year?

(a) 2007 *(b)* 2016

(c) 2001 *(d)* 1997

(e) None of the above

15. The last day of a century cannot be

(a) Thursday *(b)* Wednesday

(c) Friday *(d)* Monday

(e) None of the above

16. Today is Monday. What will be the day after 64 days?

(a) Saturday *(b)* Friday

(c) Thursday *(d)* Tuesday

(e) None of the above

17. If 5th March, 1999 was Friday, what day of the week was it on 9th March 2000?

(a) Wednesday *(b)* Saturday *(c)* Friday *(d)* Thursday

(e) None of the above

18. January 3, 2007 was Wednesday. What day of the week fell on January 3, 2008?

(a) Tuesday *(b)* Friday

(c) Thursday *(d)* Saturday

(e) None of the above

19. January 3, 2008 was Friday. What day of the week fell on January 3, 2009?

(a) Sunday *(b)* Monday

(c) Tuesday *(d)* Saturday

(e) None of the above

20. What was the day of the week on 2nd Jan, 2010, if it was Sunday on 1st Jan, 2006?

(a) Saturday *(b)* Thursday

(c) Sunday *(d)* Friday

(e) None of the above

21. On 6th March, 2005, Monday falls. What was the day of the week on 7th March, 2004?

(a) Tuesday *(b)* Monday

(c) Friday *(d)* Sunday

(e) None of the above

22.4th April, 1988 was Monday . What day of the week was it, on 6th November 1987? *(a)* Tuesday *(b)* Friday

(c) Sunday *(d)* Saturday

(e) None of the above

23. What was the day of the week on 17th July, 1776?

(a) Wednesday *(b)* Thursday *(c)* Monday *(d)* Saturday

(e) None of the above

24. What was the day of the week on 17th August, 2010?

(a) Sunday *(b)* Wednesday

(c) Tuesday *(d)* Friday

(e) None of the above

25. What was the day of the week on 5th June, 2002?

(a) Wednesday *(b)* Saturday *(c)* Friday *(d)* Monday

(e) None of the above

26. On what dates of March, 2005 did Saturday fall?

(a) 4th, 5th, 9th, 12th

(b) 5th, 12th, 19th, 26th

(c) 3rd, 10th, 17th, 25th

(d) 4th, 11th, 18th, 25th

(e) None of the above

27. Calendar for the year 2008 will be the same for which of the following years?

(a) 2017 *(b)* 2019

(c) 2020 *(d)* 2016

(e) None of the above

Answer with Solutions

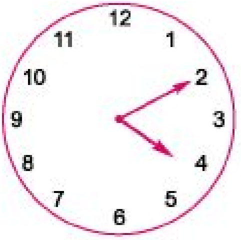
Clock

(C) Total hours from 6:14 am morning to evening 6:14 pm = 12 h and total hours from 6:14 pm evening to 8:02 pm evening = 8 :02 - 6 :14 = 1:48 h j.e.,1 h and 48 min .". Total hours and minutes

= 12h + In 48 min

= 13hand 48 min

2. (a) At 4 : 10, the hour hand is ahead of minute hand (see picture).



Given that, n = 4 and x = 10 Then, according to the formula, Recruired angle

$$= \left\{30\left(n - \frac{x}{5}\right) + \frac{x}{2}\right\}^\circ$$
$$= \left\{30\left(4 - \frac{10}{5}\right) + \frac{10}{2}\right\}^\circ$$
$$= \{(30 \times 2) + 5\}^\circ = (60 + 5)^\circ = 65^\circ$$

(d) At 3 0' clock, the hour hand is at 3 and the minute hand is at 12. It means they are 15 min spaces apart.

To be together, the minute hand must

gain 15 min over the hour hand.

As we know,

55 min is gained by minute hand in

60 min.

$$\therefore 15 \text{ min will be gained in } \left(\frac{60}{55} \times 15\right) \text{min}$$
$$= \frac{60 \times 3}{11} = \frac{180}{11} \text{ min}$$
$$= 16\frac{4}{11} \text{ min}$$

Hence, the hands will coincide at $16\frac{4}{11}$ min past 3.

Fast Track Method

Here, $n = 3$ and $(n + 1) = 4$

$\therefore$ Two hands will coincide at

$$\frac{60n}{11} \text{ min past } n.$$

[by Technique 1]

$\Rightarrow \quad \frac{60 \times 3}{11}$ min past 3

$\Rightarrow \quad \frac{180}{11}$ min past 3.

$\Rightarrow \quad 16\frac{4}{11}$ min past 3.

4. (d) Clock will make right angle at $(5n + 15) \times \frac{12}{11}$ min past n.

Given that, $n = 3$

$\therefore \ (5 \times 3 + 15) \times \frac{12}{11}$ min past 3.

[By Technique 2]

$= 30 \times \frac{12}{11}$ min past 3.

$= 32\frac{8}{11}$ min past 3.

i.e., 3 h and $32\frac{8}{11}$ min.

(C) At 9 o' clock, the hour hand is at

9 and the minute hand is at 12. It means

that the two hands are 15 min spaces

apart. To be in the same straight line (but

not together), they will be 30 min space

apart.

.•. The minute hand will have to gain

(30-15) = 15 min spaces over the hour

hand.

As we know,

55 min spaces are gained in 60 min.

$\therefore$ 15 min will be gained in

$$\left(\frac{60}{55} \times 15\right) \text{min} = \frac{180}{11} = 16\frac{4}{11} \text{ min}$$

Hence, the hands will be in the same straight line but not together at $16\frac{4}{11}$ min past 9.

Fast Track Method

Here, $n = 9$ and $n + 1 = 10\ (n > 6)$

The hands will be in the same straight line at $(5n - 30) \times \frac{12}{11}$ min past n.

[by Technique 3]

$\Rightarrow \quad (5 \times 9 - 30)\ \frac{12}{11}$ min past 9.

$\Rightarrow \quad \frac{15 \times 12}{11}$ min past 9.

$\Rightarrow \quad \frac{180}{11}$ min past 9.

$\Rightarrow \quad 16\frac{4}{11}$ min past 9.

6. (*a*) Given that $n = 2$ and $(n + 1) = 3\ (n < 6)$

The hands will be at the same straight line at

$\Rightarrow \quad (5n + 30) \times \frac{12}{11}$ min past n.

[by Technique 3]

$\Rightarrow \quad (5 \times 2 + 30) \times \frac{12}{11}$ min past 2.

$\Rightarrow \quad (10 + 30) \times \frac{12}{11}$ min past 2.

$\Rightarrow \quad \frac{40 \times 12}{11}$ min past 2.

$\Rightarrow \quad \frac{480}{11}$ min past 2.

7. (*b*) Given that, $n = 5$ and $n + 1 = 6$

The hands will make right angle at

$(5n \pm 15) \times \frac{12}{11}$ min past 5.

[by Technique 2]

$\Rightarrow \quad (5 \times 5 \pm 15) \times \frac{12}{11}$ min past 5.

$\Rightarrow \quad (25 + 15) \times \frac{12}{11}$ min past 5 and

$(25 - 15) \times \frac{12}{11}$ min past 5.

$\Rightarrow \quad 40 \times \frac{12}{11}$ min past 5

and $\quad \frac{10 \times 12}{11}$ min past 5.

$\Rightarrow \frac{480}{11}$ min past 5 and $\frac{120}{11}$ min past 5.

$\Rightarrow 43\frac{7}{11}$ min past 5 and $10\frac{10}{11}$ min past 5.

8. (*d*) Time from 7 am on Sunday to 7 pm on following Sunday

$= 7$ days 12 h $= 180$ h

$\therefore$ Watch gains $\left(5 + 5\frac{4}{5}\right)$ min or $\frac{54}{5}$ min in 180 h.

Now, $\frac{54}{5}$ min are gained in 180 h.

$\therefore$ 5 min are gained in $\left(180 \times \frac{5}{54} \times 5\right)$ h

= 83 h 20 min

= 3 days 11 h and 20 min .•. Watch is correct after 3 days 11 h and 20 min after 7 am of Sunday. /. It will be correct at 20 min past 6 pm on Wednesday.

9. (a) Time from 6 am on a day to 11 pm on 4th day = 89 h

Now, 23 h 44 min of this clock = 24 h of correct clock

89 h of this clock $= \left(24 \times \frac{15}{356} \times 89\right)$ h of correct clock $= 90$ h of correct clock

$\therefore$ Correct time will be 12 o'clock (mid-night).

10. (C) Time from 9 am in a day to 2 pm on the following day = 29 h.

24 h 10 min of this clock = 24 h of the correct clock

or $\frac{145}{6}$ h of this clock = 24 h of the correct clock

29 h of this clock $= \left(24 \times \frac{6}{145} \times 29\right)$ h of the correct clock

= 28 h 48 min of correct clock

.\ The correct time is 28 h 48 min after

9 am. This will be 48 min past 1.

11. *(b)* Given that, = 70 min

The required result

$$= \left(\frac{720}{11} - x\right)\left(\frac{60 \times 24}{x}\right) \text{min}$$

[by Technique 5]

$$= \left(\frac{720}{11} - 70\right)\left(\frac{60 \times 24}{70}\right) \text{min}$$

$$= \left(\frac{720 - 770}{11}\right)\left(\frac{60 \times 24}{70}\right) \text{min}$$

$$= -\left(\frac{50}{11} \times \frac{6 \times 24}{7}\right) \text{min} = -\frac{7200}{77} \text{ min}$$

–ve sign indicates that there is a loss.

12. *{b)* If the minute hand of a clock overtakes the hour hand at interval *x* min of the correct time, then clock losses or gains bv

$$\left(\frac{720}{11} - x\right) \times \left(\frac{60 \times 24}{x}\right) \text{min}$$

[by Technique 5] Positive for gain and negative for loss Here, x = 64

putting the value of x in above formula, we get

$$\left(\frac{720}{11} - 64\right) \times \left(\frac{60 \times 24}{64}\right)$$

$$\left(\frac{720 - 704}{11}\right) \times \left(\frac{15 \times 24}{16}\right)$$

$$\frac{16 \times 15}{11} \times \frac{3}{2} = \frac{360}{11} = 32\frac{8}{11} \text{min}$$

Calendar

13. *id)* A leap year must be divisible by 4, as every 4th year is a leap year.

14. (£>) As 2016 is completely divisible by 4.

15. (a) 100 yr have 5 odd days.

.•. Last day of 1st century is Friday. 200 yr have

(5x2)= 1 week+ 3 odd days = 3 odd days .'. Last day of 2nd century is Wednesday, 300 yr have (5 X 3) = 2 week + 1 odd day

=1 odd day .". Last day of 3rd century is Monday. 400 yr have 0 odd day. .•. Last day of 4th century is Sunday. This cycle is repeated. .•. Last day of a century cannot be TXiesday or Thursday or Saturday.

16. *id)* Each day of the week is repeated after 7 days.

So, after 63 days, it will be Monday. Hence, after 64 days, it will be Tuesday.

17. (d) 5th March, 1999 is Friday. Then, 5th March 2000

= Friday + 2 = Sunday.

{■.• 2000 is leap year and it crosses 29th

Feb 2000, so 2 is taken as odd day}

∴ 5th March 2000 = Sunday.

Then, 9th March 2000 = Thursday.

18. (c) The year 2007 is an ordinary year, so it has 1 odd day.

3rd day of the year 2007 was Wednesday. .•. 3rd day of the year 2008 will be one day beyond the Wednesday. Hence, it will be Thursday.

19. (a) The year 2008 is a leap year, so it has 2 odd days, 3rd day of the year 2008 was Friday.

∴ 3rd day of the year 2009 will be

2 days beyond the Friday

Hence, the required answer is Sunday.

20. (a) On 31st December, 2005, it was Saturday. Number of odd days from the year 2006 to the year 2009

= 1 odd day of 2006 + 1 odd day of 2007 + 2 odd days of 2008 (leap year)

+ 1 odd day of 2009 =1+1+2+1=5

∴ On 31st December 2009, it was Thursday.

Thus, on 2nd January 2010, it was Saturday.

21. *(b)* 6th March 2005 = Monday. Then, 6th March 2004

= Monday — 1 day = Sunday. {•.• 2004 is a leap year but it does not cross 29th February of 2004, so only 1 is taken as odd day}. 6th March 2004 = Sunday 7th March 2004 = Monday.

22. *{b)* Number of odd days between 6th November, 1987 and 4th April, 1988.

= (30 - 6) days of November 1987 + 31

days of December 1987 + 31 days of

January 1988 + 29 days of February

1988 + 31 days of March 1988

+ 4 days of April 1988

= {(30-6) + 31} + (31 + 29 + 31 + 4)

= 24 + 31 + 31 + 29 + 31 + 4 = 150

= 21 weeks + 3 days = 3 odd days

∴ 6th November, 1987 will be three

days backward from Monday.

.•. Required day is Friday.

23. (a) Period upto 17th July, 1776

= (1775 yr + Period form 1st January 1776 to 17th July, 1776) Counting of odd days In 1600 yr = 0 In 100yr = 5 75 yr = 18 leap years + 57 ordinary years = (18 x 2 + 57 x 1) odd days = 93 odd days

= (13 weeks + 2 days)

= 2 odd days ∴ 1775 yr have (0 + 5+2) odd days

= 7 odd days = 1 week + 0 odd day

= 0 odd day Number of days between 1.1.1776 to 17.7.1776 January + February + March

+ April + May + June + July 31 + 29 + 31 + 30 + 31 + 30 + 17 = 199 days = 28 weeks + 3 odd days ∴ Total number of odd days = (3 + 0) = 3 Hence, the required day is Wednesday.

24. (c) Period upto 17th August, 2010 = (2009yr + Period from 1.1.2010 to 17.8.2010)

Counting of odd days Odd days in 1600 yr = 0 Odd days in 400 yr = 0

9 yr = (2 leap years + 7 ordinary years) = (2x2+7xl) = l week + 4 days = 4 odd days

Number of days between 1.1.2010 to 17.8.2010

January + February + March + April + May + June + July + August. = (31 +28 + 31+30 + 31 + 30 + 31

+ 17) days = 229 days = 32 weeks + 5 odd days Total number of odd days = (0 + 0 + 4 + 5) days = 9 days = 1 week + 2 odd days Hence, the required day is Tuesday.

25. (a) Period upto 5th June, 2002

= (2001 yr + Period from 1.1.2002 to

5.6.2002)

∴ Odd days in 1600 yr = 0

Odd days in 400 yr = 0

Odd days in 1 ordinary year = 1

Odd days in 2001 year

= (0 + 0 + 1) = 1 Number of days from 1.1.2002 to 5.6.2002

January + February + March + April + May + June

= 31 + 28 + 31 + 30 + 31+5

= 156 days = 22 weeks + 2 days

= 2 odd days .•. Total number of odd days

= (1 + 2) = 3 .•. The required day is Wednesday.

26. *(b)* First, we find out the day on 1.03.2005. Period upto 1.3.2005 = (2004yr + Period from 1.1.2005 to 1.3.2005)

∴ Odd day in 1600yr = 0

Odd day in 400 yr = 0

4 yr = (1 leap year + 3 ordinary years)

= (1x2+3x1) = 5 odd days

Number of days between 1.1.2005 to

1.3.2005,

January + February + March

= 31 + 28 + 1

= 60 days = (8 weeks + 4 days)

= 4 odd days Total number of odd days

= (0 + 0 + 5+4) = 9

= 1 weak + 2 odd days

= 2 odd days ∴1.3.2005 was Tuesday. So, Saturday will fall on 5.3.2005. Hence, Saturday lies on 5th, 12th, 19th, and 26th of March 2005.

27. *(b)* Count the number of odd days from the year 2008 onwards to get the sum equal to 0 odd day.

Let us see

2008	2009	2010	2011	2012	2013
2	1	1	1	2	1

2014	2015	2016	2017	2018
1	1	2	1	1

Sum = 2+1+1+1 + 2+1 + 1 + 1

+2+1+1 = 14 days = 2 weeks = 0 odd days .". Calendar for the year 2019 will be the same as for the year 2008.

Chapter

30

Linear Equations

A linear equation is an equation for a straight line. So, the equation which has degree 1, i.e., which has linear power of the variables, is called a **linear equation.**

It is written as $ax + by + c = 0$, where a, b and c are real numbers and a and b both are not zero.

For example y = 2x +1 is a linear equation. The different values of x and y are

x	1	2	0	-1	-2	-3
y	3	5	1	-1	-3	-5

All these values of (x, y) as (1,3), (2,5), (0,1) etc., are the solutions of the given linear equation.

If we are given two equations in x and y, then we are to find those values of x and y which satisfy both the given equations.

Linear Equation in One Variable

A linear equation in which number of unknown variable is one, is known as linear equation in one variable. *For example,* $3x + 5=10$, $y + 3=5$ etc.

Linear Equation in Two Variables

A linear equation in which number of unknown variables are two, is known as linear equation in two variables. *For example,* $2x + 5y = 10$, $x + 4y = 8$ etc.

Linear Equation in Three Variables

A linear equation in which number of unknown variables are three, is known as linear equation in three variables. *For example,* $4x + 6y + 7z = 20$, $x + y + 2z = 5$ etc.

MIND IT!

1. Linear equation in one variable represents a point in number line.

2. Linear equation in two variable represents a line in *XY-p\ane* (cartesian plane).

3. Linear equation in three variables represents a plane in XVZ-coordinate system.

Methods of Solving Linear Equations

There are following methods which are useful to solve the linear equations.

Substitution Method

In this method, first the value of one variable must be represented in the form of another variable and put this value in another equation and solve it. Thus, a value of one variable is obtained and this value is used to find the value of another variable.

bx. *1* Solve the following equations with substitution method.

$2x - y = 3,4x - y = 5$ Sol. $2x - y = 3$ ...(i)

and $4x - y = 5$ •••(ii)

From Eq. (i),

$y = 2z - 3$ On putting the value of y in Eq. (ii), we get

$4x - (2x - 3) = 5 \Rightarrow 2i + 3 = 5$

$x = 1$ On putting the value of x in Eq. (i), we get 2xl-j=3 $y = -1$ Hence, $x = 1$ and $y = -1$

Elimination Method

In this method, the coefficients of one of the variables of each equation become same by multiplying a proper multiple. Solve these equations and by which we get the value of another variable and thus with the help of this value, we can find the value of another variable.

Ex. 2 Solve following equations with elimination method.

11* - 5y + 61 = 0, $3x - 20y - 2 = 0$

Sol. 11a; - $5y + 61 = 0$ ...(i)

and $3x - 20y - 2 = 0$ ...(ii)

Now, Eq. (i) is multiplied by 3 and Eq. (ii) is multiplied by 11 and then subtracting them, we get

$$\begin{array}{r} 33x - 15y + 183 = 0 \\ 33x - 220y - 22 = 0 \\ - \quad + \qquad + \quad \\ \hline 205y + 205 = 0 \end{array}$$

$$\therefore \qquad y = -1$$

On putting the value of y in Eq. (ii), we get

$3x - 20 \times (-1) - 2 = 0$

$\Rightarrow 3z + 20 - 2 = 0 \Rightarrow 3x = -18$

$x = -6$

Hence, $x = -6$ and $y = -1$

Cross Multiplication Method

Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
are two equations.

$\therefore$ By cross multiplication method,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \quad x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Ex. 3 Solve the following equations with cross multiplication method.

$$2x - 3y + 1 = 0,\ 3x + 4y - 5 = 0$$

Sol. $2x - 3y + 1 = 0$ ...(i)

and $3x + 4y - 5 = 0$ ...(ii)

By cross multiplication method,

$$\frac{x}{(-3)(-5) - 4 \times 1} = \frac{y}{1 \times 3 - (-5) \times 2} = \frac{1}{2 \times 4 - 3 \times (-3)}$$

$$\Rightarrow \quad \frac{x}{15 - 4} = \frac{y}{3 + 10} = \frac{1}{8 + 9}$$

$$\Rightarrow \quad \frac{x}{11} = \frac{y}{13} = \frac{1}{17}$$

$$\therefore \quad x = \frac{11}{17} \text{ and } y = \frac{13}{17}$$

Consistency of the System of Linear Equations

A set of linear equations is said to be **consistent,** if there exists atleast one solution for these equation. A set of linear equations is said to be **inconsistent,** if there are no solution for these equation. Let us consider a system of two linear equations as shown,

$a_1x+b_1y+c_1=0$ and $q_2x+b_2y+c_2=0$

Consistent Svstem

The above system will be consistent, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then system has unique solution and represents a pair of intersecting lines.
- If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then system has infinite solutions and represents overlapping lines.

Inconsistent System

The above system will be inconsistent, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ and do not have any solution. It represents a pair of parallel lines.

MIND IT!

Possible Solutions of Linear Equations

If system of equation is following

$$p_1x + a_1y + r_1 = 0$$

and $$p_2x + a_2y + r_2 = 0,$$

If	$\frac{p_1}{p_2} \neq \frac{a_1}{a_2}$	[unique solution]
If	$\frac{p_1}{p_2} = \frac{a_1}{a_2} \neq \frac{r_1}{r_2}$	[no solution]
If	$\frac{p_1}{p_2} = \frac{a_1}{a_2} = \frac{r_1}{r_2}$	[many solutions]

Ex. 4 Check whether the given system is consistent or not. If yes, then find the solution. *x-2y = 0,* 3x + 4y-20 = 0

Sol.

$$x - 2y = 0 \quad \text{...(i)}$$

$$3x + 4y - 20 = 0 \quad \text{...(ii)}$$

Here, $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = -\frac{2}{4} = -\frac{1}{2}; \frac{c_1}{c_2} = -\frac{0}{20} = 0$

$\therefore$ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Thus, system is consistent and has a unique solution. To find the solution of the given equations, multiply Eq. (i) by 2 and add it to Eq. (ii), we get

2x - 4?/ + 3x + 4«/ - 20 = 0 => 5x = 20 =» x = 4 Putting the value of *x* in Eq. (i),

4-22/= 0 => *V* = 4/2 = 2 Hence, given system has the solution (4, 2).

Ex. 5 For what value of *K,* the system of equations *Kx - iy* - 8 = 0 and *8x - by* -12 = 0 has a unique solution?

Sol. For a unique solution, we have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{K}{8} \neq \frac{-4}{-6} \Rightarrow \frac{K}{8} \neq \frac{2}{3}$$

$$\therefore \quad K \neq \frac{16}{3}$$

Ex. 6 For what value of *K,* the system of equations $2x + 4y + 16 = 0$ and $3x + Ky + 24 = 0$ has an infinite number of solutions.

Sol. For infinite number of solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{3} = \frac{4}{K} = \frac{16}{24}$$

$$\therefore \quad K = 6$$

Ex. 7 For what value of *K,* the system of equations Kx - 20y -6 = 0 and $6x$ - lOy -14 = 0 has no solution?

Sol. For no solution, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{K}{6} = \frac{-20}{-10} \neq \frac{-6}{-14}$$

$$\therefore \quad K = \frac{6 \times 20}{10} = 12$$

Ex. 8 If ix + 9y = 43 and $6x + iy$= 36 are the linear equations in x and y, then find the values of x and y.

Sol. Given,

$$4x + 9y = 43 \quad \text{...(i)}$$

$$6x + 4y = 36 \quad \text{...(ii)}$$

On multiplying Eq. (i) by 3 and Eq. (ii) by 2 and subtracting

$$\begin{array}{r} 12x + 27y = 129 \\ 12x + 8y = 72 \\ - \quad - \quad\quad - \\ \hline 19y = 57 \end{array}$$

$$\therefore \quad y = \frac{57}{19} = 3$$

Putting the value of y in Eq. (i), we get

$$4x + 9 \times 3 = 43$$

$$\Rightarrow \quad 4x + 27 = 43$$

$$\Rightarrow \quad 4x = 43 - 27 = 16$$

$$\therefore \quad x = \frac{16}{4} = 4$$

Hence, $x = 4$ and $y = 3$

Ex. 9 The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 20 km, the charge paid is ? 205 and for a distance of 25 km, the charge paid is ? 255. Find the fixed charges and the charge per km. How much does a person have to pay for covering a distance of 50 km? Sol. Let the fixed charges be ? x and charge per km be ? y.

Then,

$$x + 20y = 205 \quad \text{...(i)}$$
$$x + 25y = 255 \quad \text{...(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$\begin{array}{r} x + 20y = 205 \\ x + 25y = 255 \\ - \quad - \quad - \\ \hline -5y = -50 \end{array}$$

$\therefore$ $$y = 10$$

Putting the value of y in Eq. (i), we get

$$x + 20 \times 10 = 205 \Rightarrow x + 200 = 205$$

$\therefore$ $$x = 5$$

Hence, amount paid for a distance of 50 km $= x + 50y = 5 + 50 \times 10 =$ ₹ 505

Multi Concept

QUESTIONS

1. *Solve for x and y* $\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5}, \frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$

(a) 2, 3 (b) 5, 7 (c) 3, 4 (d) 1, 1

➡ (*d*) Let $\frac{1}{3x+2y} = P$ and $\frac{1}{3x-2y} = Q$

$\therefore$ $$2P + 3Q = \frac{17}{5} \quad \text{...(i)}$$

Similarly, $$5P + Q = 2 \quad \text{...(ii)}$$

On multiplying Eq. (i) by 5 and Eq. (ii) by 2 and subtracting, we get

$$10P + 15Q = 17 \quad \text{...(iii)}$$
$$10P + 2Q = 4 \quad \text{...(iv)}$$
$$- \quad - \quad -$$

$\Rightarrow$ $$13Q = 13 \Rightarrow Q = 1$$

On substituting $Q = 1$ in Eq. (ii), we get

$$5P + Q = 2 \Rightarrow 5P = 1 \Rightarrow P = \frac{1}{5}$$

Now, $$P = \frac{1}{5} = \frac{1}{3x+2y}, Q = 1 = \frac{1}{3x-2y}$$

$\Rightarrow$ $$3x + 2y = 5 \quad \text{...(v)}$$

and $$3x - 2y = 1 \quad \text{...(vi)}$$

On adding Eqs. (v) and (vi), we get

$$6x = 6$$
$$x = 1 \text{ and } y = 1$$

2. *Let a, b, c be the positive numbers. The following system of equations in x, y and z*

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1,\ \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,\ \frac{-x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \text{ has}$$

(a) no solution (b) unique solution

(c) infinitely many solutions (d) finitely many solutions

➥ (*d*) Here,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \quad \text{...(i)}$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \quad \text{...(ii)}$$

and

$$\frac{-x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \quad \text{...(iii)}$$

On adding Eqs. (i) and (ii), we get

$$\frac{x^2}{a^2}=1 \Rightarrow x=\pm a$$

Similarly, on adding Eqs. (ii) and (iii), we get

$$\frac{z^2}{c^2}=1 \Rightarrow z=\pm c$$

Similarly, $y=\pm b$

$\therefore$ *x*, *y* and *z* have finitely many solutions.

Fast Track Practice

Exercise© *Base Level Questions*

1. If *2x* + *Zy* = 29 and *y* = *x* + 3, what is the value of *x*>. [Bank PO 2006]

(a) 4 fbj 5 fc) 6 *(d)* 7 (e) None of the above

2. If 3% + *7y*= 75 and 5* - 5y = 25, what is the value of *x* + *yl*

(a) 14 *(b)* 15 *(c)* 16 (tfj 17 (e) None of the above

The cost of 2 sarees and 4 shirts is ? 16000 while 1 saree and 6 shirts cost the same. The cost of 12 shirts is *(a)* ? 12000 *(b)* ? 24000

(c) ? 48000

fc/J Cannot be determined *(e)* None of the above

^■ˉ Deepak has some hens and some goats. If the total number of animal heads is 90 and the total number of animal feet is 248, what is the total number of goats Deepak has? [PNB Mgmt. Trainee 2010]

(a) 32 *(b)* 36 *(c)* 34

(d) Cannot be determined

(e) None of the above

5. The sum of the two digits is 15 and the difference between them is 3. What is the product of the digits? [PNB Mgmt Trainee 2010]

(a) 56 *(b)* 63 *(c)* 42

(d) Cannot be determined

(e) None of the above

"• The numerator of a fraction is *6x* + 1 and the denominator is 7 - 4x *x* can have any value between - 2 and 2, both included. The values of *x* for which the numerator is greater than the denominator, are

(a) $\frac{3}{5} < x \le 2$ (b) $\frac{3}{5} \le x \le 2$

(c) $0 < x \le 2$ (d) $-2 \le x < 2$

(e) None of the above

7. If $\frac{x}{4} + \frac{y}{3} = \frac{10}{24}$ and $\frac{x}{2} + y = 1$, then the value of $(x + y)$ is

(a) 1 (b) 3/2

(c) 2 (d) 4

(e) None of the above

8. If $\frac{x+y-8}{2} = \frac{x+2y-14}{3} = \frac{3x+y-12}{11}$, then find the values of x and y, respectively.

(a) 2, 6 *(b)* 4, 8 *(c)* 3, 5 *(d)* 4, 5 *(e)* None of the above

The cost of 21 pencils and 9 clippers is ? 819. What is the total cost of 7 pencils and 3 clippers together? [DMRC CRA 2012]

(a) ? 204 *(b)* ? 409 *(c)* *X* 273 *(d)* *X* 208

1**. The value of k for which $kx+3y-k+3=0$ and 12* + ky - k, have infinite solutions, is [CLAT 2013]

(a) 0 (b) –6 (c) 6 (d) 1

11. If $6x - 10y = 10$ and $\frac{x}{x+y} = \frac{5}{7}$, then $(x - y) = ?$

(a) 6 (b) 8 (c) 12 (d) 3

(e) None of the above

12. Solve $6x + 3y = 7xy$ and $3x + 9y = 11xy$.

(a) $x = 1, y = \frac{3}{2}$ (b) $x = -1, y = \frac{2}{3}$

(c) $x = \frac{3}{2}, y = \frac{1}{2}$ (d) $x = 1, y = -\frac{3}{2}$

(e) None of the above

13. In a rare coin collection, there is one gold coin for every three non-gold coins. 10 more gold coins are added to the collection and the ratio of gold coins to non-gold coins would be 1: 2. Based on the information; the total number of coins in the collection now becomes. [CSAT 2013]

(a) 90 *(b)* 80 *(c)* 60 *(d)* 50

14. The system of equations $2x+ 4y = 6$ and

$4x + 8y = 6$ has

(a) exactly two solutions

(b) no solution

(c) infinitely many solutions

(d) a unique solution

(e) None of the above

15. If $\frac{\sqrt{3+x} + \sqrt{3-x}}{\sqrt{3+x} - \sqrt{3-x}} = 2$, then x is equal to

[SSC CGL 2010]

(a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{5}{7}$ (d) $\frac{7}{5}$

16. In an examination, a student scores 4 marks for every correct answer and losses 1 mark for every wrong answer. *A* student attempted all the 200 questions and scored 200 marks. Find the number of questions, he answered correctly.

[SSCCGL2010] *{a)* 82 *(b)* 80 *(c)* 68 *(d)* 60

The graphs of $ax + by$ = c, $dx + ey = f$ will be

I. parallel, if the system has no solution. II. coincident, if the system has finite

numbers of solutions. III. intersecting, if the system has only one

solution. Which of the above statements are correct?

[CDS 2012]

(a) Only I and II *(b)* Only II and III *(c)* Only I and III *(d)* I, II and III

18. If $3^{* \,+ y}$ = 81 and $81^{st" \, y}$ = 3, then what is the value of *xl* [CDS 2012]

(a) 17/16 *(b)* 17/8

(c) MIA (d) 15/4

19. Ten chairs and six tables together cost *X* 6200, three chairs and two tables together cost ? 1900. The cost of 4 chairs and 5 tables is [CDS 2013]

(a) ? 3000 *(b) X* 3300

(c) X 3500 *(d)* ? 3800

20. The system of equations 3x + *y* - 4 = 0 and *6x*+ 2y-8=0 has [CDS 2013]

(a) a unique solution x = 1, *y* = 1

(b) a unique solution x = 0, *y= A*

(c) no solution (d) infinite solutions

21.Hx + *y - 7* = 0and3x+ ;y - 13 = 0, then what is $4x^2 + y^2 + 4xy$ equal to? [CDS 2013] *(a)* 75 *(b)* 85 *(c)* 91 *(d)* 100

Exercise © *Higher Skill Level Questions*

1. If $2^a + 3^b = 17$ and $2^{a+2} - 3^{b+1} = 5$, then

(a) $a = 2, b = 3$ (b) $a = -2, b = 3$

(c) $a = 2, b = -3$ (d) $a = 3, b = 2$

(e) None of the above

2. The solution of the equations $\frac{p}{x} + \frac{q}{y} = m$ and $\frac{q}{x} + \frac{p}{y} = n$ is **[CDS 2001]**

(a) $x = \frac{q^2 - p^2}{mp - nq}, y = \frac{p^2 - q^2}{np - mq}$

(b) $x = \frac{p^2 - q^2}{mp - nq}, y = \frac{q^2 - p^2}{np - mq}$

(c) $x = \frac{p^2 - q^2}{mp - nq}, y = \frac{p^2 - q^2}{np - mq}$

(d) $x = \frac{q^2 - p^2}{mp - nq}, y = \frac{q^2 - p^2}{np - mq}$

3. One says "Give me a hundred friend ! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you." Find the amount of their capitals.

(a) ?40, ? 170

(b) ? 60, ? 180

(c) *X* 80, *X* 200

(d) Cannot be determined

(e) None of the above

4. The ratio of incomes of two persons is 8 : 5 and the ratio of their expenditure is 2 : 1. If each of them manages to save ? 1000 per month, find the difference of their monthly income.

(a) ? 2500 *(b)* ? 1500

(c) *X* 1000 *(d)* ? 700

(e) None of the above

5. A fraction becomes 7/8, if 5 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator, it becomes $\frac{6}{7}$. Find the fraction.

(a) $\frac{8}{11}$ (b) $\frac{9}{11}$ (c) $\frac{10}{11}$

(d) Cannot be determined

(e) None of the above

6. If $\frac{3}{x+y} + \frac{2}{x-y} = 2$ and $\frac{9}{x+y} - \frac{4}{x-y} = 1$, then what is the value of $\frac{x}{y}$? [CDS 2012]

(a) 3/2 (b) 5 (c) 2/3 (d) 1/5

7. If $\frac{a}{b} - \frac{b}{a} = \frac{x}{y}$ and $\frac{a}{b} + \frac{b}{a} = x - y$, then what is the value of x? [CDS 2012]

(a) $\frac{a+b}{a}$ (b) $\frac{a+b}{b}$

(c) $\frac{a-b}{a}$ (d) None of these

8. If $\frac{x}{2} + \frac{y}{3} = 4$ and $\frac{2}{x} + \frac{3}{y} = 1$, then what is $x + y$ equal to? [CDS 2013]

(a) 11 *(b)* 10 *(c)* 9 *(d)* 8

Answer with Solutions

Exercise © *Base Level Questions*

1. (a)2x+ 3k = 29 ...(i)

and $y = x + 3$ ... (ii)

Putting the value of y from Eq. (ii) to Eq. (i), we get

$2x + 3y = 29$ 2x + 3(x + 3) = 29 2x + 3x + 9 = 29 => 5x = 20 x = 4

2. (d) 3x+ 77 = 75 ...(i)

$5x - 5y = 25$

x — 7 = 5 ... (ii)

On multiplying Eq. (ii) by 7 and adding to Eq. (i), we get

$$\begin{aligned} 3x + 7y &= 75 \\ 7x - 7y &= 35 \\ \hline 10x &= 110 \\ \therefore \quad x &= 11 \end{aligned}$$

Putting the value of x in Eq. (ii), we get 11-7 = 5 7 = 6 x+ 7 = 6 + 11 = 17

(6) Let cost of one saree and shirt be *x* and 7, respectively.

2x +4y = 16000 ...(i)

x+67 = 16000 ...(ii)

On multiplying Eq. (ii) by 2 and subtracting from Eq. (i), we get

$$\begin{aligned} 2x + 4y &= 16000 \\ \underset{-}{2x} \underset{-}{+} 12y &\underset{-}{=} 32000 \\ \hline -8y &= -16000 \\ \therefore \quad y &= 2000 \end{aligned}$$

Putting the value of y in Eq. (ii), we get

$$\begin{aligned} x + 6 \times 2000 &= 16000 \\ \therefore \quad x &= 4000 \\ \therefore \quad \text{Cost of 12 shirts} &= 12 \times y \\ &= 12 \times 2000 = ₹\,24000 \end{aligned}$$

4. (c) Let hens = *H,* goats = *G* According to the question,

ff+G = 90 ...(i)

2H + 4G = 248 ...(ii)

On multiplying Eq. (i) by 2 and subtracting from Eq. (ii), we get

$$\begin{aligned} 2H + 2G &= 180 \\ \underset{-}{2H} \underset{+}{+} 4G &\underset{-}{=} 248 \\ \hline -2G &= -68 \\ \therefore \quad G &= 34 \end{aligned}$$

5. (*e*) Let the number be x and y.

Then, according to the question,

$$x + y = 15 \quad \dots(i)$$

$$x - y = 3 \quad \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2x = 18 \Rightarrow x = 9$$

On putting the value of x in Eq. (i), we get

$$y = 6$$

$\therefore$ Product $= xy = 9 \times 6 = 54$

6. (*a*) $6x + 1 > 7 - 4x$

$$\Rightarrow \quad x > \frac{3}{5}$$

$$\therefore \quad \frac{3}{5} < x \le 2$$

7. (*b*) Given, $\dfrac{x}{4} + \dfrac{y}{3} = \dfrac{10}{24}$

$$\Rightarrow \quad \frac{3x + 4y}{12} = \frac{10}{24}$$

$$\therefore \quad 3x + 4y = 5 \quad \dots(i)$$

and $$\frac{x}{2} + y = 1$$

$$x + 2y = 2 \quad \dots(ii)$$

On multiplying Eq. (ii) by 2 and subtracting from Eq. (i),

$$\begin{array}{r} 3x + 4y = 5 \\ 2x + 4y = 4 \\ - \quad - \quad - \\ \hline x = 1 \end{array}$$

On putting the value of x in Eq. (ii), we get

$$x + 2y = 2 \Rightarrow 1 + 2y = 2 \Rightarrow y = \frac{1}{2}$$

$$\therefore \quad x + y = 1 + \frac{1}{2} = \frac{3}{2}$$

8. (*a*) Given,

$$\frac{x + y - 8}{2} = \frac{x + 2y - 14}{3} = \frac{3x + y - 12}{11}$$

$$\Rightarrow \quad \frac{x + y - 8}{2} = \frac{x + 2y - 14}{3}$$

$$\Rightarrow \quad 3x + 3y - 24 = 2x + 4y - 28$$

$$\Rightarrow \quad 3x + 3y - 2x - 4y = -28 + 24$$

$$x - y = -4 \quad \dots(i)$$

Again, $$\frac{x + 2y - 14}{3} = \frac{3x + y - 12}{11}$$

$$\Rightarrow \quad 11x + 22y - 154 = 9x + 3y - 36$$

$$\Rightarrow \quad 2x + 19y = 118 \quad \dots(ii)$$

On multiplying Eq. (i) by 2 and subtracting from Eq. (ii), we get

$$\begin{array}{r} 2x - 2y = -8 \\ 2x + 19y = 118 \\ - \quad - \quad\quad - \\ \hline -21y = -126 \end{array}$$

$\therefore \quad y = 6$

On putting the value of y in Eq (i), we get

$$x - 6 = -4$$

$\therefore \quad x = 2$

$\therefore \quad x = 2$ and $y = 6$

9. (*c*) Let cost of 1 pencil and 1 clipper be p and c, respectively.

Now, according to the question,

$$21p + 9c = ₹\,819$$

$\Rightarrow \quad 3(7p + 3c) = ₹\,819$

$\Rightarrow \quad 7p + 3c = ₹\,273$

Cost of 7 pencils and 3 clippers = ₹ 273

10. (*c*) For infinite solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{K}{12} = \frac{3}{K} = \frac{-K+3}{-K}$$

$\Rightarrow \quad \dfrac{K}{12} = \dfrac{3}{K} \Rightarrow K^2 = 36$

$\therefore \quad K = \sqrt{36} = 6$

11. (*d*) Given, $6x - 10y = 10$ ...(i)

and $\quad \dfrac{x}{x+y} = \dfrac{5}{7}$

$\Rightarrow \quad 7x = 5x + 5y$

$\Rightarrow \quad 2x - 5y = 0$ ...(ii)

On multiplying Eq. (ii) by 2 and subtracting from Eq. (i), we get

$$\begin{array}{r} 6x - 10y = 10 \\ 4x - 10y = 0 \\ - \quad + \quad\quad - \\ \hline 2x = 10 \end{array} \qquad \therefore \quad x = 5$$

Putting the value of x in Eq. (i), we get

$$30 - 10y = 10 \Rightarrow 10y = 20$$

$\Rightarrow \quad y = 2$

$\therefore \quad (x - y) = 5 - 2 = 3$

12. (*a*) Given equations are

$$6x + 3y = 7xy \Rightarrow \frac{6}{y} + \frac{3}{x} = 7 \quad \text{...(i)}$$

and $\quad 3x + 9y = 11xy$

$\Rightarrow \quad \dfrac{3}{y} + \dfrac{9}{x} = 11$ ...(ii)

On multiplying Eq. (ii) by 2 and subtracting from Eq. (i), we get

$$\begin{array}{r} \dfrac{6}{y} + \dfrac{3}{x} = 7 \\ \dfrac{6}{y} + \dfrac{18}{x} = 22 \\ - \quad - \quad\quad - \\ \hline -\dfrac{15}{x} = -15 \end{array}$$

$\therefore \quad x = 1$

On putting this value in Eq. (i), we get

$$\frac{6}{y} + 3 = 7$$

$$\Rightarrow \quad \frac{6}{y} = 4$$

$$\therefore \quad y = \frac{6}{4} = \frac{3}{2}$$

13. (a) Let the number of gold coins initially be x and the number of non-gold coins be *y*. According to the question,

3x = y When 10 more gold coins, total number of gold coins become x + 10 and the number of non-gold coins remain the same at *y*. Now, we have 2 (10 + x) = *y* Solving these two equations, we get x = 20 and *y* = 60. Total number of coins in the collection at the end is equal to

x+ 10 + *y* = 20 + 10 + 60 = 90.

14. (*b*) Given equations $2x + 4y = 6$ and $4x + 8y = 6$.

Then, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$; $\frac{b_1}{b_2} = \frac{4}{8} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{6}{6} = 1$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, given system has no solution .

15. (*b*) Given, $\frac{\sqrt{3+x} + \sqrt{3-x}}{\sqrt{3+x} - \sqrt{3-x}} = 2$

Let $\quad \sqrt{3+x} = a$ and $\sqrt{3-x} = b$

Then, $\quad \frac{a+b}{a-b} = \frac{2}{1}$

$\Rightarrow \quad a + b = 2a - 2b \Rightarrow a = 3b$

$\therefore \quad \sqrt{3+x} = 3\sqrt{3-x}$

On squaring both sides, we get

$$(\sqrt{3+x})^2 = (3\sqrt{3-x})^2$$

$\Rightarrow \quad 3 + x = 9(3 - x)$

$\Rightarrow \quad 3 + x = 27 - 9x$

$\Rightarrow \quad 10x = 24$

$\therefore \quad x = \frac{12}{5}$

16. (*b*) Let the number of correct answers be x and number of wrong answers be y.

Then, $\quad 4x - y = 200 \quad \ldots(i)$

and $\quad x + y = 200 \quad \ldots(ii)$

On adding Eqs. (i) and (ii), we get

$$\begin{array}{r} 4x - y = 200 \\ x + y = 200 \\ \hline 5x = 400 \end{array}$$

$\therefore \quad x = 80$

17. (*c*) $ax + by = c$ and $dx + ey = f$

$$\frac{a_1}{a_2} = \frac{a}{d}, \frac{b_1}{b_2} = \frac{b}{e}, \frac{c_1}{c_2} = \frac{c}{f}$$

$\because \quad \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

$\therefore \quad \dfrac{b}{e} \neq \dfrac{c}{f}$

It represents a pair of parallel lines.

$\because \quad \dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

$\therefore \quad \dfrac{a}{d} \neq \dfrac{b}{e}$

Therefore, system has unique solution and represents a pair of intersecting lines.

18. (*b*) Given, $3^{x+y} = 81$

$\Rightarrow \quad 3^{x+y} = 3^4$

$\Rightarrow \quad x + y = 4 \quad \text{...(i)}$

and $\quad 81^{x-y} = 3 \quad$ or $\quad (3^4)^{x-y} = 3^1$

$\Rightarrow \quad x - y = \dfrac{1}{4} \quad \text{...(ii)}$

On solving the Eqs. (i) and (ii), we get

$$2x = \frac{17}{4} \Rightarrow x = \frac{17}{8}$$

19. (*a*) Let the cost of one chair be ₹ x

and cost of one table be ₹ y.

By given condition,

$10x + 6y = 6200 \quad \text{...(i)}$

and $\quad 3x + 2y = 1900$

$\Rightarrow \quad 9x + 6y = 5700 \quad \text{...(ii)}$

On subtracting Eq. (ii) from Eq. (i), we get

$$x = ₹\ 500$$

From Eq. (i),

$5000 + 6y = 6200$

$\Rightarrow \quad 6y = 1200$

$\therefore \quad y = ₹\ 200$

The cost of 4 chairs and 5 tables

$$= 4x + 5y$$
$$= 4 \times 500 + 5 \times 200$$
$$= 2000 + 1000 = ₹\ 3000$$

20. (*d*) Given equations of system

$$3x + y = 4 \quad \text{...(i)}$$
$$6x + 2y = 8 \quad \text{...(ii)}$$

Here, $a_1 = 3, b_1 = 1, c_1 = 4,$

$a_2 = 6, b_2 = 2$ and $c_2 = 8$

$\because \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

So, the system of equations has infinite solutions, because it represents a parallel line.

21. (*d*) We have, $x + y - 7 = 0$

$\Rightarrow \quad x + y = 7 \quad \text{...(i)}$

and $\quad 3x + y - 13 = 0$

$\Rightarrow \quad 3x + y = 13 \quad \text{...(ii)}$

By subtracting Eq. (i) from Eq. (ii), we get

$$\begin{array}{r} 3x + y = 13 \\ x + y = 7 \\ - \quad - \quad - \\ \hline 2x = 6 \end{array} \quad \begin{array}{l} \text{...(i)} \\ \\ \text{...(ii)} \end{array}$$

$\therefore \quad x = 3$

On putting the value of x in Eq. (i), we get

$$3 + y = 7$$

$\therefore \quad y = 4$

Now, $4x^2 + y^2 + 4xy$

$$= 4 \times (3)^2 + (4)^2 + 4 \times 3 \times 4$$
$$= 4 \times 9 + 16 + 48$$
$$= 36 + 16 + 48 = 100$$

Exercise © *Higher SkiU Level Questions*

1. (*d*) Given, $2^a + 3^b = 17$

and $2^{a+2} - 3^{b+1} = 5$

$\Rightarrow \quad 2^a \times 2^2 - 3^b \times 3^1 = 5$

$\Rightarrow \quad 4 \cdot 2^a - 3 \cdot 3^b = 5$

Let $2^a = x$ and $3^b = y$

Then, $x + y = 17$ ...(i)

$4x - 3y = 5$ ...(ii)

On multiplying Eq. (i) by 3 and adding to Eq. (ii), we get

$$\begin{array}{r} 3x + 3y = 51 \\ 4x - 3y = 5 \\ \hline 7x = 56 \end{array}$$

$\therefore \quad x = 8$

On putting the value of x in Eq. (i), we get

$8 + y = 17$

$\therefore \quad y = 9$

Now, $2^a = x$

$\Rightarrow \quad 2^a = 8 = (2)^3$

$\therefore \quad a = 3$

and $3^b = y = 9 \Rightarrow 3^b = 3^2$

$\therefore \quad b = 2$

Hence, $a = 3$ and $b = 2$

2. (*c*) $\dfrac{p}{x} + \dfrac{q}{y} = m$ ...(i)

$\dfrac{q}{x} + \dfrac{p}{y} = n$ ...(ii)

On multiplying Eq. (i) by q and Eq. (ii) by p and subtracting, we get

$$\begin{array}{r} \dfrac{pq}{x} + \dfrac{q^2}{y} = mq \\ \underset{-}{\dfrac{pq}{x}} + \underset{-}{\dfrac{p^2}{y}} = \underset{-}{np} \\ \hline \dfrac{q^2}{y} - \dfrac{p^2}{y} = mq - np \end{array}$$

$\therefore \quad q^2 - p^2 = y(mq - np)$

$\therefore \quad y = \dfrac{q^2 - p^2}{mq - np} = \dfrac{p^2 - q^2}{np - mq}$

Again, on multiplying Eq.(i) by p and Eq. (ii) by q and subtracting, we get

$$\begin{array}{r} \dfrac{p^2}{x} + \dfrac{pq}{y} = mp \\ \underset{-}{\dfrac{q^2}{x}} + \underset{-}{\dfrac{pq}{y}} = \underset{-}{nq} \\ \hline \dfrac{p^2}{x} - \dfrac{q^2}{x} = mp - nq \end{array}$$

$\Rightarrow \quad p^2 - q^2 = x(mp - nq)$

$\Rightarrow \quad x = \dfrac{p^2 - q^2}{mp - nq}$

$\therefore \quad x = \dfrac{p^2 - q^2}{mp - nq}$

and $\quad y = \dfrac{p^2 - q^2}{np - mq}$

3. (*a*) Let the capital of one is x and that of another is y.

According to the question,

$$x + 100 = 2(y - 100)$$

$$x + 100 = 2y - 200$$

or $\quad x - 2y = -300 \quad \ldots(\text{i})$

Again, according to the question

$$y + 10 = 6(x - 10)$$

$\Rightarrow \quad y + 10 = 6x - 60$

$\Rightarrow \quad 6x - y = 70 \quad \ldots(\text{ii})$

On multiplying Eq. (ii) by 2 and subtracting from Eq. (i), we get

$$\begin{array}{r} x - 2y = -300 \\ 12x - 2y = 140 \\ - \quad + \quad - \\ \hline -11x = -440 \end{array}$$

$\therefore \quad x = 40$

On putting the value of x in Eq. (i), we get

$$40 - 2y = -300$$

$\Rightarrow \quad -2y = -340$

$\therefore \quad y = 170$

So, their capitals are ₹ 40 and ₹ 170.

4. (*b*) Let the incomes of two persons be $8x$ and $5x$ and their expenditure be $2y$ and y, respectively.

$\because \quad$ Saving = Income – Expenditure

$\therefore \quad 1000 = 8x - 2y \quad \ldots(\text{i})$

and $\quad 1000 = 5x - y \quad \ldots(\text{ii})$

On multiplying Eq. (ii) by 2 and subtracting from Eq. (i), we get

$$\begin{array}{r} 8x - 2y = 1000 \\ 10x - 2y = 2000 \\ - \quad + \quad - \\ \hline -2x = -1000 \end{array}$$

$\therefore \quad x = 500$

$\therefore$ Monthly incomes are

$$8x = 8 \times 500 = ₹\,4000$$

and $\quad 5x = 5 \times 500 = ₹\,2500$

$\therefore$ Difference = ₹ 4000 – 2500 = ₹ 1500

5. (*b*) Let the fraction be $\frac{x}{y}$.

According to the question,

$$\frac{x+5}{y+5} = \frac{7}{8}$$

$$8x + 40 = 7y + 35$$

$$8x - 7y = -5 \quad \ldots(i)$$

Again, according to the question,

$$\frac{x+3}{y+3} = \frac{6}{7} \Rightarrow 7x + 21 = 6y + 18$$

$$\therefore \quad 7x - 6y = -3 \quad \ldots(ii)$$

On multiplying Eq. (i) by 6 and Eq. (ii) by 7 and subtracting, we get

$$\begin{array}{r} 48x - 42y = -30 \\ 49x - 42y = -21 \\ - \quad + \qquad + \\ \hline -x = -9 \end{array}$$

$$\therefore \quad x = 9$$

On putting the value of x in Eq. (i), we get

$$8 \times 9 - 7y = -5$$

$$\Rightarrow \quad -7y = -5 - 72$$

$$\Rightarrow \quad y = \frac{-77}{-7} = 11$$

$$\therefore \quad x = 9 \text{ and } y = 11$$

$\therefore$ Required fraction $= \frac{9}{11}$

6. (*b*) Given,
$$\frac{3}{x+y} + \frac{2}{x-y} = 2 \quad \ldots(i)$$

and
$$\frac{9}{x+y} - \frac{4}{x-y} = 1 \quad \ldots(ii)$$

Let $\quad x + y = a \quad$ and $\quad x - y = b$

On multiplying Eq. (i) by 3 and subtracting from Eq. (ii), we get

$$\frac{9}{a} - \frac{4}{b} = 1 \quad \ldots(ii)$$

$$\frac{9}{a} + \frac{6}{b} = 6 \quad \ldots(i)$$

$$\begin{array}{c} - \quad - \quad - \\ \hline \frac{-10}{b} = -5 \end{array}$$

$$\therefore \quad b = 2$$

Now, on putting the value of b in Eq. (i), we get

$$\frac{3}{a} + \frac{2}{2} = 2 \quad \Rightarrow \quad \frac{3}{a} = 2 - 1$$

$$\Rightarrow \quad a = 3 \quad \Rightarrow \quad x + y = 3 \quad \ldots(iii)$$

and $\quad x - y = 2 \quad \ldots(iv)$

On subtracting Eq. (iv) from Eq. (iii), we get

$$2y = 1$$

$$\therefore \quad y = \frac{1}{2}$$

Now, putting $y = \frac{1}{2}$ in Eq. (iii), we get

$$x + \frac{1}{2} = 3$$

$$\Rightarrow \quad x = 3 - \frac{1}{2} = \frac{6-1}{2} = \frac{5}{2}$$

$$\therefore \quad \frac{x}{y} = \frac{5/2}{1/2} = \frac{5}{2} \times \frac{2}{1} = 5$$

7. (*d*) Given equations are

$$\frac{a}{b} - \frac{b}{a} = \frac{x}{y} \quad \Rightarrow y = \frac{x}{\left(\frac{a}{b} - \frac{b}{a}\right)} \qquad \text{...(i)}$$

and $$\frac{a}{b} + \frac{b}{a} = x - y \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\frac{a}{b} + \frac{b}{a} = x - \frac{x}{\left(\frac{a}{b} - \frac{b}{a}\right)}$$

$$\Rightarrow \left(\frac{a}{b} + \frac{b}{a}\right)\left(\frac{a}{b} - \frac{b}{a}\right) = x\left(\frac{a}{b} - \frac{b}{a} - 1\right)$$

$$\Rightarrow \left(\frac{a^2}{b^2} - \frac{b^2}{a^2}\right) = x\left(\frac{a^2 - b^2 - ab}{ab}\right)$$

$$\Rightarrow \quad x = \frac{ab}{(a^2 - b^2 - ab)} \times \left(\frac{a^4 - b^4}{a^2 b^2}\right)$$

$$\Rightarrow \quad x = \frac{(a^4 - b^4)}{(a^2 - b^2 - ab)} \times \frac{1}{ab}$$

$$= \frac{(a-b)(a+b)(a^2+b^2)}{ab(a^2 - b^2 - ab)}$$

8. (*b*) $\because \quad \frac{x}{2} + \frac{y}{3} = 4$

$$\Rightarrow \quad \frac{3x + 2y}{6} = 4$$

$$\Rightarrow \quad 3x + 2y = 24 \qquad \text{...(i)}$$

and $$\frac{2}{x} + \frac{3}{y} = 1 \Rightarrow \frac{2y + 3x}{xy} = 1$$

$$\Rightarrow \quad 2y + 3x = xy \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$xy = 24$$

There are 6 possibilities for x and y, respectively.

$2 \times 12 = 24$	$6 \times 4 = 24$
$3 \times 8 = 24$	$8 \times 3 = 24$
$4 \times 6 = 24$	$12 \times 2 = 24$

2 and 12 cannot be the values of x and y as their sum is 14 and it is not given in options.

Now, we check both 3 and 8 as well as 4 and 6 as values of x and y or value of y and x. Only 4 as a value of x and 6 as a value of y satisfied the given condition $\frac{x}{2} + \frac{y}{3} = 4$.

So, $\quad x = 4$ and $y = 6$

$$\therefore \quad x + y = 4 + 6 = 10$$

Chapter

31

Quadratic Equations

A quadratic equation is an equation in which the highest power of an unknown quantity is a square that can be written as

$ax^2 + bx + c = 0$

where, a and *b* are coefficients of jr and x, respectively and c is a constant.

The factor that identifies this expression as quadratic is the exponent 2. The coefficient of x^ i.e., a cannot be zero, (a **0)*

To check whether an equation is quadratic or not, following examples will help to understand it in a better way.

S.No.	Equation	Is it quadratic?	Explanation
1.	$3x^3 - 4x + 5 = 0$	No	Here, the first term is raised to the 3rd power. It must be raised to the 2nd power in order to be quadratic.
2.	$5x^2 - 4x + 2 = 0$	Yes	This equation is in the correct form *i.e.*, $ax^2 + bx + c = 0$
3.	$7x^2 = 49$	Yes	This equation can be rewritten as $7x^2 - 49 = 0$ In this equation, b is 0. b or c can be 0, however a cannot be 0.
4.	$2x^2 = 8x - 3$	Yes	This equation can be rewritten as $2x^2 - 8x + 3 = 0$ which is in the correct form *i.e.*, $ax^2 + bx + c = 0$

MIND IT!

1. A quadratic equation has two and only two roots.

2. A quadratic equation cannot have more than two different roots.

3. If a be the root of the quadratic equation $ax^2 + bx + c = 0$, then $(x - a)$ is a factor of $ax^2 + bx + c=0$.

Important Points Related to Quadratic Equations

1. A real number **a** is said to be a root of the quadratic equation *aye* + fox + c = 0, if *act* + face + c = 0. The zeroes of the quadratic polynomial *ax^* + *bx*+ *c* and the roots of the quadratic equation ax^2 + fax + c = 0 are the same.

2. A quadratic equation ax^2 + fax + c = 0 has

(i) two distinct real roots, if $D > 0$. (ii) two equal real roots, if $D = 0$. (iii) no real roots, if $D < 0$. (iv) reciprocal roots, if $a=c$.

(v) one root = 0, if c = 0. (vi) negative and reciprocal roots, if c = - *a*.

(vii) both roots equal to 0, if fa = 0, c = 0 where, discrimenant (D) = fa - 4ac

3. **Formation of Quadratic Equation** Let **a** and (3 be two roots. Then, we can form a quadratic ecruation as

$$x^2 - (\text{Sum of roots})\, x + (\text{Product of roots}) = 0$$

$$\Rightarrow \quad x^2 - (\alpha + \beta)x + (\alpha\beta) = 0 \quad \Rightarrow \quad (x - \alpha)(x - \beta) = 0$$

Here, for standard quadratic equation $ax^2 + bx + c = 0$,

Sum of roots $= \alpha + \beta = -\dfrac{b}{a}$; Product of roots $= \alpha\beta = \dfrac{c}{a}$

4. If $ax^2 + bx + c = 0$, where a, b, c are rational, has one root $P + \sqrt{q}$, then the other root will be $P - \sqrt{q}$.

Hence, irrational roots occur in conjugate pair, if the coefficients are rational.

Methods of Solving Quadratic Equations

There are two methods for solving quadratic equations.

1. **By Factorisation Method** If or + *bx*+ *c* can be factorised as (x-oc) (x-P),

then $ax^2 + bx + c = 0$ is equivalent to (x - a) (x - P) = 0

Thus, (x - a) (x - P) = 0

(x - a) = 0 or (x - P) = 0 x = **a** or x = P Here, **a** and p are called roots of equation *ax^* + bx + c = 0,

2. **By Sridharacharya's Method** Using quadratic formula, write the equation in the standard form $ax^2 + bx + c = 0$, then roots are

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Ex. 1 Solve $5x^2$ + llx + 6 = 0.

Sol. By factorisation method,

$5x^2 + 11a + 6 = 0$

=> $5x^2 + 5x + 6x + 6 = 0$

=> 5s (a; + 1) + 6 *(x* + 1) = 0

=> (x + 1) (5s + 6) = 0

(x + 1) = 0 or (5x + 6) = 0

Ifx +1=0, thenx = - land if *5x* + 6= 0, thenx = - —

5

Alternate Method

Given equation is 5x + lis + 6 = 0.

On comparing the given equation by ox + *bx* + *c* = 0, we get

a = *5, b* = 11 and c = 6 By quadratic formula,

_ - 6 ± $7\&^2$ - 4ac _ -11 + 7121 - 120 _ -11 ± 1

2a 10 10

t, , • • -11+1-10 ,

Takme + ve sign, *x* =----------=-----= - 1

10 10

„ , . . -11-1-12 6

Taking - ve sign, *x* =----------=-----= - —

10 10 5

Ex. 2 Find two consecutive odd positive integers, sum of whose squares is 290.

Sol. Let the two consecutive odd positive integers be x and (x + 2). According to the question,

$x^2 + (x + 2)^2 = 290 \Rightarrow x^2 + x^2 + 4x + 4 = 290 \Rightarrow 2x^2 + 4x - 286 = 0 \Rightarrow x^2 + 2x - 143 = 0$ ...(i)

On comparing Eq. (i) by $ax + bx + c = 0$, we get $a = \backslash\, b = 2, c = - 143$ According to the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-2 \pm \sqrt{(2)^2 - [4 \times 1 \times (-143)]}}{2 \times 1} = \frac{-2 \pm \sqrt{4 + 572}}{2}$$

$$\Rightarrow \quad x = \frac{-2 \pm \sqrt{576}}{2} \Rightarrow x = \frac{-2 \pm 24}{2}$$

$$\Rightarrow \quad x = \frac{-2 + 24}{2} \text{ and } \frac{-2 - 24}{2} = \frac{22}{2} \text{ and } -\frac{26}{2} = 11 \text{ and } -13$$

Since, x is given to be an odd positive integer.

Therefore, x * - 13

So, two consecutive odd integers are 11 and 11 + 2, *i.e.,* 11 and 13.

Ex. 3 Two natural numbers are in the ratio of 3 :5 and their product is 2160. Find the smaller of the numbers.

Sol. Let the numbers be 3x and 5x.

Product of the numbers = 2160

$\Rightarrow 3x \text{ x } 5x = 2160 \Rightarrow 15x^2 = 2160 \Rightarrow x^2 = 144$

$$\therefore \quad x = \sqrt{144} = \pm 12$$

Since, it is a natural number, so $x = 12$.

.•. Required smaller number = 3x = 3 x 12 = 36

Ex. 4 The product of two numbers is 24 times the difference of these two numbers. If the sum of these numbers is 14, then find the larger number.

Sol. Let the two numbers be x and y. According to the question,

$x \times y = 24\ (x - y) \Rightarrow xy = 24\ (x - y)$ ...(i)

and $x + y = 14 \Rightarrow y = 14 - x$ ...(H)

On putting the value of y in Eq. (i), we get

$\Rightarrow x\ (14 - x) = 24\ (x - 14 + x) \Rightarrow$ 14a; - x^2 = 24a; - 336 + 24a;

$\Rightarrow - x^2 + 14x - 24x - 24x + 336 = 0 \Rightarrow - x^2 - 34x + 336 = 0$

$\Rightarrow$ a;^{2} + 34x - 336 = 0 $\Rightarrow x^2$ + 42a; - 8a; - 336 = 0

$\Rightarrow x\ (x + 42) - 8$ (a; + 42) = 0 $\Rightarrow$ (i + 42) (a; - 8) = 0

$x = 8$ or - 42 $\Rightarrow$ a; = 8

[ignoring negative value] On putting the value of x in Eq. (ii), we get $y = 6$ Larger number = 8

Ex. 5 Which of the following equations has/have real roots?

(i) $3x^2 + 4x + 5 = 0$ (ii) $x^2 + x + 4 = 0$

(iii) (x-l)(2x-5) = 0 (iv) $2x^2 - 3x + 4 = 0$

Sol. Roots of a quadratic equation $ax + bx + c = 0$ are real, if 6 - 4ac > 0. (i) 3a;^{2} + 4a; + 5 = 0; $b^2 - 4ac = (4)^2 - 4\ (3)\ (5) = - 44 < 0$

Hence, the roots of this equation are not real. (ii) $x^2 + x + 4 = 0$; $b^2 - Aac$ = (l)^2 - 4 (1) (4)= 1 - 16 = - 15 < 0 Hence, the roots of this equation are not real.

(iii) $(x - 1)(2x - 5) = 0 \Rightarrow x = 1$ and $x = \frac{5}{2}$

So, 1 and $\frac{5}{2} > 0$

Hence, the roots of this equation are real.

(iv) $2x^2 - 3x + 4 = 0$; $b^2 - 4ac = (-3)^2 - 4\ (2)\ (4) = 9 - 32 = -23 < 0$

Hence, the roots of this equation are not real.

Ex. 6 If one root of a quadratic equation is $2+\sqrt{5}$, then find the quadratic equation.

Sol. Since, one root $= 2+\sqrt{5}$, then another root $= 2-\sqrt{5}$

Sum of the roots $= (2+\sqrt{5}) + (2-\sqrt{5}) = 4$

Product of the roots $= (2+\sqrt{5}) \times (2-\sqrt{5}) = 4-5 = -1$

Then, required quadratic equation is

$x^2 -$ (Sum of the roots) $x +$ (Product of the roots) $= 0$

$\Rightarrow \quad x^2 - 4x - 1 = 0$

Fast Track **Formulae**

to solve the **QUESTIONS**

Formula 1

If the equation $ax^2 + bx + c = 0$ has the roots a and p, then the equation

having the roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is $cx^2 + bx + a = 0$.

Ex. 7 If roots of the equation $3x - 6x + 5 = 0$ are oc and B, then find the equation

having the roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Sol. Since, oc and Pare the roots of the equation 3x - 6x + 5 = 0. Then, according to the formula.

The equation having the roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is $5x^2 - 6x + 3 = 0$.

Formula 2

If the equation $ax^2 + bx + c = 0$ has the roots a and p, then the equation having the roots a ± A and p + A is a $(x + A)^2 + b(x + A) + c = 0$.

Ex. 8 If roots of the equation $x - 5x + 6 = 0$ are oc and P, then find the equation having the roots (a -1) and (p -1).

Sol. Given equation is $x - 5x + 6 = 0$ Then, required equation is

$(x+l)^2-5(a;+l)+6 = 0 \Rightarrow s^2 + 2x+l-5x-5+6 = 0 \Rightarrow z^2-3x + 2 = rj$

Formula 3

If a and p are the roots of the equation $ax^2 + bx + c = 0$, then the equation having the roots Aa and Ap is $ax^2 + Abx + A^2c = 0$.

Note In this technique, the equation having the roots $\frac{\alpha}{A}$ and $\frac{\beta}{A}$, is a $A^2x^2 + bA\,x + c = 0$

Ex. 9 If a and p are the roots of the equation $x - 6x + 5 = 0$, then find the equation having the roots 2a and 2p.

Sol. Given equation is a; - 6x + 5 = 0. Then, required equation is

$ic^2-2x6x + 2^2x5 = 0$

$\Rightarrow i^2 - 12x + 20 = 0$

Multi Concept

QUESTIONS

1. *If a and* p *are the roots of the equation* $ax^2 + bx+ c = 0$*, then find the values of the following expressions in terms of a, b and c*

(i)<x^2 + p^2 (ii) a^4-p"

➥ If $ax^2 + bx + c = 0$ and α, β are the roots of the equation, then

$$\alpha + \beta = -\frac{b}{a}, \ \alpha\beta = \frac{c}{a}$$

(i) By using formula $a^2 + b^2 = (a + b)^2 - 2ab$,

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right) = \frac{b^2}{a^2} - \frac{2c}{a} \Rightarrow \frac{b^2 - 2ac}{a^2}$$

(ii) $\alpha^4 - \beta^4 = (\alpha^2 + \beta^2)(\alpha + \beta)(\alpha - \beta) = [(\alpha + \beta)^2 - 2\alpha\beta](\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$$[\because a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = (a^2 + b^2)(a - b)(a + b)]$$

$$= \left(\frac{b^2 - 2ac}{a^2}\right)\left(\frac{-b}{a}\right)\sqrt{\frac{b^2}{a^2} - 4\frac{c}{a}}$$

$$= -\frac{b}{a^4}(b^2 - 2ac)\sqrt{b^2 - 4ac}$$

2. *What are the factors of* $x^2 + 4V^2 + 4y - 4xy - 2x - 81$

(a)(x-2y-4)(x-2y+2) (b)(x-y+ 2)(x-4y+ *4}*

(c) (x- y + 2) (x- 4y - 4) (d) (x+ 2y - 4) (x- 2y + *2)*

➥ (a) $x^2 + 4y^2 + 4y - 4xy - 2x - 8$

$$= x^2 + 4y^2 - 4xy - 2x + 4y - 8$$

$$= (x - 2y)^2 - 2(x - 2y) - 8$$

Let $A = x - 2y$

$$= A^2 - 2A - 8,$$

$$= (A - 4)(A + 2)$$

$$= (x - 2y - 4)(x - 2y + 2)$$

3. *The sum of the roots of the equation 5x +* (p + *q + r) x + par = 0 is equal to zero. What is the value of{*p^3 *+* $q^3 + r^3$*)l*

(a) *4pqr* (b) 3 *pqr* (c) *7 pqr* (d) *Zpqr*

➥ (b) Here, $a = 5$, $b = p + q + r$ and $c = pqr$

Sum of the roots $= \frac{-b}{a} = \frac{-(p + q + r)}{5} = 0$

$\therefore \qquad p + q + r = 0$

According to the formula,

$$a^3 + b^3 + c^3 = 3abc, \text{ if } a + b + c = 0$$

We get $\qquad p^3 + q^3 + r^3 = 3pqr \qquad [\because p + q + r = 0]$

Fast Track Practice

Exercise© *Base Level Questions*

1. Which of the following equations is a quadratic?

(a) $x^3 - x^2 - x + 5 = 0$

(b) x^4-10

(c) $7x^2 = 49$

(d) $x^4 - x^3 = 9000$

fej None of the above

2. Which of the following equations has real roots?

(a) $2x^2 - 3x + 4 = 0$

(b) $(x - 1)(2x - 5) = 0$

(c) $3x^2 + 4x + 5 = 0$

(d) Cannot be determined

(e) None of the above

3. Find the roots of the equation $2x^2 - 9x - 18 = 0$.

(a) $\frac{3}{2}$ and 6 *(b)* $-\frac{3}{2}$ and -6

(c) $-\frac{3}{2}$ and 6 *(d)* $\frac{3}{2}$ and -6

(e) None of the above

4. If $x^2 - 3x + 1 = 0$, find the value of $x + \frac{1}{x}$.

[SSC CPO 2007]

(a) 0 (b) 3

(c) 2 (d) 1

If $2x^2$ + \2x + 18 = 0, what is the value

ofx? [Bank PO 2006]

(a) 3 (b) 2

(c) - 3 (d) -2

(e) None of the above 6. If one root of x - $6kx + 5 = 0$ is 5, find the value of k. [Railway 2003]

(a) $-\frac{1}{2}$ (b) -1
(c) 1 (d) 2

7. If one of the roots of quadratic equation Ix - 50x + k = 0 is 7, then what is the value of*? [CDS 2012]

(a) 7 (b) 1
(c) $\frac{50}{7}$ (d) $\frac{7}{50}$

8. If a and b are the roots of the equation x - $6x$ + 6 = 0, find the value of 2 ($a^2 + b^2$).

(a) 40 (b) 42 (c) 48 (d) 46 (e) None of the above

9. If a and P be the roots of the equation $ax + bx + c = 0$, find the value of

$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$.

(a) $\frac{ab - b^2c}{2b^2c}$

(b) $\frac{3bc - a^3}{b^2c}$

(c) $\frac{3ac - b^2}{a^3c}$

(d) $\frac{3abc - b^3}{a^2c}$

(e) None of the above

For what values of *k*, the equation $x + 2(k - 4) x + 2k = 0$ has equal roots?

(a) 6 and 4 *(b)* 8 and 2

(c) 10 and 4 (tfj 12 and 2

fej None of the above 11. Find the roots of the equation *2x* - Liz+15=0. [Hotel Mgmt. 2001]

(a) 3 and $\frac{5}{2}$

(b) -3 and $-\frac{5}{2}$

(c) 5 and $\frac{3}{2}$

(d) -5 and $-\frac{3}{2}$

(e) None of the above

12. The quadratic equation whose roots are 3 and -*X* is [CDS 2012]

(a) $x^2 - 4x + 3 = 0$

(b) $x^2 - 2x - 3 = 0$

(c) $x^2 + 2x - 3 = 0$

(d) $x^2 + 4x + 3 = 0$

Directions *(Q.* Nos. 13-22) *In each of the following questions, there are two equations. You have to solve both equations and give answer.* [Bank PO 201 o]

(a) Ifx>y *(b)* If x> y

(c) If x< y *(d)* If x< y

(e) If x = *y* or relation cannot be established

13. $x^2 + x - 20 = 0;\quad y^2 - y - 30 = 0$

14. $225x^2 - 4 = 0;\quad \sqrt{225y} + 2 = 0$

15. $\frac{4}{\sqrt{x}} + \frac{7}{\sqrt{x}} = \sqrt{x};\quad y^2 - \frac{(11)^{\frac{5}{2}}}{\sqrt{y}} = 0$

16. $x^2 - 365 = 364;\quad y - \sqrt{324} = \sqrt{81}$

17. $3x^2 + 8x + 4 = 0;\ 4y^2 - 19y + 12 = 0$

18. $x^2 - x - 12 = 0;\quad y^2 + 5y + 6 = 0$

19. $x^2 - 8x + 15 = 0;\quad y^2 - 3y + 2 = 0$

20. $x^2 - 32 = 112;\quad y - \sqrt{169} = 0$

21. $x - \sqrt{121} = 0;\quad y^2 - 121 = 0$

22. $x^2 - 16 = 0$ $y^2 - 9y + 20 = 0$

23. If a and P are the roots of the equation 4ac - $19x + 12 = 0$, find the equation

having the roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(a) $4x^2 + 19x+12 = 0$

(b) $12x^2-19x+ 4=0$

(c) $12x^2 + 19x+ 4=0$

(d) $4x^2 + 19x-12 = 0$

(e) None of the above

If a and P are the roots of the equation $x - 11x + 24 = 0$, find the equation having the roots a + 2 and P + 2

(a) $x^2 + 15x+24 = 0$

(b) $x^2 -15x+24 = 0$

(c) $x^2 + 15x-50=0$

(d) $x^2 + 15x-60=0$

(e) $x^2-15x+ 50=0$

Exercise © *Higher SkiU Level Questions*

1. If $\left(a+\frac{1}{a}\right)^2 = 3$, what is the value of $a^3 + \frac{1}{a^3}$?

   (a) $\frac{10\sqrt{3}}{3}$ (b) 0 (c) $3\sqrt{3}$ (d) $6\sqrt{3}$

   (e) None of the above

2. If $a = \frac{x}{x+y}$ and $b = \frac{y}{x-y}$, then $\frac{ab}{a+b}$ is equal to

   (a) $\frac{xy}{x^2+y^2}$ (b) $\frac{x^2+y^2}{xy}$

   (c) $\frac{x}{x+y}$ (d) $\left(\frac{y}{x+y}\right)^2$

   (e) None of the above

3. If $x^2 = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots \infty}}}$, then what is one of the values of x equal to? [CDS 2013]

   (a) 6 (b) 5 (c) 4 (d) 3

4- If $2x^2 - 7xy + 3y^2 = 0$, then the value of *x: yis (a)* 3 : 2 fbj 2 : 3

(c) 3 : 1 and 1 : 2 (c0 5 : 6 *(e)* None of the above

5. Two numbers whose sum is 8 and difference is 4, are the roots of the equation

(a) $x^2 + 8x + 12 = 0$

(b) $x^2 + 8x - 12 = 0$+

(c) $x^2 - 8x - 12 = 0$

(d) $x^2 - 8x + 12 = 0$

(e) None of the above

6. The sum of a number and its reciporocal is 10/3, then the numbers are [CDS 2012]

(a) 3, 1/3 *(b)* 3, - 1/3

(c) - 3,1/3 *(d)* - 3, -1/3

7. If $\left\{2 \ \frac{1}{4}\right\}$ is the solution set of a quadratic equation, find that equation.

(a) $4x^2 - 9x + 2 = 0$

(b) $2x^2 - 9x + 4 = 0$

(c) $x^2-18x-6=0$

(d) $2x^2-18x+ 3=0$

(e) None of the above

8. Number of roots of the equation $3^{2*^2-7*+7} = 9$is

(a) 1 *(b)* 2 *(c)* 3 *(d)* 4

*(e)*5

9. If α and β are the roots of the equation $8x^2 - 3x + 27 = 0$, find the value of $\left(\frac{\alpha^2}{\beta}\right)^{1/3} + \left(\frac{\beta^2}{\alpha}\right)^{1/3}$.

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{7}{2}$ (d) 4

(e) $\frac{9}{2}$

10. The values of p for which the difference between the roots of the equation $x + px + 8 = 0$ is 2, are

(a) ±2 *(b)*±*A*

(c) ±6 *(d)* + 8

(e) ± 10

11. In solving a problem, one student makes a mistake in the coefficient of the first degree term and obtains - 9 and - 1 for the roots. Another student makes a mistake in the constant term of the equation and obtains 8 and 2 for the roots. The correct equation was [CDS 2013]

(a) $x^2 + 10x + 9 = 0$

(b) $x^2 - 10x + 16 = 0$

(c) $x^2 - 10x + 9 = 0$

(d) None of the above

12. The sum of the roots of the equation $x + px + q = 0$ is equal to the sum of their squares, then

(a) $p^2 - q^2 = Q$ *(b)* $p^2 + q^2 = 2q$ *(c)* $p^2 + p = 2q$ *(d)* $q^2 + q = 2p$

(e) None of the above

13. If p and q are the roots of the equation $x + px + q = 0$, then

(a) $p = 1$ and $q = - 2$

(b) $p = 0$ and $q = 1$

(c) $p = - 2$ and $q = 0$

(d) $p = - 2$ and $q = 1$

14. The difference in the roots of the

2

equation $2x$ -llx;+ 5=0 is [cos 2013]

(a) 4.5 (b) 4

(c) 3.5 (c0 3

15. If one roots of the equation $\frac{x^2}{a}+\frac{x}{b}+\frac{1}{c}=0$ is reciprocal of the other, then which one of the following is correct? [CDS 2012]

(a) $a = b$ (b) $b = c$

(c) $ac = 1$ (d) $a = c$

16. Number of solutions of the equation $\sqrt{x^2-x+1}+\frac{1}{\sqrt{x^2-x+1}}=2-x^2$ is [SSC CGL 2012]

(a) 0 (b) 1 (c) 2 (d) 4

17. If $x=\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$, then x^2-x-1 is equal to

[SSC CGL (Main) 2012]

(a) 0 *(b)* 1 fc) 2 *(d)* 5

18. If one of the roots of the equation x^2 - *bx* + c = 0 is the square of the other, then which of the following option is correct? [CDS 2013]

(a) $b^3 = 3bc + c^2 + c$

(b) $c^3 = 3bc + b^2 + b$

(c) $3bc=c^3 + b^2 + b$

(d) 3fac=$c^3 + b^3 + b^2$

19. Two students *A* and *B* solve an equation of the form *x* + *px* + *q* = 0. A starts

with a wrong value of *p* and obtains the roots as 2 and 6. *B* starts with a wrong value of *q* and gets the roots as 2 and - 9. What are the correct roots of the equation? [CDS 2012]

(a) 3 and - 4 *(b)* - 3 and - 4

(c) - 3 and 4 *(d)* 3 and 4

Answer with **Solutions**

Exercise© *Base Level Questions*

1. (*c*) Clearly, $7x^2 = 49$ or $7x^2 - 49 = 0$, which is of the form $ax^2 + bx + c = 0$, where $b = 0$.

Thus, $7x^2 - 49 = 0$ is a quadratic equation.

2. (*b*) $(x - 1)(2x - 5) = 0 \Rightarrow x = 1, \frac{5}{2}$

So, its roots are real.

3. (*c*) Given equation is

$$2x^2 - 9x - 18 = 0$$

[by factorisation method]

$$\Rightarrow \quad 2x^2 - 12x + 3x - 18 = 0$$

$$\Rightarrow \quad 2x(x - 6) + 3(x - 6) = 0$$

$$\Rightarrow \quad (2x + 3)(x - 6) = 0$$

$$\therefore \quad x = -\frac{3}{2}, 6$$

4. (*b*) Given equation is

$$x^2 - 3x + 1 = 0 \Rightarrow x^2 + 1 = 3x$$

$$\Rightarrow \quad \frac{x^2 + 1}{x} = 3$$

$$\Rightarrow \quad \frac{x^2}{x} + \frac{1}{x} = 3$$

$$\therefore \quad x + \frac{1}{x} = 3$$

5. (*c*) Given equation is

$$2x^2 + 12x + 18 = 0$$

$$\Rightarrow \quad x^2 + 6x + 9 = 0 \text{ [divide by 2]}$$

$$\Rightarrow \quad x^2 + 3x + 3x + 9 = 0$$

[by factorisation method]

$$\Rightarrow \quad x(x + 3) + 3(x + 3) = 0$$

$$\Rightarrow \quad (x + 3)(x + 3) = 0$$

$$\Rightarrow \quad (x + 3)^2 = 0$$

$$\Rightarrow \quad x + 3 = 0$$

$$\Rightarrow \quad x = -3$$

6. (*c*) Given, one roots of $x^2 - 6kx + 5 = 0$ is 5.

$$\therefore \quad x = 5 \text{ satisfies } x^2 - 6kx + 5 = 0$$

$$\Rightarrow \quad 5^2 - 6 \times k \times 5 + 5 = 0$$

$$\Rightarrow \quad 25 - 30k + 5 = 0$$

$$30 - 30k = 0$$

$$\Rightarrow \quad 30k = 30$$

$$\therefore \quad k = 1$$

7. (*a*) Given equation is $7x^2 - 50x + k = 0$.

Here, $a = 7, b = -50, c = k$

Since, $\alpha + \beta = \frac{-b}{a}$

$\therefore \quad \alpha + \beta = \frac{50}{7}$

or $\quad \beta = \frac{50}{7} - 7$

$\Rightarrow \quad \beta = \frac{1}{7} \qquad [\because \alpha = 7 \text{ (given)}]$

and $\quad \alpha\beta = \frac{c}{a}$

or $\quad 7 \times \frac{1}{7} = \frac{k}{7}$

$k = 7$

Alternate Method

Given quadratic equation is

$$7x^2 - 50x + k = 0$$

If one root is 7, then it will satisfy the equation *i.e.*, putting $x = 7$ in equation

$7 \times (7)^2 - 50 \times 7 + k = 0$

$\Rightarrow \quad 7 \times 49 - 350 + k = 0$

$\Rightarrow \quad 343 - 350 + k = 0$

$\therefore \quad k = 7$

8. (*c*) a and b are the roots of the equation

$$x^2 - 6x + 6 = 0.$$

$\therefore \quad a + b = \frac{-B}{A} = 6$ and $ab = \frac{C}{A} = 6$

We know that,

$a^2 + b^2 = (a + b)^2 - 2ab$

$= (6)^2 - 2 \times 6 = 36 - 12 = 24$

$\therefore \quad 2(a^2 + b^2) = 2 \times 24 = 48$

9. (*d*) Given, α and β are the roots of the equation $ax^2 + bx + c = 0$

$\therefore$ Sum of two roots $= -\frac{b}{a}$

$$\alpha + \beta = -\frac{b}{a}$$

and product of two roots $= \frac{c}{a}$

$$\alpha\beta = \frac{c}{a}$$

$$\therefore \quad \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta}$$

$\because \quad (a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$= \frac{\left(-\frac{b}{a}\right)^3 - \frac{3c}{a}\left(-\frac{b}{a}\right)}{\frac{c}{a}} = \frac{-\frac{b^3}{a^3} + \frac{3bc}{a^2}}{\frac{c}{a}}$$

$$= \frac{3abc - b^3}{a^2c}$$

10. (*b*) Given equation is

$x^2 + 2(k - 4)x + 2k = 0$

On comparing with $ax^2 + bx + c = 0$

Here, $a = 1$, $b = 2(k - 4)$, $c = 2k$

Since, the roots are equal, we have $D = 0$.

$$b^2 - 4ac = 0$$

$$\therefore \quad 4(k - 4)^2 - 8k = 0$$

$$4(k^2 + 16 - 8k) - 8k = 0$$

$$\Rightarrow \quad 4k^2 + 64 - 32k - 8k = 0$$

$$\Rightarrow \quad 4k^2 - 40k + 64 = 0$$

$$\Rightarrow \quad k^2 - 10k + 16 = 0$$

$$\Rightarrow \quad k^2 - 8k - 2k + 16 = 0$$

$$\Rightarrow \quad k(k - 8) - 2(k - 8) = 0$$

$$\Rightarrow \quad (k - 8)(k - 2) = 0$$

Hence, the value of k is 8 or 2.

11. (*a*) $2x^2 - 11x + 15 = 0$

[by factorisation method]

$$\Rightarrow \quad 2x^2 - (6x + 5x) + 15 = 0$$

$$\Rightarrow \quad 2x^2 - 6x - 5x + 15 = 0$$

$$\Rightarrow \quad 2x(x - 3) - 5(x - 3) = 0$$

$$\Rightarrow \quad (2x - 5)(x - 3) = 0$$

$$\therefore \quad x = \frac{5}{2}, 3$$

Hence, the roots are $\frac{5}{2}$ and 3.

12. (*b*) Given that, the roots of the quadratic equation are 3 and -1.

Let $\alpha = 3$ and $\beta = -1$

Sum of roots $= \alpha + \beta = 3 - 1 = 2$

Product of roots $= \alpha \cdot \beta = (3)(-1) = -3$

$\therefore$ Required quadratic equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow \quad x^2 - (2)x + (-3) = 0$$

$$\Rightarrow \quad x^2 - 2x - 3 = 0$$

13. (*d*) $x^2 + x - 20 = 0$

[by factorisation method]

$\Rightarrow \quad x^2 + 5x - 4x - 20 = 0$

$\Rightarrow \quad x(x+5) - 4(x+5) = 0$

$(x+5)(x-4) = 0$

$\therefore \quad x = -5$ or 4

and $\quad y^2 - y - 30 = 0$

$\Rightarrow \quad y^2 - 6y + 5y - 30 = 0$

$\Rightarrow \quad y(y-6) + 5(y-6) = 0$

$\Rightarrow \quad (y-6)(y+5) = 0$

$\therefore \quad y = 6$ or -5

Hence, $\quad y \geq x$ or $x \leq y$

14. (*e*) $225x^2 - 4 = 0$;

$\Rightarrow \quad 225x^2 = 4 \;\Rightarrow\; x^2 = \frac{4}{225}$

$\therefore \; x = \sqrt{\frac{4}{225}} = \pm\frac{2}{15}$, *i.e.*, $\frac{2}{15}$ and $-\frac{2}{15}$

and $\sqrt{225y} + 2 = 0$ or $\sqrt{225y} = -2$

On squaring both sides, we get

$(\sqrt{225y})^2 = (-2)^2$

$\Rightarrow \quad 225y = 4$

$\therefore \quad y = \frac{4}{225}$

So, relation cannot be established because $\frac{4}{225}$ lies between $\frac{2}{15}$ and $-\frac{2}{15}$.

15. (*e*) $\frac{4}{\sqrt{x}} + \frac{7}{\sqrt{x}} = \sqrt{x} \Rightarrow \frac{11}{\sqrt{x}} = \sqrt{x}$

$\therefore \quad x = 11$

and $y^2 - \frac{(11)^{5/2}}{\sqrt{y}} = 0 \Rightarrow y^2 = \frac{(11)^{5/2}}{(y)^{1/2}}$

$\Rightarrow \quad y^2 \times y^{1/2} = (11)^{5/2}$

$\Rightarrow \quad (y)^{5/2} = (11)^{5/2}$

$\therefore \quad y = 11$

$\therefore \quad x = y$

16. (*d*) $x^2 - 365 = 364$

$\Rightarrow \quad x^2 = 364 + 365$

$\therefore \quad x = \sqrt{729} = \pm 27$

and $y - \sqrt{324} = \sqrt{81} \Rightarrow y - 18 = 9$

$\therefore \quad y = 27$

So, $y \geq x$ or $x \leq y$ because $y = 27$ and $x = 27$ and -27.

17. (C) $3x^2 + 8x + 4 = 0$

=> $3x^z + 6x + 2x + 4 = 0$

=> 3x (x + 2) + 2 (x + 2) = 0

=> (x + 2) (3x + 2) = 0

$\therefore \quad x = -2, -\frac{2}{3}$

and $\quad 4y^2 - 19y + 12 = 0$

$\Rightarrow \quad 4y^2 - 16y - 3y + 12 = 0$

$\Rightarrow \quad 4y(y-4) - 3(y-4) = 0$

$\Rightarrow \quad (y-4)(4y-3) = 0$

$\therefore \quad y = 4, \frac{3}{4}$

Hence, $\quad y > x$ or $x < y$

18. (*b*) $\quad x^2 - x - 12 = 0$

$\Rightarrow \quad x^2 - 4x + 3x - 12 = 0$

$\Rightarrow \quad x(x-4) + 3(x-4) = 0$

$\Rightarrow \quad (x-4)(x+3) = 0$

$\therefore \quad x = -3, 4$

and $\quad y^2 + 5y + 6 = 0$

$\Rightarrow \quad y^2 + 3y + 2y + 6 = 0$

$\Rightarrow \quad y(y+3) + 2(y+3) = 0$

$\Rightarrow \quad (y+3)(y+2) = 0$

$\Rightarrow \quad y = -3, -2$

$\therefore \quad x \geq y$

[because $x = -3$ and $y = -3$, so $x = y$ and $x = 4$ and $y = -2$, hence $x > y$]

19. (*a*) $\quad x^2 - 8x + 15 = 0$

$\Rightarrow \quad x^2 - 5x - 3x + 15 = 0$

$\Rightarrow \quad x(x-5) - 3(x-5) = 0$

$\Rightarrow \quad (x-5)(x-3) = 0$

$\therefore \quad x = 5, 3$

and $\quad y^2 - 3y + 2 = 0$

$\Rightarrow \quad y^2 - 2y - y + 2 = 0$

$\Rightarrow \quad y(y-2) - 1(y-2) = 0$

$\Rightarrow \quad (y-2)(y-1) = 0$

$\Rightarrow \quad y = 2, 1$

$\therefore \quad x > y$

20. (e) $x^2 - 32 = 112$

$\Rightarrow \quad x^2 = 112 + 32 \quad \Rightarrow \quad x^2 = 144$

$\therefore \quad x = \pm 12$

and $\quad y - \sqrt{169} = 0$

$\Rightarrow \quad y = \sqrt{169} \quad \Rightarrow \quad y = \pm 13$

$\therefore$ Relation cannot be established.

21. (e) $x - \sqrt{121} = 0$

$\Rightarrow \quad x = \sqrt{121} \Rightarrow x = \pm 11$

and $\quad y^2 - 121 = 0 \quad \Rightarrow y^2 = 121$

$\Rightarrow \quad y = \sqrt{121} = \pm 11$

$\therefore \quad x = y$

22. (d) $x^2 - 16 = 0$

$\Rightarrow \quad x^2 = 16 \quad \Rightarrow x = \sqrt{16} = \pm 4$

and $y^2 - 9y + 20 = 0$

$y^2 - 4y - 5y + 20 = 0$

$\Rightarrow \quad y(y-4) - 5(y-4) = 0$

$\Rightarrow \quad y = 5, 4$

$\Rightarrow \quad y \geq x$ or $x \leq y$

23. (b) Given equation is $4x^2 - 19x + 12 = 0$.

Let given equation having the roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Then, required equation is

$12x^2 - 19x + 4 = 0$ [by using Formula 1]

24. (e) Given equation is

$x^2 - 11x + 24 = 0$

Then, required equation is

[by using Formula 2]

$(x-2)^2 - 11(x-2) + 24 = 0$

$\Rightarrow \quad x^2 - 4x + 4 - 11x + 22 + 24 = 0$

$\Rightarrow \quad x^2 - 15x + 50 = 0$

Exercise © *Higher Skill Level Questions*

1. (*b*) $\left(a+\frac{1}{a}\right)^2=3$

Taking square roots both sides, we get

$$a+\frac{1}{a}=\sqrt{3} \qquad \text{...(i)}$$

On cubing both sides, we get

$$\left(a+\frac{1}{a}\right)^3=(\sqrt{3})^3$$

$$\Rightarrow \quad a^3+\frac{1}{a^3}+3\cdot a\cdot\frac{1}{a}\left(a+\frac{1}{a}\right)=3\sqrt{3}$$

$$\Rightarrow \quad a^3+\frac{1}{a^3}+3\sqrt{3}=3\sqrt{3}$$

$$\therefore \quad a^3+\frac{1}{a^3}=0$$

2. (*a*) $\dfrac{ab}{a+b}=\dfrac{\frac{x}{(x+y)}\times\frac{y}{(x-y)}}{\frac{x}{(x+y)}+\frac{y}{(x-y)}}$

$$=\frac{\frac{xy}{(x+y)(x-y)}}{\frac{x(x-y)+y(x+y)}{(x+y)(x-y)}}$$

$$=\frac{xy}{x^2-xy+xy+y^2}=\frac{xy}{x^2+y^2}$$

3. (*d*) Here, $x^2 = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots \infty}}}$

So, $x^2 = 6 + \sqrt{x^2}$

$\Rightarrow \quad x^2 = 6 + x$

$\Rightarrow \quad x^2 - x - 6 = 0$

$\Rightarrow \quad x^2 + 2x - 3x - 6 = 0$

$\Rightarrow \quad x(x + 2) - 3(x + 2) = 0$

$\Rightarrow \quad (x - 3)(x + 2) = 0$

$\therefore \quad x = 3$

4. (*c*) $\quad 2x^2 - 7xy + 3y^2 = 0$

$\Rightarrow \quad 2x^2 - 6xy - xy + 3y^2 = 0$

$\Rightarrow \quad 2x(x - 3y) - y(x - 3y) = 0$

$\Rightarrow \quad (2x - y)(x - 3y) = 0$

Either, $\quad 2x - y = 0 \Rightarrow 2x = y$

$\Rightarrow \quad \frac{x}{y} = \frac{1}{2}$

or $\quad x - 3y = 0$

$\Rightarrow \quad x = 3y \Rightarrow \frac{x}{y} = \frac{3}{1}$

5. (*d*) Let the roots be α and β.

Then, $\quad \alpha + \beta = 8 \quad$ ...(i)

$\alpha - \beta = 4 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$\alpha = 6, \beta = 2$

$\therefore$ Required equation is

$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0.$

$\Rightarrow \quad x^2 - (6 + 2)x + 6 \times 2 = 0$

$\Rightarrow \quad x^2 - 8x + 12 = 0$

6. (*a*) Let the numbers be x and $\frac{1}{x}$.

Then, $\quad x + \frac{1}{x} = \frac{10}{3}$

$\Rightarrow \quad \frac{x^2 + 1}{x} = \frac{10}{3}$

$\Rightarrow \quad 3x^2 - 10x + 3 = 0$

$\Rightarrow \quad 3x^2 - 9x - x + 3 = 0$

$\Rightarrow \quad 3x(x - 3) - 1(x - 3) = 0$

$\Rightarrow \quad (3x - 1)(x - 3) = 0$

$\therefore \quad x = \frac{1}{3}, x = 3$

7. (*a*) Let $\alpha = 2, \beta = \frac{1}{4}$

Then, $\alpha + \beta = 2 + \frac{1}{4} = \frac{9}{4}$

$\alpha\beta = 2 \cdot \frac{1}{4} = \frac{1}{2}$

Equation having the roots α and β is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$$\Rightarrow \quad x^2 - \frac{9}{4}x + \frac{1}{2} = 0$$

$$\Rightarrow \quad 4x^2 - 9x + 2 = 0$$

8. (b) $3^{2x^2 - 7x + 7} = 9 = 3^2$

On comparing the exponents on both sides, we get

$$2x^2 - 7x + 7 = 2 \Rightarrow 2x^2 - 7x + 5 = 0$$

which is a quadratic equation.

$\therefore$ It has two roots.

9. (b) Since, α and β are the roots of the equation

$$8x^2 - 3x + 27 = 0$$

$$\therefore \quad \alpha + \beta = \frac{3}{8} \text{ and } \alpha\beta = \frac{27}{8}$$

$$\therefore \left(\frac{\alpha^2}{\beta}\right)^{1/3} + \left(\frac{\beta^2}{\alpha}\right)^{1/3} = \frac{(\alpha^3)^{1/3} + (\beta^3)^{1/3}}{(\alpha\beta)^{1/3}}$$

$$= \frac{\alpha + \beta}{(\alpha\beta)^{1/3}} = \frac{\frac{3}{8}}{\left(\frac{27}{8}\right)^{1/3}}$$

$$= \frac{3}{8} \times \frac{2}{3} = \frac{1}{4}$$

10. (c) $\alpha + \beta = -p, \alpha\beta = 8$

Given, $\quad \alpha - \beta = 2$

On squaring both sides, we get

$$(\alpha - \beta)^2 = 4$$

$$\Rightarrow \quad (\alpha + \beta)^2 - 4\alpha\beta = 4$$

$$\Rightarrow \quad p^2 - 32 = 4$$

$$\Rightarrow \quad p^2 = 36 \quad \Rightarrow \quad p = \pm 6$$

11. (c) When mistake is done in first degree term, the roots of the equation are - 9 and - 1.

.". Equation is

$(x + 1)(x + 9) = x^2 + 10x + 9$ ...(i)

When mistake is done in constant term, the roots of equation are 8 and 2. .•. Equation is

$(x - 2)(x - 8) = x^2 - 10x + 16$ ...(ii) /. Required equation from Eqs. (i) and (ii) is $= x^2 - 10x + 9$

Also we see in both the cases 1 st degree term is same with opposite sign i. e., in such questions we should take data from given conditions and find the correct equation.

12. (C) Let a and P be the roots of the equation

$x^2 + px + q = 0$. Then, a + P = - p, 0(0 = g

According to the question,

$$\alpha + \beta = \alpha^2 + \beta^2$$

$$\Rightarrow \quad \alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\Rightarrow \quad -p = p^2 - 2q \quad \Rightarrow \quad p^2 + p = 2q$$

13. (*a*) Since, p and q are the roots of the equation $x^2 + px + q = 0$.

Then, $p + q = -p$

and $pq = q$

Now, $pq = q$

$$\Rightarrow \quad p = 1$$

Putting the value of p in $p + q = -p$, we get

$$1 + q = -1$$

$$\Rightarrow \quad q = -2$$

14. (*a*) Let α and β be the roots of the quadratic equation $2x^2 - 11x + 5 = 0$.

$$\therefore \quad \alpha + \beta = -\frac{(-11)}{2} = \frac{11}{2} \quad \text{...(i)}$$

$$\text{and} \quad \alpha \cdot \beta = \frac{5}{2} \quad \text{...(ii)}$$

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$$= \left(\frac{11}{2}\right)^2 - 4\left(\frac{5}{2}\right) = \frac{121}{4} - \frac{20}{2}$$

$$= \frac{121 - 40}{4} = \frac{81}{4} = \left(\frac{9}{2}\right)^2$$

$\therefore$ Difference of roots $= (\alpha - \beta) = \dfrac{9}{2} = 4.5$

15. (*d*) Given quadratic equation is

$$\frac{x^2}{a} + \frac{x}{b} + \frac{1}{c} = 0 \quad \text{...(i)}$$

Now, by condition the roots of the Eq. (i) are α and $\dfrac{1}{\alpha}$.

Now, product of roots $= \dfrac{1/c}{1/a}$

$$\Rightarrow \quad \alpha \cdot \frac{1}{\alpha} = \frac{a}{c}$$

$$\Rightarrow \quad c = a$$

which is the required relation.

16. (*b*) We know that, AM $\geq$ GM

$$\therefore \quad \sqrt{a} + \frac{1}{\sqrt{a}} \geq 2$$

Here, $\sqrt{x^2 - x + 1} + \dfrac{1}{\sqrt{x^2 - x + 1}} \geq 2$

$$\Rightarrow \quad 2 - x^2 \geq 2$$

$$\Rightarrow \quad x^2 \leq 0$$

$$\Rightarrow \quad x = 0$$

Hence, the given equation has only one solution.

17. (a) Here, $x = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$

On rationalising the terms given in square root, we get

$$x = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}} = \frac{\sqrt{5}+1}{2}$$

Now, substituting the value of x in $x^2 - x - 1$.

$$\therefore x^2 - x - 1 = \left(\frac{\sqrt{5}+1}{2}\right)^2 - \left(\frac{\sqrt{5}+1}{2}\right) - 1$$

$$= \frac{5+1+2\sqrt{5}}{4} - \frac{\sqrt{5}+1}{2} - 1$$

$$= \frac{6+2\sqrt{5}-2\sqrt{5}-2-4}{4} = 0$$

18. (a) Given that, one root of the equation $x^2 - bx + c = 0$ is square of other root of this equation *i.e.*, roots (α, α^2).

$\therefore$ Sum of roots $= \alpha + \alpha^2 = -\frac{(-b)}{1}$

$\Rightarrow \quad \alpha(\alpha + 1) = b \quad$ ...(i)

and product of roots $= \alpha \cdot \alpha^2 = \frac{c}{1}$

$\Rightarrow \quad \alpha^3 = c \Rightarrow \alpha = c^{1/3} \quad$ ...(ii)

From Eqs. (i) and (ii),

$c^{1/3}(c^{1/3} + 1) = b \quad$ ...(iii)

On cubing both sides, we get

$c(c^{1/3} + 1)^3 = b^3$

$\Rightarrow \quad c\{c + 1 + 3c^{1/3}(c^{1/3} + 1)\} = b^3$

$\Rightarrow \quad c\{c + 1 + 3b\} = b^3 \quad$ [from Eq. (iii)]

$\Rightarrow \quad b^3 = 3bc + c^2 + c$

19. (6) Let a and P be the roots of the quadratic equation $x + px + q = 0$.

Given that, *A* starts with a wrong value of *p* and obtains the roots as 2 and 6. But this time *q* is correct, i.e., product of roots

= qr = (X-P = 6 X2 = 12 ...(i)

and *B* starts with a wrong value of *q* and gets the roots as 2 and -9. But this time *p* is correct. i.e., sum of roots

= p = a + P = -9 + 2 = -7 ,..(ii) (a - P)^2 = (a + P)^2 - 4ap

= (-7)^2 -412 = 49 -48 = 1

[from Eqs. (i) and (ii)] => ct-P = l ...(iii)

Now, from Eqs. (ii) and (iii), we get

(X = -3andP = -4 which are correct roots.

Chapter

32

Permutations and Combinations

In our day-to-day life, we are interested to know the number of ways, in which a particular work can be done.

For this, we will have to know all the possible ways to do that work and it can be done with the help of **permutation** and **combination.**

Permutation

Each of the different arrangements which can be made by taking some or all of a given number of things or objects at a time, is called a permutation. Permutation implies arrangement, where order of the things is important.

For example The permutations of three items a, b and c taken two at a time are ab, ba, ac, ca, cb and be. Since, the order in which the items are taken, is important, ab and ba are counted as two different permutation.

Let *1* and *n* be positive integers such that 1 < *r* <*n.* Then, the number of permutations of n different things, taken *r* at a time, is denoted by ${}^nP_r$ or p(n, r).

Formula for permutation, $${}^nP_r = \frac{n!}{(n-r)!}$$

Cases of Permutation

There are several cases of permutation

1. Formation of numbers with given digits

In these type of questions, it is asked to form numbers with some different digit. These digit can be used with repetition or without repetitions.

Ex. 1 How many numbers of four digits can be formed with the digits 1, 2, 3, 4 and 5? (Repetition of digits is not allowed.)

Sol. There are five numbers and number of places to be filled up = 4
So, required number of numbers is

$$\begin{aligned} {}^5P_4 &= \frac{5!}{(5-4)!} \\ &= \frac{5 \times 4 \times 3 \times 2 \times 1}{1} = 120 \end{aligned}$$

Ex. 2 How many numbers between 400 and 1000 can be made with the digits 2,3, 4, 5, 6 and 0?

Sol. Here, nothing is mentioned about repetition of digits, therefore we will assume that repetition of digits is not allowed.

Now, any number of three digits to be in between 400 and 1000 its hundred place must be occupied by 4 or 5 or 6 (because if it will start with 0, 2 and 3, then it will not lie between 400 to 1000).

So, remaining two places can be filled up by five digits (since, six digits 2, 3, 4, 5, 6, 0 are given and if the first place is occupied by 4, then remaining two can be filled up by five digits 2, 3, 5, 6 and 0).

So, required number of ways to fill first place = 3 and required number of ways to **fill** remaining two places = P_2

$\therefore$ Required number of numbers $= 3 \times {}^5P_2$

$$= 3 \times \frac{5!}{(5-2)!}$$

$$= \frac{3 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 60$$

2. Formation of words with given letters

These question are very much similar to previous case questions but here in place of numbers, word are formed from a set of english alphabets given in the form of a word.

Important Point

Number of permulations of n objects out of which p are alike and are of one type, q are alike and are of second type and r are alike and are of third type $\frac{n!}{p!q!r!}$.

Ex. 3 In how many ways, can the letters of the word 'DIRECTOR' be arranged, so that the three vowels are never together?

Sol. Total number of letters = 8 and total number of vowels = 3 Here, R occurs two times.

$\therefore$ **Total number of arrangements when there is no restriction $= \frac{8!}{2!} = 20160$, but when three vowels are together, regarding them as one letter, we have only $5 + 1 = 6$ letters.**
These 6 letters can be arranged in $\frac{6!}{2!}$ ways, since R occurs twice.

Also, three vowels can be arranged among themselves in 3! ways. Hence, number of arrangements when the three vowels are together

$$= 3! \times \frac{6!}{2!} = 2160$$

Number of arrangements, so that the three vowels are never together

= 20160-2160= 18000

3. Arrangement of persons in a row or at a round table

These type of question are based on arrangement of person (boy or girls etc) in a straight line facing some direction or around some circular object like table etc.

Note Number of permutations of n objects taken all at a time is $n\backslash$, when repetition is not allowed.

Ex. 4 In how many different ways 5 girls can be seated in a row? Sol. Number of ways in which 5 girls can be seated in a row = 5! = **5x4x3x2x1=** 120

4. Arrangement of books on a shelf, etc

In such questions arrangement of books is done into a shelf in a row or one over the other.

Note Questions based on sending invitation to different persons are similar to questions based on arrangement of books.

Number of permutations of n different objects taken i at a time, when repetition is allowed = n^{l}

Ex. 5 In how many ways, 3 books can be given away to 7 boys, when each boy is eligible for any of the books?

Sol. Each of the three books can be given away to anyone of the 7 boys in 3 ways. .•. Required number of ways = 7 = 343

Ex. 6 A gentleman has 6 friends to invite. In how many ways, can he send invitation cards to them, if he has three servants to carry the cards?

Sol. Invitation cards may be sent to each of the six friends by anyone of the three servants in 3 ways.

∴ Required ways =3x3x3x3x3x3 = 3^6 = 729

Some other examples of Permutation

Ex. 7 If ${}^nP_4 = 360$, find n.

Sol. Given, $${}^nP_4 = 360$$

$$\therefore \quad \frac{n!}{(n-4)!} = 360$$

$$\Rightarrow \quad \frac{(n)(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = 360 \qquad \left[\because {}^nPr = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \quad n(n-1)(n-2)(n-3) = 360 = 6 \times 5 \times 4 \times 3$$

$$\therefore \quad n = 6$$

Ex. 8 If $^{10}P_r = 720$, find r.

Sol. Given, $^{10}P_r = 720$

$$\therefore \quad \frac{10!}{(10-r)!} = 720 = 10 \times 9 \times 8$$

$$\Rightarrow \quad \frac{10!}{(10-r)!} = \frac{10 \times 9 \times 8 \times 7!}{7!} = \frac{10!}{7!}$$

$$\Rightarrow \quad (10-r)! = 7! \Rightarrow 10 - r = 7$$

$$\therefore \quad r = 3$$

Combination

Combination of things means selection of things. Here, order of things has no

importance.

For example The combination of two letters from the group of three letters *A, B* and C

would be as follows *AB, BC, AC.*

Here, we make groups. So, *AB* or *BA* as a group is same. Obviously, if order matters,

then *AB* and *BA* are not same.

Formula for combination, $^nC_r = \dfrac{n!}{r!(n-r)!}$

It signifies number of groups formed from n different things, when r things are taken into consideration.

Important Points

- $^nC_n = {}^nC_0 = 1$
- $^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$
- $^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n - 1$
- $^nC_r = {}^nC_{n-r}$ or $^nC_r = ({}^nP_r)/r!$
- $^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

Cases of Combination

There are several cases of Combination

1. Formation of committee from a given set of persons

These questions are based on formation of a committee consisting of some members (male and/ or female) from a group of persons following a certain condition.

Ex. 9 In how many ways can 5 members form a committee out of 10 be selected so that (i) two particular members must be included.

(ii) two particular members must not be included.

Sol. (i) When two particular members are included then, we have to select $5 - 2 = 3$ members out of $10 - 2 = 8$

$\therefore$ The required number of ways $= C\ (8, 3) = \dfrac{8!}{3!5!} = \dfrac{8 \times 7 \times 6}{6} = 56$

(ii) When 2 particular members are not included then, we have to select 5 members out of $10 - 2 = 8$

$\therefore$ The required number of ways $= C\ (8, 5) = \dfrac{8!}{5!3!} = \dfrac{8 \times 7 \times 6}{6} = 56$

2. Selection of questions from question paper etc

In such question, a question paper is given with one or more parts and the different ways in which some specified number of questions can be attempted is asked.

Ex. **10A** question paper has two parts, part *A* and part *B,* each containing 10 questions. If the student has to choose 8 from part *A* and 5 from part *B,* in how many ways can he choose the question?

Sol. The required number of ways

$$= C\ (10, 8) \cdot C\ (10, 5)$$
$$= \frac{10!}{8!2!} \times \frac{10!}{5!5!} = \frac{10 \times 9}{2} \times \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2}$$
$$= 5 \times 9 \times 3 \times 2 \times 7 \times 6 = 11340$$

Some other examples of Combination

Ex. 11 Find the value of the following.

(i) ${}^{15}C_{11}$ (ii) ${}^{10}C_4$

Sol. (i) $${}^{15}C_{11} = \frac{15!}{11!\,(15-11)!} \qquad \left[\because {}^{n}C_r = \frac{n!}{(n-r)!\,r!}\right]$$

$$= \frac{15 \times 14 \times 13 \times 12 \times 11!}{11! \times 4!}$$

$$= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2} = 1365$$

(ii) $${}^{10}C_4 = \frac{10!}{4!(10-4)!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{4! \times 6!}$$

$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

Ex. 12 Find the number of ways, in which 7 books can be selected out of 10 books available.

Sol. Total number of books = 10

Number of books to be selected = 7

$\therefore$ Required number of ways = $${}^{10}C_7 = \frac{10!}{7!\,(10-7)!}$$

$$= \frac{10 \times 9 \times 8 \times 7!}{7! \times 3 \times 2 \times 1} = 120$$

Factorial

Factorial of a number can be defined as the product of all natural numbers upto that number *i.* e.,

n! = nx(n-l)x(n-2)x(n-3)x(n-4)x.....xl=nx(n-l)!

41 = 4x3x2x1=4x3! 111 = 11x10x9x8x7x6x5x4x3x2x1 **Note** Factorial of negative number and integers is not defined ${}^{n}P_n = n\backslash,\ {}^{n}P_0 = l$

Fundamental Principles of Counting

Multiplication Principle

If an operation can be performed in *m* different ways, following which a second

operation can be performed in *n* different ways, then the two operations in

succession can be performed inmxn ways.

This can be extended to any finite number of mutually exclusive operations.

In general, if there are *n* jobs to perform and each can be performed in rtij- ways

(i = 1,2,3,... n), then number of ways of doing all things simultaneously is

***n\ xnfy* xm$ x... *xmn*.**

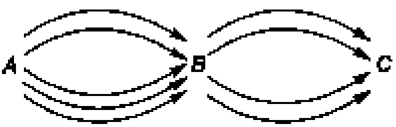
Here, the jobs performed are mutually inclusive.

Ex. 13A **hall has 12 gates. In how many ways, can a man enter the hall through one gate and come out through a different gate?**

Sol. Since, there are 12 ways of entering into the hall, the man come out through a different gate in 11 ways. Hence, by the fundamental principle of multiplication, total number of ways is **12 x** 11 = **132.**

Ex. 14 There are three stations *A, B* and C and five routes for going from station *A* to *B* and four routes from station *B* to C. Find the number of different ways through which a person can go from *A* to *C via B.*

Sol. Since, there are five routes for going from *A* to *B* and four routes from *B* to *C*. So, number of different ways to go from *A* to *CviaB*= 5 **x** 4 = 20.



Addition Principle

If an operation can be performed in *m* different ways and another operation, which is

independent of the first operation, can be performed in n different ways, then either

of the two operations can be performed in *(m* + n) ways. This can be extended to any

finite number of mutually exclusive operations.

In general, if there are *n* independent jobs, each of which can be performed in m^

ways, then the total number of ways of performing all things simultaneously is

n\ + ai2 + m^ +... + m*$_n$*.

Here, the jobs performed are mutually exclusive.

Ex. 15 There are 25 students in a class with 15 boys and 10 girls. The class teacher selects either a boy or a girl for monitor post of the class. In how many ways, the class teacher can make this selection?

Sol. As there are 15 boys and 10 girls and monitor selected can be anyone from the given students.

Hence, required number of ways = 15 + 10 = 25

Fast Track **Formulae**

to solve the QUESTIONS

Formula 1

If $^nC_x = {}^nC_y$, then $x = y$ or $x + y = n$

Ex. 16 If $^{15}C_{3r} = {}^{15}C_{r+3}$, find r.

Sol. $\because$ $^{15}C_{3r} = {}^{15}C_{r+3}$

$\Rightarrow$ Either $3r = r + 3$ or $3r + r + 3 = 15$

$\Rightarrow$ Either $r = \dfrac{3}{2}$ or $r = 3$

Since, r cannot be a fraction, so $r = 3$.

Formula 2

Number of circular permutations of n different objects $=(n-1)!$

Ex. 17 Find the number of ways, in which 10 boys can form a ring? **Sol.** Let us assume the boys *beBi,B2,B3,Bi,B5,Be, By, Bg, Bg* and *B^q.*

If we assume *B±* fixed, then other 9 boys can be arranged in 9! ways.

∴ Total number of ways = 9! = 362880

Fast Track Method Total number of boys = 10

According to the formula,

Total number of ways in which ring can be formed = (10 - 1)! = 9! = 362880

Formula 3

in a circular permutation, if clockwise and anti-clockwise arrangements are considered to be same, then Number of circular Dermutations of

$$n \text{ objects} = \frac{(n-1)!}{2}$$

Ex. 18 Find the total number of ways, in which 10 beads can be strung into a necklace?

Sol. Total number of beads = 10

According to the formula,

$$\text{Required number of ways} = \frac{(10-1)}{2} = \frac{9!}{2} = \frac{362880}{2} = 181440$$

Formula 4

Number of ways to declare the result where *'vi* match are played $=2^n$

Ex. 19 In a cricket tournament 5 matchs were played, then in how many ways result can be declared?

Sol. Total ways to declare the result = $2^n = 2^5 = 32$ ways

Formula 5

Let there are *n* persons in a hall. If every person shakes his hand with every other person only once, then total number of handshakes

$$= {}^nC_2 = \frac{n(n-1)}{2}$$

Note If in place of handshakes each person gives a gift to another person, then formula changes to = n $(n$ - 1) Number of diagonals in a polygon of n sides = ${}^{n}C_2 - n$

Ex.20 In a party, every person shakes his hand with every other person only once. If total number of handshakes is 210, then find the number of persons.

Sol. Let number of persons be n. Then, according to the question, ${}^{n}C_2 = 210$

$$\Rightarrow \quad \frac{n(n-1)}{2} = 210$$

$$\Rightarrow \quad n(n-1) = 420 = 21 \times 20$$

$$\Rightarrow \quad n = 21$$

Formula 6

If there are n non-col linear points in a plane, then (i) Number of straight lines formed = ${}^{n}C_2$ (ii) Number of triangles formed = "c3

(iii) Number of quadrilaterals formed = ${}^{n}C_4$

Ex.21 **In a plane, there are 16 non-collinear points. Find the number of straight lines formed.**

Sol. Here,n = 16.

/. Required number of straight lines formed = ™C2

$$= {}^{16}C_2 = \frac{16!}{2!(16-2)!} = \frac{16 \times 15 \times 14!}{2 \times 14!}$$

$$= 8 \times 15 = 120$$

Formula 7

if there are n points in a plane out of which m are collinear, then (i) Number of straight lines formed = "c2 - ${}^{m}C_2$ +1 (ii) Number of triangles formed = ${}^{n}C_3 - {}^{m}C_3$

Ex.22 In a plane, there are 11 points, out of which 5 are collinear. Find the number of triangles made by these points.

Sol. Here, $n = 11, m = 5$

Then, required number of triangles $= {}^nC_3 - {}^mC_3 = {}^{11}C_3 - {}^5C_3$

$$= \frac{11 \times 10 \times 9}{3 \times 2 \times 1} - \frac{5 \times 4 \times 3}{3 \times 2 \times 1}$$

$$= 165 - 10 = 155$$

Fast Track Practice

Exercise© *Base Level Questions*

1. Find the value of ${}^5P_2$.

(a) 15 *(b)* 18 *(c)* 20 *(d)* 122 *(e)* 26

2. If ${}^nP_3$= 9240, then find the value of *n.*

{*a)* 20 *(b)* 21 (CJ 22 (d) 23 r^ 24

3. If${}^{B0}C_r = {}^{50}C_{r+2}$,find7\

(a) 24 fbj 23 *(c)* 22 fdj 21

re; 20

4. If (lx 2 x 3 x 4x .... x *n) = n\,* then (14! - 13! - 12!) is equal to [SSC (10+2) 2012]

(a) 14x12 x (12!) *(b)* 14 x 12 x (13!) *(c)* 14 x 13 x (13!) *(d)* 13x12 x (12!)

5. In how many different ways, can the letters of the word 'INHALE' be arranged? [SBI Clerk 2012]

(a) 720 *(b)* 360

(c) 120 *(d)* 650

(e) None of the above

6. In how many ways, the letters of the word 'ARMOUR' can be arranged?

[Bank PO 2010]

(a) 720 *(b)* 300

(c) 640 *(d)* 350

(e) None of the above

7. In how many ways, the letters of the word 'BANKING' can be arranged?

[Bank PO 2010]

(a) 5040 *(b)* 2540

(c) 5080 *(d)* 2520

(e) None of the above

8. In how many ways, the letters of the word 'STRESS' can be arranged?

[Bank PO 2010]

(a) 360 *(b)* 240

(c) 720 *(d)* 120

(e) None of the above

9. In how many different ways, the letters of word 'FINANCE' can be arranged?

[NABARD2010]

(a) 5040 *(b)* 2040

(c) 2510 *(d)* 4080

(e) None of the above

10. In how many different ways, can the letters of the word VENTURE' be arranged? [IBPS Clerk 2011]

(a) 840 *(b)* 5040 *(c)* 1260 *(d)* 2520 *(e)* None of the above

11. How many different signals, can be made by 5 flags from 8 flags of different colours?

(a) 6270 *(b)* 1680 *(c)* 20160 *(d)* 6720 *(e)* None of the above

12. A child has four pockets and three marbles. In how many ways, the child can put the marbles in the pockets?

(a) 12 *(b)* 64 *(c)* 256 *(d)* 60 *(e)* None of the above

13. In how many ways, can the letters of the word 'ASSASSINATION' be arranged, so that all the S are together?

(a) 10! *(b)* 14!/(4!)

(c) 151200 *(d)* 3628800

(e) None of the above

14. There is a 7-digit telephone number with all different digits. If the digit at extreme right and extreme left are 5 and 6 respectively, find how many such telephone numbers are possible?

[IBACIO2012] (a) 120 *(b)* 100000

(c) 6720 *(d)* 30240

(e) None of the above

15. In a meeting between two countries, each country has 12 delegates. All the delegates of one country shake hands with all delegates of the other country. Find the number of handshakes possible?

[SSC CGL 2008]

(a) 72 *(b)* 144

(c) 288 *(d)* 234

16. Find the number of ways, in which 12 different beads can be arranged to form a necklace.

(a) $\frac{11!}{2}$ (b) $\frac{10!}{2}$ (c) $\frac{12!}{2}$

(d) Couldn't be determined
(e) None of the above

17.20 persons were invited to a party. In how many ways, they and the host can be seated at a circular table? *(a)* 18 ! *(b)* 19! *(c)* 20!

(d) Couldn't be determined

(e) None of the above

18. In how many ways, can 24 persons be seated around a circular table, if there are 13 seats?

(a) $\frac{24!}{13 \times 11!}$ (b) $\frac{22!}{14 \times 12!}$

(c) $\frac{23!}{13 \times 11!}$ (d) $\frac{24!}{12 \times 12!}$

(e) None of the above

19. In how many different ways, 5 boys and 5 girls can sit on a circular table, so that the boys and girls are alternate?

(a) 2880 *(b)* 2800 *(c)* 2680 *(d)* 2280 *(e)* None of the above

20. How many necklaces of 12 beads can be made from 18 beads of various colours?

(a) $\frac{118 \times 13!}{2}$ (b) $\frac{110 \times 14!}{2}$

(c) $\frac{119 \times 13!}{2}$ (d) $\frac{110 \times 12!}{2}$

(e) None of the above

21. A committee of 5 members is going to be formed from 3 trainees, 4 professors and 6 research associates. How many ways can they be selected, if [Bank PO 2010] (i) in committee, there are 2 trainees and 3 research associates? *(a)* 15 *(b)* 45 *(c)* 60 *(d)* 9 *(e)* None of the above (ii) there are 4 professors and 1 research associate or 3 trainees and 2 professors?

(a) 12 *(b)* 13 *(c)* 24 *(d)* 52 *(e)* None of the above

22. In how many ways, a committee of 3 men and 2 women can be formed out of a total of 4 men and 4 women? [SSC (10+2) 2007]

(a) 15 *(b)* 16 *(c)* 20 *(d)* 24

23. In how many ways, a cricket team of 11 players can be made from 15 players, if a particular player is always chosen?

(a) 1835 *(b)* 1001 *(c)* 1635 *(d)* 1365 *(e)* None of the above

24. In how many ways, a cricket team of 11 players can be made from 15 players, if a particular player is never chosen?

(a) 364 *(b)* 480 *(c)* 1365 *(d)* 640 *(e)* None of the above

25. How many straight lines can be drawn from 15 non-collinear points?

(a) 105 *(b)* 120 *(c)* 110 *(d)* 115 *(e)* 118

26. There is a polygon of 12 sides. How many triangles can be drawn using the vertices of polygon?

(a) 200 *(b)* 220 *(c)* 240 *(d)* 260 *(e)* 280

27. There are 14 points in a plane, out of which 4 are collinear. Find the number of triangles made by these points.

(a) 364 *(b)* 360 *(c)* 368 *(d)* 365

28. There are 10 points in a plane, out of which 5 are collinear. Find the number of straight lines formed by joining them.

(a) 36 *(b)* 45 *(C)* 30 *(d)* 35

29. Find the number of diagonals formed in hexagon.

(a) 12 *(b)* 10 *(c)* 6 *(d)* 9

Exercise © *Higher Skill Level Questions*

1. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800$, find ${}^{r}P_{2}$.

(a) 1840 *(b)* 2640

(c) 1640 *(d)* 820

(e) None of the above

Directions (Q. Nos. 2-5) Find *the number of permutations that can be made from the letters of the word 'OMEGA'.*

2. O and A occupying end places.

(a) 12 *(b)* 14 *(c)* 20 *(d)* 18 *(e)* None of the above

3. E being always in the middle.

(a) 18 ways *(b)* 24 ways

(c) 48 ways *(d)* 20 ways

(e) None of the above

4. Vowels occupying odd places.

(a) 12 ways *(b)* 16 ways

(c) 6 ways *(d)* 20 ways

(e) None of the above

5. Vowels being never together.

(a) 36 ways *(b)* 84 ways

(c) 120 ways *(d)* 10 ways

(e) None of the above

6. A question paper consists of two sections having respectively 3 and 5 questions. The following note is given on the paper. "It is not necessary to attempt all the questions". One question from each section is compulsory. In how many ways, a candidate can select the question?

(a) 38 *(b)* 217 *(c)* 256 *(d)* 320 *(e)* None of the above

7. Find the number of combinations that can be formed with 5 oranges,

4 mangoes and 3 bananas, when one fruit of each kind is taken.

(a) 60 *(b)* 120 *(c)* 110 *(d)* 75 *(e)* None of the above

8. In how many ways, 12 balls can be divided between 2 boys, one receiving

5 and the other 7 balls?

(a) 1784 *(b)* 1584 *(c)* 1854 *(d)* 1560 *(e)* None of the above

9. There are 10 stations on a railway line. The number of different journey tickets that are required by the authorities, is

[SNAP 2012]

(a) 92 *(b)* 90 *(C)* 91 *(d)* 93

(d) None of the above

10. The number of ways in which a committee of 3 ladies and 4 gentlemen can be appointed from a group consisting of 8 ladies and 7 gentlemen, if Mrs. *X* refuses to serve in a committee if Mr. Yis its member, is

(a) 1960 *(b)* 3240

(c) 1540 *(d)* None of these

11. There are 10 questions in a question paper. In how many ways, a student can solve these questions, if he solves one or more questions?

(a) 1024 *(b)* 1025 *(c)* 1023 *(d)* 1000

(e) None of the above

12. In how many ways, can 15 people be seated around two round tables with seating capacities of 7 and 8 people?

[IBAC1O2013]

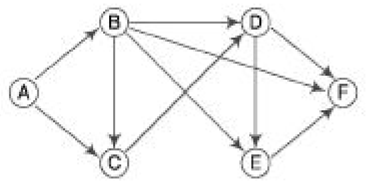
(a) 15!/(8!) *(b)* 7l/88!

(c) $^{15}C_8 x 6! x 7!$ *(d)* $^{15}C_8 * 8!$ *(e)* None of the above

13. A five-digit number divisible by 3 is to be formed using digits 0, 1, 2, 3, 4 and 5 without repetition, the total number of ways this can be done, is

(a) 122 *(b)* 210 *(c)* 216 *(d)* 217 *(e)* None of the above

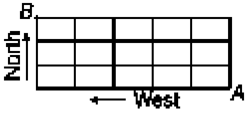
14. The figure below shows the network connecting cities *A, B, C, D, E* and *F*. The arrows indicate permissible direction of travel. What is the number of distinct Daths from *A* to *F!*



(a) 9 *(b)* 10

(c) 11 *(d)* None of these

15. In the given figure, the lines represent one way roads allowing travel only northwards or only westwards. Along how many distinct routes can a car reach point *B* from point A? *(a)* 15 *(b)* 56 *(c)* 120 *(d)* 336



16. A new flag is to be designed with six vertical stripes using some or all of the colour yellow, green, blue and red. Then, the number of ways this can be made such that no two adjacent stripes have the same colour is

(a) 12 x 81 (b) 16 x 192

(c) 20 x 125 (d) 24 x 216

17. An intelligence agency forms a code of two distinct digits selected from 0, 1, 2, ..., 9 such that the first-digit of code is non-zero. The code, handwritten on a slip, can however potentially create confusion when

read upside down, for examples the code 91 may appear as 16. How many codes are there for which no such confusion can arise?

(a) 80 *(b)* 78 *(c)* 71 *(d)* 69

18. How many numbers can be formed from 1, 2, 3, 4, 5 (without repetition), when the digit at the unit's place must be greater than that in the ten's place?

(a) 54 *(b)* 60 *(c)* $\frac{5!}{3}$ *(d)* $2 \times 4!$

19. Boxes numbered 1, 2, 3, 4 and 5 are kept in a row and they are to be filled with either a red or a blue ball, such that no two adjacent boxes can be filled with blue balls. Then, how many different arrangements are possible, given that all balls of a given colour are exactly identical in all respects? *(a)* 8 *(b)* 10 *(c)* 15 *(d)* 22

Answer with **Solutions**

Exercise© *Base Level Questions*

1. (*c*) $^5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = 5 \times 4 = 20$

2. (*c*) $^nP_3 = 9240 \Rightarrow \frac{n!}{(n-3)!} = 9240$

$\Rightarrow \quad n(n-1)(n-2) = 9240$

$\Rightarrow \quad n(n-1)(n-2) = 22 \times 21 \times 20$

$\therefore \quad n = 22$

3. (*a*) By formula, 1 $^nC_x = {}^nC_y$

$\Leftrightarrow x = y$ or $x + y = n$

Now,

$$^{50}C_r = {}^{50}C_{r+2}$$

$\Rightarrow \quad r + r + 2 = 50$ or $r = r + 2$

$\Rightarrow \quad 2r = 48$

$[\because r = r + 2$ is not possible$]$

$\therefore \quad r = 24$

4. (*a*) $14! - 13! - 12!$

$= 14 \times 13 \times 12! - 13 \times 12! - 12!$

$= 12!\,(14 \times 13 - 13 - 1)$

$= 12!\,(182 - 14) = 168 \times 12!$

$= 14 \times 12 \times 12!$

5. (*a*) The word 'INHALE' has 6 distinct letters.

$\therefore$ Number of arrangements $= n! = 6!$

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

6. (*e*) Number of arrangements $\frac{n!}{p!\,q!\,r!}$

Total letters = 6, but R has come twice.

So, required number of arrangements

$= \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 360$

7. (*d*) Total letters = 7, but N has come twice.

So, required number of arrangements

$= \frac{7!}{2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!}$

$= 2520$

8. (*d*) Required number of arrangements

$= \frac{6!}{3!}$ $\quad [\because$ S has come thrice$]$

$= \frac{6 \times 5 \times 4 \times 3!}{3!}$

$= 120$

9. (e) Total number of letters = 7, but N has come twice. So, required number of arrangements

$$= \frac{7!}{2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!}$$

$$= 2520$$

10. (*d*) The required different ways

$$= \frac{7!}{2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!} = 2520$$

11. (*d*) The number of ways taking 5 flags out of 8 flags

$$= {}^8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!} = 6720$$

12. (6) The first marble can be put into the pockets in 4 ways, so can the second and third. Thus, the number of ways in which the child can put the marbles

= 4X4X4= 64 ways

13. (c) When All S are taken together, then AS5AS5INATION are letters.

So, 10 letters in total can be arranged in 10! ways.

[∵ All 'S' are considered as 1]

But, here are 3 'A' and 2 'I' and 2 'N'.

∴ Required number of ways

$$= \frac{10!}{3! \times 2! \times 2!} = 151200$$

14. (c) There is a 7-digit telephone number but extreme right and extreme left positions are fixed.

i.e., 6XXXXX5

/. Required number of ways

= 8X7X6X5X4 = 6720

15. (6) Total number of handshakes

= 12X12 = 144

16. (a) Number of arrangements of beads = (12 - 1) ! = 11!, but it is not mentioned that either it is clockwise or anti-clockwise. So, required number of arrangements

$= \frac{1}{2}(12-1)! = \frac{11!}{2}$

17. (c) Total persons on the circular table

= 20 guests + 1 host = 21 They can be seated in (21 - 1)! = 20! ways.

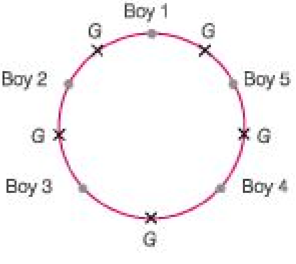
18. (a) First, we select 13 persons out of 24 persons in C_{13} ways. Now, these

13 persons can be seated in 12! ways around a table. So, required number of ways

$= {}^{24}C_{13} \times 12! = \frac{24!}{13!(24-13)!} \times 12!$

$= \frac{24!}{13!\,11!} \times 12! = \frac{24!}{13 \times 11!}$

19. (a) After fixing up one boy on the table, the remaining can be arranged in 4! ways, but boys and girls have to be alternate. There will be 5 places, one place each between two boys. These 5 places can be filled by 5 girls in 5! ways.



Hence, by the principle of multiplication, the required number of ways $= 4! \times 5! = 2880$

20. (*c*) First, we can select 12 beads out of 18 beads in ${}^{18}C_{12}$ ways. Now, these 12 beads can make a necklace in $\frac{11!}{2}$ ways as clockwise and anti-clockwise arrangements are same.

So, required number of ways

$$= {}^{18}C_{12} \cdot \frac{11!}{2!} = {}^{18}C_6 \cdot \frac{11!}{2!}$$

$$= \frac{18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 11!}{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 2!}$$

$$= \frac{17 \times 7 \times 13!}{2!} = \frac{119 \times 13!}{2}$$

21. (i) (*c*) Required number $= {}^3C_2 \times {}^6C_3$

$$= \frac{3!}{2!(3-2)!} \times \frac{6!}{3!(6-3)!}$$

$$= \frac{3 \times 2 \times 1}{2 \times 1 \times 1} \times \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 3 \times 2 \times 1} = 60$$

(ii) (*a*) Required number

$$= {}^4C_4 \times {}^6C_1 + {}^3C_3 \times {}^4C_2$$

$$= \frac{4!}{4!(4-4)!} \times \frac{6!}{1!(6-1)!} + \frac{3!}{3!(3-3)!} \times \frac{4!}{2!(4-2)!}$$

$$= 1 \times 6 + 1 \times 6 = 12$$

22. (*d*) Total number of ways

$$= {}^4C_3 \times {}^4C_2$$

$$= \frac{4!}{3! \times 1!} \times \frac{4!}{2! \times 2!}$$

$$= \frac{4 \times 4 \times 3 \times 2 \times 1}{2 \times 2}$$

$$= 4 \times 6 = 24$$

23. (*b*) Since, particular player is always chosen. It means that $11 - 1 = 10$ players are selected out of the remaining $15 - 1 = 14$ players.

$\therefore$ Required number of ways

$$= {}^{14}C_{10} = \frac{14!}{10! \times 4!}$$

$$= \frac{14 \times 13 \times 12 \times 11}{4 \times 3 \times 2 \times 1}$$

$$= 7 \times 13 \times 11$$

$$= 91 \times 11 = 1001$$

24. (*a*) Since, particular player is never chosen. It means that 11 players are selected out of $15 - 1 = 14$ players.

$\therefore$ Required number of ways

$$= {}^{14}C_{11} = 364$$

25. (*a*) Required number of lines

$$= {}^{n}C_2 = {}^{15}C_2 = \frac{15 \times 14}{2} = 105$$

26. (*b*) Required number of triangles

$$= {}^{12}C_3 = \frac{12 \times 11 \times 10}{6} = 220$$

27. (*b*) The required number of triangles

$$= {}^{n}C_3 - {}^{m}C_3$$

Here, $n = 14$, $m = 4$

$$= {}^{14}C_3 - {}^{4}C_3$$

$$= \frac{14 \times 13 \times 12 \times 11!}{3! \times 11!} - \frac{4!}{3! \times 1!}$$

$$= \frac{14 \times 13 \times 12}{6} - \frac{4}{1}$$

$$= 14 \times 26 - 4 = 364 - 4 = 360$$

28. (*a*) Required number of straight lines

$$= {}^{n}C_2 - {}^{m}C_2 + 1$$

Here, $n = 10$, $m = 5$

$$= {}^{10}C_2 - {}^{5}C_2 + 1 = 45 - 10 + 1 = 36$$

29. (*d*) Hexagon has 6 sides.

$\Rightarrow \quad n = 6$

$\therefore$ Required number of diagonals

$$= {}^{n}C_2 - n = {}^{6}C_2 - 6$$

$$= \frac{6!}{2!(6-2)!} - 6 = \frac{6!}{2!4!} - 6$$

$$= \frac{6 \times 5 \times 4!}{2 \times 4!} - 6 = 15 - 6 = 9$$

Exercise © *Higher SkiU Level Questions*

1. (c) $\frac{^{56}P_{r+6}}{^{54}P_{r+3}} = \frac{30800}{1}$

$$\Rightarrow \quad \frac{56!}{(50-r)!} \times \frac{(51-r)!}{54!} = \frac{30800}{1}$$

$$\Rightarrow \frac{56 \times 55 \times 54!(51-r)(50-r)!}{(50-r)!\,54!} = \frac{30800}{1}$$

$$\Rightarrow \quad 51 - r = \frac{30800}{56 \times 55} = 10$$

$$\Rightarrow r = 51 - 10 = 41$$

$$\therefore \quad {}^{r}P_2 = {}^{41}P_2 = \frac{41!}{39!} = 41 \times 40 = 1640$$

2. (a) When O and A occupy end places. Then, the three letters (M, E, G) can be arranged themselves by 3! = 6 ways and two letters (0, A) can be arranged among themselves in 2! = 2 ways.

∴ Total number of ways = 6x2 = 12

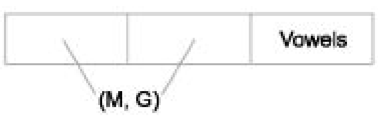
3. (6) When E is fixed in the middle, then there are four places left to be filled by four remaining letters O, M, G and A and this can be done in 4! ways.

/. Total number of ways = 4! = 24

4. (a) Three vowels (O, E, A) can be arranged in the odd places in 3! ways (1st position, 3rd position, 5th position) and two consonants (M, G) can be arranged in the even places in 2! ways (2nd place and 4th place).

∴ Total number of ways = 3! X 2! = 12

5. *(b)* Total number of words = 5! = 120



Combining the vowels at one place (OEA) with remaining 2 letters MG, letters can be arranged in 3! ways. Also, three vowels can be arranged in 3! ways.

So, when vowels are together, then number of words = 3! x 3! = 36 .•. Required number of ways, when vowels being never together = 120 - 36 = 84

6. (6) Here, we have two sections A and B. Section A has 3 questions and B has 5 questions and one question from each section is compulsory according to the given condition.

.". Number of ways selecting one or more than one question from section A

$= 2^3 - 1 = 7$

Similarly, from section B $= 2^5 - 1 = 31$

According to the rule of multiplication, the required number of ways in which a candidate can select the questions

= 7X31 =217 (a) The required number of combinations, when one fruit of each kind is taken

$= {}^5C_1 \times {}^4C_1 \times {}^3C_1 = 5 \times 4 \times 3 = 60$

8. (6) Here, order is important, then the number of ways in which 12 different balls can be divided between two boys who receives 5 and 7 balls respectively, is

$$= \frac{12!}{5!\,7!} \times 2! = 1584$$

9. (*b*) There are 10 stations on railway line.

So, the number of different journey tickets between two stations from given 10 stations from one side $= {}^{10}C_2 = \frac{10 \times 9}{2} = 45$

Similarly, number of different journey tickets from other side = 45 .'. Total number of tickets to be generated by authorities.

= 45 + 45 = 90

10. (d) If Mrs. Xis selected among the ladies in the committee, then Mr. Y is not selected or if Mrs. X is not selected then Mr. Y can be there in the committee. So, required number of ways

$$= {}^8C_3 \times {}^6C_4 + {}^7C_3 \times {}^7C_4$$
$$= \frac{8 \times 7 \times 6}{3 \times 2} \times \frac{6 \times 5}{2 \times 1} + \frac{7 \times 6 \times 5}{3 \times 2} \times \frac{7 \times 6 \times 5}{3 \times 2}$$
$$= 840 + 1225 = 2065$$

11. (C) Required number of ways

$= 2^n - 1 = 2^{10} - 1 = 1024 - 1 = 1023$

12. (C) Number of ways in which 8 persons can

15

be selected from 15 persons = C_8

Now, 8 persons can be seated around a circular table in 7! ways. Now, remaining 7 persons can be seated around a circular table in 6! ways.

.\ Required number of ways = C_8 x 7! X 6!

13. (c) A five-digit number, which is divisible by 3, is formed when sum of digits is also divisible by 3. So, combination formed using six-digits, which are divisible by 3

= 5 + 4 + 3 + 2+1 = 15 = 5 + 4 + 2+1 + 0 = 12

So, set of numbers are (5, 4, 3, 2, 1) and

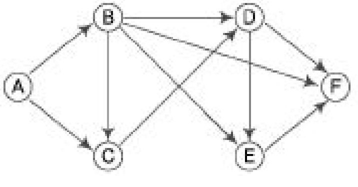
(5, 4, 2, 1, 0).

Number formed by using 1st set

= 5X4X3X2X1 = 120 Similarly, using 2nd set

= 4X4X3X2X1 = 96 Hence, using 2nd set, underlined place cannot be filled by 0, otherwise it will become a four-digit number. ∴Total numbers = 120+ 96 =216

14. (d) The maximum routes from A to F are listed below



(1) ABDF (2) ABCDF (3) ABF (4) ABEF (5) ACDF (6) ABCDEF (7) ACDEF (8) ABDEF

15. (6) Any route from A to B consists of 3 + 5 = 8 segments, where the car can move only 5 segments to the West and only 3 segments to the North. The number of distinct routes is equal to the number of ways of choosing 3 out of the 8 segments along which the car can go North or choosing 5 segments along which car can go West.

Therefore, the number of distinct routes from A to B is

$$^8C_3 = \frac{8!}{3!(8-3)!}$$

$$\frac{8!}{3!\times 5!} = \frac{8\times 7\times 6\times 5\times 4}{6\times 4\times 3\times 2\times 1}$$

$$= 56$$

Hence, there are 56 routes.

16. (a) Any of the 4 colours can be chosen for the first stripe. Any of the remaining 3 colours can be used for the second stripe. The third stripe can again be coloured in

3 ways (we can repeat the colour of the first

stripe but not use the colour of the second

stripe).

Similarly, There are 3 ways to colour each

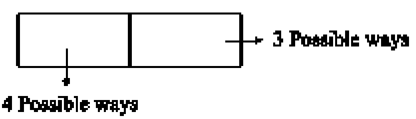
of the remaining stripes.

.•. The number of ways the flag can be

coloured is $4(3)^5 = (12)(3^4) = 12\text{x}81$

17. *{d)* The available digits are 0, 1, 2, ... 9. The first digit can be chosen in 9 ways (0 not acceptable), the second digit can be accepted in 9 ways (digit repetition not allowed). Thus, the code can be made in 9x9 =81 ways.

Now, there are only 4 digits which can create confusion 1, 6, 8, 9. The same can be given in the following ways



Total number of ways confusion can arise

= 4X3 = 12 Thus, the ways in which no such confusion arise =81-12=69

18. (6) The digit in the unit's place should be greater than that in the ten's place. Hence, if digit 5 occupies the unit place, then remaining four digits need not to follow any order, hence required numbers = 4!. However, if digit 4 occupies the unit place then 5 cannot occupy the ten's position. Hence, digit at the ten's place and it will be filled by the digit 1, 2 or 3. This can happen in 3 ways. The remaining 3 digits can be filled in the remaining three places in 3! ways.

Hence, in all, we have (3x30 numbers ending in 4. Similarly, if we have 3 in the unit's place, the ten's place can be either 1 or 2. This can happen in 2 ways. The remaining 3 digits can be arranged in the remaining 3 places in 3! ways. Hence, we will have (2x3!) number ending in 3. Similarly, we can find that there will be 3! numbers ending in 2 and no number with 1. Hence, total number of numbers

= 4!+ (3) X3!+ (2x3!) + 3!

= 4!+ 6x3! = 24+ (6x6) =60

19. (d) Total number of ways of filling the 5 boxes numbered as (1, 2, 3, 4 and 5) with either blue or red balls 2 =32.

Two adjacent boxes with blue can be obtained in 4 ways, *i.e.,* (12), (23), (34) and (45).

Three adj acent boxes with blue can be obtained in 3 ways, *i.e.,* (123), (234) and (345). Four adjacent boxes with blue can be obtained in 2 ways, *i.e.,* (1234) and (2345). And five boxes with blue can be got in 1 way. Hence, the total number of ways of filling the boxes such that adjacent boxes have blue = (4+3 + 2 + 1) = 10.

Hence, the number of ways of filling up the boxes such that no two adjacent boxes have blue = 32-10 = 22.

Chapter

33

Probability

Probability means the chances of happening/occurring of an event. So, in this chapter we discuss about the predictibility of an event to happen/occur. We usually predict about many events based on certain parameters. *For example*

♦ Getting a head or tail, when a coin is tossed

♦ Getting a number from 1 to 6, when a die is rolled

The better we know about the parameters related to an event better will be the accuracy of the result predicted. Mathematically, we can say that probability of happening an event is equal to the ratio of number of favourable outcomes to number of possible outcomes.

It is represented as shown below Probability happeninq of an event *P*

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

Terms Related to Probability

Various terms related to probability are as follows

Experiment

An action where the result is uncertain even though the all possible outcomes related to it is known in advance. This is also known as random experiment, e. *g.*, Throwing a die, tossing a coin etc.

Sample Space

A sample space of an experiment is the set of all possible outcomes of that experiment. It is denoted by S. *For example* If we throw a die, then sample space

$S = \{1, 2, 3, 4, 5, 6\}$ If we toss a coin, then sample space S = {Head, Tail}

Possible outcomes

All possibilities related to an event are known as possible outcomes.

Tossing a Coin When a coin is tossed, these are two possible outcomes.

So, we say that the probability of getting *H* is 1/2 or the probability of getting *T* is 1/2,

Throwing a Die When a single die is thrown, there are six possible outcomes 1, 2, 3,

4, 5 and 6.

The probability of getting any one of these numbers is $\frac{1}{6}$.

Ex.1 There are 5 marbles in a bag. 3 of them are red and 2 of them are blue. What

is the probability that a blue marble will be picked? **Sol.** Number of favourable outcomes = 2 (because there are 2 blue marbles) Total number of outcomes = 5 (because there are 5 marbles in total)

$$\text{So, required probability} = \frac{2}{5} = 0.4$$

Event

Event is the single result of an experiment, e. *g.,* Getting a head is an event related to tossing of a coin.

Types of Events

Various types of events are as follows

Certain and Impossible Events

A **certain event** is certain to occur, *i.e., S* (sample space) is a certain event. Probability of certain event is 1, *i.e., P[S)* = 1.

An **impossible event** has no chance of occurring, *i.e., §* is the impossible event. Probability of impossible event is 0, i. e., *P* (0) = 0.

Ex. 2 A teacher chooses a student at random from a class of 30 boys. What is the probability that the student chosen is a boy?

Sol. Since, all the students are boys, so chosen may be any one, *i.e.*,

Favourable cases = Total cases = 30

$$\therefore \quad \text{Probability} = \frac{30}{30} = 1$$

Ex. 3 A bag contains 20 black marbles, if a marble is picked at random from the bag. Find the probability that marble picked is of Red colour. **Sol.** The bag contains 20 black marbles and there is no red marble in the bag So favourable cases = 0 Total outcomes = 20

$$\therefore \text{Required probability} = \frac{0}{20} = 0$$

Equally Likely Events

Events related to an experiment are said to be equally likely events, if probability of

occurrence of each event is same.

For example When a dice is rolled the possible outcome of getting an odd number =

possible out come of getting an even number = 3.

So getting a even number or odd number are equally likely events.

Complement of an Event

The complement of an event *A* is the set of all outcomes in the sample space that are not included in the outcomes of event *A*. The complement of event *A* is represented by *A* (read as *A* bar). The probability of complement of an event can be found by subtractina the aiven probability from 1.

$$P(\bar{A}) = 1 - P(A)$$

Ex. 4 A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing a card that is not a king?

Sol. A standard deck contains 4 king.

So, probability of getting a king $= \frac{4}{52}$

Now, probability of not getting a king = 1 – Probability of getting a king

$$= 1 - \frac{4}{52} = 1 - \frac{1}{13} = \frac{13-1}{13} = \frac{12}{13}$$

Mutually Exclusive and Exhaustive Events

♦ Two events $E\pm$ and E_2 related to an experiment *E,* having sample space S are known as **mutually exclusive,** if the probability of occurrence of both events simultaneously is zero. *i. e.*,

$P(E_1nE_2)=Q$ *For example* When a coin is tossed either head or tail will appear. Head and tail con not occur simultareously. Therefore occurrence of a head or a tail are two mutually exclusive events.

■f Two events Ey and E_2 related to an experiment *E,* having sample space *S* are known as **mutually exhaustive,** if the probability of occurrence of event Ey or E_2 isl.

i.e., $P\{E_1<uE_2)=l$

For example Let *A* be probability of getting an even number when a dice is rolled and *B* be the probability of getting an odd number. The probability of occurrence of event *A* or event *B* is 1 i.e., any of the even can occur so they are mutually exhanstive. **Note** Events Ey E_2, E_3 ■■■E_n related to Sare known as

1. Mutually exclusive, if P (E_r n E_2 n£3-n£n) = 0 orE: nE_2 $r\backslash E_3$ ■•■E_n = 0

2. Mutually exhaustive, if $P(E_1$ uE_2 UE3 • • • $<jE_n)$ = 1 or E_t u E_2 u E_3 ■■■E_n = S

Dependent Events

Two events are called dependent, if the outcome or occurrence of the first affects the outcome or occurrence of the second, so that the probability is changed.

Ex.5 A card is chosen at random from a standard deck of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a queen and the second card chosen is a jack?

Sol. P (queen on first pick) $= \frac{4}{52}$

P (jack on 2nd pick given queen on 1st pick) $= \frac{4}{51}$

(why 51, because one card is already picked)

$$P \text{ (queen and jack)} = \frac{4}{52} \times \frac{4}{51} = \frac{4}{663}$$

Independent Events

Two events A and B are called independent, if occurring or non-occurring of A does not affect the occurring or non-occurring of B. If A and *B* are independent events, then

P (A and *B) = P {AnB) = P(A)-P (B) For example* getting head after tossing a coin and getting a 5 on a rolling single 6-sided die are independent events.

In general we can say that, if events *Ei,E2---En* related to an experiment are independent, then

Pft n *E2* n *E3* ■ ■ ■ *EB) = PftlP* ft) *P]&) PJE4)* ■ ■ • P *(En)* or P ft u *E2* uE3 *-En)* =1 - P ft) Pft) Pft)-Pft)

Ex. 6 A coin is tossed and a single 6-sided die is rolled. Find the probability of getting the head side of the coin and getting a 3 on the die.

Sol. Probability of getting a head when a coin is tossed $= \frac{1}{2}$

Probability of getting a 3 when a die is rolled $= \frac{1}{6}$

Now, the required probability that both occurs at the same time $= \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

Rules/Theorems Related to Probability

The various theorems related to probability are discussed below

Addition Rule of Probability

When two events *A* and *B* are mutually exclusive, the probability that *A* or *B* will occur, is the sum of the probability of each event.

P(Aor B) = P(A) + P(B) and P (AuB) = P *(A)* + *P (B)* But when two events *A* and *B* are non-mutually exclusive, the probability that *A* or *B* will occur, is

P (A or B) = P (A) + *P(B)-P(A* and B) *P(AuB)* = *P{A)* + *P(B)-P(AnB)*

Multiplication Theorem of Probability

When two events A and B are mutually exclusive, the probability that A and B will occur simultaneously is given as P (A n B) = *P(A)* ■ *P (B/A) P(AnB)* = *P(A)* ■ *P(B)* (A and B are independent event)

Ex. 7 From a well-shuffled pack of 52 cards, a card is frawer at roundom, find the probability that it is either a heart or a queen.

Sol. Let *A* be the probability of getting a hea card and *B* be the probability of getting a queen card

$$P(A) = \frac{13}{52} \quad P(B) = \frac{4}{52} \quad P(A \cap B) = \frac{1}{52} \qquad [\because \text{One heart card is a queen}]$$

$$\therefore \text{Required probability} = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{13+4-1}{52} = \frac{16}{52} = \frac{4}{13}$$

Ex. 8 Eight persons *A, B, C, D, E, F, G* and *H* appeared for an inter view. Find the probability that both *A* and D are selected in the interview?

Sol. Probability that A is selected $P(A) = \frac{1}{8}$

Probability that D is selected $P(D) = \frac{1}{8}$

$\therefore$ Required probability that both are selected $= \frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$

Law of Total Probability

If $E\backslash, E_2, E_3$---E_n *be n* mutually exclusive events related to an experiment, then probability of an event *A* which occurs with E_1 or E_2 or E_3 ■ ■ ■ E_n is given by $P(A) = P(EJ\ P\ (AIEJ + P(E_2)\ P\ (A/Ex) + - P(AIE_n)$

Conditional Probability

The conditional probability of an event *B* in relationship to an event *A* is the probability that event *B* occurs given that event *A* has already been occurred. The notation for conditional probability is *P (B/A),* It is pronounced as the probability of happening of an event *B* given that *A* has already been happened.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \text{ and } P(B \backslash A) = \frac{P(A \cap B)}{P(A)}$$

Ex. 9 A Mathematics teacher conducted two tests in her class. 25% of the students passed both tests and 42% of the students passed the first test. What per cent of the students passed the second test given that they have already passed the first test?

Sol. This problem describes a conditional probability, since it asks us to find the probability that the second test was passed given that the first test was passed. This can be solved bv multiplication rule.

i.e., $$P(B/A) = \frac{P(A \text{ and } B)}{P(A)}$$

According to the formula,

$$P(\text{second/first}) = \frac{P(\text{first and second})}{P(\text{first})} = \frac{0.25}{0.42} = 0.60 = 60\%$$

Note The probability of *'f* success in *'ri* trials of an event is given as $^nC_rP^TQ^{D\ r}$ where, *P* is the probability of success of that event in single trial and 'q" is the probability of failure of that event

Types of Questions

Various types of question asked on probability are as follows

1. Based on Coins

This types of questions are based on tossing of coin (s) and obtaining a particular face (Head/Tail) or obtaining same face on two or more coins.

Ex. 10 What is the probability of each outcome, when a coin is tossed?

Sol. Here, $S = \{H, T\}$, *i.e.*, $n(S) = 2$

$$\therefore \quad P(\text{Head}) = \frac{1}{2}; \; P(\text{Tail}) = \frac{1}{2}$$

Ex.11 A coin is tossed twice, then find the probability that a head is obtained **atleast once.**

Sol. When a coin is tossed twice, then possible outcome $n(S) = \{HH, HT, TH, TT\} = 4$ Way in which a head is obtained atleast once $n(\boldsymbol{P}) = \{HH, HT, TH\} = 3$

$$\therefore \text{Required probability} = \frac{n(P)}{n(S)} = \frac{3}{4}$$

2. Based on Dice

This type of questions are based on rolling of one or more dice and getting a particular number on the face or a particular sum on faces of the dice etc.

Ex. 12A single 6-sided die is rolled. What is the probability of getting an even number and getting an odd number?

Sol. The possible outcomes are **1,** 2, 3,4,5, and 6.

Even numbers are 2, 4 and 6, *i.e.,* 3 outcomes.

So, the number of favourable (even) outcomes = 3
and total number of outcomes = 6

So, the probability of getting an even number $= \frac{3}{6} = \frac{1}{2}$

Similarly, probability of getting an odd number $= \frac{3}{6} = \frac{1}{2}$

Ex. 13A single 6-sided die is rolled. What is the probability of getting a 2 or a 5?

Sol. Total outcomes when a die is rolled = 6

Probability of getting a number in a single throw of die = $\frac{1}{6}$

So, $P(2) = \frac{1}{6}$ and $P(5) = \frac{1}{6}$

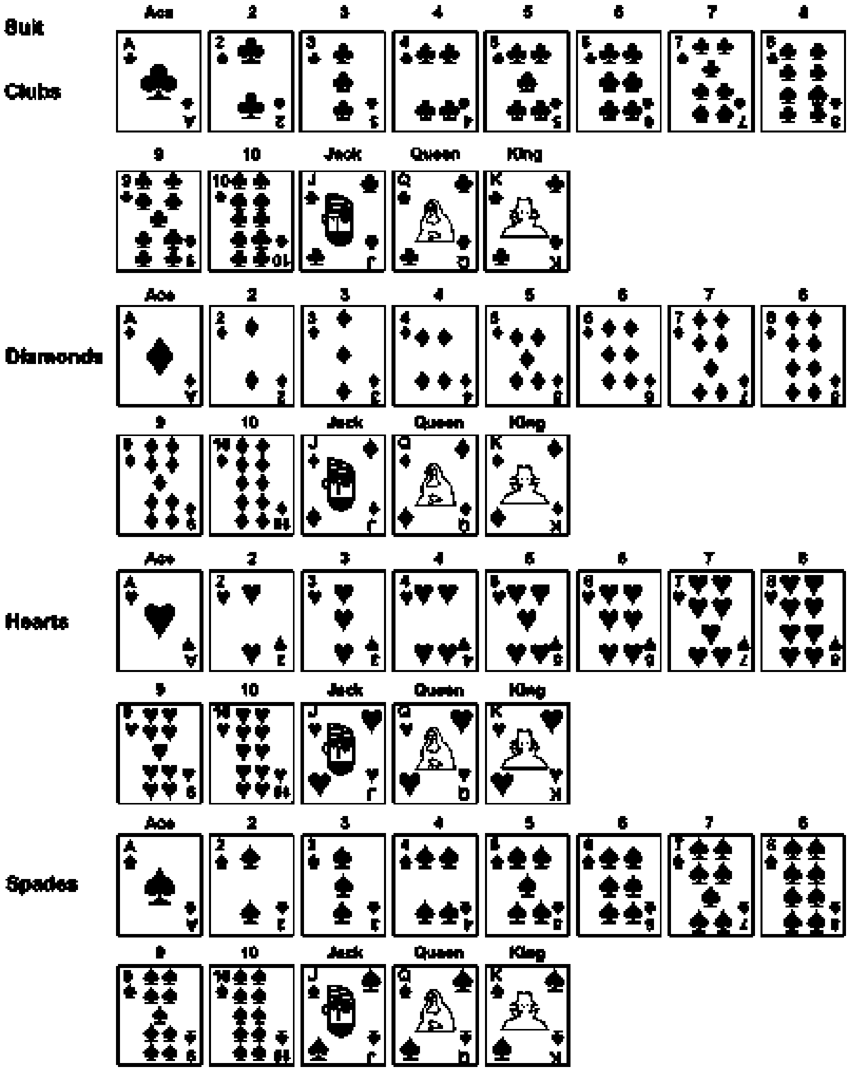
$\therefore$ Required probability of getting either 2 or 5

$$= P(2 \text{ or } 5) = P(2) + P(5) = \frac{1}{6} + \frac{1}{6}$$

$$= \frac{1+1}{6} = \frac{2}{6} = \frac{1}{3}$$

3. Based on Playing Cards

There are total of 52 cards in a deck of playing cards, which are explained below.



Important Points

(i) So, there are 13 cards of each suit Clubs, Diamonds, Hearts and Spades.

(ii) There are 4 Aces, 4 Jacks, 4 Queens and 4 Kings. (iii) There are 26 red and 26 black cards. (iv) There are 12 face cards.

Ex.14 A total of five cards are chosen at random from a standard deck of 52 playing cards. What is the probability of choosing 5 aces?

Sol. There are only 4 aces, so we cannot choose 5 aces. Thus, it is impossible event.

$\therefore$ Probability (5 aces) $= \frac{0}{52} = 0$

Ex. 15A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing a king or a club? Sol. There are 4 king in a standard deck and 13 club card.

Also 1 king is of club. so probability of getting a king $= \frac{4}{52}$

Probability of getting a club $= \frac{13}{52}$

Probability of getting a king of club $= \frac{1}{52}$

$\therefore$ Required probability of getting a king or club

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{4+13-1}{52} = \frac{16}{52} = \frac{4}{13}$$

4. Based on Marbles or Balls

These types of questions are based on choosing a ball or a marble of particular colour from one or more bag containing different coloured balls or marbles.

Ex. 16 A glass jar contains 1 red, 3 green, 2 blue and 4 yellow marbles. If a single marble is chosen at random from the jar, what is the probability that it is yellow or green?

Sol. Total marble $= 1 + 3 + 2 + 4 = 10$, *i.e.*, $n(S) = 10$

Now, probability of getting a yellow marble $= \frac{4}{10}$

Probability of getting a green marble $= \frac{3}{10}$

Since, the events are mutually exclusive

$\therefore$ P (yellow or green) $= P$ (yellow) $+ P$ (green) $= \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$

Ex. 17A person has 2 bags. He has 3 black and 4 white balls in one bag and 4 black and 3 white balls in another bag. Find the probability of getting a black ball.

Sol. 1st bag contains 3 black and 4 white balls and 2nd bag contains 4 black and 3 white ball.

Case I If 1st bag was chosen among the two bags and ball drawn is black, then

Required probability $= \frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$

$\left[\frac{1}{2}\text{ represents the probability of choosing a bag out of two}\right]$

Case II If 2nd bag was chosen among the two bags and ball drawn is black, then

Required probability $= \frac{1}{2} \times \frac{4}{7} = \frac{4}{14}$

.". Required probability of choosing the black ball among the two bag

$$= P\text{ (1st bag) or } P\text{(2nd bag)}$$
$$= \frac{3}{14} + \frac{4}{14} = \frac{3+4}{14} = \frac{7}{14} = \frac{1}{2}$$

5. Miscellaneous (Choosing a student, hitting a target, etc.)

Ex. 18 In a Mathematics class of 30 students, 17 are boys and 13 are girls. On a unit test, 4 boys and 5 girls made an *A* grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an '*A* grade student'?

Sol. Total boys = 17 and total girls = 13

Also, girls getting *A* grade = 5 and boys getting *A* grade = 4

Probability of choosing a girl $= \frac{13}{30}$

Probability of choosing a *A* grade student $= \frac{9}{30}$ $[\because 4 + 5 = 9]$

Now, *A* grade student chosen can be a girl,

So, probability of choosing it $= \frac{5}{30}$

$\therefore$ Required probability of choosing a girl or a *A* grade student

$$= \frac{13}{30} + \frac{9}{30} - \frac{5}{30}$$
$$= \frac{22-5}{30} = \frac{17}{30}$$

Ex. 19A person can hit a target 4 out of 7 shots. If he fixes 10 shots, what is the probability that he hit the target twice?

Sol. Here, $n = 10$ and $r = 2$

Success, $p = \frac{4}{7}$

Failure, $q = 1 - \frac{4}{7} = \frac{3}{7}$

$$P\text{(hit the target twice)} = {}^{10}C_2\left(\frac{4}{7}\right)^2\left(\frac{3}{7}\right)^8$$

$$= \frac{10!}{2!\,(10-2)!}\left(\frac{4}{7}\right)^2\left(\frac{3}{7}\right)^8 = 45\,\frac{(4)^2(3)^8}{7^{10}}$$

X. 20 The probability that it is Friday and a student is absent, is 0.03. Since, there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent, given that today is Friday?

Sol. Probability of knowing that it is friday and a student is absent = 0.03 Also, probability of day be a Friday = 0.2 Now, if it is given that today is friday then, it is a conditional probability

$$\text{So } P(\text{absent}/\text{Friday}) = \frac{P\text{ (Friday and absent)}}{P\text{ (Friday)}} = \frac{0.03}{0.2} = 0.15$$

Fast Track Practice

Exercise© *Base Level Questions*

1. What is the probability that a card drawn at random from a pack of 52 cards is either a king or a spade?

[Bank Clerks 2010]

(a) $\frac{17}{52}$ *(b)* $\frac{4}{13}$ *(c)* $\frac{3}{13}$ *(d)* $\frac{13}{52}$

(e) None of the above

2. A card is drawn from a well-shuffled pack of cards. The probability of getting a queen of club or a king of heart is

(a) $\frac{1}{52}$ *(b)* $\frac{1}{26}$ *(c)* $\frac{1}{13}$ *(d)* $\frac{1}{39}$

(e) None of the above

One card is drawn at random from a well-shuffled pack of 52 cards. What is the probability that the card is either a red card or a king? *(a)* 1/13 *(b)* 7/13 *(c)* 10/13 *(d)* None of these

4. A single letter is selected at random from the word 'PROBABILITY. The probability that it is a vowel, is

(a) $\frac{3}{11}$ *(b)* $\frac{4}{11}$ *(c)* $\frac{2}{11}$ *(d)* 0

(e) None of the above

5. The probability that a leap year selected at random contains 53 Sundays, is

(a) $\frac{7}{366}$ *(b)* $\frac{26}{183}$ *(c)* $\frac{1}{7}$ *(d)* $\frac{2}{7}$

(e) None of the above

6. The probability of drawing a red card from a deck of playing cards is

(a) $\frac{2}{18}$ *(b)* $\frac{1}{13}$ *(c)* $\frac{1}{4}$ *(d)* $\frac{1}{2}$

(e) None of the above

The probability of getting a composite number when a six-faced unbiased die is tossed, is

(a) $\frac{1}{4}$ *(b)* $\frac{1}{3}$ *(c)* $\frac{1}{2}$ *(d)* 1

(e) None of the above

8. If three unbiased coins are tossed simultaneously, then the probability of exactly two heads, is

(a) 1/8 *(b)* 2/8 *(c)* 3/8 *(d)* 4/8

(e) None of the above

9. Let *E* be the set of all integers with 1 at their unit places. The probability that a

number chosen from {2, 3, 4........, 50} is

an element of *E,* is

(a) $\frac{5}{49}$ *(b)* $\frac{4}{49}$

(c) $\frac{3}{49}$ *(d)* $\frac{2}{49}$

(e) None of the above

10. When two dice are rolled, what is the probability that the sum of the numbers appeared on them is 11?

(a) $\frac{1}{6}$ *(b)* $\frac{1}{18}$ *(c)* $\frac{1}{9}$ *(d)* 1
(e) None of the above

11. A committee of 3 members is to be selected out of 3 men and 2 women. What is the probability that the committee has atleast one woman? [Bank PO 2008]

(a) $\frac{1}{10}$ *(b)* $\frac{9}{20}$ *(c)* $\frac{9}{10}$ *(d)* $\frac{1}{20}$
(e) None of the above

12. A basket contains three blue and four red balls. If three balls are drawn at random from the basket, what is the probability that all the three balls are either blue or red? [Bank PO 2010]

(a) 1 *(b)* $\frac{1}{7}$
(c) $\frac{3}{14}$ *(d)* $\frac{3}{28}$
(e) None of the above

13. An urn contains 3 red and 4 green marbles. If three marbles are picked at random, what is the probability that two are green and one is red?

[New India Assurance 2009]

(a) $\frac{3}{7}$ *(b)* $\frac{18}{35}$
(c) $\frac{5}{14}$ *(d)* $\frac{4}{21}$
(e) None of the above

14. A number is selected at random from the

set {1, 2, 3, ... , 50}. The probability

that it is a prime, is

(a) 0.1 *(b)* 0.2 *(c)* 0.3 *(d)* 0.7

(e) None of the above

Directions (Q. Nos. 15-17) *Study the given information carefully and answer the questions that follow.*

A basket contains 4 red, 5 blue and 3 green marbles. [Bank P02010]

15. If two marbles are drawn at random, what is the probability that both are red?

(a) $\frac{3}{7}$ (b) $\frac{1}{2}$
(c) $\frac{2}{11}$ (d) $\frac{1}{6}$
(e) None of the above

16. If three marbles are picked at random, what is the probability that at least one is blue?

(a) $\frac{7}{12}$ (b) $\frac{37}{44}$
(c) $\frac{5}{12}$ (d) $\frac{7}{44}$
(e) None of the above

If three marbles are picked at random, what is the probability that either all are ereen or all are red?

(a) $\frac{7}{44}$ (b) $\frac{7}{12}$
(c) $\frac{5}{12}$ (d) $\frac{1}{44}$
(e) None of the above

Directions (Q. Nos. 18-19) *Study the information carefully to answer the questions that follow.*

A basket contains 3 blue, 2 green and 5 red balls. [Bank po 2007]

18. If three balls are picked at random, what is the probability that atleast one is red?

(a) $\frac{1}{2}$ (b) $\frac{7}{12}$ (c) $\frac{11}{12}$ (d) $\frac{1}{5}$
(e) None of the above

19. If four balls are picked at random, what is the probability that two are green and two are blue?

(a) $\frac{1}{18}$ (b) $\frac{1}{70}$ (c) $\frac{3}{5}$ (d) $\frac{1}{2}$
(e) None of the above

20. Three mangoes and three apples are kept in a box. if two fruits are chosen at random, find the probability that one is a mango and the other is an apple.

(a) $\frac{2}{3}$ (b) $\frac{3}{5}$ (c) $\frac{1}{3}$ (d) $\frac{1}{5}$

(e) None of the above

21. The probability that a man will be alive for 10 more years is $\frac{1}{4}$ and the probability that his wife will alive for 10 more years is $\frac{1}{3}$. The probability that none of them will be alive for 10 more years, is

(a) $\frac{5}{12}$ (b) $\frac{1}{2}$

(c) $\frac{7}{12}$ (d) $\frac{11}{12}$

(e) None of the above

22. In a lottery 10000 tickets are sold and ten prizes are awarded. What is the probability of not getting a prize, if you buy one ticket?

(a) 9/10000 (b) 9/10

(c) 999/1000 (d) 9999/10000

23. Two persons A and B appear in an interview for two vacancies. If the probabilities of their selections are $\frac{1}{4}$ and $\frac{1}{6}$, respectively, then the probability that none of them is selected, is

(a) $\frac{5}{8}$ (b) $\frac{5}{12}$

(c) $\frac{1}{12}$ (d) $\frac{1}{24}$

(e) None of the above

24. The probabilities of solving a problem by three students A, B and C are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively. The probability that the problem will be solved, is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

(e) None of the above

25. Out of 13 applicants for a job, there are 5 women and 8 men. It is desired to select 2 persons for the job. The probability that atleast one of the selected persons will be a woman, is

(a) $\frac{25}{39}$ (b) $\frac{14}{35}$ (c) $\frac{5}{13}$ (d) $\frac{10}{13}$

(e) None of the above

26. Five coins are tossed at a time. Then, the probability of obtaining atleast one tail is

(a) $\frac{31}{32}$ (b) $\frac{1}{32}$

(c) $\frac{1}{5}$ (d) $\frac{5}{32}$

(e) None of the above

Directions (Q. Nos. 27-31) *Study the given information carefully and answer the questions that follow.*

An urn contains 6 red, 4 blue, 2 green and 3 yellow marbles. [Bank po 2010]

27. If two marbles are picked at random, what is the probability that both marbles are red?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{15}$ (d) $\frac{2}{5}$

(e) None of the above

28. If three marbles are picked at random, what is the probability that two are blue and one is yellow?

(a) $\frac{3}{91}$ (b) $\frac{1}{5}$ (c) $\frac{18}{455}$ (d) $\frac{7}{15}$

(e) None of the above

29. If four marbles are picked at random, what is the probability that at least one is blue?

(a) $\frac{4}{16}$ (b) $\frac{69}{91}$ (c) $\frac{11}{15}$ (d) $\frac{22}{91}$

(e) None of the above

30. If two marbles are picked at random, what is the probability that either both are green or both are yellow?

(a) $\frac{5}{91}$ (b) $\frac{1}{35}$ (c) $\frac{1}{3}$ (d) $\frac{4}{105}$

(e) None of the above

If four marbles are picked at random, what is the probability that one is green, two are blue and one is red?

(a) $\frac{24}{455}$ (b) $\frac{13}{35}$ (c) $\frac{11}{15}$ (d) $\frac{7}{91}$

(e) None of the above

Directions (Q. Nos. 32-36) *Study the following information carefully to answer the questions that follow.* A box contains 2 blue caps, 4 red caps, 5 green caps and 1 yellow cap.

[Bank PO 2009]

32. If two caps are picked at random, what is the probability that both are blue?

(a) $\frac{1}{6}$ (b) $\frac{1}{10}$ (c) $\frac{1}{12}$ (d) $\frac{1}{45}$

(e) None of the above

33. If four caps are picked at random, what is the probability that none is green?

(a) $\frac{7}{99}$ (b) $\frac{5}{99}$ (c) $\frac{7}{12}$ (d) $\frac{5}{12}$

(e) None of the above

34. If three caps are picked at random, what is the probability that two are red and one is green?

(a) $\frac{9}{22}$ (b) $\frac{6}{19}$ (c) $\frac{1}{6}$ (d) $\frac{3}{22}$

(e) None of the above

35. If one cap is picked at random, what is the probability that it is either blue or yellow?

(a) $\frac{2}{9}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{6}{11}$

(e) None of the above

36. If two caps are picked at random, what is the probability that atleast one is red?

(a) $\frac{1}{3}$ (b) $\frac{16}{21}$ (c) $\frac{19}{33}$ (d) $\frac{7}{19}$

(e) None of the above

37. A and B are two events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$. If A and B are independent, then $P(B)$ is

(a) $\frac{2}{3}$ (b) $\frac{3}{8}$ (c) $\frac{2}{7}$ (d) $\frac{4}{7}$

(e) None of the above

38. A box contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the box, the probability that it is green is 2/3. The number of blue balls in the box is

(a) 13 (b) 12 (c) 16 (d) 8

39. From 4 children, 2 women and 4 men, 4 persons are selected. The probability that there are exactly 2 children among the selected persons, is

(a) $\frac{11}{21}$ (b) $\frac{9}{21}$ (c) $\frac{10}{21}$ (d) $\frac{5}{21}$

(e) None of the above

40. There are two bags containing white and black balls. In the first bag, there are 8 white and 6 black balls and in the second bag, there are 4 white and 7 black balls. One ball is drawn at random from any of these two bags. Find the probability of this ball being black.

(a) $\frac{5}{9}$ (b) $\frac{7}{19}$ (c) $\frac{41}{77}$ (d) $\frac{9}{17}$

(e) None of the above

41. A bag contains 3 red, 4 white and 7 black balls. The probability of drawing a red or a black ball, is

(a) $\frac{4}{7}$ (b) $\frac{1}{7}$ (c) $\frac{5}{7}$ (d) $\frac{3}{7}$

(e) None of the above

Exercise © *Higher SkiU Level Questions*

1. The probability that a man can hit a target is 3/4. He tries 5 times. The probability that he will hit the target atleast three times, is

(a) $\frac{291}{364}$ (b) $\frac{371}{464}$

(c) $\frac{471}{502}$ (d) $\frac{459}{512}$

(e) None of the above

2. *A* dice is rolled three times and sum of three numbers appearing on the uppermost face is 15. The chance that the first roll was four is

(a) 2/5 *(b)* 1/5

(c) 1/6 *(d)* None of these

3. In a ward-robe, Nitish has 3 trousers. One of them is black, second is blue and third brown. In this ward-robe, he has 4 shirts also. One of them is black and the other 3 are white. He opens his ward-robe in the dark and picks out one shirt-trouser pair without examining the colour. What is the likelihood that neither the shirts nor the trousers are black?

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

(e) None of the above

4. Murari has 9 pairs of dark blue socks and 9 pairs of black socks. He keeps them all in the same bag. If he picks out three socks at random, what is the probability that he will get a matching pair?

(a) $\frac{{}^{9}C_3 \times {}^{9}C_1}{{}^{18}C_3}$ (b) $\frac{2 \times {}^{9}C_2 \times {}^{9}C_1}{{}^{18}C_3}$

(c) 1 (d) $\frac{4}{7}$

(e) None ol the above

5. An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane at the first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1, respectively. What is the probability that the plane is hit when all the four shots are fired? *(a)* 0.4379 *(b)* 0.6872

(c) 0.6976 *(d)* 0.3507

(e) None of the above

6. Four boys and three girls stand in queue for an interview. The probability that they stand in alternate position, is

(a) $\frac{1}{34}$ (b) $\frac{1}{35}$ (c) $\frac{1}{17}$ (d) $\frac{1}{68}$

(e) None of the above

Directions (Q. Nos. 7-10) *Study the following information carefully to answer the questions.*

Five boys and five girls are sitting in a row. Find the probability that

7. All the girls are sitting together.

(a) $\frac{1}{42}$ (b) $\frac{41}{42}$ (c) $\frac{3}{35}$ (d) $\frac{1}{42}$

(e) None of the above

8. All the boys are sitting together.

(a) $\frac{3}{42}$ (b) $\frac{41}{42}$ (c) $\frac{3}{35}$ (d) $\frac{1}{42}$

(e) None of the above

9. All the girls are not sitting together.

(a) $\frac{41}{42}$ (b) $\frac{39}{42}$

(c) $\frac{31}{42}$ (d) $\frac{31}{35}$

(e) None of the above

10. All the boys are not sitting together.

(a) $\frac{3}{35}$ (b) $\frac{43}{49}$

(c) $\frac{41}{42}$ (d) $\frac{1}{42}$

(e) None of the above

11. An elevator starts with 5 passengers and stops at 8 different floors of the house. Find out the probability of all the 5 passengers alighting at different floors.

(a) $\frac{101}{512}$ (b) $\frac{105}{512}$

(c) $\frac{107}{512}$ (d) $\frac{109}{512}$

(e) None of the above

12. If a 4-digit number is formed at random using the digits 1, 3, 5, 7 and 9 without repetition, then the probability that it is divisible by 5, is

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$

(c) $\frac{1}{5}$ (d) $\frac{2}{3}$

(e) None of the above

Answer with Solutions

Exercise rt *Base Level Questions*

1. (*b*) Required probability

$$= \frac{3}{52} + \frac{13}{52} = \frac{16}{52} = \frac{4}{13}$$

[**Hint** Why $\frac{13}{52}$ because there are 13 spades and why $\frac{3}{52}$ instead of $\frac{4}{52}$ (there are four kings) because one king is already counted in spades.]

2. (*b*) Total ways = 52

There is one queen of club and one king of heart.

∴ Favourable ways = 1 + 1 = 2

∴ Required probability = $\frac{2}{52} = \frac{1}{26}$

3. (*b*) Total number of cards = 52

Total number of red cards = 26

Total number of kings = 4

But 2 red cards are also kings

∴ Here probability

$$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{26+4-2}{52} = \frac{28}{52} = \frac{7}{13}$$

4. (*b*) Total number of letters $= n(S) = 11$

whereas, number of vowels $= n(E) = 4$

∴ Required probability = $\frac{n(E)}{n(s)} = \frac{4}{11}$

5. (*d*) In a leap year, there are 366 days. It means 52 full weeks + 2 odd days. These two days can be (Mon, Tues), (Tues, Wed), (Wed, Thurs), (Thurs, Fri), (Fri, Sat), (Sat, Sun) or (Sun, Mon).

So, required probability = $\frac{2}{7}$

6. (*d*) Total number of cards $n(S) = 52$

Number of red cards $n(E) = 26$

∴ $$P(E) = \frac{n(E)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

7. (*b*) $n(S) = 6$; $n(E) = (4, 6) = 2$

∴ $$P(E) = \frac{2}{6} = \frac{1}{3}$$

8. (*c*) $n(S) = 2^3 = 8$

Let E = Event of getting exactly two heads

$= \{(H, H, T), (H, T, H), (T, H, H)\}$

$= n(E) = 3$

∴ Required probability = $\frac{3}{8}$

9. (*b*) $n(S) = 49$

Favourable numbers are 11, 21, 31, 41.

$\therefore$ Required probability $= \dfrac{4}{49}$

10. (*b*) $n(S) = 36$

$$n(E) = \{(5,6),(6,5)\} = 2$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

11. (*c*) Required probability

$$= \frac{{}^2C_1 \times {}^3C_2 + {}^2C_2 \times {}^3C_1}{{}^5C_3}$$

$$= \frac{9}{10}$$

12. (*b*) Probability to be a blue $= \dfrac{{}^3C_3}{{}^7C_3}$

Probability to be a red $= \dfrac{{}^4C_3}{{}^7C_3}$

$\therefore$ Required probability

$$= \frac{{}^3C_3}{{}^7C_3} + \frac{{}^4C_3}{{}^7C_3} = \frac{5}{35} = \frac{1}{7}$$

13. (*b*) Number of ways to select 3 marbles out of 7 marbles $= n(S) = {}^7C_3 = 35$

Probability that 2 are green and 1 is red

$$= n(E) = {}^4C_2 \times {}^3C_1 = 18$$

$\therefore$ Required probability $= \dfrac{18}{35}$

14. (*c*) $n(S) = 50$

Prime numbers are $= 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47$

$$\therefore \quad n(E) = 15$$

$$P(E) = \frac{15}{50} = \frac{3}{10} = 0.3$$

15. (*e*) Total number of possible outcomes

$$= n(S) = {}^{12}C_2 = \frac{12 \times 11}{2} = 66$$

Total number of favourable events

$$= n(E) = {}^4C_2$$

$$= \frac{4 \times 3}{2 \times 1} = 6$$

$\therefore$ Required probability $= \dfrac{6}{66} = \dfrac{1}{11}$

16. (*b*) Total number of possible outcomes

$$= {}^{12}C_3 = 220$$

Number of events which do not contain blue marbles (3 marbles out of 7 marbles)

$$= {}^{7}C_3 = 35$$

$\therefore$ Required probability

$$= 1 - \frac{35}{220} = \frac{37}{44}$$

17. (*d*) Total number of ways of selection of 3 marbles out of 12 $= n(S) = {}^{12}C_3 = 220$

Total number of favourable events

$$= n(E) = {}^{3}C_3 + {}^{4}C_3 = 1 + 4 = 5$$

$\therefore$ Required probability

$$= \frac{5}{220} = \frac{1}{44}$$

18. (*c*) Total number of outcomes $= {}^{10}C_3 = 120$

Number of outcomes not containing red balls $= {}^{5}C_3 = 10$

$\therefore$ Probability that at least one is red

$$= 1 - \frac{10}{120} = \frac{11}{12}$$

19. (*b*) Total number of outcomes $= {}^{10}C_4 = 210$

Favourable number of outcomes

$$= {}^{3}C_2 \times {}^{2}C_2 = 3 \times 1 = 3$$

$\therefore$ Required probability $= \dfrac{3}{210} = \dfrac{1}{70}$

20. (*b*) Total number of ways

$$= n(S) = {}^{6}C_2 = 15$$

Favourable number of ways

$$= n\,(E) = {}^{3}C_1 \times {}^{3}C_1 = 9$$

$\therefore$ Required probability $= \dfrac{9}{15} = \dfrac{3}{5}$

21. (*b*) Required probability $= P(\overline{A}) \times P(\overline{B})$

$$= \left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{3}\right) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$$

22. (*c*) Total lottery tickets = 10000

Total prize in the lottery = 10

$\therefore$ Probability of getting a prize

$$= \frac{10}{10000} = \frac{1}{1000}$$

Now, probability of not getting a prize

= 1 − Probability of getting a prize

$$= 1 - \frac{1}{1000} = \frac{999}{1000}$$

23. (*a*) Required probability $= P(\overline{A}) \times P(\overline{B})$

$$= \left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{6}\right) = \frac{3}{4} \times \frac{5}{6} = \frac{5}{8}$$

24. (*c*) First, we find the probability of not solving the problem.

$P(\overline{A}) \times P(\overline{B}) \times P(\overline{C})$

$$= \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right)$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

$\therefore$ Required probability $= 1 - \frac{1}{4} = \frac{3}{4}$

25. (*a*) Total ways $= {}^{13}C_2$

Favourable number ways of selecting men only $= {}^{8}C_2$

$\therefore$ Probability of selecting no woman

$$= \frac{{}^{8}C_2}{{}^{13}C_2} = \frac{14}{39}$$

$\therefore$ Probability of selecting at least one woman

$$= 1 - \frac{14}{39} = \frac{25}{39}$$

26. (*a*) Total events $= n(S) = 2^5 = 32$

$n(\overline{E})$ of getting heads $= 1$

$$P(\overline{E}) = \frac{1}{32}$$

$$\therefore \quad P(E) = 1 - P(\overline{E}) = 1 - \frac{1}{32} = \frac{31}{32}$$

27. (*e*) Total number of marbles

$$= 6 + 4 + 2 + 3 = 15$$

Ways of selection of two red marbles

$$= n(E) = {}^{6}C_2$$

Ways of selection of two marbles

$$= n(S) = {}^{15}C_2$$

So, required probability

$$= \frac{{}^{6}C_2}{{}^{15}C_2} = \frac{6 \times 5}{15 \times 14} = \frac{1}{7}$$

28. (*c*) Ways of selection of two blue marbles

$$= {}^{4}C_2$$

Ways of selection of one yellow marble $= {}^{3}C_1$

Ways of selection of three marbles $= {}^{15}C_3$

So, required probability $= \dfrac{{}^{4}C_2 \times {}^{3}C_1}{{}^{15}C_3}$

$$= \frac{\dfrac{4!}{2!(4-2)!} \times \dfrac{3!}{1!(3-1)!}}{\dfrac{15!}{3!(15-3)!}}$$

$$= \frac{\dfrac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} \times \dfrac{3 \times 2 \times 1}{1 \times 2 \times 1}}{\dfrac{15 \times 14 \times 13 \times 12!}{3 \times 2 \times 12!}}$$

$$= \frac{6 \times 3}{5 \times 7 \times 13} = \frac{18}{455}$$

29. (*b*) Probability that none is blue from four marbles

$$= \frac{^{15-4}C_4}{^{15}C_4} = \frac{^{11}C_4}{^{15}C_4} = \frac{22}{91}$$

So, probability that at least one is blue from four marbles

$$= 1 - \frac{22}{91} = \frac{69}{91}$$

30. (*d*) Ways of selection of two green marbles

$$= {^2C_2} = 1$$

Ways of selection of two yellow marbles

$$= {^3C_2} = 3$$

So, probability (both are green)

$$= \frac{1}{^{15}C_2} \quad \text{...(i)}$$

Probability (both are yellow)

$$= \frac{3}{^{15}C_2} \quad \text{...(ii)}$$

Then, required probability

$$= \frac{1}{^{15}C_2} + \frac{3}{^{15}C_2} = \frac{4}{^{15}C_2} = \frac{4}{105}$$

31. (*a*) Ways of selection of 4 marbles

$$= n(S) = {^{15}C_4}$$

Ways of selection of one green marble

$$= n(E_1) = {^2C_1}$$

Ways of selection of two blue marbles

$$= n(E_2) = {^4C_2}$$

Ways of selection of one red marble

$$= n(E_3) = {^6C_1}$$

$\therefore$ Required probability

$$= \frac{^2C_1 \times {^4C_2} \times {^6C_1}}{^{15}C_4} = \frac{24}{455}$$

32. (*e*) Total number of caps

$$= 2 + 4 + 5 + 1 = 12$$

Total number of outcomes

$$= n(S) = {^{12}C_2} = 66$$

Favourable number of outcomes

$$= n(E) = {^2C_2} = 1$$

$\therefore$ Required probability $= \dfrac{1}{66}$

33. (*a*) Total number of caps = 12

Total number of results $= n(S) = {^{12}C_4} = 495$

Out of 5 caps, number of ways to not pick a green cap $= n(E_1) = {^5C_0} = 1$

and out of 7 caps, number of ways to pick 4 caps $= n(E_2) = {^7C_4} = 35$

$\therefore$ Required probability $= \dfrac{1 \times 35}{495} = \dfrac{7}{99}$

34. (*d*) Total number of caps = 12

$\therefore \quad n(S) = {}^{12}C_3 = 220$

$n(E_1)$ = Out of 4 red caps, number of ways to pick 2 caps = ${}^4C_2 = 6$

$n(E_2)$ = Out of 5 green caps,
Number of ways to pick one cap = ${}^5C_1 = 5$

$$P(E) = \frac{n(E_1) \times n(E_2)}{n(S)} = \frac{6 \times 5}{220} = \frac{3}{22}$$

35. (*b*) Total number of caps = 12

$$n(S) = {}^{12}C_1 = 12$$

Out of (2 blue + 1 yellow) caps, number of ways to pick one cap $n(E) = {}^3C_1 = 3$

$\therefore$ Required probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

36. (*c*) Required probability $= 1 - \dfrac{{}^8C_2}{{}^{12}C_2}$

$$= 1 - \frac{28}{66} = \frac{38}{66} = \frac{19}{33}$$

37. (*e*) Let $P(B) = x$

Given, $P(A \cup B) = 0.8$ and $p(A) = 0.3$

$\Rightarrow \quad P(A) + P(B) - P(A \cap B) = 0.8$

$\Rightarrow \quad P(A) + P(B) - P(A)\,P(B) = 0.8$

[$\because$ A and B are independent]

$\Rightarrow \quad 0.3 + x - 0.3x = 0.8$

$\Rightarrow \quad 0.7x = 0.5$

$\therefore \quad x = \dfrac{5}{7}$

38. (*d*) Let the number of green marble = x.
Then, probability of getting a green marble

$$\frac{{}^xC_1}{{}^{24}C_1} = \frac{2}{3}$$

$\Rightarrow \quad \dfrac{x}{24} = \dfrac{2}{3} \Rightarrow x = 16$

So, number of blue marble = 24 − 16 = 8

39. (*b*) Total number of cases = ${}^{10}C_4$

Favourable number of cases = ${}^4C_2 \cdot {}^6C_2$

[Since, we are to select 2 children out of 4 and remaining 2 persons are to be selected from remaining 6 persons $(2W + 4M)$]

$\therefore$ Required probability

$$= \frac{{}^4C_2 \cdot {}^6C_2}{{}^{10}C_4} = \frac{\frac{4 \times 3}{2 \times 1} \times \frac{6 \times 5}{2 \times 1}}{\frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}}$$

$$= \frac{\frac{12}{2} \times \frac{30}{2}}{210} = \frac{90}{210} = \frac{9}{21}$$

40. (c) The probability of selecting first bag out of two bags and getting one black ball from

1 ^ C 3 First bag = - X-----1- = —

2 14Cl 14

The probability of selecting second bag and getting one black ball from other

2 ud 22

Hence, required probability

$$= \frac{3}{14} + \frac{7}{22} = \frac{41}{77}$$

41. (c) Here, total balls are 14.

∴ Required probability

$$= \frac{{}^{3}C_1 + {}^{7}C_1}{{}^{14}C_1} = \frac{3+7}{14} = \frac{10}{14} = \frac{5}{7}$$

Exercise © *Higher Skill Level Questions*

1. (*d*) Required probability

Given, $n = 5$ and $r = 3$,

Then, success $P = \frac{3}{4}$

Failure, $q = 1 - \frac{3}{4} = \frac{1}{4}$

Man hit the target thrice

$$= {}^5C_3 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^2 + {}^5C_4 \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right) + {}^5C_5 \left(\frac{3}{4}\right)^5$$

$$= \frac{270}{1024} + \frac{405}{1024} + \frac{243}{1024} = \frac{918}{1024} = \frac{459}{512}$$

2. (*d*) Total number of favourable outcomes $n(S) = 6^3 = 216$

Combinations of outcomes for getting sum of 15 on uppermost face = (4, 5, 6), (5, 4, 6), (6, 5, 4), (5, 6, 4), (4, 6, 5), (6, 4, 5), (5, 5, 5), (6, 6, 3), (6, 3, 6), (3, 6, 6)

Now, outcomes on which first roll was a four, $n(E)$ = (4, 5, 6), (4, 6, 5)

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{216} = \frac{1}{108}$$

3. (*d*) Probability that trousers are not black $= \frac{2}{3}$

Probability that shirts are not black $= \frac{3}{4}$

$\therefore$ Required probability $= \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

4. (c) Since, there are only two types of socks in the bag. So, if Murari picks up 3 socks, then certainly two of them are of same type. Thus, this is a certain event. Hence, required probability = 1

5. (c) First we find the probability that the plane is not hit

= (1 - 0.4) (1 - 0.3) (1 - 0.2) (1 - 0.1) = 0.6x 0.7X 0.8x 0.9 =0.3024 .'. Probability that the plane is hit = 1-0.3024=0.6976

6. (6) Total number of possible arrangements for 4 boys and 3 girls in a queue = 7!

When they occupy alternate position, then the arrangement would be like BGBGBGB. Thus, total number of possible arrangements $= 4! \times 3!$

$\therefore$ Required probability $= \frac{4! \times 3!}{7!}$

$$= \frac{4 \times 3 \times 2 \times 3 \times 2}{7 \times 6 \times 5 \times 4 \times 3 \times 2} = \frac{1}{35}$$

7. (*a*) We have, 5 boys and 5 girls.

Since, all the girls are sitting together, we consider them as one. Now, we can arrange 5 boys and 1 girl in 6! ways and these 5 girls (whom we considered as one) can also be arranged in 5! ways.

$\therefore$ Favourable number of ways $= 6!5!$

and total number of ways to arrange 5 boys and 5 girls $= 10!$

$\therefore$ Required probability $= \frac{6!5!}{10!} = \frac{1}{42}$

8. (*d*) As, we have done in solution number 6

$\therefore$ Required probability $= \frac{6!5!}{10!} = \frac{1}{42}$

9. (*a*) Required probability = 1 − Probability that all the girls sit together $= 1 - \frac{1}{42} = \frac{41}{42}$

10. (*c*) Required probability = 1 − Probability that all the boys sit together $= 1 - \frac{1}{42} = \frac{41}{42}$

11. (*b*) Required probability

$$= \frac{\text{Favourable number of cases}}{\text{Total number of cases}} = \frac{{}^8P_5}{8^5} = \frac{105}{512}$$

12. (*c*) Number of 4-digit numbers which are formed with 1, 3, 5, 7, 9

$$= {}^5P_4 = 5 \times 4 \times 3 \times 2 = 120 = n(S)$$

Number of 4-digit numbers which are formed with 1, 3, 5, 7, 9 and are divisible by 5

$$= {}^4P_3 = 4 \times 3 \times 2 = 24 = n(E)$$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{24}{120} = \frac{1}{5}$

Chapter

34

Area and Perimeter

Area and perimeter are measuring parameters related to various two-dimensional figures like triangle, rectangle, square etc.

Area

Total space enclosed by the boundary of a plane figure is called the area of that particular figure. In another words, the area of a figure is a measure associated with the part of plane enclosed in the figure. Area is measured

in square unit like, square metre,

square centimetre etc.

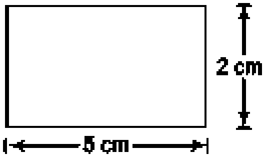
For example If length (J) of a

rectangle is 5 cm and breadth (b) is

2 cm, then area of rectangle is given

as

lxb=*5* x2 =10 sq cm



Perimeter

Perimeter is the length of border around any enclosed plane.

Therefore, sum of the sides of a plane figure is the perimeter

of that particular figure.

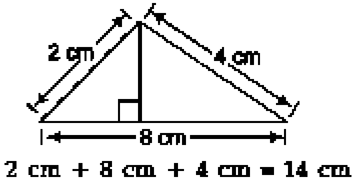
Unit of perimeter is same as the unit of sides of a given figure.

like metre, centimetre etc.

For example If the sides of a triangle are 2 cm, 8 cm and 4 cm,

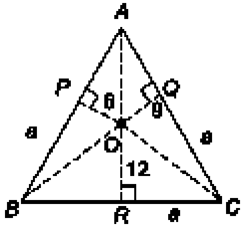
respectively, then perimeter of that particular triangle is given

as



Triangle

A figure enclosed by three sides is known as a triangle. A triangle has three angles with total sum of 180°. Adjoining figure represents a triangle with sides, *AB, BC, CA* and *ZA, ZB, ZC* are the three angles of the triangle. *Various types of triangles are discussed below.*

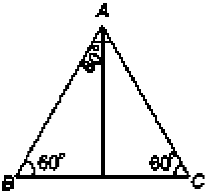


Equilateral Triangle

It has all three sides equal and each angle equal to 60°.

$$\text{(i) Area} = \frac{\sqrt{3}}{4}a^2 \approx 0.433\,a^2 \quad \text{(ii) Height} = \frac{\sqrt{3}}{2}a \approx 0.866\,a$$

(iii) Perimeter = 3a, where a = Side

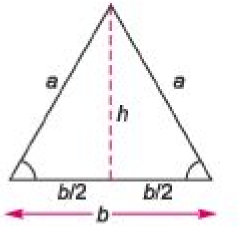


Isosceles Triangle

It has any two sides and two angles equal and altitude drawn on non-equal side bisects it.

$$\text{(i) Area} = \frac{b}{4}\sqrt{4a^2 - b^2}$$

$$\text{(ii) Height} = \sqrt{a^2 - \left(\frac{b}{2}\right)^2} = \frac{1}{2}\sqrt{4a^2 - b^2}$$



(iii) Perimeter = a + a + *b*=*2a*+*b*

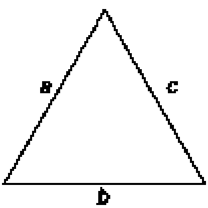
where, a = Each of two equal sides *b* = Third side

Scalene Triangle

It has three unequal sides.

$$\text{(i) Area} = \sqrt{s(s-a)(s-b)(s-c)} \ \text{[Hero's formula]}$$

$$\text{where, } s = \frac{a+b+c}{2} \text{ and } a, b \text{ and } c \text{ are the sides of the}$$



triangle and s is semi-perimeter. (ii) Perimeter = a + *b*+ *c*

Right Angled Triangle

It is a trianale with one anale eaual to 90°.

$$\text{(i) Area} = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times b \times p$$

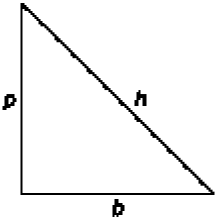
(ii) Perimeter =p+ b + h (iii) h^2=p^2 + b^2

$$\Rightarrow h = \sqrt{p^2 + b^2}$$

where, p = Perpendicular, b =Base and h = Hypotenuse

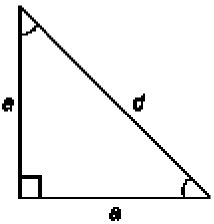
2 2 2

Note $h = p + b$ is known as Pythagoras theorem.



Isosceles Right Angled Triangle

It is a triangle with one angle equal to 90° and two sides containing the right angle are equal.



(i) Area $= \frac{1}{2} \times a^2$

(ii) Perimeter $= a + a + d = 2a + d$

Properties of Triangle

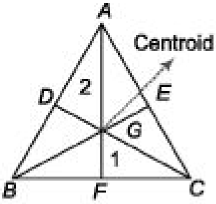
1. Sum of any two sides of a triangle is greater than the third side

2. Side opposite to the greatest angle will be the greatest and side opposite to the smallest angle will be the smallest

3. Among all the triangles that can be formed with a given perimeter, the equilateral triangle will have the maximum area

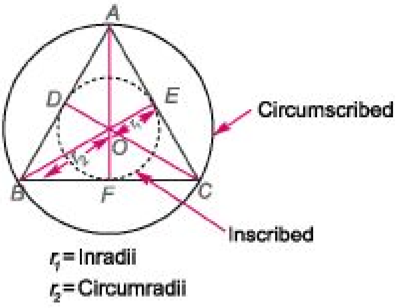
4. The lines joining the mid-points of sides of a triangle to the opposite vertex are called medians. In the given figure, *AF, BE* and *CD* are medians

5. The point where the three medians of a triangle meet is called centroid. In the given figure, *O* is the centroid. The centroid divides each of the median in the ratio of 2 : 1



6. The median of a triangle divides it into two triangles of equal areas

7. The incentre and circumcentre lies at a point that divides the height in the ratio 2:1. *i.e.,* the circumradius is always twice the median

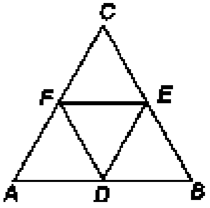


8. Radius of an incircle of an equilateral triangle of side a is $\frac{a}{2\sqrt{3}}$ and area is $\frac{\pi a^2}{12}$.

9. Radius of circumcircle of an equilateral triangle of side a will be $\frac{a}{\sqrt{3}}$ and area is $\frac{\pi a^2}{3}$.

10. The area of the triangle formed by joining the mid-points of the sides of a given triangle is

$\frac{1}{4}$th of the area of the given triangle.



Ex. 1 Find the perimeter of a triangle with sides equal to 3 cm, 8 cm and 5 cm.

Sol. Required perimeter = Sum of the sides

= (3 + 8 + 5) cm = 16 cm

Ex. 2 The perimeter of an equilateral triangle is 45 cm. Find its area.

Sol. Given that,perimeter of an equilateral triangle is 45 cm.

Let each side of triangle be *a* cm, then sum of sides = 45 cm

or $\quad 3a = 45$

$\therefore \quad a = \frac{45}{3} = 15$ cm

$\therefore \quad \text{Area} = \frac{\sqrt{3}}{4} a^2$

$$= \frac{\sqrt{3}}{4} \times 15^2 = \frac{225\sqrt{3}}{4} \text{ sq cm}$$

Ex. 3 A triangle with three equal sides has its area equal to 4V3~ sq cm. What will be the perimeter of this triangle?

Sol. We know that,

Area of equilateral triangle $= \frac{\sqrt{3}}{4} a^2$

According to the question,

$$\frac{\sqrt{3}}{4} a^2 = 4\sqrt{3}$$

$\Rightarrow \quad a^2 = 4 \times 4 = 16$

$\therefore \quad a = \sqrt{16} = 4$ cm

$\therefore$ Required perimeter sum of sides $= 3a = 3 \times 4 = 12$ cm

Ex. 4 The perimeter of an isosceles triangle is 32 cm while its equal sides together measure 18 cm. Find the third side and each of the equal sides. Sol. Let the third side be *x*.

According to the question,

Sum of all three sides = Perimeter *(x)* + 18 = 32

x = 32 - 18 = 14 cm

So, the third side = 14 cm
and each equal side = $\frac{18}{2} = 9$ cm

Ex. 5 The area of a right angled triangle is 42 sq cm. If its perpendicular is equal to 10 cm, find its base.

Sol. Given that, area = 42 sq cm, perpendicular (p) = 10 cm, base (b) = ?
According to the question,

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height (perpendicular)} = \frac{1}{2} \times b \times p$$

$$\Rightarrow \quad 42 = \frac{1}{2} \times b \times 10$$

$$\therefore \quad b = \frac{2 \times 42}{10} = \frac{84}{10} = 8.4 \text{ cm}$$

Ex. 6 Find the area of a triangle whose sides are 26 cm, 28 cm and 30 cm.

Sol. Given that, a = 26 cm, b = 28 cm, c = 30 cm
From Hero's formula, Area = $\sqrt{s(s-a)(s-b)(s-c)}$

Now, $$s = \frac{a+b+c}{2} = \frac{26+28+30}{2} = \frac{84}{2} = 42$$

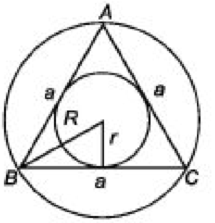
and $(s-a) = 42 - 26 = 16$ cm,
$(s-b) = 42 - 28 = 14$ cm and $(s-c) = 42 - 30 = 12$ cm

$$\therefore \quad \text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{42 \times 16 \times 14 \times 12}$$
$$= \sqrt{14 \times 3 \times 16 \times 14 \times 4 \times 3} = 14 \times 4 \times 2 \times 3 = 336 \text{ sq cm.}$$

Ex. 7 If the area of circumcircle of an equilateral triangle is $\frac{25\pi}{3}$ cm^2, then find the radius of the incircle to the triangle.

Sol. From the adjoining figure,

Let ΔABC is equilateral triangle with side 'a' cm with circum circle of radius R cm and incircle with radius r cm.



Now, area of circumcircle with side 'a' cm $= \frac{\pi a^2}{3}$

$$\frac{\pi a^2}{3} = \frac{25\pi}{3}$$

$\Rightarrow$ $$a^2 = 25$$

$\therefore$ $$a = \sqrt{25} = 5$$

Now, radius of incircle $= \frac{a}{2\sqrt{3}} = \frac{5}{2\sqrt{3}}$ cm

Quadrilateral

A figure enclosed by four sides is called a quadrilateral. A quadrilateral has four angles and sum of these angles is equal to 360°. *Various types of quadrilateral are discussed below.*

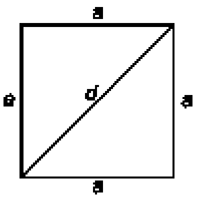
Square

It is a parallelogram with all 4 sides equal and each angle is equal to 90°.

(i) Area = $(\text{side})^2 = a^2$ or $\frac{1}{2}d^2$ (ii) Perimeter $= 4 \times$ side $= 4a$

(iii) Diagonal $(d) = a\sqrt{2}$

where, a = side, d = diagonal



Properties of Square

1. Diagonal of a square are equal and bisect each other at right angles (90°)

2. All square are rhombus but converse is not true

3. Diagonal is the diameter of the circumscribing circle that circumscribes the square and

circumradius = $\frac{a}{\sqrt{2}}$.

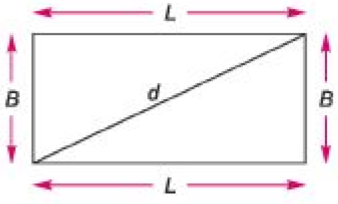
4 If area of two squares are in the ratio of $A_1 : A_2$, then ratio of their perimeter is given $\sqrt{A_1} : \sqrt{A_2}$

Rectangle

It is a parallelogram with opposite sides equal and each angle is equal to 90°.

(i) Area = Length x Breadth = *L* x *B* (ii) Perimeter = 2 (L + *B)*

(iii) Diagonal (d) = $\sqrt{L^2 + B^2}$



Properties of Rectangle

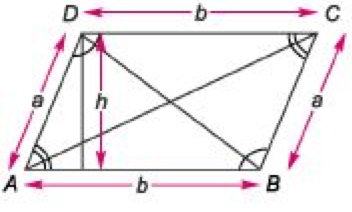
1. The diagonals of a rectangle are of equal lengths and they bisect each other

2. All rectangles are parallelograms but reverse is not true

Parallelogram

A quadrilateral, in which opposite sides are parallel is called a parallelogram.

(i) Area = Base x Height =*bxh* (ii) Perimeter = 2 *(a* + b)



Note Opposite angles are equal in a parallelogram but they are not right angle

Properties of Parallelogram

(i) Diagonals of a parallelogram bisect each other

(ii) Each diagonal of a parallelogram divides it into two triangles of equal area (iii) A parallelogram inscribed in a circle is a rectangle (iv) A parallelogram circumscribed about a circle is a rhombus (v) A parallelogram and a rectangle have equal areas if they are on the same base and

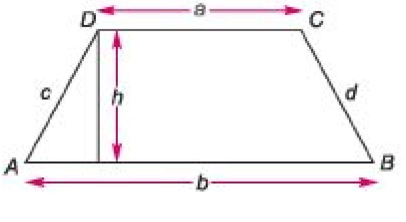
between the same parallel lines (vi) The opposite angles of parallelogram are equal. *[ZA* = *ZC* and *ZB* = *ZD)* (from above

figure) (vii) The sum of the squares of the four sides is equal to the sum of squares of diagonal AC^2 + *BE?* $=AB^2 + BC^2 + CD^2 + AD^2 = 2(AB^2 + BC^2)$ {v *AB* = *CD* and *BC* = *AD}*

Trapezium

It is a quadrilateral with any one pair of opposite sides parallel.

(i) Area $=\frac{1}{2}$ (Sum of the parallel sides × Height) $=\frac{1}{2}(a+b)h$ where, a and b are



parallel sides and *h* is the height or perpendicular distance between a and fa. (ii) Perimeter =*AB* + *BC* + *CD*+*AD* (hi) Area of trapezium, when the lengths of parallel and non-parallel sides are

given $=\frac{a+b}{k}\sqrt{s(s-k)(s-c)(s-d)}$ where, $k=a-b$ and $s=\frac{k+c+d}{2}$

(iv) Perpendicular distance 'h' between the two parallel sides

$$=\frac{2}{k}\sqrt{s(s-k)(s-c)(s-d)}$$

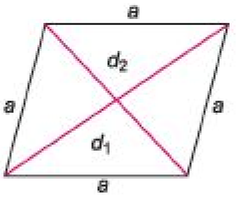
Rhombus

It is a parallelogram with all 4 sides equal. The opposite angles in a rhombus are equal but they are not right angle.

(i) Area $= \frac{1}{2} \times d_1 \times d_2$ (ii) Perimeter $= 4a$

(iii) Side $(a) = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$ (iv) $4a^2 = d_1^2 + d_2^2$

where, a = side, d_1 and d_2 are diagonals.



Properties of Rhombus

(i) A rhombus has unequal diagonal and they bisect each other at right angles (90°) (ii) All rhombus are parallelogram but reverse is not true (iii) A rhombus may or may not be a square but all squares are rhombus

Regular Polygon

In regular polygons, all sides and all interior angles are equal. A polygon is called pentagon, hexagon, heptagon, octagon, nanogon and decagon according by it contains 5,6,7,8,9 and 10 sides, respectively. If each side of a regular polygon of n sides = a. then

(i) Area of regular pentagon $= 5a^2 \frac{\sqrt{3}}{4}$

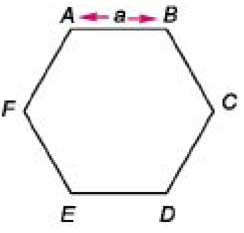
(ii) Area of regular hexagon $= 6a^2 \frac{\sqrt{3}}{4}$

(iii) Area of regular octagon $= 2(\sqrt{2} + 1)a^2$

(iv) Each exterior angle $= \frac{360°}{n}$

(v) Each interior angle $= 180°$ – Exterior angle

(vi) Number of diagonals $= \left\{\frac{n(n-1)}{2} - n\right\}$



Regular Hexagon

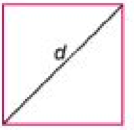
Ex. 8 A square field has its area equal to 289 sq m. Find its side and perimeter.

Sol. Given, area = 289 sq m $\Rightarrow a^2 = 289$ [a = side]

$\therefore \quad a = \sqrt{289} = 17$ cm

$\therefore$ Perimeter $= 4a = 4 \times 17 = 68$ m

Ex. 9 Find the diagonal of the square field in example 8.



Sol. We know that, $\frac{1}{2}d^2 = \text{Area} \Rightarrow d^2 = 2 \times 289$

$\therefore \quad d = \sqrt{2 \times 289} = 17\sqrt{2}$ m

Ex. 10 The area of a rectangular field is 400 sq m. If the breadth of the field is 16 m, find the length of the field.

Sol. Given, $B = 16$, $L = ?$, Area = 400sq m

According to the question, $L \times B = 400$ [L = Length, B = Breadth]

$\Rightarrow \quad L \times 16 = 400$

$\therefore \quad L = \frac{400}{16} = 25$ m

Ex. 11 The length and breadth of a rectangle are 6 cm and 4 cm, respectively. What will be its diagonal?

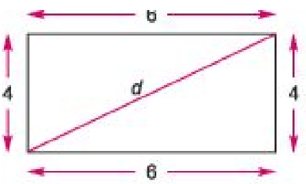
Sol. Given that, $L = 6$ cm, $B = 4$ cm, $d = ?$

According to the formula,

$$d = \sqrt{L^2 + B^2}$$

$$= \sqrt{6^2 + 4^2} = \sqrt{36 + 16}$$

$$= \sqrt{52} = \sqrt{13 \times 4} = 2\sqrt{13} \text{ cm}$$



Ex. 12 The base of a parallelogram is twice its height. If the area of the parallelogram is 144 sq cm, find its height.

Sol. Let height of the parallelogram = a

$\therefore$ Base of parallelogram = $2a$

We know,

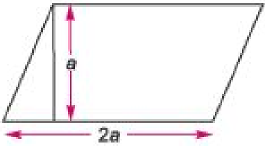
Area of parallelogram = Base × Height

According to the question,

$$2a \times a = 144$$

$\therefore \quad a^2 = \frac{144}{2} = 72$

$\therefore \quad a = \sqrt{72} = 6\sqrt{2}$ cm



Ex. 13 The difference between two parallel sides of a trapezium is 8 cm. The perpendicular distance between them is 38 cm. If the area of the trapezium is 950 cm^2, find the length of the parallel sides.

Sol. Let the two parallel sides be a and b.

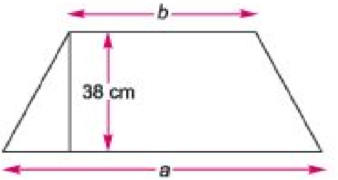
We know, Area $= \frac{1}{2} \times$ (sum of parallel sides) $\times$ height

According to the question,

$$a - b = 8 \quad \text{...(i)}$$

and $$\frac{1}{2}(a + b) \times 38 = 950$$

$$\Rightarrow \quad a + b = \frac{1900}{38} = 50 \quad \text{...(ii)}$$



From Eqs. (i) and (ii), we get

$$2a = 58$$

$$\therefore \quad a = \frac{58}{2} = 29 \text{ cm}$$

Now, from Eq. (i),

$$a - b = 8$$

$$\therefore \quad b = a - 8 = 29 - 8 = 21 \text{ cm}$$

$$\therefore \quad a = 29 \text{ cm}, b = 21 \text{ cm}$$

Ex. 14 Find the area of a regular hexagon whose side measures 18 cm.

Sol. Given that, $a = 18$ cm

Area of regular hexagon $= 6\frac{\sqrt{3}}{4}a^2 = \frac{6 \times \sqrt{3} \times 18 \times 18}{4}$

$$= 6 \times 81\sqrt{3} = 486\sqrt{3} \text{ sq cm}$$

Circle

It is a plane figure enclosed by a line on which every point is equidistant from a fixed point (centre) inside the curve.

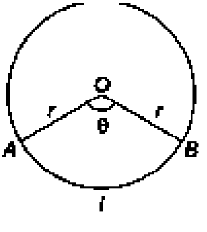
(i) Area $= \pi r^2$

(ii) Circumference (perimeter) $= 2\pi r$

(iii) Diameter $= 2r$

(iv) Length of the arc $(l) = \frac{\pi r \theta}{180°}$

(v) Area of sector $AOB = \frac{\pi r^2 \theta}{360°}$



r = radius

$\pi = \frac{22}{7}$

[Sector is a part of area of circle between two radii] where, 6 = angle enclosed between two radii

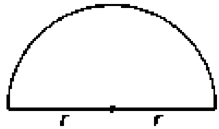
Semi-circle

A circle when separated into two parts along its diameter, then each half part is known as semi-circle.

(i) Area of semicircle $= \frac{1}{2}\pi r^2$

(ii) Perimeter $= \pi r + 2r$

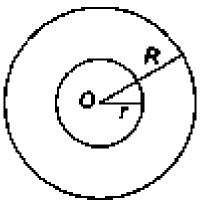
Here, r = radius of semi-circle.



In Case of Circular Ring

(i) Area = rcffi^2 - r^2)

(ii) Difference in circumference of both the rings = (27ii?-2jrr)=27r *(R-r)* where, *R* = radius of bigger ring *i* = radius of smaller ring



Ex. 15 The inner circumference of a 7 m wide circular race track, is 220 m. Find the radius of the outer circle.

Sol. R = radius of outer circle, r = radius of inner circle

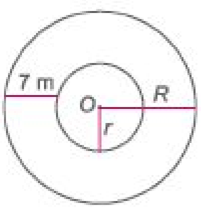
According to the question,

$$2\pi r = 220$$

$$\Rightarrow \quad r = \frac{220}{2\pi} = \frac{220}{2 \times \frac{22}{7}} = \frac{220 \times 7}{44} = 35 \text{ m}$$

Clearly, $R - r = 7 \Rightarrow R - 35 = 7$

$\therefore \quad R = 35 + 7 = 42$ m



Ex. 16 A wheel makes 2000 revolutions in covering a distance of 88 km. Find the radius of wheel.

Sol. Distance covered in 1 revolution $= \dfrac{\text{Total distance}}{\text{Total revolutions}} = \dfrac{88 \times 1000}{2000} = 44\text{ m}$

Now, circumference of wheel $= 2\pi r = 44$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 44 \Rightarrow r = 44 \times \frac{7}{44} = 7\text{ m}$$

Fast Track **Techniques**

to solve the **QUESTIONS**

Technique 1

If the length and breadth of a rectangle are increased by a% and *b%*,

respectively, then area will be increased by $\left(a + b + \dfrac{ab}{100}\right)\%$.

Ex. 17 If the length and breadth of a rectangle are increased by 10% and 8%, respectively, then by what per cent will the area of that rectangle be increased?

Sol. Given that, $a = 10, b = 8$

According to the formula,

Percentage increase in area $= \left(10 + 8 + \dfrac{10 \times 8}{100}\right)\%$

$$= \left(18 + \frac{80}{100}\right)\% = \left(18 + \frac{4}{5}\right)\% = 18\frac{4}{5}\%$$

Note If any of the two sides of rectangle is decreased, then put negative value for that in technique 1

Ex.18 If the length of a rectangle is increased by 5% and the breadth of the rectangle is decreased by 6%, then find the percentage change in area.

Sol. Given that, $a = 5, b = -6$

According to the formula,

Percentage change in area $= \left(5 - 6 - \dfrac{5 \times 6}{100}\right)\%$

$$= -1 - \frac{30}{100} = -1 - 0.30 = -1.3\% \text{ (decrease)}$$

Negative value shows that there is a decrease in area.

Technique 2

If all the sides of any two-dimensional figure are changed by *a%,* then its area will be changed by $\left(2a + \frac{a^2}{100}\right)\%$. In case of circle, radius (or diameter) is increased in place of sides.

Ex. 19 If sides of a square are increased by 5%, by what per cent, its area will be increased?

Sol. Given that, $a = 5$

According to the formula,

$$\text{Percentage increase in area} = \left(2 \times 5 + \frac{5^2}{100}\right)\%$$

$$= \left(10 + \frac{25}{100}\right)\%$$

$$= \left(10 + \frac{1}{4}\right)\% = 10.25\%$$

Ex.20 The radius of a circle is increased by 10%. Find the percentage increase in its area.

Sol. Given that, a = 10, According to the formula,

Percentage increase in area = $\left(2 \times 10 + \frac{10^2}{100}\right)\%$ = (20 + 1)% = 21%

Note Whenever there is a decrease, use negative value for a in technique 2

Technique 3

If all the measuring sides of any two-dimensional figure are changed (increased or decreased) by a%, then its perimeter also changes by a%. in case of circle such change takes place because of the change in radius (or diameter).

Ex. 21 If diameter of a circle is increased by 12.5%, find the percentage increase in its perimeter.
Sol. According to the formula,

Required percentage increase = 12.5%

Ex. 2 2 If sides of a rectangle are decreased by 15%, find the percentage decrease in its perimeter.
Sol. Given that, $a = 15$,

According to the formula,

Percentage decrease in perimeter = 15%

Technique 4

if area of a square is a sq unit, then the area of the circle formed by the

same perimeter is given by $\frac{4a}{\pi}$ sq unit.

Ex. 23 If area of a square is 44 sq cm, find the area of the circle formed by the same perimeter.

Sol. Given that, $a = 44$ sq cm,
According to the formula,

$$\text{Required area} = \frac{4a}{\pi} \text{ sq cm} = \frac{4 \times 44}{\frac{22}{7}} = \frac{4 \times 44 \times 7}{22} = 4 \times 2 \times 7 = 56 \text{ sq cm}$$

Technique 5

Area of a square inscribed in a circle of radius r is equal to $2r^2$.

Ex. 24 Find the area of a square inscribed in a circle of radius 5 cm.

Sol. From the figure,

Diameter of circle = Diagonal of square

$\therefore$ Diagonal of square = 10 cm $\quad [d = 2r]$

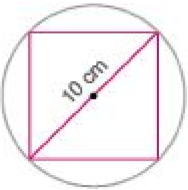
So, area of square $= \frac{1}{2} \times (\text{diagonal})^2$

$$= \frac{1}{2} \times (10)^2 = \frac{1}{2} \times 100 = 50 \text{ cm}^2$$

Alternate Method

Here, $\quad r = 5$ cm

Then, area of square $= 2r^2 = 2 \times (5)^2 = 2 \times 25 = 50 \text{ cm}^2$



Ex. 25 The circumference of a circle is 50 cm. Find the side of the square inscribed in the circle.

Sol. Circumference of the circle $= 2\pi r = 50 \Rightarrow r = \frac{25}{\pi}$

Since, area of square $= 2r^2 \Rightarrow \text{Side} = r\sqrt{2} = \frac{25\sqrt{2}}{\pi}$ cm

Technique 6

The area of the largest triangle inscribed in a semi-circle of radius r is equal to r^2.

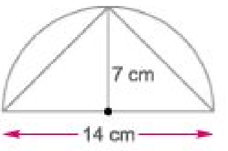
Ex.26 The largest triangle is inscribed in a semi-circle of radius 7 cm. Find the area inside the semi-circle which is not occupied by triangle.

Sol. Given that, radius = 7 cm, diameter = 14 cm

According to the formula,

Area of the triangle $= 7^2 = 49$ sq cm

Area of semi-circle $= \frac{\pi r^2}{2} = \frac{\frac{22}{7} \times 7^2}{2} = 11 \times 7 = 77$ sq cm



$\therefore$ Required answer = Area of semi-circle − Area of the largest triangle

$= (77 - 49)$ sq cm $= 28$ sq cm

(Technique;*

The length and breadth of a rectangle are / and b, then area of circle of

maximum radius inscribed in that rectangle is $\frac{\pi b^2}{4}$.

Ex.27 Find the area of circle with maximum radius that can be inscribed in the rectangle of length 12 cm and breadth 8 cm.

Sol. From the figure, * ^cm >

Diameter of largest circle that can be inscribed in a / n.

rectangle is equal to its breadth ' \

Diameter of circle = 8 cm 8 cm I

Radius = — = 4 cm \ *y*

Area of circle = *nr*

= 7t x $(4)^2$ = 1671 cm^2 **Fast Track Method**

Here, *I* = 12 and *b* = 8

.". Area of circle with maximum radius =-----

4

Jt$(8)^2$ 64ti 1ft 2

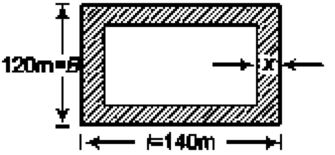
= —±^—=------= 16tc cm

4 4

if a pathway of width *x* is made inside or outside a rectangular plot of length / and breadth *b*, then area of pathway is

(i) *2x (l + b + 2x),* if path is made outside the plot. (ii) *2x(l + b-2x),* if path is made inside the plot.

Ex.28 There is a garden of 140 m x 120 m and a gravel path is to be made of an equal width all around it, so as to take up just one-fourth of the garden. What must be the breadth of the path?



Sol. Area of the garden $= l \times b = 140 \times 120 = 16800$ sq m

Area of path $= \frac{1}{4} \times$ Area of garden

$= \frac{1}{4} \times 16800 = 4200$ sq m

Area of garden without path

$= 16800 - 4200 = 12600$ sq m ...(i)

Now, length of garden without path $= (l - 2x) = 140 - 2x$ m ...(ii)

Breadth of garden without path $= (b - 2x) = 120 - 2x$ m ...(iii)

Now, from Eqs. (i), (ii) and (iii), we get

Area of garden without path = (Length × Breadth) without path

$$12600 = (140 - 2x)(120 - 2x) \Rightarrow 12600 = 16800 - 240x - 280x + 4x^2$$

$$\Rightarrow \quad 4x^2 - 520x + 4200 = 0 \Rightarrow x^2 - 130x + 1050 = 0$$

$$\therefore \quad x = 8.65 \text{ and } 121.3$$

Since, x cannot be 121.3 since greater than length.

$\therefore$ Width of path $= 8.65$ m

Fast Track Method

Since, path covers $\frac{1}{4}$th area of the garden, that means path is inside the garden.

Given, $l = 140$ m, $b = 120$ m, $x = ?$

According to the question,

$$2x(l + b - 2x) = \frac{1}{4} \times l\,b \Rightarrow 2x(140 + 120 - 2x) = \frac{1}{4} \times 140 \times 120$$

$$\Rightarrow \quad x(260 - 2x) = 2100 \Rightarrow x^2 - 130x + 1050 = 0 \Rightarrow x = 8.65 \text{ or } 121.3$$

Leaving 121.3, since width of the park cannot be greater than length.

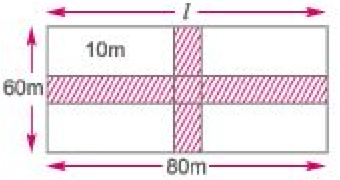
$\therefore$ Width of the park $= 8.65$ m

Technique 9

If two paths, each of width *x* are made parallel to length (/) and breadth (£>) of the rectangular plot in the middle of the plot, then area of the paths is *x{l + b-x).*

Ex.29 A rectangular grass plot 80 mx 60 m has two roads, each 10 m wide,

running in the middle of it, one parallel to length and the other parallel to breadth. Find the area of the roads.



Sol. Area of the grass plot = 80 x 60= 4800 sqm Width of the road = 10 m

New length without road = (80 - 10) = 70 m

New breath without roads = (60 - 10) = 50 m

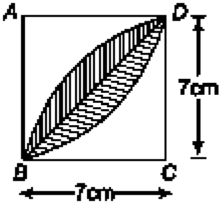
Area of park without roads = 70 X 50 = 3500 sq m

Now, area of the roads = Area of plot - Area of plot without roads = 4800 - 3500 = 1300 sq m

Fast Track Method Here, *I* = 80 m, *b* = 60 m, *x* = 10 cm

Then, according to the formula,

Required area = *x (I + b - x)*= 10 (80 + 60 - 10)= 10 x 130 = 1300 sq m



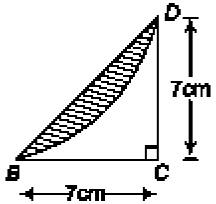
$$\Rightarrow \quad \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} \times b \times h$$

$$\Rightarrow \quad \frac{\pi (7)^2 \times 90^\circ}{360^\circ} - \frac{1}{2} \times 7 \times 7$$

$$\Rightarrow \quad \frac{77}{2} - \frac{49}{2} = \frac{28}{2} \text{ sq cm}$$

$\therefore$ Area of complete shaded portion

$$= 2 \times \frac{28}{2} = 28 \text{ sq cm.}$$



w (ft) Let the two diagonals of rhombus be x and y, respectively

x+ y = 10 m (given) ...(i)

We know, area of rhombus = — x Product of diagonals 2

1 ?

9 = — x xx *y* => xy = 18m ...(ii)

Now, $x^2 + y^2 = (x + y)^2 - 2xy$

Putting values from Eqs. (i) and (ii), we get

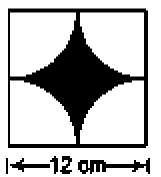
$x^2 + y^2 = (10)^2 - 2(18) = 100 - 36 = 64 m^2$

(*d*) We can see this figure as,

$\therefore$ Area of shaded region = Area of square of side 12 cm − Area of 4 quadrants of radius 6 cm

$$\Rightarrow \left((12)^2 - 4 \times \frac{1}{4} \times \pi \times (6)^2\right) cm^2$$

$$\Rightarrow (144 - 36\pi)\, cm^2 \Rightarrow 36(4 - \pi)\, cm^2$$

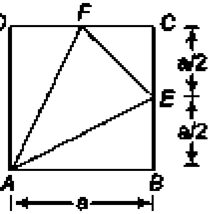


4. *ABCD is a square, E and F are the mid-points ofBC and CD. What is the ratio of area of AAEF to that of the square ABCD.*

(a) 3:8 (b)5:8 (c) 3:2 (d)7:4

➡ (a) Let the side of the square be a.

$\therefore$ Area of the square $= a \times a = a^2$ ...(i)



Now, area of ΔAEF = (Area of square) − (Area of ΔABE + Area of ΔCEF + Area of ΔADF)

$$= a^2 - \left(\frac{1}{2} \times a \times \frac{a}{2} + \frac{1}{2} \times \frac{a}{2} \times \frac{a}{2} + \frac{1}{2} \times a \times \frac{a}{2}\right)$$

$$= a^2 - \frac{a^2}{4} + \frac{a^2}{8} + \frac{a^2}{4} = a^2 - \frac{5a^2}{8} \Rightarrow \frac{3a^2}{8} \qquad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

Area of ΔAEF : Area of square $ABCD = \frac{3a^2}{8} : a^2 = 3 : 8$

5. *The three perpendicular distances of three sides of an equilateral triangle from a point which lies inside that triangle are 6 cm, 9 cm and 12 cm, respectively. The perimeter of the triangle is* [ssc ccl (Main) 2012]

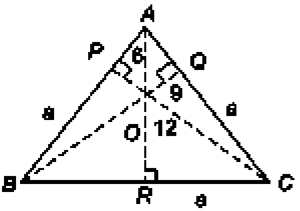
(a) $42\sqrt{2}$ cm (b) $45\sqrt{3}$ cm (c) $52\sqrt{2}$ cm (d) $54\sqrt{3}$ cm

➡ (d) From the adjoining figure let O be the point in side the equilateral ΔABC of side 'a' cm.

Then,

Area of ΔABC = Area of ΔBOC + Area of ΔAOC + Area of ΔAOB

$$\frac{\sqrt{3}}{4}a^2 = \frac{1}{2} BC \times OR + \frac{1}{2} AC \times OQ + \frac{1}{2} AB \times OP$$



$$\frac{\sqrt{3}}{4}a^2 = \frac{1}{2} \times a \times 12 + \frac{1}{2} \times a \times a + \frac{1}{2} \times a \times 6$$

$$\frac{\sqrt{3}}{4}a^2 = a\left(\frac{12}{2} + \frac{9}{2} + \frac{6}{2}\right)$$

$$\frac{\sqrt{3}}{4}a = \left(\frac{12 + 9 + 6}{2}\right)$$

$$a = \frac{4}{\sqrt{3}} \times \frac{27}{2}$$

$$a = \frac{54}{\sqrt{3}} \text{ cm}$$

$\therefore$ Perimeter $= 3 \times a = 3 \times \frac{54}{\sqrt{3}} = 54\sqrt{3}$ cm

Alternate Method

Let $l = OP = 6$, $m = OR = 12$ and $n = OQ = 9$

Then, side of equilateral triangle $= \frac{2}{\sqrt{3}}(l + m + n)$

$$= \frac{2}{\sqrt{3}}(12 + 6 + 9)$$

$$= \frac{2}{\sqrt{3}} \times 27 = \frac{54}{\sqrt{3}} \text{ cm}$$

$\therefore$ Perimeter of equilateral triangle $= 3 \times$ side $= 3 \times \frac{54}{\sqrt{3}}$

$$= 54\sqrt{3} \text{ cm}$$

Fast Track Practice

Triangle

1. Find the area of a triangle whose sides measure 8 cm, 10 cm and 12 cm.

[Hotel Mgmt. 2010]

(a) $8\sqrt{83}$ sq cm *(b)* $5\sqrt{63}$ sq cm
(c) $6\sqrt{53}$ sq cm *(d)* $7\sqrt{93}$ sq cm
(e) None of the above

2. The lengths of three line segments (in cm) are given in each of the four cases. Which one of the the following cases is not suitable to be the three sides of a triangle?

(a) 2, 3, 4 *(b)* 2, 3, 5

(c) 2, 4, 5 *(d)* 3, 4, 5

3. Find the perimeter of a triangle with sides equal to 6 cm, 4 cm and 5 cm.

(a) 14 cm *(b)* 18 cm

(c) 20 cm *(d)* 15 cm

(e) None of the above

4. The area of a right angled triangle is 40 sq cm. If its base is equal to 28 cm, find its height. [Bank Clerks 2010]

(a) $3\frac{8}{7}$ cm *(b)* $4\frac{6}{7}$ cm
(c) $2\frac{6}{7}$ cm *(d)* $5\frac{6}{7}$ cm
(e) None of the above

5. The altitude of an equilateral triangle is V3 cm.Whatisitsperimeter?

(a) 3 cm *(b)* 3V3 cm

(c) 6 cm (d) 6a/3 cm

6. The area of a right angled triangle is 10 sq cm. If its perpendicular is equal to 20 cm, find its base.

(a) 1 cm (b) 4 cm

(c) 3 cm (d) 2 cm

(e) None of the above

7. The base of a triangular wall is 7 times its height. If the cost of painting the wall at ? 350 per 100 sq m is ? 1225, then what is the base length?

(a) 50 m (b) 70 m (c) 75 m (d) 100 m

The three sides of a triangle are 15, 25 and x units. Which one of the following is correct? [CDS 2014]

(aJ10<x<40 (b)10<x<40

(cJ10<x<40 (dJ10<x<40

9. A triangle with three equal sides has its area equal to $3\sqrt{3}$ sq cm. Find its perimeter. [SSC (10+2) 2010]

(a) $6\sqrt{3}$ cm (b) $2\sqrt{3}$ cm

(c) $5\sqrt{3}$ cm (d) $7\sqrt{3}$ cm

10. The sides of a triangle area in the ratio of $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}$ and its perimeter is 94 cm. Find the length of the smallest side of the triangle. [SSC (10+2) 2010]

(a) 18 cm (b) 22.5 cm

(c) 24 cm (d) 27 cm

11. Find the length of the altitude of an equilateral triangle of side 9V3 cm.

(a) 21 cm (b) 15.5 cm

(c) 14 cm (d) 13.5 cm

(e) None of the above

12. The area of an equilateral triangle is

sq cm. Find the length of each side of the triangle [SSC CGL 2010]

(a) 3 cm *(b)* 2^2 cm

(c) 2V3 cm *(d)* 4 cm

13. The perimeter of an equilateral triangle is 90 cm. Find its area.

(a) 225V3 sq cm *(b)* 250V3 sq cm *(c)* 135V3 sq cm *(d)* 151V3 sq cm *(e)* None of the above

14. The sides of a right angled triangle are equal to three consecutive numbers expressed in centimeters. What can be the area of such a triangle? [CDS 2014] *(a)* 6 cm^2 *(b)* 8 cm^2 (cJIOcm^2 (dj 12 cm^2

15. If the area of an equilateral triangle is *x* and its perimeter is *y,* then which one of the following is correct? [CDS 2013] *(a)* $y^4 = 432\ x^2$ *(b)* $y^4 = 216x^2$

(c) $y^2 = 432x^2$ *(d)* None of these

16. The perimeter of an isosceles triangle is 26 cm while equal sides together measure 20 cm. The third side and each of the equal sides are respectively.

(a) 6 cm and 10 cm *(b)* 8 cm and 9 cm *(c)* 10 cm and 8 cm *(d)* 14 cm and 6 cm *(e)* None of the above

17. The area of an isosceles AABC with *AB* = *AC* and altitude *AD* = 3 cm is 12 sq cm. What is its perimeter?

(a) 18 cm *(b)* ^6 cm [CDS2013]

(c) 14 cm (c/J 12 cm

18. The area of a right angled triangle is 24 cm and one of the sides containing the right angle is 6 cm. The altitude on the hypotenuse is

(a) 3.6 cm *(b)* 4.8 cm

(c) 5.2 cm *(d)* 12 cm

(e) None of the above

19. The area of an equilateral triangle is $\frac{\sqrt{243}}{4}$ sq cm. Find the length of its side. [SSC CGL 2011]

(a) 3 cm *(b) 3-J3* cm

(c) 9 cm *(d)* V3 cm

20. The ratio of length of each equal side and the third side of an isosceles triangle is 3 : 4. If the area of the triangle is *18\fE* sq units, the third side is [SSC(10+2)2012]

(a) Q42 units *(b)* 12 units

(c) 16 units *(d)* 5VT0 units

21. Three sides of a triangular field are of length 15 m, 20 m and 25 m long, respectively. Find the cost of sowing seeds in the field at the rate of *X* 5 per sqm. [SSCCGL2013] *(a)* 750 *(b)* 150

(c) 300 *(d)* 600

22. Two isosceles triangles have equal vertical angles and their corresponding sides are in the ratio of 3 : 7. What is the ratio of their areas?

(a) 9 : 7 *(b)* 2 : 1

(c) 9 : 49 *(d)* 7 : 9

(e) None of the above

23. A *ADEF* is formed by joining the mid-points of the sides of AABC. Similarly, a *APQR* is formed by joining the mid-points of the sides of the *ADEF.* If the sides of the *APQR* are of lengths 1, 2 and 3 units, what is the perimeter of the AABC? [CDS 2013]

(a) 18 units

(b) 24 units

(c) 48 units

(d) Cannot be determined

Quadrilateral

24. Find the area of a rectangle having 15 m length and 8 m breadth.

[SSC (10+2) 2007]

(a) 120 sq m *(b)* 111 sq m

(c) 115 sq m *(d)* 125 sq m

25. The length of a rectangular field is 100 m and its breadth is 40 m. What will be the area of the field?

[SSC (10+2) 2009]

(a) (4×10^2)sq m

(b) (4×10)sq m

(c) (4×10^4)sq m

(d) (4×10^3)sq m

26. The area of a rectangular playground is 300 sq m. If the breadth of the field is 15 m, find the length of the field.

[Bank PO 2008]

(a) 20 m *(b)* 11 m

(c) 25 m *(d)* 10 m

(e) None of the above

27. The ratio of length and breadth of a rectangle is 5 : 3. If length is 8 m more than breadth, find the area of the rectangle. [Bank PO 2009]

(a) 300 sq m *(b)* 250 sq m

(c) 240 sq m (d) 185 sq m

(e) None of the above

28. The ratio between the length and the breadth of a rectangle is 2 : 1. If breadth is 5 cm less than the length, what will be the perimeter of the rectangle?

(a) 30 cm (b) 25 cm

(c) 35 cm (d) 40 cm

(e) None of the above

29. The perimeter of a rectangle having area equal to 144 cm and sides in the ratio4:9is [CDS 2013] (a) 52 cm (b) 56 cm

(c) 60 cm (d) 64 cm

30. One side of a rectangular field is 9 m and one of its diagonal is 20 m. Find the area of the field.

(a) $9\sqrt{319}$ sq m (b) $7\sqrt{314}$ sq m
(c) $2\sqrt{319}$ sq m (d) $5\sqrt{319}$ sq m
(e) None of the above

31. The area of a rectangle lies between 40 cm and 45 cm . If one of the sides is 5 cm, then its diagonal lies between

[CDS 2014]

(a) 8 cm and 10 cm

(b) 9 cm and 11 cm

(c) 10 cm and 12 cm

(d) 11 cm and 13 cm

32. The area of a rectangular field is 15 times the sum of its length and breadth. If the length of that field is 40 m, what is the breadth of that field?

[DMRC2012]

(a) 24 m (b) 25 m

(c) 28 m *(d)* 32 m

33. The difference between the length and breadth of a rectangle is 23 m. If its perimeter is 206 m, then its area is

[DMRC2012]

(a) 2520 m^2 *(b)* 2480 m^2

(c) 2420 m^2 *(d)* None of these

34. If the length of a rectangle decreases by 5 m and breadth increases by 3 m, then its area reduces 9 sq m. If length and breadth of this rectangle increased by 3 m and 2 m respectively, then its area increased by 67 sq m. What is the length of rectangle? [RRB 2012] *(a)* 9 m *(b)* 15.6 m (c)17m *(d)* 18.5 m

35. The area of a rectangle whose length is 5 more than twice its width is 75 sq units. What is the perimeter of the rectangle? [CDS 2012] *(a)* 40 units *(b)* 30 units

(c) 24 units *(d)* 20 units

36. The area of a rectangle, whose one side is 'a' is $2a^2$. What is the area of a square having one of the diagonals of the rectangle as side?

(a) 2 a^2 *(b)* $3a^2$ *(c)* $4a^2$ *(d)* $5a^2$

37. If the sides of a rectangle are increased by 5%, find the percentage increase in its diagonals.

(a) 6% *(b)* 4% *(c)* 5% *(d)* 9% *(e)* None of the above

38. If the sides of a rectangle are increased by 10% find the percentage increase in its diagonals.

(a) 20% *(b)* 10% *(c)* 15% *(d)* 18%

39. Area of a rectangular field is 3584 m and the length and the breadth are in the ratio 7 : 2, respectively. What is the perimeter of the rectangle?

[SBI Clerks 2012] *(a)* 246 m *(b)* 292 m *(c)* 286 m *(d)* 288 m *(e)* None of the above

40. The length and perimeter of a rectangle are in the ratio of 5:18. What will be the ratio of its length and breadth?

[SSCCCL2012] *(a)* 4 : 3 *(b)* 3 : 5 *(c) 5 : 4 (d) 4 : 7*

41. The length of a rectangle is twice its breadth. If the length is decreased by half of the 10 cm and the breadth is increased by half of the 10 cm, the area of the rectangle is increased by 5 sq cm more than 70 sq cm. Find the length of the rectangle.

(a) 30 cm *(b)* 40 cm

(c) 21 cm *(d)* 45 cm

(e) None of the above

42. Find the cost of carpeting a room 8 m long and 6 m broad with a carpet 75 cm wide at ? 20 per m.

(a) X 1300 *(b) X* 1500

(c) ? 1750 *(d)* ? 1280

(e) None of the above

43. A rectangle has 20 cm as its length and 200 sq cm as its area. If the area is increased by *IV.* times the original area by increasing its length only then the perimeter of the rectangle so formed (in cm) is [SSCCPO2013] *(a)* 72 *(b)* 60

(c) 64 *(d)* 68

44. A rectangular grassy plot 160 m x 45 m has a gravel path 3 m wide all the four sides inside it. Find the cost of gravelling the path at ? 5 per sq m.

(a) ? 5970 *(b)* ? 4970

(c) ? 6490 *(d)* ? 4970

(e) None of the above

45. A ground 100 x 80 m has, two cross roads in its middle. The road parallel to the length is 5 m wide and the other road is 4 m wide, both roads are perpendicular to each other. The cost of laying the bricks at the rate of ? 10 per m , on the roads, will be [CLAT 2013] *(a)* ? 7000 *(b)* ? 8000

(cj? 9000 (dj? 1000

46. The breadth of a rectangle is 25 m. The total cost of putting a grass bed on this field was ? 12375, at the rate of? 15 per sq m. What is the length of the rectangular field? [Bank Clerks 2011] *(a)* 27 m *(b)* 30 m *(c)* 33 m *(d)* 32 m *(e)* None of the above

47. *A* took 15 s to cross a rectangular field diagonally walking at the rate of 52 m/min and *B* took the same time to cross the same field along its sides walking at the rate of 68 m/min. Find the area of the field.

(a) 45 sq m *(b)* 35 sq m

(c) 51 sq m *(d)* 30 sq m

(e) None of the above

48. The length and the breadth of a rectangular plot are in the ratio of 5 : 3. The owner spends ? 3000 for surrounding it from all the sides at the rate of ? 7.5 per metre. What is the difference between the length and breadth of the plot? [SSC CCL 2008] *(a)* 50 m *(b)* 100 m *(c)* 75 m *(d)* 60 m

49. A rectangle has 30 cm as its length and

720 sq cm as its area. Its area is

increased to 1— times its original area

4 by increasing only its length. Its new

perimeter is

(a) 123 cm *(b)* 125 cm

(c) 119 cm *(d)* 121 cm

(e) None of the above

50. If side of a square is 20 cm, find its area. [SSC (10+2) 2007] *(a)* 100 sq cm *(b)* 150 sq cm

(c) 400 sq cm *(d)* 350 sq cm

51. What is the area of a square having perimeter 68 cm? [Bank Clerks 2009]

(a) 361 sq cm *(b)* 284 sq cm

(c) 269 sq cm *(d)* 289 sq cm

(e) None of the above

52. The diagonal of a square is 4V2 cm. The diagonal of another square whose area is double that of the first square is

[SNAP 2008]

(a) 8 cm *(b)* 8a/2 cm

(c) 4^2 cm *(d)* 6 cm

(e) None of the above

53. The diagonals of two squares are in the ratio of 3 : 2. Find the ratio of their areas.

(a) 9 : 4 *(b)* 9 : 2 *(c)* 9 : 5 *(d)* 9 : 7

54. If the sides of a square is increased by 25%, then the area of the square will be increased by [SSC CCL 2013] *(a)* 125% *(b)* 50%

(c) 56.25% *(d)* 53.75%

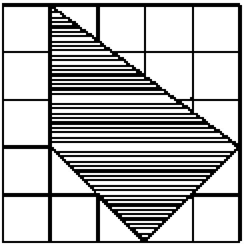
55. The perimeter of two squares is 12 cm and 24 cm. The area of the bigger square is how many times that of the smaller? [CTET2012] *(a)* 2 times *(b)* 3 times

(c) 4 times *(d)* 5 times

56. The perimeters of two squares are 68 cm and 60 cm. Find the perimeter of the third square whose area is equal to the difference of the areas of these two squares. [SSC (10+2) 2011] *(a)* 64 cm *(b)* 60 cm

(c) 32 cm *(d)* 8 cm

57. In the figure, side of each square is 1 cm. The area, in sq cm of the shaded partis [CTET2012]



(a) 8 *(b)* 9 *(c)* 10 fdJ11

58. A cost of cultivating a square field at a rate of ? 135 per hectare is ? 1215. The cost of putting a fence around it at the rate of 75 paise per metre would be (a)? 360 (b)? 810 (c)?900 *(d)l* 1800

(e) None of the above

59. The base of a parallelogram is thrice of its height. If the area of the parallelogram is 2187 sq cm, find its height.

(a) 27 cm *(b)* 35 cm *(c)* 29 cm *(d)* 26 cm *(e)* None of the above

60. Diagonals of a rhombus are 1 m and 1.5 m in lengths. The area of the rhombus is [CLAT 2013]

(a) 0.75 m^2 *(b)* 1.5 m^2

(c)1.5m^2 *(d)* 0.375 m^2

61. If the diagonals of a rhombus are 4.8 cm and 1.4 cm, then what is the perimeter of the rhombus? [CDS 2013] *(a)* 5 cm *(b)* 10 cm *(c)* 12 *cm(d)* 20 cm

62. Angles of a quadrilateral are in the ratio 3:4:5:8. The smallest angle is

[CLAT 2013] *(a)* 54° *(b)* 40° *(c)* 36° *(d)* 18°

63. Find the distance between the two parallel sides of a trapezium if the area of the trapezium is 500 sq m and the two parallel sides are equal to 30 m and 20 m, respectively. [Bank PO 2008] *(a)* 20 cm *(b)* 15 cm *(c)* 18 cm *(d)* 25 cm *(e)* None of the above

64. One side of a parallelogram is 8.06 cm and its perpendicular distance from opposite side is 2.08 cm. What is the approximate area of the parallelogram?

[CDS 2013]

(a) 12.56 cm^2 *(b)* 14.56 cm^2

(c) 16.76 cm^2 (0"; 22.56 cm^2

65. Floor of a square room of side 10 m is to be completely covered with square tiles, each having length 50 cm. The smallest number of tiles needed is [CTET 2012] *(a)* 200 *(b)* 300 *(c)* 400 *(d)* 500

66. The difference between two parallel sides of a trapezium is 8 cm. The perpendicular distance between them is 19 cm while the area of trapezium is 760 sq cm. What will be the lengths of the parallel sides?

(a) 44 cm and 36 cm

(b) 44 cm and 38 cm

(c) 36 cm and 25 cm

(d) 39 cm and 26 cm

(e) None of the above

67. The area of a trapezium is 384 cm . If its parallel sides are in the ratio 3 : 5 and the perpendicular distance between them is 12 cm, the smaller of the parallel sides is [SSC CCL 2012] *(a)* 20 cm *(b)* 24 cm

(c) 30 cm *(d)* 36 cm

68. In a trapezium, the two non-parallel sides are equal in length, each being of 5 units. The parallel sides are at a distance of 3 units apart. If the smaller side of the parallel sides is of length 2 units, then the sum of the diagonals of the trapezium is [CDS 2014] *(a)* 10V5 units *(b)* 6V5 units

(c) 5V5 units *(d)* 3V5 units

69. The ratio of the length of the parallel sides of a trapezium is 3 : 2. The shortest distance between them is 15 cm. If the area of the trapezium is 450 sq cm, find the sum of the lengths of the parallel sides. [SSC CCL 2011] *(a)* 15 cm *(b)* 36 cm

(c) 42 cm *(d)* 60 cm

70. What will be the perimeter of square. If the sum of lengths of diagonal is 144 cm.

(a) 144 cm (b) 144a/2 cm

(c) 288 cm (d) 228^" cm

71. If area of a regular pentagon is 125V3 sq cm, how long is its each side? [MBA 2008] (a) 10 cm (b) 15 cm

(c) 16 cm (d) 25 cm

(e) None of the above

72. Find each interior of a regular pentagon.

(a) 108° (b) 105°

(c) 103° (d) 101°

(e) None of the above

73. Calculate each exterior angle of a regular octagon.

(a) 35° (b) 39° (c) 40° (d) 45°

74. The sides of a parallelogram are 12 cm and 8 cm long and one of the diagonals is 10 cm long. If d is the length of other diagonal, then which one of the following is correct? [CDS 2012]

(a) $d < 8$ cm

(b) 8 cm $< d <$ 10 cm

(c) 10 cm $< d <$ 12 cm

(d) $d >$ 12 cm

75. *ABCD* is a rhombus with diagonals *AC* and *BD*. Then, which one among the following is correct?

(a) *AC* and *BD* bisect each other but not necessarily perpendicular to each other.

(b) *AC* and *BD* are perpendicular to each other but not necessarily bisect each other.

(c) *AC* and *BD* bisect each other and perpendicular to each other.

(d) *AC* and *BD* neither bisect each other nor perpendicular to each other

Circle

76. What is the diameter of the circle having 3 cm radius?

(a) 6 cm *(b)* 12 cm

(c) 1.5 cm *(d)* 8 cm

(e) None of the above

77. The radius of a circular field is 25 m. Find the area of the field.

[Bank Clerks 2008] *(a)* 125 7t sq m *(b)* 625 tc sq m *(c)* 25 *it* sq m *(d)* 135 tu sq m

(e) None of the above

78. A wheel makes 4000 revolutions in moving a distance of 44 km. Find the radius of the wheel.

(a) 15 m *(b)* 27 m

(c) 25 m *(d)* 28 m

(e) None of the above

79. If the area of a semi-circle be 77 sq m, find its perimeter.

(a) 36 m *(b)* 42 m

(c) 48 m *(d)* 54 m

(e) None of the above

80. A railing of 288 m is required for fencing a semi-circular park. Find the area of the park. (jc = 22 / 7)

[SSC Multitasking 2014] *(a)* 4928 m^2 *(b)* 9865 m^2

(c) 8956 m^2 (d) 9856 m^2

81. The area of a sector of a circle of radius 36 cm is 72tc cm^2. The length of the corresponding arc of the sector is

[CDS 2014]

(a) 7t cm (b) 2jc cm

(c) 3n cm (d) An cm

82. The inner circumference of a circular race track 7 m wide is 440 m. Find the radius of the outer circle.

(a) 57 m (b) 68 m

(c) 77 m (d) 69 m

(e) None of the above

83. If the radius of a circle is increased by 6%, find the percentage increase in its area. [RRB 2009] (a) 15% (b) 12.36%

(c) 8.39% (d) 17%

84. The area of a sector of a circle is 77 sq cm and the angle of the sector is 45°. Find the radius of the circle.

[SSC (10+2)2012]

(a) 7 cm (b) 14 cm

(c) 21 cm (d) 28 cm

85. The wheel of an engine turns 350 times round its axle to cover a distance of 1.76 km. The diameter of the wheel is

[SSC FCI 2012]

(a) 3 cm (b) 3a/3 cm

(c) 9 cm *(d)* V3 cm

86. The ratio of the areas of the circumcircle and the incircle of a square is [SSC CGL (Main) 2012] *(a)* 2 : 1 *(b)* 1 : 2

(c) a/2 :1 *(d)* 1: a/2

87. The circumferences of two circles are in the ratio 2 : 3. What is the ratio of their areas? [CDS 2012] *(a)* 2 : 3 *(b)* 4 : 9

(c) 1 : 3 *(d)* 8 : 27

88. Consider the following statements

I. Area of segment of a circle is less than area of its corresponding sector. II. Distance travelled by a circular wheel of diameter 2d cm in one revolution is greater than 6d cm.

Which of the above statements is/are correct? [CDS 2012]

(a) Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

89. The radius of a circle is so increased that its circumference increased by 5%. The area of the circle, then increases by

[SNAP 2010]

(a) 12.5% *(b)* 10.25%

fc; 10.5% *(d)* 11.25%

(e) None of the above

90. The area of a circle is increased by 22 sq cm when its radius is increased by 1 cm. Find the original radius of the circle. [SSC CGL 2007] *(a)* 6 cm *(b)* 3.2 cm

(c) 3 cm *(d)* 3.5 cm

91. Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 154 sq m. *(a)* 7 m *(b)* 8 m

(c) 12 m *(d)* 13 m

92. What is the area of the larger segment circle formed by a chord of, length 5 cm subtending an angle of 90° at the centre? [CDS 2014]

(a) $\frac{25}{4}\left(\frac{\pi}{2}+1\right)cm^2$
(b) $\frac{25}{4}\left(\frac{\pi}{2}-1\right)cm^2$
(c) $\frac{25}{4}\left(\frac{3\pi}{2}+1\right)cm^2$
(d) None of the above

93. A man riding a bicycle, completes one lap of a circular field along its circumference at the speed of 14.4 km/h in 1 min 28 s. What is the area of the field?

(a) 7958 sq m

(b) 9856 sq m

(c) 8842 sq m

(d) Cannot be determined

(e) None of the above

94. A person observed that he required 30 s time to cross a circular ground along its diameter than to cover it once along the boundary. If his speed was 30m/min, then the radius of the circular ground is (take 7i = 22/7) [SSC CGL 2013] *(a)* 10.5 m *(b)* 3.5 m

(c) 5.5 m *(d)* 7.5 m

Miscellaneous

95. What is the total area of three equilateral triangles inscribed in a semi-circle of radius 2cm? [CDS 2014]

(a) 12 cm^2 (b) $\frac{3\sqrt{3}}{4}$ cm^2

(c) $\frac{9\sqrt{3}}{4}$ cm^2 (d) $3\sqrt{3}$ cm^2

96. How many circular plates of diameter *d* be taken out of a square plate of side 2d with minimum loss of material?

[CDS 2014]

(a) 8 *(b)* 6 *(c)* 5 *(d)* 2

97. What is the area of a circle whose area is equal to that of a triangle with sides 7 cm, 24 cm and 25 cm? [CDS 2013] *(a)* 80 cm^2 *(b)* 84 cm^2

(c) 88 cm^2 *(d)* 90 cm^2

98. A circle of radius 10 cm has an equilateral triangle inscribed in it. The length of the perpendicular drawn from the centre to any side of the triangle is

[CDS 2014]

(a) 2.5V3 cm *(b) 5-J3* cm

(c) 10V3 cm *(d)* None of these

99. *AB* and *CD* are two diameters of a circle of radius *r* and they are mutually perpendicular. What is the ratio of the area of the circle to the area of the triangle *ACD?* [CDS 2014]

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) 2π

100. The area of a rectangle is 1.8 times the area of a square. The length of the rectangle is 5 times the breadth. The side of the square is 20 cm. What is the perimeter of the rectangle?

[Bank Clerks 2011] *(a)* 145 cm *(b)* 144 cm

(c) 133 cm *(d)* 135 cm

(e) None of the above

101. The side of a square is 5 cm which is 13 cm less than the diameter of a circle. What is the approximate area of the circle? [Bank Clerks 2011] *(a)* 245 sq cm *(b)* 235 sq cm

(c) 265 sq cm *(d)* 255 sq cm

(e) 275 sq cm **102.**If area of a square is 64 sq cm, then find the area of the circle formed by the same perimeter. [MBA 2009]

(a) $\frac{215}{\pi}$ sq cm
(b) $\frac{218}{\pi}$ sq cm
(c) $\frac{256}{\pi}$ sq cm
(d) $\frac{318}{\pi}$ sq cm
(e) None of the above

103. Find the area of a square inscribed in a circle of radius 4 cm.

(a) 32 sq cm *(b)* 18 sq cm

(c) 64 sq cm *(d)* 25 sq cm

(e) None of the above

104. The circumference of a circle is 25 cm. Find the side of the square inscribed in the circle.

(a) $\frac{25}{\pi\sqrt{2}}$ cm *(b)* $\frac{21}{\pi\sqrt{3}}$ cm
(c) $\frac{23}{\pi\sqrt{2}}$ *(d)* $\frac{29}{\pi\sqrt{3}}$ cm
(e) None of the above

105. A rectangle of maximum area is drawn inside a circle of diameter 5 cm. What is the maximum area of such a rectangle?

[CDS 2014]

(a) 25 cm^2 *(b)* 12.5 cm^2

(c) 12 cm^2 *(d)* None of these

106. The largest triangle is inscribed in a semi-circle of radius 4 cm. Find the area inside the semi-circle which is not occupied by the triangle.

(a) 8(ji - 2) sq cm *(b)* 7 (jc - 1) sq cm *(c)* $8(71 - 1)^2$ sq cm *(d)* 6(71 - 2) sq cm *(e)* None of the above

107. The perimeter of a square is twice the perimeter of a rectangle. If the perimeter of the square is 72 cm and the length of the rectangle is 12 cm, what is the difference between the breadth of the rectangle and the side of the square? [IBPS Clerk 2011] *(a)* 9 cm *(b)* 12 cm(c) 18 cm *(d)* 3 cm *(e)* None of the above

108. Find the area of the largest triangle that can be inscribed in a semi-circle of radius *a* cm.

(a) $2a$ sq cm *(b)* a^2 sq cm
(c) $\frac{1}{2}a^2$ sq cm *(d)* $2a^2$ sq m
(e) None of the above

109. The area of a rectangle is 4 times the area of a square. The area of the square is 729 sq cm and the length of the rectangle is 81 cm. What is the difference between the side of the square and the breadth of the rectangle?

(a) 18 cm *(b)* 27 cm

(c) 24 cm *(d)* 9 cm

(e) None of the above

110. Find the area of the largest triangle that can be inscribed in a semi-circle of radius 9 cm.

(a) 81 sq cm *(b)* 51 sq cm

(c) 91 sq cm *(d)* 75 sq cm

(e) None of the above

111. Find the area of the largest circle that can be drawn inside a rectangle with sides 18 cm and 14 cm. [SSC CCL 2007] *(a)* 49 sq cm *(b)* 154 sq cm

(c) 378 sq cm *(d)* 1078 sq cm

112.The area of a square is twice the area of a circle. The area of the circle is 392 sq cm. Find the length of the side of the square. [Bank Clerks 2011]

(a) 28 cm *(b)* 26 cm

(c) 24 cm *(d)* 22 cm

(e) None of the above

113.The area of a rectangle is equal to the area of a circle with circumference equal to 39.6 m. What is the length of the rectangle if its breadth is 4.5 m?

[IBPS Bank Clerk 2011] *(a)* 33.52 m *(b)* 21.63 m

(c) 31.77 m *(d)* 27.72 m

(e) None of the above

114. Area of circle is equal to the area of a rectangle having perimeter of 50 cm and the length is more than its breadth by 3 cm. What is the diameter of the circle? [Bank Clerks 2007] *(a)* 7 cm *(b)* 21 cm

(c) 28 cm *(d)* 14 cm

(e) None of the above

115. The area of a rectangle is 4 times of the area of a square. The length of the rectangle is 90 cm and the breadth of the rectangle is 2/3 rd of the side of the square. What is the side of the square?

[Bank Clerks 2008]

(a) 10 cm

(b) 20 cm

(c) 15 cm

(d) Couldn't be determined

(e) None of the above

116. The area of a rectangle is 4 times the area of a square. The length of the rectangle is 90 cm and the breadth of the rectangle is 2/3rd of the side of the square. What is the side of the square?

[SBI2012]

(a) 10 cm

(b) 20 cm

(c) 9 cm

(d) Cannot be determined

(e) None of the above

117. The circumference of a circle is equal to the perimeter of a rectangle. The length and the breadth of the rectangle are 45 cm and 43 cm, respectively. What is the half the radius of the circle?

[Bank Clerks 2011] *(a)* 56 cmfbj 14 cm *(c)* 28 cm *(d)* 7 cm *(e)* None of the above

118. Circumference of a circle A is 1— times perimeter of a square. Area 'of the square is 784 sq cm. What is the area of another circle *B* whose diameter is half the radius of the circle A? *(a)* 154 sq cm *(b)* 156 sq cm

(c) 35.8 sq cm *(d)* 616 sq cm *(e)* None of the above

119.If the area of a circle is equal to the area of square with side *2-Jn* units, what is the diameter of the circle ?

[CDS 2012]

(a) 1 unit *(b)* 2 units

(c) 4 units *(d)* 8 units

120. If the circumference of a circle is equal to the perimeter of square, then which one of the following is correct? [CDS 2012]

(a) Area of circle = Area of square

(b) Area of circle > Area of square

(c) Area of circle > Area of square

(d) Area of circle < Area of square

121. A square, a circle and equilateral triangle have same perimeter. Consider the following statements. I. The area of square is greater than

the area of the triangle. II. The area of circle is less than the area of triangle.

Which of the statement is/are correct? [CDS 2012]

(a) Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

122.ABCD is a rectangle. Let *E* be a point on *AB* and *F* a point on *CD* such that *DE* is parallel to *BF*. If *AE* = 3 cm and if the area of *ABFC* = 6 sq cm.

Consider the following statements. I. Area of rectangle *ABCD* can be of the

form *pq* sq cm, where *p, q* are

distinct primes. II. Area of the figure *EBFD* is of the

form r sq cm, where *r* is rational but

not an integer.

Which of the above statements is/are

correct? [CDS 2012]

(a) Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

123. The area of circle inscribed in an equilateral triangle is 154 sq cm. What is the perimeter of the triangle? *(a)* 21 cm *(b)* 42V3 cm *(c)* 21V3 *(d)* 42 cm

124. One diagonal of a rhombus is 60% of the other diagonal. Then, area of the rhombus is how many times the square of the length of the larger diagonal?

(a) $\frac{1}{5}$ *(b)* $\frac{2}{5}$ *(c)* $\frac{6}{7}$ *(d)* $\frac{3}{10}$

(e) None of the above

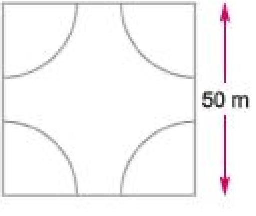
125. The cross-section of a canal is a trapezium in shape. If the canal is 20 m wide at the top and 12 m wide at the bottom and the area of the cross-section is 640 sq m, find the length of the cross-section.

(a) 80 m *(b)* 40 m *(c)* 60 m *(d)* 70 m *(e)* None of the above

126. What is the area between a square of side 10 cm and two inverted semicircular, cross-sections each of radius 5 cm inscribed in the square? [CDS 2013] *(a)* 17.5 cm^2 *(b)* 18.5 cm^2

(c) 20.5 cm^2 *(d)* 21.5 cm^2

127.A square park has each side 50 m. At each corner of the park, there is a flower bed in the form of a quadrant of radius 7 m, as shown in the figure. Find the area of remaining part of the park.

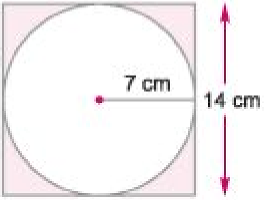


(a) 2346 sq m *(b)* 2340 sq m

(c) 2250 sq m *(d)* 2155 sq m

(e) None of the above

128. Find the area of shaded portion if each side of the square is 14 cm.



(a) 50 sq cm *(b)* 45 sq cm

(c) 62 sq cm *(d)* 42 sq cm

(e) None of the above

129.*ABC* is a triangle right angled at *AB* - 6 cm and *AC* - 8 cm. Semi-circle are drawn outside the triangle having area *x, y, z* units, respectively on the sides *AB, BC* and *AC.* What is *x* + *y* - *z* equal to?

(a) 48 cm^2 *(b)* 32 cm^2

(c) 0 *(d)* None of these

130. The diameters of two circles are the side of a square and the diagonal of the square. The ratio of the areas of the smaller circle and the larger circle is

[SSCCCL2013] *(a)* 1 : 4 *(b)* a/2 : V3

(c) 1: a/2 *(d)* 1 : 2

131. The slant height of a conical mountain is 2.5 km. The area of the base of the mountain is 49/100jt sq km. Then, the height of the mountain is *(a)* 1.7 km *(b)* 1.9 km

(c) 2.1 km *(d)* 2.4 km

(e) None of the above

132.A regular hexagon is inscribed in a circle of radius 5 cm. If *x* is the area inside the circle but outside the regular hexagon, then which one of the following is correct? [CDS 2013]

(a) 12 cm^2 < x< 15 cm^2

(b) 15 cm^2 < x < 17 cm^2

(c) $17\ cm^2 < x < 19\ cm^2$

(d) $19\ cm^2 < x < 21\ cm^2$

133. Which one of the following is a Pythagorean triple in which one side differs from the hypotenuse by two **units? [CDS 2014]**

(a) $(2n + 1, 4n, 2n^2 + 2n)$

(b) $(2n, An, n^2 + 1)$

(c) $(2n^2, 2n, 2n + 1)$

(d) $\{2n, n^2 -\backslash n^2 + 1)$,

where, *n* is a positive real number

Answer with **Solutions**

Triangle

1. (*b*) Given that,

$a = 8$ cm, $b = 10$ cm and $c = 12$ cm

We know, that

$$s = \frac{a+b+c}{2} = \frac{8+10+12}{2}$$
$$= \frac{30}{2} = 15 \text{ cm}$$

$\therefore \quad (s-a) = (15-8) = 7$ cm

$(s-b) = (15-10) = 5$ cm

$(s-c) = (15-12) = 3$ cm

$$\therefore \text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{15 \times 7 \times 5 \times 3} = \sqrt{1575} = \sqrt{25 \times 63}$$
$$= 5\sqrt{63} \text{ sq cm}$$

2. (*b*) We know that in any triangle "the sum of two sides is always greater than its third side" and "the difference of two sides is always less than its third side".

(i) $2 + 3 \not< 5$ (ii) $|5 - 2| \not< 3$

3. (*d*) Given that, $a = 6$ cm, $b = 4$ cm

and $c = 5$cm

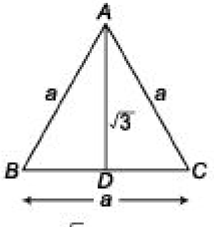
Required perimeter $= (a + b + c)$

$= (6 + 4 + 5)$ cm $= 15$ cm

4. (*c*) Given that, area $= 40$ sq cm, base $= 28$ cm and height = perpendicular = ?

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Perpendicular}$$
$$\Rightarrow \quad 40 = \frac{1}{2} \times 28 \times \text{Perpendicular}$$
$$\therefore \text{ Perpendicular} = \frac{40}{14} = \frac{20}{7} = 2\frac{6}{7} \text{ cm}$$

5. (*c*) Height of equilateral triangle



$$(AD) = \frac{\sqrt{3}}{2} \times \text{Side}$$
$$\therefore \quad \sqrt{3} = \frac{\sqrt{3} \times \text{Side}}{2} \Rightarrow 2\sqrt{3} = \sqrt{3} \times \text{Side}$$
$$\text{Side} = \frac{2\sqrt{3}}{\sqrt{3}} = 2 \text{ cm}$$

$\therefore$ Perimeter of an equilateral triangle

$$= 3a = 3 \times 2 = 6 \text{ cm}$$

6. (*a*) Given that, area = 10 sq cm,

Perpendicular = 20 cm and base = ?

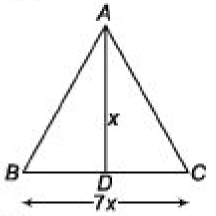
$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Perpendicular}$$

$$\Rightarrow \quad 10 = \frac{1}{2} \times \text{Base} \times 20$$

$$\therefore \quad \text{Base} = 1 \text{ cm}$$

7. (*d*) Let the height of the triangle be x.

Then, $BC = 7x$



Area of ΔABC

$$= \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times 7x \times x = \frac{7}{2}x^2$$

Now, cost ₹ 350 = 100 sq m

Cost ₹ 1225 = $\frac{100}{350} \times 1225 = 350$ sq m

It means $\frac{7}{2}x^2 = 350 \Rightarrow x = 10$ m

and base = $7x = 70$ m

8. (*a*) In a triangle,

Sum of two sides is always greater than 3rd side *i.e.*, $x < 25 + 15 = 40$ ...(i)

Difference of two sides is always less than 3rd side *i.e.*, $25 - 15 = 10 < x$ ...(ii)

Form Eqs. (i) and (ii), we get

$$10 < x < 40$$

9. (*a*) According to the question,

$$\frac{\sqrt{3}}{4}a^2 = 3\sqrt{3} \qquad \left[\begin{array}{l} a = \text{side} \\ \frac{\sqrt{3}}{4}a^2 = \text{area} \end{array}\right]$$

$$\Rightarrow \quad \frac{1}{4}a^2 = 3 \Rightarrow a^2 = 3 \times 4$$

$$\therefore \quad a = 2\sqrt{3}$$

$\therefore$ Required perimeter $= 3a = 3 \times 2\sqrt{3}$

$$= 6\sqrt{3} \text{ cm}$$

10. (*c*) Given ratio $= \frac{1}{3} : \frac{1}{4} : \frac{1}{5}$

$$= 20 : 15 : 12$$

Let lengths of the sides be $20x$, $15x$ and $12x$.
Then, according to the question,

$$20x + 15x + 12x = 94 \Rightarrow 47x = 94$$

$\therefore \quad x = \frac{94}{47} = 2$

Smallest side $= 12x = 12 \times 2 = 24$ cm

11. (*d*) We know,

Altitude (height) $= \frac{\sqrt{3}}{2} \times a = \frac{\sqrt{3} \times 9\sqrt{3}}{2}$

$$= \frac{27}{2} = 13.5 \text{ cm}$$

12. (*d*) Area of equilateral triangle $= \frac{\sqrt{3}}{4} a^2$

$\Rightarrow \quad 4\sqrt{3} = \frac{\sqrt{3}}{4} a^2 \Rightarrow a^2 = 16 \qquad [a = \text{side}]$

$\therefore \quad a = \sqrt{16} = 4$ cm

13. (*a*) Given that, perimeter = 90 cm

$\Rightarrow \quad 3a = 90 \text{ cm} \qquad [a = \text{side}]$

$\therefore \quad a = \frac{90}{3} = 30$ cm

$\therefore \quad \text{Area} = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times (30)^2$

$$= \frac{900 \times \sqrt{3}}{4} = 225\sqrt{3} \text{ sq cm}$$

14. (*a*) Since, the triangle is right angled.
$\therefore$ All the three consecutive sides must satisfy Pythagoras theorem
$\therefore$ 3, 4 and 5 are the sides of triangle which satisfy Pythagoras theorem.
$\therefore (5^2 = 4^2 + 3^2)$

$\therefore$ Area of triangle $= \frac{1}{2} \times 4 \times 3 = 6\text{cm}^2$

15. (*a*) Area of equilateral triangle

$$= \frac{\sqrt{3}\, a^2}{4} = x \qquad \ldots\text{(i)}$$

and perimeter $= 3a = y \quad \Rightarrow \quad a = \frac{y}{3} \qquad \ldots$(ii)

Now, putting the value of a from Eq. (ii) in Eq. (i), we get

$$\frac{\sqrt{3}\left(\frac{y}{3}\right)^2}{4} = x \Rightarrow x = \frac{\sqrt{3} \times y^2}{9 \times 4}$$

$\Rightarrow \quad x = \frac{y^2}{3\sqrt{3} \times 4} \Rightarrow x = \frac{y^2}{12\sqrt{3}}$

$\Rightarrow \quad 12\sqrt{3}\, x = y^2$

On squaring both sides, we get

$$y^4 = 432x^2$$

16. (*a*) Let the third side be x.

According to the question,

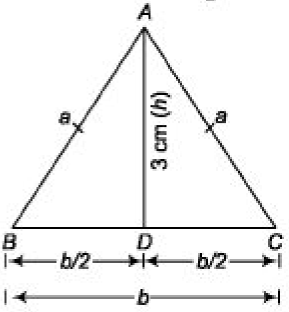
$$x + 20 = 26$$

$$\Rightarrow \quad x = 26 - 20 = 6 \text{ cm}$$

$\therefore$ Each equal side $= \dfrac{20}{2} = 10$ cm

17. (*a*) Let $AB = CA = a$ cm and base $= b$ cm

Now, area of the $\Delta ABC = \dfrac{1}{2} \times b \times h$



$$\Rightarrow \quad 12 = \frac{1}{2} \times b \times 3$$

$$\therefore \quad b = \frac{12 \times 2}{3} = 8 \text{ cm}$$

Here, $BD = CD = \dfrac{b}{2} = \dfrac{8}{2} = 4$ cm

In right angled ΔABD, by Pythagoras theorem,

$$AB = \sqrt{BD^2 + AD^2} \Rightarrow a = \sqrt{4^2 + 3^2}$$

$$= \sqrt{16 + 9} = \sqrt{25} = 5 \text{ cm}$$

Now, perimeter of an isosceles triangle

$$= 2a + b = 2 \times 5 + 8$$

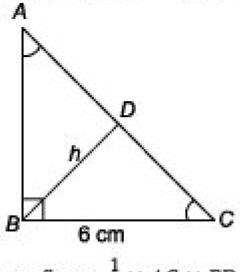
$$= 10 + 8 = 18 \text{ cm}$$

18. (*b*) Given, area of right angled triangle $= 24 \text{ cm}^2$

$$\Rightarrow \quad \frac{1}{2} \times AB \times BC = 24$$

$$\Rightarrow \quad AB = \frac{24 \times 2}{6} = 8 \text{ cm}$$

$$\therefore \quad AC = \sqrt{AB^2 + BC^2} = \sqrt{8^2 + 6^2} = 10 \text{ cm}$$



Now, from figure, $\dfrac{1}{2} \times AC \times BD = 24$

=> -*X* 10 X *h* = 24 2

.. 24 X 2

h =---------= 4.8 cm

10

19. (a) According to the question,

•*h* 2 V243 — *a* =-------

4 4

2 _ t/81 X 3 _ 9 a/3 a/3 a/3

a = V9 = 3 cm

20. (6) Let sides of isosceles triangle are 3x, 3x and 4x.

™ ,,n • ,. *a* + *b* + *c* Then, half-perimeter (s) =-------------

2 3x + 3x + 4x ,.

=---------------------= 5x

2

Given, area of isosceles triangle = 18-n/I squnits

,/s(s - a) (s - i) *{s-c)* = 18^5

^5x (5x - 3x) (5x - 3x) (5x - 4x) = 18^5

^5x X 2x X 2x X x = 18a/5

=> 2a/5x^2 = 18a/5

=> x^2 =9

=> x = 3

.". Third side of isosceles triangle = 4X3 = 12 units

21. (a) Since, $AC^2 = AB^Z + BC^2$

$^A\backslash$ 15 ^\$^{25}$

e 20 c

$$\Rightarrow \quad (25)^2 = (15)^2 + (20)^2$$
$$\Rightarrow \quad 625 = 225 + 400$$
$$\Rightarrow \quad 625 = 625$$

So, the triangular field is right angled at B.

$\therefore$ Area of the field $= \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 15 \times 20$

$= 150 \text{ m}^2$

So, the cost of sowing seed is ₹ 5 per sq m.

$\therefore$ Cost of sowing seed for

$150 \text{ m}^2 = 150 \times 5$

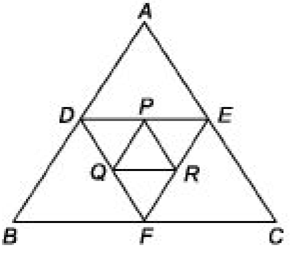
$=$ ₹ 750

22. (*c*) Triangles are equiangular and hence they are similar.

$\therefore$ Ratio of their areas = Ratio of squares of corresponding sides

$= (3)^2 : (7)^2 = 9 : 49$

23. (*b*) Perimeter of $\Delta PQR = 1 + 2 + 3 = 6$ units



Now, in ΔDEF,

$$\frac{DQ}{DF} = \frac{1}{2} = \frac{PQ}{FE}$$

So, $\quad 2PQ = FE$

Similarly, $DF = 2PR$ and $DE = 2QR$

$\therefore$ Perimeter of $\Delta DEF = 2 \times 6 = 12$ units

Similarly,

Perimeter of $\Delta ABC = 2 \times$ Perimeter of ΔDEF

$= 2 \times 12 = 24$ units

Quadrilateral

24. (a) Required area = Length x Breadth

= 15x8 = 120 sqm

25. (d) Required area = Length X Breadth

= 100 X40 =4000 sqm = (4 X 10^3) sqm

26. (a) According to the question,

Lx B = Area => LX15 = 300

[L = length and *B* = breadth]

L = ^ = 20m 15

27. (c) Let length of rectangle = 5x

and breadth of rectangle = 3x According to the question,

5x - 3x = 8 => 2x = 8

x = 4

Length = 5x = 5x4 = 20m

Breadth = 3x = 3x4 = 12m

.•. Required area = Length X Breadth

= 20 X 12 = 240 sqm

28. (G) Let length = 2x and breadth = x According to the question,

2x — x = 5 => x = 5

.'. Required perimeter = 2(2x + x) = 2 x 3x = 2X3X5 = 30 cm

29. (a) Let $l = 4x$ and $b = 9x$

Area of rectangle $= l \times b$

$$144 = 4x \times 9x \Rightarrow x^2 = \frac{144}{36} \Rightarrow x^2 = 4$$

$\therefore \quad x = 2$

$\therefore \quad l = 8$ cm and $b = 18$ cm

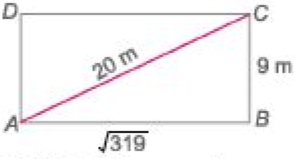
Perimeter of rectangle $= 2(l + b)$

$= 2(8 + 18)$

$= 2 \times 26$

$= 52$ cm

30. (a)



In ΔABC, by Pythagoras theorem

$$(AB)^2 = (AC)^2 - (BC)^2$$

$$(AB) = \sqrt{(20)^2 - (9)^2} = \sqrt{319}$$

$\therefore$ Required area = Length × Breadth

$= 9\sqrt{319}$ sq m

31. (b) Area of rectangle lies between 40 cm^2 and 45 cm^2.

Now, one side = 5 cm

Since, area cannot be less than 40 cm^2.

Hence, other side cannot be less than

$$= \frac{40}{5} = 8 \text{ cm}$$

Since, area cannot be greater than 45 cm^2.

Hence, other side cannot be greater than

$$= \frac{45}{5} = 9 \text{ cm}$$

$\therefore$ Minimum value of diagonal

$$= \sqrt{8^2 + 5^2} = \sqrt{89} = 9.43 \text{ cm}$$

Maximum value of diagonal

$$= \sqrt{9^2 + 5^2} = \sqrt{106} = 10.3 \text{ cm}$$

32. (a) Length of rectangle = 40 m Let breadth of = x

Then, according of the question,

(40 + x)15 = 40 Xx => 600 + 15x = 40x=> 25x = 600 x = 24 m

33. (a) Let the length of rectangle = *L* m

.". Breadth of rectangle = *B* m Using conditions from the question,

***L* - *B* = 23** ...(i)

2(L + *B*) = 206

L + *B* = 103 ...(ii)

On adding Eqs. (i) and (ii), we get 2L = 126

=> *L* = 63 m

=> B = 103 -63 = 40 m

Then, area of rectangle = *L* X. *B* = 63 X 40 = 2520 m^2.

34. (c) Let length and breadth of a rectangle is x and *y*. Then, as per first condition,

(x - 5) (*y* + 3) = xy - 9 => xy-5y+3x-15 = xy-9 => 3x-5y = 6 ...(i)

As per second condition,

(x + 3) (*y* + 2) = xy + 67 => xy+3y+2x+6 = xy+67 => 2x+3y = 61 ...(ii)

On multiplying Eq. (i) by 3 and Eq. (ii) by 5, then adding, we get

$$
\begin{aligned}
& 9x - 15y = 18 \\
\Rightarrow \quad & 10x + 15y = 305 \\
\hline
\Rightarrow \quad & 19x = 323 \\
\Rightarrow \quad & x = \frac{323}{19} = 17
\end{aligned}
$$

Hence, the length of rectangle is 17 m. 35. (a) Let the width of the rectangle = *x* units Length = (2 x + 5) units According to the question,

$$
\begin{aligned}
& \text{Area} = x(2x + 5) \\
\Rightarrow \quad & 75 = 2x^2 + 5x \\
\Rightarrow \quad & 2x^2 + 5x - 75 = 0 \\
\Rightarrow \quad & 2x^2 + 15x - 10x - 75 = 0 \\
\Rightarrow \quad & x(2x + 15) - 5(2x + 15) = 0 \\
\Rightarrow \quad & (2x + 15)(x - 5) = 0 \\
\Rightarrow \quad & x = 5 \text{ and } \frac{-15}{2}
\end{aligned}
$$

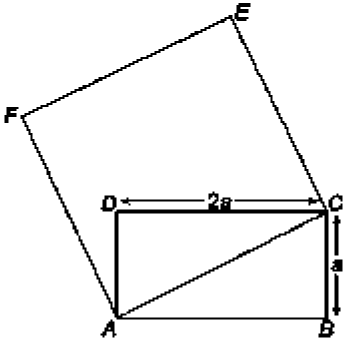
Width cannot be negative.

.'. Width = 5 units

∴ Length = 2x+5 =2x5+5 = 15units

.". Perimeter of the rectangle

= 2(15+5) =40 units 36. (d) Given that,



2

Area of rectangle = 2a = *1 X b*

=> ix jb = $2a^2$ = *lxa*

=> i = 2a

Now, In *AACD*

$AC^2 = AD^2 + CD^2 = a^2 + 4a^2 = 5a^2$ Side of square *(AC)* - av5

Hence, area of square = (av5) = 5a

37. (c) According to the formula,

Percentage increase in diagonals = 5%

38. (6) According to the formula,

■/ Percentage increase in sides = 10%

.•. Percentage increase in diagonals = 10%

39. (d) Let length of the rectangular field = 7x m and breadth of the rectangular field = *Zx* m According to the question,

Area of rectangular field = Length x Breadth => 3584 = 7x X 2x

=> $14x^2$ = 3584 => x^2 =5^5i = 256 14

=> x^2 = 256 => x = 16m

/. Length of rectangular field

= 7x 16 = 112m and breadth of rectangular field

= 2X 16 = 32m /. Perimeter of rectangle

= 2 (Length + Breadth)

= 2(112+ 32) = 2 X 144 = 288 m

40. (*c*) According to the question,

$$\frac{l}{2(l+b)} = \frac{5}{18}$$

$$\Rightarrow \quad 10l + 10b = 18l \Rightarrow 8l = 10b$$

$$\Rightarrow \quad \frac{l}{b} = \frac{10}{8} = \frac{5}{4}$$

$$\therefore \quad l : b = 5 : 4$$

Hence, ratio of length and breadth of a rectangle is 5 : 4.

41. (*b*) Given that, $l = 2b$

[Here l = length and b = breadth]

Decrease in length = Half of the 10 cm

$$= \frac{1}{2} \times 10 = 5 \text{ cm}$$

Increase in breadth = Half of the 10 cm

$$= \frac{1}{2} \times 10 = 5 \text{ cm}$$

Increase in the area = (70 + 5) = 75 sq cm

According to the question,

$$(l-5)(b+5) = lb + 75$$

$$\Rightarrow \quad (2b-5)(b+5) = 2b^2 + 75 \quad [\text{as } l = 2b]$$

$$\Rightarrow \quad 5b - 25 = 75 \Rightarrow 5b = 100$$

$$\therefore \quad b = \frac{100}{5} = 20$$

$$\therefore \quad l = 2b = 2 \times 20 = 40 \text{ cm}$$

42. (*d*) Area of the carpet = Area of the room

$= 8 \times 6 = 48$ sq m

Width of the carpet $= \frac{75}{100} = \frac{3}{4}$ m

Length of the carpet

$= \left(48 \times \frac{4}{3}\right)$

$= 16 \times 4 = 64$ m

$\therefore$ Cost of carpeting $= 64 \times 20 =$ ₹ 1280

43. (*d*) $l_1 = 20$ cm, $A_1 = 200$ sq cm

$\therefore \; b_1 = \frac{200}{20} = 10$ cm

Now, $\quad A_2 = 200 \times \frac{6}{5} = 240$ sq cm

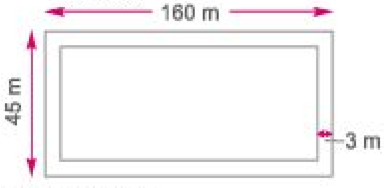
$b_2 = 10$ cm

$\therefore \quad l_2 = \frac{240}{10} = 24$ cm

$\therefore$ Perimeter of new rectangle $= 2(l_2 + b_2)$

$= 2(24 + 10) = 2 \times 34 = 68$ cm

44. (*a*) Given that, $l = 160$ m, $b = 45$ m and $x = 3$ m



By the formula

$\therefore$ Area of path $= 2x\,(l + b - 2x)$

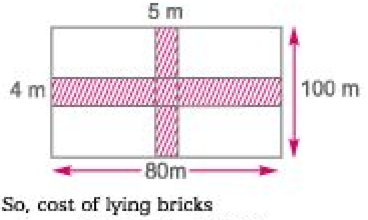
$= 6\,(160 + 45 - 6)$

$= 6 \times 199 = 1194$ sq m

$\therefore$ Required cost $= (1194 \times 4) =$ ₹ 5970

45. (*b*) Area to be paved with bricks

$= 5 \times 100 + 4 \times 80 - 4 \times 5 = 800 \text{ m}^2$



So, cost of lying bricks

$= 800 \times 10 =$ ₹ 8000

46. (*c*) Area of the rectangular field

$= \frac{12375}{15} = 825$ sq m

According to the question,

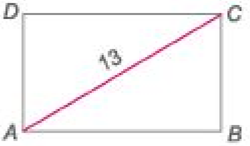
$(L \times B) = 825$ [L = length and B = breadth]

$\Rightarrow \quad L \times 25 = 825$

$\therefore \quad L = \frac{825}{25} = 33$ m

47. (e) Distance travelled by A in 15 s

$$= \frac{52 \times 15}{60} = 13 \text{ m}$$



$\therefore$ Diagonal of the field = 13 m

$$\therefore \qquad \sqrt{l^2 + b^2} = 13$$

$$l^2 + b^2 = 169 \qquad \ldots(\text{i})$$

Distance travelled by B in 15 s

$$= \frac{68 \times 15}{60} = 17 \text{ m}$$

$$\Rightarrow \qquad l + b = 17 \text{ m} \qquad \ldots(\text{ii})$$

On solving Eqs. (i) and (ii),

$$(l + b)^2 = l^2 + b^2 + 2lb$$

$$\Rightarrow \qquad 289 = 169 + 2lb$$

$$\Rightarrow \qquad 120 = 2lb$$

$$\Rightarrow \qquad lb = 60$$

$\therefore$ Area = 60 sq m

48. (*a*) Perimeter of the field $= \frac{3000}{75} = 400$ m

$$\Rightarrow \qquad 2(l + b) = 400$$

$$\Rightarrow \qquad 2(5x + 3x) = 400$$

$$\Rightarrow \qquad 8x = 200$$

$$\therefore \qquad x = \frac{200}{8} = 25$$

$\therefore$ Required difference $= (5x - 3x)$

$$= 2x = 2 \times 25 = 50 \text{ m}$$

49. (*a*) Original breadth of rectangle

$$= \frac{720}{30} = 24 \text{ cm}$$

Now, area of rectangle

$$= \frac{5}{4} \times 720 = 900 \text{ cm}^2$$

$\therefore$ New length of rectangle $= \frac{900}{24} = 37.5$ cm

$\therefore$ New perimeter of rectangle $= 2(l + b)$

$$= 2(37.5 + 24) = 2 \times 61.5 = 123 \text{ cm}$$

50. (*c*) Required area $= a^2 = (20)^2$ [a = side]

$$= 400 \text{ sq cm}$$

51. (*d*) According to the question,

$$4a = 68 \qquad [a = \text{side}]$$

$$\therefore \qquad a = \frac{68}{4} = 17 \text{ cm}$$

$\therefore$ Required area $= a^2 = (17)^2 = 289$ sq cm

52. (*a*) Diagonal of square $= \sqrt{2}\, a$ [a = side]

$$4\sqrt{2} = \sqrt{2}\, a$$

$$a = 4 \text{ cm}$$

Now, area of square = $a^2 = (4)^2 = 16$

Side of a square whose area is 2×16.

$$a_1^2 = 32 \Rightarrow a_1 = \sqrt{32} \Rightarrow a_1 = 4\sqrt{2}$$

Now, diagonal of new square

$$= \sqrt{2}a = \sqrt{2} \times 4\sqrt{2} = 8 \text{ cm}$$

53. (*a*) Let the diagonals of the squares be $3x$ and $2x$.

$\therefore$ Ratio of their areas $= \dfrac{1/2\,(3x)^2}{1/2\,(2x)^2} = \dfrac{9}{4}$

$$= 9 : 4$$

54. (*c*) Required increment [by Technique 2]

$$= \left(2a + \frac{a^2}{100}\right)\% \qquad [a = 25]$$

$$= \left(2 \times 25 + \frac{(25)^2}{100}\right)$$

$$= \left(50 + \frac{625}{100}\right)\% = 56.25\%$$

55. (*c*) We know that,

Perimeter of square = 4 × Side

$\therefore \quad 4 \times a = 12$ [for smaller square]

$\Rightarrow \quad a = 3$

$\therefore$ Area of smaller square

$$= 3 \times 3 = 9 \text{ cm}^2 \qquad \text{...(i)}$$

Now, $4 \times b = 24$ [for bigger square]

$\Rightarrow \quad b = 6$

$\therefore$ Area of bigger square

$$= 6 \times 6 = 36 \text{ cm}^2 = 4 \times 9 \text{ cm}^2$$

$= 4 \times$ Area of smaller square

[from Eq. (i)]

Hence, area of bigger square is 4 times that of smaller square.

56. (*c*) $a_1 = \dfrac{68}{4} = 17$ cm

and $\quad a_2 = \dfrac{60}{4} = 15$ cm

[a_1 and a_2 are sides]

According to the question,

Area of the third square

$$= [(17)^2 - (15)^2]$$

$$= (17 + 15)(17 - 15)$$

$$= 32 \times 2$$

$$= 64 \text{ sq cm}$$

Let a_3 = Side of the third square.

According to the question,

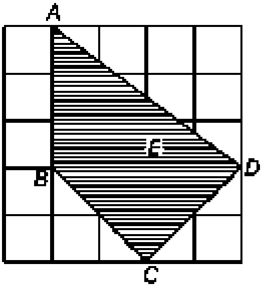
$(a_3)^2 = 64$ sq cm

$\therefore \quad a_3 = \sqrt{64} = 8$ cm

$\therefore$ Perimeter of the third square

$$= 4 \times a_3 = 4 \times 8 = 32 \text{ cm.}$$

57. (c)



Shaded part $ABCD$ can be visualised as $\Delta ABD + \Delta BCD$

$$\begin{Bmatrix}\text{where } BD \text{ is common base and} \\ AB \text{ and } CE \text{ are respective heights}\end{Bmatrix}$$

So, area can be calculated by finding the area of two triangles and adding them.

$\therefore$ Area of shaded region

$$= \text{Area of } \Delta ABD + \text{Area of } \Delta BCD$$
$$= \frac{1}{2} AB \times BD + \frac{1}{2} CE \times BD$$
$$= \frac{1}{2} \times 3 \times 4 + \frac{1}{2} \times 2 \times 4 = 6 + 4$$
$$= 10 \text{ sq cm}$$

58. (*c*) Area of the field $= \frac{1215}{135} = 9$ hec

$$= 90000 \text{ m}^2 \; [1 \text{ hec} = 10000 \text{ m}^2]$$

$\therefore$ Side of the field $= \sqrt{90000} = 300$ m

Perimeter of the field $= 4 \times 300 = 1200$ m

Now, cost of putting a fence around field

$$= 1200 \times \frac{75}{100} = ₹\, 900$$

59. (*a*) We know,

Area of parallelogram = Base × Height

Let height $= a$ and base $= 3a$

According to the question,

$$3a \times a = 2187$$
$$\Rightarrow \quad 3a^2 = 2187$$
$$\Rightarrow \quad a^2 = \frac{2187}{3} = 729$$
$$\therefore \quad a = 27 \text{ cm}$$

60. (*a*) Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

$$= \frac{1}{2} \times 1 \times 1.5 = 0.75 \text{ m}^2$$

61. (b) Perimeter of rhombus

$= 2\sqrt{d_1^2 + d_2^2} = 2\sqrt{(4.8)^2 + (1.4)^2}$

$= 2\sqrt{23.04 + 1.96} = 2\sqrt{25} = 2 \times 5 = 10$ cm

62. (a) Let First angle $= 3x$

Second angle $= 4x$

Third angle $= 5x$

and fourth angle $= 8x$

We know $3x + 4x + 5x + 8x = 360°$

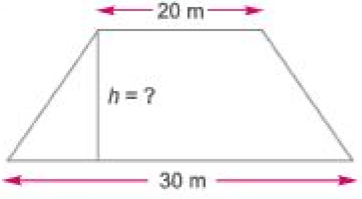
$\Rightarrow \quad 20x = 360°$

$\therefore \quad x = 18°$

Measure of smallest angle

$= 3x = 3 \times 18° = 54°$

63. (a) According to the question,



$$\text{Area} = \frac{1}{2}(30 + 20) \times h$$

$\Rightarrow \quad 50h = 500 \times 2$

$\therefore \quad h = 20$ cm

64. (c) Area of parallelogram = Base × Height

$= 8.06 \times 2.08 = 16.76 \text{ cm}^2$

65. (c) Area of square room $= (10)^2 = 100$ sq m

$= 100 \times (100)^2$ sq cm

$= 100 \times 100 \times 100$ sq cm

Now, area of tile $= (50)^2 = 50 \times 50$ sq cm

$\therefore$ Number of tiles needed

$$= \frac{\text{Area of square room}}{\text{Area of tile}}$$

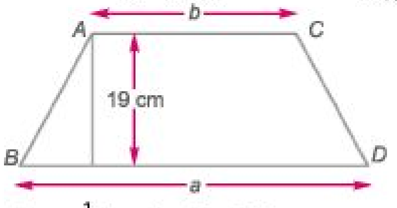
$$= \frac{100 \times 100 \times 100}{50 \times 50} = 400$$

Hence, 400 tiles will be needed.

66. (a) Let the two parallel sides be a and b.

According to the question,

$a - b = 8 \quad \ldots(i)$



and $\quad \frac{1}{2}(a + b) \times 19 = 760$

$\Rightarrow \quad a + b = 40 \times 2$

$\Rightarrow \quad a + b = 80 \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$2a = 88 \qquad \therefore a = \frac{88}{2} = 44$$

From Eq. (i), we get

$$a - b = 8$$

$\therefore \quad b = a - 8 = 44 - 8 = 36$ cm

$\therefore \quad a = 44$ cm and $b = 36$ cm

67. (*b*) Let the sides of trapezium be $5x$ and $3x$, respectively.

According to the question,

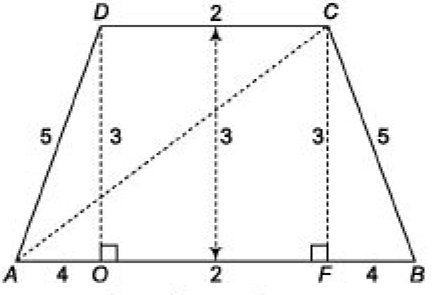
$$\frac{1}{2} \times (5x + 3x) \times 12 = 384$$

$$\Rightarrow \quad 8x = \frac{384 \times 2}{12} \Rightarrow x = \frac{64}{8} = 8 \text{ cm}$$

Length of smaller of the parallel sides

$$= 8 \times 3 = 24 \text{ cm}$$

68. (*b*) In ΔBCF, by Pythagoras theorem,



$$(5)^2 = (3)^2 + (BF)^2$$

$$BF = 4 \text{ cm}$$

$\therefore \quad AB = 2 + 4 + 4 = 10$ cm

Now, in ΔACF, $AC^2 = CF^2 + FA^2$

$\Rightarrow \quad AC^2 = 3^2 + 6^2 \Rightarrow AC = \sqrt{45}$ cm

Similarly, $BD = \sqrt{45}$ cm

$\therefore$ Sum of diagonals

$= AC + BD = \sqrt{45} + \sqrt{45} = 2\sqrt{45} = 6\sqrt{5}$ cm

69. (*d*) Let the lengths of parallel sides are $3x$ and $2x$.

We know that,

Area of trapezium $= \frac{1}{2}$ (Sum of the parallel sides) $\times$ Distance between them

$\therefore \quad \frac{1}{2}(3x + 2x)15 = 450 \Rightarrow 75x = 900$

$\therefore \quad x = \frac{900}{75} = 12$ cm

$\therefore$ Sum of the parallel sides $= (3x + 2x)$

$= 5x = 5 \times 12 = 60$ cm

70. (*b*) Let the diagonal of square $= d$

According to the question,

Sum of the diagonals $= d + d = 144$

$\Rightarrow \quad 2d = 144$

$\therefore \quad d = 72$ cm

Now, diagonal of square $= a\sqrt{2}$

$\Rightarrow \quad a\sqrt{2} = 72, \ a = \dfrac{72}{\sqrt{2}}$

$\therefore$ Perimeter of square $= 4a = 4 \times \dfrac{72}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= 144\sqrt{2}$ cm

71. (*a*) We know that,

Area of regular pentagon $= 5a^2 \dfrac{\sqrt{3}}{4}$ [a = side]

According to the question,

$\dfrac{5a^2\sqrt{3}}{4} = 125\sqrt{3} \Rightarrow a^2 = \dfrac{125\sqrt{3} \times 4}{5\sqrt{3}} = 100$

$\therefore \quad a = 10$ cm

72. (*a*) Each exterior angle of regular pentagon

$= \dfrac{360°}{5} = 72°$

$\therefore$ Each interior angle

$= 180° -$ Exterior angle

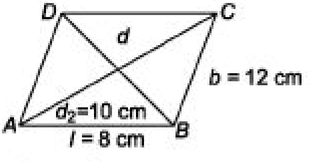
$= 180° - 72° = 108°$

73. (*d*) Each exterior angle of regular octagon

$= \dfrac{360°}{8} = 45°$

74. (*d*) In parallelogram,

$d_1^2 + d_2^2 = 2(l^2 + b^2)$



$\therefore \quad d^2 + (10)^2 = 2(64 + 144)$

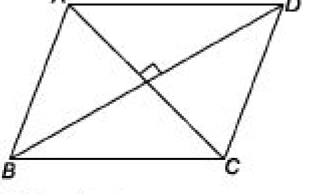
$\Rightarrow \quad d^2 = 2 \times 208 - 100$

$\Rightarrow \quad d^2 = 416 - 100 = 316$

$\Rightarrow \quad d = \sqrt{316}$

$\Rightarrow \quad d = 17.76$ cm $\Rightarrow d > 10$

75. (*c*)



ABCD is a rhombus.

$\therefore \quad AB = BC = CD = DA$

And diagonals bisect each other at right angles.

$\therefore$ Option (c) is correct.

Circle

76. (*a*) Radius $= \dfrac{\text{Diameter}}{2}$

$\therefore$ Diameter $= 2 \times$ Radius

$$= 2 \times 3 = 6 \text{ cm}$$

77. (*b*) Required area $= \pi r^2 = \pi \times 25 \times 25$

$$= \pi \times 25 \times 25 = 625\pi \text{ sq m.}$$

78. (*e*) Distance covered in 1 revolution

$$= \frac{44 \times 1000}{4000} = 11 \text{ m}$$

According to the question,

$$2\pi r = 11 \Rightarrow \frac{44}{7} \times r = 11$$

$$\therefore \quad r = \frac{11 \times 7}{44} = 1.75 \text{ m}$$

79. (*a*) According to the question,

Area of semi-circle = 77 m

$$\frac{1}{2}\pi r^2 = 77$$

$$\Rightarrow \quad r^2 = \frac{77 \times 2 \times 7}{22}$$

$$\therefore \quad r = 7 \text{ m}$$

$\therefore$ Circumference of semi-circle $= \pi r + 2r$

$$= (\pi + 2)r = \left(\frac{22}{7} + 2\right) \times 7 = 36 \text{ m}$$

80. (*a*) Let the radius of the park be r, then

$$\pi r + 2r = 288$$

$$(\pi + 2)r = 288$$

$$\Rightarrow \quad \left(\frac{22}{7} + 2\right)r = 288$$

$$\Rightarrow \quad r = \frac{288 \times 7}{36} = 56$$

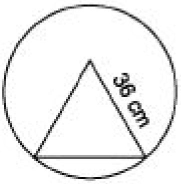
$\therefore$ Area of the park $= \dfrac{1}{2}\pi r^2$

$$= \frac{1}{2} \times \frac{22}{7} \times 56 \times 56$$

$$= 4928$$

81. (*d*) Given, area of sector $= 72\pi$ cm^2

or $\quad \dfrac{\pi r^2 \theta}{360°} = 72\pi$



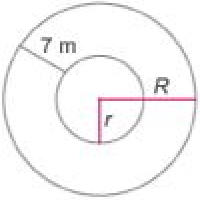
$$\theta = \frac{72 \times 360}{36 \times 36} \Rightarrow \theta = 20°$$

Now, length of arc

$$= \frac{\pi r \theta}{180°} = \frac{\pi \times 36 \times 20°}{180°} = 4\pi \text{ cm}$$

82. (*c*) R = Radius of outer circle

r = Radius of inner circle



According to the question,

$$2\pi r = 440$$

$$\Rightarrow \quad r = \frac{440}{2\pi} = \frac{440}{2 \times \frac{22}{7}} = \frac{440 \times 7}{44} = 70 \text{ m}$$

Clearly, $\quad R - r = 7$

$\Rightarrow \quad R - 70 = 7$

$\therefore \quad R = 70 + 7 = 77$ m

83. (*b*) Given that, $a = 6$

According to the formula,

Percentage increase in area

$$= \left(2a + \frac{a^2}{100}\right)\% \quad \text{[by Technique 2]}$$

$$= \left(2 \times 6 + \frac{36}{100}\right)\%$$

$$= (12 + 0.36)\% = 12.36\%$$

84. (*b*) Let the radius of circle = r cm

According to the question,

Area of sector = 77 cm^2

$$\frac{\theta}{360°} \times \pi r^2 = 77 \Rightarrow \frac{45°}{360°} \times \pi r^2 = 77$$

$$\Rightarrow \quad r^2 = \frac{77 \times 7 \times 8}{22} \Rightarrow r = 14 \text{ cm}$$

85. (*d*) Distance covered in 1 round

$$= \frac{\text{Total Distance}}{\text{Total round}}$$

$$= \frac{1.76 \times 1000}{350}$$

$$= \frac{176}{35} \text{ m}$$

$$\therefore \quad 2\pi r = \frac{176 \times 100}{35} \text{ cm}$$

$$2r = \text{Diameter} = \frac{17600 \times 7}{22 \times 35}$$

$$= 160 \text{ cm}$$

86. (*a*) Ratio of the areas of the circumcircle and incircle of a square

$$= \frac{(\text{Diagonal})^2 \pi}{(\text{Side})^2 \pi} = \frac{(\text{Side} \times \sqrt{2})^2}{(\text{Side})^2} = \frac{2}{1} \text{ or } 2:1$$

87. (*b*) Let the radii of two circles are r_1 and r_2, respectively.

Given,

$$\frac{\text{Circumference of 1st circle}}{\text{Circumference of 2nd circle}} = \frac{2}{3}$$

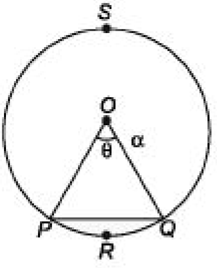
$$\Rightarrow \quad \frac{2\pi r_1}{2\pi r_2} = \frac{2}{3} \Rightarrow \frac{r_1}{r_2} = \frac{2}{3} \Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9}$$

$$\therefore \frac{\text{Area of 1st circle}}{\text{Area of 2nd circle}} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9}$$

88. (*c*) I. We know that, area of segment ($PRQP$)

= Area of sector ($OPRQO$) − Area of ΔOPQ

$$= \frac{\pi r^2 \theta}{360} - \frac{1}{2} r^2 \sin\theta$$



So, the area of a segment of a circle is always less than area of its corresponding sector.

II. Distance travelled by a circular wheel of diameter $2d$ cm in one revolution

$$= 2 \times 3.14 \times d = 6.28d$$

which is greater than $6d$ cm.

89. (*b*) Increase in circumference of circle = 5%

$\therefore$ Increase in radius is also 5%.

Now, increase in area of circle

$$= \left(2a + \frac{a^2}{100}\right)\% \text{ [by Technique 2]}$$

where, a = increase in radius

$$= \left(2 \times 5 + \frac{5 \times 5}{100}\right)\% = 10.25\%$$

90. (*c*) Let original radius be r.

Then, according to the question,

$$\pi (r+1)^2 - \pi r^2 = 22$$

$$\Rightarrow \quad \pi \times [(r+1)^2 - r^2] = 22$$

$$\Rightarrow \quad \frac{22}{7} \times (r+1+r)(r+1-r) = 22$$

$$\Rightarrow \quad 2r + 1 = 7 \Rightarrow 2r = 6$$

$$\therefore \quad r = \frac{6}{2} = 3 \text{ cm}$$

91. (*a*) Length of the rope = Radius of circle

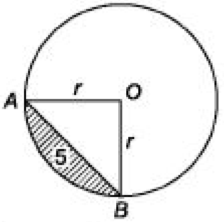
According to the question,

$$\pi r^2 = 154$$

$$\therefore \quad r^2 = 154 \times \frac{7}{22} = 7 \times 7 = 49$$

$$\therefore \quad r = \sqrt{49} = 7 \text{ m}$$

92. (*c*) In ΔAOB, $AO = OB = r$ [radius of circle]



By Pythagoras theorem,

$AB^2 = OA^2 + OB^2 \Rightarrow (5)^2 = r^2 + r^2$

$\therefore \quad r^2 = \dfrac{25}{2}$ cm

Now, area of sector $AOB = \dfrac{\theta}{360°} \times \pi r^2$

$= \dfrac{90°}{360°} \times \pi \times \dfrac{25}{2} = \dfrac{25\pi}{8}$ cm^2

Now, area of minor segment

= area of sector − area of triangle

$= \dfrac{25\pi}{8} - \dfrac{r^2}{2} = \dfrac{25\pi}{8} - \dfrac{25}{4} = \left(\dfrac{25\pi - 50}{8}\right)$

$\therefore$ Area of major segment = Area of circle − Area of minor segment

$= \pi r^2 - \left(\dfrac{25\pi - 50}{8}\right) = \dfrac{25\pi}{2} - \dfrac{(25\pi - 50)}{8}$

$= \dfrac{100\pi - 25\pi + 50}{8} = \dfrac{75\pi + 50}{8}$

$= \dfrac{25}{8}(3\pi + 2) = \dfrac{25}{4}\left(\dfrac{3\pi}{2} + 1\right)$ cm^2

93. (*b*) The man takes 3600 s for 14.4 km

The man will take 88 s for

$\dfrac{14.4 \times 88}{3600} = \dfrac{352}{1000}$ km = 352 m

Now, circumference of circular field

$= 352 \text{ m} \Rightarrow 2\pi r = 352$ m

$2 \times \dfrac{22}{7} \times r = 352 \Rightarrow r = 56$ m

Therefore, area of the field $= \pi r^2$

$= \dfrac{22}{7} \times 56 \times 56 = 8 \times 22 \times 56$ m^2

= 9856 sq m.

94. (*b*) Let the radius of circular field = r m.

Speed of person in m/s $= \dfrac{30}{60} = \dfrac{1}{2}$ m/s

According to the question,

$\dfrac{2\pi r}{1/2} - \dfrac{2r}{1/2} = 30 \Rightarrow 4\pi r - 4r = 30$

$\Rightarrow \quad \left(4 \times \dfrac{22}{7} - 4\right) r = 30$

$\Rightarrow \quad (12.5 - 4)r = 30 \Rightarrow (8.5)r = 30$

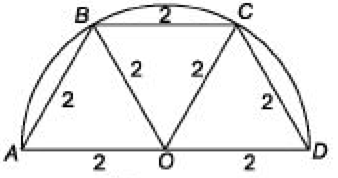
$\Rightarrow \quad r = \dfrac{30}{8.5} = 3.5$ m

Miscellaneous

95. (*d*) Since, Δ's AOB, BOC and COD are equilateral.

$\therefore$ Sides = 2 cm

Now, total area $= 3 \times \frac{\sqrt{3}}{4} (\text{Side})^2$



$= 3 \times \frac{\sqrt{3}}{4} \times 4 = 3\sqrt{3} \text{ cm}^2$

96. (*c*) Area of square plate $= (\text{Side})^2$

$= (2d)^2 = 4d^2$

Area of circular plate $= \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$

$\because$ Number of square plates

$= \dfrac{4d^2}{\frac{\pi d^2}{4}} = \dfrac{4 \times 4}{\pi} \approx 5$

Since, nearest integer value is 5.

97. (*b*) Given that, $a = 7$, $b = 24$ and $c = 25$

Semi-perimeter of triangle

$= \frac{a + b + c}{2} = \frac{7 + 24 + 25}{2} = \frac{56}{2} = 28$ cm

According to the question,

Area of circle = Area of triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{28(28-7)(28-24)(28-25)}$

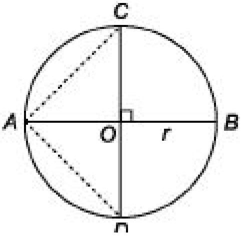
$= \sqrt{28 \times 21 \times 4 \times 3} = \sqrt{7056} = 84 \text{ cm}^2$

98. (*d*) Circumradius $= \frac{2}{3} \times$ Height

Height $= \frac{10 \times 3}{2} = 15$ cm

So, length of perpendicular drawn from centre $= 15 - 10 = 5$ cm

99. (*b*) Required ratio $= \dfrac{\text{Area of circle}}{\text{Area of } \Delta ACD}$



$$= \frac{\pi r^2}{\frac{1}{2} \times 2r \times r} = \pi$$

100. (*b*) Area of square $= (\text{Side})^2 = 20^2$

$= 400$ sq cm

$\therefore$ Area of rectangle

$= 1.8 \times 400 = 720$ sq cm

Let length and breadth of rectangle be $5x$ and x, respectively.

Then, according to the question,

$$5x \times x = 720$$

$$\Rightarrow \quad 5x^2 = 720 \Rightarrow x^2 = \frac{720}{5} = 144$$

$$\therefore \quad x = \sqrt{144} = 12 \text{ cm}$$

Perimeter of rectangle $= 2(5x + x) = 12x$

$= 12 \times 12 = 144$ cm

101. (*d*) Diameter of the circle $= 13 + 5 = 18$cm

$$\therefore \text{ Radius} = \frac{\text{Diameter}}{2} = \frac{18}{2} = 9 \text{ cm}$$

Area of the circle $= \pi r^2 = \frac{22}{7} \times 9^2$

$$= \frac{22 \times 81}{7} = \frac{1782}{7} = 254.57 \text{ sq cm}$$

$= 255$ sq cm [approx]

102. (*c*) Area of square $= 64$ sq cm

$(\text{Side})^2 = 64$ [a = side]

$$\therefore \quad \text{Side} = \sqrt{64} = 8 \text{ cm}$$

According to the question,

$$\Rightarrow \quad 2\pi r = 4 \times 8 \Rightarrow r = \frac{4 \times 8}{2\pi} = \frac{16}{\pi}$$

$\therefore$ Area of the circle

$$= \pi \times \frac{16}{\pi} \times \frac{16}{\pi} = \frac{256}{\pi} \text{ sq cm.}$$

Fast Track Method

Given that, $a = 64$ sq cm [by Technique 4]

According to the formula,

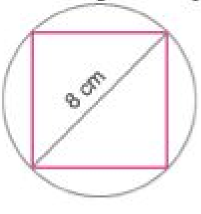
$$\text{Required area} = \frac{4a}{\pi} = \frac{4 \times 64}{\pi} = \frac{256}{\pi} \text{ sq cm}$$

103. (*a*) **Fast Track Method**

Here, $r = 4$ cm [by Technique 5]

Required area $= 2r^2 = 2 \times 4^2 = 32$ sq cm

$\because$ Diameter = Diagonal of square



$\therefore$ Area of square $= \frac{1}{2} \times (\text{diagonal})^2$

$$= \frac{1}{2} \times 8^2 = \frac{64}{2} = 32 \text{ sq cm}$$

104. (*a*) Circumference of the circle $= 2\pi r = 25$

$$\Rightarrow \quad r = \frac{25}{2\pi}$$

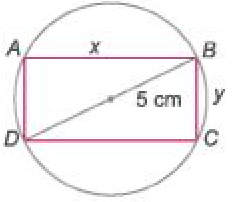
According to the formula,

Side of inscribed square

$$= r\sqrt{2} = \frac{25}{2\pi} \times \sqrt{2}$$

$$= \frac{25}{\pi\sqrt{2}} \text{ cm}$$

105. (*c*) *ABCD* be the rectangle inscribed in the circle of diameter 5 cm.



$\therefore$ Diameter = Diagonal of rectangle

Now, let x and y be the lengths and breadths of rectangle, respectively.

Now, in ΔABD

$$AB^2 + AD^2 = (5)^2$$

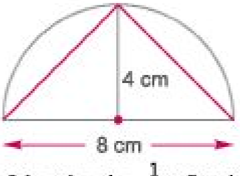
$$\Rightarrow \quad x^2 + y^2 = 25$$

Since, they form Pythagoras theorem.

So, $x = 4$ and $y = 3$

$\therefore$ Area of rectangle $= 3 \times 4 = 12 \text{ cm}^2$

106. (*a*) According to the formula,



Area of the triangle $= \frac{1}{2} \times 8 \times 4 = 16$ sq cm

Area of semi-circle $= \frac{\pi r^2}{2} = \frac{16\pi}{2} = 8\pi$

$\therefore$ Required answer $= 8\pi - 16$

$$= 8(\pi - 2) \text{ sq cm.}$$

107. (*b*) Perimeter of square = 72 cm

$\therefore$ Perimeter of rectangle

$$= \frac{72}{2} = 36 \text{ cm}$$

$$\Rightarrow \quad 2 \times (l + b) = 36$$

[l and b are dimension of rectangle]

$$2 \times (12 + b) = 36$$

$\Rightarrow \quad 12 + b = 18$

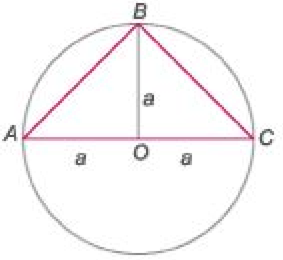
$\therefore \quad b = 18 - 12 = 6$ cm

And side of square $= \dfrac{72}{4} = 18$

$\therefore$ Difference between breadth of the rectangle and side of the square

$= 18 - 6 = 12$ cm.

108. (*b*) Area of the largest triangle



$= \left(\dfrac{1}{2} \times 2a \times a\right)$ sq cm $= a^2$ sq cm

109. (*d*) According to the question,

Area of rectangle $= 4 \times 729$

$\Rightarrow \quad 81 \times B = 4 \times 729$

$\therefore \quad B = 4 \times 9 = 36$ cm

Area of square $= 729$

$\Rightarrow \quad a^2 = 729 \qquad [a = \text{side}]$

$\therefore \quad a = \sqrt{729} = 27$

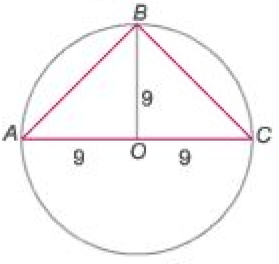
$\therefore$ Required difference $= B - a$

$= 36 - 27 = 9$ cm

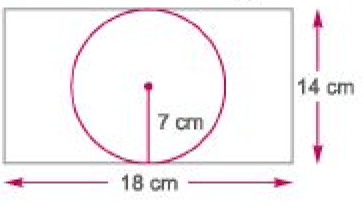
110. (*a*) Given that, $r = 9$cm

Required area $= \dfrac{1}{2} \times$ Base $\times$ Height

$= \dfrac{1}{2} \times 18 \times 9 = 81$ sq cm



111. (*b*) Radius of circle $(r) = \dfrac{1}{2} \times 14 = 7$ cm



$\therefore$ Area of the circle $= \pi r^2 = \dfrac{22}{7} \times 7 \times 7$

$= 154$ sq cm.

112. (*a*) According to the question,

Area of square $= 2 \times 392$

$\Rightarrow \quad a^2 = 784 \quad$ [a = side]

$\therefore \quad a = \sqrt{784} = 28$ cm.

113. (*d*) Let the radius of the circle be r.

Perimeter of given circle $= 39.6$ m

$2\pi r = 39.6 \Rightarrow r = \dfrac{7 \times 39.6}{22 \times 2} = 6.3$ m

Area of circle $= \dfrac{22}{7} \times 6.3 \times 6.3$

$= \dfrac{873.18}{7} = 124.74 \text{ m}^2$

Now, area of a rectangle = Area of a circle

$l \times b = 124.74 \Rightarrow l \times 4.5 = 124.74$

$\therefore \quad l = \dfrac{124.74}{4.5} = 27.72$ m.

114. (*d*) Let breadth of the rectangle be b.

Then, length $= (b + 3)$

According to the question,

$2(b + b + 3) = 50 \Rightarrow 2b + 3 = 25$

$\Rightarrow \quad 2b = 22 \Rightarrow b = \dfrac{22}{2} = 11$

$\therefore$ Breadth = 11 cm

Length $= (11 + 3) = 14$ cm

$\therefore$ Area of the circle = Area of the rectangle

$\Rightarrow \quad \pi r^2 = 14 \times 11$

$\Rightarrow \quad r^2 = \dfrac{14 \times 11 \times 7}{22} = 49$

$\therefore \quad r = \sqrt{49} = 7$ cm

$\therefore$ Diameter $= 2r = 2 \times 7 = 14$ cm.

115. (*c*) Let side of square be x.

$\therefore$ Breadth of the rectangle $= \dfrac{2}{3}x$

$\Rightarrow \quad 90 \times \dfrac{2}{3}x = 4x^2$

$\therefore \; x = 90 \times \dfrac{2}{3} \times \dfrac{1}{4} = 15$ cm.

116. (*e*) Let the side of the square be x cm.

$\therefore$ Breadth of rectangle $= \dfrac{2}{3}x$ cm

According to the question,

$4x^2 = 90 \times \dfrac{2}{3}x$

$\Rightarrow \quad 4x = 60 \Rightarrow x = \dfrac{60}{4} = 15$ cm

117. (*b*) According to the question,

$2\pi r = 2(45 + 43) = 2 \times 88$

$\Rightarrow \quad r = \dfrac{2 \times 88}{2\pi}$

$\therefore \quad \frac{1}{2}r = \frac{2 \times 88 \times 7}{2 \times 44} = 14$ cm.

118. (*a*) Given, area of the square = 784 sq cm

$(\text{Side})^2 = 784$

$\therefore$ Side $= \sqrt{784} = 28$ cm

According to the question,

Circumference of a circle A

$$= 1\frac{4}{7} \times \text{Perimeter of square}$$

$$2\pi r_1 = \frac{11}{7} \times 4 \times 28$$

$$r_1 = \frac{11 \times 4 \times 28 \times 7}{7 \times 22 \times 2} = 28 \text{ cm}$$

Now, diameter of another circle $B = \frac{1}{2} r_1$

$$= \frac{1}{2} \times 28 = 14 \text{ cm}$$

$\therefore$ Radius of circle $B = \frac{14}{2} = 7$ cm

$\therefore \quad$ Area of circle $B = \pi(7)^2 = \frac{22}{7} \times 7 \times 7$

$$= 154 \text{ sq cm.}$$

119. (*c*) Area of the circle = Area of the square $= (\text{Side})^2$

$\therefore \quad \pi r^2 = (2\sqrt{\pi})^2$

$\Rightarrow \quad \pi r^2 = 4\pi$

$\Rightarrow \quad r^2 = \frac{4\pi}{\pi} = 4$

$\Rightarrow \quad r = \sqrt{4} = 2$ units

$\therefore$ Diameter of circle $(d) = 2 \times r = 2 \times 2$

$$= 4 \text{ units}$$

120. (*c*) Let the radius of a circle is r and a be the length of the side of a square.

Given, circumference of a circle = Perimeter of a square

$\Rightarrow \quad 2\pi r = 4a$

$\Rightarrow \quad a = \frac{\pi}{2} r = 1.57r$

Now, area of the circle $(A_c) = \pi r^2 = 3.14 r^2$

and area of the square

$(A_s) = a^2 = 2.4649 r^2$

$\therefore$ Area of circle > Area of square.

121. (*a*) let the radius of circle is 'r' and a side of a square is 'a', then given condition

$2\pi r = 4a \Rightarrow a = \frac{\pi r}{2}$

$\therefore$ Area of square

$$= \left(\frac{\pi r}{2}\right)^2 = \frac{\pi^2 r^2}{4} = \frac{9.86 r^2}{4} = 2.46 r^2$$

and area of circle $= \pi r^2 = 3.14 r^2$

and let the side of equilateral triangle is x.
Then, given condition,

$$3x = 2\pi r \Rightarrow x = \frac{2\pi r}{3}$$

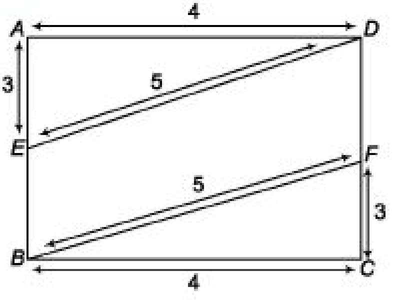
$\therefore$ Area of equilateral triangle $= \frac{\sqrt{3}}{4}x^2$

$$= \frac{\sqrt{3}}{4} \times \frac{4\pi^2 r^2}{9}$$

$$= \frac{\pi^2}{3\sqrt{3}} r^2 = 1.89 r^2$$

Hence, Area of circle > Area of square > Area of equilateral triangle.

122. (*a*)



Given that,
Area of $\Delta BFC = 6 \text{ cm}^2$

$$\frac{1}{2} \times 3 \times x \times = 6 \Rightarrow x = 4 \text{ cm}$$

In ΔBFC, $BF^2 = 4^2 + 3^2 = 16 + 9$

$\Rightarrow \quad BF^2 = 25$

$\Rightarrow \quad BF = 5$

$\therefore$ Area of rectangle $ABCD\ pq^2 = p(2)^2 \text{ cm}^2$.

Which is of the form pq^2.

While the area of $EBFD$ cannot be the form of $r^2 \text{ cm}^2$.

123. (*b*) We know that, the radius of a circle inscribed in a equilateral triangle $= \frac{a}{2\sqrt{3}}$

Where, a be the length of the side of an equilateral triangle.

Given that, area of a circle inscribed in an equilateral triangle = 154 cm^2

$$\therefore \quad \pi\left(\frac{a}{2\sqrt{3}}\right)^2 = 154$$

$$\Rightarrow \quad \left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{154 \times 7}{22} = (7)^2$$

$$\Rightarrow \quad a = 42\sqrt{3}\ cm$$

Perimeter of an equilateral triangle

$$= 3a = 3\,(14\sqrt{3})$$

$$= 42\sqrt{3} \text{ cm}$$

124. (*d*) Let one diagonal be x.

Then, other diagonal $= \left(\frac{60x}{100}\right) = \frac{3x}{5}$ cm

Area of rhombus

$$= \frac{1}{2} \times x \times \frac{3x}{5} = \frac{3}{10} x^2$$

$$= \frac{3}{10} \times \quad \text{(square of longer diagonal)}$$

Hence, area of rhombus is $\frac{3}{10}$ times.

125. (*b*) Let length of cross-section be x.

According to the question,

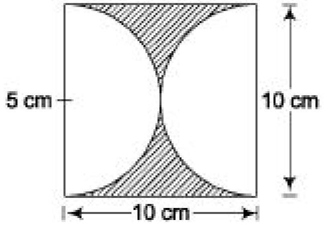
$$\frac{1}{2}(20 + 12) \times x = 640$$

$$\Rightarrow \quad \frac{1}{2}(32)\ x = 640$$

$$\Rightarrow \quad 16x = 640 \quad \therefore \quad x = 40 \text{ m}$$

126. (*d*) Area between square and semi-circles

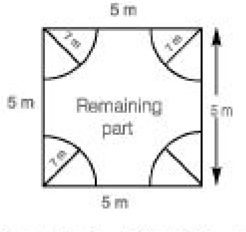
= Area of square − 2 (Area of semi-circle)



$$= (10)^2 - \frac{2}{2} \times \frac{22}{7} (5)^2$$

$$= 100 - 78.5 = 21.5 \text{ cm}^2$$

127. (*a*) As four quadrants make a circle,



∴ Area of park without flower bed

$$= \text{Area of square} - \text{Area of circle}$$

$$= \left[(50)^2 - \left(\frac{22}{7} \times 7 \times 7\right)\right]$$

$$= [2500 - 154] = 2346 \text{ sq m}$$

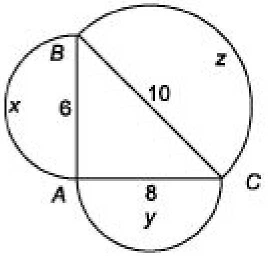
∴ Area of remaining part = 2346 sq m.

128. (*d*) Area of shaded part = Area of square − Area of in circle

$$= \left[(14)^2 - \frac{22}{7} \times 7 \times 7\right]$$

$$= (196 - 154) = 42 \text{ sq cm.}$$

129. (*c*) In ΔABC



By pythagoras theorem,

$$BC^2 = AC^2 + BA^2$$

$$AC^2 = 64 + 36$$

$$BC = 10 \text{ cm}$$

Now, area of semi-circle

$$= x = \frac{\pi(3)^2}{2}$$

$$= \frac{9\pi}{2} \text{ cm}^2$$

Area of semi-circle $= y = \frac{16\pi}{2} \text{ cm}^2$

Area of semi-circle $= z = \frac{25\pi}{2} \text{ cm}^2$

Now, value of

$$x + y - z = \left(\frac{9\pi}{2} + \frac{16\pi}{2}\right) - \frac{25\pi}{2} = 0$$

130. (*d*) Diagonal of a square $= \sqrt{2} \times$ Side

$\therefore$ Ratio of area of smaller circle to larger circle

$$= \frac{\pi r_1^2}{\pi r_2^2} = \frac{\pi \times \left(\frac{a}{2}\right)^2}{\pi \times \left(\frac{\sqrt{2}a}{2}\right)^2}$$

Here, a = Diameter of smaller circle

$$= \frac{1}{2} = 1 : 2$$

131. (*d*) Area of base of mountain $= \frac{49}{100}\pi \text{ km}^2$

$$\pi r^2 = \frac{49}{100}\pi$$

$$r = \sqrt{\frac{49}{100}} = 0.7 \text{ km}$$

$\therefore$ Height of mountain

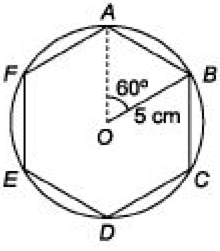
$= \sqrt{(\text{Slant height})^2 - (\text{Radius of base})^2}$

$= \sqrt{(2.5)^2 - (0.7)^2} = 2.4$ km

132. (*a*) $OB = OA =$ radius

Also, $\angle AOB = 60° \left(\frac{360°}{6} = 60°\right)$

and $\angle OAB = \angle OBA = 60°$



$\therefore$ ΔAOB is an equilateral triangle.

Then, $AB = 5$ cm

So, area x = Area of circle – Area of hexagon

$$= \pi r^2 - \frac{3\sqrt{3}(a)^2}{2}$$

$$= \frac{22}{7} \times (5)^2 - \frac{3\sqrt{3}}{2} \times (5)^2 \quad [\because r = a = 5]$$

$$= 78.57 - 64.95 = 13.62 \text{ cm}^2$$

133. (*d*) By hit and trial method.

Put, $n = 2$ in option (d)

$$= [(2 \times 2), (2)^2 - 1, (2)^2 + 1] = (4, 3, 5)$$

Which satisfy pythagoras theorem and one side differes from hypotenuse by 2 units.

Chapter

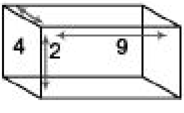
35

Volume and Surface Area

Volume and surface area are related to solids or hollow bodies. These bodies occupy space and have usually three dimensions length, breadth and height.

Volume

Space occupied by an object is called the 'volume' of that particular object. It is always measured in cube unit like cubic meter, cubic centimetre etc.



For example If length, breadth and height of a box are 9 cm, 4 cm and 2 cm respectively, then Volume of the box = Length x Breadth x Height

= 9x4x2=72 cm^3

Surface Area

Surface area of a solid body is the area of all of its surfaces

together. Surface area is measured in square unit like

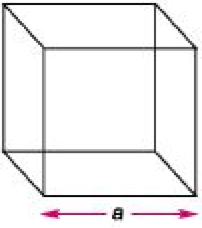
square centimetre, square metre etc.

For example A cube has 6 surfaces and each surface is in a

square like shape. Therefore, its surface area will be

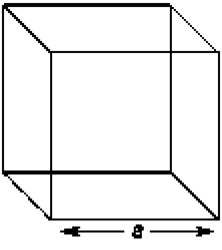
$6a^2$ sq units, where cr is the area of each surface of the

cube.



Cube

A solid body having 6 equal faces with equal length, breadth and height is called a cube. Infact, each face of a cube is a sauare.



♦ Volume of the cube = *cf*

♦ Lateral surface area of the cube = *Act*

♦ Total surface area of the cube = 6cr

♦ Diagonal of the cube = *a-j3* where, *a* - Side (edge) of the cube

Ex. 1 The diagonal of a cube is 12*y[3* cm. Find its volume and surface area.

Sol. Let the edge of cube = *a*

According to the question,

Diagonal of cube = *a-M*

So, *aj3* = 12^ => *a* = 12

∴ Volume = $a^3 = 12^3 = 1728$ cm^3

Surface area = $6a^2 = 6 \times (12)^2 = 6 \times 144 = 864$ sq cm

Ex. 2 The surface area of a cube is 486 sq cm. Find its volume.

Sol. Let the edge of cube = *a*

According to the question,

$$6a^2 = 486 \Rightarrow a^2 = \frac{486}{6} = 81$$

$$\therefore \quad a = \sqrt{81} = 9 \text{ cm}$$

$$\therefore \text{ Volume} = a^3 = 9^3 = 9 \times 9 \times 9 = 729 \text{ cm}^3$$

Ex. 3 The surface area of a cube is 1350 sq cm. Find the volume of the cube.

Sol. Let each side of the cube = a

According to the question,

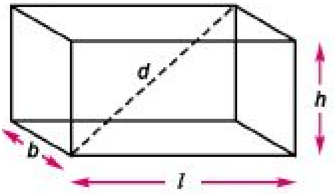
$$6a^2 = 1350 \Rightarrow a^2 = \frac{1350}{6} = 225$$

$$\therefore \quad a = \sqrt{225} = 15$$

$$\therefore \quad \text{Volume} = a^3 = 15^3 = 3375 \text{ cm}^3$$

Cuboid

A rectangular solid body having 6 rectangular faces is called a cuboid.



♦ Volume of the cuboid = *bh*

■f Lateral surface area of a cuboid = 2 (+ *b)h*

♦ Total surface area of the cuboid = 2 (*b* + *bh* + *h)*

✦ Diagonal of cuboid = $\sqrt{^2 + b^2 + h^2}$

where, = Length, *b* - Breadth and *h* - Height

Some other cube or cuboidal shaped object are as follows.

Room

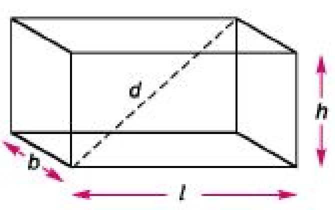
A rectangular room has 4 walls (surfaces) and opposite walls have equal areas.

♦ Total area of walls = 2 (+ b) x *h*

■f Total volume of the room = *lbh*

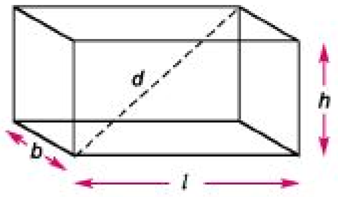
*■ Area of floor or roof = x b

where, = Length, *b* = Breadth and *h* = Height



Box

A box has its shape like cube or cuboid. The amount that a box can hold or contain, is called the capacity of the box. Infact, capacity is the internal volume.



♦ Surface area of an open box

= 2 (length + breadth) x Height + Length x Breadth = 2x(+b)xh + x b «- Capacity of box = (- 20 *{b* - 20 *(h* - 2f) where, *t* = Thickness of the box

♦ Volume of the material of the box

= External volume - Internal volume (capacity) = *bh-{ -2t)(b-* 2t) *{h - 2t)* where, = Length, b = Breadth and *h* = Height

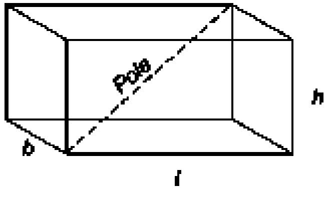
Note For calculation of any of the parameter, lenght, breadth and height should be in same unit

Ex. 4 Find the volume and surface area of a cuboid 18 m long, 14 m broad and 7 m high. **Sol.** Volume of the cuboid = Length x Breadth x Height Here, length = 18 m, breadth = 14 m and height = 7 m

= 18 X 14 X 7= 1764m^3 Surface area = 2*(lb + bh + lh)*= 2(18 x 14 + 14 x 7 + 18 x 7)

= 2(252+ 98+ 126) = (2 x 476) = 952 sq m

Ex. 5 What is the length of the largest pole that can be placed in a room 11 m lone, 8 m broad and 9 m high?



Sol. Length of the longest pole = Length of the diagonal Here,

$l = 11$ m , $b = 8$ m and $h = 9$ m

$$= \sqrt{l^2 + b^2 + h^2}$$
$$= \sqrt{11^2 + 8^2 + 9^2}$$
$$= \sqrt{121 + 64 + 81}$$
$$= \sqrt{266} = \sqrt{4 \times 66.5} = 2\sqrt{66.5} \text{ m}$$

Ex. 6 A wooden box measures 10 cm x 6 cm x 5 cm. Thickness of wood is 2 cm. Find the volume of the wood required to make the box. Sol. External volume = 10 x 6 x 5 = 300 cm^3

Internal volume (/ - *2t)(b - 2t)(h - 2t)* = (10 - 4) X (6 - 4) X (5 - 4)

= 6x2x1= 12 cm^3 .•. Volume of the wood = External volume - Internal volume = 300- 12 = 288 cm^3

Ex. 7 The length, breadth and height of a cuboid are in the ratio of 6 :5 :4 and its whole surface area is 66600 cm . What is its volume? Sol. Let length = *6x,* breadth = *5x* and height = *4x* According to the question, Total surface area = 66600

$$2(lb + bh + lh) = 66600$$
$$\Rightarrow 2(6x \times 5x + 5x \times 4x + 6x \times 4x) = 66600$$
$$\Rightarrow 148x^2 = 66600 \Rightarrow x^2 = \frac{66600}{148} = 450$$
$$\therefore x = \sqrt{450} = \sqrt{2 \times 225} = 15\sqrt{2} \text{ cm}$$
Then, length $(l) = 6 \times 15\sqrt{2} = 90\sqrt{2}$ cm

Breadth $(b) = 5 \times 15\sqrt{2} = 75\sqrt{2}$ cm

Height $(h) = 4 \times 15\sqrt{2} = 60\sqrt{2}$ cm

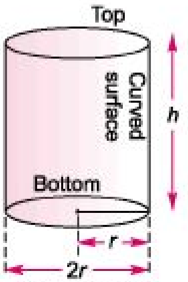
$$\therefore \text{Volume} = l \times b \times h = 90\sqrt{2} \times 75\sqrt{2} \times 60\sqrt{2}$$
$$= 2 \times 90 \times 75 \times 60\sqrt{2}$$
$$= 13500 \times 60\sqrt{2} = 810000\sqrt{2} \text{ cm}^3$$

Ex. 8 The area of 4 walls of a room is 520 sq m while its length and breadth are 15 m and 11 m, respectively. Find the height of the room. Sol. Here, *I* = 15,6 = Hand *h* = ? According to the question, Area of four walls = 520

2(1 + b)x h = 520 => 2(15+ 11) X *h* = 520=>52fc = 520 ft = 10 m

Cylinder

A cylinder is a solid or hollow body that is formed by keeping circles of equal radii one on another. Apart from this, a cylinder is formed by rolling a rectangular sheet also. ***A cylinder has three surfaces***



Solid cylinder

♦ Curved **surface**

♦ Bottom •► **Top**

Solid Cylinder

♦ Volume of cylinder = Area of base x Height =

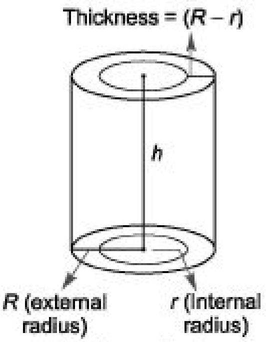
*■ Curved surface area = Perimeter of base x Height = 2 *nih*

*■ Total surface area = Curved surface area + Area of both the circles (top and bottom

surfaces) = 2 Jtrfi + 2 ki^2 = *2 nr(z + h]*

where, r = Radius of base and h = Height

Hollow Cylinder



Hollow cylinder

If cylinder is hollow, then ♦ Volume of hollow cylinder = Outer volume - Inner volume = jchCR^2 - r^2)

■f Curved surface area = Curved surface area of outer surface +

Curved surface area of inner surface

= 2 *nRh* + 2 Ttrf! = 2 *iOi(R + r)* + Total surface area of hollow cylinder

= Curved surface area + Area of both top and bottom surface

= *2nh{R + i)* + *2n(R*^{2} -r^2)

where, *R* = External radius of base, r = Internal radius of base

and *h* = Height

Ex. 9 Find the volume, curved surface area and the total

surface area of a cylinder with diameter of base 14 cm and height 80 cm.

Sol. Here, $r = \frac{d}{2} = \frac{14}{2}$ cm and $h = 80$ cm

$$\text{Volume} = \pi r^2 h = \frac{22}{7} \times \frac{14}{2} \times \frac{14}{2} \times 80 = (22 \times 1 \times 7 \times 80) = 12320 \text{ cm}^3$$

$$\text{Curved surface area} = 2\pi rh = 2 \times \frac{22}{7} \times \frac{14}{2} \times 80$$

$$= (44 \times 80) = 3520 \text{ cm}^2$$

$$\text{Total surface area} = \text{Curved surface area} + \text{Area of both top and bottom surface}$$

$$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times \frac{14}{2} \times (80 + 7) = 44 \times 1 \times 87 = 3828 \text{ cm}^2$$

Ex. 10 How many iron rods each of length 14 m and diameter 4 cm can be made out of 0.88 m^3 of iron?

Sol. Here, $r = \frac{2}{100}$ m and $h = 14$ m

Volume of 1 rod $= \pi r^2 h = \frac{22}{7} \times \frac{2}{100} \times \frac{2}{100} \times 14 = 22 \times \frac{1}{50} \times \frac{1}{50} \times 2$

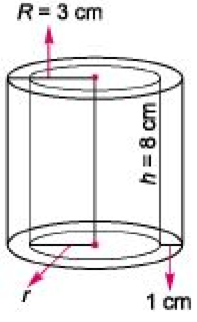
$= \frac{44}{2500} = \frac{11}{625}$ m^3

[Here, $r = \frac{d}{2}$ and is divided by 100 to convert into metre]

Volume of iron rod = 0.88 m^3

$\therefore$ Number of rods $= 0.88 \times \frac{625}{11} = 50$

Ex.11 A hollow cylinder made of wood has thickness 1 cm while its external radius is 3 cm. If the height of the cylinder is 8 cm, then find the volume, curved surface area and total surface area of the cylinder.



Sol. r = Inner radius = External radius − Thickness = 3 − 1 = 2 cm

$\therefore$ Required volume $= \pi h(R^2 - r^2) = \frac{22}{7} \times 8\ (3^2 - 2^2)$

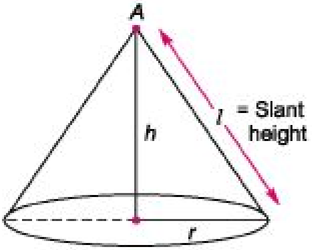
$= \frac{22}{7} \times 8 \times 5 = \frac{880}{7}$ cm^3

Curved surface area $= 2\pi h\ (R + r)$

$= 2 \times \frac{22}{7} \times 8 \times 5 = \frac{1760}{7}$ sq cm

Cone

Cone is a solid or hollow body with a round base and pointed top. It is formed by the rotation of a triangle along any of the side.



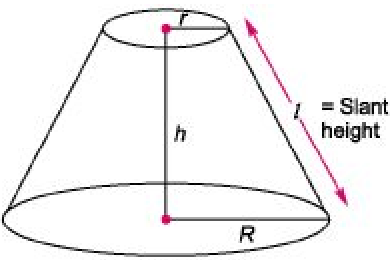
Cone

- Volume = $\frac{1}{3}$ × Base area × Height = $\frac{1}{3}\pi r^2 h$
- Slant height ( ) = $\sqrt{r^2 + h^2}$
- Curved surface area = πr = $\pi r\sqrt{r^2 + h^2}$
- Total surface area = Curved surface area + Area of base
  = $\pi rl + \pi r^2 = \pi r(l + r)$

  where, r = Radius of base, h = Height and = Slant height

Frustum of Cone

If a cone is cut by a plane parallel to the base so as to divide the cone into two parts upper part and lower part, then the lower part is called frustum.



Frustum of cone

- Volume = $\frac{\pi h}{3}(r^2 + R^2 + rR)$
- Slant height () = $\sqrt{h^2 + (R - r)^2}$

■f Curved surface area = Ji(fi + *r)*

+ Total surface area = *n{(r + R)* +r^2 + fi^2}

where, r = Radius of top, R = Radius of base, h = Height and = Slant height

Ex. 12 The diameter of a right circular cone

is 14 m and its slant height is 10 m. Find its curved surface area, total surface area and volume.

Sol. Here, $r = \frac{14}{2} = 7$ m and $l = 10$ m $\quad [\because r = \frac{d}{2}]$

Curved surface area $= \pi r l = \frac{22}{7} \times 7 \times 10 = 22 \times 10 = 220$ sq m

Total surface area $= \pi r(r + l) = \frac{22}{7} \times 7\,(7 + 10) = 22 \times 17 = 374$ sq m

$$\text{Volume} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \times \sqrt{l^2 - r^2} = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times \sqrt{10^2 - 7^2}$$

$$= \frac{1}{3} \times 22 \times 7\sqrt{100 - 49} = \frac{154}{3}\sqrt{51}\ \text{m}^3$$

Ex. 13 Find the cost of colouring the total surface of the right circular cone in example 15, if the rate of colouring is 14 paise per sq m.

Sol. Calculated surface area in example 15 = 374 sq m

$\therefore$ Required cost $= 374 \times \frac{14}{100}$

$= ₹\ 52.36$

Ex. 14A frustum of a right circular cone has a diameter of base and top 20 cm and 12 cm, respectively and a height of 10 cm. Find the area of its whole surface and volume.

Sol. Given, $R = 10\text{ cm}$, $r = 6\text{ cm}$ and $h = 10\text{ cm}$

Slant height $(l) = \sqrt{h^2 + (R - r)^2}$

$= \sqrt{10^2 + (10 - 6)^2} = \sqrt{100 + 16}$

$= \sqrt{116} = 10.77\text{ cm}$

Whole surface area $= \pi(R^2 + r^2 + Rl + rl)$

$= \frac{22}{7}(100 + 36 + 10 \times 10.77 + 6 \times 10.77)$

$= \frac{22}{7}(136 + 107.7 + 64.62)$

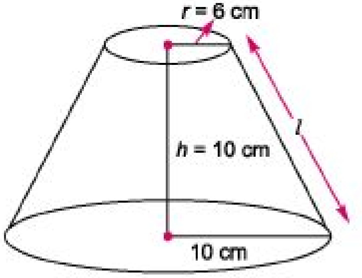
$= \frac{22}{7} \times 308.32 = \frac{6783.04}{7} = 969\text{ sq cm (approx.)}$

Volume $= \frac{\pi h}{3}(R^2 + r^2 + Rr)$

$= \frac{22}{7} \times \frac{10}{3}(100 + 36 + 60)$

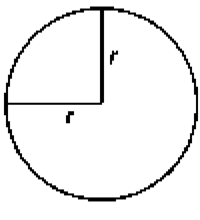
$= \frac{22}{7} \times \frac{10}{3} \times 1963$

$= \frac{43120}{21} = 205.33\text{ cm}^3$



Sphere

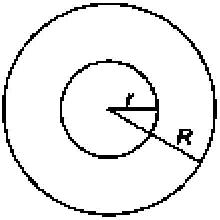
A sphere is a three-dimensional solid figure, which is made up of all points in the space, which lie at a constant distance from a fixed point. That constant distance and fixed point are respectively called the radius and centre of the sphere. Infact, a sphere is like a solid ball.



- Volume of the sphere $= \frac{4}{3}\pi r^3$
- Total surface area $= 4\pi r^2$ where, r = Radius

Hollow Sphere or Spherical Shell

Its both external and internal surfaces are spherical and both the surfaces have a common central point.

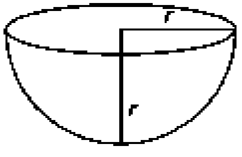


- Volume of hollow sphere $= \frac{4}{3}\pi(R^3 - r^3)$
- Internal surface area $= 4\pi r^2$
- External surface area $= 4\pi R^2$
  where, R = External radius and r = Internal radius

Hemisphere

It is the half part of a sphere.

- **Volume of the hemisphere $= \frac{2}{3}\pi r^3$**
- **Total surface area $= 3\pi r^2$**
- **Curved surface area $= 2\pi r^2$ where, r = Radius**



Ex. 15 Find the volume and surface area of a sphere of diameter 14 cm.

Sol. Here, $r = 7$ cm

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = \frac{4 \times 22 \times 49}{3} = 1437.33 \text{ cm}^3$$

$$\text{Surface area} = 4\pi r^2$$

$$= 4 \times \frac{22}{7} \times 7 \times 7 = 4 \times 22 \times 7 = 616 \text{ sq cm}$$

Ex. 16 Find the volume, curved surface area and the total surface area of a hemisphere of radius 7 cm.

Sol. Here, $r = 7$ cm

$$\text{Volume} = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = \frac{44 \times 49}{3} = 718.66 \text{ cm}^3$$

$$\text{Curved surface area} = 2\pi r^2$$

$$= 2 \times \frac{22}{7} \times 7 \times 7 = 44 \times 7 = 308 \text{ sq cm}$$

$$\text{Total surface area} = 3\pi r^2$$

$$= 3 \times \frac{22}{7} \times 7 \times 7 = 3 \times 22 \times 7 = 462 \text{ sq cm}$$

Ex. 17A hemispherical bowl is made of steel and 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved area of the bowl.

Sol. Inner radius = 5 cm

Thickness = 0.25 cm

Outer radius = 5.25 cm

$$\therefore \text{Outer curved surface area} = 2\pi r^2 = 2 \times \frac{22}{7} \times 5.25 \times 5.25 = \frac{1212.75}{7}$$

$$= 173.25 \text{ sq cm}$$

Ex. 18 The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases. **Sol.** When radius = 7 cm

Then, surface area = 47lq

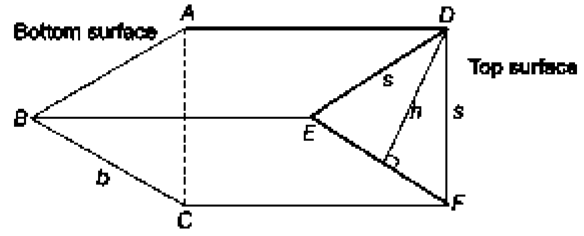
When radius = 14 cm

Then, surface area = 47ir2

.". Ratio of surface areas = 47tr[^{2} : 47tr$_2{}^2$ = q^2 : $r_2{}^2$ = $7^2 : 14^2 = 1^2 : 2^2 = 1 : 4$

Prism

A right prism is a solid whose top and bottom faces are parallel to each other and are identical polygons that are parallel. The faces joining the top and bottom faces are rectangles and are called **lateral faces.** The distance between the base and the top is called **height** or **length** of the right prism.



■f Volume of prism = Area of base x Height of the prism

♦ Lateral (vertical) surface area = Perimeter of base x Height of the prism

*■ Total surface area = Lateral surface area + 2 x Area of base

Ex. 19 The base of a right prism is a square having side of 10 cm. If its height is 8 cm, then find the total surface area and volume of the prism.

Sol. Given, side =10 cm and height = 8 cm Now,

(i) Total surface area = Lateral surface area + 2 X Area of base Now, lateral surface area = Perimeter of the base X Height

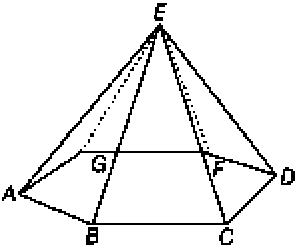
= [10 + 10 + 10 + 10] x 8 = 40 x 8 = 320 cm^2 Area of base = Area of square = 10 X 10 = 100 On putting, these values in formula, we get Total surface area = 320 + 2 x 100 = 320 + 200 = 520 cm^2 (ii) Volume of the prism = Area of base x Height = 10 x 10 x 8 = 800 cm^3

Pyramid

A solid whose base is a polygon and whose faces are triangles, is called a pyramid. The triangular faces meet at a common point called vertex,

"A pyramid whose base is regular polygon and the foot of the perpendicular from the vertex of the base, coincides with the centre of the base, is called a right pyramid."

- Volume of a pyramid = $\frac{1}{3}$ × Area of base × Height
- Lateral surface area = $\frac{1}{2}$ × Perimeter of the base × Slant height
- Total surface area = Lateral surface area + Area of the base



Ex. 20 The base of a pyramid is a square whose side is 10 cm. It's slant and vertical heights are 13 and 12. Then, find the total surface area and volume of the pyramid.

Sol. Given, side = 10 cm and height = 13 cm

(i) Total surface area = Lateral surface area + Area of the base ...(i)

Now,

$$\text{Lateral surface area} = \frac{1}{2} \times \text{Perimeter of the base} \times \text{Slant height}$$

$$= \frac{1}{2} \times (10 \times 4) \times 13$$

$$= \frac{40}{2} \times 13 = 260 \text{ cm}^2$$

and area of the base $= (\text{side})^2 = (10)^2 = 100$

On putting these above values in Eq.(i), we get

Total surface area $= 260 + 100 = 360 \text{ cm}^2$

(ii) $\text{Volume of the pyramid} = \frac{1}{3} \times \text{Area of base} \times \text{Height (vertical)}$

$$= \frac{1}{3} \times 10 \times 10 \times 12 = \frac{100 \times 12}{3} = 400 \text{ cm}^3$$

Fast Track **Techniques**

to solve the **QUESTIONS**

Technique 1

If length, breadth and height of a cuboid are changed by *x%, y%* and z% respectively, then its volume is increased by

$$\left[x + y + z + \frac{xy + xz + yz}{100} + \frac{xyz}{(100)^2}\right]\%$$

Note Increment in the value is taken as positive and decrement in value is taken as negative Positive result shows total increment and negative result shows total decrement

Ex.21 If all the dimensions of a cuboid are increased by 100%, by what per cent does the volume of cuboid increase? **Sol.** Here, $x = y = z = 100$

According to the formula, Percentage increase in volume

$$= \left[x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{100}\right]\%$$

$$= \left[100 + 100 + 100 + \frac{100 \times 100 + 100 \times 100 + 100 \times 100}{100} + \frac{100 \times 100 \times 100}{(100)^2}\right]\%$$

$$= \left[300 + \frac{10000 + 10000 + 10000}{100} + \frac{1000000}{10000}\right]\%$$

$$= \left[300 + \frac{30000}{100} + 100\right]\%$$

$$= (300 + 300 + 100)\% = 700\%$$

Technique 2

If side of a cube or radius (or diameter) of sphere is increased by $x\%$, then its volume increases by $\left[\left(1 + \frac{x}{100}\right)^3 - 1\right] \times 100\%$

Ex. 22 If side of a cube is increased by 10%, by how much per cent does its volume increase?

Sol. Here, $x = 10$

According to the formula,

Percentage increase in volume $= \left[\left(1 + \frac{x}{100}\right)^3 - 1\right] \times 100\%$

$$= \left[\left(1 + \frac{10}{100}\right)^3 - 1\right] \times 100\%$$

$$= \left[\left(1 + \frac{1}{10}\right)^3 - 1\right] \times 100\% = \left[\left(\frac{11}{10}\right)^3 - 1\right] \times 100\%$$

$$= \left[\frac{1331}{1000} - 1\right] \times 100\%$$

$$= (1.331 - 1) \times 100\%$$

$$= (0.331 \times 100)\% = 33.1\%$$

Technique 3

if height of a cylinder is changed by *x%* and radius remains the same, then the volume changes by x%.

Ex. 2 3 If height of a cylinder is increased by 4%, while radius remains unchanged, by how much per cent volume increases? Sol. Here, $x = 4$

According to the formula, Percentage increase in volume $x = 4\%$

Technique 4

If radius of a cylinder or edge of a cube or radius of sphere is changed byx% and height remains unchanged, then volume changes by

$$\left(2x + \frac{x^2}{100}\right)\%.$$

Ex.24 If radius of a cylinder is increased by 10%, while height remains unchanged, by what per cent does the volume of cylinder increase?

Sol. Here, $x = 10$

According to the formula,

Percentage increase in volume $= \left[2x + \frac{x^2}{100}\right]\% = \left(2 \times 10 + \frac{10 \times 10}{100}\right)\% = 21\%$

Ex.25 When the radius of an sphere is increased by 4%, what per cent increase takes place in surface area of sphere?

Sol. Here, $x = 4$

According to the formula,

Percentage increase in area $= \left[2x + \frac{x^2}{100}\right]\% = \left[2 \times 4 + \frac{(4)^2}{100}\right]\% = 8.16\%$

Technique 5

If radius of a cylinder or cone is changed by x% and height is changed by y%, then volume changes by $\left[2x + y + \frac{x^2 + 2xy}{100} + \frac{x^2 y}{100^2}\right]$%.

Ex.26 If radius of a cylinder is decreased by 4%, while its height is increased by 2%, then what will be effect on volume?

Sol. Here, $x = -4$ and $y = 2$

According to the formula,

$$\text{Net effect on volume} = \left[2x + y + \frac{x^2 + 2xy}{100} + \frac{x^2 y}{100^2}\right]\%$$

$$= \left[2 \times (-4) + 2 + \frac{(-4)^2 + 2 \times 2\,(-4)}{100} + \frac{(-4)^2 \times 2}{100^2}\right]\%$$

[negative sign shows that there is a decrease in volume]

$$= \left[-8 + 2 + \frac{16 - 16}{100} + \frac{32}{100^2}\right]\%$$

$$= [-6 + 0 + 0.0032]\,\% = -5.99\% \text{ (decrease)}$$

Technique 6

If in a cylinder or cone, height and radius both change by x%, then volume changes by $\left[\left(1 + \frac{x}{100}\right)^3 - 1\right] \times 100\%$.

Ex. 27 If in a cylinder, both height and radius increase by 100%, by what per cent does its volume increase?

Sol. Here, $x = 10$

According to the formula,

Percentage increase in volume

$$= \left[\left(1 + \frac{x}{100}\right)^3 - 1\right] \times 100\% = \left[\left(1 + \frac{100}{100}\right)^3 - 1\right] \times 100\%$$

$$= [(2)^3 - 1] \times 100\% = (8 - 1) \times 100\% = 700\%$$

Ex.28 By what per cent does the volume of cylinder decrease, if there is 100% decrease in its radius and height?

Sol. Here, $x = -100$

According to the formula,

Percentage decrease in volume $= \left[\left\{1 + \frac{x}{100}\right\}^3 - 1\right] \times 100\%$

$= \left[\left\{1 + \frac{(-100)}{100}\right\}^3 - 1\right] \times 100\%$

[negative signs indicates decrease]

$= [(1 - 1)^3 - 1] \times 100\% = -100\%$

If two measuring dimensions which are included in the surface area of a cube, cuboid, sphere, cylinder or cone are increased or decreased by $x\%$ and $y\%$, then the surface area of the figure will increase or decrease by $\left(x + y + \frac{xy}{100}\right)\%$

[take positive sign for increase and negative sign for decrease]

Ex. 2 9 When each side of a cube is increased by 20%, then find the increase in total surface area of a cube.

Sol. Here, $x = y = 20\%$

According to the formula,

Total surface area $= \left(x + y + \frac{xy}{100}\right)\%$

$= \left(20 + 20 + \frac{20 \times 20}{100}\right)\%$

$= \left(40 + \frac{400}{100}\right)\%$

$= 44\%$

Technique 7

Three cubes of metal whose sides are *x, y* and z respectively, are melted to form a new (bigger) cube, if there is no loss of weight in this process. Then, side of new cube will be

$$\sqrt[3]{x^3 + y^3 + z^3}$$

Ex. 30 The sides of three cubes of metal are 30 cm, 40 cm and 50 cm respectively. Find the side of new cube formed by melting these cubes together.

Sol. Here, $x = 30$, $y = 40$ and $z = 50$

$$\text{Side of new cube} = \sqrt[3]{x^3 + y^3 + z^3}$$
$$= \sqrt[3]{(30)^3 + (40)^3 + (50)^3}$$
$$= \sqrt[3]{27000 + 64000 + 125000}$$
$$= \sqrt[3]{216000}$$
$$= 60 \text{ cm}$$

Multi Concept

QUESTIONS

1. *A tank is 7 m long and 4 m wide. At what speed should water run through a pipe 5 cm broad and 4 cm deep, so that in 6 h and 18 min, water level in the tank rises by 4.5 m?*

(a) 12 km/h (b) 10 km/h (c) 14 km/h (d) 18 km/h

➥ (b) Volume of water flown into the tank

= Length of tank × Breadth of tank × Height of water rise

$= 7 \times 4 \times 4.5$

$= 126 \text{ m}^3$

$$\text{Time 6 h 18 m} = \left(6 + \frac{18}{60}\right)h$$
$$= \left(6 + \frac{3}{10}\right)h = \frac{63}{10} \text{ h}$$

Let the length of water flown through pipe be x m.

Then,

Volume of water flown through pipe = Volume of water flown in the tank

$$x \times \frac{5}{100} \times \frac{4}{100} = 126$$

$$\Rightarrow \quad x = \frac{126 \times 100 \times 100}{5 \times 4}$$
$$= 126 \times 5 \times 100 \text{ m}$$
$$= \frac{126 \times 5 \times 100}{1000} \text{ km}$$

$$\therefore \text{ Speed of water flowing} = \frac{\text{Length of water flown}}{\text{Time taken}}$$

$$= \frac{126 \times 5 \times 100}{1000 \times \frac{63}{10}} = \frac{126 \times 5}{63} = 10 \text{ km/h}$$

2. *A water supply tank as shown below with given dimensions is used in a house. 5 members use full tank for five days. If three members use the tank full, how long the tank works for proper supply?*

(a) 10.5 days (b) 12.5 days

(c) 15.5 days (d) 8 days

➥ (*b*) Water available in the tank

= Volume of cone + Volume of cylinder

$$W = \frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2$$

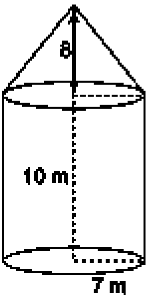
Here, $r = 7$

h_1 = Height of cone = 8

and h_2 = Height of cylinder = 10

$$= \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 8 + \frac{22}{7} \times (7)^2 \times 10$$

$$= \frac{1}{3} \times 22 \times 56 + 22 \times 7 \times 10$$



$$= 410.66 + 1540$$

$$= 1950.66 \text{ m}^3$$

∵ 5 member's use five days = 1950.66 m^3 water

∴ 5 member's use 1 day = 1950.66 / 5

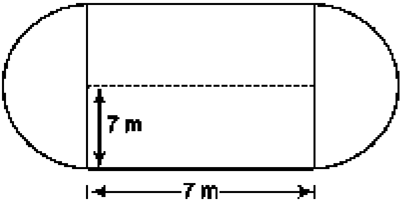
∴ 1 member use 1 day = 1950.66 / 5 × 5

∴ Daily use of 1 member = 78.0264 m^3

Number of days for two members $= \dfrac{1950.66}{78.0264 \times 2} = 12.5$ days

3. *A milk tank of following dimensions used to carry the milk to the milk booth. If* 50 *tanks used daily, then find the consumption of milk at milk booth.*

(a) 185766.66m^3 (b) 105775.66m^3 (c) 155766.66m^3 (d) 125766.66 m^3



➥ (*d*) Milk tank capacity = Volume of two hemisphere + Volume of cylinder

$$M = \frac{4}{3}\pi r^3 + \pi r^2 h$$
$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 + \frac{22}{7} \times (7)^2 \times 7$$
$$= \frac{4}{3} \times 22 \times 7 \times 7 + 22 \times 7^2$$
$$= 22 \times 7^2 \times \frac{7}{3} = 2515.33 \text{ m}^3$$

∴ Daily consumption of milk at booth

$$= (\text{Milk in one tank}) \times (\text{Number of tanks})$$
$$= 2515.33 \times 50 = 125766.66 \text{ m}^3$$

4. *Marbles of diameter 1.4 cm are dropped into a cylindrical beaker containing some water and are fully submerged. The diameter of the beaker is 7 cm. Find how many marbles have been dropped in it, if the water rises by 5.6 cm?*

(a) 50 (b) 150 (c) 500 (d) 550

➥ (*b*) ∵ Volume of raised water in cylinderical beaker

$$= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 5.6 = 215.6 \text{ cm}^3 \qquad \left[\text{here, } r = \frac{7}{2} \text{ and } h = 5.6\right]$$

and volume of a marble $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.7)^3 = \frac{4.312}{3} \text{ cm}^3 \qquad \left[r = \frac{1.4}{2} = 0.7\right]$$

Now,

Number of marbles × Volume of a marble = Volume of cylindrical beaker

$$\text{Number of marbles} = \frac{\text{Volume of cylindrical beaker}}{\text{Volume of marble}}$$

$$\therefore \text{ Number of marbles} = \frac{215.6 \times 3}{4.312} = 150$$

Fast Track Practice

Cube, Cuboid, Room or Box

1. The side of a cube measures 9 cm. What will be the diagonal of the cube (in cm)?

(a) 5^3 *(b)* 7V3 *(c)* 9V3 *(d)* 3V3 *(e)* None of the above

2. The diagonal of a cube is 2^3 cm. Find the surface area of the cube.

(a) 15 sq cm *(b)* 18 sq cm

(c) 25 sq cm *(d)* 24 sq cm

(e) None of the above

3. If the total surface area of a cube is 6 sq units, then what is the volume of the cube? [CDS 2013]

(a) 1 cu unit *(b)* 2 cu units

(c) 4 cu units *(d)* 6 cu units

4. If the volume of a cube is 729 cu cm, what is the length of its diagonal?

[CDS 2012]

(a) 9a/2 cm *(b)* 9V3 cm

(c) 18 cm *(d)* 18V3 cm

5. Find the surface area of a cuboid 10 m long, 5 m broad and 3 m high.

[Bank Clerks 2008]

(a) 105 sq m *(b)* 104 sq m

(c) 170 sq m *(d)* 190 sq m

(e) None of the above

6. The surface area of a cube is 726 sq cm. Find the volume of the cube.

[SSC (10+2) 2008]

(a) 1331 cm^3 *(b)* 1232 cm^3

(c) 1626 cm^3 *(d)* 1836 cm^3

7. The volume of a cuboid is 1989 cm , while its length and breadth are 17 cm and 13 cm, respectively. Find the height of the cuboid.

(a) 9 cm *(b)* 4 cm *(c)* 14 cm *(d)* 7 cm *(e)* None of the above

8. Three cubes of sides 1 cm, 6 cm and 8 cm are melted to form a new cube. Find half of the surface area of the new cube.

(a) 243 sq cm *(b)* 463 sq cm

(c) 486 sq cm *(d)* 293 sq cm

(e) None of the above

9. The maximum length of a pencil that can be kept in a rectangular box of dimensions 8 cm x 6 cm x 2 cm, is [SNAP 2012]

(a) 2VI3 cm *(b)* 2^14 cm

(c) 2-^26 cm fdJ10V2cm

10. Internal length, breadth and height of a rectangular box are 10 cm, 8 cm and 6 cm, respectively. How many boxes are needed which can be packed in a cube whose volume is 6240 cu. cm.? [CTET 2012]

(a) 12 *(b)* 13 *(c)* 15 *(d)* 17

11. A metal box measures 20 cm x 12 cm x 5 cm. Thickness of the metal is 1 cm. Find the volume of the metal required to make the box.

(a) 550 cm^3 *(b)* 656 cm^3

(c) 660 cm^3 *(d)* 475 cm^3

(e) None of the above

12. How many cubes of 3 cm edge can be cut out of a cube of 18 cm edge?

(a) 63 *(b)* 432

(c) 216 *(d)* None of these

13. A cube has each edge 2 cm and a cuboid is 1 cm long, 2 cm wide and 3 cm high. The paint in a certain container is sufficient to paint an area equal to 54 cm . Which one of the following is correct? [CDS 2014]

(a) Both cube and cuboid can be painted

(b) Only cube can be painted

(c) Only cuboid can be painted

(d) Neither cube nor cuboid can be painted

14. The whole surface area of a rectangular block is 8788 sq cm. If length, breadth and height are in the ratio of 4 : 3 : 2, then find the length. [SSC CCL 2008]

(a) 26 cm *(b)* 52 cm

(c) 104 cm *(d)* 13 cm

15. If the areas of three adjacent faces of a cuboidical box are 120 sq cm, 72 sq cm and 60 sq cm, respectively, then find the volume of the box.

(a) 820 cm^3 *(b)* 720 cm^3

(c) 750 cm^3 *(d)* 750 cm^3

(e) None of the above

16. The capacity of a cuboid tank of water is 50000 L. Find the breadth of the tank, if its length and depth are 2.5 m and 10 m, respectively.

(a) 2 m *(b)* 4 m

(c) 9 m *(d)* 6 m

(e) None of the above

17. The paint in certain container is sufficient to paint an area equal to 9.375 sq m. How many bricks of dimensions 22.5 cm x 10 cm x 7.5 cm can be painted out of this container?

(a) 100 *(b)* 200

(c) 50 *(d)* 175

(e) None of the above

18. What are the dimensions (length, breadth and height, respectively) of a cuboid with volume 720 cu cm, surface area 484 sq cm and the area of the base 72sqcm? [CDS 2012]

(a) 9, 8 and 10 cm

(b) 12, 6 and 10 cm

(c) 18, 4 and 10 cm

(d) 30, 2 and 12 cm

19. The volume of a cube is numerically equal to sum of its edges. What is the total surface area in square units?

[CDS 2012]

(a) 12 *(b)* 36 *(c)* 72 *(d)* 144

20. If each side of a cube is decreased by 19%, then decrease in surface area is

[Hotel Mgmt. 2010]

(a) 40% *(b)* 38.4%

(c) 35% *(d)* 34.39%

(e) None of the above

21. If the side of a cube is increased by 12%, by how much per cent does its volume increase?

(a) 40.4928% *(b)* 50.5240%

(C) 60.3292% *(d)* 30.4928%

(e) None of the above

Cylinder

22. Find the volume of a right circular cylinder of length 80 cm and diameter of the base 14 cm.

(a) 1400 cm^3 *(b)* 1553 cm^3

(c) 12320 cm^3 *(d)* 13320 cm^3 *(e)* None of the above

23. The diameter of the base of a right circular cylinder is 14 cm, while its length is 40 cm. Find the total surface area of the cylinder. [Bank PO 2007]

(a) 2068 sq cm *(b)* 1825 sq cm *(c)* 1925 sq cm *(d)* 2160 sq cm *(e)* None of the above

24. If the lateral surface area of a cylinder is 94.2 sq cm and its height is 5 cm, then find the radius of its base. (7t = 3.14)

[Hotel Mgmt. 2009] *(a)* 5 cm *(b)* 8 cm *(c)* 3 cm *(d)* 4 cm *(e)* None of the above

25. A rod of 2 cm diameter and 30 cm length is converted into a wire of 3 m length of uniform thickness. The diameter of the wire is [CLAT2013]

(a) 2/10 cm *(b)* 2 /VIO cm

(cJ1/VT0cm (a; 1/10 cm

26. What is the height of a solid cylinder of radius 5 cm and total surface area is 660 sqcm? [CDS 2012]

(a) 10 cm*(b)* 12 *cm(c)* 15 cm *(d)* 16 cm

27. What will be the curved surface area of a right circular cylinder having length 160 cm and radius of the base is 7 cm?

[Hotel Mgmt. 2010]

(a) 6020 sq cm *(b)* 5052 sq cm *(c)* 7045 sq cm *(d)* 7040 sq cm *(e)* None of the above

28. The ratio of the radii of two cylinders is 2 : 3 and the ratio of their heights is 5 : 3. The ratio of their volumes will be

[Railways 2007] *(a)* 4 : 9 *(b)* 9 : 4

(c) 20 : 27 *(d)* 27 : 20

29. The curved surface area of a cylindrical pillar is 264 sq m and its volume is 924 m . The ratio of its diameter to its height is

(a) 7 : 3 *(b)* 3 : 7 *(c)* 7 : 6 *(d)* 6 : 7 *(e)* None of the above

30. If height of cylinder is decreased by 8%, while its radius remains unchanged, by what per cent does the volume decrease?

(a) 4% *(b)* 6% *(c)* 8% *(d)* 10% *(e)* None of the above

31. If the radius of a cylinder is increased by 25% and its height remains unchanged, then find the per cent increase in volume. [Bank PO 2009]

(a) 56.25% *(b)* 52.25%

(c) 50.4% *(d)* 60.26%

(e) None of the above

32. If the radius of a cylinder is decreased by 8%, while its height is increased by 4%, what will be the effect on volume?

(a) 11.9744% (decrease)

(b) 11.9744% (increase)

(c) 12.4678% (decrease)

(d) 12.4678% (increase)

(e) None of the above

33. A drainage tile is a cylindrical shell 21 cm long. The inside and outside diameters are 4.5 cm and 5.1 cm, respectively. What is the volume of the clay required for the tile?

(a) 6.96tu cm^3 *(b)* 6.76tu cm^3

(c) 5.76ju cm^3 *(d)* None of these

34. The diameter of a roller is 84 cm and its length 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground (in sq m).

(a) 1632 *(b)* 1817

(c) 1532 *(d)* 1584

(e) None of the above

35. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of

painting the curved surface of the piller at the rate of ? 10 per sq m.

[Bank Clerks 2010] *(a)* ? 50 *(b)* ? 68

(c) f 98 *(d)* ? 55

(e) None of the above 36. A piller 14 cm in diameter is 5 m high. How much material was used to construct it? [Bank Clerks 2010]

(a) (77×10^2) cm^3 *(b)* (77×10^4) cm^3 *(c)* $(77 \times 10^5)cm^3$ *(d)* $(77 \times 10^3)cm^3$ *(e)* None of the above

Cone and Frustum of Cone

37. The curved surface area of a right circular cone of radius 14 cm is 440 sq cm. What is the slant height of the cone?

[CDS 2012]

(a) 10 cm *(b)* 11cm *(c)* 12 cm (d) 13 cm

38. The diameter of base of a right circular cone is 7 cm and slant height is 10 cm, then what is its lateral surface area?

[CDS 2012]

(a) 110 sq cm *(b)* 100 sq cm

(c) 70 sq cm *(d)* 49 sq cm

39. What is the whole surface area of a cone of base radius 7 cm and height 24 cm?

[CDS 2014]

(a) 654 sq cm *(b)* 704 sq cm

(c) 724 sq cm *(d)* 964 sq cm

40. What is the volume of a cone having a base of radius 10 cm and height 21 cm?

(a) 2200 cm^3 *(b)* 3000 cm^3

(c) 5600 cm^3 *(d)* 6600 cm^3

41. The diameter of the base of a cone is 6 cm and altitude is 4 cm. What is the approximate curved surface area of the cone?

(a) 45 cm^2 *(b)* 47 cm^2

(c) 49 cm^2 *(d)* 51 cm^2

42. The volume of a right circular cone is 100 *n* cm and its height is 12 cm. Find its slant height. [Bank PO 2008]

(a) 13 cm *(b)* 16 cm

(c) 9 cm *(d)* 26 cm

(e) None of the above

43. If the volume of a right circular cone of height 9 cm is 48 7t cm , then find the diameter of its base.

(a) 8 cm *(b)* 4 cm

(c) 7 cm (d) 11 cm

(e) None of the above

44. Shantanu's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 5 such caps.

(a) 5000 sq cm (b) 2750 sq cm (c) 3000 sq cm (d) 2700 sq cm (e) None of the above

45. The radius of the base of a right circular cone is doubled. To keep the volume fixed, the height of the cone will be

[SSC (10+2) 2012]

(a) half of the previous height
(b) one-third of the previous height
(c) one-fourth of the previous height
(d) $\frac{1}{\sqrt{2}}$ times of the previous height

46. **The diameter of a right circular cone is 14 m, while its slant height is 9 m. Find the volume of the cone.**

(a) $\frac{49\pi\sqrt{32}}{3}$ m^3 (b) $\frac{50\pi\sqrt{32}}{3}$ m^3

(c) $\frac{3}{49\pi\sqrt{32}}$ m^3 (d) $\frac{\pi\sqrt{32}}{9}$ m^3

(e) None of the above

47. The frustum of a right circular cone has the diameters of base 10 cm, of top 6 cm and a height of 5 cm. Find its slant height.

(a) i/29 cm (b) 3V3 cm

(c) VT3 cm (d) 4V3 cm

(e) None of the above

48. The frustum of a right circular cone has the radii of base 4 cm, of the top 2 cm and a height of 6 cm. Find the volume of the frustum.

(a) 115 cm^3 (b) 156 cm^3

(c) 185 cm^3 *(d)* 176 cm^3

(e) None of the above

49. A cone of radius *r* cm and height *h* cm is divided into two parts by drawing a plane through the middle point of its height and parallel to the base. What is the ratio of the volume of the original cone to the volume of the smaller cone? [CDS 2014]

(a) 4:1 *(b)* 8:1

(c) 2:1 *(d)* 6:1

50. If the ratio of volumes of two cones is 2 : 3 and the ratio of the radii of their bases is 1 : 2, then the ratio of their heights will be

(a) 3 : 8 *(b)* 8 : 3

(c) 9 : 2 *(d)* 8 : 1

(e) None of the above

51. The ratio of the radius and height of a

2 ^ cone is 5 : 12. Its volume is 314— cm . Its

7 slant height is *(a)* 18 cm *(b)* 13 cm

(c) 16 cm *(d)* 15 cm

(e) None of the above

52. If the height of the right circular cone is increased by 200% and the radius of the base is reduced by 50%, then the volume of the cone

(a) increases by 25%

(b) increases by 50%

(c) remains unchanged

(d) decreases by 25%

(e) None of the above

53. If the volumes of two right circular cones are in the ratio 1 : 3 and their diameters are in the ratio 3 : 5, then the ratio of their heights is [SSC CPO 2013]

(a) 25 : 27 *(b)* 1 : 5

(c) 3 : 5 *(d)* 5 : 27

54. The diameters of two cones are equal. If their slant heights be in the ratio of 5 : 7, then find the ratio of their curved surface areas. [SSC CPO 2007]

(a) 25 : 7 *(b)* 25 : 49

(c) 5 : 49 *(d)* 5 : 7

55. A conical cap has the base diameter 24 cm and height 16 cm. What is the cost of painting the surface of the cap at the rate of 70 paise per sq cm? [CDS 2013]

(a) ? 520 *(b)* ? 524 *(c)* ? 528 *(d)* *X* 532

56. The curved surface area and the total surface area of a cylinder are in the ratio 1 : 2. If the total surface area of the right cylinder is 616 cm , then its volume is

[SSC CCL 2013] *(a)* 1632 cm^3 *(b)* 1078 cm^3

(c) 1232 cm^3 *(d)* 1848 cm^3

Directions (Q. Nos. 57-58) *The areas of the ends of a frustum of a pyramid are P and Q where P <Q and H is its thickness.*

[CDS 2012]

57. What is the difference in radii of the ends of the frustum?

(a) $\frac{\sqrt{Q}-\sqrt{P}}{\sqrt{\pi}}$ (b) $\frac{\sqrt{Q}-\sqrt{P}}{\pi}$

(c) $\sqrt{Q}-\sqrt{P}$ (d) None of these

58. What is the volume of the frustum?

(a) $3H(P+Q+\sqrt{PQ})$

(b) $H(P+Q+\sqrt{PQ})$

(c) $H(P+Q+\sqrt{PQ})/3$

(d) $H(P+Q-\sqrt{PQ})/3$

59. The radius of the base of a right circular cone is increased by 15% keeping the height fixed. The volume of the cone will be increased by [CDS 2012]

(a) 30% *(b)* 31% *(c)* 32.25% *(d)* 34.75%

Sphere

60. What will be the surface area of the sphere having 4 cm radius? [RRB2010]

(a) 64ti sq cm *(b)* 69tu sq cm

(c) 32 *n* sq cm *(d)* 35*n* sq cm

61. What is the diameter of the largest circle lying on the surface of a sphere of surface area 616 sq cm? [CDS 2014]

(a) 14 cm *(b)* 10.5 cm

(c) 7 cm *(d)* 3.5 cm

62. The diameter of the Moon is approximately one-fourth of the diameter of the Earth. What is the ratio (approximate) of their volumes?

(a) 4:16 (b)1:64 *(c)VA* (001:128

63. A sphere and a hemisphere have the same surface area. The ratio of their volumes is [SSC (10+2) 2012]

(a) $\frac{\sqrt{3}}{4}:1$ *(b)* $\frac{3\sqrt{3}}{4}:1$

(c) $\frac{\sqrt{3}}{8}:1$ *(d)* $\frac{3\sqrt{3}}{8}:1$

64. A hemisphere has 3 cm radius. Calculate its volume.

(a) $\frac{4}{3}\pi\ cm^3$ *(b)* $13\pi\ cm^3$

(c) $18\pi\ cm^3$ *(d)* $\frac{19}{\pi}\ cm^3$

(e) None of the above

65. Find the number of lead balls of diameter 2 cm each, that can be made from a sphere of diameter 16 cm.

(a) 2048 *(b)* 2055 *(c)* 2058 *(d)* 2085 *(e)* None of the above

66. A hemispherical bowl has 3.5 cm radius. It is to be painted inside as well as outside. Find the cost of painting it at the rate of ? 5 per 10 sq cm. [Bank Clerks 2010]

(a) ? 50 *(b) X* 81 *(C) X* 56 *(d)* ? 77 *(e)* None of the above

67. If the surface area of a sphere is 616 sq cm, what is its volume? [CDS 2012]

(a) 4312/3 cm^3 *(b)* 4102/3 cm^3 *(c)* 1257 cm^3 (d) 1023 cm^3

68. If the ratio of the diameters of two spheres is 3 : 5, then what is the ratio of their surface areas? [CDS 2012]

(a) 9 : 25 *(b)* 9 : 10

(c) 3 : 5 *(d)* 27 : 125

69. If 64 identical small spheres are made out of a big sphere of diameter 8 cm, what is surface area of each small sphere?

(a) n cm^2 *(b)* 2 *n* cm^2

(c) 4it cm^2 *(d)* 8ti cm^2

70. A hemisphere has 28 cm diameter. Find its curved surface area. [SSC CGL 2009]

(a) 1232 sq cm *(b)* 1236 sq cm *(c)* 1238 sq cm *(d)* 1233 sq cm

71. What will be the difference between total surface area and curved surface area of a hemisphere having 2 cm diameter?

(a) 2it sq cm *(b) 3n* sq cm

(c) n sq cm *(d) An* sq cm

(e) None of the above

72. A metallic sphere of radius 12 cm is melted into three smaller spheres. If the radii of two smaller spheres are 6 cm and 8 cm, the radius of the third is

(a) 14 cm *(b)* 16 cm

(c) 10 cm ft/) 12 cm

73. If the radius of a sphere is increased by 3%, then what per cent increase takes place in surface area of the sphere?

[Bank Clerks 2010]

(a) 6.09% *(b) 7%*

(c) 5.06% *(d)* 9%

(e) None of the above

74. If radius of a sphere is decreased by 24%, by what per cent does its surface area decrease? [Bank Clerks 2011]

(a) 44% *(b)* 49% *(c)* 42.24% *(d)* 46.2% *(e)* None of the above

75. If the radius of a sphere is increased by 100%, by what per cent does its volume increase?

(a) 300% *(b)* 900%

(c) 500% *(d)* 700%

(e) None of the above

76. Weight of a solid metallic sphere of radius 4 cm is 4 kg. The weight of a hollow sphere made with same metal, whose outer diameter is 16 cm and inner diameter is 12 cm, is

(a) 20.5 kg *(b)* 15.5 kg

(c) 16.5 kg *(d)* 18.5 kg

Prism and Pyramid

77. K prism has the base a right angled triangle whose sides adjacent to the right angle are 10 cm and 12 cm long. The height of the prism is 20 cm. The density of the material of the prism is 6 g/cu cm. The weight of the prism is

[SSC (10+2) 2012]

(a) 3.4 kg *(b)* 4.8 kg

(c) 6.4 kg *(d)* 7.2 kg

78. The base of a right prism is a right angled isosceles triangle whose hypotenuse is a cm. If the height of the prism is *h* cm, then its volume is

[SSC (10+2) 2012]

(a) $\frac{a^2h}{4}$ cm^3 *(b)* $\frac{a^2h}{6}$ cm^3

(c) $\frac{a^2h}{8}$ cm^3 *(d)* $\frac{a^2h}{12}$ cm^3

79. The perimeter of the triangular base of a right prism is 60 cm and the sides of the base are in the ratio 5 : 12 : 13. Then, its volume will be (height of the prism being 50 cm)

(a) 6000 cm^3 *(b)* 6600 cm^3

(c) 5400 cm^3 *(d)* 9600 cm^3

80. Find the total surface area of a pyramid having a slant height of 8 cm and a base which is a square of side 4 cm (in cm)?

(a) 80 *(b)* 64

(c) 72 *(d)* 84

81. A prism and a pyramid have the same base and the same height. Find the ratio of the volumes of the prism and the pyramid.

(a) 1:1 *(b)* 1: 3 *(c)* 3 :1

(d) Cannot be determined

Miscellaneous

82. The base of a cone and a cylinder have the same radius 6 cm. They have also the same height 8 cm. The ratio of the curved surfaces of the cylinder to that of the cone is [SSC (10+2)2012]

(a) 4 : 3 *(b)* 5 : 3 *(c)* 8 : 5 *(d)* 8 : 3

83. A solid metallic cylinder of base radius 3 cm and height 5 cm is melted to form cones, each of height 1 cm and base radius 1 mm. The number of cones is

[SSCFCI 2012] *(a)* 7500 *(b)* 13500(cJ 3500 *(d)* 4500

84. How many spherical bullets can be made out of a solid cube whose edge measures 44 cm, each bullet being 4 cm in diameter?

(a) 2550 *(b)* 2541 *(c)* 2500 *(d)* 2575

85. A solid consists of circular cylinder with exact fitting right circular cone placed on the top. The height of the cone is *h*. If total volume of the solid is three times the volume of the cone, then the height of the circular cylinder is [SSC CCL 2012]

(a) $2h$ (b) $\frac{2h}{3}$ (c) $4h$ (d) $\frac{3h}{2}$

86. The volumes of a sphere and a right circular cylinder having the same radius are equal. The ratio of the diameter of the sphere to the height of the cylinder is

[SSC (10+2) 2007]

(a) 1 : 2 *(b)* 2 : 1 *(c)* 2 : 3 *(d)* 3 : 2

87. Water flows at the rate of 10 m/min from a cylindrical pipe 5 mm in diameter. How long will it take to fill up a conical vessel whose diameter at the base is 40 cm and depth is 24 cm?

(a) 51 min 12 s *(b)* 52 min 1 s *(c)* 48 min 15 s *(d)* 55 min

88. A cylindrical jar, whose base has a radius of 15 cm, is filled with water upto a height of 20 cm. A solid iron spherical ball of radius 10 cm is dropped in the jar to submerge completely in water. Find the increase in the level of water (in cm).

[SSCCPO2013]

(a) $5\frac{17}{27}$ *(b)* $5\frac{5}{7}$ *(c)* $5\frac{8}{9}$ *(d)* $5\frac{25}{27}$

89. A sphere of radius *r* has the same volume as that of a cone with a circular base of radius *r*. Find the height of the cone. [Bank Clerks 2007]

(a) $2r$ *(b)* $\frac{r}{3}$ *(c)* $4r$ *(d)* $\left(\frac{2}{3}\right)r$

(e) None of the above

90. The radii of a sphere and a right circular cylinder are equal and their curved surface areas are also equal. The ratio of their volumes is

(a) 3 : 4 *(b)* 2 : 3

(c) 3 : 2 *(d)* 4 : 3

The volume of a cuboid is twice the volume of a cube. If the dimensions of the cuboid are 9 cm, 8 cm and 6 cm, then total surface area of the cube will be *(a)* 213 sq cm *(b)* 216 sq cm

(c) 218 sq cm *(d)* 219 sq cm

(e) None of the above

92. Let A be a pyramid on a square base and B be a cube. It o, 6 and c denote the number of edges, number of faces and number of corners, respectively. Then, the result a = $b + c$ is true for [CDS 2013]

(a) only *A* *(b)* only *B*

(c) both *A* and *B* *(d)* neither *A* nor S

93. A conical flask is full of water. The flask has base radius r and height h. This water is poured into a cylindrical flask of base radius mr. The height of water in the cylindrical flask is

(a) $\frac{h}{2}m^2$ (b) $\frac{2h}{m}$ (c) $\frac{h}{3m^2}$ (d) $\frac{m}{2h}$

94. Seven equal cubes each of side 5 cm are joined end-to-end. Find the surface area of the resulting cuboid. [Bank PO 2007]

[a) 750 sq cm *(b)* 1500 sq cm

(c) 2250 sq cm *(d)* 700 sq cm *(e)* None of the above

95. A hemispherical basin of 150 cm diameter holds water 120 times as much as a cylindrical tube. If the height of the tube is 15 cm, then the diameter of the tube is [SSC CCL 2008]

(a) 27 cm *(b)* 24 cm

(c) 25 cm *(d)* 26 cm

96. A well of inner diameter 14 m is dug to a depth of 15 m. Earth taken out of it has been evenly spread all around it to a width of 7 m to form an embankment. Find The height of embankment so formed. [DMRC2012]

(a) 7 m *(b)* 5 m

(c) 14 m *(d)* None

97. In a shower, 10 cm of rain falls. What will be the volume of water that falls on 1 hect area of ground? [Delhi Police SI 2008]

(a) 500 cm^3 *(b)* 650 cm^3

(c) 1000 cm^3 *(d)* 750 cm^3

98. A large solid metallic cylinder whose radius and height are equal to each other is to be melted and 48 identical solid balls are to be recast from the liquid metal so formed. What is the ratio of the radius of a ball to the radius of the cylinder? [CDS 2012]

(a) 1 : 16 *(b)* 1 : 12

(c) 1 : 8 (d) 1 : 4

99. What is the volume of the largest sphere that can be curved out of a cube of edge 3 cm? [CDS 2012]

(a) 9tc cm^3 *(b)* 6tc cm^3

(c) 4.571 cm^3 *(d)* 37i cm^3

100.A right circular metal cone (solid) is 8 cm high and the radius is 2 cm. It is melted and recast into a sphere. What is the radius of the sphere? [CDS 2012] *(a)* 2 cm *(b)* 3 cm

(c) 4 cm *(d)* 5 cm

101. Let the largest possible right circular cone and largest possible sphere be fitted into two cubes of same length. Let C and *S* denote the volume of cone and volume of sphere respectively, then which one of the following is correct?

[CDS 2012] *(a)* C = 2S *(b)S*= 2C

(c)C=S (d) C = 3S

102.10 circular plates each of thickness 3 cm, each are placed one above the other and a hemisphere of radius 6 cm is placed on the top just to cover the cylindrical solid. What is the volume of the solid so formed? [CDS 2012]

(a) 2647t cm^3 *(b)* 25271 cm^3

(c) 236tc cm^3 *(d)* None of these

103.A cylindrical box of radius 5 cm contains 10 solid spherical balls, each of radius 5 cm. If the top most ball touches the upper cover of the box, then volume of the empty space in the box is [SSC CPO 2007]

(a) $\frac{2500}{3}\pi\ cm^3$ (b) $5000\ \pi\ cm^3$

(c) $2500\ \pi\ cm^3$ (d) $\frac{5000}{3}\pi\ cm^3$

(e) None of the above

104. A cone has height which is half of 16.8 cm, while diameter of its base is 4.2 cm. It is melted and recast into a sphere. Find the surface area of the sphere. *(a)* 55.44 sq cm *(b)* 60 sq cm *(c)* 65.58 sq cm *(d)* 59 sq cm *(e)* None of the above

105.A hospital room is to accommodate 56 patients. It should be done in such a way that every patient gets 22 m of floor and 8.8 m of space. If the length of the room is 14 m, then breadth and height of the room are respectively *(a)* 8.8 m, 4 m *(b)* 8.4 m, 4.2 m

(c) 8 m, 4 m *(d)* 7.8 m, 4.2 m

106. Water flows into a tank 100 m x 150 m x 2 m through a rectangular pipe 1.5 m x 1.25 m at a speed of 20 km/h. In what time, will the water rise by 2 m? *(a)* 96 min *(b)* 50 min

(c) 48 min *(d)* 75 min

(e) None of the above

107.What part of a ditch 48 m long, 16.5 m broad and 4 m deep can be filled by the earth got by digging a cylindrical tunnel of diameter 4 m and length 56 m? [SSC CCL 2007]

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$

(c) $\frac{7}{9}$ (d) $\frac{8}{9}$

108. Find the area of the iron sheet required to prepare a cone double the height of 12 cm with diameter of the base 14 cm.

(a) 504 sq cm *(b)* 804 sq cm

(c) 304 sq cm *(d)* 704 sq cm *(e)* None of the above

109.A copper sphere of diameter 36 cm is drawn into a wire of diameter 4 mm. Find the length of the wire. *(a)* 2000 m *(b)* 1500 m

(c) 1855 m *(d)* 1944 m

(e) None of the above

110. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of ? 10 per sq m is ? 15000, then find the height of the hall. [Hotel Mgmt. 2010]

(a) 8 m *(b)* 6 m

(c) 9 m *(d)* 7 m

(e) None of the above

111.A cylinder is surmounted by a cone at one end, a hemisphere at the other end. The common radius is 3.5 cm, the height of the cylinder is 6.5 cm and the total height of the structure is 12.8 cm. The volume V of the structure lies between [CDS 2014]

(a) 370 cm^3 and 380 cm^3

(b) 380 cm^3 and 390 cm^3

(c) 390 cm^3 and 400 cm^3

(d) None of the above

Answer with Solutions

Cube, Cuboid, Room or Box

1. (c) Given, side of cube, $(a) = 9$

$\therefore$ Diagonal of cube $= a\sqrt{3} = 9\sqrt{3}$

2. (d) Given, diagonal $= 2\sqrt{3}$

$\Rightarrow \quad a\sqrt{3} = 2\sqrt{3}$ [a = edge of the cube]

$\therefore \quad a = 2$

$\therefore$ Required surface area $= 6a^2$

$$= 6 \times 2^2 = 24 \text{ sq cm}$$

3. (a) Total surface area of a cube $= 6a^2$

$\Rightarrow \quad 6 = 6a^2 \Rightarrow a^2 = 1$

$\therefore \quad a = 1$

Now, volume of the cube $= a^3 = 1^3 = 1$ cu unit

4. (b) Volume of cube $= (\text{Side})^3$

$\therefore \quad 729 = a^3 \Rightarrow a = 9$ cm

$\therefore$ Diagonal of cube $= \text{Side} \times \sqrt{3} = 9 \times \sqrt{3}$

$$= 9\sqrt{3} \text{ cm}$$

5. (d) Given that, $l = 10$ m, $b = 5$ m, $h = 3$ m

$lb = 10 \times 5 = 50$, $bh = 5 \times 3 = 15$,

$lh = 10 \times 3 = 30$

$\therefore$ Surface area of a cuboid

$$= 2(lb + bh + lh)$$
$$= 2(50 + 15 + 30)$$
$$= 2 \times 95 = 190 \text{ sq m}$$

6. (a) According to the question,

$6a^2 = 726$ [a = edge of the cube]

$\Rightarrow \quad a^2 = \dfrac{726}{6} = 121$

$\therefore \quad a = \sqrt{121} = 11$ cm

$\therefore$ Required volume

$$= a^3 = 11^3 = 1331 \text{ cm}^3$$

7. (a) Volume $= 1989$

$\Rightarrow \quad lbh = 1989 \Rightarrow 17 \times 13 \times h = 1989$

$\therefore \quad h = \dfrac{1989}{17 \times 13} = 9$ cm

8. (a) Volume (new cube) $= (1^3 + 6^3 + 8^3)$

$$= 729 \text{ cm}^3$$
$$a^3 = 729$$
$$a = \sqrt[3]{729} \Rightarrow a = 9 \text{ cm}$$

$\therefore$ Surface area of the new cube $= 6a^2$

$$= 6 \times 9^2 = 486 \text{ sq cm}$$

$$\frac{\text{Surface area}}{2} = \frac{486}{2} = 243 \text{ sq cm}$$

9. (*c*) Length of largest pencil that can be kept in a box

$= \text{Diagonal of box} = \sqrt{l^2 + b^2 + h^2}$

where, $l = 8$ cm, $b = 6$ cm, $h = 2$ cm

$= \sqrt{64 + 36 + 4}$

$= \sqrt{104} = 2\sqrt{26}$ cm

10. (*b*) Volume of rectangular box

$= 10 \times 8 \times 6 = 480 \text{ cm}^3$

Volume of cubes $= 6240 \text{ cm}^3$

$\therefore$ Required boxes

$$= \frac{\text{Volume of cubes}}{\text{Volume of rectangular box}}$$

$$= \frac{6240}{480} = 13$$

Hence, 13 boxes are needed.

11. (*c*) External volume

$= 20 \times 12 \times 5 = 1200 \text{ cm}^3$

Internal volume

$= (20 - 2) \times (12 - 2) \times (5 - 2)$

$= 18 \times 10 \times 3 = 540 \text{ cm}^3$

$\therefore$ Volume of the metal = External volume − Internal volume

$= 1200 - 540 = 660 \text{ cm}^3$

12. (*c*) Let the number of cubes that can be cut from bigger cube = n. Then,

$n \times$ Volume of smaller cube = Volume of bigger cube

Edge of smaller cube = 3 cm

Edge of bigger cube = 18 cm

$n \times (3)^3 = (18)^3$

$\Rightarrow \quad n = \dfrac{18 \times 18 \times 18}{3 \times 3 \times 3}$

$\therefore \quad n = 6 \times 6 \times 6 = 216$ cubes

13. (a) Surface area of cube which can be painted = 6 $(\text{Side})^2 = 6(2)^2 = 24 \text{ cm}^2$ Now, surface area of cuboid which can be painted

= *2(lb+ bh* + In)

= 2(2 + 6 + 3) = 22 cm' Total surface area of both cube and cuboid

$= 22 + 24 = 46 \text{ cm}^2 < 54 \text{ cm}^2$ Therefore, both cube and cuboid can be painted.

14. (*b*) Let length, breadth and height be $4x, 3x$ and $2x$, respectively.

Whole surface area $= 2(lb + bh + lh)$

$$\Rightarrow \quad (lb + bh + lh) = \frac{8788}{2} = 4394$$

$$\Rightarrow \quad (4 \times 3 + 3 \times 2 + 2 \times 4)\, x^2 = 4394$$

$$\Rightarrow \quad 26x^2 = 4394$$

$$\Rightarrow \quad x^2 = 169$$

$$\Rightarrow \quad x = 13$$

$\therefore$ Length $= 4x = 4 \times 13 = 52$ cm

15. (*b*) Given, $lb = 120$ sq cm., $bh = 72$ sq cm, $lh = 60$ sq cm.

$$\therefore \quad lb \times bh \times lh = 120 \times 72 \times 60$$

$$\Rightarrow \quad (lbh)^2 = 120 \times 72 \times 60$$

$$\Rightarrow \quad lbh = \sqrt{120 \times 72 \times 60}$$

$$= \sqrt{12 \times 10 \times 12 \times 6 \times 10 \times 6}$$

$$= 12 \times 10 \times 6 = 720 \text{ cm}^3$$

16. (*a*) Capacity of tank $= 50000$ L $= 50$ m^3

$$= \left[\therefore 1\text{L} = \frac{1}{1000}\text{m}^3\right]$$

$$\therefore \text{ Breadth} = \frac{50}{2.5 \times 10} = 2 \text{ m}$$

17. (*a*) Surface area of 1 brick

$$= 2\,(lb + bh + lh)$$

$$= 2(22.5 \times 10 + 10 \times 7.5 + 7.5 \times 22.5)$$

$$= 2\,(225 + 75 + 168.75)$$

$$= 2 \times 468.75 = 937.50 \text{ cm}^2$$

$$= \frac{937.50}{100 \times 100} = 0.09375 \text{ sq m}$$

$$\therefore \text{ Number of bricks} = \frac{\text{Total area}}{\text{Surface area of 1 brick}}$$

$$= \frac{9.375}{0.09375} = 100$$

18. (*a*) Volume of the cuboid $= 720$ cm^3

Height of the cuboid

$$= \frac{\text{Volume of the cuboid}}{\text{Base area of the cuboid}}$$

$= \frac{720}{72} = 10$ cm [By Hit and Trial]

Surface area of the cuboid

$= 2\,(lb + bh + hl)$

$= 2\,(9 \times 8 + 8 \times 10 + 10 \times 9)$

$= 2\,(72 + 80 + 90)$

$= 2 \times 242$

$= 484\ \text{cm}^2$

∴ It is obvious that length, breadth and height of the cuboid is 9 cm, 8 cm and 10 cm.

19. (*c*) Let the edge of a square be *x*. and sum of its edges $= 12x$

Now, by condition, $x^3 = 12x$

$\Rightarrow \quad x(x^2 - 12) = 0$

$\Rightarrow \quad x^2 = 12 \quad [\because x \neq 0] \quad \ldots(\text{i})$

∴ Its total surface area

$= 6x^2 = 6(12) = 72$ sq units

20. (*d*) Here, $x = y = -19\%$

According to the formula,

[by Technique 7]

Percentage decrease in surface area

$$= \left[x + y + \frac{xy}{100}\right]\%$$

$$= \left[-19 - 19 + \frac{(-19) \times (-19)}{100}\right]\%$$

$$= \left[-38 + \frac{361}{100}\right]\%$$

$= [-38 + 3.61]\% = -34.39\%$

21. (*a*) Here, $x = 12\%$

According to the formula,

[by Technique 2]

Percentage increase in volume

$$= \left[\left(1 + \frac{x}{100}\right)^3 - 1\right] \times 100\%$$

$$= \left[\left(1 + \frac{12}{100}\right)^3 - 1\right] \times 100\%$$

$= [(1.12)^3 - 1] \times 100\%$

$= 0.404928 \times 100\% = 40.4928\%$

Cylinder

22. (C) Given, r = 7 cm, *h* = 80 cm

Volume = *nr'h* = — X7X7X80 7

$= 12320\ cm^3$

23. (a) Total surface area of cylinder

$= 2\pi r(h + r)$

14 Given that, $r = — = 7$ cm, $h = 40$ cm 2

$\therefore$ Required total surface area

$$= 2 \times \frac{22}{7} \times 7 \times (40 + 7)$$

$$= 44 \times 47 = 2068 \text{ sq cm}$$

24. (c) Given, lateral surface area = 94.2 sq cm

$$\Rightarrow \quad 2\pi rh = 94.2$$

$$\therefore \quad r = \frac{94.2}{2\pi h} = \frac{94.2}{2 \times 3.14 \times 5} = 3 \text{ cm}$$

25. (*b*) Given,

$r_1 = 1$ cm, $h_1 = 30$ cm, $h_2 = 300$ cm.

Volume of rod = Volume of wire

$$\Rightarrow \quad \pi r_1^2 h_1 = \pi r_2^2 h_2$$

$$\Rightarrow \quad \pi \times (1)^2 \times 30 = \pi \times r_2^2 \times 300$$

$$\Rightarrow \quad r_2^2 = \frac{30}{300}$$

$$\therefore \quad r_2 = \frac{1}{\sqrt{10}} \text{ cm}$$

$\therefore$ diameter $= 2r_2$

$$= 2 \times \frac{1}{\sqrt{10}} = \frac{2}{\sqrt{10}} \text{ cm.}$$

26. (*d*) Let the height and radius of solid cylinder be h and r cm respectively.

Given that, radius $(r) = 5$ cm

and total surface area $= 660 \text{ cm}^2$

$$\Rightarrow \quad 2\pi rh + 2\pi r^2 = 660$$

$$\Rightarrow \quad 2\pi r(h + r) = 660$$

$$\Rightarrow \quad (h+5) = \frac{330}{5\pi} = \frac{330}{5} \times \frac{7}{22}$$

$$\Rightarrow \quad h = \frac{66 \times 7}{22} - 5 = 21 - 5$$

$\therefore$ Required height = 16 cm

27. (*d*) Curved surface area of the cylinder

$$= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 160$$

$$= 7040 \text{ sq cm}$$

28. (*c*) Let radii be $2r$ and $3r$ and heights be $5h$ and $3h$.

$$\therefore \text{ Ratio of volumes} = \frac{\pi (2r)^2 \times 5h}{\pi (3r)^2 \times 3h}$$

$$= \frac{20}{27} = 20 : 27$$

29. (*a*) Given that, $2\pi rh = 264$

and $\quad \pi r^2 h = 924$

$$\therefore \quad \frac{\pi r^2 h}{2\pi rh} = \frac{924}{264} = \frac{7}{2} \Rightarrow \frac{r}{2} = \frac{7}{2} \Rightarrow r = 7$$

$$\therefore \quad d = 2r = 14 \text{ m}$$

Also, $2 \times \frac{22}{7} \times 7 \times h = 264$

$$\Rightarrow \quad h = \frac{264}{44} = 6$$

$$\therefore \quad \frac{d}{h} = \frac{14}{6} = \frac{7}{3} = 7:3$$

30. (C) According to the formula,

[by Technique 3] Percentage decrease in volume = 8%

31. (a) According to the formula,

[by Technique 4]

Percentage increase in volume

$$= \left(2x + \frac{x^2}{100}\right)\% = \left(2 \times 25 + \frac{25^2}{100}\right)\%$$

$$= \left(50 + \frac{625}{100}\right)\% = (50 + 6.25)\%$$

$$= 56.25\%$$

32. (*a*) Here, $x = -8\%$, $y = 4\%$

According to the formula,

[by Technique 5]

Net effect on volume

$$= \left[2x + y + \frac{x^2 + 2xy}{100} + \frac{x^2 y}{100^2}\right]\%$$

$$= \left[2 \times (-8) + 4 + \frac{(-8)^2 + 2 \times (-8) \times 4}{100} + \frac{(-8)^2 \times 4}{100^2}\right]\%$$

$$= \left[-16 + 4 + \frac{64 - 64}{100} + \frac{256}{10^4}\right]\%$$

$$= [-12 + 0 + 0.0256]\%$$

$$= -11.9744\% \text{ (decrease)}$$

33. (*d*) Volume of clay required

$$= \pi\left[\left(\frac{5.1}{2}\right)^2 - \left(\frac{4.5}{2}\right)^2\right] \times 21$$

$$= \pi[(2.55)^2 - (2.25)^2] \times 21$$

$$= \pi\,(0.3 \times 4.8) \times 21 = 30.24\,\pi \text{ cm}^3$$

34. (*d*) In one revolution,

Area covered = Curved surface area

$$\Rightarrow \quad 2\pi rh = 2 \times \frac{22}{7} \times 42 \times 120$$

$$= 31680 \text{ sq cm}$$

In 500 revolutions,

Area covered $= 31680 \times 500$

$$= (1584 \times 10^4) \text{ sq cm}$$

$$= \frac{1584 \times 10^4}{10^4} = 1584 \text{ sq m}$$

35. (*d*) Curved surface area $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 0.25 \times 3.5$$

$$= 5.5 \text{ sq m}$$

$\therefore$ Cost of painting 5.5 sq m $= 10 \times 5.5$

$= ₹\, 55$

36. (*d*) Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 500 = 77000$$

$$= (77 \times 10^3) \text{ cm}^3$$

Cone and Frustum of Cone

37. (*a*) Curved surface area of right circular cone = $\pi r l$

$$\therefore\ 440 = \frac{22}{7} \times 14 \times l$$

$$\Rightarrow \qquad l = \frac{440 \times 7}{22 \times 14} = 10 \text{ cm}$$

38. (*a*) Given that,

Diameter of a right circular cone = 7 cm

$\therefore$ Radius of a right circular cone $= \frac{7}{2}$ cm

and slant height of a right circular cone (l)

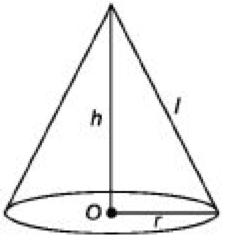
$= 10$ cm

$\therefore$ Lateral surface area of a cone $= \pi r l$

$$= \frac{22}{7} \times \frac{7}{2} \times 10$$

$$= 11 \times 10 = 110 \text{ cm}^2$$

39. (*b*) Slant height, $l = \sqrt{h^2 + r^2}$



$$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49}$$

$$= \sqrt{625} = 25$$

Total surface area $= \pi r (l + r)$

$$= \frac{22}{7} \times 7(25 + 7) \quad = \frac{22}{7} \times 7 \times 32$$

$= 704$ sq cm

40. (*a*) Volume of cone $= \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times 10 \times 21 = 2200 \text{ cm}^3$$

41. (*b*) Radius of cone (r) $= \frac{6}{2} = 3$ cm

and height of cone (h) = 4 cm

Slant height (l) $= \sqrt{h^2 + r^2}$

$$= \sqrt{(4)^2 + (3)^2}$$

$$= \sqrt{16 + 9}$$

$= 5$ cm

Now, curved surface area $= \pi r l$

$$= \frac{22}{7} \times 3 \times 5$$

$$= \frac{330}{7} \approx 47 \text{ cm}^2$$

42. (*a*) Volume $= \frac{1}{3}\pi r^2 h$

According to the question,

$$\frac{1}{3}\pi r^2 h = 100\pi \Rightarrow \frac{1}{3}\pi r^2 \times 12 = 100\pi$$

$$\Rightarrow \quad r^2 = 25$$

$$\therefore \quad r = \sqrt{25} = 5 \text{ cm}$$

$\therefore$ Slant height $(l) = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2}$

$$= \sqrt{169} = 13 \text{ cm}$$

43. (*a*) Volume $= 48\pi \text{ cm}^3$ and $h = 9$ cm

$$\therefore \quad \frac{1}{3}\pi r^2 h = 48\pi$$

$$\Rightarrow \quad \frac{1}{3} \times \pi \times r^2 \times 9 = 48\pi$$

$$\therefore \quad r^2 = \frac{48\pi \times 3}{\pi \times 9} = 16$$

$$\therefore \quad r = \sqrt{16} = 4 \text{ cm}$$

$\therefore$ Diameter $= 2r = 2 \times 4 = 8$ cm

44. (*b*) Slant height $(l) = \sqrt{r^2 + h^2}$

$$= \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25 \text{ cm}$$

Curved surface area $= \pi r l = \frac{22}{7} \times 7 \times 25$

$$= 550 \text{ sq cm}$$

$\therefore$ Area of 5 caps $= 550 \times 5 = 2750$ sq cm

45. (*c*) In first situation,

Radius $= r_1$, height $= h_1$ and volume $= v_1$

In second situation,

Radius $= 2r_1$, height $= h_2$ and volume $= v_2$

In the volume of fixed, then

$$v_1 = v_2$$

$$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3}\pi (2r_1)^2 h_2$$

$$\Rightarrow \quad h_1 = 4h_2$$

$$\therefore \quad h_2 = \frac{h_1}{4}$$

Therefore, height of the cone will be one-fourth of the previous height.

46. (*a*) Given, $l = 9$ m

and diameter $= 14$ m $\Rightarrow r = \frac{14}{2} = 7$ m

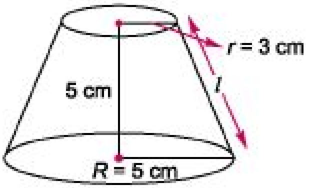
Now, volume $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi \times 49 \times \sqrt{l^2 - r^2}$$

$$= \frac{1}{3}\pi \times 49 \times \sqrt{81 - 49}$$

$$= \frac{1}{3} \times 49\pi \times \sqrt{32} = \frac{49\pi\sqrt{32}}{3} \text{ m}^3$$

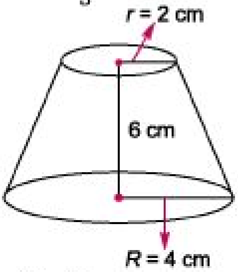
47. (*a*) Slant height $= \sqrt{h^2 + (R - r)^2}$



$= \sqrt{25 + (5 - 3)^2}$

$= \sqrt{25 + 4} = \sqrt{29}$ cm

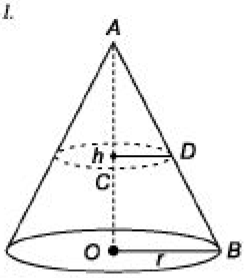
48. (*d*) Volume $= \dfrac{\pi h}{3}(R^2 + r^2 + Rr)$



$= \dfrac{22}{7} \times \dfrac{6}{3} \times (16 + 4 + 8)$

$= \dfrac{22}{7} \times 2 \times 28 = 22 \times 8 = 176 \text{ cm}^3$

49. (*b*) Let the cone is divided into two parts by a line *l*.



In ΔAOB and ΔACD

$\Delta AOB \sim \Delta ACD$

By basic proportionality theoram,

$CD = \dfrac{r}{2}$, since $AC = \dfrac{h}{2}$

$\therefore$ Ratio $= \dfrac{\text{Volume of original cone}}{\text{Volume of smaller cone}}$

$$= \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi\left(\frac{r}{2}\right)^2\left(\frac{h}{2}\right)} = \frac{8}{1}$$

$\therefore$ Ratio = 8:1

50. (*b*) $\dfrac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \dfrac{2}{3}$

$$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \frac{2}{3}$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^2 \times \frac{h_1}{h_2} = \frac{2}{3} \qquad \left[\because \frac{r_1}{r_2} = \frac{1}{2}\right]$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{8}{3}$$

$$\therefore \quad h_1 : h_2 = 8 : 3$$

51. (*b*) Let radius = $5x$, height = $12x$

According to the question,

$$\frac{1}{3} \times \frac{22}{7} \times (5x)^2 \times 12x = \frac{2200}{7}$$

$$\Rightarrow \quad x = 1$$

$$\therefore \quad r = 5, h = 12$$

$\therefore$ Slant height $(l) = \sqrt{r^2 + h^2} = \sqrt{25 + 144}$

$$= \sqrt{169} = 13 \text{ cm}$$

52. (*d*) Here, $x = -50\%$, $y = 200\%$

According to the formula,

Net effect **[by Technique 5]**

$$= \left[2x + y + \frac{x^2 + 2xy}{100} + \frac{x^2 y}{100^2}\right]\%$$

$$= \left[-100 + 200 + \frac{2500 - 20000}{100} + \frac{500000}{10000}\right]\%$$

$[\because x = -50, y = 200]$

$= [100 - 175 + 50]\,\% = -25\%$

53. (*a*) Let diameter, radius and height of first cone are d_1, r_1 and h_1, respectively and that of second cone are d_2, r_2 and h_2, respectively.

$$\frac{r_1}{r_2} = \frac{d_1}{d_2} = \frac{3}{5}, \frac{h_1}{h_2} = ?$$

Given, $\dfrac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \dfrac{1}{3} \Rightarrow \left(\dfrac{r_1}{r_2}\right)^2 \times \dfrac{h_1}{h_2} = \dfrac{1}{3}$

$$\Rightarrow \left(\frac{3}{5}\right)^2 \times \frac{h_1}{h_2} = \frac{1}{3} \Rightarrow \frac{h_1}{h_2} = \frac{1}{3} \times \frac{25}{9} = \frac{25}{27}$$

54. (*d*) Given,

$$\frac{l_1}{l_2} = \frac{5}{7}$$

Now, curved surface area of first cone

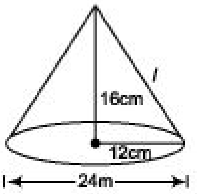
$$= \pi r l_1$$

and curved surface area of second cone

$$= \pi r l_2$$

$$\therefore \quad \text{Ratio} = \frac{\pi r l_1}{\pi r l_2} = \frac{l_1}{l_2} = 5 : 7$$

55. (c) $l = \sqrt{h^2 + r^2}$



$$= \sqrt{16^2 + 12^2} = \sqrt{256 + 144}$$
$$= \sqrt{400} = 20 \text{ cm}$$

Curved surface area $= \pi rl$
$$= \frac{22}{7} \times 12 \times 20$$

$\therefore$ Cost of painting $= \frac{22}{7} \times 12 \times 20 \times 0.70$
$$= ₹\, 528$$

56. (b) Curved surface area of cylinder
$$= 2\pi rh \text{ cm}^2$$

Total surface area of cylinder
$$= 2\pi r(h + r) \text{ cm}^2$$

Now,

According to the question,
$$\frac{2\pi rh}{2\pi r(h + r)} = \frac{1}{2}$$
$$\Rightarrow \quad \frac{h}{h + r} = \frac{1}{2}$$
$$\Rightarrow \quad 2h = h + r$$
$$\therefore \quad h = r$$

Now,

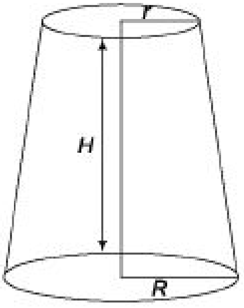
Total surface area $= 616 \text{ cm}^2$
$$\Rightarrow \quad 2\pi r\,(h + r) = 616$$
$$\Rightarrow \quad 2\pi r\,(r + r) = 616 \qquad (\because h = r)$$
$$\Rightarrow \quad 2\pi r\,(2r) = 616$$
$$\Rightarrow \quad 4\,\pi r^2 = 616$$
$$\Rightarrow \quad r = \sqrt{\frac{616}{4\pi}} = \sqrt{\frac{616 \times 7}{4 \times 22}}$$
$$= \sqrt{7 \times 7} = 7$$
$$\therefore \quad r = h = 7 \text{ cm}$$

$\therefore$ Volume $= \pi r^2 h$
$$= \frac{22}{7} \times (7 \times 7) \times 7$$
$$= 1078 \text{ cm}^3$$

57. (a) Given that,

Area of first end $= P = \pi r^2$

and area of second end $= Q = \pi R^2$



Given, $P < Q$

$\Rightarrow \qquad r = \sqrt{\dfrac{P}{\pi}}$ and $R = \sqrt{\dfrac{Q}{\pi}}$

$\therefore$ Difference in radii of the ends of the frustum $= R - r$

$$= \sqrt{\frac{Q}{\pi}} - \sqrt{\frac{P}{\pi}} = \frac{\sqrt{Q} - \sqrt{P}}{\sqrt{\pi}}$$

58. (c) We know that,

Volume of frustum

$$= \frac{\pi H}{3}(R^2 + r^2 + Rr)$$

$$= \frac{\pi}{3} H \left\{\left(\sqrt{\frac{Q}{\pi}}\right)^2 + \left(\sqrt{\frac{P}{\pi}}\right)^2 + \sqrt{\frac{Q}{\pi}} \cdot \sqrt{\frac{P}{\pi}}\right\}$$

$$= \frac{\pi H}{3}\left\{\frac{Q}{\pi} + \frac{P}{\pi} + \frac{\sqrt{PQ}}{\pi}\right\}$$

$$= \frac{H}{3}(P + Q + \sqrt{PQ})$$

59. (c) Let the fixed height of a right circular cone is h and initial radius is r.

Then, initial volume of cone, $V_1 = \dfrac{1}{3}\pi r^2 h$

After increasing 15% radius of a cone

$$= \left(r + \frac{3r}{20}\right) = \frac{23}{20} r$$

New volume becomes, $V_2 = \dfrac{1}{3}\pi\left(\dfrac{23}{20}\right)^2 r^2 h$

$\therefore$ Increasing percentage $= \left(\dfrac{V_2 - V_1}{V_1}\right) \times 100$

$$= \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi r^2 h}\left\{\left(\frac{23}{20}\right)^2 - 1\right\} \times 100$$

$$= \left(\frac{23}{20} + 1\right)\left(\frac{23}{20} - 1\right) \times 100$$

$$= \frac{43}{20} \times \frac{3}{20} \times 100 = 32.25\%$$

Sphere

60. (*a*) Required surface area $= 4\pi r^2$

$$= 4 \times \pi \times 4^2 = 64\ \pi \text{ sq cm}$$

61. (*a*) Surface area of sphere $= 616\ \text{cm}^2$

$$4\pi r^2 = 616 \Rightarrow r^2 = \frac{616 \times 7}{4 \times 22}$$

$$\Rightarrow \quad r^2 = 7 \times 7 \quad \Rightarrow \quad r = 7\ \text{cm}$$

$\therefore$ Diameter of largest circle lying on sphere

$$= 2 \times r = 2 \times 7 = 14\ \text{cm}$$

62. (*b*) Given that, the diameter of Moon is approximately one-fourth of the diameter of Earth.

Let radius on Moon $= r$

Then, radius of Earth $= 4r$

$$\therefore \quad \frac{\text{Volume of Moon}}{\text{Volume of Earth}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi (4r)^3}$$

$$= \frac{r^3}{64 r^3} = \frac{1}{64} = 1 : 64$$

63. (*d*) According to the question,

Surface area of sphere = Surface area of hemisphere

$$4\pi r_1^2 = 3\pi r_2^2 \Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{3}}{2}$$

$$\therefore \text{ Ratio in volumes} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{3\sqrt{3}}{8} : 1$$

64. (*c*) Volume $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \pi \times 3^3$

$$= \frac{2}{3} \times \pi \times 27 = 18\ \pi\ \text{cm}^3$$

65. (*e*) Radius of the sphere $= \frac{16}{2} = 8$ cm

Volume of the sphere

$$= \left(\frac{4}{3}\pi \times 8 \times 8 \times 8\right) \text{cm}^3$$

Radius of each lead ball $= \frac{2}{2} = 1$ cm

Volume of each lead ball

$$= \frac{\text{Volume of sphere}}{\text{Volume of lead ball}}$$

$$= \frac{4}{3}\pi \times 1 \times 1 \times 1 = \frac{4\pi}{3}\ \text{cm}^3$$

$\therefore$ Number of lead balls

$$= \frac{4}{3}\pi \times 8 \times 8 \times 8 \times \frac{3}{4\pi}$$

$$= 8 \times 8 \times 8 = 512$$

66. (*d*) Curved surface area of the hemisphere

$$= 2\pi r^2 = 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 77 \text{ sq cm}$$

As bowl is to be painted inside and outside.

$\therefore$ Total surface to be painted

$$= 77 \times 2 = 154 \text{ sq cm}$$

$\therefore$ Cost of painting 154 sq cm

$$= \frac{5}{10} \times 154 = \frac{1}{2} \times 154 = ₹\, 77$$

67. (*a*) Curved surface area of the sphere

$$= 4\pi r^2$$

or $\quad 616 = 4\pi r^2$

$\Rightarrow \quad \pi r^2 = \frac{616}{4} = 154$

$\Rightarrow \quad r^2 = \frac{154 \times 7}{22} = 49$

$\therefore \quad r = \sqrt{49} = 7 \text{ cm}$

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= \frac{4312}{3} \text{ cm}^3$$

68. (*a*) Let the diameter's of two sphere are d_1 and d_2, respectively.

$\therefore \quad d_1 : d_2 = 3:5$

$\therefore$ Ratio of their surface areas $= \frac{4\pi r_1^2}{4\pi r_2^2}$

$$= \frac{(2r_1)^2}{(2r_2)^2} = \frac{d_1^2}{d_2^2}$$

$$= \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25} = 9 : 25$$

69. (*c*) Volume of small spheres

$$= \frac{\text{Volume of bigger sphere}}{\text{Number of small spheres}} = \frac{\frac{4}{3}\pi(4)^3}{64}$$

$$= \frac{4}{3} \times \frac{\pi \times 4 \times 4 \times 4}{64} = \frac{4}{3}\pi \text{ cm}^3$$

Let radius of small sphere be r'.

$\therefore \quad \frac{4}{3}\pi r'^3 = \frac{4}{3}\pi \Rightarrow r' = 1 \text{ cm}$

Now, surface area of small sphere

$$= 4\pi r'^2 = 4\pi \text{ cm}^2$$

70. (*a*) Curved surface area $= 2\pi r^2$

$$= 2\pi \times 14 \times 14 = 2 \times \frac{22}{7} \times 14 \times 14$$

$$= 2 \times 22 \times 2 \times 14 = 88 \times 14$$

$$= 1232 \text{ sq cm}$$

71. (c) Given,

Diameter = 2 cm

r = 1 cm Now,

Total surface area of hemisphere = *3nr* and curved surface area = 2Jtr

2 2 2

Required difference = *3nr -2nr - nr* = Jl X 1 = 7C sq cm

72. (C) Let radius of the third sphere be r.

Then, -*n* X $(12)^3$= -*n* X $(6)^3$ + -71 3 3 3

x $(8)^3$ + - nr^3 3

=> $(12)^3 = (6)^3 + (8)^3 + r^3$

=)• r^3 =1728 -216-512

=> r^3 = 1000

r = 10 cm

73. (a) According to the formula,

[by Technique 7] Percentage increase in surface area

r $x^2$1 r $(3)^2$1

= I 2x + ------1% = I 2 X 3 + ^— 1%

100J 100J

= [6 + 0.09]% = 6.09%

74. (c) According to the formula,

[by Technique 7] Percentage decrease in surface area

$$= \left[2 \times (-24) + \frac{(-24) \times (-24)}{100}\right]\%$$

$$= [-48 + 5.76]\% = -42.24\%$$

75. (*d*) Here, $x = 100\%$

According to the formula,

[by Technique 6]

Percentage increase in volume

$$= \left[\left(1 + \frac{x}{100}\right)^3 - 1\right] \times 100\%$$

$$= \left[\left(1 + \frac{100}{100}\right)^3 - 1\right] \times 100\%$$

$$= [8 - 1] \times 100\% = 700\%$$

76. (*d*) Volume of solid sphere of radius 4 cm

$$= \frac{4}{3}\pi(4)^3 \text{ cm}^3$$

Volume of hollow sphere

$$= \frac{4}{3}\pi\,[(8)^3 - (6)^3] \text{ cm}^3$$

$\because$ Weight of $\frac{4}{3}\pi(4)^3$ cm^3 = 4 kg

$\therefore$ Weight of $\frac{4}{3}\pi\,[(8)^3 - (6)^3]$ cm^3

$$= \frac{4}{\frac{4}{3}\pi(4)^3} \cdot \frac{4}{3}\pi[(8)^3 - (6)^3]$$

$$= \frac{4(512 - 216)}{4^3} = 18.5 \text{ kg}$$

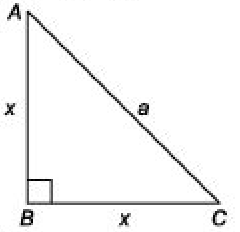
Prism and Pyramid

77. (*d*) Volume of prism = Area of base × Height

$$= \frac{1}{2} \times 10 \times 12 \times 20$$
$$= 1200 \text{ cm}^2$$

$\therefore$ Weight of prism $= 1200 \times 6$
$$= 7200 \text{ g}$$
$$= 72 \text{ kg}$$

78. (*a*) Let $AB = BC = x$



In ΔABC,

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow \quad x^2 + x^2 = a^2$$
$$\Rightarrow \quad 2x^2 = a^2$$
$$\therefore \quad x = \frac{a}{\sqrt{2}}$$

$\therefore$ Volume of prism

$$= \text{Area of base} \times \text{Height}$$
$$= \frac{1}{2} \cdot \frac{a}{\sqrt{2}} \cdot \frac{a}{\sqrt{2}} \times h = \frac{a^2 h}{4} \text{ cm}^3$$

79. (*a*) Let the sides of the base are $5x, 12x$ and $13x$, respectively.

Given, perimeter of base = 60 cm

$$\Rightarrow \quad 5x + 12x + 13x = 60$$
$$\therefore \quad x = \frac{60}{30} = 2$$

The sides of base are 10 cm, 24 cm and 26 cm.

$\therefore$ Volume of prism

$$= \frac{1}{2} \times 10 \times 24 \times 50 = 6000 \text{ cm}^3$$

80. (*a*) Total surface area of the pyramid

$$= \left[\frac{1}{2} \text{(Perimeter of the base) (Slant height)}\right] + \text{Area of the base}$$

$$= \frac{1}{2} (4)(4)(8) + 4^2 = 80 \text{ cm}^2$$

81. (*c*) Volume of the prism

$$= \text{(Area of the base)} \times \text{(Height)}$$

Volume of the pyramid

$$= \frac{1}{3} \text{(Area of the base)} \times \text{(Height)}$$

Required ratio

$$= \frac{A \times H}{\frac{1}{3} \times A \times H} = 3:1$$

Therefore, Ratio of the volumes of the prism and the pyramid = 3 : 1.

Miscellaneous

82. (*c*) Ratio of curved surface area of cylinder and cone

$$= \frac{2\pi rh}{\pi r\sqrt{h^2 + r^2}} = \frac{2 \times 6 \times 8}{6 \times \sqrt{6^2 + 8^2}}$$

$$= \frac{96}{6 \times 10} = \frac{96}{60} = \frac{8}{5} = 8 : 5$$

83. (*b*) Let required number of coins be n. Then,

$$n \times \frac{1}{3} \times \pi \left(\frac{1}{10}\right)^2 \times 1 = \pi \times (3)^2 \times 5$$

$\therefore \quad n = 9 \times 5 \times 3 \times 100 = 13500$

84. (*b*) Volume of 1 bullet $= \frac{4}{3}\pi(2)^3$

$$= \left(\frac{32}{3} \times \frac{22}{7}\right) \text{cm}^3$$

Volume of the cube $= (44)^3$

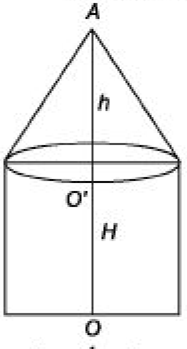
$$\therefore \text{Number of bullets} = \frac{\text{Volume of solid}}{\text{Volume of 1 bullet}}$$

$$= \frac{(44)^3 \times 3 \times 7}{32 \times 22} = 2541$$

85. (*b*) Let the height of circular cylinder $= H$

According to the question,

$$\frac{\text{Total volume of the solid}}{\text{Volume of circular cone}} = 3$$



$$\Rightarrow \qquad \frac{\pi r^2 H + \frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi r^2 h} = 3$$

$$\Rightarrow \quad \pi r^2 H + \frac{1}{3}\pi r^2 h = \pi r^2 h$$

$$\Rightarrow \quad \pi r^2 H = \frac{2}{3}\pi r^2 h$$

$$\therefore \quad H = \frac{2}{3}h$$

86. (*d*) Let r = Radius of cylinder = Radius of sphere, h = Height of the cylinder.

According to the question,

$$\frac{4}{3}\pi r^3 = \pi r^2 h$$

$$\Rightarrow \quad h = \frac{4}{3}r \quad \Rightarrow \quad 4r = 3h$$

$$\Rightarrow \quad 2r = \frac{3}{2}h \quad \Rightarrow \quad \frac{2r}{h} = \frac{3}{2}$$

$\therefore$ Required ratio = 3 : 2

87. (*a*) Given, radius of pipe $\frac{5}{2 \times 10} = \frac{5}{20}$ cm

[$\because$ 1 cm = 10 mm]

Height of pipe = 1000 cm

Radius of vessel = 20 cm and height = 24 cm

Volume of water flow in one minute from cylindrical pipe

$$= \pi \left(\frac{5}{20}\right)^2 \times 1000 = \frac{125}{2}\pi \text{ cm}^3$$

and volume of conical vessel

$$= \frac{1}{3}\pi(20)^2 \times 24 = 3200\pi \text{ cm}^3$$

$$\therefore \text{ Required time} = \frac{3200\pi \times 2}{125\pi}$$

$$= 51\frac{1}{5} \text{ or } 51 \text{ min } 12 \text{ s}$$

88. (*d*) Let level of water will be increased by h cm

$$\pi \times (15)^2 \times h = \frac{4}{3}\pi(10)^3$$

$$\therefore \quad h = \frac{4}{3} \times \frac{10 \times 10 \times 10}{15 \times 15} = 5\frac{25}{27} \text{ cm}$$

89. (*c*) Given that,

Volume of sphere = Volume of cone

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h \quad \Rightarrow \quad 4r = h$$

90. (*b*) Given, $4\pi r^2 = 2\pi rh$

$\therefore \qquad h = 2r$

Now, required ratio

$$= \frac{4}{3}\pi r^3 : \pi r^2 h = 4r : 3h$$

$$= 4r : 6r \qquad [\because h = 2r]$$

$$= 2 : 3$$

91. (*b*) Volume of the cuboid $= 9 \times 8 \times 6$

$$= 432 \text{ cm}^3$$

Volume of the cube $= \frac{1}{2} \times 432$

$$= 216 \text{ cm}^3$$

$\therefore$ Each side of cube $= \sqrt[3]{216} = 6$ cm

Total surface area of the cube

$$= 6 \times (\text{Side})^2 = 6 \times 6^2$$

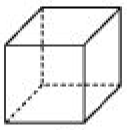
$$= 6 \times 36 = 216 \text{ sq cm}$$

92. (*d*) For cube figure,

Edges, $a = 12$

Faces, $b = 6$

Corners, $c = 8$



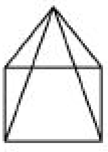
For pyramid figure,

Edges, $a = 8$

Faces, $b = 5$

Corners, $c = 5$

So, $a = b + c$ is neither true for cube nor for the pyramid.



93. (*c*) Volume of water

$=$ Volume of conical flask $= \frac{1}{3}\pi r^2 h$

Now, the water is poured into cylindrical flask.

$\therefore$ Volume of cylinder $=$ Volume of water

$$= \pi (mr)^2 \times \text{Height} = \frac{1}{3}\pi r^2 h$$

$\therefore$ Height $= \dfrac{h}{3m^2}$

94. (*a*) Given, l = Length of the cuboid

$$= 5 \times 7 = 35 \text{ cm}$$

b = Breadth of the cuboid = 5 cm

h = Height of the cuboid = 5 cm

$\therefore$ Surface area $= 2(lb + bh + lh)$

$$= 2[35 \times 5 + 5 \times 5 + 35 \times 5]$$

$$= 2[175 + 25 + 175]$$

$$= 2 \times 375 = 750 \text{ sq cm}$$

95. (*c*) Given, diameter = 150 cm

$\therefore \qquad r = \frac{150}{2}$ cm

According to the question,

$$\frac{2}{3}\pi\left(\frac{150}{2}\right)^3 = 120\ \pi r^2 \times 15$$

$$\Rightarrow \quad \frac{2}{3} \times \frac{150 \times 150 \times 150}{8} = 120 \times 15 \times r^2$$

$$\Rightarrow \quad r^2 = \frac{150 \times 150 \times 150}{12 \times 120 \times 5}$$

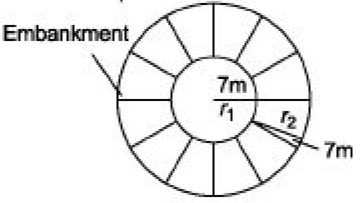
$$\Rightarrow \quad r^2 = \frac{625}{4} \Rightarrow r = \sqrt{\frac{625}{4}} = \frac{25}{2}$$

$\therefore$ Diameter $= 2r = 2 \times \frac{25}{2} = 25$ cm

96. (*b*) Let the height of embankment be h.

Volume of earth taken out $= \pi r^2 h$

$$= \frac{22}{7} \times (7)^2 \times 15$$



Volume of earth taken out

= Volume of embankment

[an embankment is a heap of stone or mud or earth to stop water]

$$\pi r^2 h = \pi(r_1 + r_2)^2 h - \pi r_1^2 h$$

$[\because r_1 = 7, r_2 = 7]$

$$\Rightarrow \quad \frac{22}{7} \times 7^2 \times 15 = \frac{22}{7}(14^2 - 7^2)h$$

$$\therefore \quad h = \frac{7^2 \times 15}{196 - 49} = 5 \text{ m}$$

97. (*c*) 1 hec $= 10000 \text{ m}^3$

Volume of water = Base area $\times$ Height

$$= 10000 \times \frac{10}{100} = 1000 \text{ m}^3$$

98. (*d*) Volume of cylinder $= \pi r_1^2 h$

Volume of sphere $= \frac{4}{3}\pi r_2^3$

Number of spheres = 48

$$\therefore \quad \frac{\text{Volume of cylinder}}{\text{Volume of sphere}} = \frac{\pi r_1^2 h}{\frac{4}{3}\pi r_2^3}$$

$$\Rightarrow \quad \frac{\pi r_1^2 h}{\frac{4}{3}\pi r_2^3} = 48$$

$$\Rightarrow \quad \frac{\pi r_1^3}{\frac{4}{3}\pi r_2^3} = 48 \qquad (\because r_1 = h)$$

$$\Rightarrow \quad \frac{3}{4}\left(\frac{r_1}{r_2}\right)^3 = 48$$

$$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^3 = \frac{48 \times 4}{3}$$

$$\left(\frac{r_1}{r_2}\right)^3 = 64$$

$$\Rightarrow \quad \frac{r_1}{r_2} = 4$$

$$\Rightarrow \quad \frac{r_2}{r_1} = \frac{1}{4}$$

99. (*c*) From figure, it is clear that

Diameter of a sphere = Side of the cube

$= 3$ cm

$\therefore$ Radius $= \frac{3}{2}$ cm

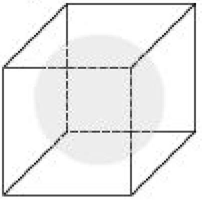
$\therefore$ Volume of the largest sphere

$$= \frac{4}{3}\pi(\text{radius})^3$$

$$= \frac{4}{3}\pi\left(\frac{3}{2}\right)^3$$

$$= \frac{4}{3}\pi \cdot \frac{27}{8}$$

$$= \frac{9}{2}\pi = 4.5\pi \text{ cm}^3$$



100. (*a*) Given that, the height and radius of a right circular metal cone (solid) are 8 cm and 2 cm, respectively.

i.e., $h = 8$ cm and $r = 2$ cm

Let the radius of the sphere is R.

Then, by condition,

$$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$$

$$\Rightarrow \quad 4 \times 8 = 4R^3$$

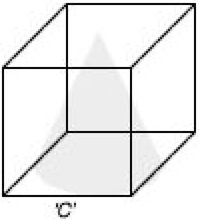
$$\Rightarrow \quad R^3 = (2)^3$$

$$\Rightarrow \quad R = 2$$

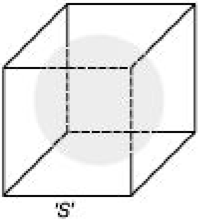
$\therefore$ Radius of the sphere $= 2$ cm

101. (*b*) Let the side of both cube is a then, the height and radius of a cone

$$r = \frac{a}{2} \quad \text{and} \quad h = a$$



'C'



'S'

and radius of sphere $R = \frac{a}{2}$

$\therefore$ Volume of cone $(C) = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi\left(\frac{a}{2}\right)^2 (a) = \frac{\pi a^3}{12} \quad \ldots(i)$$

and volume of sphere (S)

$$= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{\pi a^3}{6} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$S = 2C$$

102. (*d*) If 10 circular plates each of thickness 3 cm, each are placed one above the other, then it forms a cylinder with height ($3 \times 10 = 30$ cm) and a hemisphere of radius 6 cm is placed on the top just to cover the cylinder that means its radius is 6 cm also.

$\therefore$ Radius of hemisphere (R) = 6 cm

Radius of cylinder (r) = 6 cm

and height of cylinder (h) = 30 cm

$\therefore$ Volume of the solid = Volume of cylinder + Volume of hemisphere

$$= \pi r^2 h + \frac{2}{3}\pi R^3$$

$$= \pi (6)^2 \times 30 + \frac{2}{3}\pi (6)^3$$

$$= \pi \times 36 \times 30 + \frac{2}{3}\pi \times 216$$

$$= 1080\pi + 2\pi \times 72$$

$$= 1080\pi + 144\pi$$

$$= 1224\pi \text{ cm}^3$$

which is the required volume of solid.

103. (*a*) Height of the cylinder

= 10 × Diameter of each ball

= 10 × 10 = 100 cm

$\therefore$ Required empty space

$$= \pi(5)^2(100) - \frac{4}{3}\pi(5)^3 \times 10$$

$$= \pi(5)^2 \times 10\left[10 - \frac{20}{3}\right]$$

$$= \pi(5)^2 \times 10\left[\frac{30-20}{3}\right] = \frac{2500}{3}\pi \text{ cm}^3$$

104. (*a*) Given that,

Height = 1 m

Diameter = 140 cm

or $\frac{140}{100}$ m = 1.40

$\therefore$ $r = \frac{1.40}{2} = 0.7$ m

Now,

Required sheet = Total surface area

$$= 2\pi r(h + r)$$

$$= \frac{22}{7} \times 1.4 (1 + 0.7) = 7.48 \text{ sq m}$$

$\therefore$ Sheet required = 7.48 sq m

105. (*a*) Let the breadth and height of room be m and h m, respectively.

Then, according to the question,

$l \times b = n \times$ Area occupied by one patient

$\Rightarrow \quad 14 \times b = 56 \times 2.2$

$\therefore \quad b = \dfrac{56 \times 2.2}{14} = 8.8 \text{ m}$

and volume $= 8.8 \text{ m}^3$

$\therefore$ $h \times$ Area of occupied by one patient $= 8.8$

$$h = \frac{8.8}{2.2}$$

$$h = 4 \text{ m}$$

106. (*c*) Volume of water in the tank

$$= 100 \times 150 \times 2 = 30000 \text{ m}^3$$

Speed of water $= \dfrac{20 \times 1000}{60}$

$$= \frac{1000}{3} \text{ m/min}$$

Water flow per minute $= \dfrac{15}{10} \times \dfrac{125}{100} \times \dfrac{1000}{3}$

$$= 625 \text{ m}^3$$

Time taken $= \dfrac{30000}{625} = 48$ min

107. (*b*) Volume of the earth dugout as a tunnel

$$= \pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 56 = 704 \text{ m}^3$$

Volume of the ditch $= 48 \times \dfrac{33}{2} \times 4$

$$= 24 \times 33 \times 4 = 3168 \text{ m}^3$$

$\therefore$ Part required $= \dfrac{704}{3168} = \dfrac{2}{9}$

108. (*d*) $r =$ Radius of the cone $= \dfrac{14}{2} = 7$ cm

$h =$ Height of the cone $= 12 \times 2 = 24$ cm

$\therefore$ Slant height $(l) = \sqrt{r^2 + h^2}$

$$= \sqrt{(7)^2 + (24)^2} = \sqrt{49 + 576}$$

$$= \sqrt{625} = 25 \text{ cm}$$

Area of the sheet = Total surface area

$$= (\pi r l + \pi r^2)$$

$$= \pi r (l + r)$$

$$= \frac{22}{7} \times 7 \times (25 + 7) = 704 \text{ sq cm}$$

109. (*d*) Let length of the wire be h.

According to the question,

Volume of sphere = Volume of wire

$$\left(\frac{4}{3} \pi \times 18 \times 18 \times 18\right)$$

$$= \left(\pi \times \frac{2}{10} \times \frac{2}{10} \times h\right)$$

$\therefore \quad h = (100 \times 1944)$ cm

$$= \frac{100 \times 1944}{100} = 1944 \text{ m}$$

$$\left[\because 1 \text{ cm} = \frac{1}{100} \text{ m}\right]$$

110. (*b*) Area of 4 walls = Lateral surface area

$$\text{Perimeter} = 2\,(l + b) = 250$$

$$\Rightarrow \quad l + b = \frac{250}{2} = 125 \text{ m}$$

$$\text{Area to be painted} = \frac{\text{Cost}}{\text{Rate}} = \frac{15000}{10}$$

$$= 1500 \text{ sq m}$$

Area of 4 walls $= 2\,(l + b)\,h = 250\ \text{h}$

$$250h = 1500$$

$$\therefore \quad h = \frac{1500}{250} = 6 \text{ m}$$

Note In painting related problems notice whether roof is painted or not.

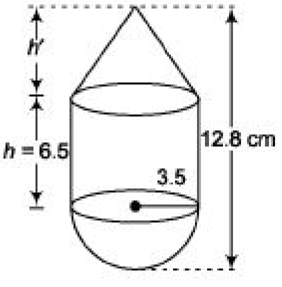
111. (*a*) Let common radius be r cm.

Then, height of cylinder $= h = 6.5$ cm

and height of cone $= h' = (12.8 - 6.5 - 3.5)$

$= 2.8$ cm

$\therefore$ Volume of the complete structure



= (Volume of cylinder) + (Volume of cone)
+ (Volume of hemisphere)

$$= \frac{1}{3}\pi r\ \ h + \pi r\ \ h + - \ \pi r$$

$$= \pi r^2\left(\frac{h'}{3} + h + \frac{2}{3}r\right)$$

$$= \pi(3.5)^2\left(\frac{2.8}{3} + 6.5 + \frac{2}{3} \times 3.5\right)$$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 9.76$$

$$= 375.76 \text{ cm}^3$$

Hence, Volume (V) of the structure lies between 370 cm^3 and 380 cm^3.

Chapter

36

Geometry

Geometry is a branch of Mathematics which deals with the questions and concepts of shape, size, relative position of figures, their angles etc. *It can be broadly divided into two parts*

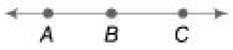
Plane Geometry Plane geometry is about flat shapes like line, circle and triangle. These are two dimensional figure which can easily be drawn on paper. **Solid Geometry** Solid geometry is about three dimensional objects like cube, prism and sphere. These are three dimensional figure.

Point

A figure of which length, breadth and height cannot be measured is called a point. It is infinitesimal.

Line

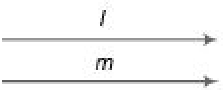
A line is defined by its length but has no breadth. A line contains infinite points. Three or more points are said to be **collinear,** if there is a line which contains all of them.



Plane It is a flat surface having length and breadth both but no thickness. It is a 2-dimensional figure.

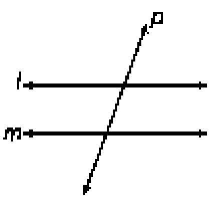
Parallel Lines

Two lines in the same plane are said to be parallel, if they don't have any intersection point, however how far they are extended in either direction. They remain at same distance for the whole length. The sign of parallel is '| |'.Here, *1* and *m* are called parallel lines.



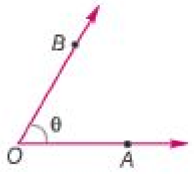
Transversal Lines

A straight line that cuts two or more straight lines at distinct points is called a transversal. In the figure given below *1* and *m* are parallel lines and p is a transversal.



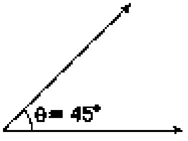
Angle

An angle is formed by two rays with a common initial point. Let O is the initial point, then O is called the vertex. e.g., *ZAOB=Q*

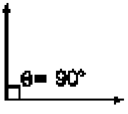


Types of Angle

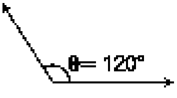
1. **Acute angle** The angle whose value lies between 0° and 90° is called an acute angle. (0°< 0 < 90°)



2. **Right angle** The angle whose value is 90° is called a right angle. (6=90°)

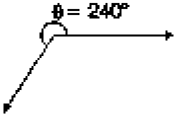


3. **Obtuse angle** The angle whose value lies between 90° and 180° is called an obtuse angle. (90°<8<180°)

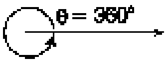


4. **Straight angle** The angle whose value is 180° is called a straight angle. (0=180°)

5. **Reflex angle** The angle whose value lies between 180° and 360° is called a reflex angle. (18O°<0<36O°)

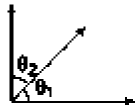


6. **Complete angle** The angle whose value is 360° is called a complete angle. (0=360°)

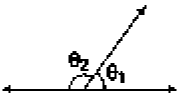


7. **Complementary angle** If the

sum of two angles is 90°, then they are called complementary. Let ©! and 02 be two angles, then % +02=9O°.



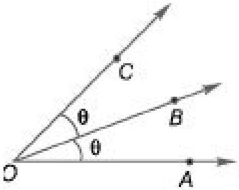
8. **Supplementary angle** If the sum of two angles is 180°, then they are called supplementary. Let 0i and 02 be two angles, then % +02=180°.



Angle Bisector

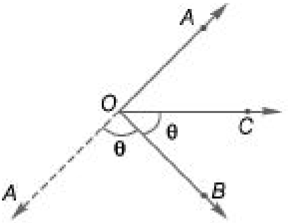
A line which cuts an angle into two equal angles is called an angle bisector. An angle bisector can be internal or external.

Internal angle bisector Here, two angles are formed *ZAOE* and *ZBOC.* Both angles are equal (0) because *OB* is the internal angle bisector.



i.e., $\theta = \frac{1}{2}\angle AOC = \angle AOB = \angle BOC$

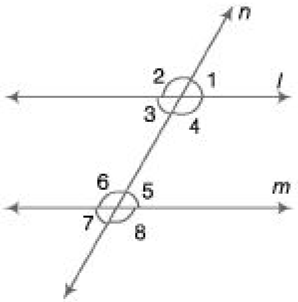
External angle bisector Here, *ZA'OB* and *ZBOC* are equal and external bisector is *OB*.



$$i.e., \theta = \frac{1}{2} \angle A'OC = \angle A'OB = \angle BOC`.$$

Some other Angles

Let *1* and *m* are parallel lines and *n* is the transversal which cuts these parallel lines. *The different angles formed are as follows*



1. **Corresponding angles** Corresponding angle pairs are

Z1 and *Z* 5, *Z* 2 and *Z* 6, *Z* 4 and *Z* 8, *Z* 3 and *Z* 7 and all the corresponding pair

are equal

i.e., *Z\ = Z5, Z2* = Z6, Z4 = Z8 and Z3 = *Zl.*

2. **Vertically opposite angles** Vertically opposite angles pairs are

Zl and Z3, Z4 and Z2, Z8 and Z6, Z5 and *Zl* and all vertically opposite

angles are equal

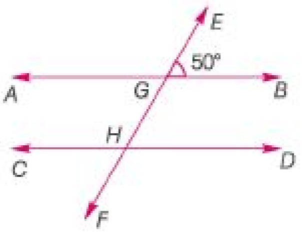
i.e., Zl = Z3, Z4 = Z2, Z8 = Z6 and Z5 = *Zl.*

3. **Alternate angles** Alternate angles pairs are Z3 and Z5, Z4 and Z6 and they are equal

i.e., Z3 = Z5 and Z4 = Z6.

Note 1. The sum of interior angles on the same side of transversal is equal to 180 ° i.e., Z3 + Z6 = 180°*and* Z4 + Z5 = 180° 2. The sum of exterior angles on the same side of transversal is equal to 180° *i.e.,* Z2 + *Zl* = 180° and Zl + Z8 = 180°

Ex. 1 In the given figure, *AB* and *CD* are parallel lines. If *ZEGB* = 50°, find *ZCHG.*



Sol. $\angle AGH = \angle EGB$ [Vertically opposite angles]

$$\angle AGH = 50°$$

Now, $\angle AGH + \angle CHG = 180°$

[Interior angles on the same side of the transversal are supplementary.]

$\therefore$ $50° + \angle CHG = 180°$

$\Rightarrow$ $\angle CHG = 180° - 50° = 130°.$

Ex. 2 An angle 0° is one-fourth of its supplementary angle. What is the measure of the angle 0°?

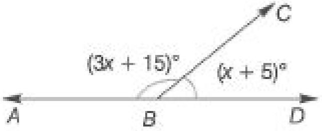
Sol. If the sum of two angles is 180°, the angles are said to be supplementary.

$\therefore$ The supplementary angle of $\theta°$ is $(180° - \theta°)$.

Given that, $\theta° = \frac{1}{4}(180° - \theta°)$

$\Rightarrow$ $4\,\theta° = 180° - \theta° \Rightarrow 5\,\theta° = 180° \Rightarrow \theta° = \frac{180°}{5} = 36°$

Ex. 3 In the given figure, find *x.*

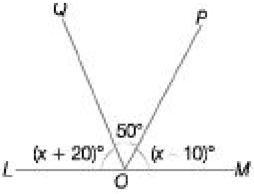


Sol. $\angle ABC + \angle DBC = 180°$

$\Rightarrow \quad (3x + 15)° + (x + 5)° = 180°$

$\Rightarrow \quad 4x = 160° \Rightarrow x = 40°$

Ex. 4 In the figure given below, *LOM* is a straight line. What is the value of *x?*



Sol. Prom the given figure,

ZLOQ + *ZQOP* + *ZPOM* = 180 (straight line) (z°+20°) + 50^{o}+(z^{o}-10°) = 180° => *2x*°+ 60° = 180° => *2x*° = 120° => i°= 60°

Ex. 5 If AB||CD and the line *EF* cuts these lines at the points *M* and *N,*

respectively. Bisectors of angles *ZBMN* and *ZMND* meet at the point Q. Find the value of *ZMQN.*

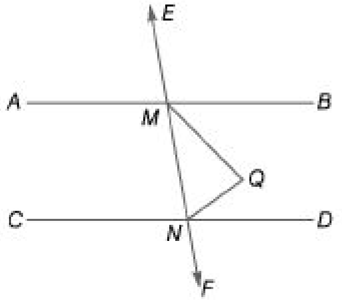
Sol. $\angle BMN + \angle DNM = 180°$

$$\frac{1}{2}\angle BMN + \frac{1}{2}\angle DNM = 90°$$

$\Rightarrow \quad \angle QMN + \angle MNQ = 90°$

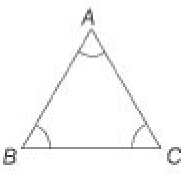
Now,

$$\angle MQN = 180° - (\angle QMN + \angle MNQ)$$



Triangle

A triangle is a three sided closed plane figure which is formed by joining 3 non-collinear points. In figure A *B, C are three non-collinear points which forms AABC.* Now, in a A *ABC,*



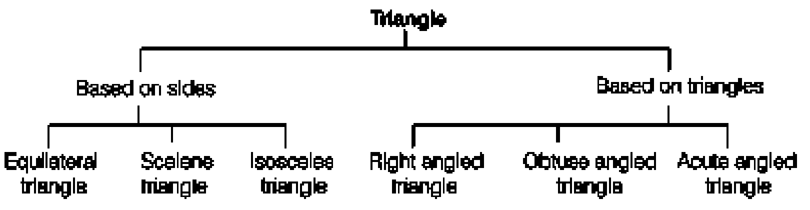
There are **three vertices *A, B*** and **C,**

Three sides *AB, BC* and *AC* and three angles *Z A, Z B* and *Z C* and

sum of these three angles is 180°.

i.e., ZA+ZB + ZC =180°

Types of Triangle



1. **Equilateral triangle** A triangle having all sides equal is called an equilateral triangle. In this triangle each angle is congruent and is equal to 60°. (Figure 1)

2. **Scalene triangle** A triangle having all sides of different length is called a scalene triangle. (Figure 2)

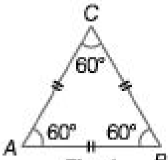


Fig. 1

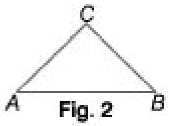
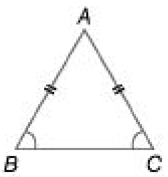
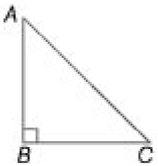


Fig. 2

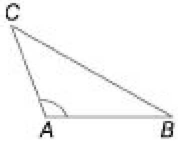
3. **Isosceles triangle** A triangle having two sides equal is called an isosceles triangle. In this triangle angles opposite to congruent sides are also equal.



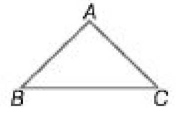
4. **Right angled triangle** A triangle one of whose angles measures 90° is called a right angled triangle. In *AABC, ZB* = 90°



5. **Obtuse angled triangle** A triangle one of whose angles lies between 90° and 180° is called an obtuse angled triangle. In AABC, ZA>90°



6. **Acute angled triangle** A triangle whose each angle is less than 90° is called an acute angled triangle. In AABC,(ZA, *ZB, ZC)* <90°



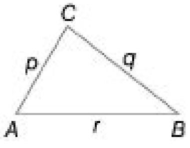
Properties of Triangles

1. Sum of two sides is always more than third side

2. Difference of two sides is always less than third side

3. Greater angle has greater side opposite to it and smaller angle has smaller side opposite to it

4. In the adjoining figure, let *p, q* and *i* be the sides of a *AABC* and *i* is the largest side, then



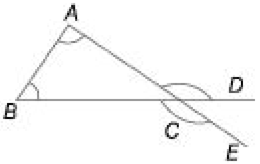
(a) If $r < p+ <$r, then triangle is acute angle triangle

(b) If $r = p + qr,$ then triangle is right angle triangle,

(c) If $r > pi + cr,$ then triangle is obtuse angle triangle

5. The exterior angle is equal to the sum of two interior angles not adjacent to it,

$ZACD = ZBCE = ZA+ ZB$

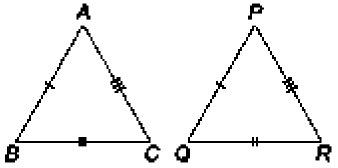


Congurency of Triangles

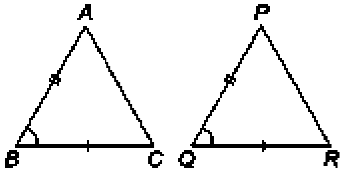
7wo *tnanqles are con.qru.ent if they satisfy the following conditions.*

1. SSS **congruency** (Side - Side - Side)

If the three sides of one triangle are equal to the corresponding three sides of other triangle, then two triangles are congruent. In AABC and *APQR, AB* = *PQ, BC* = *QR* and *AC* = *PR,* then AABC = *PQR*



2. **SAS conguruency** (Side - Angle - Side) If two sides and the angle included between them are equal to the corresponding side and angle included of other triangle are equal, then the two triangles are congruent.

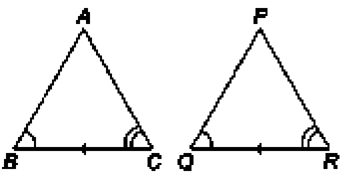


In AABC and *APQR,*

AB = *PQ,BC* = *QR* and *ZB* = *ZQ,* then AABC = *APQR*

3. **ASA congruency** (Angle - Side - Angle)

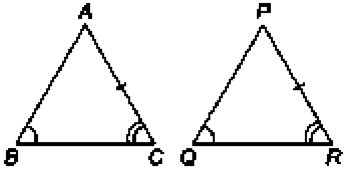
If two angles and the side included between them are equal to the corresponding angles and side included of other triangle are equal, then two triangles are congruent.



In AABC and *APQR, ZB* = *ZQ, ZC* = *ZR* and *BC* = *QR* then, AABC = *APQR*

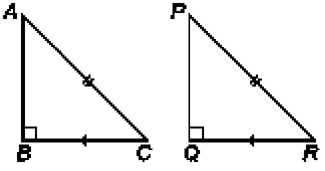
4. **AAS congruency** (Angle - Angle - Side)

If two angles and the side other than the included side of one triangle are equal to the corresponding angles and the side other than included side of other triangle are equal, then the two triangles are congruent.



In AABC and *APQR, ZB* = *ZQ, ZC* = *ZR* and *AC* = *PR* then, AABC = *APQR*

5. **RHS congruency** (Right angle-Hypotenuse-Side) If hypotenuse and one side of right angled triangle are equal to hypotenuse and corresponding side of other triangle, then the two triangles are congruent.



In AABC and *APQR,BC* =*QR AC*=*PR* and *ZB* = *ZQ*= 90°, then AABC = *APQR*

Similarity of Triangles

Two triangles are said to be similar if they satisfy the following conditions.

1. **A A similarity** If two angles of one triangle are equal to the two angles of the other triangle, then the two triangles are similar.

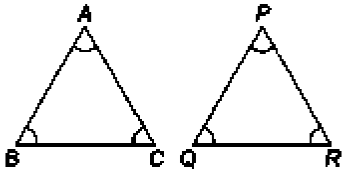
2. **SAS similarity** If two sides of one triangle are proportional to the corresponding side of other triangle and the angle included by them are equal, then the two triangles are similar.

3. **SSS similarity** If three sides of one triangle are proportional to the corresponding three sides of other triangle, then the two triangles are similar.

Properties of Similar Triangle

(i) Ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides (ii) Ratio of the areas of two similar triangles is equal to the ratio of the squares of

corresponding altitudes and medians (iii) In two similar triangles *ABC* and *PQR,*



$$\frac{AB}{PQ}=\frac{AC}{PR}=\frac{BC}{QR}=\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR}$$

Some Important Terms Related to Triangles

Altitude

A

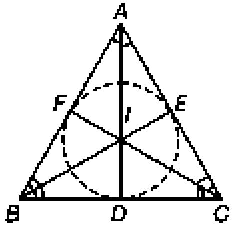
A

B D C

It is the perpendicular drawn to a side of a triangle from the vertex opposite to that side. All the three altitudes of a triangle meet a point called the **orthocentre** of the triangle. In *AABC, AD, BE* and *CF* are altitudes and meet at orthocentre O.

Angle Bisector

It is the bisector of an angle contained in the vertex of a triangle. All the three angle bisectors of a triangle meet at a point called the **incentre** of the triangle. The incentre is the centre of a circle which can be perfectly **inscribed** in the trianale.

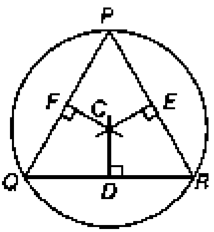


In radius $= ID = IE = IF$. Also, $\angle BIC = 90 + \dfrac{\angle A}{2}$.

In AABC, *AD, BE* and *CF* are angle bisectors and meet at incentre J.

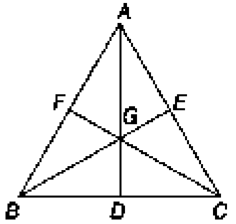
Perpendicular Bisector

It is the line passing through the mid-point of the side of a triangle and perpendicular to it. All the three perpendicular bisectors of a triangle meet at a point called the **circumcentre** of the triangle.The circumcentre is the centre of a circle which can be perfectly **circumscribed** about the triangle. The circumradius = *PC=QC=RC* and *ZQCR* = *2ZP.* In *APQR,* the angle bisectors meet at circumcentre C. Also, *D, E* and *F* are the mid-points of OR, *RP* and *PQ,* respectively.



Median

It is the line joining the mid-point of a side of a triangle with the vertex opposite to that side. All the three medians of a triangle meet at a point called the **centroid** of the triangle. They also intersect each other such that each median is split in the ratio of 1 : 2 from the base side. In AABC, *AD, BE* and *CF* are the medians and meet at the centroid G. Also,



$$\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = 2$$

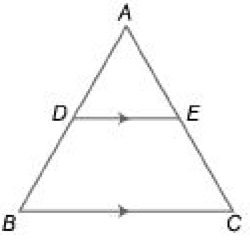
and *D, E* and *F* are mid-points of *BC, CA* and *AB,* respectively.

Important Points

1. The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of sides containing the angle.

2. The line joining the mid point of any two sides of a triangle is parallel to the third side and equal to half of it.

3. Any line parallel to one side of a triangle divides the other two sides proportionally. If *DE* is drawn parallel to *BC,* then



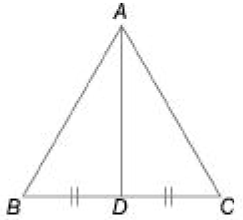
$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{or} \quad \frac{AD}{AB} = \frac{AE}{AC}$$

or

$$\frac{AD}{DE} = \frac{AB}{BC} = \frac{AE}{DE} = \frac{AC}{BC}$$

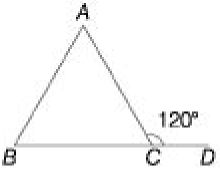
4. **Apollonius theorem** In a triangle the sum of the

squares of any two sides of a triangle is equal to twice the sum of square of the median to the third side and square of **half the** third side,



In *AABC, AD* is median, then $AB^2 + AC^2 = 2\{AD^2 + BD^2)$

Ex. 6 In the figure given, Z *BAC* : Z *ABC* = 2:3. Find the measure of *AABC.*



Sol. Let Z *A* = *2x* and Z *B* = 3s

Then, *2x* + *3x* = 120°

[Exterior angle is equal to the sum of the interior opposite angles] => 5x = 120° => *x* = 24°

Z *ABC* = *3x* = 3 x 24° = 72°

Ex. 7 In A *PQR,* if *PQ* = 6 cm, *PR* = *8* cm, *QS* = 3 cm and *PS* is the bisector of Z *QPR,* what is the length of *QR?*

Sol. Since, *PS* is the angle bisector of Z *QPR, Ql* = *PQ*

SR PR

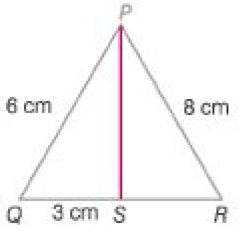
3 6 „D *(3 x* 8"l .

=> ----= - => *SR* = \------- cm = 4 cm

SR 8 I 6 J

$QR = QS + SR$

$= 3+4$ $QR = 7$cm



Ex. 8 The angles of triangle are in the ratio 3:5:7. The triangle is

Sol. Let the angle measure (3x)°, (5x)° and (7x)° Then, 3i + $5x + 7x = 180$

=> 15i = 180 => $x = 12$

∴ These angles are 36° ,60° and 84° Hence, the triangle is acute angled.

Ex. 9 In a *AABC, ZBCA* = 90° and *CD* is perendicular to *AB*. If *AD* = 4 cm and *BD* = 9 cm, then the value of *DC* will be

Sol. In ΔABC and ΔACD,

$\because \quad \dfrac{AC}{AB} = \dfrac{4}{AC}$

$\therefore \quad AC^2 = 4 \times 13 = 52 \quad$ ... (i)

In ΔABC and ΔBCD,

$\dfrac{BC}{AB} = \dfrac{9}{BC}$

$\Rightarrow \quad BC^2 = 9 \times 13 = 117 \quad$ ...(ii)

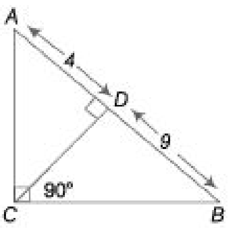
Now, $\quad \dfrac{1}{CD^2} = \dfrac{1}{AC^2} + \dfrac{1}{BC^2}$

$= \dfrac{1}{52} + \dfrac{1}{117}$

$= \dfrac{9+4}{13 \times 4 \times 9}$

$\Rightarrow \quad \dfrac{1}{CD^2} = \dfrac{1}{36}$

$\therefore \quad D = 6$ cm

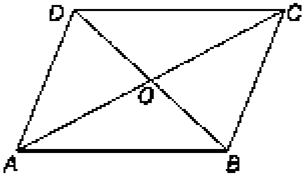


Quadrilateral

It is a plane figure bounded by four straight lines. It has four sides and four internal angles. The sum of the internal angles of a quadrilateral is equal to 360°.

Parallelogram

A quadrilateral in which the opposite sides are equal and parallel, is called a parallelogram. In a parallelogram,



(i) The sum of any two adjacent interior angles is equal to 180°.

$Z A+ZB = ZB + ZC=ZC + ZD = ZD+Z \mathrm{A} =180°$ (ii) The opposite angles are equal in magnitudes $Z A = Z C$ and $Z B = Z D$. (iii) line joining the mid-points of the adjacent sides of a quadrilateral form a

parallelogram. (iv) line joining the mid-points of the adjacent sides of a parallelogram is a

parallelogram. (v) The parallelogram inscribed in a circle is a rectangle and circumscribed about

a circle is a rhombus. (vi) $AC^2 + BD^1 = 2\ (\mathrm{AB}^2 + BC^2)$

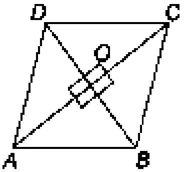
Rhombus

A parallelogram in which all the sides are equal, is called a rhombus.

(i) The opposite sides are parallel and all the sides are of equal

lengths. $AB = BC=CD = DA$.

(ii) The sum of any two adjacent interior angles is equal to 180°. $ZA + ZB = ZB + ZC=ZC + ZD = ZD+Z \mathrm{A} =180°$



(iii) The opposite angles are equal in magnitudes, i.e., *Z A* = *ZC* and *Z B* = *Z D*. (iv) The diagonals bisect each other at right angles and form four right angled triangles.

(v) Area of the four right triangles, $\Delta AOB = \Delta BOC = \Delta COD = \Delta DOA$ and each equals $\frac{1}{4}$th the area of the rhombus.

(vi) Figure formed by joining the mid-points of the adjacent sides of a rhombus is a rectangle.

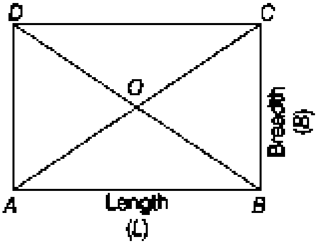
Rectangle

A parallelogram in which the adjacent sides are perpendicular to each other and opposite side are equal is called a rectangle.

(i) The diagonals of a rectangle are of equal

magnitudes and bisect each other i.e., $AC = BD$

and $OA=OB=OC= OD$.



(ii) The figure formed by joining the mid-points of

adjacent sides of a rectangle is a rhombus. (iii) The quadrilateral formed by joining the

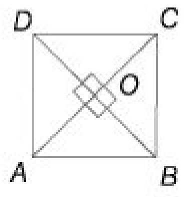
mid-points of intersection of the angle bisectors of a parallelogram is a rectangle.

Square

A parallelogram in which all the sides are equal and perpendicular to each other, is called a square.

(i) The diagonals bisect each other at right angles and form

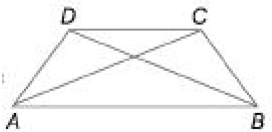
four isosceles right angled triangles. (ii) The diagonals of a square are of equal magnitudes i.e., $AC = BD$.



(iii) The figure formed by joining the mid-points of adjacent sides of a square is a square.

Trapezium

It is a quadrilateral where only one pair of opposite sides are parallel.



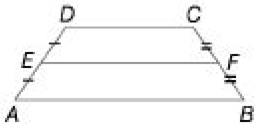
ABCD is a trapezium as *AB* 11 *DC*.

(i) If the non-parallel sides i.e., *(AD* and *BC)* are equal, then diagonals will also be equal to each other,

(ii) Diagonals intersect each other in the ratio of lengths

of parallel sides. (hi) line joining the mid-points of obhque (non-parallel)

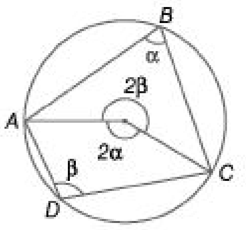
sides is half the sum of parallel sides and is called the



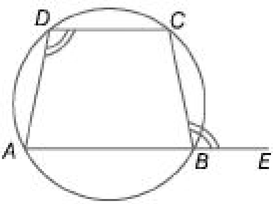
median. *i.e.*, median, $EF = \frac{1}{2}(AB + DC)$

Cvclic Quadrilateral

A quadrilateral whose vertices are on the circumference of a circle, is called a cyclic quadrilateral. The opposite angles of a cyclic quadrilateral are supplementary, i.e., **a** + P=180°.



If the side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle. *i.e., ZADC = ZCBE*



Polygons

A polygon is a closed plane figure bounded by straight lines.

Convex polygon A polygon in which none of its interior angles is more than 180° is called convex polygon.

Concave polygon A polygon in which atleast one angle is more than 180° is called concave polygon.

Irregular polygon A polygon in which all the sides or angles are not of the same

measure.

Regular polygon A regular polygon has all its sides and angles equal.

Number of sides	Name of the Polygon	Sum of all the angles	Number of diagonals
3	Triangle	180°	0
4	Quadrilateral	360°	2
5	Pentagon	540°	5
6	Hexagon	720°	9
7	Heptagon	900°	14
8	Octagon	1080°	20
9	Nanogon	1260°	27
10	Decagon	1440°	35

(i) Each exterior angle of a regular polygon $= \dfrac{360°}{\text{Number of sides}}$

(ii) Each interior angle = 180 — Exterior angle. (iii) Sum of all interior angles =*{2n* -4) x 90°
(iv) Sum of all exterior angles =360°

(v) Number of diagonals of polygon on n sides $= \dfrac{n(n-3)}{2}$

Ex. 10 The angles of a quadrilateral are in the ratios 3:4:5:6. The smallest of these angles is

Sol. Let the angles of the quadrilateral be (3x)°, (4z)°, (5x)° and (6i)°.

Then, 3x + 4x + *5x* + 6x = 360 => 18x = 360 => *x* = 20

∴ Smallest angle = (3 x 20)° = 60°

Ex. 11 In the given figure, *ABCD* is a parallelogram. *E* and F are the centroids of *AABD* and ABCD, respectively. *EF* is equal to

Sol. Since, .Eisthecentroidof A .A.RD and .40 is its median.

$$\therefore \quad AE : EO = 2 : 1$$

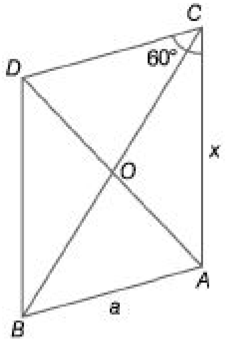
$$\therefore \quad EO = \frac{1}{3} OA$$

Similarly, $$FO = \frac{1}{3} OC$$

$$\therefore \quad EF = EO + OF = \frac{1}{3} OA + \frac{1}{3} OC = \frac{1}{3} AC = AE$$

$$\Rightarrow \quad EF = AE$$

Ex. 12 *ABCD* is a rombus in which ZC = 60 °. Then, *AC: BD* is equal to



Sol. *ABCD* is a rhombus. So, its all sides are equal.

Now, $$BC = DC$$

$$\Rightarrow \quad \angle BDC = DBC = x^\circ \quad \text{[let]}$$

Also, $$\angle BCD = 60^\circ \quad \text{[given]}$$

$$\therefore \quad x^\circ + x^\circ + 60^\circ = 180^\circ \Rightarrow 2x = 120^\circ \Rightarrow x = 60^\circ$$

$$\therefore \quad \angle BCD = \angle DBC = \angle BCD = 60^\circ$$

So, ΔBCD is an equilateral triangle.

$$\therefore \quad BD = BC = a \quad \text{[let]}$$

In ΔAOB

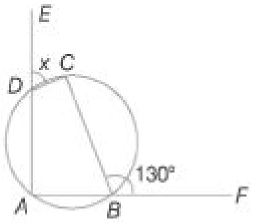
$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow \quad OA^2 = AB^2 - OB^2 = a^2 - \left(\frac{a}{2}\right)^2$$

$$a^2 - \frac{a^2}{4} = \frac{3a^2}{4} \Rightarrow OA = \frac{\sqrt{3}a}{2} \Rightarrow AC = \left(2 \times \frac{\sqrt{3}a}{2}\right) = \sqrt{3}a$$

$$\therefore \quad AC : BD = \sqrt{3}a : a = \sqrt{3} : 1$$

Ex. 13 In the following figure, *A, B, C* and D are the concyclic points. Find the value of *x.*

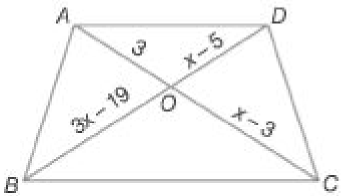


Sol. *Z ABC* + *Z CBF* = 180°

Z ABC = 180° - 130° = 50° Now, in *ABCD Z ABC* + *Z ADC* = 180° => *Z ADC* = 180° - 50° = 130° Now, *Z ADC* + *Z CDE* = 180°

=> *x* = 180° - 130° = 50°

Ex. 14 In the give figure, *AD* *BC*. Find the value of *x*.



Sol. Here, $AD \parallel BC$,

$$\therefore \quad \frac{OA}{OC} = \frac{OD}{OB}$$

$$\Rightarrow \quad \frac{3}{x-3} = \frac{x-5}{3x-19}$$

$$\Rightarrow \quad 9x - 57 = x^2 - 8x + 15$$

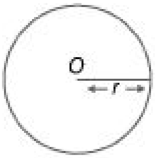
$$\Rightarrow \quad x^2 - 17x + 72 = 0$$

$$\Rightarrow \quad (x-8)(x-9) = 0$$

$$\Rightarrow \quad x = 8, 9$$

Circle

A circle is a set of points which are equidistant from a given point. The given point is known as the centre of that circle. In the given figure O is the centre of circle and *r* is the radius of circle and represented as C (10, r).

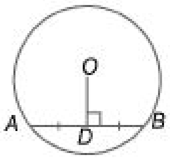


Chord

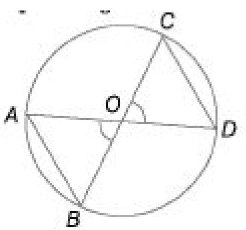
A line segment whose end points lie on the circle. AB is the chord in the given figure.

The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord. $AD = DB$ and $OD \pm AB$.

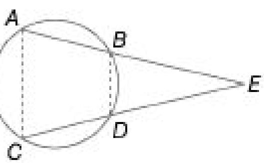
(i) Equal chords of a circle are equidistant from the centre and *vice-versa.*

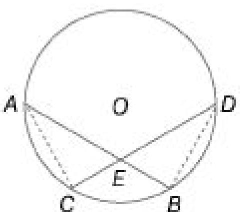


fiil Eaual chords substends eaual anales at the centre and *vice-versa.*



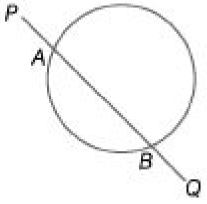
(iii) In a circle or in conguruent circles equal chords are made by equal arcs. (iv) If two chords AB and CD of a circle, intersect inside a circle (outside the circle when produced at point E) then, $AE \times BE = CE \times DE$





Secant

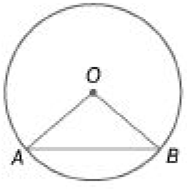
A line segment which intersects the circle at two distinct points, is called as secant. In the given diagram *PQ* intersects circle at two points at *A* and *B*.



Sector of Circle

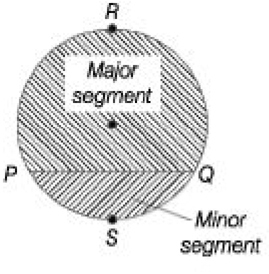
The region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.

Thus, in the adjoining figure, *OABO* is the sector of the circle C (o, r).



Segment of a Circle

A chord divides the circle into two regions. These two regions are called the segments of a circle.



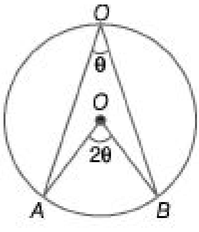
In the figure, *PSR* is the major segment and *PQR* is minor segment. Angles made in the same segment are equal.

Angles and Central Angles

The angle subtended at the centre by any two point on the circumference of the circle is called central angle.

(i) Angle in a semi-circle is a right angle.

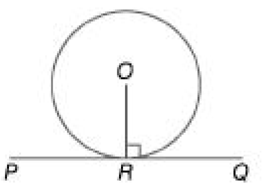
(ii) The angle subtended by an arc at the centre of the circle is twice the angle subtended by the arc at any point on the remaining part of the circle. In the figure given below *ZAOB* is the central angle and *ZAO'B* is angle subtended at the circle. So, *ZAOB=2ZA<yB.*



Tangent

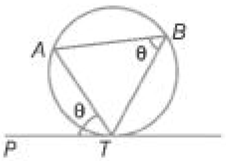
A line segment which has one common point with circumference of a circle i.e., it touches only at one point is called as tangent of circle.

In the given figure *PQ* is tangent which touches the circle at point *R*.

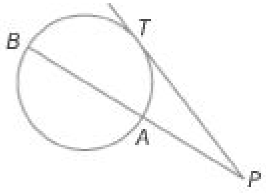


(i) Radius is always perpendicular to tangent. (ii) The length of two tangents drawn from the external point to the circle are equal.

(iii) The angle which a chord makes with a tangent at its point of contact is equal to any angle in the alternate segment. *Z.PTA* = *ZABT,* where *AT* is the chord and *PT* is the tangent to the circle.

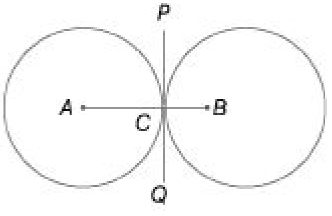


(iv) If PT is a tangent (with P being an external point and T being the point of contact) and PAB is a secant to circle (with A and B as the points, where the secant cuts the circle), then $PT^2 = PA x$ PB.

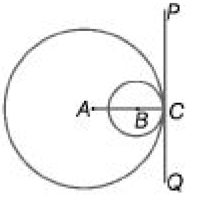


Pair of Circle

(i) (a) When two circles touch externally, then the distance between their centres is equal to the sum of their radii then, $AB = AC + BC$

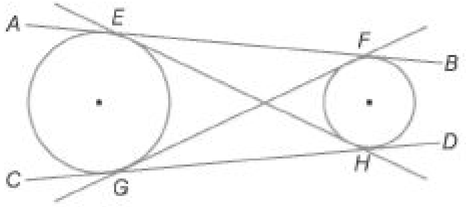


(b) When two circles touch internally the distance between their centres is equal to the difference between their radii



$AB = AC - BC$

(ii) In a given pair of circles, there are two types of tangents. The direct tangents and the cross (or transverse) tangents. In the figure, *AB* and *CD* are the direct tanaents and *EH* and *GF* are the transverse tanaents.



When two circles of radii r_1 and r_2 have their centres at a distance d.

- **The length of the direct common tangent = $\sqrt{d^2 - (r_1 - r_2)^2}$.**
- **The length of transverse tangent = $\sqrt{d^2 - (r_1 + r_2)^2}$.**

Ex. 15A chord *AB* is drawn in a circle with centre *O* and radius 5 cm. If the shortest distance between centre and chord is 4 cm, find the length of chord *AB?*

Sol. In the adjoining figure

$AO = 5$ cm (radius)

$OC = 4$ cm (shortest distance between centre and chord)

Let length of chord AB be $2x$, then $AC = x$.

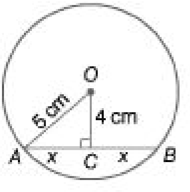
In ΔAOB, $AO^2 = AC^2 + OC^2$

$\Rightarrow \quad (5)^2 = x^2 + (4)^2$

$\Rightarrow \quad 25 = x^2 + 16$

$\therefore \quad x = \sqrt{25 - 16} = \sqrt{9} = 3$ cm

$\therefore$ Length of chord $AB = 2x = 2 \times 3 = 6$ cm



Ex. 16 Find the value of x in the given figure.

Sol. Since, *PT* is a tangent and *PAB* is a secant to the circle. ***PT*^{2} =*PAPB***

=> 144 = *x (x* + 7**)**

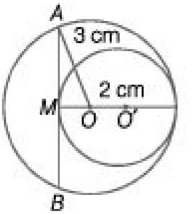
=> x^2 + *7x* - 144 = **0**

=> *{x* + **16)** (s - **9)** = **0**

=> a; = **9** [vi = - **16** is **not** possible.]

Ex. 1 7 Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the greater circle which is outside the inner circle is of length.

Sol. Let O and O' be the centres of greater and smaller circles, respectively.



$\therefore \quad OM = O'M - OO'$
$= 2 - 1 = 1$

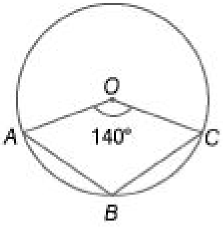
In ΔAOM,

$\because \quad AM^2 = OA^2 - OM^2$
$= 9 - 1 = 8$

$\Rightarrow \quad AM = 2\sqrt{2}$

$\therefore$ Length of bigest chord, $AB = 2 \cdot AM$
$= 2 \cdot 2\sqrt{2} = 4\sqrt{2}$ cm

Ex. 18 In the given figure, it is given that O is the centre of the circle and $ZAOC$ = 140°, find $ZABC$.



Sol. Join A and C to any point D on the circle. We know that, the angle made by an arc at anv point on the circle is half of the centre.

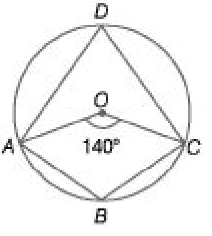
$\therefore \quad \angle ADC = \frac{1}{2} \angle AOC = \frac{1}{2} \times 140° = 70°$

Since, $ABCD$ is a cyclic quadrilateral.

$\therefore \quad \angle ADC + \angle ABC = 180°$

$\Rightarrow \quad 70° + \angle ABC = 180°$

$\Rightarrow \quad \angle ABC = 110°$



Fast Track Practice

Line and Angles

1. An angle which is less than 360° and more than 180°, is called

(a) a reflex angle *(b)* a straight angle *(c)* an acute angle *(d)* an obtuse angle *(e)* None of the above

2. If every interior angle of regular octagon is 135°, then find the external angle of it.

[RRBCGL2012]

(a) 65° *(b)* 75°

(c) 45° *(d)* 55°

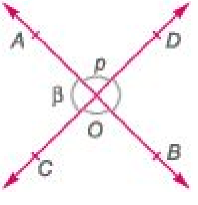
3. The supplement of 80° is

(a) 10° *(b)* 100°

(c) 280° *(d)* 120°

(e) None of the above

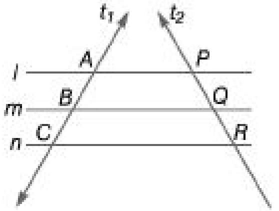
4. In the given figure, straight lines *AB* and *CD* intersect at *O*. If ZB = 3 *Zp,* then *Zp* is equal to



(a) 40° *(b)* 45°

(c) 50° *(d)* 55°

fej None of the above 5. In the given figure, *l*,*m* and n are three parallel lines and t_1 and t_2 are two transversal lines which cut *l*,*m* and *n* at A, *B, C* and P, Q, *R,* respectively. Which of the following options is correct?

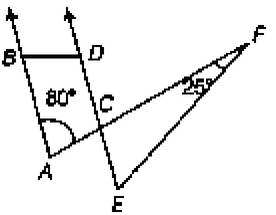


(a) $\frac{BC}{PQ} = \frac{AB}{QR}$ (b) $\frac{AP}{BQ} = \frac{BQ}{CR}$

(c) $\frac{AB}{BC} = \frac{PQ}{QR}$ (d) $\frac{BQ}{AP} = \frac{PQ}{AB}$

(e) None of the above

6. In the given figure, *AB**CD*. If *ZCAB* = 80f and *ZEFC* = 25°, then *ZCEF* is equal to

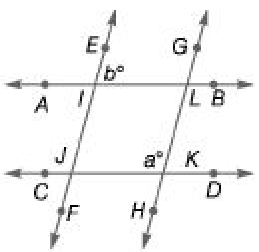


(a) 65° *(b)* 55°

(c) 45° fd) 75°

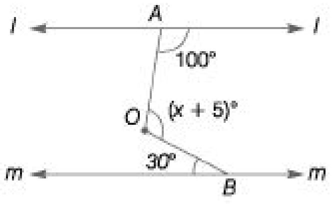
7. In the given figure, *AB* \\ *CD* and

EF 11 Gff. Find the relation between a and 6.



(a) 2a +b = 180= fbj a + fa = 180= (c)a-o = 180° fdj a+ 2o = 180° (ej None of the above

In the given figure, if *I*11 m, then find the value of x (in degrees).

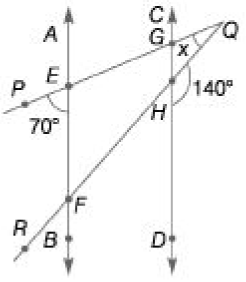


(a) 105° (b) 100°

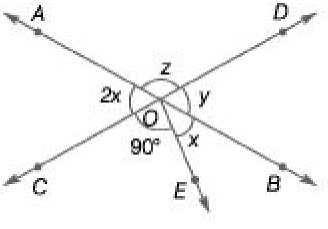
(c) 110° fd) 115°

fej None of the above

9. In the given figure, *AB* 11 *CD* and they cut *PQ* and Qii at *E,F* and G,ff, respectively. If *ZPQR* = *x,* then find the value of *x* (in degrees).

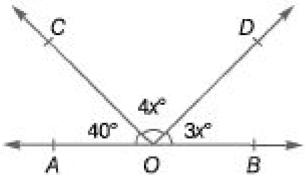


(a) 20° *(b)* 30° *(c)* 24° *(d)* 32° *(e)* None of the above 10. In the given figure, *ZCOE* = 90°. Find the value of *x.*

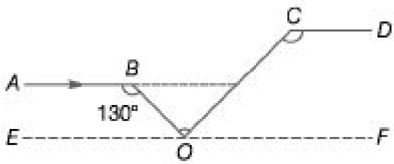


(a) 120° *(b)* 60° *(c)* 45° *(d)* 30°

fej None of the above 11. In the given figure, *AOB* is straight line If *ZAOC* = 40°, *ZCOD* = *4x°* and *ZBOD* = *3x°,* then *ZCOD* is equal to



(a) 80° *(b)* 100° (c) 120° (tfj 140° 12. In the given figure, *AB*11 *CD.* If *ZABO* = 130° and ZOCZ) = 110°, then *ZBOC* is equal to



{a) 50° (b) 60° *(c)* 70° (a; 90=

13. Two transversals *S* and T cut a set of distinct parallel lines. S cuts the parallel lines in points *A, B, C, D* and *T* cuts the parallel lines in points *E, F, G* and *H,* respectively. If *AB* = 4, *CD* = 3 and *EF* = 12, then what is the length of Gfl?

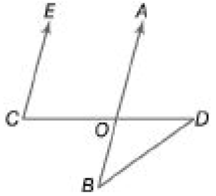
(a) 4 *(b)* 6 (cj 8 *(d)* 9

14. Consider the following statements

I. The locus of points which are equidistant from two parallel lines is a line parallel to both of them and drawn midway between them. II. The perpendicular distances of any point on this locus line from two original parallel lines are equal. Further, no point outside this locus line has this property. Which of the above statements is/are correct? *(a)* Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

15. In the figure given below, *EC* is parallel to *AB, ZECD* = 70° and *ZBDO* = 20°. What is the value of *ZOBD1*



***(a)* 20° *(b)* 30° *(c)* 40° *(d)* 50°**

Triangle

16.In a AABC, *ZA: ZB : ZC = 2: 4:3*. The shortest side and the longest side of the triangle are respectively [SSC CPO 2013] *(a) AC* and *AB (b) AC* and *BC (c) BC* and *AC (d) AB* and *AC*

17. In a ΔABC, $\angle A = 90^\circ$, $\angle C = 55^\circ$ and $\overline{AD} \perp \overline{BC}$. What is the value of $\angle BAD$?

[SSC CGL 2013]

(a) 60° *(b)* 45° *(c)* 55° *(d)* 35°

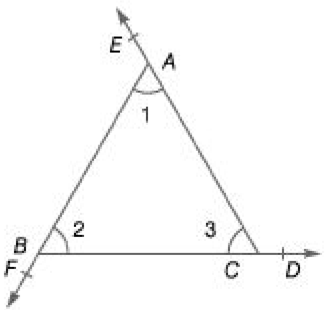
18.0 is the circumcentre of the *AABC.* If *ZBAC* = 50^3, then *ZOBC* is equal to

[SSC (10+2) 2012]

***(a)* 30° *(b)* 60°**

***(c)* 40° *(d)* 50°**

19. The sides *BC,CA* and *AB* of AABC have been produced to *D, E* and *F* respectively as shown in the figure, forming exterior angles *ZACD, ZBAE* and *ZCBF.* Then, *ZACD* + *ZBAE* + *ZCBF* is equal to



(a) 240° *(b)* 300° (c) 320° fdj 360°

20. *ABC* is a right angled triangle such that *AB* = a - *b*, *BC* = a and CA = a + 6 . D is a point on BC such that *BD* = *AB*. The ratio of *BD* : DC for any value of a and *b* is given by [CDS 2013]

(a) 3 : 2 fbj 4 : 3 (c) 5 : 4 fdj 3 : 1

21. ABC is a triangle, where BC = 2AB , ZB = 30° and ZA = 90°. The magnitude of the side AC is [CDS 2013]

(a) $\frac{2\ BC}{3}$ (b) $\frac{3BC}{4}$

(c) $\frac{BC}{\sqrt{3}}$ (d) $\frac{\sqrt{3}\ BC}{2}$

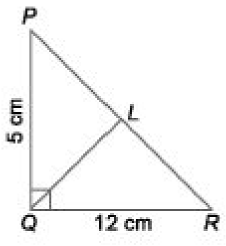
22. The bisectors *BI* and *CI* of ∠*B* and ∠*C* of a Δ*ABC* meet in *I*. What is ∠*BIC* equal to? [CDS 2013]

(a) $90° - \frac{A}{4}$ (b) $90° + \frac{A}{4}$

(c) $90° - \frac{A}{2}$ (d) $90° + \frac{A}{2}$

23. In the figure given below, *ZPQR* = 90° and *QL* is a median, *PQ* = 5 cm and *QR* = 12 cm. Then, *QL* is equal to

[CDS 2013]



(a) 5 cm *(b)* 5.5 cm (c) 6 cm *(d)* 6.5 cm

24. ABC and .XYZ are two similar triangles with ZC = *ZZ,* whose areas are respectively 32 ^{2} and 60.5 ^{2}. If XY = 7.7 cm, then what is *AB* equal to?

[CDS 2013]

(a) 5.6 cm *(b)* 5.8 cm

(c) 6.0 cm fdj 6.2 cm

25. *ABC* is a triangle right angled at A and a perpendicular *AD* is drawn on the hypotenuse *BC.* What is *BCAD* equal to? [CDS 2013]

(a)AB-AC (b)AB-AD

(c)CA-CD (d)AD-DB

26. The side *AC* of a AABC is extended to *D* such that *BC* = *CD.* If ZACB is 70°, then what is ZADB equal to? [CDS 2013]

(a) 35° fbj 45° (c) 70° (dj 110°

27. £ is the mid-point of the median *AD* of a *AABC.* If BiJ is extended it meets the side *AC* at F, then *CF* is equal to

[CDS 2013]

(a) AC/3 (b) 2AC/3

(c) AC/2 *(d)* None of these

28. Consider the following statements

I. If *G* is the centroid of *AABC,* then

GA = *GB* = *GC.* II. If i7 is the orthocentre of AABC, then *HA* = *HB* = *HC.*

Which of the statements given above is/are correct? [CDS 2013]

(a) Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor I

29. The three sides of a triangle are 15, 25 and *x* units. Which one of the following is correct? [CDS 2014]

(a)10<x<40 (b)10<x<40

(c) 10 <x< 40 fdJ10<x<40

30. If AD is the internal angle bisector of AABC with AB = 3 cm and AC = 1 cm, then what is $BD : BC$ equal to? [CDS 2014]

(a) 1 : 3 *(b)* 1 : 4

(c) 2 : 3 *(d)* 3 : 4

In a AABC, if ZA = 115°, ZC = 20° and D is a point on BC such that AD 1 BC and BD = 7 cm, then AD is of length

[SSCCPO2013]

(a) 15 cm fbj 5 cm

(c) 7 cm *(d)* 10 cm

32. In AABC, DE || BC, where DE intersects AB and AC at the points D and E, respectively. If AD = 6 cm, DB = 12* - 6 cm, AB = $2x$ cm and CB = 16 - $2x$ cm, then the value of re is

[SSCCCL (Main) 2012] *(a)* 6 cm *(b)* 4 cm *(c)* 2 cm fdj 8 cm

33. In *AABC, D* and E are points on sides AB and AC, such that *DE\\BC.* If AD = *x, DB* = x - 2, AE = x + 2 and BC = *x*- *%* then the value of x is

[SSCCPO2013] (a) 4 fbj 2 (cj1 fdj 8

34. In a *AABC, AB* = AC and D is a point on *AB,* such that *AD* = *DC* = *BC.* Then, *ZBAC* is [SSCFCI2012]

(a) 40° fbj 45°

(cj 30° *(d)* 36°

35. In a AABC, ZA : ZB : AC = 2: 3: 4 A line CD drawn parallel to *AB,* then *ZACD* is [SSCCGL2013]

(a) 80° fbj 20°

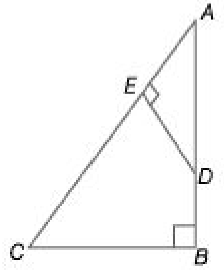
(cj 40° *(d)* 60°

36. The mid-points of AB and AC of a AABC are respectively X and Y. If BC + XY = 12 units, then the value of BC - XY is [SSC FCI 2012]

(a) 6 *(b)* 8

(c) 4 ray 12

37. In the figure given below, ZABC = *ZAED* = 90°.



Consider the following statements I. *ABC* and *ADE* are similar triangles. II. The four points *B, C, E* and *D* may lie on a circle.

Which of the above statements is/are correct? [CDS 2012]

(a) Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

38. Consider the following statements in respect of an equilateral AABC.

I. There is a point *P* inside the AABC, such that each of its sides subtends an angle of 120° at *P*. II. There is a point *P* inside the *AABC,* such that the *APBC,* is obtuse angled and A is the orthocentre of *APBC.* Which of the above statements is/are correct? *(a)* Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

39. In a AABC, *Z BCA* = 60° and $AB^2 = BC^2 + CA^2 + X$. What is the value of *X7* [CDS 2012]

(a) *{BC)[CA)* *(b)* - *{BC){CA)*

(c) *{AB)(BC)* *(d)* Zero

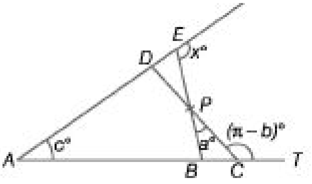
40. In a AABC, *XY* is drawn parallel to *BC,* cutting sides at *X* and *Y,* where *AB* = 4.8 cm, *BC* = 7.2 cm and *BX* = 2 cm. What is the length of *XY?* [CDS 2012]

(a) 4 cm *(b)* 4.1 cm

(c) 4.2 cm *(d)* 4.3 cm

41. The angles $x°$, a", c° and (7t-6)° are indicated in the figure given below. Which one of the following is correct?

[CDS 2012]



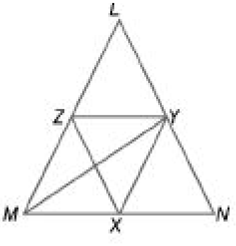
(a) x = a + c - b

(b) x = *b* - *a* - c

(c) x = a + *b* + c

(d) x = a - *b* + *c*

42. In the figure given below, *YZ* is parallel to *MN, XY* is parallel to *LM* and *XZ* is parallel to *LN.* Then, *MY* is [CDS 2012]



(a) the median of ALMA/

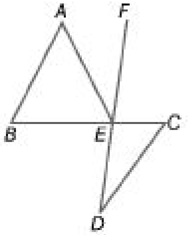
(b) the angular bisector of *ZLMN*

(c) perpendicular to *LN*

(d) perpendicular bisector of *LN*

43. In the figure given below, *AB* is parallel to *CD, ZABC* =65°, *ZCDE* = 15° and *AB* = *AE.* What is the value of ZAEF?

[CDS 2012]

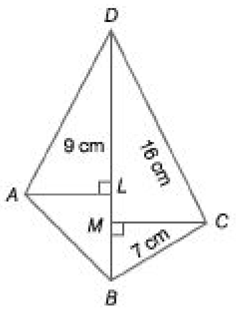


(a) 30° *(b)* 35°

(c) 40° *(d)* 45

Quadrilateral

44. In a quadrilateral *ABCD,* it is given that *BD* = 16 cm. If *AL*±*BD* and *CMLBD* such that *AL* = 9 cm and *CM* = 7 cm, then area (quadrilateral *ABCD)* is equal to



(a) 256 cm^2 (dJ 128 cm^2

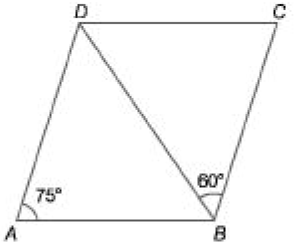
(cj 64 cm^2 (d)96cm^2

45. Three angles of a quadrilateral are 80°, 95° and 112°. Its fourth angle is

(a) 78° *(b)* 73°

(c) 85° *(d)* 100°

46. In the given figure, *ABCD* is a parallelogram in which *ZBAD* = 75° and ZCBD = 60°. Then, *ZBDC* is equal to



(a) 60° (b) 75° *(c)* 45° (tfj 50°

47. If one angle of a parallelogram is 24° less than twice the smallest angle, then the largest angle of the parallelogram is

(a) 68° *(b)* 102°

(c) 112° *(d)* 136°

48. External angle of a regular polygon is 72°. Find the sum of all the internal angles of it. [RRB CGL 2012]

(a) 360° *(b)* 480° *(c)* 352° *(d)* 540°

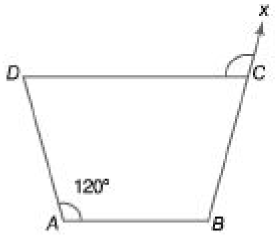
The ratio of the numbers of sides of two regular polygons is 1:2. If each interior angle of the first polygon is 120°, then the measure of each interior angle of the second polygon is [SSC CGL (Main) 2012]

(a) 140° *(b)* 135° *(c)* 150° *(d)* 160°

50. If difference between exterior and interior angles of a polygon is 60°, then find the number of sides in the polygon.

(a) 4 *(b)* 5 *(c)* 6 *(d)* 7 *(e)* 8

51. In the quadrilateral *ABCD* shown below, *ZDAB* = *ZDCX* = 120°. If *ZABC* = 105°, then what is the value of ZADC?[CDS 2012]



(a) 45° *(b)* 60° *(c)* 75° *(d)* 95°

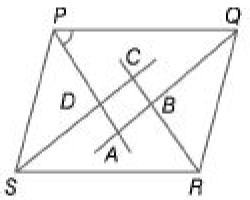
52. If the diagonals of a quadrilateral are equal and bisect each other at right angles, then the quadrilateral is a

[CDS 2012]

(a) rectangle *(b)* square

(c) rhombus *(d)* trapezium

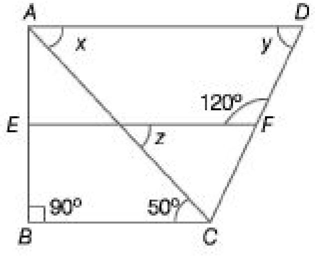
53. In the figure given below, *PQRS* is a parallelogram. *HAP, AQ, CR* and *CS* are bisectors of *ZP,ZQ,ZR* and *ZS* respectively, then *ABCD* is a [CDS 2013]



(a) square *(b)* rhombus

(c) rectangle *(d)* None of these

54.



In the figure given above, $ABCD$ is a trapezium. EF is parallel to AD and BC. Then, Zy is equal to [CDS 2012]

{*a)* 30° *(b)* 45° *(c)* 60° *(d)* 65°

55. A quadrilateral $ABCD$ is inscribed in a circle. If AB is parallel to CD and $AC = BD$, then the quadrilateral must be a

[CDS 2013]

(a) parallelogram *(b)* rhombus

(c) trapezium *(d)* None of these

56. Let $ABCD$ be a parallelogram. Let m and n be positive integers such that $n < m < 2n$. Let $AC = 2\ mn$ and $BD = m^2 - n^2$. Let $AB = (m^2 + n^2)l2$.

[CDS 2012]

Statement I $AC > BD$

Statement II $ABCD$ is a rhombus. Which one of the following is correct in respect of the above statements?

(a) Both statements I and II are true and statement II is the correct explanation of statement I

(b) Both statements I and II are true but statement II is not the correct explanation of statement

(c) Statement I is true but statement II is false

(d) Statement II is true but statement I is false

57. If $ABCD$ be a rectangle and P, Q, R, S be the mid-points of $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$

$\overline{DA}$ respectively, then the area of the quadrilateral $PQRS$ is equal to

[SSC CGL 2013]

(a) $\frac{1}{3}$ area $(ABCD)$ (b) $\frac{3}{4}$ area $(ABCD)$

(c) $\frac{1}{2}$ area $(ABCD)$ (d) area $(ABCD)$

58. Let *X* be any point within a square *ABCD*. On *AX*, a square *AXYZ* is described such that *D* is within it. Which one of the following is correct? [CDS 2012]

(a) AX = DZ (b) ZADZ ZBAX

(c) AD = DZ (d) BX = DZ

59. ABCD is a quadrilateral such that *BC* = *BA* and *CD* > *AD*. Which one of the following is correct?

(a) ZBAD = ZBCD (b) ZBAD < ZBCD (c) ZBAD > ZBCD (d) 2ZBAD = ZBCD

60.Two light rods *AB* = *a* + *b* and *CD* = *a-b* symmetrically lying on a horizontal. There are kept intact by two strings *AC* and *BD*. The perpendicular distance between rods is *a*. The length of *AC* is given by

(a) $a^2 + b^2$ (b) $a^2 - b^2$
(c) $\sqrt{a^2 - b^2}$ (d) $\sqrt{a^2 + b^2}$

Circle

61. *AB* is the diameter of a circle with centre *0* and *P* is a point on it. If *ZPOA* = 120=, then the value of *ZPBO* is

[SSCCGL (Main) 2012] *(a)* 30° *(b)* 50° *(c)* 60° *(d)* 40°

62. Two circles of same radius 5 cm, intersect each other at A and *B*. If *AB* = 8 cm, then the distance between the centres is [SSC CCL 2013]

(a) 10 cm (to) 4 cm *(c)* 6 cm *(d)* 8 cm

63.*0* is the centre of a circle. *AC* and *BD* are two chords of the circle intersecting each other at *P*. If *ZAOB* = 15° and *ZAPB* = 30", then *ZCOD* is equal to

[SSCCGL (Main) 2012]

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{10}{3}$

64. *R* and *r* are the radii of two circles *(R* > 1). If the distance between the centres of the two circles be *d*, then length of common tangent of two circles is

(a) $\sqrt{r^2 - d^2}$ (b) $\sqrt{d^2 - (R - r^2)}$

(c) $\sqrt{(R - r)^2 - d^2}$ (d) $\sqrt{R^2 - d^2}$

65. P is a point outside a circle and is 13 cm away from its centre. A secant drawn from the point P intersects the circle at points A and B in such a way that $PA = 9$ cm and $AB = 7$ cm. The radius of the circle is [SSC CCL 2012]

(a) 5.5 cm *(b)* 5 cm *(c)* 4 cm *(d)* 4.5 cm

66. Consider the following statements

I. The tangent of a circle is a line that meets the circle in one and only one point.

II. The tangent of a circle at the end point of the diameter is perpendicular to the diameter.

Which of the above statements is/are correct?

(a) Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

67. Consider the following statements

[CDS 2013]

I. The perpendicular bisector of a chord

of a circle does not pass through the

centre of the circle.

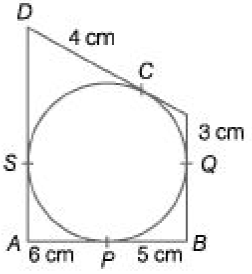
II. The angle in a semi-circle is a right

angle.

Which of the statements given above is/are correct?

(a) Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II 68. In the given figure, quadrilateral *ABCD* is circumscribed, touching the circle at *P, Q, R* and S. If *AP* = 6 cm *BP* = 5 cm, *CQ* = 3 cm and *DR* = 4 cm, then perimeter of quadrilateral *ABCD* is



(a) 18 cm *(b)* 27 cm

(c) 36 cm *(d)* 32 cm

69. From the circumcentre *I* of the *AABC,* perpendicular ID is drawn on £C. If *ABAC* = 60°, then the value *oiZBID* is

[SSCCGL2012]

(a) 75° (b) 60° (c) 45° *(d)* 80°

70. In a AABC, O is its circumcentre and *ZBAC* = 50°. The measure of *ZOBC* is

[SSCCPO2013]

(a) 60° fbj 30° *(c)* 40° fdj 50°

71. The diagonals *AC* and fiD of a cyclic quadrilateral *ABCD* intersect each other at the point *P.* Then, it is always true

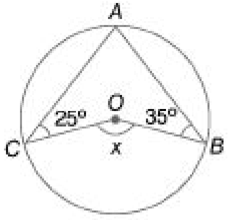
that [SSCCCL2013]

(a) AP.CP = BP.DP (b) AP.BP = CP.DP (c) AP.CD = AB.CP (d) BP.AB = CD.CP

72. A, B, C and *D* are four points on a circle. *AC* and *BD* intersect at a point *E* such that *ZBEC* = 130= and *ZECD* = 20=. Then, *ZBAC* is equal to [SSC CCL 2013]

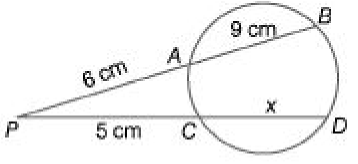
(a) 90° (b) 100° (C) 110° (d) 120°

73. From the circumcentre *I* of the *AABC,* perpendicular *ID* is drawn on *BC.* If ZBAC = 60°, then the value of *ZBID* is. Find the value of X [SSC CCL (Main) 2012]



(a) 75° (b) 60° (c) 45° (d) 80°

Find i in the given figure.



(a) 13 cm (b) 12 cm

(c)16cm (dj 15 cm

(e) None of the above

75. An equilateral *ATQR* is drawn inside a square *PQRS.* The value of the *ZPTS* (in degrees) is [SSC (10+2) 2012]

(a) 75 (b) 90

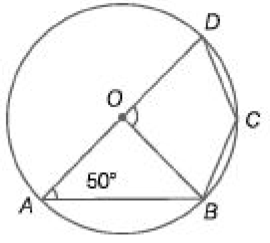
(c) 120 (d) 150

76. If the sides of a quadrilateral *ABCD* touch a circle and *AB* = 6 cm, *CD* = 5 cm BC = 7 cm, then the length of *AD* (in cm) is [SSC CCL (Main) 2012]

(a) 4 (b) 6

(c) 8 (of) 9

77. In the given figure, *O* is the centre of a circle and *ZOAB* = 50°. Then, *ZBOD* is equal to



(a) 130° *(b)* 50°

(c) 100° *(d)* 80°

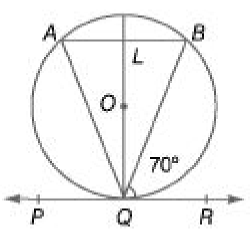
78. If a regular hexagon is inscribed in a circle of radius *r,* then find the perimeter of the hexagon.

(a) 5r (b) 6r (c) %r (d) 9r (e) 12r

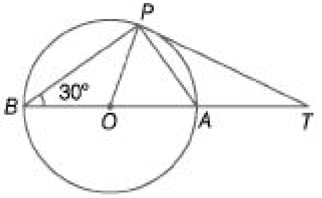
79. Two concentric circles having common centre *O* and chord *AB* of the outer circle intersect the inner circle at points C and *D.* If distance of chord from the centre is 3 cm, outer radius is 13 cm and inner radius is 7 cm, then length of *AC* (in cm) is [SSC CCL (Main) 2012]

(a) $8\sqrt{10}$ (b) $6\sqrt{10}$
(c) $4\sqrt{10}$ (d) $2\sqrt{10}$

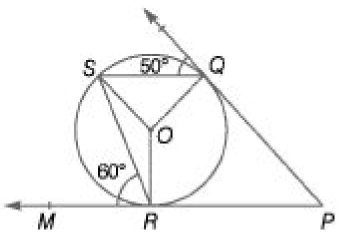
In the given figure, *PQR* is a tangent to the circle at *Q,* whose centre is *O* and *AB* is a chord parallel to *PR* such that *ZBQR* = 70P. Then, *ZAQB* is equal to



(a) 20° *(b)* 35° *(c)* 40° *(d)* 45° 81. In the given figure, *O* is the centre of a circle, *BOA* is its diameter and the tangent at the point P meets *BA* extended at *T.* If *ZPBO* = 30°, then *ZPTA* is eaual to



(a) 60° *(b)* 30° *(c)* 15° *(d)* 45° 82. In the given figure, *O* is the centre of a circle, PQL and *PRM* are the tangents at the points *Q* and *R* respectively and S is a point on the circle such that *ZSQL* = 50° and *ZSRM* = 60°. Then, *ZQSR* is equal to



(a) 40° (b) 50° (cj 60° *(d)* 70°

83. ABC is an isosceles triangle with *AB* = *AC.* A circle through *B* touching *AC* at the middle point inter-sects *AB* at *P.* Then, AP : AB is [SSC CGL 2013]

(a) 3 : 5 fbj 1: 4 (cj 4 :1 (dj 2 : 3

84. A chord *AB* of a circle C_x of radius (VI+1) cmtouchesacircleC_2 ofradius (VI -1) cm, then the length of AB is

[SSC CGL 2013]

(a)8VIcm (b) 4 #3 cm

(c) 4 VI cm *(d)* 2^/1 cm

85. P and Q are two points on a circle with centre at *O. R* is a point on the minor arc of the circle between the points *P* and *Q.* The tangents to the circle from the point

S are drawn which touch the circle at *P* and *Q.* If *ZPSQ* = 20°, then *ZPRQ* is equal to [SSC CGL 2013]

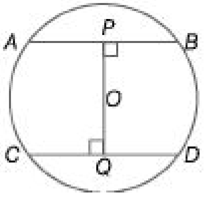
(a) 200° *(b)* 160° (c) 100° *(d)* 80°

86. The length of the diagonal of a square is 8 cm. A circle has been drawn circumscribing the square. The area of the portion between the circle and the square (in sq cm) is [SSC FCI 2012]

(a) $16\frac{2}{7}$ (b) $18\frac{2}{7}$ (c) $10\frac{2}{7}$ (d) $12\frac{2}{7}$

87. In the given figureAB and *CD* are two parallel chords of a circle with centre *O* and radius 5 cm. Also, AB = 8 cm and *CD* = *6* cm. If OP 1 AB and *OQ 1 CD,* then determine the length of *PQ*

[DMRC2012]



(a) 7 cm *(b)* 10 cm

(c) 8 cm *(d)* None of these

Suppose AB is a diameter of a circle, whose centre is at *O* and C be any point on the circle. If *CD* ± *AB* and *CD* = 12 cm, AD = 16 cm, then *BD* is equal to [SSC FCI 2012]

(a) 10 cmfbj 12 cmb) 8 cm *(d)* 9 cm

89. Three circles of radii 4 cm, 6 cm and 8 cm touch each other pairwise externally. The area of the triangle formed by the line segments joining the centres of the three circles is [SSC (10+2) 2012]

(a) 6VI sq cm *(b)* 24VI sq cm

(c) 144VTI sq cm *(d)* 12V105 sq cm

90. A, *B* and *C* are three points on a circle. The tangent at *C* meets *BA* extended at *T.* Given, *ZATC* = 36° and *ZACT* = 48°, the angle subtended by AB at the centre of the circle is [SSC FCI 2012]

(a) 84° *(b)* 48° *(c)* 96° *(d)* 72°

91. Consider the following statements

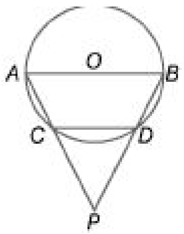
I. The angular measure in radian of a circular arc of fixed length subtending at its centre decreases, if the radius of the arc increases. II. 1800° is equal to *5n* radian.

Which of the statements is/are correct?

(a) Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

92. In the figure given below, *AO* = *CD* where *O* is the centre of the circle. What is the value of ZAPS? [CDS 2012]



(a) 60° *(b)* 50° *(c)* 45° *(d)* 30°

93. A circular ring with centre *O* is kept in the vertical position by two weightless thin strings *TP* and *TQ* attached to the ring at *P* and *Q*. The line *OT* meets the ring at *E* whereas a tangential string at *E* meets *TP* and *TQ* at A and *B,* respectively. If the radius of the ring is 5 cm and *OT* = 13 cm, then what is the length of AB? [CDS 2012]

(a) 10/3 cm *(b)* 20/3 cm

(c)lOcm (dj 40/3 cm

94. Consider the following statements in respect of two chords *XY* and *ZT* of a circle intersecting at *P*. [CDS 2013]

I. *PXPY* = *PZPT* II. *PXZ* and *PTY* are similar triangles. Which of the statements given above is/are correct? *(a)* Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

95. A regular hexagon is inscribed in a circle of radius 5 cm. If x is the area inside the circle but outside the regular hexagon, then which one of the following is correct? [CDS 2013]

(a) 13 cm^2 < x < 15 cm^2

(b) 15 cm^2 < x < 17 cm^2

(c) 17 cm^2 < x < 19 cm^2

(d) 19 cm^2 <x<21 cm^2

96. The diameter of a circle with centre at C is 50 cm. *CP* is a radial segment of the circle. *AB* is a chord perpendicular to *CP* and passes through *P*. *CP* produced intersects the circle at *D*. If *DP* = 18 cm, then what is the length of AB? [CDS 2013]

(a) 24 cmfbj 32 cm(c) 40 cm *(d)* 48 cm

97. ABC is an equilateral triangle inscribed in a circle. *D* is any point on the arc *BC*. What is *ZADB* equal to? [CDS 2013]

(a) 90° *(b)* 60°

(c) 45° *(d)* None of these

98. AB and *CD* are two chords of a circle meeting externally at *P*. Then, which of the following is/are correct? [CDS 2014]

I. *PAxPD* = *PCxPB* II. *APAC* and *APDB* are similar.

Select the correct answer using the codes given below. *(a)* Only I *(b)* Only II

(c) Both I and II *(d)* Neither I nor II

99. *ABC* is triangle right angled at *B*. If AB = 6 cm and *BC* = 8 cm, then what is the length of the circum-radius of the AABC? [CDS 2014]

(a) 10 cm *(b)* 7 cm

(c) 6 cm *(d)* 5 cm

100. *PQ* is chord of length 6 cm of a circle of radius 5 cm. Tangents to the circle at *P* and *Q* meet at *T*. Length of TP is

[SSCCGL2013]

(a) 4.75 cm *(b)* 2.75 cm

(c) 3.75 cm *(d)* 4.25 cm

10*l*.*ABCD* is a rhombus. AB is produced to *F* and *BA* is produced to *E* such that $AB = AE = BF$, then [SSC CGL 2013]

(a) $ED^2 + CF^2 = EF^2$ *(b)* ED\\CF *(c)* ED > CF *(d)* ED 1 CF

102.AB and *CD* are two parallel chords of a circle such that AB = 10 cm and *CD* = 24 cm. If the chords are of the opposite sides of the centre and distance between them is 17 cm, then the radius of the circle is [SSC CGL 2013] *(a)* 12 cm *(b)* 13 cm

(c) 10 cm *(d)* 11cm

103. Consider the following statements

I. Let *ABCD* be a parallelogram which

is not a rectangle.

Then, $2(AB^2 + BC^2) * AC^2 + BD^2$ II. If ABCD is a rhombus with AB = 4

cm, then $AC + BD = n$ for some

positive integer *n*.

Which of the above statements is/are

correct? [CDS 2014]

(a) Only I *(b)* Only II

(b) Both I and II *(d)* Beither I nor I **104.** Each of the two circles of same radius a

passes through the centre of the other. If the circles cut each other at the points A, B and *0,0′ be* their centres, then area of the quadrilateral *AOBO′* is **[SSC CGL 2013]**

(a) $\frac{1}{4}a^2$ (b) $\frac{1}{2}a^2$ (c) $\frac{\sqrt{3}}{2}a^2$ (d) a^2

Answer with Solutions

Line and Angles

1. (*a*) An angle which is less than 360° and more than 180°, is called a reflex angle.

2. (*c*) Every external angle of octagon

$$= 180° - \text{Interior angle}$$
$$= 180° - 135° = 45°$$

3. (*b*) Supplement of $80° = 180° - 80° = 100°$

4. (*b*) $\angle p + \angle \beta = 180°$

$\Rightarrow \quad \angle p + 3\angle p = 180°$

$\Rightarrow \quad 4\angle p = 180°$

$\therefore \quad \angle p = 45°$

5. (*c*) Given that, $l \parallel m \parallel n$ and t_1 and t_2 are the transversal lines, therefore $\dfrac{AB}{BC} = \dfrac{PQ}{QR}$

6. (*b*) Let $\angle CEF = x°$

Now, $AB \parallel CD$ and AF is a transversal.

$\therefore \quad \angle DCF = \angle CAB = 80°$

[corresponding angles]

In ΔCEF, side EC has been produced to D.

$\Rightarrow \quad x + 25 = 80°$

$\Rightarrow \quad x = 55°$

7. (*b*) Since, $AB \parallel CD, \angle EIL = \angle IJK$

[corresponding angles]

$\Rightarrow \quad \angle IJK = b \quad [\because \angle EIL = b]$

Since, $EF \parallel GH$,

$\angle IJK + \angle JKL = 180°$

$\Rightarrow \quad b + a = 180°$

[sum of the interior angles on the same side of the transversal]

Hence, the required relation is

$$a + b = 180°.$$

8. (*a*) Draw a line n passing through O and parallel to l and m.

Since, $l \parallel n$,

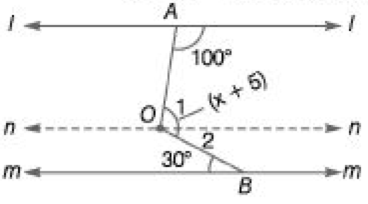
$\angle 1 + 100° = 180°$

[sum of the interior angles on the same side of the transversal]

$\Rightarrow \quad \angle 1 = 80°$

Since, $n \parallel m$

$\angle 2 = 30° \quad$ [alternate angles]



Now, $\angle AOB = \angle 1 + \angle 2 = (80 + 30)^\circ$
$= 110^\circ$
But $\angle AOB = (x + 5)^\circ = 110^\circ$
$\Rightarrow x = (110 - 5)^\circ = 105^\circ$

9. (*b*) Since, $AB \parallel CD$ and PQ is transversal.
$\angle PEF = \angle EGH$ [corresponding angles]
$\Rightarrow \angle EGH = 70^\circ$ [$\because \angle PEF = 70^\circ$]
Now, $\angle EGH + \angle HGQ = 180^\circ$ [linear pair]
$\Rightarrow \angle HGQ = (180^\circ - 70)^\circ = 110^\circ$
Also, $\angle DHQ + \angle GHQ = 180^\circ$ [linear pair]
$\Rightarrow \angle GHQ = (180 - 140)^\circ = 40^\circ$
In ΔGQH,
$\angle GQH + \angle GHQ + \angle HGQ = 180^\circ$
[sum of the angles in a triangle is 180°]
$\Rightarrow x + 40^\circ + 110^\circ = 180^\circ$
$\Rightarrow x + 150^\circ = 180^\circ$
$\Rightarrow x = 180^\circ - 150^\circ = 30^\circ$

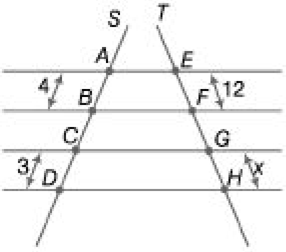
10. (*d*) Here, $\angle BOD = \angle AOC$
$\Rightarrow 2x = y$
Now, $\angle COE + \angle EOB + \angle BOD = 180^\circ$
$90^\circ + x + 2x = 180^\circ$
$\Rightarrow 3x = 90^\circ$
$\therefore x = \frac{90^\circ}{3} = 30^\circ$

11. (*a*) Since, AOB is a striaght line.
We have, $\angle AOC + \angle COD + \angle BOD = 180^\circ$
$\Rightarrow 40 + 4x + 3x = 180$
$\Rightarrow 7x = 140$
$\Rightarrow x = 20$
$\therefore \angle COD = (4 \times 20)^\circ = 80^\circ$

12. (*b*) Through O, draw $EOF \parallel AB \parallel CD$.
Now, $AB \parallel EO$ and BO is the transversal.
$\angle ABO + \angle EOB = 180^\circ$
$\Rightarrow 130^\circ + \angle EOB = 180^\circ$
$\Rightarrow \angle EOB = 50^\circ$
Again,
$CD \parallel OF$ and CO is the transversal.
$\therefore \angle OCD + \angle COF = 180^\circ$
$\Rightarrow 110^\circ + \angle COF = 180^\circ$
$\Rightarrow \angle COF = 70^\circ$
Let $\angle BOC = x^\circ$
Then,
$\angle EOB + \angle BOC + \angle COF = 180^\circ$
$\Rightarrow 50^\circ + x^\circ + 70^\circ = 180^\circ$
$\Rightarrow x = (180^\circ - 120^\circ) = 60^\circ$
$\therefore \angle BOC = 60^\circ$

Triangle

13. (*d*) From figure,



Let $\quad GH = x$

By proportionality law,

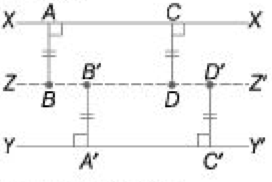
$$\frac{AB}{CD} = \frac{EF}{GH}$$

$\Rightarrow \quad \frac{4}{3} = \frac{12}{x}$

$\Rightarrow \quad x = 3 \times 3 = 9$

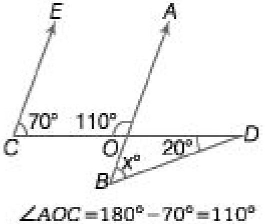
14. (*c*) Statements I and II are both true, because the locus of points which are equidistant from two parallel lines is a line parallel to both of them and draw mid way between them.

Also, it is true that the perpendicular distances of any point on this locus line from two original parallel lines are equal. Further, no point outside this locus line has this property.



15. (*d*) Given that, $EC \parallel AB$

$\therefore \quad \angle ECO + \angle AOC = 180°$



$\Rightarrow \quad \angle AOC = 180° - 70° = 110°$

$\therefore \quad \angle BOD = \angle AOC = 110°$

[alternate angle]

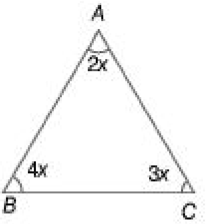
Now, in ΔOBD,

$$\angle BOD + \angle ODB + \angle DBO = 180°$$

$\therefore \quad 110° + 20° + x° = 180°$

$\Rightarrow \quad x° = 50°$

16. (*c*)



Let $\angle A = 2x$

$\angle B = 4x$

and $\angle C = 3x$

We know, $\angle A + \angle B + \angle C = 180°$

$\therefore \quad 2x + 4x + 3x = 180°$

$\Rightarrow \quad 9x = 180°$

$\Rightarrow \quad x = 20°$

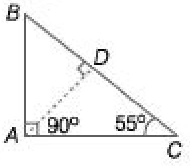
Now, $\angle A = 40°$

$\angle B = 80°$

and $\angle C = 60°$

Hence, the shortest side of triangle = side opposite to the smallest angle = BC and the longest side of triangle = side opposite to the longest angle = AC.

17. (*c*)



In ΔBAC,

$\angle B = 180° - (90° + 55°) = 35°$

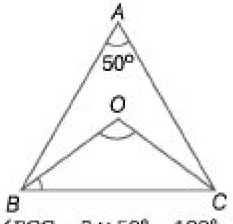
Now, in ΔADB,

$\angle ADB = 90°$

$\therefore \angle ADB + \angle DBA + \angle BAD = 180°$

$\angle BAD = 180° - 90° - 35° = 55°$

18. (*c*)



$\angle BOC = 2 \times 50° = 100°$

In ΔOBC, $OB = OC$

$\Rightarrow \quad \angle OBC = \angle OCB$

$\therefore$ Sum of three angles of a triangle = $180°$

$\Rightarrow \quad \angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow \quad 2\angle OBC + 100° = 180°$

$\Rightarrow \quad \angle OBC = \frac{80°}{2} = 40°$

19. (*d*) Clearly, $\angle 1 + \angle BAE = 180^\circ$

$$\angle 2 + \angle CBF = 180^\circ$$
$$\angle 3 + \angle ACD = 180^\circ$$

$\therefore$ $(\angle 1 + \angle 2 + \angle 3)$

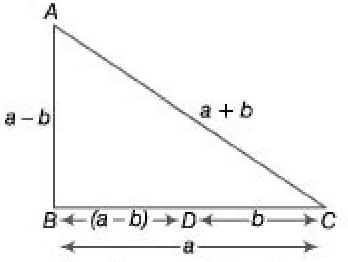
$+(\angle BAE + \angle CBF + \angle ACD) = 540^\circ$

$\Rightarrow$ $180^\circ + (\angle BAE + \angle CBF + \angle ACD) = 540^\circ$

$\Rightarrow$ $\angle ACD + \angle BAE + \angle CBF = 360^\circ$

20. (*d*) In right angled ΔABC,

$$(a+b)^2 = (a-b)^2 + a^2$$



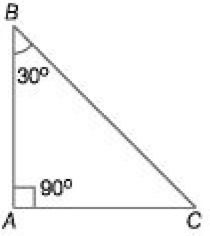
$\Rightarrow$ $a^2 + b^2 + 2ab = a^2 + b^2 - 2ab + a^2$

$\Rightarrow$ $4ab = a^2 \Rightarrow 4b = a$

Now, $\dfrac{BD}{DC} = \dfrac{a-b}{b} = \dfrac{4b-b}{b}$

$$= \frac{3b}{b} = \frac{3}{1}$$
$$= 3:1$$

21. (*d*) Given that, $\angle A = 90^\circ$ and $\angle B = 30^\circ$



In ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow$ $\angle C = 180^\circ - 90^\circ - 30^\circ$

$\Rightarrow$ $\angle C = 60^\circ$

and $BC = 2AB$ ...(i)

From Pythagoras theorem,

$$BC^2 = AC^2 + AB^2$$

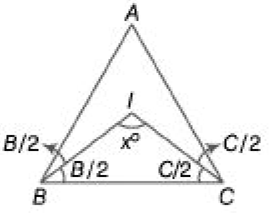
$\Rightarrow$ $(2AB)^2 = AC^2 + AB^2$

$\Rightarrow$ $AC^2 = 4AB^2 - AB^2 = 3AB^2$

$\Rightarrow$ $AC = \sqrt{3} \cdot AB = \dfrac{\sqrt{3}}{2} \cdot (2AB)$

$\Rightarrow$ $AC = \dfrac{\sqrt{3}}{2} \cdot BC$ [from Eq.(i)]

22. (*d*) Given that, BI and CI are angle bisectors of $\angle B$ and $\angle C$, respectively.



Now, in ΔBIC

$$x^\circ + \frac{B}{2} + \frac{C}{2} = 180^\circ$$

[let $\angle BIC = x^\circ$]

$$\Rightarrow \quad x^\circ = 180^\circ - \frac{1}{2}(B + C)$$

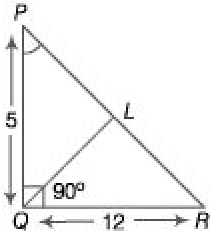
$$\Rightarrow \quad x^\circ = 180^\circ - \frac{1}{2}(180^\circ - A)$$

[$\because$ in ΔABC, $A + B + C = 180^\circ$]

$$x^\circ = 180^\circ - 90^\circ + \frac{A}{2}$$

$$\therefore \quad \angle BIC = x^\circ = 90^\circ + \frac{A}{2}$$

23. (*d*) Given that, $PQ = 5$ cm, $QR = 12$ cm and QL is a median.



$$\therefore \quad PL = LR = \frac{PR}{2} \quad \text{...(i)}$$

In ΔPQR,

$$(PR)^2 = (PQ)^2 + (QR)^2$$

[by Pythagoras theorem]

$$= (5)^2 + (12)^2$$

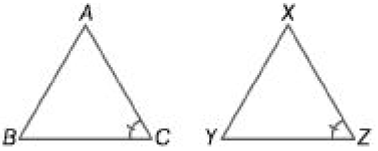
$$= 25 + 144 = 169 = (13)^2$$

$$\Rightarrow \quad PR^2 = (13)^2 \Rightarrow PR = 13$$

Now, by theorem, if L is the mid-point of the hypotenuse PR of a right angled ΔPQR, then

$$QL = \frac{1}{2}PR = \frac{1}{2}(13) = 6.5 \text{ cm}$$

24. (*a*) For similar triangles, ratio of areas is equal to the ratio of the squares of any two corresponding sides.



Here, $\dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta XYZ} = \dfrac{AB^2}{XY^2}$

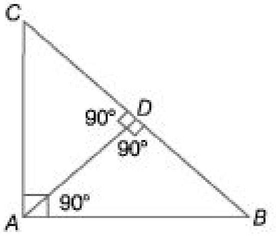
$\Rightarrow \quad \dfrac{32}{60.5} = \dfrac{AB^2}{(7.7)^2}$

$\Rightarrow \quad \dfrac{32 \times 59.29}{60.5} = AB^2$

$\Rightarrow \quad 31.36 = AB^2$

$\therefore \quad AB = \sqrt{31.36} = 5.6 \text{ cm}$

25. (a) In case of a right angled triangle, if we draw a perpendicular from the vertex containing right angle to the hypotenuse, we get three triangles, two smaller and one original and these three triangles are similar triangles.

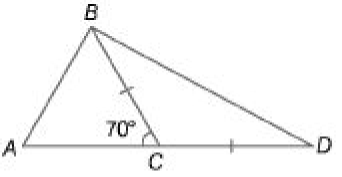


So, $\quad \Delta ABC \sim \Delta ABD \sim \Delta ADC$

$\therefore \quad BC \cdot AD = AB \cdot AC$

26. (*a*) $\angle ACB + \angle BCD = 180°$ [linear pair]

$$\angle BCD = 180° - 70° = 110°$$



In ΔBCD,

$$BC = CD$$

$$\angle CBD = \angle CDB \quad \text{...(i)}$$

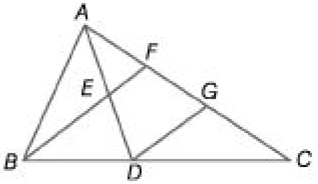
[angles opposite to equal sides]

Also, $\angle BCD + \angle CBD + \angle CDB = 180°$

$$2\angle CDB = 180° - \angle BCD = 180° - 110° = 70°$$

$\therefore \quad \angle CDB = \angle ADB = \frac{70°}{2} = 35°$

27. (6) Draw line segment *DG* parallel to *BE*



Then, in ΔADG,

$$EF \parallel DG$$

and $\quad AE = ED$

$\therefore \quad AF = GC \quad$ ...(i)

Similarly, in ΔBCF,

$$DG \parallel BF$$

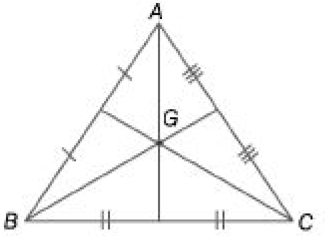
and $\quad BD = DC$

$\therefore \quad FG = GC \quad$ ...(ii)

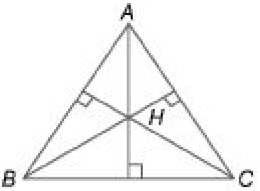
From Eqs. (i) and (ii),

$$CF = \frac{2}{3} AC$$

28. (*d*)



$GA = GB = GC$ is true only and only for equilateral triangle and here it is not given that ABC is an equilateral triangle. So, statement I is not correct. Similarly, statement II will also hold only for equilateral triangle. Hence, it is also not correct.



29. (*a*) In a triangle, sum of two sides is always greater than 3rd side.

i.e., $\quad x < 40 \quad$ ...(i)

Difference of two sides is asways greater than 3rd side.

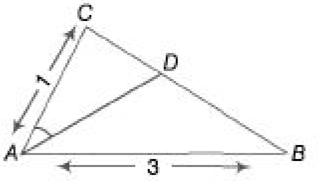
i.e., $\quad 10 < x \quad$ ...(ii)

Form Eqs. (i) and (ii),

$$10 < x < 40$$

30. (*d*) In ΔABC, AD is the internal angle bisector of $\angle A$. Using property of internal angle bisector,

$$\frac{BD}{CD} = \frac{AB}{AC}$$



$$\Rightarrow \quad \frac{CD}{BD} = \frac{AC}{AB}$$

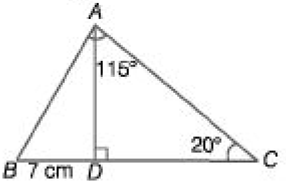
$$\Rightarrow \quad \frac{CD}{BD} + 1 = \frac{AC}{AB} + 1$$

$$\Rightarrow \quad \frac{CD + BD}{BD} = \frac{AC + AB}{AB}$$

$$\Rightarrow \quad \frac{BC}{BD} = \frac{3+1}{3} \Rightarrow \frac{BD}{BC} = \frac{3}{4}$$

$$BD : BC = 3 : 4$$

31. (c) In ΔABC,



$$\angle A = 115°, \angle C = 20°$$

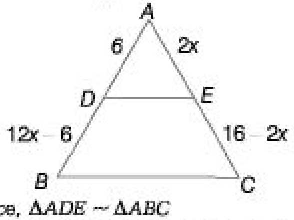
$$\therefore \quad \angle B = 180° - (115° + 20°) = 45°$$

Now, in ΔABD

$$\frac{AD}{BD} = \tan 45°$$

$$\Rightarrow \quad AD = BD = 7 \text{ cm}$$

32. (c) Given, $DE \parallel BC$



Since, $\Delta ADE \sim \Delta ABC$

If any line parallel to one side of triangle devides the other two sides proportionally

Then, $\quad \dfrac{AB}{AD} = \dfrac{AC}{AE}$

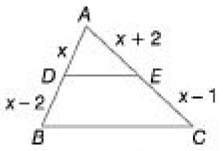
$$\Rightarrow \quad \frac{12x}{6} = \frac{16}{2x}$$

$$\Rightarrow \quad x^2 = \frac{16 \times 6}{12 \times 2} = 4$$

$$\Rightarrow \quad x = 2 \text{ cm}$$

33. (a) $\because \; DE \parallel BC$

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$



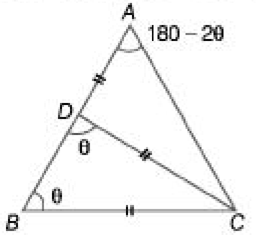
$\Rightarrow \quad \dfrac{x}{x-2} = \dfrac{x+2}{x-1}$

$\Rightarrow \quad x^2 - x = x^2 - 4$

$\Rightarrow \quad x = 4$

34. (*d*) Given that

$AB = AC$ and $AD = CD = BC$



Let $\quad \angle ABC = \theta$

Then, $\quad \angle ACB = \theta \quad [\because AB = AC]$

$\Rightarrow \quad \angle BAC = 180° - 2\theta$

$\Rightarrow \quad \angle ACD = 180 - 2\theta \quad [\because AD = CD]$

$\angle BCD = \angle ACB - \angle ACD$

$\Rightarrow \quad \angle BCD = \theta - (180° - 2\theta)$

$= 3\theta - 180°$

and $\quad \angle BDC = \theta \quad [\because CD = BC]$

Now, in ΔBCD

$\angle CBD + \angle BDC + \angle BCD = 180°$

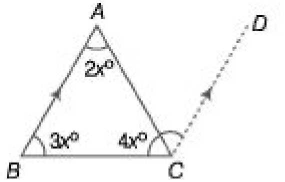
$\Rightarrow \quad \theta + \theta + 3\theta - 180° = 180°$

$\Rightarrow \quad 5\theta = 360° \Rightarrow \theta = 72°$

$\therefore \quad \angle BAC = 180° - 2\theta$

$= 180° - 144 = 36°$

35. (*c*) Let the angles be $2x, 3x$ and $4x$.



Then, $2x + 3x + 4x = 180°$

$9x = 180° \Rightarrow x = 20°$

$\therefore \quad \angle A = 2x = 2 \times 20 = 40°$

$\angle B = 3x = 3 \times 20 = 60°$

$\angle C = 4x = 4 \times 20 = 80°$

Now, $AB \parallel CD$ and AC be the transversal.

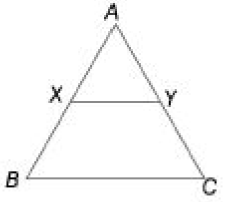
Then, $\angle BAC = \angle ACD$

[alternate interior angles]

$\therefore \quad \angle ACD = 40°$

36. (c) Since, X and Y are the mid-points of AB and AC, respectively.

Therefore, $XY = \frac{1}{2} BC$



Now, $BC + XY = 12$

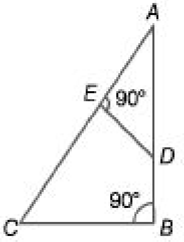
or $BC + \frac{1}{2} \cdot BC = 12$ $\quad \left[\because XY = \frac{1}{2} BC\right]$

$\Rightarrow \frac{3}{2} \cdot BC = 12 \Rightarrow BC = 12 \times \frac{2}{3} = 8$

and $XY = \frac{1}{2} \times 8 = 4$

$\therefore BC - XY = 8 - 4 = 4$

37. (c)



Given, $\angle AED = 90°$

$\Rightarrow \angle DEC = 90°$

$\Rightarrow \angle ACB = \angle ADE$

$\therefore \Delta ABC \sim \Delta ADE$

Now, $EDCB$ is a quadrilateral.

$\therefore$ In $EDCB$,

$\angle E + \angle D + \angle B + \angle C = 360°$

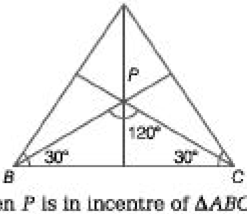
$\angle E + \angle B = 180°$

$\Rightarrow \angle C + \angle D = 180°$

$\therefore \angle C$ and $\angle D$ are supplementry

$\therefore$ $EDCB$ may lie on a circle, since, $EDCB$ is a cyclic quadrilateral.

38. (a) In an equilateral ΔABC, centroid, orthocentre and incentre are coincide.



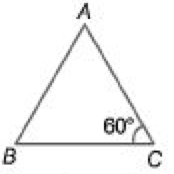
$\therefore$ When P is in incentre of ΔABC, then $AP = BP = CP$

and every side of an equilateral triangle make $120°$ angle at point P.

While A is not an orthocentre of triangle .

39. (*b*) By cosine law,

$$\cos 60° = \frac{AC^2 + BC^2 - AB^2}{2 \cdot AC \cdot BC} = \frac{1}{2}$$

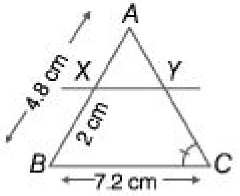


$$\Rightarrow \quad AC^2 + BC^2 - AB^2 = AC \cdot BC$$

$\therefore$ By comparing, we get

$$X = -(AC)(BC)$$

40. (*c*) Given that, $AB = 4.8$ cm, $BC = 7.2$ cm and $BX = 2$ cm



$$\therefore \quad AX = AB - BX$$
$$= 4.8 - 2 = 2.8 \text{ cm}$$

From figure, $\Delta AXY \approx \Delta ABC$

$$\therefore \quad \frac{XY}{BC} = \frac{AX}{AB}$$

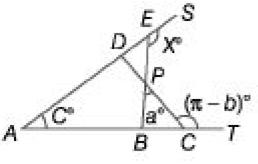
$$\Rightarrow \quad XY = \frac{AX}{AB} \cdot BC$$
$$= \frac{2.8}{4.8} \times 7.2$$

$$\therefore \quad XY = 4.2 \text{ cm}$$

41. (*c*) $\because \angle PCT + \angle PCB = \pi$ [linear pair]

$\angle PCB = \pi - (\pi - b°) = b°$ ...(i)

In ΔBPC,



$$\angle PCB + \angle BPC + \angle PBC = \pi$$

$$\Rightarrow \quad \angle PBC = \pi - \angle PCB + \angle PBC = \pi$$
$$= \pi - b° - a° \quad \text{...(ii)}$$

$$\because \quad \angle ABE + \angle EBC = \pi$$

[$\because \angle PBC = \angle EBC$]

[linear pair]

$\Rightarrow \quad \angle ABE = \pi \angle PBC = \pi - (\pi - b^o - a^o)$

$= a^o + b^o \quad \ldots\text{(iii)}$

Now, in ΔABE,

Sum of two interior angles = Exterior angle

$$\angle EAB + \angle ABE = \angle BES$$

$$\Rightarrow \quad c^o + b^o + a^o = x^o$$

$$\therefore \quad x^o = a^o + b^o + c^o$$

42. (*a*) Since,

$$ZY \parallel MN \text{ and } ZX \parallel YN$$

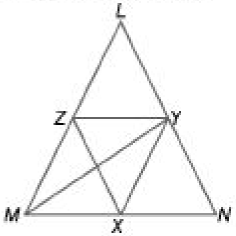
$\therefore$ $XNYZ$ is a parallelogram.

$$\Rightarrow \quad ZX = YZ \quad \ldots\text{(i)}$$

Also,

$$ZX \parallel YN \text{ and } XY \parallel ZL$$

$\therefore$ $XYLZ$ is a parallelogram.



$$\therefore \quad XZ = YL \quad \ldots\text{(ii)}$$

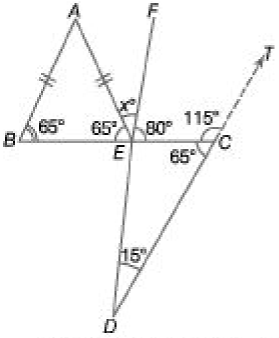
From Eqs. (i) and (ii), we get

$$YN = LY$$

$\therefore$ MY is a median of ΔLMN.

43. (*b*) Given that,

$\angle ABC = 65^\circ$ and $\angle CDE = 15^\circ$



Here, $\angle ABC + \angle TCB = 180°$ [$\because AB \| CD$]

$\therefore \quad \angle TCB = 180° - 65° = 115°$

$\because \quad \angle TCB + \angle DCB = 180°$ [linear pair]

$\therefore \quad \angle DCB = 65°$

Now, in ΔCDE,

$\angle CED = 180° - (\angle ECD + \angle EDC)$

[$\because \angle ECD = \angle BCD$]

$= 180° - (65° + 15°) = 100°$

$\because \quad \angle DEC + \angle FEC = 180°$ [linear pair]

$\Rightarrow \quad \angle FEC = 180° - 100° = 80°$

Given that, $AB = AE$

i.e., ΔABE an isosceles triangle.

$\therefore \quad \angle ABE = \angle AEB = 65°$

$\because \angle AEB + \angle AEF + \angle FEC = 180°$

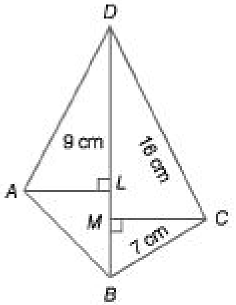
[straight line]

$\Rightarrow \quad 65° + x° + 80° = 180°$

$\therefore \quad x° = 180° - 145° = 35°$

Quadrilateral

44. (*b*)



Area (Quadrilateral $ABCD$)

Area (ΔABD) + Area (ΔBCD)

$$= \left(\frac{1}{2} \times BD \times AL\right) + \left(\frac{1}{2} \times BD \times CM\right)$$

$$= \left(\frac{1}{2} \times 16 \times 9\right) + \left(\frac{1}{2} \times 16 \times 7\right)$$

$$= (72 + 56) = 128 \text{ cm}^2$$

45. (*b*) Let the fourth angle be $x°$.

Then, $80 + 95 + 112 + x = 360$

$\Rightarrow \quad 287 + x = 360$

$\Rightarrow \quad x = (360 - 287) = 73°$

46. (*c*) $\angle C = A = 75°$

[opposite angles of parallelogram]

In ΔBCD, $\angle CBD + \angle BCD + \angle BDC = 180°$

$\Rightarrow \quad 60° + 75° + \angle BDC = 180°$

$\Rightarrow \quad 135° + \angle BDC = 180°$

$\angle BDC = 45°$

47. (*c*) Let the smallest angle be $x°$,

Then, its adjacent angle $= (2x - 24)°$

$\therefore \quad x + 2x - 24 = 180$

$\Rightarrow \quad 3x = 204 \Rightarrow x = 68$

$\therefore$ Largest angle $= (2 \times 68 - 24)°$

$= (136 - 24)° = 112°$

48. (*d*) External angle of any polygon

$$72° = \frac{360°}{n}$$

$\Rightarrow \quad n = 5$

$\therefore$ Given, polygon is regular pentagon.

$\therefore$ Every interior angle of it

$= 180° -$ External angle

$= 180° - 72° = 108°$

$\therefore$ Sum of total interior angles of it

$= 5 \times 108^\circ = 540^\circ$

49. (*c*) Given, interior angle of the first polygon $= 120^\circ$

Let number of sides in first polygon be n_1.

Then, $\frac{n_1 - 2}{n_1} \times 180^\circ = 120^\circ$

$\Rightarrow \quad 3n_1 - 6 = 2n_1$

$\Rightarrow \quad n_1 = 6$

$\therefore$ Sides of the second polygon $= 6n$

$= 6 \times 2 = 12$

$\therefore$ Interior angle of the second polygon

$= \frac{12-2}{12} \times 180^\circ = 150^\circ$

50. (*c*) Exterior angle − Interior angle $= 60^\circ$

$$\Rightarrow \frac{(n-2) \times 180^\circ}{n} - \frac{360^\circ}{n} = 60^\circ$$

$$\Rightarrow \frac{1}{n}[(n-2) \times 180^\circ - 360^\circ] = 60^\circ$$

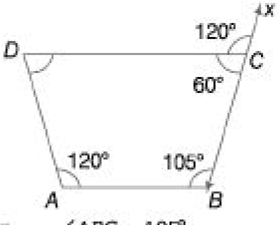
$$\Rightarrow \frac{1}{n}[180^\circ n - 360^\circ - 360^\circ] = 60^\circ$$

$$\Rightarrow 180^\circ n - 720^\circ = 60^\circ n$$

$$\Rightarrow 120^\circ n = 720^\circ$$

$$n = \frac{720^\circ}{120^\circ} = 6$$

51. (*c*)



Given, $\angle ABC = 105^\circ$

$\angle DAB = 120^\circ$

$\angle DCX = 120^\circ$

$\Rightarrow \quad \angle DCB = 180^\circ - 120^\circ = 60^\circ$

Angles of a quadrilateral is equal to 360°.

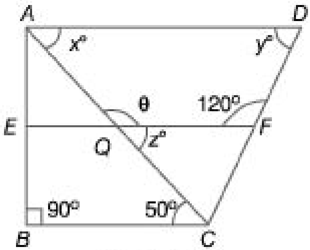
$\therefore \quad \angle ADC = 360^\circ - [120^\circ + 105^\circ + 60^\circ]$

$= 360^\circ - 285^\circ = 75^\circ$

52. (6) We know that, in a square, diagonals are equal and bisect each other at 90°. Hence, the required quadrilateral is a square.

53. (C) Clearly, *AB* \ \ *DC* and *AD* \ \ *BC*. Therefore, *ABCD* is a parallelogram but it is not necessary that *AB* - *BC*. Thus, *ABCD* is a rectangle.

54. (c) From figure,



$$x^\circ = z^\circ = 50^\circ$$

[alternate interior angles]

$$\theta + z^\circ = 180^\circ \quad \text{[linear pair]}$$

$$\theta = 180^\circ - 50^\circ = 130^\circ$$

Now, in quadrilateral *AQFD*,

$$x^\circ + y^\circ + 120^\circ + \theta = 360^\circ$$

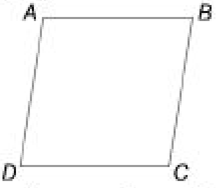
$$\Rightarrow \quad 50^\circ + y^\circ + 120^\circ + 130^\circ = 360^\circ$$

$$\therefore \quad y = 360^\circ - 300^\circ = 60^\circ$$

55. (c) The quadrilateral must be trapezium because a qudrilateral where only one pair of opposite sides are parallel (in this case *[AB* I I *CD)* is trapezium.

56. (*b*) Given that, $AC = 2mn$, $BD = m^2 - n^2$ and $AB = \dfrac{m^2 + n^2}{2}$

We know that,



$$(AC^2 + BD^2) = 2(AB^2 + BC^2)$$

$$\Rightarrow (4m^2n^2 + m^4 + n^4 - 2m^2n^2) = 2\left\{\frac{1}{4}(m^2 + n^2)^2 + BC^2\right\}$$

$$\Rightarrow \quad (m^2 + n^2)^2 = \frac{1}{2}(m^2 + n^2)^2 + 2BC^2$$

$$\Rightarrow \quad 2BC^2 = \frac{1}{2}(m^2 + n^2)^2$$

$$\Rightarrow \quad BC^2 = \frac{(m^2 + n^2)^2}{4}$$

$$\Rightarrow \quad BC = \frac{m^2 + n^2}{2}$$

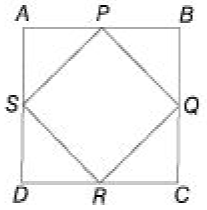
$\therefore$ $ABCD$ is a rhombus.

Let $AC > BD \Rightarrow 2mn > m^2 - n^2$

$$\Rightarrow \quad (m + n)^2 > 2m^2$$

which is always true for every positive integers m, n, where $n < m < 2n$.

57. (*c*) Let length and breadth of rectangle $ABCD$ are $2x$ and $2y$, respectively.



Then, area of $ABCD = 2x \times 2y = 4xy$

Area of all the four triangles

$$= 4 \times \frac{1}{2} \times x \times y = 2xy$$

Area of $PQRS = 4xy - 2xy = 2xy$

$$= \frac{1}{2}(4xy) = \frac{1}{2} \text{ area } (ABCD)$$

58. (*d*) In ΔABX and ΔACZ,

$AB = AD$ [side of square $ABCD$]

and $AX = AZ$ [side of a square $AXYZ$]

Let $\angle BAX = \theta$

$\therefore$ $\angle XAD = 90° - \theta$

Also, $AXYZ$ is a square.

$\therefore$ $\angle ZAX = 90°$

$\Rightarrow$ $\angle ZAD + \angle XAD = 90°$

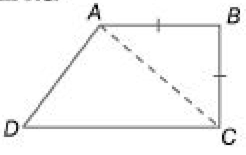
$\Rightarrow$ $ZAD = 90° - (90° - \theta) = \theta$

i.e., $\angle BAX = \angle ZAD$

$\therefore$ $\Delta ABX \cong \Delta ADZ$

$\therefore$ $BX = DZ$ [by CPCT]

59. (*c*) Join AC.



Now, in ΔABC,

$\because \quad AB = BC$

$\therefore \quad \angle BAC = \angle BCA \quad \ldots(i)$

[angles opposite to equal side]

In ΔADC,

$\because \quad CD > AD$

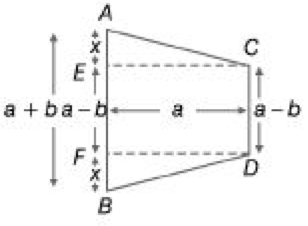
$\therefore \quad \angle DAC > \angle DCA \quad \ldots(ii)$

[since, in a triangle, angle opposite to greater side is bigger than the angle opposite to smaller side]

On adding Eqs. (i) and (ii), we get

$$\angle BAD > \angle BCD$$

60. (*d*) Since, they are symmetrically lying on horizontal plane.



$\therefore \quad AC = BD$

$\therefore \quad AE = BF = x$

Now, $\quad AB = (a - b) + 2x$

i.e., $\quad a + b = a - b + 2x$

$\Rightarrow \quad 2b = 2x$

$\therefore \quad x = b$

Now, in ΔACE,

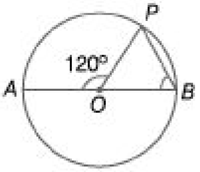
$$x^2 + a^2 = AC^2$$

$\Rightarrow \quad AC^2 = b^2 + a^2$

$\therefore \quad AC = \sqrt{b^2 + a^2}$

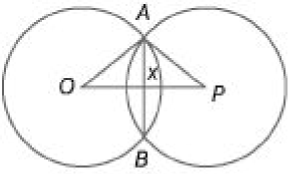
Circle

61. (c) The angle subtended by an arc at the centre is double the angle subtended by the same arc at the circumference of the circle.



$$\therefore \quad \angle PBO = \frac{1}{2} \times \angle POA$$
$$= \frac{1}{2} \times 120°$$
$$= 60°$$

62. (c)



From the figure, on joining AO and AP.

In ΔAOX,

$$AO^2 = AX^2 + OX^2 \Rightarrow (5)^2 = \left(\frac{AB}{2}\right)^2 + (OX)^2$$

$$\Rightarrow \quad 25 = \left(\frac{8}{2}\right)^2 + (OX)^2$$

$$\Rightarrow \quad 25 = (4)^2 + (OX)^2$$

$$\Rightarrow \quad (OX)^2 = 25 - 16 = 9$$

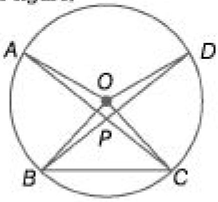
$\therefore \quad OX = \sqrt{9} = 3$ cm

Similarly, in ΔAPX, $PX = 3$ cm

$\therefore$ Distance between the centre

$$= OX + PX = 3 + 3 = 6 \text{ cm}$$

63. (*c*) From figure,



$\angle AOB + \angle COD = 2\angle ACB + 2\angle DBC$

$\Rightarrow 15^\circ + \angle COD = 2(\angle ACB + \angle DBC)$

$\Rightarrow 15^\circ + \angle COD = 2\angle APB$

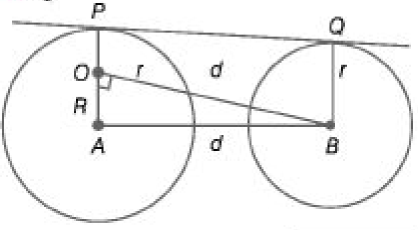
$\Rightarrow \quad \angle COD = 2 \times 30^\circ - 15^\circ$

$\therefore \quad \angle COD = 60^\circ - 15^\circ = 45^\circ$

$\therefore \quad \tan^2 \angle APB + \cot^3 \angle COD$

$$= \tan^2 30^\circ + \cot^3 45^\circ = \frac{1}{3} + 1 = \frac{4}{3}$$

64. (*b*) Let the common tangent of both circle is PQ.

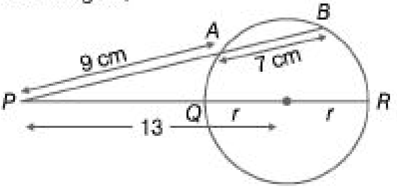


$\therefore$ From figure, $PQ = OB = \sqrt{(AB)^2 - (OA)^2}$

$$= \sqrt{d^2 - (R - r)^2}$$

65. (*b*) Let the radius of circle $= r$

From figure,



$\Rightarrow \quad PA \times PB = PQ \times PR$

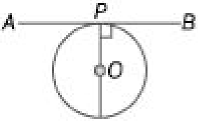
$\Rightarrow \quad 9 \times (9 + 7) = (13 - r)(13 + r)$

$\Rightarrow \quad 169 - r^2 = 144 \Rightarrow r^2 = 25$

$\Rightarrow \quad r = 5$ cm

66. (c) By definition of tangent,

A tangent to a circle is a straight line that touches the circle at a single point. Also, tangent at the end points of a diameter of a circle is perpendicular to the diameter. So, both statements are correct.

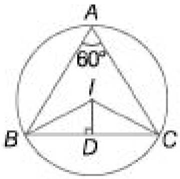


67. (6) The perpendicular bisector of the chord of a circle always pass through the centre. So, Statement I is wrong. The angle in a semi-circle is a right angle. So, Statement II is correct.

68. (C) V *CR* = *CQ* = *3 cm,BQ* = *BP* = 5 cm *AS* = *AP* = 6 cm and *DS* = *DR* = 4 cm /. Perimeter of quadrilateral *ABCD*

= [(6 + 5) + (5 + 3) + (3 + 4) + (4 + 6)] = (11 + 8 + 7+ 10) = 36 cm

69. (6) From figure



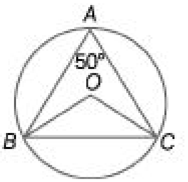
$\angle BIC = 2 \times \angle BAC = 120^\circ$ and $IB = IC$

$\therefore \quad \angle IBD = \angle ICD$

$= \dfrac{180^\circ - 120^\circ}{2} = 30^\circ$

Now, $\angle BID = 90^\circ - 30^\circ = 60^\circ$

70. (c) The angle subtended by an arc at the centre of the circle is twice the angle subtended by the arc at any point on the remaining part of the circle. ∴ *ZBOC* = 2 *ZBAC* = 2 X 50° = 100°



Now, in ΔBOC

$OB = OC$ [radii of circumcentre]

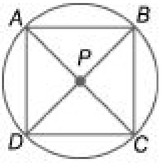
$\therefore \angle OBC = \angle OCB = x$ [let]

$\therefore x + x + 100^\circ = 180^\circ$

$\Rightarrow 2x = 80^\circ$

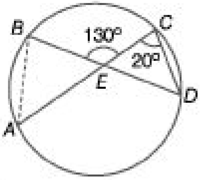
$\Rightarrow x = 40^\circ$

71. (*b*)



So, $AP.BP = CP.DP$

72. (*c*)



$$\angle CED = 180° - 130° = 50°$$

Now, in ΔCED

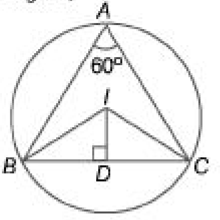
$$\angle ECD + \angle CED + \angle CDE = 180°$$

$\Rightarrow \quad \angle CDE = 180° - 50° - 20° = 110°$

$\therefore \quad \angle BAC = \angle CDE = 110°$

[angles in same segment are equal]

73. (*b*) From figure,

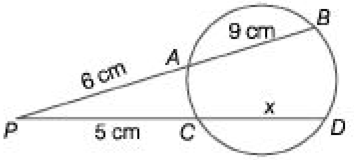


$\angle BIC = 2 \times \angle BAC = 120°$ and $IB = IC$

$\therefore \quad \angle IBD = \angle ICD = \dfrac{180° - 120°}{2} = 30°$

Now, $\angle BID = 90° - 30° = 60°$

74. (*a*) If two chord of a circle, intersect inside a circle (outside a circle) at any point. Then,

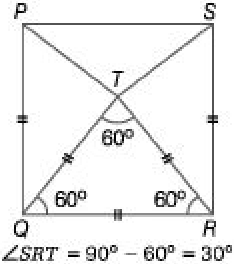


$$PA \times PB = PC \times PD$$

$\Rightarrow \quad 6 \times 15 = 5 \times (x + 5)$

$\Rightarrow \quad x + 5 = 18 \Rightarrow x = 13$ cm

75. (*d*) In ΔSRT,



$$\angle SRT = 90° - 60° = 30°$$

$\therefore \quad \angle RTS = \dfrac{1}{2}(180° - 30°) = 75°$

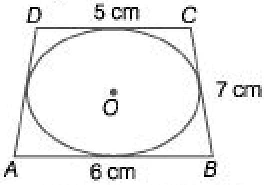
$\therefore \quad \angle PTQ = \angle RTS = 75°$

$\because \angle PTS + \angle PTQ + \angle QTR + \angle RTS = 360°$

$\Rightarrow \quad \angle PTS + 75° + 60° + 75° = 360°$

$\Rightarrow \quad \angle PTS = 360° - 210° = 150°$

76. (*a*) From figure,



$\Rightarrow \qquad AB + CD = BC + AD$

[Since, sides of quadrilateral touches the circle]

$\Rightarrow \qquad 6 + 5 = 7 + AD$

$AD = 11 - 7 = 4$ cm

77. (*c*) $OA = OB \Rightarrow \angle OBA = \angle OAB = 50°$

In ΔOAB,

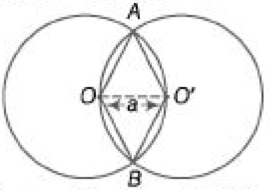
$$\angle OAB + \angle OBA + \angle AOB = 180°$$

$$\Rightarrow \qquad 50° + 50° + \angle AOB = 180°$$

$$\Rightarrow \qquad \angle AOB = 80°$$

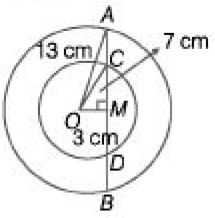
$$\therefore \qquad \angle BOD = (180° - 80°) = 100°$$

78. (*b*) Here, $OA = OB = AB = r$



$\therefore$ Perimeter of hexagon $= 6 \times AB = 6r$

79. (*d*) From figure,



In ΔOCM, $CM^2 = 7^2 - 3^2 = 40$

$\therefore \qquad CM = 2\sqrt{10}$ cm

In ΔAOM,

$$AM = \sqrt{13^2 - 3^2} = 4\sqrt{10}$$

Now, $\quad AC = AM - CM = 4\sqrt{10} - 2\sqrt{10}$

$= 2\sqrt{10}$ cm

80. (*c*) Since, $AB \mid\mid PR$ and $QOL \perp AB$

[$\because OQ \perp PR \Rightarrow LOQ \perp PR$]

Since, OL bisects chord AB.

So, ΔAQB is isosceles.

$\therefore \qquad \angle LQA = \angle LQB$

But $\qquad \angle LQB = (90° - 70°) = 20°$

$\Rightarrow \qquad \angle LQA = \angle LQB = 20°$

$\therefore \qquad \angle AQB = 40°$

81. (*b*) Join OP. Now, $\angle BPA = 90^\circ$

[angle in semi-circle]

In ΔPBA,

$$\angle BPA + \angle PBA + \angle BAP = 180^\circ$$

$\Rightarrow$ $90^\circ + 30^\circ + \angle BAP = 180^\circ$

$\therefore$ $\angle BAP = 60^\circ$

But, BAT is a straight angle.

$\Rightarrow$ $\angle BPA + \angle PAT = 180^\circ$

$\Rightarrow$ $60^\circ + \angle PAT = 180^\circ$

$\Rightarrow$ $\angle PAT = 120^\circ$

$OA = OP \Rightarrow \angle OPA = \angle OAP$

$= \angle BAP = 60^\circ$

Now, $\angle OPT = 90^\circ$

$\Rightarrow$ $\angle OPA + \angle APT = 90^\circ$

$\Rightarrow$ $60^\circ + \angle APT = 90^\circ$

$\therefore$ $\angle APT = 30^\circ$

In ΔPAT, we have

$$\angle PAT + \angle APT + \angle PTA = 180^\circ$$

$\therefore$ $\angle PTA = 30^\circ$

82. (*d*) Since, PQL is a tangent and OQ is a radius, so $\angle OQL = 90^\circ$

$$\angle OQS = (90^\circ - 50^\circ) = 40^\circ$$

Now, $OQ = OS$

$\Rightarrow$ $\angle OSQ = \angle OQS = 40^\circ$

Similarly, $\angle ORS = (90^\circ - 60^\circ) = 30^\circ$

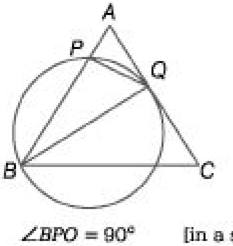
and $OR = OS$

$\Rightarrow$ $\angle OSR = \angle ORS = 30^\circ$

$\Rightarrow$ $\angle QSR = \angle OSQ + \angle OSR$

$= (40^\circ + 30^\circ) = 70^\circ$

83. (*b*)



$\angle BPO = 90^\circ$ [in a semi-circle]

Let $AB = AC = 2x$

Then, $AQ = QC = x$

In ΔABQ, $BQ^2 = AB^2 - AQ^2$

$= (2x)^2 - x^2 = 3x^2$

$\Rightarrow$ $BO = \sqrt{3}x$

Now, in ΔBPQ, $BQ^2 = BP^2 + PQ^2$ ...(i)

In ΔAPQ, $AQ^2 = AP^2 + PQ^2$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$$AQ^2 - BQ^2 = AP^2 - BP^2$$

$\Rightarrow$ $BP^2 - AP^2 = BQ^2 - AQ^2$

$\Rightarrow$ $(BP + AP)(BP - AP) = 3x^2 - x^2$

$\Rightarrow$ $2x.(BP - AP) = 2x^2$

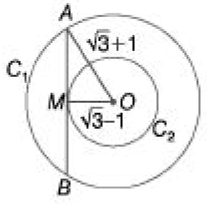
$\Rightarrow$ $BP - AP = x$ ...(iii)

and $BP + AP = 2x$ ...(iv)

From Eqs. (iii) and (iv), $AP = \frac{x}{2}$

$\therefore$ Required ratio $= \frac{AP}{AB} = \frac{2}{2x} = \frac{1}{4} = 1:4$

84. (*b*) Let the chord AB of circle C_1 touches the circle C_2 at point M.



Then, $OA = \sqrt{3} + 1$ and $OM = \sqrt{3} - 1$

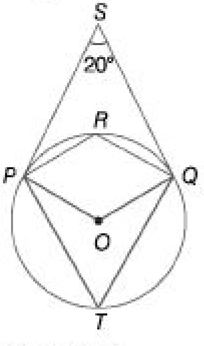
Now, in right angled ΔOAM,

$$AM^2 = OA^2 - OM^2 = (\sqrt{3}+1)^2 - (\sqrt{3}-1)^2$$
$$= (3 + 1 + 2\sqrt{3}) - (3 + 1 - 2\sqrt{3})$$

$\Rightarrow \quad AM^2 = 4\sqrt{3} \Rightarrow AM = 2\sqrt[4]{3}$

$\therefore \quad AB = 2AM = 4\sqrt[4]{3}$

85. (*c*) Join P and Q with an another point, say T, on the major arc.



Also, join PO and QO.

In quadrilateral $POQS$,

$$\angle PSQ = 20°$$
$$\angle OPS = \angle OQS = 90°$$

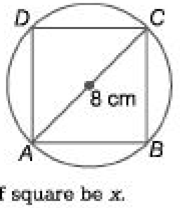
$\therefore \quad \angle POQ = 360° - (90° + 90° + 120°)$
$= 160°$

$\therefore \quad \angle PTQ = \frac{1}{2}\angle POQ = \frac{1}{2} \times 160° = 80°$

Now, $PTQR$ is a cyclic quadrilateral.

$\therefore \quad \angle PRQ = 180° - \angle PTQ$
$= 180° - 80° = 100°$

86. (*b*)



Let side of square be x.

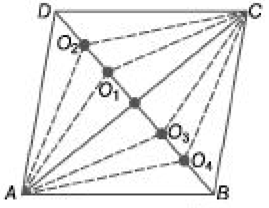
$$x^2 + x^2 = 8^2$$

$\Rightarrow \quad 2x^2 = 64 \Rightarrow x^2 = \frac{64}{2} \Rightarrow x = \frac{8}{\sqrt{2}}$

$\therefore$ Area of square $= \left(\frac{8}{\sqrt{2}}\right)^2 = 32 \text{ cm}^2$

Area of circle

$$= \frac{22}{7} \times \left(\frac{8}{2}\right)^2 \text{ cm}^2 = \frac{22}{7} \times 16 = \frac{352}{7}$$



$\therefore$ Required difference $= \frac{352}{7} - 32$

$$= \frac{352 - 224}{7} = \frac{128}{7} = 18\frac{2}{7} \text{ cm}^2$$

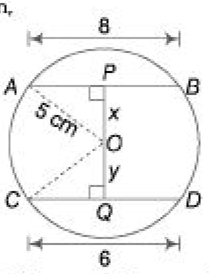
$\because$ Sum of corresponding angles of a cyclic quadrilateral $= 180°$

$$\angle Q + \angle S = 180°$$

$\Rightarrow \quad 45° + \angle S = 180°$

$\therefore \quad \angle S = 180° - 45° = 135°$

87. (*a*) Given,



Applying Pythagoras theorem in $\angle AOP$ and ΔCOQ.

In ΔAOP, $\quad AO^2 = AP^2 + PO^2$

$\Rightarrow \quad AO^2 = \left(\frac{AB}{2}\right)^2 + PO^2$

$\Rightarrow \quad (5)^2 = (4)^2 + x^2$

$\Rightarrow \quad 25 - 16 = x^2$

$\therefore \quad x = \sqrt{9} = 3 \text{ cm}$

In ΔCOQ,

$$CO^2 = OQ^2 + CQ^2$$

$\Rightarrow \quad CO^2 = OQ^2 + \left(\frac{CD}{2}\right)^2$

$\Rightarrow \quad (5)^2 = y^2 + (3)^2$

$\Rightarrow \quad 25 - 9 = y^2$

$\Rightarrow \quad y = \sqrt{16} = 4 \text{ cm}$

$\therefore PQ = PO + OQ = 3 + 4 = 7 \text{ cm}$

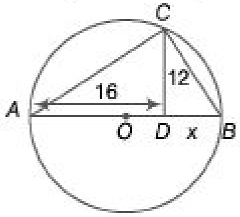
88. (*d*) Let $BD = x$

In ΔACD,

$$AC = \sqrt{16^2 + 12^2}$$
$$= \sqrt{256 + 144}$$
$$= \sqrt{400} = 20 \text{ cm}$$

In ΔBCD,

$$BC = \sqrt{12^2 + x^2} = \sqrt{144 + x^2}$$



Now, in ΔABC

$$\angle ACB = 90^\circ$$

[angle subtended by semi-circle]

$$\therefore \quad (AB)^2 = (AC)^2 + (BC)^2$$

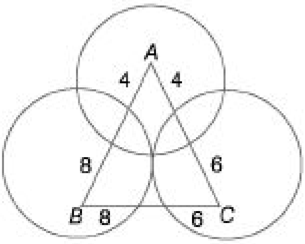
$$(16 + x)^2 = (20)^2 + (\sqrt{144 + x^2})^2$$

$$\Rightarrow \quad 256 + x^2 + 32x = 400 + 144 + x^2$$

$$\Rightarrow \quad 32x = 288$$

$$\Rightarrow \quad x = 9 \text{ cm}$$

89. (*b*) From figure,



$$AB = 12 \text{ cm} = a$$
$$BC = 14 \text{ cm} = b$$
and $$CA = 10 \text{ cm} = c$$

Area of ΔABC

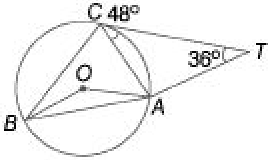
$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{AB + BC + CA}{2}$$
$$= \frac{12 + 14 + 10}{2}$$
$$= \frac{36}{2} = 18$$

$$s - a = 18 - 12 = 6$$
$$s - b = 18 - 14 = 4$$
$$s - c = 18 - 10 = 8$$

$$\Rightarrow \quad \sqrt{18 \times 4 \times 6 \times 8} = 24\sqrt{6} \text{ cm}^2$$

90. (c) $\angle ACT = 48°, \angle ATC = 36°$



$\therefore \quad \angle CAT = 180° - (48° + 36°)$

$= 180° - 84° = 96°$

$\therefore \quad \angle CAB = 180° - 96° = 84°$

Now, $\angle ABC = \angle ACT = 48°$

[angles made in alternate segments]

$\therefore \quad \angle BCA = 180° - (\angle ABC + \angle CAB)$

$= 180° - (48° + 84°) = 48°$

$\therefore \quad \angle BOA = 2 \times \angle BCA = 2 \times 48° = 96°$

91. (a) I. We know that,

Radius $= \dfrac{\text{Arc}}{\text{Angle}}$ [given arc length is constant]

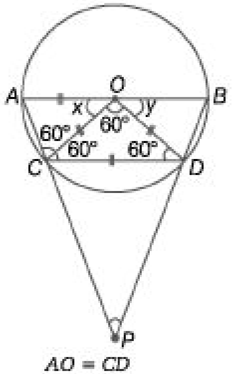
Radius $\propto \dfrac{1}{\text{Angle}}$

$\therefore$ Angular measure in radian decreases, if the radius of the arc increases.

II. $1800° \times \dfrac{\pi}{180°} = 10\,\pi$

$\therefore$ Only Statement I is correct.

92. (a)



$AO = CD$

$\Rightarrow \quad OC = OD = CD$

[$\because AO = OC = OD =$ radii]

ΔCOD is equilateral.

$\angle x + \angle y = 180° - 60°$

and $\quad \angle x = \angle y$

$\therefore \quad \angle 2x = 120°$

$\angle x = 60°$

and $\angle AOC$ is equilateral.

$\therefore \quad \angle DCP = 180° - 120° = 60°$

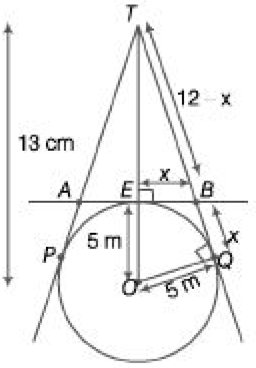
and $\quad \angle CDP = 60°$

$\therefore \angle APB = 360° - (60° + 120° + 120°) = 60°$

93. (b) In ΔOTO,

$OT^2 = OQ^2 + TQ^2$

$\Rightarrow \qquad (13)^2 = (5)^2 + (TQ)^2$



$\Rightarrow \qquad TQ^2 = 169 - 25 = 144$

$\Rightarrow \qquad TQ = 12 \text{ cm}$

Then, in ΔTEB,

$TB^2 = EB^2 + TE^2$

$\Rightarrow \qquad (12 - x)^2 = BQ^2 + TE^2$

$\because \qquad (EB = BQ)$

$\Rightarrow \qquad 144 + x^2 - 24x = x^2 + (8)^2$

$\Rightarrow \qquad 144 + x^2 - 24x = x^2 + 64$

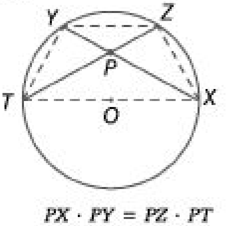
$\Rightarrow \qquad 24x = 80 \Rightarrow x = \dfrac{20}{6} = \dfrac{10}{3} \text{ cm}$

$\therefore \qquad AB = 2EB$

$= 2x = 2 \times \dfrac{10}{3}$

$\Rightarrow \qquad AB = \dfrac{20}{3} \text{ cm}$

94. (*c*) When two chords of a circle intersects internally, then they are divided in a proportion.



i.e., $\qquad PX \cdot PY = PZ \cdot PT$

In ΔPXZ and ΔPTY,

$\angle ZPX = \angle YPT$

[vertically opposite angles]

$\angle PZX = \angle PYT$

[angles in same segment]

$\angle PXZ = \angle PTY$

[angles in same segment]

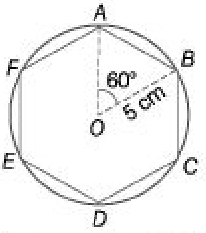
$\therefore \qquad \Delta PXZ \cong \Delta PTY$

Hence, the both statements are correct.

95. (*a*) $OB = OA$ = radius

Also, $\angle AOB = 60^\circ \left(\dfrac{360^\circ}{6} = 60^\circ\right)$

and $\angle OAB = \angle OBA = 60^\circ$



$\therefore$ ΔAOB is an equilateral triangle.

Then,

$$AB = 5 \text{ cm}$$

So, Area,

$$x = \text{Area of circle} - \text{Area of hexagon}$$

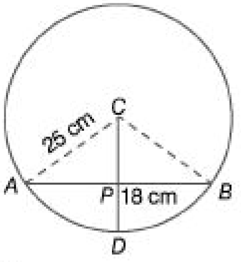
$$= \pi r^2 - \frac{3\sqrt{3}\,(a)^2}{2}$$

$$= \frac{22}{7} \times (5)^2 - \frac{3\sqrt{3}}{2} \times (5)^2$$

$[\because r = a = 5]$

$$= 78.57 - 64.95 = 13.62 \text{ cm}^2$$

96. (*d*)



In ΔACP,

$$CP = CD - PD = 25 - 18 = 7$$

Now, $AC^2 = CP^2 + AP^2$

$$\therefore\ AP = \sqrt{AC^2 - CP^2} = \sqrt{(25)^2 - (7)^2}$$

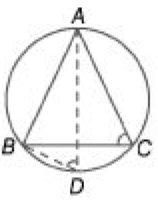
$$= \sqrt{625 - 49} = \sqrt{576} = 24 \text{ cm}$$

Similarly, $PB = 24$ cm

$\therefore$ $AB = AP + PB$

$$= 24 + 24 = 48 \text{ cm}$$

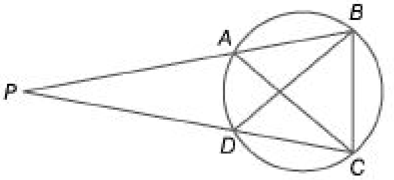
97. (*b*)



$$\angle ADB = \angle ACB = 60°$$

[angles in the same segment are equal]

98. (*d*) AB and CD are chords when produced meet extremelly at P

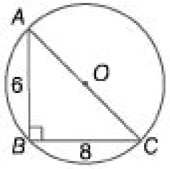


$\therefore \quad AP \times BP = DP \times DP$

Now,

As $ACNBD$ and ΔPAC is not similar to ΔPDB.

99. (*d*) Since, ΔABC is right angled at B.



Using Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$\Rightarrow \quad AC^2 = 6^2 + 8^2$

$\Rightarrow \quad AC^2 = 36 + 64$

$\quad = 100$

$\therefore \quad AC = 10$ cm

and in case of right angled triangle, radius lies of *h* hypotenuse and is the circumcircle of ΔABC.

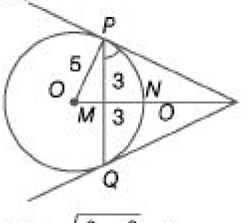
$\therefore$ Radius of circumcircle

$$= \frac{10}{2}$$

$$= 5 \text{ cm}$$

100. (*c*) Let distance from M point to T point $= x$ cm.

In ΔOPM,



$$OM = \sqrt{5^2 - 3^2} = 4$$

$\therefore \quad MN = ON - OM = 5 - 4 = 1$ cm

In ΔPOT,

$$PT^2 = OT^2 - OP^2 = (5 + x)^2 - 5^2$$

$$= x^2 + 10x \quad \ldots(i)$$

In ΔPMT,

$$PT^2 = PM^2 + TM^2$$
$$= 3^2 + (1 + x)^2$$
$$= x^2 + 2x + 10 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

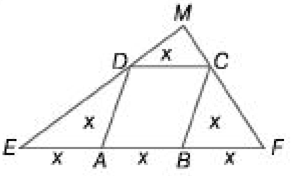
$$x^2 + 2x + 10 = x^2 + 10x$$

$$\Rightarrow \quad x = \frac{10}{8} = \frac{5}{4}$$

From Eq. (i),

$$PT = \sqrt{x^2 + 10x}$$
$$= \sqrt{\left(\frac{5}{4}\right)^2 + 10\left(\frac{5}{4}\right)}$$
$$= 3.75 \text{ cm}$$

101. (*d*)



Produce EC and FC to meet at a point M.
Let each side of the rhombus be x. Then,
$AB = BC = CD = AD = BF = AE = x$

$\because \quad \angle DAB + \angle CBA = 180°$

$\therefore \quad \angle DAE + \angle CBF = 180°$

In ΔADF,

$$\angle EAD + \angle AED + \angle ADE = 180° \quad \ldots(i)$$

In ΔBCF,

$$\angle CBE + \angle CFB + \angle BCF = 180° \quad \ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$(\angle EAD + \angle CBF) + (\angle AED + \angle ADE) + (\angle CFB + \angle BCF)$$

$$\Rightarrow \quad 180° + \angle 2\ AED + 2\angle CFB = 360°$$
$$[\because AE = AD \text{ and } BC = BF]$$

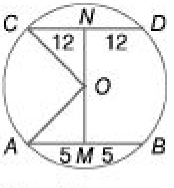
$$\Rightarrow \quad \angle AED + \angle CFB = 90°$$

$$\Rightarrow \quad \angle EMF = 90°$$

$$\Rightarrow \quad EM \perp FM$$

$$\therefore \quad ED \perp CF$$

102. (*b*) Let radius of the circle be r.



Given, $\quad MN = 17$ cm

Let $\quad ON = x$

Then, $\quad OM = 17 - x$

In ΔAOM,

$$OA^2 = OM^2 + AM^2$$

$\Rightarrow \quad r^2 = (17 - x)^2 + 5^2$

$= 289 + x^2 - 34x + 25$

$r^2 = 314 + x^2 - 34x \quad \ldots(i)$

$OC^2 = ON^2 + CN^2$

$\Rightarrow \quad r^2 = x^2 + 12^2 = x^2 + 144 \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$314 + x^2 - 34x = x^2 + 144$

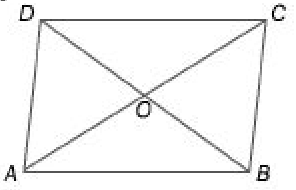
$\Rightarrow \quad 34x = 170$

$\Rightarrow \quad x = 5$

From Eq. (ii), $r^2 = 25 + 144 = 169$

$r = 13$ cm

103. (*b*) **Statement I** $ABCD$ is a parallelogram, then



In ΔABC

$AC^2 = AB^2 + BC^2 \quad \ldots(i)$

and in ΔCBD

$BD^2 = BC^2 + CD^2 \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$AC^2 + BD^2 = AB^2 + BC^2 + BC^2 + CD^2$

$= 2AB^2 + 2BC^2 \quad [\because AB = CD]$

$AC^2 + BD^2 = 2(AB^2 + BC)^2$

Statement II $ABCD$ is a rhombus and diagonals AC and BD bisect each other.

$\therefore \quad AO = OC$ and $OB = OD$

In ΔAOB,

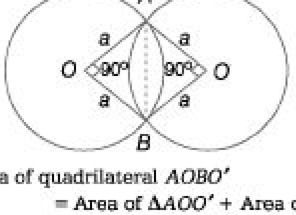
$AB^2 = AO^2 + OB^2$

$\Rightarrow \quad (4)^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$

$\therefore \quad AC^2 + BD^2 = 64 = (4)^3$

i.e., n^3 only Statement II is correct.

104. (*c*) From the figure,



Area of quadrilateral $AOBO'$

$= \text{Area of } \Delta AOO' + \text{Area of } \Delta BOO'$

Now, $AO = OO' = AO' = a$

So, AOO' is a equilateral triangle.

Similarly, BOO' is also a equilateral triangle

$= \frac{\sqrt{3}}{4}a^2 + \frac{\sqrt{3}}{4}a^2 = \frac{2\sqrt{3}a^2}{4} = \frac{\sqrt{3}}{2}a^2$

Chapter

37

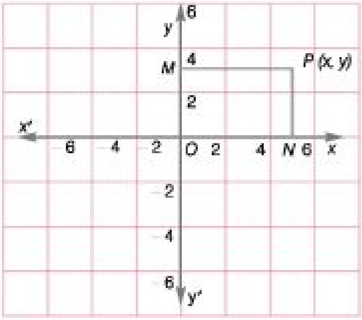
Coordinate Geometry

It is a system of geometry, where the position of points on the plane is described by using an ordered pair of numbers.

Rectangular Coordinate Axes

The lines XOX' and YOY' are mutually perpendicular to each other and they meet at point O which is called the origin.

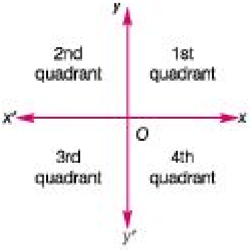
Line XOX' represents X-axis and line YOY' represents Y-axis and together taken, they are called coordinate axes. Any point in coordinate axis can be represented by specifing the position of ***x*** and **y-coordinates.**



Quadrants

The X and Y-axes divide the cartesian plane into four regions referred to quadrants.

Quadrant	Region	Sign of (x, y)	Example
I	XOY	$(+, +)$	$(2, 3)$
II	YOX'	$(-, +)$	$(-2, 4)$
III	$X'OY'$	$(-, -)$	$(-1, -2)$
IV	$Y'OX$	$(+, -)$	$(1, -3)$



♦ The coordinates of point O (origin) are taken as (0,0).

♦ The coordinates of any point on X-axis are of the form (x, 0). •f The coordinates of any point on Y-axis are of the form (0, y).

Distance Formula

Distance between Two Points If $A\ (x_1, y_1)$ and $B(x_2, y_2)$ are two points, then

$$AB = V(x_2-x_1)^2 + (y_2-y_1)^2$$

A (xi,y0

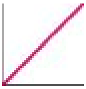
-*- x

For example The distance between *A* (1,2) and *B* (5,6) is

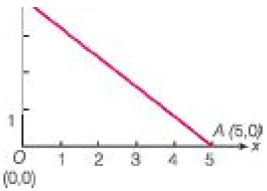
AB = ^(5 -l)^2 + (6 -2)^2 = ^4^2 + 4^2 = V32; AB=W2 units **Distance of a Point from the Origin** The distance of a point *A* (x, y) from the origin O (0, 0) is given by *OA* = -Jx^2 + y^2

M

(0,0) 0 *For example* The distance of point *A* (5,3) from origin is



OA = ^5^2 + 3^2 = ^25 +9 = V34 units



Area of a Triangle y|

4 .8 (0,4J

If *A* (x^yJ.B (x2,y2) and C(x3,y3) are three vertices of a *AABC,* then its area is given by 3

1 Area of AABC = - xa (y2 - y3) + x2(y3 - yj + x3 *(y1* - y2) 2

For example If we are to find the area of a triangle having the vertices (0, 0), (5, 0) and (0, 4), then (xj.yi) = (0,0) (x2, y2) = (5,0) and (x3, y3) = (0,4)

∴ Area of Z =-{0(0-4) + 5(4-0) + 0(0-0)} =ix20=10

Collinearity of Three Points

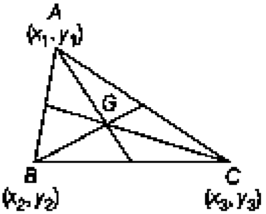
Three points A(x_1,y_1),B(x_2,y_2) *and* C(x_3,y_3) *are collinear, if* (i) Area of triangle *ABC* is 0

i.e., Xi(y_2 -y_3) + x_2(y_3 -y_a) + Xgfyj -y_2) =0 (ii) Slope of *AB* = Slope of *BC* = Slope of *AC*

(iii) Distance between A and B + Distance between B and C = Distance between A andC.

Centroid of a Triangle

Centroid is the point of intersection of all the three medians of a triangle. If *A* (x-j, *Y\),* B (x_2, *Y2)* an^ C (x_3, y_3) are the vertices of *AABC,* then the coordinates of its centroid are



$$\left(\frac{1}{3}(x_1 + x_2 + x_3), \frac{1}{3}(y_1 + y_2 + y_3)\right)$$

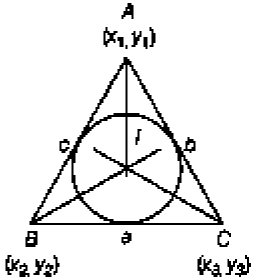
Circumcentre

The circumcentre of a triangle is the point of inter section of the perpendicular bisectors of its sides and is equidistance from all three vertices. If *A[xi,Yi)i B(x_2,Y2)* and C(x_3,y_3) are the vertices of triangles and 0(x,y) is the circumcentre of AABC, then *OA* = *OB*= *OC*

$$\sqrt{(x - x_1)^2 + (y - y_1)^2} = \sqrt{(x - x_2)^2 + (y - y_2)^2} = \sqrt{(x - x_3)^2 + (y - y_3)^2}$$

Incentre

The centre of the circle, which touches the sides of a triangle, is called its incentre. Incentre is the point of intersection of internal angle bisectors of triangle.



If *A*(xlr yj, *B*(x2,y2) and C (x3,y3) are the vertices of a AABC such that *BC* = *a, CA* = *b* and *AB* = *c,* then coordinates of its incentre *1* are

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

Section formulae

Let Afx^yi) and B(x2,y2) be two points on the cartesian plane. Let point P(x,y) divides the line *AB* in the ratio of *m : n* internally. Then,

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

If P divides AB externally, then

$$x = \frac{mx_2 - nx_1}{m-n}, y = \frac{my_2 - ny_1}{m-n}$$

If P is the mid-point of AB, then

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

Basic Points Related to Straight Lines

1. General form of equation of straight line is *ax* + by + c = 0. Where, *a, b* and c are real constants and x and y are two unknowns.

2. The equation of a line having slope *m* and intersects at c on x-axis is y = mx + c.

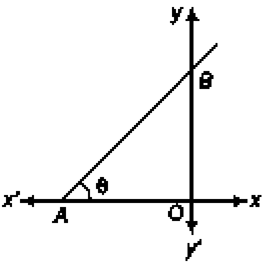
3. **Slope (gradient) of a line $ax + by + c = 0$**

$$by = -ax - c$$

$$y = -\frac{a}{b}x - \frac{c}{b}$$

Comparing with $y = mx + c$, where m is slope

$$\therefore \qquad m = \tan\theta = -\frac{a}{b}$$



Slope of the line is always measured in anti-clockwise direction.

4. **Point slope form** A line in terms of coordinates of any two points on it, if (x^yi) and (x_2,y_2) are coordinates of any two points on a line, then its slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Difference of ordinates}}{\text{Difference of abscissae}}$$

5. **Two point form** a line the equation of a line passing through the points *A* (xj, yj) and B(x_2,y_2)is

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow (y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

Slope of such a line m is $\frac{y_2 - y_1}{x_2 - x_1}$.

6. **Condition of parallel lines** If the slopes of two lines i.e., m_1 and m_2 are equal then lines are parallel.

♦ ***mi = w.2***

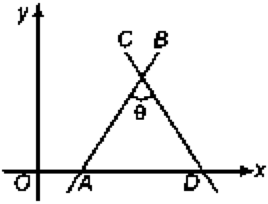
+ Equation of line parallel to ax + by + c = 0 is ax + by + q =0.

7. **Condition of perpendicular lines** If the multiplication of slopes of two lines *i.e.,* m-L and m_2 is equal to —% then lines are perpendicular.

•f m_1-m_2=-l

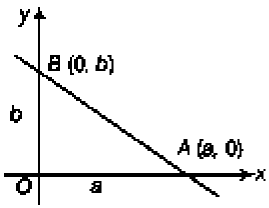
■f Equation of line perpendicular to *ax* + by + c = 0 is bx - ay + q =0.

8. Angle between the two lines $\tan\theta = \pm\left(\dfrac{m_2 - m_1}{1 + m_1 m_2}\right)$



9. **Intercept form** Equation of line *L* intersects at *a* and b on x and y-axes,

respectively is $\dfrac{x}{a} + \dfrac{y}{b} = 1.$

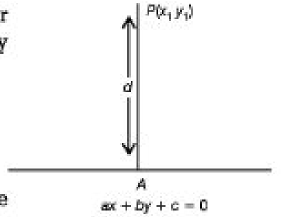


10. Condition of concurrency of three lines Let the equation of three lines are qx + *fyy* + q = 0, *aqx* + *bfl* + <% = 0 and a_3x + *b^y* + c_3 =0. Then, three lines will be concurrent, if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

11. **Distance of a point from the line** Let ax + by + c = 0 beany equation of line and

$P(x_1\ y_1)$ be any point in space. Then, the perpendicular Distance (d) of a point P from a line is given by
$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

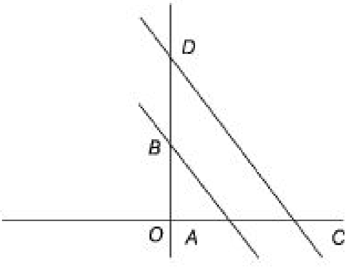


12. The length of the perpendicular from the origin to the line $ax + by + c_2 = 0$, is $\dfrac{|c|}{\sqrt{a^2 + b^2}}$

13. Area of triangle by straight line $ax + by + c = 0$ $a \neq 0$, $b \neq 0$ with coordinate axes is $\left|\dfrac{1}{2}\dfrac{c^2}{ab}\right|$.

14. Distance between parallel lines $ax + by + c = 0$ and $ax + by + d = 0$ is equal to $\left|\dfrac{d - c}{\sqrt{a^2 + b^2}}\right|$.

15. Area of trapezium, between two parallel lines and axes,
Area of trapezium $ABCD$ = Area of OCD –
Area of $OAB = \dfrac{1}{2}\left|\dfrac{d^2}{ab}\right| - \dfrac{1}{2}\left|\dfrac{c^2}{ab}\right|$.
$$= \frac{1}{2}\left(\left|\frac{d^2}{ab}\right| - \left|\frac{c^2}{ab}\right|\right)$$



Note Don't write it as $\dfrac{1}{2}\left|\dfrac{d^2 - c^2}{ab}\right|$

Ex. 1 In which quadrant do the given points lie?

(i)(-8,-6) (II) (8,-4) (IN) (-4, 6) (iv)(4,4)

Sol. (i) Since, abscissa (- 8) and ordinate (- 6) are negative, therefore (- 8, - 6) lies in 3rd quadrant.

(ii) Since, abscissa (8) is positive and ordinate (- 4) is negative, therefore (8, - 4) lies in 4th quadrant.

(iii) Since, abscissa (- 4) is negative and ordinate (6) is positive, therefore (- 4, 6) lies in 2nd quadrant.

(iv) Since, abscissa (4) and ordinate (4) are positive, therefore (4, 4) lies in 1st quadrant. Ex. 2 Find the distance between the points *A* (- 6, 8) and *B* (4, - 8).

Sol. Here, $A(-6, 8) = A(x_1, y_1)$ and $B(4, -8) = B(x_2, y_2)$

$\therefore$ $x_1 = -6$, $y_1 = 8$, $x_2 = 4$ and $y_2 = -8$

$\therefore$ Required distance $= AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{\{4 - (-6)\}^2 + (-8 - 8)^2} = \sqrt{(4 + 6)^2 + (-16)^2}$$

$$= \sqrt{100 + 256} = \sqrt{356} = 18.86 \text{ units}$$

Ex. 3 Find the distance of the point *A* (8, - 6) from the origin.

Sol. Distance of a point $A(8, -6)$ from the origin $O(0, 0)$ is

$$OA = \sqrt{(8)^2 + (-6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10 \text{ units}$$

Ex. 4 Find the area of *AABC,* whose vertices are *A* (8, - 4), *B* (3, 6) and C (- 2, 4).

Sol. Here, $A(8, -4)$ so, $x_1 = 8$, $y_1 = -4$

$B(3, 6)$ so, $x_2 = 3$, $y_2 = 6$

$C(-2, 4)$ so, $x_3 = -2$, $y_3 = 4$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

$$= \frac{1}{2}\{8(6 - 4) + 3(4 - (-4)) + (-2)(-4 - 6)\}$$

$$= \frac{1}{2}\{16 + 24 + 20\} = \frac{1}{2} \times 60 = 30 \text{ sq units}$$

Ex. 5 Find the value of *k* for which the points *A(*-1, 3), *B* (2, *k)* and C (5, -1) are collinear.

Sol. Here, $x_1 = -1$, $x_2 = 2$, $x_3 = 5$, $y_1 = 3$, $y_2 = k$ and $y_3 = -1$ Since, points are collinear. Area (A) = 0

=> zi (2/2 - 2/3) + x2 iV3 ~ Vl) + Z3 (Vi - 3/2) = 0 => - 1. (k + 1) + 2 (- 1 - 3) + 5 (3 - k) = 0

=> -fc-l-8+ 15-5fc = 0

=> 6fe = 6=>fc = l

Ex. 6 What is the slope of the line perpendicular to the line passing through the points (3,5) and (-4, 2) ?

Sol. Let m_1 be the slope of the line passing through the points $A(3, 5)$ and $B(-4, 2)$.

$\therefore$ $$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{-4-3} = \frac{3}{7}$$

Let the line having slope m_2 is perpendicular to the line.

Then, $$m_1 \cdot m_2 = -1$$

$\therefore$ $$\frac{3}{7} \times m_2 = -1 \Rightarrow m_2 = \frac{-7}{3}$$

$\therefore$ Slope of perpendicular line is $\frac{-7}{3}$.

Ex. 7 If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, - 3) and (3,4). Find the coordinates of the centroid.

Sol. Let $P(1, 1)$, $Q(2, -3)$ and $R(3, 4)$ be the mid-points of sides AB, BC and CA, respectively of triangle ABC.

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of triangle ABC.

P is mid-point of AB,

$$\frac{x_1 + x_2}{2} = 1, \quad \frac{y_1 + y_2}{2} = 1$$

$$x_1 + x_2 = 2, \quad y_1 + y_2 = 2 \qquad \text{...(i)}$$

Q is mid-point of BC,

$$\frac{x_2 + x_3}{2} = 2, \quad \frac{y_2 + y_3}{2} = -3$$

$$x_2 + x_3 = 4, \quad y_2 + y_3 = -6 \qquad \text{...(ii)}$$

R is mid-point of AC, $$\frac{x_1 + x_3}{2} = 3, \quad \frac{y_1 + y_3}{2} = 4$$

$$x_1 + x_3 = 6, \quad y_1 + y_3 = 8 \qquad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$(x_1, y_1) = (2, 8), (x_2, y_2) = (0, -6) \text{ and } (x_3, y_3) = (4, 0)$$

Then, coordinates of the centroid

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right) = \left(\frac{2+0+4}{3}, \frac{8-6+0}{3}\right) = \left(2, \frac{2}{3}\right)$$

Ex. 8 If *A* (-2,1), B(2, 3) and C(-2, -4) are three points, then find the angle between *AB* and *BC*.

Sol. Let m_1 and m_2 be the slopes of line AB and BC, respectively.

$\therefore$ $$m_1 = \frac{3-1}{2-(-2)} = \frac{2}{4} = \frac{1}{2} \text{ and } m_2 = \frac{-4-3}{-2-2} = \frac{-7}{-4} = \frac{7}{4}$$

Let θ be the angle between AB and BC

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\frac{7}{4} - \frac{1}{2}}{1 + \frac{7}{4}\cdot\frac{1}{2}}\right| = \pm\frac{2}{3}$$

$\therefore$ $$\theta = \tan^{-1}\left(\frac{2}{3}\right)$$

Fast Track Practice

1. In which quadrant, does the point (- 5, 7) lie? [Railways 2004]

(a) 1st ft) 2nd

(c) 3rd ft) 4th

2. On which axis does the point (6, 0) lie?

(a) x-axis ft) y-axis

ft) Either x or y *(d)* At origin *(e)* None of the above

3. The distance of the point *P* (8, - 6) from the origin is

(a) 2 units *(b)* 8 units

(c) 10 units ft) 16 units

(e) None of the above

4. If the distance between the points *(x,* 0) and (-7, 0) be 10 units, then the possible values of *x* are [SSC FCI 2012]

(a) 3 and 17 ft)-3 and 17

ft) 3 and -17 ft) -3 and -17

5. If the distance of the point *P (x, y)* from *A(a,* 0) is $a + x$, then $y^2 = ?$

(a) 2ax ftj 4ax ft) 6ax *(d)* 8ax (e) None of the above

6. The intersection point on the x-axis of *7x - Zy* = 2 is [SSC (10+2) 2012]

(a) $\frac{2}{5}$ (b) $\frac{2}{7}$ (c) $\frac{3}{4}$ (d) $\frac{3}{7}$

7. Locus of the points equidistant from the points (- 1,-1) and (4, 2) is

(a) $5x - 3y - 9 = 0$ *(b)* $5x + 3y + 9 = 0$ *(c)* $5x + 3y - 9 = 0$ ft) $5x - 3y + 9 = 0$ *(e)* None of the above

8. Coordinates of a point is (0, 1) and ordinate of an another point is - 3. If distance between both the points is 5, then abscissa of second point is

(a) 3 *(b)* -3

ftj±3 ft)1

(e) -1

9. The vertices of a triangle are A (4, 4), *B* 3,- 2 and *C* (- 3,16). The area of the triangle is

(a) 30 sq units *(b)* 36 sq units ft) 27 sq units ft) 40 sq units *(e)* None of the above

10. Two vertices of an equilateral triangle are origin and (4, 0). What is the area of the triangle?

(a) 4 sq units *(b)* V3 sq units

ftj 4^3 sq units ft) 2V3 sq units *(e)* 3 sq units

11. If the points A 1,-1 B 5, 2 andCf &,5 are collinear, then k = ?

(a) 2 *(b)* 4 *(c)* 6 *(d)* 9 *(e)* None of the above

12. Slope of x-axis is

(a) 0 *(b)* 1 *(c)* -1 *(d)* ~ (e) None of the above

13. Slope of y-axis is

(a) 0 *(b)* 1 *(c)* -1 (dj -fe) None of the above

14. If two vertices of a triangle are (5, 4) and (-2, 4) and centroid is (5, 6), then third vertex is

(a) (12,10) *(b)* (10,12)

(c) (-10,12) (a; (12,-10)

(e; (-10,-12)

15. The points (2, 2) (6, 3) and (4, 11) are the vertices of

(a) an equilateral

(b) a right angled triangle

(c) an isosceles triangle

(d) a scalene triangle

(e) do not form triangle

16. A point C divides the line *AC,* where A(l, 3) and *B* (2, 7) in the ratio of 3 : 4. The coordinates of C are [Railways 2006]

(a) $\left(\frac{5}{3}, 5\right)$ *(b)* $(-2, -9)$

(c) $\left(\frac{3}{5}, 5\right)$ *(d)* $\left(\frac{10}{7}, \frac{33}{7}\right)$

17. The slope of a line passing through the points A (4, - 3) and *B* (6, - 3) is

(a) 4 *(b)* 0 *(c)* - *(d)* 5 fe) None of the above

18. If the inclination of a line joining the points A (x, - 3) and *B* (2, 5) is 135°, then z=?

(a) 10 ft) 15 ft; 20 *(d)* 25 fe) None of the above

19. If a point *(x, y)* in a OXY-plane is equidistant from (-1,1) and (4, 3), then

[CLAT2013] *(a)* 10x + 4y = 23 ft) 6x + 4y = 23 (c)-x+y = 7 fdJ4x+3y=0

20. If the graph of the equation *2x+ 3y* = 6 form a triangle with coordinates axis, then the area of triangle will be

[SSC (10+2) 2012]

(a) 2 sq units ftj 3 sq units

ft) 6 sq units ft) 1 sq units

21.If A -5, 7) B -4,-5) C-1,-6) and Z) (4,5) are the vertices of a quadrilateral, then find the area of the quadrilateral $ABCD$. *(a)* 72 sq units *(b)* 80 sq units (c) 90 sq units *(d)* 92 sq units *(e)* None of the above

22. Find the equation of a line parallel to y-axis and passing through the point (- 3, 4)

(a) x + 3 = 0 *(b)* x - 3 = 0

(c) x + 4 = 0 *(d)* x - 4 = 0

fe) None of the above

23. Find the equation of a line of unit slope and passing through the origin.

(a) $y = x$ (b) $x + y = 0$

(c) $x = y + 1$ (d) $x = \frac{1}{2}y + 1$

(e) None of the above

24. The value of k for which the lines $x + 1y = 9$ and $kx + Ay = - 5$ are parallel, is

(a) k = 2 *(b)* k = 1

(c) fc = - 1 *(d)* k = 3

fej None of the above

25. Find the value of k for which the lines $5x + 3y + 2 = 0$ and 3* - ky + 6 = 0 are perpendicular.

(a) 5 *(b)* 4

(c) 3 fdj 2

re; 1

26. The length of the equation $4x+ Zy$ - 12 = 0 that intersects two coordinate axes is

[a) 2.5 units *(b)* 7 units (c) 5 units fdj 6 units

27. Find the distance of the intercept at point of x-axis and the line $5x+ 9y$ - 45 from origin.

(a) 5 *(b)* 9

(c) 5/9 *(d)* 9/5

(e) None of the above

28. Find the equation of perpendicular bisector of the line made by joining the points (1, 1) and (3, 5).

(a) x + 2y + 8 = 0 *(b)* x - 2y + 8 = 0 *(c)* x - 2y - 8 = 0 *(d)* x + 2y - 8 = 0 *(e)* None of the above

29. Find the equation of the line which makes equal intercepts on the axis and passes through the point (4, 5).

(a) x+ y=9 (b) x+2y = 7

(c) 2x+2y = 7 (d) 2x + y = 7

30. In what ratio, the line made by joining the points *A* (- 4,- 3) and *B* (5,2) intersects x-axis?

(a) 3 : 2 *(b)* 2 : 3 *(c)* -3 : 2 *(d)* -2:3 *(e)* None of the above

31. Find the ratio in which the line 3x + *y* - 9 divides the line made by joining the points (1, 3) and (2, 7).

(a) 3:4 external *(b)* 3:4 internal *(c)* 1 : 4 internal fa") 4:3 internal *(e)* None of the above

32. If the point *(x, y)* is equidistant from points (7, 1) and (3, 5), find *(x - y).*

(a) 2 *(b)* 4 *(c)* 6 *(d)* 8 *(e)* None of the above

33. The area of the region bounded by *y* = I *x*\ - 5 with the coordinate axis is

[SSCCGL2012] *(a)* 25 sq units *(b)* 52 sq units

(c) 50 sq units *(d)* 20 sq units

34. If three vertices of a parallelogram are A (3,5) £(-5,-4) and C(7, 10), then fourth vertex is

(a) (10,19) *(b)* (15,10)

(c) (19,10) *(d)* (10,15)

(e) (15,19)

35. Three vertices of a rhombus are (2,-1), (3, 4) and (-2, 3). Find the fourth vertex.

(a) (1,2) *(b)* (3,2)

(c) (-2,-3) *(d)* (-3,-2)

(e) (2,3)

36. Do the points (4, 3), (- 4,- 6) and (7, 9) form a triangle ? If yes, then find the longest side of the triangle.

(a) 18.6 *(b)* 16.5

(c) 24 fdj 34

fej Triangle can't be formed

37. Are the points (1, 7), (4, 2), (- 1,- 1) and (- 4, 4) vertices of a square? If yes, what is the length of the side of square?

(a) 34 *(b)* V34

(c) -*M* *(d)* 68

(e) None of the above

38. If the points *A* (6, 1), B(8, 2), C(9, 4) and *D(P,3)* are the vertices of a parallelogram, taken in order, find the value of P.

(a) 3 *(b)* 4 *(c)* 5 *(d)* 6 *(e) 7*

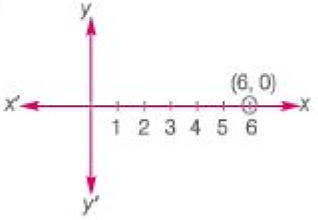
39. If $P(a,0)$, $Q(0,b)$ and $R(1,1)$ are collinear. Then, find the value of $\frac{1}{a}+\frac{1}{b}$.

(a) 2 *(b)* 1 *(c)* −1 *(d)* 0

Answer with Solutions

1. (*b*) For the given point $(-5, 7)$, the value of abscissa is negative and value of ordinate is positive. So, $(-5, 7)$ lies in 2nd quadrant.

2. (*a*)



So, point $(6, 0)$ lies on x-axis.

3. (*c*) Required distance $= \sqrt{x^2 + y^2}$

$$= \sqrt{(8)^2 + (-6)^2}$$

$$= \sqrt{64 + 36} = \sqrt{100} = 10 \text{ units}$$

4. (*c*) Distance between the points $(x, 0)$ and $(-7, 0) = 10$

Given, $x_1 = x, y_1 = 0$

$x_2 = -7$, and $y_2 = 0$

Required distance

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow \quad \pm\sqrt{(-7 - x)^2 + (0 - 0)^2} = 10$$

$$\Rightarrow \quad \pm(x + 7) = 10$$

if $x + 7 = 10$, then $x = 3$

if $-(x + 7) = 10$, then $x = -17$

5. (*b*) Given, $AP = a + x$

$$\sqrt{(x - a)^2 + (y - 0)^2} = a + x$$

$$\Rightarrow \quad (x - a)^2 + y^2 = (a + x)^2$$

$$\Rightarrow \quad x^2 - 2ax + a^2 + y^2 = a^2 + 2ax + x^2$$

$$\Rightarrow x^2 + a^2 + y^2 - a^2 - x^2 = 2ax + 2ax$$

$$\therefore \quad y^2 = 4ax$$

6. (*b*) Equation of the line is $7x - 3y = 2$

$$\Rightarrow \quad \frac{7x}{2} - \frac{3y}{2} = 1$$

$$\Rightarrow \quad \frac{x}{2/7} - \frac{y}{2/3} = 1 \quad \ldots \text{(i)}$$

Comparing Eq. (i) with $\frac{x}{a} + \frac{y}{b} = 1$

Intersection point on x-axis $a = \frac{2}{7}$

7. (c) Let $A(-1,-1)$, $B(4,2)$ and $P(x,y)$ be the points such that

$$AP^2 = BP^2$$

$\therefore (x+1)^2 + (y+1)^2 = (x-4)^2 + (y-2)^2$

$\Rightarrow \quad (x^2 + 1 + 2x + y^2 + 1 + 2y)$

$\quad = (x^2 + 16 - 8x + y^2 + 4 - 4y)$

$\Rightarrow \quad 2 + 2x + 2y = 20 - 8x - 4y$

$\Rightarrow \quad 10x + 6y - 18 = 0$

$\Rightarrow \quad 5x + 3y - 9 = 0$

8. (c) Let abscissa be x. Then,

$(x-0)^2 + (-3-1)^2 = 5^2$

$\Rightarrow \quad x^2 + 16 = 25$

$\Rightarrow \quad x^2 = 9$

$\therefore \quad x = \pm 3$

9. (c) Let $x_1 = 4, x_2 = 3, x_3 = -3$

$y_1 = 4, y_2 = -2$, and $y_3 = 16$

$\therefore$ Area of triangle

$$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$= \frac{1}{2}\{4(-2-16) + 3(16-4) + (-3)(4-(-2))\}$$

$$= \frac{1}{2}\{4 \times (-18) + 3 \times 12 + (-3)(6)\}$$

$$= \frac{1}{2}\{-72 + 36 - 18\}$$

$$= \frac{1}{2} \times (-54)$$

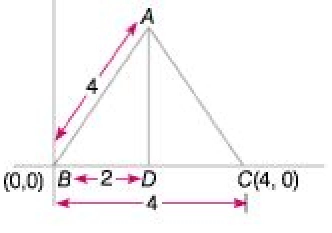
$$= \frac{1}{2} \times 54 = 27 \text{ sq units}$$

[neglecting negative sign]

10. (c) Since, triangle is equilateral.

$\therefore \quad AB = BC = CA = 4$

$\therefore \quad BD = \frac{4}{2} = 2$



In ΔADC,

$AD^2 = 4^2 - 2^2 = 16 - 4 = 12, \Rightarrow AD = 2\sqrt{3}$

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AD$$
$$= \frac{1}{2} \times 4 \times 2\sqrt{3}$$
$$= 4\sqrt{3} \text{ sq units}$$

11. (*d*) Given, $x_1 = 1, \quad x_2 = 5, x_3 = k$

$y_1 = -1, y_2 = 2, \text{ and } y_3 = 5$

$\because$ A, B and C are collinear.

$$\therefore \quad \Delta = 0$$
$$\Rightarrow \quad \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = 0$$
$$\Rightarrow \quad \{1(2-5) + 5(5-(-1) + k(-1-2)\} = 0$$
$$\Rightarrow \quad \{-3 + 30 - 3k\} = 0$$
$$\Rightarrow \quad 3k = 27$$
$$\therefore \quad k = 9$$

12. (*a*) Slope of x-axis $(m) = \tan\theta$

$= \tan 0° = 0$

13. (*d*) Slope of y-axis $= \tan 90° = \infty$

14. (*a*) Let third vertex be (x, y).

Coordinates of centroid

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Given, $x_1 = x, x_2 = 5, x_3 = -2$

$y_1 = y, y_2 = 4, y_3 = 4$ and centroid $= (5, 6)$

$$5 = \frac{x + 5 - 2}{3}$$
$$\Rightarrow \quad x = 12$$
$$\text{and} \quad 6 = \frac{y + 4 + 4}{3}$$
$$\Rightarrow \quad y = 10$$

15. (*b*) Let $A(2,2)$, $B(6,3)$ and $C(4,11)$.

$AB^2 = (6-2)^2 + (3-2)^2 = 16 + 1 = 17$

$BC^2 = (4-6)^2 + (11-3)^2 = 4 + 64 = 68$

$CA^2 = (4-2)^2 + (11-2)^2 = 4 + 81 = 85$

Given, $\quad AB^2 + BC^2 = AC^2$

Hence, ΔABC is right angled triangle.

16. (*d*) Coordinates of C are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Given,

$m = 3, n = 4$

$x_1 = 1, x_2 = 2, y_1 = 3, \text{ and } y_2 = 7$

$$= \left(\frac{3 \times 2 + 4 \times 1}{3 + 4}, \frac{3 \times 7 + 4 \times 3}{3 + 4}\right)$$
$$= \left(\frac{10}{7}, \frac{33}{7}\right)$$

17. (*b*) Given, $x_1 = 4$, $x_2 = 6$, $y_1 = -3$, and $y_2 = -3$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-3+3}{6-4} = \frac{0}{2} = 0$$

18. (*a*) Slope of $AB = \frac{5+3}{2-x} = \frac{8}{2-x} = \tan 135°$

$$\Rightarrow \tan(180° - 45°) = \frac{8}{2-x}$$
$$\Rightarrow \quad -\tan 45° = \frac{8}{2-x}$$
$$\Rightarrow \quad -1 = \frac{8}{2-x}$$
$$\Rightarrow \quad -2 + x = 8$$
$$\therefore \quad x = 10$$

19. (*a*) Distance between (x, y) and $(-1, 1)$

$$= \sqrt{(y-1)^2 + (x+1)^2}$$

Distance between (x, y) and $(4, 3)$

$$= \sqrt{(y-3)^2 + (x-4)^2}$$

$\because$ Points are equidistant

$$\therefore \sqrt{(y-1)^2 + (x+1)^2} = \sqrt{(y-3)^2 + (x-4)^2}$$

On squaring both side, we get

$$(y-1)^2 + (x+1)^2 = (y-3)^2 + (x-4)^2$$
$$\Rightarrow \quad y^2 + 1 - 2y + x^2 + 1 + 2x$$
$$= y^2 + 9 - 6y + x^2 + 16 - 8x$$
$$\Rightarrow \quad 2x - 2y + 2 = -6y - 8x + 25$$
$$\Rightarrow \quad 10x + 4y = 23$$

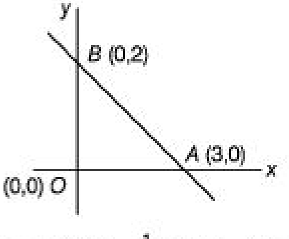
20. (*b*) $2x + 3y = 6$

$$\Rightarrow \quad \frac{2x}{6} + \frac{3y}{6} = 1$$
$$\Rightarrow \quad \frac{x}{3} + \frac{y}{2} = 1$$

Comparing with equation of line.

$$\frac{x}{a} + \frac{y}{b} = 1$$

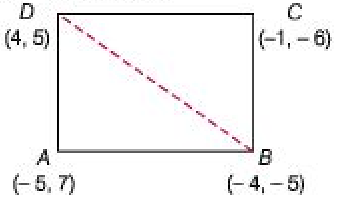
$\therefore$ Intercept at x-axis $= 3$ and intercept at y-axis $= 2$.



Area of $\Delta OAB = \frac{1}{2} \times 3 \times 2 = 3$ sq units

21. (*a*) By joining B and D, we get two triangles $\Delta\, ABD$ and $\Delta\, BCD$.

Given, $x_1 = -5$, $x_2 = -4$, $x_3 = 4$, $y_1 = 7$, $y_2 = -5$, and $y_3 = 5$



Area of $\Delta ABD = \frac{1}{2}[x_1(y_2 - y_3)$

$+ x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-5(-5-5) + (-4)(5-7) + 4(7+5)]$

$= \frac{1}{2}[50 + 8 + 48]$

$= \frac{106}{2} = 53$ sq units

Now in ΔBCD

Given, $x_1 = -4$, $x_2 = -1$, $x_3 = 4$, $y_1 = -5$, $y_2 = -6$, and $y_3 = 5$

Area of $\Delta BCD = \frac{1}{2}[-4(-6-5)$

$-1(5+5) + 4(-5+6)]$

$= \frac{1}{2}(44 - 10 + 4)$

$= \frac{1}{2} \times 38 = 19$ sq units

Hence, area of quadrilateral $ABCD$ = Area of $\Delta\, ABD$ + Area of $\Delta\, BCD$

$= 53 + 19$

$= 72$ sq units

22. (*a*) Required equation of the line is

$$\frac{y-4}{x+3} = \tan 90° = \infty$$

$$\Rightarrow \quad \frac{y-4}{x+3} = \frac{1}{0}$$

$$\Rightarrow \quad x + 3 = 0$$

23. (*a*) Govem, $m = 1$ and $c = 0$ (as it passes through origin.)

$\therefore$ Equation of line is

$$y = 1 \times x + 0 \Rightarrow y = x$$

24. (*a*) $x + 2y = 9$

$$y = \frac{-1}{2}x + 9$$

$$kx + 4y = -5$$

$$y = \frac{-k}{4}x - \frac{5}{4}$$

On comparing with $y = mx + c$, we get

$$m_1 = \frac{-1}{2}$$

and $$m_2 = \frac{-k}{4}$$

Since, the given lines are parallel

$\therefore$ $$m_1 = m_2$$

$$\frac{-1}{2} = -\frac{k}{4}$$

$$k = 2$$

25. (*a*) $5x + 3y + 2 = 0$

$$y = -\frac{5x}{3} - \frac{2}{3}$$

$$3x - ky + 6 = 0$$

$$y = +\frac{3}{k}x + \frac{6}{k}$$

On comparing with $y = mx + c$

$$m_1 = -\frac{5}{3} \quad \text{and} \quad m_2 = \frac{3}{k}$$

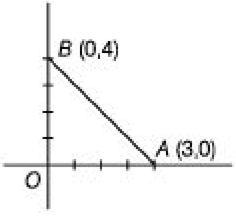
Since, both the lines are perpendicular,
then, $m_1 m_2 = -1$

$\Rightarrow$ $$-\frac{5}{3} \times \frac{3}{K} = -1$$

$\therefore$ $$K = 5$$

26. (*c*) $4x + 3y - 12 = 0$

$\Rightarrow$ $$\frac{x}{3} + \frac{y}{4} = 1$$



Therefore, the line cuts x-axis at the point $(3, 0)$ and y-axis at the point $(0, 4)$.

$\therefore$ $OA = 3$ units, $OB = 4$ units

$\therefore$ $AB = \sqrt{3^2 + 4^2} = 5$ units

27. (*b*) We know that, $y = 0$ on x-axis

$\therefore$ Putting $y = 0$ in the line $5x + 9y = 45$

$\Rightarrow$ $$5x = 45 \Rightarrow x = 9$$

Now, distance between $(9,0)$ and $(0,0)$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0 - 9)^2 + (0 - 0)^2}$$

$$= \sqrt{81} = 9$$

28. (*d*) Slope of the line made by joining the points $A(1,1)$ and $B(3,5) = \frac{5-1}{3-1} = \frac{4}{2} = 2$

and mid-point of $AB = \left(\frac{1+3}{2}, \frac{1+5}{2}\right)$

$$= (2, 3)$$

Now, slope of the line perpendicular to the line $AB = -\frac{1}{2}$

$\therefore$ Required equation of perpendicular bisector

$$y - 3 = -\frac{1}{2}(x - 2)$$

$$\Rightarrow \quad x + 2y - 8 = 0$$

29. (*a*) Let intercepts made by line = a

$\therefore$ Equation of line is

$$\frac{x}{a} + \frac{y}{a} = 1$$

$$\Rightarrow \quad x + y = a \quad \text{...(i)}$$

But this line passes through (4, 5).

$$\therefore \quad 4 + 5 = a \Rightarrow a = 9$$

Putting this value in Eq. (i), we get

$$x + y = 9$$

which is the required equation.

30. (*a*) We know that, y-coordinate is zero on x-axis.

Given, $\quad y_1 = -3, y_2 = 2$

$$\therefore \quad y = \frac{my_2 + ny_1}{m + n}$$

$$\Rightarrow \quad 0 = \frac{m(2) + n(-3)}{m + n}$$

$$\Rightarrow \quad 2m - 3n = 0$$

$$\therefore \quad \frac{m}{n} = \frac{3}{2}$$

31. (*b*) Let required ratio be $K : 1$. Then, coordinates of section point are $\left(\frac{2K+1}{K+1}, \frac{7K+3}{K+1}\right)$.

But this point lies on $3x + y - 9 = 0$

$$\therefore \quad 3\left(\frac{2K+1}{K+1}\right) + \left(\frac{7K+3}{K+1}\right) - 9 = 0$$

$$\Rightarrow \quad 6K + 3 + 7K + 3 - 9K - 9 = 0$$

$$\Rightarrow \quad 4K - 3 = 0$$

$$\therefore \quad K = \frac{3}{4}$$

$\therefore$ Required ratio $= K : 1 = 3/4 : 1$

$= 3 : 4$ internal

32. (*a*) Let $P(x, y)$ be equidistant from points $A(7, 1)$ and $B(3, 5)$.

Then, $\quad AP = BP \quad$ So, $\quad AP^2 = BP^2$

$$\therefore \quad (x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$$

$$x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$$

$\therefore$ After solving, $(x - y) = 2$

33. (a) Given, $y = |x| - 5$

$\Rightarrow \quad y = -x - 5$

$\Rightarrow \quad y = x - 5$

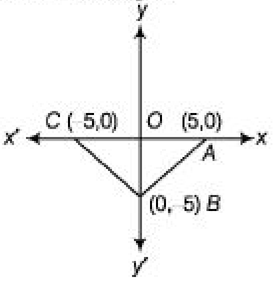
$\therefore \quad x + y = -5$

$\Rightarrow \quad x - y = 5$

$$\frac{x}{(-5)} + \frac{y}{(-5)} = 1$$

and $\quad \dfrac{x}{5} + \dfrac{y}{(-5)} = 1$

Area of bounded region



$$= \frac{1}{2} \times AC \times OB$$

$$= \frac{1}{2} \times 10 \times 5 = 25 \text{ sq units}$$

34. (e) Let fourth vertex be $D(x, y)$.

We know that, diagonals of a parallelogram intersect at mid-point. Therefore,

Mid-point of AC = Mid-point of BD

$$\left(\frac{3+7}{2}, \frac{5+10}{2}\right) = \left(\frac{-5+x}{2}, \frac{-4+y}{2}\right)$$

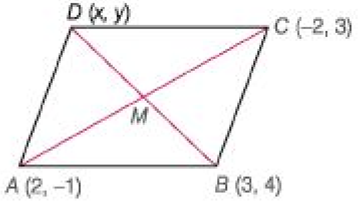
$\Rightarrow \quad \dfrac{-5+x}{2} = 5 \quad \Rightarrow \quad x = 10 + 5 = 15$

and $\quad \dfrac{-4+y}{2} = \dfrac{15}{2}$

$\therefore \quad y = 15 + 4 = 19$

Hence, fourth vertex is $(15, 19)$.

35. (d) Let coordinates of D be (x, y) and M be the intersection point of diagonals AC and BD.



Since, M is the mid-point of AC.

$\therefore$ Coordinate of M

$$= \left(\frac{-2+2}{2}, \frac{-1+3}{2}\right) = (0, 1)$$

Also, M is the mid-point of BD.

$\therefore$ Coordinates of $M = \left(\frac{x+3}{2}, \frac{y+4}{2}\right)$

$\therefore$ $$\frac{x+3}{2} = 0 \Rightarrow x = -3$$

and $$\frac{y+4}{2} = 1 \Rightarrow y = -2$$

Hence, fourth vertex is $(-3, -2)$.

36. (*a*) Let $P(4, 3)$, $Q(-4, -6)$ and $R(7, 9)$ are given points.

$$PQ = \sqrt{(-4-4)^2 + (-6-3)^2}$$

$$= \sqrt{(-9)^2 + (-9)^2} = \sqrt{64 + 81} = 12.04$$

$$QR = \sqrt{(7-(-4))^2 + (9-(-6))^2}$$

$$= \sqrt{11^2 + 15^2} = \sqrt{121 + 225} = 18.6$$

$$PR = \sqrt{(7-4)^2 + (9-3)^2}$$

$$= \sqrt{9 + 36} = 6.7$$

Since, the sum of 12.04 and 6.7 is greater than 18.6, so it will from a triangle, whose longest side is 18.6.

37. (*b*) $AB = \sqrt{(4-1)^2 + (2-7)^2}$

$$= \sqrt{9 + 25} = \sqrt{34}$$

$$BC = \sqrt{(-1-4)^2 + (-1-2)^2}$$

$$= \sqrt{25 + 9} = \sqrt{34}$$

$$CD = \sqrt{(-4+1)^2 + (4+1)^2}$$

$$= \sqrt{9 + 25} = \sqrt{34}$$

$$DA = \sqrt{(1+4)^2 + (7-4)^2}$$

$$= \sqrt{25 + 9} = \sqrt{34}$$

$$AC = \sqrt{(-1-1)^2 + (-1-7)^2}$$

$$= \sqrt{4 + 64} = \sqrt{68}$$

$$BD = \sqrt{(-4-4)^2 + (4-2)^2}$$

$$= \sqrt{64 + 4} = \sqrt{68}$$

Given, $AB = BC = CD = DA$ and $AC = BD$, *i.e.*, all the four sides of the quadrilateral $ABCD$ are equal and its diagonals AC and BD are also equal.

Therefore, $ABCD$ is a square.

Hence, length of the side of square is $\sqrt{34}$.

38. (*e*) Diagonals of a parallelogram bisect each other.

So, the coordinates of mid-point of AC
= Coordinates of mid-point of BD

i.e., $\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+P}{2}, \frac{2+3}{2}\right)$

$$\left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+P}{2}, \frac{5}{2}\right)$$

$$\therefore \quad \frac{8+P}{2} = \frac{15}{2} \Rightarrow 8 + P = 15$$

$$\therefore \quad P = 7$$

39. (*b*) Since, P, Q and R are collinear.

$\therefore$ Area of $\Delta PQR = 0$

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

Given, $x_1 = a, x_2 = 0,$
$x_3 = 1, y_1 = 0, \quad y_2 = b,$ and $y_3 = 1$

$$\Rightarrow \frac{1}{2}[a \times (b-1) + 0 \times (1-0) + 1 \times (0-b)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[ab - b - a] = 0 \Rightarrow ab = a + b$$

$$\Rightarrow \quad 1 = \frac{a+b}{ab}$$

$$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} = 1$$

Chapter

38

Trigonometry

Trigonometry is a branch of Mathematics that studies triangles and relationship between their sides and angles and these relationships can be made using trigonometric functions.

Measurement of Angles

1. Degree Measure (Sexagesimal System)

In this system, an angle is measured in degrees, minutes and seconds.

1° = *W* (60 min); 1' = 60" (60 s)

2. Radian Measure (Circular System)

Angle subtended at the centre by any arc of length 1 unit in a circle of radius 1 unit, is said to have a measure of 1 radian.

271 radian =360° or *n* radian =180°

1 radian =57° 16'22"

Relation between Radian and Degrees

$$\text{Radian measure} = \frac{\pi}{180} \times \text{Degree measure}$$

$$\text{Degree measure} = \frac{180}{\pi} \times \text{Radian measure}$$

We can simply put 7r=180° in radian measure to convert radian measure into degree measure,

Ex. 1 Convert 40° into radian measure.

Sol. $40^\circ = 40 \times \frac{\pi}{180} \text{ radian} = \frac{2\pi}{9} \text{ radian}$

Ex. 2 Convert $\frac{3\pi}{2}$ into degree measure.

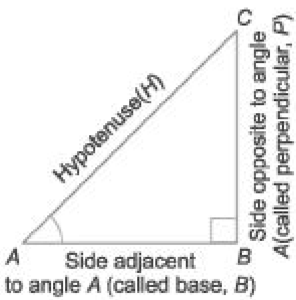
Sol. $\frac{3\pi}{2} \text{ radian} = \frac{3\pi}{2} \times \frac{180}{\pi} = 270^\circ$

Alternate Method

$$\frac{3\pi}{2} = \frac{3 \times 180^\circ}{2} = 270^\circ$$

Trigonometric Ratios

The ratios between different sides of a right angled triangle with respect to its acute angles are called **trigonometric ratios.** Trigonometric ratios for right angled *AABC* with respect to angle *A* are given below.



$$\sin A = \frac{BC}{AC} = \frac{P}{H}, \qquad \cos A = \frac{AB}{AC} = \frac{B}{H}$$

$$\tan A = \frac{BC}{AB} = \frac{P}{B}, \qquad \operatorname{cosec} A = \frac{AC}{BC} = \frac{H}{P}$$

$$\sec A = \frac{AC}{AB} = \frac{H}{B}, \qquad \cot A = \frac{AB}{BC} = \frac{B}{P}$$

By Pythagoras theorem $H^2 = B^2 + D^2$

Reciprocal Relations

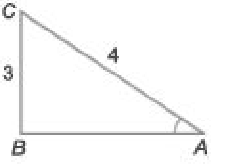
$\sin A = \dfrac{1}{\cos A}$ or $\cos A = \dfrac{1}{\sin A}$ or $\sin A \cos A = 1$

$\cos A = \dfrac{1}{\sec A}$ or $\sec A = \dfrac{1}{\cos A}$ or $\sec A \cos A = 1$

$\tan A = \dfrac{\sin A}{\cos A}$ or $\cot A = \dfrac{\cos A}{\sin A}$ or $\tan A \cot A = 1$

Ex. 3 If $\sin A = \dfrac{3}{4}$, calculate $\cos A$ and $\tan A$.

Sol. $\sin A = \dfrac{BC}{AC} = \dfrac{3}{4}$



In a right angled ΔABC.

$$(AC)^2 = (AB)^2 + (BC)^2$$

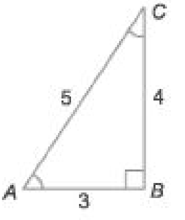
$\Rightarrow$ $4^2 = (AB)^2 + 3^2 \Rightarrow 16 - 9 = (AB)^2$

$\therefore$ $AB = \sqrt{7}$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{7}}{4} \text{ and } \tan A = \frac{BC}{AB} = \frac{3}{\sqrt{7}}$$

Ex. 4 If $\sin A = \dfrac{4}{5}$, then find the value of $(4 + \tan A)(2 + \cos A)$

Sol. Given, $\sin A = \dfrac{4}{5} = \dfrac{BC}{AC} = \dfrac{P}{H}$



Now in ΔABC, using Pythagoras theorem

$$(AC)^2 = (AB)^2 + (BC)^2$$

$\Rightarrow$ $(5)^2 = (AB)^2 + (4)^2$

$\Rightarrow$ $25 = (AB)^2 + 16$

$\Rightarrow$ $AB = \sqrt{25 - 16} = \sqrt{9} = 3$

So, $\tan A = \dfrac{BC}{AB} = \dfrac{P}{B} = \dfrac{4}{3}$

$\cos A = \dfrac{AB}{AC} = \dfrac{B}{H} = \dfrac{3}{5}$

Now, $(4 + \tan A)(2 + \cos A)$

$$= \left(4 + \frac{4}{3}\right)\left(2 + \frac{3}{5}\right) = \left(\frac{12+4}{3}\right) \times \left(\frac{10+3}{5}\right) = \frac{16}{3} \times \frac{13}{5} = \frac{208}{15}$$

Trigonometric Ratios of Some Specific Angles

Trigonometric Ratios	Angle (Degree/Radian)							
	$0°$ 0	$30°$ $\pi/6$	$45°$ $\pi/4$	$60°$ $\pi/3$	$90°$ $\pi/2$	$180°$ π	$270°$ $3\pi/2$	$360°$ 2π
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0
$\cot A$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1	∞	1
$\operatorname{cosec} A$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	-1	∞

Trigonometric Identities

An equation involving trigonometric ratios of an angle is called a trigonometric identity, if it is true for all values of the angles involved. *For example,* in a right angled triangle, we have the following identities

1. $\sin^2 A + \cos^2 A = 1$ or $\sin^2 A = 1 - \cos^2 A$ or $\cos^2 A = 1 - \sin^2 A$
2. $1 + \tan^2 A = \sec^2 A$ or $\tan^2 A = \sec^2 A - 1$ or $\sec^2 A - \tan^2 A = 1$
3. $1 + \cot^2 A = \operatorname{cosec}^2 A$ or $\cot^2 A = \operatorname{cosec}^2 A - 1$ or $\operatorname{cosec}^2 A - \cot^2 A = 1$

Ex. 5 If $\cot\theta = \frac{7}{8}$, evaluate $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$.

Sol. $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$$= \frac{(1)^2 - (\sin\theta)^2}{(1)^2 - (\cos\theta)^2} \qquad [\because (a-b)(a+b) = a^2 - b^2]$$

$$= \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2 = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

Ex. 6 Find the value of $2\tan^2 45° + \cos^2 30° - \sin^2 60°$.

Sol. $2\tan^2 45° + \cos^2 30° - \sin^2 60° = 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2$

Ex. 7 Find the value of $\frac{2\tan 30°}{1-\tan^2 30°}$.

Sol. $\frac{2\tan 30°}{1-\tan^2 30°} = \tan 60° = \sqrt{3}$ $\qquad \left[\because \tan 2x = \frac{2\tan x}{1-\tan^2 x}\right]$

Sign of Trigonometric Functions

Trigonometric ratios of complementary and supplementary angles of angle 0 are shown below.

$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ $\quad$ $\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$

$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$ $\quad$ $\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$

$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$ $\quad$ $\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$

$\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$ $\quad$ $\cot\left(\frac{\pi}{2}+\theta\right)=-\tan\theta$

$\sec\left(\frac{\pi}{2}-\theta\right)=\operatorname{cosec}\theta$ $\quad$ $\sec\left(\frac{\pi}{2}+\theta\right)=-\operatorname{cosec}\theta$

$\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)=\sec\theta$ $\quad$ $\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)=\sec\theta$

$\sin(\pi-\theta)=\sin\theta$ $\quad$ $\sin(\pi+\theta)=-\sin\theta$

$\cos(\pi-\theta)=-\cos\theta$ $\quad$ $\cos(\pi+\theta)=-\cos\theta$

$\tan(\pi-\theta)=-\tan\theta$ $\quad$ $\tan(\pi+\theta)=\tan\theta$

$\cot(\pi-\theta)=-\cot\theta$ $\quad$ $\cot(\pi+\theta)=\cot\theta$

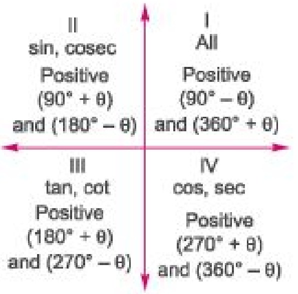
$\sec(\pi-\theta)=-\sec\theta$ $\quad$ $\sec(\pi+\theta)=-\sec\theta$

$\operatorname{cosec}(\pi-\theta)=\operatorname{cosec}\theta$ $\quad$ $\operatorname{cosec}(\pi+\theta)=-\operatorname{cosec}\theta$

$\sin(-x)=-\sin x$ $\quad$ $\operatorname{cosec}(-x)=-\operatorname{cosec} x$

$\cos(-x)=\cos x$ $\quad$ $\sec(-x)=\sec x$

$\tan(-x)=-\tan x$ $\quad$ $\cot(-x)=-\cot x$



For $(90°\pm\theta)$ and $(270\pm\theta)$ Change

$\sin\theta \leftrightarrow \cos\theta$,

$\tan\theta \leftrightarrow \cot\theta$,

$\operatorname{cosec}\theta \leftrightarrow \sec\theta$

and for $(180\pm\theta)$ and $(360\pm\theta)$ they do not change.

Note Positive or negative sign is used according to the quadrant.

Ex. 8 Find the value of $\frac{\sec 65°}{\operatorname{cosec} 25°}$.

Sol. $\frac{\sec 65°}{\operatorname{cosec} 25°}=\frac{\sec(90°-25°)}{\operatorname{cosec} 25°}=\frac{\operatorname{cosec} 25°}{\operatorname{cosec} 25°}=1$

Ex. 9 If $\sin 3A=\cos(A-26°)$, where $3A$ is an acute angle, find the value of A.

Sol. Given, $\sin 3A=\cos(A-26°)$

or $\cos(90°-3A)=\cos(A-26°)$ $\quad$ $[\because \cos(90°-\theta)=\sin\theta]$

Eliminating cos from both sides, we get

$$90°-3A=A-26° \Rightarrow 4A=116°$$

$$\therefore \quad A=\frac{116°}{4}=29°$$

Ex. 10 Find the value of $\frac{\sin^2 63°+\sin^2 27°}{\cos^2 17°+\cos^2 73°}$.

Sol. $\frac{\sin^2 63°+\sin^2 27°}{\cos^2 17°+\cos^2 73°}=\frac{\sin^2(90°-27°)+\sin^2 27°}{\cos^2(90°-73°)+\cos^2 73°}$

$$=\frac{\cos^2 27°+\sin^2 27°}{\sin^2 73°+\cos^2 73°}=1 \qquad [\because \sin^2\theta+\cos^2\theta=1, \text{ here } \theta=27° \text{ and } 73°]$$

Ex. 11 Find the value of tan 15°.

Sol. $\because \tan 15^\circ = \tan(45^\circ - 30^\circ) = \dfrac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ} = \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$

On rationalising,

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - (1)^2} = \frac{3 + 1 - 2\sqrt{3}}{3 - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

Ex. 12 Find the value of sin 2295°

Sol. sin 2295°

As 2295° is not directly measurable, So we will divide 2295° by 360°

$$360^\circ \overline{)\,2295^\circ\,}(6$$
$$\underline{2160}$$
$$135$$

Now, 2295° is 6 complete angles and 135°, So sin 2295° can be written as sin 135°.

$$\sin 135^\circ = \sin(90 + 45^\circ) = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Trigonometric Ratios of Combined Angles

Sum and Difference Formulae

1. sin (x + y) = sin x cos y + cos x sin y

2. cos (x + v) = cos x cos v + sin x sin v

3. $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

4. sin (x + y) • sin(x - y) = (sin x - sin y) or (cos y - cos x)

5. cos (x + y) cos (x - y) = (cos x - sin y) or (cos y - sin x)

6. sin (x + y + z) = cos xcos ycos z (tanx + tany + tanz) - tan x tan y tan z

7. cos(x + y + z) = cos x cosy cos z(l - tan x tan y - tan y tan z - tan x tan z)

8. $\tan(x+y+z) = \dfrac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y \tan z - \tan x \tan z}$

9. $\dfrac{\cos x \pm \sin x}{\cos x \mp \sin x} = \tan(45^\circ \pm x)$

A, B formulae or Product to sum formulae

1. $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

2. $2 \sin A \sin B = \cos(A-B) - \cos(A + B)$

3. $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

4. $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

C, D formulae or Sum to Product formulae

1. $\sin C + \sin D = 2 \sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$
2. $\sin C - \sin D = 2 \cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$
3. $\cos C + \cos D = 2 \cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$
4. $\cos C - \cos D = 2 \sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{D-C}{2}\right)$

Trigonometric Ratios of Multiple of an Angle

1. $\sin 2x = 2\sin x \cos x = \dfrac{2\tan x}{1+\tan^2 x}$

2. $\cos 2x = \cos^2 x - \sin^2 x$ or $2\cos^2 x - 1$ or $1 - 2\sin^2 x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

3. $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$ or $\cot 2x = \dfrac{\cot^2 x - 1}{2\cot x}$

4. $\sin 3x = 3\sin x - 4\sin^3 x$

5. $\cos 3x = 4\cos^3 x - 3\cos x$

6. $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

7. $\cos A.\cos 2A.\cos 4A = \dfrac{1}{4}[\cos(4A - 2A + A)] = \dfrac{1}{4}\cos 3A$

8. $\sin A.\sin 2A.\sin 4A = \dfrac{1}{4}[\sin(4A - 2A + A)] = \dfrac{1}{4}\sin 3A$

9. $\tan A.\tan 2A.\tan 4\alpha = \tan[4A - 2A + A] = \tan 3A$

Trigonometric Ratios of Half Angles

1. $\sin x = 2\sin\dfrac{x}{2}\cos\dfrac{x}{2}$ or $\dfrac{2\tan x/2}{1+\tan^2(x/2)}$

2. $\cos x = \left(\cos^2\dfrac{x}{2} - \sin^2\dfrac{x}{2}\right)$ or $\left(1 - 2\sin^2\dfrac{x}{2}\right)$ or $\left(2\cos^2\dfrac{x}{2} - 1\right)$ or $\dfrac{1-\tan^2 x/2}{1+\tan^2 x/2}$

3. $1 + \cos x = 2\cos^2\dfrac{x}{2}$

4. $1 - \cos x = 2\sin^2\dfrac{x}{2}$

5. $\tan x = \dfrac{2\tan x/2}{1-\tan^2 x/2}$

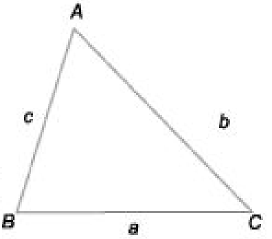
6. $\cot x = \dfrac{\cot^2(x/2) - 1}{2\cot x/2}$

7. $\sin\dfrac{x}{2} = \sqrt{\dfrac{1-\cos x}{2}}$

8. $\cos\dfrac{x}{2} = \sqrt{\dfrac{1+\cos x}{2}}$

9. $\tan\dfrac{x}{2} = \sqrt{\dfrac{1-\cos x}{1+\cos x}}$

Sine Rule



In ΔABC,

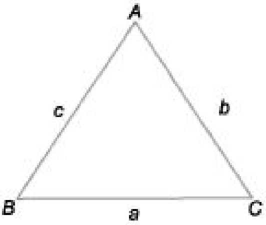
If $AB = c$,

$BC = a$,

$AC = b$

Then $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = K$ [Constant value]

Cosine Rule



In ΔABC,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

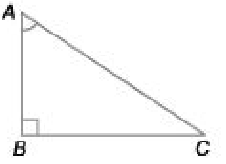
$$\cos B = \frac{c^2 + a^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 = b^2 - c^2}{2ab}$$

Important Results

1. If $\sin x = 0$ or $\tan x = 0$, then $x = n\pi$
2. If $\cos x = 0$ or $\cot x = 0$, then $x = (2n+1)\dfrac{\pi}{2}$
3. $\sin 22\dfrac{1^\circ}{2} = \dfrac{\sqrt{2-\sqrt{2}}}{2}$
4. $\cos 22\dfrac{1^\circ}{2} = \dfrac{\sqrt{2+\sqrt{2}}}{2}$
5. $\tan 22\dfrac{1^\circ}{2} = \sqrt{2} - 1$
6. $\cot 22\dfrac{1^\circ}{2} = \sqrt{2} + 1$
7. $\sin 18^\circ = \cos 72^\circ = \dfrac{\sqrt{5}-1}{4}$
8. $\cos 18^\circ = \sin 72^\circ = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$
9. $\sin 36^\circ = \cos 54^\circ = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$
10. $\cos 36^\circ = \sin 54^\circ = \dfrac{\sqrt{5}+1}{4}$

Ex. 13 In A *ABC,* right angled at *B,* find the value of sin *A* cos C + cos *A* sin C.



Sol. $\because \angle B = 90^\circ$

$\therefore \angle A + \angle C = 90^\circ$

Now, $\sin A \cos C + \cos A \sin C = \sin(A + C)$

$= \sin 90^\circ = 1$

Ex.14 If $x + \dfrac{1}{x} = 2\cos\theta$, then find the value of $x^3 + \dfrac{1}{x^3}$

Sol. We know that, $x^3 + \dfrac{1}{x^3} = \left(x + \dfrac{1}{x}\right)^3 - 3\left(x + \dfrac{1}{x}\right)$

$= (2\cos\theta)^3 - 3 \times 2\cos\theta$ $\quad\left[\because x + \dfrac{1}{x} = 2\cos\theta\right]$

$= 8\cos^3\theta - 6\cos\theta = 2[4\cos^3\theta - 3\cos\theta]$

$= 2\cos 3\theta$ $\quad[\because 4\cos^3 x - 3\cos x = \cos 3x]$

Ex. 15 Find the value of cos 20° cos 40° cos 60° cos 80°.

Sol. $\cos 20° \cos 40° \cos 60° \cos 80°$

$$= \cos 60° [\cos 20° \cos 40° \cos 80°]$$

$$= \frac{1}{2}\left[\frac{1}{4}\cos(3 \times 20°)\right] \qquad \left[\because \cos\theta \cos 2\theta \cos 4\theta = \frac{1}{4}\cos 3\theta\right]$$

$$= \frac{1}{2}\left[\frac{1}{4}\cos 60°\right] = \frac{1}{2} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{16}$$

Ex. 16 If $x + y = z$, then find the value of $\cos^2 x + \cos^2 y + \cos^2 z$.

Sol. $x + y = z$

Now, $\cos^2 x + \cos^2 y + \cos^2 z$

$$= 1 + (\cos^2 x - \sin^2 y) + \cos^2 z$$

$$= 1 + \cos(x + y) \cdot \cos(x - y) + \cos^2 z$$

$$= 1 + \cos z \cos(x - y) + \cos^2 z$$

$$= 1 + \cos z\, [(x - y) + \cos(x + y)]$$

$$= 1 + \cos z \left[2 \cos \frac{(x - y + x + y)}{2} \cdot \cos \frac{(x - y - x - y)}{2}\right]$$

$$= 1 + 2 \cos z \cdot \cos x \cdot \cos y$$

$$= 1 + 2 \cos x \cdot \cos y \cdot \cos z$$

Ex. 17 If (a^2 - b^2) sinG + *lab* cos 9 = a^2 + b^2, then find the value of tanO.

Sol. $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$

$$\Rightarrow \frac{a^2 - b^2}{a^2 + b^2} \cdot \sin\theta + \frac{2ab}{a^2 + b^2} \cdot \cos\theta = 1$$

In ΔABC,

$$\cos\alpha \cdot \sin\theta + \sin\alpha \cdot \cos\theta = \sin 90°$$

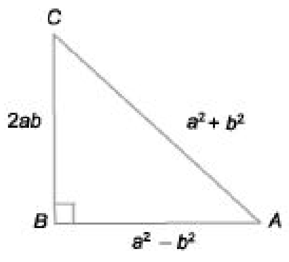
$$\Rightarrow \sin(\theta + \alpha) = \sin 90°$$

$$\Rightarrow \theta + \alpha = 90°$$

$$\Rightarrow \theta = 90° - A$$

$$\therefore \tan\theta = \tan(90° - \alpha) = \cot\alpha$$

$$= \frac{a^2 - b^2}{2ab}$$



Ex. 18 If (1 + tan *A)* (1 + tan B) = 2, then find the value of (A + *B)*.

Sol. Given, $1 + \tan A + \tan B + \tan A \tan B = 2$

$$\Rightarrow \quad \tan A + \tan B = 1 - \tan A \tan B$$

$$\Rightarrow \quad \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1 = \tan 45^\circ$$

$$\Rightarrow \quad \tan (A + B) = \tan 45^\circ \quad \left[\because \tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \quad A + B = 45^\circ = \frac{\pi}{4}$$

Maximam and Minimum Values of Trignometric Angles

1. -l<sin8orcos6<l

2. -cc<tan8orcot6<l

3. sec8orcosec8 >1

4. sec 0 **or** cosecG < -1

Maximum and Minimum Value of Some Particular Expressions

(a) $m \sin\theta \pm n \cos\theta \sqrt{m^2 + n^2} - \sqrt{m^2 + n^2}$

(b) $m \sin\theta \pm n \cos\theta \sqrt{m^2 + n^2} - \sqrt{m^2 + n^2}$

(c) $m \cos \pm n \cos \sqrt{m^2 + n^2} - \sqrt{m^2 + n^2}$

Note To obtain the maximum or minimum value of an expression try to convert it into sinG orcosG

Ex. 19 Find **the maximum value** of 5 **tan** 8 **cos8** + 4 **cos8** .

Sol. Given, $5 \tan\theta \cos\theta + 4 \cos\theta$

$$= \frac{5 \sin\theta}{\cos\theta} \cos\theta + 4 \cos\theta \quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= 5 \sin\theta + 4 \cos\theta$$

So, $m = 5, n = 4$

$\therefore$ Maximum value $= \sqrt{5^2 + 4^2}$

$= \sqrt{25 + 16}$

$= \sqrt{41}$

Ex.20 Find the minimum value of 2**sin** 8+3cos 8

Sol. 2 $\sin^2$ 6+ 3 $\cos^2$ 8

$= 2 \sin^2 8 + 2 \cos^2 8 + \cos^2 8$

$= 2 (\sin^2 6 + \cos^2 8) + \cos^2 8$

$= 2 + \cos^2 G$ [•/ $\sin^2 G + \cos^2 G = 1$]

Minimum value of cos 8 = - 1

Minimum value of expression = 2 + (-1) = 2 + 1 = 3

Some Important Results

1. tan1°-tan2°.........tan 89° =1

2. cot1°-cot2°..........cot89° = 1

3. cos 1° • cos 2°- cos 3°.........cos 90° = 0

4. cos1°■ cos2°cos3°........(abovecos 90°).....= 0

5. sinl°• sin2°• sin3°........sin!80° = 0

6. sin1°- sin2°- sin3°.......(Above sin180°) = 0

7. $\frac{\sin A}{\cos B} = 1$, when $A + B = 90°$

8. tan/\ tanS = 1 = cot *A* cot *B* , when *A* + *B* = 90

Ex.21 Find the value of sinl°- sin2°- sin 3°.......sinl80°......sinl96°

Sol. sin 1°- sin 2°- sin 3°.......sin 180°......sin 196°

As sin 180° = 0 So, when a value is multiplied with zero (0), then whole expression will result in zero. sinl° • sin2° • sin3°.....sin 180°......sin 196° = 0

Ex.22 If $\sin\theta + \cos\theta = \sqrt{2}$, then find the value of θ.

Sol. $\sin\theta + \cos\theta = \sqrt{2}$

$$\sin\theta - \cos\theta = \sqrt{2 - x^2}$$

where, $$x^\circ = \sqrt{2}$$

$$\sin\theta - \cos\theta = \sqrt{2 - (\sqrt{2})^2}$$

$$\sin\theta - \cos\theta = \sqrt{2 - 2}$$

$$\sin\theta - \cos\theta = 0$$

$$\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1$$

$$\tan\theta = 1$$

$\Rightarrow$ $$\theta = 45^\circ$$

Note When $\sin\theta + \cos\theta = x$, then $\sin - \cos\theta = \sqrt{2 - x^2}$

Ex.23 If sinG + cosec 9 = 2 then find the value of $\sin^{36}$G + cosec^{36} G

Sol. If sin 6 + cosec 9 = 2

Then, $\sin^{36}$ 9 + cosec^{36}9 = 2

Note When sin 9 + cosec 9 = 2, then always sin" 0 + cosec" 9 = 2 for any positive value of *n*

Ex.24 Find the value of $\cot\frac{\pi}{20}\cdot\cot\frac{3\pi}{20}\cdot\cot\frac{5\pi}{20}\cdot\cot\frac{7\pi}{20}\cdot\cot\frac{9\pi}{20}$

Sol. $\cot\frac{\pi}{20}\cdot\cot\frac{3\pi}{20}\cdot\cot\frac{5\pi}{20}\cdot\cot\frac{7\pi}{20}\cdot\cot\frac{9\pi}{20}$

Converting them into degree measure

$$\cot\frac{180}{20}\cdot\cot\frac{3\times180}{20}\cdot\cot\frac{5\times180}{20}\cdot\cot\frac{7\times180}{20}\cdot\cot\frac{9\times180}{20}$$

$$\cot 9^\circ . \cot 27^\circ . \cot 45^\circ . \cot 63^\circ . \cot 81^\circ$$

Rewritting the above expression

$$\cot 9^\circ . \cot 81^\circ . \cot 27^\circ . \cot 63^\circ . \cot 45^\circ$$

We known that, $\cot A \cot B = 1$

When $A + B = 90^\circ$

So, $(\cot 9^\circ \cot 81^\circ)(\cot 27^\circ \cot 63^\circ)\cot 45^\circ$

There, $$9^\circ + 81^\circ = 90^\circ$$

$$27^\circ + 63^\circ = 90^\circ$$

Then, $\cot 9^\circ \cot 81^\circ = 1$ and $\cot 27^\circ \cot 63^\circ = 1$

$\therefore$ Expression will become $1 \times 1 \times 1$ $[\because \cot 45^\circ = 1]$

Fast Track Practice

Exercise© *Base Level Questions*

1. If ΔABC is right angled at C, then what is $\cos(A+B)+\sin(A+B)$ equal to? [CDS 2013]

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

2. If α, β and γ are acute angled such that $\sin\alpha = \frac{\sqrt{3}}{2}$ $\cos\beta = \frac{\sqrt{3}}{2}$ and $\tan\gamma = 1$, then what is $\alpha+\beta+\gamma$ equal to? [CDS 2013]

(a) 105° (b) 120° (c) 135° (d) 150°

3. If $\tan\theta = 3/4$ and θ is acute, then what is the value of $\sin\theta$? [CDS 2012]

(a) $-\frac{3}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $-\frac{4}{5}$

4. If $\sin\theta = \frac{a^2-1}{a^2+1}$, then the value of $\sec\theta + \tan\theta$ will be [SSC FCI 2012]

(a) $\frac{a}{\sqrt{2}}$ (b) $\frac{a}{a^2+1}$

(c) $\sqrt{2}a$ (d) a

5. If $\alpha+\beta = 90°$ and $\alpha:\beta = 2:1$, then the value of $\sin\alpha : \sin\beta$ is [SSC FCI 2012]

(a) $\sqrt{3}:1$ (b) 2 : 1

(c) 1 : 1 (d) $\sqrt{2}:1$

6. If $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta} = \frac{5}{3}$, then $\sin\theta$ is equal to

(a) 2/3 (b) 3/4 (c) 1/4 (d) 1/3

(e) None of the above

7. If $\sin\theta+\cos\theta = 1$, what is the value of $\sin\theta\cos\theta$? [CDS 2009]

(a) 2 (b) 0 (c) 1 (d) 1/2

8. If $\sin\theta + \operatorname{cosec}\theta = 2$, then the value of $\sin^7\theta + \operatorname{cosec}^7\theta$ is

(a) 1 (b) 1/2 (c) 2 (d) 0

9. If $\tan\theta = 3/4$ and θ is acute, then $\operatorname{cosec}\theta$ is equal to [SSC CGL 2013]

(a) $\frac{5}{3}$ (b) $\frac{5}{4}$ (c) $\frac{4}{3}$ (d) $\frac{4}{3}$

10. The value of $\left(\frac{1}{(1+\tan^2\theta)} + \frac{1}{(1+\cot^2\theta)}\right)$ is [SSC CGL 2013]

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

11. The minimum value of $\sin^2\theta + \cos^4\theta$ is **[SSC CPO 2013]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

12. If $\sec\theta + \tan\theta = p$, then $\cos\theta$ is

(a) $\frac{p^2+1}{p^2-1}$ (b) $\frac{p^2-1}{(p^2+1)^2}$

(c) $\frac{2p}{p^2+1}$ (d) $\frac{4p^2}{(p^2+1)^2}$

(e) None of the above

13. If $\tan 62° = \frac{P}{Q}$, then $\tan 28°$ is equal to

(a) $\frac{P}{Q}$ (b) $\frac{Q}{P}$

(c) $\frac{P^2-Q^2}{P}$ (d) $\frac{Q}{P^2}$

(e) None of the above

14. If $\sec\theta + \tan\theta = 2$, what is the value of $\sec\theta$? **[CDS 2014]**

(a) $\frac{3}{2}$ (b) $\sqrt{2}$ (c) $\frac{5}{2}$ (d) $\frac{5}{4}$

15. What is $\sin 25° \sin 35° \sec 65° \sec 55°$ equal to? **[CDS 2013]**

(a) -1 (b) 0 (c) $\frac{1}{2}$ (d) 1

16. Consider the following

I. $\sin^2 1° + \cos^2 1° = 1$

II. $\sec^2 33° - \cot^2 57° = \operatorname{cosec}^2 37° - \tan^2 53°$

Which of the above statement is/are correct?

(a) Only I (b) Only II

(c) Both 1 and II (d) Neither I nor II

17. If $p = a\sin x + b\cos x$ and $q = a\cos x - b\sin x$, then what is the value of $p^2 + q^2$? **[CDS 2012]**

(a) $a + b$ (b) ab

(c) $a^2 + b^2$ (d) $a^2 - b^2$

18. What is the value of $\sec(90° - \theta) \cdot \sin\theta \sec 45°$? **[CDS 2012]**

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

19. If $\tan\theta = \frac{4}{3}$, then the value of $\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$ is **[SSC (10 + 2) 2011]**

(a) 0.5 (b) – 0.5
(c) 3 (d) – 3.0

20. If $\sec\theta - \text{cosec}\,\theta = 0$, then the value of $\tan\theta + \cot\theta$ is **[SSC (10+2) 2013]**

(a) 0 (b) 1 (c) – 1 (d) 2

21. If $\tan 7\theta \tan 2\theta = 1$, then the value of $\tan 3\theta$ is **[SSC CGL 2012]**

(a) $\sqrt{3}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $-\sqrt{3}$

22. The expression $\frac{\tan 57° + \cot 37°}{\tan 33° + \cot 53°}$ is equal to **[SSC CGL 2012]**

(a) tan 33° cot 57° (b) tan 57° cot 37°
(c) tan 33° cot 53° (d) tan 53° cot 37°

23. The numerical value of $\frac{5}{\sec^2\theta} + \frac{1}{1+\cot^2\theta} + 3\sin^2\theta$ is **[SSC (10+2) 2012]**

(a) 5 (b) 2 (c) 3 (d) 4

24. The value of

$\tan 4° \cdot \tan 43° \cdot \tan 47° \cdot \tan 86°$ is **[SSC CPO 2011]**

(a) 2 (b) 3 (c) 1 (d) 4

25. Consider the following equations

(i) $\text{cosec}^2 x + \sec^2 x = \text{cosec}^2 x \sec^2 x$

(ii) $\sec x + \tan^2 x = \sec x \tan^2 x$

(iii) $\text{cosec}\, x + \tan^2 x = \cot^2 x + \sec x$

Which of the above equations are correct? **[CDS 2009]**

(a) (i) and (ii) (b) (ii) and (iii)
(c) (i) and (iii) (d) (i), (ii) and (iii)

26. If $\frac{\cos x}{1+\text{cosec}\, x} + \frac{\cos x}{\text{cosec}\, x - 1} = 2$, which one of the following is one of the value of x? **[CDS 2009]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

27. $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2$ is satisfied by which one of the following values of θ? **[CDS 2009]**

(a) $\pi/2$ (b) $\pi/3$
(c) $\pi/4$ (d) $\pi/6$

28. What is $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ equal to? **[CDS 2010]**

(a) $\sec\theta - \tan\theta$ (b) $\sec\theta + \tan\theta$
(c) $\text{cosec}\,\theta + \cot\theta$ (d) $\text{cosec}\,\theta - \cot\theta$

29. If $7\cos^2\theta + 3\sin^2\theta = 4$ and $0 < \theta < \pi/2$, what is the value of $\tan\theta$? **[CDS 2010]**

(a) $\sqrt{7}$ (b) $\frac{7}{3}$ (c) 3 (d) $\sqrt{3}$

30. The value of $\frac{\tan 27° + \cot 63°}{\tan 27° (\sin 25° + \cos 65°)}$.

[SSC CPO 2013]

(a) cosec 25° (b) 2 tan27°
(c) sin 25° (d) tan 65°

31. If $\cos\theta \geq 1/2$ in the first quadrant, which one of the following is correct? **[CDS 2010]**

(a) $\theta \leq \frac{\pi}{3}$ (b) $\theta \geq \frac{\pi}{3}$ (c) $\theta \leq \frac{\pi}{6}$ (d) $\theta \geq \frac{\pi}{6}$

32. If $x = a \sec\theta \cos\phi$, $y = b \sec\theta \sin\phi$ and $z = c \tan\theta$, then the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$ is

[Railways 2006, SSC CGL 2013]

(a) 9 (b) 0 (c) 1 (d) 4

33. If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $1 - 2\sin^2\theta$ is **[SSC CGL 2013, SSC (10+2) 2011]**

(a) 0 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{4}{3}$

34. If $7\sin^2\theta + 3\cos^2\theta = 4$, then the value of $\cos\theta\,(0° \leq \theta \leq 90°)$ is **[SSC CPO 2013]**

(a) $\frac{\sqrt{6}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\frac{\sqrt{5}}{2}$

35. If $\tan 8\theta = \cot 2\theta$, where $0 < 8\theta < \frac{\pi}{2}$, then what is the value of $\tan 5\theta$?

[CDS 2014]

(a) $\frac{1}{\sqrt{3}}$ (b) 1 (c) $\sqrt{3}$ (d) 0

36. If $\sin\theta - \cos\theta = 0$, then what is $\sin^4\theta + \cos^4\theta$ equal to? **[CDS 2013]**

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

37. The expression $\sin^2 x + \cos^2 x - 1 = 0$ is satisfied by how many values of x?

[CDS 2012]

(a) Only one value of x
(b) Two values of x
(c) Infinite values of x
(d) No value of x

38. Consider the following

I. $\frac{\cot 30° + 1}{\cot 30° - 1} = 2(\cos 300° + 1)$

II. $2\sin 45° \cos 45° - \tan 45° \cot 45° = 0$

Which of the above identities is/are correct? **[CDS 2012]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

39. What is the value of $\sin A \cos A \tan A + \cos A \sin A \cot A$? **[CDS 2012]**

(a) $\sin^2 A + \cos A$ (b) $\sin^2 A + \tan^2 A$
(c) $\sin^2 A + \cot^2 A$ (d) $\text{cosec}^2 A - \cot^2 A$

40. If $2(\cos^2\theta - \sin^2\theta) = 1$ (θ is a positive acute angle), then $\cot\theta$ is equal to **[SSC (10+2) 2013]**

(a) $\frac{1}{\sqrt{3}}$ (b) 1
(c) $\sqrt{3}$ (d) $-\sqrt{3}$

41. The minimum value of $\cos^2\theta + \sec^2\theta$ is **[SSC (10+2) 2013]**

(a) 0 (b) 1 (c) 2 (d) 3

42. If $\cos x + \cos y = 2$, then the value of $\sin x + \sin y$ is **[SSC FCI 2012]**

(a) 0 (b) 1 (c) 2 (d) -1

43. If $\sec^2\theta + \tan^2\theta = \frac{7}{12}$, then $\sec^4\theta - \tan^4\theta$ is equal to **[SSC FCI 2012]**

(a) $\frac{7}{12}$ (b) $\frac{1}{2}$ (c) $\frac{5}{12}$ (d) 1

44. The numerical value of

$\left(\frac{1}{\cos\theta} + \frac{1}{\cot\theta}\right)\left(\frac{1}{\cos\theta} - \frac{1}{\cot\theta}\right)$ is **[SSC (10+2) 2012]**

(a) 0 (b) -1 (c) 1 (d) 2

45. If $A = \tan 11° \tan 29°$ and $B = 2 \cot 61° \cot 79°$, then **[SSC (10+2) 2011]**

(a) $A = 2B$ (b) $A = -2B$
(c) $2A = B$ (d) $2A = B$

46. If $\tan 2\theta \cdot \tan 4\theta = 1$, then the value of 3θ is **[SSC (10+2) 2011]**

(a) $\sqrt{3}$ (b) 0 (c) 1 (d) $\frac{1}{\sqrt{3}}$

47. If $\sin\theta + \text{cosec}\,\theta = 2$, then the value of $\sin^5\theta + \text{cosec}^5\theta$, when $0° \le \theta \le 90°$, is **[SSC (10+2) 2011]**

(a) 0 (b) 1 (c) 10 (d) 2

48. If $A = \sin^2\theta + \cos^4\theta$, for any value of θ, then the value of A is **[SSC (10+2) 2011]**

(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le 1$
(c) $\frac{13}{16} \le A \le 1$ (d) $\frac{3}{4} \le A \le \frac{13}{16}$

49. The minimum value of $2\sin^2\theta + 3\cos^2\theta$ is **[SSC CPO 2011]**

(a) 0 (b) 3 (c) 2 (d) 1

50. The value of $\cot 10° \cdot \cot 20° \cdot \cot 60° \cdot \cot 70° \cot 80°$ is **[SSC (10+2) 2011]**

(a) 1 (b) -1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

51. If $\cos\theta \text{ cosec } 23° = 1$, then the value of θ is **[SSC (10+2) 2012]**

(a) 23° (b) 37° (c) 63° (d) 67°

52. If θ is a positive acute angle and $\tan 2\theta \tan 3\theta = 1$, then the value of $\left(2\cos^2\frac{5\theta}{2} - 1\right)$ is **[SSC CGL 2012]**

(a) $-\frac{1}{2}$ (b) 1 (c) 0 (d) $\frac{1}{2}$

53. If $0° < \theta < 90°$, then the value of $\sin\theta + \cos\theta$ is **[SSC CGL 2012]**

(a) equal to 1 (b) greater than 1
(c) less than 1 (d) equal to 2

54. If $\sin\theta + \text{cosec}\,\theta = 2$, then value of $\sin^{100}\theta + \text{cosec}^{100}\theta$ is equal to

[SSC (10+2) 2011]

(a) 1 (b) 2 (c) 3 (d) 100

55. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$, then the value of $\sin\theta - \cos^4\theta$ is **[SSC (10+2) 2011]**

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

56. If θ be acute and $\tan\theta + \cot\theta = 2$, then the value of $\tan^5\theta + \cot^{10}\theta$ is

[SSC (10+2) 2011]

(a) 1 (b) 2 (c) 3 (d) 4

57. If $0° < \theta < 90°$, then all the trigonometric ratios can be obtained when **[CDS 2012]**

(a) only $\sin\theta$ is given
(b) only $\cos\theta$ is given
(c) only $\tan\theta$ is given
(d) any one of the six ratios is given

58. What is the value of $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$? **[CDS 2012]**

(a) $2\text{ cosec }\theta$ (b) $2\sec\theta$
(c) $\sec\theta$ (d) $\text{cosec }\theta$

59. If $\sin\theta\cos\theta = \sqrt{3}/4$, then the value of $\sin^4\theta + \cos^4\theta$ is **[CDS 2012]**

(a) 7/8 (b) 5/8 (c) 3/8 (d) 1/8

60. If $2\cot\theta = 3$, then what is $\frac{2\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$ equal to? **[CDS 2014]**

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

61. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta$ is **[SSC CGL 2013]**

(a) $-\sqrt{2}\cos\theta$ (b) $-\sqrt{2}\sin\theta$
(c) $\sqrt{2}\sin\theta$ (d) $\sqrt{2}\tan\theta$

62. If $3\sin x + 5\cos x = 5$, then what is the value of $(3\cos x - 5\sin x)$? **[CDS 2012]**

(a) 0 (b) 2 (c) 3 (d) 5

63. If $0 < x < \frac{\pi}{2}$ and $\sec x = \operatorname{cosec} y$, then the value of $\sin(x + y)$ is **[SSC CGL 2013]**

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{3}}$

64. If $\sin 17° = \frac{x}{y}$, then the value of $\sec 17° - \sin 73°$ is **[SSC FCI 2012]**

(a) $\frac{y^2 - x^2}{xy}$ (b) $\frac{x^2}{\sqrt{y^2 - x^2}}$

(c) $\frac{x^2}{y\sqrt{y^2 + x^2}}$ (d) $\frac{x^2}{y\sqrt{y^2 - x^2}}$

65. If $2\sin\left(\frac{\pi x}{2}\right) = x^2 + \frac{1}{x^2}$, then the value of $\left(x - \frac{1}{x}\right)$ is **[SSC CGL 2012]**

(a) -1 (b) 2 (c) 1 (d) 0

66. The equation $\cos^2\theta = \frac{(x + y)^2}{4xy}$ is only possible when **[SSC (10+2) 2013]**

(a) $x > y$ (b) $x = y$
(c) $x < y$ (d) $x = -y$

67. The value of $\sin^2 1° + \sin^2 3° + \sin^2 5° + \cdots + \sin^2 87° + \sin^2 89°$ is **[SSC (10 + 2) 2013]**

(a) 22 (b) $22\frac{1}{2}$ (c) 23 (d) $22\frac{1}{4}$

68. If $\angle A$ and $\angle B$ are complementary to each other, then the value of $\sec^2 A + \sec^2 B - \sec^2 A \sec^2 B$ is **[SSC CGL 2012]**

(a) 1 (b) -1 (c) 2 (d) 0

69. The value of $\sin^2 5° + \sin^2 10° + \sin^2 15° + \cdots + \sin^2 85° + \sin^2 90°$ is **[SSC (10 + 2) 2012, 2011]**

(a) $7\frac{1}{2}$ (b) $8\frac{1}{2}$ (c) $10\frac{1}{2}$ (d) $9\frac{1}{2}$

70. If $\sin(3x - 20°) = \cos(3y + 20°)$, then the value of $(x + y)$ is

[SSC (10 + 2) 2012]

(a) 20° (b) 30° (c) 40° (d) 45°

71. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{5}{4}$, the value of $\frac{\tan^2\theta + 1}{\tan^2\theta - 1}$ is

[SSC (10 + 2) 2012]

(a) $\frac{25}{16}$ (b) $\frac{41}{9}$ (c) $\frac{41}{40}$ (d) $\frac{40}{41}$

72. The value of $\frac{\sin 39°}{\cos 51°} + 2\tan 11°$ $\tan 31° \tan 45° \tan 59° \tan 79° - 3(\sin^2 21° + \sin^2 69°)$ is

[SSC (10 + 2) 2011]

(a) 2 (b) −1 (c) 1 (d) 0

73. If $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$ and $0° < \theta < 90°$, then the value of θ is

[SSC (10 + 2) 2011]

(a) 30° (b) 45°

(c) 60° (d) None of these

74. The numerical value of $\cot 18° \left(\cot 72° \cos^2 22° + \frac{1}{\tan 72° \sec^2 68°}\right)$ is

[SSC (10 + 2) 2011]

(a) 1 (b) $\sqrt{2}$

(c) 3 (d) $\frac{1}{\sqrt{3}}$

75. If θ be an acute angle and $7\sin^2\theta + 3\cos^2\theta = 4$, then the value of $\tan\theta$ is

[SSC (10 + 2) 2011]

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) 1 (d) 0

Exercise © *Higher SkiU Level Questions*

1. If $(\sin x + \sin y) = a$ and $(\cos x + \cos y) = b$, what is the value of $\sin x \sin y + \cos x \cos y$?

(a) $a + b - ab$ (b) $a + b + ab$

(c) $a^2 + b^2 - 2$ (d) $\dfrac{a^2 + b^2 - 2}{2}$

(e) None of the above

2. Consider the following [CDS 2009]

(i) $\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \cos^2\theta(1 + \tan\theta)$

$(1 - \tan\theta)$

(ii) $\dfrac{1 + \sin\theta}{1 - \sin\theta} = (\tan\theta + \sec\theta)^2$

Which of the equations given above is/are correct?

(a) Only (i) (b) Only (ii)
(c) Both (i) and (ii) (d) Neither (i) nor (ii)

3. The value of
(tan 1° tan 2° tan 3° .. tan 89°) is **[SSC CGL 2013]**

(a) undefined (b) 0
(c) 1 (d) 89

4. If $\sec\theta + \tan\theta = p$, then $\sin\theta$ is

(a) $\dfrac{1}{p^2+1}$ (b) $\dfrac{1}{p^2-1}$
(c) $\dfrac{p^2+1}{p^2-1}$ (d) $\dfrac{p^2-1}{p^2+1}$

5. If $\sin\theta + \cos\theta = x$, then the value of $\cos^6\theta + \sin^6\theta$ is equal to

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{4}(1+6x^2)$
(c) $\dfrac{1}{4}(1+6x^2-3x^4)$ (d) $\dfrac{1}{2}(5-3x^2)$
(e) None of the above

6. If $(1+\tan A)(1+\tan B) = 2$, then $(A+B)$ is equal to

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{6}$
(e) None of the above

7. If $\sin(10^\circ\ 6'\ 32'') = a$, then the value of $\cos(79^\circ\ 53'\ 28'') + \tan(10^\circ\ 6'\ 32'')$ is **[SSC CGL (Main) 2012]**

(a) $\dfrac{a(1+\sqrt{1-a^2})}{\sqrt{1-a^2}}$ (b) $\dfrac{1+\sqrt{1-a^2}}{\sqrt{1-a^2}}$
(c) $\dfrac{\sqrt{1-a^2}+a}{\sqrt{1-a^2}}$ (d) $\dfrac{a\sqrt{1-a^2}+1}{\sqrt{1-a^2}}$

8. If $x + y = z$, find the value of $\cos^2 x + \cos^2 y + \cos^2 z$.

(a) $1 + 2\sin x \sin y \sin z$
(b) $1 - 2\sin x \sin y \sin z$
(c) $1 + 2\cos x \cos y \cos z$
(d) $1 - 2\cos x \cos y \cos z$
(e) None of the above

9. If $\cos\theta + \sec\theta = 2$, then the value of $\cos^6\theta + \sec^6\theta$ is **[SSC (10 + 2) 2012]**

(a) 1 (b) 2 (c) 4 (d) 8

10. If $\sin\theta - \cos\theta = 0$, find the value of $\sin\left[\dfrac{\pi}{2}-\theta\right] + \cos\left(\dfrac{\pi}{2}-\theta\right)$ **[SSC (10 + 2) 2012]**

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

11. The maximum value of $\sin^8\theta + \cos^{14}\theta$, for all real values of θ is **[SSC CGL (Main) 2012]**

(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 0

12. If $l\cos^2\theta + m\sin^2\theta = \frac{\cos^2\theta(\text{cosec}^2\theta + 1)}{\text{cosec}^2\theta - 1}, 0° < \theta < 90°$, then $\tan\theta$ is equal to **[SSC CGL (Main) 2012]**

(a) $\sqrt{\frac{l-2}{1-m}}$ (b) $\sqrt{\frac{2-l}{1-m}}$

(c) $\sqrt{\frac{l-2}{m-1}}$ (d) $\sqrt{\frac{l-1}{2-m}}$

13. ΔABC is a right-angled triangle, where $\angle ABC = 90°$. If $AC = 2\sqrt{5}$ and $AB - BC = 2$, then the value of $\cos^2 A - \sin^2 C$ is **[SSC CGL (Main) 2012]**

(a) $\frac{1}{\sqrt{5}}$ (b) $\sqrt{5}$

(c) $\frac{1}{2}$ (d) $\frac{3}{5}$

14. If $\cos x + \cos^2 x = 1$, then the numerical value of $(\sin^{12}x + 3\sin^{10}x + 3\sin^8 x + \sin^6 x - 1)$ is **[SSC CGL 2013]**

(a) 0 (b) 1 (c) -1 (d) 2

15. If $a\sin\theta + b\cos\theta = c$, then the value of $a\cos\theta - b\sin\theta$ is **[SSC CGL 2013]**

(a) $\pm\sqrt{a^2 - b^2 - c^2}$

(b) $\pm\sqrt{a^2 - b^2 + c^2}$

(c) $\pm\sqrt{-a^2 + b^2 + c^2}$

(d) $\pm\sqrt{a^2 + b^2 - c^2}$

16. If $\tan\alpha = n\tan\beta$ and $\sin\alpha = m\sin\beta$, then $\cos^2\alpha$ is **[SSC CGL 2013]**

(a) $\frac{m^2}{n^2}$ (b) $\frac{m^2-1}{n^2-1}$

(c) $\frac{m^2+1}{n^2+1}$ (d) $\frac{m^2}{n^2+1}$

17. If $\sin\theta\cos\theta = 1/2$, then what is $\sin^6\theta + \cos^6\theta$ equal to? **[CDS 2014]**

(a) 1 (b) 2 (c) 3 (d) $\frac{1}{4}$

18. What is $\text{cosec}(75° + \theta) - \sec(15° - \theta) - \tan(55° + \theta) + \cot(35° - \theta)$ **[CDS 2014]**

(a) -1 (b) 0 (c) 1 (d) $3/2$

19. If $\cos x + \sec x = 2$, then what $\cos^n x + \sec^n x$ equal to, where n is a positive integer? **[CDS 2014]**

(a) 2 (b) 2^{n-2} (c) 2^{n-1} (d) 2^n

20. If α and β are complementary angles, then what is $\sqrt{\cos\alpha \operatorname{cosec}\beta - \cos\alpha \sin\beta}$ equal to?

[CDS 2014]

(a) $\sec\beta$ (b) $\cos\alpha$ (c) $\sin\alpha$ (d) $-\tan\beta$

21. If $\sin\theta + 2\cos\theta = -1$, where $0 < \theta < \frac{\pi}{2}$, what is $2\sin\theta - \cos\theta$ equal to? **[CDS 2014]**

(a) -1 (b) $\frac{1}{2}$ (c) 2 (d) 1

22. If $\sin(A + B) = 1$, where $0 < B < 45^\circ$, what is $\cos(A - B)$ equal to? **[CDS 2014]**

(a) $\sin 2B$ (b) $\sin B$
(c) $\cos 2B$ (d) $\cos B$

23. If $\sin\theta + \cos\theta = \sqrt{3}$, then what is $\tan\theta + \cot\theta$ equal to? **[CDS 2013]**

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $\sqrt{3}$

24. If $\tan\theta + \sec\theta = m$, then what is $\sec\theta$ equal to? **[CDS 2013]**

(a) $\frac{m^2-1}{2m}$ (b) $\frac{m^2+1}{2m}$
(c) $\frac{m+1}{m}$ (d) $\frac{m^2+1}{m}$

25. If $5\sin\theta + 12\cos\theta = 13$, then what is $5\cos\theta - 12\sin\theta$ equal to? **[CDS 2013]**

(a) -2 (b) -1 (c) 0 (d) 1

26. Conside the following statements

I. $\tan\theta$ increases faster than $\sin\theta$ as θ increases.

II. The value of $\sin\theta + \cos\theta$ is always greater than 1.

Which of the statemetns given above is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

27. What is $\frac{(\sin\theta + \cos\theta)(\tan\theta + \cot\theta)}{\sec\theta + \operatorname{cosec}\theta}$ equal to? **[CDS 2013]**

(a) 1 (b) 2 (c) $\sin\theta$ (d) $\cos\theta$

28. What is $\dfrac{(1+\sec\theta-\tan\theta)\cos\theta}{(1+\sec\theta+\tan\theta)(1-\sin\theta)}$ equal to? [CDS 2013]

(a) 1 (b) 2 (c) $\tan\theta$ (d) $\cot\theta$

29. If $x\sin\theta - y\cos\theta = \sqrt{x^2+y^2}$ and $\dfrac{\cos^2\theta}{a^2}+\dfrac{\sin^2\theta}{b^2}=\dfrac{1}{x^2+y^2}$, then the correct relation is [SSC (10 + 2) 2013]

(a) $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ (b) $\dfrac{x^2}{b^2}+\dfrac{y^2}{a^2}=1$

(c) $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ (d) $\dfrac{x^2}{b^2}-\dfrac{y^2}{a^2}=1$

30. $2\operatorname{cosec}^2 23^\circ \cot^2 67^\circ - \sin^2 23^\circ - \sin^2 67^\circ - \cot^2 67^\circ$ is equal to [SSC (10 + 2) 2013]

(a) $\sec^2 23^\circ$ (b) $\tan^2 23^\circ$

(c) 0 (d) 1

31. The value of $\cot\theta\cdot\tan(90^\circ-\theta) - \sec(90^\circ-\theta)\operatorname{cosec}\theta + (\sin^2 25^\circ + \sin^2 65^\circ) + \sqrt{3}(\tan 5^\circ \tan 15^\circ\cdot\tan 30^\circ\cdot\tan 75^\circ\cdot\tan 85^\circ)$ is [SSC (10 + 2) 2012]

(a) 1 (b) -1

(c) 2 (d) 0

32. In a right angled ΔXYZ, right angled at Y, if $XY = 2\sqrt{6}$ and $XY - YZ = 2$, then $\sec X + \tan X$ is [SSC CGL 2012]

(a) $\dfrac{1}{\sqrt{6}}$ (b) $\sqrt{6}$

(c) $2\sqrt{6}$ (d) $\dfrac{\sqrt{6}}{2}$

33. The simplified value of $(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y \tan x \sec y)^2$ is [SSC (10 + 2) 2011]

(a) -1

(b) 0

(c) $\sec^2 x$

(d) 1

34. If $\tan 15^\circ = 2-\sqrt{3}$, then the value of $\tan 15^\circ \cot 75^\circ + \tan 75^\circ \cot 15^\circ$ is [SSC (10 + 2) 2011]

(a) 14 (b) 12 (c) 10 (d) 8

Answer with **Solutions**

Exercise© *Base Level Questions*

1. (*c*) In ΔABC, if $\angle C$ is $90°$, then

$$\angle A + \angle B = 180° - 90° = 90°$$

Now, $\cos(A + B) + \sin(A + B)$

$$= \cos 90° + \sin 90° = 0 + 1 = 1$$

2. (*c*) $\sin \alpha = \dfrac{\sqrt{3}}{2}$

$\therefore \quad \alpha = 60° \qquad \left(\because \sin 60 = \dfrac{\sqrt{3}}{2}\right)$

$\cos \beta = \dfrac{\sqrt{3}}{2}$

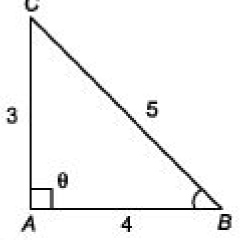
$\therefore \quad \beta = 30° \qquad \left(\because \cos 30° = \dfrac{\sqrt{3}}{2}\right)$

$\tan \gamma = 1$

$\therefore \quad \gamma = 45° \qquad (\because \tan 45° = 1)$

So, $\quad \alpha + \beta + \gamma = 60° + 30° + 45° = 135°$

3. (*b*) Given that, $\tan\theta = \dfrac{P}{B} = \dfrac{3}{4}$ and $0° < \theta < 90°$ (acute)



In ΔABC,

By Pythagoras theorem

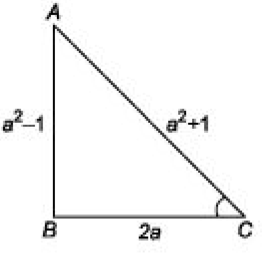
$$BC^2 = AC^2 + AB^2$$

$$= 9 + 16 = 25$$

$$BC = 5$$

$\therefore \quad \sin\theta = \dfrac{P}{H} = \dfrac{3}{5}$

4. (*d*) $\sin\theta = \dfrac{a^2 - 1}{a^2 + 1}$



In ΔABC,

$$BC = \sqrt{AC^2 - AB^2}$$
$$= \sqrt{(a^2 + 1)^2 - (a^2 - 1)^2}$$
$$= \sqrt{a^2 + 1 + 2a^2 - a^2 - 1 + 2a^2}$$
$$= \sqrt{4a^2} = 2a$$

$$\therefore \sec\theta + \tan\theta = \frac{a^2 + 1}{2a} + \frac{a^2 - 1}{2a}$$
$$= \frac{2a^2}{2a} = a$$

5. (*a*) Given

$$\alpha + \beta = 90° \text{ and } \alpha : \beta = 2 : 1$$
$$\alpha = \frac{2}{2 + 1} \times 90°$$
$$= \frac{2}{3} \times 90° = 60°$$

Similarly, $\beta = \frac{1}{3} \times 90° = 30°$

$$\therefore \sin\alpha : \sin\beta = \sin 60° : \sin 30°$$
$$= \frac{\sqrt{3}}{2} : \frac{1}{2} = \sqrt{3} : 1$$

6. (*c*) $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{5}{3}$

$$\Rightarrow \quad \frac{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{\frac{1 + \sin\theta}{\cos\theta}}{\frac{1 - \sin\theta}{\cos\theta}} = \frac{5}{3} \Rightarrow \frac{1 + \sin\theta}{1 - \sin\theta} = \frac{5}{3}$$

$$\Rightarrow \quad (1 + \sin\theta) \times 3 = 5(1 - \sin\theta)$$
$$\Rightarrow \quad 3 + 3\sin\theta = 5 - 5\sin\theta$$
$$\Rightarrow \quad 3\sin\theta + 5\sin\theta = 5 - 3$$
$$\Rightarrow \quad 8\sin\theta = 2$$
$$\therefore \quad \sin\theta = \frac{2}{8} \text{ or } \sin\theta = \frac{1}{4}$$

7. (*b*) $\sin\theta + \cos\theta = 1$

On squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = 1$$
$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$$
$$1 + 2\sin\theta\cos\theta = 1$$
$$2\sin\theta\cos\theta = 0$$
$$\therefore \quad \sin\theta\cos\theta = 0$$

8. (c) $\sin\theta + \operatorname{cosec}\theta = 2$

$$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2 \Rightarrow \sin^2\theta + 1 = 2\sin\theta$$

$$\sin^2\theta - 2\sin\theta + 1 = 0$$

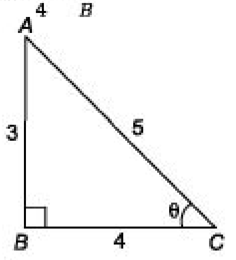
$$\Rightarrow \quad (\sin\theta - 1)^2 = 0$$

$$\Rightarrow \quad \sin\theta = 1 = \sin 90^\circ$$

$$\therefore \quad \theta = 90^\circ$$

$$\therefore \quad \sin^7\theta + \operatorname{cosec}^7\theta = 1 + 1 = 2$$

9. (a) $\tan\theta = \frac{3}{4} = \frac{P}{B}$



In ΔABC,

$$H^2 = P^2 + B^2$$

$$\Rightarrow \quad H^2 = 3^2 + 4^2$$

$$\Rightarrow \quad H = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\therefore \quad \operatorname{cosec}\theta = \frac{H}{P} = \frac{5}{3}$$

10. (a) $\frac{1}{1 + \tan^2\theta} + \frac{1}{1 + \cot^2\theta}$

$$\frac{1}{1 + \frac{\sin^2\theta}{\cos^2\theta}} + \frac{1}{1 + \frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\cos^2\theta}{\cos^2\theta + \sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta + \cos^2\theta}$$

$$\cos^2\theta + \sin^2\theta = 1$$

11. (c) $\therefore$ Minimum value of

$\sin^2\theta + \cos^4\theta$ will be at $\theta = 45^\circ$

$\therefore$ Required minimum value

$$\sin^2 45 + \cos^4 45 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

12. (c) $\sec\theta + \tan\theta = p$ or $\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = p$

or $\quad 1 + \sin\theta = p\cos\theta \quad$ ...(i)

On squaring both sides, we get

$$1 + \sin^2\theta + 2\sin\theta = p^2\cos^2\theta$$

$$\Rightarrow \quad 1 + 1 - \cos^2\theta + 2\sin\theta = p^2\cos^2\theta$$

$$\Rightarrow \quad 2 + 2\sin\theta - \cos^2\theta = p^2\cos^2\theta$$

$$\Rightarrow \quad 2(1 + \sin\theta) - \cos^2\theta = p^2\cos^2\theta$$

$$\Rightarrow \quad 2 \times p\cos\theta - \cos^2\theta = p^2\cos^2\theta$$

[from Eq. (i)]

$\Rightarrow \quad 2p\cos\theta = p^2\cos^2\theta + \cos^2\theta$

$\Rightarrow \quad 2p\cos\theta = \cos^2\theta\,(1+p^2)$

$\therefore \quad \cos^2\theta = \dfrac{2p\cos\theta}{(1+p^2)} \Rightarrow \cos\theta = \dfrac{2p}{1+p^2}$

13. (*b*) $\tan 62° = \dfrac{P}{Q}$

$\Rightarrow \quad \tan(90° - 28°) = \dfrac{P}{Q} \Rightarrow \quad \cot 28° = \dfrac{P}{Q}$

$\Rightarrow \quad \dfrac{1}{\tan 28°} = \dfrac{P}{Q} \Rightarrow \tan 28° = \dfrac{Q}{P}$

14. (*d*) By trigonometric identity,

$$\sec^2\theta - \tan^2\theta = 1$$

$\Rightarrow \quad (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \quad \sec\theta - \tan\theta = \dfrac{1}{2} \quad \ldots\text{(i)}$

and given, $\sec\theta + \tan\theta = 2 \quad \ldots\text{(ii)}$

On adding Eqs. (i) and (ii), we get

$$2\sec\theta = \frac{1}{2} + 2$$

$\therefore \quad \sec\theta = \dfrac{5}{4}$

15. (*d*) $\sin 25° \sin 35° \sec 65° \sec 55°$

$\Rightarrow \quad \sin 25° \sin 35° \cdot \dfrac{1}{\cos 65°} \cdot \dfrac{1}{\cos 55°}$

$\Rightarrow \quad \sin 25° \cdot \sin 35° \times \dfrac{1}{\cos(90° - 25°)} \times \dfrac{1}{\cos(90° - 35°)}$

$\Rightarrow \sin 25° \times \sin 35° \times \dfrac{1}{\sin 25°} \times \dfrac{1}{\sin 35°}$

$\Rightarrow 1$

16. (*d*) We know that, $\sin^2\theta + \cos^2\theta = 1$ is true

$\therefore$ I. $\sin^2 1° + \cos^2 1° = 1$ which is not true.

II. $\sec^2 33° - \cot^2 57°$

$= \text{cosec}^2 37° - \tan^2 53°$

Now, $\quad \sec^2(90° - 57°) = \text{cosec}^2 57°$

and $\quad \cot^2 57° = \cot^2(90° - 33°) = \tan^2 33°$

$\therefore \quad \sec^2 33° - \cot^2 57°$

$= \text{cosec}^2 57 - \tan^2 33°$

II is not true.

17. (*c*) Given, $p = a\sin x + b\cos x \quad \ldots\text{(i)}$

and $q = a\cos x - b\sin x \quad \ldots\text{(ii)}$

On squaring both the equations

$$p^2 = a^2\sin^2 x + b^2\cos^2 x + 2ab\sin x\cos x$$

and $\quad q^2 = a^2\cos^2 x + b^2\sin^2 x - 2ab\sin x\cos x$

$$P^2 + q^2 = a^2\sin^2 x + b^2\cos^2 x + 2ab\sin x\cos x$$
$$+ a^2\cos^2 x + b^2\sin^2 x - 2ab\sin x\cos x$$
$$p^2 + q^2 = a^2(\sin^2 x + \cos^2 x) + b^2$$
$$(\cos^2 x + \sin^2 x)$$
$$= a^2 + b^2$$

18. (*c*) Given, $\sec(90-\theta)^\circ \sin\theta \sec 45$

$$= \operatorname{cosec}\theta \sin\theta \sec 45^\circ$$
$$= \frac{1}{\sin\theta}\cdot\sin\theta\cdot(\sqrt{2}) = \sqrt{2}$$

19. (*c*) $\tan\theta = \dfrac{4}{3}$ [given]

$$\therefore \quad \frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta} = \frac{3\tan\theta + 2}{3\tan\theta - 2}$$
$$= \frac{3\times\frac{4}{3} + 2}{3\times\frac{4}{3} - 2} = \frac{4+2}{4-2} = 3$$

20. (*d*) $\sec\theta - \operatorname{cosec}\theta = 0$

$$\Rightarrow \quad \sec = \operatorname{cosec}\theta$$
$$\frac{1}{\cos\theta} = \frac{1}{\sin\theta}$$
$$\Rightarrow \quad \tan\theta = 1$$
$$\Rightarrow \quad \tan\theta = \tan 45^\circ$$
$$\Rightarrow \quad \theta = 45^\circ$$
$$\therefore \tan\theta + \cot\theta = \tan 45^\circ + \cot 45^\circ$$
$$= 1 + 1 = 2$$

21. (*c*) Given, $\tan 7\theta\cdot\tan 2\theta = 1$

$$\Rightarrow \quad \tan 7\theta = \cot 2\theta$$
$$\Rightarrow \quad \tan 7\theta = \tan(90^\circ - 2\theta)$$
$$\Rightarrow \quad 7\theta = 90^\circ - 2\theta$$
$$\Rightarrow \quad 9\theta = 90^\circ \Rightarrow \theta = 10^\circ$$
$$\therefore \quad \tan 3\theta = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

22. (*b*) $\dfrac{\tan 57^\circ + \cot 37^\circ}{\tan 33^\circ + \cot 53^\circ} = \dfrac{\tan 57^\circ + \cot 37^\circ}{\cot 57^\circ + \tan 37^\circ}$

$$= \frac{\tan 57^\circ + \frac{1}{\tan 37^\circ}}{\frac{1}{\tan 57^\circ} + \tan 37^\circ}$$
$$= \frac{1 + \tan 57^\circ \tan 37^\circ}{1 + \tan 37^\circ \tan 57^\circ} \times \frac{\tan 57^\circ}{\tan 37^\circ}$$
$$= \frac{\tan 57^\circ}{\tan 37^\circ} = \tan 57^\circ\cdot\cot 37^\circ$$

23. (*a*) $\dfrac{5}{\sec^2\theta} + \dfrac{2}{1+\cot^2\theta} + 3\sin^2\theta$

$$= 5\cos^2\theta + \frac{2}{\operatorname{cosec}^2\theta} + 3\sin^2\theta$$
$$[\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta]$$

$= 5 \cos^2\theta + 2\sin^2\theta + 3\sin^2\theta$

$= 5\cos^2\theta + 5\sin^2\theta$

$= 5(\sin^2\theta + \cos^2\theta)$

$\{\because \sin^2\theta + \cos^2\theta = 1\}$

$= 5 \times 1 = 5$

24. (*b*) $\dfrac{5\sin 75^\circ \sin 77^\circ + 2\cos 13^\circ \cos 15^\circ}{\cos 15^\circ \sin 77^\circ} - \dfrac{7\sin 81^\circ}{\cos 9^\circ}$

$= \dfrac{5[\sin(90^\circ - 15^\circ)\sin(90^\circ - 13^\circ)] + 2\cos 13^\circ \cos 15^\circ}{\cos 15^\circ \sin(90^\circ - 13^\circ)} - \dfrac{7[\sin(90^\circ - 9^\circ)]}{\cos 9^\circ}$

$= \dfrac{5\cos 15^\circ \cos 13^\circ + 2\cos 13^\circ \cos 15^\circ}{\cos 15^\circ \cos 13^\circ} - \dfrac{7\cos 9^\circ}{\cos 9^\circ}$

$= \dfrac{7\cos 15^\circ \cos 13^\circ}{\cos 15^\circ \cos 13^\circ} - 7$

$= 7 - 7 = 0$

25. (*c*) (i) LHS $= \operatorname{cosec}^2 x + \sec^2 x$

$= \dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x} = \dfrac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$

$= \dfrac{1}{\sin^2 x \cos^2 x} = \dfrac{1}{\sin^2 x} \cdot \dfrac{1}{\cos^2 x}$

$= \operatorname{cosec}^2 x \sec^2 x =$ RHS

(ii) LHS $= \sec^2 x + \tan^2 x$

$= \dfrac{1}{\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x}$

$= \dfrac{1 + \sin^2}{\cos^2 x} \neq \sec^2 x \tan^2 x =$ RHS

(iii) LHS $= \operatorname{cosec}^2 x + \tan^2 x$

$= 1 + \cot^2 x + \sec^2 x - 1$

$\because \quad \operatorname{cosec}^2 x = 1 + \cot^2 x$

$\Rightarrow \quad \sec^2 x = 1 + \tan^2 x$

$= \cot^2 x + \sec^2 x =$ RHS

So, Eqs. (i) and (iii) are correct.

26. (*c*) $\dfrac{\cos x}{\operatorname{cosec} x + 1} + \dfrac{\cos x}{\operatorname{cosec} x - 1} = 2$

$\Rightarrow \dfrac{\cos x(\operatorname{cosec} x - 1) + \cos x(\operatorname{cosec} x + 1)}{(\operatorname{cosec} x + 1)(\operatorname{cosec} - 1)} = 2$

$\Rightarrow \dfrac{\cos x \operatorname{cosec} x - \cos x + \cos x \operatorname{cosec} x + \cos x}{\operatorname{cosec}^2 x - 1} = 2$

$$\Rightarrow \quad \frac{2\cos x \cdot \dfrac{1}{\sin x}}{\cot^2 x} = 2 \quad \Rightarrow \quad \frac{2\cot x}{\cot^2 x} = 2$$

$$\therefore \quad \cot x = 1 = \cot 45°$$

$$\therefore \quad x = 45° = \frac{\pi}{4}$$

27. (*c*) $\dfrac{\cos\theta}{1-\sin\theta} - \dfrac{\cos\theta}{1+\sin\theta} = 2$

$$\Rightarrow \frac{\cos\theta(1+\sin\theta) - \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 2$$

$$\Rightarrow \frac{\cos\theta + \cos\theta\sin\theta - \cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta} = 2$$

$$\Rightarrow \quad \frac{2\cos\theta\sin\theta}{\cos^2\theta} = 2$$

$$\Rightarrow \quad 2\tan\theta = 2$$

$$\Rightarrow \quad \tan\theta = 1 = \tan 45°$$

$$\therefore \quad \theta = 45° = \frac{\pi}{4}$$

28. (*b*) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta} \times \dfrac{1+\sin\theta}{1+\sin\theta}}$

$$= \sqrt{\frac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$= \sqrt{\left(\frac{1+\sin\theta}{\cos\theta}\right)^2} = \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta + \tan\theta$$

29. (*d*) $7\cos^2\theta + 3\sin^2\theta = 4$

Dividing by $\cos^2\theta$ at both sides, we get

$$\frac{7\cos^2\theta}{\cos^2\theta} + \frac{3\sin^2\theta}{\cos^2\theta} = \frac{4}{\cos^2\theta}$$

$$\Rightarrow \quad 7 + 3\tan^2\theta = 4\sec^2\theta$$

$$\Rightarrow \quad 7 + 3\tan^2\theta = 4(1+\tan^2\theta)$$

$$\Rightarrow \quad 7 + 3\tan^2\theta = 4 + 4\tan^2\theta$$

$$\Rightarrow \quad 3 = \tan^2\theta$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

30. (*a*) $\dfrac{\tan 27° + \cot 63°}{\tan 27°(\sin 25° + \cos 65°)}$

$$= \frac{\tan 27° + \cot(90° - 27°)}{\tan 27°(\sin 25° + \cos(90° - 25°))}$$

$$= \frac{\tan 27° + \tan 27°}{\tan 27°[\sin 25° + \sin 25°]}$$

$$= \frac{2}{2\sin 25°} = \text{cosec } 25°$$

31. (*a*) $\cos\theta \geq \frac{1}{2}$ means the value of θ lies between $0°$ and $\frac{\pi}{3}$.

$\therefore$ θ is less than or equal to $\frac{\pi}{3}$, *i.e.*, $\theta \leq \frac{\pi}{3}$

32. (*c*) Given,

$$x = a\sec\theta\cos\phi$$
$$y = b\sec\theta\cdot\sin\phi$$
$$z = c\tan\theta$$

Now, $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

On putting the values of x, y and z, we get

$$= \frac{a^2\sec^2\theta\cos^2\phi}{a^2} + \frac{b^2\sec^2\theta\sin^2\phi}{b^2} - \frac{c^2\tan^2\theta}{c^2}$$

$= \sec^2\theta\,[\cos^2\phi + \sin^2\phi] - \tan^2\theta$

$= \sec^2\theta - \tan^2\theta \qquad [\because \sec^2\theta - \tan^2\theta = 1]$

$= 1$

33. (*b*) $\cos^4\theta - \sin^4\theta = \frac{2}{3}$

$\Rightarrow \qquad (\cos^2\theta)^2 - (\sin^2\theta)^2 = \frac{2}{3}$

$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = \frac{2}{3}$

$\cos^2\theta - \sin^2\theta = \frac{2}{3} \Rightarrow \cos 2\theta = \frac{2}{3}$

$1 - 2\sin^2\theta = \frac{2}{3}$

34. (*b*) $7\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow \quad 4\sin^2\theta + 3\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow \quad 4\sin^2\theta + 3(\sin^2\theta + \cos^2\theta) = 4$

$\Rightarrow \quad 4\sin^2\theta + 3 = 4$

$\Rightarrow \quad 4\sin^2\theta = 1$

$\Rightarrow \quad \sin\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 30°$

$\Rightarrow \quad \cos\theta = \cos 30° = \frac{\sqrt{3}}{2}$

35. (*b*) $\tan 8\theta = \cot 2\theta$

$$\tan 8\theta = \tan(90° - 2\theta)$$
$$8\theta = 90° - 2\theta \Rightarrow \theta = 9°$$

$\therefore \quad \tan 5\theta \Rightarrow \tan 45° = 1$

36. (*c*) $\because \sin\theta - \cos\theta = 0$

$$\sin\theta = \cos\theta$$

Since, $\sin\theta$ and $\cos\theta$ are equal for $\theta = 45°$

So, $\sin^4\theta + \cos^4\theta = (\sin 45°)^4 + (\cos 45°)^4$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$$

$= \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$

37. (*c*) Given that, $\sin^2 x + \cos^2 x - 1 = 0$

$$\Rightarrow \qquad \sin^2 x + \cos^2 x = 1$$

Which is an identity of trigonometric ratio, and always true for every real value of x.

$\therefore$ The equation have an infinite solution.

38. (*c*) Statement I

$$\frac{\cot 30^\circ + 1}{\cot 30^\circ - 1} = 2(\cos 30^\circ + 1)$$

$$\frac{\sqrt{3}+1}{\sqrt{3}-1} = 2\left(\frac{\sqrt{3}}{2} + 1\right)$$

$$\Rightarrow \quad \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 2\left(\frac{\sqrt{3}+2}{2}\right)$$

$$\Rightarrow \quad \frac{3+1+2\sqrt{3}}{3-1} = \sqrt{3} + 2$$

$$\Rightarrow \quad \frac{4+2\sqrt{3}}{2} = \sqrt{3} + 2$$

$$\Rightarrow \quad \frac{2(2+\sqrt{3})}{2} = \sqrt{3} + 2$$

$$\Rightarrow \quad \sqrt{3} + 2 = \sqrt{3} + 2$$

$\therefore$ It is true.

Statement II

$$2\sin 45^\circ \cos 45^\circ - \tan 45^\circ \cot 45^\circ = 0$$

$$\Rightarrow \quad 2 \times \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right) - 1 \times 1 = 0$$

$$\Rightarrow \quad 2 \times \frac{1}{2} - 1 \times 1 = 0$$

$$1 - 1 = 0$$

$\therefore$ Both Statements I and II are true.

39. (*d*) $\sin A \cdot \cos A \cdot \tan A + \cos A \cdot \sin A \cdot \cot A$

$$= \sin A \cdot \cos A \cdot \frac{\sin A}{\cos A} + \cos A \cdot \sin A \cdot \frac{\cos A}{\sin A}$$

$$= \sin^2 A + \cos^2 A$$

$$= 1 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \operatorname{cosec}^2 A - \cot^2 A$$

$$[\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta]$$

40. (*c*) $2(\cos^2\theta - \sin^2\theta) = 1$

$$\Rightarrow \quad \cos^2\theta - \sin^2\theta = \frac{1}{2} \qquad \ldots\text{(i)}$$

We know that,

$$\cos^2\theta + \sin^2\theta = 1 \qquad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$$2\cos^2\theta = \frac{3}{2} \Rightarrow \cos^2\theta = \frac{3}{4}, \sin^2\theta = \frac{1}{4}$$

$$\therefore \quad \cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{\frac{3}{4}}{\frac{1}{4}} = 3$$

$$\Rightarrow \quad \cot\theta = \sqrt{3}$$

41. (*c*) Minimum value of $\cos\theta = -1$

and minimum value of $\sec\theta = -1$

$\therefore\ \cos^2\theta + \sec^2\theta = (-1)^2 + (-1)^2 = 2$

42. (*a*) $\cos x + \cos y = 2$

$\because \quad (\cos x)_{max} \le 1$

$\Rightarrow \quad \cos x = 1, \cos y = 1$

At $\quad x = y = 0$

$\therefore\ \sin x + \sin y = 0$

43. (*a*) $\sec^2\theta - \tan^2\theta = 1$ [Identity]

$\sec^2\theta + \tan^2\theta = \frac{7}{12}$ [given]

$\therefore$ Value of $\sec^4\theta - \tan^4\theta$

$= (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$

$= 1 \times \frac{7}{12} = \frac{7}{12}$

44. (*c*) $\left(\frac{1}{\cos\theta} + \frac{1}{\cot\theta}\right)\left(\frac{1}{\cos\theta} - \frac{1}{\cot\theta}\right)$

$= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)$

$= \frac{1+\sin\theta}{\cos\theta} \cdot \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} = 1$

45. (*b*) $A = \tan 11° \cdot \tan 29°$

$B = 2\cot 61° \cdot \cot 79°$

$= 2\cot(90° - 29°)\cot(90° - 11°)$

$= 2\tan 29° \cdot \tan 11°$

$[\because \cot(90° - \theta) = \tan\theta]$

$\therefore\ B = 2A$

46. (*c*) $\tan 2\theta = \frac{1}{\tan 4\theta}$

$\tan 2\theta = \cot 4\theta$

$\Rightarrow \quad \tan 2\theta = \tan(90° - 4\theta)$

$\Rightarrow \quad 2\theta = 90° - 4\theta$

$\Rightarrow \quad 6\theta = 90°$

$\Rightarrow \quad \theta = 15°$

$\therefore\ \tan 3\theta = \tan(3 \times 15°) = \tan 45° = 1$

47. (*d*) $\sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow \quad \sin\theta + \frac{1}{\sin\theta} = 2$

$\sin^2\theta + 1 = 2\sin\theta$

$\Rightarrow \quad \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow \quad (\sin\theta - 1)^2 = 0$

$\Rightarrow \quad \sin\theta = 1$

$\Rightarrow \quad \operatorname{cosec}\theta = 1$

Value of $\sin^5\theta + \operatorname{cosec}^5\theta$

$= (1)^5 + (1)^5$

$= 1 + 1 = 2$

48. (*b*) When $\theta = 0°$

$$\sin^2\theta + \cos^4\theta = 1$$

When $\theta = 45°$

$$\sin^2\theta + \cos^4\theta = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

When $\theta = 30°$

$$\sin^2\theta + \cos^4\theta = \frac{1}{4} + \frac{9}{16} = \frac{13}{16}$$

Average value of A

$$\Rightarrow \quad \frac{3}{4} \le A \le 1$$

49. (*b*) $2\sin^2\theta + 3\cos^2\theta$

$$= 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta$$

$$= 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta$$

$$= 2 + \cos^2\theta$$

$\because$ Minimum value of $\cos\theta = -1$

$\therefore$ Required minimum value

$$= 2 + (-1)^2 = 2 + 1 = 3$$

50. (*d*) $\cot 10°\cdot\cot 80°\cdot\cot 20°\cdot\cot 70°\cdot\cot 60°$

$= \cot 10°\cdot\tan 10°\cdot\cot 20°\cdot\tan 20°\cdot\cot 60°$

$[\because \tan(90° - \theta) = \cot\theta]$

$$= 1\cdot 1\cdot\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

51. (*d*) $\cos\theta\cdot\operatorname{cosec} 23° = 1$

$$\Rightarrow \quad \operatorname{cosec} 23° = \frac{1}{\cos\theta} = \sec\theta$$

$$\Rightarrow \quad \operatorname{cosec} 23° = \operatorname{cosec}(90° - \theta)$$

$$\Rightarrow \quad 23° = 90° - \theta$$

$$\Rightarrow \theta = 90° - 23° = 67°$$

52. (*c*) $\tan 2\theta\cdot\tan 3\theta = 1$

$$\Rightarrow \quad \tan 3\theta = \frac{1}{\tan 2\theta} = \cot 2\theta$$

$$\Rightarrow \quad \tan 3\theta = \tan(90° - 2\theta)$$

$$\therefore \quad 3\theta = 90° - 2\theta$$

$$\Rightarrow \quad 5\theta = 90° \Rightarrow \theta = \frac{90°}{5} = 18°$$

$$\therefore \quad 2\cos^2\frac{5\theta}{2} - 1 = 2\cos^2\frac{5\times 18}{2} - 1$$

$$= 2\cos^2 45° - 1$$

$$= 2\times\left(\frac{1}{\sqrt{5}}\right)^2 - 1$$

$$\Rightarrow \quad 2\times\frac{1}{2} - 1 = 0$$

53. (*c*) Let $X = \sin\theta + \cos\theta$

On squaring both side, we get

$$X^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta$$

$$= 1 + 2\sin\theta\cdot\cos\theta$$

$$\because \quad 0 < \theta < 90°$$

Value of both $\sin\theta$ and $\cos\theta$ will be less than 1.

$\therefore \quad 2\sin\theta \cdot \cos\theta < 1$

$\therefore \quad X^2 < 2 \Rightarrow X < \sqrt{2}$

$X < 1.414$

54. (b) $\sin\theta + \operatorname{cosec}\theta = 2$

$$\Rightarrow \quad \sin\theta + \frac{1}{\sin\theta} = 2$$

$$\Rightarrow \quad \sin^2\theta - 2\sin\theta + 1 = 0$$

$$\Rightarrow \quad (\sin\theta - 1)^2 = 0$$

$$\Rightarrow \quad \sin\theta - 1 = 0$$

$$\Rightarrow \quad \sin\theta = 1$$

$$\Rightarrow \quad \operatorname{cosec}\theta = 1$$

$$\therefore \quad \sin^{100}\theta + \operatorname{cosec}^{100}\theta$$

$$= (1)^{100} + (1)^{100}$$

$$= 1 + 1 = 2$$

55. (c) $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$

$$\Rightarrow \quad \sin\theta + \cos\theta = 3\sin\theta - 3\cos\theta$$

$$\Rightarrow \quad 4\cos\theta = 2\sin\theta$$

$$\Rightarrow \quad \tan\theta = 2$$

Value of $\sin^4\theta - \cos^4\theta$

$$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \sin^2\theta - \cos^2\theta$$

$$= \cos^2\theta(\tan^2\theta - 1)$$

$$= \frac{\tan^2\theta - 1}{\sec^2\theta}$$

$$= \frac{\tan^2\theta - 1}{1 + \tan^2\theta} = \frac{4 - 1}{1 + 4} = \frac{3}{5}$$

56. (b) $\tan\theta + \cot\theta = 2$

$$\Rightarrow \quad \tan\theta + \frac{1}{\tan\theta} = 2$$

$$\Rightarrow \quad \tan^2\theta + 1 = 2\tan\theta$$

$$\Rightarrow \quad \tan^2\theta - 2\tan\theta + 1 = 0$$

$$\Rightarrow \quad (\tan\theta - 1)^2 = 0$$

$$\Rightarrow \quad \tan\theta = 1$$

and $\quad \cot\theta = 1$

$$\therefore \tan^5\theta + \cot^{10}\theta = (1)^5 + (1)^{10} = 1 + 1 = 2$$

57. (d) If $0° < \theta < 90°$, then all the trigonometric ratios can be obtained when any one of the six ratios is given.

$\because$ We use any of the following identity to get any trigonometric ratios

$$\sin^2\theta + \cos^2\theta = 1, \; 1 + \tan^2\theta = \sec^2\theta$$

and $\quad 1 + \cot^2\theta = \operatorname{cosec} \theta$

58. (*a*) Let $f(\theta) = \dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{2 + 2\cos\theta}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{2\,(1+\cos\theta)}{\sin\theta\,(1+\cos\theta)} = 2\operatorname{cosec}\theta$$

59. (*b*) Given that, $\sin\theta\cdot\cos\theta = \dfrac{\sqrt{3}}{4}$ ...(i)

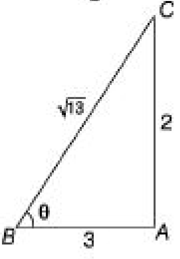
Now, we have $\sin^4\theta + \cos^4\theta =$

$$(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cdot\cos^2\theta$$

$$= (1)^2 - 2\,(\sin\theta\cdot\cos\theta)^2$$

$$= 1 - 2\left(\frac{\sqrt{3}}{4}\right)^2 = 1 - 2\cdot\frac{3}{16} = 1 - \frac{3}{8} = \frac{5}{8}$$

60. (*c*) In ΔABC, $\cot = \dfrac{3}{2}$



$\therefore AB = 3$ and $AC = 2$

By Pythagoras theorem, $BC^2 = (2)^2 + (3)^2$

$$BC = \sqrt{13}$$

Now, $\cos\theta = \dfrac{3}{\sqrt{13}}$ and $\sin\theta = \dfrac{2}{\sqrt{13}}$

$$\frac{2\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = \frac{\dfrac{6}{\sqrt{13}} - \dfrac{2}{\sqrt{13}}}{\dfrac{6}{\sqrt{13}} + \dfrac{2}{\sqrt{13}}} = \frac{4}{8} = \frac{1}{2}$$

61. (*c*) Given, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ ...(i)

On squaring both sides, we get

$$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$$

$$\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$$

$$2\sin\theta\cos\theta = \cos^2\theta - \sin^2\theta$$

$$2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$$

$$\cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{(\cos\theta + \sin\theta)} = \frac{2\sin\theta\cos\theta}{\sqrt{2}\cos\theta}$$

[from Eq. (i)]

$$= \sqrt{2}\sin\theta$$

62. (*c*) Given that, $3\sin x + 5\cos x = 5$

On squaring both side, we get

$9\sin^2 x + 25\cos^2 x + 30\sin x\cos x = 25$

$$\Rightarrow \quad 9(1-\cos^2 x) + 25(1-\sin^2 x) + 30\sin x\cos x = 25$$

$$\Rightarrow \quad 9 - 9\cos^2 x + 25 - 25\sin^2 x + 30\sin x \cdot \cos x = 25$$

$$\Rightarrow \quad 9 + 25 - \{9\cos^2 x + 25\sin^2 x - 30\sin x\cos x\} = 25$$

$$\Rightarrow \quad 9 = (3\cos x - 5\sin x)^2$$

$$\Rightarrow \quad 3\cos x - 5\sin x = 3$$

63. (*b*) $\sec x = \operatorname{cosec} y \Rightarrow \dfrac{1}{\cos x} = \dfrac{1}{\sin y}$

$$\therefore \quad \cos x = \sin y$$

$$\therefore \quad \sin\left(\frac{\pi}{2} - x\right) = \sin y$$

$$\therefore \quad \frac{\pi}{2} - x = y$$

$$(x + y) = \frac{\pi}{2}$$

Now, value of $\sin(x + y) = \sin\dfrac{\pi}{2} = 1$

64. (*d*) Given, $\sin 17° = \dfrac{x}{y}$

Now, $\sec 17° - \sin 73°$

$$= \sec 17° - \sin(90° - 17°)$$

$$= \sec 17° - \cos 17°$$

$$= \frac{1}{\cos 17°} - \cos 17°$$

$$= \frac{1 - \cos^2 17°}{\cos 17°} = \frac{\sin^2 17}{\cos 17°}$$

$$= \frac{\frac{x^2}{y^2}}{\sqrt{1 - \frac{x^2}{y^2}}} \quad [\because \cos\theta = \sqrt{1 - \sin^2\theta}]$$

$$= \frac{\frac{x^2}{y^2}}{\sqrt{1 - \frac{x^2}{y^2}}} = \frac{\frac{x^2}{y^2}}{\frac{\sqrt{y^2 - x^2}}{y}} = \frac{x^2}{y\sqrt{y^2 - x^2}}$$

65. (*d*) $x^2 + \dfrac{1}{x^2} = 2\sin\left(\dfrac{\pi x}{2}\right)$

$$[\because a^2 + b^2 = (a - b)^2 + 2ab]$$

$$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 + 2 = 2\sin\left(\frac{\pi x}{2}\right)$$

$$\therefore \quad x - \frac{1}{x} = 0$$

$$\left\{\sin\frac{\pi x}{2} = 1 \text{ for all integer values of } x\right\}$$

66. (*b*) **By Hit and Trial**

The equation $\cos^2\theta = \dfrac{(x+y)^2}{4xy}$ is possible,

only when $x = y$

On putting $x = y$ in the equation,

$$\cos^2\theta = \frac{(2x)^2}{4x^2}$$

$$= \frac{4x^2}{4x^2} = 1$$

$$\Rightarrow \quad \theta = 0°$$

67. (*b*) $\sin^2 1° + \sin^2 3° + \cdots + \sin^2 45°$

$$+ \cdots + \sin^2(90 - 3) + \sin^2(90° - 1°)$$

$$= \sin^2 1° + \sin^2 3° + \cdots + \sin^2 45° + \cdots + \cos^2 3° + \cos^2 1°$$

$$= (1 \times 22) + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 22 + \frac{1}{2} = 22\frac{1}{2}$$

68. (*d*) $A + B = 90°$

[$\because$ they are complementary]

$$\Rightarrow \quad B = 90° - A$$

$\therefore$ $\sec^2 A + \sec^2 B - \sec^2 A \cdot \sec^2 B$

$$= \sec^2 A + \sec^2(90° - A) - \sec^2 A \sec^2 (90° - A)$$

$$= \sec^2 A + \operatorname{cosec}^2 A - \sec^2 A \cdot \operatorname{cosec}^2 A$$

$$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} - \frac{1}{\sin^2 A \cdot \cos^2 A}$$

$$= \frac{\sin^2 A + \cos^2 A - 1}{\sin^2 A \cdot \cos^2 A}$$

$$= \frac{1 - 1}{\sin^2 A \cdot \cos^2 A} = 0$$

69. (*d*) $\sin^2 5° + \sin^2 10° + \cdots + \sin^2 45° + \cdots$

$$+ \sin^2 85° + \sin^2 90°$$

$$= \sin^2 5° + \sin^2 10° + \cdots + \sin^2 45° + \cdots + \sin^2(90° - 5) + \sin^2 90°$$

$$= 8 \times [\sin^2 5° + \cos^2 5° \cdots] + \sin^2 45° + \sin^2 90°$$

$$= 8 + \left(\frac{1}{\sqrt{2}}\right)^2 + 1 = 8 + 1 + \frac{1}{2} = 9\frac{1}{2}$$

70. (*b*) $\sin(3x - 20°) = \cos(3y + 20°)$

$$\Rightarrow \sin(3x - 20°) = \sin(90° - (3y + 20°))$$

$$\sin(3x - 20°) = \sin(70° - 3y)$$

$$\therefore \quad 3x - 20° = 70° - 3y$$

$$\Rightarrow \quad 3x + 3y = 90°$$

$$\Rightarrow \quad 3(x + y) = 90°$$

$$\Rightarrow \quad x + y = 30°$$

71. (*c*) Given, $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \dfrac{5}{4}$

$$\Rightarrow \quad \frac{\cos\theta\left(\dfrac{\sin\theta}{\cos\theta} + 1\right)}{\cos\theta\left(\dfrac{\sin\theta}{\cos\theta} - 1\right)} = \frac{5}{4}$$

$$\Rightarrow \quad \frac{\tan\theta + 1}{\tan\theta - 1} = \frac{5}{4}$$

$$\Rightarrow \quad 4\tan\theta + 4 = 5\tan\theta - 5$$

$$\tan\theta = 9$$

Putting volume of $\tan\theta$,

$$\frac{\tan^2\theta + 1}{\tan^2\theta - 1} = \frac{(9)^2 + 1}{(9)^2 - 1}$$

$$= \frac{81 + 1}{81 - 1} = \frac{82}{80} = \frac{41}{40}$$

72. (*d*) $\dfrac{\sin 39°}{\cos 51°} + 2\tan 11° \cdot \tan 79° \cdot$

$$\tan 31° \cdot \tan 59° \cdot \tan 45° - 3(\sin^2 21 + \sin^2 69°)$$

$$= \frac{\sin 39°}{\cos(90° - 39°)} + 2\tan 11° \cdot \tan(90° - 11°)$$

$$\tan 31° \cdot \tan(90° - 31°) \cdot 1 - 3(\sin^2 21° + \sin^2(90° - 21°)$$

$[\because \tan(90° - \theta) = \cot\theta, \cos(90° - \theta) = \sin\theta]$

$$= \frac{\sin 39°}{\sin 39°} + 2\tan 11° \cdot \cot 11° \cdot \tan 31° \cdot$$

$$\cot 31° - 3(\sin^2 21° + \cos^2 21°)$$

$$= 1 + 2 - 3 = 0$$

$[\because \tan\theta \cdot \cot\theta = 1 \text{ and } \sin^2\theta + \cos^2\theta = 1]$

73. (*c*) $\dfrac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$

$$\Rightarrow \quad \cos^2\theta = 3\cot^2\theta - 3\cos^2\theta$$

$$\Rightarrow \quad 4\cos^2\theta = 3\cot^2\theta$$

$$\Rightarrow \quad 4\cos^2\theta - \frac{3\cos^2\theta}{\sin^2\theta} = 0$$

$$\Rightarrow \quad \cos^2\theta\left(4 - \frac{3}{\sin^2\theta}\right) = 0$$

$$\therefore \quad 4 - \frac{3}{\sin^2\theta} = 0$$

$$\Rightarrow \quad 4\sin^2\theta - 3 = 0$$

$$\sin^2\theta = \frac{3}{4}$$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2} = \sin 60°$$

$$\Rightarrow \quad \theta = 60°$$

74. (*a*) $\cot 18°$

$$\left(\cot 72° \cdot \cos^2 22° + \frac{1}{\tan 72° \cdot \sec^2 68°}\right)$$

$$= \cot 18° \cdot \cot 72° \cdot \cos^2 22° + \frac{\cot 18°}{\tan 72° \cdot \sec^2 68°}$$

$[\cot(90° - \theta) = \tan\theta]$

$$= \cot 18° \cdot \tan 18° \cdot \cos^2 22° + \frac{\cot 18°}{\cot 18°} \cdot \cos^2 68°$$

$$= \cos^2 22° + \cos^2 68°$$

$$= \cos^2 22° + \sin^2 22° = 1$$

$[\sin(90° - \theta) = \cos\theta]$

75. (*b*) $7\sin^2\theta + 3\cos^2\theta = 4$

$$\Rightarrow \quad 7\frac{\sin^2\theta}{\cos^2\theta} + 3 = \frac{4}{\cos^2\theta}$$

$$\Rightarrow \quad 7\tan^2\theta + 3 = 4\sec^2\theta$$

$$\Rightarrow \quad 7\tan^2\theta + 3 = 4(1 + \tan^2\theta)$$

$$\Rightarrow \quad 7\tan^2\theta - 4 - 4\tan^2\theta + 3 = 0$$

$$\Rightarrow \quad 3\tan^2\theta = 1$$

$$\Rightarrow \quad \tan^2\theta = \frac{1}{3}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

Exercise © *Higher Skill Level Questions*

1. (*d*) $(\sin x + \sin y) = a$ and $(\cos x + \cos y) = b$

Squaring both the equations, we get

$(\sin x + \sin y)^2 = a^2$

$$\sin^2 x + \sin^2 y + 2\sin x \sin y = a^2 \qquad \ldots\text{(i)}$$

and $(\cos x + \cos y)^2 = b^2$

$$\cos^2 x + \cos^2 y + 2\cos x \cos y = b^2 \qquad \ldots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$(\sin^2 x + \sin^2 y + 2\sin x \sin y) + (\cos^2 x + \cos^2 y + 2\cos x \cos y) = a^2 + b^2$$

$$\Rightarrow \sin^2 x + \cos^2 x + \sin^2 y + \cos^2 y + 2(\sin x \sin y + \cos x \cos y) = a^2 + b^2$$

$$\Rightarrow 1 + 1 + 2(\sin x \sin y + \cos x \cos y) = a^2 + b^2$$

$$\therefore \quad \sin x \sin y + \cos x \cos y = \frac{a^2 + b^2 - 2}{2}$$

2. (*c*) (i) RHS $= \cos^2\theta (1 + \tan\theta)(1 - \tan\theta)$

$$= \cos^2\theta (1 - \tan^2\theta)$$

$$= \cos^2\theta \left(1 - \frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$= \frac{\cos^2\theta - \sin^2\theta}{1} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \text{LHS} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

(ii) RHS $= (\tan\theta + \sec\theta)^2$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)^2 = \frac{(1 + \sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1 + \sin\theta)^2}{1 - \sin^2\theta} = \frac{(1 + \sin\theta)(1 + \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}$$

$$= \frac{1 + \sin\theta}{1 - \sin\theta} = \text{LHS}$$

3. (*c*) tan1° tan2° tan3° ... tan89°

or

tan1° tan2° tan3° ...... tan88° tan89°

Now, tan1° tan89° tan2° tan88° ......

$$= \tan 1°[\tan(90 - 1°)] \tan 2° \tan (90 - 2°) \ldots\ldots$$

$$= \tan 1° \cot 1° \tan 2° \cot 2° \ldots\ldots$$

$$= \tan 1° \times \frac{1}{\tan 1°} \tan 2° \times \frac{1}{\tan 2°} \times \ldots\ldots = 1$$

4. (*d*) $\sec\theta + \tan\theta = p$ ...(i)

$$\Rightarrow \quad \frac{1}{\sec\theta + \tan\theta} = \frac{1}{P}$$

$$\Rightarrow \quad \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta} = \frac{1}{P}$$

$$\Rightarrow \quad \frac{\sec\theta - \tan\theta}{1 + \tan^2\theta - \tan^2\theta} = \frac{1}{P}$$

$$\sec\theta - \tan\theta = \frac{1}{p} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$2\sec\theta = p + \frac{1}{p}$$

$$\Rightarrow \quad \sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right) = \frac{p^2 + 1}{2p}$$

$$\Rightarrow \quad \cos\theta = \frac{2p}{p^2 + 1}$$

Now, in ΔABC,

$$(BC)^2 = (AC)^2 - (AB)^2$$

$$= (P^2 + 1)^2 - (2P)^2$$

$$= P^4 + 1 + 2P^2 - 4P^2$$

$$= P^4 + 1 - 2P^2$$

$\Rightarrow \quad (BC)^2 = (P^2 - 1)^2$

$\Rightarrow \quad BC = (P^2 - 1)$

$\therefore \quad \sin\theta = \dfrac{p^2 - 1}{p^2 + 1}$

5. (c) $\sin\theta + \cos\theta = x$

On squaring both sides, we get

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = x^2$$

$1 + 2\sin\theta\cos\theta = x^2$

$\therefore \quad \sin\theta\cos\theta = \dfrac{x^2 - 1}{2} \quad \ldots\text{(i)}$

$\cos^6\theta + \sin^6\theta = (\cos^2\theta)^3 + (\sin^2\theta)^3$

$= (\cos^2\theta + \sin^2\theta)$

$(\cos^4\theta - \cos^2\theta\sin^2\theta + \sin^4\theta)$

$= \{(\cos^2\theta)^2 + (\sin^2\theta)^2 - \cos^2\theta\sin^2\theta\}$

$= \{(\cos^2\theta + \sin^2\theta)^2 - 2\cos^2\theta\sin^2\theta - \cos^2\theta\sin^2\theta\}$

$= 1 - 3\cos^2\theta\sin^2\theta = 1 - 3\left(\dfrac{x^2 - 1}{2}\right)^2$

[from Eq. (i)]

$= 1 - \dfrac{3(x^4 - 2x^2 + 1)}{4} = \dfrac{4 - 3x^4 + 6x^2 - 3}{4}$

$= \dfrac{1 - 3x^4 + 6x^2}{4} = \dfrac{1}{4}(1 + 6x^2 - 3x^4)$

6. (c) Given,

$$1 + \tan A + \tan B + \tan A\tan B = 2$$

$$\tan A + \tan B = 1 - \tan A\tan B$$

or $\quad \dfrac{\tan A + \tan B}{1 - \tan A\tan B} = 1$

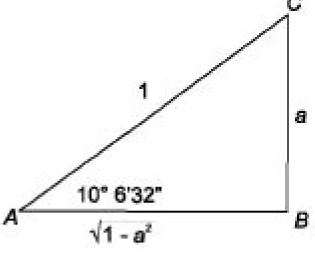
$= \tan 45°$

$\tan(A + B) = \tan 45°$

$\left[\because \tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}\right]$

$\therefore \quad A + B = 45° = \dfrac{\pi}{4}$

7. (a) $\sin(10° 6' 32'') = \dfrac{a}{1}$



$$\sin(10°6'32'') = \frac{a}{1}$$

$$\sin(90° - 79°\,53'\,28'') = a$$

$$\cos 79°\,53'\,28'' = a$$

$\therefore$ $\cos(79°\,53'\,28'') + \tan(10°\,6'\,32'')$

$$= a + \frac{a}{\sqrt{1-a^2}}$$

$$= \frac{a(1+\sqrt{1-a^2})}{\sqrt{1-a^2}}$$

8. (*c*) $x + y = z$

Now, $\cos^2 x + \cos^2 y + \cos^2 z = ?$

$= 1 + (\cos^2 x - \sin^2 y) + \cos^2 z$

$= 1 + \cos(x+y) \cdot \cos(x-y) + \cos^2 z$

$= 1 + \cos z \cos(x-y) + \cos^2 z$

$= 1 + \cos z\,[\cos(x-y) + \cos(x+y)]$

$= 1 + \cos z$

$$\left[2\cos\frac{(x-y+x+y)}{2} \cdot \cos\frac{(x-y-x-y)}{2}\right]$$

$= 1 + 2\cos z \cdot \cos x \cdot \cos y$

$= 1 + 2\cos x \cdot \cos y \cdot \cos z$

9. (*b*) $\cos\theta + \sec\theta = 2$

$$\Rightarrow \quad \cos\theta + \frac{1}{\cos\theta} = 2$$

On squaring both sides, we get

$$\cos^2\theta + \frac{1}{\cos^2\theta} + 2 = 4$$

$$\Rightarrow \quad \cos^2\theta + \frac{1}{\cos^2\theta} = 2$$

On cubing both sides, we get

$$\cos^6\theta + \frac{1}{\cos^6\theta} + 3\cos^2\theta + \frac{3}{\cos^2\theta} = 8$$

$$\Rightarrow \quad \cos^6\theta + \frac{1}{\cos^6\theta} + 3(2) = 8$$

$$\left[\because \cos^2\theta + \frac{1}{\cos^2\theta} = 2\right]$$

$$\Rightarrow \quad \cos^6\theta + \frac{1}{\cos^6\theta} = 2$$

$$\Rightarrow \quad \cos^6\theta + \sec^6\theta = 2$$

10. (*c*) Given, $\sin\theta - \cos\theta = 0$

On squaring both sides, we get

$$(\sin\theta - \cos\theta)^2 = 0$$

$$\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = 0$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$$2\sin\theta\cos\theta = 1 \qquad \ldots\text{(i)}$$

$$\therefore \quad \sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)$$

$$= \sin\theta + \cos\theta = \sqrt{(\sin\theta + \cos\theta)^2}$$

$= \sqrt{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}$

$= \sqrt{1+1} = \sqrt{2}$ [from Eq. (i)]

11. (*a*) $f(\theta) = \sin^8\theta + \cos^{14}\theta$

$= \sin^8\theta + (1-\sin^2\theta)^7$

[$\because$ maximum value of $\sin^2\theta = 1$]

$= 1 + 0 = 1$

12. (*d*) Given,

$$l\cos^2\theta + m\sin^2\theta = \frac{\cos^2\theta\,(\text{cosec}^2\theta + 1)}{\text{cosec}^2\theta - 1}$$

$$= \frac{\cos^2\theta\,(1+\sin^2\theta)}{1-\sin^2\theta}\cdot\frac{\sin^2\theta}{\sin^2\theta}$$

$$= \frac{\cos^2\theta\,(1+\sin^2\theta)}{\cos^2\theta}$$

$= 1 + \sin^2\theta = \cos^2\theta + \sin^2\theta + \sin^2\theta$

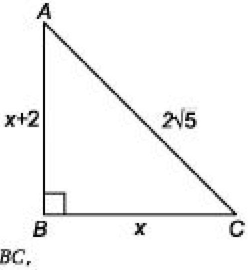
$= \cos^2\theta + 2\sin^2\theta$

$\Rightarrow (l-1)\cos^2\theta = (2-m)\sin^2\theta$

$\Rightarrow \quad \tan^2\theta = \dfrac{l-1}{2-m}$

$\therefore \quad \tan\theta = \sqrt{\dfrac{l-1}{2-m}}$

13. (*d*) Let $BC = x$, then $AB = x + 2$



In ΔABC,

$(x+2)^2 + x^2 = (2\sqrt{5})^2$

$\Rightarrow \quad x^2 + 4 + 4x + x^2 = 20$

$\Rightarrow \quad 2x^2 + 4x - 16 = 0$

$\Rightarrow \quad x^2 + 2x - 8 = 0$

$\Rightarrow \quad (x-2)(x+4) = 0 \Rightarrow x = 2$

$AB = 4,\ BC = 2$

$\therefore \quad \cos^2 A - \cos^2 C$

$$= \left(\frac{4}{2\sqrt{5}}\right)^2 - \left(\frac{2}{2\sqrt{5}}\right)^2 = \frac{3}{5}$$

14. (*a*) $\cos x + \cos^2 x = 1$

$\Rightarrow \quad \cos x = 1 - \cos^2 x$

$\Rightarrow \quad \cos x = \sin^2 x$ ...(i)

Now, again $\cos x + \cos^2 x = 1$

On cubing both side, we get

$(\cos x + \cos^2 x)^3 = (1)^3$

$\cos^3 x + (\cos^2 x)^3 + 3\cos^2 x \cdot \cos^2 x$

$+ 3 \cos x \cdot \cos^4 x = 1$

$\cos^3 x + \cos^6 x + 3\cos^4 x + 3 \cos^5 x = 1$

Put $\cos x = \sin^2 x$ [from Eq. (i)]

$\sin^6 x + \sin^{12} x + 3 \sin^8 x + 3 \sin^{10} x = 1$

$\therefore \sin^{12} x + 3 \sin^{10} x + 3 \sin^8 x + \sin^6 x - 1 = 0$

15. (*d*) $a\sin\theta + b\cos\theta = c$

On squaring both sides, we get

$a^2 \sin^2\theta + b^2 \cos^2\theta + 2ab\sin\theta\cos\theta = c^2$

$\Rightarrow a^2(1 - \cos^2\theta) + b^2(1 - \sin^2\theta) + 2 ab\sin\theta\cos\theta = c^2$

$\Rightarrow a^2 - a^2\cos^2\theta + b^2 - b^2\sin^2\theta + 2ab\sin\theta\cos\theta = c^2$

On rearranging, we get

$a^2 + b^2 - c^2 = a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta$

$\Rightarrow a^2 + b^2 - c^2 = (a\cos\theta - b\sin\theta)^2$

$\Rightarrow a\cos\theta - b\sin\theta = \pm\sqrt{a^2 + b^2 - c^2}$

16. (*b*) $\tan\alpha = n\tan\beta \Rightarrow \dfrac{\sin\alpha}{\cos\alpha} = n\dfrac{\sin\beta}{\cos\beta}$

$$\frac{m\sin\beta}{\cos\alpha} = n\frac{\sin\beta}{\cos\beta}$$

$$\Rightarrow \quad \cos\alpha = \frac{m}{n}\cos\beta$$

On squaring both sides, we get

$$\cos^2\alpha = \frac{m^2}{n^2}\cos^2\beta \qquad \ldots(i)$$

Also, $\sin\alpha = m\sin\beta$

On squaring both sides, we get

$$\sin^2\alpha = m^2\sin^2\beta$$

$$\Rightarrow \quad 1 - \cos^2\alpha = m^2(1 - \cos^2\beta)$$

$$\Rightarrow \quad 1 - \cos^2\alpha = m^2 - m^2\cos^2\beta$$

$$\Rightarrow \quad -\frac{(1 - \cos^2\alpha - m^2)}{m^2} = \cos^2\beta$$

$$\frac{(\cos^2\alpha + m^2 - 1)}{m^2} = \cos^2\beta \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\cos^2\alpha = \frac{m^2}{n^2} \times \frac{(\cos^2\alpha + m^2 - 1)}{m^2}$$

$$\Rightarrow \quad n^2\cos^2\alpha = \cos^2\alpha + m^2 - 1$$

$$\Rightarrow \quad (n^2 - 1)\cos^2\alpha = m^2 - 1$$

$$\therefore \quad \cos^2\alpha = \frac{m^2 - 1}{n^2 - 1}$$

17. (*d*) Given, $\sin\theta \cdot \cos\theta = \frac{1}{2}$

$\sin^6\theta + \cos^6\theta = (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta$

$= (1 - 3\sin^2\theta\cos^2\theta) \quad \left[\because \sin\theta\cdot\cos\theta = \frac{1}{2}\right]$

$= 1 - 3 \times \frac{1}{4} = 1 - \frac{3}{4} = \frac{1}{4}$

18. (*b*) $\operatorname{cosec}(75^\circ + \theta) - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot(35^\circ - \theta)$

$\Rightarrow \operatorname{cosec}(75^\circ + \theta) - \operatorname{cosec}[90^\circ - (75^\circ - \theta)] - \tan(55^\circ + \theta) + \cot[90^\circ - (35^\circ - \theta)]$

$\Rightarrow \operatorname{cosec}(75^\circ + \theta) - \operatorname{cosec}(75^\circ + \theta) - \tan(55^\circ + \theta) + \tan(55^\circ + \theta) = 0$

19. (*a*) $\cos x + \sec x = 2$ ...(i)

On squaring both sides,

$\cos^2 x + \sec^2 x + 2 = 4$

$\cos^2 x + \sec^2 x = 2$

On cubing Eq. (i)

$\cos^3 x + \sec^3 x + 3(\cos x + \sec x) = 8$

$\Rightarrow \cos^3 x + \sec^3 x + (3 \times 2) = 8$

$\Rightarrow \cos^3 x + \sec^3 x = 2$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$\cos^n x + \sec^n x = 2$

20. (*c*) Since, α and β are complementary angle.

$\alpha = 90^\circ - \beta$

Now, $\sqrt{\cos\alpha \operatorname{cosec}\beta - \cos\alpha\sin\beta}$

$= \sqrt{\frac{\cos\alpha}{\sin\beta} - \cos\alpha\sin\beta}$

$= \sqrt{\frac{\cos\alpha}{\cos(90^\circ - \beta)} - \cos\alpha\cos(90^\circ - \beta)}$

$= \sqrt{\frac{\cos\alpha}{\cos\alpha} - \cos\alpha\cdot\cos\alpha}$

$= \sqrt{1 - \cos^2\alpha} \Rightarrow \sqrt{\sin^2\alpha} = \sin\alpha$

21. (*c*) $\sin\theta + 2\cos\theta = -1$

On squaring both sides, we get

$(\sin\theta + 2\cos\theta)^2 = (-1)^2$

$\Rightarrow \sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = 1$

$\Rightarrow (1 - \cos^2\theta) + 4(1 - \sin^2\theta) + 4\sin\theta\cos\theta = 1$

$\Rightarrow -(\cos^2\theta + 4\sin^2\theta) + 4\sin\theta\cos\theta = 1 - 5$

$$\Rightarrow \quad \cos^2\theta + 4\sin^2\theta - 4\sin\theta\cos\theta = 4$$

$$\Rightarrow \quad (2\sin\theta - \cos\theta)^2 = 4$$

$$\Rightarrow \quad 2\sin\theta - \cos\theta = 2$$

22. (*a*) $\because \sin(A+B) = 1$

$$\Rightarrow \quad (A+B) = \sin^{-1} 1$$

$$\Rightarrow \quad (A+B) = 90^\circ$$

$\therefore \quad B = 90^\circ - A$ or $A = 90^\circ - B$

Now, $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= \cos(90^\circ - B)\cos B + \sin(90^\circ - B)\sin B$

$= \sin B \cos B + \cos B \sin B$

$= 2\sin B \cos B = \sin 2B$

23. (*a*) $\sin\theta + \cos\theta = \sqrt{3}$

On squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$$

$$\Rightarrow \quad \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$

$$\Rightarrow \quad 1 + 2\sin\theta\cos\theta = 3$$

$$\Rightarrow \quad \sin\theta\cos\theta = \frac{3-1}{2} = \frac{2}{2} = 1 \quad \ldots\text{(i)}$$

Now, $\tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

From Eq. (i),

$$\tan\theta + \cot\theta = \frac{1}{1} = 1$$

24. (*b*) $\tan\theta + \sec\theta = m \quad \ldots\text{(i)}$

$$\Rightarrow \quad \sec\theta = m - \tan\theta$$

On squaring both sides, we get

$$(\sec\theta)^2 = (m - \tan\theta)^2$$

$$\Rightarrow \quad \sec^2\theta = m^2 + \tan^2\theta - 2m\tan\theta$$

$$\Rightarrow \quad \sec^2\theta - \tan^2\theta = m^2 - 2m\tan\theta$$

$$\Rightarrow \quad 1 = m^2 - 2m\tan\theta$$

$$[\because \sec^2\theta - \tan^2\theta = 1]$$

$$\Rightarrow \quad \tan\theta = \frac{m^2-1}{2m}$$

On putting the value of $\tan\theta$ in initial equation, we get

$$\frac{m^2-1}{2m} + \sec\theta = m$$

$$\Rightarrow \quad \sec\theta = m - \left(\frac{m^2-1}{2m}\right)$$

$$\therefore \quad \sec\theta = \frac{2m^2 - m^2 + 1}{2m} = \frac{m^2+1}{2m}$$

25. (*c*) $\because 5\sin\theta + 12\cos\theta = 13$

On squaring both sides, we get

$$25\sin^2\theta + 144\cos^2\theta + 120\sin\theta\cos\theta = 169$$

$$\Rightarrow \quad 25(1-\cos^2\theta) + 144(1-\sin^2\theta) + 120\sin\theta\cos\theta = 169$$

$$\Rightarrow \quad 25 - 25\cos^2\theta + 144 - 144\sin^2\theta + 120\sin\theta\cos\theta = 169$$

$$\Rightarrow \quad 25\cos^2\theta + 144\sin^2\theta - 120\sin\theta\cos\theta = 169 - 169$$

$$\Rightarrow \quad (5\cos\theta - 12\sin\theta)^2 = 0$$

$$\Rightarrow \quad 5\cos\theta - 12\sin\theta = 0$$

26. (*a*) Only Statement I is correct as $\tan\theta$ increases faster than $\sin\theta$ as θ increases while Statement II is wrong as the value of $\sin\theta + \cos\theta$ is not always greater than 1. It may also be equal to 1.

27. (*a*) $\dfrac{(\sin\theta + \cos\theta)(\tan\theta + \cot\theta)}{\sec\theta + \operatorname{cosec}\theta}$

$$= \frac{(\sin\theta + \cos\theta)\left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)}{\dfrac{1}{\cos\theta} + \dfrac{1}{\sin\theta}}$$

$$= \frac{(\sin\theta + \cos\theta)\left(\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)}{\dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}}$$

$$= \frac{(\sin\theta + \cos\theta)\left(\dfrac{1}{\sin\theta\cos\theta}\right)}{\dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{\dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}}{\dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}} = 1$$

28. (*a*) $\dfrac{(1 + \sec\theta - \tan\theta)\cos\theta}{(1 + \sec\theta + \tan\theta)(1 - \sin\theta)}$

$$= \frac{\left(1 + \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\right)\cos\theta}{\left(1 + \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta)}$$

$$= \frac{\left(\dfrac{\cos\theta + 1 - \sin\theta}{\cos\theta}\right)\cos\theta}{\dfrac{(\cos\theta + 1 + \sin\theta)(1 - \sin\theta)}{\cos\theta}}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\dfrac{\cos\theta + 1 + \sin\theta - \sin\theta\cos\theta - \sin\theta - \sin^2\theta}{\cos\theta}}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\dfrac{\cos\theta + 1 - \sin^2\theta - \sin\theta\cos\theta}{\cos\theta}}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\dfrac{\cos\theta + \cos^2\theta - \sin\theta\cos\theta}{\cos\theta}}$$

$$[\because 1 - \sin^2\theta = \cos^2\theta]$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\dfrac{\cos\theta(\cos\theta + 1 - \sin\theta)}{\cos\theta}} = 1$$

29. (*b*) Given,

$$\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} = \frac{1}{x^2 + y^2}$$

and $x\sin\theta - y\cos\theta = \sqrt{x^2 + y^2}$

On squaring both sides, we get

$$x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta = x^2 + y^2$$

$$\Rightarrow \quad x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta - x^2 - y^2 = 0$$

$$\Rightarrow \quad x^2(\sin^2\theta - 1) + y^2(\cos^2\theta - 1) - 2xy\sin\theta\cos\theta = 0$$

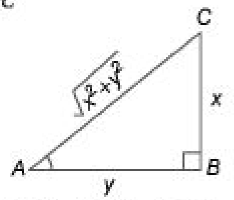
$$\Rightarrow \quad x^2\cos^2\theta + y^2\sin^2\theta + 2xy\sin\theta\cos\theta = 0$$

$$\Rightarrow \quad (x\cos\theta + y\sin\theta)^2 = 0$$

$$x\cos\theta + y\sin\theta = 0$$

$$\Rightarrow \quad x\cos\theta = -y\sin\theta \Rightarrow \tan\theta = -\frac{x}{y}$$

In ΔABC



$$(AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow \quad (AC)^2 = (-y)^2 + (x)^2$$

$$\Rightarrow \quad AC^2 = y^2 + x^2$$

$$\text{or} \quad AC = \sqrt{x^2 + y^2}$$

$$\Rightarrow \quad \sin\theta = \frac{x}{\sqrt{x^2 + y^2}} \text{ or } \frac{-x}{\sqrt{x^2 + y^2}}$$

$$\text{and} \quad \cos\theta = -\frac{y}{\sqrt{x^2 + y^2}} \text{ or } \frac{y}{\sqrt{x^2 + y^2}}$$

Now, $\dfrac{\cos^2\theta}{a^2} + \dfrac{\sin^2\theta}{b^2} = \dfrac{1}{x^2 + y^2}$

$$\Rightarrow \frac{y^2}{(x^2+y^2)\cdot a^2} + \frac{x^2}{(x^2+y^2)\cdot b^2} = \frac{1}{x^2+y^2}$$

$$\Rightarrow \quad \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \quad \Rightarrow \quad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

30. (*a*) $2\ \text{cosec}^2 23° \cot^2 67° - \sin^2 23°$

$$- \sin^2 67^2 - \cot^2 67°$$

$= \cot^2 67° (2\ \text{cosec}^2 23° - 1)$

$$- \sin^2 23° - \cos^2 23°$$

$= \cot^2 67° (\text{cosec}^2 23° + \text{cosec}^2 23° - 1)$

$$- (\sin^2 23° + \cos^2 23°)$$

$$[\because \cos\theta = \sin(90° - \theta)]$$

$= \cot^2 67° (\text{cosec}^2 23° + \cot^2 23°) - 1$

$= \cot^2 67°\ \text{cosec}^2 23° + \cot^2 67° \cot^2 23° - 1$

$= \cot^2 67°\ \text{cos ec}^2 23° + \cot^2 67° \tan^2 67° - 1$

$$\tan^2 23°\ \text{cosec}^2 23° + 1 - 1$$

$$= \frac{\sin^2 23°}{\cos^2 23°} \times \frac{1}{\sin^2 23°} = \frac{1}{\cos^2 23°} = \sec^2 23°$$

31. (*a*) $\cot\theta \cdot \tan(90° - \theta) - \sec(90° - \theta) \cdot$ $\text{cosec}\,\theta + (\sin^2 25° + \sin^2 65°) + \sqrt{3}$ $(\tan 5° \cdot \tan 15° \cdot \tan 30° \cdot \tan 75° \cdot \tan 85°)$

$= \cot\theta \cdot \cot\theta - \text{cosec}\,\theta \cdot \text{cosec}\,\theta$ $+ (\sin^2 25° + \cos^2 25°) + \sqrt{3}\ (\tan 5° \cdot$ $\cot 5° \cdot \tan 15° \cdot \cot 15° \cdot \tan 30°)$

$$\left[\begin{array}{l} \because \sec(90° - \theta) = \text{cosec}\,\theta \\ \sin(90° - \theta) = \cos\theta \\ \tan(90° - \theta) = \cot\theta \end{array}\right]$$

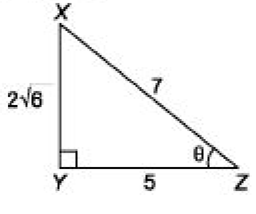
$$= (\cot^2\theta - \text{cosec}^2\theta) + (\sin^2 25° + \cos^2 25°) + \sqrt{3} \times \frac{1}{\sqrt{3}}$$

$[\because \sin^2\theta + \cos^2\theta = 1$

and $\cot^2\theta - \text{cosec}^2\theta = -1]$

$= -1 + 1 + 1 = 1$

32. (*b* $XZ - YZ = 2$ ...(i)



By pythagoras theorem,

$$XY^2 + YZ^2 = XZ^2$$

$\Rightarrow \quad (2\sqrt{6})^2 = XZ^2 - YZ^2$

$\Rightarrow \quad 24 = (XZ - YZ)\,(XZ - YZ)$

$\Rightarrow \quad \dfrac{24}{2} = (XZ + YZ)$

$\Rightarrow \quad XZ + YZ = 12 \qquad \ldots\text{(ii)}$

Adding Eqs. (i) and (ii), we get

$$2XZ = 14 \Rightarrow XZ = 7$$

$\therefore \quad YZ = 7 - 2 = 5$

$\therefore \quad \sec X = \dfrac{7}{2\sqrt{6}}$ and $\tan x = \dfrac{5}{2\sqrt{6}}$

$\therefore \sec X + \tan X$

$$= \frac{7}{2\sqrt{6}} + \frac{5}{2\sqrt{6}} = \frac{12}{2\sqrt{6}} = \sqrt{6}$$

33. (*d*) $(\sec x \cdot \sec y + \tan x \cdot \tan y)^2$

$$- (\sec x \cdot \tan y + \tan x \cdot \sec y)^2$$

$= (\sec^2 x \cdot \sec^2 y + \tan^2 x \cdot \tan^2 y + 2$

$\sec x \cdot \sec y \cdot \tan x \cdot \tan y) - (\sec^2 x \cdot \tan^2$

$+ \tan^2 x \cdot \sec^2 y + 2 \sec x \cdot \sec y \cdot \tan x \cdot \tan y)$

$= \sec^2 x \cdot \sec^2 y + \tan^2 x \cdot \tan^2 y$

$- \sec^2 x \cdot \tan^2 y - \tan^2 x \cdot \sec^2 y$

$= \sec^2 x \cdot \sec^2 y - \sec^2 x \cdot \tan^2 y$

$- \tan^2 x \cdot \sec^2 y + \tan^2 x \cdot \tan^2 y$

$= \sec^2 x\,(\sec^2 y - \tan^2 y) - \tan^2 x$

$(\sec^2 y - \tan^2 y)$

$= \sec^2 x - \tan^2 x = 1 \qquad [\because \sec^2 - \tan^2 \theta = 1]$

34. (*a*) $\tan 15^\circ \cdot \cot 75^\circ + \tan 75^\circ \cdot \cot 15^\circ$

$= \tan 15^\circ \cdot \cot (90^\circ - 15^\circ)$

$+ \tan (90^\circ - 15^\circ) \cdot \cot 15^\circ$

$= \tan^2 15^\circ + \cot^2 15^\circ \qquad \ldots\text{(i)}$

$[\because \tan (90^\circ - \theta) = \cot \theta,\ \cot (90^\circ - \theta) = \tan \theta]$

Now, $\cot 15^\circ = \dfrac{1}{2 - \sqrt{3}} = \dfrac{2 + \sqrt{3}}{(2 - \sqrt{3})\,(2 + \sqrt{3})}$

$= 2 + \sqrt{3}$

or $\tan 15^\circ = 2 - \sqrt{3}$

$\therefore \tan^2 15^\circ + \cot^2 15^\circ = (2 - \sqrt{3})^2 + (2 + \sqrt{3})^2$

$= 2\,[(2)^2 + (\sqrt{3})^2] = 2\,(4 + 3)$

$= 2 \times 7 = 14$

Chapter

38

Trigonometry

Trigonometry is a branch of Mathematics that studies triangles and relationship between their sides and angles and these relationships can be made using trigonometric functions.

Measurement of Angles

1. Degree Measure (Sexagesimal System)

In this system, an angle is measured in degrees, minutes and seconds.

1° = *W* (60 min); 1' = 60" (60 s)

2. Radian Measure (Circular System)

Angle subtended at the centre by any arc of length 1 unit in a circle of radius 1 unit, is said to have a measure of 1 radian.

271 radian =360° or *n* radian =180°

1 radian =57° 16'22"

Relation between Radian and Degrees

$$\text{Radian measure} = \frac{\pi}{180} \times \text{Degree measure}$$

$$\text{Degree measure} = \frac{180}{\pi} \times \text{Radian measure}$$

We can simply put 7r=180° in radian measure to convert radian measure into degree measure,

Ex. 1 Convert 40° into radian measure.

Sol. $40° = 40 \times \frac{\pi}{180} \text{ radian} = \frac{2\pi}{9} \text{ radian}$

Ex. 2 Convert $\frac{3\pi}{2}$ into degree measure.

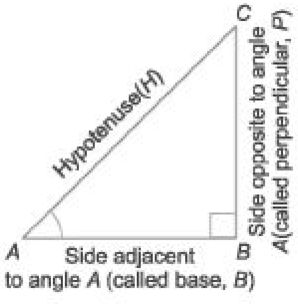
Sol. $\frac{3\pi}{2} \text{ radian} = \frac{3\pi}{2} \times \frac{180}{\pi} = 270°$

Alternate Method

$$\frac{3\pi}{2} = \frac{3 \times 180°}{2} = 270°$$

Trigonometric Ratios

The ratios between different sides of a right angled triangle with respect to its acute angles are called **trigonometric ratios.** Trigonometric ratios for right angled *AABC* with respect to angle *A* are given below.



$$\sin A = \frac{BC}{AC} = \frac{P}{H}, \qquad \cos A = \frac{AB}{AC} = \frac{B}{H}$$

$$\tan A = \frac{BC}{AB} = \frac{P}{B}, \qquad \operatorname{cosec} A = \frac{AC}{BC} = \frac{H}{P}$$

$$\sec A = \frac{AC}{AB} = \frac{H}{B}, \qquad \cot A = \frac{AB}{BC} = \frac{B}{P}$$

By Pythagoras theorem $H^2 = B^2 + D^2$

Reciprocal Relations

$$\sin A = \frac{1}{\cos A} \quad \text{or} \quad \cos A = \frac{1}{\sin A} \quad \text{or} \quad \sin A \cos A = 1$$

$$\cos A = \frac{1}{\sec A} \quad \text{or} \quad \sec A = \frac{1}{\cos A} \quad \text{or} \quad \sec A \cos A = 1$$

$$\tan A = \frac{\sin A}{\cos A} \quad \text{or} \quad \cot A = \frac{\cos A}{\sin A} \quad \text{or} \quad \tan A \cot A = 1$$

Ex. 3 If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Sol. $\sin A = \frac{BC}{AC} = \frac{3}{4}$

In a right angled ΔABC.

$$(AC)^2 = (AB)^2 + (BC)^2$$

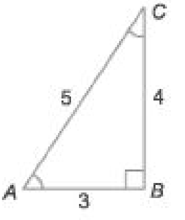
$\Rightarrow$ $4^2 = (AB)^2 + 3^2 \Rightarrow 16 - 9 = (AB)^2$

$\therefore$ $AB = \sqrt{7}$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{7}}{4} \text{ and } \tan A = \frac{BC}{AB} = \frac{3}{\sqrt{7}}$$

Ex. 4 If $\sin A = \frac{4}{5}$, then find the value of $(4 + \tan A)(2 + \cos A)$

Sol. Given, $\sin A = \frac{4}{5} = \frac{BC}{AC} = \frac{P}{H}$



Now in ΔABC, using Pythagoras theorem

$$(AC)^2 = (AB)^2 + (BC)^2$$

$\Rightarrow$ $(5)^2 = (AB)^2 + (4)^2$

$\Rightarrow$ $25 = (AB)^2 + 16$

$\Rightarrow$ $AB = \sqrt{25 - 16} = \sqrt{9} = 3$

So, $\tan A = \frac{BC}{AB} = \frac{P}{B} = \frac{4}{3}$

$$\cos A = \frac{AB}{AC} = \frac{B}{H} = \frac{3}{5}$$

Now, $(4 + \tan A)(2 + \cos A)$

$$= \left(4 + \frac{4}{3}\right)\left(2 + \frac{3}{5}\right) = \left(\frac{12+4}{3}\right) \times \left(\frac{10+3}{5}\right) = \frac{16}{3} \times \frac{13}{5} = \frac{208}{15}$$

Trigonometric Ratios of Some Specific Angles

Trigonometric Ratios	Angle (Degree/Radian)							
	0° 0	30° $\pi/6$	45° $\pi/4$	60° $\pi/3$	90° $\pi/2$	180° π	270° $3\pi/2$	360° 2π
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0
$\cot A$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1	∞	1
$\text{cosec } A$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	-1	∞

Trigonometric Identities

An equation involving trigonometric ratios of an angle is called a trigonometric identity, if it is true for all values of the angles involved. *For example,* in a right angled triangle, we have the following identities

1. $\sin^2 A + \cos^2 A = 1$ or $\sin^2 A = 1 - \cos^2 A$ or $\cos^2 A = 1 - \sin^2 A$
2. $1 + \tan^2 A = \sec^2 A$ or $\tan^2 A = \sec^2 A - 1$ or $\sec^2 A - \tan^2 A = 1$
3. $1 + \cot^2 A = \text{cosec}^2 A$ or $\cot^2 A = \text{cosec}^2 A - 1$ or $\text{cosec}^2 A - \cot^2 A = 1$

Ex. 5 If $\cot\theta = \frac{7}{8}$, evaluate $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$.

Sol. $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$$= \frac{(1)^2 - (\sin\theta)^2}{(1)^2 - (\cos\theta)^2} \qquad [\because (a-b)(a+b) = a^2 - b^2]$$

$$= \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2 = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

Ex. 6 Find the value of $2\tan^2 45° + \cos^2 30° - \sin^2 60°$.

Sol. $2\tan^2 45° + \cos^2 30° - \sin^2 60° = 2\times(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2$

Ex. 7 Find the value of $\frac{2\tan 30°}{1-\tan^2 30°}$.

Sol. $\frac{2\tan 30°}{1-\tan^2 30°} = \tan 60° = \sqrt{3}$ $\qquad \left[\because \tan 2x = \frac{2\tan x}{1-\tan^2 x}\right]$

Sign of Trigonometric Functions

Trigonometric ratios of complementary and supplementary angles of angle 0 are shown below.

$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ $\quad$ $\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$

$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$ $\quad$ $\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$

$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$ $\quad$ $\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$

$\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$ $\quad$ $\cot\left(\frac{\pi}{2}+\theta\right)=-\tan\theta$

$\sec\left(\frac{\pi}{2}-\theta\right)=\operatorname{cosec}\theta$ $\quad$ $\sec\left(\frac{\pi}{2}+\theta\right)=-\operatorname{cosec}\theta$

$\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)=\sec\theta$ $\quad$ $\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)=\sec\theta$

$\sin(\pi-\theta)=\sin\theta$ $\quad$ $\sin(\pi+\theta)=-\sin\theta$

$\cos(\pi-\theta)=-\cos\theta$ $\quad$ $\cos(\pi+\theta)=-\cos\theta$

$\tan(\pi-\theta)=-\tan\theta$ $\quad$ $\tan(\pi+\theta)=\tan\theta$

$\cot(\pi-\theta)=-\cot\theta$ $\quad$ $\cot(\pi+\theta)=\cot\theta$

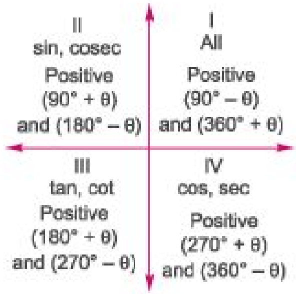
$\sec(\pi-\theta)=-\sec\theta$ $\quad$ $\sec(\pi+\theta)=-\sec\theta$

$\operatorname{cosec}(\pi-\theta)=\operatorname{cosec}\theta$ $\quad$ $\operatorname{cosec}(\pi+\theta)=-\operatorname{cosec}\theta$

$\sin(-x)=-\sin x$ $\quad$ $\operatorname{cosec}(-x)=-\operatorname{cosec}x$

$\cos(-x)=\cos x$ $\quad$ $\sec(-x)=\sec x$

$\tan(-x)=-\tan x$ $\quad$ $\cot(-x)=-\cot x$



For $(90^\circ \pm \theta)$ and $(270 \pm \theta)$
Change
$\sin\theta \leftrightarrow \cos\theta$,
$\tan\theta \leftrightarrow \cot\theta$,
$\operatorname{cosec}\theta \leftrightarrow \sec\theta$
and for $(180 \pm \theta)$ and $(360 \pm \theta)$ they do not change.

Note Positive or negative sign is used according to the quadrant.

Ex. 8 Find the value of $\frac{\sec 65^\circ}{\operatorname{cosec} 25^\circ}$.

Sol. $\frac{\sec 65^\circ}{\operatorname{cosec} 25^\circ}=\frac{\sec(90^\circ-25^\circ)}{\operatorname{cosec} 25^\circ}=\frac{\operatorname{cosec} 25^\circ}{\operatorname{cosec} 25^\circ}=1$

Ex. 9 If $\sin 3A=\cos(A-26^\circ)$, where $3A$ is an acute angle, find the value of A.

Sol. Given, $\sin 3A=\cos(A-26^\circ)$

or $\cos(90^\circ-3A)=\cos(A-26^\circ)$ $\quad$ $[\because \cos(90^\circ-\theta)=\sin\theta]$

Eliminating cos from both sides, we get

$$90^\circ-3A=A-26^\circ \Rightarrow 4A=116^\circ$$

$$\therefore \quad A=\frac{116^\circ}{4}=29^\circ$$

Ex. 10 Find the value of $\frac{\sin^2 63^\circ+\sin^2 27^\circ}{\cos^2 17^\circ+\cos^2 73^\circ}$.

Sol. $\frac{\sin^2 63^\circ+\sin^2 27^\circ}{\cos^2 17^\circ+\cos^2 73^\circ}=\frac{\sin^2(90^\circ-27^\circ)+\sin^2 27^\circ}{\cos^2(90^\circ-73^\circ)+\cos^2 73^\circ}$

$$=\frac{\cos^2 27^\circ+\sin^2 27^\circ}{\sin^2 73^\circ+\cos^2 73^\circ}=1 \quad [\because \sin^2\theta+\cos^2\theta=1, \text{ here } \theta=27^\circ \text{ and } 73^\circ]$$

Ex. 11 Find the value of tan 15°.

Sol. $\because \tan 15^\circ = \tan(45^\circ - 30^\circ) = \dfrac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ} = \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$

On rationalising,

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - (1)^2} = \frac{3 + 1 - 2\sqrt{3}}{3 - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

Ex. 12 Find the value of sin 2295°

Sol. sin 2295°

As 2295° is not directly measurable, So we will divide 2295° by 360°

$$360^\circ \overline{)\,2295^\circ\,}(6$$
$$\underline{2160}$$
$$135$$

Now, 2295° is 6 complete angles and 135°, So sin 2295° can be written as sin 135°.

$$\sin 135^\circ = \sin(90 + 45^\circ) = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Trigonometric Ratios of Combined Angles

Sum and Difference Formulae

1. sin (x + y) = sin x cos y + cos x sin y

2. cos (x + v) = cos x cos v + sin x sin v

3. $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

4. sin (x + y) • sin(x - y) = (sin x - sin y) or (cos y - cos x)

5. cos (x + y) cos (x - y) = (cos x - sin y) or (cos y - sin x)

6. sin (x + y + z) = cos xcos ycos z (tanx + tany + tanz) - tan x tan y tan z

7. cos(x + y + z) = cos x cosy cos z(l - tan x tan y - tan y tan z - tan x tan z)

8. $\tan(x+y+z) = \dfrac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y \tan z - \tan x \tan z}$

9. $\dfrac{\cos x \pm \sin x}{\cos x \mp \sin x} = \tan(45° \pm x)$

A, B formulae or Product to sum formulae

1. 2 sin A cos *B* = sin (A + *B)* + sin (A - *B)*

2. 2 sin A sin *B* = cos *(A-B)*- cos (A + *B)*

3. 2 cos A sin *B* = sin (A + *B)* - sin (A - *B)*

4. 2 cos A cos B = cos *(A+ B)* + cos (A - *B)*

C, D formulae or Sum to Product formulae

1. $\sin C + \sin D = 2 \sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$
2. $\sin C - \sin D = 2 \cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$
3. $\cos C + \cos D = 2 \cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$
4. $\cos C - \cos D = 2 \sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{D-C}{2}\right)$

Trigonometric Ratios of Multiple of an Angle

1. $\sin 2x = 2\sin x\cos x = \dfrac{2\tan x}{1+\tan^2 x}$

2. $\cos 2x = \cos^2 x - \sin^2 x$ or $2\cos^2 x - 1$ or $1 - 2\sin^2 x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

3. $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$ or $\cot 2x = \dfrac{\cot^2 x - 1}{2\cot x}$

4. $\sin 3x = 3\sin x - 4\sin^3 x$

5. $\cos 3x = 4\cos^3 x - 3\cos x$

6. $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

7. $\cos A.\cos 2A.\cos 4A = \dfrac{1}{4}[\cos(4A - 2A + A)] = \dfrac{1}{4}\cos 3A$

8. $\sin A.\sin 2A.\sin 4A = \dfrac{1}{4}[\sin(4A - 2A + A)] = \dfrac{1}{4}\sin 3A$

9. $\tan A.\tan 2A.\tan 4a = \tan[4A - 2A + A] = \tan 3A$

Trigonometric Ratios of Half Angles

1. $\sin x = 2\sin\dfrac{x}{2}\cos\dfrac{x}{2}$ or $\dfrac{2\tan x/2}{1+\tan^2(x/2)}$

2. $\cos x = \left(\cos^2\dfrac{x}{2} - \sin^2\dfrac{x}{2}\right)$ or $\left(1 - 2\sin^2\dfrac{x}{2}\right)$ or $\left(2\cos^2\dfrac{x}{2} - 1\right)$ or $\dfrac{1-\tan^2 x/2}{1+\tan^2 x/2}$

3. $1 + \cos x = 2\cos^2\dfrac{x}{2}$

4. $1 - \cos x = 2\sin^2\dfrac{x}{2}$

5. $\tan x = \dfrac{2\tan x/2}{1-\tan^2 x/2}$

6. $\cot x = \dfrac{\cot^2(x/2) - 1}{2\cot x/2}$

7. $\sin\dfrac{x}{2} = \sqrt{\dfrac{1-\cos x}{2}}$

8. $\cos\dfrac{x}{2} = \sqrt{\dfrac{1+\cos x}{2}}$

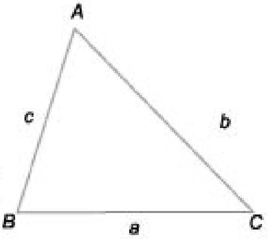
9. $\tan\dfrac{x}{2} = \sqrt{\dfrac{1-\cos x}{1+\cos x}}$

Sine Rule

In ΔABC,

If $AB = c$,
$BC = a$,
$AC = b$

Then $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = K$ [Constant value]



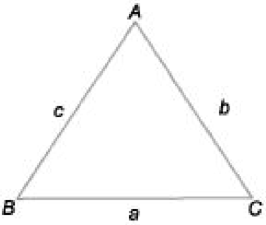
Cosine Rule

In ΔABC,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 = b^2 - c^2}{2ab}$$



Important Results

1. If $\sin x = 0$ or $\tan x = 0$, then $x = n\pi$
2. If $\cos x = 0$ or $\cot x = 0$, then $x = (2n+1)\dfrac{\pi}{2}$
3. $\sin 22\dfrac{1^\circ}{2} = \dfrac{\sqrt{2-\sqrt{2}}}{2}$
4. $\cos 22\dfrac{1^\circ}{2} = \dfrac{\sqrt{2+\sqrt{2}}}{2}$
5. $\tan 22\dfrac{1^\circ}{2} = \sqrt{2} - 1$
6. $\cot 22\dfrac{1^\circ}{2} = \sqrt{2} + 1$
7. $\sin 18^\circ = \cos 72^\circ = \dfrac{\sqrt{5}-1}{4}$
8. $\cos 18^\circ = \sin 72^\circ = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$
9. $\sin 36^\circ = \cos 54^\circ = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$
10. $\cos 36^\circ = \sin 54^\circ = \dfrac{\sqrt{5}+1}{4}$

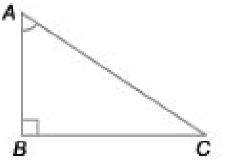
Ex. 13 In A ABC, right angled at B, find the value of $\sin A \cos C + \cos A \sin C$.

Sol. $\because \angle B = 90^\circ$

$\therefore \angle A + \angle C = 90^\circ$

Now, $\sin A \cos C + \cos A \sin C = \sin (A + C)$

$= \sin 90^\circ = 1$



Ex. 14 If $x + \dfrac{1}{x} = 2\cos\theta$, then find the value of $x^3 + \dfrac{1}{x^3}$

Sol. We know that, $x^3 + \dfrac{1}{x^3} = \left(x + \dfrac{1}{x}\right)^3 - 3\left(x + \dfrac{1}{x}\right)$

$= (2\cos\theta)^3 - 3 \times 2\cos\theta$ $\quad\left[\because x + \dfrac{1}{x} = 2\cos\theta\right]$

$= 8\cos^3\theta - 6\cos\theta = 2\,[4\cos^3\theta - 3\cos\theta]$

$= 2\cos 3\theta$ $\quad[\because 4\cos^3 x - 3\cos x = \cos 3x]$

Ex. 15 Find the value of cos 20° cos 40° cos 60° cos 80°.

Sol. $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$

$$= \cos 60^\circ [\cos 20^\circ \cos 40^\circ \cos 80^\circ]$$

$$= \frac{1}{2}\left[\frac{1}{4}\cos(3 \times 20^\circ)\right] \qquad \left[\because \cos\theta \cos 2\theta \cos 4\theta = \frac{1}{4}\cos 3\theta\right]$$

$$= \frac{1}{2}\left[\frac{1}{4}\cos 60^\circ\right] = \frac{1}{2} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{16}$$

Ex. 16 If $x + y = z$, then find the value of $\cos^2 x + \cos^2 y + \cos^2 z$.

Sol. $x + y = z$

Now, $\cos^2 x + \cos^2 y + \cos^2 z$

$$= 1 + (\cos^2 x - \sin^2 y) + \cos^2 z$$

$$= 1 + \cos(x + y) \cdot \cos(x - y) + \cos^2 z$$

$$= 1 + \cos z \cos(x - y) + \cos^2 z$$

$$= 1 + \cos z\,[(x - y) + \cos(x + y)]$$

$$= 1 + \cos z\left[2\cos\frac{(x - y + x + y)}{2} \cdot \cos\frac{(x - y - x - y)}{2}\right]$$

$$= 1 + 2\cos z \cdot \cos x \cdot \cos y$$

$$= 1 + 2\cos x \cdot \cos y \cdot \cos z$$

Ex. 17 If (a^2 - b^2) sinG + *lab* cos 9 = $a^2 + b^2$, then find the value of tanO.

Sol. $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$

$$\Rightarrow \frac{a^2 - b^2}{a^2 + b^2} \cdot \sin\theta + \frac{2ab}{a^2 + b^2} \cdot \cos\theta = 1$$

In ΔABC,

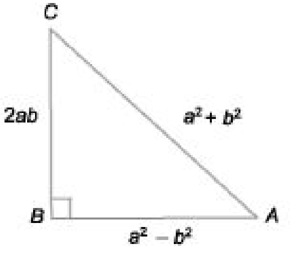
$$\cos\alpha \cdot \sin\theta + \sin\alpha \cdot \cos\theta = \sin 90^\circ$$

$$\Rightarrow \sin(\theta + \alpha) = \sin 90^\circ$$

$$\Rightarrow \theta + \alpha = 90^\circ$$

$$\Rightarrow \theta = 90^\circ - A$$

$$\therefore \tan\theta = \tan(90^\circ - \alpha) = \cot\alpha = \frac{a^2 - b^2}{2ab}$$



Ex. 18 If (1 + tan *A)* (1 + tan B) = 2, then find the value of (A + *B)*.

Sol. Given, $1 + \tan A + \tan B + \tan A \tan B = 2$

$$\Rightarrow \quad \tan A + \tan B = 1 - \tan A \tan B$$

$$\Rightarrow \quad \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1 = \tan 45^\circ$$

$$\Rightarrow \quad \tan (A + B) = \tan 45^\circ \quad \left[\because \tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \quad A + B = 45^\circ = \frac{\pi}{4}$$

Maximam and Minimum Values of Trignometric Angles

1. -l<sin8orcos6<l

2. -cc<tan8orcot6<l

3. sec8orcosec8 >1

4. sec 0 **or** cosecG < -1

Maximum and Minimum Value of Some Particular Expressions

(a) $m \sin\theta \pm n \cos\theta \ \sqrt{m^2 + n^2} \ - \sqrt{m^2 + n^2}$

(b) $m \sin\theta \pm n \cos\theta \ \sqrt{m^2 + n^2} \ - \sqrt{m^2 + n^2}$

(c) $m \cos \pm n \cos \sqrt{m^2 + n^2} \ - \sqrt{m^2 + n^2}$

Note To obtain the maximum or minimum value of an expression try to convert it into sinG orcosG

Ex. 19 Find **the maximum value** of 5 **tan** 8 **cos8** + 4 **cos8** .

Sol. Given, $5 \tan\theta \cos\theta + 4 \cos\theta$

$$= \frac{5 \sin\theta}{\cos\theta} \cos\theta + 4 \cos\theta \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= 5 \sin\theta + 4 \cos\theta$$

So, $\quad m = 5, n = 4$

$\therefore$ Maximum value $= \sqrt{5^2 + 4^2}$

$$= \sqrt{25 + 16}$$

$$= \sqrt{41}$$

Ex.20 Find the minimum value of 2**sin** 8+3cos 8

Sol. 2 $\sin^2$ 6+ 3 $\cos^2$ 8

$= 2 \sin^2 8 + 2 \cos^2 8 + \cos^2 8$

$= 2 (\sin^2 6 + \cos^2 8) + \cos^2 8$

$= 2 + \cos^2 G$ [•/ $\sin^2 G + \cos^2 G = 1$]

Minimum value of cos 8 = - 1

Minimum value of expression = 2 + (-1) = 2 + 1 = 3

Some Important Results

1. tan1°-tan2°.........tan 89° =1

2. cot1°-cot2°..........cot89° = 1

3. cos 1° • cos 2°- cos 3°.........cos 90° = 0

4. cos1°■ cos2°cos3°........(abovecos 90°).....= 0

5. sinl°• sin2°• sin3°........sin!80° = 0

6. sin1°- sin2°- sin3°.......(Above sin180°) = 0

7. $\frac{\sin A}{\cos B} = 1$, when $A + B = 90°$

8. tan/\ tanS = 1 = cot *A* cot *B* , when *A* + *B* = 90

Ex.21 Find the value of sinl°- sin2°- sin 3°.......sinl80°......sinl96°

Sol. sin 1°- sin 2°- sin 3°.......sin 180°......sin 196°

As sin 180° = 0 So, when a value is multiplied with zero (0), then whole expression will result in zero. sinl° • sin2° • sin3°.....sin 180°......sin 196° = 0

Ex.22 If $\sin\theta + \cos\theta = \sqrt{2}$, then find the value of θ.

Sol. $\sin\theta + \cos\theta = \sqrt{2}$

$$\sin\theta - \cos\theta = \sqrt{2 - x^2}$$

where, $$x^\circ = \sqrt{2}$$

$$\sin\theta - \cos\theta = \sqrt{2 - (\sqrt{2})^2}$$

$$\sin\theta - \cos\theta = \sqrt{2 - 2}$$

$$\sin\theta - \cos\theta = 0$$

$$\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1$$

$$\tan\theta = 1$$

$\Rightarrow$ $$\theta = 45^\circ$$

Note When $\sin\theta + \cos\theta = x$, then $\sin - \cos\theta = \sqrt{2 - x^2}$

Ex.23 If sinG + cosec 9 = 2 then find the value of $\sin^{36}$G + cosec^{36} G

Sol. If sin 6 + cosec 9 = 2

Then, $\sin^{36}$ 9 + cosec^{36}9 = 2

Note When sin 9 + cosec 9 = 2, then always sin" 0 + cosec" 9 = 2 for any positive value of *n*

Ex.24 Find the value of $\cot\frac{\pi}{20}\cdot\cot\frac{3\pi}{20}\cdot\cot\frac{5\pi}{20}\cdot\cot\frac{7\pi}{20}\cdot\cot\frac{9\pi}{20}$

Sol. $\cot\frac{\pi}{20}\cdot\cot\frac{3\pi}{20}\cdot\cot\frac{5\pi}{20}\cdot\cot\frac{7\pi}{20}\cdot\cot\frac{9\pi}{20}$

Converting them into degree measure

$$\cot\frac{180}{20}\cdot\cot\frac{3\times180}{20}\cdot\cot\frac{5\times180}{20}\cdot\cot\frac{7\times180}{20}\cdot\cot\frac{9\times180}{20}$$

$$\cot 9^\circ . \cot 27^\circ . \cot 45^\circ . \cot 63^\circ . \cot 81^\circ$$

Rewritting the above expression

$$\cot 9^\circ . \cot 81^\circ . \cot 27^\circ . \cot 63^\circ . \cot 45^\circ$$

We known that, $\cot A \cot B = 1$

When $A + B = 90^\circ$

So, $(\cot 9^\circ \cot 81^\circ)(\cot 27^\circ \cot 63^\circ) \cot 45^\circ$

There, $$9^\circ + 81^\circ = 90^\circ$$

$$27^\circ + 63^\circ = 90^\circ$$

Then, $\cot 9^\circ \cot 81^\circ = 1$ and $\cot 27^\circ \cot 63^\circ = 1$

$\therefore$ Expression will become $1 \times 1 \times 1$ $[\because \cot 45^\circ = 1]$

Fast Track Practice

Exercise© *Base Level Questions*

1. If ΔABC is right angled at C, then what is $\cos(A+B)+\sin(A+B)$ equal to? [CDS 2013]

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

2. If α, β and γ are acute angled such that $\sin\alpha = \frac{\sqrt{3}}{2}$ $\cos\beta = \frac{\sqrt{3}}{2}$ and $\tan\gamma = 1$, then what is $\alpha+\beta+\gamma$ equal to? [CDS 2013]

(a) 105° (b) 120° (c) 135° (d) 150°

3. If $\tan\theta = 3/4$ and θ is acute, then what is the value of $\sin\theta$? [CDS 2012]

(a) $-\frac{3}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $-\frac{4}{5}$

4. If $\sin\theta = \frac{a^2-1}{a^2+1}$, then the value of $\sec\theta + \tan\theta$ will be [SSC FCI 2012]

(a) $\frac{a}{\sqrt{2}}$ (b) $\frac{a}{a^2+1}$

(c) $\sqrt{2}a$ (d) a

5. If $\alpha+\beta = 90°$ and $\alpha:\beta = 2:1$, then the value of $\sin\alpha : \sin\beta$ is [SSC FCI 2012]

(a) $\sqrt{3}:1$ (b) $2:1$

(c) $1:1$ (d) $\sqrt{2}:1$

6. If $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta} = \frac{5}{3}$, then $\sin\theta$ is equal to

(a) 2/3 (b) 3/4 (c) 1/4 (d) 1/3

(e) None of the above

7. If $\sin\theta+\cos\theta = 1$, what is the value of $\sin\theta\cos\theta$? [CDS 2009]

(a) 2 (b) 0 (c) 1 (d) 1/2

8. If $\sin\theta + \operatorname{cosec}\theta = 2$, then the value of $\sin^7\theta + \operatorname{cosec}^7\theta$ is

(a) 1 (b) 1/2 (c) 2 (d) 0

9. If $\tan\theta = 3/4$ and θ is acute, then $\operatorname{cosec}\theta$ is equal to [SSC CGL 2013]

(a) $\frac{5}{3}$ (b) $\frac{5}{4}$ (c) $\frac{4}{3}$ (d) $\frac{4}{3}$

10. The value of $\left(\frac{1}{(1+\tan^2\theta)} + \frac{1}{(1+\cot^2\theta)}\right)$ is [SSC CGL 2013]

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

11. The minimum value of $\sin^2\theta + \cos^4\theta$ is **[SSC CPO 2013]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

12. If $\sec\theta + \tan\theta = p$, then $\cos\theta$ is

(a) $\frac{p^2+1}{p^2-1}$ (b) $\frac{p^2-1}{(p^2+1)^2}$

(c) $\frac{2p}{p^2+1}$ (d) $\frac{4p^2}{(p^2+1)^2}$

(e) None of the above

13. If $\tan 62° = \frac{P}{Q}$, then $\tan 28°$ is equal to

(a) $\frac{P}{Q}$ (b) $\frac{Q}{P}$

(c) $\frac{P^2-Q^2}{P}$ (d) $\frac{Q}{P^2}$

(e) None of the above

14. If $\sec\theta + \tan\theta = 2$, what is the value of $\sec\theta$? **[CDS 2014]**

(a) $\frac{3}{2}$ (b) $\sqrt{2}$ (c) $\frac{5}{2}$ (d) $\frac{5}{4}$

15. What is $\sin 25° \sin 35° \sec 65° \sec 55°$ equal to? **[CDS 2013]**

(a) -1 (b) 0 (c) $\frac{1}{2}$ (d) 1

16. Consider the following

I. $\sin^2 1° + \cos^2 1° = 1$

II. $\sec^2 33° - \cot^2 57° = \text{cosec}^2 37° - \tan^2 53°$

Which of the above statement is/are correct?

(a) Only I (b) Only II

(c) Both 1 and II (d) Neither I nor II

17. If $p = a\sin x + b\cos x$ and $q = a\cos x - b\sin x$, then what is the value of $p^2 + q^2$? **[CDS 2012]**

(a) $a + b$ (b) ab

(c) $a^2 + b^2$ (d) $a^2 - b^2$

18. What is the value of $\sec(90° - \theta) \cdot \sin\theta \sec 45°$? **[CDS 2012]**

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

19. If $\tan\theta = \frac{4}{3}$, then the value of $\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$ is **[SSC (10 + 2) 2011]**

(a) 0.5 (b) – 0.5
(c) 3 (d) – 3.0

20. If $\sec\theta - \operatorname{cosec}\theta = 0$, then the value of $\tan\theta + \cot\theta$ is **[SSC (10+2) 2013]**

(a) 0 (b) 1 (c) – 1 (d) 2

21. If $\tan 7\theta \tan 2\theta = 1$, then the value of $\tan 3\theta$ is **[SSC CGL 2012]**

(a) $\sqrt{3}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $-\sqrt{3}$

22. The expression $\frac{\tan 57° + \cot 37°}{\tan 33° + \cot 53°}$ is equal to **[SSC CGL 2012]**

(a) tan 33° cot 57° (b) tan 57° cot 37°
(c) tan 33° cot 53° (d) tan 53° cot 37°

23. The numerical value of $\frac{5}{\sec^2\theta} + \frac{1}{1+\cot^2\theta} + 3\sin^2\theta$ is **[SSC (10+2) 2012]**

(a) 5 (b) 2 (c) 3 (d) 4

24. The value of

$\tan 4° \cdot \tan 43° \cdot \tan 47° \cdot \tan 86°$ is **[SSC CPO 2011]**

(a) 2 (b) 3 (c) 1 (d) 4

25. Consider the following equations

(i) $\operatorname{cosec}^2 x + \sec^2 x = \operatorname{cosec}^2 x \sec^2 x$

(ii) $\sec x + \tan^2 x = \sec x \tan^2 x$

(iii) $\operatorname{cosec} x + \tan^2 x = \cot^2 x + \sec x$

Which of the above equations are correct? **[CDS 2009]**

(a) (i) and (ii) (b) (ii) and (iii)
(c) (i) and (iii) (d) (i), (ii) and (iii)

26. If $\frac{\cos x}{1+\operatorname{cosec} x} + \frac{\cos x}{\operatorname{cosec} x - 1} = 2$, which one of the following is one of the value of x? **[CDS 2009]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

27. $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2$ is satisfied by which one of the following values of θ? **[CDS 2009]**

(a) $\pi/2$ (b) $\pi/3$
(c) $\pi/4$ (d) $\pi/6$

28. What is $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ equal to? **[CDS 2010]**

(a) $\sec\theta - \tan\theta$ (b) $\sec\theta + \tan\theta$
(c) $\operatorname{cosec}\theta + \cot\theta$ (d) $\operatorname{cosec}\theta - \cot\theta$

29. If $7\cos^2\theta + 3\sin^2\theta = 4$ and $0 < \theta < \pi/2$, what is the value of $\tan\theta$? **[CDS 2010]**

(a) $\sqrt{7}$ (b) $\frac{7}{3}$ (c) 3 (d) $\sqrt{3}$

30. The value of $\frac{\tan 27^\circ + \cot 63^\circ}{\tan 27^\circ(\sin 25^\circ + \cos 65^\circ)}$. **[SSC CPO 2013]**

(a) cosec 25° (b) 2 tan27°
(c) sin 25° (d) tan 65°

31. If $\cos\theta \geq 1/2$ in the first quadrant, which one of the following is correct? **[CDS 2010]**

(a) $\theta \leq \frac{\pi}{3}$ (b) $\theta \geq \frac{\pi}{3}$ (c) $\theta \leq \frac{\pi}{6}$ (d) $\theta \geq \frac{\pi}{6}$

32. If $x = a\sec\theta\cos\phi$, $y = b\sec\theta\sin\phi$ and $z = c\tan\theta$, then the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$ is **[Railways 2006, SSC CGL 2013]**

(a) 9 (b) 0 (c) 1 (d) 4

33. If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $1 - 2\sin^2\theta$ is **[SSC CGL 2013, SSC (10+2) 2011]**

(a) 0 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{4}{3}$

34. If $7\sin^2\theta + 3\cos^2\theta = 4$, then the value of $\cos\theta\,(0^\circ \leq \theta \leq 90^\circ)$ is **[SSC CPO 2013]**

(a) $\frac{\sqrt{6}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\frac{\sqrt{5}}{2}$

35. If $\tan 8\theta = \cot 2\theta$, where $0 < 8\theta < \frac{\pi}{2}$, then what is the value of $\tan 5\theta$? **[CDS 2014]**

(a) $\frac{1}{\sqrt{3}}$ (b) 1 (c) $\sqrt{3}$ (d) 0

36. If $\sin\theta - \cos\theta = 0$, then what is $\sin^4\theta + \cos^4\theta$ equal to? **[CDS 2013]**

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

37. The expression $\sin^2 x + \cos^2 x - 1 = 0$ is satisfied by how many values of x? **[CDS 2012]**

(a) Only one value of x
(b) Two values of x
(c) Infinite values of x
(d) No value of x

38. Consider the following

I. $\frac{\cot 30^\circ + 1}{\cot 30^\circ - 1} = 2(\cos 300^\circ + 1)$

II. $2\sin 45^\circ\cos 45^\circ - \tan 45^\circ\cot 45^\circ = 0$

Which of the above identities is/are correct? **[CDS 2012]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

39. What is the value of $\sin A \cos A \tan A + \cos A \sin A \cot A$? **[CDS 2012]**

(a) $\sin^2 A + \cos A$ (b) $\sin^2 A + \tan^2 A$

(c) $\sin^2 A + \cot^2 A$ (d) $\text{cosec}^2 A - \cot^2 A$

40. If $2(\cos^2\theta - \sin^2\theta) = 1$ (θ is a positive acute angle), then $\cot\theta$ is equal to **[SSC (10+2) 2013]**

(a) $\frac{1}{\sqrt{3}}$ (b) 1

(c) $\sqrt{3}$ (d) $-\sqrt{3}$

41. The minimum value of $\cos^2\theta + \sec^2\theta$ is **[SSC (10+2) 2013]**

(a) 0 (b) 1 (c) 2 (d) 3

42. If $\cos x + \cos y = 2$, then the value of $\sin x + \sin y$ is **[SSC FCI 2012]**

(a) 0 (b) 1 (c) 2 (d) -1

43. If $\sec^2\theta + \tan^2\theta = \frac{7}{12}$, then $\sec^4\theta - \tan^4\theta$ is equal to **[SSC FCI 2012]**

(a) $\frac{7}{12}$ (b) $\frac{1}{2}$ (c) $\frac{5}{12}$ (d) 1

44. The numerical value of

$\left(\frac{1}{\cos\theta} + \frac{1}{\cot\theta}\right)\left(\frac{1}{\cos\theta} - \frac{1}{\cot\theta}\right)$ is

[SSC (10+2) 2012]

(a) 0 (b) -1 (c) 1 (d) 2

45. If $A = \tan 11^\circ \tan 29^\circ$ and $B = 2\cot 61^\circ \cot 79^\circ$, then **[SSC (10+2) 2011]**

(a) $A = 2B$ (b) $A = -2B$

(c) $2A = B$ (d) $2A = B$

46. If $\tan 2\theta \cdot \tan 4\theta = 1$, then the value of 3θ is **[SSC (10+2) 2011]**

(a) $\sqrt{3}$ (b) 0 (c) 1 (d) $\frac{1}{\sqrt{3}}$

47. If $\sin\theta + \text{cosec}\,\theta = 2$, then the value of $\sin^5\theta + \text{cosec}^5\theta$, when $0^\circ \le \theta \le 90^\circ$, is

[SSC (10+2) 2011]

(a) 0 (b) 1 (c) 10 (d) 2

48. If $A = \sin^2\theta + \cos^4\theta$, for any value of θ, then the value of A is **[SSC (10+2) 2011]**

(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le 1$

(c) $\frac{13}{16} \le A \le 1$ (d) $\frac{3}{4} \le A \le \frac{13}{16}$

49. The minimum value of $2\sin^2\theta + 3\cos^2\theta$ is **[SSC CPO 2011]**

(a) 0 (b) 3 (c) 2 (d) 1

50. The value of $\cot 10^\circ \cdot \cot 20^\circ \cdot \cot 60^\circ \cdot \cot 70^\circ \cot 80^\circ$ is **[SSC (10+2) 2011]**

(a) 1 (b) -1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

51. If $\cos\theta \operatorname{cosec} 23° = 1$, then the value of θ is **[SSC (10+2) 2012]**

(a) 23° (b) 37° (c) 63° (d) 67°

52. If θ is a positive acute angle and $\tan 2\theta \tan 3\theta = 1$, then the value of $\left(2\cos^2\frac{5\theta}{2} - 1\right)$ is **[SSC CGL 2012]**

(a) $-\frac{1}{2}$ (b) 1 (c) 0 (d) $\frac{1}{2}$

53. If $0° < \theta < 90°$, then the value of $\sin\theta + \cos\theta$ is **[SSC CGL 2012]**

(a) equal to 1 (b) greater than 1
(c) less than 1 (d) equal to 2

54. If $\sin\theta + \operatorname{cosec}\theta = 2$, then value of $\sin^{100}\theta + \operatorname{cosec}^{100}\theta$ is equal to

[SSC (10+2) 2011]

(a) 1 (b) 2 (c) 3 (d) 100

55. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$, then the value of $\sin\theta - \cos^4\theta$ is **[SSC (10+2) 2011]**

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

56. If θ be acute and $\tan\theta + \cot\theta = 2$, then the value of $\tan^5\theta + \cot^{10}\theta$ is

[SSC (10+2) 2011]

(a) 1 (b) 2 (c) 3 (d) 4

57. If $0° < \theta < 90°$, then all the trigonometric ratios can be obtained when **[CDS 2012]**

(a) only $\sin\theta$ is given
(b) only $\cos\theta$ is given
(c) only $\tan\theta$ is given
(d) any one of the six ratios is given

58. What is the value of $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$? **[CDS 2012]**

(a) $2\operatorname{cosec}\theta$ (b) $2\sec\theta$
(c) $\sec\theta$ (d) $\operatorname{cosec}\theta$

59. If $\sin\theta\cos\theta = \sqrt{3}/4$, then the value of $\sin^4\theta + \cos^4\theta$ is **[CDS 2012]**

(a) 7/8 (b) 5/8 (c) 3/8 (d) 1/8

60. If $2\cot\theta = 3$, then what is $\frac{2\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$ equal to? **[CDS 2014]**

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

61. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta$ is **[SSC CGL 2013]**

(a) $-\sqrt{2}\cos\theta$ (b) $-\sqrt{2}\sin\theta$
(c) $\sqrt{2}\sin\theta$ (d) $\sqrt{2}\tan\theta$

62. If $3\sin x + 5\cos x = 5$, then what is the value of $(3\cos x - 5\sin x)$? [CDS 2012]

(a) 0 (b) 2 (c) 3 (d) 5

63. If $0 < x < \frac{\pi}{2}$ and $\sec x = \operatorname{cosec} y$, then the value of $\sin(x + y)$ is [SSC CGL 2013]

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{3}}$

64. If $\sin 17° = \frac{x}{y}$, then the value of $\sec 17° - \sin 73°$ is [SSC FCI 2012]

(a) $\frac{y^2 - x^2}{xy}$ (b) $\frac{x^2}{\sqrt{y^2 - x^2}}$

(c) $\frac{x^2}{y\sqrt{y^2 + x^2}}$ (d) $\frac{x^2}{y\sqrt{y^2 - x^2}}$

65. If $2\sin\left(\frac{\pi x}{2}\right) = x^2 + \frac{1}{x^2}$, then the value of $\left(x - \frac{1}{x}\right)$ is [SSC CGL 2012]

(a) -1 (b) 2 (c) 1 (d) 0

66. The equation $\cos^2\theta = \frac{(x + y)^2}{4xy}$ is only possible when [SSC (10+2) 2013]

(a) $x > y$ (b) $x = y$

(c) $x < y$ (d) $x = -y$

67. The value of $\sin^2 1° + \sin^2 3° + \sin^2 5° + \cdots + \sin^2 87° + \sin^2 89°$ is [SSC (10 + 2) 2013]

(a) 22 (b) $22\frac{1}{2}$ (c) 23 (d) $22\frac{1}{4}$

68. If $\angle A$ and $\angle B$ are complementary to each other, then the value of $\sec^2 A + \sec^2 B - \sec^2 A \sec^2 B$ is [SSC CGL 2012]

(a) 1 (b) -1 (c) 2 (d) 0

69. The value of $\sin^2 5° + \sin^2 10° + \sin^2 15° + \cdots + \sin^2 85° + \sin^2 90°$ is [SSC (10 + 2) 2012, 2011]

(a) $7\frac{1}{2}$ (b) $8\frac{1}{2}$ (c) $10\frac{1}{2}$ (d) $9\frac{1}{2}$

70. If $\sin(3x - 20°) = \cos(3y + 20°)$, then the value of $(x + y)$ is

[SSC (10 + 2) 2012]

(a) 20° (b) 30° (c) 40° (d) 45°

71. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{5}{4}$, the value of $\frac{\tan^2\theta + 1}{\tan^2\theta - 1}$ is

[SSC (10 + 2) 2012]

(a) $\frac{25}{16}$ (b) $\frac{41}{9}$ (c) $\frac{41}{40}$ (d) $\frac{40}{41}$

72. The value of $\frac{\sin 39°}{\cos 51°} + 2\tan 11°$ $\tan 31° \tan 45° \tan 59° \tan 79° - 3(\sin^2 21° + \sin^2 69°)$ is [SSC (10 + 2) 2011]

(a) 2 (b) −1 (c) 1 (d) 0

73. If $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$ and $0° < \theta < 90°$, then the value of θ is

[SSC (10 + 2) 2011]

(a) 30° (b) 45°
(c) 60° (d) None of these

74. The numerical value of $\cot 18° \left(\cot 72° \cos^2 22° + \frac{1}{\tan 72° \sec^2 68°}\right)$ is

[SSC (10 + 2) 2011]

(a) 1 (b) $\sqrt{2}$
(c) 3 (d) $\frac{1}{\sqrt{3}}$

75. If θ be an acute angle and $7\sin^2\theta + 3\cos^2\theta = 4$, then the value of $\tan\theta$ is [SSC (10 + 2) 2011]

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) 1 (d) 0

Exercise © *Higher SkiU Level Questions*

1. If $(\sin x + \sin y) = a$ and $(\cos x + \cos y) = b$, what is the value of $\sin x \sin y + \cos x \cos y$?

(a) $a + b - ab$ (b) $a + b + ab$

(c) $a^2 + b^2 - 2$ (d) $\dfrac{a^2 + b^2 - 2}{2}$

(e) None of the above

2. Consider the following [CDS 2009]

(i) $\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \cos^2\theta\,(1 + \tan\theta)$

$(1 - \tan\theta)$

(ii) $\dfrac{1 + \sin\theta}{1 - \sin\theta} = (\tan\theta + \sec\theta)^2$

Which of the equations given above is/are correct?

(a) Only (i) (b) Only (ii)
(c) Both (i) and (ii) (d) Neither (i) nor (ii)

3. The value of
$(\tan 1^\circ \tan 2^\circ \tan 3^\circ .. \tan 89^\circ)$ is

[SSC CGL 2013]

(a) undefined (b) 0
(c) 1 (d) 89

4. If $\sec\theta + \tan\theta = p$, then $\sin\theta$ is

(a) $\frac{1}{p^2+1}$ (b) $\frac{1}{p^2-1}$
(c) $\frac{p^2+1}{p^2-1}$ (d) $\frac{p^2-1}{p^2+1}$

5. If $\sin\theta + \cos\theta = x$, then the value of $\cos^6\theta + \sin^6\theta$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{4}(1+6x^2)$
(c) $\frac{1}{4}(1+6x^2-3x^4)$ (d) $\frac{1}{2}(5-3x^2)$
(e) None of the above

6. If $(1+\tan A)(1+\tan B) = 2$, then $(A+B)$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$
(e) None of the above

7. If $\sin(10^\circ\ 6'\ 32'') = a$, then the value of $\cos(79^\circ\ 53'\ 28'') + \tan(10^\circ\ 6'\ 32'')$ is

[SSC CGL (Main) 2012]

(a) $\frac{a(1+\sqrt{1-a^2})}{\sqrt{1-a^2}}$ (b) $\frac{1+\sqrt{1-a^2}}{\sqrt{1-a^2}}$
(c) $\frac{\sqrt{1-a^2}+a}{\sqrt{1-a^2}}$ (d) $\frac{a\sqrt{1-a^2}+1}{\sqrt{1-a^2}}$

8. If $x+y=z$, find the value of $\cos^2 x + \cos^2 y + \cos^2 z$.

(a) $1+2\sin x \sin y \sin z$
(b) $1-2\sin x \sin y \sin z$
(c) $1+2\cos x \cos y \cos z$
(d) $1-2\cos x \cos y \cos z$
(e) None of the above

9. If $\cos\theta + \sec\theta = 2$, then the value of $\cos^6\theta + \sec^6\theta$ is **[SSC (10 + 2) 2012]**

(a) 1 (b) 2 (c) 4 (d) 8

10. If $\sin\theta - \cos\theta = 0$, find the value of $\sin\left[\frac{\pi}{2}-\theta\right] + \cos\left(\frac{\pi}{2}-\theta\right)$ **[SSC (10 + 2) 2012]**

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

11. The maximum value of $\sin^8\theta + \cos^{14}\theta$, for all real values of θ is [SSC CGL (Main) 2012]

(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 0

12. If $l\cos^2\theta + m\sin^2\theta = \frac{\cos^2\theta(\text{cosec}^2\theta + 1)}{\text{cosec}^2\theta - 1}$, $0° < \theta < 90°$, then $\tan\theta$ is equal to [SSC CGL (Main) 2012]

(a) $\sqrt{\frac{l-2}{1-m}}$ (b) $\sqrt{\frac{2-l}{1-m}}$

(c) $\sqrt{\frac{l-2}{m-1}}$ (d) $\sqrt{\frac{l-1}{2-m}}$

13. ΔABC is a right-angled triangle, where $\angle ABC = 90°$. If $AC = 2\sqrt{5}$ and $AB - BC = 2$, then the value of $\cos^2 A - \sin^2 C$ is [SSC CGL (Main) 2012]

(a) $\frac{1}{\sqrt{5}}$ (b) $\sqrt{5}$

(c) $\frac{1}{2}$ (d) $\frac{3}{5}$

14. If $\cos x + \cos^2 x = 1$, then the numerical value of $(\sin^{12}x + 3\sin^{10}x + 3\sin^8 x + \sin^6 x - 1)$ is [SSC CGL 2013]

(a) 0 (b) 1 (c) -1 (d) 2

15. If $a\sin\theta + b\cos\theta = c$, then the value of $a\cos\theta - b\sin\theta$ is [SSC CGL 2013]

(a) $\pm\sqrt{a^2 - b^2 - c^2}$

(b) $\pm\sqrt{a^2 - b^2 + c^2}$

(c) $\pm\sqrt{-a^2 + b^2 + c^2}$

(d) $\pm\sqrt{a^2 + b^2 - c^2}$

16. If $\tan\alpha = n\tan\beta$ and $\sin\alpha = m\sin\beta$, then $\cos^2\alpha$ is [SSC CGL 2013]

(a) $\frac{m^2}{n^2}$ (b) $\frac{m^2-1}{n^2-1}$

(c) $\frac{m^2+1}{n^2+1}$ (d) $\frac{m^2}{n^2+1}$

17. If $\sin\theta\cos\theta = 1/2$, then what is $\sin^6\theta + \cos^6\theta$ equal to? [CDS 2014]

(a) 1 (b) 2 (c) 3 (d) $\frac{1}{4}$

18. What is $\text{cosec}(75° + \theta) - \sec(15° - \theta) - \tan(55° + \theta) + \cot(35° - \theta)$ [CDS 2014]

(a) -1 (b) 0 (c) 1 (d) $3/2$

19. If $\cos x + \sec x = 2$, then what $\cos^n x + \sec^n x$ equal to, where n is a positive integer? **[CDS 2014]**

(a) 2 (b) 2^{n-2} (c) 2^{n-1} (d) 2^n

20. If α and β are complementary angles, then what is $\sqrt{\cos \alpha \operatorname{cosec} \beta - \cos \alpha \sin \beta}$ equal to? **[CDS 2014]**

(a) $\sec \beta$ (b) $\cos \alpha$ (c) $\sin \alpha$ (d) $-\tan \beta$

21. If $\sin \theta + 2 \cos \theta = -1$, where $0 < \theta < \frac{\pi}{2}$, what is $2 \sin \theta - \cos \theta$ equal to? **[CDS 2014]**

(a) -1 (b) $\frac{1}{2}$ (c) 2 (d) 1

22. If $\sin (A + B) = 1$, where $0 < B < 45°$, what is $\cos (A - B)$ equal to? **[CDS 2014]**

(a) $\sin 2B$ (b) $\sin B$
(c) $\cos 2B$ (d) $\cos B$

23. If $\sin \theta + \cos \theta = \sqrt{3}$, then what is $\tan \theta + \cot \theta$ equal to? **[CDS 2013]**

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $\sqrt{3}$

24. If $\tan \theta + \sec \theta = m$, then what is $\sec \theta$ equal to? **[CDS 2013]**

(a) $\frac{m^2 - 1}{2m}$ (b) $\frac{m^2 + 1}{2m}$
(c) $\frac{m + 1}{m}$ (d) $\frac{m^2 + 1}{m}$

25. If $5 \sin \theta + 12 \cos \theta = 13$, then what is $5 \cos \theta - 12 \sin \theta$ equal to? **[CDS 2013]**

(a) -2 (b) -1 (c) 0 (d) 1

26. Conside the following statements

I. $\tan \theta$ increases faster than $\sin \theta$ as θ increases.

II. The value of $\sin \theta + \cos \theta$ is always greater than 1.

Which of the statemetns given above is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

27. What is $\frac{(\sin \theta + \cos \theta)(\tan \theta + \cot \theta)}{\sec \theta + \operatorname{cosec} \theta}$ equal to? **[CDS 2013]**

(a) 1 (b) 2 (c) $\sin \theta$ (d) $\cos \theta$

28. What is $\dfrac{(1+\sec\theta-\tan\theta)\cos\theta}{(1+\sec\theta+\tan\theta)(1-\sin\theta)}$ equal to? **[CDS 2013]**

(a) 1 (b) 2 (c) $\tan\theta$ (d) $\cot\theta$

29. If $x\sin\theta - y\cos\theta = \sqrt{x^2+y^2}$ and $\dfrac{\cos^2\theta}{a^2}+\dfrac{\sin^2\theta}{b^2}=\dfrac{1}{x^2+y^2}$, then the correct relation is **[SSC (10 + 2) 2013]**

(a) $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ (b) $\dfrac{x^2}{b^2}+\dfrac{y^2}{a^2}=1$

(c) $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ (d) $\dfrac{x^2}{b^2}-\dfrac{y^2}{a^2}=1$

30. $2\operatorname{cosec}^2 23^\circ \cot^2 67^\circ - \sin^2 23^\circ - \sin^2 67^\circ - \cot^2 67^\circ$ is equal to **[SSC (10 + 2) 2013]**

(a) $\sec^2 23^\circ$ (b) $\tan^2 23^\circ$

(c) 0 (d) 1

31. The value of $\cot\theta\cdot\tan(90^\circ-\theta) - \sec(90^\circ-\theta)\operatorname{cosec}\theta + (\sin^2 25^\circ + \sin^2 65^\circ) + \sqrt{3}(\tan 5^\circ \tan 15^\circ\cdot\tan 30^\circ\cdot\tan 75^\circ\cdot\tan 85^\circ)$ is **[SSC (10 + 2) 2012]**

(a) 1 (b) -1

(c) 2 (d) 0

32. In a right angled ΔXYZ, right angled at Y, if $XY = 2\sqrt{6}$ and $XY - YZ = 2$, then $\sec X + \tan X$ is **[SSC CGL 2012]**

(a) $\dfrac{1}{\sqrt{6}}$ (b) $\sqrt{6}$

(c) $2\sqrt{6}$ (d) $\dfrac{\sqrt{6}}{2}$

33. The simplified value of $(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y \tan x \sec y)^2$ is **[SSC (10 + 2) 2011]**

(a) -1

(b) 0

(c) $\sec^2 x$

(d) 1

34. If $\tan 15^\circ = 2-\sqrt{3}$, then the value of $\tan 15^\circ \cot 75^\circ + \tan 75^\circ \cot 15^\circ$ is **[SSC (10 + 2) 2011]**

(a) 14 (b) 12 (c) 10 (d) 8

Answer with **Solutions**

Exercise© *Base Level Questions*

1. (*c*) In ΔABC, if $\angle C$ is $90°$, then

$$\angle A + \angle B = 180° - 90° = 90°$$

Now, $\cos(A + B) + \sin(A + B)$

$$= \cos 90° + \sin 90° = 0 + 1 = 1$$

2. (*c*) $\sin\alpha = \frac{\sqrt{3}}{2}$

$\therefore \quad \alpha = 60° \qquad \left(\because \sin 60 = \frac{\sqrt{3}}{2}\right)$

$\cos\beta = \frac{\sqrt{3}}{2}$

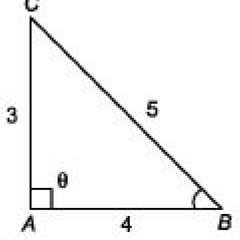
$\therefore \quad \beta = 30° \qquad \left(\because \cos 30° = \frac{\sqrt{3}}{2}\right)$

$\tan\gamma = 1$

$\therefore \quad \gamma = 45° \qquad (\because \tan 45° = 1)$

So, $\quad \alpha + \beta + \gamma = 60° + 30° + 45° = 135°$

3. (*b*) Given that, $\tan\theta = \frac{P}{B} = \frac{3}{4}$ and $0° < \theta < 90°$ (acute)



In ΔABC,

By Pythagoras theorem

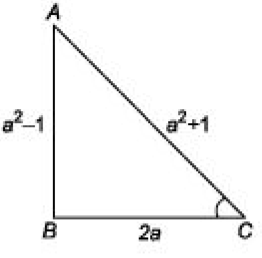
$$BC^2 = AC^2 + AB^2$$

$$= 9 + 16 = 25$$

$$BC = 5$$

$\therefore \quad \sin\theta = \frac{P}{H} = \frac{3}{5}$

4. (*d*) $\sin\theta = \frac{a^2 - 1}{a^2 + 1}$



In ΔABC,

$$BC = \sqrt{AC^2 - AB^2}$$

$$= \sqrt{(a^2+1)^2 - (a^2-1)^2}$$

$$= \sqrt{a^2 + 1 + 2a^2 - a^2 - 1 + 2a^2}$$

$$= \sqrt{4a^2} = 2a$$

$$\therefore \ \sec\theta + \tan\theta = \frac{a^2+1}{2a} + \frac{a^2-1}{2a}$$

$$= \frac{2a^2}{2a} = a$$

5. (*a*) Given

$$\alpha + \beta = 90° \text{ and } \alpha : \beta = 2 : 1$$

$$\alpha = \frac{2}{2+1} \times 90°$$

$$= \frac{2}{3} \times 90° = 60°$$

Similarly, $\beta = \frac{1}{3} \times 90° = 30°$

$$\therefore \sin\alpha : \sin\beta = \sin 60° : \sin 30°$$

$$= \frac{\sqrt{3}}{2} : \frac{1}{2} = \sqrt{3} : 1$$

6. (*c*) $\dfrac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \dfrac{5}{3}$

$$\Rightarrow \quad \frac{\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}}{\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{\dfrac{1+\sin\theta}{\cos\theta}}{\dfrac{1-\sin\theta}{\cos\theta}} = \frac{5}{3} \Rightarrow \frac{1+\sin\theta}{1-\sin\theta} = \frac{5}{3}$$

$$\Rightarrow \quad (1+\sin\theta) \times 3 = 5(1-\sin\theta)$$

$$\Rightarrow \quad 3 + 3\sin\theta = 5 - 5\sin\theta$$

$$\Rightarrow \quad 3\sin\theta + 5\sin\theta = 5 - 3$$

$$\Rightarrow \quad 8\sin\theta = 2$$

$$\therefore \quad \sin\theta = \frac{2}{8} \text{ or } \sin\theta = \frac{1}{4}$$

7. (*b*) $\sin\theta + \cos\theta = 1$

On squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = 1$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$$

$$1 + 2\sin\theta\cos\theta = 1$$

$$2\sin\theta\cos\theta = 0$$

$$\therefore \quad \sin\theta\cos\theta = 0$$

8. (c) $\sin\theta + \operatorname{cosec}\theta = 2$

$$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2 \Rightarrow \sin^2\theta + 1 = 2\sin\theta$$

$$\sin^2\theta - 2\sin\theta + 1 = 0$$

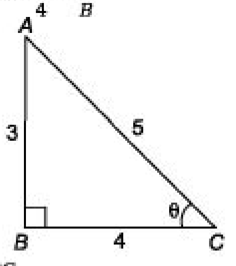
$$\Rightarrow \quad (\sin\theta - 1)^2 = 0$$

$$\Rightarrow \quad \sin\theta = 1 = \sin 90°$$

$$\therefore \quad \theta = 90°$$

$$\therefore \quad \sin^7\theta + \operatorname{cosec}^7\theta = 1 + 1 = 2$$

9. (a) $\tan\theta = \frac{3}{4} = \frac{P}{B}$



In ΔABC,

$$H^2 = P^2 + B^2$$

$$\Rightarrow \quad H^2 = 3^2 + 4^2$$

$$\Rightarrow \quad H = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\therefore \quad \operatorname{cosec}\theta = \frac{H}{P} = \frac{5}{3}$$

10. (a) $\frac{1}{1 + \tan^2\theta} + \frac{1}{1 + \cot^2\theta}$

$$\frac{1}{1 + \frac{\sin^2\theta}{\cos^2\theta}} + \frac{1}{1 + \frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\cos^2\theta}{\cos^2\theta + \sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta + \cos^2\theta}$$

$$\cos^2\theta + \sin^2\theta = 1$$

11. (c) $\therefore$ Minimum value of

$\sin^2\theta + \cos^4\theta$ will be at $\theta = 45°$

$\therefore$ Required minimum value

$$\sin^2 45 + \cos^4 45 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

12. (c) $\sec\theta + \tan\theta = p$ or $\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = p$

or $\quad 1 + \sin\theta = p\cos\theta \quad \ldots$(i)

On squaring both sides, we get

$$1 + \sin^2\theta + 2\sin\theta = p^2\cos^2\theta$$

$$\Rightarrow \quad 1 + 1 - \cos^2\theta + 2\sin\theta = p^2\cos^2\theta$$

$$\Rightarrow \quad 2 + 2\sin\theta - \cos^2\theta = p^2\cos^2\theta$$

$$\Rightarrow \quad 2(1 + \sin\theta) - \cos^2\theta = p^2\cos^2\theta$$

$$\Rightarrow \quad 2 \times p\cos\theta - \cos^2\theta = p^2\cos^2\theta$$

[from Eq. (i)]

$\Rightarrow \quad 2p\cos\theta = p^2\cos^2\theta + \cos^2\theta$

$\Rightarrow \quad 2p\cos\theta = \cos^2\theta(1+p^2)$

$\therefore \quad \cos^2\theta = \frac{2p\cos\theta}{(1+p^2)} \Rightarrow \cos\theta = \frac{2p}{1+p^2}$

13. (*b*) $\tan 62° = \frac{P}{Q}$

$\Rightarrow \quad \tan(90° - 28°) = \frac{P}{Q} \Rightarrow \quad \cot 28° = \frac{P}{Q}$

$\Rightarrow \quad \frac{1}{\tan 28°} = \frac{P}{Q} \Rightarrow \tan 28° = \frac{Q}{P}$

14. (*d*) By trigonometric identity,

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \quad (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \quad \sec\theta - \tan\theta = \frac{1}{2} \quad \ldots(\text{i})$

and given, $\sec\theta + \tan\theta = 2 \quad \ldots(\text{ii})$

On adding Eqs. (i) and (ii), we get

$2\sec\theta = \frac{1}{2} + 2$

$\therefore \quad \sec\theta = \frac{5}{4}$

15. (*d*) $\sin 25° \sin 35° \sec 65° \sec 55°$

$\Rightarrow \quad \sin 25° \sin 35° \cdot \frac{1}{\cos 65°} \cdot \frac{1}{\cos 55°}$

$\Rightarrow \quad \sin 25° \cdot \sin 35° \times \frac{1}{\cos(90° - 25°)} \times \frac{1}{\cos(90° - 35°)}$

$\Rightarrow \sin 25° \times \sin 35° \times \frac{1}{\sin 25°} \times \frac{1}{\sin 35°}$

$\Rightarrow 1$

16. (*d*) We know that, $\sin^2\theta + \cos^2\theta = 1$ is true

$\therefore$ I. $\sin^2 1° + \cos^2 1° = 1$ which is not true.

II. $\sec^2 33° - \cot^2 57°$

$= \text{cosec}^2 37° - \tan^2 53°$

Now, $\sec^2(90° - 57°) = \text{cosec}^2 57°$

and $\cot^2 57° = \cot^2(90° - 33°) = \tan^2 33°$

$\therefore \quad \sec^2 33° - \cot^2 57°$

$= \text{cosec}^2 57 - \tan^2 33°$

II is not true.

17. (*c*) Given, $p = a\sin x + b\cos x \quad \ldots(\text{i})$

and $q = a\cos x - b\sin x \quad \ldots(\text{ii})$

On squaring both the equations

$p^2 = a^2\sin^2 x + b^2\cos^2 x + 2ab\sin x\cos x$

and $q^2 = a^2\cos^2 x + b^2\sin^2 x - 2ab\sin x\cos x$

$$P^2 + q^2 = a^2 \sin^2 x + b^2 \cos^2 x$$
$$+ 2ab \sin x \cos x$$
$$+ a^2 \cos^2 x + b^2 \sin^2 x - 2ab \sin x \cos x$$
$$p^2 + q^2 = a^2(\sin^2 x + \cos^2 x) + b^2$$
$$(\cos^2 x + \sin^2 x)$$
$$= a^2 + b^2$$

18. (*c*) Given, $\sec(90-\theta)^\circ \sin\theta \sec 45$
$$= \operatorname{cosec}\theta \sin\theta \sec 45^\circ$$
$$= \frac{1}{\sin\theta} \cdot \sin\theta \cdot (\sqrt{2}) = \sqrt{2}$$

19. (*c*) $\tan\theta = \frac{4}{3}$ [given]
$$\therefore \quad \frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta} = \frac{3\tan\theta + 2}{3\tan\theta - 2}$$
$$= \frac{3 \times \frac{4}{3} + 2}{3 \times \frac{4}{3} - 2} = \frac{4+2}{4-2} = 3$$

20. (*d*) $\sec\theta - \operatorname{cosec}\theta = 0$
$$\Rightarrow \quad \sec = \operatorname{cosec}\theta$$
$$\frac{1}{\cos\theta} = \frac{1}{\sin\theta}$$
$$\Rightarrow \quad \tan\theta = 1$$
$$\Rightarrow \quad \tan\theta = \tan 45^\circ$$
$$\Rightarrow \quad \theta = 45^\circ$$
$$\therefore \tan\theta + \cot\theta = \tan 45^\circ + \cot 45^\circ$$
$$= 1 + 1 = 2$$

21. (*c*) Given, $\tan 7\theta \cdot \tan 2\theta = 1$
$$\Rightarrow \quad \tan 7\theta = \cot 2\theta$$
$$\Rightarrow \quad \tan 7\theta = \tan(90^\circ - 2\theta)$$
$$\Rightarrow \quad 7\theta = 90^\circ - 2\theta$$
$$\Rightarrow \quad 9\theta = 90^\circ \quad \Rightarrow \quad \theta = 10^\circ$$
$$\therefore \quad \tan 3\theta = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

22. (*b*) $\frac{\tan 57^\circ + \cot 37^\circ}{\tan 33^\circ + \cot 53^\circ} = \frac{\tan 57^\circ + \cot 37^\circ}{\cot 57^\circ + \tan 37^\circ}$
$$= \frac{\tan 57^\circ + \frac{1}{\tan 37^\circ}}{\frac{1}{\tan 57^\circ} + \tan 37^\circ}$$
$$= \frac{1 + \tan 57^\circ \tan 37^\circ}{1 + \tan 37^\circ \tan 57^\circ} \times \frac{\tan 57^\circ}{\tan 37^\circ}$$
$$= \frac{\tan 57^\circ}{\tan 37^\circ} = \tan 57^\circ \cdot \cot 37^\circ$$

23. (*a*) $\frac{5}{\sec^2\theta} + \frac{2}{1 + \cot^2\theta} + 3\sin^2\theta$
$$= 5\cos^2\theta + \frac{2}{\operatorname{cosec}^2\theta} + 3\sin^2\theta$$
$$[\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta]$$

$$= 5\cos^2\theta + 2\sin^2\theta + 3\sin^2\theta$$
$$= 5\cos^2\theta + 5\sin^2\theta$$
$$= 5(\sin^2\theta + \cos^2\theta)$$
$$\{\because \sin^2\theta + \cos^2\theta = 1\}$$
$$= 5 \times 1 = 5$$

24. (*b*) $$\frac{5\sin 75^\circ \sin 77^\circ + 2\cos 13^\circ \cos 15^\circ}{\cos 15^\circ \sin 77^\circ} - \frac{7\sin 81^\circ}{\cos 9^\circ}$$

$$= \frac{5[\sin(90^\circ - 15^\circ)\sin(90^\circ - 13^\circ)] + 2\cos 13^\circ \cos 15^\circ}{\cos 15^\circ \sin(90^\circ - 13^\circ)} - \frac{7[\sin(90^\circ - 9^\circ)}{\cos 9^\circ}$$

$$= \frac{5\cos 15^\circ \cos 13^\circ + 2\cos 13^\circ \cos 15^\circ}{\cos 15^\circ \cos 13^\circ} - \frac{7\cos 9^\circ}{\cos 9^\circ}$$

$$= \frac{7\cos 15^\circ \cos 13^\circ}{\cos 15^\circ \cos 13^\circ} - 7$$

$$= 7 - 7 = 0$$

25. (*c*) (i) LHS $= \operatorname{cosec}^2 x + \sec^2 x$

$$= \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$$

$$= \frac{1}{\sin^2 x \cos^2 x} = \frac{1}{\sin^2 x} \cdot \frac{1}{\cos^2 x}$$

$$= \operatorname{cosec}^2 x \sec^2 x = \text{RHS}$$

(ii) LHS $= \sec^2 x + \tan^2 x$

$$= \frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$= \frac{1 + \sin^2}{\cos^2 x} \neq \sec^2 x \tan^2 x = \text{RHS}$$

(iii) LHS $= \operatorname{cosec}^2 x + \tan^2 x$

$$= 1 + \cot^2 x + \sec^2 x - 1$$

$\because \quad \operatorname{cosec}^2 x = 1 + \cot^2 x$

$\Rightarrow \quad \sec^2 x = 1 + \tan^2 x$

$$= \cot^2 x + \sec^2 x = \text{RHS}$$

So, Eqs. (i) and (iii) are correct.

26. (*c*) $$\frac{\cos x}{\operatorname{cosec} x + 1} + \frac{\cos x}{\operatorname{cosec} x - 1} = 2$$

$$\Rightarrow \frac{\cos x(\operatorname{cosec} x - 1) + \cos x(\operatorname{cosec} x + 1)}{(\operatorname{cosec} x + 1)(\operatorname{cosec} - 1)} = 2$$

$$\Rightarrow \frac{\cos x \operatorname{cosec} x - \cos x + \cos x \operatorname{cosec} x + \cos x}{\operatorname{cosec}^2 x - 1} = 2$$

$$\Rightarrow \quad \frac{2\cos x \cdot \frac{1}{\sin x}}{\cot^2 x} = 2 \quad \Rightarrow \quad \frac{2\cot x}{\cot^2 x} = 2$$

$$\therefore \quad \cot x = 1 = \cot 45^\circ$$

$$\therefore \quad x = 45^\circ = \frac{\pi}{4}$$

27. (*c*) $\dfrac{\cos\theta}{1-\sin\theta} - \dfrac{\cos\theta}{1+\sin\theta} = 2$

$$\Rightarrow \frac{\cos\theta(1+\sin\theta) - \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 2$$

$$\Rightarrow \frac{\cos\theta + \cos\theta\sin\theta - \cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta} = 2$$

$$\Rightarrow \quad \frac{2\cos\theta\sin\theta}{\cos^2\theta} = 2$$

$$\Rightarrow \quad 2\tan\theta = 2$$

$$\Rightarrow \quad \tan\theta = 1 = \tan 45^\circ$$

$$\therefore \quad \theta = 45^\circ = \frac{\pi}{4}$$

28. (*b*) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta} \times \dfrac{1+\sin\theta}{1+\sin\theta}}$

$$= \sqrt{\frac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$= \sqrt{\left(\frac{1+\sin\theta}{\cos\theta}\right)^2} = \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta + \tan\theta$$

29. (*d*) $7\cos^2\theta + 3\sin^2\theta = 4$

Dividing by $\cos^2\theta$ at both sides, we get

$$\frac{7\cos^2\theta}{\cos^2\theta} + \frac{3\sin^2\theta}{\cos^2\theta} = \frac{4}{\cos^2\theta}$$

$$\Rightarrow \quad 7 + 3\tan^2\theta = 4\sec^2\theta$$

$$\Rightarrow \quad 7 + 3\tan^2\theta = 4(1+\tan^2\theta)$$

$$\Rightarrow \quad 7 + 3\tan^2\theta = 4 + 4\tan^2\theta$$

$$\Rightarrow \quad 3 = \tan^2\theta$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

30. (*a*) $\dfrac{\tan 27^\circ + \cot 63^\circ}{\tan 27^\circ(\sin 25^\circ + \cos 65^\circ)}$

$$= \frac{\tan 27^\circ + \cot(90^\circ - 27^\circ)}{\tan 27^\circ(\sin 25^\circ + \cos(90^\circ - 25^\circ))}$$

$$= \frac{\tan 27^\circ + \tan 27^\circ}{\tan 27^\circ[\sin 25^\circ + \sin 25^\circ]}$$

$$= \frac{2}{2\sin 25^\circ} = \operatorname{cosec} 25^\circ$$

31. (*a*) $\cos\theta \geq \frac{1}{2}$ means the value of θ lies between $0°$ and $\frac{\pi}{3}$.

$\therefore$ θ is less than or equal to $\frac{\pi}{3}$, *i.e.*, $\theta \leq \frac{\pi}{3}$

32. (*c*) Given,

$$x = a\sec\theta\cos\phi$$
$$y = b\sec\theta\cdot\sin\phi$$
$$z = c\tan\theta$$

Now, $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

On putting the values of x, y and z, we get

$$= \frac{a^2\sec^2\theta\cos^2\phi}{a^2} + \frac{b^2\sec^2\theta\sin^2\phi}{b^2} - \frac{c^2\tan^2\theta}{c^2}$$

$$= \sec^2\theta\,[\cos^2\phi + \sin^2\phi] - \tan^2\theta$$

$$= \sec^2\theta - \tan^2\theta \qquad [\because \sec^2\theta - \tan^2\theta = 1]$$

$$= 1$$

33. (*b*) $\cos^4\theta - \sin^4\theta = \frac{2}{3}$

$$\Rightarrow \quad (\cos^2\theta)^2 - (\sin^2\theta)^2 = \frac{2}{3}$$

$$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = \frac{2}{3}$$

$$\cos^2\theta - \sin^2\theta = \frac{2}{3} \Rightarrow \cos 2\theta = \frac{2}{3}$$

$$1 - 2\sin^2\theta = \frac{2}{3}$$

34. (*b*) $7\sin^2\theta + 3\cos^2\theta = 4$

$$\Rightarrow \quad 4\sin^2\theta + 3\sin^2\theta + 3\cos^2\theta = 4$$

$$\Rightarrow \quad 4\sin^2\theta + 3(\sin^2\theta + \cos^2\theta) = 4$$

$$\Rightarrow \quad 4\sin^2\theta + 3 = 4$$

$$\Rightarrow \quad 4\sin^2\theta = 1$$

$$\Rightarrow \quad \sin\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30°$$

$$\Rightarrow \quad \cos\theta = \cos 30° = \frac{\sqrt{3}}{2}$$

35. (*b*) $\tan 8\theta = \cot 2\theta$

$$\tan 8\theta = \tan(90° - 2\theta)$$

$$8\theta = 90° - 2\theta \Rightarrow \theta = 9°$$

$$\therefore \quad \tan 5\theta \Rightarrow \tan 45° = 1$$

36. (*c*) $\because \sin\theta - \cos\theta = 0$

$$\sin\theta = \cos\theta$$

Since, $\sin\theta$ and $\cos\theta$ are equal for $\theta = 45°$

So, $\sin^4\theta + \cos^4\theta = (\sin 45°)^4 + (\cos 45°)^4$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$$

37. (*c*) Given that, $\sin^2 x + \cos^2 x - 1 = 0$

$$\Rightarrow \quad \sin^2 x + \cos^2 x = 1$$

Which is an identity of trigonometric ratio, and always true for every real value of x.

$\therefore$ The equation have an infinite solution.

38. (*c*) Statement I

$$\frac{\cot 30° + 1}{\cot 30° - 1} = 2(\cos 30° + 1)$$

$$\frac{\sqrt{3}+1}{\sqrt{3}-1} = 2\left(\frac{\sqrt{3}}{2} + 1\right)$$

$$\Rightarrow \quad \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 2\left(\frac{\sqrt{3}+2}{2}\right)$$

$$\Rightarrow \quad \frac{3+1+2\sqrt{3}}{3-1} = \sqrt{3} + 2$$

$$\Rightarrow \quad \frac{4+2\sqrt{3}}{2} = \sqrt{3} + 2$$

$$\Rightarrow \quad \frac{2(2+\sqrt{3})}{2} = \sqrt{3} + 2$$

$$\Rightarrow \quad \sqrt{3} + 2 = \sqrt{3} + 2$$

$\therefore$ It is true.

Statement II

$$2\sin 45° \cos 45° - \tan 45° \cot 45° = 0$$

$$\Rightarrow \quad 2 \times \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right) - 1 \times 1 = 0$$

$$\Rightarrow \quad 2 \times \frac{1}{2} - 1 \times 1 = 0$$

$$1 - 1 = 0$$

$\therefore$ Both Statements I and II are true.

39. (*d*) $\sin A \cdot \cos A \cdot \tan A + \cos A \cdot \sin A \cdot \cot A$

$$= \sin A \cdot \cos A \cdot \frac{\sin A}{\cos A} + \cos A \cdot \sin A \cdot \frac{\cos A}{\sin A}$$

$$= \sin^2 A + \cos^2 A$$

$$= 1 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \operatorname{cosec}^2 A - \cot^2 A$$

$$[\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta]$$

40. (*c*) $2(\cos^2\theta - \sin^2\theta) = 1$

$$\Rightarrow \quad \cos^2\theta - \sin^2\theta = \frac{1}{2} \qquad \ldots\text{(i)}$$

We know that,

$$\cos^2\theta + \sin^2\theta = 1 \qquad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$$2\cos^2\theta = \frac{3}{2} \Rightarrow \cos^2\theta = \frac{3}{4}, \sin^2\theta = \frac{1}{4}$$

$$\therefore \quad \cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{\frac{3}{4}}{\frac{1}{4}} = 3$$

$$\Rightarrow \quad \cot\theta = \sqrt{3}$$

41. (*c*) Minimum value of $\cos\theta = -1$

and minimum value of $\sec\theta = -1$

$\therefore\ \cos^2\theta + \sec^2\theta = (-1)^2 + (-1)^2 = 2$

42. (*a*) $\cos x + \cos y = 2$

$\because \quad (\cos x)_{max} \le 1$

$\Rightarrow \quad \cos x = 1, \cos y = 1$

At $\quad x = y = 0$

$\therefore\ \sin x + \sin y = 0$

43. (*a*) $\sec^2\theta - \tan^2\theta = 1$ [Identity]

$$\sec^2\theta + \tan^2\theta = \frac{7}{12} \quad \text{[given]}$$

$\therefore$ Value of $\sec^4\theta - \tan^4\theta$

$$= (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$$

$$= 1 \times \frac{7}{12} = \frac{7}{12}$$

44. (*c*) $\left(\frac{1}{\cos\theta} + \frac{1}{\cot\theta}\right)\left(\frac{1}{\cos\theta} - \frac{1}{\cot\theta}\right)$

$$= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{1+\sin\theta}{\cos\theta} \cdot \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} = 1$$

45. (*b*) $A = \tan 11° \cdot \tan 29°$

$B = 2\cot 61° \cdot \cot 79°$

$= 2\cot(90° - 29°)\cot(90° - 11°)$

$= 2\tan 29° \cdot \tan 11°$

[$\because \cot(90° - \theta) = \tan\theta$]

$\therefore\ B = 2A$

46. (*c*) $\tan 2\theta = \dfrac{1}{\tan 4\theta}$

$\tan 2\theta = \cot 4\theta$

$\Rightarrow \quad \tan 2\theta = \tan(90° - 4\theta)$

$\Rightarrow \quad 2\theta = 90° - 4\theta$

$\Rightarrow \quad 6\theta = 90°$

$\Rightarrow \quad \theta = 15°$

$\therefore\ \tan 3\theta = \tan(3 \times 15°) = \tan 45° = 1$

47. (*d*) $\sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow \quad \sin\theta + \dfrac{1}{\sin\theta} = 2$

$\sin^2\theta + 1 = 2\sin\theta$

$\Rightarrow \quad \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow \quad (\sin\theta - 1)^2 = 0$

$\Rightarrow \quad \sin\theta = 1$

$\Rightarrow \quad \operatorname{cosec}\theta = 1$

Value of $\sin^5\theta + \operatorname{cosec}^5\theta$

$= (1)^5 + (1)^5$

$= 1 + 1 = 2$

48. (*b*) When $\theta = 0°$

$$\sin^2\theta + \cos^4\theta = 1$$

When $\theta = 45°$

$$\sin^2\theta + \cos^4\theta = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

When $\theta = 30°$

$$\sin^2\theta + \cos^4\theta = \frac{1}{4} + \frac{9}{16} = \frac{13}{16}$$

Average value of A

$\Rightarrow \quad \frac{3}{4} \le A \le 1$

49. (*b*) $2\sin^2\theta + 3\cos^2\theta$

$$= 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta$$

$$= 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta$$

$$= 2 + \cos^2\theta$$

$\because$ Minimum value of $\cos\theta = -1$

$\therefore$ Required minimum value

$$= 2 + (-1)^2 = 2 + 1 = 3$$

50. (*d*) $\cot 10°\cdot\cot 80°\cdot\cot 20°\cdot\cot 70°\cdot\cot 60°$

$= \cot 10°\cdot\tan 10°\cdot\cot 20°\cdot\tan 20°\cdot\cot 60°$

$[\because \tan(90° - \theta) = \cot\theta]$

$= 1\cdot 1\cdot\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

51. (*d*) $\cos\theta\cdot\operatorname{cosec} 23° = 1$

$\Rightarrow \quad \operatorname{cosec} 23° = \frac{1}{\cos\theta} = \sec\theta$

$\Rightarrow \quad \operatorname{cosec} 23° = \operatorname{cosec}(90° - \theta)$

$\Rightarrow \quad 23° = 90° - \theta$

$\Rightarrow \theta = 90° - 23° = 67°$

52. (*c*) $\tan 2\theta\cdot\tan 3\theta = 1$

$\Rightarrow \quad \tan 3\theta = \frac{1}{\tan 2\theta} = \cot 2\theta$

$\Rightarrow \quad \tan 3\theta = \tan(90° - 2\theta)$

$\therefore \quad 3\theta = 90° - 2\theta$

$\Rightarrow \quad 5\theta = 90° \Rightarrow \theta = \frac{90°}{5} = 18°$

$\therefore \quad 2\cos^2\frac{5\theta}{2} - 1 = 2\cos^2\frac{5\times 18}{2} - 1$

$$= 2\cos^2 45° - 1$$

$$= 2\times\left(\frac{1}{\sqrt{5}}\right)^2 - 1$$

$\Rightarrow \quad 2\times\frac{1}{2} - 1 = 0$

53. (*c*) Let $X = \sin\theta + \cos\theta$

On squaring both side, we get

$$X^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta$$

$$= 1 + 2\sin\theta\cdot\cos\theta$$

$\because \quad 0 < \theta < 90°$

Value of both $\sin\theta$ and $\cos\theta$ will be less than 1.

$\therefore \quad 2\sin\theta \cdot \cos\theta < 1$

$\therefore \quad X^2 < 2 \Rightarrow X < \sqrt{2}$

$X < 1.414$

54. (*b*) $\sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow \quad \sin\theta + \dfrac{1}{\sin\theta} = 2$

$\Rightarrow \quad \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow \quad (\sin\theta - 1)^2 = 0$

$\Rightarrow \quad \sin\theta - 1 = 0$

$\Rightarrow \quad \sin\theta = 1$

$\Rightarrow \quad \operatorname{cosec}\theta = 1$

$\therefore \quad \sin^{100}\theta + \operatorname{cosec}^{100}\theta$

$= (1)^{100} + (1)^{100}$

$= 1 + 1 = 2$

55. (*c*) $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$

$\Rightarrow \quad \sin\theta + \cos\theta = 3\sin\theta - 3\cos\theta$

$\Rightarrow \quad 4\cos\theta = 2\sin\theta$

$\Rightarrow \quad \tan\theta = 2$

Value of $\sin^4\theta - \cos^4\theta$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \sin^2\theta - \cos^2\theta$

$= \cos^2\theta(\tan^2\theta - 1)$

$= \dfrac{\tan^2\theta - 1}{\sec^2\theta}$

$= \dfrac{\tan^2\theta - 1}{1 + \tan^2\theta} = \dfrac{4 - 1}{1 + 4} = \dfrac{3}{5}$

56. (*b*) $\tan\theta + \cot\theta = 2$

$\Rightarrow \quad \tan\theta + \dfrac{1}{\tan\theta} = 2$

$\Rightarrow \quad \tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow \quad \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow \quad (\tan\theta - 1)^2 = 0$

$\Rightarrow \quad \tan\theta = 1$

and $\quad \cot\theta = 1$

$\therefore \tan^5\theta + \cot^{10}\theta = (1)^5 + (1)^{10} = 1 + 1 = 2$

57. (*d*) If $0° < \theta < 90°$, then all the trigonometric ratios can be obtained when any one of the six ratios is given.

$\because$ We use any of the following identity to get any trigonometric ratios

$\sin^2\theta + \cos^2\theta = 1,\ 1 + \tan^2\theta = \sec^2\theta$

and $\quad 1 + \cot^2\theta = \operatorname{cosec}\ \theta$

58. (*a*) Let $f(\theta) = \dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{2 + 2\cos\theta}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{2\,(1+\cos\theta)}{\sin\theta\,(1+\cos\theta)} = 2\operatorname{cosec}\theta$$

59. (*b*) Given that, $\sin\theta\cdot\cos\theta = \dfrac{\sqrt{3}}{4}$ ...(i)

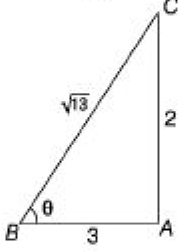
Now, we have $\sin^4\theta + \cos^4\theta =$

$$(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cdot\cos^2\theta$$

$$= (1)^2 - 2\,(\sin\theta\cdot\cos\theta)^2$$

$$= 1 - 2\left(\frac{\sqrt{3}}{4}\right)^2 = 1 - 2\cdot\frac{3}{16} = 1 - \frac{3}{8} = \frac{5}{8}$$

60. (*c*) In ΔABC, $\cot = \dfrac{3}{2}$



$\therefore AB = 3$ and $AC = 2$

By Pythagoras theorem, $BC^2 = (2)^2 + (3)^2$

$$BC = \sqrt{13}$$

Now, $\cos\theta = \dfrac{3}{\sqrt{13}}$ and $\sin\theta = \dfrac{2}{\sqrt{13}}$

$$\frac{2\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = \frac{\frac{6}{\sqrt{13}} - \frac{2}{\sqrt{13}}}{\frac{6}{\sqrt{13}} + \frac{2}{\sqrt{13}}} = \frac{4}{8} = \frac{1}{2}$$

61. (*c*) Given, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ ...(i)

On squaring both sides, we get

$$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$$

$$\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$$

$$2\sin\theta\cos\theta = \cos^2\theta - \sin^2\theta$$

$$2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$$

$$\cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{(\cos\theta + \sin\theta)} = \frac{2\sin\theta\cos\theta}{\sqrt{2}\cos\theta}$$

[from Eq. (i)]

$$= \sqrt{2}\sin\theta$$

62. (*c*) Given that, $3\sin x + 5\cos x = 5$

On squaring both side, we get

$9\sin^2 x + 25\cos^2 x + 30\sin x\cos x = 25$

$$\Rightarrow \quad 9(1-\cos^2 x) + 25(1-\sin^2 x) + 30\sin x\cos x = 25$$

$$\Rightarrow \quad 9 - 9\cos^2 x + 25 - 25\sin^2 x + 30\sin x \cdot \cos x = 25$$

$$\Rightarrow \quad 9 + 25 - \{9\cos^2 x + 25\sin^2 x - 30\sin x\cos x\} = 25$$

$$\Rightarrow \quad 9 = (3\cos x - 5\sin x)^2$$

$$\Rightarrow \quad 3\cos x - 5\sin x = 3$$

63. (*b*) $\sec x = \operatorname{cosec} y \Rightarrow \dfrac{1}{\cos x} = \dfrac{1}{\sin y}$

$$\therefore \quad \cos x = \sin y$$

$$\therefore \quad \sin\left(\frac{\pi}{2} - x\right) = \sin y$$

$$\therefore \quad \frac{\pi}{2} - x = y$$

$$(x + y) = \frac{\pi}{2}$$

Now, value of $\sin(x + y) = \sin\dfrac{\pi}{2} = 1$

64. (*d*) Given, $\sin 17° = \dfrac{x}{y}$

Now, $\sec 17° - \sin 73°$

$$= \sec 17° - \sin(90° - 17°)$$

$$= \sec 17° - \cos 17°$$

$$= \frac{1}{\cos 17°} - \cos 17°$$

$$= \frac{1 - \cos^2 17°}{\cos 17°} = \frac{\sin^2 17}{\cos 17°}$$

$$= \frac{\dfrac{x^2}{y^2}}{\sqrt{1 - \dfrac{x^2}{y^2}}} \quad [\because \cos\theta = \sqrt{1 - \sin^2\theta}]$$

$$= \frac{\dfrac{x^2}{y^2}}{\sqrt{1 - \dfrac{x^2}{y^2}}} = \frac{\dfrac{x^2}{y^2}}{\dfrac{\sqrt{y^2 - x^2}}{y}} = \frac{x^2}{y\sqrt{y^2 - x^2}}$$

65. (*d*) $x^2 + \dfrac{1}{x^2} = 2\sin\left(\dfrac{\pi x}{2}\right)$

$$[\because a^2 + b^2 = (a - b)^2 + 2ab]$$

$$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 + 2 = 2\sin\left(\frac{\pi x}{2}\right)$$

$$\therefore \quad x - \frac{1}{x} = 0$$

$$\left\{\sin\frac{\pi x}{2} = 1 \text{ for all integer values of } x\right\}$$

66. (*b*) **By Hit and Trial**

The equation $\cos^2\theta = \frac{(x+y)^2}{4xy}$ is possible,

only when $x = y$

On putting $x = y$ in the equation,

$$\cos^2\theta = \frac{(2x)^2}{4x^2}$$

$$= \frac{4x^2}{4x^2} = 1$$

$$\Rightarrow \qquad \theta = 0°$$

67. (*b*) $\sin^2 1° + \sin^2 3° + \cdots + \sin^2 45°$

$$+ \cdots + \sin^2(90 - 3) + \sin^2(90° - 1°)$$

$$= \sin^2 1° + \sin^2 3° + \cdots + \sin^2 45° + \cdots + \cos^2 3° + \cos^2 1°$$

$$= (1 \times 22) + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 22 + \frac{1}{2} = 22\frac{1}{2}$$

68. (*d*) $A + B = 90°$

[$\because$ they are complementary]

$\Rightarrow \qquad B = 90° - A$

$\therefore \quad \sec^2 A + \sec^2 B - \sec^2 A \cdot \sec^2 B$

$$= \sec^2 A + \sec^2(90° - A) - \sec^2 A \sec^2(90° - A)$$

$$= \sec^2 A + \operatorname{cosec}^2 A - \sec^2 A \cdot \operatorname{cosec}^2 A$$

$$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} - \frac{1}{\sin^2 A \cdot \cos^2 A}$$

$$= \frac{\sin^2 A + \cos^2 A - 1}{\sin^2 A \cdot \cos^2 A}$$

$$= \frac{1-1}{\sin^2 A \cdot \cos^2 A} = 0$$

69. (*d*) $\sin^2 5° + \sin^2 10° + \cdots + \sin^2 45° + \cdots + \sin^2 85° + \sin^2 90°$

$$= \sin^2 5° + \sin^2 10° + \cdots + \sin^2 45° + \cdots + \sin^2(90° - 5) + \sin^2 90°$$

$$= 8 \times [\sin^2 5° + \cos^2 5° \cdots] + \sin^2 45° + \sin^2 90°$$

$$= 8 + \left(\frac{1}{\sqrt{2}}\right)^2 + 1 = 8 + 1 + \frac{1}{2} = 9\frac{1}{2}$$

70. (*b*) $\sin(3x - 20°) = \cos(3y + 20°)$

$\Rightarrow \sin(3x - 20°) = \sin(90° - (3y + 20°))$

$\sin(3x - 20°) = \sin(70° - 3y)$

$\therefore \qquad 3x - 20° = 70° - 3y$

$\Rightarrow \qquad 3x + 3y = 90°$

$\Rightarrow \qquad 3(x + y) = 90°$

$\Rightarrow \qquad x + y = 30°$

71. (*c*) Given, $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \dfrac{5}{4}$

$$\Rightarrow \quad \frac{\cos\theta\left(\dfrac{\sin\theta}{\cos\theta} + 1\right)}{\cos\theta\left(\dfrac{\sin\theta}{\cos\theta} - 1\right)} = \frac{5}{4}$$

$$\Rightarrow \quad \frac{\tan\theta + 1}{\tan\theta - 1} = \frac{5}{4}$$

$$\Rightarrow \quad 4\tan\theta + 4 = 5\tan\theta - 5$$

$$\tan\theta = 9$$

Putting volume of $\tan\theta$,

$$\frac{\tan^2\theta + 1}{\tan^2\theta - 1} = \frac{(9)^2 + 1}{(9)^2 - 1}$$

$$= \frac{81 + 1}{81 - 1} = \frac{82}{80} = \frac{41}{40}$$

72. (*d*) $\dfrac{\sin 39°}{\cos 51°} + 2\tan 11° \cdot \tan 79° \cdot$

$$\tan 31° \cdot \tan 59° \cdot \tan 45° - 3(\sin^2 21 + \sin^2 69°)$$

$$= \frac{\sin 39°}{\cos(90° - 39°)} + 2\tan 11° \cdot \tan(90° - 11°)$$

$$\tan 31° \cdot \tan(90° - 31°) \cdot 1 - 3(\sin^2 21° + \sin^2(90° - 21°)$$

$[\because \tan(90° - \theta) = \cot\theta, \cos(90° - \theta) = \sin\theta]$

$$= \frac{\sin 39°}{\sin 39°} + 2\tan 11° \cdot \cot 11° \cdot \tan 31° \cdot$$

$$\cot 31° - 3(\sin^2 21° + \cos^2 21°)$$

$$= 1 + 2 - 3 = 0$$

$[\because \tan\theta \cdot \cot\theta = 1 \text{ and } \sin^2\theta + \cos^2\theta = 1]$

73. (*c*) $\dfrac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$

$$\Rightarrow \quad \cos^2\theta = 3\cot^2\theta - 3\cos^2\theta$$

$$\Rightarrow \quad 4\cos^2\theta = 3\cot^2\theta$$

$$\Rightarrow \quad 4\cos^2\theta - \frac{3\cos^2\theta}{\sin^2\theta} = 0$$

$$\Rightarrow \quad \cos^2\theta\left(4 - \frac{3}{\sin^2\theta}\right) = 0$$

$$\therefore \quad 4 - \frac{3}{\sin^2\theta} = 0$$

$$\Rightarrow \quad 4\sin^2\theta - 3 = 0$$

$$\sin^2\theta = \frac{3}{4}$$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2} = \sin 60°$$

$$\Rightarrow \quad \theta = 60°$$

74. (*a*) $\cot 18^\circ \left(\cot 72^\circ \cdot \cos^2 22^\circ + \dfrac{1}{\tan 72^\circ \cdot \sec^2 68^\circ}\right)$

$$= \cot 18^\circ \cdot \cot 72^\circ \cdot \cos^2 22^\circ + \frac{\cot 18^\circ}{\tan 72^\circ \cdot \sec^2 68^\circ}$$

$[\cot (90^\circ - \theta) = \tan \theta]$

$$= \cot 18^\circ \cdot \tan 18^\circ \cdot \cos^2 22^\circ + \frac{\cot 18^\circ}{\cot 18^\circ} \cdot \cos^2 68^\circ$$

$$= \cos^2 22^\circ + \cos^2 68^\circ$$

$$= \cos^2 22^\circ + \sin^2 22^\circ = 1$$

$[\sin (90^\circ - \theta) = \cos \theta]$

75. (*b*) $7 \sin^2 \theta + 3 \cos^2 \theta = 4$

$$\Rightarrow \quad 7\frac{\sin^2 \theta}{\cos^2 \theta} + 3 = \frac{4}{\cos^2 \theta}$$

$$\Rightarrow \quad 7 \tan^2 \theta + 3 = 4 \sec^2 \theta$$

$$\Rightarrow \quad 7 \tan^2 \theta + 3 = 4 (1 + \tan^2 \theta)$$

$$\Rightarrow \quad 7 \tan^2 \theta - 4 - 4 \tan^2 \theta + 3 = 0$$

$$\Rightarrow \quad 3 \tan^2 \theta = 1$$

$$\Rightarrow \quad \tan^2 \theta = \frac{1}{3}$$

$$\Rightarrow \quad \tan \theta = \frac{1}{\sqrt{3}}$$

Exercise © *Higher Skill Level Questions*

1. (*d*) $(\sin x + \sin y) = a$ and $(\cos x + \cos y) = b$

Squaring both the equations, we get

$(\sin x + \sin y)^2 = a^2$

$\sin^2 x + \sin^2 y + 2\sin x \sin y = a^2$ ...(i)

and $(\cos x + \cos y)^2 = b^2$

$\cos^2 x + \cos^2 y + 2\cos x \cos y = b^2$ ...(ii)

Adding Eqs. (i) and (ii), we get

$$(\sin^2 x + \sin^2 y + 2\sin x \sin y) + (\cos^2 x + \cos^2 y + 2\cos x \cos y) = a^2 + b^2$$

$$\Rightarrow \sin^2 x + \cos^2 x + \sin^2 y + \cos^2 y + 2(\sin x \sin y + \cos x \cos y) = a^2 + b^2$$

$$\Rightarrow 1 + 1 + 2(\sin x \sin y + \cos x \cos y) = a^2 + b^2$$

$$\therefore \sin x \sin y + \cos x \cos y = \frac{a^2 + b^2 - 2}{2}$$

2. (*c*) (i) RHS $= \cos^2\theta(1 + \tan\theta)(1 - \tan\theta)$

$$= \cos^2\theta(1 - \tan^2\theta)$$

$$= \cos^2\theta\left(1 - \frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$= \frac{\cos^2\theta - \sin^2\theta}{1} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$= \text{LHS}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

(ii) RHS $= (\tan\theta + \sec\theta)^2$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)^2 = \frac{(1 + \sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1 + \sin\theta)^2}{1 - \sin^2\theta} = \frac{(1 + \sin\theta)(1 + \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}$$

$$= \frac{1 + \sin\theta}{1 - \sin\theta} = \text{LHS}$$

3. (*c*) $\tan 1° \tan 2° \tan 3° \ldots \tan 89°$

or

$\tan 1° \tan 2° \tan 3° \ldots\ldots \tan 88° \tan 89°$

Now, $\tan 1° \tan 89° \tan 2° \tan 88° \ldots\ldots$

$$= \tan 1° [\tan (90 - 1°)] \tan 2° \tan (90 - 2°) \ldots\ldots$$

$$= \tan 1° \cot 1° \tan 2° \cot 2° \ldots\ldots$$

$$= \tan 1° \times \frac{1}{\tan 1°} \tan 2° \times \frac{1}{\tan 2°} \times \ldots\ldots = 1$$

4. (*d*) $\sec\theta + \tan\theta = p$ ...(i)

$$\Rightarrow \quad \frac{1}{\sec\theta + \tan\theta} = \frac{1}{P}$$

$$\Rightarrow \quad \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta} = \frac{1}{P}$$

$$\Rightarrow \quad \frac{\sec\theta - \tan\theta}{1 + \tan^2\theta - \tan^2\theta} = \frac{1}{P}$$

$$\sec\theta - \tan\theta = \frac{1}{p} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$2 \sec\theta = p + \frac{1}{p}$$

$$\Rightarrow \quad \sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right) = \frac{p^2 + 1}{2p}$$

$$\Rightarrow \quad \cos\theta = \frac{2p}{p^2 + 1}$$

Now, in ΔABC,

$$(BC)^2 = (AC)^2 - (AB)^2$$

$$= (P^2 + 1)^2 - (2P)^2$$

$$= P^4 + 1 + 2P^2 - 4P^2$$

$$= P^4 + 1 - 2P^2$$

$$\Rightarrow \quad (BC)^2 = (P^2 - 1)^2$$

$$\Rightarrow \quad BC = (P^2 - 1)$$

$$\therefore \quad \sin\theta = \frac{p^2 - 1}{p^2 + 1}$$

5. (c) $\sin\theta + \cos\theta = x$

On squaring both sides, we get

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = x^2$$

$$1 + 2\sin\theta\cos\theta = x^2$$

$$\therefore \quad \sin\theta\cos\theta = \frac{x^2 - 1}{2} \qquad \ldots\text{(i)}$$

$$\cos^6\theta + \sin^6\theta = (\cos^2\theta)^3 + (\sin^2\theta)^3$$

$$= (\cos^2\theta + \sin^2\theta)$$

$$(\cos^4\theta - \cos^2\theta\sin^2\theta + \sin^4\theta)$$

$$= \{(\cos^2\theta)^2 + (\sin^2\theta)^2 - \cos^2\theta\sin^2\theta\}$$

$$= \{(\cos^2\theta + \sin^2\theta)^2 - 2\cos^2\theta\sin^2\theta - \cos^2\theta\sin^2\theta\}$$

$$= 1 - 3\cos^2\theta\sin^2\theta = 1 - 3\left(\frac{x^2 - 1}{2}\right)^2$$

[from Eq. (i)]

$$= 1 - \frac{3(x^4 - 2x^2 + 1)}{4} = \frac{4 - 3x^4 + 6x^2 - 3}{4}$$

$$= \frac{1 - 3x^4 + 6x^2}{4} = \frac{1}{4}(1 + 6x^2 - 3x^4)$$

6. (c) Given,

$$1 + \tan A + \tan B + \tan A\tan B = 2$$

$$\tan A + \tan B = 1 - \tan A\tan B$$

$$\text{or} \quad \frac{\tan A + \tan B}{1 - \tan A\tan B} = 1$$

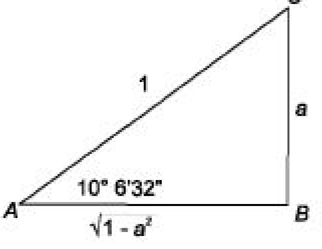
$$= \tan 45^\circ$$

$$\tan(A + B) = \tan 45^\circ$$

$$\left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}\right]$$

$$\therefore \quad A + B = 45^\circ = \frac{\pi}{4}$$

7. (a) $\sin(10^\circ\, 6'\, 32'') = \dfrac{a}{1}$



$$\sin(10°6'32'') = \frac{a}{1}$$
$$\sin(90° - 79°\,53'\,28'') = a$$
$$\cos 79°\,53'\,28'' = a$$
$\therefore$ $\cos(79°\,53'\,28'') + \tan(10°\,6'\,32'')$
$$= a + \frac{a}{\sqrt{1-a^2}}$$
$$= \frac{a(1+\sqrt{1-a^2})}{\sqrt{1-a^2}}$$

8. (c) $x + y = z$

Now, $\cos^2 x + \cos^2 y + \cos^2 z = ?$

$= 1 + (\cos^2 x - \sin^2 y) + \cos^2 z$

$= 1 + \cos(x+y) \cdot \cos(x-y) + \cos^2 z$

$= 1 + \cos z \cos(x-y) + \cos^2 z$

$= 1 + \cos z\,[\cos(x-y) + \cos(x+y)]$

$= 1 + \cos z$
$$\left[2\cos\frac{(x-y+x+y)}{2} \cdot \cos\frac{(x-y-x-y)}{2}\right]$$

$= 1 + 2\cos z \cdot \cos x \cdot \cos y$

$= 1 + 2\cos x \cdot \cos y \cdot \cos z$

9. (b) $\cos\theta + \sec\theta = 2$

$\Rightarrow$ $$\cos\theta + \frac{1}{\cos\theta} = 2$$

On squaring both sides, we get
$$\cos^2\theta + \frac{1}{\cos^2\theta} + 2 = 4$$

$\Rightarrow$ $$\cos^2\theta + \frac{1}{\cos^2\theta} = 2$$

On cubing both sides, we get
$$\cos^6\theta + \frac{1}{\cos^6\theta} + 3\cos^2\theta + \frac{3}{\cos^2\theta} = 8$$

$\Rightarrow$ $$\cos^6\theta + \frac{1}{\cos^6\theta} + 3(2) = 8$$
$$\left[\because \cos^2\theta + \frac{1}{\cos^2\theta} = 2\right]$$

$\Rightarrow$ $$\cos^6\theta + \frac{1}{\cos^6\theta} = 2$$

$\Rightarrow$ $$\cos^6\theta + \sec^6\theta = 2$$

10. (c) Given, $\sin\theta - \cos\theta = 0$

On squaring both sides, we get
$$(\sin\theta - \cos\theta)^2 = 0$$
$$\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = 0$$
$$[\because \sin^2\theta + \cos^2\theta = 1]$$
$$2\sin\theta\cos\theta = 1 \quad \ldots(i)$$

$\therefore$ $$\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)$$

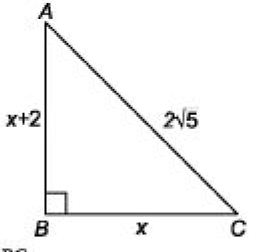
$= \sin\theta + \cos\theta = \sqrt{(\sin\theta + \cos\theta)^2}$

$$= \sqrt{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}$$
$$= \sqrt{1+1} = \sqrt{2} \quad \text{[from Eq. (i)]}$$

11. (a) $f(\theta) = \sin^8\theta + \cos^{14}\theta$
$$= \sin^8\theta + (1-\sin^2\theta)^7$$
[$\because$ maximum value of $\sin^2\theta = 1$]
$$= 1 + 0 = 1$$

12. (d) Given,
$$l\cos^2\theta + m\sin^2\theta = \frac{\cos^2\theta\,(\text{cosec}^2\theta + 1)}{\text{cosec}^2\theta - 1}$$
$$= \frac{\cos^2\theta\,(1+\sin^2\theta)}{1-\sin^2\theta}\cdot\frac{\sin^2\theta}{\sin^2\theta}$$
$$= \frac{\cos^2\theta\,(1+\sin^2\theta)}{\cos^2\theta}$$
$$= 1 + \sin^2\theta = \cos^2\theta + \sin^2\theta + \sin^2\theta$$
$$= \cos^2\theta + 2\sin^2\theta$$
$$\Rightarrow (l-1)\cos^2\theta = (2-m)\sin^2\theta$$
$$\Rightarrow \tan^2\theta = \frac{l-1}{2-m}$$
$$\therefore \tan\theta = \sqrt{\frac{l-1}{2-m}}$$

13. (d) Let $BC = x$, then $AB = x + 2$



In ΔABC,
$$(x+2)^2 + x^2 = (2\sqrt{5})^2$$
$$\Rightarrow x^2 + 4 + 4x + x^2 = 20$$
$$\Rightarrow 2x^2 + 4x - 16 = 0$$
$$\Rightarrow x^2 + 2x - 8 = 0$$
$$\Rightarrow (x-2)(x+4) = 0 \Rightarrow x = 2$$
$$AB = 4,\ BC = 2$$
$$\therefore \cos^2 A - \cos^2 C$$
$$= \left(\frac{4}{2\sqrt{5}}\right)^2 - \left(\frac{2}{2\sqrt{5}}\right)^2 = \frac{3}{5}$$

14. (a) $\cos x + \cos^2 x = 1$
$$\Rightarrow \cos x = 1 - \cos^2 x$$
$$\Rightarrow \cos x = \sin^2 x \quad \text{...(i)}$$
Now, again $\cos x + \cos^2 x = 1$

On cubing both side, we get

$$(\cos x + \cos^2 x)^3 = (1)^3$$

$$\cos^3 x + (\cos^2 x)^3 + 3\cos^2 x \cdot \cos^2 x + 3\cos x \cdot \cos^4 x = 1$$

$$\cos^3 x + \cos^6 x + 3\cos^4 x + 3\cos^5 x = 1$$

Put $\cos x = \sin^2 x$ [from Eq. (i)]

$$\sin^6 x + \sin^{12} x + 3\sin^8 x + 3\sin^{10} x = 1$$

$$\therefore \sin^{12} x + 3\sin^{10} x + 3\sin^8 x + \sin^6 x - 1 = 0$$

15. (*d*) $a\sin\theta + b\cos\theta = c$

On squaring both sides, we get

$$a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = c^2$$

$$\Rightarrow a^2(1-\cos^2\theta) + b^2(1-\sin^2\theta) + 2ab\sin\theta\cos\theta = c^2$$

$$\Rightarrow a^2 - a^2\cos^2\theta + b^2 - b^2\sin^2\theta + 2ab\sin\theta\cos\theta = c^2$$

On rearranging, we get

$$a^2 + b^2 - c^2 = a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta$$

$$\Rightarrow a^2 + b^2 - c^2 = (a\cos\theta - b\sin\theta)^2$$

$$\Rightarrow a\cos\theta - b\sin\theta = \pm\sqrt{a^2 + b^2 - c^2}$$

16. (*b*) $\tan\alpha = n\tan\beta \Rightarrow \dfrac{\sin\alpha}{\cos\alpha} = n\dfrac{\sin\beta}{\cos\beta}$

$$\frac{m\sin\beta}{\cos\alpha} = n\frac{\sin\beta}{\cos\beta}$$

$$\Rightarrow \quad \cos\alpha = \frac{m}{n}\cos\beta$$

On squaring both sides, we get

$$\cos^2\alpha = \frac{m^2}{n^2}\cos^2\beta \quad \ldots\text{(i)}$$

Also, $\sin\alpha = m\sin\beta$

On squaring both sides, we get

$$\sin^2\alpha = m^2\sin^2\beta$$

$$\Rightarrow \quad 1 - \cos^2\alpha = m^2(1-\cos^2\beta)$$

$$\Rightarrow \quad 1 - \cos^2\alpha = m^2 - m^2\cos^2\beta$$

$$\Rightarrow \quad -\frac{(1-\cos^2\alpha - m^2)}{m^2} = \cos^2\beta$$

$$\frac{(\cos^2\alpha + m^2 - 1)}{m^2} = \cos^2\beta \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\cos^2\alpha = \frac{m^2}{n^2} \times \frac{(\cos^2\alpha + m^2 - 1)}{m^2}$$

$$\Rightarrow \quad n^2\cos^2\alpha = \cos^2\alpha + m^2 - 1$$

$$\Rightarrow \quad (n^2-1)\cos^2\alpha = m^2 - 1$$

$$\therefore \quad \cos^2\alpha = \frac{m^2-1}{n^2-1}$$

17. (*d*) Given, $\sin\theta \cdot \cos\theta = \frac{1}{2}$

$\sin^6\theta + \cos^6\theta = (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta$

$= (1 - 3\sin^2\theta\cos^2\theta) \quad \left[\because \sin\theta \cdot \cos\theta = \frac{1}{2}\right]$

$= 1 - 3 \times \frac{1}{4} = 1 - \frac{3}{4} = \frac{1}{4}$

18. (*b*) $\text{cosec}(75° + \theta) - \sec(15° - \theta) - \tan(55° + \theta) + \cot(35° - \theta)$

$\Rightarrow \text{cosec}(75° + \theta) - \text{cosec}[90° - (75° - \theta)] - \tan(55° + \theta) + \cot[90° - (35° - \theta)]$

$\Rightarrow \text{cosec}(75° + \theta) - \text{cosec}(75° + \theta) - \tan(55° + \theta) + \tan(55° + \theta) = 0$

19. (*a*) $\cos x + \sec x = 2$ ...(i)

On squaring both sides,

$$\cos^2 x + \sec^2 x + 2 = 4$$

$$\cos^2 x + \sec^2 x = 2$$

On cubing Eq. (i)

$\cos^3 x + \sec^3 x + 3(\cos x + \sec x) = 8$

$\Rightarrow \quad \cos^3 x + \sec^3 x + (3 \times 2) = 8$

$\Rightarrow \quad \cos^3 x + \sec^3 x = 2$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$\cos^n x + \sec^n x = 2$$

20. (*c*) Since, α and β are complementary angle.

$$\alpha = 90° - \beta$$

Now, $\sqrt{\cos\alpha\ \text{cosec}\,\beta - \cos\alpha\sin\beta}$

$= \sqrt{\frac{\cos\alpha}{\sin\beta} - \cos\alpha\sin\beta}$

$= \sqrt{\frac{\cos\alpha}{\cos(90° - \beta)} - \cos\alpha\cos(90° - \beta)}$

$= \sqrt{\frac{\cos\alpha}{\cos\alpha} - \cos\alpha \cdot \cos\alpha}$

$= \sqrt{1 - \cos^2\alpha} \Rightarrow \sqrt{\sin^2\alpha} = \sin\alpha$

21. (*c*) $\sin\theta + 2\cos\theta = -1$

On squaring both sides, we get

$$(\sin\theta + 2\cos\theta)^2 = (-1)^2$$

$\Rightarrow \quad \sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = 1$

$\Rightarrow \quad (1 - \cos^2\theta) + 4(1 - \sin^2\theta) + 4\sin\theta\cos\theta = 1$

$\Rightarrow -(\cos^2\theta + 4\sin^2\theta) + 4\sin\theta\cos\theta = 1 - 5$

$\Rightarrow \quad \cos^2\theta + 4\sin^2\theta - 4\sin\theta\cos\theta = 4$

$\Rightarrow \quad (2\sin\theta - \cos\theta)^2 = 4$

$\Rightarrow \quad 2\sin\theta - \cos\theta = 2$

22. (*a*) $\because \sin(A+B) = 1$

$\Rightarrow \quad (A+B) = \sin^{-1} 1$

$\Rightarrow \quad (A+B) = 90°$

$\therefore \quad B = 90° - A$ or $A = 90° - B$

Now, $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$= \cos(90° - B)\cos B + \sin(90° - B)\sin B$

$= \sin B\cos B + \cos B\sin B$

$= 2\sin B\cos B = \sin 2B$

23. (*a*) $\sin\theta + \cos\theta = \sqrt{3}$

On squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$$

$\Rightarrow \quad \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow \quad 1 + 2\sin\theta\cos\theta = 3$

$\Rightarrow \quad \sin\theta\cos\theta = \dfrac{3-1}{2} = \dfrac{2}{2} = 1 \quad \ldots(i)$

Now, $\tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

From Eq. (i),

$$\tan\theta + \cot\theta = \frac{1}{1} = 1$$

24. (*b*) $\tan\theta + \sec\theta = m \quad \ldots(i)$

$\Rightarrow \quad \sec\theta = m - \tan\theta$

On squaring both sides, we get

$$(\sec\theta)^2 = (m - \tan\theta)^2$$

$\Rightarrow \quad \sec^2\theta = m^2 + \tan^2\theta - 2m\tan\theta$

$\Rightarrow \quad \sec^2\theta - \tan^2\theta = m^2 - 2m\tan\theta$

$\Rightarrow \quad 1 = m^2 - 2m\tan\theta$

$[\because \sec^2\theta - \tan^2\theta = 1]$

$\Rightarrow \quad \tan\theta = \dfrac{m^2-1}{2m}$

On putting the value of $\tan\theta$ in initial equation, we get

$$\frac{m^2-1}{2m} + \sec\theta = m$$

$\Rightarrow \quad \sec\theta = m - \left(\dfrac{m^2-1}{2m}\right)$

$\therefore \quad \sec\theta = \dfrac{2m^2 - m^2 + 1}{2m} = \dfrac{m^2+1}{2m}$

25. (*c*) $\because 5\sin\theta + 12\cos\theta = 13$

On squaring both sides, we get

$$25\sin^2\theta + 144\cos^2\theta + 120\sin\theta\cos\theta = 169$$

$$\Rightarrow \quad 25(1-\cos^2\theta) + 144(1-\sin^2\theta) + 120\sin\theta\cos\theta = 169$$

$$\Rightarrow \quad 25 - 25\cos^2\theta + 144 - 144\sin^2\theta + 120\sin\theta\cos\theta = 169$$

$$\Rightarrow \quad 25\cos^2\theta + 144\sin^2\theta - 120\sin\theta\cos\theta = 169 - 169$$

$$\Rightarrow \quad (5\cos\theta - 12\sin\theta)^2 = 0$$

$$\Rightarrow \quad 5\cos\theta - 12\sin\theta = 0$$

26. (*a*) Only Statement I is correct as $\tan\theta$ increases faster than $\sin\theta$ as θ increases while Statement II is wrong as the value of $\sin\theta + \cos\theta$ is not always greater than 1. It may also be equal to 1.

27. (*a*) $\dfrac{(\sin\theta + \cos\theta)(\tan\theta + \cot\theta)}{\sec\theta + \operatorname{cosec}\theta}$

$$= \frac{(\sin\theta + \cos\theta)\left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)}{\dfrac{1}{\cos\theta} + \dfrac{1}{\sin\theta}}$$

$$= \frac{(\sin\theta + \cos\theta)\left(\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)}{\dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}}$$

$$= \frac{(\sin\theta + \cos\theta)\left(\dfrac{1}{\sin\theta\cos\theta}\right)}{\dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}}$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{\dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}}{\dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}} = 1$$

28. (*a*) $\dfrac{(1 + \sec\theta - \tan\theta)\cos\theta}{(1 + \sec\theta + \tan\theta)(1 - \sin\theta)}$

$$= \frac{\left(1 + \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\right)\cos\theta}{\left(1 + \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta)}$$

$$= \frac{\left(\dfrac{\cos\theta + 1 - \sin\theta}{\cos\theta}\right)\cos\theta}{\dfrac{(\cos\theta + 1 + \sin\theta)(1 - \sin\theta)}{\cos\theta}}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\dfrac{\cos\theta + 1 + \sin\theta - \sin\theta\cos\theta - \sin\theta - \sin^2\theta}{\cos\theta}}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\dfrac{\cos\theta + 1 - \sin^2\theta - \sin\theta\cos\theta}{\cos\theta}}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\dfrac{\cos\theta + \cos^2\theta - \sin\theta\cos\theta}{\cos\theta}}$$

$[\because 1 - \sin^2\theta = \cos^2\theta]$

$$= \frac{\cos\theta + 1 - \sin\theta}{\dfrac{\cos\theta(\cos\theta + 1 - \sin\theta)}{\cos\theta}} = 1$$

29. (*b*) Given,

$$\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} = \frac{1}{x^2 + y^2}$$

and $x\sin\theta - y\cos\theta = \sqrt{x^2 + y^2}$

On squaring both sides, we get

$$x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta = x^2 + y^2$$

$$\Rightarrow \quad x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta - x^2 - y^2 = 0$$

$$\Rightarrow \quad x^2(\sin^2\theta - 1) + y^2(\cos^2\theta - 1) - 2xy\sin\theta\cos\theta = 0$$

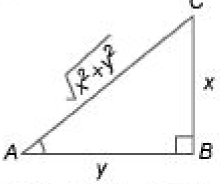
$$\Rightarrow \quad x^2\cos^2\theta + y^2\sin^2\theta + 2xy\sin\theta\cos\theta = 0$$

$$\Rightarrow \quad (x\cos\theta + y\sin\theta)^2 = 0$$

$$x\cos\theta + y\sin\theta = 0$$

$$\Rightarrow \quad x\cos\theta = -y\sin\theta \Rightarrow \tan\theta = -\frac{x}{y}$$

In ΔABC



$$(AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow \quad (AC)^2 = (-y)^2 + (x)^2$$

$$\Rightarrow \quad AC^2 = y^2 + x^2$$

$$\text{or} \quad AC = \sqrt{x^2 + y^2}$$

$$\Rightarrow \quad \sin\theta = \frac{x}{\sqrt{x^2 + y^2}} \text{ or } \frac{-x}{\sqrt{x^2 + y^2}}$$

$$\text{and} \quad \cos\theta = -\frac{y}{\sqrt{x^2 + y^2}} \text{ or } \frac{y}{\sqrt{x^2 + y^2}}$$

Now, $\dfrac{\cos^2\theta}{a^2} + \dfrac{\sin^2\theta}{b^2} = \dfrac{1}{x^2 + y^2}$

$$\Rightarrow \frac{y^2}{(x^2+y^2)\cdot a^2}+\frac{x^2}{(x^2+y^2)\cdot b^2}=\frac{1}{x^2+y^2}$$

$$\Rightarrow \quad \frac{y^2}{a^2}+\frac{x^2}{b^2}=1 \quad \Rightarrow \quad \frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

30. (*a*) $2 \operatorname{cosec}^2 23° \cot^2 67° - \sin^2 23°$

$- \sin^2 67^2 - \cot^2 67°$

$= \cot^2 67° (2 \operatorname{cosec}^2 23° - 1)$

$- \sin^2 23° - \cos^2 23°$

$= \cot^2 67° (\operatorname{cosec}^2 23° + \operatorname{cosec}^2 23° - 1)$

$- (\sin^2 23° + \cos^2 23°)$

$[\because \cos\theta = \sin(90° - \theta)]$

$= \cot^2 67° (\operatorname{cosec}^2 23° + \cot^2 23°) - 1$

$= \cot^2 67° \operatorname{cosec}^2 23° + \cot^2 67° \cot^2 23° - 1$

$= \cot^2 67° \operatorname{cos\,ec}^2 23° + \cot^2 67° \tan^2 67° - 1$

$\tan^2 23° \operatorname{cosec}^2 23° + 1 - 1$

$$= \frac{\sin^2 23°}{\cos^2 23°} \times \frac{1}{\sin^2 23°} = \frac{1}{\cos^2 23°} = \sec^2 23°$$

31. (*a*) $\cot\theta \cdot \tan(90° - \theta) - \sec(90° - \theta) \cdot$
$\operatorname{cosec}\theta + (\sin^2 25° + \sin^2 65°) + \sqrt{3}$
$(\tan 5° \cdot \tan 15° \cdot \tan 30° \cdot \tan 75° \cdot \tan 85°)$

$= \cot\theta \cdot \cot\theta - \operatorname{cosec}\theta \cdot \operatorname{cosec}\theta$
$+ (\sin^2 25° + \cos^2 25°) + \sqrt{3}\,(\tan 5° \cdot$
$\cot 5° \cdot \tan 15° \cdot \cot 15° \cdot \tan 30°)$

$$\begin{bmatrix} \because \sec(90° - \theta) = \operatorname{cosec}\theta \\ \sin(90° - \theta) = \cos\theta \\ \tan(90° - \theta) = \cot\theta \end{bmatrix}$$

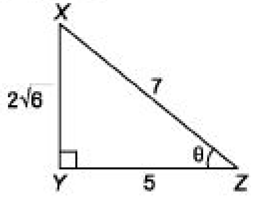
$$= (\cot^2\theta - \operatorname{cosec}^2\theta) + (\sin^2 25° + \cos^2 25°) + \sqrt{3} \times \frac{1}{\sqrt{3}}$$

$[\because \sin^2\theta + \cos^2\theta = 1$

and $\cot^2\theta - \operatorname{cosec}^2\theta = -1]$

$= -1 + 1 + 1 = 1$

32. (*b* $XZ - YZ = 2$ ...(i)



By pythagoras theorem,

$$XY^2 + YZ^2 = XZ^2$$

$\Rightarrow \quad (2\sqrt{6})^2 = XZ^2 - YZ^2$

$\Rightarrow \quad 24 = (XZ - YZ)(XZ - YZ)$

$\Rightarrow \quad \frac{24}{2} = (XZ + YZ)$

$\Rightarrow \quad XZ + YZ = 12 \quad \ldots(ii)$

Adding Eqs. (i) and (ii), we get

$$2XZ = 14 \Rightarrow XZ = 7$$

$\therefore \quad YZ = 7 - 2 = 5$

$\therefore \quad \sec X = \frac{7}{2\sqrt{6}}$ and $\tan x = \frac{5}{2\sqrt{6}}$

$\therefore \sec X + \tan X$

$$= \frac{7}{2\sqrt{6}} + \frac{5}{2\sqrt{6}} = \frac{12}{2\sqrt{6}} = \sqrt{6}$$

33. (*d*) $(\sec x \cdot \sec y + \tan x \cdot \tan y)^2 - (\sec x \cdot \tan y + \tan x \cdot \sec y)^2$

$= (\sec^2 x \cdot \sec^2 y + \tan^2 x \cdot \tan^2 y + 2 \sec x \cdot \sec y \cdot \tan x \cdot \tan y) - (\sec^2 x \cdot \tan^2 + \tan^2 x \cdot \sec^2 y + 2 \sec x \cdot \sec y \cdot \tan x \cdot \tan y)$

$= \sec^2 x \cdot \sec^2 y + \tan^2 x \cdot \tan^2 y - \sec^2 x \cdot \tan^2 y - \tan^2 x \cdot \sec^2 y$

$= \sec^2 x \cdot \sec^2 y - \sec^2 x \cdot \tan^2 y - \tan^2 x \cdot \sec^2 y + \tan^2 x \cdot \tan^2 y$

$= \sec^2 x (\sec^2 y - \tan^2 y) - \tan^2 x (\sec^2 y - \tan^2 y)$

$= \sec^2 x - \tan^2 x = 1 \quad [\because \sec^2 - \tan^2 \theta = 1]$

34. (*a*) $\tan 15^\circ \cdot \cot 75^\circ + \tan 75^\circ \cdot \cot 15^\circ$

$= \tan 15^\circ \cdot \cot (90^\circ - 15^\circ) + \tan (90^\circ - 15^\circ) \cdot \cot 15^\circ$

$= \tan^2 15^\circ + \cot^2 15^\circ \quad \ldots(i)$

$[\because \tan (90^\circ - \theta) = \cot \theta, \cot (90^\circ - \theta) = \tan \theta]$

Now, $\cot 15^\circ = \frac{1}{2 - \sqrt{3}} = \frac{2 + \sqrt{3}}{(2 - \sqrt{3})(2 + \sqrt{3})}$

$= 2 + \sqrt{3}$

or $\tan 15^\circ = 2 - \sqrt{3}$

$\therefore \tan^2 15^\circ + \cot^2 15^\circ = (2 - \sqrt{3})^2 + (2 + \sqrt{3})^2$

$= 2 [(2)^2 + (\sqrt{3})^2] = 2 (4 + 3)$

$= 2 \times 7 = 14$

Chapter

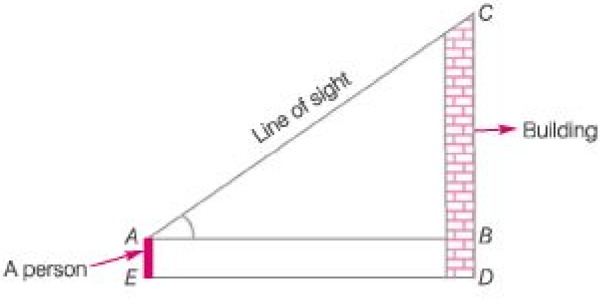
39

Height and Distance

It is an important application of trigonometry which helps us to find the height of any object and distance of that object from any point which are not directly measurable. If the angle of elevation/ depression from a point is known.

Line of Sight

A line of sight is the line drawn from the eye of an observer to the point, where the object is viewed by the observer.

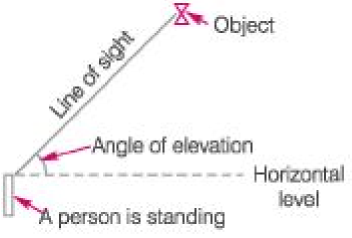


Horizontal Line

The line of sight which is parallel to ground level is known as horizontal line.

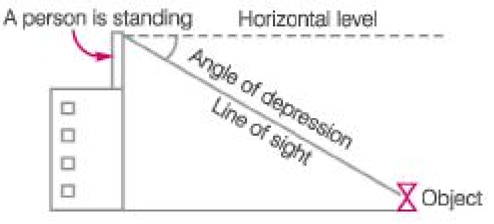
Angle of Elevation

The angle of elevation of the point viewed is the angle formed by the line of sight with the horizontal, when the point being viewed is above the horizontal level.



Angle of Depression

When the line of sight is below the horizontal level, the angle so formed by the line of sight with the horizontal is called the angle of depression.



MIND IT!

(i) Angle of elevation and depression are always acute angles. (ii) Unless stated, it is assumed that the height of the observer is not considered.

Ex. 1 A tower stands vertically on the ground. From a point on the ground which is 30 m away from the foot of a tower, the angle of elevation of the top of the tower is found to be 45°. Find the height of the tower.

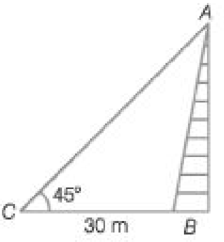
Sol. Given, angle of elevation

$\angle ACB = 45°$; $BC = 30$ m; $AB = ?$

In ΔABC, $\tan 45° = \dfrac{AB}{BC}$ $\left[\because \tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}\right]$

$$1 = \frac{AB}{30}$$

$\therefore$ $AB = 30$ m



Ex. 2 The shadow of a building standing on a level ground is found to be 40 m longer when the Sun's altitude becomes 30° from 45°. Find the height of the tower.

Sol. Given, $DC = 40$ m; $\angle ACB = 30°$ and $\angle ADB = 45°$

In ΔADB,

$$\tan 45° = \frac{AB}{BD}$$

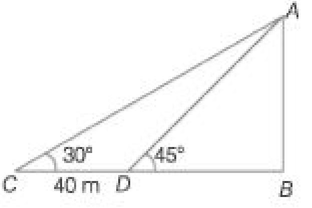
$\therefore$ $AB = BD$ $[\because \tan 45° = 1]$ ...(i)

In ΔACB,

$$\tan 30° = \frac{AB}{BC} = \frac{AB}{BD + DC}$$

$\Rightarrow$ $\dfrac{1}{\sqrt{3}} = \dfrac{AB}{BD + DC}$

$\therefore$ $\sqrt{3}AB = BD + DC$ ...(ii)



On putting the value of Eq. (i) in Eq. (ii), we get

$\sqrt{3}AB = AB + 40$

$\Rightarrow$ $\sqrt{3}AB - AB = 40$

$\Rightarrow$ $AB(\sqrt{3} - 1) = 40$

$\therefore$ $AB = \dfrac{40}{\sqrt{3} - 1} = \dfrac{40}{0.732} = 54.644$ m $[\because \sqrt{3} = 1.732]$

Alternate Method

If height is h then,

$$h = \frac{CD}{\cot 30° - \cot 45°} = \frac{40}{\sqrt{3} - 1}$$

$$= \frac{40}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{40(\sqrt{3} + 1)}{2}$$

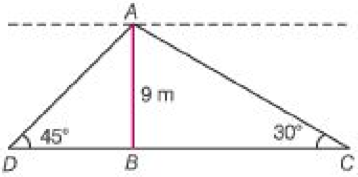
$$= 20(\sqrt{3} + 1)$$

$$= 20 \times 2.732 = 54.64 \text{ m}$$

Ex. 3 From a point A on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 9 m from the surface of river, then find the width of the river.

Sol. The width of the river = DC

In ΔABC,



$$\tan 30^\circ = \frac{AB}{BC}$$

$$BC = \frac{9}{\tan 30^\circ} = 9\sqrt{3} \text{ m}$$

Now, in ΔABD,

$$\tan 45^\circ = \frac{AB}{BD}$$

$$AB = BD$$

$$BD = 9 \text{ m} \qquad [\because AB = 9 \text{ m}]$$

$$\therefore \quad DC = DB + BC = 9 + 9\sqrt{3} = 9(\sqrt{3} + 1) = 24.588 \text{ m}$$

Ex. 4 A vertical post 15 ft high is broken at certain height and its upper part, not completely seperated, meet the ground at an angle of 30°. Find the height at which the post is broken.

Sol. Given that, height of post = 15 ft

Let the post breaks at point C and the length of lower part is h ft

or $\quad BC = h$

So, $\quad AC = AB - BC = 15 - h$

In ΔBCD,

$$\sin 30 = \frac{BC}{CD} \qquad \left[\because \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}\right]$$

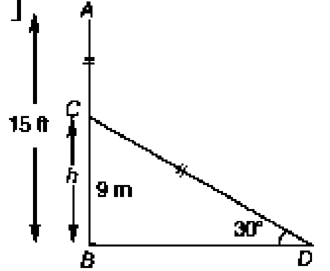
$$\Rightarrow \quad \frac{1}{2} = \frac{h}{15 - h} \qquad [\because CD = AC]$$

$$\Rightarrow \quad 2h = 15 - h$$

$$\Rightarrow \quad 2h + h = 15 \quad 3h = 15$$

$$h = \frac{15}{3} = 5 \text{ ft}$$

$\therefore$ The height at which the post is broken = 5 ft.



Ex. 5 The angles of depression of two ships from the top of a lighthouse are 45° and 30°. If the ships are 120 m apart, then find the height of the lighthouse.

Sol. Let AB, the height of the lighthouse $= x$ m

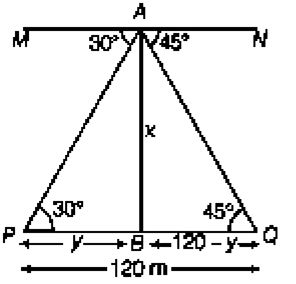
Since, $MN \parallel PQ$

$\therefore$ $\angle MAP = \angle APB = 30^\circ$ [$\because$ alternate angles]

and $\angle NAQ = \angle AQB = 45^\circ$

Let the length between P and B be y m.

So, the length between B and Q is $(120 - y)$ m.



In ΔABP,

$$\tan 30^\circ = \frac{AB}{BP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y}$$

$$y = x\sqrt{3} \quad \text{...(i)}$$

In ΔABQ,

$$\tan 45^\circ = \frac{AB}{BQ}$$

$$\Rightarrow \quad 1 = \frac{x}{120 - y}$$

$$x = 120 - y \quad \text{...(ii)}$$

From Eq. (i) and (ii), we get

$$x = 120 - x\sqrt{3}$$

$$\Rightarrow \quad x = \frac{120}{1 + \sqrt{3}}$$

$$\therefore \quad x \approx 44 \text{ m}$$

Alternate Method

When height of any object from horizontal plane is *h*.

The angle of depression of two consecutive milestone in opposite direction at object is a and B respectively.

Then, the height of the object is

$$h = \frac{\tan\alpha \cdot \tan\beta}{\tan\alpha + \tan\beta} \times \text{Distance between the object}$$

Here, $\alpha = 30^\circ$ and $\beta = 45^\circ$

Let height of the lighthouse $= h$

Then, $h = \dfrac{\tan\alpha \cdot \tan\beta}{\tan\alpha + \tan\beta} \times$ Distance between both the ships

$$h = \frac{\tan 30^\circ \cdot \tan 45^\circ}{\tan 30^\circ + \tan 45^\circ} \times 120$$

$$= \frac{\frac{1}{\sqrt{3}} \times 1}{\frac{1}{\sqrt{3}} + 1} \times 120 = \frac{\frac{\frac{1}{\sqrt{3}}}{1 + \sqrt{3}} \times 120}{\sqrt{3}}$$

$$h = \frac{120}{\sqrt{3} + 1}$$

$$\therefore \quad h \approx 44 \text{ m}$$

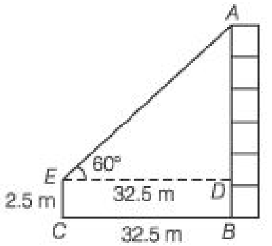
Multi Concept

QUESTIONS

1. *A man 2.5 m tall is 32.5 m away from a building. The angle of elevation of the top of the building from his eyes is 60°. What is the height of the building? Also, calculate the distance between the eye of man and top point of the building.*

(a) 5879 m, 65 m (b) 60 m, 59.5 m

(c) 58 m, 60 m (d) 59.5 m, 60.5 m



(a) Given, $EC = DB = 2.5\text{ m}$, $BC = DE = 32.5\text{ m}$,

$\angle AED = 60°$, $AB = ?$ and $AE = ?$

In ΔAED, $\tan 60° = \dfrac{AD}{DE}$

$\Rightarrow \sqrt{3} = \dfrac{AD}{32.5}$ $[\because \tan 60° = \sqrt{3}]$

$\therefore AD = 32.5 \times \sqrt{3} = 32.5 \times 1.732 = 56.29\text{ m}$

$\therefore AB = AD + DB = 56.29 + 2.5 = 58.79\text{ m}$

Now, $\sin 60° = \dfrac{AD}{AE} = \dfrac{56.29}{AE}$

$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{56.29}{AE}$

$\therefore AE = 65\text{ m}$

2. *From a point A on the ground, the angle of elevation of the top of a 20 m tall building is 45°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from A is 60°. Find the length of the flag staff and the distance of the building from the point A.*

(a) 20 m, 15 m (b) 20 m, 14.64 m

(c) 25 m, 15 m (d) 20 m, 19 m

(b) Given, $BC = 20$ m, $\angle BAC = 45°$

and $\angle DAC = 60°$

In ΔBAC, $\tan 45° = \dfrac{BC}{AC} = \dfrac{20}{AC}$

$\therefore$ $AC = 20$ m [$\because \tan 45° = 1$]

Now, in ΔDAC,

Let $DB = x$ m

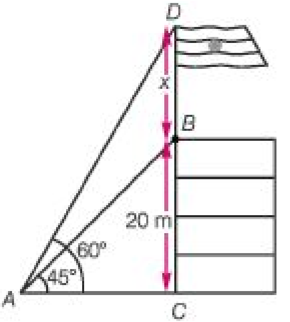
$\therefore$ $DC = DB + BC = (x + 20)$ m

$\therefore$ $\tan 60° = \dfrac{DC}{AC} = \dfrac{x + 20}{20}$

$\sqrt{3} = \dfrac{x + 20}{20}$

$\therefore$ $x + 20 = 20\sqrt{3} = 34.64$

$\therefore$ $x = 34.64 - 20 = 14.64$ m



Fast Track Practice

1. What is the angle of elevation of the Sun when the shadow of a pole is V3 times the length of the pole? [CDS 2012]

(a) 30° *(b)* 45°

(c) 60° *(d)* None of these

The shadow of a tower is 15 m, when the Sun's elevation is 30°. What is the length of the shadow, when the Sun's elevation is 60"? [CDS 2010]

(a) 3 m *(b)* 4 m *(c)* 5 m *(d)* 6 m

3. What is the angle of elevation of the Sun, when the shadow of a pole of height *x* m is —m? [CDS 2013]

V3

(a) 30° *(b)* 45° *(c)* 60° *(d)* 75°

4. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time, a tower casts a shadow of 40 m long on the ground. The height of the tower is [SSCCGL2012]

(a) 60 m *(b)* 65 m *(c)* 70 m *(d)* 72 m

5. The tops of two poles of height 24 m and 36 m are connected by a wire. If the wire makes an angle of 60° with the horizontal, then the length of the wire is

[SSCCCL2012]

(a) 8V3 *m(b)* 8 m *(c)* 6V3 m *(d)* 6 m

6. The shadow of a tower standing on a level plane is found to be 50 m longer when the Sun's elevation is 30°. When it is 60°, then what is the height of the tower? [CDS 2014]

(a) 25 m *(b)* $25\sqrt{3}$ m *(c)* $\frac{25}{\sqrt{3}}$ m *(d)* 30 m

7. The angle of elevation of the tip of a tower from a point on the ground is 45°. Moving 21 m directly towards the base of the tower, the angle of elevation changes to 60°. What is the height of the tower, to the nearest metre? [CDS 2012]

(a) 48 m *(b)* 49 m *(c)* 50 m *(d)* 51 m

8. The angles of depression from the top of a light house of two boats are 45° and 30° towards the west. If the two boats are 5 m apart, then the height of the light house is [SSCFCI2012]

(a) (2.5V3 - 1) m *(b)* 2.5(V3 - 1) m *(c)* (2.5V3 + 1) m *(d)* 2.5(V3 + 1) m

The angle of elevation of the top of an unfinished pillar at a point 150 m from its base is 30°. If the angle of elevation at the same point is to be 45°, then the pillar has to be raised to a height of how many metres? [CDS 2009]

(a) 59.4 m *(b)* 61.4 m

(c) 62.4 m *(d)* 63.4 m

10. From the top of a cliff 200 m high, the angles of depression of the top and bottom of a tower are observed to be 30° and 45°, respectively. What is the height of the tower? [CDS 2012]

(a) 400 m *(b)* 400^3 m

(c) 400/V3 m *(d)* None of these

11. On walking 120 m towards a chimney in a horizontal line through its base the angle of elevation of tip of the chimney changes from 30° to 45°. The height of the chimney is [CDS 2012]

(a) 120 m (b) 60(V3 - 1) m

(c) 60(V3 + 1) m (d) None of these

12. A man standing at a point P is watching the top of elevation of 30°. The man walks some distance towards the tower and then his angle of elevation of the top of the tower is 60°. If the height of the tower is 30 m, then the distance he moves is [SSC (10+2)2012]

(a) 20 m (b) 20V3 m

(c) 22 m (d) 22 V3 m

13. The angle of elevation of the top of a tower from the bottom of a building is twice that from its top. What is the height of the building, if the height of the tower is 75 m and the angle of elevation of the top of the tower from the bottom of the building is 60°? [CDS 2011]

(a) 25 m (b) 37.5 m

(c) 50 m (d) 60 m

14. The angles of elevation of the top of a tower from two points which are at distances of 10 m and 5 m from the base of the tower and in the same straight line with it are complementary. The height of the tower is [CDS 2012]

(a) 5 m (b) 15 m

(c) VT5 m (d) -J75 m

15. The angles of elevation of e top of an inaccessible tower from two points on the same straight line from the base of the tower are 30° and 60°, respectively. If the points are separated at a distance of 100 m, then the height of the tower is close to

[CDS 2012] (a) 86.6 m (b) 84.6 m

(c) 82.6 m (d) 80.6 m

16. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops? [CDS 2012]

(a) 13 m (b) 17 m (c) 18 m (d)23 m

Directions (Q. Nos. 17-20) *Read the following information carefully to answer the questions that follow.*

As seen from the top and bottom of a building of height h m, the angles of elevation of the top of a tower of height $\frac{(3+\sqrt{3})h}{2}$ m are α and β, respectively. [CDS 2013]

17. If P = 30°, then what is the value of tan a?

(a) 1/2 *(b)* 1/3

(c) 1/4 *(d)* None of these

18. If a = 30°, then what is the value of tanP?

(a) 1 *(b)* 1/2

(c) 1/3 *(d)* None of these

19. If a = 30° and *h* = 30 m, then what is the distance between the base of the building and the base of the tower?

(a) 15 + 15V3 m *(b)* 30 + 15V3 m *(c)* 45 + 1 5a/3 m *(d)* None of these

20. If P = 30° and if 6 is the angle of depression of the foot of the tower as seen from the top of the building, then what is tan 9 equal to?

(a) $\frac{(3-\sqrt{3})}{3\sqrt{3}}$ *(b)* $\frac{(3+\sqrt{3})}{3\sqrt{3}}$

(c) $\frac{(2-\sqrt{3})}{3\sqrt{3}}$ *(d)* None of these

21. If the angle of elevation of a tower from two distant points *a* and *b (a > b)* from its foot and in the same straight line and on the same side of it are 30° and 60°, then the height of the tower is [SSC CPO 2013]

(a) $\sqrt{\frac{a}{b}}$ *(b)* $\sqrt{a+b}$

(c) $\sqrt{ab}$ *(d)* $\sqrt{a-b}$

22. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. The distance between the two towers is n times the height of the shorter tower. What is n equal to?

[CDS 2014]

(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

23. At the foot of a mountain, the elevation of its summit is 45°. After ascending 2 km towards the mountain upon an incline of 30°, the elevation changes to 60°. The height of the mountain is

[SSC CCL 2012]

(a) $(\sqrt{3} - 1)$ km (b) $(\sqrt{3} + 1)$ km
(c) $(\sqrt{3} - 2)$ km (d) $(\sqrt{3} - 2)$ km

24. A man standing in one corner of a square football field observes that the angle subtended by a pole in the corner just diagonally opposite to this corner is 60°. When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30°. The length of the field is [SSC CCL 2013]

(a) 20 m (b) $40\sqrt{2}$ m
(c) 40 m (d) $20\sqrt{2}$ m

25. A spherical balloon of radius r subtends angle 60° at the eye of an observer. If the angle of elevation of its centre is 60° and h is the height of the centre of the balloon, then which one of the following is correct? [CDS 2013]

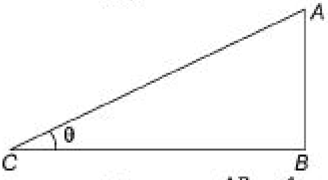
(a) h = r (b) h = J2r

(c) h = 4Zr (d)h = 2r

Answer with Solutions

Exercise © *Base Level Questions*

1. (a) $\therefore \tan\theta = \frac{AB}{BC}$



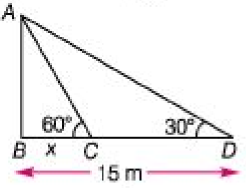
$$BC = \sqrt{3}\,AB \Rightarrow \quad \frac{AB}{BC} = \frac{1}{\sqrt{3}}$$

$\therefore \quad \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$

2. (c) $BC = ?$

Let the length of the new shadow = x m

In ΔABC, $\tan 60° = \frac{AB}{BC}$



$$AB = x\sqrt{3} \qquad [\because BC = x] \qquad \ldots(i)$$

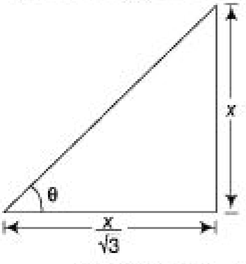
In ΔABD, $\tan 30° = \frac{AB}{15} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{15}$

$\therefore \qquad AB = \frac{15}{\sqrt{3}} \qquad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$x\sqrt{3} = \frac{15}{\sqrt{3}} \Rightarrow x = 5\text{ m}$$

3. (c) Here, θ is the angle of elevation.



$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{x}{\frac{x}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}x}{x} = \sqrt{3}$$

Here, $\tan\theta = \sqrt{3}$

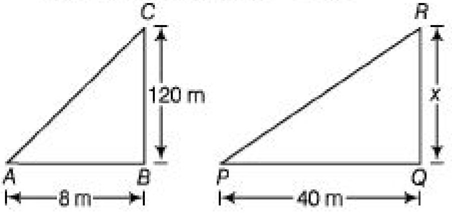
So, $\theta = 60°$ $[\because \tan 60° = \sqrt{3}]$

4. (*a*) Given height of first vertical stick = 12 m

and length of its shadow = 8 m

Let heigh of second vertical stick = x m

and length of its shadow = 40 m



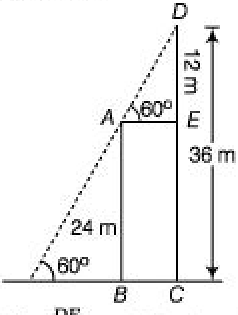
Now, in ΔABC and ΔPQR

By proportion $\frac{12}{8} = \frac{x}{40}$

$\Rightarrow \quad \frac{12 \times 40}{8} = x$

$\therefore \quad x = 60$ m

5. (*a*) So, in ΔADE,



$\sin 60° = \frac{DE}{AD}$ [AD = length of wire]

$\Rightarrow \quad AD = \frac{DE}{\sin 60°} = \frac{12}{\frac{\sqrt{3}}{2}}$

$= \frac{12 \times 2}{\sqrt{3}}$ [on multiply and divide by $\sqrt{3}$]

$= \frac{24 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}$ m

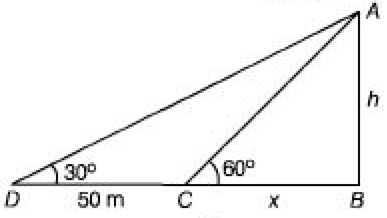
6. (*b*) Let h be the height of the tower and $BC = x$ m.

In ΔBCA,

$$\tan 60° = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x}$$

$$h = x\sqrt{3} \qquad \text{... (i)}$$

Now, in ΔABD, $\tan 30° = \dfrac{h}{50 + x}$

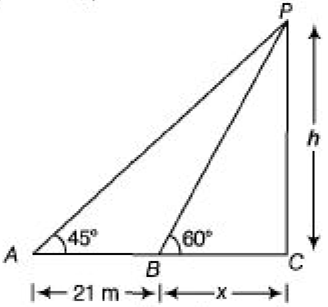


$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{50 + x} \qquad [\because h = x\sqrt{3}]$$

$$\Rightarrow \qquad 50 + x = 3x \Rightarrow x = 25 \text{ m}$$

$$\therefore \qquad h = 25\sqrt{3} \text{ m} \qquad \text{[from Eq. (i)]}$$

7. (*c*) In ΔPBC,



$$\tan 60° = \frac{h}{x} \Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow x = \frac{h}{\sqrt{3}}$$

In ΔPAC, $\tan 45° = \dfrac{h}{21 + x} = 1$

$$\Rightarrow \qquad h = 21 + x$$

$$\Rightarrow \qquad h = 21 + \frac{h}{\sqrt{3}} \qquad \text{[from Eq. (i)]}$$

$$\Rightarrow \qquad h\left(1 - \frac{1}{\sqrt{3}}\right) = 21$$

$$\Rightarrow \qquad h = \frac{21\sqrt{3}}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$$

$$\Rightarrow \qquad h = \frac{21\sqrt{3}\,(\sqrt{3} + 1)}{2} = 49.68 \approx 50 \text{ m}$$

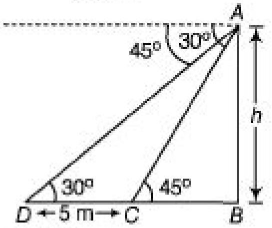
Alternate Method $h = \dfrac{21}{\cot 45° - \cot 60°}$

$$= \frac{21}{1 - \frac{1}{\sqrt{3}}} = \frac{21\sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= 49.68 \approx 50$$

8. (*d*) Let height of the light house

$$AB = h$$



Then, in ΔABC,

$$\tan 45° = \frac{AB}{BC} \Rightarrow AB = BC = hz$$

Now, in ΔABD,

$$\frac{1}{\sqrt{3}} = \frac{h}{DC + CB}$$

$$\tan 30° = \frac{AB}{DB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{DC + CB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{h+5} \Rightarrow h + 5 = \sqrt{3}h$$

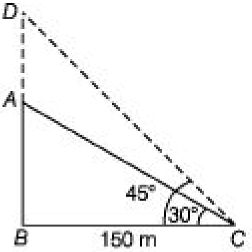
$$\Rightarrow \quad h(\sqrt{3} - 1) = 5$$

$$\Rightarrow \quad h = \frac{5}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{5}{2}(\sqrt{3} + 1) \text{ m} = 2.5(\sqrt{3} + 1) \text{ m}$$

9. (*d*) Given, $BC = 150$ m

$$\angle ACB = 30° \text{ and } \angle DCB = 45°$$



Then, $AD = ?$

In ΔABC, $\tan 30° = \frac{AB}{BC}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{150}$$

$$\therefore \quad AB = 86.6 \text{ m}$$

In ΔDBC, $\tan 45° = \frac{DB}{BC}$

$$\Rightarrow \quad 1 = \frac{AD + AB}{BC}$$

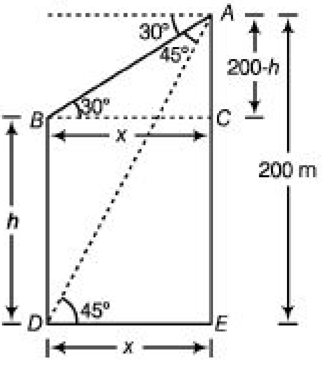
$$\Rightarrow \quad BC = AD + 86.6$$

$\Rightarrow \quad 150 - 86.6 = AD$

$\therefore \quad AD = 63.4$ m

10. (*d*) Let $AE = 200$ m be the height of the cliff and $BD = h$ be the height of the tower. and x is the distance between cliff and tower.

In ΔABC,



$$\tan 30° = \frac{200 - h}{x} = \frac{1}{\sqrt{3}} = \frac{200 - h}{x}$$

$\Rightarrow \quad x = (200 - h)\sqrt{3}$ ...(i)

And in ΔADE, $\tan 45° = \dfrac{200}{x}$

$\Rightarrow \quad 1 = \dfrac{200}{x} \Rightarrow x = 200$ m

From Eq. (i), $\quad 200 = (200 - h)\sqrt{3}$

$\Rightarrow \quad h = 200\left(\dfrac{\sqrt{3} - 1}{\sqrt{3}}\right)$ m

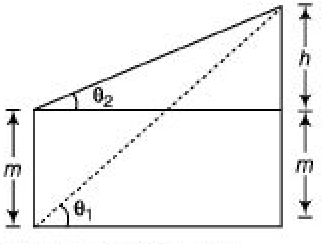
Alternate Metod

If we know about the height of any object and angles θ_1 and θ_2.

Then, height of another object

$$h = \frac{m \cot \theta_1}{\cot \theta_2 - \cot \theta_1}$$

and distance $(x) = \dfrac{m}{\tan \theta_1 - \tan \theta_2}$



Here, $\theta_1 = 45°$ and $\theta_2 = 30°$

$m = h$ and $h = 200 - h$

Now, height of the tower

$$h = \frac{m \cot \theta_1}{\cot \theta_2 - \cot \theta_1}$$

$$\Rightarrow \quad 200 - h = \frac{h \times \cot 45°}{\cot 30° - \cot 45°}$$

$$\Rightarrow \quad 200 - h = \frac{h}{\sqrt{3} - 1}$$

$$\Rightarrow \quad 200\sqrt{3} - 200 - h\sqrt{3} + h = h$$

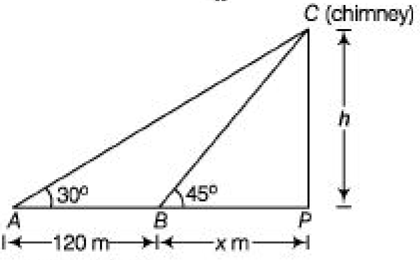
$$\Rightarrow \quad h\sqrt{3} = 200(\sqrt{3} - 1)$$

$$h = \frac{200(\sqrt{3} - 1)}{\sqrt{3}} \text{ m}$$

11. (*c*) Let h be the height of the chimney

In ΔBPC,

$$\tan 45° = \frac{h}{x} = 1 \; h = x \qquad \ldots \text{(i)}$$



Now, In ΔAPC,

$$\tan 30° = \frac{h}{120 + x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{h}{120 + h} = \frac{1}{\sqrt{3}} \qquad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad \sqrt{3}\, h = 120 + h$$

$$\Rightarrow \quad h = \frac{120}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{120(\sqrt{3} + 1)}{2}$$

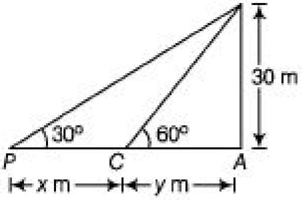
$\therefore$ Required height of chimney

$$(h) = 60(\sqrt{3} + 1) \text{ m}$$

12. (*b*) Let $PC = x$ m and $AC = y$ m

In ΔABC, $\tan 60° = \dfrac{30}{y}$

$$\Rightarrow \quad y = \frac{30}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$



In ΔPAB, $\tan 30° = \dfrac{30}{x + y}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{30}{(x + y)} \Rightarrow (x + y) = 30\sqrt{3}$$

$\Rightarrow \quad x = 30\sqrt{3} - y = 30\sqrt{3} - 10\sqrt{3} = 20\sqrt{3}$

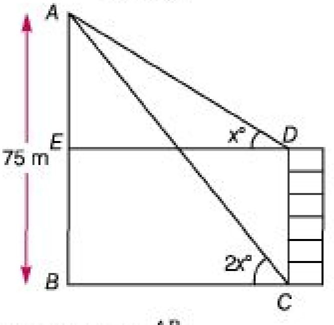
$\Rightarrow \quad x = 20\sqrt{3}$ m

$\therefore$ Required distance, he moves $= 20\sqrt{3}$ m.

13. (c) We have to find DC.

Given, $\quad 2x = 60°$

$\therefore \quad x = 30°$



In ΔABC, $\tan 60° = \dfrac{AB}{BC}$

$$\frac{\sqrt{3}}{1} = \frac{75}{BC}$$

$$\therefore \quad BC = \frac{75}{\sqrt{3}} = \frac{75 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = 25\sqrt{3} \text{ m}$$

In ΔAED,

$$\tan 30° = \frac{AE}{ED} = \frac{AE}{25\sqrt{3}} \quad [\because BC = ED]$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AE}{25\sqrt{3}}$$

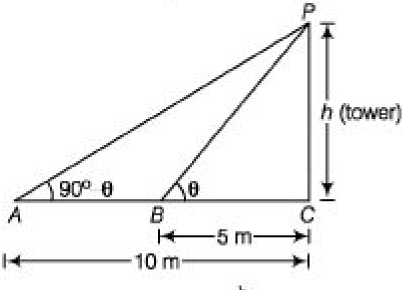
$\therefore \quad AE = 25$ m

$\therefore \quad DC = EB = AB - AE$

$= 75 - 25 = 50$ m

14. (c) Given that, angles are complementary.

Let h be the height of the tower.



Now, in ΔPBC $\tan\theta = \dfrac{h}{5}$ $\quad$ ...(i)

and in ΔPAC

$$\tan(90° - \theta) = \frac{h}{10}$$

$$\Rightarrow \quad \cot\theta = \frac{h}{10} \quad \text{...(ii)}$$

On multiplying Eqs. (i) and (ii), we get

$$\tan\theta \cdot \cot\theta = \frac{h}{5} \times \frac{h}{10}$$

$\Rightarrow$ $$\frac{h^2}{50} = 1$$

$\Rightarrow$ $$h = \sqrt{50} \text{ m}$$

Which is the required height of the tower.

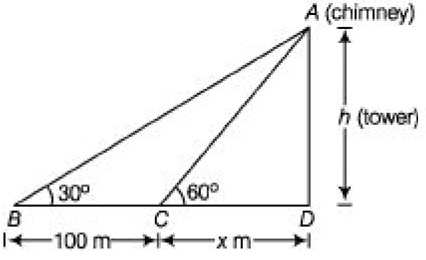
15. (*a*) Let h be the height of inaccessible tower.

Now, in ΔACD

$$\tan 60° = \frac{h}{x} = \sqrt{3}$$

$\Rightarrow$ $$x = \frac{h}{\sqrt{3}} \quad \ldots\text{(i)}$$

and in ΔABD



$$\tan 30° = \frac{h}{100 + x} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $$\sqrt{3}h = 100 + x = 100 + \frac{h}{\sqrt{3}}$$

[from Eq. (i)]

$\Rightarrow$ $$\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) h = 100$$

$\Rightarrow$ $$\frac{2}{\sqrt{3}} h = 100$$

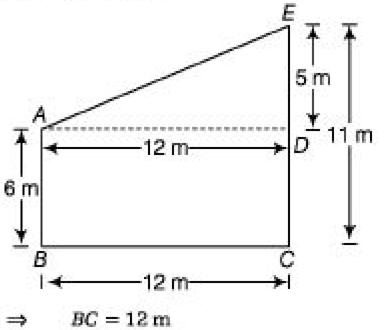
$\Rightarrow$ $$h = 50\sqrt{3}$$

$\Rightarrow$ $$h = 50 \times 1732 = 86.6 \text{ m}$$

So, required height is 86.6 m.

16. (*a*) Given, $AB = 6$ m

and $EC = 11$ m



$\Rightarrow$ $BC = 12$ m

$\because$ $BC = AD = 12$ m

and $ED = EC - CD = EC - AB$ [$\because AB = CD$]

$= 11 - 6 = 5$ m

In ΔAED,

$$(AE)^2 = (AD)^2 + (ED)^2$$

[by Pythagoras theorem]

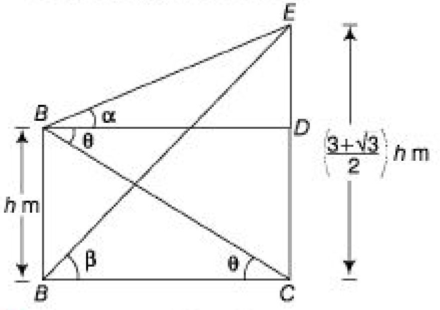
$$= (12)^2 + (5)^2 = 144 + 25$$

$$= 169 = (13)^2$$

$\therefore \quad AE = 13$ m

So, the distance between their tops = 13 m

Solutions (Q. Nos. 17-20)



17. (*b*) Given that, $\beta = 30°$

In ΔADE,

$$\tan\beta = \tan 30° = \frac{AE}{DE} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad DE = \sqrt{3}\ AE = \sqrt{3}\left(\frac{3+\sqrt{3}}{2}\right)\text{h}$$

$$\Rightarrow \quad BC = DE = \frac{3}{2}(1+\sqrt{3})\text{ h}$$

$(\because BD = DE)$ ...(i)

Now, In ΔABC,

$$\tan\alpha = \frac{AC}{BC}$$

$$\Rightarrow BC\ \tan\alpha = (AE - CE) = (AE - BD)$$

$[\because BD = CE]$

$$\Rightarrow \quad BC\ \tan\alpha = \left(\frac{3+\sqrt{3}}{2}\right)h - h$$

$$\Rightarrow \quad \frac{3}{2}(1+\sqrt{3})\ h\ \tan\alpha = \left(\frac{1+\sqrt{3}}{2}\right)h$$

$$\Rightarrow \quad \tan\alpha = \frac{1}{3}$$ [From Eq. (i)]

18. (*a*) Given that, $\alpha = 30°$

In ΔABC,

$$\tan\alpha = \tan 30° = \frac{AC}{BC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BC = \sqrt{3}\ AC = \sqrt{3}\ (AE - CE)$$

$$= \sqrt{3}\ (AE - BD)$$ $[\because BD = CE]$

$$= \sqrt{3}\left(\frac{3+\sqrt{3}}{2} - 1\right)h$$

$$= \frac{\sqrt{3}}{2}(1+\sqrt{3})\ h$$

Now, in ΔADE,

$$\tan\beta = \frac{AE}{DE}$$

$$\Rightarrow \quad \tan\beta = \frac{AE}{BC} \qquad [\because DE = BC]$$

$$= \frac{\left(\frac{3+\sqrt{3}}{2}\right)h}{\frac{\sqrt{3}}{2}(1+\sqrt{3})\ h} = \frac{\frac{\sqrt{3}\,(1+\sqrt{3})}{2}\ h}{\frac{\sqrt{3}\,(1+\sqrt{3})}{2}\ h}$$

$$\therefore \quad \tan\beta = 1$$

19. (*c*) Given that, $\alpha = 30°$ and $h = 30$ m.

In ΔABC,

$$\tan\alpha = \tan 30° = \frac{AC}{BC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{BC}{\sqrt{3}} = (AE - CE) = (AE - BD)$$

$$[\because BD = CE]$$

$$\Rightarrow \quad BC = \sqrt{3}\left(\frac{3+\sqrt{3}}{2} - 1\right)h$$

$$\Rightarrow \quad BC = \sqrt{3}\,\frac{(1+\sqrt{3})}{2}\cdot 30 = (\sqrt{3}+3)\cdot 15$$

$$\therefore \quad DE = BC = (45 + 15\sqrt{3})\ \text{m}$$

$$[\because DE = BC]$$

20. (*a*) Given that, $\beta = 30°$

In $\quad \Delta BDE, \tan\theta = \frac{BD}{DE} = \frac{h}{DE}$

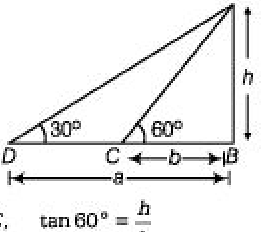
$$\Rightarrow \quad \tan\theta = \frac{h}{\frac{3}{2}(1+\sqrt{3})\ h} \qquad \text{[from Eq. (i)]}$$

$$= \frac{2}{3}\,\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{2(\sqrt{3}-1)}{3\cdot 2}$$

$$= \frac{(\sqrt{3}-1)}{3} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{(3-\sqrt{3})}{3\sqrt{3}}$$

21. (*c*) Let AB be the tower.

Where, $AB = h$ m



In ΔABC, $\quad \tan 60° = \frac{h}{b} \qquad$ ...(i)

In ΔABD, $\quad \tan 30° = \frac{h}{a} \qquad$ ...(ii)

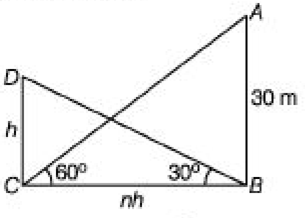
On multiplying Eqs. (i) and (ii), we get

$$\tan 60^\circ \tan 30^\circ = \frac{h^2}{ab}$$

$$\Rightarrow \quad \sqrt{3} \times \frac{1}{\sqrt{3}} = \frac{h^2}{ab} \Rightarrow h^2 = ab$$

$$\therefore \quad h = \sqrt{ab}$$

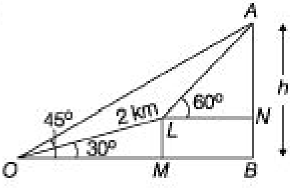
22. (*b*) Let h be the height of shorter tower. Then, the distance between the two towers is given by nh m.



In ΔBCD, $\tan 30^\circ = \dfrac{h}{nh}$

$$\frac{1}{\sqrt{3}} = \frac{1}{n} \Rightarrow n = \sqrt{3}$$

23. (*b*)



Let $AB = h$ km

In ΔOAB, $\tan 45^\circ = \dfrac{AB}{OB}$

$$\Rightarrow \quad OB = h \text{ km}$$

In ΔOLM,

$$\Rightarrow \quad \frac{OM}{OL} = \cos 30^\circ$$

$$OM = 2\cos 30^\circ \; [OL = 2 \text{ km}] = \sqrt{3} \text{ km}$$

$\therefore LN = BM = OB - OM = (h - \sqrt{3})$ km.

In ΔOLM, $\sin 30^\circ = \dfrac{LM}{OL}$

$$LM = 2 \sin 30^\circ = 1 \text{ km}$$

$$\therefore \quad BN = LM = 1 \text{ km}$$

In ΔALN, $\tan 60^\circ = \dfrac{AN}{LN}$

$$\Rightarrow \quad \sqrt{3} = \frac{AB - BN}{LN}$$

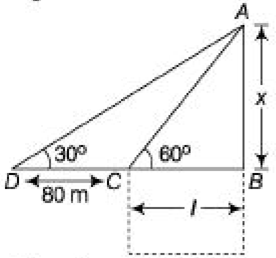
$$\Rightarrow \quad \sqrt{3} = \frac{h-1}{h-\sqrt{3}} \Rightarrow \sqrt{3}h - 3 = h - 1$$

$$\Rightarrow \quad h = \frac{2}{\sqrt{3}-1}$$

$$\therefore \quad h = \frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = (\sqrt{3}+1) \text{ km}$$

24. (*c*) From the figure,

Let the length of football field = l m



Height of the pole = x m

$\therefore$ In ABC, $\tan 60 = \dfrac{x}{l}$

$$\sqrt{3} = \frac{x}{l}; \quad x = \sqrt{3}l \qquad \text{... (i)}$$

Now, in ΔABD

$$\tan 30° = \frac{x}{l + 80} \Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{l + 80}$$

$$l + 80 = \sqrt{3}x$$

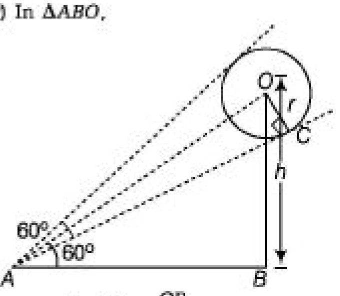
Now, from Eq. (i), we get

$$l + 80 = \sqrt{3}\,(\sqrt{3}\ l)$$

$$\Rightarrow \quad l + 80 = 3l \Rightarrow 80 = 3l - l$$

$$\therefore \quad l = \frac{80}{2} = 40 \text{ m}$$

25. (*c*) In ΔABO,



$$\sin 60° = \frac{OB}{AO}$$

$$\Rightarrow \quad AO = \frac{OB}{\sin 60°} \qquad \text{... (i)}$$

Now, in ΔAOC,

$$\sin \frac{60°}{2} = \frac{OC}{AO}$$

$$\Rightarrow \quad AO = \frac{OC}{\sin 30°} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{OB}{\sin 60°} = \frac{OC}{\sin 30°}$$

$$\Rightarrow \quad \frac{h}{\frac{\sqrt{3}}{2}} = \frac{r}{\frac{1}{2}}$$

$$\therefore \quad h = \sqrt{3}r$$

Chapter

40

Data Table

A Data Table is a chart of facts and figures represented in **horizontal rows** and **vertical columns.** These facts and figures can be of imports, exports, income of employees in a factory students applying for and qualifying a certain field of study.

The amount of data that can be presented on data table is much higher than that which can be presented on any other type of graph or chart.

For example

The data table given below shows profit of 3 companies from year 2010 to 2012.

	Year / Company	2010 (Profit%)	2011 (Profit%)	2012 (Profit%)	Row
Column	SATYAM	30	20	15	1
	TCS	15	20	10	2
	L&T	15	10	35	3
		1	2	3	

Here, to calculate different value related to a data table we must have the knowledge of rows and columns.

(i) *Profit of Satyam in year 2011 can be calculated as* Satyam is in first column and year 2011 is in second row so the intersection point is at 20 ∴ Profit of Satyam in 2011 is 20%. (ii) *Sum of profit of TCS and L&T in year 2012 can calculated as*

TCS is in second column and year 2012 is in third row and their intersection point is at 10 in third row and L&T is in third row and year 2012 is in third row and their intersection point is at 35. So, the required sum of profit = 10 + 35 = 45%

Directions (Examples 1-3) *Study the given table carefully and answer the questions that*

follow. **[Bank PO 2010]**

Percentage of marks obtained by five students in five different subjects in a school

Subject / Student	English (100)	Science (125)	Mathematics (150)	Social Studies (75)	Hindi (50)
Rahul	67	84	70	64	90
Veena	59	72	74	88	84
Soham	66	90	84	80	76
Shreya	71	66	80	66	86
Varun	63	76	88	68	72

Figures in brackets indicate maximum marks for a particular subject.

Ex. 1 What is Varun's overall percentage in the examination?

Sol. Total marks of Varun

= Sum of marks in English + Science + Mathematics + Social Studies + Hindi

= 63% of 100 + 76% of 125 + 88% of 150 + 68% of 75 + 72% of 50

$$= 63 + \frac{76 \times 125}{100} + \frac{88 \times 150}{100} + \frac{68 \times 75}{100} + \frac{72 \times 50}{100}$$

= 63 + 95 + 132 + 51 + 36 = 377

$$\therefore \quad \text{Required percentage} = \frac{\text{Total marks of Varun}}{\text{Total marks}} \times 100\% = \frac{377}{500} \times 100\% = 75.4\%$$

Ex. 2 If in order to pass the exam, a minimum of 95 marks are needed in Science, how many students pass in the exam?

Sol. Let pass percentage in Science be x, then

$$x\% \text{ of } 125 = 95$$

$$\Rightarrow \quad \frac{125 \times x}{100} = 95$$

$$\Rightarrow \quad x = \frac{95 \times 100}{125} = 76$$

Only three students will pass. (Rahul, Soham and Varun)

Ex. 3 What is the respective ratio of total marks obtained by Veena and Shreya together in Mathematics to the marks obtained by Rahul in the same subject?

Sol. Total marks obtained by Veena and Shreya together in Mathematics

$$= 74\% \text{ of } 150 + 80\% \text{ of } 150$$

$$= \frac{74 \times 150}{100} + \frac{80 \times 150}{100}$$

$$= 111 + 120 = 231$$

Marks obtained by Rahul in Mathematics $= 70\%$ of $150 = 150 \times \frac{70}{100} = 105$

$$\therefore \quad \text{Ratio} = \frac{231}{105} = \frac{11}{5} = 11 : 5$$

Directions (Examples 4-7) *Study the given tables carefully and answer the questions that follow.* **[Bank PO 2010]**

Number of candidates (in lakh) appearing in an entrance examination from six different cities and the ratio of candidates passing and failing in the same

City	A	B	C	D	E	F
Number of candidates	1.25	3.14	1.08	2.27	1.85	2.73

Ratio of candidates passing and failing within the city

City	Passing	Failing
A	7	3
B	5	3
C	4	5
D	1	3
E	3	2
F	7	5

Ex. 4 The number of candidates appearing for the exam from city C is what per cent of the number of candidates appearing for the exam from city B? (rounded off to be nearest integer.)

Sol. Required percentage

$$= \frac{1.08 \times 100}{3.14}\% \approx 34\%$$

Ex. 5 What is the respective ratio of the numbers of candidates failing in the exam from city D to those failing in the exam from city *A?*

Sol. Required ratio

$$= \frac{3}{4} \times 2.27 : \frac{3}{10} \times 1.25$$

$$= 1.7025 : 0.375 = 227:50$$

Ex. 6 Number of candidates passing the exam from city *F* is what per cent of the total number of candidates appearing from all the cities together? (rounded off to two digits after the decimal.)

Sol. Required percentage

$$= \frac{2.73 \times \frac{7}{12}}{1.25 + 3.14 + 1.08 + 2.27 + 1.85 + 2.73} \times 100\%$$

$$= \frac{1.5925}{12.32} \times 100\% = 12.93\%$$

Ex. 7 What is the number of unsuccessful candidates in city D?

Sol. Number of unsuccessful candidates in city *D*

$$= \left(2.27 \times \frac{3}{4}\right) \text{lakh}$$

$$= 1.7025 \text{ lakh}$$

Directions (Examples 8-11) *Study the following table carefully and answer the questions that*

follows. **[Bank PO 2010]**

Number of entertainment shows (in hundreds) held in various cities in a year

City	Show				
	Dance	Music	Drama	Stand up comedy	Mimicry
M	15	21	24	0.8	0.9
N	12.4	13	26	2	0.5
O	5.7	8	12	0.3	0.2
P	11.3	6	18	1	1.5
Q	17	12.4	11	3	0.4
R	14	10.5	9.8	0.7	0.1

Ex. 8 The mimicry shows held in city M are what per cent of the drama shows held in city O?

Sol. Mimicry shows held in city $M = 0.9$

Drama shows held in city $O = 12$

$\therefore$ Required percentage $= \frac{0.9 \times 100}{12}\% = 7.5\%$

Ex. 9 What is the average number of entertainment shows held in city *P?*

Sol. Average number of entertainment shows held in city P

$$= \left(\frac{11.3 + 6 + 18 + 1 + 1.5}{5}\right) \times 100$$

$$= 7.56 \times 100 = 756$$

Ex. 10 If the number of music shows in cities *N* and Q is increased by 5%, what will be the total number of music shows in both the cities together?

Sol. Music shows in cities N and Q

$$= (13 + 12.4)100 = 2540$$

After increasing by 5%, number of music shows

$$= 2540 \times \frac{105}{100} = 2667$$

Ex. 11 What is the number of stand up comedy shows held in all the cities together?

Sol. Total number of stand up comedy shows held in **all** the cities together = (0.8 + **2** + 0.3 + **1** + **3** + **0.7)** x **100** = 7.8 x **100** = 780

Fast Track Practice

Exercise Q *Base Level Questions*

Directions (Q. Nos. 1-4) *Study the following table and answer the questions based*

on it. [SSC CCL 2008]

The table given below depicts the export

of a commodity through four ports

in the year 1998 and 1999.

Port	Export in 1998 (in ₹ crore)	Export in 1999 (in ₹ crore)
A	57	61
B	148	160
C	229	234
D	146	150

1. The percentage increase in the export of the commodity from the year 1998 to 1999 was the highest from which port?

(a) A *(b)* B

(c) C *(d)* D

(e) None of the above

2. What was the change in the aggregate export of the commodity in the year 1999 as compared to the year 1998?

(a) Nearly 4.3% increase

(b) Nearly 4.3% decrease

(c) Nearly 0.04% increase

(d) Nearly 0.04% decrease

3. What was the average increase in the export of the commodity from the ports in the year 1999 as compared the year 1998?

(a) *X* 82500000

(b) ? 80000000

(c) ? 75000000

(d) ? 62500000

4. The percentage increase in the export of the commodity from the year 1998 to 1999 was the lowest from which port?

(a) A *(b)* B

(c) C *(d)* D

(e) None of the above

Directions (Q. Nos. 5-9) *Study the following table carefully and answer the questions given below.*

[Bank Clerks 2009]

Population (in lakh) of five states over the years

Year \ State	A	B	C	D	E
2003	15.6	22.6	18.4	16.6	24.2
2004	16.8	20.8	19.2	18.2	23.8
2005	18.8	24.2	19.8	17.8	25.8
2006	18.4	26.4	20.8	19.8	26.4
2007	20.2	28.2	22.6	22.4	28.2

5. What is the average population of state *B* (in lakh) for all the years together?

(a) 25.24 *(b)* 24.44 *(c)* 24.24 *(d)* 25.44 *(e)* None of the above

6. What is the ratio of the population of state *A* in 2003 to the population of state *E* in 2007?

(a) 37 : 22 *(b)* 22 : 37

(c) 47 : 26 *(d)* 26 : 47

(e) None of the above

7. Population of state *E* in 2004 is approximately what per cent of the population of state *D* in 2006?

(a) 85 *(b)* 95 *(c)* 110 *(d)* 130

(e) 120 What is the total population of all the states together (in lakh) in 2005?

(a) 106.4 *(b)* 98.4 *(c)* 96.8 *(d)* 102.8

(e) None of the above 9. Population of state *E* in 2007 is what per cent of the total population of all the states together in that year? (rounded off to nearest integer.)

(a) 26 *(b)* 25 *(C)* 23 *(d)* 21

(e) None of the above

Directions (Q. Nos. 10-14) *Study the following table carefully and answer the questions given below.*

[SBI Specialist Officer 2009]

Number of students enrolled with five colleges over the years

Year \ College	A	B	C	D	E
2004	450	320	400	480	520
2005	480	350	380	500	540
2006	420	300	410	520	460
2007	460	360	430	470	480
2008	470	340	390	530	530

10. If from college *B* in 2007, 80% of the students enrolled appeared in a competitive examination, out of which 75% students passed, how many students passed the examination?

(a) 180 *(b)* 216

(c) 270 *(d)* 240

(e) None of the above

11. In 2005, from all the colleges together on overall 40% of the students enrolled for a computer course. How many students enrolled for the course?

(a) 800 *(b)* 850

(c) 950 *(d)* 900

(e) None of the above

12. What is the ratio between the average number of students enrolled with all colleges together in 2007 and 2008, respectively?

(a) 108 : 113

(b) 108 : 117

(c) 110 : 113

(d) 111 : 113

(e) None of the above

13. Average number of students enrolled from college *B* for all the years together is approximately what per cent of the average number of students enrolled from college *C* for all the years together?

(a) 73 *(b)* 120

(c) 128 *(d)* 70

(e) 83

14. In 2006, from all colleges together, 8% of the students enrolled went abroad. Approximately, how many students went abroad?

(a) 170 *(b)* 210

(c) 220 *(d)* 190

(e) 150

Directions (Q. Nos. 15-18) *Study the following table carefully and answer the*

questions. [Bank Clerks 2009]

Number of students studying five different disciplines from five institutes

Disci-pline/ Institute	Discipline				
	Art	Com-merce	Science	Manage ment	Com-puter Science
A	350	260	450	140	300
B	240	320	400	180	320
C	460	300	360	160	380
D	440	480	420	120	340
E	280	360	340	200	330

15. What is the average number of students studying commerce from all the institutes together?

(a) 356 *(b)* 360 *(c)* 348 *(d)* 340 *(e)* None of the above

16. Total number of students studying Art from institutes *A* and *B* together is approximately, what per cent of total number of students studying Computer Science from these two institutes?

(a) 84% *(b)* 85% *(c)* 88% *(d)* 90% *(e)* 95%

17. Number of students studying Commerce from institute *D* is what per cent of the total number of students studying all the disciplines together from this institutes?

(a) $28\frac{1}{3}$% *(b)* $28\frac{2}{3}$% *(c)* $24\frac{2}{3}$% *(d)* $24\frac{1}{3}$%

(e) None of the above

18. What is the ratio of the total number of students studying Science from institutes *C* and *D* together to the total number of students studying Computer Science from these two institutes together? *(a)* 13 : 12 *(b)* 12 : 13

(c) 13 : 15 *(d)* 15 : 13

(e) None of the above

Directions (Q. Nos. 19-23) *Read the information carefully and answer the questions given below.* [ssc **ccl** 1999]

A survey of film watching habits of people living in five cities *P, Q, R, S* and *T* is summarised below in a table. The Column I in the table gives percentage of film-watchers in each city who see only one film in a week. The Column II gives the total number of film-watchers who see two or more films per week.

City	I	II
P	60	24000
Q	20	30000
R	85	24000
S	55	27000
T	75	80000

19. How many film-watchers in city *R* see only one film a week?

(a) 24850 *(b)* 36000

(c) 136000 *(d)* 160000

20. Which city has the maximum number of film-watchers who see only one film a week?

(a) P (b) R (c) S (d) T

21. A city with the minimum number of film-watchers is

(a) P (b) Q (c) S (d) T

22. The maximum number of film-watchers in any given city is

(a) Q (b) R (c) S (d) T

23. The total number of all film-watchers in the five cities who see only one film in a week is

(a) 113000 *(b)* 425200

(c) 452500 *(d)* 500000

Directions (Q. Nos 24-27) *Read the table and answer the questions.* [ssc cpo 2013]

The following table gives demand and supply of sugar in million tons, for the period 2007 to 2012. Surplus is defined as excess of supply over demand

Year	Demand	Supply
2007	43.3	46.4
2008	47.2	47.8
2009	49.5	50.7
2010	53.4	54.2
2011	54.5	57.3
2012	62.7	63.4

24. In how many years, the supply (in million tons) of sugar was more than the average supply from 2007 to 2012?

(a) 5 *(b)*2 (cJ3 *(d)A*

25. The average surplus (in million tons) of sugar for the period 2008 to 2012 is

(a) 1.44 *(b)* 1.84 *(c)* 1.53 *(d)* 1.22

26. The surplus of sugar was lowest for the year

(a) 2012 *(b)* 2008 *(c)* 2009 *(d)* 2010

27. The percentage increase in demand of sugar was the lowest as compared to its previous year in

(a) 2011 *(b)* 2008

(c) 2009 *(d)* 2010

Directions (Q. Nos. 28-32) *Read the information carefully and answer the questions given below.* [SSCCGL2000]

The table given below shows production of five types of cars by a company in the years 1989 to 1994

Type \ Year	1989	1990	1991	1992	1993	1994	Total
P	8	20	16	17	21	6	88
Q	16	10	14	12	12	14	78
R	21	17	16	15	13	8	90
S	4	6	10	16	20	31	87
T	25	18	19	30	14	27	133
Total	74	71	75	90	80	86	476

28. In which year, the production of cars of all types taken together was approximately equal to the average of the total production during the period?

(a) 1989 *(b)* 1991 *(C)* 1993 (d) 1994

29. In which year, the total production of cars of types *P* and *Q* together was equal to the total production of cars of types *R* and *S* together?

(a) 1990 *(b)* 1991

(c) 1994 *(d)* 1993

30. During the period 1989-94, which type of cars had a continuous increase in production?

(a) P (b) Q *(c) R (d) S*

31. The production of which type of cars was 25% of the total production of all types of cars during 1993?

(a) P (b) R (c) Q *(d) S*

32. The per cent increase in total production of all types of cars in 1992 to that in 1991 was

(a) 15 *(b)* 20 *(c)* 25 *(d)* 30

Directions (Q. Nos. 33-37) *Read the following table and answer the questions that follows.* [ssc cgl 1997]

Number of males and females who read different magazines and who are of different age groups

Age group (in year)	Magazines Read						Total sample surveyed (including non-readers)	
	Sport		Film		Both			
	Male	Female	Male	Female	Male	Female	Male	Female
10-15	40	30	30	20	10	15	100	120
16-35	160	120	180	100	80	65	240	150
36-60	50	40	40	50	30	20	200	430

33. The number of people who read at least one type of magazine and are over 35 yr in age, is

(a) 36 *(b)* 130

(c) 230 (d) 180

34. The number of people in the age group 10-15 who reads only one type of magazine, is

(a) 25 (b) 70

(c) 95 (d) 120

35. The number of females in the age group 16-35 who do not read sports magazine, is

(a) 120 (b) 90

(c) 60 (d) 30

36. The number of males in the age group 16-35 who do not read film magazines, is

(a) 60 (b) 80

(c) 140 (d) 190

37. What per cent of people over 35 yr do not read either type of magazine?

(a) 14% (b) 50.27%

(c) 54% (d) 63.49%

Directions (Q. Nos. 38-40) *The table below shows the number of people who responded to a survey about their favourite style of music. Use this information to answer the following questions to the nearest whole percentage.* umap 2012]

Age	15-20	21-30	31+
Classical	6	4	17
Pop	7	5	5
Rock	6	12	14
Jazz	1	4	11
Blues	2	3	15
Hip-Hop	9	3	4
Ambient	2	2	2
	33	33	68

38. What percentage of respondents under 31 indicated that Blues is their favourite style of music?

(a) 7.1% (b) 7.6%

(c) 8.3% (d) 14.1

39. What percentage of respondents aged 21-30 indicated a favourite style other than Rock music?

(a) 64% (b) 60%

(c) 75% (d) 36%

40. What percentage of the total sample indicated that Jazz is their favourite style of music?

(a) 6% (b) 8%

(c) 22% (d) 12%

Directions (Q. Nos. 41-44) *Read the information carefully and answer the Questions aiven below.* rssc ccl 20041

The table given below shows a survey carried out at a railway station for the arrivals/ departures of trains for the month of January, 2000

Delay (in min)	Number of arrivals	Number of departures
0	1250	1400
0-30	114	82
30-60	31	5
Over 60	5	3
Total	1400	1490

The total number of late arrivals of trains is *(a)* 90 *(b)* 95 *(C)* 145 *(d)* 150

42. The total number of late departures of trains is

(a) 85 (b) 87 (c) 90 (d) 150

43. The percentage of number of trains arriving late at the station is

(a) 6% (b) 10.4%(cJ 10.7% (d) 10.9%

44. If the punctuality of railways is defined as the number of occasions on which trains arrived or departed in time as a percentage of total number of arrivals and departures from the station, then the punctuality for the month under observation is

(a) 94.3% *(b)* 91.7%(cj 89.2% *(d)* 75%

Directions (Q. Nos. 45-49) *Study the table and answer the questions that follow.*

[SSC CGL 2004]

Yearly production (in thousand) of scooters in different factories

Factory \ Year	1985	1986	1987	1988	1989
P	20	15	24	13	17
Q	16	23	41	20	15
R	14	21	30	16	12
S	25	17	15	12	22
T	40	32	39	41	35
Total	115	108	149	102	101

45. In which year, the production of scooters of all factories was equal to the yearly average number of scooters produced during 1985-89? *(a)* 1985 *(b)* 1986 *(C)* 1987 *(d)* 1988

46. Which factory/factories showed a decrease of 25% in the production of scooters in 1989 as compared to 1988?

(a) P (b) S

(c) Q and *R (d) P* and 7

47. The ratio of the production of scooters by factory *P* to that by factory *T* in 1985 is

(a) 2 : 3 *(b)* ^ :2

(c) 3 : 2 *(d)* 2 : 1

48. In which year was the total production of scooters the maximum?

(a) 1989 *(b)* 1986 *(c)* 1987 *(d)* 1985

49. In which year was the total production of scooters of all factories 20% of the total production of scooters during 1985-1989?

(a) 1988 *(b)* 1985 *(c)* 1986 *(d)* 1989

Directions (Q. Nos. 50-54) *Study the following table carefully and answer the questions given below.* [Bank po 201 o]

Number of tickets sold in a week of five

movies in the multiplexes in six different

cities [number in thousands]

City \ Movie	A	B	C	D	E
Mumbai	20	15	35	26	18
Delhi	17	19	21	25	28
Kolkara	32	24	19	21	17
Chennai	18	21	32	28	34
Hyderabad	16	34	26	29	22
Lucknow	15	27	20	35	26

50. The number of tickets of movie *B* sold in Hyderabad was approximately, what per cent of the total number of tickets of the same movie sold in all the cities together?

(a) 15 *(b)* 18

(c) 12 *(d)* 20

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51. What is the difference between the number of tickets of movie *D* sold in Kolkata and the number of tickets of movie *B* sold in Lucknow?

(a) 700 *(b)* 7000

(c) 14000 *(d)* 9000

(e) None of the above

52. What is the average number of tickets of movie C sold in all the six cities?

(a) 15500 *(b)* 2550

(c) 24000 *(d)* 25500

(e) None of the above

53. The number of tickets of movie *E* sold in Chennai is what per cent of number of tickets of movies *A* sold in Mumbai?

(a) 170 *(b)* 70 *(c)* 30 *(d)* 130 *(e)* None of the above

54. In which city was the total number of tickets of all the five movies together sold the minimum?

(a) Delhi *(b)* Chennai

(c) Lucknow *(d)* Kolkata

(e) None of the above

Exercise© *Higher Skill Level Questions*

Directions (Q. Nos. 1-5) *Study the following table carefully to answer the questions*

that follow. [Bank PO 2010]

Number (N) of six type of electronic products sold by six different stores in a month and the price per product (p) (price in ₹ 1000) charged by each store

Store	Product											
	A		B		C		D		E		F	
	N	P	N	P	N	P	N	P	N	P	N	P
L	54	135	48	112	60	104	61	124	40	136	48	126
M	71	4.5	53	3.8	57	5.6	49	4.9	57	5.5	45	4.7
N	48	12	47	18	52	15	54	11.5	62	10.5	56	11
O	52	53	55	48	48	50	54	49	59	47	58	51
P	60	75	61	68	56	92	44	84	46	76	59	78
Q	43	16	44	15	45	14.5	48	15.6	55	18.2	55	14.9

1. What is the total amount earned by store *C* through the sale of *M* and O type products together?

(a) X 2719.2 lakh *(b)* X 271.92 lakh *(c)* X 2.7192 lakh *(d)* X 27.192 lakh

2. Number of *L* type product sold by store *F* is what percent of the number same type of products sold by store *El*

(a) 76.33% *(b)* 124%

(c) 83.33% *(d)* 115%

(e) None of the above

3. What is the difference in the amount earned by store *A* through the sale of *P* type products and that earned by store *B* through the sale of *Q* type products?

(a) X 38.4 lakh *(b)* X 0.384 lakh *(c)* X 3.84 lakh *(d)* X 384 lakh

4. What is the respective ratio of total number of *N* and *L* type products together sold by store *D* and the same products sold by store A?

(a) 119 : 104 *(b)* 102 : 115

(c) 104 : 115 *(d)* 117 : 103

(e) None of the above

5. What is the average price per product charged by all the stores together for product *Q?*

(a) ? 14700 *(b)* ? 15700

(c) ? 15200 *(d)* X 14800

(e) None of the above

Directions (Q. Nos. 6-10) *Study the following table carefully to answer the questions that follow.* [Bank po 2010]

Number (N) of candidates tin lakhs] appearing for an entrance examination

from six different states and the

percentage *(.pi* of candidates clearing

the same over the years

St.	A		B		C		D		E		F	
Yr	N	P	N	P	N	P	N	P	N	P	N	P
2004	1.23	42	1.04	51	1.11	32	1.32	24	1.23	36	1.33	31
2005	1.05	43	1.12	62	1.07	47	1.15	49	1.18	55	1.24	24
2006	2.04	38	1.48	32	1.08	28	1.96	35	1.42	49	1.58	26
2007	1.98	41	2.07	43	1.19	30	1.88	46	1.36	47	1.79	29
2008	1.66	53	1.81	50	1.56	42	1.83	60	1.73	57	1.86	34
2009	1.57	39	1.73	36	1.64	52	2.01	56	1.69	55	1.95	37

6. In which year did the highest number of candidates clear the entrance exam from state *Dl*

(a) 2008 *(b)* 2006 *(c)* 2009 *(d)* 2007 *(e)* None of the above

7. What is the respective ratio of total number of candidates clearing the entrance exam from state *B* in the year 2004 to those clearing the entrance exam from state *C* in the same year?

(a) 221 : 148 *(b)* 218 : 143

(c) 148 : 221 *(d)* 143 : 218

(e) None of the above

8. What is the total number of candidates clearing the entrance exam from states *E* and *F* together in the year 2006?

(a) 16160 *(b)* 110660

(c) 1.1066 *(d)* 1106600

(e) None of the above

9. What is the average number of candidates appearing for the entrance exam from state *D* in the years 2007, 2008 and 2009 together?

(a) $1.907\frac{2}{3}$ (b) $18666\frac{1}{3}$

(c) $1.866\frac{1}{3}$ (d) $190666\frac{2}{3}$

(e) None of the above

10. What is the number of candidates not clearing the entrance exam from state *A* in the year 2007? *(a)* 186820 *(b)* 11682

(c) 1868200 *(d)* 116820

(e) None of the above

Directions (Q. Nos. 11-15) *Study the table carefully and answer the questions that follow.*

Profit (in ₹ 1000) made by six different shopkeepers over the months

Shopkeeper \ Month	Oct. 2009	Nov. 2009	Dec. 2009	Jan. 2010	Feb. 2010	March 2010
P	5.25	6.04	5.84	6.10	5.95	6.02
Q	4.84	4.28	4.97	4.88	5.04	5.12
R	4.99	5.82	5.48	5.45	5.68	5.36
S	5.06	5.11	5.28	5.38	5.44	5.59
T	5.28	4.96	5.31	5.69	4.93	5.72
U	5.94	6.23	5.87	6.07	6.19	6.23

Which shopkeeper's profit kept increasing continuously over the given months?

(a) R *(b)* Q *(c)* T *(d)* U

(e) None of the above

12. What was the average profit earned by shopkeeper *R* in the months of October

2009 and November 2009 together? *(a)* ? 5405 *(b)* ? 5040

(c) ? 4825 *(d)* ? 4950

(e) None of the above

13. What is the percentage increase in profit of shopkeeper *S* in the month of December 2009 over the previous month?

(rounded off to two-digits after decimal.) *(a)* 3.15% *(b)* 2.67%

(c) 2.18% *(d)* 3.33%

(e) None of the above What is the respective ratio between the profit earned by shopkeeper *U* in the months of February 2010 and March

2010 together to that earned by shopkeeper *Q* in the same months?

b) 637 : 512 *(b)* 621 : 508

(c) 512 : 637 *(d)* 508 : 621

(e) None of the above

15. What is the difference in profit earned by shopkeeper *T* in January 2010 from the previous month?

fa; *X* 640 *(b) X* 420 *(c) X* 380 fd; *X* 760

(e) None of the above

Directions (Q. Nos. 16-20) *Study the following tables carefully and answer the questions given below them.*

[Bank PO 2010]

Number of candidates appeared in a competitive examination from five centres over the years

Centre / Year	Mumbai	Delhi	Kolkata	Hyderabad	Chennai
2001	35145	65139	45192	51124	37346
2002	17264	58248	52314	50248	48932
2003	24800	63309	56469	52368	51406
2004	28316	70316	71253	54196	52315
2005	36503	69294	69632	58360	55492
2006	29129	59216	64178	48230	57365
2007	32438	61345	56304	49178	58492

Approximate percentage of candidates qualified to appeared in the competitive examination from five centres over the years

Mumbai	Delhi	Kolkata	Hyderabad	Chennai
12	24	18	17	9
10	28	12	21	12
15	21	23	25	10
11	27	19	24	8
13	23	16	23	13
14	20	21	19	11
16	19	24	20	14

Approximately, what was the difference between the number of candidates qualified from Hyderabad in 2001 and 2002?

fa; 1680 *(b)* 2440

(c) 1450 *(d)* 2060

(e) 1860

17. Approximately, what was the total number of candidates qualified from Delhi in 2002 and 2006 together?

(a) 27250 *(b)* 25230

(c) 30150 *(d)* 28150

(e) 26250

18. In which of the following years, was the difference of number of candidates appeared from Mumbai over the previous year the minimum?

(a) 2004 (b) 2006

(c) 2007 (d) 2002

fe) None of the above

19. In which of the following years, was the number of candidates qualified from Chennai, the maximum among the given years?

(a) 2007 (b) 2006 (c) 2005 (d) 2003

(e) None of the above

20. Approximately, how many candidates appearing from Kolkata in 2004 qualified in the competitive examination?

fa; 13230fb; 13540(c; 15130 (d) 15400

(e) 19240

Directions (Q. Nos. 21-25) *Study the following table carefully and answer the questions given below it.* [Bank po 2010]

Percentage of marks obtained by various students in various subjects in an examination

Student	Marks					
	English (out of 100)	Hindi (out of 100)	Science (out of 150)	History (out of 60)	Mathematics (out of 150)	Geography (out of 40)
A	68	75	82	60	96	55
B	88	73	85	65	88	65
C	75	56	72	75	75	80
D	70	66	80	80	72	62
E	72	60	68	74	68	75
F	85	70	90	70	74	70

21. What is approximate overall percentage obtained by C in the examination?

(a) 78 (b) 69 (c) 75 (d) 71

(e) 65

22. What is the difference in the marks obtained by *B* in English and Mathematics together and the marks obtained by *F* in the same subjects?

(a) 24 *(b)* 17 *(c)* 15 *(d)* 28

(e) None of the above

23. The marks obtained by *E* in Geography are what per cent of the marks obtained by *E* in Hindi?

fa; 45% *(b)* 55% *(c)* 50% *(d)* 60% *(e)* None of the above

24. What is the overall percentage marks obtained by *D* in History and Geography together?

(a) 73.40 *(b)* 72.80 *(c)* 70.50 *(d)* 68.80

(e) None of the above

25. What is the average marks obtained by all the students together in Science?

fa; 77.16 *(b)* 120.50

(c) 118 *(d)* 121

(e) None of the above

Answer with Solutions

Exercise *Base Level Questions*

1. (*b*) Percentage increase from port *A*

$$= \frac{61 - 57}{57} \times 100\% = 7.01\ \%$$

Percentage increase from port *B*

$$= \frac{160 - 148}{148} \times 100\% = 8.10\%$$

Percentage increase from port *C*

$$= \frac{234 - 229}{229} \times 100\% = 2.18\%$$

Percentage increase from port *D*

$$= \frac{150 - 146}{146} \times 100 = 2.74\ \%$$

So, the highest increment is from port *B*.

2. (*a*) Total export in year 1998

$$= 57 + 148 + 229 + 146 = 580 \text{ crore}$$

Total export in year 1999

$$= 61 + 160 + 234 + 150 = 605 \text{ crore}$$

$$\text{Required percentage} = \frac{605 - 580}{580} \times 100\%$$

$$= 4.3\% \text{ increase}$$

3. (*d*) Total increase = 605 – 580 = 25 crore

$$\therefore \text{ Average increase} = \frac{25}{4} \text{ crore}$$

$$= 62500000$$

4. (*c*) It is clear from solution 1 that lowest increment is from port *C*.

5. (*b*) Required average

$$= \frac{22.6 + 20.8 + 24.2 + 26.4 + 28.2}{5}$$

$$= \frac{122.2}{5} = 24.44$$

6. (*d*) Required ratio

$$= \frac{\text{Population of State } A \text{ in 2003}}{\text{Population of State } E \text{ in 2007}}$$

$$= \frac{15.6}{28.2} = 26 : 47$$

7. (*e*) Required percentage

$$= \frac{23.8}{19.8} \times 100\% \approx 120\%$$

8. (*a*) Total population

= (18.8 + 24.2 + 19.8 + 17.8 + 25.8) lakh

= 106.4 lakh

9. (*c*) Required percentage

$$= \frac{28.2}{20.2 + 28.2 + 22.6 + 22.4 + 28.2} \times 100\%$$

$\approx 23\%$

10. (*b*) In year 2007, number of students enrolled in college *B* is 360.

Number of students passed the examination

$\Rightarrow \quad 80\%$ of 75% of 360

$$\Rightarrow \quad 360 \times \frac{80}{100} \times \frac{75}{100} = 36 \times 6 = 216$$

11. (*d*) In the year 2005,

Total number of students in all the colleges

$= 480 + 350 + 380 + 500 + 540 = 2250$

Number of students enrolled for a Computer course

$$= 40\% \text{ of } 2250$$

$$= \frac{40}{100} \times 2250 = 900$$

12. (*c*) In the year 2007,

Total number of students in all colleges

$$= 460 + 360 + 430 + 470 + 480$$
$$= 2200$$

$$\text{Average number of students} = \frac{2200}{5} = 440$$

In year 2008,

Total number of students in all the colleges

$$= 470 + 340 + 390 + 530 + 530$$
$$= 2260$$

$$\text{Average number of students} = \frac{2260}{5} = 452$$

So, ratio

$$= \frac{440}{452} = \frac{110}{113} = 110 : 113$$

13. (*e*) Total number of students in college *B* in all the years

$\Rightarrow 320 + 350 + 300 + 360 + 340 = 1670$

$\therefore$ Average number of students

$$= \frac{1670}{5} = 334$$

Total number of students in college *C* in all the years

$\Rightarrow 400 + 380 + 410 + 430 + 390 = 2010$

$\therefore$ Average number of students

$$= \frac{2010}{5} = 402$$

So, required percentage

$$= \frac{\text{Average number of students in } B}{\text{Average number of students in } C} \times 100\%$$

$$= \frac{334}{402} \times 100\% = 83.08\% \approx 83\%$$

14. (*a*) In the year 2006, total number of students in all the colleges

$= 420 + 300 + 410 + 520 + 460 = 2110$

So, number of students went abroad

$$= 8\% \text{ of } 2110$$

$$= \frac{8}{100} \times 2110 = \frac{1688}{10} \approx 170$$

15. (*e*) Required average number of students

$$= \frac{260 + 320 + 300 + 480 + 360}{5}$$

$$= \frac{1720}{5} = 344$$

16. (*e*) Total number of students studying Art from in stitutes A and $B = 350 + 240 = 590$

Total number of students studying Comp -uter Science from institutes A and B

$$= 300 + 320 = 620$$

$\therefore$ Required percentage $= \frac{590}{620} \times 100\%$

$$= 95.16\% \approx 95\%$$

17. (*b*) Number of students studying commerce from institute $D = 480$

Total number of students studying all discipline from institute D

$$= 440 + 480 + 420 + 120 + 340$$
$$= 1800$$

$\therefore$ Required percentage $= \frac{480}{1800} \times 100\%$

$$= \frac{480}{18}\% = \frac{80}{3}\% = 26\frac{2}{3}\%$$

18. (*a*) Total number of students studying science from institutes C and D

$$= 360 + 420 = 780$$

Total number of students studying Computer Science from institutes C and D

$$= 380 + 340 = 720$$

$\therefore$ Required ratio $= \frac{780}{720} = 13 : 12$

19. (*c*) Percentage of film-watcher who see only one film in a week (Coloum I) from city $R = 85\%$

$\therefore$ Percentage of film-watcher who see two or more films in a week (Coloum II) from city $R = (100 - 85) = 15\%$

$\therefore \quad 15\% = 24000$

$\therefore \quad 85\% = \frac{24000}{15} \times 85 = 136000$

20. (*d*) The number of film-watchers who see only one film in a week in city

$$P = \frac{24000}{40} \times 60 = 36000$$

$$Q = \frac{30000}{80} \times 20 = 7500$$

R = 5^x85 = 136000 15

S = ^{2}*Jmx55* = *33Q00* 45

T = ?522?. x 75 = 240000 25

Note

Total = 100%

_______ I II

P 60 40

Q 20 80

R 85 15

S 55 45

T 75 25

Hence, maximum number of one film-watchers is in *T*.

21. (6) *p* = ?i°°° x ioo = 60000

40

Q = 52^x100 = 37500 80

R = ^2°^ x 100 = 160000

15 s = 27000_xl00=60000

45

T = ?525P_ x 100 = 320000 25

Hence, *Q* has minimum number of film-watchers.

22. (d) From solution 21, it is clear that T has maximum number of film-watchers.

23. (c) From solution 20, required values

= 36000 + 7500 + 136000

+ 33000 + 240000 = 452500

24. (c) Average supply of sugar from the year 2007 to 2012

_ 46.4 + 47.8 + 50.7 + 54.2 + 57.3 + 63.4

6

319 8

=-----— = 53.3 millions tons

6 ∴ In three years 2010, 2011 and 2012, the supply of sugar was more than average supply.

25. (d) Surplus of sugar

in 2008 = 47.8 - 47.2 = 0.6 in 2009 = 50.7 - 49.5 = 1.2 in 2010 = 54.2-53.4 =0.8 in 2011 = 57.3-54.5 = 2.8 in 2012 = 63.4 -62.7 = 0.7 .". Required average surplus

_ 0.6 + 1.2 + 0.8 + 2.8 + 0.7

5

= —^- = 1.22 million tons

5

26. (*b*) It is clear from solution number (25) that surplus of sugar was lowest for the year 2008.

27. (*a*) Percentage increase in demand sugar as compared to its previous year

in 2008 $= \dfrac{47.2 - 43.3}{43.3} \times 100\% = 9\%$

in 2009 $= \dfrac{49.5 - 47.2}{47.2} \times 100\% = 4.87\%$

in 2010 $= \dfrac{53.4 - 49.5}{49.5} \times 100\% = 7.88\%$

in 2011 $= \dfrac{54.5 - 53.4}{53.4} \times 100\% = 2.05\%$

Thus, percentage increase was lowest in the year 2011.

28. (*c*) Average of total production $= \dfrac{476}{6}$

≈ 80 which was in year 1993.

29. (*d*) It is clear from the table that year 1993 satisfies the given condition.

30. (*d*) Clearly, the cars of type S had continuous increase in production.

31. (*d*) $80 \times \dfrac{25}{100} = 20$ (S in 1993)

32. (*b*) Required percentage

$$= \frac{90 - 75}{75} \times 100\% = 20\%$$

33. (*c*) Required number of people

$50 + 40 + 40 + 50 + 30 + 20 = 230$

34. (*d*) Required number of people

$40 + 30 + 30 + 20 = 120$

35. (*d*) Required number of people

$150 - 120 = 30$

36. (*a*) Required number of people

$240 - 180 = 60$

37. (*d*) Total people who are above 35 and non-readers $= 200 + 430 = 630$

Total readers over 35 = 230

$\therefore$ Total readers over 35 yr do not read either type of magazine

$$= 630 - 230 = 400$$

Now, $\quad 630 \times \dfrac{x}{100} = 400$

$\therefore \quad x = \dfrac{400 \times 100}{630} = 63.49\ \%$

38. (*b*) Number of respondents under 31 who's favourite style of music is blues

$$= 2 + 3 = 5 \quad \begin{cases} 2 \text{ from age group of } 15-20 \\ 3 \text{ from age group of } 21-30 \end{cases}$$

Total respondents below 31

$= 33 + 33 = 66$

$\therefore$ Percentage of respondents

$$= \frac{5}{66} \times 100 = 7.57 = 7.6$$

39. (*a*) Total respondents between 21-30 who's favourite style is other than rock

$= (4 + 5 + 4 + 3 + 3 + 2) = 21$

and total respondent $= 33$

$\therefore$ Per cent of respondents who's favourite style is other than rock

$$= \frac{21}{33} \times 100 = 64\%$$

40. (*d*) Total respondent who's favourite style of music is Jazz $= (1 + 4 + 11) = 16$

Total respondent

$= (33 + 33 + 68) = 134$

$\therefore$ Percentage of respondent

$$= \frac{16}{134} \times 100 = 12\%$$

41. (*d*) Total number of late arrivals

$= 114 + 31 + 5 = 150$

42. (*c*) Total number of late departures

$= 82 + 5 + 3 = 90$

43. (*c*) Required percentage

$$= \frac{150}{1400} \times 100 = 10.7\%$$

44. (*b*) Rquired percentage

$$= \frac{1250 + 1400}{1400 + 1490} \times 100$$

$$= \frac{2650}{2890} \times 100 = 91.7\%$$

45. (*a*) Average number of scooters produced during 1985-1989

$$= \frac{115 + 108 + 149 + 102 + 101}{5}$$

$$= \frac{575}{5} = 115$$

= Average production in year 1985

46. (*c*) $Q = \dfrac{20 - 15}{20} \times 100 = 25\%$

and $R = 25\%$ also.

47. (*b*) Required ratio $= 20 : 40 = 1 : 2$

48. (*c*) 1987

49. (*b*) Total production of scooter during 1985-1989

$\Rightarrow \quad 115 + 108 + 149 + 102 + 101$

$\Rightarrow \quad \dfrac{575 \times 20}{100} = 115$

= year 1985

50. (e) Total number of tickets of movie B sold in all the cities together

$$= 15 + 19 + 24 + 21 + 34 + 27$$
$$= 140 \text{ thousand}$$

Number of tickets sold of movie B in Hyderabad = 34 thousand

So, required percentage

$$= \frac{34}{140} \times 100 \approx 24$$

(e) Number of tickets sold of film D in

Kolkata = 21000

Number of tickets sold of film B in Lucknow

=27000 So, required difference

= 27000-21000 = 6000

52. (d) Total number of tickets of movie C sold in all cities

= 35 + 21 + 19 + 32+ 26 + 20 = 1 Fir? thnnsanri

$$\text{So, required average} = \frac{153000}{6} = 25500$$

53. (a) Number of tickets sold of movie E in Chennai = 34 thousand

Number of tickets sold of movie A in Mumbai = 20 thousand So, required percentaqe

$$= \frac{34}{20} \times 100 = 170$$

54. (a) Total number of tickets sold in Mumbai

= 114 thousand Total number of tickets sold in Delhi

= 110 thousand Total number of tickets sold in Kolkata

= 113 thousand Total number of tickets sold in Chennai

= 133 thousand Total number of tickets sold in Hyderabad

= 127 thousand Total number of tickets sold in Lucknow

= 123 thousand So, lowest tickets sold in Delhi.

Exercise © *Higher Skill Level Questions*

1. (*d*) Total amount earned by store C through the sales of M and O type products together

$= ₹\,(57 \times 5.6 + 48 \times 50)$ thousand

$= ₹\,(319.2 + 2400)$ thousand

$= ₹\,27.192$ lakh

2. (*e*) Number of L type products sold by store $F = 48$

Number of L type product sold by store $E = 40$

$\therefore$ Required percentage

$$= \frac{48}{40} \times 100\% = 120\%$$

3. (*a*) Required difference

$= ₹\,(60 \times 75 - 44 \times 15)$ thousand

$= ₹\,(4500 - 660)$ thousand

$= ₹\,38.4$ lakh

4. (*e*) Required ratio $= (61 + 54) : (54 + 48)$

$= 115 : 102$

5. (*b*) Required average

$$= ₹\left(\frac{16+15+14.5+15.6+18.2+14.9}{6}\right)$$

$$= ₹\left(\frac{94.2}{6}\right) \text{thousand} = ₹\,15700$$

6. (c) Number of students passed in 2004 from state *D* = 132000 x 0.24 = 31680 Number of students passed in 2005

= 115000 X0.49 = 56350 Number of students passed in 2006

= 196000 x 0.35 = 68600

Number of students passed in 2007

$= 188000 \times 0.46 = 86480$

Number of students passed in 2008

$= 183000 \times 0.60 = 109800$

Number of student passed in 2009

$= 201000 \times 0.56 = 112560$

So, maximum number was in 2009.

7. (*a*) Required ratio $= \frac{104000 \times 0.51}{111000 \times 0.32}$

$= \frac{53040}{35520} = \frac{221}{148} = 221 : 148$

8. (*b*) Required number

$= 142000 \times 0.49 + 158000 \times 0.26$

$= 69580 + 41080 = 110660$

9. (*d*) Required average

$= \frac{188000 + 183000 + 201000}{3}$

$= \frac{572000}{3} = 190666\frac{2}{3}$

10. (*d*) Percentage of candidates not clearing the entrance exam from state *A* in the year 2007 = (100 − 41)% = 59%

So, required number of candidates

$= 198000 \times 0.59 = 116820$

11. (*e*) It is clear from table that profit of *S* is increasing continuously.

12. (*a*) Required average $= \frac{4.99 + 5.82}{2} \times 1000$

$= ₹\ 5405$

13. (*d*) Required percentage

$= \frac{5.28 - 5.11}{5.11} \times 100\% = 3.33\%$

14. (*b*) Required ratio $= \frac{6.19 + 6.23}{5.04 + 5.12} = \frac{12.42}{10.16}$

$= \frac{621}{508} = 621 : 508$

15. (c) Required difference

= ? (5.69 - 5.31) X 1000 = ? 380

16. (e) Number of candidates qualified from Hyderabad in 2001

= 51124 X 0.17 = 8691 Number of candidates qualified from Hyderabad in 2002

= 50248 X0.21 = 10552 So, required difference

= 10552 - 8691 = 1861«1860

17. (d) Number of candidates qualified from Delhi in 2002 = 58248 X 0.28 = 16309 Number of candidates qualified from Delhi in 2006 = 59216 X 0.20« 11843

So, required number

= 16309 + 11843 = 28152= 28150

18. (c) Candidates appeared from Mumbai Difference in 2002 = 35145 - 17264

=17881 Difference in 2003 = 24800 - 17264

= 7536 Difference in 2004 = 28316 - 24800

= 3516 Difference in 2005 = 36503 - 28316

= 8187 Difference in 2006 = 36503 - 29129 = 7374 Difference in 2007 = 32438 - 29129 = 3309 So, least difference was in 2007.

19. (a) Number of candidates qualified from Chennai

In 2003 = 51406 X0.10 => 5141 In 2005 = 55492 X0.13 « 7214 In 2006 = 57365 x 0.11 <= 6310 In 2007 = 58492 X 0.14 => 8189 So, maximum number was in 2007.

20. (6) Number of candidates qualified from Kolkata in 2004

= 71253X0.19 = 13539=13540

21. (*d*) Marks obtained by *C* (in percentge)

$$= \left(100 \times \frac{75}{100} + 100 \times \frac{56}{100} + 150 \times \frac{72}{100} \right.$$

$$+ 60 \times \frac{75}{100} + 150 \times \frac{75}{100} + 40 \times \frac{80}{100}\Bigg) \times \frac{100}{600}\%$$

$$= \frac{75 + 56 + 108 + 45 + 112.5 + 32}{6}\%$$

$$= \frac{428.5}{6}\% = 71.41\% = 71\%$$

22. (*a*) Marks obtained by B in Mathematics and English $= 100 \times \frac{88}{100} + 150 \times \frac{88}{100}$

$$= 88 + 132 = 220$$

Marks obtained by F in Mathematics and English

$$= 100 \times \frac{85}{100} + 150 \times \frac{74}{100}$$

$$= 85 + 111 = 196$$

$\therefore$ Difference $= 220 - 196 = 24$

23. (*c*) Marks obtained by E in Geography

$$= 40 \times \frac{75}{100} = 30$$

Marks obtained by E in Hindi

$$= 100 \times \frac{60}{100} = 60$$

Now, $60 \times x\% = 30$

$\therefore \quad x = 50\%$

24. (*b*) Marks obtained by D in History

$$= 60 \times \frac{80}{100} = 48$$

and in Geography $= 40 \times \frac{62}{100} = 24.8$

Total marks obtained $48 + 24.8 = 72.8$

Total marks $= 60 + 40 = 100$

$\therefore$ Percentage marks

$$= \frac{72.8}{100} \times 100\% = 72.8\%$$

25. (*e*) Average marks of all students in Science

$$= \frac{150 \times \frac{82}{100} + \frac{85}{100} \times 150 + 150 \times \frac{72}{100} + 150 \times \frac{80}{100} + 150 \times \frac{68}{100} + 150 \times \frac{90}{100}}{6}$$

$$= \frac{150\left(\frac{82}{100} + \frac{85}{100} + \frac{72}{100} + \frac{80}{100} + \frac{68}{100} + \frac{90}{100}\right)}{6}$$

$$= \frac{477 \times 150}{600} = 119.25$$

Chapter

41

Pie Chart

Pie chart is a circular chart divided into sectors in which the arc length, its central angle and area are proportional to the quantities that it represents.

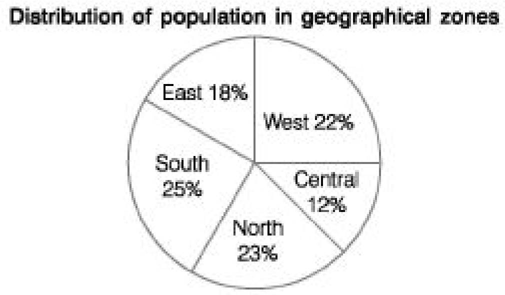
♦ Pie chart is called so because of its shape. Each slice of pie is allowed to each category and shows the portion of the entire pie.

♦ In the questions of pie chart, the total quantity is distributed over a total angle of 360° or 100%. Here, the data can be plotted with respect to only one parameter.

♦ Uses of pie charts are restricted to represent limited type of information.

♦ Pie chart is also useful for representing proportions or percentages of various elements with respect to the total quantity.

The following pie chart gives the distribution of the population in different aeoaraDhical zones.



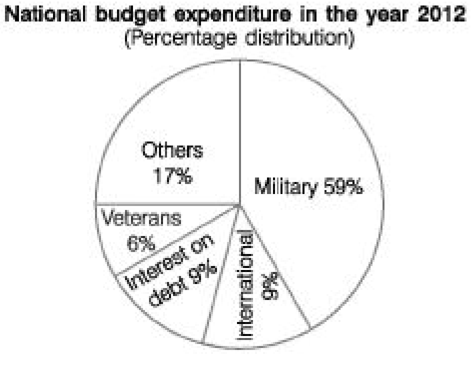
From the above pie chart, we *can calculate the following*

♦ *Population in any zone when the total population is given.*

♦ *Population of any zone as a percentage of that of another zone.*

Percentage increase in the total population, given that percentage increase in the population of one or more zones.

Directions (Examples 1-2) *Refer to the following pie chart and answer the questions that follow.*



Ex. 1 In year 2012, if India had a total expenditure of ? 120 billion, then how many billions did it spend on interest on debt?

Sol. Total expenditure = 120 billion

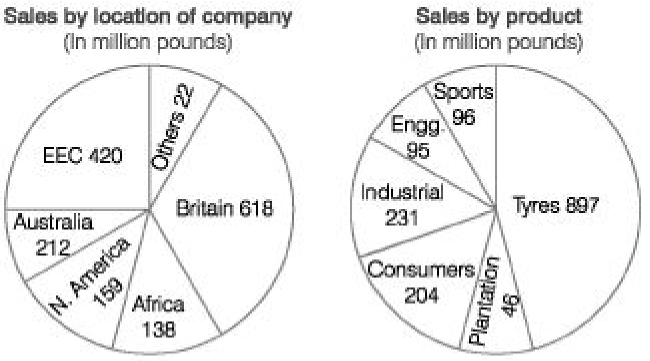
$\therefore$ Expenditure of interest on debt = 9% of 120 = $\frac{9}{100} \times 120$ = ₹ 10.8 billion

Ex. 2 If ? 9 billion were spent in year 2012 for veterans, then what would have been the total expenditure for that year (in billions)?

Sol. ₹ 9 billion were spent for veterans. It has 6% of the total expenditure for year 2012.

Hence, total expenditure $= \frac{9}{6} \times 100 =$ ₹ 150 billion

Directions (Examples 3-5) *Refer to the following pie charts and answer the questions that follow.*



Total sales = 1569 million pounds

Ex. 3 If in the next year, the sales of sports goods were expected to double assuming that the total sales do not change, what would be the percentage share of sports goods in the total sales?

Sol. Total sales = 1569 million pounds

Sports goods sales next year $= 2 \times 96 = 192$ million pounds

Therefore, percentage share of goods sales $= \frac{192}{1569} \times 100\% \approx 12\%$

Ex. 4 If in the subsequent year, consumers are to increase their shares by 7%, then assuming that the total sales remain constant, the consumer sales would have to increase by how many million of pounds?

Sol. Currently, share of consumer products $= \frac{204}{1569} \times 100\% = 13\%$

Let increase in consumer products sales be x million.

Therefore, if the share of consumer products increases by 7%, then

$$\frac{204 + x}{1569} \times 100 = 20 \Rightarrow 20400 + 100x = 31380$$

$$100x = 31380 - 20400$$

$$100x = 10980 \Rightarrow x = 110 \text{ million pounds (approx)}$$

Ex. 5 If 20% of the tyre sales were in the EEC countries, then what was the value of sales of other products in the EEC countries in million of pounds?

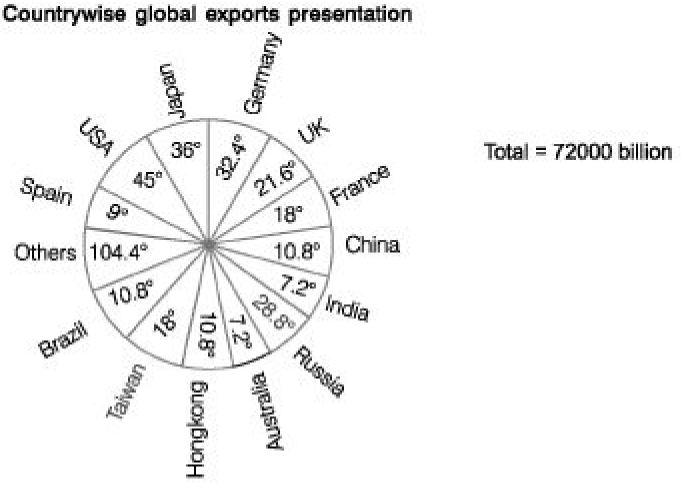
Sol. 20% of tyre sales $= \frac{20}{100} \times 897 = 179.4$ million

EEC sales = 420 million = Sales of tyre + Sales of other products

420 million = 179.4 + Sales of other products in EEC

Sales of other products in EEC = 420 - 179.4 = 240.6 million pounds (approx)

Directions (Examples 6-10) *Refer to the following pie chart and answer the questions that follow.*



Ex. 6 By how much does the value of the exports of USA exceed that of Germany?

Sol. The difference in the angles subtended by USA and Germany $= 45° - 32.4° = 12.6°$

$\therefore$ Difference in the exports of USA and Germany (in billion)

$$= 72000 \times \frac{12.6°}{360°} = 2520 \text{ billion}$$

Ex. 7 The difference in the values of the exports of Japan and France is how many times that of UK and Taiwan? Sol. The difference in the angles of the export of Japan and Prance

= 36°- 18°= 18° ...(i)

The difference in the angles subtended by UK and Taiwan

= 21.6° - 18°= 3.6° ...(ii)

Clearly, Eq. (i) is 5 times of Eq. (ii).

Ex. 8 The value of the exports of the OPEC countries is how much more than the value of the exports of India and Australia put together, given that OPEC has a 20% share in the value of the exports of others?

Sol. Value of the exports of India and Australia

$$= 7.2° + 7.2° = 14.4° \quad ...(i)$$

Value of exports OPEC countries

$$= 104.4° \times \frac{20}{100} = 20.88° \quad ...(ii)$$

$$\text{Difference} = 20.88° - 14.4° = 6.48°$$

$$\therefore \quad \text{Required value} = 72000 \times \frac{6.48°}{360°} = 1296 \text{ billion}$$

Ex. 9 If exports of developing countries accounted for 36% of the total worldwide exports, then what is the value of the exports of Japan as a percentage of the exports of the developing countries?

Sol. Exports of developing countries = 36% of total exports

$$\text{Exports of Japan} = \frac{36}{360} \times 100\% = 10\% \text{ of total exports}$$

$$\therefore \quad \text{Required percentage} = \frac{10}{36} \times 100\% = 0.2777 \times 100\% = 27.77\%$$

Ex. 10 Considering 'others' as a single country, what is the number of countries whose exports are more than the average exports per country?

Sol. The total number of countries = 14

$$\text{Average angle subtended by each country} = \frac{360°}{14} \approx 25.7°$$

Only USA, Japan, Germany, Russia and others are greater than 25.7.

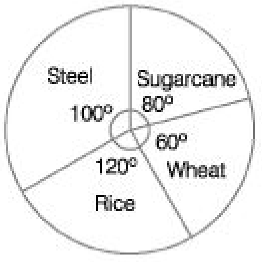
Hence, our answer is 5.

Fast Track Practice

Exercise© *Base Level Questions*

Directions (Q. Nos. 1-2) *The following two questions are based on the given pie chart which shows the annual agricultural yield of a certain place.*

[CPF Assistant Commandant 2012]



1. The yield of rice is how much per cent more than that of sugarcane?

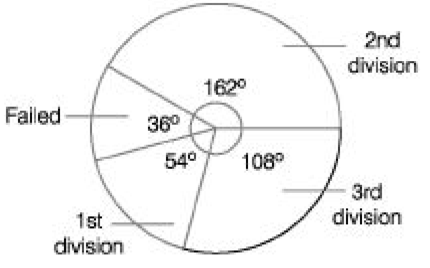
(a) 40% *(b)* 50% *(c)* 60% *(d)* 75%

2. If the yield of rice and wheat is 9000 tonnes, then the yield of excess product of others in composition with sugarcane is

(a) 1000 tonnes *(b)* 2000 tonnes *(c)* 3000 tonnes *(d)* 4000 tonnes

Directions (Q. Nos. 3-7) *The following pie chart shows the performance in an examination in a particular year for* 360 *students. Study the pie chart and answer the questions given below.*

[SSC (10+2) 2012]



3. The number of students who passed in 1st division is *(a)* 45 *(b)* 54

(c) 64 *(d)* 74

4. The number of students who passed in 2nd division is more than those in 1st division by

(a) 111 *(b)* 112 *(c)* 109 *(d)* 108

5. The ratio of successful students to the failed students is

(a) 9 : 1 *(b)* 5 : 1

(c) 1 : 9 *{d) 2:1*

6. The percentage of students who have failed in the examination is

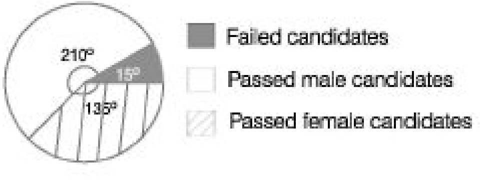
(a) 20% *(b)* 36% *(c)* 10% *(d)* 30%

The total number of students who have passed in 2nd or 3rd division is *(a)* 162 *(b)* 270

(c) 108 *(d)* None of these

Directions (Q. Nos. 8-12) *The following pie chart shows the analysis of the result of an examination in which 5 candidates have failed. Study the chart and answer the questions given*

below. [SSC CGL 2012]



8. The total number of examinees was

(a) 100 *(b)* 120 *(c)* 135 *(d)* 150

9. The number of passed male examinees is

(a) 80 *(b)* 75 *(c)* 70 *(d)* 60

10. Percentage of passed female candidates with respect to total examinees is

(a) 30% *(b)* 37.5%(c) 40% *(d)* 45%

11. Percentage of passed male candidates of total passed candidates is

(a) 60.8% *(b)* 56%

(c) 71% *(d)* 58%

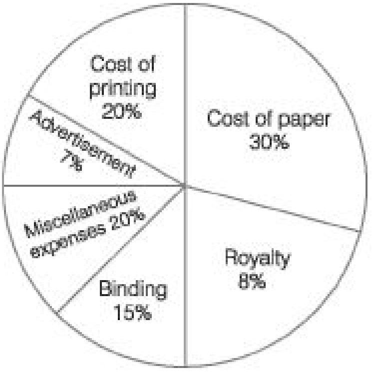
12. Ratio of passed male candidates to the successful female candidates is

(a) 9 : 1 *(b)* 1 : 14

(c) 14 : 9 *(d)* 9 : 14

Directions (Q. Nos. 13-14) *In the following pie chart, percentage expenses on various items during the production of a book are given. Based upon the information given in the pie chart, answer the questions given below.*

[CGPSC2013]



13- If the cost of paper is ? 150000, then the expense on advertisement is *(a) X* 35000 *(b) X* 3500

(c) ? 40000 *(d)* ? 25000

(e) None of the above 14. The central angle corresponding to the cost of printing is *(a)* 60° *(b)* 72° *(c)* 45° *(d)* 102° *(e)* None of the above

Directions (Q. Nos. 15-19) *The following pie chart represents a total expenditure of* ? 540000 *on different items in constructing a flat in a town. Study the pie chart and answer the questions.*

[SSCCGL2013]

15. The expenditure (in ?) on bricks is *(a)* 75000 *(b)* 67500

(C) 150000 *(d)* 70000

!■"• The expenditure on bricks is less than the expenditure on timber (in ?) by *(a)* 10000 *(b)* 12500

(c) 60000 *(d)* 65000

17. The percentage of the total expenditure spent on steel and cement is

(a) 33.23% *(b)* 25% *(c)* $33\frac{1}{3}$% *(d)* 30%

18. Which is the item of maximum expenditure?

(a) Cement *(b)* Steel

(c) Timber *(d)* Labour

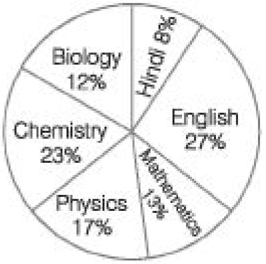
*■'• The expenditure (in *T)* on cement is

(a) 75000 *(b)* 90000

(c) 135000 *(d)* 112500

Directions (Q. Nos. 20-24) *Study the following pie chart carefully to answer the questions that follow.* [Bank po 2010]

Percentagewise distribution of teachers who teach six different subjects



Total number of teachers = 1800

20. If two-ninth of the teachers who teach Physics, are female, then number of male Physics teachers is approximately what per cent of the total number of teachers who teach Chemistry?

(a) 57% *(b)* 42% *(c)* 63% *(d)* 69%

(e) 51% 21«What is the total number of teachers teaching Chemistry, English and Biology?

(a) 1226 *(b)* 1116 *(c)* 1176 *(d)* 998

(e) None of the above

22. What is the difference between the total number of teachers who teach English and Physics together and the total number of teachers who teach Mathematics and Biology together?

(a) 352 *(b)* 342 *(c)* 643 *(d)* 653 *(e)* None of the above

23. What is the respective ratio of the number of teachers who teach Mathematics and the number of teachers who teach Hindi?

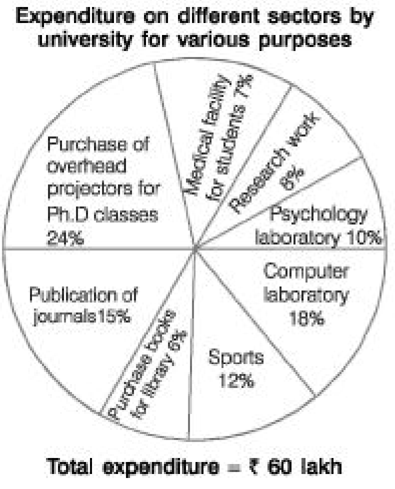
(a) 13 : 7*(b)* 7 : 13(cj 7 : 26 *(d)* 8 : 15 *(e)* None of the above

24. If the percentage of Mathematics teachers is increased by 50% and percentage of Hindi teachers is decreased by 25%, then what will be the total number of Mathematics and Hindi teachers together?

(a) 390 *(b)* 379 *(c)* 459 *(d)* 480

(e) None of the above

Directions (Q. Nos. 25-29) *Study the following pie chart carefully to answer the questions that follow.* [Bank po 2013]



25. What is total sum of expenditures on research work, purchase of overhead projectors for Ph.D classes and purchase of books for library together?

(a) X 22.6 lakh *(b)* X 22.8 lakh

(c) X 23.4 lakh *(d)* X 20.8 lakh *(e)* None of the above

26. What is the difference between the expenditures made by university on publication of journals and psychology laboratory?

(a) X 4 lakh *(b)* X 3 lakh

(c) X 4.2 lakh *(d)* X 3.8 lakh

(e) None of the above

27. What is the respective ratio between the expenditures made by university on research work and purchase of books for library?

(a) 4 : 5 *(b)* 5 : 4 *(c)* 8 : 3 *(d)* 8 : 5

(e) None of the above

28. If the expenditure on purchase of overhead projectors for Ph.D students is decreased by 7%, then what will be the expenditure on the same after the decrease?

(a) X 133920 *(b)* X 1339200

(C) X 102000 *(d)* X 108000

(e) None of the above

29. Which of the following is definitely true?

(a) Ratio between expenditure of university on purchase of library books and expenditure on computer laboratory is 3 : 1 respectively

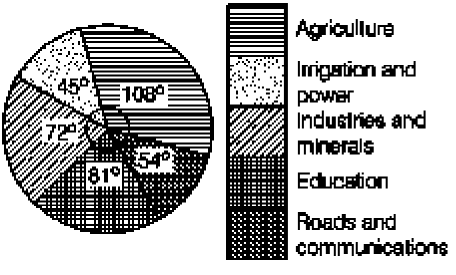
(b) Expenditure on medical facilities for students is X 4.6 lakh

(c) Difference between the expenditure on research and medical facilities for students is X 60000

(d) All are true

(e) None of the above

Directions (Q. Nos. 30-31) *The adjoining pie chart represents the proposed outlay of the fifth five year plan* 0/? 40000 crore. Examine the chart and answer the questions. [SSC Multitasking 2011]



30. The amount proposed on agriculture is more than that on industries and minerals by *(a)* 7.5% *(b)* 10%

(c) 12% *(d)* 12.5%

(e) None of the above

31« The amount (in *X* crore) proposed on irrigation and power is less than that on industries and minerals by

(a) 3000 *(b)* 3500

(c) 2000 *(d)* 2500

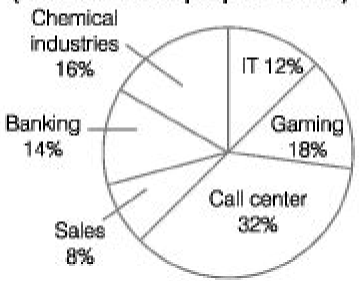
(e) None of the above

Exercise © *Higher SkiU Level Questions*

Directions (Q. Nos. 1-5) *Study the following pie chart carefully and answer the questions given below.*

[SBI Clerk 2012]

Percentage of people in a city working in night shifts from various industries (Total number of people = 40250)



Percentage of females from various industries working in night shifts

Industries	Females
IT	20%
Gaming	20%
Call center	45%
Sales	60%
Banking	40%
Chemical industries	15%

1. What is the respective ratio of the men to the women working in night shifts from the call center industry?

(a) 9 : 11 *(b)* 7 : 5 *(c)* 8 : 13 *(d)* 11 : 7 *(e)* None of the above

2. What is the approximate average number of females working in night shifts from all the industries together?

(a) 2227 *(b)* 4481 *(c)* 3326 *(d)* 2823 *(e)* 4107

*• What is the total number of men working in night shifts from all the industries together? *(a)* 28297 *(b)* 25788

(c) 28678 *(d)* 26887

(e) None of the above 4. The number of women working in night shifts from the gaming industry is what per cent of the total number of people working in the night shifts from all the industries together?

(a) 5.6% *(b)* 3.6% *(c)* 3.2% *(d)* 4.4% *(e)* None of the above

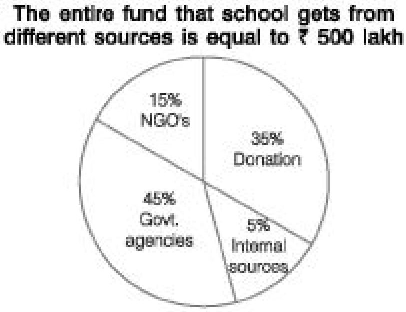
5. What is the difference between the total number of men and the total number of women working in night shifts from all the industries together?

(a) 13254(b) 13542 *(c)* 13524 (dj 13363

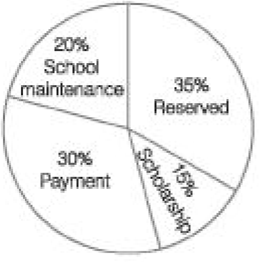
(e) None of the above

Directions (Q. Nos. 6-10) *Study the following pie charts carefully and answer the questions given below it.*

[Bank PO 2010]



Sources of funds in school



Uses of funds by school

What is the difference between the funds acquired by school from NGO's and internal sources? *(a)* *X* 50 lakh *(b)* *X* 45 lakh

(c) ? 75 lakh *(d)* ? 25 lakh

(e) None of the above 7. If the school managed school maintenance from the government agencies fund only, how much fund from government agencies would still left for other use? *(a)* ? 120 lakh *(b)* *X* 150 lakh

(c) *X* 110 lakh *(d)* ?95 lakh

(e) None of the above

8. If scholarship has to be paid out of the donation fund, then what is the approximate per cent of donation fund used for this purpose?

(a) 43% *(b)* 53% *(c)* 37% *(d)* 45%

(e) 32% What is the total amount used by the school for payment?

(a) ? 100 lakh *(b)* 7 110 lakh

(c) 7 150 lakh (d) 7 140 lakh

(e) None of the above 10. What amount of the fund is acquired by the school from government agencies?

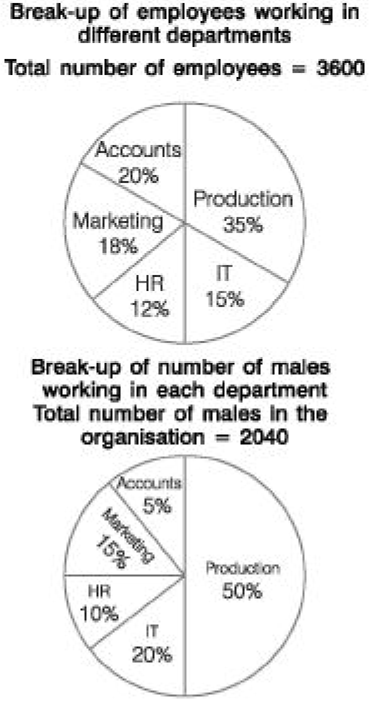
(a) ? 220 lakh (b) 7 310 lakh

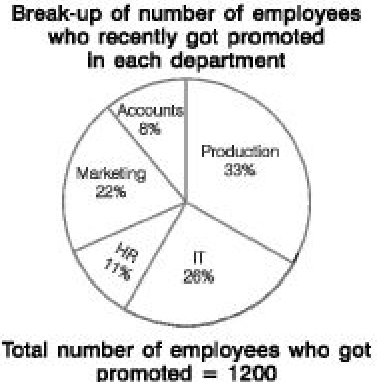
(c) 7 255 lakh (d) 7 225 lakh

(e) None of the above

Directions (Q. Nos. 11-15) *Study the given pie charts carefully to answer the questions that follow.* [Bank po 2010]

Break-up of number of employees working in different departments of an organisation, the number of males and the number of employees who recently got promoted in each department





The number of employees who got promoted from the HR department was what per cent of the total number of employees working in that department? (Rounded off to two digits after decimal.) *(a)* 36.18% *(b)* 30.56%

(c) 47.22% *(d)* 28.16%

(e) None of the above

12. The total number of employees who got promoted from all the departments together was what per cent of the total number of employees working in all the departments together? (Rounded off the nearest integer.)

(a) 56% *(b)* 21%

(c) 45% *(d)* 33%

(e) 51%

13. What is the total number of females working in the production and marketing departments together?

(a) 468 *(b)* 812

(c) 582 *(d)* 972

(e) None of the above

14. If half of the number of employees who promoted from the IT department were males, then what was the approximate percentage of males who got promoted from the number of working males in IT department?

(a) 61% *(b)* 29%

(c) 54% *(d)* 42%

(e) 38%

15. How many females work in the accounts department?

(a) 618 *(b)* 592

(c) 566 *(d)* 624

(e) None of the above

Answer with Solutions

Exercise© *Base Level Questions*

1. (*b*) Given, rice = 120° and Sugarcane = 80°

Rice compared to sugarcane

$$= \frac{120 - 80}{80} \times 100 = \frac{40}{80} \times 100 = \frac{1}{2} \times 100 = 50\%$$

So, rice's yield is 50% higher than sugarcane

2. (*a*) By the pie chart, Rice + Wheat

$$= 120° + 60°$$
$$= 180° = \text{Half circle}$$

If half circle is 9000 tonnes (given in the question), then full circle (total production) is 18000 tonnes.

The difference between others and Sugarcane = 100 − 80 = 20°

$\because \quad 360° = 18000 \qquad$ [full circle = 360°]

$$= \frac{18000 \times 20}{360} = 50 \times 20 = 1000 \text{ tonnes}$$

3. (*b*) The number of students passed in 1st division $= \frac{54°}{360°} \times 360 = 54$

4. (*d*) The number of students passed in 2nd division $= \frac{162°}{360°} \times 360 = 162$

$\therefore \quad$ Required number = 162 − 54 = 108

5. (*a*) Required ratio $= \frac{54° + 108° + 162°}{36°}$

$$= \frac{324}{36} = \frac{9}{1} = 9 : 1$$

6. (*c*) The percentage of failed students in the examination $= \frac{36°}{360°} \times 100 = 10\%$

7. (*b*) The total number of students passed in 2nd and 3rd division

$$= \frac{108° + 162°}{360°} \times 360 = 270$$

8. (*b*) Let total number of candidates be x.

By pie chart, $\frac{15°}{360°} \times x = 5$

$\therefore \qquad x = 24 \times 5 = 120$

9. (*c*) Number of passed male candidates

$$= \frac{210°}{360°} \times 120 = 70$$

10. (*b*) Number of passed female candidates

$$= \frac{135°}{360°} \times 100\% = 37.5\%$$

11. (*d*) Percentage of passed male candidates

$$= \frac{210°}{360°} \times 100 = 58\% \text{ (approx)}$$

12. (*c*) Required ratio $= \frac{210°}{135°} = 14 : 9$

13. (*a*) Let the expense on paper = x

$$\Rightarrow \quad \frac{x \times 30}{100} = 150000$$

$$\Rightarrow \quad x = \frac{150000 \times 10}{3} \Rightarrow x = 500000$$

$\therefore$ Expense of advertisement

$$= \frac{500000 \times 7}{100} = ₹\ 35000$$

14. (*b*) The central angle corresponding to the cost of printing $= \frac{20}{100} \times 360 = 72°$

15. (*a*) Angle made by the expenditure on bricks

$$= 360° - (45° + 100° + 90° + 75°)$$
$$= 360° - 310° = 50°$$

Thus, expenditure on bricks

$$= \frac{50°}{360°} \times 540000 = ₹\ 75000$$

16. (*c*) Required value

$$= \frac{(90° - 50°)}{360°} \times 540000 = ₹\ 60000$$

17. (*c*) Expenditure on steel and cement

$$= 75° + 45° = 120°$$

$\therefore$ Required percentage

$$= \frac{120°}{360°} \times 100\% = \frac{100}{3}\% = 33\frac{1}{3}\%$$

18. (*d*) From the given pie chart, it is clear that 'Labour' has maximum expenditure.

19. (*d*) Expenditure on cement

$$= \frac{75°}{360°} \times 540000 = ₹\ 112500$$

20. (*a*) Number of teachers in Physics

$$= 1800 \times \frac{17}{100} = 306$$

Female teachers in Physics

$$= 306 \times \frac{2}{9} = 68$$

Male teachers in Physics = 306 − 68 = 238

Number of teachers in Chemistry

$$= 1800 \times \frac{23}{100} = 414$$

$\therefore$ Required percentage

$$= \frac{238}{414} \times 100 \approx 57\%$$

21. (*b*) Total number of teachers teaching Chemistry, English and Biology

$$= 1800 \times \frac{(23 + 27 + 12)}{100}$$

$$= 1116$$

22. (*b*) Required difference

$$= 1800 \times \frac{(27 + 17)}{100} - 1800 \times \frac{(13 + 12)}{100}$$

$$= 792 - 450 = 342$$

23. (*e*) Required ratio $= \dfrac{1800 \times \dfrac{13}{100}}{1800 \times \dfrac{8}{100}}$

$$= 13 : 8$$

24. (*c*) Required number

$$= \left(1800 \times \frac{13}{100} \times \frac{150}{100}\right) + \left(1800 \times \frac{8}{100} \times \frac{75}{100}\right)$$

$$= 351 + 108 = 459$$

25. (*b*) Total sum of expenditures

$$= \frac{(8 + 24 + 6)}{100} \times 60$$

$$= \frac{38}{100} \times 60 = 22.8 \text{ lakh}$$

26. (*b*) Required difference

$$= \frac{(15-10)}{100} \times 60 = \frac{5}{100} \times 60 = 3 \text{ lakh}$$

27. (*e*) Required ratio = 8 : 6 = 4 : 3

28. (*e*) Required sum of expenditures

$$= \frac{(24-7)}{100} \times 60 \text{ lakh}$$

$$= \frac{17 \times 60}{100} \text{ lakh} = ₹\ 1020000$$

29. (*c*) Required difference

= Expenditure on research work

− Expenditure on medical facilities

$$= \frac{8-7}{100} \times 60 \text{ lakh} = ₹\ 60000$$

30. (*b*) Amount spend on agriculture

$$= \frac{108°}{360°} \times 40000 = ₹\ 12000 \text{ crore}$$

Amount spend on industries and minerals

$$= \frac{72°}{360°} \times 40000 = ₹\ 8000 \text{ crore}$$

∴ Required percentage

$$= \frac{12000 - 8000}{40000} \times 100 = 10\%$$

31. (*a*) Required amount

$$= \frac{72° - 45°}{360°} \times 40000$$

$$= \frac{27}{360} \times 40000 = ₹\ 3000 \text{ crore}$$

Exercise © *Higher Skill Level Questions*

1. (*e*) Required ratio = 55 : 45

= 11 : 9

2. (*a*) Required average

$$= \frac{40250}{6}\left[\frac{12}{100} \times \frac{20}{100} + \frac{18}{100} \times \frac{20}{100} + \frac{32}{100} \times \frac{45}{100} + \frac{8}{100} \times \frac{60}{100} + \frac{14}{100} \times \frac{40}{100} + \frac{16}{100} \times \frac{15}{100}\right]$$

$$= \frac{40250}{6 \times 10000}[240 + 360 + 1440 + 480 + 560 + 240]$$

$$= \frac{40250 \times 3320}{60000}$$

= 2227

3. (*d*) Required number

$$= 40250\left[\frac{12 \times 80}{100 \times 100} + \frac{18 \times 80}{100 \times 100} + \frac{32 \times 55}{100 \times 100} + \frac{8 \times 40}{100 \times 100} + \frac{14 \times 60}{100 \times 100} + \frac{16 \times 85}{100 \times 100}\right]$$

$$= \frac{40250}{10000}[960 + 1440 + 1760 + 320 + 840 + 1360]$$

$$= \frac{40250}{10000} \times 6680 = 26887$$

4. (*b*) Number of people working in night shifts

$= 40250$

Number of women working in night shifts in gaming

$$= \frac{20}{100} \times \frac{18}{100} \times 40250$$

$\therefore$ Required percentage

$$= \frac{20 \times 18 \times 40250}{100 \times 100} \times \frac{100}{40250} = 3.6\%$$

5. (*c*) Number of women

$= 40250 - 26887 = 13363$

$\therefore$ Required difference

$= 26887 - 13363 = 13524$

6. (*a*) Required difference = (Percentage of fund acquired from NGO − Percentage of fund acquired from internal sources) of 500 lakh

$= (15 - 5)\%$ of 500 lakh $= \dfrac{500 \times 10}{100}$ lakh

$= ₹\ 50$ lakh

7. (*e*) Fund from government agencies

$= \dfrac{500 \times 45}{100} = ₹\ 225$ lakh

Expenses in school maintenance

$= \dfrac{500 \times 20}{100} = ₹\ 100$ lakh

$\therefore$ Remaining fund $= (225 - 100)$ lakh

$= ₹\ 125$ lakhs

8. (*a*) Fund from donation $= \dfrac{500 \times 35}{100}$

$= ₹\ 175$ lakh

Scholarship amount $= \dfrac{15 \times 500}{100} = ₹\ 75$ lakh

$\therefore$ Required percentage

$= \dfrac{75}{175} \times 100 = 42.85\%$

$= 43\%$ (approx)

9. (*c*) Total amount used by the school for payment

$= \dfrac{500 \times 30}{100} = ₹\ 150$ lakh

10. (*d*) Fund acquired from government agencies

$= \dfrac{500 \times 45}{100} = ₹\ 225$ lakh

11. (*b*) Number of promoted employees in HR department = 1200 × 0.11 = 132

Number of employees working in HR department = 3600 × 0.12 = 432

∴ Required percentage

$$= \frac{132}{432} \times 100\% = 30.56\%$$

12. (*d*) Number of promoted employees in all departments = 1200

Number of working employees in all departments = 3600

∴ Required percentage

$$= \frac{1200}{3600} \times 100\% \approx 33\%$$

13. (*c*) Number of employees working in production and marketing

$$= 3600 \times (0.35 + 0.18) = 1908$$

Number of male employees in production and marketing

$$= 2040 \times (0.50 + 0.15) = 1326$$

∴ Number of female employees in production and marketing

$$= 1908 - 1326 = 582$$

14. (*e*) Number of promoted employees in IT department = 1200 × 0.26 = 312

Number of promoted male employees in IT department = 156

Number of employees working male in IT department = 2040 × 0.20 = 408

∴ Required percentage

$$= \frac{156}{408} \times 100\% \approx 38\%$$

15. (*a*) Number of employees working in accounts department

$$= 3600 \times (0.20)$$
$$= 720$$

Number of male employees in accounts department = 2040 × (0.05) = 102

∴ Number of female employees working in accounts department = 720 − 102

$$= 618$$

Chapter

42

Bar Chart

A bar chart is a chart with **rectangular bars** with lengths proportional to the values that they represent. Bar charts are diagramatic representation of discrete data.

A bar is a thick line whose width is shown merely for attention. In this method of data representation, the data is plotted on the x and y-axes as bars.

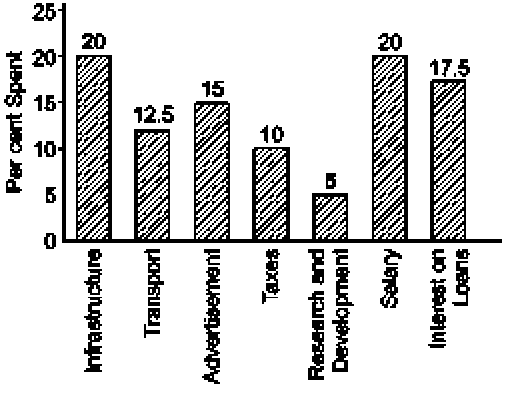
One of the axes (normally the x-axis) of the bar diagram represents a discrete variable, while the other axis represents the scale for continuous variable.

Each bar diagram has a title indicating the subject matter represented in the diagram.

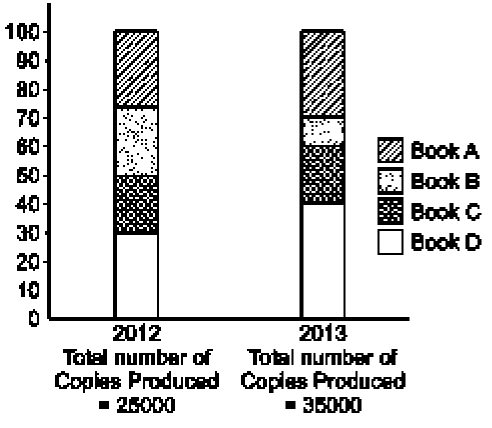
Types of Bar Chart

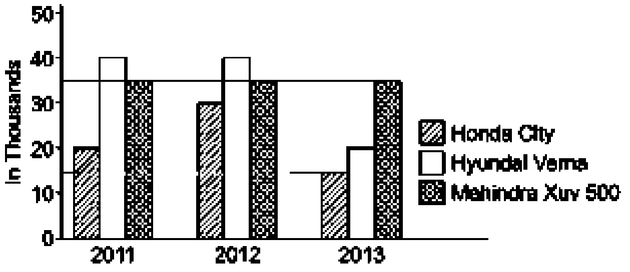
Following are the main bar charts

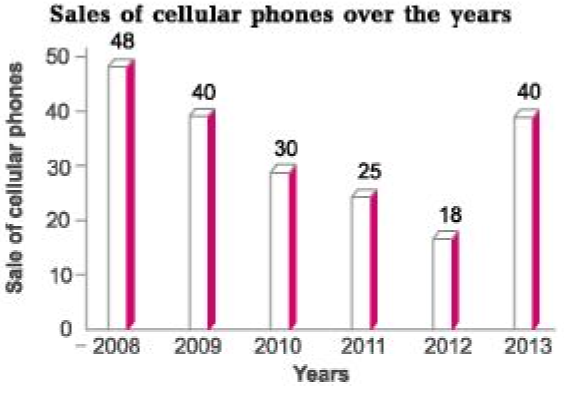
1. **Simple bar chart** A simple bar chart relates to only one variable. *For example* Given below chart shows the percentage distribution of total expenditure of a company



2. **Sub-divided bar chart** A sub-divided bar chart is used to represent various parts of total magnitude of a given variable.







Ex. 1 Find the percentage increase in sales from 2012 to 2013.

Sol. Percentage increase in sales $= \frac{40-18}{18} \times 100\% = \frac{22}{18} \times 100\% \approx 122\%$

Ex. 2 Find the two years between which the rate of change of cellular phones is minimum.

Sol. Rate of change during years

$2008\text{-}09 = \frac{8}{48} \times 100\% = 16.6\%$, $\quad 2009\text{-}10 = \frac{10}{40} \times 100\% = 25\%$

$2010\text{-}11 = \frac{5}{30} \times 100\% = 16.6\%$, $\quad 2011\text{-}12 = \frac{7}{25} \times 100\% = 28\%$

$2012\text{-}13 = \frac{22}{18} \times 100\% \approx 122\%$

Hence, it is minimum for years 2008-09 and 2010-11.

Directions (Examples 3-4) *Refer to the following bar chart and answer the questions that follow.*

Ex. 3 What is the percentage increase in import between 2008-09 and 2012-13?

Sol. Percentage increas $\dfrac{\text{Import in 2012–13 – Import In 2008–09}}{\text{Impartin2008 – 09}} \times 100\%$

$$= \frac{14000 - 7000}{7000} \times 100\%$$

$$= \frac{7000}{7000} \times 100\% = 100\%$$

Ex. 4 If oil imports constituted 20% of the total imports in 2010-11, then what percentage of the trade gap was due to oil (assuming that no oil is exported)?

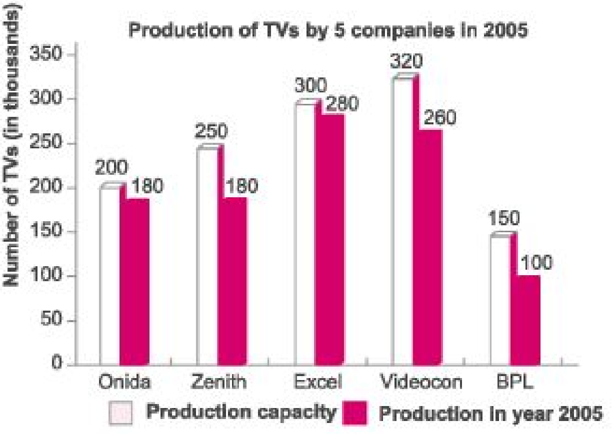
Sol. Oil import in 2010-11 $= \dfrac{20}{100} \times 13000 =$ ₹2600 crore

Trade gap in 2010-11 $= (13000 - 7000) =$ ₹6000 crore

Hence, percentage of trade gap due to oil $= \dfrac{\text{Oil import}}{\text{Tradegap}} \times 100\%$

$$= \frac{2600}{6000} \times 100\% \approx 43\% \text{ (approx)}$$

Directions (Examples 5-7) *Read the following par chart carefully and answer the questions that follows*



Ex. 5 In year 2005, which company had the maximum percentage unutilised capacity?

Sol. Percentage of unutilised capacity $= \dfrac{\text{Production capacity} - \text{Production}}{\text{Production capacity}} \times 100\%$

$$\text{Onida} = \frac{20}{200} \times 100\% = 10\%$$

$$\text{Zenith} = \frac{70}{250} \times 100\% = 28\%$$

$$\text{Excel} = \frac{20}{300} \times 100\% = 6.67\%$$

$$\text{Videocon} = \frac{60}{320} \times 100\% = 18.75\%$$

$$\text{BPL} = \frac{50}{150} \times 100\% = 33.3\%$$

Hence, BPL had maximum unutilised capacity.

Ex. 6 The TVs produced by Excel form what percentage of the total production?

Sol. Total production $= 180 + 180 + 280 + 260 + 100 = 1000$

$\therefore$ Required percentage $= \dfrac{280}{1000} \times 100\% = 28\%$

Ex. 7 A new company CASINO was set up in 2006 and sold 122000 pieces in that year. Due to this, the other five given companies together reduced their production by the same number of sets sold by CASINO in the ratio of their production capacities. What is the production of Excel (in thousand sets) in 2006?

Sol. Total number of TVs sold by CASINO = 122000

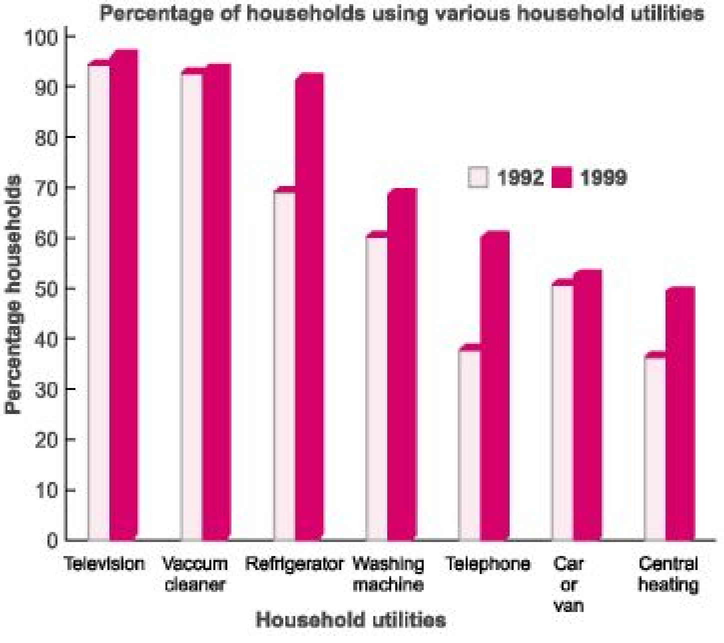
Total production capacity of all the companies put together

$$= 200 + 250 + 300 + 320 + 150 = 1220$$

$$\text{Decrease in production of Excel} = \frac{300}{1220} \times 122000 = 30000$$

$\therefore$ Actual production of Excel in year 2006 = 280000 − 30000 = 250000

Directions (Examples 8-9) *These questions are based on the fallowing bar chart.*



Ex. 8 In 1992, if the total number of households was 403 million, then what would be the number of households having washing machine (in million)? Sol. Percentage of households having washing machine in 1992 = 65% Hence, number of households having washing machine in 1992 = Total number of households x 65% = 403 million x 65% = 261.95 million « 262 million

Ex. 9 If the number of households increased from 403 million in 1992 to 600 million in 1999, then what would be the number of new telephones installed (in million)? (Assuming one telephone per household)

Sol. Number of telephones in 1992 $= \frac{40}{100} \times 403$ million $= 161.2$ million

Number of telephones in 1999 $= \frac{63}{100} \times 600$ million $= 378$ million

Hence, number of new telephones installed

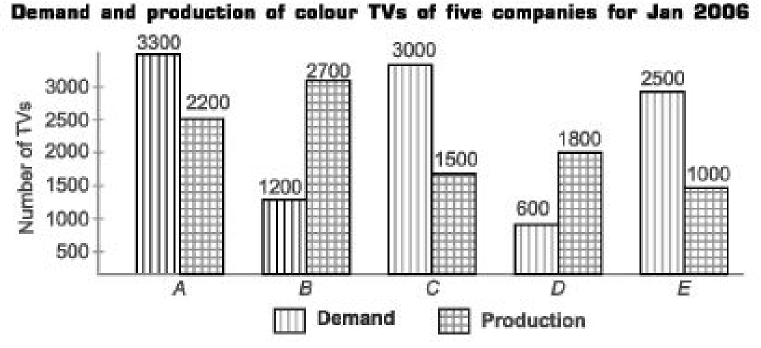
$= 378 - 161.2 = 216.8$ million

Fast Track Practice

Exercise© *Base Level Questions*

Directions (Q. Nos. 1-4) *Study the following graph and answer the questions that follow.*

[SSC CPO 2009]



1. What is the ratio of the number of companies having more demand than production to the number of companies having more production than demand?

(a) 2 : 3

(b) 4 : 1

(c) 2 : 2

(d) 3 : 2

2. What is the difference between average demand and average production of the five companies taken together?

(a) 1400 *(b)* 400

(c) 280 *(d)* 138

3. Demand of company *D* is approximately what per cent of demand of company *El*

(a) 12% *(b)* 20%

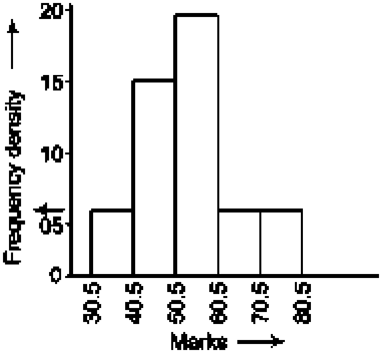
(c) 24% *(d)* 30%

4. What is the ratio of average demand to average production of companies *B* andD?

(a) 1 : 5 *(b)* 2 : 5

(c) 3 : 5 *(d)* 4 : 5

Directions (Q. Nos. 5-9) *The following histogram shows the distribution of marks of* 50 *students in a college. Study the graph and answer the questions given below.* **[SSC (10+2) 2012]**



The number of students, whose marks lie between 40.5 and 50.5 is

(a) 15 *(b)* 10

(c) 5 *(d)* None of these

6. The class, in which the marks of maximum number of students lies, is

(a) 40.5 to 50.5 *(b)* 50.5 to 60.5

(C) 60.5 to 70.5 *(d)* 70.5 to 80.5 The percentage of the students, whose marks lie between 40.5 and 60.5 is

(a) 60 *(b)* 70

(c) 75 *(d)* None of these

8. A student is marked as 'qualified', if he obtain marks above 40.5 Here, the ratio of the number of qualified students to those who are not qualified, is

(a) 6 : 1 *(b)* 7 : 1

(c) 8 : 1 *(d)* 9 : 1

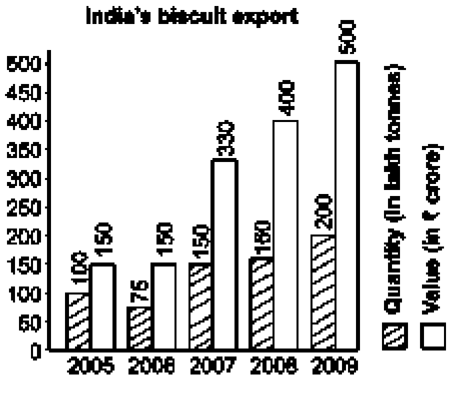
9. The average marks of the students, number is

(a) 43.5 *(b)* 44.5

(c) 45.5 *(d)* 74.7

Directions (Q. Nos. 10-13) *Study the following bar diagram and answer the questions that*

follow. [SSCCGL2013]



10. In which two years, was the value per ton equal?

(a) 2006 and 2007

(b) 2005 and 2006

(c) 2008 and 2009

(d) 2007 and 2008

11. The year, in which the percentage increase in export was maximum from its preceding year, is

(a) 2009

(b) 2007

(c) 2008

(d) None of the above

12. In which year, was the value per ton minimum?

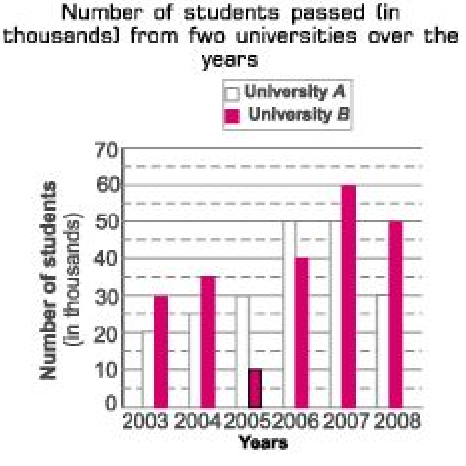
(a) 2007 *(b)* 2009 *(c)* 2005 *(d)* 2006

13. What was the percentage drop in export quantity from 2005 to 2006?

(a) 75% *(b) 0%*

(c) 25% *(d)* 50%

Directions (Q. Nos. 14-18) *Study the following graph carefully and answer the questions that follow.* [Bank Clerks 2010]



14. What is the respective ratio of the number of students passed from university A in the year 2007 and the number of students passed from university B in the year 2004?

(a) 5 : 4 *(b)* 4 : 5

(c) 7 : 10 *(d)* 10 : 7

(e) None of the above

15. What is the difference between the total number of students passed from both the universities in the year 2007 together and the total number of students passed in year 2005 from both the universities together?

(a) 70000 *(b)* 37000

(c) 7000 *(d)* 3700

(e) None of the above

16. What is the sum of students passed from university B in the years 2003, 2005 and 2006 together?

(a) 80000 *(b)* 8000

(c) 800000 *(d)* 75000

(e) None of the above

17. Number of students passed from university B in 2008 is approximately what per cent of the total number of students passed from university A over the years?

(a) 30% *(b)* 25%

(c) 20% *(d)* 35%

(e) 40%

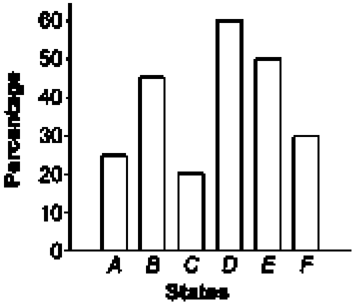
18. What is the respective ratio of the number of students passed in 2007, 2008 and 2005 from university At

(a) 5 : 3 : 2 *(b)* 3 : 5 : 5

(c) 5 : 3 : 3 *(d)* 5 : 1 : 1

(e) None of the above

Directions (Q. Nos. 19-20) *The following two questions are based on the given histogram that shows the percentage of villages in the States, which are not electrified* [CPF Assistant 2012]



19. Which of the following States has twice the percentage of villages electrified in comparison to State *IP*.

***(a)* A *(b)* C *(c)* E *(d)* F**

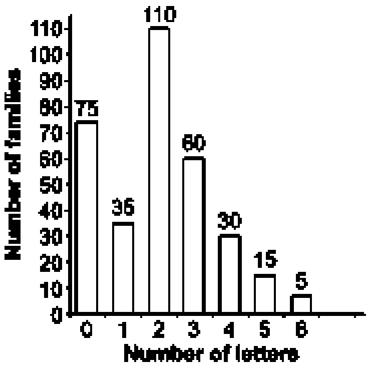
20. How many State have atleast 50% electrified villages?

***(a)* 1 *(b)* 2 *(c)* 3 *(d)* 5**

Directions (Q. Nos. 21-22) *Following bar diagram shows the number of families of a certain locality with various number of letters*

received by those families in a certain month, as shown in a survey by local post-office. Read the graph and answer the questions below.

[SSC Multitasking 2013]



21. The total number of families that came under the survey is

(a) 110 *(b)* 250 *(c)* 300 *(d)* 330

22. The average number of letters received per family in that month is

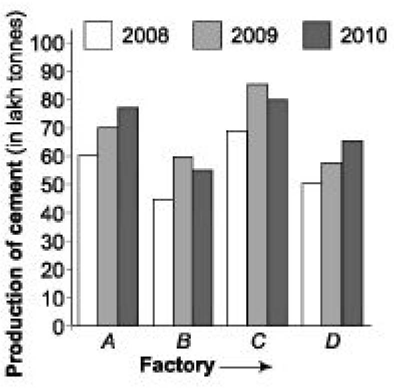
(a) $31\frac{3}{7}$ (b) 3

(c) 2 (d) $2\frac{10}{17}$

Exercise © *Higher SkiU Level Questions*

Directions (Q. Nos. 1-5) *The following graph shows the production of cement (in lakh tonnes) of four factories A, B, C and D over the years.*

[SSCCPO2013]



1. The percentage increase in production of cement by factory *B* from 2008 to 2009 is about *(a)* 30.8 *(b)* 16.7 *(c)* 18.2 *(d)* 22.2

The production of cement by factory *B* in 2009 and production of cement by factory *D* in 2010 together is what per cent of production by factory *A* in 2008? *(a)* 200 *(b)* 50 *(c)* 100 *(d)* 150

3. Which of the four factories has recorded the maximum percentage growth in production of cement from

2008 to 2009?

(a) D (b) A (c) B (d) C

4. Which of the given factories has recorded the maximum percentage growth in production of cement from

2009 to 2010?

(a) D (b) A (c) B (d) C

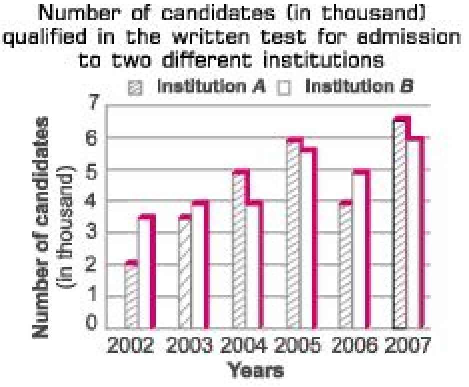
5. The difference (in lakh tonnes) between the average production of cement by four factories in 2009 and average production by the same factories in 2008 is

(a) 12.75 *(b)* 11.50

(c) 11.75 *(d)* 12.50

Directions (Q. *Nos.* 6-10) *Study the following graph and answer the questions that follow.*

[Bank PO 2010]



6. What was the respective ratio between the number of candidates qualified in the written test in the year 2002 for admission in institution *B* and the number of candidates qualified in the written test in the year 2006 for admission to institution A?

(a) 8 : 5 *(b)* 7 : 4 *(c)* 7 : 8 *(d)* 7 : 5 *(e)* 8 : 5

7. What was the approximate average number of candidates qualified in the written test for admission to institution *B* over all the year?

(a) 4555 *(b)* 4200 *(c)* 4160 *(d)* 4888 *(e)* 4667

8. In which year, the total number of candidates qualified in the written test for admission to both the institutions together was the second highest?

(a) 2003 *(b)* 2004 *(c)* 2005 *(d)* 2006 *(e)* 2007

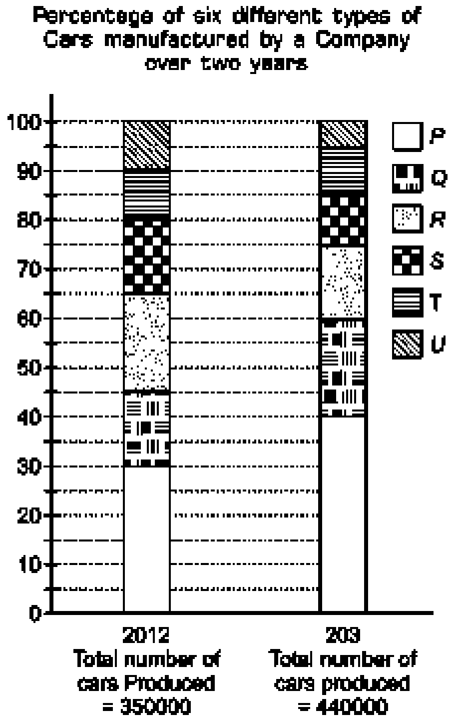
9. What is the difference between the total number of candidates qualified in written test in the year 2006 for admission to institutions *A* and *B* together and the number of candidates qualified in written test in the year 2003 for admission to institution *A1*

(a) 5000 *(b)* 3500 *(c)* 1500 *(d)* 5500 *(e)* None of the above

10. What was the total number of candidates qualified in the written test for admission to institution *A* over all the years together?

(a) 27000 *(b)* 26500 *(c)* 26000 *(d)* 27500 *(e)* None of the above

Directions (Q. Nos. 11-15) *Read the following bar chart carefully and answer the questions that follows.*



11. Total number of cars of models *P, Q* and *T* manufacured in 2012 is

(a) 245000 *(b)* 227500

(c) 210000 *(d)* 192500

(e) 157500

12. For which model the percentage rise/fall in production from 2012 to 2013 was minimum?

(a) Q (b) R (c) S (d) T (e) U

13. What was the difference in the number of *Q* type cars produced in 2012 and that produced in 2013?

(a) 35500 *(b)* 27000

(c) 22500 *(d)* 17500

(e) 16000

14. If the percentage production of *P* type cars in 2013 was the same as that in 2012, then the number of *P* type cars produced in 2013 would have been

(a) 140000 *(b)* 132000

(c) 117000 *(d)* 105000

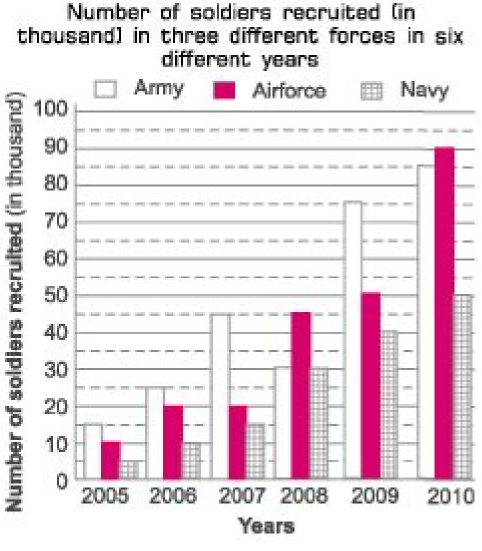
(e) 97000

15. If 85% of the *S* type cars produced in each year were sold by the company, how many *S* type cars remained unsold? *(a)* 7650 *(b)* 9350

(c) 118500 *(d)* 12250

(e) 13350

Directions (Q. Nos. 16-20) *Study the following graph carefully to answer the questions that follow.* [Bank PO 2011]



16. What was the average number of soldiers recruited in the navy over all the years together?

(a) 25000 *(b)* 24000 *(c)* 2400 *(d)* 28000 *(e)* None of the above

17. Number of soldiers recruited in navy in the year 2009 was what percentage of number of soldiers recruited in army in the year 2006?

(a) 140 *(b)* 150 *(C)* 160 *(d)* 180 *(e)* None of the above

18. If 30 % of soldiers recruited in airforce in the year 2010 was female, what is the number of males recruited in airforce in that year?

(a) 63000 *(b)* 6300 *(c)* 61000 *(d)* 6100 *(e)* None of the above

19. What was the respective ratio between the number of soldiers recruited for airforce in the year 2005 and the number of soldiers recruited in army in the year 2009?

(a) 2 : 15*(b)* 5 : 13(cJ 2 : 17 (d) 15 : 4 *(e)* None of the above

20. What was the approximate percentage decrease in number of soldiers recruited in army in the year 2008 as compared to the previous year?

(a) 20 *(b)* 23 *(c)* 38 *(d)* 30 *(e)* 33

Answer with Solutions

Exercise © *Base Level Questions*

1. (*d*) Companies which have more demand than production = *A*, *C* and *E*, *i.e.*, 3 companies.

Compaines which have more production than demand = *B*, *D* *i.e.*, 2 companies

$\therefore$ Required ratio = 3 : 2

2. (*c*) Average demand

$$= \frac{\text{Sum of all demands}}{\text{Number of all demands}}$$

$$= \frac{3300 + 1200 + 3000 + 600 + 2500}{5}$$

$$= \frac{10600}{5} = 2120$$

Similarly, average production

$$= \frac{2200 + 2700 + 1500 + 1800 + 1000}{5}$$

$$= \frac{9200}{5} = 1840$$

$\therefore$ Required difference = 2120 – 1840 = 280

3. (*c*) Required percentage $= \frac{600}{2500} \times 100\%$ = 24%

4. (*b*) Average demand of *B* and *D*

$$= \frac{1200 + 600}{2} = \frac{1800}{2} = 900$$

Average production of *B* and *D*

$$= \frac{2700 + 1800}{2} = \frac{4500}{2} = 2250$$

$\therefore$ Required ratio $= \frac{900}{2250} = 2 : 5$

5. (a) Number of students obtained marks between 40.5 and 50.5 = 15

6. (6) The maximum number of students lie in class interval (50.5 to 60.5)

7. (6) Number of students obtained marks between 40.5 and 60.5 = 15 + 20 = 35

$\therefore$ Required percentage $= \frac{35}{50} \times 100 = 70\%$

8. (*d*) Number of passed students

= 15 + 20 + 5 + 5 = 45

Required ratio = 45 : 5 = 9 : 1

9. (*d*) Average marks of the students

$$= \frac{30.5 \times 5 + 40.5 \times 15 + 50.5 \times 20 + 60.5 \times 20 + 70.5 \times 5 + 80.5 \times 5}{50}$$

$$= \frac{152.5 + 607.5 + 1010 + 1210 + 352.5 + 402.5}{50}$$

$$= \frac{3735}{50} = 74.7$$

10. (*c*) Value of biscuit per ton

in 2005 $= \frac{150 \times 10000000}{100 \times 100000} =$ ₹ 150

in 2006 $= \frac{150 \times 10000000}{75 \times 100000} =$ ₹ 200

in 2007 $= \frac{330 \times 10000000}{150 \times 100000} =$ ₹ 220

in 2008 $= \frac{400 \times 10000000}{160 \times 100000} =$ ₹ 250

in 2009 $= \frac{500 \times 10000000}{200 \times 100000} =$ ₹ 250

Thus, value per ton is equal in the years 2008 and 2009.

11. (*b*) Percentage increase in export from preceding year

in 2007 $= \frac{150 - 75}{75} \times 100\% = 100\%$

in 2008 $= \frac{160 - 150}{150} \times 100\% = 6.66\%$

in 2009 $= \frac{200 - 160}{160} \times 100\% = 25\%$

Thus, required year is 2007.

12. (*c*) From the solution number (10), it is clear that value per ton is minimum in the year 2005.

13. (*c*) The percentage drop in export quantity from 2005 to 2006

$$= \frac{100 - 75}{100} \times 100\%$$

$$= 25\%$$

14. (*d*) Required ratio $= \frac{50000}{35000} = 10 : 7$

15. (*a*) Required difference

$= (50000 + 60000) - (30000 + 10000)$

$= 110000 - 40000 = 70000$

16. (*a*) Required sum

$$= (30000 + 10000 + 40000)$$
$$= 80000$$

17. (*b*) Required percentage

$$= \frac{50000}{20000 + 25000 + 30000 + 50000 + 50000 + 30000} \times 100\%$$

$$= \frac{50000}{205000} \times 100\% \approx 25\%$$

18. (C) Required ratio = 50 : 30 : 30 = 5: 3 : 3

19. (6) State *D* has 40% villages electrified and State C has 80% villages electrified. Thus, correct options is (b).

20. (d) From the question, we have 6 States and only one State *D* doesn't meet this condition, because it has less than 50% villages electrified, it has 40% electrification.

Thus, 6 − 1 = 5 states have atleast 50% villages electrified.

21. (*d*) Total number of families

$$= (75 + 35 + 110 + 60 + 30 + 15 + 5)$$
$$= 330 \text{ families}$$

22. (*c*) Average number of letters received per family

$$= \frac{\text{Sum of letters}}{\text{Total number of families}}$$

$$= \frac{\begin{pmatrix} 75 \times 0 + 35 \times 1 + 110 \times 2 + 60 \times 3 \\ + 30 \times 4 + 15 \times 5 + 5 \times 6 \end{pmatrix}}{330}$$

$$= \frac{0 + 35 + 220 + 180 + 120 + 75 + 30}{330}$$

$$= \frac{660}{330} = 2$$

∴ Average number of letters received per family = 2

Exercise © *Higher SkiU Level Questions*

1. (*d*) Required percentage increase

$$= \frac{55-45}{45} \times 100\% = 22.2\%$$

2. (*a*) Production of cement by factory B in 2009 and by factory D in 2010 together

$= 55 + 65 = 120$ lakh ton

Production of cement by factory A in 2008

$= 60$ lakh ton

$\therefore$ Required percentage

$$= \frac{120}{60} \times 100\% = 200\%$$

3. (*d*) Percentage increase in the production of cement from the year 2008 to the year 2009

for factory $A = \dfrac{70-60}{60} \times 100\% = 16.66\%$

for factory $B = \dfrac{55-45}{45} \times 100\% = 22.2\%$

for factory $C = \dfrac{85-65}{65} \times 100\% = 30.77\%$

for factory $D = \dfrac{60-50}{50} \times 100\% = 20\%$

Thus, percentage increase was maximum for factory C.

4. (*a*) Percentage increase in the production of cement from 2009 to 2010

for factory $A = \dfrac{75-70}{70} \times 100\% = 7.14\%$

for factory $B = -ve$

for factory $C = -ve$

for factory $D = \dfrac{65-60}{60} \times 100\% = 8.33\%$

Thus, percentage increase was maximum for factory *D*.

5. (*d*) Required difference

$$= \left(\frac{70 + 55 + 85 + 60}{4}\right) - \left(\frac{60 + 45 + 65 + 50}{4}\right)$$

$$= \frac{270}{4} - \frac{220}{4} = \frac{50}{4}$$

= 12.5 lakh ton

6. (*c*) Number of students qualified in the written test in the year 2002 for admission in institution $B = 3500$

Number of students qualified in the written test in the year 2006 for admission in institution $A = 4000$

Required ratio $= \frac{3500}{4000} = \frac{7}{8} = 7 : 8$

7. (*e*) Average number of candidates

$$= \frac{3500 + 4000 + 4000 + 5500 + 5000 + 6000}{6}$$

$$= \frac{28000}{6} \approx 4667$$

8. (*c*) Number of students in

2002 is 2000 + 3500 = 5500

2003 is 3500 + 4000 = 7500

2004 is 5000 + 4000 = 9000

2005 is 6000 + 5500 = 11500

2006 is 4000 + 5000 = 9000

2007 is 6500 + 6000 = 12500

Hence, 2005, year was the second highest.

9. (d) Required difference

= (4000 + 5000) - 3500 = 5500

10. (a) Required number of candidates

(2 + 3.5 + 5 + 6 + 4 + 6.5) X 1000 = 27000

11. (c) We shall first determine the number of cars for each model produced by the Company during the two years.

In 2012.

Total number of cars produced = 350000 P = (30-0)% of 350000

= 30% of 350000 = 105000 Q = (45 - 30)% of 350000

= 15% of 350000 = 52500 *R* = (65 - 45)% of 350000

= 20% of 350000 = 70000 *S* = (75 - 65)% of 350000

= 10% of 350000 = 35000 *T* = (90 - 75)% of 350000

= 15% of 350000 = 52500 *U* = (100 - 90)% of 350000 = 10% of 350000 = 35000 In 2013 : Total number of cars produced = 440000.

P = (40 - 0)% of 440000

= 40% of 440000 = 176000 Q = (60 - 40)% of 440000

= 20% of 440000 = 88000 *R* = (75 - 60)% of 440000

= 15% of 440000 = 66000 S = (85 - 75)% of 440000

= 10% of 440000 = 44000 *T* = (95 - 85)% of 440000

= 10% of 440000 = 44000 *U* = (100 - 95)% of 440000 = 5% of 440000 = 22000 Total number of cars of modesl *P, Q* and *T* manufactured in 2012 = 105000 + 52500 + 52500 = 210000

12. (6) Using the above calcualtion, the percentage change (rise/fall) in production from 2012 to 2013 for various models is

$$\text{For } P = \left[\frac{(176000 - 105000)}{105000} \times 100\right]\%$$
$$= 67.62\%, \text{ rise}$$
$$\text{For } Q = \left[\frac{(88000 - 52500)}{52500} \times 100\right]\%$$
$$= 67.62\%, \text{ rise}$$

$$\text{For } R = \left[\frac{(70000 - 66000)}{70000} \times 100\right]\%$$

$$= 5.71\%, \text{ fall}$$

$$\text{For } S = \left[\frac{(44000 - 35000)}{35000} \times 100\right]\%$$

$$= 25.71\%, \text{ rise}$$

$$\text{For } T = \left[\frac{(52500 - 44000)}{52500} \times 100\right]\%$$

$$= 16.19\%, \text{ fall}$$

$$\text{For } U = \left[\frac{(35000 - 22000)}{35000} \times 100\right]\%$$

$$= 37.14\%, \text{ fall}$$

.•. Minimum percentage rise/ fall in production is in the case of model R.

13. (a) Required difference

= 88000 -52500 = 35500 (using calculations from above questions No. 11)

14. (6) If the percentage production of *P* type cars in 2013 = Percentage production of *P* type cars in 2012 = 30% then, number of *P* type cars produced in 2013 = 30% of 440000 = 132000.

15. (c) Number of *S* type cars which remained unsold in 2012 = 15% of 350000

and number of *S* type cars which remained unsold in 2013 = 15% of 440000 /. Total number of *S* type cars which remains unsold

= 15% of (350000 + 440000) = 15% 790000 = 118500

16. (*a*) Required average

$$= \frac{5000 + 10000 + 15000 + 30000 + 40000 + 50000}{6}$$

$$= \frac{150000}{6} = 25000$$

17. (*c*) Required percentage

$$= \frac{40000}{25000} \times 100\% = 160\%$$

18. (*a*) Required number

$$= 90000 \times \frac{70}{100} = 63000$$

19. (*a*) Required ratio $= \frac{10000}{75000} = 2 : 15$

20. (*e*) Required percentage

$$= \frac{45000 - 30000}{45000} \times 100\% = 33\%$$

Chapter

43

Line Graph

A line graph (cartesian graph) indicates the variation of a quantity with respect to the two parameters caliberated (plotted) on X and Y-axes, respectively.

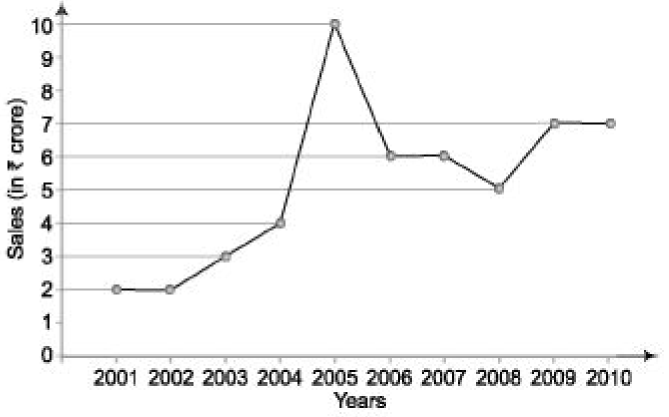
A line graph shows the quantitative information or a relationship between two changing quantities (variables) with a **line or curve** that connects a series of successive data points.

Types of Line Graph

Different types of line graph are discussed below

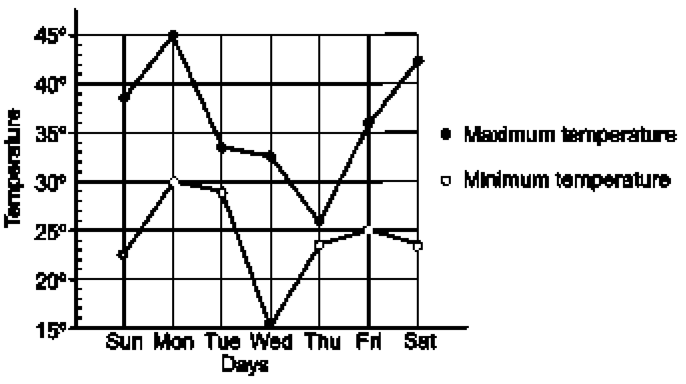
Single Line Graph Used for single variable representation.

For example The following single line graph represents the yearly sales figure of a company in the years 2001-2010.

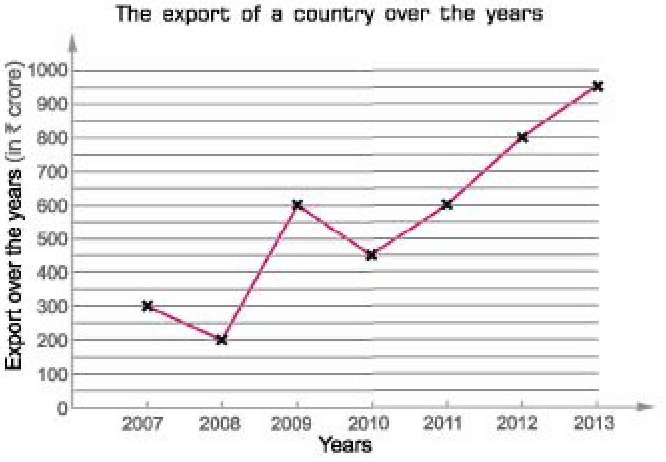


Multiple Line Graph Used for more than one variable representation.

For example The following multiple line graph represents the maximum and minimum temperature recorded everyday in a certain week.



Directions (Examples 1-5) *Study the following graph carefully and answer the given questions.*



Ex. 1 Which year has the highest per cent increase/decrease in exports as compared to the preceding year?

Sol. The per cent increase/decrease in exports as compared to the preceding year for the year

$$2008 = \frac{300 - 200}{300} \times 100\% = 33.33\% \text{ decrease}$$
$$2009 = \frac{600 - 200}{200} \times 100\% = 200\% \text{ increase}$$
$$2010 = \frac{600 - 450}{600} \times 100\% = 25\% \text{ decrease}$$
$$2011 = \frac{600 - 450}{450} \times 100\% = 33.33\% \text{ increase}$$
$$2012 = \frac{800 - 600}{600} \times 100\% = 33.33\% \text{ increase}$$
$$2013 = \frac{950 - 800}{800} \times 100\% = 18.75\% \text{ increase}$$

Thus, the calculations show that

(i) Highest per cent increase is in year 2009 *i.e.,* 200%. (ii) Highest per cent decrease is in year 2008 *i.e.,* 33.33%.

Ex. 2 What is the difference in exports in the years 2009 and 2010? **Sol.** Required difference = 600 - 450 = ? 150 crore

Ex. 3 What is the percentage increase in exports from the lowest to the highest for the given years?

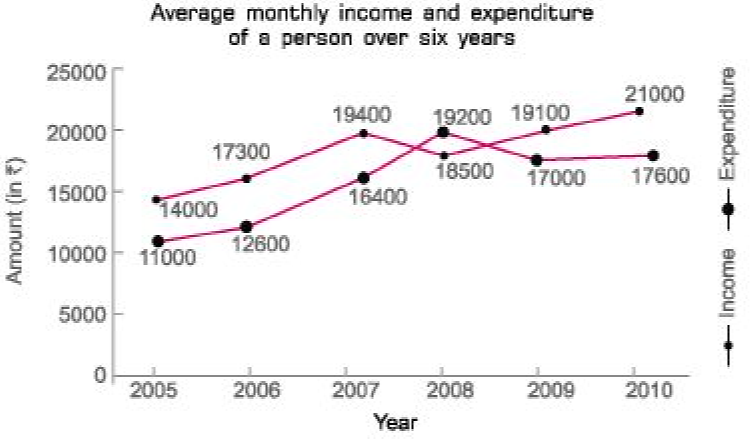
Sol. Required percentage $= \frac{950 - 200}{200} \times 100\% = 375\%$

Ex. 4 Exports in 2009 is approximately what per cent of that of year 2010?

Sol. Required percentage $= \frac{600}{450} \times 100\% = 135\%$

Ex. 5 What is the total exports (in ?/ crore) for the given years?

Sol. Total exports = 300 + 200 + 600 + 450 + 600 + 800 + 950 = ? 3900 crore **Directions** (Examples 6-8) *Refer to the data given below and answer the questions that follow.*



Note Data may be denoted by cross point or dot point

Ex. 6 In the year 2008, by how much did the person's saving get depleted? **Sol.** Average monthly depletion in savings for the year 2008 = 19200 - 18500 = ? 700 Hence, annual depletion = 700 X 12 = ? 8400

Ex. 7 What was percentage increase in expenditure of the person between years 2005 and 2010?

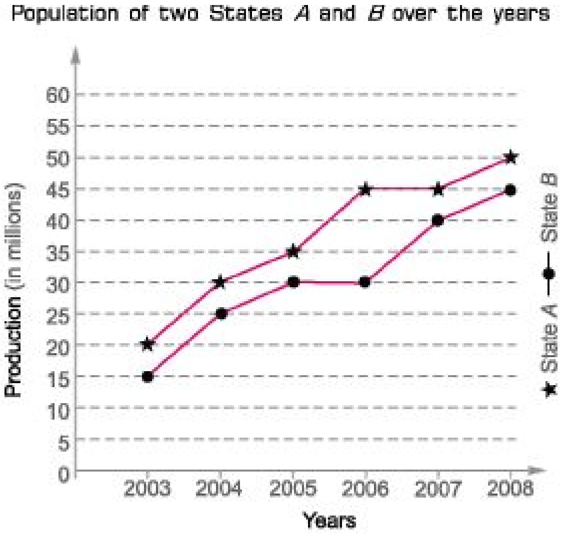
Sol. Required percentage increase $= \frac{(17600 - 11000) \times 100}{11000} \% = \frac{6600 \times 100}{11000} \% = 60\%$

Ex. 8 What was the percentage increase in income from 2006 to 2008?

Sol. Required percentage increase $= \left(\frac{18500 - 17300}{17300}\right) \times 100 = 6.93\% \approx 7\%$

Fast Track Practice

Directions (Q. Nos. 1-5) *Study the following graph carefully and answer the questions that follow.* [SBI Clerk 2009]



1. For State *B,* the percentage rise in population from the previous year was the highest in which of the following years?

(a) 2008 *(b)* 2006 *(C)* 2005 *(d)* 2004 *(e)* 2007

2. What was the average population of State *B* (in millions) for the years together?

(a) 38.5 *(b)* 28.5

(c) 35 *(d)* 26

(e) 30.83

3. What was the per cent rise in population of State *A* in 2007 from the previous year?

(a) 25 (b) $33\frac{1}{3}$

(c) 33 (d) $25\frac{1}{3}$

(e) None of the above

4. What was the ratio of the total population of State *A* to that of State *B*, for all the years together?

(a) 37 : 45 *(b)* 37 : 43

(c) 43 : 37 *(d)* 45 : 37

(e) None of the above

5. Population of State *A* in 2005 was what per cent of its total population for all the years together?

(a) $17\frac{8}{33}\%$

(b) $16\frac{5}{33}\%$

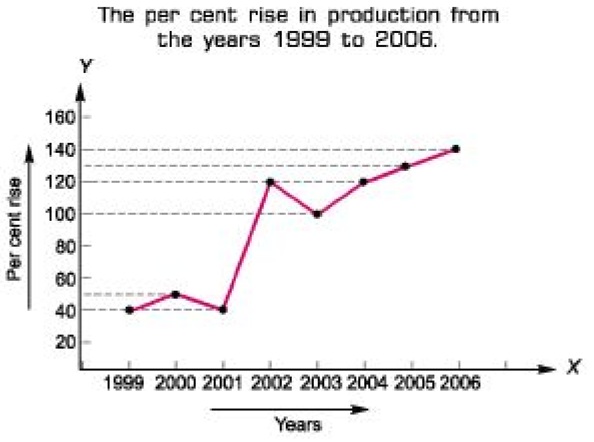
(c) $16\frac{8}{37}\%$

(d) $17\frac{8}{37}\%$

(e) None of the above

Directions (Q. Nos. 6-10) *Study the fallowing graph carefully and answer the questions that*

follow. **[Bank PO 2007]**



6. For how many years, the per cent rise was more than 100%?

(a) One *(b)* Two

(c) Three *(d)* Five

(e) None of the above

7. What was the average of per cent rise over the given years?

(a) 110% *(b)* 105%

(c) 100.5% *(d)* 92.5%

(e) None of the above

8. For how many years, the per cent rise was lower than the average of the per cent rise over the given years?

(a) Two *(b)* One

(c) Five *(d)* Three

(e) None of the above

9. For which of the given years, the per cent rise (from the previous year) was the least? [Years 2001 and 2003 are not to be considered.]

(a) 2000 *(b)* 2004 *(c)* 2006

(d) Cannot be determined

(e) None of the above

10. If the production in year 1998 was 1000 units, how much was the production in year 2002?

(a) 35280 units

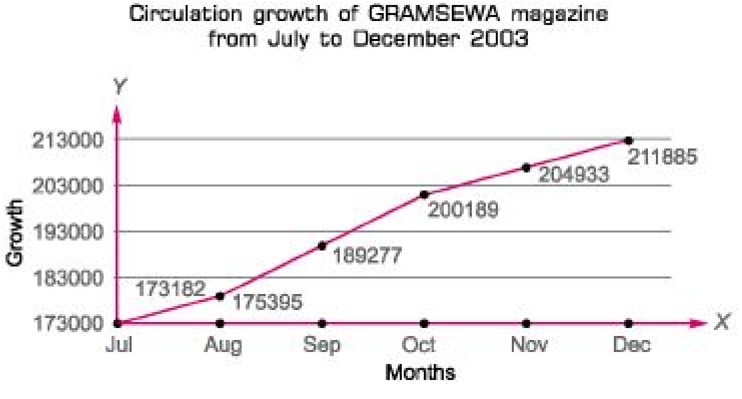
(b) 64680 units

(c) 46200 units

(d) Cannot be determined

(e) None of the above

Directions (Q. Nos. 11-15) *Study the following graph carefully and answer the questions that follow.*



11. During November and December, there is an even growth rate, the average of which is

(a) 2.36% *(b)* 2%

(c) 2.88% *(d)* 3.36%

(e) None of the above

12. The circulation in October is ... times than that of July.

(a) 1.5 *(b)* 2 *(c)* 1 *(d)* 1.15 *(e)* None of the above

13. The growth rate is very marginal during the month of

(a) August *(b)* October

(c) November *(d)* December

(e) None of the above

14. What is the total circulation of magazine from July to December?

(a) 1154681 *(b)* 1154861

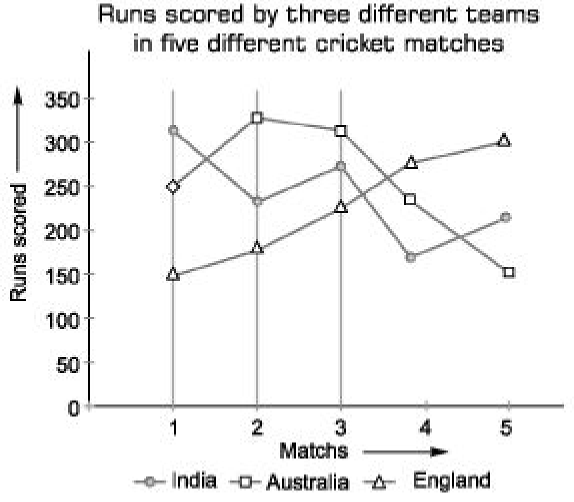
(c) 1145861 *(d)* 1150862

(e) None of the above

15. The circualtion of magazine in November is.........more than that of September.

(a) 15566 *(b)* 16556(c) 15656 *(d)* 16000 *(e)* None of the above

Directions (Q. Nos. 16-20) *Study the following graph carefully and answer the questions that follow.* [1DB1SO2012]



16. Total runs scored by India and Australia in Match 4 together is approximately, what percentage of the total runs scored by England in all the five matches together?

(a) 42 *(b)* 18 *(c)* 36 *(d)* 24 *(e)* 28

17. In which match, is the difference between the runs scored by Australia and England second lowest?

(a) 1 *(b)* 2 *(c)* 3 *(d)* 4

(e) 5

18. In which match the total runs scored by India and England together is the third highest/lowest?

(a) 1 *(b)* 2 *(c)* 3 *(d)* 4

(e) 5

19. What is the respective ratio between the runs scored by India in Match 5 Australia in Match 1 and England in Match 2?

(a) 11 : 13 : 7

(b) 11 : 7 : 13

(c) 11 : 3 : 9

(d) 11 : 13 : 9

(e) None of the above

20. What are the average runs scored by all the three teams in Match 3 together?

(a) 280

(b) 270

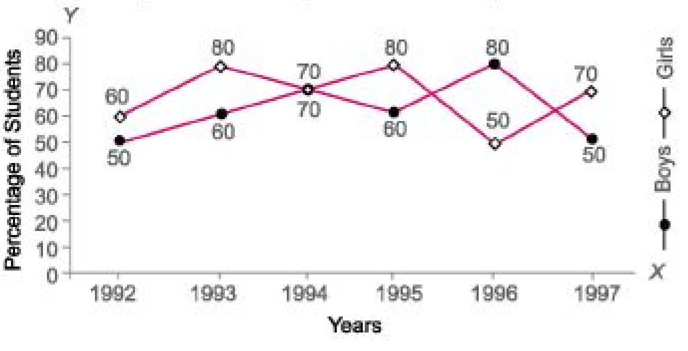
(c) 275

(d) 285

(e) None of the above

Directions (Q. Nos. 21-25) *Study the following graph carefully and answer the questions that follows.* [Bank Clerk 2010]

Percentage of students passed over the period shown.



21. What is the over all percentage of failed boys over the period shown?

(a) $61\frac{2}{3}$ *(b)* 35 *(c)* $35\frac{1}{3}$ *(d)* $38\frac{1}{3}$

(e) None of the above

22. In which of the following years was the improvement of girls maximum as compared to that of previous years?

(a) 1992 *(b)* 1993 *(c)* 1997 *(d)* 1995 *(e)* None of the above

23. For how many times was the gap constant for the percentage of successful boys and girls?

(a) Nil *(b)* 1 *(c)* 2 *(d)* 3

(e) None of the above

24. For how many years was there a paired rise or fall in the percentage of boys and girls passed over the period shown?

(a) Nil *(b)* 1

(c) 2 *(d)* 3

(e) None of the above

25. What is the ratio of the average of failed girls and that of failed boys?

(a) 17 : 21

(b) 23 : 19

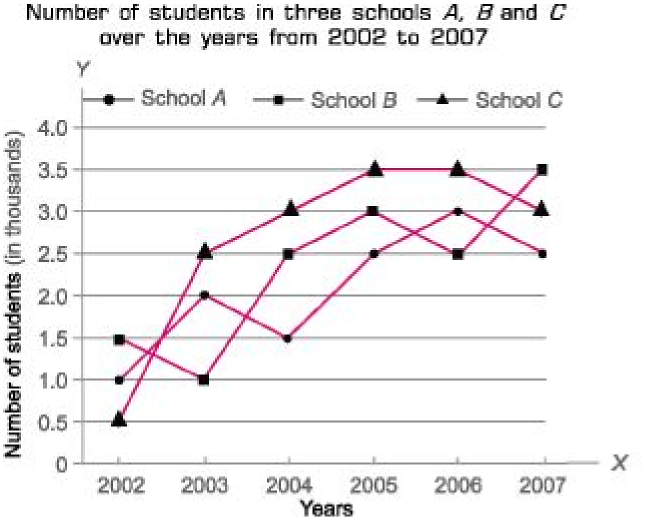
(c) 21 : 17

(d) 7 : 8

(e) None of the above

Directions (Q. Nos. 26-30) *Study the following graph carefully and answer the questions that*

follow. **[Bank Clerks 2010]**



26. What was the average number of students in all the schools together in the year 2006?

(a) 30000 *(b)* 9000

(c) 3000 *(d)* 6000

(e) None of the above

27. How many times the total number of students in all the three schools *A, B* and C together was exactly equal among the given years?

(a) 2 *(b)* 5

(o) 4 *(d)* 3

(e) None of the above

28. Total number of students in school *B* and school C together in the year 2004 was approximately what per cent of the total number of students in school *B* and school C together in the year 2007?

(a) 85 *(b)* 80

(c) 75 *(d)* 184

(e) 131

29. What was the difference between the total number of students in all the schools together in the year 2003 and the number of students in school *B* in the year 2005?

(a) 2000 (b) 3000

(c) 3500 (d) 2500

(e) None of the above

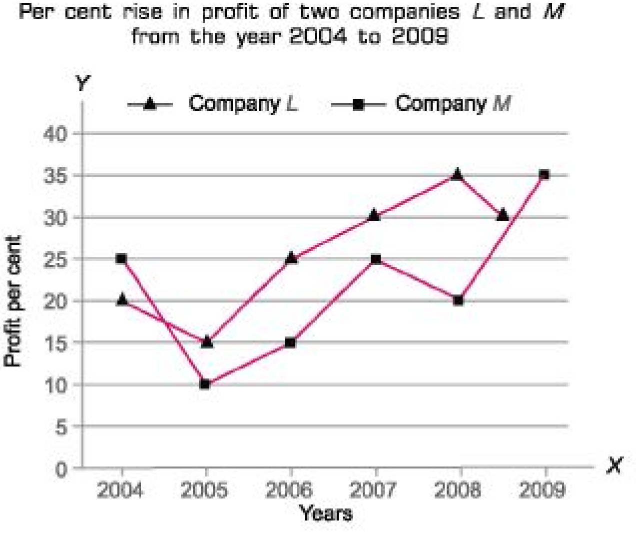
30. What was the approximate average number of students in school *A* over all the years together?

(a) 1990 (b) 2090

(c) 2300 (d) 1800

(e) 2700

Directions (Q. Nos. 31-35) *Study the following graph carefully and answer the questions that follow.* [Bank PO 2010]



31. If the profit earned by company *L* in the year 2005 was ? 1.84 lakh, what was the profit earned by the company in the year 2006?

(a) ? 2.12 lakh

(b) ? 2.3 lakh

(c) ? 2.04 lakh

(d) Cannot be determined

(e) None of the above

32. Which of the following statements is true with respect to the above graph?

(a) Company *M* made the highest profit in the year 2009

(b) Company *L* made the least profit in the year 2005

(c) The ratio of the profits earned by company *L* and company *M* in the year 2006 was 6 : 5

(d) Company *L* made the highest profit in the year 2008

(e) All of the above

33. What was the percentage increase in per cent rise in profit of company *M* in the year 2009 from the previous year?

{*a)* 25 *(b)* 15

(c) 50 *(d)* 75

(e) None of the above

34. If the profit earned by company *M* in the year 2008 was ? 3.63 lakh, what was the amount of profit earned by it in the year 2006?

(a) ? 2.16 lakh

(b) ? 1.98 lakh

(c) ? 2.42 lakh

(d) Cannot be determined

(e) None of the above

35. What was the average per cent rise in profit of company L over all the years together?

(a) $15\frac{1}{3}$ (b) $25\frac{1}{3}$

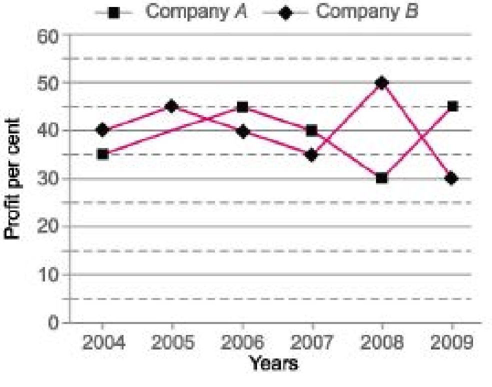
(c) $18\frac{5}{6}$ (d) $21\frac{5}{6}$

(e) None of the above

Directions (Q. Nos. 36-38) *Study the following graph carefully and answer the questions that follow.*

Percentage profit made by two companies A and B from the year 2004 to 2009

$$\text{Per cent profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100\ \%$$



Note If expenditure made by company is 100%, then income of company can be written as (100 + profit)%

36. If the amount of profit earned by company A in the year 2007 was ? 1.5 lakh, what was its expenditure in that year?

(a) ? 1.96 lakh

(b) f 2.64 lakh

(c) ? 1.27 lakh

(d) ? 3.75 lakh

(e) None of the above

37. What is the respective ratio of the amount of profit earned by companies *A* and *B* in the year 2009?

(a) 2:3

(b) 4:7

(c) 11 : 15

(d) Cannot be determined

(e) None of the above

38. In the year 2004, the expenditure incurred by companies *A* and *B* was the same, what was respective ratio of the incomes of companies *A* and *B* in that year?

(a) 27 : 28

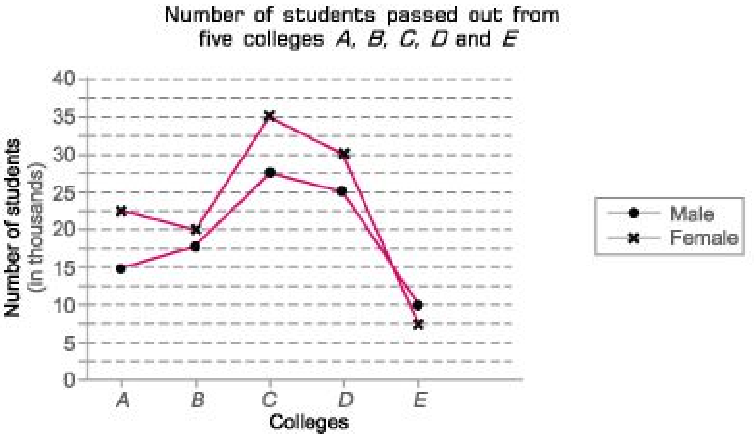
(b) 28 : 27

(c) 29 : 30

(d) 31 : 35

(e) None of the above

Directions (Q. Nos. 39-41) *Study the following graph carefully and answer the questions that follow.*



39. What is the sum of average number of males and average number of females passed out from all the colleges together?

(a) 38000 *(b)* 48000

(c) 42000 *(d)* 51000

(e) None of the above

40. The number of females passed out from college *C* is approximately what per cent of the total number of females passed out from all the colleges together?

(a) 28 *(b)* 30 *(c)* 36 *(d)* 25

(e) 40

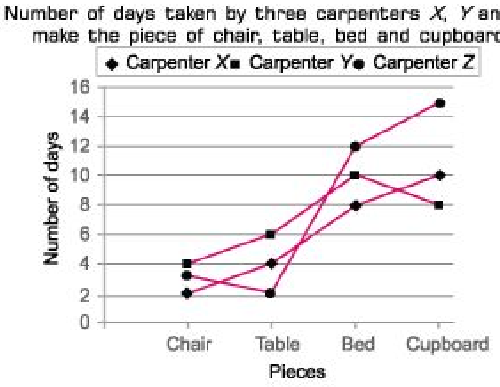
41. What is the difference between the total number of students passing out from college *A* and the total number of students passing out from college *El*

(a) 20500 *(b)* 21000

(c) 10500 *(d)* 10000

(e) None of the above

Directions (Q. Nos. 42-43) *Study the following graph carefully and answer the questions that follow.*



42. If carpenter *X* and carpenter *Y* were to make a chair together, then how many days would they take? *(a)* 1 day *(b)* 4 days

(c) 3 days *(d)* 2 days

(e) None of the above

43. What is the total number of days that carpenter *Z* will take to make one piece each of all the four items together?

(a) 32 days *(b)* 24 days

(c) $1\frac{1}{59}$ days *(d)* $1\frac{1}{32}$ days

Answer with Solutions

1. (*d*) When we see the graph carefully we observe that the slope or change is highest for year 2004 and 2007. So,

In the year 2004

$$= \frac{25 - 15}{15} \times 100\% = 66.66\%$$

In the year 2007

$$= \frac{40 - 30}{30} \times 100\% = 33.33\%$$

Hence, the percentage rise is highest in the year 2004.

2. (*e*) Required average

$$= \frac{15 + 25 + 30 + 30 + 40 + 45}{6}$$

$$= \frac{185}{6} = 30.83 \text{ million}$$

3. (*e*) Required percentage

$$= \frac{45 - 45}{45} \times 100\% = 0\%$$

4. (*d*) Required ratio

$$= \frac{20 + 30 + 35 + 45 + 45 + 50}{15 + 25 + 30 + 30 + 40 + 45}$$

$$= \frac{225}{185} = 45 : 37$$

5. (*e*) Required percentage

$$= \frac{35}{225} \times 100\% = 15\frac{5}{9}\%$$

6. (*e*) For 4 yr, *i.e.*, 2002, 2004, 2005 and 2006, the percentage rise was more than 100%.

7. (*d*) Required average

$$= \frac{\begin{pmatrix}40 + 50 + 40 + 120 + 100 \\ + 120 + 130 + 140\end{pmatrix}}{8}$$

$$= \frac{740}{8} = 92.5\%$$

8. (*d*) The relevant years were 1999, 2000 and 2001.

9. (*c*) In the year 2000,

$$= \frac{50 - 40}{40} \times 100\% = 25\%$$

In the year 2004, $\frac{120 - 100}{100} \times 100\% = 20\%$

In the year 2006, $\frac{140 - 120}{120} \times 100\% = 16.67\%$

Clearly, per cent rise was the least for the year 2006.

10. (*e*) Production in year 2002

$$= 1000 \times \frac{140}{100} \times \frac{150}{100} \times \frac{140}{100} \times \frac{220}{100}$$

$= 6468$

11. (*c*) Growth in November

$$= \frac{204933 - 200189}{200189} \times 100\% = 2.36\%$$

Growth in December

$$= \frac{211885 - 204933}{204933} \times 100\% = 3.39\%$$

$$\text{Average} = \frac{2.36 + 3.39}{2}\% = \frac{5.75}{2}\%$$

$= 2.875\% = 2.88\%$

12. (*d*) Let the circulation in October is x times to that of July.

Then, $200189 = x \times 173182$

$\therefore \quad x = \frac{200189}{173182} = 1.15$

13. (*a*) In the month of August, the growth rate is least.

14. (*b*) Total circulation

$= 173182 + 175395 + 189277 + 200189 + 204933 + 211885 = 1154861$

15. (*c*) 204933 – 189277 = 15656

(Q. Nos. 16-20)

	Runs		
Match	India	Australia	England
1	320	260	160
2	240	330	180
3	270	310	230
4	190	220	270
5	220	150	300

16. (*c*) Required percentage

$$= \frac{190 + 220}{160 + 180 + 230 + 270 + 300} \times 100$$

$$= \frac{410}{1140} \times 100$$

$= 36\%$

17. (c) Difference between the runs scored by Australia and England in Match 1 = 260 - 160 = 100 Match 2 = 330 -180 = 150 Match 3 = 310 -230 =80 Match 4 = 270 - 220 = 50 Match 5 = 300 - 150 = 150 .•. Second lowest difference in Match 3.

18. (a) Total runs scored by India and England in Match 1 = 320 + 160 = 480 Match 2 = 240 + 180 = 420 Match 3 = 270 + 230 = 500 Match 4 = 190 + 270 = 460 Match 5 = 220 + 300 = 520 /. Third highest/lowest score in Match 1.

19. (*d*) Required ratio

$$= 220 : 260 : 180 = 11 : 13 : 9$$

20. (*b*) Required average runs scored by all three teams in match 3.

$$= \frac{270 + 310 + 230}{3} = 270$$

21. (*d*) Over all percentage of passed boys

$$= \frac{50 + 60 + 70 + 60 + 80 + 50}{6}$$

$$= \frac{370}{6} = \frac{185}{3}$$

∴ Percentage of failed boys

$$= 100 - \frac{185}{3} = \frac{300 - 185}{3}$$

$$= \frac{115}{3} = 38\frac{1}{3}$$

22. (*c*) In the year 1993,

Percentage of girls

$$= \frac{80 - 60}{60} \times 100\% = 33.3\%$$

In the year 1997,

Percentage of girls

$$= \frac{70 - 50}{50} \times 100\% = 40\%$$

In 1997, the improvement of girls is maximum as compare to the previous years.

23. (*d*) For the years 1993, 1995 and 1997.

24. (*b*) Year 1993

25. (*e*) Average percentage of passed girls

$$= \frac{60 + 80 + 70 + 80 + 50 + 70}{6}$$

$$= 68\frac{1}{3}$$

∴ Average of percentage of failed girls

$$= 100 - 68\frac{1}{3} = 31\frac{2}{3}$$

and average percentage of failed boys

$$= 38\frac{1}{3}$$

∴ Ratio $= 31\frac{2}{3} : 38\frac{1}{3} = \frac{95}{3} \times \frac{3}{115} = 19 : 23$

26. (*c*) Required average number of students in 2006

$$= \frac{(2.5 + 3 + 3.5) \times 1000}{3}$$

$$= 3000$$

27. (d) Total number in the year 2002

= (0.5 + 1 + 1.5) X1000

= 3 X 1000 = 3000 Total number of students in the year 2003

= (1 + 2 + 2.5) X100

= 5.5X 1000 = 5500 Total number of students in the year 2004

= (1.5+ 2.5+ 3)x 100

= 7 X 1000 = 7000 Total number of students in the year 2005

= (2.5+ 3+ 3.5) X 100

= 9x 1000= 9000 Total number of students in the year 2006

= (2.5 + 3 + 3.5) X100

= 9 X1000 =9000 Total number of students in the year 2007

= (2.5 + 3 + 3.5) X100

= 9 X1000 =9000 Hence, the total number of students is same in the years 2005, 2006 and 2007.

28. (*a*) In the year 2004,

School B + School $C = (2.5 + 3)$

$= 5.5 \times 1000 = 5500$

In the year 2007,

School B + School $C = (3.5 + 3)$

$= 6.5 \times 1000 = 6500$

Required percentage

$$= \frac{5500}{6500} \times 100 = 84.61\% \approx 85\%$$

29. (*d*) $(1 + 2 + 2.5) - 3 = 2.5 \times 1000 = 2500$

30. (*b*) Required average

$$= \frac{(1 + 2 + 1.5 + 2.5 + 3 + 2.5)}{6} \times 1000$$

$$= \frac{12.5 \times 1000}{6}$$

$= 2083 \approx 2090$

31. (*b*) Profit of the company L in the year 2005 $= 1.84$ lakh

In the year 2006, 25% rise in the profit of the company L.

So, the profit of the company L in the year 2006 $= 1.84 \times \frac{(100 + 25)}{100}$

$= 1.84 \times 1.25 = ₹\ 2.3$ lakh

32. (a) According to the graph, it is clear that the profit of companies *L* and *M* increased every year.

Minimum profit in the year 2004 and maximum profit in the year 2009 by both companies.

Ratio cannot be determined because numerical values are not given. So, only Statement a is true.

33. (*d*) Required percentage in profit of company *L* in 2009

$$= \frac{35 - 20}{20} \times 100\% = 75\%$$

34. (*c*) Profit of company *M* in the year 2008

$$= 3.63 \text{ lakh}$$

$\therefore$ Profit of company *M* in the year 2006

$$= 3.63 \times \frac{100}{120} \times \frac{100}{125} = 2.42 \text{ lakh}$$

35. (*e*) Required average

$$= \frac{20 + 15 + 25 + 30 + 35 + 30}{6}$$

$$= \frac{155}{6} = 25\frac{5}{6}$$

36. (*d*) Suppose, the expenditure of company *A* in the year 2007 was ₹ x lakh.

According to the formula,

Per cent profit

$$= \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$

$$\therefore \quad 40 = \frac{1.5}{x} \times 100 \Rightarrow \quad 40x = 150$$

$$\therefore \quad x = \frac{150}{40} = 3.75 \text{ lakh}$$

37. (*d*) Data is insufficient. So, cannot be determined.

38. (*a*) Suppose, in year 2004, expenditure by companies *A* and *B* each was ₹ I and their incomes are I_1 and I_2, respectively.

For company *A*,

$$35 = \frac{I_1 - I}{I} \times 100$$

$$\Rightarrow \quad 35\,I = 100\,I_1 - 100\,I$$

$$\Rightarrow \quad 35\,I + 100\,I = 100\,I_1$$

$$\Rightarrow \quad 135\,I = 100 I_1 \quad \text{...(i)}$$

For company *B*,

$$40 = \frac{I_2 - I}{I} \times 100$$

$$\Rightarrow \quad 40\,I = 100\,I_2 - 100\,I$$

$$\Rightarrow \quad 40\,I + 100\,I = 100\,I_2$$

$$\Rightarrow \quad 140\,I = 100 I_2 \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{I_1}{I_2} = \frac{135}{140} = \frac{27}{28} = 27:28$$

39. (*c*) Average number of males

$$= \frac{15000 + 17500 + 27500 + 25000 + 10000}{5}$$

$$= \frac{95000}{5} = 19000$$

∴ Average number of females

$$= \frac{22500 + 20000 + 35000 + 30000 + 7500}{5}$$

$$= \frac{115000}{5} = 23000$$

∴ Required number of students

$$= 19000 + 23000 = 42000$$

40. (*b*) Number female passed out from collage $C = 35000$

Total students (female) = 115000

Required percentage

$$= \frac{35000}{115000} \times 100\% \approx 30\%$$

41. (*e*) Required difference

$$= (15000 + 22500) - (10000 + 7500)$$

$$= 37500 - 17500 = 20000$$

42. (*e*) Number of days taken by carpenter $X = 2$ days

Number of days taken by carpenter $Y = 4$ days

Total number of days taken by both carpenters X and $Y = \frac{2 \times 4}{2 + 4} = \frac{8}{6} = \frac{4}{3}$

$$= 1\frac{1}{3} \text{ days}$$

43. (*a*) Total number of days that carpenter Z will take to make one piece of all four items together

$$= 3 + 2 + 12 + 15 = 32 \text{ days}$$

Chapter

44

Mixed Graph

Mixed graph is used when the desired parameter is a function of two or three variables and the data are to be presented as a combination of two or more forms of data presentation.

In mixed graph, we study various types of graph based questions i.e. based on data tables, pie chart, bar chart, line graph etc., when the data are represented by any two of them, it is called **mixed graph.**

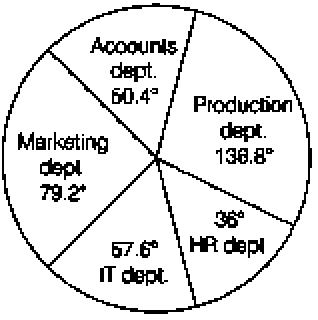
It may be the combination of data table and pie chart or data table and line graph or data table and bar chart or pie chart and line graph or bar chart and line graph,

Such type of questions requires a very careful analysis of data because the result is dependent on two or more graphs. *Examples given below will give a clear idea about the types of questions.*

Directions (Examples 1-5) *Study the graph carefully and answer the questions that follow.* [RBI Grade b 2011]

Degreewise distribution of employees working in various departments of an organisation and the ratio of number of men to number of women.

Total number of employees = 3250



Respective ratio of number of men to number of women in each department.

Department	Men	Women
Production	4	1
HR	12	13
IT	7	3
Marketing	3	2
Accounts	6	7

Note Above mixed graph is combination of pie chart and data table

Ex. 1 What is the number of men working in the marketing department?

Sol. Number of men working in the marketing department

$$= 3250 \times \frac{79.2}{360} \times \frac{3}{3+2} = 715 \times \frac{3}{5} = 143 \times 3 = 429$$

Ex. 2 What is the ratio of the number of women working in the HR department to the number of men working in the IT department?

Sol. Number of women working in the HR department

$$= 3250 \times \frac{36}{360} \times \frac{13}{13+12} = 169$$

Number of men working in the IT department

$$= 3250 \times \frac{57.6}{360} \times \frac{7}{7+3} = 364$$

$\therefore$ Required ratio $= \frac{169}{364} = \frac{13}{28} = 13 : 28$

Ex.3 The number of men working in the production department of the organisation is what per cent of the total number of employees working in that department?

Sol. Total number of employees in production department

$$= 3250 \times \frac{136.8}{360} = 1235$$

Number of men working in the production department

$$= 1235 \times \frac{4}{4+1} = 988$$

$$\therefore \text{ Required percentage} = \frac{988}{1235} \times 100\% = 80\%$$

Ex. 4 The number of women working in the IT department of the organisation is what per cent of the total number of employees in the organisation from all the departments together?

Sol. Number of women working in the IT department

$$= 3250 \times \frac{57.6}{360} \times \frac{3}{7+3} = 156$$

Total number of employees = 3250

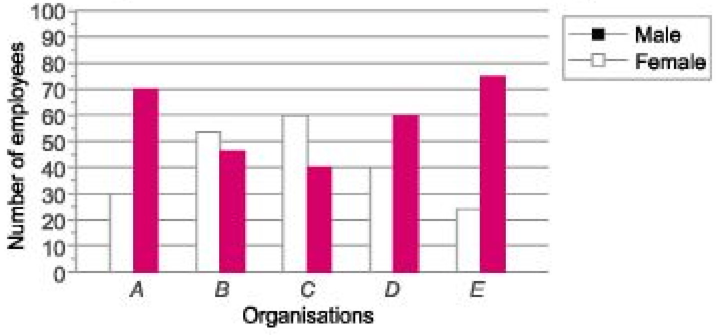
$$\therefore \text{ Required precentage} = \frac{156}{3250} \times 100\% = 4.8\%$$

Fast Track Practice

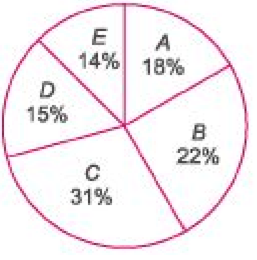
Exercise© *Base Level Questions*

Directions (Q. Nos. 1-5) *Study the following graph and pie chart carefully and answer the Questions that follow.*

Percentage of males and females in the five organisations



Percentage of employees in five different organisations
[Total number of employees = 35000]



1. Total number of employees in organisation C is approximately what per cent of total number of employees in organisation *D?*

(a) 147% *(b)* 279%

(c) 312% *(d)* 207%

(e) 183%

2. What is the total number of males in all the organisations together?

(a) 13350 *(b)* 14700

(c) 15960 *(d)* 16280

(e) None of the above

3. What is the total number of males in organisations *A* and *C* together?

(a) 6125 *(b)* 8400

(C) 8025 *(d)* 7400

(e) None of the above

4. What is the difference between the number of females in organisation *B* and the number of females in organization .E?

(a) 2100 *(b)* 3010

(c) 1700 *(d)* 3000

(e) None of the above

5. What is the number of females in organisation D?

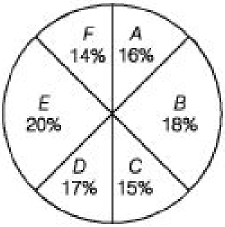
(a) 3855 *(b)* 3250

(c) 3300 *(d)* 3675

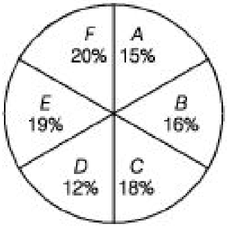
(e) None of the above

Directions (Q. Nos. 6-15) *Study the following pie charts and table to answer the questions that follow. Statewise details of adult population of a country* [**RBI** 2009]

Graduate and above
Total number = 24 lakh



Upto XII std. pass
Total number = 32 lakh



M : F Ratio [M = Male and F = Female]

State	Graduate and above		Upto XII Std. pass	
	M	F	M	F
A	7	5	7	9
B	5	3	3	5
C	5	4	4	5
D	9	8	5	7
E	9	7	9	10
F	4	3	3	2

6. What is the difference between graduate male population and XII Std. male population from State *At*

(a) 24000 *(b)* 14000

(c) 28000 *(d)* 36000

(e) None of the above

7. What is the ratio of graduate female population of State *E* to XII Std. female population of State *D,* respectively?

(a) 7 : 5 *(b)5:7*

(c) 16 : 15 *(d)* 15 : 16

(e) None of the above

8. Graduate female population of state C is what per cent of the XII Std. female population of that state?

(a) 40% *(b)* 62.5%

(c) 50% *(d)* 52.5%

(e) None of the above

9. Class XII pass male population of State C is what per cent of the total XII Std. population of all the states together?

(a) 8% *(b)* 12% *(c)* 11% *(d)* 9% *(e)* None of the above

10. What is the ratio of graduate male population of State *E* to XII Std. female population of that state?

(a) 28 : 35 *(b)* 35 : 28

(c) 32 : 45 *(d)* 45 : 32

(e) None of the above

Total graduate population of State *F* is what per cent of the total XII Std. population of State *At*

(a) 56% *(b)* 72% *(c)* 68% *(d)* 76%

(e) None of the above

12. XII Std. male population of state *E* is what per cent of XII Std. male population of State *Ft*

(a) 70% *(b)* 75% *(c)* 68% *(d)* 72% *(e)* None of the above

13. What is the ratio of the total graduate and XII Std. male population of State *A* to the total graduate and XII Std. female population of that state?

(a) 215 : 216 *(b)* 214 : 215

(c) 217 : 215 *(d)* 215 : 217

(e) None of the above

14. What is the ratio of the total graduate population of State *D* to total XII Std. population of that state?

(a) 17 : 16 *(b)* 16 : 17

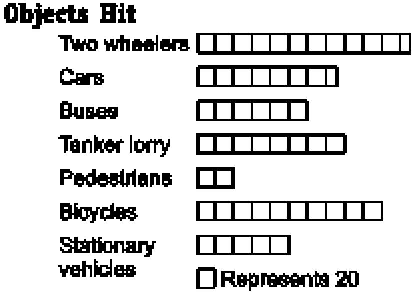
(C) 64 : 51 *(d)* 51 : 64

(e) None of the above

15. Graduate female population of State *B* is what per cent of the graduate female population of State *Et* (rounded off to nearest integer.)

(a) 129% *(b)* 82% *(c)* 77% *(d)* 107% *(e)* None of the above

Directions (Q. Nos. 16-20) *The fallowing is a horizontal bar diagram showing the accidents in which two wheelers are involved with other objects. Study the diagram and answer the questions.* [ssc (10 + 2) 2012]



16. The percentage of accidents in pedestrians and cyclists are involved is

(a) 60% *(b)* 20.4%*(c)* 24% *(d)* 6%

17. The percentage by which the accidents involving buses is less than the accidents involving tankers lorry is

(a) 40% *(b)* 28% *(c)* 6% *(d)* 4%

18. The difference in percentage between the accidents involving two wheelers and other objects is respectively

(a) 54, more *(b)* 54, less

(c) 77, more *(d)* 77, less

19. 60% of the accidents are caused due to

(a) two wheelers, cars, buses and stationary vehicles

(b) two wheelers, cars buses and tanker lorry

(c) cars, buses, tanker lorry and pedestrians

(d) cars, tanker lorry, bicycles and stationary vehicles

20. If the data of the bar diagram is represented by a pie chart and the angle of a sector of the pie chart is 36°, then this sector represents the accidents involving

(a) buses

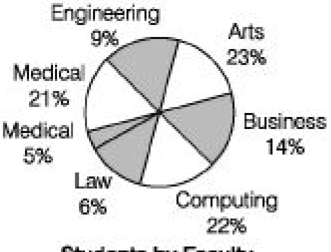
(b) stationary vehicles

(c) pedestrians

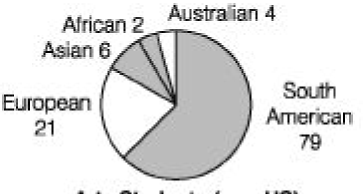
(d) bicycles

Exercise © *Higher Skill Level Questions*

Directions (Q. Nos. 1-5) *The pie charts below show the percentage of students in each faculty at North-West University and the number of non-US students in the Arts faculty. These percentages have been rounded to the nearest whole number. There are a total 1049 students in Arts faculty. Use this information to answer the fallowing questions.* [SNAP 2012]



Students by Faculty



Arts Students (non-US)

1. What percentage of students in the Arts faculty are non-US students?

(a) 14% *(b)* 9% *(c)* 30% *(d)* 11% *(e)* None of the above

2. How many students are there in the Engineering faculty?

(a) 420 *(b)* 410 *(c)* 390 *(d)* 440 *(e)* None of the above

3. How many students are there at the University?

(a) 4650 *(b)* 4560 *(c)* 4640 *(d)* 4450 *(e)* None of the above

4. If 6% of Science students are Asian, how many Asian students are studying Science?

(a) 48 *(b)* 66

(c) 120 *(d)* 57

(e) None of the above

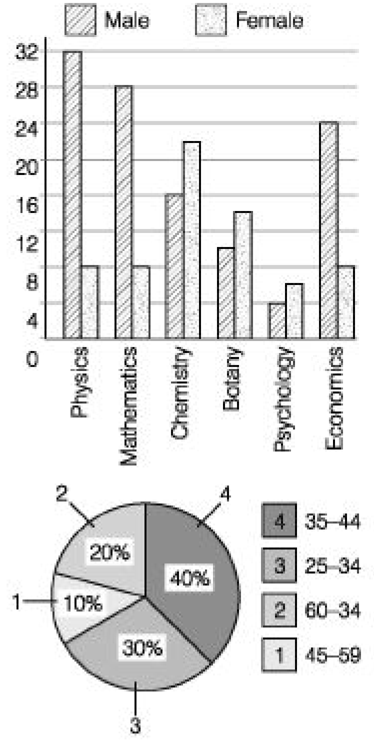
5. There are 34 European medical students. What percentage of the faculty does this represent?

(a) 14% *(b)* 18%

(c) 16% *(d)* 15%

(e) None of the above

Directions (Q. Nos. 6-10) *Items Study the two figures given below and answer the five items that follow.*



How many Physics professors belong to the age group 35-44? *(a)* 18 *(b)* 16 *(c)* 14 *(d)* 12

7. Which one of the following disciplines has the highest ratio of males to females?

(a) Physics *(b)* Mathematics

(c) Chemistry *(d)* Economics

8. What percentage of all Psychology professors are females?

(a) 40% *(b)* 50% *(c)* 60% *(d)* 70%

9. If the number of female Physics professors in the age group 25-34 equals 25% of all the Physics professors in that age group, then what is the number of male Physics professors in the age group 25-34?

(a) 9 *(b)* 6 *(c)* 3 *(d)* 2

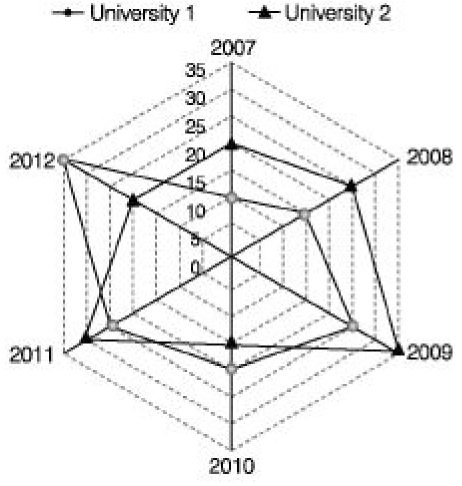
10. If the Psychology professors in the university constitute 2% of all the professors in the university, then what is the number of professors in the university?

(a) 400 *(b)* 500 *(c)* 600 *(d)* 700

Directions (Q. Nos. 11-15) *Study the radar graph carefully and answer the questions that follow.* [SBi po 2013]

Number of students (in thousand) in two

different universities in six different years.



11. What was the difference between the number of students in University 1 in the year 2010 and the number of students in University 2 in the year 2012?

(a) 0 *(b)* 5000 *(c)* 15000 *(d)* 10000 *(e)* 1000

12. What is the sum of the number of students in University 1 in the year 2007 and the number of students in University 2 in the year 2011 together?

(a) 50000 *(b)* 55000 *(c)* 45000 *(d)* 57000 *(e)* 40000

13. If 25% of the students in University 2 in the year 2010 were females, what was the number of male students in the University 2 in the same year?

(a) 11250(b) 12350(c) 12500(d) 11500 *(e)* 11750

What was the per cent increase in the number of students in University 1 in the year 2011 as compared to the previous year?

(a) 135 *(b)* 15 *(c)* 115 *(d)* 25

(e) 35 15. In which year was the difference between the number of students in University 1 and the number of students in University 2 highest?

(a) 2008 *(b)* 2009 *(c)* 2010 *(d)* 2011

(e) 2012

Answer with **Solutions**

Exercise © *Base Level Questions*

1. (*d*) Total No of employers = 35000 and
Total number of employees in organisation
$C = 31\%$ of 35000

$$= 35000 \times \frac{31}{100} = 10850$$

Similarly,
Total number of employees in organisation
$D = 15\%$ of 35000

$$= 35000 \times \frac{15}{100} = 5250$$

∴ Required percentage

$$= \frac{10850}{5250} \times 100\% \approx 207\%$$

2. (*e*) Total number of employees = 35000
Then, Number of males in organisation

$$A = 18\% \text{ of } 35000 \times 70\%$$

$$= \frac{35000 \times 18}{100} \times \frac{70}{100} = 4410$$

Similarly,

$$B = \frac{35000 \times 22}{100} \times \frac{45}{100} = 3465$$

$$C = \frac{35000 \times 31}{100} \times \frac{40}{100} = 4340$$

$$D = \frac{35000 \times 15}{100} \times \frac{60}{100} = 3150$$

$$E = 35000 \times \frac{14}{100} \times \frac{75}{100} = 3675$$

Hence, total number of males
= 4410 + 3465 + 4340 + 3150 + 3675
= 19040

3. (*e*) Number of males in organisations *A* and *C* together

$$= 35000 \times \frac{18}{100} \times \frac{70}{100} + 35000 \times \frac{31}{100} \times \frac{40}{100}$$

$$= 4410 + 4340 = 8750$$

4. (*b*) Required difference

$$= \left(35000 \times \frac{22}{100} \times \frac{55}{100}\right) - \left(35000 \times \frac{14}{100} \times \frac{25}{100}\right)$$

$$= 4235 - 1225 = 3010$$

5. (*e*) Number of females in organisation *D*

$$= 35000 \times \frac{15}{100} \times \frac{40}{100} = 2100$$

6. (*b*) Graduate male population in State *A*

$$= \left(24 \times \frac{16}{100} \times \frac{7}{12}\right) \text{lakh} = 2.24 \text{ lakh}$$

Now,

XII Std. male population

$$= \left(32 \times \frac{15}{100} \times \frac{7}{16}\right) \text{lakh} = 2.1 \text{ lakh}$$

$\therefore$ Required difference

$$= (2.24 - 2.1) \text{ lakh} = 14000$$

7. (*d*) Graduate female population of State *E*

$$= 24 \times \frac{20}{100} \times \frac{7}{16} = 2.1 \text{ lakh}$$

XII Std. female population of State *D*

$$= 32 \times \frac{12}{100} \times \frac{7}{12} = 2.24 \text{ lakh}$$

$\therefore$ Required ratio $= 2.1 : 2.24$

$$= 210 : 224 = 15 : 16$$

8. (*c*) Graduate female population of State *C*

$$= 24 \times \frac{15}{100} \times \frac{4}{9} = 1.6 \text{ lakh}$$

XII Std. female population of State *C*

$$= 32 \times \frac{18}{100} \times \frac{5}{9} = 3.2 \text{ lakh}$$

$\therefore$ Required percentage

$$= \frac{1.6}{3.2} \times 100\% = 50\%$$

9. (*a*) XII Std. pass male population of State *C*

$$= 32 \times \frac{18}{100} \times \frac{4}{9} = 2.56 \text{ lakh}$$

$\therefore$ Required percentage $= \dfrac{2.56}{32} \times 100 = 8\%$

10. (*e*) Graduate male population of State *E*

$$= 24 \times \frac{20}{100} \times \frac{9}{16} = 2.7 \text{ lakh}$$

XII Std. female population of State *E*

$$= 32 \times \frac{19}{100} \times \frac{10}{19} = 3.2 \text{ lakh}$$

$\therefore$ Required ratio $= 27 : 32$

11. (*e*) Total graduate population of State *F*

$$= 24 \times \frac{14}{100} = 3.36 \text{ lakh}$$

XII Std. population of State *A*

$$= 32 \times \frac{15}{100} = 4.8 \text{ lakh}$$

$\therefore$ Required percentage $= \dfrac{3.36}{4.8} \times 100 = 70\%$

12. (*b*) XII Std. pass male population of State E

$$= 32 \times \frac{19}{100} \times \frac{9}{19} = 2.88 \text{ lakh}$$

XII Std. pass male population of State F

$$= 32 \times \frac{20}{100} \times \frac{3}{5} = 3.84 \text{ lakh}$$

$\therefore$ Required percentage

$$= \frac{2.88}{3.84} \times 100\% = 75\%$$

13. (*c*) Graduate male population of State A

$$= 24 \times \frac{7}{12} \times \frac{16}{100} = 2.24 \text{ lakh}$$

XII Std. pass male population of state A

$$= 32 \times \frac{15}{100} \times \frac{7}{16} = 2.1 \text{ lakh}$$

Sum = (2.24 + 2.1) lakh = 4.34 lakh

Graduate female population of State A

$$= 24 \times \frac{5}{12} \times \frac{16}{100} = 1.6 \text{ lakh}$$

XII Std. pass female population of State A

$$= 32 \times \frac{15}{100} \times \frac{9}{16} = 2.7 \text{ lakh}$$

Sum = (1.6 + 2.7) lakh = 4.3 lakh

$\therefore$ Required ratio = 434 : 430 = 217 : 215

14. (*a*) Required ratio $= 24 \times \frac{17}{100} : 32 \times \frac{12}{100}$

$$= 24 \times 17 : 32 \times 12 = 17 : 16$$

15. (*c*) Graduate female population of State B

$$= 24 \times \frac{18}{100} \times \frac{3}{8} = 1.62 \text{ lakh}$$

Graduate female population of State E

$$= 24 \times \frac{20}{100} \times \frac{7}{16} = 2.1 \text{ lakh}$$

$\therefore$ Required percentage

$$= \frac{1.62}{2.1} \times 100\% \approx 77\%$$

16. (*c*) Total number of accidents = 50

Number of accidents of pedestrians and cyclists = 12

$\therefore$ Required percentage $= \frac{12}{50} \times 100 = 24\%$

17. (*d*) Percentage of accidents by buses

$$= \frac{6}{50} \times 100 = 12\%$$

Percentage of accidents by tanker lorry

$$= \frac{8}{50} \times 100 = 16\%$$

$\therefore$ Required difference = 16 − 12 = 4%

18. (*b*) Percentage of two wheelers in accidents

$$= \frac{11.5}{50} \times 100 = 23\%$$

Percentage of two wheelers and other vehicles in accidents

$$= \frac{(50 - 115)}{50} \times 100 = 77\%$$

$\therefore$ Required difference = 77 − 23 = 54 (less)

19. (*a*) Percentage of accidents by two wheelers

$$= 23\%$$

Percentage of accidents by cars $= \frac{7.5}{50} \times 100$

$$= 15\%$$

Percentage of accidents by buses

$$= \frac{6}{50} \times 100 = 12\%$$

Percentage of accidents by stationary vehicles $= \frac{5}{50} \times 100 = 10\%$

Total percentage

$$= 23\% + 15\% + 12\% + 10\% = 60\%$$

60% of the accidents occurs due to two wheelers, cars, buses and stationary vehicles.

20. (*b*) The angle of a sector of the pie chart = 36° or 10%

$\because$ $[360° = 100\%\ 1° = \frac{100}{360}$

and $36° = \frac{100}{360} \times 36 = 10\%]$

$\therefore$ This sector represents the accidents involving stationary vehicles.

Exercise © *Higher SkiU Level Questions*

1. (*c*) Total non-US Arts faculty

$$= 21 + 6 + 2 + 4 = 33$$

Total Arts faculty

$$= 21 + 6 + 2 + 4 + 79 = 112$$

$\therefore$ Percentage of non-US Arts faculty

$$= \frac{33}{112} \times 100 = 30\%$$

2. (6) As their are 1049 students in Arts which constitutes 23% of total students by faculty Let the total No. of students in the university = x.

$$\text{So,}\quad 23\%\text{ of } x = 1049$$

$$\text{or,}\quad \frac{23 \times x}{100} = 1049 \Rightarrow x = \frac{1049 \times 100}{23}$$

$$\therefore\quad x = 4560$$

$$\text{Total students} = 4580$$

Percentage of Engineering students = 9% Total number of Engineering students

$$= \frac{9 \times 4560}{100} = 410 \text{ students}$$

3. (*b*) Total number of students = 4560

(calculated in question number 2.)

4. (*d*) Total number of Science students

$= 21\%$ of total students

$= \dfrac{21 \times 4560}{100} \approx 958$ students

Now, there are 6% Asian students.

$\therefore$ Total Asian students

$= 6\%$ of total Science students

$= \dfrac{6 \times 958}{100} \approx 57$ students

5. (*d*) Total number of Medical students = 5% of total students $= \dfrac{5 \times 4560}{100} = 228$ students

Let $x\%$ faculty represents 34 medical students.

$\therefore \quad x\%$ of $228 = 34$

$$x = \frac{34}{228} \times 100 = 15\%$$

6. (*b*) From figure 1, the total number of Physics professors are $32 + 8 = 40$. From figure 2, 40% of Physics professors are in the age group 35-44. Hence, number of Physics professors in the 35-44 age group equals $40 \times \dfrac{40}{100} = 16$.

7. (*a*) Chemistry, Botany and Psychology have more females than males, so we don't calculate the ratios for them.

For Physics, male/female ratio $= \dfrac{32}{8} = 4$

For Mathematics, male/female ratio o

$= \dfrac{28}{8} < 4.$

For Economics, male/female ratio $= \dfrac{24}{8} = 3.$

Hence, Physics has the highest male/female ratio.

8. (*c*) Number of female Psychology professors is 6 from figure 1. Total number of Psychology professors is $6 + 4 = 10$, again from figure 1. Hence, percentage of all Psychology professors, who are female

$= \dfrac{6}{10} \times 100 = 60\%$

9. (*a*) Total number of Physics professors is 40, from figure 1. From figure 2, the percentage of Physics professors in the age group 25-34 is 30%.

Hence, total number of Physics professors in the 25-34 range is $40 \times \dfrac{30}{100} = 12.$

Out of these 12, 25% *i.e.*, $\left(\dfrac{25}{100}\right)$ are females. Hence, the number of male

Physics professors in the age group 25-34 is

$$12 - \frac{12 \times 25}{100}$$
$$= 12 - 3 = 9.$$

10. (*b*) From figure 1, total number of Psychology professors is 6 + 4 = 10. Let total number of professors in the university be *p*. We are given that 2% of *P* = 10.

Hence, $\frac{2P}{100} = 10$, which gives us $P = 500$.

11. (*a*) Number of students in University 1 in 2010 = 20000

Number of students in University 2 in 2012 = 20000

∴ Required difference

$$= 20000 - 20000 = 0$$

12. (*e*) Number of students in University 1 in 2007 = 10000

Number of students in University 2 in 2011 = 30000

∴ Required sum

$$= 10000 + 30000 = 40000$$

13. (*a*) Total students in University 2 in 2010 = 15000

∴ Number of females

$$= 15000 \times \frac{25}{100} = 3750$$

and number of males = 15000 − 3750 = 11250

14. (*d*) Number of students in University 1 in 2011 = 25000 and number of students in University 1 in 2010 = 20000

∴ Required percentage

$$= \frac{25000 - 20000}{20000} \times 100$$
$$= \frac{5000}{20000} \times 100 = 25\%$$

15. (e) Difference between the students of University 1 and University 2 in 2007

= 20000 - 10000 = 10000 Difference in number of students in 2008

= 25000 -15000 = 10000 Difference in number of students in 2009

= 35000 - 25000 = 10000 Difference in number of students in 2010

= 20000 -15000 = 5000 Difference in number of students in 2011

= 30000 - 25000 = 5000 Difference in number of students in 2012

= 35000 -20000 = 15000 It is clear from above that the difference between the number of students in University 1 and the number of students in University 2 is highest in the year 2012.

Chapter

45

Data Sufficiency

Introduction

Analysis of given data to reach conclusion is known as **Data Sufficiency.** Given data may or may not be sufficient for a definite conclusion. You have to decide whether the problem can be solved by using the information from the given statements combined or individually or it cannot be answered using these statement. These type of questions basically checks the analytical ability of a candidate and his basic knowledge in different topics.

Data sufficiency questions are not new topics in themselves. Hence, to solve the questions on Data Sufficiency, basic knowledge of algebra, geometry, arithmetic and statistics is prerequisite. The problems based on this topic consists of a mathematical or logical problem followed by two or more than two statements containing the information related to it.

Directions (Examples 1-5) *Each question below is followed by two Statements* I *and* II. *You are to determine whether the data given in the statement is sufficient for answering the question. You should use the data and your knowledge of Mathematics to* choose between the possible answers. *Give answer*

[Bank PO 2008]

(a) If the Statement I alone is sufficient to answer the question but the Statement II alone is not sufficient.

(b) If the Statement II alone is sufficient to answer the question but the Statement I alone is not sufficient.

(c) If both Statements I and II together are needed to answer the question.

(d) If either the Statement I alone or Statement II alone is sufficient to answer the question

(e) If answer cannot be obtained from the Statements I and II together but need even more data.

Ex. 1 Is A an odd number?

I. *A* multiplied by an odd number is equal to an odd number. II. *A* is not divisible by 2.

Sol. *(d)* From Statement I, *A* multiplied by an odd number is equal to an odd number. *i.e.,* Odd number X Odd number = Odd number .'. *A* is odd.

From Statement II, *A* is not divisible by 2, *i. e., A* is odd. Hence, both Statements I and II are alone sufficient to get the answer.

Ex. 2 The ages of P and *G* are in the ratio of 7 : 5. What is the age of P?

I. The ages of P and *N* are in the ratio of 3 : 1.

II. After 7 yr, the ratio of ages of P and *A* will be 4 : 3.

Sol. (e) We have to find the age of *P* and given data is not sufficient in both Statements I and II. Given informations are insufficient.

Ex. 3 What is the salary of *B* in a group of *A, B, C* and D whose average salary is ? 62880?

I. Total of the salaries of *A* and C is exact multiple of 8.

II. Average of the salary of *A, C* and D is ? 61665.

Sol. (6) From Statement II, salary of *B*= (62880 x 4) - (61665 x 3)

= 251520 - 184995 = 66525 Hence, only Statement II is sufficient to give the answer.

Ex. 4 What is the three-digit number?

I. The three-digit number is divisible by 9.

II. The first and the third digits are 6.

Sol. (*c*) From Statement II, $\boxed{6}\boxed{}\boxed{6}$

$\Rightarrow$ From Statement I, $\boxed{6}\boxed{6}\boxed{6}$

The number is divisible by 9, so both statements are required.

Ex. 5 What is the profit earned by selling a printer for ? 3000.

I. The cost price of 6 such printers is equal to selling price of 5 such printers. II. 20% profit is earned by selling each printer.

Sol. (*d*) From Statement I, profit per cent $= \frac{6-5}{5} \times 100\% = 20\%$

$$CP = \frac{3000 \times 100}{100 + 20} = ₹\ 2500$$

Similarly, CP can be find out from Statement II also.

So, we can get the amount of profit either by Statement I or II.

Directions (Examples 6-9) *Each of these questions is followed by information in Statements* I, II *and* III. *You are to study the questions and statements and decide which of the statements is/ are necessary to answer the questions.*

Ex. 6 What is the capacity of the cylindrical tank?

I. Radius of the base is half of its height.

II. Area of the base is 616 sq m.

III. Height of the cylinder is 28 m.

(a) I and II (b) II and III (c) I and III (d) All I, II and III

(e) Any two of the three

Sol. (e) To know the capacity, we have to find the volume of the cylinder *i.e.*, $\pi r^2 h$.

For this, any two of the three are enough *e.g.*,

Take Statements I and III,

$$h = 28 \text{ m, then } r = \frac{28}{2} = 14 \text{ m}$$

Then, capacity of cylindrical tank $\pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 28 = (22 \times 28 \times 28) \text{ m}^3$

From Statements II and III,

$$\text{Area of base} = \pi r^2 = 616 \text{ sq m}, h = 28$$

$\because$ Capacity = Area $\times h = \pi r^2 h = (616 \times 28) \text{ m}^3$

From Statements I and II,

$$\text{Capacity} = \left(616 \times 2 \times \sqrt{\frac{616}{\pi}}\right) \text{m}^3$$

Ex. 7 What is the speed of the train?

I. The train crosses a signal pole in 18 s.

II. The train crosses a platform whose length is equal to that of train in 36 s.

III. Length of the train is 300 m.

(a) I and III (b) II and III (c) I and II (d) III and either I or II (e) Any two of the three

Sol. (*d*) To know the speed of the train, we have to know distance and time. So, Statement III is essentially required. Either Statement I or II with Statement III provides the answer, *e.g.*, take Statements II and III.

Total distance covered to cross the platform = 300 + 300 = 600 m

600 m in 36 s. So, speed = $\frac{600}{36} \times \frac{18}{5} = 60$ km/h $\quad$ [1 m/s = $\frac{18}{5}$ km/h]

Ex. 8 What is staff strength of company X?

I. Male and female employees are in the ratio of 2 : 3, respectively.

II. Of the officer employees, 80% are males.

III. Total number of officers is 132.

(a) I and III (b) II and either III or I (c) All I, II and HI

(d) Any two of the three (e) Data is insufficient **Sol.** (e) Data is insufficient because combinely all statement fail to provide sufficient information required to answer.

Ex. 9 What is the two digit number?

I. Number obtained by interchanging the digits is more than the original number by 9.

II. Sum of the digits is 7.

III. Difference between the digits is 1.

(a) I and III (b) I and II (c) II and III

(d) II and either I or III (e) Any two of I, II and III

Sol. *(d)* Let unit's and ten's digits be xand *y,* respectively. Then, Number = $Wy + x$

Prom Statement I, (10a; + *y)* - (10j/ + *x)* = 9=>x - *y* = 1

Prom Statement **II,** $x + y = 7$ Prom Statement III, $x - y = 1$

.•. Number can be found using Statements II and either I or **III.** As two number of variables require two equation for knowing unknowns.

Fast Track Practice

Directions (Q. Nos. 1-5) *Each of the questions below consists of two statements numbered* I *and* II *given below it. You are to decide whether the data provided in the statements are sufficient to answer the questions. Read both the statements and give the answer* [Bank po 201 o]

(a) If the data in Statement I alone are sufficient to answer the question while the data in Statement II alone are not sufficient to answer the question

(b) If the data in Statement II alone are sufficient to answer the question while the data in Statement I alone are not sufficient to answer the question

(c) If the data in Statement I alone or in Statement II alone are sufficient to answer the question

(d) If the data in both the Statements I and II are not sufficient to answer the question

(e) If the data in both the Statements I and II together are necessary to answer the question

1. What is the perimeter of a semi-circle?

I. The radius of the semi-circle is equal to half the side of a square.

II. The area of the square is 196 sq cm

2. What is the exact average of *n,* 35, 39, 42, *p* and *w?*

I. *n* is six more than *w,*

II. *w* is four less than *p*

3. What was the per cent profit/loss earned incurred by selling an article for ? 24000?

I. The ratio of the selling price to the cost price of the article is 5 : 3

II. The difference between the cost price and the selling price is ? 9600.

What will be the difference between two numbers? I. The square of the first number is 9 times

the second number I. The ratio of the first number to the second number is 3 : 4

5. What is the ratio of two numbers *x* and *yi*

I. 40% of x is 20% of 50.

II. 30% of y is 25% of 72

Directions (Q. Nos. 6-10) *Each of the following questions is followed by information in three statements. You are to find out which statement(s) is/are sufficient to answer the question and mark your answer accordingly.*

6. What is the average age of the six members *A, B, C, D, E* and *F* in the family?

. Average age of *D* and £ is 14 yr

II. Average age of *A, B, C* and *F* is 50 yr

III. Average age of *A, B, D* and *E* is 40 yr (a) I and II (b) I and III

(c) II and III (d) I, II and III

(e) None of the above

7. What is the area of the right angled triangle?

I. Base of the triangle is X cm

II. Height of the triangle is / cm

III. Hypotenuse of the triangle is Z cm, *(a)* I and II *(b)* Only II

(c) II and III *(d)* Any two of three

(e) None of the above

8. In how many days will *B* alone complete the work?

I. *A* and *B* together can complete the work

in 8 days. IS and C together can complete the work

in 10 days. III. *A* and C together can complete the work

in 12 days. *(a)* Only I and II *(b)* Only II and II

(c) I, II and III

(d) Data is insufficient

(e) None of the above

9. What is the rate of interest percentage per annum?

I. An amount doubles itself at simple interest in 10 yr.

II. Difference between the compound interest and simple interest on an amount of? 15000 in 2yr is? 150.

III. The compound interest calculated after 8 yr is more than the amount principle *(a)* Only I *(b)* Only II

(c) II and III *(d)* I and III

(e) Either I or II

10. What are the marks scored by Abhijit in English?

. Marks scored by Abhijit in Mathematics are more than his marks in Science by 20

II. Total marks scored by Abhijit in Mathematics, Science and English are 197.

III. Marks scored by Abhijit in Science are more than his marks in English by 12

(a) Any two of the three

(b) II and III

(c) I, II and III

(d) Data is insufficient

(e) None of the above

Directions (Q. Nos. 11-15) *In each of the following questions, a question followed by two statements numbered* I *and* II *are given. You are to read both the statements and then give the answer*

(a) If the data given in Statement I alone are sufficient to answer the question whereas the data given in Statement II alone are not sufficient to answer the question

(b) If the data given in Statement II alone are sufficient to answer the question whereas the data given in Statement alone are not sufficient to answer the question

(c) If the data in either Statement I alone or in statement II alone are sufficient to answer the question

(d) If the data in both the Statements I and II are not sufficient to answer the question

(e) If the data given in both the Statements and II are necessary to answer the question

11. What is Supriya's present age?

I. Supriya is 3 yr older than Priya

II. The ratio of Priya's and Reshma's age is 3 : 4.

12. What is the rate of interest percentage per annum?

I. An amount of ?6200 fetches simple interest of ? 1736 in 2 yr.

II. An amount of ? 4500 fetches compound interest of ? 1348.20 in 2 yr.

13. How many marks did Nigam get in Biology?

I. Nigam got 42 marks in English which were half the marks he got in Biology

II. Nigam's marks in Biology were 14 % of the total marks he got in all the subjects together.

14. What is the speed of the boat in still water?

I. The boat travels at the speed of 4 km/h upstream.

II. The boat travels at the speed of 6 km/h downstream.

15. What is the two-digit number?

I. The difference between the two digits of the number is 0.

II. The sum of the two-digits of the number is 18.

Directions (Q. Nos. 16-20) *Each question below is followed by two statements* I *and* II. *You are to determine whether the data given in the statement is sufficient for answering the question. You should use the data and your knowledge of Mathematics to choose the possible answers. Give the answer*

[Bank PO 2009]

(a) If the Statement I alone is sufficient to answer the question but the Statement II alone is not sufficient

(b) If the Statement II alone is sufficient to answer the question but the Statement alone is not sufficient

(c) If both Statements I and II together are needed to answer the question

(d) If either the Statement I alone or Statement II alone is sufficient to answer the question

(e) If you cannot get the answer from the Statements I and II together but need even more data

16. What is the number of teachers in the school?

I. Each teacher takes atleast three lectures in a day

II. There are 45 lectures in a week

17. In how many years can a simple interest of *X* 6570 be obtained on an amount of ? 36500?

I. The rate of simple interest is 6% per annum.

II. The difference between the simple interest and compound interest is ? 402.084.

18. What is the three digit number?

I. Two-fifth of the number is half of 204

II. 20% of the number is 51.

19. What is Raveena's age?

I. Raveena is half as old as Karishma

II. Raveena's age is three-fifth of her mother's age who is 45 yr old

20. What is the area of the rectangular plot?

I. The length of the plot is 375 m.

I. The length of the plot is thrice its breadth

Directions (Q. Nos. 21-25) *Each of the questions below consists of a question and two statements numbered* I *and* II *given below it. You have to decide whether the data provided in the statement are sufficient to answer the question.* [IDBISO2012]

(a) If the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question

(b) If the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question

(c) If the data in either Statement I alone or in Statement II alone are sufficient to answer the question

(d) If the data in both the Statements I and I are not sufficient to answer the question

(e) If the data in both the Statements I and II together are necessary to answer the question

What is the minimum passing percentage in a test?

. Raman scored 25% marks in the test and Sunil scored 288 marks which is 128 more than Raman II. Raman scored 64 marks less than the minimum passing marks

22. What is the value of $x^2 + y + z$?

I. $4x + 3y + 5z = 60, 2x = y, 2y = z$

II. $3x + 3y + 2z = 34, 2x + 5y + 6z = 72$

23. Whose body weight is second highest among the five boys Arun, Vinay. Suraj, Raju and Pratap?

I. Average weight of Arun, Suraj and Vinay is 68 kg and average weight of Raju and Pratap is 72 kg. Also, Suraj is 78 kg, Raju is 68 kg and Vinay is 46 kg.

II. Average weight of Arun, Suraj, Vinay and Raju is 68 kg and also Suraj is 78 kg, Raju is 68 kg and Vinay is 46 kg. All of them have different weight.

24. What is the respective ratio between the length of a rectangle and side of a square?

I. Area of the square is 576 sq cm and the area of the rectangle is 600 sq cm

I. Breadth of the rectangle is half the side of the square.

25. What is the smaller angle of a parallelogram?

I. Ratio between the angles of a triangle is 3:5:4 and the larger angle, of the

parallelogram is 34° more than the largest angle of the triangle. II. Larger angle of the parallelogram is 38° more than its smaller angle.

Directions (Q. Nos. 26-30) *Each of the questions below is followed by two statements* I *and* n. *You are to determine whether the data given in the statement are sufficient for answering the question. You should use the data and your knowledge of Mathematics to choose between the possible answers. Give answer*

(a) If the Statement I alone is sufficient to answer the question but the Statement II alone is not sufficient

(b) If the Statement II alone is sufficient to answer the question but the Statement alone is not sufficient

(c) If both the Statements I and II together are needed to answer the question

(d) If either the Statement I alone or Statement II alone is sufficient to answer the question

(e) If you cannot get the answer from the Statements I and II together but need even more data [Bank PO 2008]

26. The ages of Anand and Sujeet are in the ratio of 6 : 5. What is the age of Anand?

The ages of Anand and Sandeep are in the ratio of 10 : 7. II. After 5 yr, the ratio of Anand's and Sujeet's ages will be 7 : 6

27. What is the three digit number?

Three-fifth of that number is less than that

number by 90 II. One-fourth of that number is 25% of that number

28. In how many days 14 men can complete a piece of work?

I. If 18 women can complete the same piece of work in 24 days

II. If 28 children can complete the same piece of work in 56 days

29. What is the salary of *A,* in a group of *A, B, C, D* and *E* whose average salary is ? 65780?

. Total of the salaries of *B* and C is ? 88545 11. Total of the salaries of *D* and *E* is *X* 59020

30. What is the profit earned by selling a watch for ? 15675?

I. Total cost price of 5 such watches is equal to selling price of 4 such watches

II. 25% profit is earned by selling each watch.

Answer with Solutions

1. (*e*) From Statement II,
Suppose, side of a square be x.
Hence, $x^2 = 196$
$\therefore$ $x = 14$ cm
From Statements I and II,
Radius of semi-circle, $r = 7$ cm
$\therefore$ Circumference of semi-circle
$$= \pi r = \frac{22}{7} \times 7 = 22 \text{ cm}$$
Hence, both statements are required.

2. (*d*) From Statement I,
$$n = W + 6$$
From Statement II,
$$p = w + 4$$
The value of w is not given.
So, exact average can't be determined.

3. (*a*) Selling price of an article = ₹ 24000
From Statement I,
Cost price $= \frac{3}{5} \times 24000 =$ ₹ 14400
$$\therefore \text{ Profit \%} = \frac{24000 - 14400}{14400} \times 100$$
$$= 66.66\%$$
From Statements II,
It is not clear that cost price is more than selling price or *vice-versa*.

4. (*e*) Suppose, numbers be x and y.
Statement I, $x^2 = 9y$
Statement II, $\frac{x}{y} = \frac{3}{4}$
From both statements, we get
$$x = 12 \text{ and } y = 16$$

5. (*e*) Statement I,
$$\frac{40}{100} \times x = \frac{20}{100} \times 50 \Rightarrow x = 25$$
Statement II,
$$\frac{30}{100} \times y = \frac{25}{100} \times 72 \Rightarrow y = 60$$
From both statements,
$$x : y = 25 : 60 = 5 : 12$$

6. (*a*) From Statement I,
$$D + E = 14 \times 2 = 28$$
From Statment II,
$$A + B + C + F = 4 \times 50 = 200$$
$$\frac{A + B + C + D + E + F}{6} = \frac{28 + 200}{6}$$
$$= \frac{228}{6} = 38$$
$\therefore$ Statements I and II are sufficient to answer the question.

7. (*d*) Area of triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times x \times y = \frac{1}{2} xy$$

So, to find the area, the measure of height and base is required which can be obtained by any of the two statement.

8. (*c*) $(A + B)$'s 1 day's work $= \frac{1}{8}$ ...(i)

$(B + C)$'s 1 day's work $= \frac{1}{10}$ ...(ii)

$(C + A)$'s 1 day's work $= \frac{1}{12}$ ...(iii)

$2(A + B + C)$'s 1 day's work $= \frac{1}{8} + \frac{1}{10} + \frac{1}{12}$

$(A + B + C)$'s 1 day's work

$$= \frac{1}{2}\left[\frac{1}{8} + \frac{1}{10} + \frac{1}{12}\right] \quad \text{...(iv)}$$

Subtracting Eq. (iii) from Eq. (iv), we get B's 1 day's work.

Then, required number of days

$$= \frac{1}{B\text{'s 1 day's work}}$$

9. (*e*) From Statement I,

$$\text{SI} = \frac{PRT}{100}$$

$$\Rightarrow \quad P = \frac{P \times R \times 10}{100}$$

$$\Rightarrow \quad R = 10\%$$

From Statement II,

$$\text{Difference } (D) = \frac{PR^2}{(100)^2}$$

$$\Rightarrow \quad 150 = \frac{15000 \times R^2}{10000}$$

$$\Rightarrow \quad R = 10\%$$

Thus, either Statement I or II is sufficient.

10. (*c*) From Statements I and III,

Let marks in English be x

Marks in Science $= x + 12$

and marks in Mathematics $= x + 32$

From Statement II,

$E + S + M = 197$

$$x + x + 12 + x + 32 = 197$$

$$\Rightarrow \quad 3x + 44 = 197$$

$$\Rightarrow \quad 3x = 197 - 44 = 153$$

$$\Rightarrow \quad x = 51$$

$\therefore$ Marks in English $= 51$

11. (*d*) Data are not sufficient even from Statements I and II both.

12. (*c*) From Statement I,

$$R = \frac{SI \times 100}{P \times T}$$

$$\Rightarrow \quad \frac{1736 \times 100}{6200 \times 2} = 14\%$$

From Statement II,

$$1348.2 = 4500\left(1 + \frac{r}{100}\right)^2 - 4500$$

$$\Rightarrow 1348.2 + 4500 = 4500\left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \quad 5848.2 = 4500\left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \quad \frac{5848.2}{4500} = \left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \quad \frac{3249}{2500} = \left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \quad \left(\frac{57}{50}\right)^2 = \left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \quad 1 + \frac{r}{100} = \frac{57}{50}$$

$$\Rightarrow \quad \frac{r}{100} = \frac{57}{50} - 1 = \frac{7}{50}$$

$$\Rightarrow \quad r = \frac{7 \times 100}{50}$$

$$\therefore \quad r = 14\%$$

Both statements are alone sufficient.

13. (*a*) From Statement I,

Marks in $\frac{\text{Biology}}{2}$ = Marks in English

$\Rightarrow$ Marks in Biology

$$= 2 \times 42 = 84 \text{ marks}$$

$$\therefore \quad B = 84 \text{ marks}$$

In Statement II, total marks is not given.

14. (*e*) We know that,

Speed of boat in still water

$$= \frac{\text{Speed upstream} + \text{Speed downstream}}{2}$$

From Statements I and II,

$$\text{Speed} = \frac{6+4}{2} = \frac{10}{2} = 5 \text{ km/h}$$

So, answer can be determined from both statements.

15. (*e*) From Statements I and II,

$$9 - 9 = 0 \text{ and } 9 + 9 = 18$$

So, number = 99

16. (*e*) Data is insufficient, so we cannot get answer from all two statements.

17. (*a*) From Statement I,

$$T = \frac{SI \times 100}{P \times R} = \frac{6570 \times 100}{36500 \times 6} = 3 \text{ yr}$$

18. (*d*) Suppose the number be x.

From Statement I,

$$x \times \frac{2}{5} = \frac{1}{2} \times 204$$

$$\Rightarrow \quad x \times \frac{2}{5} = 102$$

$$\Rightarrow \quad x = 255$$

From Statement II,

$$x \times \frac{20}{100} = 51$$

$$\Rightarrow \quad x = \frac{51 \times 100}{20}$$

$$x = 255$$

Both statements alone are sufficient.

19. (*b*) From Statement II,

Age of Raveena $= 45 \times \frac{3}{5} = 27$ yr

20. (*c*) From Statements I and II,

Length of plot = 375m

Breadth of plot $= \frac{375}{3}$ m = 125m

Area of rectangular plot $= l \times b$

$= 375 \times 125$ sq m = 46875 sq m

21. (*e*) From Statement I,

Marks scored by Sunil

= 288 and marks scored by Raman

$= 288 - 128 = 160$

$\therefore$ Maximum marks $= \frac{160 \times 100}{25} = 640$

From Statement II,

Minimum passing Marks

$= 160 + 64 = 224$

$\therefore$ Minimum passing percentage

$$= \frac{224}{640} \times 100 = 35\%$$

Hence, the data in both the statements are necessary to answer the question.

22. (*a*) From Statement I

$$4x + 3y + 5z = 60 \quad \text{...(i)}$$

$$2x = y$$

$$\Rightarrow \quad x = \frac{y}{2}$$

$$\Rightarrow \quad 2y = z$$

From Eq. (i), $4 \times \frac{y}{2} + 3y + 5 \times 2y = 60$

$$15y = 60 \Rightarrow y = 4$$

$$x = 2, z = 8$$

$$\therefore \quad x^2 + y + z = (2)^2 + 4 + 8$$

$$= 16 \quad \text{...(i)}$$

From Statement II,

$$3x + 3y + 2z = 34$$

and $\quad 2x + 5y + 6z = 72 \quad$ ...(ii)

We cannot find the value of x, y and z from Eqs. (i) and (ii).

23. (c) From Statement I,

Total weight of Arun, Suraj and Vinay

= 3 x 68 kg .". Arun's weight

= 3 x 68 - (78 + 46) = 80 kg Now, total weight of Raju and Pratap

= 2 x 72 = 144 kg ∴ Pratap's weight = 144 - 68 = 76 kg /. Second highest weight = 78 kg

(Suraj's weight) From statement II, Total weight of Arun, Suraj, Vinay and Raju

= 4 X 68 = 272 kg /. Arun's weight

= 272-(78 + 68 + 46) = 272- 192 = 80 kg /. Second highest weight

= 78 kg (Suraj's weight)

24. (e) From Statement I,
Area of square = 576 sq cm
∴ Side of square = 24 cm and area of rectangle = 600 sq cm

$$l \times b = 600$$

We cannot find the length of rectangle.
From Statement II,
Breadth of rectangle

$$= \frac{1}{2} \times \text{Side of square}$$

$$= \frac{1}{2} \times 24 = 12 \text{ cm}$$

∴ Length of rectangle

$$= \frac{600}{12} = 50 \text{ cm}$$

$$\therefore \quad \text{Ratio} = \frac{\text{Length of rectangle}}{\text{Side of square}} = \frac{50}{24}$$

$$= \frac{25}{12}$$

25. (c) From Statement I,
Larger angle of triangle

$$= \frac{5}{3+5+4} \times 180$$

$$= 75°$$

∴ Larger angle of parallelogram

$$= 75° + 34°$$

$$= 109°$$

∴ Smaller angle of parallelogram

$$= 180° - 109°$$

$$= 71°$$

From Statement II,
Let the smaller angle of parallelogram be $= x$
and the larger angle of parallelogram
$$= x + 38°$$
Sum of two opposite angles of a parallelogram $= 180°$
$$x + x + 38° = 180°$$
$$\therefore \quad x = \frac{180° - 38°}{2}$$
$$= \frac{142°}{2} = 71°$$

26. (*b*) From Statement II,
The ratio of their ages after 5 yr is 7 : 6
So, $$\frac{6x + 5}{5x + 5} = \frac{7}{6}$$
$$\Rightarrow \quad 36x + 30 = 35x + 35$$
$$\Rightarrow \quad 36x - 35x = 35 - 30$$
$$\Rightarrow \quad x = 5$$
$\therefore$ Age of Anand $= 6 \times 5 = 30$ yr
So, Statement II is alone sufficient.

27. (*a*) From Statement I,
Suppose the number be x.
$$x - \frac{3}{5}x = 90, \frac{2}{5}x = 90$$
$$x = 225$$
From Statement II,
$$x \times \frac{1}{4} = \frac{25}{100}x \Rightarrow \frac{x}{4} = \frac{x}{4}$$
So, Statement I is sufficient.

28. (*e*) More data is required.

29. (*c*) From the Statement I and II,
Salary of $A = 5 \times 65780 - (88545 + 59020)$
$= 328900 - 147565 =$ ₹ 181335
Both Statements are required.

30. (*d*) From Statement I,
Selling price of a watch = ₹ 15675
and the cost price of a watch $= 15675 \times \frac{4}{5}$
= ₹ 12540
$\therefore$ Profit $= 15675 - 12540 =$ ₹ 3135
From Statement II,
Cost price of a watch
$$= \frac{15675}{100 + 25} \times 100 = ₹12540$$
$\therefore$ Profit $= 15675 - 12540 =$ ₹ 3135

Practice Set 1

1. The value of
$\sin^2\theta\cos^2\theta(\sec^2\theta + \text{cosec}\ \theta)$ is

(a) 2 (b) 4 (c) 1 (d) 3

2. If (cosec θ – cot θ) , then (cosec θ cot θ) is equal to

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{2}$

3. If $4x = \sec\theta$ and $\frac{4}{x} = \tan\theta$, then $8\left(x^2 - \frac{1}{x^2}\right)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

4. If $A = \sin^4\theta + \cos^4\theta$, then

(a) $0 < A < \frac{1}{2}$ (b) $\frac{1}{2} \le A \le 1$

(c) $1 < A \le \frac{3}{2}$ (d) $\frac{3}{2} \le A \le 2$

5. If sec θ tan $\theta = P$, then cos θ is equal to

(a) $\frac{P^2+1}{P^2-1}$ (b) $\frac{P^2-1}{(P^2+1)^2}$

(c) $\frac{2P}{P^2+1}$ (d) $\frac{4P^2}{(P^2+1)^2}$

6. If $(a-b) = 3$, $(b-c) = 5$ and $(c-a) = 1$, then value of $\frac{a^3+b^3+c^3-3abc}{a+b+c}$ is

(a) 10.5 (b) 15.5 (c) 17.5 (d) 20.5

7. If $x^4 + \frac{1}{x^4} = 119$, then value of $x^3 - \frac{1}{x^3}$ is equal to

(a) 27 (b) 36 (c) 45 (d) 54

8. Simplified value of
$\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ldots\left(1-\frac{1}{99}\right)\left(1-\frac{1}{100}\right)$ is

(a) $\frac{2}{99}$ (b) $\frac{1}{25}$

(c) $\frac{1}{50}$ (d) $\frac{1}{100}$

9. If $1^3 + 2^3 + 3^3 + \ldots + 10^3 = 3025$, then find the value of $2^3 + 4^3 + 6^3 + \ldots + 20^3$ *(a)* 6050 *(b)* 9075 *(c)* 12100 *(d)* 24200 10. If 25^{25} is divided by 26, then remainder is

(a) 1 *(b)* 2 *(c)* 24 *(d)* 25

11. The simplified value of

$$\frac{\sqrt{8+\sqrt{28}}-\sqrt{8-\sqrt{28}}}{\sqrt{8+\sqrt{28}}+\sqrt{8-\sqrt{28}}} \text{ is}$$

(a) $\frac{1}{\sqrt{7}}$ (b) $\sqrt{7}$ (c) $-2\sqrt{7}$ (d) $\frac{1}{-2\sqrt{7}}$

12. Find the value of x in

$$\sqrt{x+2\sqrt{x+2\sqrt{x+2\sqrt{3x}}}}=x.$$

(a) 3 *(b)* 2 *(c)* 4 *(d)* 6

13. Product of three natural numbers is 24000 and their HCF is 10. How many such triplets of numbers are there?

(a) 5 ft) 4 *(c)7 (d)6*

14. The average of squares of consecutive odd numbers from 1 to 13 is

(a) 70 ft) 65 *(c)* 75 *(d)* 66

15. The average temperature of Delhi for Monday, Tuesday and Wednesday was 25°C, while for Thursday, Wednesday and Tuesday, it was 26°C. If the temperature on Thursday was 27°C, them what was the temperature on Monday?

(a) 21 °C *(b)* 24°C *(c)* 27°C *(d)* 30°C

16. In a class of 60 students, the average age of 25 girls is 12 yr and boys is 14 yr. Two of the students aged 16 and 14 yr leave the class and are replaced by 3 students of ages 12,11 and 10 yr, Find the change in average age of the class.

fa; 0.167 yr ft) 0.45 yr

(c) 1.2 yr *(d)* 1 yr

17. In the famous Mauritius Island, there are four men for every three women and five children for every three men, How many children are there in Island, if it has 531 women?

(aj 454 ftj 1180 *(c)* 1070 *(d)* 389

18. What will be the ratio of petrol and kerosene in the final solution formed by mixing petrol and kerosene that are present in three vessels in the ratio 4:1, 5 : 2 and 6:1, respectively?

(a) 166 : 22 *(b)* 83 : 22

(c) 83 : 44 *(d)* None of these

19. A sample of 50 L of glycerine is found to be adulterated to the extent of 20%. Find how much pure glycerine should be added to bring down percentage of impurity to 5%?

(a) 100 L *(b)* 130 L *(c)* 140 L *(d)* 150 L

20. In an examination, there were 1000 boys and 800 girls. 60% of the boys and 50% of the girls passed. Find the per cent of candidates failed.

(a) 46.4% *(b)* 48.4%

(c) 44.4% *(d)* 49.6%

If the price of sugar rises from 7 6 per kg to 7 7.50 per kg, a person having no increase in his expenditure on sugar, will have to reduce his consumption of sugar by *(a)* 15% *(b)* 20% *(c)* 25% *(d)* 30%

22. The monthly incomes of two persons are in the ratio 4 : 7 and their expenses are in the ratio 11 : 20. If each saves 7 400 per month, then their monthly incomes must be respectively

(a) 7 3600 and 7 4200

(b) 7 4000 and 7 7000

(c) 7 4200 and 7 7350

(d) 7 4800 and 7 8400

23. A man buys 5 horses and 10 cows for 7 1600. He sells horses at a profit of 15% and cows at a loss of 10%. If his over all profit was 7 90, what was the cost price of a horse and of a cow?

(a) 7 150, 7 85 *(b)* 7 200, 7 60

(c) 7 220, 7 40 (b9 7 180, 7 70

24. If on selling 12 notebooks, any seller makes a profit equal to the selling price of 4 notebooks, what is his per cent profit?

(a) 50 *(b)* 25 *(c)* $16\frac{2}{3}$

(d) Data Inadequate

25. A person buys some pencils at 5 for a rupee and sells them at 3 for a rupee This aain Der cent will be

(a) $66\frac{2}{3}\%$ *(b)* $76\frac{2}{3}\%$ *(c)* $56\frac{2}{3}\%$ *(d)* $46\frac{2}{3}\%$

26. Pipe *A* alone can fill a tank in 8 h. Pipe S alone can fill it in 6 h. If both the pipes are opened and after 2 h pipe *A* is closed, then the other pipe will fill the tank in

(a) 4 h *(b)* $2\frac{1}{2}$h *(c)* 6 h *(d)* $3\frac{1}{2}$h

27. What is difference between compound interest on 7 5000 for *VA* yr at 4% per annum when the interest is compounded yearly and half-yearly?

(a) 7 2.0 *(b)* 7 3.06

(c) 7 8.30 *(d)* 7 4.80

28. *A* can do a job in 15 days, fi in 10 days and C in 30 days. If *A* is helped by 6 and C on every third day, then the job will be comDleted in

(a) $6\frac{1}{3}$ days *(b)* $8\frac{1}{3}$ days

(c) 8 days *(d)* 9 days

29. The river flows at 4 km/h. A boat can go downstream thrice as fast as upstream The speed of boat in still water is

(a) 12 km/h *(b)* 16 km/h

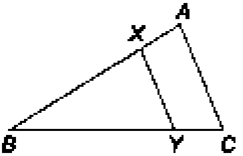
(c) 8 km/h *(d)* 10 km/h

30. A dealer offered a machine for sale for

7 27500 but even, if he had charged 10% less, he would have made a profit of 10%, The actual cost of the machine is *(a)* 7 22000 *(b)* 7 24250

(c) 7 22500 *(d)* 7 22275

31. In the figure given below, XV parallel to *AC.* If XY divides the triangle into equal Darts, then value of *AX/ AB* is eaual to



(a) $\frac{1}{2}$ *(b)* $\frac{1}{\sqrt{2}}$ *(c)* $\frac{\sqrt{2}+1}{\sqrt{2}}$ *(d)* $\frac{\sqrt{2}-1}{\sqrt{2}}$

32. Let *BE* and *CF* be the two medians of a *AABC* and G be the intersection. Also, let *EF* cut *AG* at O, then *AO* : *AG* is

(a) 1 : 1 *(b)* 1 : 2 *(c)* 2 : 1 *(d)* 3 : 1

33. When a pendulum of length 50 cm oscillates, it produces an arc of 16 cm The angle so formed in degree measure is (approx.)

(a) 18°25' *(b)18°35'*

(c) 18°20' *(d)* 18°08'

34. If *ABCD* is a rhombus, then

(a) $AC^2 + BD^2 = 6\ AB^2$

(b) $AC^2 + BD^2 = 5AB^2$

(c) $AC^2 + BD^2 = 4AB^2$

(d) $AC^2 + BD^2 = 3AB^2$

35. Two circles of radii 4 cm and 9 cm respectively touch each other externally at a point and a common tangent touches them at a point *P* and Q respectively Then, area of square with one side *PQ* is

(a) 72 sq cm *(b)* 144 sq cm

(c) 97 sq cm *(d)* 194 sq cm

36. If / is the incentre of *AABC* and *ZA* = 60°, then the value of *ZBIC* is

(a) 100° *(b)* 120°

(c) 150° (of; 110°

37. The perimeter of square and a circular field are the same. If the area of the circular field is 3850 m^2, then what is the area (in m^2) of the square?

(a) 4225 *(b)* 3025

(c) 2500 *(d)* 2025

38. If the length of a rectangle is increased by 25% and the width is decreased by 20%, then the area of rectangle will be

(a) increase by 5%

(b) decrease by 5%

(c) remains unchanged

(d) increased by 10%

39. If base of a prism is a square of side 4 cm, If the height of prism is 10 cm, then what will be total surface area of that prism?

(a) 192 cm^2

(b) 212 cm^2

(c) 214 cm^2

(d) None of the above

40. A conical vessel whose interval radius is 12 cm and height 50 cm, is full of liquid The content are emptied into a cylindrical vessel with radius (interval) 10 cm. The height of which the liquid rises in the cylinder vessel is

(a) 25 cm *(b)* 20 cm

(c) 24 cm *(d)* 22 cm

41. A tree breaks due to strorm and the broken part bends, so to that the top of the tree touches the ground making an angle 30° with it. The distance between the feet of the tree to the point, where the top touches the ground, is 8 m. Find the height of the tree

(a) 8 m *(b) 8Sm(c)* 24 m *(d)* 12 m

42. Two points *A* (-3, b) and *B* (1, b + 4) and the coordinates of the middle point of AS are (-1,1). The value of b is

(a) 1 *(b)* -1 *(c)* 2 *(d)* 0

43. If $x^3 - \frac{1}{x^3} = 14$,then the value of $x - \frac{1}{x}$ will be

(a) 2 *(b)* 3

(c) 4 *(d)* 5

44. If $a^2 = by + cz$, $b^2 = cz + ax$ and $c^2 = ax + by$, then the value of $\frac{x}{a+x} + \frac{y}{b+y} + \frac{z}{c+z}$ will be

(a) $a + b + c$ *(b)* $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

(c) 1 *(d)* 0

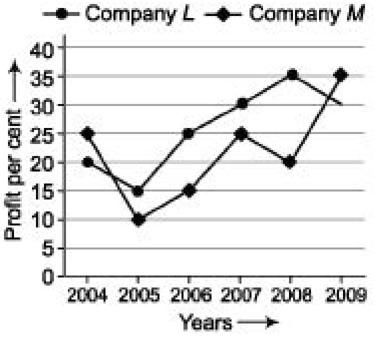
45. If $\tan(x + y)\tan(x - y) = 1$, then the value of $\tan\left(\frac{2x}{3}\right)$ is

(a) $\frac{1}{\sqrt{3}}$ *(b)* $\frac{2}{\sqrt{3}}$ *(c)* $\sqrt{3}$ *(d)* 1

Directions (Q. Nos. 46-50) *Study the following graph carefully and answer the questions that follow.*

The following figure shows the per cent rise in profit of two companies I and *M* from the year 2004 to 2009.

46. If the profit earned by company *L* in the year 2005 was ? 1.84 lakh, what was the profit earned by the company in the year 2006?



(a)? 2.12 lakh *(b)* ? 2.3 lakh

(c) ? 2.04 lakh

(d) Cannot be determined

47. Which of the following statements is true with respect to the above graph?

(a) Company *M* made the highest profit in the year 2009

(b) Company *L* made the least profit in the year 2005

(c) The ratio of the profits earned by company *A* and company *M* in the year 2006 was 6 : 5

(d) Company *L* made the highest profit in the year 2008

48. What was the percentage increase in per cent rise in profit of company *M* in the year 2009 from the previous year?

(a) 25 *(b)* 15 *(c)* 50 *(d) lb*

49. If the profit earned by company *M* in the year 2008 was ? 3.63 lakh, what was the amount of profit earned by it in the year 2006?

(a) X 2.16 lakh *(b)* ? 1.98 lakh

(c) X 2.42 lakh

(d) Cannot be determined

50. What was the average per cent rise in profit of company *L* over all the years toqether?

(a) $15\frac{1}{3}$
(b) $25\frac{1}{3}$
(c) $18\frac{5}{6}$
(d) None of the above

Answers

1. (c) 2. (b) 3. (a) 4. (b) 5. (c) 6. (c) 7. (b) 8. (c) 9. (d) 10. (d)

11. (a) 12. (b) 13. (c) 14. (b) 15. (b) 16. (a) 17. (b) 18. (b) 19. (d) 20. (c)

21. (b) 22. (d) 23. (b) 24. (a) 25. (a) 26. (b) 27. (a) 28. (d) 29. (c) 30. (c)

31. (d) 32. (d) 33. (c) 34. (c) 35. (b) 36. (b) 37. (b) 38. (c) 39. (a) 40. (c)

41. (b) 42. (b) 43. (a) 44. (c) 45. (a) 46. (b) 47. (a) 48. (d) 49. (c) 50. (d)

Practice Set 2

Directions (Q. Nos. 1-5) *What should come in place of the question mark* (?) in *the following questions?*

1. 34.36- 45.67 + 86.56 = ? + 37.96

(a) 31.29 *(b)* 32.29

(c) 38.49 *(d)* 37.49

(e) None of the above

2. $(5x6)^2x(9x6)*(4x6)$ = ?

(a) 2055 ft) 2505

ft) 2205 ft) 2025

fe) None of the above

3. ?% Of 350 - $(6)^2$ = 48

(a) 12 *(b)* 24 ft) 42 ft) 54 fe) 62

4. $2\frac{1}{3}$ of $1\frac{1}{4}$ of (?) = 280

(a) 116 *(b)* 124 *(c)* 48 *(d)* 96

(e) None of the above

5. $(6)^4$-$(36)^3$x216=$6^{\{?-2)}$

(a) 3 © 6 ft) 1 ft) 4 fe) None of the above

Directions (Q. Nos. 6-10) *In each of the following questions, one number is missing in the series. You have to understand the pattern of the series and then insert the missing number.*

B. 4,7, 11,18,29,47........123,199

(a) 76 *(b)* 70 ft) 84 *(d)* 102 *(e)* 95

7. 2,6, 12,20........42,56,72,90

(a) 20 *(b)* 21 ft) 30 *(d)* 12 *(e)* 23

17,7,24, 19,9,28........8,31,27,10,37

(a) 20 *(b)* 21 ft) 18 *(d)* 12 **fe) 23**

9. 6, 126........9, 108, 12, 7, 133, 19, 12, 72,

6 *(a)* 21 ft; 23 ft) 30 ft) 35 fe) 40

10.8,4,12,6, 18........27

(a) 10 *(b)* 12 *(c)* 18 ft) 24 fe) None of the above

11. Out of four consecutive prime numbers, the product of first three numbers is 385 and the product of the last three numbers is 1001. Find the last number

(a) 1 ft) 11

(c) 13 *(d)* 17

fe) None of the above

12. The largest three-digit number which is a perfect cube, is

(a) 986 ftj 729 ft) 981 *(d)* 864 *(e)* None of the above

13. It is required to change a rupee coin into 2 paise and 5 paise coins with the total number of coins equal to 26. Find the number of each type of coins

(a) 10 and 16 ftj 12 and 14

(c) 11 and 15 *(d)* 13 and 13

(e) None of the above

14. The LCM of two numbers is 280 and the ratio of the numbers is 7:8. Find the numbers

(a) 70 and 48 ftj 42 and 48

fe) 35 and 40 *(d)* 28 and 32

(e) None of the above

15. A certain number of men complete a work in 45 days. If there were 5 men more, the work could be finished in 9 days less How many men were originally there?

(a) 30 ftj 15 ftj 25 *(d)* 20 fe) None of the above

16. A rope makes 125 rounds of a cylinder with base radius 15 cm. How many times can it go round a cylinder with base radius 25 cm?

(a) 100 ftj 75 ftj 80 ft; 65 fe) None of the above

17. The average age of a group of four men whose ages are in the ratio of 2 :3: 4 :5 is 42 yr, what is the age of the eldest person in this group?

(a) 60 yr *(b)* 48 yr

ft) 36 yr *(d)* 24 yr

fe) None of the above

18. The price of an article decreased by 20% as a result of which the sale increased by 10%. What will be the effect on the total revenue of the shop?

(a) 10% increase ft) 12% increase *(c)* 12% decrease *(d)* 10% decrease *(e)* None of the above

19. A mixture of 20 L of milk and water contains 20% water. How much water should be added to this mixture so that the new mixture contains 25% water?

(a) $1\frac{1}{3}$ L *(b)* $1\frac{1}{2}$ L *(c)* $1\frac{1}{4}$ L *(d)* $1\frac{1}{5}$ L

(e) None of the above

20. Three students *A, B* and C hired a computer for a month. *A* runs 27 floppy discs for 19 days, 6 runs 21 for 17 days and C runs 24 for 23 days. If at the end of the month, the rent amounts to ?23700, how much ought to be paid by *A?*

(a) ? 8550 ft) ? 5950 ft) ?9200 ft; ?6750 *(e)* None of the above

21. In what proportion must water be mixed with spirit to gain $12^1/2$ by selling it at CP?

(a) 2:1 (b) 1 : 8 *(c)* 1 : 9 *(d)* 2 : 9 *(e)* None of the above

22. In a zoo, there are some pigeons and some rabbits. If their heads are counted, these are 100 and if their legs are counted, these are 320. How many pigeons are there?

(a) 66 ftj 60 *(c)* 40 *(d)* 45 *(e)* None of the above

23. A and *B* together can complete a work in 35 days while *A* alone can complete the same work in 60 days. In how many days, 6 alone will be able to complete the same work?

(a) 80 days *(b)* 84 days

(c) 88 days *(d)* 92 days

(e) None of the above

24. A can do a piece of work in 15 days and fi alone can do it in 10 days. *B* works at it for 5 days and then leaves. In how many days, *A* alone can finish the remaining work?

(a) 7 days *(b)* $7\frac{1}{2}$ days

(c) 8 days *(d)* $8\frac{1}{2}$ days

(e) None of the above

25. Working together, *P* and Q can do a job in 6 days, Q and *R* can do the same job in 10 days while P and *R* can do it in 5 days How long will it take, if all of them work together to complete the job?

(a) $4\frac{2}{7}$ days *(b)* $4\frac{3}{7}$ days

(c) $4\frac{4}{7}$ days *(d)* $4\frac{5}{7}$ days

(e) None of the above

26. A long distance runner runs 9 laps of a 400 m track everyday. His timings (in min) for four consecutive days are 88, 96, 89 and 87, respectively. On an average, how many m/min does the runner cover?

(a) 39 m/min ft) 40 m/min

(c) 41 m/min *(d)* 43 m/min

(e) None of the above

27. A man performs 2/25 of his total journey by bus, 21/50 by car and the remaining 2 km on foot. Find the total journey.

(a) 2.5 km ft) 2.7 km

(c) 3.4 km *(d)* 3.8 km

(e) None of the above

28. A sum becomes 10/9 times itself in 1 yr Find the rate of simple interest.

(a) $11\frac{1}{2}$% (b) $11\frac{1}{9}$% (c) $12\frac{1}{2}$% (d) $12\frac{1}{9}$%
(e) None of the above

29. A sum of *x* 390200 is to be paid back in three equal annual instalments. How much is each instalment, if the rate of interest charged is 4% per annum compounded annually?

(a) ? 140608 ftj ? 120560

(c) ? 10000 *(d) X*18000

(e) None of the above

30. A sum ?2400 deposited at CI, doubled after 5 yr, After 20 yr, it will become

(a) ? 24000 ft; ? 38400

(c) ? 19200

(d) Couldn't be determined fe) None of the above

31. Two numbers are less than a third number by 25% and 30%, respectively How much per cent is the second number less than first number?

(a) $6\frac{1}{3}$% (b) $6\frac{1}{4}$% (c) $6\frac{2}{3}$% (d) $6\frac{3}{4}$%
(e) None of the above

32. The ratio of the areas of a square to that of the square drawn on its diagonal is

(a) 1 : 1 *(b)* 1 : 2 *(c)* 1 : 3 (ft) 1 : 4 (e) None of the above

33. The length of longest pole that can be placed in the floor of a room is 12 m and the length of longest pole that can be placed in the room is 15 m. The height of the room is

(a) 3 m *(b)* 6 m *(c)* 9 m *(d)* 12 m (e) None of the above

34. The ratio of heights of two cylinders is 3:2 and the ratio of their radii is 6:7 What is the ratio of their curved surface areas?

(a) 9 : 7 *(b)* 1 : 1 fc) 7:9 *(d) 1*: 4 *(e)* None of the above

35. Three unbiased coins are tossed. Find the probability of getting atleast two heads.

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$
(e) None of the above

36. Find the probability that a vowel selected at random from the 5 vowels is an 7'

(a) $\frac{4}{5}$ (b) $\frac{1}{3}$
(c) $\frac{1}{5}$ (d) $\frac{1}{2}$
(e) None of the above

37. The calendar for the year 2002 is the same as for the year.

(a) 2006 *(b)* 2008 *(c)* 2009 *(d)* 2010 *(e)* None of the above

38. What will be the day on 8th September 1998?

(a) Monday *(b)* Tuesday

(c) Wednesday *(d)* Friday *(e)* None of the above

39. A watch which gains uniformly is 2 min slow at 10 am on Wednesday and is $2^1/2$ min fast at 1 pm the following day When was it correct?

(a) 10:30 pm Wednesday

(b) 9 :15 pm Wednesday

(c) 10 :45 pm Wednesday

(d) 10 pm Wednesday

(e) None of the above

40. How many numbers upto 700 are divisible by both 3 and 5? *(a)* 42 *(b)* 46

(c) 39 (d) 52

(e) None of these

Directions (Q. Nos. 41-45) *Each question below is followed by two Statements* I *and* II. *You are to determine whether the given data in the statement is sufficient for answering the question. You should use the data and your knowledge of Mathematics to choose between the possible answers. Give the answer*

(a) If the statement I alone is sufficient to answer the question but the statement II alone is not sufficient

(b) If the statement II alone is sufficient to answer the question but the statement I alone is not sufficient

(c) If both statements I and II together are needed to answer the question

(d) If either the statement I alone or statement II alone is sufficient to answer the question

(e) If cannot get the answer from the statements I and II together and need even more data

41. Find x.

I. 20% of x is equal to the $\frac{1}{5}$th part of an another number.

II. $\frac{7}{20}$th part of x is equal to the 35% of an another number.

42. Find the ratio of men, women and children the city

I. The population of the city is 93280 in

which 56100 are men, I. Ratio of the numbers of men and

children is 5 : 2 and the number of

women is double the number of

children

43. Find the SP of rice,

I. 50 kg rice are bought for ?3350 and

?150 are spent in travelling I. Profit was 5%.

44. What is the speed of train?

. Length of train is 120 m I. It crosses an another train of length 180 m, in 4 s.

45. What is the CI after 3 yr?

I. Rate is 5%.

I. After two years, difference between CI and SI is *X* 20.

Directions (Q. Nos. 46-50) *Study the following table carefully and answer the questions given below.*

Percentage of marks obtained by different students in different subjects

	Subjects (Maximum marks)						
Students	Hindi (150)	English (150)	Math (150)	S.St. (150)	Science (75)	Marathi (50)	Physical education (75)
A	88	85	86	74	78	80	85
B	92	80	79	82	70	70	97
C	75	89	85	90	83	91	85
D	63	66	69	71	85	64	62
E	80	76	89	95	79	70	73
F	69	81	86	76	69	85	76

46. How many marks did E get in all the subjects together?

(a) 659 *(b)* 600 *(c)* 625 *(d)108 (e)* None of the above

47. What are the average marks obtained by all the students together in English?

(a) 110.27 *(b)* 113.76

(c) 121.52 *(d)* 119.25

(e) None of the above

48. How many students have scored the highest marks in more than one subject?

(a) One *(b)* Two *(c)* Three *(d)* Four *(e)* None of the above

49. Who has scored the highest marks in all the subjects together?

(a) E (b) C (c) F (d) A (e) None of the above

50. Marks obtained by *F* in S.St, are what per cent of marks obtained by *E* in the same subject?

(a) 74 *(b)* 85 *(c)* 76 *(d)* 80 (e) None of these

Answers

1. (e) 2. (d) 3. (b) 4. (d) 5. (a) 6. (a) 7. (c) 8. (e) 9. (a) 10. (e)

11. (c) 12. (b) 13. (a) 14. (c) 15. (d) 16. (b) 17. (a) 18. (c) 19. (a) 20. (a)

21. (b) 22. (c) 23. (b) 24. (b) 25. (a) 26. (b) 27. (a) 28. (b) 29. (a) 30. (b)

31. (c) 32. (b) 33. (c) 34. (a) 35. (d) 36. (c) 37. (b) 38. (c) 39. (d) 40. (b)

41. (e) 42. (b) 43. (c) 44. (e) 45. (c) 46. (a) 47. (d) 48. (c) 49. (a) 50. (d)

Practice Set 3

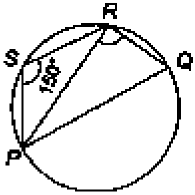
1. The value of $\frac{0.796 \times 0.796 - 0.204 \times 0.204}{0.796 - 0.204}$ is

(a) 0.408 *(b)* 0.59 *(c)* 0.592 *(d)* 1

2. The LCM of two numbers is 20 times their HCF. The sum of HCF and LCM is 2520. If one of the numbers is 480, the other number is

(a) 400 *(b)* 480 *(c)* 520 *(d)* 600

3. *PQ* is a diameter and *PQRS* is a cyclic quadrilateral, if *ZPSR* = 150=, then measure of *ZRPQ* is



(a) 90° (b) 60°
(c) 30° (d) None of these

4. In ΔABC, *AD* and *BE* are perpendiculars from *A* and *B* to the sides *BC* and *AC*, then

(a) $CA \cdot CE = CB \cdot CD$ (b) $CB \cdot CA = CE \cdot CD$
(c) $\frac{AD}{BE} = \frac{AB}{DE}$ (d) None of these

5. If $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$, then the value of $\frac{\tan A - \tan B}{1 + \tan A \tan B}$ is equal to

(a) $\frac{23}{16}$ (b) $\frac{16}{63}$ (c) $\frac{1}{63}$ (d) $\frac{13}{63}$

B. *ABCD* is a cyclic quadrilateral whose diagonals intersect at *E.* If *ZBEA* = 80°, *ZDBC* = 60° and *ZBCD* = 40=, which of the following statements is true?

(a) BD bisects *ZADC*

(b) AB = BC

(c) DA = DC

(d) AC bisects *ZBCD*

7. If one of the interior angles of a regular polygon is found to be equal to (9/8) times of one of the interior angles of a regular hexagon, then the number of sides of the polygon is

(a) 4 *(b)* 5

(c) 1 (d) 8

8. The cost of carpeting a room is ? 120. If the width had been 4 m less, the cost of the carpet would have been ? 20 less The width of the room is

(a) 24 m *(b)* 20 m *(c)* 25 m *(d)* 18.5 m

9. The length of one side of a rhombus is 6.5 cm and its altitude is 10 cm. If the length of its diagonal be 26 cm, then length of the other diagonal will be

(a) 5 cm *(b)* 10 cm *(c)* 6.5 cm *(d)* 26 cm

10. A wire, bent in the form of a square,

encloses an area of 484 cm^2. If the same

wire is bent so as to form a circle, then

the areas enclosed will be [use *n* = 22/71

(a) 484 cm^2 *(b)* $638\frac{2}{7}$ cm^2

(c) 616 cm^2 *(d)* 644 cm^2

11. If the volume and surface area of a sphere are numerically the same, then its radius is

(a) 1 unit *(b)* 2 units

(c) 3 units *(d)* 4 units

12. The angle of elevation of the top of an

unfinished pillar at a point 150 m from its base is 30°. If the angle of elevation at the same point is to be 45°, then the pillar has to be raised to a height of how many metres? *(a)* 59.4 m *(b)* 61.4 m *(c)* 62.4 m *(d)* 63.4 m

13. In a circle with centre O, *AOC* is a diameter of the circle, *BD* is a chord and *OB* and *CD* are joined. If ZAOS = 130^P, then *ZBDC* is equal to

(a) 30° *(b)* 25° *(c)* 50° *(d)* 60°

14. If $x = 7 - 4\sqrt{3}$, then $\sqrt{x} + \frac{1}{\sqrt{x}}$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

15. The simplified form of

$$\frac{2}{\sqrt{7}+\sqrt{5}} + \frac{7}{\sqrt{12}-\sqrt{5}} - \frac{5}{\sqrt{12}-\sqrt{7}}$$ is

(a) 5 (b) 2 (c) 1 (d) 0

16. The square root of $\dfrac{\left(3\frac{1}{4}\right)^4 - \left(4\frac{1}{3}\right)^4}{\left(3\frac{1}{4}\right)^2 - \left(4\frac{1}{3}\right)^2}$ is

(a) $7\frac{1}{12}$ (b) $5\frac{5}{12}$ (c) $1\frac{1}{12}$ (d) $1\frac{7}{12}$

17. A number which when divided by 10 leaves a remainder of 9, when divided by 9 leaves a remainder of 8 and when divided by 8 leaves a remainder of 7, is b) 1539 ft) 539 ft) 359 foO 1359

18. If $\tan A = \sqrt{2} - 1$, then the value of cosec A. sec A is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{2}{\sqrt{2}}$ (c) $2\sqrt{2}$ (d) $\frac{\sqrt{3}}{2}$

19. The seventh term of the sequence 1, 3, 6, 10, ... is

(a) 20 (b) 26 (c) 28 (d) 32

20. At a point 20 m away from the foot of a tower, the angle of elevation of the top of the tower is 30°. The height of the tower is

(a) $20\sqrt{3}$ m (b) $\frac{20}{\sqrt{3}}$ m

(c) $\frac{\sqrt{3}}{20}$ m (d) None of these

21. If A, B, C are three angles of a triangle, then which one of the following relations is not correct?

(a) $\sin A = \sin(B + C)$

(b) $\tan(A - B - C) = \tan 2A$

(c) $\cos\left(\frac{B+C}{2}\right) = \sin\left(\frac{A}{2}\right)$

(d) $\cos(B + C) = \cos A$

22. The value of $\cos 10^\circ \cdot \cos 20^\circ \cdot \cos 40^\circ$ is euqal to

(a) $\frac{1}{4}\tan 10^\circ$ (b) $\frac{1}{8}\cot 10^\circ$

(c) $\frac{1}{8}\operatorname{cosec} 10^\circ$ (d) $\frac{1}{8}\sec 10^\circ$

23. If $\sin\theta + \sqrt{\sin\theta + \sqrt{\sin\theta + \sqrt{\sin\theta + \quad \infty}}} = \sec^4$ α, then $\sin\theta$ is equal to

(a) $\sec^2\alpha$ (b) $\tan^2\alpha$

(c) $\sec^2\alpha\tan^2\alpha$ (d) $\cos^2\alpha$

24. In a school' 40% of the students play football and 50% play cricket. If 18% of the students neither play football nor cricket, the percentage of the students playing both is

(a) 40% ftj 32%

(c) 22% *(d)* 8%

25. A reduction of 20% in the price of sugar enables a purchaser to obtain 3 kg more for ? 120. The original price (per kg) of sugar is

(aj? 15 ftj? 12 ftj? 10 *(d)18*

26. A certain sum of money becomes three times of itself in 20 yr at simple interest, In how many years, does it become double of itself at the same rate of simple interest?

(a) 8 yr ftj 10 yr *(c)* 12 yr *(d)* 14 yr

27. The compound interest on a certain sum of money at 5% for 2 yr is ? 328. The simple interest on that sum at the same rate and for the same period of time will be

(a) 7 320 ft) ? 322 *(c)* 7 325 *(d)* ? 326

28. For $0 \le x \le \frac{\pi}{2}$, the number of values of x satisfying the equation $\tan x + \sec x = 2 \cos x$ is

(a) 0 *(b)* 1 *(c)* 2 *(d)* 3

29. The value of $\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$ is equal to

(a) cosec x + cot x *(b)* cosec x + tan x

(c) sec x + tan x *(d)* cosec x − cot x

30. If $\theta + \cot \theta = 2$, then what is the value of $\tan^7 \theta + \cot^7 \theta$?

(a) 2^7 *(b)* 2^{-7} *(c)* 1/2 *(d)* 2

31. Each exterior angle of a regular polygon of n sides is equal to

(a) $4n \times 90°$ *(b)* $\frac{360°}{n}$

(c) $(2n-4) \times 90°$ *(d)* $4 \times 90°$

32. The average of 11 numbers is 10.8. If the average of the first six be 10.4 and that of the last six is 11.5, then the middle (6th) number is

(a) 10.3 ftj 12.6 *(c)* 13.5 *(d)* 15.5

33. The average age of 12 players of a team is 25 yr. If the captain's age is included, the average age increases by 1 yr. The age of the captain is

(a) 25 yr ft) 38 yr *(c)* 36 yr *(d)* 26 yr

34. The length of the diagonal of a square is

a cm. Which of the following represents the area of the square (in sq cm)?

(a) $2a$ *(b)* $\frac{a}{\sqrt{2}}$ *(c)* $a^2/2$ *(d)* $a^2/4$

35. If the volumes of two cubes are in the ratio 27 :1, then ratio of their edges is

(a) 3 : 1 *(b)* 27 : 1 ft) 1 : 3 ft) 1 : 27

36. The sides of a rectangular plot are in the ratio 5 : 4 and its area is equal to 500 sq m. The perimeter of the plot is

(a) 80 m *(b)* 100 m ft) 90 m ft) 95 m

37. The radius of a wheel is 21 cm. How many revolutions will it make in travelling 924 m?

[use ji = 22/7] *(a) 1* ft) 11 ft) 200 ft) 700

38. If (1 + tan *A)* (1 + tan S) = 2, then *{A + B)* is equal to

(a) $\frac{\pi}{2}$ *(b)* $\frac{\pi}{3}$ *(c)* $\frac{\pi}{4}$ *(d)* $\frac{\pi}{6}$

39. If a + b + c = 2s, then the value of $(s - a)^2 + (s - b)^2 + (s - c)^2$ will be

fe)s^2+fl^z+6^z + c^z(fc;$a^2 + 6^2 + c^2$-s^2 ft;s^2-a^2-6^2-c^2ft)4s^2-a^2-6^2-c^2

40. *APQR* and A/JWN are similar. If 3PQ = ZJW and *MN* = 9 cm, then *QR* is equal to

(aj 3 cm *(b)* 6 cm ft) 9 cm (ft) 12 cm

41. The height of a conical tank is 60 cm and the diameter of its base is 64 cm. The cost of painting it from outside at the rate of ? 35 per sq m is

(a) X 52.00 (approx) *(b) X* 39.20 (approx) ft) *X* 35.20 (approx) ft) *X* 23.94 (approx)

42. *A* and 6 can do a piece of work in 72 days 6 and C can do it in 120 days. *A* and C can do it in 90 days. In how many days, all the three together can do the work?

(a) 80 days *(b)* 100 days

ft) 60 days ft) 150 days

43. The equation of a line passing through the points *A* (-1,1)and S (2, - 4) is

(a)3x + 5y + 2 = 0 (b) 5x + 3y + 2 = 0 ft) *2x* + 3y + 5 = 0 *(d)* None of these

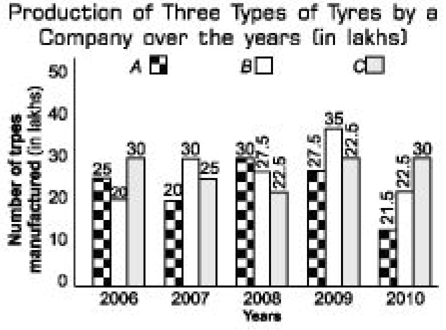
44. If the slope of a line passing through the points *A* (2, 5) and 6 (x, 3) is 2, then x is equal to

ft)1 ft) 2 fe)-1 ft)-2

45. Two vertices of a *AABC are B* (- 3,1) and C (0,-2) and its centroid is at the origin The third vertex A is

femi) ft; (2,3) ft; (-1,2) ft; (-2,3)

Directions (Q. Nos. 46-50) *Study the following graph carefully and answer the questions given below.*



46. What was the percentage drop in the number of C type tyres manufactured from 2007 to 2008?

(a) 25% ft; 10% ft; 15% ft; 2.5%

47. What was the difference between the number of *B* type tyres manufactured in 2008 and 2009?

(a) 100 000 ft; 2000000

ft) 1000000 ft; None of these

48. The total number of all the three types of tyres manufactured was the least in which of the following years?

(a) 2009 ft; 2010 ft; 2006 *(d)* 2008

49. In which of the following years, was the percentage production of S type to C type the max-imum?

(a) 2008 ft; 2009 ft; 2010 *(d)* 2007

50. The total production of C type tyres in 2006 and 2007 together was what percentage of production of 6 type tyres in 2008?

(a) 50% ft; 100% ft; 150% ft; 200%

Answers

1. (d) 2. (d) 3. (b) 4. (a) 5. (b) 6. (d) 7. (d) 8. (a) 9. (a) 10. (c)

11. (c) 12. (d) 13. (b) 14. (d) 15. (d) 16. (b) 17. (c) 18. (c) 19. (c) 20. (b)

21. (d) 22. (b) 23. (c) 24. (d) 25. (c) 26. (b) 27. (a) 28. (b) 29. (a) 30. (d)

31. (b) 32. (b) 33. (b) 34. (c) 35. (a) 36. (c) 37. (d) 38. (c) 39. (b) 40. (a)

41. (d) 42. (C) 43. (b) 44. (a) 45. (a) 46. (b) 47. (d) 48. (b) 49. (b) 50. (d)

Practice Set 4

1. A contractor has to complete his contract in 92 days. 234 men were set to do the work, each of them works for 16 h per day. After 66 days, 4/7 of the work is completed. How many additional men may be employed, so that the work may be completed in time, given that each of additional men work 18 h per day?

(a) 396 ft) 162

(c) 200 *(d)* None of these

2. If 17^{200} is divided by 18, the remainder is

(a) 0 ft) 1 *(c)* 2 *(d)* 16

3. In the following number series, a number is wrong. Find that wrong number

316, 156, 86, 36, 18, 8, 3

(a) 86 (b) 36 (c) 156 (d) 16

4. Simplify $\dfrac{\dfrac{3}{2+\sqrt{3}} - \dfrac{2}{2-\sqrt{3}}}{2-5\sqrt{3}}$.

(a) 1 (b) -1 (c) $\sqrt{3}$ (d) $\dfrac{\sqrt{3}}{2}$

5. A fraction becomes 2 when 1 is added to both the numerator and denominator and it becomes 3 when 1 is subtracted from both numerator and denominator. Find the difference of numerator and denominator

(a) 3 ft) 4 ft) 6 ft) 7

6. The difference of two numbers is 20 and their product is 56.25 times their difference. Find the LCM of the numbers

(a) 225 *(b)* 625 *(c)* 1125 *(d)* 825

7. There were 42 students in a hostel. Due to the admission of 13 new students, the expenses of mess increase by ? 31 per day while the average expenditure per head diminished by ? 3. What was the original expenditure of the mess?

(a) ? 633 ft) ? 583 *(c)* ? 623 *(d)* ? 533

8. The mid proportion between smaller and larger number is twelve times the product of these two numbers, The square of smaller number is half of the larger number. What is the sum of these two numbers?

(a) 12 *(b)* 14

(c) Cannot be determined

(d) None of the above

9. ? 180 contained in a box of one rupee, 50 paise and 25 paise coins in the ratio 2:3:4. What is the number of 50 paise coins? *(a)* 80 *(b)* 160 *(c)* 120 *(d)* 180 10.10 gallons are drawn from a cask full of wine, It is then filled with water, 10 gallons of the mixture are drawn and the cask is again filled with water. After such four operations, the quantity of wine now left in the cask to that of water in it is 256: 369. How much does the cask hold? *(a)* 50 gallons *(b)* 55 gallons

(c) 60 gallons *(d)* 65 gallons

11. Sonu invested 10% more than the investment of Mona and Mona invested 10% less than the investment of Raghu. If the total investment of all the three persons is ? 5780, then find the investment of Raghu

(a) X 1850 ft) ? 2000 *(c)* ? 2200 *(d) X* 2320

12. X can do 20% of a work in a day, Y can do 25% of the same work in a day and X, Y and Z together can do 50% of the same work in a day. How many per cent of work can be done by Z in a day?

(a) 5% ft) 7% *(c)* 10% (b? 13%

13. The price of rice is reduced by 5%, How many kilogrames of rice can now be bought for the money which was sufficient to buy 50 kg of rice earlier?

(a) 52.63 kg *(b)* 50.52 kg

(c) 54.44 kg *(d)* 56.36 kg

14. *ABCD* is a square with sides of length 10 units. *OCD* is an isosceles triangle with base *CD. OC* cuts *AB* at point Q and *OD* cuts *AB* at point *P.* The area of trapezoid *PQCD* is 80 sq units. The length of altitude from O of the *AOPQ* (in units) is

(a) 12 ftj 13

(c) 14 *(d)* None of these

15. Water flows out through a circular pipe whose internal diameter is 2 cm, at the rate of 6 m/s into a cylindrical tank, the radius of whose base is 60 cm, By how much will the level of water rise in 30 min?

(a) 3 m *(b)* 3.5 m *(c)* 4 m *(d)* 4.5 m

16. A hollow garden roller 63 cm wide with a girth of 440 cm is made of iron 4 cm thick. The volume of iron used is

(a) 107712 cm^3 *(b)* 170112 cm^3 *(c)* 102217 cm^3 *(d)* 107212 cm^3

17. A village having the population of 4000, requires 150 L of water per head per day It has a tank measuring 20x15x6 m^3 For how many days will the water of this tank last?

(a) 2 days *(b)* 3 days

(c) 1 day *(d)* 4 days

18. The area of a trapezium is 384 cm^2. If its parallel sides are in the ratio 3: 5 and the perpendicular distance between them is 12 cm, the smaller of the parallel sides is

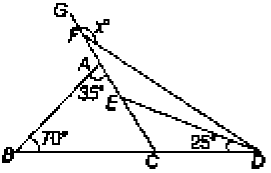
(a) 20 cm *(b)* 24 cm *(c)* 30 *cm(d)* 36 cm

19. The perimeter of the triangular base of a right prism is 60 cm and the sides of the base are in the ratio 5:12 :13. Then, its volume will be (height of the prism being 50 cm)

(a) 6000 cm^3 *(b)* 6600 cm^3

(c) 5400 cm^3 *(d)* 9600 cm^3

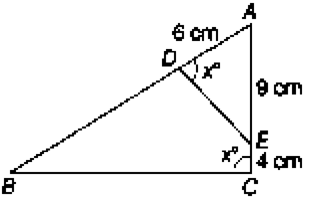
20. In the adjoining figure, *DE* is the internal bisector of *ZD.* Find *Zx*



(a) 130° *(b)* 155°

(c) 160° *(d)* 145°

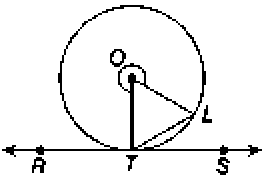
21. In the given figure, find the value of *BD.*



(a) 13.5 cm *(b)* 11 cm

(c) 3.5 cm (d) 9.5 cm

22. If *ZLTS*= 38°, find the external *ZTOL*



(a) 256° *(b)* 284°

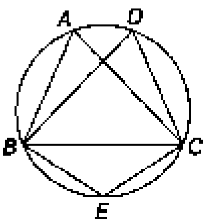
(c) 302° *(d)* 310°

23. *PQ* is a chord of length 8 cm of a circle of radius 5 cm. The tangents at *P* and Q intersect at a point *T.* Find the length of *TP.*

(a) $\frac{16}{3}$ cm (b) $\frac{20}{3}$ cm

(c) $\frac{18}{5}$ cm (d) $\frac{19}{6}$ cm

24. In the given figure, *AABC* is an isosceles triangle in which AS = *AC* and *ZABC* = 50°, then *ZBDC* is equal to



(a) 60° *(b)10°*

(c)80° (d)90°

25. The ratio between the number of sides of two regular polygons is 1:2 and ratio between their interior angles is 2 : 3, The number of sides of these polygons are respectively *(a)* 7, 14 *(b)* 6, 12 *(c)* 5, 10 *(d)* 4, 8

26. $\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}$ is equal to

(a) $\sec\theta$ (b) $2\sec\theta$

(c) $2\cos\theta$ (d) $\sin\theta$

27. If $7\cos^2\theta+3\sin^2\theta=4$ and $0<\theta<\frac{\pi}{2}$, then find the value of $\tan\theta$.

(a) $\sqrt{3}$ (b) 3 (c) $\frac{1}{3}$ (d) $\frac{1}{\sqrt{3}}$

28. In a rectangle *ABCD*, $AB=15$ cm, $\angle BAC=60^\circ$, then *BC* is equal to

(a) $15\sqrt{3}$ (b) $\frac{15\sqrt{3}}{2}$

(c) $\frac{15}{\sqrt{3}}$ (d) $\frac{15}{2\sqrt{3}}$

29. If $3\tan\theta=4$, then find $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$.

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) None of these

30. If $\tan\theta+\frac{1}{\tan\theta}=2$, then $\tan^2\theta+\frac{1}{\tan^2\theta}$ is equal to

(a) 1 (b) 2

(c) 4 (d) None of these

31. From the top of a 10 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower

(a) 17.42 m *(b)* 27.32 m

(c) 23.62 m *(d)* None of these

32. Reena had ? 10000 with her. Out of this money, she lent some money to Akshay for 2 yr at 15% of SI. She lent remaining money to Brijesh for an equal number of years at the rate of 18%. After 2 yr, Reena found that Akshay had given her ? 360 more as interest at compared to Brijesh The amount of money which Reena had lent to Brijesh must be

(a) ? 4000 *(b)* ? 4200

(c) ? 4500 *(d)* None of these

33. Find the least number of complete years in which a sum of money put out at 20% compound interest, will be more than double

(a) 2 yr *(b)* 3 yr *(c)* 4 yr *(d)* 6 yr

34. If $\left(a+\frac{1}{a}\right)^2=3$, then what is the value of $a^3+\frac{1}{a^3}$?

(a) $3\sqrt{3}$ *(b)* 3 *(c)* 0 *(d)* 9

35. If $x^2-3x+1=0$, then find the value of $x+\frac{1}{x}$.

(a) 0 *(b)* 2
(c) 3 *(d)* None of these

36. If $x-y=-1$, then find the value of x^3-y^3+3xy.

(a) 1 *(b)* -1 *(c)* 3 *(d)* -3

37. If $x+\frac{1}{y}=1$ and $y+\frac{1}{z}=1$, then find the value of $z+\frac{1}{x}$.

(a) 0 *(b)* 1 *(c)* 2 *(d)* 4

38. If $x^4+\frac{1}{x^4}=727$, then find the value of $x^3-\frac{1}{x^3}$.

(a) 140 *(b)* 120 *(c)* 190 *(d)* 160 39. By selling 32 oranges for ? 30, a man losses 25%. How many oranges should be sold for *X* 24, so as to gain 20% in the transaction?

(a) 20 *(b)* 22 *(c)* 16 *(d)* 19 40.Anush purchased 120 reams of paper at ? 100 per ream and the expenditure on transport was ? 480. He had to pay an extra duty of 50 paise per ream and the coolie charges were ? 60. What should he charge per ream to gain 40%?

(a) ? 147 *(b)* ? 135 *(c)* ? 142 *(d)* ? 137 41. A dishonest dealer sells his goods at 10% loss on cost price and uses 30% less weiaht. What is his nrofit or loss Der cent?

(a) $28\frac{4}{7}$% loss *(b)* $28\frac{4}{7}$% profit
(c) $28\frac{3}{7}$% loss *(d)* $28\frac{3}{7}$% profit

42. A train with 90 km/h crosses a bridge in 36 s. Another train, 100 m shorter crosses the same bridge at 45 km/h, Find the time taken by the second train to cross the bridge.

(a) 30 s *(b)* 32 s *(c)* 60 s *(d)* 64 s

43. The speeds of two trains *A* and 6 are 42 km/h and 33 km/h, respectively. Length of

5 is 3/2 times the length of *A.* Train *A* takes 50 s to cross train fi, if they are moving in the same direction. How long will they take to cross each other if they are moving in opposite directions?

(a)Ss (b) 17 s fc) 12 s *(d)* 9 s

44. 25 men and 15 women can complete a piece of work in 12 days. All of them start working together and after working for 8 days, the women stopped working 25 men completed the remaining work in

6 days. How many days will it take for completing the entire job, if only 15 women are put on the job?

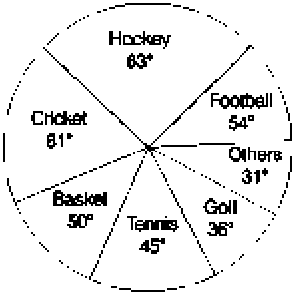
(a) 48 days *(b)* 24 days

(c) 36 days *(d)* 60 days

45. If (12" + 1) is divisible by 13, then *n* is

(a) any even integer® only 1 *(c)* any odd integer *(d)* only 12

Directions (Q. Nos. 46-50) *The circle graph given here shows the spendings of a country on various sports during a particular year, study the graph carefully and answer the questions given below it.*



tB.What per cent of the total spendings is spent on tennis?

(a) $12\frac{1}{2}\%$ (b) $22\frac{1}{2}\%$

(c) 25% (d) 45%

47. How much per cent more is spent on Hockey than that on golf?

(a) 27% *(b)* 35%

(c) 37.5% *(d)* 75%

48. How much per cent less is spent on Football than that on cricket?

(a) $22\frac{2}{9}\%$ (b) 27%

(c) $33\frac{1}{3}\%$ (d) $37\frac{1}{2}\%$

49. If the total amount spent on sports during the year was 7 2 crore, the amount spent on cricket and hockey together was

(a) 7 800000 *(b)* 7 8000000

fc) r 12000000 *(d)1* 16000000

50. If the total amount spent on sports during the year be 7 18000000, the amount spent on basketball exceeds that on tennis by

(a) 7 250000 *(b)* 7 360000

fc) 7 375000 *(d)* 7 410000

Answers

1. (a) 2. (b) 3. (a) 4. (a) 5. (b) 6. (a) 7. (a) 8. (c) 9. (c) 10. (a)

11. (b) 12. (a) 13. (a) 14. (d) 15. (a) 16. (a) 17. (b) 18. (b) 19. (a) 20. (b)

21. (a) 22. (b) 23. (b) 24. (c) 25. (d) 26. (b) 27. (a) 28. (a) 29. (a) 30. (b)

31. (b) 32. (d) 33. (c) 34. (c) 35. (c) 36. (b) 37. (b) 38. (a) 39. (c) 40. (a)

41. (b) 42. (d) 43. (a) 44. (c) 45. (c) 46. (a) 47. (d) 48. (c) 49. (b) 50. (a)

Practice Set 5

Directions (Q. Nos. 1-6) *What will come in place of question mark* (?) in *the following number series'!*

1. 311 300 278 245 201 146?

(a) 70 *(b)* 90 *(c)* 80 *(d)* 110 *(e)* None of the above

2. 17 22 32 47 67 92?

(a) 112 *(b)* 132 *(c)* 111 *(d)* 122 *(e)* None of the above

3. 123 183 213 228 235.5?

(a) 238.25 *(b)* 239.25

(c) 275.50 *(d)* 238.50

(e) None of the above

4. 9 204275119?

(a) 174 *(b)* 170 fc) 168 *(d)* 180 (e) None of the above

5. 23 32 45 62 83 ?

(a) 116 *(b)* 106

fc) 102 *(d)* 118

fe,) None of the above

6. 17 23 35 59 107?

(a) 217 *(b)* 223 fc) 203 *(d)* 227 fe,) None of the above

7. The average speed of a tractor is two-fifth the average speed of a car. The car covers 450 km in 6 h. How much distance will the tractor cover in 8 h?

(a) 210 km/h

(b) 240 km/h

(c) 420 km/h (o5 480 km/h

fe) None of the above

8. The marks of six boys in a group are 48, 59, 87, 37, 78 and 57. What are the average marks of all six boys?

(a) 62 *(b)* 64 *(c)* 61 *(d)* 63 *(e)* None of the above

9. What is 74% of five-eighth of 1200?

(a) 555 *(b)* 565

(c) 445 *(d)* 455

(e) None of the above

10. Sumit purchased an item for *X* 6500 and sold it at the gain of 24%. From that

amount, he purchased another item and sold it at the loss of 20%. What is his over all gain/loss? *(a)* Loss of *X* 42 *(b)* Gain of *X* 42

(c) Loss of *X* 52

(d) Neither gain nor loss

(e) None of the above

How many sacks are required for filling 1026 kg of rice, if each sack is filled with 114 kg of rice?

(a) 19 *(b)* 15 *(c)* 7 *(d)* 9 *(e)* None of the above

12. Mani's monthly income is three-fourth Rakhi's monthly income. Rakhi's monthly income is ? 38000, What is Mani's annual income? *(a)* *X* 4.32 lakh *(b)* *X* 3.42 lakh

(c) X 3.22 lakh *(d)* X 4.22 lakh

(e) None of the above

13. What will come in place of both the question marks (?) in the following questions

$$\frac{(?)^{3/5}}{33} = \frac{3}{(?)^{2/5}}$$

(a) $(99)^2$ *(b)* $\sqrt{93}$ *(c)* $3\sqrt{11}$ *(d)* 99

(e) None of the above

What value will be obtained if the square of 22 is subtracted from the cube of 12?

(a) 1244 *(b)* 1344 *(c)* 1454 *(d)* 1354 *(e)* None of the above

15. The ratio between the present ages of Tarun and Varun is 3 :7, respectively After 4 yr, Varun's age will be 39 yr. What was Tarun's age 4 yr ago?

(a) 12 yr *(b)* 13 yr *(c)* 19 yr *(d)* 18 yr *(e)* None of the above

16. Ravi consistently runs 350 m everyday except on Tuesday when he runs 500 m How many kilometres will he run in two weeks? (In this question, week starts from Monday.)

(a) 4.5 km *(b)* 4.8 km

(c) 5.2 km *(d)* 5.4 km

(e) None of the above

17. The simple interest accrued in 3 yr on a principal of ? 25000 is three-twentieth the principal. What is the rate of simple interest per cent per annum?

(a) 5 *(b)* 4 *(c)* 6 *(d)* 3 (e) None of the above

18. The sum of five consecutive even numbers is equal to 170. What is the sum of the second largest number amongs them and the square of the smallest number amongs them together?

(a) 940 *(b)* 932 *(c)* 938 *(d)* 934 (e) None of the above

19. The area of a square is four times the area of a rectangle. The length of the rectangle is 25 cm and its breadth is 1 cm less than one-fifth its length. What is the perimeter of the square?

(a) 40 cm ft) 60 cm ft) 160 cm

(d) Couldn't be determined

(e) None of the above

20.Sohan got 54 marks in Hindi, 65 marks in Science, 89 marks in Maths, 69 marks in Social Science and 68 marks in English. The maximum marks of each subject are 100. How much over all percentage of marks did he get?

(a) 74 *(b)* 69 *(c)* 68 *(d)* 72

(e) None of the above

21. 8 women can complete a work in 15 h. In how many hours, will 12 women complete the same work?

(a) 12 *(b)* 6 *(c)* 8 *(d)* 10 *(e)* None of the above

22. If 3 men or 9 boys can finish a work in 21 days. In how many days, can 5 men and 6 boys together do the same work?

(a) 12 days *(b)* 8 days

(c) 14 days

(d) Couldn't be determined

(e) None of the above

23. In a test, Rajesh got 112 marks which is 32 more than the passing marks. Sonal got 75% marks which is 70 more than the passing marks. What is the minimum passing percentage of the test?

(a) 35 *(b)* 45 *(c)* 40 *(d)* 30 *(e)* None of the above

24.25% of Reena's yearly income is equal to 75% of Anubhav's monthly income. If

Anubhav's yearly income is ? 240000, what is Reena's monthly income? *(a)* ? 60000 *(b)* *X* 12000

(c) f. 5200

(d) Couldn't be determined

(e) None of the above

25. What will be the compound interest obtained on a principal amount of ? 5500 at the rate of 3% per annum after 2 yr?

(a) ? 343.95 (b) ? 324.95

(c) ? 354.95 (d) X 314.95 (e) None of the above

26. The average speed of a bus is 67 km/h The bus was scheduled to start at 12:00 pm. It was scheduled to reach a destination 335 km away from its starting point at 7:00 pm and a halt was scheduled on the way. For how long was the halt scheduled?

(a) 3 h ft) 1 h (c) 2 h

(d) Couldn't be determined

(e) None of the above

27. A man sold 15 units of wood and the cost of each unit of wood was ? 32. Then, he bought 2 kg of rice with his earnings at the rate of ? 28 per kg, How much money was the man left with after buying the rice?

(a) ? 442 (b) ? 434

(c) ?424 (d) ? 414

(e) f 454

28. The average of five numbers is 371.8. The average of the first and second numbers is 256,5 and the average of the fourth and fifth numbers is 508. Which of the following is the third number?

(a) 360 ftj 310 (c) 430 (d) 380 (e) 330

29. Every month Priyanka spends 15% of her monthly income for paying loan instalment Her annual income is ? 4,2 lakh, What is the total amount that Priyanka spends on loan instalments in 6 months together?

(a) ? 32500 ftj ? 35100

(c) ? 30500 (d) ? 31500

fe; None of the above

30. What will be the average of the following set of scores?

313, 252, 161, 727, 525, 818

(a) 463 ft; 466 (c) 456 (d) 476

(e) 446

31. Find the sum of the squares of first 50 natural numbers, (a) 42925 (b) 42900

(c) 42860 (d) 42875

(e) None of the above

32. Simplify $\dfrac{\frac{3}{2} + \frac{1}{2} \times \frac{3}{2}}{\frac{3}{2} + \frac{1}{2} \text{ of } \frac{3}{2}} + \frac{1}{8}$.

(a) 18 (b) 21 (c) 16 (d) 24 (e) None of the above

Directions (Q. Nos. 33-35) *What should come in place of the question mark* (?) in *the following questions?*

33.$(6)^2 + (8)^2 x (2)^2 - (9)^2 = ?$

(a) 215 (b) 209 (c) 221 (d) 211 (e) None of the above

34.7008 * 24+ 6208-s-16 = ?

(a) 640 ft) 720 (c) 700 ((/J 690 (p) None nf the ahnve

35. $\frac{3}{4}$th of $\frac{3}{5}$th of $\frac{2}{3}$rd of ? = 3174

(a) 10550 (b) 10540 (c) 10580 (d) 10500

(e) None of the above

Directions (Q. Nos. 36-39) *Each question below is followed by two statements* I *and* II. *You are to determine whether the given data in the statement is sufficient for answering the question. You should use the data and your knowledge of Mathematics to choose between the possible answers. Give the answer.*

(a) If the Statement I alone is sufficient to answer the question but the statement

I alone is not sufficient

(b) If the Statement 11 alone is sufficient to answer the question but the statement

alone is not sufficient

(c) If both Statements I and II together are needed to answer the question

(d) If either the statement I alone or Statement II alone is sufficient to answer the question

(e) If can't get the answer from the Statements I and II together and need even more data

36. What is the population of state *A* in year 1999?

I. The population of stated is 120000 in 1997 and it increases 20% every year

I. Population of state *A* is double the population of state *B* in 1997

37. What is the number of students in the class?

I. Ratio of boys and girls is 3:4 I. Girls are 18 more than boys,

38. What is the cost of covering the floor of a rectangular hall by a carpet?

I. Cost of carpet is ?450 per sq m I. The perimeter of a hall is 50 m,

39. Find the number of two digits

. Sum of the digits is 15,

I. Difference between the digits is 3,

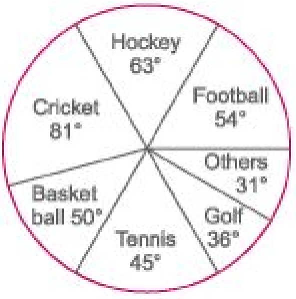
40. A school hall has the dimensions 30 m, 12 m 6 m, Find the number of children who can be accommodated, if each child should get 8 m^3 of space

(a) 240 *(b)* 270 *(c)* 250 *(d)* 150 (e) None of the above

41. Three unbiased coins are tossed. Find the probability of getting one head and two tails,

(a) $\frac{3}{8}$ *(b)* $\frac{1}{8}$ *(c)* $\frac{5}{8}$ *(d)* $\frac{7}{8}$
(e) None of the above

Directions (Q. Nos. 42-46) *The circle-graph given here shows the spendings of a country on various sports during a particular year. Study the graph carefully and answer the questions given below it.*



42. What per cent of the total spendings is SDent on Tennis?

(a) $12\frac{1}{2}$% *(b)* $22\frac{1}{2}$%
(c) 25% *(d)* 45%
(e) None of the above

43. How much percent more is spent on Hockey than that on Golf? *(a)* 27% *(b)* 35%

(c) 37,5% *(d)* 75%

(e) None of the above

44. How much per cent less is spent on Football than that on Cricket?
(a) $22\frac{2}{9}$% *(b)* 27% *(c)* $33\frac{1}{3}$% *(d)* $37\frac{1}{2}$%
(e) None of the above

45. If the total amount spent on sports during the year was ? 2 crore, then amount spent on Cricket and Hockey together was

(a) ?800000 *(b)* ?8000000

(c) ? 12000000 *(d)* ?16000000

(e) None of the above

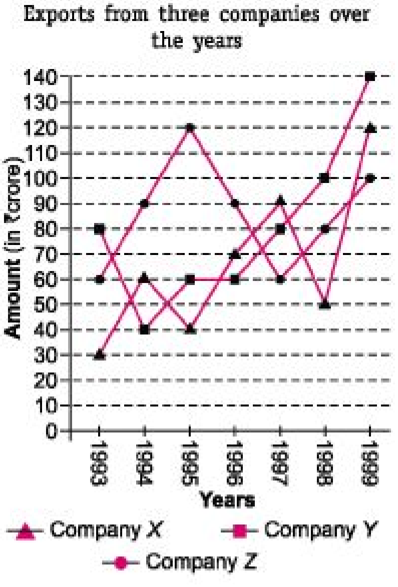
46. If the total amount spent on sports during the year be ? 18000000, then amount spent on Basketball exceeds that on Tennis by

(a) *X* 250000 *(b)* *X* 360000

(c) ?375000 *(d)* ? 410000

(e) None of the above

Directions (Q. Nos. 47-50) *Study the following graph and answer the questions based on it.*



47. Average annual exports during the given period for company V is approximately what per cent of the average annual exports for company Z?

(a) 87.12%

(b) 89.64%

(*c*) 91.21%

(*d*) 93.33%

(*e*) None of the above

48. In how many of the given years, were the exports from Company Z more than the average annual exports over the given years?

(*a*) 2

(*b*) 3

(*c*) 4

(*d*) 5

(*e*) None of the above

49. What was the difference between the average exports of the three companies in year 1993 and the average exports in year 1998?

(*a*) *X* 15.33 crore

(*b*) ? 18.67 crore

(*c*) *X* 20 crore

(*d*) ? 22.17 crore

(*e*) None of the above

50. In which year was the difference between the exports from companies X and *Y* the minimum?

(*a*) 1994

(*b*) 1995

(*c*) 1996

(d) 1997

(e) None of the above

Answers

1. (c) 2. (d) 3. (b) 4. (a) 5. (e) 6. (e) 7. (b) 8. (c) 9. (a) 10. (c)

11. (d) 12. (b) 13. (d) 14. (a) 15. (e) 16. (c) 17. (a) 18. (e) 19. (e) 20. (b)

21. (d) 22. (e) 23. (C) 24. (a) 25. (e) 26. (b) 27. (c) 28. (e) 29. (d) 30. (b)

31. (a) 32. (a) 33. (d) 34. (e) 35. (c) 36. (a) 37. (c) 38. (e) 39. (e) 40. (b)

41. (a) 42. (a) 43. (d) 44. (c) 45. (b) 46. (a) 47. (d) 48. (c) 49. (c) 50. (c)

Why This book of Fast Track Objective Arithmetic

All the recruitment tests and competitive exams conducted in India, like *UPSC, CSAT, SSC (10+2, CGL, CPO) SBI and IBPS Bank (PO and Clerk), LIC AAO, CDS, Management and Hospitality Entrances, Railways and Police recruitments* etc. essentially contain a significant number of objective questions based on Arithmetic to check the candidates capabilities to answer them. If a candidate desire to achieve 'success' in these tests, he must be highly skilled in performing accurate and fast arithmetic calculations. Accordingly, this book aims to develop two specific skills in a candidate : *How to solve an arithmetic problem with accuracy and How to perform this calculation speedily?*

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