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APTITUDE

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(D) REASONING

Numerical Ability

CHAPTER

[1]

Number System

TYPES OF NUMBERS

Natural Numbers

Counting numbers are called natural numbers. Thus 1, 2, 3, 4, 5, 6, ... are all natural numbers.

Whole Numbers

All counting nubers and 0 from the set of whole numbers. Thus 0, 1, 2, 3, 4, 5,..... etc. are whole numbers. Clearly, every natural number is whole number and 0 is a whole number which is not a natural number.

Integers

All counting numbers, zero and negatives of counting numbers form the set of integers.

Thus, ..., -3, -2, -1, 0, 1, 2, 3, ... are all integers.

Set of positive integers = {1, 2, 3, 4, 5, 6,....}

Set of negative integers = {-1, -2, -3, -4, ...}

Set of all non-negative integers = {0, 1, 2, 3, 4, 5, ...}

Even Numbers

A counting number divisible by 2 is called an even number. Thus 0, 2, 4, 6, 8, 10, 12.... etc. are all even numbers.

Odd Numbers

A counting number not divisible by 2 is called a odd number.

Thus 1, 3, 5, 7, 9 11, 13, 15,... etc. are all odd numbers.

Rational Numbers

A number 'r' is called a rational number, if it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

For example 2/3, 4/5, 7/8, -3/4 are all examples of rational numbers.

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A number 'r' is called a rational number, if it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

For example 2/3, 4/5, 7/8 , -3/4 are all examples of rational numbers.

Also, -25 can be written as $\frac{-25}{1}$ here p = -25 and q = 1. Therefore, the rational numbers also include the natural numbers, whole numbers and integers. there are infinitely many rationals numbers.

Irrational Numbers

A number 's' is called irrational, if it cannot be written in;,'t:he;{ irm $\frac{p}{q}$, where p and q are integers and q =/:0.

There are infinitely many irrational numbers too. Some exairiple·s are:

.2, 3, 15,yr. 0.10110111011}10...

It turns out that.the collection of all rational numbers ahd irtiiional numbers together make up what we call the collection of real numbers, w;p.ich is denoted by R.

Therefore, a real number is either rational or irratio:hal. So, we can' say that **every real number is represented by a unique point on th€; timber line. Also every point on the number line respresents a unique r,: .nlllller.** This is why we call the number line, the real number line.

Primer Numbers

A number other than 1 is called a priiri \1pnil:ier if it is divisible only by 1 and itself.

Example 1.

All prime numbers less than 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Sol.

To test whether a given number is prime number or not If you want to test whether any number is a prime number or not, take an integer larger than the approximate square root of that number. Let it be 'x'. Test the divisibility of the given number by every prime number less than 'x'. If it is not divisible by any of them, then it is prime number; otherwise it is a composite number (other than prime). See the examples given below.

Example 2.

Is 349 a prime number?

Sol.

The squre root of 349 is approximately 19. The prime numbers less than 19 are 2, 3, 5, 7, 11, 13, 17. Clearly, 349 is not divisible by any of them. Therefore, 349 is a prime number.

Example 3.

Is 979 a prime number?

Sol.

The approximate square roote of 979 is 32. Prime numbers less than 32 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31. We observer that 979 is divisible be 11, so it is not a prime number.

Composite Numbers

A number, other than 1, which is not a prime number is calied a composite number.

e.g., 4, 6, 8, 9, 12, 14.

Example 4.

The number(>f positive i1:1te'ge':rs 1,i'in the r rtge 12 K 4()'$uc;li:,tha:,tJhe p,:todui:lt, (n :_ 1ffo).,. 2:) .(n - 3).. 3.2. i/is llC>f ;ivis. 9le C)5yri•i

SQl.

The P!o11:ct(11_- I).(1:1 --} <tt/?X;;:3· :•t illnothe dgis e byn_dfly n.thi prod gpdoe s not <;0ntam factorsdf n,.1.e., u1s pmne n11- . The prtme ;1um ,ers th a t s1;1t1$fy th. ahov e condition§t are! 13/17, 19, 28, 293f31; and 37.

H:err:ce tll;re ,;; 7; prim'inu ber. 'bet i n ,i2 aiid 4.J,..

BASIC REPRESENTATION OF NUMBERS

The number system involving numbers 0 to 9 is called the decimal number system (as there are 10 numbers in this system). In decimal number system the base is 10.

Example 5.

There is tW0 digit number pq in decimalsystem. Both p and q are .natural numbers. What is tl:ie value ofpq?

Sol.

F i om the ba ie cou11ting,tUl{};{WE::l knovvh that p is in ten's placB•)llld.q is i11 µnit's plac , Hence, the value.of pq is 10(p)+ q,

Example 6.

Atwo;digif,p.mnber pg: is atrtleq:to tlie nu per formed by reversing the origfoal digit;'.'!£ t9,· u -sum is divisible b f 11,,,9, 11:t\ .2,,findthe number of pairs of (p, q).

i o L -

Let the original numH t be pq. 1he v'alue bf the number = lOp +q.

The nulJ:1ber form d byjeversi11;g the digi!s = qp: Value of this num,l:ir =]0q + p:

Sum of thetwo.numbers = l1p + l1q ;= ll(p + q).

Now if the .sum is divis.ihre by11, 9, 2, it means that (p+q) mu(¼t be divisible by;>bQth. 9and 2: Henqe, p + q = 18. .So i:Lnieans p = q <c 9. The original nufuper :fa 99,.. ,

Example 7.

1n atWb dig-ifprifu'e number", if 18 is adqed,-)Ve get another priin, b'umlie:ic,, ith;,:i:evets :S: How,w,a y'su-0ltnufuberst tre pf)ssible?

Sol.

Let a two-digit number be pq.

10 p + q + 18 = 10q + p

or −9p ;q=}Ɉ..

∴ q − p = 2

Jj:'.satit i thilc tiditi: n,,.· nd.·a1i ·•th&conditidn,, of being a prime number (pq and qp both). :v whe.te;are 2 numb rs, 13 and '79 ar.e t11e pq:ssibl ones.

Notetliata b .)nt ke 0111{pdd,yal ,;s, i.e., 1,3,5, 7,9. Representation of Rational Numbers

REPRESENTATION OF RATIONAL NUMBERS

Take a fraction $\frac{13}{5}$. It can be converted into a decimafrept,!en.tz tion by dividing 13 by 5, i.e., $\frac{13}{5}$ = 2.6. So, conversely, 2.6 is equivalent to $\frac{13}{5}$. Also 2'i6 c n be written $\frac{26}{10}=\frac{13}{5}$.

Rational numbers when converted into decimai·f rm.;i::an 'be either a recurring and non-terminating or a terminating decimal.

For example, terminating decimal= 2.6; no:g::i:ter Jnating and recurring decimal= 2.636363...

USEFUL TECHNIQUE

The short cut to convert decimal intg a,tati9:µ;:l number is as follows. Write the complete number in the numerator and subtract'.'i\9nlre,c::tirring part from this. Divide this by denominator which have number of 9s = Number of te<;:tirring digits and number of 0s = Number of non-recurring digit after decimal.

$$\frac{\text{Number - (Non-recurring part of the number)}}{\text{9s followed by 0s}}$$

Number of 9s = Number of recurring digits

Number of 0s = Number of non-recurring digits in decimal part.

Example 8.

Convert $0.53\overline{467}$ into a rational number.

Sol. $x = 0.53\overline{467}$

$100x = 53.\overline{467}$

$100000x = 53467.\overline{467}$

$99900x = 53414$

$x = \frac{53414}{99900}$

This question can also be solved by applying the above formula. As we have $\frac{53467-53}{99900} = \frac{53414}{99900}$.

Example 9.

Convert 4.333333... into a rational number.

Sol.

Let $x = 4.333333....$

Then $10x = 43.33333333...$

$10x - x = (43.33333...) - (4.33333...)$

So $9x = 39.$

Therefore, $x = \frac{39}{9} = \frac{13}{3}$

This is the rational form of a non-terminating, recurring decimal 4.3333... .

Example 10.

Convert $6.3\overline{467}$ into a rational number.

Sol.

Applying the stated formula above, we have $x = \frac{63467-63}{9990} = \frac{63404}{9990} = \frac{31702}{4995}$

TESTS OF DIVISIBILITY

Divisibility By 2

A number is divisible by 2 1:0t1 1nit digit is any of 0, 2, 4, 6, 8.

Ex. 58694 is divisible by 2, while Si!H5 is not divisible by 2.

Divisibility By 3

A number is divisible"by:;fronly whell' the sum of its digits is divisible by 3.

Ex. (i) In the numbe?:6 4 21, the sum of digits = 27, which is divisible by 3.

∴ 69542J{fa divisilJIJ by 3.

(ii) In the fiumh T 948653, the sum of digits = 35, which is not divisible by 3.

∴ 948653/iSndf divisible by 3.

Divisible By':g

A number is diviifble by 9 only when the sum of its digits is divisible by 9.

Ex. (i) In the number 246591, the sum of digits = 27, which is divisible by 9.

∴ 246591 is divisible by 9.

(ii) In the number 734519, the sum of digits = 29, which is not divisible by 9.

∴ 734519 is not divisible by 9.

Divisible By 4

A number is divisible by 4 if last two digits is divisible by 4.

Ex. (i) 6879376 is divisible by 4, since 76 is divisible by 4.

(ii) 496138 is not divisible by 4, since 38 is not divisible by 4.

Divisible By 8

A number is divisible by 8 if the last three digit number formed i.e. hundred's ten's and unit's digit of the given number is divisible by 8.

Ex. (i) In the number 16789852, the number formed by last 3 digits, namely 352 is divisible by 8.

∴ 16789352 is divisible by 8.

(ii) In the number 576484, the number formed by last 3 digits, namely 484 is not divisible by 8.

∴ 576484 is not divisible by 8.

Divisible By 10

A number is divisible by 10 only when its unit digit is 0.

Ex. (i) 7849320 is divisible by 10, since its unit digit is 0.

(ii) 678405 is not divisible by 10, since its unit digit it not 0.

Divisible By 5

A number is divisible by 5 only when its unit digit is 0 or 5.

Ex. (i) Each of the numbers 7689[illegible] and [illegible]8790 is divisible by 5.

Divisible By 11

A number is divisible by 11 if the difference between the sum of its digits at odd places and the sum of its digits at even places is either 0 or a number divisible by 11.

Ex. (i) Consider the number 29435417.

(Sum of its digits at odd places) - (Sum of its digits at even places)

(7 + 4 + 3 + 9) – (1 + 5 + 4 + 2) = (23 - 12) = 11, which is divisible by 11.

∴ 29435417 is divisible by 11.

(ii) Consider the number 57463822.

(Sum of its digits at odd places) - (Sum of digits at even places)

= (2 + 8 + 6 + 7) - (2 + 3 + 4 + 5) = (23 - 14) = 9, which is not divisible by 11.

∴ 57463822 is not divisible by 11.

Example 11.

Find the least value of x for which $7x5462$ is divisible by 9.

Sol.

Let the required value be x. Then

$(7 + x + 5 + 4 + 6 + 2) = (24 + x)$ is divisible by 9. $\Rightarrow x = 3$

Example 12.

Find the least value of *for which 4832* 18 is divisible by 11.

Sol.

Let the digit in place of * is x .

(Sum of digits at odd places) – (Sum of digits at even places)

Here we are counting the even and odd digits taking unit digit as first digit (odd digit)

$= (8 + x + 3 + 4) - (1 + 2 + 8) = (4 + x)$, which should be divisible by 11.

$\therefore \quad x = 7$.

Any six-digit, twelve-digit, or eighteen-digit, or any such number with number of digits equal to multiple of 6, is divisible by each of 7, 11 and 13 if all of its digits are same.

For example 666666, 888888, 333333333333 are all divisible by 7, 11, 13

As 666666 can be written as $666 \times 1000 + 666$

or $666\ (1000+1) = 666 \times (1001)$

$= 666 \times (7 \times 11 \times 13)$

Hence 666666 is divisible by all of 7, 11, 13, 77, 91, 143 and 1001.

THE LAST DIGIT OF ANY POWER

The last digits of the powers of any number follow a cyclic pattern - i.e., they repeat after certain number of steps. If we find out after how many steps the last digit of the powers of a number repeat, then we can find out the last digit of any power of any number.

Let us look at the powers of 2.

Last digit of 2^1 is 2

Last digit of 2^2 is 4

Last digit of 2^3 is 8

Last digit of 2^4 is 6

Last digit of 2^5 is 2

Since last digit the 2^5 is the same as the last digit of 2^1, then onwards the last digit will start repeating, i.e., digits of 2^5, 2^6, 2^7, 2^8 will be the same as those of 2^1, 2^2, 2^3, 2^4. Then the last digit of 2^9 is agin the same as the last digit of 2^1 and so on. So we have been able to identify that for powers of 2 the last digits repeat after every 4 steps. **In other words whenever the power is a multiple of 4, the last digit of that number will be the same as the last digit of 2^4.**

Suppose we want to find out the last digit of 2^{66}, we should look at a multiple of 4 which is less than of equal to the power 66. Since 64 is a multiple of 4, the last digit of 2^{64} will be the same as the last digit of 2^4.

Then the last digits of 2^{65}, 2^{66} will be the same as the last digits of 2^1, 2^2 respectively. Hence the last dight of 2^{66} is the same as the last digit of 2^2 i.e., 4.

Similarly, we can find out the last digit of 3^{75} by writing down the pattern of the powers of 3.

Last digit of 3^1 is 3

Last digit of 3^2 is 9

Last digit of 3^3 is 7

Last digit of 3^4 is 1

Last digit of 3^5 is 3

The last digit repeats after 4 steps (like in the case of powers of 2).

To find the last digit of 3^{75}, we look for a multiple of 4 which is less than or equal to 75. Since 72 is multiple of 4, the last digit of 3^{72} will be the same as that of 3^4. Hence the last digit of 3^{75} will be the same as the last digit of 3^3 i.e., 7.

LAST DIGIT OF A SUM OR PRODUCT

The last digit of 243 + 456 will be the same as the sum of the last digits of the two numbers, i.e., the sum of 3 and 6, which is 9.

Example 13

Find the last digit of $2^{416} \times 4^{430}$.

Sol.

Writing down the powers of 2 and 4 to check the pattern of the last digits, we have

Last digit of $2^1 = 2$

Last digit of $2^2 = 4$

Last digit of $2^3 = 8$

Last digit of $2^4 = 6$

Last digit of $2^5 = 2$

Last digit of $4^1 = 4$

Last digit of $4^2 = 6$

Last digit of $4^3 = 4$

Last digit of $4^4 = 6$

We find that the last digit of powers of 2 repeat after 4 steps, the last digit of any power of 4 is 4 for an odd power and 6 for an even power. The last digit of 2^{412} will be the same as 2^4 as 416 is a multiple of 4. So the last digit of 2^{416} is 6. Last digit of 4^{430} is 6. Since the power of 4 is even.

Hence the last digit of $2^{416} \times 4^{430}$ will be equal to the last digit of $6 \times 6 = 6$.

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Example 14.

What is the remainder when the product 1991 × 1992 × 2000 is divided by 7?

Sol.

The remainder when 1991, 1992, and 2000 are divided by 7 are 3, 4, and 5 respectively. Hence the final remainder is the remainder when the product 3 × 4 × 5 = 60 is divided by 7. Therefore, remainder = 4.

Suppose the numbers, N_1, N_2, N_3... give quotients Q_1, Q_2, Q_3.... and remainders R_1, R_2, R_3..., respectively, when divided by a common divisor D.

Therefore

$N_1 = D \times Q_1 + R_1$,

$N_2 = D \times Q_2 + R_2$,

$N_3 = D \times Q_3 + R_3$.... abd so on..

Let P be the product of N_1, N_2, N_3

Therefore, $P = N_1N_2N_3...(D\times Q_2+ R_1)(D\times Q_2+R_2)(D\times Q_3+ R_3) = D \times K + R_1R_2R_3$....where K is some number ...(1)

In the above equation, since only the product $R_1R_2R_3$.... is free of D, therefore the remainder when P is divided by D is the remainder when the product $R_1R_2R_3$.... is divided by D.

Let S be the sum of N_1, N_2, N_3....

Therefore, $S = (N_1)+(N_2)+(N_3)+... = (D\times Q_1 + R_1)+(D\times Q_2 + R_2)+(D\times Q_3 + R_3)$

$= D\times K + R_1 + R_2 + R_3$... where K is some number ... (2)

Hence the remainder when S is divided by D is the remainder when $\mathbf{R_1 + R_2 + R_3}$ is divided by D.

Example 15.

What is the remainder when 2^{2010} is divided by 7?

Sol.

2^{2010} is again a product (2 × 2 × 2...(2010 times)). Since 2 is a number less than 7 we try to convert the product into product of numbers higher than 7. Notice that 8 = 2 × 2 × 2. Therefore we convert the product in the following manner- $2^{2010} = 8^{670} = 8 \times 8 \times 8$... (670 times.) The remainder when 8 is divided by 7 is 1. Hence the remainder when 8^{670} is divided by 7 is the remainder obtained when the product 1 × 1 × 1... is divided by 7. Therefore, remainder = 1.

Example 16.

What is the remainder when 2^{2012} is divided by 7?

Sol.

This problem is like the previous one, except that 2012 is not an exact multiple of 3 so we cannot covert it completely into the form 8^x. We will write it in following manner-$2^{2012} = 8^{670} \times 4$. Now, 8^{670} gives the remainder 1 when divided by 7 as we have seen in the previous problem. And 4 gives a remainder of 4 only when divided by 7. Hence the remainder when 2^{2012} is divided by 7 is the remainder when the product 1 × 4 is divided by 7. Therefore, remainder = 4.

Example 17.

Find the remainder when 7^{52} is divided by 2402.

Sol.

$7^{52} = (7^4)^{13} = (2401)^{13} = (2402 - 1)^{13} = 2402K + (-1)^{13} = 2402K - 1$. ($\because$ using Binomial theorem)

Hence, the remainder when 7^{52} is divided by 2402 is equal to -1 or $2402 - 1 = 2401$.

Remainder = 2401.

RULES OF DIVISIBIITY WHEN DIVIDEND IS OF THE FORM $a^n + b^n$ OR $a^n - b^n$.

Theorem 1: $a^n + b^n$ is divisible by $a + b$ when n is **ODD**.

Theorem 2: $a^n - b^n$ is divisible by $a + b$ when n is **EVEN**.

Theorem 3: $a^n - b^n$ is always divisible by $a - b$.

Hence $a^n - b^n$ is divisible by both $(a + b)$ and $(a - b)$ when n is even and $a^n - b^n$ is divisible by only $(a - b)$ when n is odd.

Example 18.

What is the remainder when $3^{444} + 4^{333}$ is divided by 5?

Sol.

The dividend is in the form $a^x + b^y$. We need to change it into the form $a^n + b^n$.

$3^{444} + 4^{333} = (3^4)^{111} + (4^3)^{111}$. Now $(3^4)^{111} + (4^3)^{111}$ will be divisible by $3^4 + 4^3 = 81 + 64 = 145$. Since the number is divisible by 145 it will certainly be divisible by 5. Hence, the remainder is 0.

Example 19.

What is the remainder when $(5555)^{2222} + (2222)^{5555}$ is divided by 7?

Sol.

The remainders when 5555 and 2222 are divided by 7 are 4 and 3 respectively. Hence, the problem reduces to finding the remainder when $(4)^{2222} + (3)^{5555}$ is divided by 7.

Now $(4)^{2222} + (3)^{5555} = (4^2)^{1111} + (3^5)^{1111} = (16)^{1111} + (243)^{1111}$.

Now $(16)^{1111} + (243)^{1111}$ is divisible by $16 + 243$ or it is divisible by 259, which is a multiple of 7.

Hence the remainder when $(5555)^{2222} + (2222)^{5555}$ is divided by 7 is zero.

POWERS OF A NUMBER CONTAINED IN A FACTORIAL

Highest power of prime number p in

$n! = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \left[\frac{n}{p^4}\right] + ...$ where [x] denotes the greatest integer less than or equal to x.

Example 20.

Find the highest power of 2 in 50!

Sol.

The highest power of 2 in 50! $= \left[\frac{50}{2}\right] + \left[\frac{50}{4}\right] + \left[\frac{50}{8}\right] + \left[\frac{50}{16}\right] + \left[\frac{50}{32}\right] = 25 + 12 + 6 + 3 + 1 = 47$

Example 21.

Find the highest power of 30 in 50!

Sol.

$30 = 2 \times 3 \times 5$. Now 5 is the largest prime factor of 30, therefore, the powers of 5 in 50! will be less than those of 2 and 3. Therefore, there cannot be more 30s than there are 5 in 50! So we find the highest power of 5 in 50! The highest power of 5 in 50! = $\left[\frac{50}{5}\right]+\left[\frac{50}{25}\right] = 10 + 2 =$ 12. Hence the highest power of 30 in 50! = 12

Example 22.

Find the number of zeroes present at the end of 100!

Sol.

We get a zero at the end of a number when we multiply that number by 10. So, to calculate the number of zeroes at the end of 100!, we have to find highest power of 10 present in the number. SInce $10 = 2 \times 5$, we have to find the highest power of 5 in 100! The highest power of 5 in 100! $= \left[\frac{100}{5}\right]+\left[\frac{100}{25}\right] = 20 + 4 = 24$

Therefore, the number of zeroes at the end of 100! = 24.

SUCCESSIVE DIVISION

If the quotient of a division is taken and this is used as the dividend in the next division, such a division is called "successive division. A successive division process can continue upto any number of steps - until the quotient in a division becomes zero for the first time. i.e, the quotient in the first division is taken as dividend in the second division; the quotient in the second division is taken as the dividend in the third division; the quotient in the third division is taken as the dividend in the fourth division and so on.

If we say that 3305 is divided successively by 4,5, 7 and 2, then the quotients and remainder are as follows in the successive division.

Dividend	Divisor	Quotient	Remainder
3305	4	826	1
826	5	165	1
165	7	23	4
23	2	11	1

Here we say that when 3305 is successively divided by 4, 5, 7 and 2 the respective remainders are 1, 1, 4, and 1.

◆◆◆◆

Objective Questions

1. The last digit of the number obtained by multiplying the numbers.
$91 \times 92 \times 93 \times 94 \times 95 \times 96 \times 97 \times 98 \times 98$
(a) 8 (b) 4
(c) 0 (d) 2

2. What is the unit digit of the product
$(3^{69} \times 6^{59} \times 7^{75})$?
(a) 1 (b) 2
(c) 4 (d) 6

3. If the number 715 * 324 is completely divisible by 3, then the smallest whole number in place of * will be:
(a) 2 (b) 1
(c) 0 (d) none of these

4. If the number 27915 k 6 is completely divisible by 11, then the smallest whole number in place of k will be:
(a) 3 (b) 1
(c) 2 (d) 5

5. If the number 24573k is exactly divisible by 72, then the minimum value of k is:
(a) 5 (b) 7
(c) 6 (d) 8

6. The largest 5 digit number exactly divisible by 91 is :
(a) 99918 (b) 99921
(c) 99981 (d) 99978

7. $(x^n - a^n)$ is divisible by $(x - a)$
(a) for all even of n (b) only for even values of n
(c) only for odd values of n (d) only for prime values of n

8. On dividing a number by 56, we get 27 as remainder, on dividing the same number by 14 what will be the remainder?
(a) 7 (b) 12
(c) 11 (d) 13

9. On dividing a certain number by 5, we get 3 as remainder, what will be the remainder when cube of this number is divided by 5?

(a) 3
(b) 2
(c) 1
(d) 4

10. It is being given that $(2^{16} + 1)$ is completely divisible by at whole number. Which of the following number is completely divisible by $2^{16} + 1$?

(a) $2^{16}-1$
(b) $2^{32}-1$
(c) $2^{48}+1$
(d) both (b) and (c)

11. A number when divided successively by 4, 5 and 6 leaves remainders 2, 3 and 5 respectively. What is the minimum such number?

(a) 214
(b) 224
(c) 234
(d) 354

12. The number of digits in $(2abc)^4$ where 2abc is a four digits number is:

(a) 13
(b) 14
(c) 15
(d) can't say

13. Find the value of $0.4\overline{59}$.

(a) $\frac{91}{298}$
(b) $\frac{99}{198}$
(c) $\frac{91}{99}$
(d) $\frac{91}{198}$

14. When the sum of the digits of a number is subtracted from the number, the result will be always divisible by:

(a) 3
(b) 9
(c) 9
(d) both (a) and (b)

15. $11^{105} + 14^{105}$ is divisible by:

(a) 3
(b) 25
(c) 42
(d) none of these

16. The greatest number that will always divides $37^{2n} - 12^{2n}$ is: (where n is a natural no.)

(a) 1813
(b) 49
(c) 588
(d) 1225

17. Find the unit digit of $(1387)^{3265}$

(a) 9
(b) 7
(c) 3
(d) 1

18. Find the number of zeros at the end of 150!

(a) 45
(b) 50
(c) 37
(d) 35

19. The product of 10 consecutive even natural numbers is always divisible by

(a) $2^{10} \times 11!$
(b) $2^{10} \times 12!$
(c) $2^{10} \times 10!$
(d) none of these

20. If 7^{103} is divided by 25, then the remainder is

(a) 20
(b) 16
(c) 18
(d) 15

21. What will be the remainder when 17^{200} is divided by 18?

(a) 17
(b) 16
(c) 1
(d) 2

22. The symbol 25_b represents a two-digit number in base b. If the number 52_b is double the number 25_b, then b is :

(a) 7
(b) 8
(c) 9
(d) 11

23. Srinivas was able to find the two natural numbers, both greater than one as soon as their product was told to him and no other information was given. The sum of the two numbers is 16. What was their product, if it is greater than 40?

(a) 48
(b) 55
(c) 60
(d) 63

24. Which one of the following numbers is exactly divisible by 11?

(a) 235641
(b) 245642
(c) 315624
(d) 415624

25. Two numbers are such that their difference, their sum, and their product are in the ratio of 1 : 7 : 24. The product of the two numbers is:

(a) 6
(b) 12
(c) 24
(d) 48

26. The difference of two numbers is 1365. On dividing the larger number by the smaller, we get 6 as quotient and 15 as remainder. What is the smaller number?

(a) 240
(b) 270
(c) 295
(d) 360

27. If n is a natural number, then $(6n^2 + 6n)$ is always divisible by:

(a) 6 only
(b) 6 and 12 both
(c) 12 only
(d) by 18 only

28. If the square of a number of two digits is decreased by the square of the number formed by reversing the digits, then the result is not always divisible by:

(a) 9
(b) the product of the digits
(c) the sum of the digits
(d) the difference of the digits

29. If the digit 1 is placed after a two digit number whose ten' digit is t and units's digit is u, the new number is:

(a) 10t + u + 1
(b) 100t + 10u + 1
(c) 1000t + 10u + 1
(d) t + u + 1

30. Let x, y and z be distinct positive integers satisfying x < y < z and x + y + z = k. What is the smallest value of K that does not determine x, y, z uniquely?

(a) 9
(b) 6
(c) 7
(d) 8

31. The remainder when 2^{60} is divided by 5 equals

(a) 0
(b) 1
(c) 2
(d) None of these.

32. 2! + 4! +6! +8! + 10! + ... 100! when divided by 3, would leave remainder of :

(a) 0
(b) 1
(c) 2
(d) 3

33. What is the greatest positive power of 5 that divides 30! exactly?

(a) 5
(b) 6
(c) 7
(d) 8

34. The sum of two natural numbers is 85 and their LCM is 102. Find the numbers.

(a) 51 and 34
(b) 50 and 35
(c) 60 and 25
(d) 45 and 40

35. By what smallest number, 21600 must be multiplied or divided in order to make it a perfect square?

(a) 6
(b) 5
(c) 8
(d) 10

36. A number when divided by 238 leaves a remainder 79. What will be the remainder when that number is divided by 17?

(a) 8
(b) 9
(c) 10
(d) 11

37. What is the remainder when 17^{23} is divided by 16?

(a) 0
(b) 1

(c) 2 (d) 3

38. It is given that $2^{32}+1$ is exactly divisible by a certain number. Which one of the following is also divisible by $2^{32}+1$?

(a) $2^{96}+1$ (b) $2^{16}-1$

(c) $2^{16}+1$ (d) 7×2^{33}

39. In 1966, the age of Ram was same as the last digits of his year of birth. What was his age then?

(a) 32 (b) 48

(c) 64 (d) Cannot be determined

40. A number when divided by 5 leaves a remainder 3. What is the remainder when the square of this number is divided by 5?

(a) 4 (b) 3

(c) 9 (d) Cannot be determined

41. A number formed by writing any digit 6 times (say as 444444 or 999999) is always divisible by

(a) 1001 (b) 7

(c) 13 (d) All of these

42. Three consecutive whole numbers are such that the square of the middle number is greater than the product of the other two by 1. Find the middle number.

(a) 6 (b) 18

(c) 12 (d) All of these

43. How many zeros are there at the end of the product $33 \times 175 \times 180 \times 12 \times 44 \times 80 \times 66$?

(a) 2 (b) 4

(c) 5 (d) 6

44. Let $u_{n+1} = 2u_n + 1, (n = 0, 1, 2, ...)$ and $u_0 = 0$. Then $u_{10} =$

(a) 1023 (b) 2047

(c) 4095 (d) 8195

45. Convert 1234 from base 6 to base 10.

(a) 3100 (b) 3010

(c) 301 (d) None of these

46. $N = 148 \times 293 \times 581 \times 874$. What will the remainder when N is divided by 29?

(a) 7 (b) 6

(c) 5 (d) None of these

47. What is the highest power of 31 in 1001! ?

(a) 31 (b) 32

(c) 33 (d) None of these

48. $N = 56^{56} + 56$. What would be the remainder when N is divided by 57?

(a) 0 (b) 56

(c) 55 (d) None of these

49. $9^6 + 1$ when divided by 8, would leave a remainder

(a) 0 (b) 1

(c) 2 (d) 3

50. What is the last digit of the number 23457^{194321}?

(a) 9 (b) 1

(c) 3 (d) 7

51. Find the units digit of $34563^{20359} + 2358^{784}$

(a) 3 (b) 9

(c) 5 (d) 1

52. Let $N = (1331) \times (1335) \times (1337)$. What is the remainder when N is divided by 12?

(a) 1 (b) 3

(c) 5 (d) 7

53. Let a, b and c be distinct integers that are odd and positive. Which one of the following statements can be true?

(a) abc is even (b) ab + bc + ca is even

(c) ab + bc + a + b is odd (d) None of these

54. A number 'N' when divided by a certain divisor 'D' leaves a remainder of 43. When thrice the number 'N' is divided by 'D' the remainder is 9. Find the value of D?

(a) 60 (b) 56

(c) 110 (d) 60 or 120

55. If $A = 1^1 \times 2^2 \times 3^3 \times 4^4 \times$ --------------- $\times 100^{100}$, then now many zeroes will be there.

(a) 1300 (b) 1305

(c) 1310 (d) 1315

56. What will be the remainderwhen $25^{12} - 1$ is divided by 601.

(a) 1 (b) 0

(c) 600 (d) 559

57. 2 different numbers when divided by the same divisor leave 11 and 12 as remainders respectively and when their sum was divided by the same divisor, remainder was 4. What is the divisor.

(a) 36
(b) 28
(c) 14
(d) 19

58. A number 344 ab 5, is divisible by both 9 and 22. Find the number? (Given a + b < 8).

(a) 344025
(b) 344205
(c) 344275
(d) 344075

59. A seven digit number is such that both its end digits are 1 and the rest of the digits are 0 except the midelle digit also it is known that the number is diviseble by 13 what is the middle digit of the number?

(a) 3
(b) 2
(c) 7
(d) 6

60. A number is formed by writing the first 24 natural numbers consecutively. What will be the remainder if this number is divided by 9?

(a) 0
(b) 1
(c) 2
(d) 3

61. The number of values of n for which $n^2 + n + 1$ is divisible by 35 is

(a) 1
(b) 2
(c) Infinite
(d) None of these

62. A gardener plants suplings in such a way that every row had as many suplings as every column, of is al there were 729 trees now many suplings were there is each row?

(a) 13
(b) 27
(c) 17
(d) None of these

63. In the previous problem if the decides to plant a new supling ---------------- any two suplings new many new suplings would be have to plant?

(a) 2080
(b) 2085
(c) 2081
(d) 2083

64. If $(a^2 + b^2)^2 = (a^3 + b^3)^2$ and ab $\neq 0$, then $\frac{a}{b} + \frac{b}{a}$.

(a) 0
(b) 7/3
(c) 2/3
(d) None of these

65. If a and b are grater than 0, then

(a) $\left(\frac{a^2}{b}\right)^{1/2} + \left(\frac{b^2}{a}\right)^{1/2} \geq (a)^{1/2} + (b)^{1/2}$

(b) $\left(\frac{a^2}{b}\right)^{1/2} + \left(\frac{b^2}{a}\right)^{1/2} \le (a)^{1/2} + (b)^{1/2}$

(c) $\left(\frac{a^2}{b}\right)^{1/2} + \left(\frac{b^2}{a}\right)^{1/2} = (a)^{1/2} + (b)^{1/2}$

(d) Depends on the value of a and b

66. The unit's digit of $(2^{1000} - 2) \times 17^{999}$ is

(a) 2 (b) 4

(c) 5 (d) 6

67. Find the remainder when $7^{13} + 1$ is divided by 6.

(a) 4 (b) 1

(c) 3 (d) 2

Answers (Objective Questions)

1. (c)	2. (c)	3. (a)	4. (a)	5. (c)	6. (a)	7. (a)
8. (d)	9. (b)	10. (d)	11. (c)	12. (b)	13. (d)	14. (d)
15. (b)	16. (d)	17. (b)	18. (c)	19. (c)	20. (c)	21. (c)
22. (b)	23. (b)	24. (d)	25. (d)	26. (b)	27. (b)	28. (b)
29. (b)	30. (d)	31. (b)	32. (c)	33. (c)	33. (a)	35. (a)
36. (d)	37. (b)	38. (a)	39. (d)	40. (a)	41. (d)	42. (d)
43. (b)	44. (a)	45. (d)	46. (a)	47. (c)	48. (a)	49. (c)
50. (d)	51. (a)	52. (b)	53. (d)	54. (d)	55. (a)	56. (b)
57. (d)	58. (a)	59. (b)	60. (d)	61. (d)	62. (b)	63. (a)
64. (c)	65. (a)	66. (a)	67. (d)			

Solutions:—

1.

Sol. As unit digit of 92 and 95 will be zero, so 0 will be the unit digit of this whole multiplication.

2.

Sol. 6 raised to any power have unite digit = 6

3 has cycle of 4 means $3^1 = 3$

$3^2 = 9$

$3^3 = 27 \Rightarrow$ unit digit = 7

$3^4 = 81 \Rightarrow$ unit digit = 1

$3^5 = 243$ unit digit = 3

and so on

$\Rightarrow 3^8 =$ unit digit = unit digit of $3^4 = 1$

$\Rightarrow 3^{69} = (3^4)^{17}.\ 3^1 \Rightarrow \quad 1.3 =$ unit digit = 3

Now similarly 7 has a cycle of 4

$7^{75} = (7^4)^{18}\ 7^3$ OR

(K 1) . (343) $\Rightarrow$ unit digit = 3

(Where k is the number)

Hence the unit digit

$3^{69} \times 6^{59} \times 7^{75} = 3 \times 6 \times 3 = 4$

3.

Sol. For number to be divisible by 3 sum of digits should be divisible by 3.

$\Rightarrow$ 7 + 1 + 5 + * + 3 + 2 + 4

$\Rightarrow$ 22 + * should be divisible by 3, which means * can be 2, 5 or 8. but smallest value = 2

4.

Sol. So number to be divisible by 11

[sum of the digits at odd place (starting from unit digit)- sum of the digit at even places should be a multiple of 11]

i.e. (6 + 5 + 9 + 2) – (7 + 1 + k) should be multiple of 11

$\Rightarrow$ 22 – 8 – k should be multiple of 11

$\Rightarrow$ 14 – k should be multiple of 11

i.e. k = 3

5.

Sol. Theorem If the number N is divisible by both x and y where x and y are co-primes, the number N will always be divisible by $x \times y$.

So here in this question for the number 2 4573 k to be divisible by 72 it should be divisible by both 8 and 9 (as you can see 8 and 9 are co-prime means HCF of 8 and 9 = 1)

To be divisible by 8 last 3 digits must be divisible by 8 *i.e.* 73k should be divisible by 8 $\Rightarrow$ k = 6

Now sum of the digits by putting k = 6

$\Rightarrow$ 2 + 4 + 5 + 7 + 3 + 6 = 27 is divisible by 9 so no. 245736 is divisible by 9.

Therefore k = 6

6.

Sol. For the number to be divisible by 91, the number should be divisible by both 13 and 7. check one by one.

7.

Sol. $\therefore$ $(x - a)$ divides $x^n - a^n$ for all values of n.

8.

Sol. Let the number be N

$\Rightarrow$ N = 56 × Q + 27 (Given that 27 is remainder)

Now $\dfrac{N}{14} = \dfrac{56Q + 27}{14}$, Remainder = 13

As 56 Q is divisible by 14

9.

Sol. Let the number = N

$\Rightarrow$ N = 5 × Q + 3 ($\because$ Given remainder = 3)

Now $N^3 = (5Q + 3)^3 = 25Q^3 + 27 + 3 \times 3 \times 5Q\,(3 + 5Q)$

$\because$ $(a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$

$$\Rightarrow \frac{N^3}{5} = \frac{25Q^3 + 45Q(3+5Q)+27}{5}$$

$\Rightarrow$ Remainder = remainder of $\frac{27}{5} = 2$

$\because$ $25Q^3 + 45Q\,(3 + 5Q)$ is divisible by 5

10.

Solution: Assume $2^{16} = x$

Now $2^{16} + 1 = (x + 1)$

Option (b) $= 2^{32} - 1 \Rightarrow (2^{16})^2 - 1 = x^2 - 1 = (x - 1)(x + 1)$

Option (c) $= 2^{48} + 1 = (2^{16})^3 + 1 \Rightarrow (2^{16})^3 + (1)^3 \Rightarrow (2^{16} + 1)((2^{16})^2 + (1)^2 + 2^{16}.1)$

$\because$ $a^3 + b^3 = (a + b)(a^2 + b^2 + ab) = (x + 1)(x^2 + 1 + x)$

$\Rightarrow$ Both options (b) and (c) are divisible by $(x + 1)$ or $2^{16} + 1$

11.

Sol. Here in this question there are three divisions. We know that in successive division.

$(\text{Dividend})_3 = Q_3 \times (\text{Divisor})_3 + R_3$

$\Rightarrow$ Assume $Q_3 = k$

$\Rightarrow (DIV)_3 = k \times 6 + 5 = 6k + 5$

Now this $(DIV)_3$ will become Q_2

$(DIV)_2 = Q_2 \times D_2 + R_2$

$\Rightarrow (DIV)_2 = (6k + 5) \times 5 + 3 \Rightarrow 30k + 28$

Now this $(DIV)_2$ will become Q_1

$\Rightarrow (DIV)_1 = Q_1 \times D_1 + R_1$

$(DIV)_1$ = Number = $(30k + 28) \times 4 + 2$

N $\Rightarrow$ 120k + 114

Minimum number (put k = 1) = 120 + 114 $\Rightarrow$ 234

12.

Sol. $(2abc)^4$ must be at least $(2000)^4$ and less than $(3000)^4$

Now both $(2000)^4$ and $(3000)^4$ has 14 digits.

$\therefore$ $(2abc)^4$ has 14 digits

13.

Sol. According to the formula that we explained in theory

$$0.4\overline{59} = \frac{459-4}{990} = \frac{455}{990} = \frac{91}{198}$$

or If $x = 0.4\overline{59} = 0.4595959..........$

$10x = 4.595959................$ I

$1000x = 459.595959............$ II

Now II – I

$\Rightarrow 990x = 459 - 4 = 455$

$$\Rightarrow x = \frac{455}{990} = \frac{91}{198}$$

14.

Sol. Let take a three digit number a b c

Now a b c can be written as 100a + 10b + c

According to question

$\Rightarrow (100a + 10b + c) - (a + b + c) = 99a + 9b$

Which is always divisible by 9 and 3

15.

Sol. $x^m + y^m$ is divisible by $(x + y)$ if m is odd.

$\Rightarrow$ $11^{105} + 14^{105}$ is divisible by 25 *i.e.* (11 + 14)

16.

Sol. $x^n - a^n$ is always divisible by both

$(x - a)$ and $(x + a)$ when n is even

$\Rightarrow 37^{2n} - 12^{2n}$ will be divisible by both (37 – 12) and (37 + 12) *i.e.* by 25 and 49

As both of these numbers i.e. 25 and 49 are co-prime

$37^{2n} - 12^{2n}$ will be divided by 25×49 by *i.e.* 1225

17.

Sol. Unit digit of $(1387)^{3265}$ will be same of unit digit of $(7)^{3265}$

Now 7 has a cycle of 4 with 7^4 has unit digit 1

$\Rightarrow$ 7^{4n} will have unit digit 1

Now $(7)^{3265} = (7^4)^{816}\, 7^1 \Rightarrow (K1).7 = 7$ (Where k is any number)

18.

Sol. No. of zero in any number is total number of 2's and 5's. Also number of 2's > 5's in any factorial greater than 5.

$\Rightarrow$ No. of zeros is the highest power of 5 in the factorial.

So total no. of 5's in $150! = \left[\frac{150}{5}\right] + \left[\frac{150}{5^2}\right] + \left[\frac{150}{5^3}\right] = 30 + 6 + 1 = 37$

Hence 150! has 37 zeros.

Where [ ] is greatest integer function.

19.

Sol. Product of n consecutive natural numbers is always divisible by n!

So product of 10 consecutive natural numbers will be divisible by 10!

As if is given that all consecutive numbers are even numbers so 2 will be common in each no.

Hence product will be divisible by $2^{10} \times 10!$

20.

Sol.

$$\frac{7^{103}}{25} = \frac{7(7^2)^{51}}{25} = \frac{7(49)^{51}}{25} \Rightarrow \frac{7(-1)^{51}}{25} = \frac{7(-1)}{25} = \frac{-7}{25} \Rightarrow 18$$

$\left(\text{as } \frac{49}{25} \text{ there mainderis } 49 \text{ or } -1\right)$

21.

Sol. When n is even, $(x^n - a^n)$ is completely divisible by $(x - a)$

$(17^{200} - 1^{200})$ is completely divisible by (17 + 1), i.e., 18.

$\Rightarrow (17^{200} - 1)$ is completely divisible by 18.

$\Rightarrow$ On dividing 17^{200} by 18, we get 1 as remainder.

22.

Sol. $5 \times b^2 + 2 \times b = 2(2 \times b^2 + 5 \times b)$

$b^2 - 8b = 0$

$b = 0, 8$

But b cannot be zero. $\Rightarrow b = 8$

23.

Sol. If by knowing the product Srinivas is able to find the two numbers then the numbers must necessarily be prime numbers.

Hence the sum of two numbers = 16 gives that there are two cases when both the numbers are prime that is (3, 13) and (5, 11.)

Hence for the given condition the answer has to be 55.

24.

Sol. $(4 + 5 + 2) - (1 + 6 + 3) = 1$, not divisible by 11.

$(2 + 6 + 4) - (4 + 5 + 2) = 1$, not divisible by 11.

$(4 + 6 + 1) - (2 + 5 + 3) = 1$, not divisible by 11.

$(4 + 6 + 1) - (2 + 5 + 4) = 0$, So 415624 divisible by 11.

25.

Sol. Let the two numbers be x and y

Then $(x - y) : (x + y) : xy = 1 : 7 : 24$

i.e, if $x - y = k$ ...(i)

Then $x + y = 7k$ ...(ii)

and $xy = 24k$ ...(iii)

$\therefore\ x = 4k,\ y = 3k$ (solving (i) & (ii))

$\therefore\ xy = 24k \Rightarrow 12k^2 = 24k$

$\Rightarrow k = 2$ (as $k \neq 0$)

$\therefore\ xy = 24k = 48.$

26.

Sol. Let the smaller number be x. Then larger number $= (x + 1365)$.

$\therefore\ x + 1365 = 6x + 15$

$5x = 1350$

$x = 270$ or Smaller number = 270

27.

Sol. $(6n^2 + 6n) = 6n(n + 1)$, which is always divisible by 6 and 12 both, since $n(n + 1)$ is always even.

28.

Sol. $(10x + y)^2 - (10y + x)^2$

$= (100x^2 + 20xy + y^2) - (100y^2 + 20xy + x^2)$

$= 100(x^2 - y^2) - 1(x^2 - y^2)$

$= 99(x^2 - y^2)$
$= 99(x + y)(x - y)$

Which is always divisible by 9. Sum of the digits and also by difference of the digits.

29.

Sol. Let the original number be 10t + u

10 + u + 1

30.

Sol. In this case since x,y and z are distinct positive integers, our aim is figure out which of the answer choices cannot be expressed as the sum of 3 integers uniquely. For eg. 6 can only be expressed as (1 + 2 + 3). 7 can only be expressed as (1 + 2 + 4). But 8 can be expressed as either (1,2,5) or (1,3,4).

31.

Sol. The last digit of the powers of 2 repeat in the order 2, 4, 8, 6, 2, 4, 8, 6.... Thus every power of 2 which is a multiple of 4 has last digit 6. The 60 power will hence have 6 as the last digit, and hence the remainder when divided by 5 is 1.

32.

Sol. All the terms in the series except 2! are divisible by 3, therefore remainder w be 2.

33.

Sol. Greatest power of 5 in 30!

$= \left[\frac{30}{5}\right] + \left[\frac{30}{5^2}\right] = 6 + 1 = 7$ where [ ] is greatest integer function.

34.

Sol. Since LCM = 102 = 2 × 3 × 17

Therefore, 17 has to be a component of at least one of the numbers. Only choice (a) fits.

35.

Sol. $21600 = 2^3 \times 3^3 \times 10^2$; we need another pair of 2 × 3 so that it becomes a perfect square. Hence, it needs to be multiplied by 2 × 3 = 6.

36.

Sol. N = 238a + 79
= 17 × 14a + 17 × 4 + 11

Hence, on dividing N by 17, the remainder is 11.

Note: This method would have failed had 17 not been a factor of 238.

37.

Sol. $17^{23} = (16 + 1)^{23}$

$^{23}C_0 \times (16)^{23} \times (1)^0 + {}^{23}C_1(16)^{22} \times (1)^1 + ... + {}^{23}C_{23}(1)^{23} \times (16)^0$

All the terms except the last one are multiples of 16 and last term = 1

Hence, remainder when 17^{23} is divided by 16 = 1

38.

Sol. If $2^{32} + 1 = a + b$

Since $a^3 + b^3 = (a + b)^3$
$= (a + b)^3 - 3ab\ (a + b)$
$= (a + b)\ [(a + b)^2 - 3ab]$

So $(a^3 + b^3)$ is divisible by $(a + b)$.

39.

Sol. Two possible years of his birth are 1948 and 1898.

Note: The century in which he was born is not specified!

So, his age cannot be uniquely determined.

Alternative method:

Since the century of birth of Ram is not given and we have the properties of only 2 digits of the year of birth, we will have more than one solution which are given by the equation $x + x = 100(n) + 96$, where

n = 0, 1, 2, ... give different values of x.

For n = 0, $x = 48$, year = 1948

For n = 1, $x = 98$, year = 1898

Further n gives irrelevant years as we get a 3 digit x.

40.

Sol. The number would be $(5x + 3)$ where x would be any integer which comes to be the quotient when the number is divided by 5.

$$\frac{(5x+3)^2}{5} = \frac{25x^2+30x+9}{5}$$

$$\frac{25x^2+30x+5+4}{5} = 5x^2+6x+1+\frac{4}{5}$$

Hence, remainder = 4

41.

Sol. aaaaaa = aaa × 1000 + aaa = aaa (1000 + 1) = 1001 (aaa)= 7 × 11 × 13 (aaa)

This is obviously divisible by 7, 11 and 13.

Note: Any 6,12,18.... digit number contaning same digits is always divisible by 1001. also 1001 = 7 × 11 × 13

42.

Sol. Let us assume the middle number to be x.

Then the three consecutive numbers are

$(x - 1)$, x and $(x + 1)$

$x^2 = (x - 1)(x + 1) + 1$

$x^2 = x^2 - 1 + 1$ have become equal.

Hence, this condition will hold for very number.

43.

Sol. We know that $2 \times 5 = 10$ (one zero)

The given product can be written as

$33 \times (5^2 \times 7) \times (2^2 \times 5 \times 9) \times (2^2 \times 3) \times (2^2 \times 11) \times (2^4 \times 5) \times (2 \times 11 \times 3)$

The powers of 2 and 5 are $2^{11} \times 5^4$

Only when one 5 is multiplied with one 2 a zero will be produced.

So, there are 4 zeros.

44.

Sol. $U_0 = 2^0 - 1 = 0$

$U_1 = 2^1 - 1 = 1$

$U_2 = 2^2 - 1 = 3$

$U_3 = 2^3 - 1 = 7$ and so on.

$\therefore \ U_{10} - 2^{10} - 1 = 1023.$

45.

Sol. To convert from base 6 to base 10, multiply each digit in base 6 number.

$(1234)_6$

$= (4 \times 6^0 + 3 \times 6^1 + 2 \times 6^2 + 1 \times 6^3)_{10}$

$= (4 + 18 + 72 + 216)_{10}$

$= (310)_{10}$

46.

Sol. $N = (29 \times 5 + 3) \times (29 \times 10 + 3) \times (29 \times 20 + 1) \times (29 \times 30 + 4)$ in expansion of this only term, i.e. $3 \times 3 \times 1 \times 4 = 36$ is not divisible by 29 and will leave a remainder of 7

47.

Sol. The total multiples of 31 in

$1000! = \frac{1000}{31} = 32$

(31,62, 93, ..., 992)

The total number of multiples of $31^2 = 961$

In $(1000)! = \frac{1000}{961} = 1$

$\therefore$ Total power of 31 in $1000! = 32 + 1 = 33$

48.

Sol. $56^{56} + 56$ can be written as

$(57-1)^{56} + 56$

All the terms except $(-1)^{56}$ of $(57-1)^{56}$ will be divisible by 57, so remainder will be 1.

Once 56 is added to $(57-1)^{56}$ the sum will be divisible by 57. So remainder will be 0.

49.

Sol. $x^n - 1$ is always divisible by $x - 1$

$9^6 + 1 = 9^6 - 1 + 2$

$9^6 - 1$ will always be divisible by $(9 - 1)$ i.e. by 8

$\therefore$ When $(9^6 - 1) + 2$ is divided by 8 remainder will be 2.

50.

Sol. We just need to check the last digit of 7^{194321}

194321 is of the form $4k + 1$, where k is a natural number, so the last digit will be $7^1 = 7$

51.

Sol. Units digit of 34563^{20359} = units digit of 3^{20359}

20359 leaves a remainder of 3 when divided by 4

$\therefore$ Units digit of 3^{20359} = Units digit of 3^3 i.e. 7

Similarly, units digit of 2358^{784} is 6

Hence the unit digit = 6 + 7 = 13 or 3 is the unit digit of sum.

52.

Sol. Remainder

Now $\frac{1333}{12}$ = Remainder = 1 $\Rightarrow$ for $\frac{1335}{12}$, Remainder = 3 and for $\frac{1337}{12}$, Remainder = 5

$= 1 \times 3 \times 5 = 15$

Now 15 > 12 So final remainder = $\frac{15}{12} = 3$

53.

Sol. abc is always odd while ab + bc + ca is odd and ab + bc + a + b is even because all the four terms are odd

54.

Sol. Since 'N' leaves a remainder of 43, 3N must leave a remainder of $3 \times 43 = 129$. Since it is given that 3N leaves a remainder 9. the maximum possible value of D is 129 – 9 ie. 120. In fact the factors of 120 greater than 43 could also be the values of D i.e. in this case 60 is also a possible value of D.

Hence D can be either 120 or 60

55.

Sol.5 + 10 + 15 + 20 + 25 × 2 + 30 + 33 --------- 100 × 2 = 1300

56.

Sol. $25^{12} - 1$

$= (25^6)^2 - 1$

$= (25^6 - 1)(25^6 + 1)$

$= (25^3) - 1 (25^3 + 1)(25^6 + 1)$

$= (25^3 - 1)(25 + 1)(25^2 - 25 + 1)(25^6 + 1)$

$= (25^3 - 1)(25)(601)(25^6 + 1)$

57.

sol. x = km + 11, y = lm + 12

$= \frac{x+y}{m} = \frac{(k+l)m}{m} + \frac{23}{m}$

$= k + l + \frac{23}{m}$

= m = 19

58.

Sol. b should be either 2 or 7

3 + 4 + 4 + a + b + 5 should be diviseble by 9.

16 + a + b

a + b < 8 ∴ b = 2 a = 0

59

Sol. 100x 001

N 00, $\underset{(1)}{001}$ $\underset{(2)}{00x}$ $\underset{(3)}{001}$

001 + 001 = 002

∴ x = 2

60.

Sol. The no is 1 2 3 ------ 24.

= (1 + 2 + 3 ---- 9) + 1 × 10 + 2 × 5 + (1 + 2 + 3 + 4)

$= \frac{9 \times 10}{2} \times 2 + 10 + 10 + \frac{4 \times 5}{2}$

= 90 + 10 + 10 + 10 = 120 =3

61.

sol. 35 = 5 × 7

n^2 can have last digit as 0, 1, 4, 9, 6, 5.

and $n^2 + n$ can have last digit as 0, 2, 6

Therefore $n^2 + n + 1$ has its last digit as 1, 3, 7

62.

sol. $n^2 = 729$

$n = 27$

63.

sol. $2n(n - 1) + (n - 1)^2$

$= 2 \times 27 \times 26 + 26^2$

$= 2080$

64.

sol. $(a^2 + b^2)^3 = (a^3 + b^2)^2$

$= \cancel{a^6} + \cancel{b^6} + 3a^2b^2(a^2 + b^2) = \cancel{a^6} + \cancel{b^6} + 2a^3b^3.$

$= \dfrac{a^2b^2 + 3a^2b^2(a^2 + b^2)}{a^3b^3} = \dfrac{2}{3}$

$= \dfrac{a^2 + b^2}{ab} = \dfrac{2}{3}$

65.

Sol. $a^{1/2} = x \quad b^{1/2} = y$

L.H.S $= \dfrac{x^2}{y} + \dfrac{y^2}{x} = \dfrac{x^3 + y^3}{xy}$

R.H.S $= x + y$

L.H.S – R.H.S

$= \dfrac{x^3 + y^3}{xy} - (x + y)$

$= \dfrac{x^3 + y^3 - x^2y - y^2x}{xy}$

$= \dfrac{(x - y)^2(x + y)}{xy} > 0$

66.

Sol. $(2^{1000} - 2) \times 17^{999}$.

= Unit digit of $2^{1000} - 2$ is 4

= Unit digit of 17^{999} is 3.

$\therefore$ unit digit is 2

67.

Sol. $\frac{7^{13}+1}{6}$

$= \frac{1+1}{6}$

$= \frac{2}{6}$

CHAPTER

2

HCF & LCM

HIGHEST COMMON FACTOR (HCF)

The largest number that divides two or more given numbers is called the highest common factor (HCF) of those numbers. There are two methods to find HCF of the given numbers:

Prime Factorization Method: When a number is written as the product of prime numbers, the factorization is called the prime factorization of that number. For example,

$$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

To find the HCF of given numbers by this method, we perform the prime factorization of all the numbers and then check for the common prime factors. For every prime factor common to all the numbers, we choose the least index (power) of that prime factor among the given number. The HCF is product of all such prime factors with their respective least indices.

Example 1.

Find the HCF of 72, 288, and 1080

Sol.

$$72 = 2^3 \times 3^2$$
$$288 = 2^5 \times 3^2$$
$$1080 = 2^3 \times 3^3 \times 5$$

The prime factors common to all the numbers are 2 and 3. The lowest indices of 2 and 3 in the given numbers are 3 and 2 respectively.

Hence, $\text{HCF} = 2^3 \times 3^2 = 72$

Example 2.

Find the HCF of $36x^3y^2$ and $24x^4y$.

Sol.

$$36x^3y^2 = 2^2 \times 3^2 \times x^3 \times y^2$$
$$24x^4y = 2^3 \times 3 \times x^4 \times y$$

The least index of 2, 3, x and y in the numbers are 2, 1, 3 and 1 respectively.

Hence the $HCF = 2^2 \times 3 \times x^3 \times y = 12x^3y$

Division method: To find HCF of two numbers by division method, we divide the higher number by the lower number. Then we divide the lower number by the first remainder, the first remainder by the second remainder... and so on, till the remainder is 0. The last divisor is the required HCF.

Example 3.

Find the HCF of 288 and 1080 by the division method.

Sol.

```
288) 1080 (3
     864
     216) 288 (1
          216
           72) 216 (3
               216
                 0
```

Hence, the last divisor 72 is the HCF of 288 and 1080.

Concept of Co-Prime Numbers

Two numbers are co-prime to each other if they have no common factor except 1. For example, 15 and 32, 16 and 5, 8 and 27 are the pairs of co-prime numbers. if the HCF of two numbers N_1 and N_2 be H, then, the numbers left after dividing N_1 and N_2 by H are co-prime to each other.

Therefore, if the HCF of two numbers be A, the numbers can be written as Ax and Ay, where x and y will be co-prime to each other.

Example 4.

Three company of soldiers containing 120, 192, and 144 soldiers are to be broken down into smaller groups such that each group contains soldiers from one company only and all the groups have equal number of soldiers. What is the least number of total groups formed?

Sol.

The least number of groups will be formed when each group has number of soldiers equal to the HCF. The HCF of 120, 192 and 144 is 24. Therefore, the numbers of groups formed for the three companies will be 5, 8 and 6 respectively. Therefore, the least number of total groups formed = 5 + 8 + 6 = 19

Example 5.

The number 2604, 1020 and 4812 when divided by a number N give the same remainder of 12. Find the highest such number N.

Sol. Since all the numbers give a remainder of 12 when divided by N, hence (2604 –12), (1020–12) and (4812 –12) are all divisible by N. Hence, N is the HCF of 2592, 1008 and 4800.

Now $2592 = 2^5 \times 3^4$, $1008 = 2^4 \times 3^2 \times 7$ and $4800 = 2^6 \times 3 \times 5^2$.

Hence, the number $N = HCF = 2^4 \times 3 = 48$.

Example 6.

The numbers 400, 536 and 645, when divided by a number N, give the remainders of 22, 23 and 24 respectively. Find the greatest such number N.

Sol. N will be the HCF of (400 – 22), (536 – 23) and (645 – 24).

Hence, N will be the HCF of 378, 513 and 621,... N = 27.

Example 7.

The HCF of two numbers is 12 and their sum is 288. How many pairs of such numbers are possible?

Sol.

If the HCF is 12, the numbers can be written as 12x and 12y, where x and y are co-prime to each other.

Therefore, $12x + 12y = 288$

$$x + y = 24.$$

The pair of numbers that are co-prime to each other and sum up to 24 are (1, 23), (5, 19), (7, 17) and (11, 13).

Hence, only four pairs of such numbers are possible. The numbers are (12, 276), (60, 228), (84, 204) and (132, 156).

Example 8.

The HCF of two numbers is 12 and their product is 31104. How many such numbers are possible?

Sol.

Let the numbers be 12x and 12y, where x and y are co-prime to each other.

Therefore, $12x \times 12y = 31104$

$$xy = 216.$$

Now we need to find co-prime pairs whose product is 216.

$$216 = 2^3 \times 3^3.$$

Therefore, the co-prime pairs will be (1, 216) and (8, 27). Therefore, only two such numbers are possible.

Least Common Multiple (LCM)

The least common multiple (LCM) of two or more number which is divisible by all the given numbers.

To calculate the LCM of two or more numbers, we use the following two methods:

Prime Factorization method

After performing the prime factorization of the numbers, i.e. breaking the numbers into product of prime numbers, we find the highest index, among the given numbers, of all the prime numbers. The LCM is the product of all these prime numbers with their respective highest indices.

Example 9.

Find the LCM of 72, 288 and 1080.

Sol.

$72 = 2^3 \times 3^2$,

$288 = 2^5 \times 3^2$,

$1080 = 2^3 \times 3^3 \times 5$

The prime numbers present are 2, 3 and 5. The highest indices (powers) of 2, 3 and 5 are 5, 3 and 1, respectively.

Hence the LCM = $2^5 \times 3^3 \times 5 = 4320$.

Example 10.

Find the LCM of $36x^3y^2$ and $24x^4y$.

Sol.

$36x^3y^2 = 2^2 \times 3^2 \times x^3 \times y^2$, $24x^4y = 2^3 \times 3 \times x^4 \times y$

The highest indices of 2, 3, x and y are 3, 2, 4 and 2 respectively.

Hence, the LCM = $2^3 \times 3^2 \times x^4 \times y^2 = 72x^4y^2$

Division Method

To find the LCM of 72, 196 and 240, we use the division method in the following way:

2	72, 240, 196
2	36, 120, 98
2	18, 60, 49
3	9, 30, 49
	3, 10, 49

L.C.M. of the given numbers = product of divisors and the remaining numbers

$= 2 \times 2 \times 2 \times 3 \times 3 \times 10 \times 49 = 72 \times 10 \times 49 = 35280$

Remember

For Two numbers, HCF × LCM = product of the two numbers

For example, the HCF of 288 and 1020 is 72 and the LCM of these two numbers is 4320. We can see that $72 \times 4320 = 288 \times 1080 = 311040$.

Note

This formula is applicable only for two numbers.

Properties of HCF and LCM

- The HCF of two or more numbers is smaller than or equal to the smallest of those numbers.
- The LCM of two or more numbers is greater than or equal to the largest of those numbers
- If the HCF of numbers N_1, N_2, N_3,... is H, then N_1, N_2, N_3 ... can be written as multiples of H (Hx, Hy, Hz...). Since the HCF divides all the numbers, every number will be a multiple of the HCF.
- If the HCF of two numbers N_1 and N_2 is H, then, the numbers $(N_1 + N_2)$ and $(N_1 - N_2)$ are also divisible by H. Let $N_1 = Hx$ and $N_2 = Hy$, since the numbers will be multiples of H. Then, $N_1 + N_2 = Hx + Hy = H(x + y)$, and $N_1 - N_2 = Hx - Hy = H(x - y)$. Hence both the sum and differences of the two numbers are divisible by the HCF.
- If numbers N_1, N_2, N_3, N_4 etc. give an equal remainder when divided by the same number P, then P is a factor of $(N_1 - N_2)$, $(N_2 - N_3)$, $(N_3 - N_4)$...
- If L is the LCM of N_1, N_2, N_3, N_4... all the multiples of L are divisible by these numbers.
- If a number P always leaves a remainder R when divided by the numbers N_1, N_2, N_3, N_4 etc., then P = LCM (or a multiple of LCM) of N_1, N_2, N_3,N_4... + R.

Example 11.

Find the highest four-digit number that is divisible by each of the numbers 24, 36, 45 and 60.

Sol.

$24 = 2^3 \times 3,$

$36 = 2^2 \times 3^2,$

$45 = 3^2 \times 5$

and $60 = 2^2 \times 3 \times 5$

Hence, the LCM of 24, 36, 45 and 60 $= 2^3 \times 3^2 \times 5 = 360$.

The highest four-digit number is 9999. 9999 when divided by 360 gives the remainder 279. Hence, the number (9999 – 279 = 9720) will be divisible by 360.

Hence the highest four-digit number divisible by 24, 36, 45 and 60 = 9720.

Example 12.

Find the highest number less than 1800 that is divisible by each of the numbers 2, 3, 4, 5, 6 and 7.

Sol.

The LCM of 2, 3, 4, 5, 6 and 7 is 420, and every multiple of 420, is divisible by each of these numbers. Hence, the number 420, 840, 1260, and 1680 are all divisible by each of these numbers. We can see that 1680 is the highest number less than 1800 which is multiple of 420.

Hence, the highest number divisible by each one of 2, 3, 4, 5, 6 and 7, and less than 1800 is 1680.

Example 13.

Find the lowest number which gives a remainder of 5 when divided by any of the numbers 6, 7, and 8.

Sol.

The LCM of 6, 7 and 8 is 168. Hence, 168 is divisible by 6, 7 and 8. Therefore, 168 + 5 = 173 will give a remainder of 5 when divided by these numbers.

Example 14.

What is the smallest number which when divided by 9, 18, 24 leaves a remainder of 5, 14 and 20 respectively?

Sol.

The common difference between the divisor and the remainder is 4, (9 – 5 = 4, 18 – 14 = 4, 24 – 20 = 4). Now the LCM of 9, 18 and 24 is 72.

Now 72 – 4 = 72 – 9 + 5 = 72 –18 + 14 = 72 – 24 + 20. Therefore, if we subtract 4 from 72, the resulting number will give remainders of 5, 14, and 20 with 9, 18 and 24.

Hence, the number = 72 – 4 = 68

Example 15.

A number when divided by 3, 4, 5 and 6 always leaves a remainder of 2, but leaves no remainder when divided by 7. What is the lowest such number possible?

Sol.

The LCM of 3, 4, 5 and 6 is 60. Therefore, the number is of the form 60k + 2, i.e. 62, 122, 182, 242 etc. We can see that 182 is divisible by 7. Therefore, the lowest such number possible = 182.

Example : 16

There is a number which when divided by 4, 5 and 6 always leaves the same remainder 3. Find a number such that it is largest < 1000.

Sol. L.C.M of (4, 5, 6) = 60

$$\begin{array}{r|l} & 16 \\ 60 & 1000 \\ & \underline{60} \\ & 400 \\ & \underline{360} \\ & 40 \end{array}$$

1000 – 40 = 960

Number is 960 +3 = 963

FACTORISATION

It is the process of splitting any number into the form where it is expressed only in terms of the most basic prime factors.

For example, $12 = 2^2 \times 3^1$. 12 is expressed in the factorised form in terms of its basic prime factors. This is the factorised form of 12. It is possible to find the number of factors of a composite number without listing all those factors, from its factorised form. Take 12 for instance, it can be expressed as $12 = 2^2 \times 3^1$.

The factors of 12 are:

$(2^0 \times 3^1),(2^0 \times 3^0),(2^1 \times 3^1),(2^1 \times 3^1),(2^1 \times 3^0),(2^2 \times 3^0)$

Here the powers of 2 can be one of (0, 1, 2) and the powers of 3 can be one of (0, 1). So the total possibilities if you take the two as combination is $3 \times 2 = 6$. Each combination of the powers of 2 and 3 gives a distinctly different factor. Hence, since there are 6 different combinations of the powers of 2 and 3 there are 6 distinctly different factors of 12.

Let N be a comosite number such that $N = (x)^a (y)^b (z)^c$ .. where x, y, z.. are prime factors. Then, the number of divisors (factors) of N = (a +1) (b + 1) (c + 1)...

Here factors and divisors means the same

Example 14.

Find the total number of factors of 576.

Sol.

The factorised form of 576 is $(24 \times 24) = (2^3 \times 3)(2^3 \times 3) = (2^6 \times 3^2)$.

So the total number of factors is (6 + 1) (2 +1) =21.

Example 15.

Find the number of divisors of 21600.

Sol. $21600 = 2^5 \times 3^3 \times 5^2$

Number of divisors = $(5 +1) \times (3 + 1) \times (2 + 1) = 6 \times 4 \times 3 = 72$.

Example 16.

How many divisors of 21600 are odd numbers?

Sol. An odd number does not have a factor of 2 in it. Therefore, we will consider all the divisors having powers of 3 and 5 but not 2. Therefore, ignoring the powers of 2, the number of odd divisors

$= (3 + 1) \times (2 + 1) = 4 \times 3 = 12$.

Example 17.

How many divisors of 21600 are even numbers?

Sol. Total number of divisors of 21600 = 72.

Number of odd divisors of 21600 = 12.

Number of even divisors of 21600 = 72 – 12 = 60.

Example 18.

If N= $2^3 \times 3^7$, then

(a) What is the smallest number that you need to multiply with in order to make it a perfect square?

(b) What is the smallest number that you need to divide by in order to make it a perfect square?

Sol.

(a) Any perfect square shall have in its factorised form, even powers for the prime numbers. So in order to make $2^3 \times 3^7$ a perfect square, the smallest number that we need to multiply it with would be 2×3. The resulting perfect square would be $2^4 \times 3^8$, i.e., the square root $2^2 \times 3^4$.

(b) Similarly, in order to arrive at a perfect square by dividing the number, we need to do so by dividing the number by 2×3. The resulting perfect square would be $2^2 \times 3^6$.

Example 19.

How many divisors of 21600 are prefect squares?

Sol. In a perfect square, all the prime factors have even powers. For example, $2^5 \times 6^8$ will not be a perfect square as the power of 2 is odd whereas $2^4 \times 6^8$ will be a perfect square because all the prime factors have even powers. $21600 = 2^5 \times 3^3 \times 5^2$ therefore, all the divisors made by even powers of 2, 3 and 5 will be perfect squares.

The even powers of 2 are 2^0, 2^2, 2^4 even powers of 3 are 3^0 and 3^2 and even powers of 5 are 5^0 and 5^2. We can select and even power of 2 in 3 ways, even power of 3 in 2 ways, and even power of 5 in 2 ways. Therefore, the number of combinations = $3 \times 2 \times 2 = 12$.

Let N be a composite number such that N = $(x)^a(y)^b(z)^c$... where x, y, z... are prime factors. Then, the sum of divisors of N = $\frac{x^{a+1}-1}{x-1} \times \frac{y^{b+1}-1}{y-1} \times \frac{z^{c+1}-1}{z-1} \ldots$

Example 20.

What is the sum of divisors of 60?

Sol.

$60 = 2^2 \times 3 \times 5$

Sum of the divisors = $\frac{2^3-1}{2-1} \times \frac{3^2-1}{3-1} \times \frac{5^2-1}{5-1} = 168$

Objective Questions

Exercise — (I)

1. The product of two numbers is 2700 and their H.C.F is 15. Find all the possible pairs of such numbers.

   (a) 2 (b) 4
   (c) 3 (d) 6

2. The product of three numbers is 1620. If the H.C.F of any two out of three number is 3, what is their L.C.M

   (a) 180 (b) 240
   (c) 360 (d) 480

3. V is a factor of 720. V it self has exactly 3 factors how many values of V are possible?

   (a) 1 (b) 2
   (c) 6 (d) 3

4. What is the least number which when divided by 6, 7, and 9 leaves remainder 4 in each care but is eactly divisible by 11.

   (a) 2101 (b) 1012
   (c) 1210 (d) None of these

5. Three cakes weighing $4\frac{1}{2}lbs$, $6\frac{3}{4}lbs$ and $7\frac{1}{5}lbs$ has to be divided into parts of equal weights. Further, each part must be as heavy as possible. If one such part is served to each guest, then what is the maximum number of guests the could be entertained

   (a) 54 (b) 72
   (c) 20 (d) None of these

6. If is a possitive integer, then now many values of n willgive an integral value of $\frac{16n^2+7n+6}{n}$?

   (a) 2 (b) 3
   (c) 4 (d) None of these

7. How many divisors of 360 are not divisors of 540 and how many divisors of 540 are not divisors of 360.

   (a) 6 (b) 8
   (c) 10 (d) 7

8. How many divisors of 36^{36} are perfect cuves?

(a) 625 (b) 576

(c) 676 (d) 484

9. Find the smallest number with 15 divisor?

(a) 144 (b) 136

(c) 120 (d) 180

Exercise — II

10. Determine the L.C.M of 2/5, 3/10 and 6/25.

(a) 6/5 (b) 11/5

(c) 9/5 (d) None of these

11. Determine the H.C.F of of 4/9, 10/21, and 20/63

(a) 4/189 (b) 6/63

(c) 2/63 (d) None of these

12. The H.C.F of two number is 12 and their difference is 12. The number are.

(a) 66,78 (b) 70, 82

(c) 94, 104 (d) 84, 96

13. Merchent has three different kinds of milk, 435 lit, 493 lit and 551 litres. Find the least number of casks of equal size required to store all the milk with out mixing

(a) 51 (b) 61

(c) 47 (d) 45

14. How often will five bells toll together in one hour if they start tgogether and toll at intervals of 5, 6, 8, 12, 20 seconds respectively?

(a) 29 (b) 30

(c) 31 (d) 120

15. The traffic lights at three different road crossings changes after even 48 sec, 72 sec and 108 sec, respectively. If they all change simult anuously at 8:20:00 hrs; then they will again change simultaneously at;

(a) 8:27:12 hrs (b) 8:27:24 hrs

(c) 8:27:36 hrs (d) 8:27:45 hrs

16. The number of prime factors in the expession $6^{10} \times 7^{17} \times 11^{27}$ is;

(a) 4 (b) 64

(c) 71 (d) 81.

17. A wholesaler tea dealer has 408 kg, 468 kg and 516 kg of three different quantities of tea, he want it all to be packed into boxes of equal size without mixing. Find the capacity of the largest possible box.

(a) 50
(b) 36
(c) 24
(d) 12

18. A room is 4 metres 37 cm long and 3m also 23 cm broad. It is required to pane the floor with minimum square slabs. Find the number of slabs required for this purpose?

(a) 485
(b) 431
(c) 391
(d) 381

19. Three men start together to evance the same around a circular track of 11 kilometer in circumference. Their speeds are 4, $5\frac{1}{2}$ and 8 kilometer per hour, respectively. When will they meet at the starting point

(a) 11 hrs
(b) 12 hrs
(c) 23 hrs
(d) 22 hrs

20. The L.C.M and G.C.D of two numbers are 1530 and 51, respectively. Find how many such pairs are possible.

(a) 2
(b) 3
(c) 4
(d) only one

21. Three different containers contain different quantities of mixture of milk and water, whose measurements are 403 kg, 434 kg and 465 kg. What biggest measure must be there to measure all the different quantities exactly.

(a) 1 kg
(b) 7 kg
(c) 31 kg
(d) 41 kg

22. A red light flashes 3 times per minute and a green light flashes 5 times in two minutes at regular intervals. If both lights start flashing at the same time, how many times do they flash together in each hour.

(a) 30
(b) 24
(c) 20
(d) 60

23. The LCM of two numbers is 72 and their HCF is 12. If one of the numbers is 24, what is the other number?

(a) 38
(b) 26
(c) 36
(d) 42

24. The largest natural number which exactly divides the product of any four cansecutine natural numbers is

(a) 6
(b) 12
(c) 24
(d) 120

25. What is the smallest number which when increased by 3 is divisible by 16, 24, 30 and 32

(a) 480
(b) 475
(c) 472
(d) 477

26. Find the least number when divided by 2, 3, 4, 5 and 6 leaves 1, 2, 3, 4 and 5 as remainders, respectively but when divided by 7 leaves no remainder

(a) 210
(b) 119
(c) 126
(d) 154

27. Find the least number of five digits which when divided by 8, 12, 16 and 20 leaves remainders 1, 5, 9 and 13 respectively.

(a) 10003
(b) 10093
(c) 10073
(d) 10073

28. The H.C.F of two number is 11 and their L.C.M is 693. If one of the number is 77, find the other

(a) 909
(b) 119
(c) 66
(d) 99

29. A heap of stones can be made up into groups of 21. Which made up into groups of 16, 20, 25 and 45, there are 3 stones left in each case how many stones at least can there be in the heap?

(a) 7203
(b) 2403
(c) 3603
(d) 4803

30. If LCM (x, y, z) = (x) (y) (z). Find their H.C.F

(a) 1
(b) 2
(c) *x*
(d) y

31. A number has an odd number of factors. Is it a perfect square?

(a) Yes
(b) No
(c) can't say
(d) 000

32. The sum of two numbers is 528 and their HCF is 33. The number of pairs of numbers satisfying the above conditions is:

(a) 6
(b) 4
(c) 9
(d) 12

33. The product of two numbers is 4107. If the HCF of these numbers is 37, then the greater number is:

(a) 107
(b) 101
(c) 111
(d) 185

34. The greatest number that exactly dividers 105, 1001 and 2436 is:

(a) 3 (b) 7

(c) 11 (d) 21

35. The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively is:

(a) 123 (b) 127

(c) 305 (d) 127

36. The least number which when divided by 48, 60, 72, 108 and 140 leaves 38, 50, 62, 98 and 130 as remainders respectively is:

(a) 15120 (b) 15130

(c) 15110 (d) 15115

37. How many natural numbers less than 70 have odd number of factors? (Including 1 and these numbers themselves)

(a) 8 (b) 15

(c) 12 (d) 21

38. H.C.F. of 3240, 3600 and a third number is 36 and their L.C.M is $2^4 \times 3^5 \times 5^2 \times 7^2$. The third number is:

(a) $2^2 \times 3^5 \times 7^2$ (b) $2^2 \times 5^3 \times 7^2$

(c) $2^5 \times 5^3 \times 7^2$ (c) $2^3 \times 3^5 \times 7^2$

39. The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is:

(a) 1 (b) 2

(c) 3 (d) 4

40. The smallst number which when divided by 4, 6 or 7 leaves a remainder of 2, is

(a) 44 (b) 62

(c) 80 (d) 86

41. Let the least number of six digits, which when divided by 4, 6, 10 and 15, leaves in each case the same remainder of 2, be N. The sum of the digits in N is :

(a) 3 (b) 4

(c) 5 (d) 6

42. What is the smallest number which when increased by 5 is completely divisible by 8, 11 and 24?

(a) 264 (b) 259

(c) 269 (d) None of these

43. A number which when divided by 10 leaves a remainder of 9, when divided by 9 leaves a remainder of 8, by 8 leaves a remainder of 7, etc., down to where, when divided by 2, it leaves a remaider of 1, is:

(a) 59
(b) 419
(c) 1259
(d) 2519

44. What is the least perfect square divisible by 8, 9 and 10?

(a) 4000
(b) 6400
(c) 3600
(d) 14641

45. Product of two positive integers is 15210 and their HCF is 39. How many such pairs are possible?

(a) 1
(b) 2
(c) 3
(d) None of these

46. Which is the least number that must be subtracted from 1856, so that the remainder when divided by 7, 12, and 16 is 4.

(a) 137
(b) 1361
(c) 140
(d) 172

Answers (Objective Questions)

1.	(a)	**2.**	(a)	**3.**	(b)	**4.**	(b)	**5.**	(d)
6.	(c)	**7.**	(a)	**8.**	(a)	**9.**	(a)	**10.**	(a)
11.	(c)	**12.**	(d)	**13.**	(a)	**14.**	(c)	**15.**	(a)
16.	(a)	**17.**	(d)	**18.**	(c)	**19.**	(d)	**20.**	(b)
21.	(c)	**22.**	(a)	**23.**	(c)	**24.**	(c)	**25.**	(d)
26.	(b)	**27.**	(c)	**28.**	(d)	**29.**	(a)	**30.**	(a)
31.	(a)	**32.**	(b)	**33.**	(d)	**34.**	(b)	**35.**	(b)
36.	(c)	**37.**	(a)	**38.**	(a)	**39.**	(b)	**40.**	(d)
41.	(c)	**42.**	(b)	**43.**	(d)	**44.**	(c)	**45.**	(b)
46.	(d)								

Exercise — (I)

1.

Sol. Let the nos. be 15 a and 15b

$15a \times 15b = 2700$

$ab = 12$

$a = 1 \qquad b = 12$

$a = 3 \qquad b = 4$

2.

Sol. Let the nos be 3a 3b 3c

$3a \times 3b \times 3c = 1620$

L.C.M = 3 abc

$3abc = \frac{1620}{2} = 180$

3.

sol. 3 factor means perfect sequence of a prime number i.e. 2, 3, 5.

4, 9, 25

4, 8, 89 possible

4.

sol. L.C.M (6, 7, 9) $k_1 + 4 = 11k_2$

$126\ k_1 + 4 = 11k_2$

$k_1 = 8$

Hence number = **1012**

5.

Sol. H.C.F of $\left(\frac{9}{2}, \quad \frac{27}{4}, \quad \frac{36}{5} = \frac{9}{20}\right) lbs.$

maximum guest $\frac{9/2}{9/20} + \frac{27/4}{9/20} + \frac{36/5}{9/20}$

$= 10 + 15 + 16$

$= 41.$

6.

sol. $\frac{16n^2 + 7n + 6}{n}$

$= 16n + 7 + \frac{6}{n}$

6 = 1, 2, 3, 6

7.

sol. $360 = 2^3 \times 3^2 \times 5$

$540 = 2^2 \times 3^3 \times 5$

H.C.F $= 2^2 \times 3^2 \times 5$

No. of factors common $= 3 \times 3 \times 2$

$= 18$

No. of factors of $360 = 4 \times 3 \times 2 = 24$

No. of factors of $540 = 3 \times 4 \times 2 = 24$

$24 - 18 = 6$

8.

sol. $36^{36} = (2^2 \times 3^2)^{36}$

$= 2^{72}\ 3^{72}$

No. of factors perfect cubes $= 25 \times 25 = 625$

9.

Sol. The number should be of the form = a^2b^4

a = 3 b = 2

$2^4 \times 3^2 = 144$

10.

sol. $\text{L.C.M}\left(\frac{2}{5},\frac{3}{10},\frac{6}{25}\right) = \frac{L.C.M\ of\ Numeratar}{H.C.F\ of\ Denomenotar}$

$= \frac{6}{5}$

11.

Sol. $\frac{H.C.F\ of\ Numeratar}{L.C.M\ of\ Denomenotar} = \frac{2}{63}$

13.

Sol. H.C.F of (435, 493, 551) = 29

$= \frac{435}{29} = 15 \qquad \frac{493}{29} = 17 \qquad \frac{551}{29} = 19$

15 + 17 + 19 = 51

14.

sol. L.C.M of 5, 6, 8, 12, 20

2	5,6,8,12,20
2	5, 3, 4, 6, 10
3	5, 3, 2, 3, 5
5	5, 1, 2, 1, 5
	1, 1, 2, 1, 1

L.C.M = 2 × 2 × 2 × 3 × 5 = 120 sec.

$\frac{\overset{30}{\cancel{3600}}}{\cancel{120}}$ = 30 + 1 = 31 times (1 at the initial)

15.

Sol L.C.M of (48, 72, 108) sec = 432

16.

Sol. $6^{10} \times 7^{17} \times 11^{27}$

$\Rightarrow (2 \times 3)^{10} \times 7^{17} \times 11^{27}$.

$\Rightarrow 2^{10} \times 3^{10} \times 7^{17} \times 11^{27}$.

$\therefore$ The no. of prime factors = 4

17.

Sol. The capacity of the box is H.C.F of 408, 468 and 516 i.e. 12.

18.

Sol. Length = 437 cm and breadth = 323 cm

the side of the squence slab is the H.C.F of 437 and 328 i.e. 19 cm.

The number of slab, $= \dfrac{437 \times 323}{17 \times 19} = 391$

19.

Sol. L.C.M $\left(\dfrac{11}{4}, \quad \dfrac{11}{11/2}, \ \dfrac{11}{8}\right)$

L.C.M. $\left(\dfrac{11}{4}, \quad 2, \quad \dfrac{11}{8}\right)$

$= 22$

20.

sol. Let the nos be 51 a and 51 b

51 ab = 1530

abn = 30

ab = 6 × 5

= 5 × 6

= 15 × 2

= 2 × 15

= 30 × 1

= 1 × 30

21.

Sol. H.C.F of (403, 434 and 465) kg = 31 kg

22.

Sol. $\frac{60}{3}$ = 20 sec, $\frac{120}{5} = 24$ sec.

$\frac{\overset{30}{\cancel{3600}}}{\cancel{120}} = 30$

23.

Sol. $72 \times 12 = 24 \times N \Rightarrow N = 36$

25.

Sol. L.C.M of (16, 24, 30, 32) – 3

= 480 – 3 = 477

26.

Sol. 2 – 1 = 3 – 2 = 4 – 3 = 5 – 4 = 6 – 5 = 1 = k

L.C.M (2, 3, 4, 5, 6) 19 – 1 27 k_2

= 60 k_1 – 127 k_2

$k_2 = \frac{60k_1 - 1}{7}$

= 119.

27.

Sol. 8 – 1 = 12 – 5 = 16 – 9 = 20 – 13 = 7

L.C.M (8, 12, 16, 20) = 240

```
      41
240)10000
     960
     400
     240
     160
```

10000 – 160 + 240 = 10080 – 7 = 10073

28.

Sol. The nos. 11 a and 11b.

11 ab = 693

11a = 77

b = 9

other is 99.

29.

sol. L.C.M of (16, 20, 25, 45) $k_1 + 3 = 21k_2$

$3600\ k_1 + 3 = 21k_2$

30.

Sol. LCM of three number x, y, z is product the numbers when numbers are co-prime to each other or when their HCF is 1.

31.

Sol. Say if a No. is $N = x^a\ y^b\ z^c$

where x, y, z are prime factors of number.

Then total no. factors = (a + 1) (b + 1) (c + 1)

Now (a + 1) (b + 1) (c + 1) is odd only when all the three terms are odd

i.e. (a + 1), (b + 1) and (c + 1) must be all odd

or a, b, c should be even number or $N = x^a\ y^b\ z^c$ or $x^{2k}\ y^{2k}\ 2^{2m}$ or number N is a perfect square

32.

Sol. Given the two numbers have HCF 33. So assume numbers are 33a and 33b where a and b are co-prime. (why co-prime, ∵ If a and b will not co-prime, HCF will become more than 33)

Given that 33a + 33b = 528

⇒ a + b = 16

Now pairs that satisfy the condition are

a	+	b	= 16
1		15	= 16
2		14	= 16
3		13	= 16
4		12	= 16
5		11	= 16
6		10	= 16
7		9	= 16
8		8	= 16

Out of these pairs

Only 4 pairs (1,15) (3,13) (5,11) and (7,9) are 10-prime.

Hence 4 pairs (33 × 1,33 × 15), (33×3,33 × 13), (33 × 5, 33 × 11) and (33 × 7, 33 × 9) are possible.

33.

Sol. Let the number be 37a and 37b

∵ (HCF is 37) [Note that a and b are co-prime]

Given 37a × 37b = 4107

ab = 3 ⇒ a × b = 3

1 × 3 = 3

So numbers are 37a and 37b

37 × 1 and 37 × 3

37 and 111

So greater number is 111

34.

Sol. The greatest number = HCF of 105, 1001 and 2436

HCF of 2436 and 1001 is = 7

Also HCF is 105 and 7 is = 7 , Hence the HCF of 150, 1001 and 2346 is = 7

35.

Sol. Required number is HCF of (1657 – 6) and (2037 – 5) = 127

36.

Sol. Here you can see that

(48 – 38) = 10, (60 – 50) = 10, (72 – 62) = 10, (108 – 98) = 10 and (140 – 130) = 10

Now required number is = (LCM of 48, 60, 72, 108 and 140 – 10)

= 15120 – 10 = 15110

37.

Sol. Only perfect squares has odd number of factors. So number of perfect squares less than 70.

1, 4, 9, 16, 25, 36, 49, 64 = 8

38.

Sol. 3240 $= 2^3 \times 3^4 \times 5$; 3600 $= 2^4 \times 3^2 \times 5^2$;

H.C.F.= $36 = 2^2 \times 3^2$.

Since H.C.F is the product of lowest powers of common factors, so the third number must have $(2^2 \times 3^2)$ as its factor.

Since L.C.M is the product of highest powers of common prime factors, so the third number must have 3^5 and 7^2 as its factors.

$\therefore$ Third number = $2^2 \times 3^5 \times 7^2$.

39.

Sol. Let the numbers be 13a and 13b.

Then, $13a \times 13b = 2028$

$ab = 12.$

Now, co-primes with product 12 are (1, 12) and (3, 4).

So, the required numbes are (13 × 1, 13 × 12) and (13 × 3, 13 × 4).

Clearly, there are 2 such pairs.

40.

Sol. Required number = LCM (4,6,7) + 2 = 86.

41.

Sol. Least number of 6 digits is 100000. L.C.M. of 4, 6, 10 and 15 = 60.

On dividing 100000 by 60, the remainder obtained is 40.

$\therefore$ Least number of 6 divisible by 4, 6, 10 and 15 = 100000 + (60 – 40) = 100020.

$\therefore$ N = (100020 + 2) = 100022. Sum of digits in N = (1 + 2 + 2) = 5.

42.

Sol. Required number = LCM (8, 11, 24) – 5 = 259.

43. Required number is [LCM of (1,2,3,4,5,6,7,8,9,10)]-1=2520-1 = 2519

44.

Sol. The answer should be a multiple of LCM(8, 9, 10)

$= 360 = 2^2 \times 3^2 \times 2 \times 5$

In order to make it a perfect square, we need to multiply

(2 × 5) to it, i.e. 360 × 2 × 5 = 3600

45.

Sol. Since HCF is 39

$\therefore$ Let numbers be 39a and 39b (a and b are co-primes)

We have, 39a × 39b = 15210, or ab = 10

There are only two pairs of natural numbers whose product will be 10.

(1, 10) and (2, 5) are the pairs.

Hence the numbers are (39×1, 39×10) and (39×2, 39×5).

46.

Sol. The LCM of 7, 12 and 16 is 336. The closest multiple of 336 to 1856 is 1680. So 1684 when divided by 7, 12 and 16 leaves a remainder of 4. This is the closest such number to 1856. Hence the number to be subtracted from 1856 to get 1684, must be the least such number. So the answer is $(1856 - 1684) = 172$.

HINT : Students please note that in case you are not able to figure this method out, you can go with reverse substitution by subtracting each of the answer choices from 1856 starting with the least of the answer choices and going higher up and thus finding which of them fits into the given condition.

CHAPTER

3

Exponent, Surds & Logarithm

EXPONENTS

If a number 'a' is added three times to itself. then we write it as 3a. Instead of adding. if we multiply 'a' three times within itself, we write as a^3.

we say that 'a' is expressed as an exponent. Here, 'a' is called the 'base' and 3 is called the 'power' of **'index' or 'exponent'.**

Similary, 'a' can be expressed to any exponent 'n' and accordingly wirtten as a^n. This is read as "a to the power n" or "a raised to the power n."

$a^n = a \times a \times a \times a \times$ .... n times

For example:

$2^3 = 2 \times 2 \times 2 = 8$ and $3^4 = 3 \times 3 \times 3 \times 3 = 81$

If we have powers in the manner of "steps", then such a number is evaluated by starting at the topmost of the "steps" and coming down one "step" in each operation.

For example, 2^{4^3} is evaluated by starting at the topmost level '3'. thus we first calculate 4^3, as equal to 64. Since 2 is raised to the power 4^3. we now have 2^{64}.

Similary, 2^{3^2} is equal to "2 raised to the power 3^2" or "2 raised to the power 9" or 2^9 which is equal to 512.

Table of Rules/Laws of Indices

	Rule/Law	Example		Rule/Law	Example
(1)	$a^m \times a^n = a^{m+n}$	$5^2 \times 5^7 = 5^9$	(2)	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{7^5}{7^3} = 7^2 = 49$
(3)	$\left(a^m\right)^n = a^{mn}$	$\left(4^2\right)^3 = 4^6$	(4)	$a^{-m} = \frac{1}{a^m}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$
(5)	$\sqrt[m]{a} = a^{1/m}$	$\sqrt[3]{64} = 64^{1/3} = \left(4^3\right)^{1/3} = 4$	(6)	$(ab)^m = a^m \cdot b^m$	$(2 \times 3)^4 = 2^4 \times 3^4$
(7)	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$	(8)	$a^0 = 1, (a \neq 0)$	$3^0 = 1$
(9)	$a^1 = a$	$4^1 = 4$	(10)	**Note:** $a^{m^n} \neq \left(a^m\right)^n$	

For example, $2^{3^4} \neq \left(2^3\right)^4 \Rightarrow 2^{81} \neq 2^{3+3+3+3} \Rightarrow 2^{81} \neq 2^{12}$.

Example 1.

Find the value of

$$\frac{1}{(216)^{-2/3}} + \frac{1}{(256)^{-3/4}} + \frac{1}{(243)^{-1/5}}$$

Sol.

We have $216 = 6^3$, $256 = 4^4$, $243 = 3^5$

$$\frac{1}{(216)^{-2/3}} = 216^{2/3}$$

$$\because \quad \frac{1}{a^{-m}} = a^m$$

$$= \frac{1}{(216)^{-2/3}} + \frac{1}{(256)^{-3/4}} + \frac{1}{(243)^{-1/5}}$$

$$= (216)^{2/3} + (256)^{2/3} + (243)^{1/5}$$

$$= (6^3)^{2/3} + (4^4)^{3/4} + (3^5)^{1/5}$$

$$= 6^{3\times\frac{2}{3}} + 4^{4\times\frac{3}{4}} + 3^{5\times\frac{1}{5}}$$

$$= 6^2 + 4^3 + 3^1$$

$$= 36 + 64 + 3 = 103$$

Example 2.

Find the value of $\dfrac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$.

Sol.

The above expression

$$\left[\frac{2^{n-1}(2+1)}{2^n(2-1)}\right] = \left(\frac{3}{2}\right) \times 2^{n-1-n} = \left(\frac{3}{2}\right) \times 2^{-1} = \frac{3}{1} \times \frac{1}{2} = \frac{3}{2}$$

Example 3.

Evaluate : (i) $(0.00032)^{3/5}$ (ii) $(256)^{0.16} \times (16)^{0.18}$

Sol.

(i) $$(0.00032)^{3/5} = \left(\frac{32}{100000}\right)^{3/5} = \left(\frac{2^5}{10^5}\right)^{3/5}$$

$$\left\{\left(\frac{2}{10}\right)^5\right\}^{\frac{3}{5}} = \left(\frac{1}{5}\right)^{\left(5\times\frac{3}{5}\right)} = \left(\frac{1}{5}\right)^3 = \frac{1}{125}$$

(ii) $(256)^{0.16} \times (16)^{0.18}$

$$= \{(16)^2\}^{0.16} \times (16)^{0.18}$$
$$= (16)^{(2 \times 0.16)} \times (16)^{0.18}$$
$$= (16)^{0.32} \times (16)^{0.18}$$
$$= (16)^{(0.32 + 0.18)}$$
$$= (16)^{0.5}$$
$$= (16)^{1/2} = 4$$

Example 4.

If $2^{x-1} + 2^{x+1} = 1280$, then find the value of x.

Sol.

$$2^{x-1} + 2^{x+1} = 1280$$
$$2^{x-1}(1 + 2^2) = 1280$$
$$2^{x-1} = \frac{1280}{5} = 256$$
$$2^{x-1} = 2^8$$
$$x-1 = 8$$
$$x = 9$$

Example 5.

Find the value of $\left[5^2\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{5}}$

Sol.

$$\left[5^2\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{5}} = \left[5^2\left\{\left(2^3\right)^{\frac{1}{3}} + \left(3^3\right)^{\frac{1}{3}}\right\}^3\right]^{\frac{1}{5}}$$

$$\left[5^2\left\{2^{\left(3\times\frac{1}{3}\right)} + 3^{\left(3\times\frac{1}{3}\right)}\right\}^3\right]^{\frac{1}{5}} = \left\{5^2(2+3)^3\right\}^{\frac{1}{5}} = \left(5^2 \times 5^3\right)^{\frac{1}{5}} = \left(5^5\right)^{\frac{1}{5}}$$

$$= 5^{\left(5\times\frac{1}{5}\right)} = 5^1 = 5$$

Example 6.

Find the value of $\dfrac{(81)^{\frac{n}{4}} \times 3^{2n+1}}{9^n \times 3^{n-1}}$.

Sol.

$$\frac{(81)^{\frac{n}{4}} \times 3^{2n+1}}{9^n \times 3^{n-1}} = \frac{\left(3^4\right)^{\frac{n}{4}} \times 3^{2n+1}}{\left(3^2\right)^n \times 3^{n-1}}$$

$$= \frac{3^{\left(4\times\frac{n}{4}\right)} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = \frac{3^n \times 3^{2n+1}}{3^{2n} \times 3^{n-1}}$$

$$= \frac{3^{n+(2n+1)}}{3^{2n+n-1}} = \frac{3^{(3n+1)}}{3^{(3n-1)}}$$

$$= 3^{(3n+1)-(3n-1)} = 3^2 = 9$$

Example 7.

If $(1024)^{x-3} = (32)^x$, then find the value of x.

Sol.

The given equation can be rewritten as

$$(2^{10})^{x-3} = (2^5)^x$$

$$10x - 30 = 5x$$

$$5x = 30$$

$$x = 6$$

Example 8.

If $\left(\dfrac{1}{16\times 81}\right)^{3-\frac{x}{2}} = (16^2 \times 3^2)^{x-6}$ then find the value of x.

Sol.

The given equation can be rewritten as

$$\left(3^{-4}\times 2^{-4}\right)^{3-\frac{x}{2}} = \left(2^4\right)^{2x-12} \times 3^{2x-12}$$

$$3^{2x-12} \times 2^{2x-12} = 2^{8x-48} \times 3^{2x-12}$$

$$2^{2x-12} = 2^{8x-48}$$

$$2x-12 = 8x-48$$

$$x = 6$$

Example 9.

Arrange the following number in ascending order $(125)^{10}$, $(625)^9$ and $(25)^{16}$

Sol.

Each of the given numbers can be expressed with 5 as the base.

Hence of the given numbers can be written as $(5^3)^{10}$, $(5^4)^9$ and $(5^2)^{16}$

$\Rightarrow 5^{30}$, 5^{36} and 5^{32}

As the bases are equal, the values can be compared on the basis of the powers.

Hence $5^{30} < 5^{32} < 5^{36}$

i.e., $(125)^{10} < (25)^{16} < (625)^9$.

Example 10.

Arrange the following numbers in descending order $(144)^3$, $(256)^2$ and $(36)^6$.

Sol.

Each of the given bases can be expressed in exponential form.

Hence the given numbrs are 12^6, 16^4 and 6^{12} i,e, $(12^{1/2})^{12}$,$(16^{1/3})^{12}$ and $(6)^{12}$ All these numbers have the same power. Hence, they can be compared on the basis of the bases of the numbers.

As $12^{1/2}$ lies between 3 and 4 and $16^{1/3}$ lies between 2 and 3, $16^{1/3} < 12^{1/2} < 6$.

Thus $6^{12} > 12^6 > 16^4$

$\Rightarrow 36^6 > 144^3 > 256^2$ is the descending order.

Example 11.

Simplify: $\left(\dfrac{x^a}{x^b}\right)^{(a^2+b^2+ab)} \times \left(\dfrac{x^b}{x^c}\right)^{(b^2+c^2+bc)} \times \left(\dfrac{x^c}{x^a}\right)^{(c^2+a^2+ca)}$

Sol.

Given Expression

$$= \left\{x^{(a-b)}\right\}^{(a^2+b^2+ab)} \times \left\{x^{(b-c)}\right\}^{(b^2+c^2+bc)} \times \left\{x^{(c-a)}\right\}^{(c^2+a^2+ca)}$$

$$= x^{(a-b)(a^2+b^2+ab)} \times x^{(b-c)(b^2+c^2+bc)} \times x^{(c-a)(c^2+a^2+ca)}$$

$$= x^{(a^3-b^3)} \times x^{(b^3-c^3)} \times x^{(c^3-a^3)} = x^{(a^3-b^3+b^3-c^3+c^3-a^3)} = x^0 = 1.$$

Surds

Any number of the form p/q. where p and q are integers and $q \neq 0$ is called a rational number. Any real number which is not a rational number is an irrational number. Amongst irrational numbers, of particular interest to us are SURDS. Amongst surds, we will specifically be looking at 'quadratic surds' - surds of the type $a+\sqrt{b}$ and $a+\sqrt{b}+\sqrt{c}$, where the terms involve only square roots and not any higher roots. We do not need to go very deep into the area of surds - what is required is a basic understanding of some of the operations on surds. If there is a surd of the form $\left(a+\sqrt{b}\right)$, then a surd of the form $\pm\left(a-\sqrt{b}\right)$ is called the conjugate of the surd $\left(a+\sqrt{b}\right)$. The product of a surd and its conjugate will always be a rational number.

Example 12.

Simplify $\frac{1}{4+\sqrt{2}}-\frac{1}{4-\sqrt{2}}$

Sol.

$$\frac{1}{4+\sqrt{2}}-\frac{1}{4-\sqrt{2}}=\frac{\left(4-\sqrt{2}\right)-\left(4+\sqrt{2}\right)}{\left(4+\sqrt{2}\right)\left(4-\sqrt{2}\right)}$$

$$=\frac{-2\sqrt{2}}{14}=\frac{-\sqrt{2}}{7}$$

Example 13.

Rationalize the denominator of the surd $\frac{1}{4+\sqrt{6}-10}$.

Sol.

Here first take $4+\sqrt{6}$ as one term and $\sqrt{10}$ as the second term and carry out the rationalization by multiplying the numerator and denominator with $\left(4+\sqrt{6}+\sqrt{10}\right)$

$$=\frac{\left(4+\sqrt{6}+\sqrt{10}\right)}{\left(4+\sqrt{6}-\sqrt{10}\right)\left(4+\sqrt{6}-\sqrt{10}\right)}=\frac{\left(4+\sqrt{6}+\sqrt{10}\right)}{\left(4+\sqrt{6}\right)^2-\left(\sqrt{10}\right)^2}$$

$$=\frac{4+\sqrt{6}+\sqrt{10}}{16+6+8\sqrt{6}-10}=\frac{4+\sqrt{6}+\sqrt{10}}{12+8\sqrt{6}}$$

As the denominator has still irrational part, it should be rationalized one more time by multiplying the numerator and denominator with its conjugate surd.

$$=\frac{\left(4+\sqrt{6}+\sqrt{10}\right)\left(3-2\sqrt{6}\right)}{4\left(3+2\sqrt{6}\right)\left(3-2\sqrt{6}\right)}$$

$$=\frac{12+3\sqrt{6}+3\sqrt{10}-8\sqrt{6}-12-2\sqrt{60}}{4(9-24)}$$

$$=\frac{-5\sqrt{6}-3\sqrt{10}-2\sqrt{60}}{-60}=\frac{5\sqrt{6}-3\sqrt{10}+4\sqrt{15}}{60}$$

Example 14.

Find the value of $\sqrt{2\sqrt{2\sqrt{2 \ldots\ldots \infty}}}$

Sol.

In such problems, we can assume the whole quantity to be a.

Let $a = \sqrt{2\sqrt{2\sqrt{2 \ldots\ldots \infty}}} \quad \Rightarrow a = \sqrt{(2a)}$

Squaring both sides, we get

$$a^2 = 2a \quad \Rightarrow \quad a^2 - 2a = 0$$

$$a(a-2) = 0$$

$\Rightarrow$ $a = 0$ or 2

But a cannot be zero. Hence, a = 2.

Example 15.

$y = \sqrt{\sqrt{7}+7+\sqrt{(8+2\sqrt{7})}} - \sqrt{7}$, what is the value of y.

Sol.

$$8+2\sqrt{7} = 7+1+2\sqrt{7} = (\sqrt{7}+1)^2,$$

so $$y = \sqrt{\sqrt{7}+7+\sqrt{(\sqrt{7}+1)^2}} - \sqrt{7}$$

$$y = \sqrt{\sqrt{7}+7+\sqrt{(8+2\sqrt{7})}} - \sqrt{7} = \sqrt{1+7+2\sqrt{7}} - \sqrt{7}$$

$$= \sqrt{(\sqrt{7}+1)^2} - \sqrt{7}$$

$\Rightarrow$ $\sqrt{7}+1-\sqrt{7} = 1$

Hence, $y = 1$.

Logarithm

The logarithmic function is the inverse of exponential function.

Definition

If any number N is expressed in the form a^x, then index x is called the logarithm of the number N to the base a.

Thus, if $N = a^x$, then $x = \text{lag}_a N$., which can be told as log of Number N to the base a.

(a) a is a positive number not equal to 1.

(b) Logarithms of negative numbers are not defined.

Logarithmic Rules

Concept

1. $\log_a (xy) = \log_a x + \log_a y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a (x^k) = k \log_a x$,

   $\log_a \sqrt[k]{x} = \log_a (x)^{1/k} = \frac{1}{k}\log_a x$
4. $\log_a 1 = 0$ [As $a^0 = 1$]
5. $\log_x x = 1$
6. $\log_a x = \frac{1}{\log_x a}$
7. $\log_a x = \frac{\log_b x}{\log_b a}$
8. $a^{(\log_a x)} = x$
9. When base is not mentioned, it will be taken as 10
10. $a^{\log_{10}^{b}} = b^{\log_{10}^{a}}$

Example 16.

Find the values of the following.

(a) $\log_2 4$

(b) $\log_{25} 5$

(c) $\log_8 16$

(d) $\log_{36} \sqrt{216}$

Sol.(a)Let $Y = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2(1) = 2.$

(b) Let $Y = \log_{25} 5$

$$= \frac{\log_{10} 5}{\log_{10} 25}\left(\text{as } \log_3 a = \frac{\log_{10} a}{\log_{10} 3}\right)$$

$$y = \frac{\log_{10} 5}{\log_{10} 5^2} = \frac{\log_{10} 5}{2\log_{10} 5} = \frac{1}{2}$$

(c) Let $x = \log_8 16$

$$\Rightarrow \quad x = \frac{\log 16}{\log 8} \text{ (base assumed 10)}$$

$$\frac{\log 2^4}{\log 2^3} = \frac{4\log 2}{3\log 2} = \frac{4}{3}$$

$\Rightarrow x \log 8 = \log 16$ $\Rightarrow x = \frac{4}{3}$

(d) Let $Y = \log_{36}\sqrt{216} = \frac{\log\sqrt{216}}{\log 36}$

$$= \frac{\frac{1}{2}\log(6)^3}{\log(6)^2}$$

$$= \frac{\frac{3}{2}\log(6)}{2\log(6)}$$

$$= \frac{3}{4}$$

Example 17.

Find the values of the following.

(a) $\log_5 10.\ \log_{10} 15.\ \log_{15} 20.\ \log_{20} 25$ (b) $\log_9 16\ .\ \log_{32} 27$

Sol.

(a) $\log_5 10.\ \log_{10} 15. \log_{15} 20.\ \log_{20} 25.$

$$= \frac{\log 10}{\log 5} \times \frac{\log 15}{\log 10} \times \frac{\log 20}{\log 15} \times \frac{\log 25}{\log 20}$$

$$= \frac{\log 25}{\log 5}$$

$$= \frac{2\log 5}{\log 5} = 2$$

(b) $\log_9 16.\ \log_{32} 27$

$$= \frac{\log 16}{\log 9} \times \frac{\log 27}{\log 32}$$

$$= \frac{4\log 2}{2\log 3} \times \frac{3\log 3}{5\log 2} = \frac{6}{5}$$

Example 18.

If $2\ \log_x(x - 2) = \log_x 4$, find the the value of x.

Sol.

$\log_x(x - 2)^2 = \log_x 4$. Hence, $(x - 2)^2 = 4$

$\because \quad \log_x{}^a = \log_x{}^b \Rightarrow a = b$

$\Rightarrow \quad x^2 + 4 - 4x = 4$

$\Rightarrow \quad x^2 - 4x = 0$

$x(x - 4) = 0$

Therefore, $x = 4$ or $x = 0$. But $x > 0$, we find that $x = 4$ is the only valid solution. Because value of the base cannot be negative.

Example 19.

If $a^x = b$, $b^y = c$, $c^z = a$, then find the value of xyz.

Sol.

$$a = c^z = (b^y)^z = (b)^{yz} = (a^x)^{yz}.$$

Now $a^1 = a^{xyz}$

Hence, $xyz = 1$.

Example 20.

How many digits are there in 2^{38}. Given (log 2 = 0.30103)

Sol.

Take log of 2^{38}

$$\log 2^{38} = 38 \log 2 = 11.43914$$

Number of digits = 11 + 1 = 12.

$\because$ (Number of digits in a given number = Integral part of the log value of the number +1.)

Example 21.

Simplify: $\left[\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}\right]$

Sol.

Given expression

$$= \log_{xyz}(xy) + \log_{xyz}(yz) + \log_{xyz}(zx) \qquad \left[\because \log_a x = \frac{1}{\log_x a}\right]$$

$$= \log_{xyz}(xy \times yz \times zx) = \log_{xyz}(xyz)^2$$

$$= 2\log_{xyz}(xyz) = 2 \times 1 = 2$$

Objective Questions

1. Let a, b be any positive integers and $x = 0$ or 1, then
   (a) $a^x b^{(1-x)} = xa + (1-x)b$
   (b) $a^x b^{(1-x)} = (1-x)a + xb$
   (c) $a^x b^{(1-x)} = a^{(1-x)} bx$
   (d) None of the above is necessarily true.

2. Find the number of different values of x that satisfy the equation
   $2[27 \log_3 (\log_4 x)] = (\log_4 x)^2 + 5(\log_4 x) + 2$
   (a) 1
   (b) 2
   (c) 3
   (d) 4
   (e) No real values of x exist

3. $\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1} = ?$
   (a) 1
   (b) abc
   (c) 0
   (d) None of these

4. Solve for x, if 3 log (x – 1)
   $\frac{x+4}{8} = \frac{x+4}{8}$
   (a) 4
   (b) 3
   (c) 2
   (d) 1
   (e) No solution

5. Find the possible value(s) of
   $\sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots\ldots\infty}}}$
   (a) –3
   (b) 12
   (c) 4
   (d) 7

6. If $x = 2^{\frac{1}{3}} + 2^{\frac{1}{3}}$, then $2x^3 - 6x =$
   (a) 1
   (b) 3
   (c) 2
   (d) 5

7. If $3^x = y^2$, $y^x = 9$, $x > 0$ and $y > 0$, then find $x – y$.
   (a) 1
   (b) 0
   (c) 2
   (d) –1

8. Find the value of $\log_{b^2} a$, if $\log_{b^2} a = \log_{a^2} b$

(a) $\frac{1}{2}$
(b) $-\frac{1}{2}$
(c) $\frac{1}{4}$
(d) none of these

9. If $\log_q p = 3$ and $\log_q 8p = 4$, then find p.

(a) $\sqrt[3]{2}$
(b) $\sqrt[3]{4}$
(c) $\sqrt[3]{16}$
(d) 512

10. If log 15 = a and log 20 = b, then find log 12 in terms of a and b.

(a) 3a + b − 4
(b) a + 3b −4
(c) 3a + b −3
(d) a + 3b − 3

11. If $\log_7 \log_5 \left(\sqrt{x} + 5 + \sqrt{x}\right) = 0$ find the value of x.

(a) 1
(b) 0
(c) 2
(d) None of these

12. If a + b + c = 0, where $a \neq b \neq c$, then $\frac{a^2}{2a^2 + bc} + \frac{b^2}{2b^2 + ac} + \frac{c^2}{2c^2 + ab}$ is equal to

(a) zero
(b) 1
(c) −1
(d) abc

13. The value of $\log_5 \frac{(125)\times(625)}{25}$ is equal to :

(a) 725
(b) 6
(c) 3125
(d) 5

14. The values of a in the equation : $\log_{10} (a^2 - 15a) = 2$ are:

(a) $\frac{15 \pm \sqrt{233}}{2}$
(b) 20, −5
(c) $\frac{15 \pm \sqrt{305}}{2}$
(d) ± 20

15. $\frac{2^{n+4} - 2\left(2^n\right)}{2\left(2^{n+3}\right)}$ when simplified is:

(a) $\frac{2^{n+4} - 2\left(2^n\right)}{2\left(2^{n+3}\right)}$
(b) $2^{n+1} - \frac{1}{8}$
(c) $1 - 2n$
(d) 7/8

16. If $\log x - 5\log 3 = -2$, then x equals:

(a) 1.25
(b) 0.81
(c) 2.43
(d) 0.8

17. Simplify; $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$ the result is :

(a) a^{16}
(b) a^{12}
(c) a^8
(d) a^4

18. The expression $2+\sqrt{2}+\frac{1}{2+\sqrt{2}}+\frac{1}{2-\sqrt{2}}$ equals:

(a) 2
(b) $2-\sqrt{2}$
(c) $2+\sqrt{2}$
(d) $2\sqrt{2}$

19. If $\log_{10} 2 = a$ and $\log_{10} 3 = b$, then $\log_5 12$ equals :

(a) $\frac{a+b}{1+a}$
(b) $\frac{2a+b}{1+a}$
(c) $\frac{a+2b}{1+a}$
(d) $\frac{2a+b}{1-a}$

20. The expression $\sqrt{\frac{4}{3}}-\sqrt{\frac{3}{4}}$ is equal to :

(a) $\sqrt{3}/6$
(b) $-\sqrt{3}/6$
(c) $\sqrt{-3}/6$
(d) $5\sqrt{3}/6$

21. If $a = \log_8 225$ and $b = \log_2 15$ then a, in terms of b, is :

(a) $b/2^8$
(b) $2b/3$
(c) b
(d) $3b/2$

22. What is the value of $[\log_{10}(5\log_{10}100)]^2 = ?$

(a) 25
(b) 10
(c) 2
(d) 1

23. Given $2^x = 8^{y+1}$ and $9^y = 3^{x-9}$; the value of $x + y$ is :

(a) 18
(b) 21
(c) 24
(d) 27

24. The number of real values of x satisflying the equation $2^{2x^2-7x-5} = 1$ is :

(a) 1
(b) 2
(c) 4
(d) more than 4

25. If $\log_7 \log_5\left(\sqrt{x+5}+\sqrt{x}\right) = 0$, what is the value of x ?

(a) 2
(b) 3
(c) 4
(d) 5

26. If $x^{1/p} = y^{1/q} = z^{1/r}$ and xyz = 1, then the value of p + q + r would be:

(a) 2 (b) A rational number

(c) 1 (d) 0

27. If a = $\log_{24}12$, b = $\log_{36}24$ and c = $\log_{48}36$, then the value of 1 + abc would be:

(a) 2ac (b) 2bc

(c) bc (d) 2ab

28. The value of $\log\left[1-\left\{1-\left(1-x^2\right)^{-1}\right\}^{-1}\right]^{-\frac{1}{2}}$ can be expressd as :

(a) $\log\left(1-x^2\right)^{-\frac{1}{2}}$ (b) $\log\left(1-x^2\right)^{\frac{1}{2}}$

(c) $\log x^2$ (d) $\log x$

29. If $a = b^2 = c^3 = d^4$ then the value of $\log_a$ (abcd) would be :

(a) $\log_a 1 + \log_a 2 + \log_a 3 + \log_a 4$ (b) $\log_a 24$

(c) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$ (d) $1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}$

30. If a = log 2, b = log 3 and c = log 7 then find the value of $\log_6 7$ in terms of a, b and c.

(a) $\frac{b}{a+c}$ (b) $\frac{c}{a+b}$

(c) $\frac{a}{b+c}$ (d) None of these

31. Evaluate the expression : $\frac{1^3+2^3+3^3\ldots+12^3}{1^2+2^2+3^2\ldots+12^2} = ?$

(a) $\frac{234}{25}$ (b) $\frac{224}{35}$

(c) $\frac{324}{35}$ (d) $\frac{335}{24}$

32. Evaluate : $\log_3 4 \times \log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 8 \times \log_8 9$

(a) 0 (b) 1

(c) –1 (d) None of these

33. How many real values of 'x' satisfy the equation

$81^{\log_{10}^{x}} = 3 + 2x^{\log_{10}^{3}}$

(a) 0 (b) 1 (c) 2 (d) 5 (e) 4

34. If $\log(x-7)+\log(x+1)=1$ then which of the following is correct?

(a) $x^2-7x-6-e=0$

(b) $x^2-7x-6+e=0$

(c) $x^2-6x-7-e=0$

(d) $x^2-6x-7+e=0$

35. Solve the equation for x : $2x^{\frac{1}{3}}+2.x^{-\frac{1}{3}}=5$

(a) $2,\frac{1}{2}$

(b) $4,\frac{1}{4}$

(c) $6,\frac{1}{6}$

(d) $8,\frac{1}{8}$

36. $\sqrt[3]{\sqrt{0.000064}}$ simplifies to

(a) 0.2

(b) 0.02

(c) 0.4

(d) 0.04

37. The value of $\left(x^a/x^b\right)^{(a+b)}.\left(x^b/x^c\right)^{(b+c)}.\left(x^c/x^a\right)^{(c+a)}$ is :

(a) 0

(b) x^{abc}

(c) x^{a+b+c}

(d) 1

38. $\log\left(a^2/bc\right)+\log\left(b^2/ac\right)+\log\left(c^2/ab\right)$ is :

(a) 1

(b) 0

(c) 39

(d) abc

39. The value of $\left(\frac{1}{216}\right)^{-2/3}+\left(\frac{1}{256}\right)^{-3/4}+\left(\frac{1}{243}\right)^{-1/5}$ is :

(a) 107

(b) 105

(c) 103

(d) None of thse

40. If $\log_{10}(x^2-6x+45)=2$, then the values of x are :

(a) 6, 9

(b) 9, –5

(c) 10, 5

(d) 11, –5

41. The value of $\frac{3^{(12+n)}\times 9^{(2n-7)}}{3^{5n}}$ is :

(a) $\frac{1}{3}$

(b) $\frac{9}{13}$

(c) $\frac{1}{9}$

(d) $\frac{2}{3}$

42. $\sqrt{2}, \sqrt[3]{4}$ and $\sqrt[4]{6}$ in ascending order are:

(a) $\sqrt{2}, \sqrt[3]{4}, \sqrt[4]{6}$
(b) $\sqrt[4]{6}, \sqrt[3]{4}, \sqrt{2}$
(c) $\sqrt{2}, \sqrt[4]{6}, \sqrt[3]{4}$
(d) $\sqrt[4]{6}, \sqrt{2}, \sqrt[3]{4}$

43. If $\log_5(3^x - 17)$, $\log_5(3^x - 65)$ and $\log_5 4$ are in arithmetic progression, x can be

(a) The square of an integer.
(b) An integer but neither the square nor the cube of an integer.
(c) A rational number but not an integer.
(d) An irrational number.

44. Simplify: $\dfrac{1}{\sqrt{1}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{5}} + \dfrac{1}{\sqrt{5}+\sqrt{7}} + \cdots$ up to 50 terms.

(a) $\dfrac{\sqrt{101}-1}{2}$
(b) $\sqrt{109} - \sqrt{99}$
(c) $1 - \dfrac{1}{\sqrt{101}}$
(d) $\dfrac{1}{\sqrt{99}} - \dfrac{1}{\sqrt{101}}$

45. Which of the following is the greatest?

$2^{\frac{1}{3}}, 3^{\frac{1}{4}}, 5^{\frac{1}{6}}, 7^{\frac{1}{6}}$, and $11^{\frac{1}{12}}$

(a) $2^{\frac{1}{3}}$
(b) $3^{\frac{1}{4}}$
(c) $5^{\frac{1}{6}}$
(d) $11^{\frac{1}{12}}$

46. Find the value of

$\sqrt[3]{3\sqrt[3]{3\sqrt[3]{3\sqrt[3]{3\sqrt[3]{3\ldots\ldots\infty}}}}}$

(a) $3^{\frac{1}{3}}$
(b) $3^{\frac{2}{3}}$
(c) 3
(d) $3^{\frac{1}{2}}$

47. If $\dfrac{\log a}{y-z} = \dfrac{\log b}{z-x} = \dfrac{\log c}{x-y}$ then the values of $a^{y+z}.b^{z+x}.c^{x+y}$ is

(a) 0
(b) 1
(c) 2
(d) 4

Answers (Objective Questions)

1. (a)	2. ()	3. (a)	4. ()	5. (c)	6. (d)
7. (d)	8. (d)	9. (d)	10. (b)	11. (b)	12. (b)
13. (d)	14. (b)	15. (b)	16. (c)	17. (b)	18. (a)
19. (d)	20. (a)	21. (b)	22. (d)	23. (d)	24. (b)
25. (c)	26. (d)	27. (b)	28. (d)	29. (c)	30. (b)
31. (a)	32. (d)	33. ()	34. (c)	35. (d)	36. (a)
37. (d)	38. (b)	39. (c)	40. (d)	41. (c)	42. (c)
43. (a)	44. (a)	45. (b)	46. (d)	47. (b)	

Solutions (Objective Questions)

1.

Sol. Students please note that the best way to solve this example is the method of reverse substitution. put $x = 0$ and $x = 1$ you will find that the answer is (a).

2.

Sol.

3.

Sol. $\frac{1}{\log_a bc+1} = \frac{1}{\log_a bc+\log_a a} = \frac{1}{\log_a abc}$

$= \log_{abc} a$

Similarly, simplifying others and adding, we get

$\log_{abc} a + \log_{abc} b + \log_{abc} c = \log_{abc} abc = 1$

4.

Sol.

5.

Sol. Let $\sqrt{12+\sqrt{12+\sqrt{12+........\infty}}} = x$

$\Rightarrow \quad \sqrt{12-x} = x$

$\Rightarrow \quad x^2 - x - 12 = 0$ or

$(x-4)(x+3) = 0$

$x = 4$ or -3.

As the given expression is always positive, $x = 4$.

6.

Sol. Given

$$x = 2^{1/3} + \frac{1}{2^{1/3}}$$

$$x^3 = 2 + \frac{1}{2} + 3x \text{ or}$$

$$x^3 - 3x = \frac{5}{2}$$

$$\Rightarrow \quad 2x^3 - 6x = 5$$

7.

Sol. Given $\quad 3^x = y^2$ and $y^x = 9$

$\Rightarrow \quad 3^x = y^2$ and $y^x = 3^2$

$\Rightarrow \quad x = 2$ and $y = 3$

$\therefore \quad x - y = -1$

Alternate Method

$3^x = y^2$ .... (1)

$y^x = 3^2$ .... (2)

$\Rightarrow \quad (3^{x/2})^x = 3^2$

from (1) $\Rightarrow \quad 3^{\frac{x^2}{2}} = 3^2$

$\Rightarrow \quad \frac{x^2}{2} = 2 \quad \Rightarrow x = 2 \quad [x > 0]$

$\therefore \quad y = 3$ [from (1)]

$\therefore \quad x - y = -1$

8.

Sol. Given $\frac{\log a}{\log b^2} = \frac{\log b}{\log a^2} \Rightarrow \frac{\log a}{2\log b} = \frac{\log b}{2\log a}$

$(\log a)^2 = (\log b)^2 \Rightarrow \log a = \pm(\log b)$

$\frac{\log a}{\log b} = \pm 1, \quad \frac{\log a}{2\log b} = \pm\frac{1}{2}$

or $\quad \log_{b^2} a = \pm\frac{1}{2}$

9.

Sol. Given $\log_q p = 3$, $\log_q 8p = 4$

$\frac{\log_q 8p}{\log_q p} = \frac{4}{3}$

$\Rightarrow \quad \log_p 8p = \frac{4}{3}$

$\Rightarrow \quad \log_p 8p = \frac{4}{3}$

$\Rightarrow \quad \log_p^8 + \log_p^p = \frac{4}{3}$

$\log_p^8 = \frac{4}{3} - 1$

$\log_p^8 = \frac{1}{3}$

$8 = p^{1/3}$

$p = 512$

10.

Sol. $\log 3 + \log 5 = a$ .... (1)

$2\log 2 + \log 5 = b$ .... (2)

Also log 10 = log 2 + log 5 = 1 .... (3)

We can express log 2 and log 3 in terms of a and b as follows

(2) $\Rightarrow$ 2log 2 + 1 – log 2 = b

$\Rightarrow$ log 2 = b – 1

(3) $\Rightarrow$ log 5 = 2 – b and

(1) = log 3 = a + b – 2

Now log 12 = 2log2 + log3 = 2(b – 1) + (a + b – 2) = a + 3b – 4

11.

Sol. $\log_7 \log_5 \left(\sqrt{x}+5+\sqrt{x}\right) = 0,$

$\therefore$ $\log_5\left(\sqrt{x}+5+\sqrt{x}\right) = 7^0 = 1,$

or $\left(\sqrt{x}+5+\sqrt{x}\right) = 5^1 = 5$

$\therefore$ $2\sqrt{x} = 0,$ or $x = 0$

12.

Sol. Students please note that the fastest way to solve such sums is the method of simulation. In other words, assume some values of a, b & c such that a + b + c =0 and , c b a and find the value of the expression that is given. So let a = 1, b = –1 and c = 0. So we find that :

$a \neq b \neq c$

Then $\frac{a^2}{2a^2+bc}+\frac{b^2}{2b^2+ac}+\frac{c^2}{2c^2+ab}$

$= \frac{1}{2}+\frac{1}{2}+0 = 1$

Hence the answer.

13.

Sol. $\log_5 \frac{(125)\times(625)}{25} = \log_5 \frac{5^3\times 5^4}{5^2} = \log_5 5^5 = 5\ \log_5 5 = 5$

14.

Sol. $\log_{10}\left(a^2-15a\right) = 2$

$a^2 - 15a = 10^2 = 100$ or

$a^2 - 15a - 100 = 0$ or

$(a-20)(a+5) = 0$

Which means a = 20, – 5

15.

Sol. $\frac{2^{n+4}-2\left(2^n\right)}{2\left(2^{n+3}\right)} = \frac{2^n[16-2]}{2^n[2\times 8]} = \frac{14}{16} = \frac{7}{8}$

16.

Sol. $\log x - 5 \log 3 = -2$

or, $\log x = 5 \log 3 - 2$ OR $\log x = 5 \log 3 - 2 \log 10$

or, $\log x = \log (3)^5 - \log 100$ or, $\log x = \log 243 - \log 100$

or, $\log x = \log \frac{243}{100}$ or,

$\log x = \log 2.43$

$x = 2.43$

17.

Sol. $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4 = \left[\sqrt[6]{a^9}\right]^{4/3} \left[\sqrt[3]{a^9}\right]^{2/3}$

$= [a^2][a^2] = a^2 \times a^2 = a^4$

18.

Sol. $2+\sqrt{2}+\frac{1}{2+\sqrt{2}}+\frac{1}{\sqrt{2}-2} = 2+\sqrt{2}+\frac{1}{2+\sqrt{2}}-\frac{1}{2-\sqrt{2}}$

$= 2+\sqrt{2}+\left(\frac{2-\sqrt{2}-2-\sqrt{2}}{(2)^2-\left(\sqrt{2}\right)^2}\right) = 2+\sqrt{2}+\left(\frac{-2\sqrt{2}}{4-2}\right) = 2+\sqrt{2}-\sqrt{2} = 2$

19.

Sol. $\log_{10} 2 = a, \ \log_{10} 3 = b$

$\log_5 12 = \frac{\log_{10} 12}{\log_{10} 5} = \frac{\log_{10}\left(2^2 \times 3\right)}{\log_{10}(10/2)}$

$= \frac{2\log_{10} 2 + \log_{10} 3}{1 - \log_{10} 2}$

$\Rightarrow \frac{2a+b}{1-a}$

20.

Sol. $\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}} = \frac{\sqrt{4}}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{4}}$

$= \frac{4-3}{\sqrt{3}\times\sqrt{4}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

21. (b)

Sol. $a = \log_8 225 = \frac{\log_2 225}{\log_2 8}$

$$= \frac{\log_2 15^2}{\log_2 2^3}$$

$$a = \frac{2\log_2 15}{3} = \frac{2}{3}b$$

22.

Sol. $[\log_{10}(5 \log_{10} 100)]^2$

$$= [\log_{10}(5 \times 2)]^2 = 1^2 = 1$$

23.

Sol.

$$2^x = 8^{y+1}$$

$$2^x = 2^{3y+3}$$

$$x = 3y + 3 \quad \text{.................(i)}$$

Also,

$$9^y = 3^{x-9}$$

$$3^{2y} = 3^{x-9}$$

$$2y = x - 9 \quad \text{.................(ii)}$$

solving (i) & (ii), we get,

$$x = 21, y = 6,$$

Hence sum = 27

24.

Sol.

$$2^{2x^2-7x+5} = 1 = 2^0$$

$$\therefore \quad 2x^2 - 7x + 5 = 0$$

$$2x^2 - 2x - 5x + 5 = 0$$

$$(2x-5)(x-1) = 0$$

$$x = 1 \text{ or } \frac{5}{2}.$$

So, there are two solutions.

25.

Sol. Given, $\log_7 \log_5\left(\sqrt{x+5}+\sqrt{x}\right) = 0$

$$\log_5\left(\sqrt{x+5}+\sqrt{x}\right) = 7^0 = 1$$

$$(\because \log_a b = 0 \Rightarrow b = a^0)$$

$$\sqrt{x+5}+\sqrt{x} = 5^1 = 5$$

$$\Rightarrow \quad \sqrt{x+5} = 5-\sqrt{x}$$

Squaring both the side, we get

$$(x+5) = 25 + x - 10\sqrt{x}$$

$$10\sqrt{x} = 20$$
$$\sqrt{x} = 2$$
$$x = 4$$

26.

Sol. Let $x^{1/p} = y^{1/q} = z^{1/r} = k$

$\Rightarrow$ $x = k^p$, $y = k^q$ and $z = k^r$

Given $xyz = 1$ (i)

By puting value of x, y, z in (i), we get

$$k^p \cdot k^q \cdot k^r = 1$$
$$= k^{p+q+r} = k^0$$
$$p + q + r = 0$$
$$= k^{p+q+r} = k^0$$
$$p + q + r = 0$$

27.

Sol. Let $a = \log_{24}12$, $b = \log_{36}24$ and $c = \log_{48}36$,

Consider, $1 + abc = 1 + (\log_{24} 12)(\log_{36} 24)(\log_{48} 36)$

$$= 1 + \frac{\log 12}{\log 24} \times \frac{\log 24}{\log 36} \times \frac{\log 36}{\log 48} \quad \left(\text{By using } \log_a b = \frac{\log b}{\log a}\right)$$

$$= 1 + \frac{\log 12}{\log 48} = \frac{\log 48 + \log 12}{\log 48}$$

$$= \log_{48}(48 \times 12) = \log_{48}(12 \times 12 \times 4)$$
$$= \log_{48}(12 \times 2)^2 = 2\log_{48} 24$$
$$= 2 \log_{36} 24 \ \log_{48} 36 = 2b.c$$

28.

Sol. Consider, $\log\left[1 - \left\{1 - \left(1 - x^2\right)^{-1}\right\}^{-1}\right]^{-1/2}$

$$= \log\left[1 - \left\{1 - \frac{1}{1 - x^2}\right\}^{-1}\right]^{-1/2} = \log\left[1 - \left\{\frac{1 - x^2 - 1}{1 - x^2}\right\}^{-1}\right]^{-1/2} = \log\left[1 - \left\{\frac{-x^2}{1 - x^2}\right\}^{-1}\right]^{-1/2}$$

$$= \log\left[1 + \frac{1 - x^2}{x^2}\right]^{-1/2} = \log\left[\frac{1}{x^2}\right]^{-1/2} = \log x$$

29.

Sol. Let $a = b^2 = c^3 = d^4$

$$a = b^2$$

$\Rightarrow \quad \log_a a = 2\log_a b$

$\log_a b = \frac{1}{2}$

$a = c^2$

$\log_a a = 3\log_a c$

$\log_a c = \frac{1}{3}$

Similarly, $\log_a d = \frac{1}{4}$

Consider, $\log_a(abcd) = \log_a a + \log_a b + \log_a c + \log_a d$

$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ (By putting values)

30.

Sol. $\log_6 7 = \frac{\log 7}{\log 6} = \frac{\log 7}{\log(2.3)}$

$\frac{\log 7}{\log 2 + \log 3} = \frac{c}{a+b}$

$\because \quad a = \log 2, \quad b = \log 3$ and $c = \log 7$

31.

Sol. Given expression is $\frac{1^3 + 2^3 + 3^3 ... + 12^3}{1^2 + 2^2 + 3^2 ... + 12^2}$

sum of cubes of first n natural numbers $= \left[\frac{n(n+1)}{2}\right]^2$

sum of squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$

$\therefore$ By using above results, $\frac{1^3 + 2^3 + 3^3 ... + 12^3}{1^2 + 2^2 + 3^2 ... + 12^2} = \frac{\left[\frac{12(12+1)}{2}\right]^2}{\frac{12(12+1)(12\times 2+1)}{6}}$

$\frac{468}{50} = \frac{234}{25}$

Here, $n = 12$

32.

Sol. $\log_3 4 \times \log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 8 \times \log_8 9$

$= \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \frac{\log 7}{\log 6} \times \frac{\log 8}{\log 7} \times \frac{\log 9}{\log 8}$ $\left(\because \log_a b = \frac{\log b}{\log a}\right)$

$$= \frac{\log 9}{\log 3} = \log_3 9 = \log_3 3^2 = 2$$

33.

Sol.

34.

Sol. Given log $(x - 7)$ + log $(x + 1) = 1$

$\log[(x-7)(x+1)] = 1 = \log_e e$ $\quad (\because \log mn = \log m + \log n \text{ and } \log_e e = 1)$

$(x-7)(x+1) = e$

$x^2 - 6x - 7 - e = 0$

35.

Sol. The given equation is $2x^{\frac{1}{3}} + 2x^{-\frac{1}{3}} = 5$

Let $x^{\frac{1}{3}} = y$, By putting the value of $x^{\frac{1}{3}}$ we get the equation as

$$2y + \frac{2}{y} = 5$$

$$2y^2 - 5y + 2 = 0$$

$$2y^2 - 4y - y + 2 = 0$$

$$(2y-1)(y-2) = 0$$

$$y = \frac{1}{2}, 2$$

Thus, $\quad x^{\frac{1}{3}} = \frac{1}{2}, \; x^{\frac{1}{3}} = 2$

$$x = \frac{1}{8}, 8$$

36.

Sol. $\sqrt[3]{\sqrt{0.000064}} = \left[\left(64 \times 10^{-6}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}$

$$\left[\left[\left(8 \times 10^{-3}\right)^2\right]^{\frac{1}{2}}\right]^{\frac{1}{3}} = \left(8 \times 10^{-3}\right)^{\frac{1}{3}}$$

$$= \left[\left(2 \times 10^{-1}\right)^3\right]^{\frac{1}{3}} = 2 \times 10^{-1} = 0.2$$

37.

Sol. $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \left(\frac{x^c}{x^a}\right)^{c+a} = \left(x^{a-b}\right)^{a+b} . \left(x^{b-c}\right)^{b+c} . \left(x^{c-a}\right)^{c+a}$ $\left(\text{using } \frac{a^m}{a^n} = a^{m-n}\right)$

$= x^{(a-b)(a+b)} . x^{(b-c)(b+c)} . x^{(c-a)(c+a)} = x^{a^2-b^2} . x^{b^2-c^2} . x^{c^2-a^2}$ (using $a^2 - b^2 = (a - b)(a + b)$)

$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$ (using $a^m.a^n = a^{m+n}$)

$= x^0 = 1$

38.

Sol. Consider $\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ac}\right) + \log\left(\frac{c^2}{ab}\right) = \log\left[\frac{a^2}{bc} \times \frac{b^2}{ac} \times \frac{c^2}{ab}\right]$ [using log m + log n = log mn]

$= \log 1 = 0$

39.

Sol. $\left(\frac{1}{216}\right)^{-2/3} + \left(\frac{1}{256}\right)^{-3/4} + \left(\frac{1}{243}\right)^{-1/5} = \left(\frac{1}{6^3}\right)^{-2/3} + \left(\frac{1}{4^4}\right)^{-3/4} + \left(\frac{1}{3^5}\right)^{-1/5}$

$\left(\frac{1}{216}\right)^{-2/3} + \left(\frac{1}{256}\right)^{-3/4} + \left(\frac{1}{243}\right)^{-1/5} = \left(\frac{1}{6^3}\right)^{-2/3} + \left(\frac{1}{4^4}\right)^{-3/4} + \left(\frac{1}{3^5}\right)^{-1/5} = \left(6^{-3}\right)^{-2/3} + \left(4^{-4}\right)^{-3/4} + \left(3^{-5}\right)^{-1/5}$

$= 6^2 + 4^3 + 3^1 = 36 + 64 + 3 = 103$

40.

Sol. If $a^y = n$ then $\log_a n = y$

$\therefore \log_{10}(x^2 - 6x + 45) = 2$

$x^2 - 6x + 45 = 10^2$

$x^2 - 6x - 55 = 0$

$x^2 - 11x + 5x - 55 = 0$

$x(x - 11) + 5(x - 11) = 0$

$(x - 11)(x + 5) = 0$

$x = 11, -5$

41.

Sol. $\frac{3^{12+n} \times 9^{2n-7}}{3^{5n}} = \frac{3^{12+n} \times 3^{4n-14}}{3^{5n}}$

$= \frac{3^{5n-2}}{3^{5n}} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

42. **(c)**

Sol. $\sqrt{2} = (2)^{1/2} = \sqrt[12]{2^6} = \sqrt[12]{64}$

$\sqrt[3]{4} = (4)^{1/3} = \sqrt[12]{4^4} = \sqrt[12]{256}$

$\sqrt[4]{6} = (6)^{1/4} = \sqrt[12]{6^3} = \sqrt[12]{216}$

$\therefore$ Ascending order $\sqrt{2}$, $\sqrt[4]{6}$, $\sqrt[3]{4}$

43.

$$2\log_5(3^x-65) = \log_5(3^x-17)+\log_5 4$$
$$\log_5(3^x-65)^2 = \log_5 4(3^x-17)$$
$$(3^x-65)^2 = 4(3^x-17)$$
$$(3^x)^2 - 134(3^x) + 4293 = 0$$
$$(3^x-81)(3^x-53) = 0$$
$$3^x = 81, 53$$

But $\log_5(3^x-17)$ and $\log_5(3^x-65)$ are defined only when $(3^x-17), (3^x-65) > 0$, i.e., when $3^x > 65$.

$$3^x = 81$$
$$x = 4 \quad \text{i.e., the square of an integer.}$$

44.

Sol. The 50th term $= \dfrac{1}{\sqrt{99}+\sqrt{101}}$

The given series $= \dfrac{\sqrt{1}-\sqrt{3}}{(\sqrt{1}-\sqrt{3})(\sqrt{1}+\sqrt{3})}$

$$+\frac{\sqrt{3}-\sqrt{5}}{(\sqrt{3}-\sqrt{5})(\sqrt{3}+\sqrt{5})}+\ldots$$

$$+\frac{\sqrt{99}-\sqrt{101}}{(\sqrt{99}-\sqrt{101})(\sqrt{99}-\sqrt{101})}$$

$$= \frac{\sqrt{1}-\sqrt{3}}{-2}+\frac{\sqrt{3}-\sqrt{5}}{-2}+\frac{\sqrt{5}-\sqrt{7}}{-2}+\ldots+\frac{\sqrt{99}-\sqrt{101}}{-2}$$

$$= \frac{1}{-2}\left[\sqrt{1}-\sqrt{3}+\sqrt{3}-\sqrt{5}+\ldots+\sqrt{99}-\sqrt{101}\right]$$

$$-\frac{1}{2}\left[\sqrt{1}-\sqrt{101}\right] = \frac{\sqrt{101}-\sqrt{1}}{2}$$

$$= -\frac{1}{2}\left[\sqrt{1}-\sqrt{101}\right] = \frac{\sqrt{101}-\sqrt{1}}{2}$$

45.

Sol. $2^{\frac{1}{3}} = \left(2^8\right)^{\frac{1}{24}} = (256)^{\frac{1}{24}}$,

$3^{\frac{1}{4}} = \left(3^6\right)^{\frac{1}{24}} = (729)^{\frac{1}{24}}$,

$5^{\frac{1}{6}} = \left(5^4\right)^{\frac{1}{24}} = (625)^{\frac{1}{24}}$,

$7^{\frac{1}{6}} = \left(7^3\right)^{\frac{1}{24}} = (343)^{\frac{1}{24}}$,

$11^{\frac{1}{12}} = \left(11^2\right)^{\frac{1}{24}} = (121)^{\frac{1}{24}}$

Hence, $3^{\frac{1}{4}}$ is the greatest.

46.

Sol. Let $\sqrt[3]{3\sqrt[3]{3\sqrt[3]{3\sqrt[3]{3\sqrt[3]{3 \ldots\ldots\ldots\infty}}}}} = y$

$= \sqrt[3]{3y} = y$

$3y = y^3,\ y[y^2-3] = 0$

As the given expression is positive, $y = \sqrt{3}$

47.

Sol. Let $\frac{\log a}{y-z} = \frac{\log b}{z-x} = \frac{\log x}{x-y} = k$

$\Rightarrow$ $a = 10^{k(y-z)}$, $b = 10^{k(z-x)}$ and $c = 10^{k(x-y)}$

Now $a^{y+z}b^{z+x}c^{x+y} = 10^{k\left(y^2-z^2\right)}10^{k\left(z^2-x^2\right)}10^{k\left(x^2-y^2\right)} = 10^0 = 1$

Algebra

CHAPTER 4

Linear Equation

What is Inequation

A statement involving variable (s) and the **sign of inequality such as >, <, ≥, ≤** is called an **inequation or an inequality.**

An inequation may contain one or more variables. Also, it may be linear or quadratic or cubic etc.

Following are some examples of inequations

(i) $3x - 5 < 0$ (ii) $2x + 7 \leq 0$

(iii) $5x - 6 > 0$ (iv) $4x + 8 \geq 0$

(v) $x^2 + 3x + 5 < 0$ (vi) $x^2 - 5x + 8 \leq 0$

(vii) $x^3 - 9x^2 + 11x - 6 > 0$ (viii) $x^3 + 6x^2 + 11x + 9 \leq 0$

LINEAR INEQUATION IN ONE VARIABLE

Let 'a' be a non zero real number and x be a variable. Then inequations of the form $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$ and $ax + b \geq 0$ are known as linear inequations in one variable x.

For example, $9x–16 > 0$, $7x - 4 \geq 0$, $8x + 2 < 0$ and $2x - 6 \leq 0$ are linear inequations in one variable.

LINEAR INEQUATIONS IN TWO VARIABLES

Let a, b be non-zero real numbers and x, y be variables. Then inequations of the form $ax + by < c$, $ax + by \leq c$, $ax + by > c$ an $ax + by \geq c$ are known as linear inequations in two variables x and y.

For example, $3x + 5y \leq 6$, $7x - 4y \geq 12$, $2x + 3y < 4$, $5x + 2y > 6$ are linear inequations in two variables x and y.

QUADRATIC INEQUATION

Let a be a non-zero real number. Then an inequation of the form $ax^2 + bx + c < 0$, or $ax^2 + bx + c \leq 0$, or $ax^2 + bx + c > 0$, or $ax^2 + bx + c \geq 0$ is known as a quadratic inequation.

For example, $2x^2 + 5x - 6 < 0$, $x^2 - 6x + 4 \geq 0$, $2x^2 + 8x + 1 > 0$ and $3x^2 - 5x + 4 \leq 0$ are quadratic inequations.

SOLUTIONS OF AN INEQUATION

A solution of an inequation is the value (s) of the variable (s) that makes it a true statement.

SOLVING LINEAR INEQUATIONS IN ONE VARIABLE

Solving an inequation is the process of obtaining its all possible solutions. In the process of solving an inequation, we use mathematical simplifications which have the following rules:

Rule 1

Same number (positive or negative) added to (or subtracted from) both sides of an inequation does not change the sign of the inequation.

Rule 2

When both sides of an inequation are multiplied (or divided) by the same positive real number there is no sign change in the inequality. **However, the sign of inequality is reversed when both sides of an inequation are multiplied or divided by a negative number.**

Rule 3

Any term of an inequation can be taken to the other side with its sign changed without affecting the sign of inequality.

How to solve the linear Inequation or Inequality:

For a linear inequation in one variable is of the form $ax + b < 0$ or, $ax + b \leq 0$ or, $ax + b > 0$ or, $ax + b \geq 0$.We follow the following algorithm to solve a linear inequation in one variable.

Step

(i) Obtain the linear inequation .

(ii) Collect all terms involving the variable on one side of the inequation and the constant terms on the other side.

(iii) Simplify both sides of inequality in their simplest forms to reduce the inequation in the form.

$ax < b$ or, $ax \leq b$ or, $ax > b$ or, $ax \geq b$.

(iv) Solve the inequation obtained in step III by dividing both sides of the inequation by the coefficient of the variables.

(v) Write the solution set obtained in step IV in the form of an interval on the real line.

Following examples will help you to understand the above mentioned rules

Example 1.

Solve the linear inequations $3x - 4 \leq 0$

Sol.

We have, $3x - 4 \leq 0$

$(3x - 4) + 4 \leq 0 + 4$ [Adding 4 on both sides]

$3x \le 4$

$\frac{3x}{3} \le \frac{4}{3} \Rightarrow x \le \frac{4}{3}$, Now, $x \le \frac{4}{3}$ means that inequality is satisfied for every value of x (real number) $\le \frac{4}{3}$, Hence the solution set of the given inequation is $(-\infty, \frac{4}{3}]$

Example 2.

Solve $7x - 3 < 5x + 1$, When (i) x is a real number (ii) x is integer number (iii) x is a natural number.

Sol.

We have,

$7x - 3 < 5x + 1$

$7x - 5x < 3 + 1$ [Transposing 3x on LHS and – 3 on RHS]

$2x < 4$

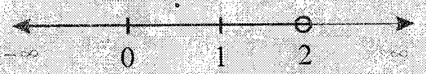
$\frac{2x}{2} < \frac{4}{2}$ [Dividing both sides by 2]

$x < 2$

(i) If $x \in R$, then

$x < 2 \Rightarrow x \in (-\infty, 2)$

Hence, the solution set is $(-\infty, 2)$ as shown in Fig.



(ii) If $x \in Z$, then

$x < 2 \Rightarrow x = 1, 0, -1, -2, -3, -4, -5....$

So, the solution set is $\{.... ....-5, -4, -3, -2, -1, 0, 1\}$

(iii) If $x \in N$, then

$x < 2 \Rightarrow x = 1$, So the solution set is $\{1\}$

Example 3.

Solve the following inequations:

(i) $\frac{1}{x-3} < 0$ (ii) $\frac{x+1}{x+2} \ge 1$

Sol.

(i) We have $\frac{1}{x-3} < 0 \Rightarrow x - 3 < 0$ $[\because \frac{a}{b} < 0$ and $a > 0 \Rightarrow b < 0]$

$\Rightarrow x < 3 \Rightarrow x \in (-\infty, 3)$ Hence, the solution set of the given inequation is $(-\infty, 3)$.

(ii) We have, $\frac{x+1}{x+2} \ge 1 \Rightarrow \frac{x+1}{x+2} - 1 \ge \Rightarrow \frac{x+1-x-2}{x+2} \ge 0 \Rightarrow \frac{-1}{x+2} > 0$

$\Rightarrow x + 2 < 0$ $\quad\quad [\because \frac{a}{b} > 0 \text{ and } a < 0 \Rightarrow b < 0]$

$\Rightarrow x < -2 \Rightarrow x \in (-\infty, 2)$, Hence, the solution set of the given inequation is $(-\infty, 2)$

SOLUTION OF SYSTEM OF LINEAR INEQUATIONS IN ONE VARIABLE

Solution set of a system of linear inequations in one variable is the intersection of the solution sets of the linear inequations in the given system.

We use the following method to solve a system of linear inequations in one variable.

Method

Step (i) Obtain the system of linear inequations.

Step (ii) Solve each inequation and obtain their solution sets. Also, represent them on real number line.

Step (iii) Find the intersection of the solution sets obtained in step II by taking the help of the graphical representation of the solution sets in step II.

Step (iv) The set obtained in step III is the required solution set of the given system of inequations.

Following examples will illustrate the above method

Example 4.

Solve the following system of linear inequations:

$3x - 6 \geq 0$

$2x - 5 \leq 3$

Sol.

The given system of inequations is

$3x - 6 \geq 0$ ...(i)

$2x - 5 \leq 3$ ...(ii)

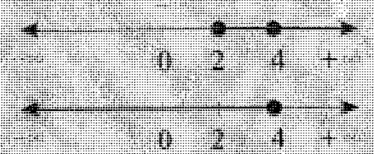
Now, $3x - 6 \geq 0 \Rightarrow 3x \geq 6 \Rightarrow \Rightarrow x \geq 2$

$\therefore$ Solution set of inequation (i) is $[2, \infty)$

and, $2x - 5 \leq 3 \Rightarrow 2x \leq 8 \Rightarrow x \leq 4$

$\therefore$ Solution set of inequation (ii) is $(-\infty, 4]$

The solution sets of inequations (i) and (ii) are represented graphically on real line in Figs.



Clearly, the intersection of these solution sets is the set [2, 4]. Hence, the solution set of the given system of inequations is the interval [2, 4]

Example 5.

Solve $-13 \le 4x - 1 \le 11$

Sol. We have,

$$-13 \le 4x - 1 \le 11$$

$$\Leftrightarrow \quad -13 \le 4x - 1 \text{ and } 4x - 1 \le 11$$

Thus, we have two inequations and we wish to solve them simultaneously. Instead of solving these inequations by using the method discussed in above examples, let us solve them directly in a different way as given below;

We have,

$-13 \le 4x - 1 \le 11 \Rightarrow -13 + 1 \le 4x - 1 + 1 \le 11 + 1$ [Adding 1 throughout]

$\Rightarrow -12 \le 4x \le 12$ [Dividing by 4 throughout]

$\Rightarrow -3 \le x \le 3 \Rightarrow x \in [-3, 3]$ Hence, the interval $[-3, 3]$ is the solution set of the given system of inequations.

$-13 + 1 \le 4x \le 12$

$-12 \le 4x \le 12$

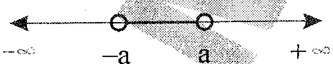
$-3 \le x \le 3$

SOME IMPORTANT RESULTS

Result I

If a is a positive real number, then

(i) $|x| < a \Leftrightarrow -a < x < a$ i.e. $x \in (-a, a)$



(ii) $|x| \le a \Leftrightarrow -a \le x \le a$ i.e. $x \in [-a, a]$

(i) We know that: $|x| = \begin{cases} x, \text{ if } x > 0 \\ -x, \text{ if } x = 0 \\ -0, \text{ if } x = 0 \end{cases}$

So, we consider the following cases:

Case (i) When $x \ge 0$

In the case, $|x| = x$.

$\therefore |x| < a \Rightarrow x < a$

Thus, in this case the solution set of the given inequation is given by

$x \ge 0$ and $x < a$

$\Rightarrow 0 \le x < a$ ...(i)

Case (ii) When $x < 0$:

In this case, $|x| = -x$

$\therefore |x| < a \Rightarrow -x < a$

$\Rightarrow x > -a$

Thus, in this case the solution set of the given inequation is given by

$x < 0$ and $x > -a$

$\Rightarrow -a < x < 0$ ...(ii)

Combining (i) and (ii), we get

$|x| < a \Leftrightarrow -a < x < 0$ or $0 \le x < a$

$\Rightarrow \mathbf{|x| < a \Leftrightarrow -a < x < a}$

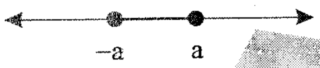
(ii) In a similar way we can solve that when $\mathbf{|x| \le a \Leftrightarrow -a \le x \le a}$

Result II

If a is a positive real number, then

(i) $|x| > a \Leftrightarrow x < -a$ or $x > a$

(ii) $|x| \ge a \Leftrightarrow x \le -a$ or $x \ge a$



Case (i) When $x \ge 0$:

In this case,

$|x| = x$

$\therefore |x| > a \Rightarrow x > a$

Thus, in this case the solution set of the inequation $|x| > a$ is given by

$x \ge 0$ and $x > a$

$\Rightarrow x > a$ $[\because a > 0]$...(i)

Case (II) When $x < 0$

In this case,

$|x| = -x$

$\therefore |x| > a \Rightarrow -x > a$

$\Rightarrow x < -a$

Thus, in this case the solution set of the given inequation is given by

$x < 0$ and $x < -a$

$\Rightarrow x < -a$ $[\because a > 0]$...(ii)

Combining (i) and (ii), we get when $\mathbf{|x| > a \Leftrightarrow x < -a}$ **or** $\mathbf{x > a}$

(ii) Similary we can prove that when $\mathbf{|x| \ge a}$ **then** $\mathbf{x \le -a}$ **or** $\mathbf{x \ge a.}$

Result III

Let r be a positive real number and a be a fixed real number. Then,

(i) $|x-a| < r \Leftrightarrow a - r < x < a + r$ i.e. $x \in (a - r, a + r)$

(ii) $|x-a| \le r \Leftrightarrow a - r \le x \le a + r$ i.e. $x \in [a - r, a + r]$

(iii) $|x-a| > r \Leftrightarrow x < a - r$ or, $x > a + r$

(iv) $|x-a| \ge r \Leftrightarrow x \le a - r$, or $x \ge a + r$

(i) We have, $|x-a| < r \Leftrightarrow -r < x - a < r \Leftrightarrow a - r < x - a + a < a + r$

$\Leftrightarrow a - r < x < a + r$

(ii) Using result 1 (ii), we have

$|x-a| \le r \Leftrightarrow -r \le x - a \le r$

$\Leftrightarrow a-r \le x - a + a \le a + r$

$\Leftrightarrow a-r \le x \le a + r$

(iii) Using result 2 (i), we have

$|x-a| > r \Leftrightarrow x - a < r$, or $x - a > r$

$\Leftrightarrow x < a-r$, or $x > a + r$

(iv) Using result 2 (ii), we have

$|x-a| \ge r \Leftrightarrow x - a \le -r$, or $x - a \ge r$

$\Leftrightarrow x \le a - r$, or $x \ge a + r$

These results may be used directly for solving linear inequations involving absolute values.

Result IV

Let a, b be positive real numbers. Then

(i) $a < |x| < b \Leftrightarrow x \in (-b, -a) \cup (a, b)$

(ii) $a \le |x| \le b \Leftrightarrow x \in [-b, -a] \cup [a, b]$

(iii) $a \le |x - c| \le b \Leftrightarrow x \in [-b + c, -a + c] \cup [a+c, b+c]$

(iv) $a < |x - c| < b \Leftrightarrow x \in (-b + c, -a+c) \cup (a+c, b+c)$

(i) $a < |x| < b \Leftrightarrow |x| > a$ and $|x| < b \Leftrightarrow (x < -a$ or $x > a)$ and $(-b < x < b)$

$\Leftrightarrow x \in (-b, -a) \cup (a, b)$

Similarly, we can prove other results.

Example 6.

$|3x-2| \le \frac{1}{4}$

Sol.

We know that

$|x - a| \le r \Leftrightarrow a - r \le x \le a + r$

$\therefore \quad |3x - 2| \le \frac{1}{4} \Leftrightarrow 2 - \frac{1}{4} \le 3x \le 2 + \frac{1}{4}$

$\Leftrightarrow \frac{7}{4} \le 3x \le \frac{9}{4} \Leftrightarrow \frac{7}{12} \le x \le \frac{3}{4} \Leftrightarrow$ **$x \in [7/12, 3/4]$**

Example 7.

Solve: $|x-2| \ge 5$

Sol.

We know that:

$x - 2 \ge 5 \Rightarrow x \ge 7$

$x - 2 \le -5 \Rightarrow x \le -3$

$|x-a| \ge r \Leftrightarrow x \le a - r$, or $x \ge a + r$

$\therefore \quad |x-2| \ge 5 \Leftrightarrow x \le 2 - 5$, or $x \ge 2 + 5$

$\Leftrightarrow x \le -3$, or $x \ge 7$

$\Leftrightarrow x \in (-\infty, -3]$ or $x \in [7, \infty)$

$\Leftrightarrow x \in (-\infty, -3] \cup [7, \infty)$

Hence the solution set of the given inequation is $(-\infty, -3] \cup [7, \infty)$

Example 8.

Solve: $1 \le |x - 2| \le 4$

Sol.

We know that:

$a \le |x-c| \le b.$

$\Leftrightarrow x \in [-b + c, -a + c] \cup [a + c, b + c]$

$\therefore 1 \le |x-2| \le 4 \Leftrightarrow x \in [-4 + 2, -1 + 2] \cup [1 + 2, 4 + 2]$

$\Leftrightarrow x \in [-2, 1] \cup [3, 6]$

Hence, the solution set of the given inequation is $[-2, 1] \cup [3, 6]$

SOME APPLICATIONS OF LINEAR INEQUATIONS IN ONE VARIABLE

Example 9.

Find all pairs of consecutive odd positive integers, both of which are smaller than 16, such that their sum is more than 18.

Sol.

Let x be the smaller of the two consecutive odd positive integers. Then, the other odd integer is

$x + 2$.

It is given that both the integers are smaller than 16 and their sum is more than 18.

Therefore,

$x + 2 < 16$ and, $x + (x + 2) > 18$

$\Rightarrow x < 14$ and $2x + 2 > 18$

$\Rightarrow x < 14$ and $2x > 16$

$\Rightarrow x < 14$ and $x > 8$

$\Rightarrow 8 < x < 14$

$\Rightarrow x = 9, 11, 13,$ [$\because$ x is an odd integer]

Hence, the required pairs of odd integers are (9,11), (11, 13), (13, 15).

Example 10.

A man wants to cut three lengths from a single piece of board of length 92 cm. The second length is to be 4 cm longer than the shortest and third length is to be twice as long as the shortest. What are the possible lengths for the shortest board if third piece is to be at least 6 cm longer than the second?

Sol.

Let the length of the shortest piece be x cm. Then, the lengths of the second and third piece are x + 4 cm and 2x cm respectively. Then,

$x + (x + 4) + 2x \leq 92$ and $2x \geq (x + 4) + 6$

$\Rightarrow \quad 4x + 4 \leq 92$ and $2x \geq x + 10$

$\Rightarrow \quad 4x \leq 88$ and $x \geq 5$

$\Rightarrow \quad x \leq 22$ and $x \geq 5$

$\Rightarrow \quad 5 \leq x \leq 22.$

Hence, the shortest piece must be at least 5 cm long but not more than 22 cm long.

GRAPHICAL SOLUTION OF LINEAR INEQUATIONS IN TWO VARIABLES

If a, b, c are real numbers, then the equation $ax + by + c = 0$ is called a linear equation in two variables x and y whereas the inequalities $ax + by \leq c$, $ax + by \geq c$, $ax + by < c$ and $ax + by > c$ are called linear inequations in two variables x and y.

Example 11.

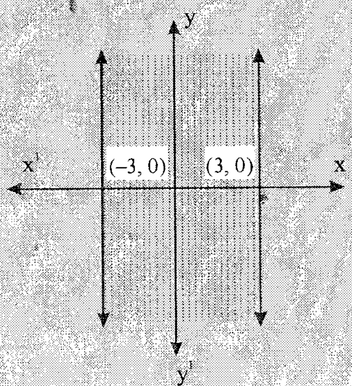
Solve the following inequations graphically:

(i) $|2x| \le 6$ (ii) $|3y-3x| \le 9$ (iii) $|x-y| \ge 1$

Sol.

(i) Converting the given inequation into equation, we obtain $\mathbf{2x = 6}$ **or** $\mathbf{x = 3}$. This equation represents a line parallel to y-axis at a distance of 3 units from it. The line given by $x = 3$ divides the xy-plane into two regions.

Clearly, the point O(0, 0) satisfies $x \le 3$. So, the graph of $x \le 3$ is as shown in Fig. The shaded region represents the solution set of this inequation.



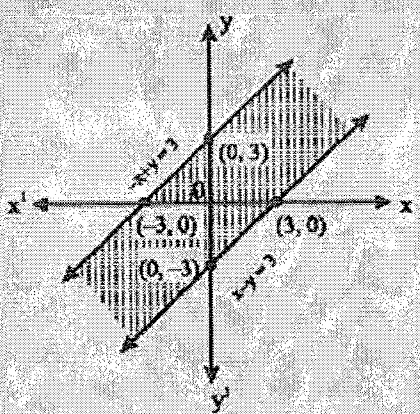
(ii) We have, $|3y-3x| \le 9$. This inequation is equivalent to

$-9 \le 3y - 3x \le 9$ or $-3 \le y - x \le 3$ $[\because |x| \le a \Leftrightarrow -a \le x \le a]$

$\Leftrightarrow -3 \le y - x$ and $y - x \le 3$

$\Leftrightarrow x - y - 3 \le 0$ and $x - y + 3 \ge 0$

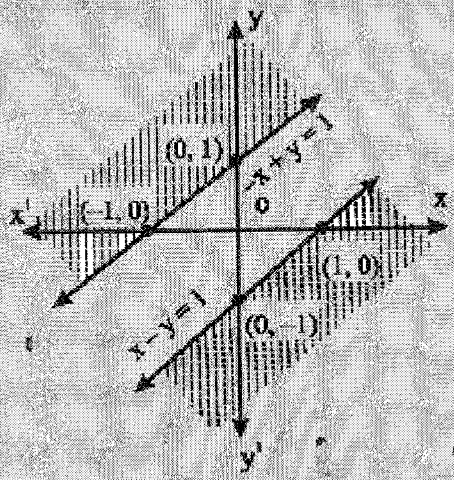
The region represented by $|y - x| \le 3$ is the region common to the regions represented by $x - y - 3 \le 0$ and $x - y + 3 \ge 0$ as shown in Fig. This shaded region represents the solution set of the given inequation.



(iii) We have, $|x - y| \ge 1 \Leftrightarrow x - y \ge 1$ or $x - y \le -1$

$\Leftrightarrow x - y - 1 \ge 0$ or $x - y + 1 \le 0$

The required region is the union of regions represented by $x - y - 1 \geq 0$ and $x - y + 1 \leq 0$ as shown in Fig. The shaded region represents the solution set of the given inequation.



Objective Questions

1. If $xy + yz + zx = 0$, then $(x + y + z)^2$ equals

   (a) $(x + y)^2 + xz$
   (b) $(x + z)^2 + xy$
   (c) $x^2 + y^2 + z^2$
   (d) $2(xy + yz + xz)$

2. The sum of the roots of equation $4x^2 + 5 - 8x = 0$ is equal to:

   (a) –5
   (b) $-\frac{5}{4}$
   (c) –2
   (d) none of the above.

3. If one root of $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is

   (a) 49/4
   (b) 4/49
   (c) 4
   (d) ¼

4. The sum of all the roots of $4x^3 - 8x^2 - 63x - 9 = 0$ is:

   (a) 8
   (b) 2
   (c) –8
   (d) –2

5. The values of y which will satisfy the equations:

   $2x^2 + 6x + 5y + 1 = 0$ and $2x + y + 3 = 0$ may be found by solving:

   (a) $y^2 + 14y - 7 = 0$
   (b) $y^2 + 8y + 1 = 0$
   (c) $y^2 + 10y - 7 = 0$
   (d) $y^2 + y - 12 = 0$

6. In solving a problem that reduces to a quadratic equation one student makes a mistake only in the constant term of the equation and obtains 8 and 2 the roots. Another studens makes a mistake only in the coefficient of the first degree term and finds – 9 and – 1 for the roots, the correct equation is :

(a) $x^2 - 10x + 9 = 0$
(b) $x^2 + 10x + 9 = 0$
(c) $x^2 - 10x + 16 = 0$
(d) $x^2 - 8x - 9 = 0$

7. The numbers x, y, z proportional to 2, 3, 5. The sum of x, y and z is 100. The number y is given by the equation y = ax – 10. then a is :

(a) 2
(b) 3/2
(c) 3
(d) 5/2

8. If $3x^3 - 9x^2 + kx - 12$ is divisible by $x - 3$, then it is also divisible by:

(a) $3x^2 - 4$
(b) $3x^2 + 4$
(c) $3x - 4$
(d) $3x + 4$

9. For any real value of x the maximum value of $8x - 3x^2$ is:

(a) $\frac{8}{3}$
(b) 4
(c) 5
(d) $\frac{16}{3}$

10. For what value(s) of k does the pair of equations $y = x^2$ and $y = 3x + k$ have two identical solutions?

(a) 1
(b) 2
(c) –9/4
(d) 3

11. If $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$ then the value of x is:

(a) $\frac{6}{13}$ or $\frac{4}{143}$
(b) $\frac{3}{2}$ or $\frac{2}{3}$
(c) $\frac{5}{2}$ or $\frac{2}{3}$
(d) $\frac{9}{13}$ or $\frac{4}{13}$

12. If 'p' and 'q' are the roots of $x^2 + x + 1 = 0$, then the value of $p^3 + q^3$ becomes:

(a) 4
(b) – 4
(c) 2
(d) –2

13. The root of the equation $\frac{x+4}{4} + \frac{x-5}{3} = 11$ is

(a) 12
(b) 20
(c) 2
(d) 10

14. The condition for which the system of equations kx – y = 2 and 6x – 2y = 8 has a unique solution, is :

(a) k = 3
(b) $k \neq 3$
(c) $k \neq 0$
(d) k = 0

15. The number of positive integer valued pairs (x, y) satisfying $7x - 10y = 12$ and $x \leq 500$ is :

(a) 44 (b) 46

(c) 48 (d) 50

16. If $x + \frac{1}{x} = 2$, then the value of $x^2 + \frac{1}{x^2} - 2$ is:

(a) 6 (b) 4

(c) 2 (d) 0

17. Two oranges three bananas and four apples cost Rs. 15. Three oranges, two bananas and one apple cost Rs. 10. I bought 3 oranges, 3 bananas and 3 apples. How much did I pay?

(a) Rs. 10 (b) Rs. 8

(c) Rs 15 (d) cannot be determined

18. If $x^4 + \frac{1}{x^4} = 47$, find the value of $x^3 + \frac{1}{x^3}$

(a) 18 (b) 20

(c) 22 (d) 24

19. If x is real, what is the maximum possible value of the expression $\frac{x+2}{2x^2+4x+8}$?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{12}$

20. One junior students is asked to divide half one of the given number by 6 and the other half by 4 and then add the quanitites. Instead of doing so, the students divides the given number by 5. If the answer is 4 short of the correct answer, then he actual answer is:

(a) 320 (b) 360

(c) 480 (d) 400

21. In a group of cows and chickens, the number of legs was 14 more than twice the number of heads. The number of cows was:

(a) 5 (b) 7

(c) 10 (d) 12

Answers (Objective Questions)

1.	(c)	**2.**	(d)	**3.**	(a)	**4.**	(b)	**5.**	(c)
6.	(a)	**7.**	(a)	**8.**	(b)	**9.**	(d)	**10.**	(c)
11.	(d)	**12.**	(c)	**13.**	(b)	**14.**	(b)	**15.**	(d)
16.	(d)	**17.**	(c)	**18.**	a)	**19.**	(c)	**20.**	(c)
21.	(b)								

Solutions :—

1.

Sol. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2\,(xy + yz + xz) = x^2 + y^2 + z^2 + 2 \times 0 = x^2 + y^2 + z^2$.

2.

Sol. Sum of the roots of a quadratic equation $= \frac{-(-8)}{4} = 2$

3.

Sol. If one root of $x^2 + px + 12 = 0$ is 4, then $4^2 + 4p + 12 = 0$, i.e. $p = -7$. $x^2 - 7x + q = 0$ has equal roots.

$\therefore$ If the roots are α each, $2\alpha = -(-7)/1 = 7$, i.e. $\alpha = 7/2$, and $\alpha^2 = (q/1)$

$q = 49/4$.

4.

Sol. Sum of all the roots of $ax^3 + bx^2 + cx + d = 0$ is $-\frac{b}{a}$, So the sum of all the roots of $4x^3 - 8x^2 - 63x - 9 = 0$

is $\frac{-(-8)}{4} = 2$

5.

Sol. $2x + y + 3 = 0$

$$x = -\frac{(y+3)}{2}$$

Also $2x^2 + 6x + 5y + 1 = 0$

$$\frac{2(y+3)^2}{4} - 6\left(\frac{y+3}{2}\right) + 5y + 1 = 0$$

or $2(y + 3)^2 - 12(y + 3) + 4(5y + 1) = 0$

Thus $y^2 + 10y - 7 = 0$

6.

Sol. Original quadratic equation : $ax^2 + bx + c = 0$

If 8 and 2 are the roots, corresponding equation: $(x-8)(x-2)= 0$ or $x^2 - 10x + 16 = 0$. As given that in this equation mistake is only in the constant term. so term containing coefficient of x is correct. which means correct coffiecient of x term = –10

Another student find roots as – 9 and – 1 and he did mistake only in the first degree term. it means he founded the constant term correct, which means correct product of the roots or constant term

$= -9 \times -1 = 9$

Therefore correct equation: $x^2 - 10x + 9 = 0$.

7.

Sol. $x = 2k$, $y = 3k$, $z = 5k$

$x + y + z = 100$ or $10k = 100$ which means $k = 10$

so $x = 20$, $y = 30$ and $y = ax - 10$ or $30 = 20a - 10$ hence $a = 2$

8.

Sol. $fx = 3x^3 - 9x^2 + kx - 12$

$f(3) = 0$

$27 - 27 + 3k - 12 = 0$

$K = 4$

9.

Sol. Let $Z = 8x - 3x^2$

$$Z = -3\left[x^2 - \frac{8}{3}x\right]$$

$$Z = -3\left[x^2 - 2\times x\times\frac{4}{3} + \left(\frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^2\right]$$

$$Z = -3\left(x - \frac{4}{3}\right)^2 + 3\times\left(\frac{4}{3}\right)^2$$

So the maximum value occurs when $x = \frac{4}{3}$

10.

Sol. $x^2 = y = 3x + k$

$x^2 - 3x - k = 0$, for the roots to be identical, discriminant $D = 0$

$(-3)^2 - 4 \times 1 \times (-k) = 0$ $\quad\left[\sqrt{b^2 - 4ac} = 0\right]$

$$k = \frac{-9}{4}$$

11.

Sol. Given $\sqrt{\dfrac{x}{1-x}}+\sqrt{\dfrac{1-x}{x}} = \dfrac{13}{6}$

$$\sqrt{\frac{x}{x\left(\frac{1}{x}-1\right)}}+\sqrt{\frac{x\left(\frac{1}{x}-1\right)}{x}} = \frac{13}{6}$$

$$\sqrt{\frac{1}{\frac{1}{x}-1}}+\sqrt{\frac{1}{x}-1} = \frac{13}{6}$$

$$\frac{1}{\sqrt{\frac{1}{x}-1}}+\sqrt{\frac{1}{x}-1} = \frac{13}{6}$$

$$1+\frac{1}{x}-1 = \frac{13}{6}\left(\sqrt{\frac{1}{x}-1}\right)$$

$$\frac{1}{x} = \frac{13}{6}\left(\sqrt{\frac{1}{x}-1}\right)$$

By squaring on both side, we get $169x^2-169x+36=0$

$$x = \frac{169\pm 65}{338}$$

$$x = \frac{9}{13} \text{ or } \frac{4}{13}$$

12.

Sol. Let p and q be the roots of equation $x^2+x+1=0$ $\therefore$ $p+q=-1$ and $pq=1$

Consider $p^3+q^3=(p+q)^3-3pq(p+q)=(-1)^3-3(1)(-1)=-1+3=2$

13.

Sol. Consider the equation $\dfrac{x+4}{4}+\dfrac{x-5}{3}=11$ By taking L.C.M., we have

$$\frac{3(x+4)+4(x-5)}{12}=11$$

$3x+12+4x-20=132$

$7x=132+8=140$

$x=20$

Hence, root of the given equation is 20.

14.

Sol. As we know, the system $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$ has a unique solution, if $\dfrac{a_1}{a_2}\neq\dfrac{b_1}{b_2}$

So, given system of equation is $Kx-y=2$ and $6x-2y=8$

This system has unique solution, if $\dfrac{K}{6}\neq\dfrac{-1}{-2}\Rightarrow K\neq 3$.

15.

Sol. 10y always ends with 0; hence, 7x should end with 2 to satisfy the given equation.
Therefore, the given equation is satisfied only when x ends with 6.
Substituting x = 6 we get y = 3.
Similarly, if x = 16, y = 10; x = 26, y = 17 and so on.
Since x < 500, the possible values of x are 6, 16, 26, 36, ..., 496, i.e., 50 values.

16.

Sol. $x+\frac{1}{x} = 2$

squaring both the sides we get

$$x^2+\frac{1}{x^2}+2 = 4$$

$$x^2+\frac{1}{x^2} = 2$$

17.

Sol. The two equations are : 2o + 3b + 4a = 15, 3o + 2b + a = 10. Adding the two equations we get, 5o + 5b + 5a = 25 or o + b + a = 5 i.e. 3o + 3b + 3a = 15.

18.

Sol. $x^4+\frac{1}{x^4} = 47$ or

$$\left(x^2\right)^2+\left(\frac{1}{x^2}\right)^2 = 47$$

$$\left(x^2+\frac{1}{x^2}\right)^2-2 = 47 \Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 = 49$$

$$x^2+\frac{1}{x^2} = 7$$

Similarly,

$$x+\frac{1}{x} = 3,$$

Now, $x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3-3x\frac{1}{x}\left(x+\frac{1}{x}\right)$

$= 3^3 - 3 \times 3 = 27 - 9 = 18$

19.

Let y = $\frac{x+2}{2x^2+4x+8} \Rightarrow 2yx^2 + 4yx + 8y = x + 2$

$\Rightarrow 2yx^2 + (4y - 1)x + 8y - 2 = 0$

For the roots to be real, $(4y-1)^2 - 8y(8y-2) \geq 0$ or $(4y-1)^2 - 16y(4y-1) \geq 0$

or $(1-4y)(1+12y) \geq 0$; $-\frac{1}{12} \leq y \leq \frac{1}{4}$

20.

Sol. Let the required number be $2x$.

Now, according to the ques.

$$\left(\frac{x}{6}+\frac{x}{4}\right)-4=\frac{2x}{5}$$

$$\frac{x}{6}+\frac{x}{4}-\frac{2x}{5}=4$$

$$10x + 15x - 24x = 240$$

$$x = 240$$

Hence, the actual answer is 2(240) = 480.

21.

Sol. let the number of cows be x and number of chickens be y

	Heads	Legs
Cows	x	$4x$
Chicken	y	$2y$

Number of legs were 14 more than twice the number of heads.

i.e., $4x + 2y = 14 + 2(x + y)$

$4x + 2y = 14 + 2x + 2y$

$2x = 14$

$x = 7$

CHAPTER

5

Permutation and Combination

Let us start this chapter with a simple example:

Example 1.

Ravi has IIM Ahamedabad an interview two days later and he has come to a garment store to buy a dress for his interview. In the store, Ravi notices 20 trousers and 30 shirts that he can buy. In how many ways can Ravi buy

1. 1 shirt and 1 trouser?
2. only one garment out of these?

Ans.

Let the trouser be $T_1, T_2, T_3, \ldots, T_{19}, T_{20}$ and the shirts be $S_1, S_2, S_3, \ldots, S_{29}, S_{30}$.

1. One shirt and one trouser: How many pairs of one shirt and one trouser can we make. The pairs will be from $(T_1S_1, T_1S_2, T_1S_3, \ldots T_1S_{30})$ to $(T_{20}S_1, T_{20}S_2, T_{20}S_3, \ldots, T_{20}S_{30})$. For every trouser, Ravi can make 30 pairs. As there are 20 trousers, the number of pairs will be $20 \times 30 = 600$.
2. There are $20 + 30 = 50$ garments in all, out of which we have to choose one garment. Since we can choose any one of the 50 garments, the number of ways is = 50

Note:

That when we were choosing shirt AND trouser we multiplied and when were choosing shirt OR trouser we added. This principle is known as fundamental principle of counting.

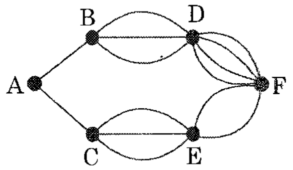
Fundamental Principle of Counting

If one operation can be performed in m ways, and a second operation can be performed in n ways, the number of ways of performing both the operations will be $m \times n$ and the number of ways of performing either one of the operations will be $m + n$.

Tips

1. In case of OR we add.
2. In case of AND we multiply.

The number of routes between few cities A, B, C, D, E and F are shown in the figure. Find



Example 2.

In how many ways can you go from city A to city F via city B?

Sol.

We can go from A to B in one way, from B to D in 3 ways, and from D to F in 4 ways, Since we have to go from A to B, and from B to D, and from D to F the number of ways = $1 \times 3 \times 4 = 12$

Example 3.

In how many ways can you go from city A to city F via city C?

Ans.

$1 \times 3 \times 2 = 6$

Example 4.

Find the total number of ways you can go from city A to city F.

Sol.

We can go from A to F either via city B OR via city C. Hence the number of ways = $12 + 6 = 18$

Example 5.

In how many ways can you post 10 letters in 4 letterboxes?

Sol.

The first letter can be posted in 4 ways, the second can be posted in 4 ways, the third can be posted in 4 ways ... and so on. Since we have to post all the letters (the case of AND) the number of ways = $4 \times 4 \times 4 \ldots. = 4^{10}$.

Example 6.

How many numbers can be formed using the digits 1, 3, 4, 5, 6, 8 and 9 if no repetition is allowed?

Sol.

The number of digits is not specified in this problem so we can form one-digit numbers, two-digit numbers, or three digit numbers, etc But since no repetitions are allowed and we have only the 7 numbers to work with, the maximum number of digits would have to be 7.

Applying the product rule, we see that we may form 7 one-digit numbers, $7 \times 6 = 42$ two-digit numbers $7 \times 6 \times 5 = 210$ three digit numbers, $7 \times 6 \times 5 \times 4 = 840$ four digit numbers, $7 \times 6 \times 5 \times 4 \times 3 = 2520$ five digit numbers, $7 \times 6 \times 5 \times 4 \times 3 \times 2 = 5040$ six-digit numbers, $7 \times 6 \times 5 \times$

$4 \times 3 \times 2 \times 1 = 5040$ seven digit number. The events of forming one-digit numbers, two digit numbers, three digit numbers, etc., are mutually exclusive events so we apply the sum rule to see that there are $7 + 42 + 210 + 840 + 2520 + 5040 + 5040 = 13699$ different numbers we can form according to the problem.

Example 7.

How many different words, with or without meaning can you form by arranging the letters of the word ROCKET?

Sol.

We are going to form a six-letter word. The first letter can be filled in 6 ways, the second letter can be filled in 5 ways, the third letter can be filled in 4 ways.... and so on. Therefore, the total number of ways of filling up 6 places with the alphabets of ROCKET $= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

Example 8.

How many arrangements can be made out of the letters of the word **"draught"**, such that vowels are never separated?

Sol.

Assume that we tie a string around the vowels A and U so that they are together. If we now consider A and U as one letter, we have 6 letters in all. Now we need to arrange these 6 letters in 6 places.

As seen earlier, the number of arrangements is $= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$. Now inside the string A and U can be arranged in two ways, i.e., AU and UA. Therefore, the total number of ways $= 2 \times 720 = 1440$.

Example 10.

The letters of the word FIGMENT are to be arranged in the following manner.

(a) There is no restriction. (b) Start with F

(c) All vowels together (d) Vowels at first and last positions

Sol.

(a) There are 7 letters which can be arranged at 7 positions in 7! ways = 5040 ways.

(b) Starting with F, remaining 6 letters can be arrnged in 6! ways = 720 ways.

(c) Tying all vowels with a string we have F, G, M, N, T, (IE) i.e. as 6 letters. These can be arranged in 6! ways, 2 vowels can exchange their positions in 2 ways.

Total number of ways $= 6! \times 2! = 1440$ ways.

(d) For vowels at first and last positions, first place can be taken by I and last by E, or vice versa, Remaining 5 positions can be filled by 5 letters in 5! ways. So total words formed $= 5! \times 2! = 240$

Permutation of Things when some are identical

Above was the case when all letters in the word were different. What if some letters are identical?

⇒ If out of n things, p are exactly alike of one kind, q exactly alike of second kind and r exactly alike of third kind and the rest are different, then the number of permutations of n things taken all at a time

$$= \frac{n!}{p!q!r!}$$

Example 12.

In how many ways can the letters of the word SUCCESSFUL be arranged? In how many of them will

(i) all S's come together,

(ii) all S's not together,

(iii) the S's come together and U's also come together?

Sol.

The word contains 10 letters of which 3 are S's, 2 are C's and 2 are U's and the rest all are different.

∴ The letters of the word SUCCESSFUL can be arranged in $= \frac{10!}{3!2!2!} = 151200$ ways.

I. Since the S's are to come together, treat 3 S's as one letter. Now with this restriction there will be 8 letters of which 2 are C's and 2 are U's and the rest all the different.

∴ The arrangement in which S's will come together $= \frac{8!}{2!2!} = 10080$

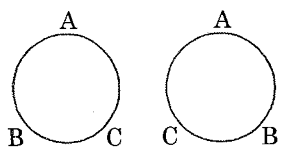
II The arrangements in which all S's will not come together
= Total number of arrangements – The number of arrangements in which all the S's will come together=151200 – 10080 = 141120

III Since the S's and U's are to come together, treat 3 S's as one letter and 2 U's as one letter. Now there will be 7 letters of which 2 are C's and the rest all are different.

∴ The arrangements in which S's and U's will come together $\frac{7!}{2!} = 2520$

Circular Permutation

Number of circular permutation of n different things taken all at a time = (n –1)! ways. Three persons around a circular table can be arranged in 2 ways, i.e. (3–1)! ways



Necklace: In case of the necklace or garland as anticlockwise and clockwise arrangements are same. So total number of arrangements of n beads of forming a necklace is $\frac{1}{2}(n-1)!$

Example 13.

There are 25 gangsters including 2 brothers, 'Ram' and 'Shyam'. In how many ways can they be arranged around the circular table if

(a) there is exactly one person between these 2 brothers,

(b) the 2 brothers are always separated?

Sol.

(a) One person between 2 brothers can be selected in 23 ways.

Now assume these 2 brother and one Gangster between them as one unit, So this unit (Ram-Gangster-Shyam) and remaining 22 persons can be arranged in (23 –1)! = 22! ways.

2 brothers can interchange their positions.

So total number of ways = 2 × 23 × 22!= 2 × 23! ways

(b) Total ways of arranging 25 people = 24!

Subtract those ways in which 2 brothers are together = 2 × 23!

∴ Number of ways when 2 brothers are always separated = 24! – 2 × 23!

Selection

Suppose we want to take four people, Bhuvan, Mangal, Shyam and Aangad, and arrange them in groups of three at a time where order matters. The number of arrangements will be $^4P_3 = \frac{4!}{(4-3)!}$ = 24. The following demonstrates all the possible arrangements:

BMS,	BSM,	BAS,	BSA,	BMA,	BAM
MBS,	MSB,	MAS,	MSA,	MBA,	MAB
SBM,	SMB,	SBA,	SAB,	SMA,	SAM
ABM,	AMB,	ABS,	ASB,	AMS,	ASM

Now suppose we only wanted to select 3 people out of the four people, Bhuvan, Mangal, Shyam and Aangad. Then what?

Then many of the arrangements are same to us. For example, the arrangements BMS, BSM, MBS, MSB, SBM, SMB are the same to us where selection is concerned because we in effect are selecting the same three people, Bhuvan, Mangal and Shyam!

Therefore, there are only 4 selections of three people- BMS, MSA, SAB, and MAB. What happened to those 24 earlier cases?

We can see that there are 3! = 6 ways of arranging three people. All these 3! ways became one way where selection was concerned.

Therefore, the number of selection of 3 people out of 4

$$= \frac{\text{number of arrangements of three people out of four}}{3!} = \frac{4!}{3!(4-3)!}$$

Combination

Example 14.

In how many ways can you select r things out of n different things?

Sol.

As we see start in the previous example, the number of arrangements of r things would be ^nP_r

The number of selections would be $\frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$

Selection of r things out of n dissimilar things

$= \frac{\text{Arrangements of r things out of n dissimilar things}}{r!} = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$. This is known as combination and denoted by $^nC_r = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$.

Permutation or Combination?

In permutation (arrangement) the order matters whereas in combination (selection) the order does not matter.

The arrangements of four letters a, b, c, d taken two at a time are twelve in total, namely, ab, ac, ad, bc, bd, cd, ba, ca, da, cb, db, dc

The selections of four letters a, b, c, d taking two letters at a time are six in number: namely, ab, ac, ad, bc, bd, cd

Understand selection and arrangement in a different manner: Suppose we need to seat r people out of n people. We can first select r people out of n people and then arrange this group of r people in order to seat them. Therefore, selection x r! = arrangement $\Rightarrow {}^nC_r \times r! = {}^nP_r$

Meanings of ^nC_r and ^nP_r:

$^nC_r \rightarrow$ Number of **combinations** of n different thins taken r at a time

$= \frac{n!}{r!(n-r)!}$ = Selection of r things out of n things

$^nP_r \rightarrow$ Number of **arrangements** of n different things taken r at a time

$= \frac{n!}{(n-r)!}$ = Selecting r things out of n things and then arranging these r things

Factorial Notation $\rightarrow$ Factorial of a natural number n is the product of all natural numbers from 1 to n.

$n! = 1 \times 2 \times 3 \times \ldots \times n$

$0! = 1$

Example 15.

A baseball team has 13 members. How many lineups of 9 players are possible?

Sol. We can first choose 9 players in $^{13}C_9$ ways and then arrange them in 9! Ways. Therefore, the number of ways = $^{13}C_9 \times 9!$ or they can straightaway arrange them in $^{13}P_9$ ways.

Example 16.

A multiple-choice test has 30 questions, each with five choices. How many answer keys are possible?

Sol. For the first question, 5 answers are possible. For the second question also, 5 answers are possible, and so on. Therefore, total number of possible answer keys = $5 \times 5 \times 5 \times 5 \ldots \times 5 = 5^{30}$.

Example 17.

In how many ways can four persons be seated out of 5 boys and 3 girls on four different seats?

Sol. This question is not simple formulae based permutation or combination question. We'll have to break the problem in cases

Case 1: four boys-number of ways of selection = $^5C_4 = 5$

Case 2: Three boys and one girl-number of ways of selection = $^5C_3 \times {}^3C_1 = 30$

Case 3: Two boys and two girls-number of ways of selection = $^5C_2 \times {}^3C_2 = 30$

Case 4: One boy and three girls-number of ways of selection = $^5C_1 \times {}^3C_3 = 5$

Now for every case we'll do the selection first and then arrange them in 4! ways. For each case the multiplication law of the principal of counting will apply since we're both selecting and arranging. For finding the total number of ways we'll add up all the cases since only one of the cases can happen (either-or principal)

Hence the total number of ways = $(5 + 30 + 30 + 5) \times 4! = 1680$.

Example 18.

A group of 6 students comprised of 3 boys and 3 girls. In how many ways could they be arranged in a straight line such that

(a) the girls and the boys occupy alternate positions?

(b) no two boys were sitting together?

Sol.

(a) The positions could be

BG BG BG OR GB GB GB

Hence, the number of arrangements is $3! \times 3! + 3! \times 3! = 2 \times 3! \times 3!$

(b) First of all we will arrange 3 girls in 3! ways.

Now we have 4 positions for 3 boys that can be filled in $^4P_3$ ways.

Hence, the total number of arrangements $^4P_3 \times 3!$

Example 19.

If there is a party and every person shakes hands with each other once, and there are 45 handshakes, how many people are there at the party?

Sol. let there be n persons in the party. A handshake takes place every time we chose 2 persons out of these n people. Therefore, total number of handshakes = total number of ways of choosing two people out of n people = $^nC_2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2} \Rightarrow \frac{n(n-1)}{2} = 45 \Rightarrow n = 10$. Therefore, there are 10 people in the party.

Example 20.

How many diagonals does an n-sided regular polygon have?

Sol. A diagonal is formed by a line joining two vertices of a polygon. Hence, the number of lines joining two vertices of a polygon = number of ways of selecting two vertices out of n vertices = $^nC_2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$.

Out of these $\frac{n(n-1)}{2}$ lines, n of them are forming the sides of the polygons. Therefore, the number of diagonals = $\frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$.

Total number of ways of selection by taking some or all of n dissimilar things

If we are given n things for selection without given the condition about how many things we should select, we can select none of them, one of them, two of them, three of them, ... or n of them. Therefore, total number of ways of selection = $^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n = 2^n$.

We can also understand it this way: For each of the n things, we have 2 options, either to select it or not to select it. Therefore, the number of selections = $2 \times 2 \times 2 \times .... \times 2 = 2^n$

The number of selections when it is necessary to select at least one of the n things = $2^n - 1$

Some important facts

(i) $^nC_0 = 1 = {}^nC_n$

(ii) $^nP_r = r!\,{}^nC_r$ = Arrangement of r things means first selecting r things and arranging these r things.

(iii) $^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

(iv) $^nC_0 + {}^nC_1 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

Example 21.

A man has 7 friends. In how many ways can he invite one or more of them to dinner?

Sol. Since the man has to select at least one of the 7 friends, the number of possible selections $= 2^7 - 1 = 127$

■ ■ ■

Objective Questions

1. A total of 28 handshakes was exchanged at the conclusion of a party. Assuming that each participant was equally polite toward all the others, the number of people present was:
   (a) 14
   (b) 28
   (c) 56
   (d) 8

2. In counting a coloured balls, some red and some black, it was found that 49 of the first 50 counted were red. Thereafter, 7 out of every 8 counted were red. If , in all, 90% or more of the balls counted were red, the maximum value of n is :
   (a) 225
   (b) 210
   (c) 200
   (d) 180

3. In an academic programme there are 6 foreign students of whom 2 are Chinese. 2 Americans and the remainings two from SAARC countries. They have to stand in a row for a photograph so that the two Chinese are together the two Americans are together and so also the two SAARC citizens. The number of ways in which the students could do so is
   (a) 24 ways
   (b) 48 ways
   (c) 6 ways
   (d) 12 ways

4. There are 10 trains runnings between Delhi and Jammu Tawi. The number of ways in which a person could go from Delhi to Jammu Tawi and return by a different train is:
   (a) 80
   (b) cannot be determind
   (c) 99
   (d) 90

5. Four brothers and three sisters sit in a single row, facing the photographer's camera. If the three sisters always sit together, how many different photographs, having all of them can be clicked?
   (a) 840
   (b) 126
   (c) 120
   (d) 720

6. In how many ways can four letters of the word 'SERIES' be arranged?
   (a) 24
   (b) 42
   (c) 84
   (d) 102

7. If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, then n =
   (a) 20
   (b) 22
   (c) 15
   (d) None of the above.

8. The value of $\sum_{r=1}^{n} \frac{{}^nP_r}{r!}$ is
   (a) 2^n
   (b) 2^n-1
   (c) 2^{n-1}
   (d) 2^{n+1}

9. Boxes numbered 1, 2, 3, 4 and 5 are kept in a row and they are to be filled with either a red or a blue ball, such that no two adjacent boxes can be filled with blue balls. Then how many different arrangements are possible, given that all balls of a given colour are exactly identical in all respects?

(a) 8 (b) 10

(c) 15 (d) 22

10. A man has nine friends – four boys and five girls. In how many ways can he invite them, if there have to be exactly three girls in the invitees?

(a) 320 (b) 160

(c) 80 (d) 200

11. In how many ways can the eight directors, the vice-chairman and the chairman of a firm be seated at a round-table, if the chairman has to sit between the vice-chairman and the director?

(a) $9! \times 2$ (b) $2 \times 8!$

(c) $2 \times 7!$ (d) None of these

12. ABC is a three-digit number in which $A > 0$. The value of ABC is equal to the sum of the factorials of its three digits. What is the value of B?

(a) 9 (b) 7

(c) 4 (d) 2

13. How many numbers can be formed from 1, 2, 3, 4 and 5 (without repetition), when the digit at the units place must be greater than that in the tenth place?

(a) 54 (b) 60

(c) $\frac{5!}{3}$ (d) $2 \times 4!$

14. How many five digit numbers can be formed using 2, 3, 8, 7, 5 exactly once such that the number is divisible by 125? (Repeatation not allowed)

(a) 0 (b) 1

(c) 4 (d) 3

15. There are 10 points on a line and 11 points on another line, which are parallel to each other. How many triangles can be drawn taking the vertices on any of the line?

(a) 1,050 (b) 2,550

(c) 150 (d) 1,045

16. One red, three white and two blue flags are to be arranged in such a way that no two flags of the same colour are adjacent and the flags at the two ends are of different colours. The number of ways in which this can be done is

(a) 6 (b) 8

(c) 4 (d) 12

17. Let n be the number of different 5 digit numbers, divisible by 4 with the digits 1, 2, 3, 4, 5 and 6, no digit being repeated in the numbers. What is the value of n?

(a) 144 (b) 168
(c) 192 (d) None of these

18. 10 straight lines, no two of which are parallel and no three of which pass through any common point, are drawn on a plane. The total number of regions (including finite and infinite regions) into which the plane would be divided by the lines is

(a) 56 (b) 255
(c) 1024 (d) not unique

19. How many four-letter computer passwords can be formed using only the symmetric letters (no repetition allowed)?

(a) 7920 (b) 330
(c) 14640 (d) 419430

20. How many three-letter computer passwords can be formed (no repetition allowed) with at least one symmetric letter?

(a) 990 (b) 2730
(c) 12870 (d) 15600

21. In how many ways is it possible to choose a white square and a black square on a chess board so that the squares must not lie in the same row or column?

(a) 56 (b) 896
(c) 60 (d) 768

22. How many numbers greater than 0 and less than a million can be formed with the digits of 0, 7 and 8?

(a) 486 (b) 1086
(c) 728 (d) None of these

23. How many three digit positive integers, with digits x, y and z in the hundred's, ten's and unit's place respectively, exist such that $x < y$, $z < y$ and $x \neq 0$?

(a) 245 (b) 285
(c) 240 (d) 320

24. There are 6 boxes numbered 1, 2,.........6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is

(a) 5 (b) 21
(c) 33 (d) 60

25. A graph may be defined as a set of points connected by lines called edges. Every edge connects a pair of points. Thus, a triangle is a graph with 3 edges and 3 points. The degree of a point is the number of edges connected to it. For example, a triangle is a graph with three points of degree 2 each. Consider a graph with 12 points. It is possible to reach any point from any other point through a sequence of edges. The number of edges, e, in the graph must satisfy the condition

(a) $11 \leq e \leq 66$ (b) $10 \leq e \leq 66$
(c) $11 \leq e \leq 65$ (d) $0 \leq e \leq 11$

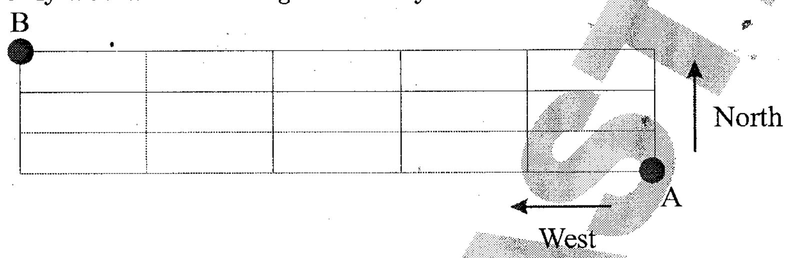
26. There are 12 towns grouped into four zones with three towns per zone. It is intended to connect the towns with telephone lines such that every two towns are connected with three direct lines if they belong to the same zone, and with only one direct line otherwise. How many direct telephone lines are required?

(a) 72
(b) 90
(c) 96
(d) 144

27. N persons stand on the circumference of a circle at distinct points. Each possible pair of persons, not standing next to each other, sings a two-minute song one pair after the other. If the total time taken for singing is 28 minutes, what is N?

(a) 5
(b) 7
(c) 9
(d) None of the above

28. In the adjoining figure, the lines represent one-way roads allowing travel only northwards or only westwards. Along how many distinct routes can a car reach point B from point A?



(a) 15
(b) 56
(c) 120
(d) 336

29. A new flag is to be designed with six vertical stripes using some or all of the colours yellow, green, blue and red. Then the number of ways this can be done such that no two adjacent stripes have the same colour out is

(a) 12×81
(b) 16×125
(c) 20×125
(d) 24×216

30. There are 6 tasks and 6 persons. Task 1 cannot be assigned either to person 1 or to person 2; task 2 must be assigned to either person 3 or person 4. Every person is to be assigned one task. In how many ways can the assignment be done?

(a) 144
(b) 180
(c) 192
(d) 360

31. How many two-digit natural numbers are there so that ten's digit is never less than the unit's digit?

(a) 44
(b) 55
(c) 54
(d) None of these

32. How many three-digit positive integers have their hundreds tens and units digits in ascending order?

(a) 70
(b) 77
(c) 84
(d) 91

Answers (Objective Questions)

1. (d)	2. (b)	3. (b)	4. (d)	5. (d)	6. (d)	7. (d)
8. (b)	9. (d)	10. (b)	11. (b)	12. (c)	13. (b)	14. (c)
15. (d)	16. (a)	17. (c)	18. (a)	19. (a)	20. (c)	21. (d)
22. (c)	23. (c)	24. (b)	25. (a)	26. (b)	27. (b)	28. (b)
29. (a)	30. (a)	31. (c)	32. (c)			

Solutions :—

1.

Sol. Let the number of people present in a party be x.

Thus the number of handshakes= ${}^{x}C_2$

$$= \frac{x(x-1)}{2}$$

$$\therefore \frac{x(x-1)}{2} = 28$$

OR x(x −1) = 8 × 7 or x = 8

2.

Sol. $49 + \frac{7}{8}(n-50) \geq \left(\frac{90}{100}\right)n$

$49 \times 8 + 7n - 350 \geq \frac{9}{10} \times 8n$

$1960 + 35n - 1750 \geq 36n$

$n \leq 210$

3.

Sol. Total number of students = 6, Number of chinese = 2, number of Americans = 2 and number of SAARC members = 2

They have to stand in a row such that all 3 are together i.e., C C A A S S

Consider two chinese as 1, two Americans as 1 and two SAARC members as 1. We can arrange 2 chinese, Americans and SAARC itself.

$\therefore$ Total no. of ways = 3! × 2! × 2! × 2! = 48

4.

Sol. Total number of trains between Delhi and Jammu = 10

So, a person can choose any one out of 10 trains to go from Delhi to Jammu. But he will return by a different train.

So, be has choice for 9 trains.

$\therefore$ Total no. of ways = 10 × 9 = 90

5.

Sol. $B_1\ B_2\ B_3\ B_4\quad S_1\ S_2\ S_3$

Total entities are 7,

Taking the three sisters as a single entity. Now, we have to arrange 5 entities & then we have to arrange three sisters, internally. so, we can arrange 5 entities in 5! ways and three sisters in 3! ways. Hence, total number of different photographs, that can be taken = (5!) × (3!) = 720.

6.

Sol. SERIES is a six lettered word where 4 are of distinct type and 2 are repeated twice.

$\therefore$ No. of arrangements when all 4 are distinct (SERI) = 4! = 24.

No. of arrangements when 2 are same = $\frac{4!\times 6}{2!} = 72$

No. of arrangements when 2 sets of repated letters are taken = $\frac{4!}{2!2!} = 6$ (S, S, E, E)

Total no. of arrangements = 24 + 72 + 6 = 102

7.

Sol. $\frac{{}^{n+2}C_8}{{}^{n-2}P_4} = \frac{57}{16}$

$$\frac{\frac{(n+2)!}{(n-6)!8!}}{\frac{(n-2)!}{(n-6)!}} = \frac{57}{16}$$

$$\frac{(n+2)(n+1)(n-1)}{8\times 7\times 6\times 5\times 4\times 3\times 2} = \frac{57}{16}$$

$$(n+2)(n+1).n.(n-1) = 21 \times 20 \times 19 \times 18$$

$\therefore\ n = 19$

8.

Sol. $\sum_{r=1}^{n} \frac{{}^nP_r}{r!} = \sum_{r=1}^{n} \frac{\frac{n!}{(n-r)!}}{r!} = \sum_{r=1}^{n} \frac{n!}{(n-r)!r!} = \sum_{r=1}^{n} {}^nC_r$

$= \left(\sum_{r=1}^{n} {}^nC_r\right) - {}^nC_0 = 2^n - 1$

9.

Sol. Each box can be filled in 2 ways.

Hence, total no. of ways = $2^5 = 32$

Blue balls cannot be filled in adjacent boxes

Total no. of such cases in which blue ball is filled in 2 adjacent boxes is

2 blue + 3 blue + 4 blue + 5 blue = 4 ways (12, 23, 34, 45) + 3 ways (123, 234, 345) + 2 ways (1234, 2345) + 1 way = 10 ways.

Hence, total cases in which blue balls can not be filled in adjacent boxes = 32 – 10 = 22

10.

Sol. Out of five girls, he has to invite exactly 3. This can be done in ${}^5C_3$. Out of 4 boys he may invite either one or two or three or four or none of them.

This may be done in ${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 + {}^4C_0$

$= {}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = (1+1)^4 = 2^4$ ways.

Hence, the total number of ways in which he can invite his friends are ${}^5C_3 \times 2^4 = 10 \times 16 =$ 160 ways.

11.

Sol. Let the vice-chairman and the chairman from 1 unit along with the eight directors, we now have to arrange 9 different units in a circle.

This can be done in 8! ways.

At the same time, the vice-Chairman & the chairman can be arranged in two different ways. Therefore, the total number of ways = 2 × 8!.

12.

Sol. Seeing the options. Since 7! = 5040 and 9! = 362880 are four and 6 digit numbers respectively, hence, B can't be 7 or 9.

3 digits number will be of the form 100 A + 10 B + C.

∴ 100 A + 10 B + C = A! + B! + C!

Now by hit & trial method

i.e., with B = 2 and B = 4 find which one is better.

Hence, B = 2, A = 1, C = 5 can be the answer as 125= 1!+2!+5!

13.

Sol. The numbers should be formed from 1, 2, 3, 4 and 5 (without repetition), such that the digit at the units place must be greater than in the tenth place. Tenth place has five options.

If 5 is at the tenth place then the digit at the unit's place cannot be filled by the digit greater than that at the tenth place.

If 4 is at the tenth place, then the unit's place has only option of 5, while the three places can be filled up in 3! ways.

If 3 is at the tenth place, then the units' place can be filled up by 4 or 5, i.e. in 2 ways. While other three places can filled up in 3! ways.

If 2 is at the tenth place, then the unit's place can be filled up by 3, 4 or 5 i.e. in 3 ways. While other three places can be filled up in 3! ways.

If 1 is at the tenth place, then any other four places can be filled up in 4! ways.

Thus the total number of numbers satisfying the given conditions is

$0 + 3! + 2(3!) + 3(3!) + 4! = 60.$

14.

Sol. Only those numbers which last 3 digits are divisible by 125 are divisible by 125.

So, those contain 375 & 875 at the end will be divisible by 125 and the remaining two digits at the thousand's and 10 thousand's places can be arranged in two different ways.

Therefore, 4 such number can be formed.

15.

Sol. For a triangle, two points on one line and one on the other has to be chosen.

No. of ways = $^{10}C_2 \times {}^{11}C_1 + {}^{11}C_2 \times {}^{10}C_1 = 1,045$

16.

Sol. There are three white flags and ends having different colours, the only possibility at the ends are red and white or white and blue.

W___W___W___ the empty spaces can be filled in 3 ways by one red and two blue flags.

___W___W___W again the empty spaces can be filled in 3 ways so, the total number of ways will be 6.

17.

Sol. For a number to be divisible by 4, its last two digits should be divisible by 4.

i.e., the last two digits should be 12, 16, 24, 32, 36, 52, 56 or 64

No. of numbers end with 12 = $^4C_3 \times 3! = 24$

Similarly number of numbers end with 16, 24, 32, 36, 52, 56 or 64 each = 24.

Thus, value of n = 24 × 8 = 192

18.

Sol. For 2 such lines, no. of regions formed are 4

For 3 lines no. of regions formed are (4 + 3 = 7)

For 4 lines no. of regions formed are (7 + 4 = 11)

For 5 lines no. of regions formed are (11 + 5 = 16)

Similarly for 6, 7, 8, 9 and 10 lines, no. of regions are

16 + 6 = 22
22 + 7 = 29
29 + 8 = 37
37 + 9 = 46
46 + 10 = 56

$\therefore$ For 10 lines no. of regions = $^{10}C_2 + 10 + 1 = 45 + 11 = 56$

Each of the 11 letters A, H, I, M, O, T, U, V, W, X and Z appears same when looked at in a mirror. They are called symmetric letters. Other letters in the alphabet are asymmetric letters.

19.

For questions 19 & 20:

Sol. Four letter passwords have four places of which
1st place can be filled in 11 ways
2nd place can be filled in 10 ways
3rd place can be filled in 9 ways
4th place can be filled in 8 ways

Total passwords formed = $11 \times 10 \times 9 \times 8 = 7920$

20.

Sol. Total three - letter computer passwords from any of the 26 letters = $26 \times 25 \times 24$

Again, three letter passwords from asymmetric letters = $15 \times 14 \times 13$

$\therefore$ Passwords with at least one symmetric letter = $26 \times 25 \times 24 - 15 \times 14 \times 13 = 12870$

21.

Sol. Then no. of ways in choosing one white and one black square on the chess = $^{32}C_1 \times {}^{32}C_1 = 32 \times 32 = 1024$

Suppose we choose a white square = $32\ C_1$ then choosen of a black square which is not in the same new or column as white square is $24\ C_1$

$\therefore\ 32 \times 24 = 268$

22.

Sol. No. of 1 digit numbers = 2

No. of 2 digit numbers = $3 \times 2 = 6$

No. of 3 digit numbers = $3 \times 3 \times 2 = 18$

No. of 4 digit numbers = $3^3 \times 2 = 54$

No. of 5 digit numbers = $3^4 \times 2 = 162$

No. of 6 digit numbers (less than a million) = $3^5 \times 2 = 486$

$\therefore$ Required numbers = $486 + 162 + 54 + 18 + 6 + 2 = 728$

23.

Sol. Consider the number : x y z where

$x < y$, $z < y$ and $x \neq 0$.

If y = 9, x can be between 1 to 8 and z can be between 0 to 8.

Total combination = $9 \times 8 = 72$

If y = 8, x can be between 1 to 7 and z can be between 0 to 7.

Combinations = $7 \times 8 = 56$

Similarly, we add all combinations

$8 \times 9 + 7 \times 8 + 6 \times 7 + 5 \times 6 + 4 \times 5 + 3 \times 4 + 2 \times 3 + 1 \times 2 = 240$ ways

24.

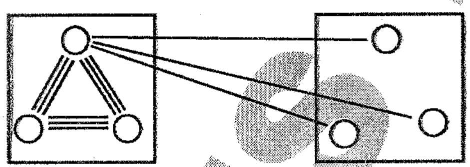
Sol. As the boxes containing the green balls are consecutively numbered total number of ways will be $6 + 5 + 4 + 3 + 2 + 1 = 21$ ways.

25.

Sol. There are 12 points. Since they can be reached from any other point, the edges will be $^{12}C_2 = 66$. Also the minimum number of edges will be 11.

26.

Sol.



Consider zone 1

No. of lines for internal connections in each zone = 9

Total number of lines for internal connections in four zones = $9 \times 4 = 36$

No. of lines for external connections between any two zones = $3 \times 3 = 9$ (as shown in figure)

$\therefore$ Total no. of lines required for connecting towns of different zones

$= {}^4P_2 \times 9 = 6 \times 9 = 54$

$\therefore$ Total no. of lines in all = $54 + 36 = 90$

27.

Sol. There are 28 minutes, hence total no. of songs are 14.

Since each pair sings one song. Hence, total number of pairs is 14

Since, in each possible pair persons are not standing next to each other.

$\therefore {}^nC_2 - n = 14 \Rightarrow n = 7$

Hence, total number of people = 7.

28.

Sol. The car requires 3 steps north and 5 steps west so as to reach B.

Hence, total no. of ways = $\frac{8!}{5!3!} = \frac{8 \times 7 \times 6}{6} = 56$

29.

Sol. There are 6 stripes. The first stripe can be coloured in 4 ways, 2nd in 3, 3rd in 3, and so on.

Thus total no. of ways = $4 \times 3 \times 3 \times 3 \times 3 \times 3 = 12 \times 81$

30.

Sol. Task 2 can be assigned to 3 or 4.

So, there are only 2 options for task 2.

Now, task 1 can not be assigned to 1 or 2 i.e. there are 3 options.

So required no. of ways = (2 options for task 2) × (3 options for task 1)× (4 options for task 3) × (2 options for task 5) × (1 option for task 6) × (3 options for task 4)

$= 2 \times 3 \times 4 \times 3 \times 2 \times 1 = 144.$

31.

Sol. If ten's digit is 1, we have two such numbers viz. 10 and 11.

When ten's digit is 2, we have three such numbers, viz 20, 21 and 22.

.....

.....

When ten's digits is 9, we have 10 such numbers viz. 90, 91.....99.

$\therefore$ Total number of such numbers is 2 + 3 + 4 + ... + 10 = 54

32.

Sol. Let the three digit numbers satisfying the giving conditions be denoted by xyz. $x < y < z$.

If x = 1, y has possibilities. For each value of y, z can have values from y + 1 to 9.

Number of number satisfying $x < y < z$ is

7 + 6 + 5 .... + 1 = 28

If x = 2, xyz can be similarly shown to have 21 possibilities it can be seen that if y has k possibilities, xyz has

$\frac{(k)(k+1)}{2}$ possibilities

Total number of possibilities

$$= 28 + 21 + \frac{(5)(6)}{2} + \frac{(4)(5)}{2} + \frac{(3)(4)}{2} + \frac{(2)(3)}{2} + \frac{(1)(2)}{2} = 84$$

Alternate Method

Let xyz be the number if x = 1, we can select any 2 numbers from 2 to 9 for y and z in $^9C_2$ ways.

Similarly, if x = 2, we can select any 2 numbers from 3 to 9 for y and z in $^7C_2$. Similarly for x = 3 we will have $^6C_2$ ways, for x = 4 $^5C_2$ ways, for x=5, $^4C_2$ ways, $^4C_2$ ways for x = 6, $^3C_2$ ways, for x = 7, $^2C_2$ i.e. only 1 way i.e. y = 8 and z = 9.

Hence total number of numbers will be

$^8C_2 + {^7C_2} + {^6C_2} + {^5C_2} + {^4C_2} + {^3C_2} + {^2C_2} = 84$

CHAPTER 6

Probability

Introduction

Probability involves the study of events in which one of several outcomes is possible. For example, tossing a coin will result in one of the two outcomes-head or tail. Rolling a standard die results in one of six outcomes: 1, 2, 3, 4, 5, or 6. Drawing a card from a standard deck results in one of 52 outcomes. In each of these examples, we can assign to each of the possible outcome a number which measures the likelihood that the outcome will occur. A probability is a number $0 \leq p \leq 1$, with the convention that p = 0 means an outcome is impossible and p = 1 means an outcomes is certain.

The probability p can be defined in a number of ways, For example,

$$\text{Probability p of an event} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

$$\text{or Probability p of an event} = \frac{m}{m+n}$$

where,

m = number of ways the event can happen in the given outcomes and

n = the number of ways the event cannot happen in the given outcomes.

For example, if we toss three coins, the total outcomes are 8 in all (HHH), (HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT). If we are trying to calculate the probability that we shall have exactly two heads, the probability is equal to $\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{8}$

Now suppose we are asked the following questions-From a pack of cards, a card is drawn. What is the probability of drawing a

(a) Jack or a queen?

(b) Jack or a red card?

If we make all the possible cases, the favorable cases in case

(a) would be J(♦) or J(♣) or J(♥) or J(♠) and Q(♦) or Q(♣) or Q(♥) or Q(♠) i.e. 8 favorable cases out of 52. Therefore, the probability of drawing a Jack or a queen is $\frac{8}{52} = \frac{2}{13}$

(b) would be 26 red cards (¨ and♥) and two extra jacks – J(♣) and J(♠). Therefore, the probability of drawing a red card or a jack is $\frac{28}{52} = \frac{7}{13}$

What is the difference between example (a) and (b)? Simply this-there is no card in common between jacks and queens in (a) whereas there are two cards in common between jacks and red cards – J(♦) and J(♥). The difference is whether two events are mutually exclusive or not.

Two events are mutually exclusive or disjoint if they have no outcomes is common. In other words, two mutually exclusive events cannot occur simultaneously. If A and B mutually exclusive, the probability that either event A or event B occurs is

P(A or B) = P(A) + P(B).

This result extends to multiple mutually exclusive events. For example, in rolling two dice, the probability of rolling a sum of 4 is $\frac{3}{36}=\frac{1}{12}$, and the probability of rolling a sum of 7 is $\frac{1}{6}$. These events are mutually exclusive, and so P(sum of 4 or 7) = P(sum of 4) + P(sum of 7) $=\frac{1}{12}+\frac{1}{6}=\frac{1}{4}$

For non-mutually exclusive events, we need to be more careful. In this case, we have

P(A or B) = P(A) + P(B) – P(A and B).

For example, in rolling two dice, the probability of rolling an even sum is $\frac{1}{2}$, and the probability of rolling a sum which is a multiple of 3 is $\frac{12}{36}=\frac{1}{3}$. These events are non-mutually exclusive, and so P(sum multiple of 2 or 3) = P(sum multiple of 2) + P(sum multiple of 3) – P(sum multiple of 2 and 3) = $\frac{1}{2}+\frac{1}{3}-\frac{1}{6}=\frac{2}{3}$.

Now what is the difference between the following two questions.

Independent Events

Two events A and B independent in the probability of one event's occuring is not affected by the other event occurs. If A and B are independent, the probability that event A and B occur is

P(A and B) = P(A) × P(B).

For example, in rolling a die three times, the outcome of one roll is independent of the other rolls. Therefore, the probability of rolling three 6's = P(6 on 1st roll) × P(6 on 2nd roll) × P(6 on 3rd roll) = $\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}=\frac{1}{216}$

So in example 2, as the cards are drawn with replacement, the two events are independent of each other. Since we can have two favorable cases-(QJ) or (JQ), the probability = $2\times\frac{4}{52}\times\frac{4}{52}=\frac{2}{169}$

Understand that we are drawing the two cards one by one in the above problem. If the problem was-From a pack of cards, two cards are drawn simultanously. What is the probability of drawing a Jack and a queen?- the probability would have been $\frac{{}^4C_1\times{}^4C_1}{{}^{52}C_2}=\frac{8}{663}$

Example 1.

A and B are throwing with two dice. If A throws a sum of 8, find the probability that B will throw a higher number.

Sol.

The total number of outcomes are $6 \times 6 = 36$. The favorable cases are that B throws a sum of 9, 10, 11 or 12. The numbers of ways B can throw 9, 10, 11, and 12 are 4, 3, 2, 1 respectively. So

Probability that B will throw a higher No. $\frac{4+3+2+1}{36} = \frac{10}{36} = \frac{5}{18}$

Do we always need to consider the favorable cases to calculate the probability? Not really, sometimes we use the following concept

If p is the probability of happening of an event, then the probability of that event not happening is 1 –p.

Example 2.

10 persons are sitting around a round table. What is the probability that two particular persons are NOT sitting next to each other?

Sol.

The probability that two particular persons are NOT sitting next to each other = 1 – probability that two particular persons are sitting next to each other = $1 - \frac{2 \times 8!}{9!} = \frac{7}{9}$

Example 3.

The probability that a man speak a true statement is $\frac{3}{4}$. If he gives three statements in a row, what is the probability that he told at least one lie?

Sol.

Without further ado, the probability that he told at least one lie = 1 – probability that he spoke all true statements = $1 - \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{37}{64}$

Example 4.

From a pack of cards, two cards are drawn one after another without replacement. What is the probability of drawing a Jack and a queen?

Sol.

Probability of drawing a Jack and a queen = Probability of drawing a jack first × probability of drawing a queen second + Probability of drawing a queen first × probability of drawing a jack second

Probability of drawing a Jack and a queen $= \frac{4}{52} \times \frac{4}{51} + \frac{4}{52} \times \frac{4}{51} = 2 \times \frac{1}{13} \times \frac{4}{51} = \frac{8}{663}$

Example 5.

There are three bags- bag 1 contains 3 red balls and 2 white balls, bag 2 contains 4 red balls and 2 blue balls, and bag 3 contains 3 blue balls and 5 white balls. A bag is selected at random and a ball is drawn. What is the probability that it is red?

Sol.

We can see here that drawing a red ball here is dependent on the selection of the bag. So we have two dependent events here-selection of bag and selection of red ball from the bag. We can see that

Probability of drawing a red ball = probability of selecting bag × probability of drawing a red ball from bag

Probability of drawing a red ball = $\frac{1}{3}\times\frac{3}{5}+\frac{1}{3}\times\frac{4}{6}+\frac{1}{3}\times 0=\frac{19}{45}$

Example 6.

Three unbiased coins are tossed. What is the probability of getting the following?

(i) All heads (ii) 2 heads (iii) Exactly 1 head

(iv) At least 1 head (v) At least 2 heads

Sol.

If 3 coins are tossed together, we can obtain any one of the following as an outcome.

HHH, HHT, HTH, THH, TTH, THT, HTT, TTT

So exhaustive number of cases = 8

(i) All heads can be obtained in only one way, i.e. HHH

So the favourable number of cases = 1

Thus, the required probability $=\frac{1}{8}$

(ii) Two heads can be obtained in any one of the following ways: HHT, THH, HTH.

So favourable number of cases = 3. Thus, required probability $=\frac{3}{8}$

(iii) Required probability $=\frac{3}{8}$

(iv) Required probability $=1-\frac{1}{8}=\frac{7}{8}$

(v) Required probability $=\frac{3}{8}+\frac{1}{8}=\frac{1}{2}$

Example 7.

An bag contains 9 red, 7 white and 4 black balls. If 2 balls are drawn at random, find the probability that (i) both the balls are red, (ii) one ball is white.

Sol.

There are 20 balls in the bag out of which 2 balls can be drawn in ${}^{20}C_2$ ways. So the exhaustive number of cases = ${}^{20}C_2$ = 190

(i) There are 9 red balls out of which 2 balls can be drawn in $^9C_2$ ways. Therefore, favourable number of cases = $^9C_2 = 36$. So the required probability = $\frac{36}{190} = \frac{18}{95}$

(ii) There are 7 white balls out of which one white can be drawn in $^7C_1$ ways. One ball from the remaining 13 balls can be drawn in $^{13}C_1$ ways. Therefore, one white and one other colour ball can be drawn in $^7C_1 \times {}^{13}C_1$ ways. So the favourable number of cases = $^7C_1 \times {}^{13}C_1 = 91$

So the required probability = $\frac{91}{190}$

Example 8.

Ramesh and Geeta were in the same class. The probability of Ramesh attending the class is 0.6. The probability of Geeta attending The class is 0.4.

(a) What is the probability that both of them attended the class?

(b) What is the probability that at least one of them attended the class?

Sol.

(a) Since the 2 events happen independent of each other, the probability of both Ramesh and Geeta attending the class simultaneously is $0.6 \times 0.4 = 0.24$

(b) The probability of at least one of them attending the class is

P(Ramesh attends) + P(Geeta attends) – P (Both attend)

$\Rightarrow 0.6 + 0.4 - 0.24 = 0.76$

Example 9.

Two machines A and B produce 100 and 200 items every day. Machine A produce 10 defective items and machine B produces 40 defective items. On one particular day the supervisor of the shop floor picked up an item and found that it was defective. Find the probability that it came from machine A.

Sol.

Method 1

The total number of defective items produced on any single day = 50

The number of defective items from machine A = 10

Hence, probability of that item having come from machine A = $\frac{10}{50} = \frac{1}{5}$

Method 2

Probability of finding a defective item = Probability that it is from machine A and is defective + Probability that it is from machine B and is defective

$\Rightarrow \frac{100}{300} \times \frac{10}{100} + \frac{200}{300} \times \frac{40}{200} = \frac{50}{300} = \frac{1}{6}$

Hence, probability that the defective item is from machine A= $\dfrac{\frac{100}{300} \times \frac{10}{100}}{\frac{1}{6}} = \frac{1}{5}$

Example 10.

If n persons are seated on a round table, what is the probability that 2 of them are always together?

Sol.

Total number of ways in which n persons can sit on a round table is (n –1)! Therefore, exhaustive number of cases = (n –1)! Considering 2 individuals as one persons there are (n –1) persons who can sit on a round table in (n –2)! ways. But the 2 individuals can be seated together in 2! ways. Therefore, favourable number of cases = (n –2)! × 2!

So required probability = $\frac{(n-2)!\times 2!}{(n-1)!} = \frac{2}{n-1}$

Example 11.

Three different prizes have to be distributed among 4 different students. Each student could get 0 to 3 prizes. If all the prizes were distributed, find

(a) the number of ways the prizes are distributed.

(b) the probability that exactly 2 students did not receive a prize.

Sol.

(a) Each of the prizes could have been given to any of the 4 students.

Hence, the total number of ways of distributing the prizes = 4^3

Note: This will include all the cases when the prizes are distributed among 3 or 2 or only 1 student.

(b) The total number of ways of distributing the prizes among exactly 2 students is $({}^4C_2)(2^3 - 2)$ ways = 6 × 6 = 36 ways.

${}^4C_2$ gives the selection of 2 boys.

$2^3 - 2$ gives the total number of ways of distributing 3 prizes among those 2 students. The subtraction of the 2 cases is to take care of those cases when all the prizes are distributed to only one among the two.

∴ The required probability = $\frac{36}{64} = \frac{9}{16}$

Example 12.

A and B are two players, each throwing two dice on his turn, with A starting first and players taking turns alternately. A wins if he throws an 8 and B wins if he throws a 7. What is the probability that A wins?

Sol.

Probability that A wins = P(A throws 8 on his first chance) + P(A doesn't throw 8 on his first chance) × P(B doesn't throws 7 on his first chance) ×P(A throw 8 on his 2nd chance) + P(A doesn't throw 8 on his first chance) × P(B doesn't throws 7 on his first chance) × P(A doesn't throws 8 on his second chance) × P(B doesn't throws 7 on his second chance) × P(A throws 8 on his third chance) + ... And so on.

Probability that A throws 8 = 5/36, So probability that A doesn't throws 8 = 1– 5/36 = 31/36

Probability that B throws 7 = 6/36. So probability that B doesn't throws 7 = 1– 6/36 = 31/36

Therefore, probability that A wins

$$= \frac{5}{36} + \frac{31}{36} \times \frac{31}{36} \times \frac{5}{36} + \frac{31}{36} \times \frac{31}{36} \times \frac{31}{36} \times \frac{31}{36} \times \frac{5}{36} + \ldots$$

$$= \frac{5}{36}\left[1 + \left(\frac{31}{36}\right)^2 + \left(\frac{31}{36}\right)^4 + \left(\frac{31}{36}\right)^6 \ldots\ldots\infty\right]$$

[This is an infinite G.P. with r = 31/36 which is less than 1]

$$= \frac{5}{36} \times \frac{1}{1 - \frac{961}{1296}} = \frac{5}{36} \times \frac{1296}{335} = \frac{36}{67}$$

Example 13.

If we throw a dice 10 times what is the probability of it showing 6 exactly 3 times?

Sol.

Out of 10 throws, we can choose the 3 throws in which it will show 6 in ${}^{10}C_3$ ways. The probability of the dice showing 6 in these three throws is $\left(\frac{1}{6}\right)^3$. Also, it should not show 6 in the rest of the 7 throws for which the probability is $\left(\frac{5}{6}\right)^7$.

Therefore, the probability of it showing 6 exactly 3 times = ${}^{10}C_3 \times \left(\frac{1}{6}\right)^3 \times \left(\frac{5}{6}\right)^7$.

If you have understood what we did, you will understand the following formula also

Note

If the probability of an event happening in a single trial is p, then the proobbility that event will happen in exactly r trials out of n trials is ${}^nC_r \times p^r \times (1-p)^{n-r}$.

Compound Probability and Conditional Probability

The probability of the joint occurrence of two or more events is called **Compound Probability** and is denoted by $P(A \cap B)$ for two events A and B, and by $P(A_1 \cap A_2 \cap \ldots \cap A_r)$ for r events $A_1, A_2, \ldots, A_r$.

The probability of the occurrence of the event B assuming that the event A has actually occurred is called the **Conditional Probability** of B and is denoted by $P(B/A)$, which is read as the probability of B given A.

Theorem of Compound Probability (The Rule of Multiplication)

$P(A \cap B)$ = The probability of the joint occurrence of A and B

= P(A) . P(B/A), where P(B/A) is the conditional probability of B, assuming that A has actually occurred.

Extension: For three events, A, B and C, we have

$P(A \cap B \cap C) = P(A) \cdot P(B/A) \cdot P(C/A \cap B)$

Compound Probability for independent events

If two events are independent; the occurrence of one of them will not affect the probability of occurrence of the other.

$$\therefore \quad P(B/A) = \frac{P(A \cap B)}{P(A)} \text{ and } P(B/A) = \frac{P(A \cap B)}{P(B)}$$

Thus for two independent events A and B, the theorem of Compound Probability becomes

$$P(A \cap B) = P(A) \cdot P(B)$$

For r mutually independent events A_1, A_2, ..., A_r, we have

$$P(A_1 \cap A_2 \cap .... \cap A_r) = P(A_1) \cdot P(A_2) ..... P(A_r)$$

Dependent and Independent Events

Two events A and B are said to be independent if the occurrence of one of them, say A, does not affect the probability of occurrence of the other event B. In other words, A and B are independent if and only if

P(B/A) = P(B) and P(A/B) = P(A), i.e., $P(A \cap B) = P(A) \cdot P(B)$

Otherwise, they are said to be dependent (or interdependent) events. For dependent events, the occurrence or non-occurrence of one of them affects the probability of occurrence of the other events. If A and B are two dependent events, then

$P(A \cap B) = P(A)P(B/A)$, or P(B)P(A/B)

Example 16.

In an examination, 30% of the students have failed in Mathematics, 20% of the students have failed in Chemistry and 10% have failed in both mathematics and Chemistry. A student is selected at random.

(i) What is the probability that the student has failed in Mathematics if it is known that he has failed in Chemistry?

(ii) What is the probability that the student has failed either in Mathematics or in Chemistry?

Sol.

Let M and C be the events that a student selected at random 'Fail in Mathematics' and 'Fail in Chemistry' respectively.

Then $P(M) = \frac{30}{100} = 0.3$, $P(C) = \frac{20}{100} = 0.2$ and $P(M \cap C) = \frac{10}{100} = 0.1$

(i) The required probability that the student selected has failed in Mathematics, given that he has failed in Chemistry

$$= P(M/C) = \frac{P(M \cap C)}{P(C)} = \frac{0.1}{0.2} = \frac{1}{2} = 0.5$$

(ii) The required probability that the student selected at random has failed either in Mathematics or in Chemistry

$$= P(M \cup C) = P(M) + P(C) - P(M \cap C) = 0.3 + 0.2 - 0.1 = 0.4$$

Example 17.

A can solve 80 per cent of the problems given in statistics and B can solve 60 per cent. What is the probability that at least one of them will solve a problem selected at random?

Sol.

$= 1 - P(A) = 1 - \frac{4}{5} = \frac{1}{5}$ Similarly, $P(B') = 1 - P(B) = 1 - \frac{3}{5} = \frac{2}{5}$

∴ Probability that none of A and B can solve the problem (i.e. both A and B cannot solve the problem)

$= P(A' \cap B') = P(A') \cdot P(B') = \frac{1}{5} \times \frac{2}{5} = \frac{2}{25}$

[Since A and B are independent events, A and B are also independent]

Hence the required probability that at least one of A and B will solve the problem

= 1 – (Probability that none of A and B can solve the problem)

$= 1 - \frac{2}{25} = \frac{23}{25} = 0.92$

Example 18.

The probabilities of solving a problem by three students A, B, C are $\frac{2}{7}, \frac{3}{8}$ and $\frac{1}{2}$ respectively. If all of them try independently, find the probability that the problem is solved. Find also the probability that the problem could not be solved.

Sol.

Let A be the event that student A will solve the problem. Similarly, events B and C for the students B and C solving the problem. Then

$P(A) = \frac{2}{7}$, $P(B) = \frac{3}{8}$, $P(C) = \frac{1}{2}$

P(A') = Probability that A will not solve the problem

$= 1 - P(A) = 1 - \frac{2}{7} = \frac{5}{7}$ Similarly, $P(B') = 1 - \frac{3}{8} = \frac{5}{8}$ and $P(C') = 1 - \frac{1}{2} = \frac{1}{2}$

The probability that none of A, B, C can solve the problem

$= A(A' \cap B' \cap C') = P(A') \cdot P(B') \cdot P(C') = \frac{5}{7} \times \frac{5}{8} \times \frac{1}{2} = \frac{25}{112}$

[Since A', B, C' are independent events.]

Hence the required probability that the problem will be solved

= 1 – (Probability that none of A, B, C can solve the problem)= $1 - \frac{25}{112} = \frac{87}{112}$

The probability that the problem could not be solved = $\frac{25}{112}$

Example 19.

A bag contains 4 red and 3 blue balls. Two drawings of 2 balls are made. Find the probability of drawing first 2 red balls and second 2 blue balls.

(i) If the balls are returned to the bag after the first draw.

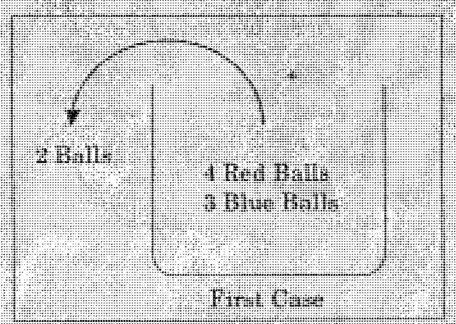
(ii) If the balls are not returned after the first draw.

Sol.

Let A be the event that first 2 balls drawn are red and B the event that second 2 balls drawn are blue.

(i) P(A) = Probability of drawing 2 red balls in the first draw

$$= \frac{^4C_2}{^7C_2} = \frac{\frac{4\cdot 3}{2\cdot 1}}{\frac{7\cdot 6}{2\cdot 1}} = \frac{2}{7} \qquad \text{.........(1)}$$



Since the 2 red balls drawn in the first draw are returned to the bag before the second draw, the total no. of balls remains the same.

$\therefore$ P(B) = Probability of drawing 2 blue balls in the second draw

$$= \frac{^3C_2}{^7C_2} = \frac{\frac{3\cdot 2}{2\cdot 1}}{\frac{7\cdot 6}{2\cdot 1}} = \frac{1}{7}$$

In this case, A and B are two independent events.

Hence the required probability of drawing first 2 red and second 2 blue balls

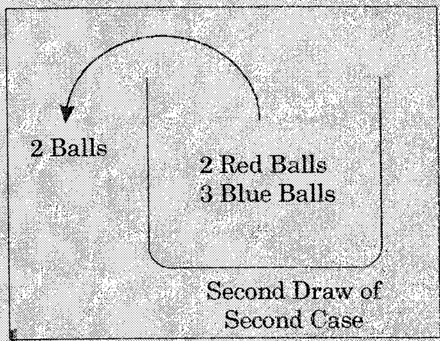
$$= P(A \cap B) = P(A)\cdot P(B) = \frac{2}{7} \times \frac{1}{7} = \frac{2}{49}$$

(ii) If the 2 red balls drawn in the first draw are not returned before the second draw, then we are left with 2 red and 3 blue balls, the total number of balls being 2 + 3 i.e. 5. Clearly, the probability of the first event (i.e. first drawing) affects the probability of the second drawing.

$\therefore$ P(B/A) = Probability that 2 blue balls are drawn in the second draw

$$= \frac{^3C_2}{^5C_2} = \frac{\frac{3\cdot 1}{2\cdot 1}}{\frac{5\cdot 4}{2\cdot 1}} = \frac{3}{10}$$

Hence the required probability of drawing 2 red balls in the first draw and 2 blue balls in the second draw when the 2 balls not returned after the first draw.



$$= P(A \cap B) = P(A) \cdot P(B/A) = \frac{2}{7} \times \frac{3}{10} = \frac{3}{35}$$

Bayes' Theorem

Statement: An event A can occur only if any one of the set of exhaustive and mutually exclusive events $B_1, B_2, \ldots, B_n$ occurs. The probabilities $P(B_1), P(B_2), \ldots, P(B_n)$ and the conditional probabilities $P(A/B_i)$; $i = 1, 2, \ldots, n$, for A to occur are known. Then the conditional probability $P(B_i/A)$ when A has actually occurred is given by

$$P(B_i/A) = \frac{P(B_i)P(A/B_i)}{P(B_1)\cdot P(A/B_1) + P(B_2)\cdot P(A/B_2) + \ldots + P(B_n)\cdot P(A/B_n)}$$

Example 20.

Two identical boxes contain respectively 4 white and 3 red balls, and 3 white and 7 red balls. A box is chosen at random and a ball is drawn from it. If the ball is white, what is the probability that it is from the first box?

Sol.

Let A be the event that the ball drawn is white and let B_1, B_2 be the events of choosing the first and the second boxes respectively.

Then $P(B_1)$ = Probability of selecting the first box $= \frac{1}{2}$

and $P(B_2)$ = Probability of selecting the second box $= \frac{1}{2}$

$P(A/B_1)$ = Probability of selecting 1 white ball from the first box $= \frac{{}^4C_1}{{}^7C_1} = \frac{4}{7}$

Similarly, $P(A/B_2) = \frac{3}{10}$

By Bayes' Theorem, we get

$P(B_1/A)$ = the required probability that if the ball is white, then it is from the first box

$$= \frac{P(B_1)\cdot P(A/B_1)}{P(B_1)\cdot P(A/B_1)+P(B_2)\cdot P(A/B_2)}$$

$$= \frac{\frac{1}{2}\times\frac{4}{7}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{3}{10}} = \frac{4}{14}\times\frac{140}{61} = \frac{40}{61}$$

Note

$P(B_2/A)$ = Probability that if the ball is white, it is from the 2nd box

$$= \frac{P(B_2)\cdot P(A/B_2)}{P(B_1)\cdot P(A/B_1)+P(B_2)\cdot P(A/B_2)} = \frac{\frac{1}{2}\times\frac{3}{10}}{\frac{61}{140}} = \frac{21}{61}$$

Example 21.

In a bolt factory, machines M_1, M_2, M_3 manufacture respectively 25, 35 and 40 per cent of the total output. Of their output, 5, 4, and 2 per cent respectively, are defective bolts. One bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured in the machine M_2?

Sol.

Let B_1, B_2, B_3 be the events that the bolt drawn at random is manufactured by machines M_1, M_2, M_3 respectively, and let A be the event that the bolt drawn is defective.

Then $P(B_1)$ = Probability that the bolt drawn was manufactured by $M_1 = \frac{25}{100} = \frac{1}{4}$.

Similarly, $P(B_2) = \frac{35}{100} = \frac{7}{20}$ and $P(B_3) = \frac{40}{100} = \frac{2}{5}$

$P(A/B_1)$ = the probability of drawing a defective bolt from the lot of bolts manufactured by M_1

$= \frac{5}{100} = \frac{1}{20}$

Similarly, $P(A/B_2) = \frac{4}{100} = \frac{1}{25}$ and $P(A/B_3) = \frac{2}{100} = \frac{1}{50}$

By Bayes' Theorem, we have

$$P(B_2/A) = \frac{P(B_2)\cdot P(A/B_2)}{P(B_1)\cdot P(A/B_1)+P(B_2)\cdot P(A/B_2)+P(B_3)\cdot P(A/B_3)}$$

$$= \frac{\frac{7}{20}\times\frac{1}{25}}{\frac{1}{4}\times\frac{1}{20}+\frac{7}{20}\times\frac{1}{25}+\frac{2}{5}\times\frac{1}{50}} = \frac{\frac{7}{500}}{\frac{25+28+16}{2000}} = \frac{7}{500}\times\frac{2000}{69} = \frac{28}{69}$$

Hence the required probability that the defective bolt drawn is manufactured by machine M_2

$= \frac{28}{69}$

Example 22.

The chance that doctor A will diagnose disease B correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease B, died. What is the chance that his disease was correctly diagnosed?

Sol.

Let B_1 be the event that Dr. A will diagnose disease B correctly, B_2 the event that Dr. A will diagnose disease B wrongly and A the event that a patient of Dr. A who had disease B will die.

Then $P(B_1) = \frac{60}{100} = 0.6$, $P(B_2) = 1 - P(B_1) = 1 - 0.6 = 0.4$,

$P(A/B_1) = \frac{40}{100} = 0.4$ and $P(A/B_2) = \frac{70}{100} = 0.7$

By Bayes' Theorem, we get

$$P(B_1/A) = \frac{P(B_1)\cdot P(A/B_1)}{P(B_1)\cdot P(A/B_1) + P(B_2)\cdot P(A/B_2)} = \frac{0.6\times 0.4}{0.6\times 0.4 + 0.4\times 0.7} = \frac{0.24}{0.24+0.28} = \frac{0.24}{0.52}$$

$$= \frac{24}{52} = \frac{6}{13}$$

Hence the chance that his disease was correctly diagnosed $= \frac{6}{13}$

Example 23.

Three horses A, B and C are in a race: A is twice as likely to win as B and B is twice as likely to win as C.

Sol.

Let the probability of the event that C will win = p, i.e., P(C) = p

Then the probability that B will win = 2p, i.e., P(B) = 2p

and the probability that A will win = 2 × 2p = 4p i.e., P(A) = 4p

Since either A or B or C will win the race;

$\therefore$ p+2p+4p = 1, or, 7p = 1, or $p = \frac{1}{7}$

Example 24.

There are 100 students in a college class of which 36 are boys studying Statistics and 13 are girls not studying Statistics. If there are 55 girls in all, find the probability that a boy picked up at random is not studying Statistics.

Sol.

Total number of boys in the college class = 100 − 55 = 45

1 boy can be selected out of 45 boys in $^{45}C_1$ ways = 45 ways.

∴ Total number of all possible cases for the event = 45.

Now 1 boy studying Statistics can be selected out of 36 in $^{36}C_1$ ways = 36 ways.

∴ No. of cases favourable to the event A that the boy picked up at random is studying Statistics = 36.

∴ P(A) = Probability that the boy picked up is studying Statistics = $\frac{36}{45} = \frac{4}{5}$

Hence the required probability that the boy picked up at random is not studying Statistics

$$= P(\bar{A}) = 1 - P(A) = 1 - \frac{4}{5} = \frac{1}{5}$$

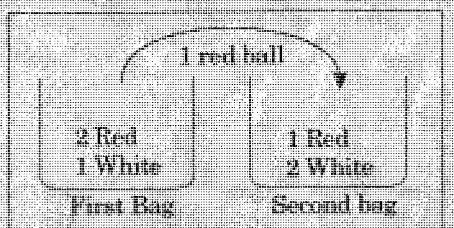
Example 25

There are two bags. The first contains 2 red and 1 white balls wherease the second bag has only 1 red and 2 white balls. One ball is taken out at random from the first bag and is put in the second bag. Then a ball is chosen at random from the second bag. What is the probability that this last ball is red?

Sol.

The ball transferred from the first to the second bag may either be red or white.

Case 1: 1 red ball is transferred from the first to the second bag.

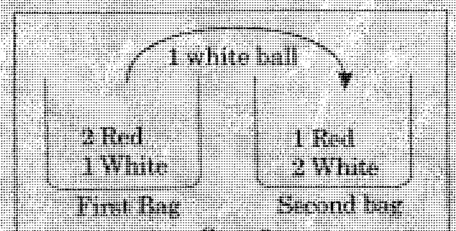


Case 1

Probability of transferring 1 red ball from the first to the second bag = $\frac{^2C_1}{^3C_1} = \frac{2}{3}$

After 1 red ball is transferred, the no. of red balls in the second bag = 1 + 1 = 2 and the total no. of balls in the second bag = 2 + 2 = 4.

∴ Probability of drawing 1 red ball from the second bag = $\frac{^2C_1}{^4C_1} = \frac{2}{4} = \frac{1}{2}$



Case 2

∴ Probability of first transferring 1 red ball and then drawing 1 red ball from the second bag

$$= \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

Case 2: White ball is transferred from the first to the second bag.

Probability of transferring 1 white ball from the first to the second bag = $\frac{^1C_1}{^3C_1} = \frac{1}{3}$

As in Case 1, probability of drawing 1 red ball from the second bag = $\frac{^1C_1}{^4C_1} = \frac{1}{4}$

$\therefore$ Probability of the first transferring 1 white ball and then drawing 1 red ball from the second bag

$$= \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

Hence by the theorem of total probability (mutually exclusive events), the required probability.

$$= \frac{1}{3} + \frac{1}{12} = \frac{4+1}{12} = \frac{5}{12}$$

Objective Questions

1. A manufacturer claims that only 2% items are defective in a shipment of 200 items sent by him. A random sample of two items is drawn from the shipment of 200 items. what is the probability that both the items drawn are defective?

(a) $\frac{3}{19900}$ (b) $\frac{6}{19900}$

(c) $\frac{9}{19900}$ (d) none of these

2. The independent probabilities that the three sections of an account department will encounter error are 0.2, 0.3 and 0.1 per week respectively. What is the probability that there would be at least one computer error per week?

(a) 0.504 (b) 0.006

(c) 0.60 (d) 0.496

3. From amongst the available 15 cricket players, of whom 5 are bowlers, a team of 11 is to be selected. what is the probability that the selected team will have at least 3 bowlers?

(a) $\frac{7}{13}$ (b) $\frac{5}{13}$

(c) $\frac{12}{13}$ (d) $\frac{9}{13}$

4. A managing committee of 7 members is to be constitumed from a group comprising 8 gentlemen and 5 ladies. What is the probability that the comittee would comprise 2 ladies.

(a) 280/429 (b) 70/429

(c) 210/429 (d) 140/429

5. A card is drawn at random from a well shuffled pack of 52 cards:

X : The card drawn is black or a king.

Y : The card drawn is a club or a heart or a jack.

Z : The card drawn is an ace or a diamond or a queen.

Then which of the following is correct?

(a) $p(X)>p(Y)>p(Z)$ (b) $p(X)>p(Y)=p(Z)$

(c) $p(X)=p(Y)>p(Z)$ (d) $p(X)=p(Y)=p(Z)$

6. The odds against a certain event are 5 : 2 and the odds in favour of another independent event are 6 : 5. the probability that at least one of the events will happen is:

(a) $\frac{12}{77}$ (b) $\frac{25}{77}$

(c) $\frac{52}{77}$ (d) $\frac{65}{77}$

7. Three screws are drawn at random from a lot of 100 screws, 10 of which are defective. The probability of the event that all 3 screws drawn are non-defective, assuming that the screws are drawn without replacement, is:

(a) 70.65% (b) 71.65%

(c) 74.65% (d) 72.65%

8. In a single throw of three dice, the probaility of not getting the same number on any two dice is:

(a) 0.45 (b) 0.66

(c) 0.56 (d) 0.83

Answers Objective Questions

1.	(b)	2.	(d)	3.	(c)	4.	(d)	5.	(c)
6.	(c)	7.	(d)	8.	(d)				

Solutions :—

1.

Sol. total no. of items = 200, No. of defective items = $200\times\frac{2}{100}=4$

P(1st item is defective) = $\frac{4}{200}$, P(2nd item is defective) = $\frac{3}{199}$

So, Required Prob = $\frac{4}{200}\times\frac{3}{199} = \frac{2\times2}{200}\times\frac{3}{199} = \frac{6}{19900}$

2.

Sol. Given probabilities of encounter error are 0.2, 0.3 and 0.1.

$\therefore$ Probabilities of non-encounter are 1 - 0.2 = 0.8, 1 – 0.3 = 0.7 and 1 – 0.1 = 0.9

Hence, P(at least one error) =1 – P(no error will be there in any section)

= 1 – [0.8 × 0.7 × 0.9]= 1 – 0.504 = 0.496 ($\because$ all the probalities are independent)

3.

Sol. Total players = 15

Total no. of selected playeres = 11 $\therefore$ P (select 11 players out of 15) = ${}^{15}C_{11}$

P (atleast 3 bowlers) = P (3 bowlers) + P (4 bowlers) + P (5 bowlers)

($\because$ Total no. of bowlers = 5)

$$= \frac{{}^5C_3 \times {}^{10}C_8}{{}^{15}C_{11}} + \frac{{}^5C_4 \times {}^{10}C_7}{{}^{15}C_{11}} + \frac{{}^5C_5 \times {}^{10}C_6}{{}^{15}C_{11}} = \frac{10 \times 45}{1365} + \frac{5 \times 210}{1365} + \frac{1 \times 120}{1365} = \frac{252}{273} = \frac{12}{13}$$

4.

Sol. Total members = 8 + 5 = 13

We have to select 7 members out of 13.

$\therefore$ No. of ways to choose 7 members out of 13 is ${}^{13}C_7$ and we have 5 ladies out of which 2 ladies have to be selected.

$$\therefore \text{ Required prob} = \frac{{}^8C_5 \times {}^5C_2}{{}^{13}C_7} = \frac{\frac{8!}{5!3!} \times \frac{5!}{2!3!}}{\frac{13!}{7!6!}} = \frac{560}{1716} = \frac{140}{429}$$

5.

Sol. $p(X) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52}$

$p(Y) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52}$

$p(Z) = \frac{13}{52} + \frac{8}{52} - \frac{2}{52} = \frac{19}{52}$

$\therefore p(X) = p(Y) > p(Z)$

6.

Sol. Odds against a certain event = 5 : 2

$\therefore$ Probability of happening $= \frac{2}{2+5} = \frac{2}{7}$

Odds in favour of another event = 6 : 5

$\therefore$ Probability of happening $= \frac{6}{6+5} = \frac{6}{11}$

Hence, the probility that at least one of the events will happen $= 1 - \left[\left(1 - \frac{2}{7}\right)\left(1 - \frac{6}{11}\right)\right] = \frac{52}{77}$

7.

Sol. Probability of all non defective screws = $\frac{^{90}C_3}{^{100}C_3} = \frac{90 \times 89 \times 88}{100 \times 99 \times 98} = 0.7265 = 72.65\%$

8.

Sol. Required probability = 1 – (2 same numbers) = $1 - \frac{(6 \times 6)}{216} = 0.83$

CHAPTER

7

Arithmetic & Geometric Progression

Arithmetic Progression

Numbers are said to be in Arithmetic Progression (A.P.) when the difference between any two consecutive numbers increase or decrease by a constant difference.

Each of the following series forms an Arithmetical Progression:

2, 6, 10, 14,.......

10, 7, 4, 1, –2,.......

a, a + d, a + 2d, a + 3d,.......

If a is the first term and d is the common difference.

nth term of the series $T_n = a + (n - 1)d$

Sum of first n terms $S_n = n[2a + (n - 1)d]/2$

Example 1.

(a) If the 7th term of an Arithmetical Progression is 23 and 12th term is 38 find the first term and the common difference.

(b) How many numbers of the series -9, -6, -3 should we take so that their sum is equal to 66?

Sol.

(a) 7th term = 23 = a + 6d ....(1)

12th term = 38 = a + 11d ....(2)

Solving (1) and (2) we get a = 5 and d = 3

(b) $n[-18 + (n - 1)3]/2 = 66$

$n^2 - 7n - 44 = 0$

$\rightarrow n = 11$ or -4 $n \neq -4$ So n = 11

The series is -9, -6, -3, 0, 3, 6, 9, 12, 15, 18, 21...

Example 2.

Determine the number of terms in the A.P. 3, 7, 11, ..., 407. Also, find its 20th term from the end.

Sol.

Clearly, the given sequence is an A.P. with first term 3 and the common difference 4. Let there be *n* terms in the given A.P. Then,

$407 = \text{nth term} \Rightarrow 407 = 3 + (n-1) \times 4 \Rightarrow 4n = 408 \Rightarrow n = 102$

Now,

20th term from the end Note: $(n - p + 1)$th term for begining

$= [102 - 20 + 1]$th term from the beginning

$= 83$ rd term from the beginning $= 3 + (83 - 1) \times 4 = 331$

Selection of Terms in an A.P.

Sometimes we require certain number of terms in A.P. The following ways of selecting terms are generally very convenient.

Number of terms	Terms	Common difference
3	a – d, a, a + d	d
4	a –3d, a – d, a + d, a + 3d	2d
5	a –2d, a –d, a, a + d, a + 2d	d
6	a –5d, a –3d, a –d, a +d, a +3d, a +5d	2d

It should be noted that in case of an odd number of terms, the middle term is a and the common difference is d while in case of an even number of terms the middle terms are a –d, a +d and the common differences is 2d.

The following examples will illustrate the use of such representations.

Example 3.

The sum of three numbers in A.P. is –3, and their product is 8. Find the numbers

Sol.

Let the numbers be (a –d), a, (a +d). Then,

Sum $= -3 \Rightarrow (a - d) + a + (a + d) = -3 \Rightarrow 3a = -3 \Rightarrow a = -1$

Product = 8

$\Rightarrow (a - d)(a)(a + d) = 8$

$\Rightarrow a(a^2 - d^2) = 8$

$\Rightarrow (-1)(1 - d^2) = 8 \quad [\because a = -1]$

$\Rightarrow d^2 = 9 \Rightarrow d \pm 3$

If d = 3, the numbers are –4, –1, 2. If d = –3, the numbers are 2, –1, –4. So, the numbers are –4, –1, 2 or 2, –1, –4.

Example 4.

Find four numbers in A.P. whose sum is 20 and the sum of whose squares is 120.

Sol.

Let the numbers be (a –3d), (a –d), (a +d), (a + 3d). Then,

Sum = 20 $\Rightarrow$ (a –3d) +(a –d) + (a +d) + (a + 3d) = 20 $\Rightarrow$ 4a = 20 $\Rightarrow$ a = 5

sum of the squares = 120

$\Rightarrow (a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120$

$\Rightarrow 4a^2 + 20d^2 = 120$

$\Rightarrow a^2 + 5d^2 = 30$

$\Rightarrow 25 + 5d^2 = 30$

$\Rightarrow 5d^2 = 5 \Rightarrow d = \pm 1$

If d = 1, then the numbers are 2, 4, 6, 8. If d = –1, then the numbers are 8, 6, 4, 2.

Thus, the numbers are 2, 4, 6, 8 or 8, 6, 4, 2.

Example 5.

Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.

Sol.

Clearly, the numbers between 250 and 1000 which are divisible by 3 are 252, 255, 258, ..., 999. This is an A.P. with first term a = 252, common difference = 3 and last term = 999. Let there be n terms in this A.P. Then,

$a_n = 999 \Rightarrow a + (n-1)d = 999 \Rightarrow 252 + (n-1) \times 3 = 999 \Rightarrow n = 250$

$\therefore$ Required sum $= S_n = \frac{n}{2}[a + l] = \frac{250}{2}[252 + 999] = 156375$

Example 6.

If the sum of n terms of an A.P. is $3n^2 + 5n$ and its mth term is 164, find the value of m.

Sol.

Let S_n denote the sum of n terms and a_n be the nth term of the given A.P. Then

$S_n = 3n^2 + 5n$ [On replacing n by (n –1) in S_n]

$\Rightarrow S_{n-1} = 3(n-1)^2 + 5(n-1) = 3n^2 - n - 2$

$a_n = S_n - S_{n-1}$

$\Rightarrow a_n = (3n^2 + 5n) - (3n^2 - n - 2)$

$\Rightarrow \quad a_n = 6n + 2$

Now, $\quad a_m = 164 = 6m + 2$

$\Rightarrow \quad 6m + 2 = 164$

$\Rightarrow \quad 6m = 162$

$\Rightarrow \quad m = 27$

Example 7.

1 + 6 + 11 + 16 + ... + x = 148.

Sol.

Clearly, terms of the given series form an A.P. with first term a = 1 and common difference d = 5. Let there be n terms in this series. Then,

1 + 6 + 11 + 16 + ... + x = 148

$\Rightarrow$ Sum of n terms = 148

$\Rightarrow \quad \frac{n}{2}\left[2a+(n-1)d\right] = 148$

$\Rightarrow \quad \frac{n}{2}\left[2+(n-1)\times 5\right] = 148$

$\Rightarrow \quad 5n^2 - 3n - 296 = 0$

$\Rightarrow \quad (n - 8)(5n + 37) = 0$

$\Rightarrow \quad n = 8 \quad [\because n \text{ is not negative}]$

Now, $\quad x = n^{th}$ term

$\Rightarrow \quad x = a + (n - 1)d$

$\Rightarrow \quad x = 1 + (8 - 1) \times 5 = 36$

Example 8.

The sum of n terms of two arithmetic progressions are in the ratio (3n + 8) : (7n + 5). Find the ratio of their 12th terms.

Sol.

Let a_1, a_2 be the first terms and d_1, d_2 the common differences of the two given A.P.'s. Then, the sums of their n terms are given by

$$S_n = \frac{n}{2}\left[2a_1+(n-1)d_1\right] \text{ and } S_n' = \frac{n}{2}\left[2a_2+(n-1)d_2\right]$$

It is given that

$$\frac{S_n}{S_n'} = \frac{3n+8}{7n+15}$$

$$\Rightarrow \quad \frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]} = \frac{3n+8}{7n+15}$$

$$\Rightarrow \quad \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{3n+8}{7n+15}$$

$$\Rightarrow \quad \frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_2+\left(\frac{n-1}{2}\right)d_2} = \frac{3n+8}{7n+15}$$

Replacing $\frac{n-1}{2}$ by 11 i.e. n by 23 on both sides, we get

$$\frac{a_1+11d_1}{a_2+11d_2} = \frac{3\times23+8}{7\times23+15} = \frac{77}{176} = \frac{7}{16}$$

Hence, the required ratio is 7 : 16.

Example 9.

The interior angles of a polygon are in A.P. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon.

Sol. let there be n sides of the polygon. Then, the sum of its interior angles is given by S_n = (2n – 4) right angles = (n – 2) × 180°

Hence, the interior angles form an A.P. with first term a = 120° and common difference d = 5°.

$$\therefore \quad S_n = \frac{n}{2}\left[2\times120°+(n-1)\times5°\right]$$

From (i) and (ii), we get

$$(n-2)\times180° = \frac{n}{2}\left[2\times120°+(n-1)\times5°\right]$$

$$\Rightarrow \quad (n-2)\times360 = n[5n+235]$$

$$\Rightarrow \quad n^2 - 25n + 144 = 0$$

$$\Rightarrow \quad (n-16)(n-9) = 0 \Rightarrow n = 16 \text{ or, } n = 9$$

For n = 16, we have

Last angle = a_n = a + (n – 1)d = 120° + (16 – 1) × 5 = 195°, which is not possible.

Hence, n = 9

Insertion of Arithmetic means

If between two given quantities a and b we have to insert n quantities $A_1, A_2, ..., A_n$ such that a, $A_1, A_2, ... A_n$, b form an A.P., then we say that $A_1, A_2 ..., A_n$ are arithmetic means between a and b.

Example 10.

Insert three arithmetic means between 3 and 19.

Sol. Let A_1, A_2, A_3 be 3 A.M.'s between 3 and 19. Then 3, A_1, A_2, A_3, 19 are in A.P.

Now 19 is 5th term = a + 4d = 19

$\Rightarrow$ 3 + 4d = 19 $\Rightarrow$ d = 4

$\therefore$ $A_1 = 3 + d \Rightarrow A_1 = 7$, $A_2 = 3 + 2d \Rightarrow A = 11$; $A_3 = 3 + 3d \Rightarrow A_3 = 15$.

Hence, the required A.M.'s are 7, 11, 15.

Geometric Progression

A sequence of non-zero numbers is called a geometric progression (abbreviated as G.P.) if the ratio of a term and the term preceding to it is always a constant quantity.

The constant ratio is called the common ratio of the G.P.

In other words, a sequence, $a_1, a_2, a_3, ..., a_n, ...$ is called a geometric progression if

$\frac{a_{n+1}}{a_n}$ = constant for all $n \in N$

Illustration 1 The sequence 4, 12, 36, 108, ... is a G.P., because

$\frac{12}{4} = \frac{36}{12} = \frac{108}{36} = ... = 3$, which is constant.

Clearly, this sequence is a G.P. with first term 4 and common ratio 3.

General Term of A G.P.

nth term of a G.P. with first term a and common ratio r is given by $a_n = ar^{n-1}$

Selection of terms in G.P.

Sometimes it is required to select a finite number of terms in G.P. It is always convenient if we select the terms in the following manner:

No. of terms	Terms	Common ratio
3	$\frac{a}{r}, a, ar$	r
4	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$	r^2
5	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$	r

Example 11.

Which term of the G.P. 2, 1, $\frac{1}{2}, \frac{1}{4}, \ldots$ is $\frac{1}{128}$?

Sol.

Clearly, the given progression is a G.P. with first term a = 2 and common ratio r = 1/2.

Let the nth term be $\frac{1}{128}$. Then,

$$a_n = \frac{1}{128}$$

$$\Rightarrow \quad ar^{n-1} = \frac{1}{128}$$

$$\Rightarrow \quad 2\left(\frac{1}{2}\right)^{n-1} = \frac{1}{128}$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^{n-2} = \left(\frac{1}{2}\right)^{7} \Rightarrow n - 2 = 7 \Rightarrow n = 9$$

Thus, 9th term of the given G.P. is $\frac{1}{128}$.

Example 12.

If the 4th and 9th terms of a G.P. be 54 and 13122 respectively, find the G.P.

Sol.

Let a be the first term and r the common ratio of the given G.P. Then,

$$a_4 = 54 \text{ and } a_9\ 13122$$

$$\Rightarrow \quad ar^3 = 54 \text{ and } ar^8 = 13122$$

$$\Rightarrow \quad \frac{ar^8}{ar^3} = \frac{13122}{54} \Rightarrow r^5 = 243 \Rightarrow r^5 = 3^5 \Rightarrow r = 3$$

Putting r = 3 in $ar^3 = 54$, we get $a(3)^3 = 54 \Rightarrow a = 2$

Thus, the given G.P. is 2, 6, 18, 54, ...

Example 13

Find a G.P. for which the sum of first two terms is –4 and the fifth term is 4 times the third term.

Sol. Let a be the first term and r be the common ratio of the given G.P. Then,

$$a_1 + a_2 = -4 \Rightarrow a + ar = -4 \qquad \text{.................(1)}$$

and $a_5 = 4a_3 \Rightarrow ar^4 = 4ar^2 \Rightarrow r^2 = 4 \Rightarrow r \pm 2$

Putting $r = 2$ and -2 respectively in (1), we get

$$a = -\frac{4}{3} \text{ and } a = 4 \text{ respectively.}$$

Thus, the required G.P. is $-\frac{4}{3}, -\frac{8}{3}, -\frac{16}{3}, ...$ or 4, –8, 16, –32,........

Example 14

The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and nth hour?

Sol. Clearly, number of bacteria at the end of different hours forms a G.P. with first term $a = 30$ and common ratio $r = 2$.

Number of bacteria present at the end of 2nd hour

= Third term of the G.P. with first term $a = 30$ and common ratio $r = 2$

$$= ar^2 = 30 \times 2^2 = 120$$

Number of bacteria present at the end of 4th hour

= 5th term of the G.P. with first term $a = 30$ and common ratio $r = 2$

$$= ar^4 = 30 \times 2^4 = 480$$

Number of bacteria present at the end of nth hour

= $(n + 1)^{th}$ term of the G.P. with first term $a = 30$ and common ratio $r = 2$

$$= ar^n = 30 \times 2^n$$

Example 15

If the sum of three numbers in G.P. is 38 and their product is 1728, find them.

Sol. Let the numbers be $\frac{a}{r}, a, ar$. Then

Product = 1728 $\Rightarrow \frac{a}{r}.a.ar = 1728 \Rightarrow a^3 = 1728 \Rightarrow a = 12$

Sum = 38

$$\Rightarrow \quad \frac{a}{r} + a + ar = 38$$

$$\Rightarrow \quad a\left(\frac{1}{r} + 1 + r\right) = 38$$

$$\Rightarrow \quad 12\left(\frac{1 + r + r^2}{r}\right) = 38$$

$$\Rightarrow \quad 6 + 6r + 6r^2 = 19r$$

$$\Rightarrow \quad 6r^2 - 13r + 6 = 0 \Rightarrow (3r - 2)(2r - 3) = 0 \Rightarrow r = 3/2 \text{ or, } r = 2/3$$

Hence, putting the values of a and r, the required numbers are 8, 12, 18 or 18, 12, 8.

Sum of n terms of a G.P.

Theorem Prove that the sum of n terms of a G.P. with first term 'a' and common ratio 'r' is given by

$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$ or, $S_n = a\left(\frac{1 - r^n}{1 - r}\right)$, $r \neq 1$

Example 16.

The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Find the sum of n terms of the G.P.

Sol.

Let a be the first term and r the common ratio of the G.P. Then,

$$a + ar + ar^2 = 16 \quad \text{.............(i)}$$

$$\text{and } ar^3 + ar^4 + ar^5 = 128 \quad \text{.............(ii)}$$

$$\Rightarrow \quad a(1 + r + r^2) = 16 \text{ and } ar^3(1 + r + r^2) = 128$$

$$\Rightarrow \quad \frac{ar^3(1+r+r^2)}{a(1+r+r^2)} = \frac{128}{16}$$

$$\Rightarrow \quad r^3 = 8 \Rightarrow r = 2$$

Putting $r = 2$ in (i), we get $a = \frac{16}{7}$

$$\therefore \quad S_n = a\left(\frac{r^n - 1}{r - 1}\right)$$

$$\Rightarrow \quad S_n = \frac{16}{7}\left(\frac{2^n - 1}{2 - 1}\right) = \frac{16}{7}(2^n - 1)$$

Sum of an Infinite G.P.

Theorem The sum of an infinite G.P. with first term a and common ratio r ($-1 < r < 1$ i.e., $|r| < 1$) is

$$S = \frac{a}{1-r}$$

Example 17.

Prove that: $6^{1/2} \cdot 6^{1/4} \cdot 6^{1/8} \ldots \infty = 6$.

Sol.

We have $6^{1/2}, 6^{1/4}, 6^{1/8} \ldots \infty = 6^{(1/2 + 1/4 + 1/8 + \ldots\ldots \infty)}$

$= 6^{\{(1/2)/(1-1/2)\}} = 6^1 = 6$ $\quad \left[\because \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \text{ to } \infty = \frac{1/2}{1-1/2} = 1\right]$

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Example 18

If each term of an infinite G.P. is twice the sum of the terms following it, then find the common ratio of the G.P.

Sol. Let a be the first term and r be the common ratio of the G.P. Then,

$$a_n = 2[a_{n+1} + a_{n+2} + a_{n+3} + \ldots \infty] \text{ for all } n \in N$$

$$\Rightarrow \quad ar^{n-1} = 2[ar^n + ar^{n+1} + \ldots \infty]$$

$$\Rightarrow \quad ar^{n-1} = \frac{2ar^n}{1-r}$$

$$\Rightarrow \quad 1 = \frac{2r}{1-r}$$

$$\Rightarrow \quad r = \frac{1}{3}$$

■ ■ ■

Objective Questions

1. Fourth term of an arithmetic progression is 8. What is the sum of the first 7 terms of the arithmetic progression?
   (a) 7
   (b) 64
   (c) 56
   (d) Cannot be determined

2. The sum of series infinity of $\frac{1}{7}+\frac{2}{7^2}+\frac{1}{7^2}+\frac{2}{7^4}+......$ is
   (a) $\frac{1}{24}$
   (b) $\frac{5}{48}$
   (c) $\frac{1}{16}$
   (d) none of these

3. If the sum of the first ten terms of an arithmetic progression is four times the sum of the first five terms, then the ratio of the first term to the common difference is :
   (a) 1 : 2
   (b) 2 : 1
   (c) 1 : 4
   (d) 4 : 1

4. The angles of a pentagon are in arithmetic progression. One of the angles, in degress, must be:
   (a) 108
   (b) 90
   (c) 72
   (d) 54

5. If a, b, c are in G.P. and $a^x = b^y = c^z$. Then
   (a) $\frac{1}{x}+\frac{1}{z}=\frac{2}{y}$
   (b) $\frac{1}{x}+\frac{1}{z}=-\frac{2}{y}$
   (c) $\frac{1}{x}+\frac{1}{y}=\frac{2}{z}$
   (d) $\frac{1}{x}+\frac{1}{y}=-\frac{2}{z}$

6. The number of the term of the series : $10+9\frac{2}{3}+9\frac{1}{3}+9+....$ that would amount to 155 is:
   (a) 32 terms
   (b) 33 terms
   (c) 30 terms
   (d) 31 terms

7. The sum of the ages of 'n' siblings of a family is equal to 140 years. If the ages of these 'n' siblings are integers that form an arithmetic progression with a common difference of 'd' years, which of the following is a valid pair of values of 'n' and 'd'?
   (a) 4, 7
   (b) 7, 7
   (c) 8, 4
   (d) 10, 2

8. The number of common terms of the two sequences 17, 21, 25,......417 and 16, 21, 26,.....466 is:

(a) 21 (b) 19

(c) 20 (d) 22

9. 11, 15, 19, 23, 27, ... and 12, 15, 18, 21, 24, 27,... are two arithmetic progressions. Find the number of terms common to both progressions, if the last term for both the progressions is 555.

(a) 45 (b) 48

(c) 46 (d) 49

10. The sum of n terms of two arithmetic progressions are in the ratio (4n – 3) : (5n + 2). The ratio of their n^{th} terms is

(a) $\frac{4n+7}{5n+3}$ (b) $\frac{4n-7}{5n+3}$

(c) $\frac{8n+7}{10n+3}$ (d) $\frac{8n-7}{10n-3}$

11. If $S = -1^3 + 2^3 - 3^3 + 4^3 - ..... + 14^3 - 15^3 + 16^3$ then the value of S is:

(a) 2368 (b) 2638

(c) 2842 (d) 2240

12. Find the sum of all the integers from 1 to 100 that are not divisible by 3 or 4.

(a) 1683 (b) 1632

(c) 2019 (d) 2499

13. Let $S_1 = \{1\}$, $S_2 = \{3, 5\}$, $S_3 = \{7, 9, 11\}$, $S_4 = \{13, 15, 17, 19\}$ and so on. Find the sum of the elements of S_{50}.

(a) 122500 (b) 115500

(c) 105000 (d) 125000

Answers (Objective Questions)

1.	(c)	2.	(d)	3.	(a)	4.	(a)	5.	(a)
6.	(c)	7.	(d)	8.	(b)	9.	(c)	10.	(d)
11.	(d)	12.	(d)	13.	(d)				

Solutions :—

1.

Sol. Students please note that you need not apply any formula in this case. The middle term of an AP is always the average of all the terms. Hence, if we multiply the middle term by the number of terms, we should get the sum of all the terms of that AP. In our problem, we have to find the sum of first 7 terms and we have been given the 4th term (which is the middle term). Hence the required answer is 8 × 7 = 56.

2.

Sol.

$$\text{Given series}: \frac{1}{7}+\frac{2}{7^2}+\frac{1}{7^2}+\frac{2}{7^4}\ldots\ldots\infty$$

$$=\left(\frac{1}{7}+\frac{1}{7^3}+\ldots..\infty\right)+\left(\frac{2}{7^2}+\frac{2}{7^4}+\ldots.\infty\right)$$

$$\text{For series 1}: \frac{a}{1-r}=\frac{1/7}{1-\frac{1}{7^2}}=\frac{7}{48}$$

$$S_\infty = \frac{a}{1-r}=\frac{1/7}{1-\frac{1}{7^2}}=\frac{7}{48}$$

$$\text{and for} = \frac{\frac{2}{7^2}}{1-\frac{1}{7^2}}=\frac{2}{48}$$

Thus the final sum

$$=\frac{7}{48}+\frac{2}{48}=\frac{9}{48}=\frac{3}{16}$$

3.

Sol.

$$S_{10} = \frac{10}{2}\left[2a+(10-1)d\right]=5\left[2a+9d\right] \text{ and } S_5=\frac{5}{2}\left[2a+(5-1)d\right]=5\left[a+2d\right]$$

Given that,

$S_{10} = 4S_5$ or

$5(2a + 9d) = 20(a + 2d)$ or

$2a + 9d = 4(a + 2d) = 4a + 8d$ or $d = 2a$

Hence a:d = 1:2

4.

Sol. Let the angles of the pentangon be a – 2d, a – d, a, a + d, a + 2d.

Then

$$(a - 2d) + (a - d) + a + (a + d) + (a + 2d)$$
$$= (n - 2) \times 180°$$
$$\Rightarrow 5a = (5 \times 2) \times 180°$$

[∵ for pentagon, n = 5]

$$\Rightarrow a = \frac{3}{5} \times 180° = 108°$$

5.

Sol. Given, a, b, c, are in G.P.

$$\Rightarrow b^2 = ac$$

By taking log on both side, we get

$$2\log b = \log(ac) = \log a + \log c \quad ....(i)$$

Also, given $a^x = b^y = c^z$

So let $a^x = b^y = c^z = k$ (say)

Now, $a^x = k$

$\Rightarrow$ $x \log a = \log k$

$\Rightarrow$ $\log a = \dfrac{\log k}{x}$

$\log b = \dfrac{\log k}{y}$

$\log c = \dfrac{\log k}{z}$

By putting these values in (i) get

$$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

6.

Sol. Given sum of series can be re-written as $10 + \frac{29}{3} + \frac{28}{3} + 9 ....$

This is an A.P with first term, a = 10 and common difference, $d = \frac{-1}{3}$. Also, given $S_n = sum\ of\ A.P. = 155$

we know, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$$\Rightarrow \quad 155 = \frac{n}{2}\left[2(10)+(n-1)\left(-\frac{1}{3}\right)\right]$$

$$\Rightarrow \quad 3 \times 310 = n[60 + (n-1)(-1)]$$

$$\Rightarrow \quad 930 = n[61 - n]$$

$$\Rightarrow \quad n^2 - 61n + 930 = 0$$

$$\Rightarrow \quad (n-30)(n-31) = 0$$

$$n = 30, 31$$

So, if we put n = 30, we get the sum = 155.

Hence, number of terms = 30

7.

Sol. Let the age of 'n' siblings be a, a + d, a + 2d,........ a + (n– 1)d

then $\frac{n}{2}[2a+(n-1)d] = 140$

$$\Rightarrow \quad 2a + (n-1)d = \frac{280}{n}$$

$$\Rightarrow \quad a = \frac{140}{n} + \frac{(1-n)d}{2}$$

For 'a' to be an integer, 'n' must divide 140 and $\frac{(1-n)d}{2}$ must be an integer.

From the options, n = 7 or 10.

For n = 7 and d = 7, a = –1, which is not possible.

For n = 10 and d = 2 $a = 14 + \frac{(-9)\times 2}{2} = 5$, which is possible.

8.

Sol. S_1 17 **21** 25 29 33 37 **41** 45 49 53 57 **61**.......417

S_2 16 **21** 26 31 36 **41** 46 51 56 **61**......466

Hence, the common term will be of the form

21, 41, 61, 81, 101, 121, 141, 161, 181, 201.......401

(first 4 terms bracketed: 4; next 5 terms bracketed: 5)

∴ Total number of common terms will be equal to 19.

9.

Sol. The common terms of the AP's are 15, 27, 39,...... The last number in this series will not be more than 555...$t_n = 15 + (n-1) \times 12 \le 555$

$\Rightarrow \quad n \le \dfrac{540}{12} + 1$

$\Rightarrow \quad n \le 46$

$\therefore$ The number of common terms is 46.

10.

Sol. Let the sums of the first and second series be S_1, and S_2 respecitvely. Let the nth terms of the first and second series be t_n and T_n respectively.

$$\frac{S_1}{S_2} = \frac{\frac{n}{2}\left(2a_1 + (n-1)d_1\right)}{\frac{n}{2}\left(2a_2 + (n-1)d_2\right)} = \frac{4n-3}{5n+2}$$

$$= \left(\frac{-3+4+4(n-1)}{2+5(n-1)+5}\right) = \left(\frac{1+(n-1)4}{7+(n-1)5}\right)$$

$$= \frac{\frac{1}{2}+\left(\frac{1}{2}+(n-1)4\right)}{\frac{7}{2}+\left(\frac{7}{2}(n-1)5\right)} = \frac{t_1+t_n}{T_1+T_n}$$

$$\therefore \quad \frac{t_n}{T_n} = \frac{\frac{1}{2}+(n-1)4}{\frac{7}{2}+(n-1)5} = \frac{8n-7}{10n-3}$$

11.

Sol. $S = -(1^3 + 3^3 + 5^3 + 7^3 + 9^3 + 11^3 + 13^3 + 15^3) + (2^3 + 4^3 + 6^3 + 8^3 + 10^3 + 12^3 + 14^3 + 16^3)$

$= 2(2^3 + 4^3 + 6^3 + ... + 16^3) - (1^3 + 2^3 + 3^3 + ...+ 16^3)$

$$= 16\sum_{n=1}^{8} n^3 - \sum_{n=1}^{16} n^3 = 16\left[\frac{8(8+1)}{2}\right]^2 - \left[\frac{16(16+1)}{2}\right]^2 = 16.4^2.9^2 - 8^2.17^2$$

$= 8^2\,[324 - 289] = 64(35) = 2240$

12.

Sol. The sum of the numbers from 1 to 100 which are divisible by 3

$= 3 + 6 + 9 + 12 + ... + 99 = 3(1 + 2 + 3 + ... + 33)$

$$\Rightarrow \quad S_3 = 3 \times \frac{33 \times 34}{2} = 1683$$

The sum of the numbers from 1 to 100 which are divisible by 4 = 4 + 8 + 12 + ... 100

$= 4\,(1 + 2 + 3 + ... + 25)$

$$\Rightarrow \quad S_4 = \frac{4 \times 25 \times 26}{2} = 1300$$

The sum of the numbers from 1 to 100, which are divisible by 3 and 4 i.e. by 12 = 12 + 24 + 36 + ... + 96 = 12(1 + 2 + 3 + ... + 8)

$$\Rightarrow \quad S_{12} = \frac{12 \times 8 \times 9}{2} = 432$$

$\therefore$ The sum of the numbers divisible by 3 or 4

$$= S_3 + S_4 - S_{12} = 1683 + 1300 - 432 = 2551$$

The sum of the numbers from 1 to 100

$$= \frac{100(100+1)}{2} = 5050$$

$\therefore$ The sum of the numbers from 1 to 100 which are not divisible by 3 or 4 = 5050 – 2551 = 2499

13.

Sol. Clearly set S_1 contains 1 odd number, S_2 contains next 2 odd numbers, S_3 contains next 3 odd numbers and so on. Hence the total number of odd numbers used till S_{49} is 1 + 2 + 3 + ... + 49 = 1225.

$\therefore$ The first term of S_{50} is 1226[th] odd term i.e. 2451.

$\therefore$ S_{50} contains 50 odd terms starting with 2450.

Hence the sum of its terms is $\frac{50}{2}[4902 + 49 \times 2] = 25 \times 5000 = 125000$

Alternate method:

The sum of elements in $S_1 = 1$

The sum of elements in $S_2 = 3 + 5 = 8 = 2^3$

The sum of elements in $S_3 = 7 + 9 + 11 = 27 = 3^3$

$\therefore$ The sum of elements in $S_{50} = (50)^3 = 125000$

CHAPTER

8

Functions

INTRODUCTION

Functions are relationships defined between a set of variables. The functional operator works on the domain and maps it onto the range.

Take an example $f(x) = 3x + 4$ for all $x \in R$.

This is a function that maps any value of x, say 2 to a value $y = f(x) = 3x + 4 = 10$.

For f to be a functions operator, it maps one value of x to one value of y.

Understand a functional operation as a operation on a machine. For any input, the output is defined by the relationship it stands for.

For instance, if $f(x) = 5x^2 + 4x + 3$, then $f(1) = 5(1)^2 + 4(1) + 3 = 12$ which means that the input is 1 and the output is 9.

There are some basic operations on functions. They are as follows:

COMPOSITION OF FUNCTINS

$f(x) = x + 1$

$g(x) = x^2 + 2x + 3$

Find fog and gog

$fog = f(g(x)) = f(x^2 + 2x + 3)$

$= x^2 + 2x + 3 + 1 = x^2 + 2x + 4$

Now $gof = g(f(x)) = g(x + 1) = (x + 1)^2 + 2(x + 1) + 3 = x^2 + 2x + 1 + 2x + 2 + 3$

$= x^2 + 4x + 6$

INVERSE OF A FUNCTION

Simply put, an inverse of a function $y = f(x)$ is a function in the reverse direction $x = f^{-1}(y)$, i.e., with range as the input and domain as the output.

How to find the inverse of function ? For example, how do I find the inverse of the function

$f(x) = y = \dfrac{2x+3}{x-5}$?

(i) write the original functin $y = \frac{2x+3}{x-5}$.

(ii) Solve for $x \Rightarrow x = \frac{5y+3}{y-2}$

(iii) Switch x and y $\Rightarrow f(x) = y = \frac{5x+3}{x-2}$

The operation of function has the same rule as that of real numbers-

$(f+q)(x) = f(x) + q(x)$

$(f-q)(x) = f(x) - q(x)$

$(fg)(x) = f(x)\, q(x)$

Example 1.

If $f(x) = \frac{1}{x}, x \neq 0$, and $f(g(x)) = g(x)$, then what is the value of $f(g(x)) \times g(x)$?

Sol.

As $f(x) = \frac{1}{x}$, $f(g(x)) = \frac{1}{g(x)} \Rightarrow f(g(x)) \times g(x) = 1$

Example 2.

If $f(x+y) = f(x) + f(y) + f(x) \times f(y)$ and $f(1) = 3$ then find $f(3)$.

(a) 63 (b) 48

(c) 53 (d) 36

Sol.

Keeping x = y = 1 in the equation, we get

$f(2) = f(1) + f(1) + f(1) \times f(1) \Rightarrow f(2) = 15$. Now keeping $x = 2$, $y = 1$ in the equation

$f(3) = f(2) + f(1) + f(2) \times f(1) \Rightarrow f(3) = 63$.

Example 3.

Let $f(x)$ be a function satisfying $f(x)\, f(y) = f(xy)$ for all real x,y. if $f(2) = 4$, then what is the value of $f\left(\frac{1}{2}\right)$?

Sol.

Keeping $x = 1$ and $y = 2$, we get $f(1) \times f(2) = f(2) \Rightarrow f(1) = 1$ Now keeping $x = \frac{1}{2}$ and $y = 2$ in the equation we get $f\left(\frac{1}{2}\right) \times f(2) = f(1) \Rightarrow f\left(\frac{1}{2}\right) = \frac{1}{4}$

PERIODIC FUNCTION

A function $f(x)$ is called a periodic function if there is a positive number p such that $f(x+p) = f(x)$ for every x.

The smallest such value of p is known as the **period** of $f(x)$. For example, the period of trigonometric functions-sinx, cos$x$, tan$x$ etc. = 2π

sin x and cos is 2π where tan x = π

Example 5.

Let $g(x)$ be a function such that $g(x+1) + g(x-1) = g(x)$ for every real x. then for what value of p is the relation $g(x+p) = g(x)$ necessarily true for every real x?

(a) 5 (b) 3 (c) 2 (d) 6

Sol.

Let $g(x-1) = a$ and $g(x) = b \Rightarrow g(x+1) = b - a$. Now, keeping $x+1$ in place of x we get $\Rightarrow g(x+2) + g(x) = g(x+1) + b = b - a \Rightarrow g(x+2) = -a$.

Again, keeping $x + 2$ in place of x we get

$\Rightarrow g(x+3) + g(x+1) = g(x+2) \Rightarrow g(x+3) + b - a = -a \Rightarrow g(x+3) = -b$.

Now we can see that $g(x) = b$ and $g(x+3) = -b$, therefore, $g(x+3+3) = -(-b) = b \Rightarrow g(x+6) = b$, therefore, p = 6.

Example 6.

$f(x) = x^2$; $g(x) = 2x + 4$

Find (a) fof (x), (b) gof (x), (c) fog(x).

Sol.

(a) $fof(x) = f(f(x)) = f(x^2) = x^4$

(b) $gof(x) = g(f(x)) = g(x^2) = 2(x^2) + 4$

(c) $fog(x) = f(g(x)) = f(2x+4) = (2x+4)^2 = 4x^2 + 16x + 16$

Example 7.

For a function $f(x)$, $f(x) + f(x-1) = x^2$ and $f(19) = 94$. Find $f(94)$

Sol.

$f(x) = x^2 - f(x-1)$

$f(94) = 94^2 - f(93) = 94^2 - (93^2 - f(92)) = 94^2 - 93^2 + (92^3 - f)91) = ...$

$= 94^2 - 93^2 + 92^2 - 91^2 + 90^2 - 89^2 + ... 22^2 - 21^2 + 20^2 - f(19)$

$= [(94 + 93)(94 - 93)] + [(92 - 91)(92+91)] + ... [(22 - 21)(22+21] + 400- 94$

$= 94 + 93 + 92 + 91$ ------------------- $22 + 21 + 302$

= Now 21 to 94 is an AP with common difference = 1 and a = 21 with 74(n) terms in total

so Sum = $\frac{74}{2}[2\times21+(74-1)\times1] = 37[42 + 73] = 37 \times 115 = 4255$

GREATEST INTEGER FUNCTION

The greatest integer function, denoted by [x], gives the greatest interger less than or equal to the given number x.

To put it simply, if the given number is an integer, then the greatest integer gives the number itself, otherwise it gives the first integer towards the left of the number of x on the number line.

For example,

$[1.4] = 1$

$[4] = 4$

$[3.4] = 3$

$[-2.3] = -3$

$[-5.3] = -6$, and so on.

Example 8.

What is the value of $\left[\sqrt{1}\right] + \left[\sqrt{2}\right] + \left[\sqrt{3}\right] + ... \left[\sqrt{49}\right] + \left[\sqrt{50}\right]$ where [x] denotes the greatest integer function?

Sol. It can be seen that

$$\left[\sqrt{1}\right] = 1, \left[\sqrt{2}\right] = [1.41] = 1, \left[\sqrt{3}\right] = [1.73] = 1, \left[\sqrt{4}\right] = 2 \text{ and so on.}$$

Therefore, from $\left[\sqrt{1}\right]$ to $\left[\sqrt{3}\right]$, the value will be 1, from $\left[\sqrt{4}\right]$ to $\left[\sqrt{8}\right]$ the value will be 2, from $\left[\sqrt{9}\right]$ to $\left[\sqrt{15}\right]$ the value will be 3 and so on..

Therefore, the total value $= 3 \times 1 + 5 \times 2 + 7 \times 3 + ... + 13 \times 6 + 2 \times 7 = 217$.

Example 9

What is the value of x for which $x[x] = 29$?

Sol. If the value of x is 5, $x[x] = 25$, and if the value of x is 6, $x[x] = 36$.

Therefore, the value of x lies between 5 and 6.

If x lies between 5 and 6, and $[x] = 5$.

$$\Rightarrow \quad x = \frac{29}{[x]} = \frac{29}{5} = 5.8$$

Example 10

If $f(x) = 2[x]$, $g(x) = x^2$, find $f(g(x))$ for $x = 4.5$.

Sol. $f(g(x)) = f(x^2) = f(4.5^2) = f(20.25) = 2[20.25] = 2(20) = 40$

Example 11

Solve for $[x]^2 \le 9$ and $2x + 3 \le 5$.

from (1)

$[x^2] \le 9$ then $-3 \le [x] \le 3$ or $-3 \le x < 4$ ....(i)

from (2)

$2x + 3 \le 5$ or $x \le 1$ ....(ii)

Hence, the value of x satisflying both is $-3 < x \le 1$.

■ ■ ■

Objective Questions

Direction for Q1-Q2: A function $f(x)$ is said to be even if $f(-x)=f(x)$, and odd if $f(-x) = -f(x)$. Thus, for example, the function given by $f(x) = x^2$ is even, while the function given by $f(x) = x^3$ is odd. Using this definition, answer the following questions.

1. The function given by $f(x)=|x|^3$, is

(a) even (b) odd

(c) neither (d) both

2. The sum of two odd functions

(a) is always an even function

(b) is always an odd function

(c) is sometimes odd and sometimes even

(d) may be neither odd nor even

3. If $f(x)=\dfrac{x(x-1)}{2}$, then $f(x+2)$ equals:

(a) $f(x)+f(2)$ (b) $(x+2)(f(x)$

(c) $x(x+2)f(x)$ (d) $\dfrac{(x+2)f(x+1)}{2}$

4. $f(x)$ is a real valued function such that $f(x) - 3f\left(\dfrac{72}{x}\right) = 4x\ (x \neq 0)$. Find the value of f(3).

(a) $-\dfrac{52}{3}$ (b) $-\dfrac{75}{2}$

(c) $\dfrac{75}{4}$ (d) $\dfrac{52}{3}$

5. Find the value of

$$\left[\frac{3}{4}\right]+\left[\frac{3}{4}+\frac{1}{100}\right]+\left[\frac{3}{4}+\frac{2}{100}\right]+\ldots\ldots\ldots\ldots\ldots\left[\frac{3}{4}+\frac{99}{100}\right]$$

(a) 100 (b) 75

(c) 74 (d) 105

6. Given y = 2[x] + 3 once y = 3 [x – 2] + 5, then find the value of [x + y]

(a) 15 (b) 16

(c) 17 (d) 18

7. Let $f(x) = \frac{9^x}{9^x + 3}$ Hence evaluate $f\left(\frac{1}{1996}\right)+f\left(\frac{2}{1996}\right)+f\left(\frac{3}{1996}\right)+\ldots\ldots\ldots+f\left(\frac{1995}{1996}\right)$

(a) 997 (b) 998

(c) 997.5 (d) 998.5

8. Find the possible positive integrs x, y, z satisfying $x^{y^z} . y^{z^x} . z^{x^y}$ = 5xyz.

(a) 5, 2, 1 (b) 5, 3, 1

(c) 3, 4, 1 (d) 4, 5, 1

9. If $f(x) = 3x+1$ for $-\infty < x < \infty$ and $g(x) = 4x^2 + 1$ for $0 < x \le 10$. Find the maximum value of $f[g(x)]$

(a) 1200 (b) 1204

(c) 1224 (d) 1225

10. The indian patsman are given the instruction by the coach that they name to score the runs in each over defined by the following function

$f(x)$ = n, $0 < n \le 10$

= n – i, $10 < n \le 20$, i = 1, 2, 3 ...... 10.

= n – f (n – 10), $20 < n \le 30$

$= \left[\frac{n}{3}\right]$ $30 < m \le 50$

$f(x)$ is number of runs being scored in n^{th} over and $[x]$ is greatest integer less than or equal to x. If they played all the 50 overs following the instruction, then the total runs scored by indian batsman are.

(a) 425 (b) 512

(c) 573 (d) 640

11. A quadratic function $f(x)$ attains a maximum of 3 at x = 1. the value of the function at x = 0 is 1. What is the value of $f(x)$ at x = 10?

(a) – 119 (b) – 159

(c) –110 (d) –180

12. Given that f(x) = $\log \frac{1+x}{1-x}$ and $g(x) = \frac{3x+x^3}{1+3x^2}$, then $f[g(x)]$ equals.

(a) $f(x)$ (b) $3[f(x)]$

(c) $[f(x)]^3$ (d) $[f(x)]^2$

13. If a function f is such that $f(0)=2,\ f(1)=3,\ f(n+2)=2f(x)-f(n+1)$, then $f(5)$ equals.

(a) – 7 (b) – 3

(c) 7 (d) 13

14. If g(x) = $\frac{p^x + p^{-x}}{2}$, then g(x + y) + g (x – y) equals.

(a) $\frac{g(x)}{g(y)}$ (b) $2\,g(x)g(y)$

(c) $\frac{g(x)g(y)}{2}$ (d) $g(x)\times g(y)$

15. If $f(x)$ is function such that – $2f(x) + f(1-x) = x^2$ for all x, then $f(x)$ is equal to

(a) $\frac{x^2-3x+1}{2}$ (b) $\frac{x^2-8x-3}{2}$

(c) $\frac{4x^2+3x-2}{6}$ (d) $\frac{x^2+2x-1}{2}$

16. If f is a polynomial function for all real x, So that $f(x^2+1)=x^4+5x^2+3$, then the value of $f(x^2-1)$ is

(a) x^4+x^2+3 (b) x^4-5x^2+1

(c) x^4+x^2-3 (d) x^4+5x^2+1

Answers (Objective Questions)

1. (a)	2. (b)	3. (d)	4. (b)	5. (b)	6. (a)
7. (c)	8. (*)	9. (b)	10. (*)	11. (b)	12. (b)
13. (d)	14. (c)	15. (b)	16. (c)		

Solutions:—

1.

Sol. $f(x) = |x|^3$

$\therefore$ $f(-x) = |-x|^3 = |x|^3 = f(x)$.

So the function is even.

2.

Sol. Let $f(x) = g(x) + h(x)$ where g and h are odd functions.

$\therefore$ $f(-x) = g(-x) + h(-x) = -g(x) - h(x) = -f(x)$. So f(x) is odd.

3.

Sol. $f(x) = \dfrac{x(x-1)}{2}$, $f(x+1) = \dfrac{(x+1)x}{2}$

and $f(x+2) = = \dfrac{(x+2)(x+1)}{2}$

$$= \frac{x+2}{2} \times \frac{(x+1)x}{2} = \frac{x+2}{2} f(x+1)$$

4.

Sol. For $x = 3$, $f(3) - 3f\left(\dfrac{72}{3}\right) = 4 \times 3$

$\therefore$ $f(3) - 3f(24) = 12$ ... (i)

For $x = 24$, $f(24) - 3f\left(\dfrac{72}{24}\right) = 4 \times 24$

$\therefore \quad f(24) - 3f(3) = 96 \quad \ldots(ii)$

Multiplying (ii) by 3 and adding it to (i), we get, $-8f(3) = 300$.

$$\Rightarrow f(3) = -\frac{300}{8} = \frac{-75}{2}$$

5.

Sol. $\left[\frac{3}{4} + \frac{23}{100}\right] = 1$

So from 25 to 99 there are 75 term

6.

Sol. 2 [x] + 3 = 3 [x −2] + 5

2 [x] + 3 = 3 [x] − 6 + 5

[x] = 4.

$4 \leq x < 5$

x = 4 + f f → fraction

y = 2 [x] + 3

= 2 × 4 + 3

= 11.

[x + y] = [4 + f + 11]

= [15 + f]

= 15.

7.

Sol. $f(x) + f(1-x) = 1$

i.e. $f\left(\frac{1}{1996}\right) + f\left(\frac{999}{1996}\right) = 1$

$f\left(\frac{997}{1996}\right) + f\left(\frac{999}{1996}\right) = 1$

$997 + f\left(\frac{998}{1996}\right) = 997.5$

8.

Sol. $\frac{x^{y^z} . y^{z^x} . z^{x^y}}{xyz} = 5$

$\Rightarrow \quad x^{y^z} . y^{z^x - 1} . z^{x^y - 1} = 5$

$\Rightarrow \quad x^{y^z} = 5 \quad y^{z^x - 1} = 1, \quad z^{x^y - 1} = 1$

$x = 5 \qquad y^z - 1 = 2 \qquad z = 1$

$y^z = 2$

$y = 2$

9.

sol. $f\,[g\ (x)] = (3(g(x))+1)$

$= 3\ (4x^2 + 1) + 1$

$= 12x^2 + 4$

$x = 10$

$= 1204$

11.

Sol. Let $f(x) = ax^2 + bx + c$

$f(1) = a + b + c = 3$

$f\ (0) = c = 1$

$\therefore \ f(x) = ax^2 + bx + 1$

or $a + b + 1 = 3$

$-\frac{b}{2a} = 1$

$b = -2a$

$a - 2a + 1 = 3.$

$-a = 2$

$a = -2$

$b = 4$

$f(x) = -2x^2 + 4x + 1.$

$f(10) = -200 + 40 + 1$

$= -159.$

13.

Sol. $f(n+2) = 2f(n) - f(n+1).$

n = 0

$f(2) = 2f(0) - f(1).$

$= 2 \times 2 - 3$

$= 1$

n = 1

$f(3) = 2f(1) - f(2)$

$= 2 \times 3 - 1$

$= 5$

n = 2

$f(4) = 2f(2) - f(3)$

$= 2 \times 1 - 5$

$= -3$

n = 3

$f(5) = 2f(3) - f(4)$

$= 2 \times 5 + 3$

$= 13$

14.

Sol. $g(x+y) = \dfrac{p^{x+y} + p^{-(x+y)}}{2}$

$g(x-y) = \dfrac{p^{x+y} + p^{-(x+y)}}{2}$

$g(x+y) + g(x-y) = \dfrac{p^{x+y} + p^{-x-y} + p^{x-y} + p^{-x+y}}{2}$

$= \dfrac{p^{x}(p^{y} + p^{-y})p^{-x}(p^{y} + p^{-y})}{2}$

$$= \frac{(p^x + p^{-x})(p^y + p^{-y})}{2}$$

$$= \frac{g(x)\,g(y)}{2}$$

15.

Sol. $2f(x) + f(1 - x) = x^2 \rightarrow$ (i) × 2

$2f(1 - x) + f(x) = (1 - x)^2 \rightarrow$ (ii) × 1.

$4f(x) + 2f(1 - x) - 2x^2$

$2f(1 - x) + f(x) = (1 - x)^2$

By sub. ( )

$$3f(x) = 2x^2 - (1-x)^2$$

$$3f(x) = 2x^2 - (1 - 2x + x^2)$$

$$f(x) = \frac{x^2 + 2x - 1}{3}$$

16.

Sol. $f(x^2+1) = x^4 + 2x^2 + 1 + 3x^2 + 2$

$$= (x^2+1)^2 + 3x^2 + 3 - 1$$

$$= (x^2+1)^2 + 3(x^2+1) - 1$$

$$f(x^2-1) = (x^2-1)^2 + 3(x^2-1) - 1$$

$$= x^4 + x^2 - 3$$

♦ ♦ ♦ ♦

ARITHMETIC

CHAPTER

9

Percentage, Profit and Loss

The word "Percentage" literally means "per hundred" or "for every hundred." Therefore, whenever you calculate something as a part of 100, that part is numerically termed as percentage.

In other words, percentage is a ratio whose second term is equal to 100. For example, 1:4 can be written as 25 : 100 or 25%, 3 : 8 can be written as 37.5 : 100 or 37.5%, 3 : 2 can be written as 150 : 100 or 150%, and so on.

Important concepts Associated with Percentage

Basic formula of percentage:

p% of a number N is = $N \times \frac{p}{100}$

Example 1.

What is 45% of 500?

Sol. 45% of 500 = $500 \times \frac{45}{100} = 225$

Example 2.

What is 20% of 50% of 60% of 200?

Sol. Required percentage = $\left(\left(\left(200 \times \frac{60}{100}\right) \times \frac{50}{100}\right) \times \frac{20}{100}\right) = 12$

Percentage of

Example 3.

What percentage of 240 is 90?

Sol. Percentage = $\frac{90}{240} \times 100 = 37.5\%$

Example 4.

What percentage of 75 is 125?

Sol. Percentage = $\frac{125}{75} \times 100 = 166.66\%$

Percentage increase/decrease

Percentage increase/decrease when a quantity a increase/decreases to become another quantity b is

$$\text{Percentage Increase/Decrease} = \frac{\text{increase/Decrease}}{\text{Initial Value}} \times 100 = \begin{cases} \frac{b-a}{a} \times 100 \text{ when } b - a (\text{increase}) \\ \frac{a-b}{a} \times 100 \text{ when } a - b (\text{decrease}) \end{cases}$$

Multiplication factors for percentage increase/decrease

We saw in percentage increase/decrease that new quantity b = $\begin{cases} a\left(1 + \frac{\text{percentage increase}}{100}\right) \\ a\left(1 - \frac{\text{percentage decrease}}{100}\right) \end{cases}$

For example, for a 20% increase the new quantity = old quantity $\left(1 + \frac{20}{100}\right)$ = old quantity $\times 1.2$

Therefore, to find the final quantity after a 20% increase, we can directly multiply the old quantity by a factor of 1.2 and get the new quantity. The factors to be multiplied for various percentage increase/decrease are given below:

Percentage Increase	New quantity = old quantity multiplied by a factor of	Percentage Decrease	New quantity = old quantity multiplied by a factor of
5%	1.05	5%	0.95
10%	1.1	10%	0.9
15%	1.15	15%	0.85
20%	1.2	20%	0.8
25%	1.25	25%	0.75
30%	1.3	30%	0.7
40%	1.4	40%	0.6
50%	1.5	50%	0.5
60%	1.6	60%	0.4

The biggest advantage of using the factors is that for subsequent percentage increase/decrease, we just keep on multiplying the corresponding factors and get the final quantity.

Example 5.

The performance bonus of a salesman increases by 10% in the first year, by 20% in the second year, and by 30% in the third year. What is the overall percentage increase in his bonus in 3 years?

Sol. Let the bonus at the start of the first year be Rs 100.

Therefore, to find the final bonus we just multiply by factors

Final bonus = 100(1.1)(1.2)(1.3) = 171.6

Therefore, overall percentage increase = 71.6%

Example 6.

If the price of petrol increases by 10%, by what percentage must the consumption be decreased to keep the expenditure constant?

Sol. Let the original price be 100Rs/litre and the original consumption be 100 litres.

Therefore, expenditure = 100 × 100 = Rs10,000.

New price = Rs 110. Since expenditure remains constant

110 × new consumption = 10000 → new consumption = $\frac{10000}{110}$ = 90.90 litres. Therefore, decrease in consumption = 9.1%.

Example 7.

In an election between two candidates, 75% of the voters cast their votes, out of which 2% of the votes were declared invalid. A candidate got 9261 votes which were 75% of the total valid votes. Find the total number of votes enrolled in that election.

Sol. Let the total number of votes enrolled be x. Then.

Number of votes cast = 75% of x. Valid votes = 98% of (75% of x).

$\therefore$ 75% of [98% of (75% of x)] = 9261

$$\Leftrightarrow \left(\frac{75}{100}\times\frac{98}{100}\times\frac{75}{100}\times x\right) = 9261 \Leftrightarrow x = \left(\frac{9261\times100\times100\times100}{75\times98\times75}\right) = 16800$$

Note

Whenever there are successive changes for a particular value, the net change could be expressed as a single percentage $\left(a+b+\frac{ab}{100}\right)\%$. *Here a and b are the first and the second percentage changes in that order.*

Example 8.

If the price of petrol increases successively by 20% and then by 10%, what is the net change in percentage terms?

Sol. 32% [Hint: Apply the $a+b+\frac{ab}{100}$ formula.]

$\because \left[20+10+\frac{20\times 10}{100}\right] = 32\%$

Note

The $a+b+\frac{ab}{100}$ *formula is applicable*

(a) When the successive changes are on the same parameter
 Example: Two successive changes in the price of an item

(b) When two parameters are being multiplied to get a product and we want to find the percentage change in the product, if one of the parameter changes by a% and the other by b%.
 Example: The length and breadth of a rectangle change by a% and b% respectively.

The product, which is the area, changes by $a+b+\frac{ab}{100}$ %.

Example 9.

The radius of a circle has increased by 20%. By what percentage does

(a) the circumference increase?

(b) the area increase?

Sol. (a) Circumference of a circle = $2\pi r$. Since r increases by 20%, the circumference also increases by 20%.

(b) Area of a circle = πR^2

Since $r^2 = r \times r$ as r increases by 20%, r^2

increases by $20+20+\frac{(20\times 20)}{100} = 44\%$.

Example 10.

The length of a rectangle is increased by 10%. What will be the percentage decrease in its breadth so as to have the same area?

Sol. Applying percent change = $a+b+\frac{ab}{100}$

Let decrease in breadth be x%, then

$0 = 10 - x - \frac{10\times x}{100} \Rightarrow \frac{11x}{10} = 10$

$\Rightarrow x = \frac{100}{11} = 9\frac{1}{11}\%$

Example 12.

5% of income of A is equal to 15% of income of B and 10% of income of B is equal to 20% of income of C. If income of C is Rs. 2,000, then total income of A, B and C is

Sol. $\frac{5}{100}A = \frac{15}{100}B$ and $\frac{10}{100}B = \frac{20}{100}C$

$\therefore$ A = 3B and B = 2C = 2 × 2000 = 4000

$\therefore$ A = 3 × 4000 = 12000

A + B + C = (12000 + 4000 + 2000) = 18000

Example 13.

Arvind spends 75% of his income. His income is increased by 20% and he increases his expenditure by 10%. His savings are increased by how many percent?

Sol. Let the income be 100.

Expenditure = 75 and savings = 25

New income = 120

New expenditure = $\left(\frac{110}{100}\times 75\right) = \frac{165}{2}$;

New savings = $\left(120 - \frac{165}{2}\right) = \frac{75}{2}$

Increase in savings = $\left(\frac{75}{2} - 25\right) = \frac{25}{2}$

Percentage increase = $\left(\frac{25}{2}\times\frac{1}{25}\times 100\right) = 50\%$

Example 14.

What quantity of water must be added to a 9 L solution of 50% milk to make it 30% milk?

Sol. The volumeof the original solution is equal to 9 L. Therefore, volume of milk in the original solution is equal to 4.5 L. Now this volume of milk remains constant in the new solution.

In the new solution, 4.5 of milk = 30%. Therefore, total volume = $\frac{4.5}{30}\times 100 = 15$ L

Therefore, water added = new volume – old volume = 6 L

Profit & Loss

Cost Price: CP of an item is the expenditure incurred to purchase (or to produce).

Selling Price: SP of an item is the revenue realized when the item is sold.

Profit/Loss: This is the difference between the selling price and the cost price. If the difference is positive, it is called profit; and if negative, it is called loss.

Profit/Loss % : This is the profit/loss as a percentage of the CP.

Margin: Normally, used in percentage terms only. This is the profit as a percentage of SP.

Marked Price: This is the price of the product as displayed on the label.

Discount: This is the reduction given on the marked price before selling it to a customer.

Mark-up: This is the increment on the cost price before being sold to a customer.

Using these terminologies, we can arrive at the following:

$$\text{Profit\%} = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$\text{SP} = \text{CP} + \text{P\% of CP} = \text{CP}\left(1+\frac{\text{P}}{100}\right)$$

Example 15.

Amit bought equal quantities of two varieties of oranges, one variety at a rate of Rs. 200 for 4 kgs and other variety at a rate of Rs. 400 for 10 kgs. He mixed the two varieties and sold them at a rate of Rs. 50 per kg. What is his profit/loss%?

Sol. Important point to note here is that Amit purchases equal quantities of both varieties. Let him purchase 1 kg of each variety. Thus, the amount he spent on first variety was Rs. 50 and on second variety was Rs. 40, incurring a total investment of Rs. 90. However he receives Rs. 100 for 2 kgs at the rate of Rs. 50 per kg. Thus he makes a profit of Rs. 10 and a profit % of $\frac{10}{90} \times 100 = 11.11\%$.

Example 16.

Krishnan has 12 eggs with him. He sells x at a profit of 10% and remaining at a loss of 10%. He gains 5% on the whole. What is the value of x?

Sol. Let each egg costs Rs. 10. Then the overall profit = Rs. 6. If x eggs are sold at a profit. Hence, total revenue = x (11) + (12 – x)9 = 2x + 108 = Rs. 126.

So 2x = 18 and x = 9

Example 17.

Aditya purchases toffees at Rs. 10 per dozen and sells them at Rs. 12 for every 10 toffees. Find the gain or loss percentage.

Sol. Assume that he buys 120 toffees. Aditya spends Rs. 100.

Revenue generated = Rs. 144.

Hence, profit percentage = 44%.

Example 18.

Anirudh bought 8 lemons for a rupee, but sells only 6 lemons for a rupee. Find his profit percentage.

Sol. Suppose Anirudh purchases 24 lemons (LCM of 8 and 6).

Therefore, cost price of 24 lemons = Rs 3.

Selling price of 24 lemons = Rs. 4.

Gain = Rs 1.

Profit percentage = $\frac{1}{3} \times 100 = 33\frac{1}{3}\%$

Example 19.

Paresh sells 40 pencils and gains the selling price of 10 pencils. What is his profit percentage?

Sol. Selling price of 40 pencils–Cost price of 40 pencils = Selling price of 10 pencils

Selling price of 30 pencils = Cost price of 40 pencils

Let CP of 40 pencils = Re 1.

SP of 30 pencils = Re 1.

SP of 40 pencils = $1 \times \frac{4}{3}$

Gain percentage = $\frac{\left(\frac{4}{3}-1\right)}{1} \times 100 = 33\frac{1}{3}\%$

Example 20.

Nilmani buys 10 pens and sells 8 of them at the cost price of 10 pens. What is his profit percentage?

Sol. To calculate profit percentage, sell all the pens purchased by Nilmani. Now, let the cost price of each pen be 1 rupee. Therefore, Nilmani purchased 10 pens for Rs 10. He sells 8 of them for Rs 10.

Nilmani sells 8 pens for Rs 10.

He will sell 10 pens for Rs $\frac{10}{8} \times 10 =$ Rs 12.5.

Therefore, Nilmani spent Rs 10 and got back Rs 12.5.

Therefore, his profit is 25%.

Example 21.

A shopkeeper sells one-third of his goods at a profit of 10%, another one-third at a profit of 20%, and the rest at a loss of 6%. What is his overall profit percentage?

Sol. Let the shopkeeper buy 300 g for Rs 300. Now he sells 100 g for Rs 110, another 100 g for Rs 120, and the rest 100 g for Rs 94. Therefore, the total amount he receives is = Rs 110 + Rs 120 + Rs 94 = 324.

Therefore, his profit percentage = $\frac{24}{300} \times 100 = 8\%$

Mark-up and Discount

Please note that the mark-up percentage is a percentage of CP.

Let's now see the entire process:

The shopkeeper incurs a cost of CP and then marks the item up by m% (of CP) and puts a price tag of MP.

$$CP \xrightarrow{m\%} MP$$

Next the customer bargains a discount of d% on the MP and thus arrives at the SP

$$CP \xrightarrow{m\%} MP \xrightarrow{d\%} SP$$

Example 22.

A shopkeeper gives 2 items free with every 3 items purchased. In effect, what discount percentage is being offered? [Assume that all the items are identical]

Sol. Please realise that the discount is not 2/3 or 66.66%.

Discount percent is expressed as a percentage of marked or list price. The list price of the 5 items will be 5x and the selling price will be 3x. thus discount percent is 2/5 or 40%

Example 23.

A woman buys apples at 25 for a rupee and the same number at 40 a rupee. She mixes and sells them at 65 for 2 rupees. What is her gain or loss percentage?

Sol. Suppose the woman buys (LCM of 25, 40 and 65) 2600 apples each at 25 apples a rupee and 40 apples a rupee.

Cost at the rate of 25 per rupee = Rs. 104

Cost at the rate of 40 per rupee = Rs. 65

Total cost for 5200 apples = Rs. 169

S.P. for 5200 apples = $\frac{5200 \times 2}{65} = 160$

Loss percentage = $\frac{(169-160) \times 100}{169} = 5.32\%$

Example 24.

Goods are purchased for Rs. 450 and one-third is sold at a loss of 10%. At what profit percentage should the remainder be sold so as to gain 20% on the whole transaction?

Sol. Total cost price of goods = Rs. 450

S.P. of total goods = $\frac{120}{100} \times 450 =$ Rs. 540

C.P. of one-third goods = $\frac{450}{3} = 150$

S.P. of one-third goods = $\frac{90}{100} \times 150$ = Rs. 135

S.P. of remaining goods = (540 – 135) = Rs. 405

C.P. of remaining (two-third) goods = Rs. 300

Hence, profit percentage = $\frac{405-300}{300} \times 100 = \frac{105}{300} \times 100 = 35\%$

Example 25.

Vendor bought bananas at 6 for 5 rupees and sold at 4 for 3 rupees. Find his gain or loss percentage.

Sol. Let number of bananas be 12(LCM of 6 and 4)

Cost Price = $\frac{12}{6} \times 5 =$ Rs. 10

Selling Price = $\frac{12}{4} \times 3 =$ Rs. 9

Loss percentage = $\frac{1}{10} \times 100 = 10\%$

Note

When two items are sold for the same price, and if one gives a profit of a% and other a loss of a%, the net result is always a loss, irrespective of the SP and the loss percentage $= \frac{a^2}{100}$.

and net loss $= \left[\frac{2a^2}{100^2 - a^2}\right] \times$ *SP times*

Example 26.

A man sold two flats for Rs. 6,75,958 each. On one he gains 16% while on the other he loses 16%. How much does he gain or lose in the whole transaction?

Sol. Remember: In such a case, there is always a loss. The selling price is immaterial.

$\therefore$ Loss % $= \left(\frac{\text{Common Loss and Gain\%}}{10}\right)^2 = \left(\frac{16}{10}\right)^2 = \left(\frac{64}{25}\right) = 2.56\%$

Example 27.

A trader has a weighing balance that shows 1,200 g for a kilogram. He further marks up his CP by 10%. Find the net profit percentage.

Sol. Due to defect in weighing balance initially there is a profit of 20%.

Now there is a mark-up of 10% in CP.

Therefore, net profit $= 10 + 20 + \frac{10 \times 20}{100} = 32\%$ profit.

Example 28.

A woman goes to market with Rs 500 to buy oranges. The prices of the oranges have decreased by 10% so she could buy 2 kg more with the amount she had. What was the original price of the oranges?

Sol. If the price of the oranges decreases by 10%, the woman would save 10% of the money that is Rs 50 to buy the same amount as before. Now in Rs 50 she can buy 2 kg more, therefore the current price of 1 kg of oranges is Rs 25. Now this current price is after 10% reduction.

Therefore, the original price $= \frac{25}{0.9} =$ Rs 27.77

Objective Questions

1. A shopkeeper used to make 5% profit on an item selling at usual marked price. One day, he tripled the marked price of the item and finally offered a discount of 30%. Find the percentage profit he made on the item that day.

(a) 120.5% (b) 100%

(c) 99.5% (d) 94.5%

2. Two numbers A and B are such that the sum of 5% of A and 4% of B is two-third of the sum of 6% of A and 8% of B. Find the ratio of A : B.

(a) 2 : 3 (b) 1 : 1

(c) 3 : 4 (d) 4 : 3

3. In a competitive examination in State A, 6% candidates got selected from the total appeared candidates. State B had an equal number of candidate appeared and 7% candidates got selected with 80 more candidates got selected than A. What was the number of candidates appeared from each State?

(a) 7600 (b) 8000

(c) 8400 (d) Data inadequate

4. 10% of the voters did not cast their vote in an election between two candidates. 10% of the votes polled were found invalid. The successful candidate got 54% of the valid votes and won by a majority of 1620 votes. The number of voters enrolled on the voter's list was :

(a) 25000 (b) 33000

(c) 35000 (d) 40000

5. In a certain school, 20% of students are below 8 years of age. The number of students above 8 years of age is 2/3 of the number of students of 8 years age which is 48. What is the total number of students in the school?

(a) 72 (b) 80

(c) 120 (d) None of these

6. A manufacturer undertakes to supply 2000 pieces of a particular component at Rs. 25 per piece. According to his estimates, even if 5% fail to pass the quality tests, then he will make a profit of 25%. However, as it turned to, 50% of the components were rejected . What is the loss to the manufacturer ?

(a) Rs. 12,000 (b) Rs.13,000

(c) Rs. 14,000 (d) Rs.15,000

7. If selling price is doubled, the profit triples. Find the profit percent :

(a) $66\frac{2}{3}$ (b) 100

(c) $105\frac{1}{3}$ (d) 120

8. In a certain store, the profit is 320% of the cost. If the cost increase by 25% but, the selling price remains constant, approximately what percentage of the selling price is the profit ?

(a) 30% (b) 70%

(c) 100% (d) 250%

9. The percentage profit earned by selling an article for Rs. 1920 is equal to the percentage loss incurred by selling the same article for Rs. 1280. At what price article should be sold to make 25% profit?

(a) Rs. 2000 (b) Rs. 2200

(c) Rs. 2400 (d) Data inadequate

10. Arun purchased 30 kg of wheat at the rate of Rs. 11.50 per kag and 20 kg of wheat at the rate of Rs. 14.25 per kg. He mixed the two and sold the mixture. Approximately what price per kg should he sell the mixture to make 30% profit ?

(a) ₹ 14.80 (b) ₹ 15.40

(c) ₹ 15.60 (d) ₹ 16.30

11. By mixing two brands of tea and selling the mixture at the rate of ₹ 177 per kg, a shopkeeper makes a profit of 18%. If every 2 kg of one brand costing ₹ 200 per kg, 3 kg of the other brand is added, then how much per kg does the other brand cost ?

(a) Rs. 110 (b) Rs. 120

(c) Rs. 140 (d) None of these

12. A shopkeeper purchased 150 identical pieces of calculators at the rate of Rs. 250 each. He spent an amount of Rs. 2500 on transport and packing. He fixed the labelled price of each calculator at Rs. 320. However, he decided to give a discount of 5% on the labelled price. What is the percentage profit earned by him ?

(a) 14% (b) 15%

(c) 16% (d) 20%

13. A trader marked his goods at 20% above the cost price. He sold half the stock at the marked price, one quarter at a discount of 20% on the marked price and the rest at a discount of 40% on the marked price. His total gain is :

(a) 2% (b) 4.5%

(c) 13.5% (d) 15%

14. Gopal went to a fruit market with certain amount of money. With this money he can buy either 50 oranges or 40 mangoes. He retains 10% of the money for taxi fare. If he buys 20 mangoes, then the number of oranges he can buy is

(a) 25 (b) 18

(c) 20 (d) None of these.

15. A shopkeeper sells product at 12% discount on his marked price. he has to pay a 10% tax. If he gives a discount of Rs. 36. How much money is he left with after tax.

(a) Rs. 234.3 (b) Rs. 233.70

(c) Rs. 237.60 (d) Rs. 239.40

16. A fruit vendor gives two successive discounts of 10% and 14%. What is the overall discount given?

(a) 22.4% (b) 22.6%

(c) 23.4% (d) 23.6%

17. The average weight of 5 members of a family is 70 kg. If the weight of each member decreases by 5%, what is the new average?

(a) 66.5 kg (b) 67.5 kg

(c) 68.5 kg (d) 69.5 kg

18. A quantity of 30 ml of 20% alcohol is mixed with 20 ml of 25% alcohol. What is the strength of alcohol in the mixture?

(a) 20% (b) 25%

(c) 22% (d) 22.5%

19. A merchant buys Rs. 20,000 worth of goods. In the transit 40% of the goods got damaged. He is forced to sell them at a 10% loss. What profit percentage should he make on the rest of the items to make an overall profit of 20%?

(a) 20% (b) 25%

(c) 35% (d) 40%

20. In a village, 18% of the population are children and 10% of children are female. If the number of female children is 90, what is the population?

(a) 500 (b) 5,000

(c) 600 (d) 6,000

21. A dishonest seller uses a weight of 800 g in place of 1 kg and adds 20% impurities in sugar. What would be his profit percentage if he claims to be selling at cost price?

(a) 50% (b) 40%

(c) 45.5% (d) None of these

22. Two-third of a consignment was sold at a profit of 5% and the remainder at a loss of 2% If the total profit was Rs. 400, the value of the consignment (in rupees)

(a) 20,000 (b) 15,000

(c) 12,000 (d) 10,000

23. One kilogram tea and 1 kg sugar together cost Rs. 95. If the price of tea falls by 10% and that of sugar rises by 20%, the price of 1 kg each combined comes to Rs. 90. The original price of tea per kilogram is

(a) Rs. 72 (b) Rs. 55

(c) Rs. 60 (d) Rs. 80

24. If CP of 36 books is equal of SP of 30 books, then the gain percentage is (You have to assume that all the books cost same.)

(a) 20%
(b) $16\frac{4}{6}\%$
(c) 16%
(d) $8\frac{2}{5}\%$

25. A man buys 6 dozen eggs for Rs. 10.80, and 12 eggs are found rotten and the rest are sold at 5 eggs per rupee. Find his percentage gain or loss.

(a) $11\frac{1}{9}\%$ gain
(b) $11\frac{1}{9}\%$ loss
(c) $9\frac{1}{11}\%$ gain
(d) $9\frac{1}{11}\%$ loss

26. A shopkeeper has certain number of eggs of which 5% are found to be broken. He sells 93% of the remainder and still has 266 eggs left. How many eggs did he originally have?

(a) 3800
(b) 4000
(c) 4200
(d) None of these

27. Two articles are sold by a shopkeeper. One of them is sold for 10% profit its selling price equals the cost price of the other article, which is sold for 10% loss. Find the effective profit/loss percentage approximately

(a) 1% profit
(b) 1% loss
(c) 0.5% profit
(d) 0.5% loss

28. If the selling price of an article was 25% more the profit/loss made would change by Rs. 1000. If its cost price was 20% more, the profit/loss made would change by Rs. 600. find the actual profit/loss made on selling the article (in Rs.)

(a) 800 profit
(b) 800 loss
(c) 1000 profit
(d) 1000 loss

29. There are two distinct numbers. If each is increased individually by 10 and then increased by the same percentage as each was initially increased by, each result is 62.5. What percent of the larger number is the smaller number?

(a) 10%
(b) 20%
(c) 6.25%
(d) 3.125%

30. The owner of a shop increases the price of the article by x% then decreases it by x%. As a result the price of the article is reduced by Rs. 180. After one more such change the price is further reduced by Rs. 160. Find the original price of the article.

(a) Rs. 1600
(b) Rs. 1610
(c) Rs. 1620
(d) Rs. 1640

31. The per price of vehicular fuel has increased by 25%. If the transportation cost is still the same, then what is the ratio of the reduced fuel consumption to the previous fuel consumption?

(a) 1 : 5 (b) 1 : 4
(c) 1 : 3 (d) 1 : 6

32. John bought five mangoes and ten oranges together for forty rupees. Subsequently, he returned one mango and got two oranges in exchange. The price of an orange would be

(a) 1 (b) 2
(c) 3 (d) 4

33. A space research company wants to sell its two products A and B. If the product A is sold at 20% loss and the product B at 30% gain, the company will not lose anything. If the product A is sold at 15% loss and the product B at 15% gain, the company will gain Rs. 6 million in the deal. What is the cost of product B?

(a) Rs. 140 million (b) Rs. 120 milion
(c) Rs. 100 milion (d) Rs. 80 milion

34. The present value of an optical instrument is Rs. 20,000. If its value will depreciate 5% in the first year, 4% in the second year and 2% in the third year, what will be its value after three years?

(a) ₹16534.5 (b) ₹16756.5
(c) ₹17875.2 (d) ₹17556.8

35. During a landscaping operation, the length of a rectangular garden is increased by 10% and its breadth is decreased by 10%. The area of the new rectangular garden is:

(a) Decreased by 1% (b) Decreased by 10%
(c) Neither increased nor decreased (d) Increased by 1%

36. 70 per cent of the employees in a multinational corporation have VCD players, 75 per cent have microwave ovens, 80 per cent have ACs and 85 per cent have washing machines. At least what percentage of employees has all four gadgets?

(a) 15 (b) 5
(c) 10 (d) Cannot be determined

37. The market price of a machine depreciates at an annual compound rate of 10%. If the current market price of the machine is Rs. 8748 then what was its market price 3 years ago?

(a) Rs. 12500 (b) Rs. 12000
(c) Rs. 20000 (d) Rs. 20500

38. The profit by selling an item was 25%. If the item was marked 40% above the selling price then what is the ratio of the marked to the cost price of the item?

(a) $\frac{5}{4}$ (b) $\frac{7}{4}$
(c) $\frac{3}{4}$ (d) $\frac{1}{4}$

Answers (Objective Questions)

1. (a)	2. (d)	3. (b)	4. (a)	5. (d)	6. (b)	7. (b)
8. (b)	9. (a)	10. (d)	11. (d)	12. (a)	13. (a)	14. (c)
15. (c)	16. (b)	17. (a)	18. (d)	19. (c)	20. (c)	21. (a)
22. (b)	23. (d)	24. (a)	25. (a)	26. (b)	27. (d)	28. (c)
29. (c)	30. (c)	31. (b)	32. (b)	33. (d)	34. (c)	35. (a)
36. (c)	37. (b)	38. (b)				

Solutions:—

1.

Sol. Let the earlier cost price of the item = ₹ 100

Earlier marked price = ₹ 105

New Marked price = 3 × 105 = ₹ 315,

Now on that day, 30% discount is offered on ₹ 315

Discount = 30 % of 315 = ₹ 94.5

Selling price = New MRP – Discount

= 315 – 94.5 = ₹ 220.50

Profit = 220.5 – 100 = ₹ 120.5

New Profit percentage = 120.50%

2.

Sol. 5% of A + 4% of B = $\frac{2}{3}$ (6% of A + 8% of B)

$$\frac{5}{100}A + \frac{4}{100}B = \frac{2}{3}\left(\frac{6}{100}A + \frac{8}{100}B\right)$$

$$\frac{1}{20}A + \frac{1}{25}B = \frac{1}{25}A + \frac{4}{75}B$$

$$\left(\frac{1}{20} - \frac{1}{25}\right)A = \left(\frac{4}{75} - \frac{1}{25}\right)B$$

$$\frac{1}{100}A = \frac{1}{75}B$$

$$\frac{A}{B} = \frac{100}{75} = \frac{4}{3}$$

3.

Sol. Let the number of candidates appeared from each sate be x.

Then, 7% of x – 6% of x = 80

$$1\% \text{ of } x = 80$$

$$x = 80 \times 100 = 8000$$

4.

Sol. Let the total number of voters be x. Then, Votes polled = 90% of x, Valid votes = 90% of (90% of x)

∴ 54% of [90% of (90% of x – 46% of [90% of (90% of x)] = 1620

8% of[90% of (90% of x)] = 1620

$$\frac{8}{100}\times\frac{90}{100}\times\frac{90}{100}\times x = 1620$$

$$x = \left(\frac{1620\times100\times100\times100}{8\times90\times90}\right) = 25000$$

5.

Sol. Let the number of students be x. Then,

Number of students of 8 or above 8 years = (100 – 20)% of x = 80% of x.

$$\therefore \quad 80\% \text{ of } x = 48+\frac{2}{3}\text{ of } 48$$

$$\frac{80}{100}x = 80$$

$$x = 100$$

6.

Sol. Total cost incurred

$$= \left[\frac{100}{125}\times25\times(95\% \text{ of } 2000)\right]$$

$$= \left[\frac{100}{125}\times25\times1900\right] = ₹\,38000.$$

Loss to the manufacturer

$$= [38000 - (25 \times 1000)] = ₹\,13000.$$

7.

Sol. Let C.P. be Rs. x and S.P. be Rs. y.

Then, $3(y - x) = (2y - x)$

$$y = 2x.$$

Profit = (y – x)= (2x – x) = ₹ x.

$$\therefore \text{profit\%} = \left(\frac{x}{x}\times100\right)\% = 100\%$$

8.

Sol. Let C.P.= ₹ 100.

Then, Profit = ₹ 320, S.P. = ₹ 420 ,

New C.P. = 125% of 100 = ₹ 125;

New S.P. = ₹ 420.

Profit = (420 − 125) = ₹ 295.

∴ Required percentage = $\left(\frac{295}{420}\times 100\right)\%$

$= \frac{1475}{21}\% = 70\%$

9.

Sol. Let C.P. be Rs. x.

Then, $\frac{1920-x}{x}\times 100 = \frac{x-1280}{x}\times 100$

$1920 - x = x - 1280$

$2x = 3200$

$x = 1600$

∴ Required S.P. = 125% of 1600

$= \left(\frac{125}{100}\times 1600\right) =$ ₹ 2000

10.

Sol. C.P. of 50 kg wheat

= (30 × 11.50 + 20 × 14.25)

= Rs. (345 + 285) = ₹ 630

S.P. of 50 kg wheat = 130% of 630

$= \left(\frac{130}{100}\times 630\right) =$ ₹ 819

∴ S.P. per kg = $\left(\frac{819}{50}\right)$ = ₹ 16.38 = ₹ 16.30

11.

Sol. Let the cost of the other brand be Rs. x per kg.

C.P. of 5 kg= (2 × 200 + 3 × x) = (400 + 3x)

and S.P. of 5 kg = (5 × 177) = ₹ 885

∴ $\frac{885-(400+3x)}{400+3x}\times 100 =$ ₹ 18

$\frac{485-3x}{400+3x} = \frac{9}{50}$

$$24250 - 150x = 3600 + 27x$$
$$177x = 20650$$
$$x = \left(\frac{350}{3}\right) = 116\frac{2}{3}$$

So, cost of the other brand = ₹ 116.66.

12.

Sol. Cost of each calculator = $\left(250 + \frac{2500}{150}\right)$ = ₹ $266\frac{2}{3}$

S.P. of each calculator = $\left(\frac{95}{100} \times 320\right)$ = ₹ 304

∴ Profit % = $\left(\frac{112}{3} \times \frac{3}{800} \times 100\right)\%$ = 14%

13.

Sol. Let C.P. of whole stock = ₹ 100.

Then, Marked Price of whole stock = ₹ 120.

M.P. of $\frac{1}{2}$ stock = ₹ 60,

M.P. of $\frac{1}{4}$ stock = ₹ 30.

Total S.P. = [60 + (80% of 30) + (60% of 30)]

= (60 + 24 +18) = ₹ 102.

Hence, gain% = (102 – 100)% = 2%.

14.

Sol. Let Gopal have Rs. 400 (a multiple of 50 and 40) .

According to the question price of 50 oranges = price of 40 mangoes = Rs. 400

Then the price of an orange is then Rs. 8 and that of a mango is Rs. 10.

If he keeps 10% of the money for taxi fare, he is left with Rs. 360.

Now if he buys 20 mangoes i.e. if he spends 200 Rs., he is left with Rs. 160, in which he can buy 20 oranges.

15.

Sol. 12% of MP = Rs. 36

MP = Rs. 300

Value after tax deduction = 0.9 × 0.88 × 300 = Rs. 237.6

Alternative Method:

12% discount on MP = Rs. 36

∴ Marked price = 300

Price after discount = 300 – 36 = 264

Remainder = 264×0.9

Option (c) is the answer since the number must end in 6.

16.

$$\text{Discount} = -10 - 14 + \frac{(-10 \times -14)}{100} = 22.6\%$$

17.

Sol. If the weight of each member is decreased by 5%, the average will be decreased by 5%

New average = 70 – 5% of 70 kg = 66.5 kg

18.

Sol. In a 30 ml 20% is alcohol

So, alcohol = 6ml

In a 20 ml 25% solution, alcohol = 5 ml

$$\text{Strength of alcohol in the mixture} = \frac{11}{50} \times 100 = 22\%$$

19.

40% of 20,000 = 8,000

So, SP of goods worth Rs. 8,000 = Rs. $0.9 \times 8{,}000$ = Rs. 7,200

For an overall profit of 20%, SP = $1.2 \times 20{,}000$ = Rs. 24,000

Now he has to sell goods worth Rs. 12,000 at (24,000 – 7,200) = Rs. 16,800

$$\therefore \quad \text{Profit} = \frac{48{,}00}{12{,}000} = 40\%$$

Alternative Method 1.

$$\frac{x-20}{20+10} = \frac{40}{60} = \frac{2}{3}$$

$$\Rightarrow \quad x = 20 + 20 = 40\%$$

Alternative Method 2:

We have incurred 10% loss for 40% items and need 20% profit for 100% items.

Let us have x% profit for 60% of items.

$$\Rightarrow -\left(\frac{40}{100}\right) \times 10 + \left(\frac{60}{100} \times x\right) = \left(\frac{100}{100} \times 20\right)$$

$$\Rightarrow \quad (0.6)x = 24$$

$$x = 40\%$$

20.

Number of female children = 10% of 18% of population.

Therefore, $0.1 \times 0.18 \times P = 90$;

$P = 5{,}000$

21.

Sol. Assume that cost of 1 kg is Rs. 1000.

If the shopkeeper bought sugar at Rs. 800, he would sell this at Rs. 1,000

Since there is 20% impurities also in this, sugar whose CP = Rs. 800 is sold at 1000 + 20% of 1000 = Rs. 1,200.

Hence, profit = $\frac{400}{800} \times 100 = 50\%$ OR

If only the faulty weight were considered, profit would be $\frac{(1000-800)}{800 \times 100} \times 25\%$

Since 20% impurities is added, it is a successive increase in profit.

Hence, net profit = $25 + 20 + \frac{25 \times 20}{100} = 50\%$

22.

Sol. Let the value of consignment be x.

When $\frac{2}{3}$ rd of consignment was sold at a profit of 5%, then Profit = $\frac{\frac{2}{3}x \times 5}{100}$

when the remaining which is $\frac{x}{3}$ consignment was sold at a loss of 2%, then according to

Ques, we have $\frac{\frac{2x}{3} \times 5}{100} - \frac{\frac{x}{3} \times 2}{100} = 400$ $(\therefore \text{Total Profit} = 400)$

$$10x - 2x = 120000$$
$$x = 15000$$

23.

Sol. Let cost of 1 kg of tea be Rs. x and that of sugar be Rs. y.

$\therefore$ $x + y = 95$ .... (i)

$0.9x + 1.2y = 90$ .... (ii)

Solving (i) and (ii), we get

$x = 80$.

24.

Sol. Let the total investment = 36 × 30 = Rs. 1080.

CP of one book = 30

SP of one book = 36

$$\text{Profit} = \frac{36-30}{30} \times 100 = \frac{6}{30} \times 100 = 20\%$$

Alternative Method:

$$36 \times CP = 30 \times SP$$

$$\Rightarrow \frac{SP}{CP} = \frac{36}{30} = 120\%$$

$\therefore$ Profit = 20%

25.

Sol. 6 dozen eggs cost = Rs. 10.80

Since one dozen is rotten, he sells only 5 dozen at 5 eggs per rupee.

Hence, SP = Rs. 12

$$\text{His gain} = \frac{(12-10.8)}{10.8}\times 100 = 11.11\%$$

26.

Sol. He is left with $x \times (0.95 \times 0.07) = 266$

(Assuming that initially he had x number of eggs)

So $$x = \frac{266}{0.07\times 0.95} = 4000$$

27.

Sol. Let the cost price of the article sold for 10% profit be Rs. 100.

$$\text{Its selling price} = \text{Rs. } \frac{110}{100}(100) = 110$$

Cost price of the other article = Rs. 110

$$\text{Its selling price} = \frac{90}{100}(110) = \text{Rs. } 99$$

Total selling price = Rs. 209

Loss = Rs. 1

$$\text{Loss\%} = \frac{1}{210}(100) = 0.5\%$$

There is no loss

Hence is Profit percentage = 3y% = 25%

28.

Sol. If the selling price was 25% more and profit was made, an additional amount of Rs. $\frac{25}{100}$ S.P. will be made.

Given $\frac{25}{100}$ S.P. = 1000

∴ S.P. 4000

Similarly

$$\frac{20}{100}\text{C.P.} = 600$$

∴ CP = 3000

Actually a profit of Rs. 1000 is made.

29.

Sol. Let one of the numbers be denoted by a variable x.

Given that $x + 10 + \frac{10}{x}(x+10) = 62.5$

$$(x + 10)(x + 10) = 62.5x$$

$$x^2 - 42.5x + 100 = 0$$
$$(x - 40)(x - 2.5) = 0$$
$$x - 40 = 0 \text{ or } x - 2.5 = 0$$
$$x = 40,\ 2.5$$

∴ The smaller number is 6.25% of the larger number

Hence the two numbers are 40 and 2.5.

30.

Sol. Let 'P' be the original price and let y% be the net decreased after increasing the price by x% and then decreasing by x %

Given
$$y\%\ (P) = 180$$
$$y\%\ (P - 180) = 160$$
$$y\% = \frac{1}{9}$$
$$\frac{1}{9}(P) = 180$$
$$P = 1620$$

31.

Sol. Fuel Consumption × Fuel Price = Transportation Cost = constant

$$\frac{\text{Current Fuel Consumption}}{\text{Previous Fuel Consumption}} = \frac{\text{Previous Fuel Price}}{\text{Current Fuel Price}} = \frac{100}{125}$$

$$\Rightarrow \frac{\text{Reduced Fuel Consumption}}{\text{Previous Fuel Consumption}} = \frac{(125-100)}{100}$$

$$= \frac{25}{100} = \frac{1}{4}$$

32.

Sol. The price on 1 mango is equal to the price of 2 oranges. Hence 5 mangoes will be equivalent to 10 oranges. So 20 oranges cost Rs. 40, or one orange will cost Rs. 2.

33.

Sol. since, selling price of both the products is same

∴ % loss = % gain

⇒ 20% of A = 30% of B ⇒ A/B = 3/2

Let cost of product A = 3x and cost of product B = 2x.

According to the question,

$$3x \times \frac{15}{100} - 2x \times \frac{15}{100} = 6$$
$$45x - 30x = 600$$
$$x = \frac{600}{15} = 40$$

Hence, cost of product B = 2 × 40 = 80 million.

34.

Sol. Present value of instrument = Rs. 20,000

In the 1st year value remains after depreciation of 5%

$$20,000 - 20,000 \times \frac{5}{100} = ₹\,19000$$

In the 2nd year value remains after depreciantion of 4%

$$19000 - 19000 \times \frac{4}{100} = ₹\,18240$$

In the 3rd year value remains = $18240 - 18240 \times \frac{2}{100}$ = Rs. 17875.2

35.

Sol. Let the length and breadth of a rectangular garden be 100 and 100 respectively.

∴ Area of garden = 100 × 100 = 10,000 sq. unit

Increased length = $100 + 100 \times \frac{10}{100} = 110$

and decreased breadth = $100 - 100 \times \frac{10}{100} = 90$

So, area of new garden = 110 × 90 = 9900

Change in area $\frac{100}{10,000} \times 100 = 1\%$

So, new area is decreased by 1%

36.

Sol. Employees who doesn't have VCD = 100 – 70 = 30%

Employess who doesn't have MWO = 100 – 75 = 25%

Employess who doesn't have AC = 100 – 80 = 20%

Employees who doesn't have WM = 100 – 85 = 15%

∴ Total employees who doesn't have atleast one of the four equipments = 30 + 25 + 20 +15 = 90%

∴ Percentage of employees having all four gadgets = 100 – 90 = 10%.

37.

Sol. Final price, A = Rs. 8748

Time = n = 3 years and Depreciation rate = r = –10%

Let the price, 3 years ago be, P

Then, $$A = P\left(1 + \frac{r}{100}\right)^n$$

By putting the values we get

$$8748 = P\left(1 - \frac{10}{100}\right)^3$$

$$\Rightarrow \quad .8748 = P\left(\frac{9}{10}\right)^3$$

$$\Rightarrow \quad \frac{8748\times10\times10\times10}{9\times9\times9} = P$$

$$\Rightarrow \quad P = 12{,}000$$

38.

Sol. Let the cost price of an item = Rs. 100,

Then, selling price = Rs. 125

($\because$ Profit by selling is 25%)

Now, marked price is 40% above the selling price.

$$\therefore \quad M.P = 125+125\times\frac{40}{100}$$

$$M.P. = 125\left(1+\frac{40}{100}\right) = 175$$

$$\text{Hence,} \quad \frac{\text{Marked Price}}{\text{Cost Price}} = \frac{175}{100} = \frac{7}{4}$$

CHAPTER

10

Average and Problems on Ages

Average

1. Average = $\frac{\text{sum of quantities}}{\text{number of quantities}}$

2. Sum of quantities = Average × Number of quantities

3. Number of quantities = $\frac{\text{sum of quantities}}{\text{Average}}$

Example 1

Total temperature for the month of september is 840°C. If the average temperature of that month is 28°C, find of how many days in the month of september.

Sol: Number of days in the month of september

$$= \frac{\text{total temperature}}{\text{Average temperature}}$$

$$= \frac{840}{28} = 30 \text{ days.}$$

1. Average of two or more groups taken together

Some shortcut methods

(a) If the number of quantities in two groups be n_1 and n_2 and their average is x and y respectively, the combined average is

$$= \frac{n_1x + n_2y}{n_1 + n_2}$$

(b) If the average of n_1 quantities is x and the average of n_2 quantities out of them is y, the average of remaining group is

$$= \frac{n_1x - n_2y}{n_1 - n_2}$$

Example 2

The average weight of 24 students of section A of a class is 58 kg whereas the average weight of 26 students of section B of the same class is 60.5 kg. Find the average weight of all the 50 students of the class

Sol. Average weight of all the 50 students

$$= \frac{n_1 x + n_2 y}{n_1 + n_2}$$

$$= \frac{24 \times 58 + 26 \times 60.5}{50}$$

$$= \frac{1392 + 1573}{50}$$

$$= \frac{2965}{50}$$

= 59.3 kg

Example 3

Average salary of all 50 employees including 5 officers of a company is Rs 850. If the average salary of the officers is Rs. 2500, find the average salary of the remening staff of the company.

Sol. Average salery of the remaning staff

$$= \frac{n_1 x - n_2 y}{n_1 - n_2}$$

$$= \frac{50 \times 850 - 2 \times 2500}{50 - 5} = \frac{30000}{45}$$

= 667 approx.

2. If is the average of x_1, x_2, ----- x_n then

(a) The average of $x_1 + a$, $x_2 + a$ --------- $x_n + ax$ is $\bar{x} + a$

(b) The average of $x_1 - a$, $x_2 - a$ ------ $x_n - ax$ is $\bar{x} - a$

(c) The average of ax_1, ax_2 ---- ax_n is $a\bar{x}$ provided $a \neq 0$

(d) The average of $\frac{x_1}{a}, \frac{x_2}{a} --- \frac{x_n}{a}$ is $\frac{\bar{x}}{a}$ provided $a \neq 0$

Example 4

The average value of six numbers 7, 12, 17, 24, 26 and 28 is 19. If 8 added to each number, what will be the new average?

Sol. The new average $= \bar{x} + a$

$= 19 + 8 = 27.$

Example 5

The average of x numbers is 5x. If x – 2 is substreated from each given number, what will be the new average?

Sol. The new average $= \bar{x} + a$

$= 5x - (x - 2)$

$= 4x + 2$

Example 6

The average of 8 numbers is 21. If each of the numbers is multiplied by 8, find the average of a new set of number.

Sol. The average of a new set of number $= a\bar{x} = 8 \times 21 = 168$

3 The average of n quantities is equal to x. If one of the given quantities whose value is p, is replaced by a new quantity having value q, the average becomes y, then $q = p + n(y - x)$.

Example 7

The average weightr of 25 persons is increased by 2 kg when one of them whose weight is 60 kg, is replaced by a new person. What is tghe weight of the new person.

Sol. The weight of the new person

$= P + n(y - x)$

$= 60 + 25 \times 2$

$= 110$ kg.

4.(a) The average of n quantities is equal to x when a quantity is removed, the average becomes y. The value of the removed quantity is $n(x - y) + y$

(b) The average of n quantities is equal to x. When a quantity is added, the average becomes y. the value of the new quantity is $n(y - x) + y$.

Example 8

The avearage age of 24 students and the class teacher is 16 years. If the class teacher's age is excluded, the average age reduces by 1 year what is the age of the class teacher.

Sol. The age of class teacher

$$= n(x - y) + y$$
$$= 25(16 + 5) + 15$$
$$= 40 \text{ years}$$

Example 9

The average age of 30 children in a class is 9 years. If the teacher's age be included, the average age becomes 10 years. Find the teacther's age

Sol. The teacher's age

$$= n(y - x) + y$$
$$= 30(10 - 9) + 10 = 40 \text{ years}$$

5. (a) The average of first n natural number is $\frac{n+1}{2}$

(b) The average of square of natural number till n is $\frac{(n+1)(2n+1)}{6}$

(c) The average of cubes of natural numbers till n is $\frac{n(n+1)^2}{4}$

(d) The average of odd number from 1 ato n is $\frac{\text{last odd number} + 1}{2}$

(e) The average of even number from 1 to n is $\frac{\text{last even number} + 2}{2}$

Example 10

Find the average of first 81 natual numbers.

Sol. The required average $= \frac{n+1}{2} = \frac{81+1}{2} = 41$

Example 11

What is the average of squares of the natural number from 1 to 41?

Sol. The required average $= \frac{(n+1)(2n+1)}{6}$

$$= \frac{42 \times 83}{6} = 581.$$

Example 12

Find the average of cubes of natural number from 1 to 27.

Sol. The required average

$$= \frac{n(n+1)^2}{4} = \frac{27(28)^2}{4}$$

$$= 5292$$

Example 13

What is the average of odd number from 1 to 40

Sol. The required average = $\frac{\text{last odd number}+1}{2} = \frac{39+1}{2} = 20$

Example 14

What is the average of even numbers from 1 to 81?

Sol. The required averagve = $\frac{\text{last even number}+2}{2} = \frac{80+2}{2} = 41$

6. Geometric mean or Geometric average

Geometric mean of x_1, x_2, ----- x_n is denoted by

$$G.M = n\sqrt{x_1 \times x_2 \times ------ x_n}$$

Geometric mean is useful in calculating averages of ratios such as average increase and so on.

Example 15

The production of a company for there successive years has increased by 10%, 20% and 40% respectively. What is the average annual increase of production.

Sol. Geometric mean of x, y and z = $(xyz)^{1/3}$

$\therefore$ Average increase = $(10 \times 20 \times 40)^{1/3}$ = 20%.

Example 16

The population of a city in two successiveyears increases at the rates of 16% and 4% respectively. Find the average increase of two years.

Sol. In case of population increase, the goemetric mean is required

$\therefore$ Geometric mean of 16% an 4% is

= $(16 \times 4)^{1/2}$ i.e. 8%.

7. Harmonic Mean or Harmonic Average

Harmonic mens of x_1, x_2, -------- x_n is denoted by

$$H\,M = \frac{1}{\frac{1}{n}\left(\frac{1}{x_1}+\frac{1}{x_2}+..........+\frac{1}{x_n}\right)}.$$

Harmonic mean is useful for finding out average speed of a vehicle average production per day and so on.

Example 17

If half of the journey is travelled at a speed of 15 km/hr. and the next half at a speed of 12 km/hr. find the average speed during the entire journey.

Sol. The average speed $= \frac{2xy}{x+y} = \frac{2\times15\times12}{15+12}$

$= 13^{1/3}$ km/hr.

$= \frac{40}{3}$ km/hr

$= 13^{1/3}$ km/hr

Objective Questions

Exercise — I

1. The average weight of 5 persons sitting in a boat is 38 kg. If the average weight of the boat and the person sitting in the boat is 52 kg, what is the weight of the boat?

(a) 228 kg (b)122 kg

(c) 232 kg (d)242 kg

2. There are 35 students in a hostel. If the number of students increased by 7, the expenses of the mass were increased by Rs 42 per day while the average expenditure per head diminished by Re 1. Find the original expenditure of the mass.

(a) Rs 480 (b)Rs 440

(c) Rs 520 (d) Rs 420

3. The daily maximum temperature in Delhi for 7 consecetive days in May 1998 were 42.7°C, 44.6°C, 42.0°, 39.1°C, 42.5°C and 38.5°C.

(a) 42-63°C (b)45-65°C

(c) 41-77°C (d)39-60°C

4. The average weight of 24 students of section A of a class is 58 kg whereas the average weight of 26 students of section B of the same class is 60.5 kg. Find the average weight of all the 50 students of the class.

(a) 57.4 kg (b)59.3 kg

(c) 58.9 kg (d)59.7 kg

5. A batsman is his17th inning make a score of 85, and there by increases his average by 3. What is his average after the 17th innings? he had never been 'not out'

(a) 47 (b)37

(c) 39 (d)43.

6. The average age of A, B, C, D five years ago was 45 years. By including x, the present average age of all the five is 49 years. The present age of x is

(a) 64 yrs. (b)48 yrs

(c) 45 yrs (d)40 yrs.

7. The agerage weight of 3 men A, B and C is 84 kg. Another man D joins the group and the average now becomes 80 kg. If another man E, whose weight is 3 kg more them that of D, replaces A, then average weight of B, C, D and E becomes 74 kg. The weight of A is

(a) 70 kg (b)72 kg

(c) 75 kg (d)80 kg

8. The average weight of 36 students is 50 kg. If was found later that the figure of 37 kg was misread as 73 kg. What is the corsect average.

(a) 49 kg (b)51 kg

(c) 50 kg (d)None of these

9. The average of marks obtained by 120 candidates was 35. If the average of marks of passed candidates was 39 and that of failed candidates was 15, the number of candidates who passed the examination.

(a) 100 (b)110

(c) 120 (d)150

10. In a coconut grove, (x + 2) trees yield 60 nuts per year, x trees yield 120 nuts per year and (x – 2) trees yield 180 nuts per year. If the average yield per year per tree be 100, find x.

(a) 4. (b)2

(c) 8 (d)6

11. Out of three numbers, the first is twice the second and is half of the third. If the average of the three numbers is 56, the three number in order are

(a) 48, 96, 24 (b)48, 24, 96

(c) 96, 24, 48 (d)96, 24, 48

12. If the mean of a, b, c is M and ab + bc + ca = 0, then the mean of a^2, b^2, c^2 is

(a) M^2 (b) $3m^2$

(c) 8 M (d) $9M^2$

Exercise — II

13. What is the average of n, n + 1, n + 2, n + 3, n + 4, n + 5.

(a) n + 2 (b) $n+2\frac{1}{2}$

(c) 6n + 15 (d) $n+3\frac{1}{2}$

14. There is a sequence of 11 consecutive odd numbers, if the average of first 7 number is x, then find the average of all the 11 integers.

(a) x + 3 (b) x + 4

(c) x + 5 (d) x + 7

15. Average of ten positive numbers is $\bar{x}$. If each number increases by 10%, then $\bar{x}$.

(a) Remains unchanged

(b) Is increased by 10%

(c) May decrease

(d) May either increase or decrease

16. The average of 5 consecutive numbers is n. If the next two numbers are also included, the average will be.

(a) Increase by 1 (b) Remain the same

(c) Increase by 1.4 (d) Increase by 2

17. The average of nine numbers is M and the average of three of these is P. If the average of remaining number is N, then

(a) N = N + P (b) 2 M = N + P

(c) 3M = 2N + P (d) 3M = 2P + N

18. An aeroplane files along the four sides of a square field at the speeds of 200, 400, 600 and 800 km/hr. The average speed of the plane around the field in km/hr is

(a) 384 (b) 400

(c) 200 (d) 284

19. A student on his birthday distributed on an average 5 choclates per student. If on the arrival of the teacher and the head master to whom the student gives 10 and 15 choclates respectively, the average choclates distributed per head is 5.5, then what is the strength of the class?

(a) 28 (b) 30

(c) 32 (d) None of these

20. On an 800 miles trip, car w travelled half the distance at 80 miles per hour and the other half at 100 miles per hour. what was the average speed of the car?

(a) (b)180.00

(c) 90.00 (d)88 8/9

21. For ten hours, a train travels at a constant speed of 20 miles per hour and during the next 15 hours, it travels 240 miles. What is the average speed of the train for the whole joining?

(a) 17.5 miles/hr (b)20.8 miles/hr

(c) 130 miles/hr (d)176 miles/hr

22. A person tranels from x to y at a speed of 40 km/hr and reterns increasing the speed by 50%. What is his average speed for both the trips?

(a) 36 km/hr (b)45 km/hr

(c) 48 km/hr (d)None of these

Answers Objective Questions

1.	(b)	2.	(d)	3.	(c)	4.	(b)	5.	(b)
6.	(c)	7.	(c)	8.	(a)	9.	(a)	10.	(a)
11.	(b)	12.	(b)	13.	(b)	14.	(b)	15.	(b)
16.	(a)	17.	(c)	18.	(a)	19.	(a)	20.	(a)
21.	(d)	22.	(d)						

1.

sol. Avg weight of 5 persons = 38 kg

∴ total weight of these five persons

= 38 × 5 = 190 kg

Now, average weight of (the boat + 5 persons)

= 52

∴ total weight of (the boat + 5 person)

= 52 × 6 = 312 kg

∴ weight of the boat = 312 − 190 = 122 kg

2.

Let the original expenditure = Rs x

original average expenditure = $\frac{x}{35}$

New avearage expenditure = $\frac{x+42}{42}$

$$\Rightarrow \quad \frac{x}{35} - \frac{x+42}{42} = 1$$

$$x = 420$$

3.

Average daily maximum temperature

$$= \frac{42.7 + 44.6 + 42.0 + 39.1 + 43.0 + 42.5 + 38.5}{7}$$

$$= \frac{292.4}{7} = 41.77°C$$

4.

Average weight of 24 students of section

A = 58 kg and section B = 60.5 kg

Total weight of 50 students = 58 × 24 + 60.5 × 26

= 2965 kg

Average weight of students in the class = $\frac{2965}{50} = 59.3kg$

5.

Let x be the Average score before the 17th innings

$$\therefore \quad \frac{16 \times x + 85}{17} = x + 3$$

$$x = 34$$

Average score after 17th innengs = 34+3 = 37

6. Present age of x.

= 49 × 5 – (4 × 45 + 4 × 5)

= 45 yrs

7.

weight of D = 80 × 4 – 84 × 3 = 68 kg

weight of E = 68 + 3 = 71 kg

(B + C + D + E)'S weight = 79 × 4 = 316 kg

∴ (B + C)'s weight = [316 – (68 + 71)] kg

= 177 kg

Hence, A,s weight = [84 × 3 – 177] kg = 75 kg

8.

correct Average

$$= \frac{50\times36-73+37}{36} = \frac{1764}{36} = 49kg$$

9.

Let the number of cendidates who passed = x.

then, 39 × x + 15 (120 – x) = 120 × 35

∴ 24 x = 4200 – 1800

x = 2400/24 = 100

10

$$\frac{(x-2)\times60+x\times120+(x-2)\times180}{x+2+x+x-2}=100$$

11. Let the third no be 2x

∴ the first = x

and the second = $\frac{x}{2}$

$$\frac{x+\frac{x}{2}+2x}{3}=56$$

$$\frac{7x}{6}=56$$

x = 48

12.

$$\frac{a+b+c}{3} = M$$

$$\Rightarrow \quad (a+b+c)^2 = (3M)^2$$

$\Rightarrow \quad a^2 + b^2 + c^2 + 2(ab + bc + ca) = 9M^2$

$\Rightarrow \quad a^2 + b^2 + c^2 = 9M^2$

$\Rightarrow \quad \dfrac{a^2 + b^2 + c^2}{3} = \dfrac{9M^2}{3} = 3M^2$

13.

$$\frac{6n+15}{6} = \frac{2n+5}{2} = n + \frac{5}{2} = n + 2\frac{1}{2}$$

14.

Average of first 7 numbers will be the 4th number = x (given)

Average of the 11 numbers will be the 6th number i.e. x + 4.

16.

Let the consecutive numebrs be x, x + 1, x + 2, x + 3, x + 4.

$$\text{Average} = \frac{5x+10}{5} = x+2$$

$$\text{Average of 7 number} = \frac{5x+10+x+5+x+6}{7}$$

$$= \frac{7x+21}{7} = x+3$$

∴ The average increased by 1

18. Let each side of the square be 2400 km (L.C.M of 800, 600,400,200)

∴ Time taken are 3, 4, 6 and 12 hrs

$$\therefore \quad \text{Average speed} = \frac{4\times 2400}{25} = 384 \text{ km/hr}$$

$= 384$ km/hr.

19.

Suppose strength of the class = x

∴ 5x + 10 + 15 = 5.5 (x + 2)

$\Rightarrow \quad 0.5x = 14$

$\Rightarrow \quad x = 28.$

20.

The distance of 800 miles was covered in 5 + 4 = 9 hrs.

$\therefore$ Average speed of w = $\frac{800}{9} = 88\frac{8}{9}$ miles per hrs.

21.

$$\frac{20\times10+240}{25} = \frac{200+240}{25} = \frac{440}{25}$$

$$= 17.6 \text{ miles/hour}$$

22.

$\therefore$ Average speed = $\frac{2x}{\frac{x}{40}+\frac{x}{60}}$ = 48 km/hr.

Problems on Ages

Introduction

Problems based on ages are generally asked in most of the competitive examinations. To solve these problems, the knowledge of linear equations is essential. In such problems, there may be three situations.

(i) Age some years ago

(ii) Present age

(iii) Age some years hence

Example 1

The age of father is 4 times the age of his son. If 5 years ago father's age was 7 times the age of his son at that time, what is father's present age?

Sol. F = 4S

5 years ago

F – 5 = 7 (S – 5)

⇒ F – 5 = 7 S – 35

7S – F = 30

7S – 4S = 30

3S = 30

S = 10

F = 40

Example 2

The age of Mr. Gupta is four times the age of his son. After ten years, the age of Mr. Gupta will be only twice the age of his son. Find the present age of Mr. Gupta's son

Sol. F = 4S

F + 10 = 2 (S + 10)

F + 10 = 2S + 20

F – 2S = 10

4S – 2S = 10

2S = 10

S = 5

Example 3

10 years ago Anu's mother was 4 times older than daughter. After 10 years the mother will be twice older than the daughter. Find the present age of Anu.

Sol. $M - 10 = 4(D - 10)$

$M - 10 = 4D - 40$

$4D - M = 30$ ---- (i)

$M + 10 = 2(D + 10)$

$M + 10 = 2D + 20$

$2D + M = 10$ ---- (ii)

$4D - M = 30$

$2D - M = -10$

$(-) \quad (+) \quad (+)$

$2D = 40$

$D = 20$

Example 4

The ratio of the age of father and son at present is 6:1. After 5 years, the ratio will become 7:2. Find the present age of the son.

Sol. $F = 6x, \quad S = x$

$$\frac{6x+5}{x+5} = \frac{7}{2}$$

$\Rightarrow \quad 12x + 10 = 7x + 35$

$\Rightarrow \quad x = 5.$

Objective Questions

1. Sachin was twice as old as Ajay 10 years back. How old is Ajay today if Sachin will be 40 years old 10 years hence?

(a) 20 years (b) 10 years
(c) 30 years (d) None of these

2. A demographic survey of 100 families in which two parents revealed that the average age of A, of the oldest child, is 20 years less than 1/2 the sum of the ages of the two parents. If F represent the age of one parent and M, the age of the other parent, then will of the following equivalent to A?

(a) $\frac{F+M-20}{2}$ (b) $\frac{F+M}{2}+20$

(c) $\frac{F+M}{2}-20$ (d) F + M − 10

3. If 10 years are substracted from the present age of Ram and the remainder divided by 14, then you would get the present age of his grandson Shyam. If Shyam is 9 years younger to Sunder whose age is 14, then what is the present age of Ram?

(a) 80 years (b) 70 years

(c) 60 years (d) None of these

4. Two groups of students, whose average ages are 20 years and 30 years, combine to forma third group whose average age is 23 years. What is the ratio of the number of students in the first group to the number students in the second group?

(a) 5 : 2 (b) 2 : 5

(c) 7 : 3 (d) None of these

5. A year ago, a father was 4 times his son's age. In 6 years, his age will be 9 more than twice his son's age. What is the present age of the son?

(a) 10 years (b) 9 years

(c) 20 years (d) None of these

6. The average of three boys is 16 years. If their ages in the ratio 4:5:7, then the age of the youngest boy is

(a) 8 years (b) 9 years

(c) 12 years (d) 16 years

7. Namrata's father is now four times her age. In five years, he will be three times her age. In now many years, will be twice her age?

(a) 5 (b) 20

(c) 25 (d) 15

8. A father is twice as old as his son 20 years back he was 12 times as old as the son what are their present age?

(a) 24, 12 (b) 44, 22

(c) 48, 24 (d) None of these

9. The present ages of three persent are in the proportion of 4 : 7 : 9. 8 years ago, the sum of wheir ages was 56. Find their present ages.

(a) 20, 35, 45 (b) 8, 20, 28

(c) 16, 28, 36 (d) None of these

10. If 6 years is subtracted from the present age of Randheer and the remainder is divided by 18, then the present age of his grandson Anup is obtained. If Anup is 2 years younger to Mahesh, whose age is 5 years, the what is the age of Randheer?

(a) 84 yrs (b) 96 yrs
(c) 48 yrs (d) 60 yrs

11. The ratio of Ashok's age to Pardeep's is 4 : 3. Ashok will be 26 years old after 6 years. How old is Pradeep now?

(a) 18 yrs (b) 21 yrs
(c) 15 yrs (d) 24 yrs

12. Ten years ago, the ages of the members of a joint family of eight people added up to 231 years. Three years later, one member died at the age of 60 years and a child was born during the same year. After another three years, one more member died, again at 60 and a child was born during the same year. The current average age of this eight member family is nearest to

(a) 21 yrs. (b) 25 yrs
(c) 24 yrs (d) 23 yrs.

Answers (Objective Questions)

1. (a)	2. (c)	3. (a)	4. (c)	5. (b)	6. (c)
7. (b)	8. (b)	9. (c)	10. (d)	11. (c)	12. (d)

1. (a)
Present age of Sachin is 30 yrs.

$$S - 10 = 2(A - 10)$$

$$\Rightarrow 30 - 10 = 2(A - 10)$$

$$\Rightarrow \frac{20}{2} = A - 10$$

$$\Rightarrow A = 20 \text{ yrs.}$$

2. (c)

$$A = \frac{F+M}{2} - 20$$

3. (a)

$$\frac{Ram - 10}{14} = Shyam$$

$\Rightarrow$ Present age of Shyam = 5 yrs.

$$\Rightarrow \frac{Ram - 10}{14} = 5$$

$\Rightarrow$ Ram − 10 = 70

$\Rightarrow$ Ram = 80 yrs.

4. (c)

Let the number of students in 1st group be a and that of the students in 2nd group be b

$$\Rightarrow \quad \frac{20a+30b}{a+b} = 23$$

$$\Rightarrow \quad 20\,a + 30\,b = 23\,a + 23\,b$$

$$\Rightarrow \quad 30\,b - 23\,b = 23\,a - 20\,a$$

$$\Rightarrow \quad 7b = 3a$$

$$\frac{a}{b} = \frac{7}{3}$$

5. (b) Let the present age of father be F and son's age be S

$$F - 1 = 4\,(S - 1) \quad \text{---- (i)}$$

$$F - 16 = 2\,(S + 6) + 9. \quad \text{---- (ii)}$$

From (i) and (ii), S = 9 yrs.

6. Let the ages be 4x, 5x and 7x

$$\frac{4x+5x+7x}{3} = 16$$

$$\Rightarrow \quad 16\,x = 48$$

$$x = 3$$

$\therefore$ age of youngest boy = 12 yrs.

7. Let the present age of Father be F and Namrata's age be N.

$$F = 4\,N \quad \text{---- (i)}$$

$$F + 5 = 3\,(N + 5\,)$$

$$F + 5 = 3N + 15$$

$$F - 3\,N = 10$$

$$N = 40$$

$$F = 40$$

$$F + x = 2\,(N + x)$$

$$40 + x = 2\,(10 + x)$$

$$40 + x = 20 + 2x$$

$$x = 20$$

8. (b) Let the present age of Father be F and the son be S.

$$F = 2S$$

$$F - 20 = 12\,(S - 20)$$

$$F - 20 = S - 240$$

$$F - S = -\,240 + 20$$

$$F - S = -\,220$$

$$25 - 125 = -220$$

$$-\,105 = -\,220$$

$$[S = 22]$$

$$[F = 44]$$

9. (c) $4x-8+7x-8+9x-8 = 56$

$20x = 80$

$x = 4$

$\therefore$ 16, 28, 36

10. (d)

$$\frac{Randheer-6}{18} = \text{Anup}$$

Present age of Anup = 3 yrs

$\Rightarrow$ Ramdheer = 18 × 3 + 6

$\Rightarrow$ Ramdheer = 60

11. Let Ashok's age be 4x.
and Pardeep's age be 3x

Present age of Ashok = 20 yrs

4x = 20

x = 5.

$\therefore$ persent age of Pardeep is 15 yrs.

12. 10 years ago, total age of 8 members = 231
After three years, sum of the ages

= 231 + 8 × 3 − 60

= 195

After three years = 195 + 8 × 3 − 60

= 159

Current sum of ages = 159 + 8 × 4

= 191

Current average age = $\frac{191}{8} \approx 24$. (23 × 875).

CHAPTER

11

Ratio, Proportion and Partnership

INTRODUCTION

If the values of two quantities A and B, 4 and 6 respectively, then we say that they are in the ratio 4 : 6 (read as "four is to six"). Ratio is the relation which one quantity bears to another of the same kind. The ratio of two quantities "a" and "b" is represented as a : b and read as "a is to b". Here, "a" is called antecedent, "b" is the consequent. Since the ratio expresses the number of times one quantity contains the other, it's an abstract quantity.

If two quantities whose values are A and B respectively are in the ration a : b, since we know tha some common factor k (>0) would have been removed from A and B to get the ratio a : b, we can write the original values of the two quantities (i.e., A and B) as ak and bk respectively. for example if the salaries of two persons are in the ratio 7 : 5, we can write their individual salaries as 7k and 5k respectively.

Some Important Properties of Ratios

1. If we muliply the numerator and the denominator of a ratio by the same number, the ratio remains unchanged.

   That is, $\frac{a}{b} = \frac{ma}{mb}$

2. If we divide the numerator and the denominator of a ratio by the same number, the ratio remains unchanged. thus a/b = $\frac{(a/d)}{(b/d)}$

3. When the ratio a/b is compounded with itself, the resulting ratio is a^2/b^2 and is called the duplicate ratio. similarly, a^3/b^3 is the triplicate ratio and $a^{0.5}/b^{0.5}$ is the sub-duplicate ratio of a/b.

Example 1.

A bag contains 50 p, 25 p and 10 p coins in the ratio 5 : 9 : 4, amounting to Rs. 206. Find the number of coins of each type.

Sol.

Let the number of 50 p, 25 p and 10 p coins be 5x, 9x and 4x respectively.

Then, $\frac{5x}{2}+\frac{9x}{4}+\frac{4x}{10}=206$

$\Leftrightarrow 50x + 45x + 8x = 4120 \Leftrightarrow 103x = 4120 \quad \Leftrightarrow \quad x = 40.$

$\therefore$ Number of 50 p coins = (5 × 40) = 200;

Number of 25 p coins = (9 × 40) = 360;

Number of 10 p coins = (4 × 40) = 160.

Example 2.

The number of red balls and green balls in a bag are in the ratio 16 : 7. If there are 45 more red balls than green balls, find the number of green balls in the bag.

Sol.

Since the ratio of number of red and green balls is 16 : 7, let the number of red balls and green balls in the bag be 16x and 7x. So, the difference of red and green balls is 9x.

$16x - 7x = 9x = 45 \Rightarrow x = 5$

$\Rightarrow$ Hence the number of green balls

= 7x i.e., 35. Hence there are 35 green balls are there.

Example 3.

What least number must be added to each of a pair of numbers which are in the ratio 7 : 16 so that the ratio between the terms becomes 13 : 22?

Sol.

Let the number to be added to each number be a. Let the actual values of the numbers be $7x$ and $16x$. since their ratio is 7 : 16

$$\frac{7x+a}{16x+a}=\frac{13}{22}$$

$\Rightarrow 154x + 22a = 208x + 13a$

$\Rightarrow 9a = 54x$

$\Rightarrow a = 6x$, when $x = 1$, a is the least number required and is equal to 6

Example 4.

A number is divided into four parts such that 4 times the first part, 3 times the second part, 6 times the third part and 8 times the fourth part are all equal. In what ratio is the number divided?

Sol.

Let the pats into which the number is divided be a, b, c and d.

4a = 3b = 5c = 8d. Let the value of each of these equal to e.

$a = \frac{e}{4}$, $b = \frac{e}{3}$, $c = \frac{e}{5}$, and $d = \frac{e}{8}$

Hence a : b : c : d = $\frac{e}{4}:\frac{e}{3}:\frac{e}{5}:\frac{e}{8}$

$= \frac{30}{120}:\frac{60}{120}:\frac{24}{120}:\frac{15}{120}$

(where 120 is the LCM of the denominators)

= 30 : 60 : 24 : 15 = 10 : 20 : 8 : 5.

Hence the ratio of the parts into which the number is divided is 10 : 20 : 8 : 5.

Example 5.

A dog pursues a cat and takes 5 leaps for every 6 leaps of the cat, but 4 leaps of the dog are equal to 5 leaps of the cat. Compare the speeds of the dog and the cat.

Sol.

4 leaps of the dog = 5 leaps of the cat.

1 leaps of the dog = $\frac{5}{4}$ leaps of the cat

5 leaps of the dog = $\frac{25}{4}$ leaps of the cat.

Now given that is time t dog takes 5 leaps and cat takes 6 leaps

So speed × t = 5 dog leaps = $\frac{25}{4}$ cat leaps

Now Ratio

$$\frac{\text{Cat speed}}{\text{Dog speed}} = \frac{6 \text{ cat leaps}}{\frac{25}{4} \text{ cat leaps}} = \frac{24}{25}$$

Ratio of the speed of the cat to the dog is 24 : 25 respectively.

Example 6.

The income of A and B are in the ratio 3 : 2 and the expenditures in the ratio 5 : 3. If each saves Rs. 2,000, what are their incomes?

Sol.

Let 3a, 2a be the incomes, and 5b and 3b be the expenditures of A and B respectively.

We have 3a – 5b = 2a – 3b = 2000

Solving, a = 4000, b = 2000

∴ A's income = Rs. 12,000, B's income = Rs. 8,000

PROPORTION

When two ratio are equal, the four quantities comprising them are said to be proportionals. thus if a/b = c/d, then a, b,c,d are proportionals. this is expressed by saying that a is to b as c is d, and proportion is written as a : b :: c : d or a : b = c : d

The terms a and d are called the extremes while the terms b and c are called the means.

If four quantities are in proportion, the product of the extremes is equal to the product of the means.

Let a, b, c, d be the proportionals.

Then by definition $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$

Hence if any three terms of proportion are given, the fourth may be found. thus if a, c, d are given, then b = $\frac{ad}{b}$.

If three quantities a, b and c are in continued proportion, then a : b = b : c

$\therefore \quad ac = b^2$

In this case, b is said to be a mean proportional between a and c; and c is said to be a third proportional to a and b.

That is when a, b and c are in continued proportion. $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$

If four quantities a, b, c and d form a proportion, many other proportions may be deduced by the properties of fractions. The results of these operations are very useful. These operations are

1. **Invertendo:** If a/b = c/d then b/a = d/c
2. **Alternando:** If a/b = c/d, then a/c = b/d
3. **Componendo:** If a/b = c/d, then $\left(\frac{a+b}{b}\right) = \left(\frac{c+d}{d}\right)$
4. **Dividendo:** If a/b = c/d, then $\left(\frac{a-b}{b}\right) = \left(\frac{c-d}{d}\right)$
5. **Componendo and Dividendo:** If a/b = c/d, then (a + b)/(a –b) = (c + d)/(c –d).

VARIATION

(i) We say that x is directly proportional to y, if x = ky for some constant k and we write, $x \propto y$.

(ii) We say that x is inversely proportional to y, if xy = k for some constant k and we write, $x \propto \frac{1}{y}$.

Example 7.

A can do a piece of work in 12 days, B is 50% more efficient than A. Find the number of days that B takes to do the same piece of work.

Sol:

Method 1: Ratio of the efficiencies is A : B = 100 : 150 = 2 : 3.

Since efficiency is inversely porportional to the number of days, the ratio of days taken to complete the job is 3 : 2. That is when A takes 3 days to do a work, B will take 2 days. Now when A takes 12 days.

So number of days taken by B = $\frac{2}{3} \times 12 = 8$ days.

Method 2: The time taken is inversely porportional to the efficiency of the person.

So if B was as efficient as A, he would have taken 12 days to finish the job. Since he is 1.5 time as efficient as A he would take $\frac{12}{1.5}$ times the days that A takes to finish the job.

Hence, B finishes the job in $\frac{12}{1.5}$ days = 8 days

Example 8.

A precious stone worth Rs. 6800 is accidentally dropped and it breaks into three pieces. The weights of these three pieces are in the ratio 3 : 9 : 8. The value of the stone is proportional to the square of its weight. Calculate the loss in values if any incurred because of the breakage.

Sol.

let the weights of three pieces be 3 g, 9 g and 8 g.

So the total weight of the unbroken stone = 3 + 9 + 8 = 20 g.

The price of the stone is directly proportional to the $(\text{weight})_2$.

So 6800 = K × $(20)^2$,. Hence, K = 17

So value of the broken pieces are k × $(3)^2$, k × $(9)^2$ and k × (8^2). i.e 9k, 81k and 64k

i.e. Rs 153, Rs 1377, and Rs. 1,088 respectively.

Hence, the total value of the broken pieces = (153 + 1377 + 1088) = Rs. 2,618.

So the total loss = 6800 – 2618 = Rs. 4,182.

Example 9.

The height of Rajan is of the same proportion as the square root of his age (between 5 and 17 years). What will be the height of Rajan after 7 years, if he is 4 ft tall at the age of 9 years?

Sol.

Height is proportional to age.

Or H = k$\sqrt{A}$; H = 4 ft at A = 9.

Hence, k = $\frac{4}{3}$.

So when A = 16 years, H = $\frac{4}{3}\sqrt{16} = 5\frac{1}{3}$ ft or 5 ft 4 inches.

Example 10.

There are three parameters: X, Y and Z. X is directly proportional to Y^2 and inversely proportional to Z^3. when X = 20, Y = 10 and Z = 5. What is the value of Y when X = 10 and Z = 10?

Sol.

$$X \propto Y^2$$

$$X \propto \frac{1}{Z^3}\ ;\ X \propto \frac{Y^2}{Z^3}$$

$$\Rightarrow X = k\frac{Y^2}{Z^3}$$, wehre k is a constant

$$k = \frac{XZ^3}{Y^2}$$

$$\text{So } \frac{20\times5^3}{10^2} = \frac{10\times10^3}{Y^2}$$

$$Y^2 = \frac{10^5}{2\times5^3} \Rightarrow Y = \frac{100}{5} \Rightarrow Y = 20$$

Example 11.

The cost of organizing a party varies directly as the number of invites. If there are 40 invites, the cost works out to Rs. 200 per head. If there are 50 invites, the cost works out to Rs. 180 per head. Find

(a) the variable cost, fixed cost of conducting the party,

(b) the total cost if 60 people attended the party.

Sol.

(a) TC = VC (N) + FC

TC = Total cost

VC = Variable cost

FC = Fixed cost

200 × 40 = VC (40) + FC

180 × 50 = VC(50) + FC

VC (10) = 1000

∴ VC = Rs. 100;

Hence, fixed cost = Rs. 4,000.

(b) The total cost when there are 60 participants is 60 × 100 + 4000 = Rs. 10,000.

Example 12.

A part of the monthly expenses of Amar, a marketing executive are fixed and the remaining part varies with the distance travelled by him. If he travels 200 km in a month, his total expenditure is Rs. 3300. If he travels 500 km in a month his total expenditure is Rs. 3900. Find his total expenditure, if he travels 800 km in a month.

Sol.

Let the total expeneses be Rs. T, Rs F be the fixed part and Rs. V be the varible part Given that, T = F + V.

As V varies with the distance travelled, if distance travelled is denoted by d.

$\Rightarrow \quad V \alpha d$

V = R × d where 'R' is the proportionality constant.

Hence T = F + Rd.

From the given data

$3300 = F + 200\ R$ ....(1)

$3900 = F + 500\ R$ ....(2)

Subtracting (1) from (2)

$600 = 300\ R \Rightarrow R = 2$, put the value of R in (1) $\Rightarrow F = 2900$,

Total expenditure if he travels 800 km

$= F + 800\ R = 2900 + 800 \times 2 = 2900 + 1600 = 4500$

PARTNERSHIP

Partnership: When two or more than two persons run a business jointly, they are called partners and the deal is known as pertnership.

Ratio of Division of Gains:

(i) When investments of all the partners are for the same time, the gain or loss is distributed among the partners in the ratio of their investment.

Suppose A and B invest Rs. X and Rs. Y respectively for a year in a business, then at the end of the year:

(A's share of profit) : (B's share of profit) = X : Y.

(ii) When investment are for different time periods, then equivalent capitals are calculate for a unit of time by taking (capital × number of units of time). Now gain or loss is divided in the ratio of these capitals.

Suppose A invests Rs. X for P months and B invests Rs. Y for Q months, then (A's share of profit) : (B's share of profit) = XP : YQ.

Example 13.

Alfred started a business investing Rs. 45,000. After 3 months, Peter joined him with a capital of Rs. 60,000. After another 6 months, Ronald joined them with a capital of Rs. 90,000. At the end of the year, they made a profit of Rs. 16,500. Find the share of each.

Sol.

Clearly, Alfred invested his capital for 12 months, Peter for 9 months and Ronald for 3 months.

So, ratio of their capitals = (45000 × 12) : (60000 × 9) : (90000 × 3)

= 540000 : 540000 : 270000 = 2 : 2 : 1

$\therefore$ Alfred's share $= \left(16500 \times \frac{2}{5}\right)$ = Rs. 6600;

Peter's share $= \left(16500 \times \frac{2}{5}\right)$ = Rs. 6600;

Ronald's share $= \left(16500 \times \frac{1}{5}\right)$ = Rs. 3300.

Example 14.

A and B enter into a partnership. A invested Rs. 2,000 and B invested Rs. 3,000. After 6 months, B withdrew from the business. At the end of the year, the profit was Rs. 4,200. How much would B get out of this profit?

Sol.

In partnership problems, the ratio in which the profit is shared is $I_A \times T_A : I_B \times T_B$,

Where I and T stand for investment and time respectively.

So the ratio in which A and B would share the profits is 2000 (12) : 3000 (6) = 4 : 3.

So B receives $\frac{3}{7} \times 4200 =$ Rs. 1,800 from the total profits made in the investment.

Example 15.

Three people decided to start a partnership firm. The ratio of their investments at the outset of a project was $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. The amounts were invested for time periods $1:\frac{1}{2}:\frac{1}{3}$. If the profits were shared in the direct proportion to both the amount invested and the time for which it is invested find A's share, if B's share is Rs. 20,000.

Sol.

Ratio of investments is $I_A : I_B : I_C = \frac{1}{2}:\frac{1}{3}:\frac{1}{4} = 6 : 4 : 3$.

Ratio of the times for which it is invested is $T_A : T_B : T_C = 1:\frac{1}{2}:\frac{1}{3} = 6 : 3 : 2$.

Ratio of profits shared $I_A T_A : I_B T_B : I_C T_C = 36 : 12 : 6 = 6 : 2 : 1$

Hence, if B's share is Rs. 20,000, A's share = 3 × Rs. 20,000 = Rs 60,000.

Example 16.

A, B and C enter into a partnership by investing in the ratio of 3 : 2 : 4. After one year, B invests another Rs. 2,70,000 and C, at the end of 2 years, also invests Rs. 2,70,000. At the end of three years, profits are shared in the ratio of 3 : 4 : 5. Find the initial investment of each.

Sol.

Let the initial investments of A, B and C be Rs. $3x$, Rs. $2x$ and Rs. $4x$ respectively.

Then,

$(3x \times 36) : [(2x \times 12) + (2x + 270000) \times 24] : [(4x \times 24) + (4x + 27000) \times 12]$

$= 3 : 4 : 5$

$\Rightarrow 108x : (72x + 6480000) : (144x + 3240000)$

$= 3 : 4 : 5$

$$\therefore \frac{108x}{72x + 6480000} = \frac{3}{4}$$

$432x = 216x + 19440000$

$216x = 19440000$

$x = 90000$

Hence, A's initial investment = $3x$ = Rs. 2,70,000;

B's initial investment = $2x$ = Rs. 1,80,000;

C's initial investment = $4x$ = Rs. 3,60,000

Objective Questions

1. The salaries of A, B, C are in the ratio 2 : 3 : 5. If the increments of 15%. 10% and 20% are allowed respectively in their salaries, then what will be the new ratio of their salaries?
   (a) 3 : 3 : 10 (b) 10 : 11 : 20
   (c) 23 : 33 : 60 (d) Can't be determined

2. Salaries of A and B are in the ratio 2 : 3. If the salary of each is increased by Rs. 4000, the new ratio becomes 40 : 57. What is B's persent salary?
   (a) Rs. 17,000 (b) Rs. 20,000
   (c) Rs. 25,500 (d) None of these.

3. Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively, what will be ratio of increased seats?
   (a) 2 : 3 : 4 (b) 6 : 7 : 8
   (c) 6 : 8 : 9 (d) None of these.

4. Ratio of the earnings of A and B is 4 : 7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earnings becomes 8 : 7. What are A's earnings?

(a) Rs. 21,000 (b) Rs.26,000
(c) Rs. 28,000 (d) Data inadequate.

5. A student finishes the first half of an examination in two-thirds of the time it takes him to finish the second half. If the whole examination takes him an hour and half, how many minutes does he spend on the second half of the examination?

(a) 36 min (b) 54 min
(c) 60 min (d) 44 min

6. A person buys some apples and mangoes from the market at a rate-such that a mango is twice as costly as an apple, and he sells them such that a mango is thrice the price of an apple. By selling an apple at twice of its CP, he makes 150% profit on the whole. Find the proportion of mangoes to apples.

(a) 3 : 5 (b) 3 : 4
(c) 1 : 2 (d) Cannot be determined

7. The incomes of A and B are in the ratio 3 : 2 and their expenditures are in the ratio 5 : 3. If each saves Rs. 1000, their incomes are

(a) Rs. 6000 and Rs. 4000 (b) Rs. 12000 and Rs. 8000
(c) Rs. 15000 and Rs. 10000 (d) None of these

8. A sporting goods store ordered an equal number of white and yellow tennis balls. The tennis ball company delivered 45 extra white balls, making the ratio of white balls to yellow balls $\frac{1}{5} : \frac{1}{6}$. How many white tennis balls did the store originally order for?

(a) 450 (b) 270
(c) 225 (d) None of these

9. The ratio of the present ages of a man and his wife is 5 : 4. After 20 years, the ratio of the ages of the man and his wife could be:

(a) 11 : 8 (b) 9 : 7
(c) 15 : 13 (d) 19 : 15

10. If $a + b - c : b + c - a : a + c - b = 3 : 4 : 5$, find a : b : c.

(a) 1 : 2 : 3 (b) 8 : 7 : 9
(c) 5 : 7 : 8 (d) 1 : 3 : 2

11. A three digit number has the ratio of its units digit is the number itself as 1 : 27. Find its tens digit.

(a) 2 (b) 3
(c) 4 (d) 5

12. The quantity y varies directly as x^n. For x = 3, 4, 5 and 6, the corresponding values of y are 40.5, 96, 187.5 and 324. Find n.

(a) 2 (b) 2.5
(c) 3 (d) 3.5

13. The ratio of the number of students in three sections A, B and C is 5 : 4 : 3. If 175 students from each of the sections B and C shift to A, the ratio becomes 40 : 19 : 13. Find the total strength of the three sections.

(a) 2500 (b) 2480
(c) 2540 (d) 2520

14. The expenditure of a hostel is partly fixed and partly proportional to the number of students. If there are 25 students the average expenditure per student is Rs. 600. And if there are 50 students the average expenditure per student is Rs 500. Find the average expenditure (in Rs.) per student if there are 100 students

(a) 400 (b) 420
(c) 350 (d) 450

15. The number of Re. 1 coins, 50 paise coins and 25 paise coins in a bag are in the ratio 5 : 4 : 3. If the difference in the amounts contributed by Re. 1 coins and 25 paise coins is Rs. 119 find the contribution of Rs 1 coins.

(a) 56 (b) 70
(c) 84 (d) 140

16. Sarita, Anupama and Vijay started off with a game according to which the loser of the game distributes half of his money amongst the other two in the ratio of the money they are holding. If after playing three rounds, Saritha ends up with Rs. 150, Vijay ends up with Rs. 100 and Anupama ends up with Rs. 50, find the total amount of money they were holding initially.

(a) Rs. 600 (b) Rs. 300
(c) Rs. 400 (d) Cannot be determined

17. If x varies as the cube of y, and y varies as the fifth root of z, then x vaires as the nth power of z, where n is:

(a) $\frac{1}{15}$ (b) $\frac{5}{3}$
(c) $\frac{3}{5}$ (d) 15

18. If $\frac{m}{n} = \frac{4}{3}$ and $\frac{r}{t} = \frac{9}{14}$, the value of $\frac{3mr - nt}{4nt - mr}$ is:

(a) $-5\frac{1}{2}$ (b) $-\frac{11}{14}$
(c) $-1\frac{1}{4}$ (d) 11/14

19. The sum of three numbers is 98. The ratio of the first to the second is 2/3 and the ratio of the second to the third is 5/8. The second number is :

(a) 15 (b) 20
(c) 30 (d) 32

20. Amber Chew opened a departmental store at Great India Palace in Noida by investing Rs. 20 milliion. After a few months her brother Sheesh Chew joined the business and shared in the ratio of 3 : 2. After how many months did Amber's brother join the busines?

(a) 4 months (b) 6 months
(c) 7 months (d) 8 months.

21. If the heights of two cones are in the ratio 7 : 3 and their diameters are in the ratio 6 : 7, what is the ratio of their volumes?

(a) 6 : 14 (b) 12 : 7

(c) 3 : 7 (d) 5 : 7

22. Two cylindrical blocks have their diameters in the ratio 3 : 1 and their heights in the ratio 1 : 3. Their volumes would thus, be in the ratio of :

(a) 3 : 1 (b) 3 : 4

(c) 1 : 2 (d) 2 : 3

23. If $\frac{a}{4}=\frac{b}{5}$ then:

(a) $\frac{a-4}{a+4}=\frac{b+5}{b-5}$ (b) $\frac{a-4}{a+4}=\frac{b-5}{b+5}$

(c) $\frac{a+4}{a-4}=\frac{b-5}{b+5}$ (c) $\frac{a+4}{a-4}=\frac{b+5}{b-5}$

24. Three numbers are in the ratio 3 : 4 : 5. The sum of the largest and the smallest equals the sum of the second and 52. the smallest number is:

(a) 20 (b) 27

(c) 39 (d) 52

25. Four milkmen rented a pasture. A grazed 18 cows for 4 months, and B 25 cows for 2 months, C 28 cows for 5 months and D 21 cows for 3 months. If A's share of rent is Rs. 360, the total rent of the field (in rupees) is:

(a) 1,500 (b) 1,600

(c) 1,625 (d) 1,650

Answers Objective Questions

1.	(c)	**2.**	(d)	**3.**	(a)	**4.**	(d)	**5.**	(b)
6.	(c)	**7.**	(a)	**8.**	(c)	**9.**	(c)	**10.**	(b)
11.	(b)	**12.**	(c)	**13.**	(d)	**14.**	(d)	**15.**	(d)
16.	(b)	**17.**	(c)	**18.**	(b)	**19.**	(c)	**20.**	(c)
21.	(b)	**22.**	(a)	**23.**	(b)	**24.**	(c)	**25.**	(c)

Solutions :—

1. Let A = 2k, B = 3k and C = 5k.

A's new salary $= \frac{115}{100}$ of $2k = \left(\frac{115}{100} \times 2k\right) = \frac{23}{10}k$

B's new salary $= \frac{110}{100}$ of $3k = \left(\frac{110}{100} \times 3k\right) = \frac{33}{10}k$

C's new salary $= \frac{120}{100}$ of $5k = \left(\frac{120}{100} \times 5k\right) = 6k$

$\therefore$ New ratio $= \frac{23k}{10} : \frac{33k}{10} : 6k = 23 : 33 : 60$

OR let their salaries be 200, 300 and 500.

After the respective % increases of 15%, 10% and 20% their salaries will be 230, 330 and 600 respectively. Hence Ratio = 23 :33: 60

2. Let the original slaries of A and B be Rs. 2x and Rs. 3x respectively.

Then, $\frac{2x + 4000}{3x + 4000} = \frac{40}{57}$

$57(2x + 4000) = 40(3x + 4000)$

$6x = 68000$

$3x = 34000$

B's present salary = $(3x + 4000)$ = Rs. $(34000 + 4000)$ = Rs. 38,000.

3. Let the Seats in Mathematics, Physics and chemistry be 50, 70 and 80 respectively. So after the given percentage increase the seats in Mathematics, Physics and chemistry will be (50+20), (70+35) and (80+60) hence the ratio will be 70: 105:140 or 2:3:4

4. Let the original earnings of A and B be Rs. $4x$ and Rs. $7x$.

New earnings of A = 150% of Rs. $4x$ = ₹$\left(\frac{150}{100} \times 4x\right)$ = ₹ $6x$

New earnings of B = 75% of Rs. $7x$ = ₹$\left(\frac{75}{100} \times 7x\right)$ = ₹$\frac{21x}{4}$

$\therefore \quad 6x : \frac{21x}{4} = 8 : 7$

$\frac{6x \times 4}{21x} = \frac{8}{7}$

As x will be cancel out from numerator and denominator, the data given is inadequate to calcultate the A's earnings.

5. If he spends 't' minutes on the second half, then the spends $\frac{2}{3}t$ on the first half.

Therefore, $\frac{2}{3}t + t = 90$ or $t = 54$

6. Suppose the person buys A apples and M mangoes and cost price of an apple is Rs. x.

Therefore, cost price of a mango will be $2x$.

Total cost price = $Ax + 2Mx$.

Now selling price of an apple is $2x$.

$\therefore$ SP of a mango will be $6x$.

Total SP = $2Ax + 6Mx$.

Now we have $2Ax + 6Mx = \frac{5}{2}(Ax + 2Mx)$ or $\frac{M}{A} = \frac{1}{2}$

7. Let the incomes are 3x and 2x and the expenditures are 5y and 3y.

Therefore, $3x - 5y = 1000$ and $2x - 3y = 1000$.

$y = 1000$, $x = 2000$. Therefore, incomes are Rs. 6000 and Rs. 4000.

8. Rates of white to yellow balls = 6 : 5.

Difference in the number of white and yellow balls = $6x - 5x = x = 45$.

Therefore, number of white balls now available = 45×6.

Number of white balls ordered = $(45 \times 6) - 45 = 225$.

9. Let the present ages of the man and his wife be 5x years and 4x years respectively. Ratio of their ages after 20 years =

$\frac{5x+20}{4x+20} = \frac{5(x+4)}{4(x+5)}$ a quantity less than 5/4

As $(x+4)$ is less than $(x+5)$ so the fraction $\frac{(x+4)}{(x+5)}$ will be less than 1 . Hence the ratio will be less than $\frac{5}{4}$

10. Let $a + b - c = 3x$, $b + c - a = 4x$ and $a + c - b = 5x$

Adding these equations

$$a + b + c = 12x, \quad a + b + c - (b + c - a) = 8x \text{ or } 2a = 8x$$

$$a + b + c - (a + c - b) = 7x \text{ or } 2b = 7x$$

$$a + bc - (a + b + c) = 9x \text{ or } 2c = 9x$$

$$a : b : c = 2a : 2b : 2c = 8 : 7 : 9$$

11. Let the number be xyz

$z = \frac{1}{27}(100x + 10y + z) \Rightarrow 26z = 100x + 10y$

The R.H.S. of the above equation is divisible by 10, Its L.H.S. must be divisible by 10

Hence z must be divisible by 5.

$\therefore$ z = 0 or 5

If z = 0, the R.H.S. would be 0. As $x \geq 1$, the R.H.S. cannot be 0.

Hence z = 5 so $100x + 10y = 130$ or $10x + y = 13$ or y = 3

12. Given $y = kx^n$

$$40.5 = k3^n \quad \text{.........(1)}$$
$$96 = k4^n \quad \text{.........(2)}$$
$$187.5 = k5^n \quad \text{.........(3)}$$
and
$$324 = k6^n \quad \text{.........(4)}$$

(1) + (2) + (3)

$$324 = k(3^n + 4^n + 5^n) \quad \text{..........(5)}$$

from (4) and (5) $3^n + 4^n + 5^n = 6^n$

By trial, $n = 3$. Choice (c)

13. Given that the Initial ratio of A, B & C is 5 : 4 : 3. If 175 students shift from each of the sections B and C, there will be a change in the strength of the individual sections but the total strength remains same. Since the total strength is represented by 72 parts in the second ratio, the same 72 parts must be used to represent the total strength in the first ratio as well

i.e. A : B : C = 5 : 4 : 3 = 30 : 24 : 18

Initial ratio is 30 : 24 : 18 and the final ratio is 40 : 19 : 13

Clearly from the above ratios we can conclude that 5 parts correspond to 175 students.

Total strength is $\frac{72}{5} \times 175 = 2520$

14. Given that when there are 25 people the total expenditure is Rs. 15000 and when there are 50 people the total expenditure is Rs. 25000. Hence for every increase of 25 members the expenditure increases by 10,000. If there are 100 people the total expenditure is 45000 and hence the average expenditure per student is Rs. 450. Choice (d)

15. Given that the ratio of the number of Re. 1, 50 paise and 25 paise coins is 5 : 4 : 3

Let the number of coins be 5x, 4x and 3x respectively, Given that 5x – 1/4(3x) = 119

i.e. $\frac{17x}{4} = 119 x = 28$. Contribution of 1 Rs. coins is $5x$ i.e. 5 × 28 = Rs. 140

16. Since the money only changes hands, the total amount of money, they have at the beginning and the end will be the same.

∴ Total amount of money at the beginning = 50 + 100 + 150 = Rs. 300

17. $x \propto y^3 \rightarrow x = ky$ and

$y \propto (z)^{\frac{1}{5}}$

$$y = k_1 z^{\frac{1}{5}} (k, k_1 - \text{constants})^3$$

$$\therefore \quad x = k\left[k_1 (z)^{\frac{1}{5}}\right]^3 = \left(kk_1^3\right)(z)^{\frac{3}{5}}$$

$$= k_2 (z)^{\frac{3}{5}} \Rightarrow x \propto (z)^{\frac{3}{5}}$$

Thus $n = \frac{3}{5}$

18. $\frac{m}{n} = \frac{4}{3}$ and $\frac{r}{t} = \frac{9}{14}$ and Given expression: $\frac{3mr - nt}{4nt - 7mr}$

$$= \frac{3\frac{mr}{nt} - 1}{4 - 7\frac{mr}{nt}} = \frac{3\left(\frac{m}{n}\right)\left(\frac{r}{t}\right) - 1}{4 - 7\left(\frac{m}{n}\right)\left(\frac{r}{t}\right)} = \frac{\frac{18}{7} - 1}{4 - 6} = \frac{\frac{18-7}{7}}{-2} = -\frac{11}{14}$$

19. Since the ratio of the first and the second is 2 : 3 and the ratio of second and third is 5 : 8. Let the second number by 15 k.

1st	2nd	3rd	Sum
10k	15k	24k	49k

Given 49k = 98k = 2, Thus second number is 15k = 15 × 2 = 30

20. Let Sheesh chew joined the busines after x month.

then, $20 \times 12 : 30 \times (12 - x) = 3 : 2$

$$\Rightarrow \quad \frac{240}{30(12-x)} = \frac{3}{2}$$

$$\Rightarrow \quad 480 = 90(12-x)$$

$$480 = 1080 - 90x$$

$$90x = 600$$

$$x = \frac{60}{9} = 6.66 \approx 7 \text{ months}$$

21. Given, heights of two cones are in the ratio 7 : 3.

$\therefore$ Heights are $\frac{7}{10}$ and $\frac{3}{10}$.

Similarly, given diameters are in the ratio 6 : 7.

Diameters are $\frac{6}{13}$ and $\frac{7}{13}$

Radius are $\frac{6}{26}$ and $\frac{7}{26}$

$$\text{Now, Ratio of volumes} = \frac{\frac{1}{3} \times \pi \times \frac{6}{26} \times \frac{6}{26} \times \frac{7}{10}}{\frac{1}{3} \times \pi \times \frac{7}{26} \times \frac{7}{26} \times \frac{3}{10}}$$

$$= \frac{12}{7} = 12 : 7$$

22. Given, Ratio of diameters = 3 : 1

$\therefore$ Radius of 1st cylinder = $\frac{3x}{2}$ and radius of second = $\frac{x}{2}$

Ratio of second = 1 : 3

$\therefore$ Height of 1st cylinder = y and height of second = 3y

So, $\frac{\text{Volume of 1}^{st}\text{ cylinder}}{\text{Volume of 2}^{nd}\text{ cylinder}} = \frac{\pi\left(\frac{3x}{2}\right)^2 y}{\pi\left(\frac{x}{2}\right)^2 .3y}$

$$= \frac{\frac{9x^2}{4}.y}{\frac{x^2}{4}.3y} = \frac{3}{1} = 3 : 1$$

23. Given $\frac{a}{4} = \frac{b}{5}$,

By componendo and dividendo, we have $\frac{a-4}{a+4} = \frac{b-5}{b+5}$

24. Let the three numbes be $3x$, $4x$ and $5x$

According to the Question $5x + 3x = 4x + 52$

$$8x - 4x = 52$$
$$x = 13$$

$\therefore$ The smallest number = 3 (13) = 39

25. Ratio of rent shared = 18 × 4 : 25 × 2 : 28 × 5 : 21 × 3

= 72 : 50 : 140 : 63

Let A's share be $72x$, [A's share = 360]

$$72x = 360$$
$$x = 5$$

Thus, A's share = 360, B's share = 50 (x) = 50 (5) = 250 and C's share = 140 (x) = 140 (5) = 700 and D's share = 63 (x) = 63 (5) = 315

Hence, total rent of the field = 360 + 250 + 700 + 315 = 1625 (in rupees).

CHAPTER

12

Mixture and Alligation

Mixture

Concept

- Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid left = $\left[x\left(1-\frac{y}{x}\right)^n\right]$ units.

Example 1.

The concentration of an acid soulation is inveresely proportional to the volume of the solution if the amount of acid is not changed. A 40% hydrochloric acid solution becomes 30% solution when 40 L of water is added to it. Find the original volume of the solution.

Sol.

The initial concentration = 40%

The final concentration = 30%

$I_C \times I_V = F_C \times F_V$

Where I and F indicate initial, final concentrations respectively.

C and V indicate concentration and volume respectively.

$\therefore \frac{30}{40} = \frac{I_V}{F_V} \Rightarrow F_V = \frac{4}{3}I_V$

There is an increase of $\frac{1}{3}I_V$, which is 40 L. Hence, initial volume = 40 × 3 = 120 L.

Example 2.

In a mixture of 42 L, the ratio of milk to water is 5 : 2. Another 12 L of water is added to the mixture. Find the ratio of milk to water in the resultant mixtue.

Sol. Amount of milk in 42 L of mixture = $\frac{5}{7} \times 42 = 30L$

Amount of water = 12 L.

Since 12 L of water is being added, the new mixture now contains 30L of milk and 24 L of water and hence, M : W = 30 : 24 = 5 : 4.

Example 3.

A container has 80 L of milk. From this container 10 L of milk was taken out and replaced by water. The process was further repeated twice.

How much milk is in the container now?

Sol. Method 1: Let the container originally contain x units of liquid.

Liquid taken out in each case is y units.

The final quantity of the component from the original mixtrue that is not being replaced, if this operation is repeated n times, is $x\left(1-\frac{y}{x}\right)^n$ units, where x is the original amount of that component in the mixture.

Remember that the original amount of milk need not be equal to the volume of the container.

In the this case, the original quantity of milk is 80 L.

the total number of operations of drawing the liquid, i.e., n = 3.

$$= 80\left(1-\frac{10}{80}\right)^3$$

The amount of milk left at the end of three operations = $80\left(1-\frac{1}{8}\right)^3$ = 53.59 L.

Example 4.

From a cask containing water, 9 L is taken out. It is replaced with an equal quantity of pure milk. This process is done twice. The ratio of water to milk in the cask now is 16 : 9. What is the volume of the cask?

Sol.

Let three be x litres in the cask.

After n number of process,

$$\frac{\text{Water left in the vessel after n time process}}{\text{Original quantity of water in vessel}} = \left(1-\frac{9}{x}\right)^n$$

$$\Rightarrow \left(1-\frac{9}{x}\right)^2 = \frac{16}{25}$$

$$\therefore x = 45 \text{ L}.$$

Example 5.

A container contains 200 L solution of milk and water in the ratio 1 : 3. You take out 50 L of solution and replace it with milk and do this operation twice. At the end of the second operation, find the ratio of milk to water in the solution.

Sol.

Final ratio of water to total = Initial ratio of water to total × $\left[\frac{V-x}{V}\right]^n$,

Where,

V = Initial solution

x = Amount replaced

n = Number of times an operation is repeated

$\therefore$ Final water to total $= \frac{3}{4}\left[\frac{200-50}{200}\right]^2$

$= \frac{3}{4}\times\left[\frac{3}{4}\right]^2 = \frac{27}{64}$

So final ratio of milk to water will be 37 : 27.

Example 6.

In two alloys the ratios of copper to tin are 3 : 4 and 1 : 6 respectively. If 7 kg of the first alloy and 21 kg of the second alloy are mixed together to form a new alloy, then what will be the ratio of copper to tin in the new alloy?

Sol.

$\frac{\text{Copper in new alloy}}{\text{Tin in new alloy}}$

$= \frac{\text{Copper in 1st + Copper in 2nd}}{\text{Tin in 1st + Tin in 2nd}}$

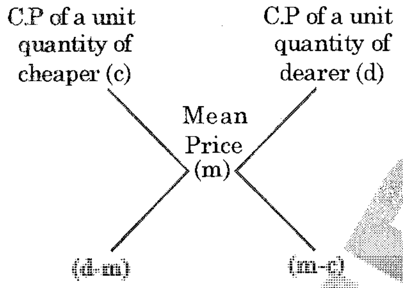
$= \frac{\frac{3}{7}\times 7+\frac{1}{7}\times 21}{\frac{4}{7}\times 7+\frac{6}{7}\times 21} = \frac{3+3}{4+18} = \frac{6}{22} = \frac{3}{11}$

Alligation

1. The word Alligation literally means linking. The rule takes its name from the lines or links used in working questions on mixture.
2. Cost price of unit quantity of the mixture is called the mean price.
3. **Rule of Alligation:** If two ingradients are mixed in a ratio, then,

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{CP of Dearer–Mean Price}}{\text{Mean Price – CP of Cheaper}}$$

We present it as under:



Cheaper Quantity : Dearer Quantity = (d – m) : (m – c)

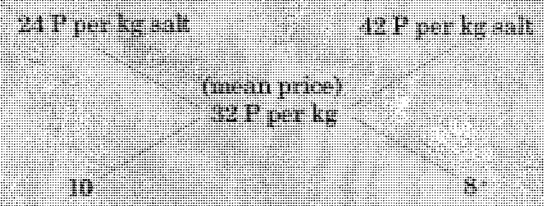
Example 7.

How many kg of salt at 42 P per kg must a man mix with 25 kg of salt at 24P per kg, so that he may, on selling the mixture at 40 p per kg, gain 25% on the outlay?

Sol. Cost Price of mixture (mean price)

$\Rightarrow$ (Mean Price) × 1.25 = 40 P

Mean Price = 32 P per kg....



$\Rightarrow$ 5 : 4

Thus for every 5 kg of salt at 24 P, 4 kg of salt at 42 P is used.

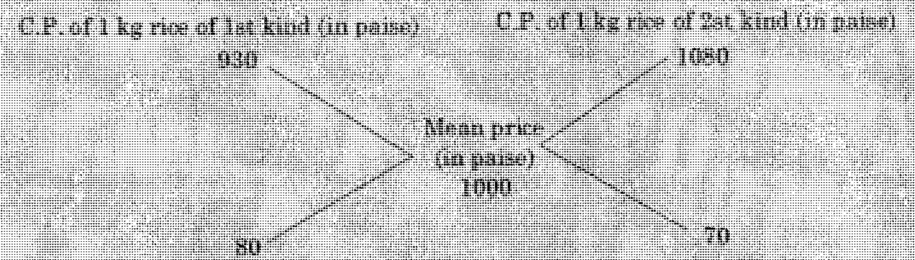
$\therefore$ The required no. of kg = $25 \times \frac{4}{5}$ = 20 kg of 42 p salt.

Example 8.

In what ratio must rice at Rs. 9.30 per kg be mixed with rice at Rs. 10.80 per kg so that the mixture be worth Rs. 10 per kg?

Sol.

By the rule of alligation, we have:



$\therefore$ Required ratio = 80 : 70 = 8 : 7.

Example 9.

In what proportion must water be mixed with spirit to gain $16\frac{2}{3}\%$ by selling it at cost price?

Sol. Let CP of spirit be Re. 1 per litre.

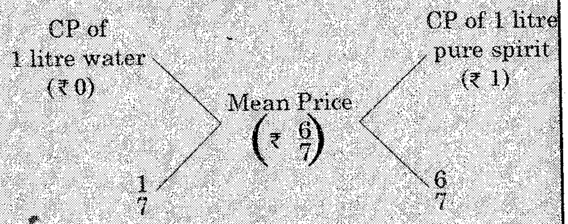
Then SP of 1 litre of mixture = Re. 1.

CP of mixture × (1 + gain%) = SP of mixture

$$\text{CP of mixture}\left(1+\frac{50}{300}\right) = \text{SP of mixture} = 1$$

CP of mixture = ₹ 1

$$\text{CP of mixture} = ₹\frac{6}{7} = \text{Mean Price}$$



$$\frac{\text{Quantity of water}}{\text{Quantity of spirit}} = \frac{1/7}{6/7}\ \frac{1}{6}$$

or Ratio of water and spirit = 1 : 6

Example 10.

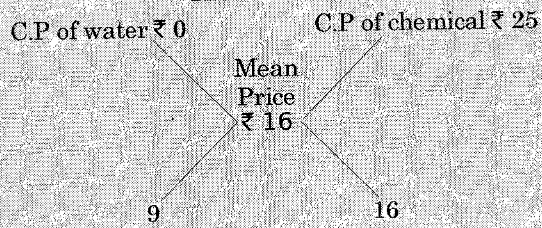
A person has a chemical of Rs 25 per litre. In what ratio should water be mixed in that chemical so that after selling the mixture at Rs 20/- litre he may get a profit of 25%.

Sol.

In this question the alligation method is applicable on prices, so we should get the average price of mixture.

SP of mixture = Rs 20/litre, profit = 25%

$$\therefore\ \text{C.P of mixture} = \text{Mean Price} = 20\times\frac{100}{125} = \text{Rs } 16/\text{litre}$$



So water and chemical should be mixed in the ratio of 9 : 16

Example 11.

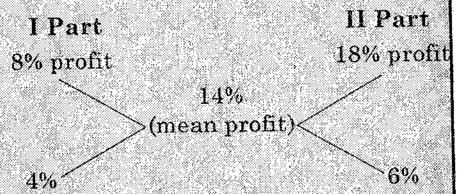
A trader has 50 kg of pulses, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. What is the quantity sold at 18% profit?

By Alligation Method:

= 4 : 6 = 2 : 3

Therefore the quantity sold at 18% profit

$$= \left(\frac{3}{5}\times 50\right) = 30 \text{ kg}$$



Example 12.

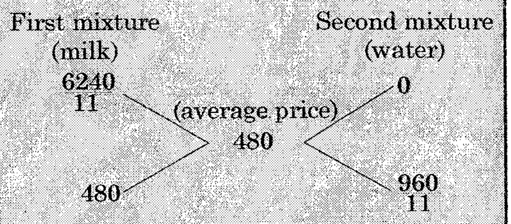
A man mixes 5 kilolitres of milk at Rs 600 per kilolitre with 6 kilolitres at Rs 540 per kilolitre. How many kilolitres of water should be added to make the average value of the mixture Rs 480 per kilolitre?

Sol. This question should be solved by the method of alligation.

Cost of milk when two qualities are mixed

$$\frac{5\times600+6\times540}{5+6} = \text{Rs } \frac{6240}{11} \text{ per kilolitre.}$$

Cost of water = Rs 0/ kilolitre,



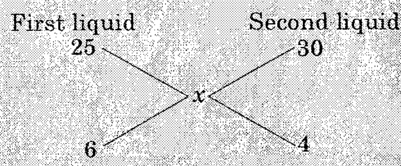
$\therefore$ Ratio of milk and water $= 480 : \frac{960}{11} = 1 : \frac{2}{11} = 11 : 2$

Which implies that 11 kilolitres of milk should be mixed with 2 kilolitres of water. Thus 2 kilolitres of water should be added.

Example 13.

One type of liquid contains 25% of milk, the other contains 30% of milk. A can is filled with 6 parts of the first liquid and 4 parts of the second liquid. Find the percentage of milk in the new mixture.

Sol. This equation can be solved by the method of Alligation



$\frac{30-x}{x-25} = \frac{6}{4} = \frac{3}{2}$ $\frac{30-x}{x-25} = \frac{6}{4} = \frac{3}{2}$

or, $60 - 2x = 3x - 75$

or, $5x = 60 + 75$

$\therefore$ $x = 27\%$

Example 14.

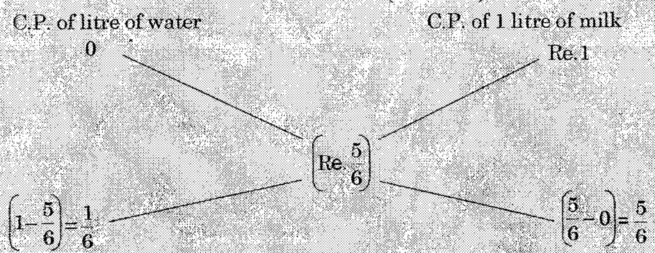
In what ratio water must be mixed with milk to gain 20% by selling the mixture at cost price?

Sol. Let C.P. of milk be Re. 1 per litre.

Then; S.P. of 1 litre of mixture = Re. 1.

Gain obtained = 20%.

$\therefore$ C.P. of 1 litre of mixture $= \text{Re.}\left(\frac{100}{120}\times1\right) = \text{Re.}\frac{5}{6}$ $\left(\frac{100}{120}\times1\right) = \text{Re.}\frac{5}{6}$.



$\therefore$ Ratio of water and milk $= \frac{1}{6} : \frac{5}{6} = 1 : 5$

Objective Questions

1. Three people A, B and C weigh themselves in a particular order. First A, B, C weigh themselves individually and then AB, BC, CA and ABC together respectively. The recorded weight for the last measure is 180 kgs. The average of the 7 measures is

(a) 320 kgs
(b) $\frac{360}{7}$ kgs
(c) $\frac{720}{7}$
(d) Cannot be determined

2. Cask A contains wine and water in the ratio 6 : 7 and cask B in the ratio 9 : 4. In what ratio must the contents of the two casks be mixed to give a mixture of wine and water in the ratio 8 : 5?

(a) 1 : 3
(b) 2 : 3
(c) 1 : 2
(d) None of these

3. In a mixture of wheat and barley the wheat is 60%. To 400 quintals of the mixture a quantity of barley is added and then the wheat is $53\frac{1}{3}\%$. How many quintals of barley are added?

(a) 25 kg
(b) 50 kg
(c) 30 kg
(d) 40 kg

4. Two solutions of 90% and 97% purity are mixed resulting in 21 litres of mixture of 94% purity. How much is the quantity of the first solution in the resulting mixture.

(a) 15 litre
(b) 12 litre
(c) 9 litre
(d) 6 litre

5. A man buys milk at Rs. 5 a litre and after adding water, sells it at Rs. 6 a litre, thereby making a profit of $33\frac{1}{3}\%$. Find the proportion of water to milk in the mixture.

(a) 1 : 9
(b) 1 : 10
(c) 5 : 9
(d) 4 : 9

6. Dhanna Seth, a rice merchant, sells different varieties of rice. He mixes one variety of rice costing ₹ 16 per kg. with another variety of rice costing ₹ 21 per kg. In what ratio should these two varieties of rice be mixed such that the resultant mixture, upon selling at ₹ 21.45 per kg, will give a 10% profit?

(a) 2 : 9
(b) 3 : 7
(c) 1 : 3
(d) 2 : 3

7. Two varieties of oil are mixed in the ratio 4 : 5 to produce first quality oil and in the ratio 1 : 2 to produce second quality oil. How many litres of first quality oil should be mixed with 15 litres of second quality oil so that third quality oil having the two varieties in the ratio of 7 : 11 may be produced?

(a) 15 litres
(b) 12 litres
(c) 18 litres
(d) 24 litres

8. A man buys spirit at Rs. 60 per litre, adds water to it and then sells it at Rs. 75 per litre. What is the ratio of spirit to water if his profit in the deal is 37.5%?
(a) 9 : 1
(b) 10 : 1
(c) 11 : 1
(d) None of these

9. An alloy contains zinc and copper in the ratio 5 : 8 and another alloy contains zinc and copper in the ratio 5 : 3. If equal amounts of both the alloys are melted together, then the ratio of zinc and copper in the resulting alloy is :
(a) 25 : 24
(b) 3 : 8
(c) 103 : 105
(d) 105 : 103

10. Two vessels A and B contain milk and water mixed in the ratio 5 : 3 and 2 : 3. When these mixtures are mixed to form a new mixture containing half milk and half water, they must be taken in the ratio:
(a) 2 : 5
(b) 3 : 5
(c) 4 : 5
(d) 7 : 3

11. A mixtures of 40 litres of milk and water contains 10% water. How much water should be added to it so that water may be 20% in the new mixture?
(a) 5 L
(b) 4 L
(c) 6.5 L
(d) 7.5 L

12. A vessel is filled to its capacity with pure milk. Ten litres are withdrawn from it and replaced by water. This procedure is repeated again. The vessel now has 32 litres of milk. Find the capacity of the vessel (in litres).
(a) 40
(b) 45
(c) 50
(d) 55
(e) 44

13. From a vessel containing 50 litres of pure milk, 10 litres is taken out and replaced with water. From the resulting solution 5 litres is taken out and replaced with water. From the resulting solution 20 litres is taken out and replaced with water. Find the quantity of milk in the finalsolution.
(a) 22 L
(b) 20.3 L
(c) 18.9 L
(d) 21.6 L

14. Two varieties of wheat are mixed together in the ratio 2 : 3. The cost price of one kg of the first variety of wheat is Rs. 5 more than the cost price of one kg of the second variety of wheat. The mixture is sold at 20% profit at Rs. 30 per kg. Find the cost price (in Rs/kg) of a mixture formed by interchanging the quantities of the varieties mixed.
(a) 24
(b) 27
(c) 28
(d) 26

15. A litre of milk sold by a milkman to a housewife contains milk and water in the ratio of 3 : 1. The housewife add 250 ml of water and then uses 250 ml of the mixture to make curd. How much pure milk is she left with?
(a) 600 ml
(b) 750 ml
(c) 1 litre
(d) 150 ml

Answers (Objective Questions)

1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(a)
6.	(b)	**7.**	(a)	**8.**	(a)	**9.**	(d)	**10.**	(c)
11..	(a)	**12.**	(c)	**13.**	(d)	**14.**	(d)	**15.**	(a)

Solutions :—

1.

Sol. The order of measures is A, B, C, A + B, B + C, C + A, A + B + C

Given A + B + C = 180

Hence, average of the 7 measures

$$= \left[\frac{[(A)+(B)+(C)+(A+B)+(B+C)+(C+A)+(A+B+C)]}{7}\right]$$

$$= \frac{4}{7}(A+B+C) = \frac{4}{7}\times 180 = \frac{720}{7} \text{ kgs.}$$

2.

Sol.

I		II
Wine = $\frac{6}{13}$		Wine = $\frac{9}{13}$
	Wine = $\frac{8}{13}$	
Difference = $\frac{1}{13}$		Difference = $\frac{2}{13}$

3.

Sol. Quantitiy of wheat in 400 kg of mixture = 0.6 × 400 = 240 kg

Quantity of barely = 400 – 240 = 160 kg.

Let, x kg of barley be added to 400 kg of the mixture

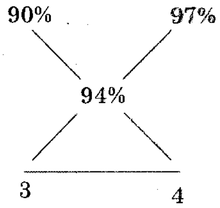
$$\frac{240}{160+x} = \frac{53\frac{1}{3}}{46\frac{2}{3}} = \frac{160}{140} = \frac{8}{7}$$

$$\therefore \quad 240 \times 7 = 8 \times 160 + 8x$$

$$\therefore \quad x = 50 \text{ kg.}$$

4.

Sol



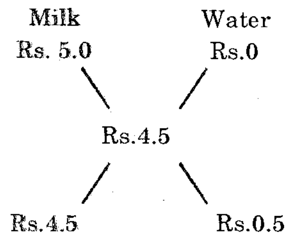
Ratio = 3 : 4. Let the common multiple be x

$\therefore$ $3x + 4x = 21 \therefore x = 3$

$\therefore$ Quantity of the first solution = $3 \times 3 = 9$ litre.

5.

Sol. Cost price of milk = Rs. 5/litre

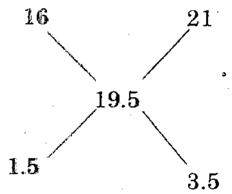


Cost price of mixture = $\frac{100}{133\frac{1}{3}} \times 6$ = Rs. 4.5/litre

$\therefore \frac{\text{Quantity of water}}{\text{Quantity of milk}} = \frac{5-4.5}{4.5-0} = \frac{0.5}{4.5} = \frac{1}{9}$ i.e.,

$\therefore$ Milk : Water = 9 : 1.

6.



At a selling price of ₹21.45 per kg and a profit of 10%, the cost price of the mixture must be

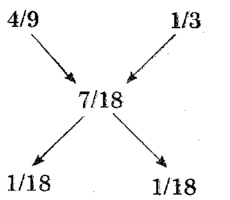
$\frac{21.45}{1.1}$ = ₹19.5 per kg.

Using rule of alligations

i.e., 3 : 7

7.

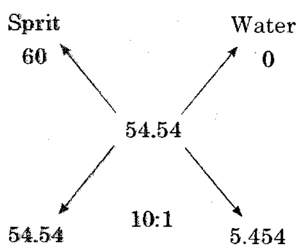
	V_1	V_2
Q_1	4	5
Q_2	1	2
Q_3	7	11



$\therefore$ 1 : 1

15 litres.

8. Since his SP of (spirit + water) = Rs.75/L and he ultimately makes a profit of 37.5%, his CP of (spirit + water) = 75/1.375 = Rs.54.54. This should indeed be the weighted average of the costs of spirit and water. So if we alligate, we can get the ratio of spirit : water (assuming that cost of water is 0).



9.

	Zinc	**Copper**
First alloy:	$\frac{5}{13}$	$\frac{8}{13}$
Another alloy:	$\frac{5}{8}$	$\frac{3}{8}$

Since, equal amounts of both the alloys are melted together.

$\therefore$ Quantity of zinc in resulting alloy = $\frac{5}{13}+\frac{5}{8}=\frac{105}{104}$

and quantity of copper = $\frac{8}{13}+\frac{3}{8}=\frac{103}{104}$

$\therefore$ Ratio of zinc and copper = $\frac{105/104}{103/104}=\frac{105}{103}$

10. Let the total mixture be x in vessel A and total mixture be y in vessel B.

Given ratio of milk and water in vessel A is 5 : 3 and 2 : 3 in B.

i.e, Milk in A = $\frac{5x}{8}$ and in B = $\frac{2y}{5}$

and water in A = $\frac{3x}{8}$ and in B = $\frac{3y}{5}$

Ratio of milk and water in new mixture = 1 : 1

$\therefore$ We have $\quad \frac{\frac{5x}{8}+\frac{2y}{5}}{\frac{3x}{8}+\frac{3y}{5}}=\frac{1}{1}$

$\Rightarrow \quad \frac{5x}{8}+\frac{2y}{5}=\frac{3x}{8}+\frac{3y}{5}$

$\Rightarrow \quad \frac{x}{y}=\frac{4}{5}=4:5$

11. Let x litre water should be added.

water present in mixture of 40 litre = $40\times\frac{10}{100}=4$ L

Now, 20% water should be present in new mixture.

$\therefore$ According to question Ratio of water and total mixture $=\frac{4+x}{40+x}=20\%$

$\Rightarrow \quad \frac{4+x}{40+x}=\frac{20}{100}=\frac{1}{5}$

$\Rightarrow \quad 20+5x=40+x$

$\Rightarrow \quad 4x=20$

$\Rightarrow \quad x=5$ L

12. Let the capacity of the vessel be x litres.

Given $\left(\frac{x-10}{x}\right)^2 x = 32$

$\Rightarrow (x-10)^2 = 32x$

$\Rightarrow x^2 - 52x + 100 = 0$

$(x-50)(x-2) = 0$

$\Rightarrow x = 50, 2$

As the vessel has 32 litres of milk after 2 replacements $x > 32$.

$\therefore x = 50$

13. Clearly the quantity of milk in the final solution

$$50\left(\frac{50-10}{50}\right)\left(\frac{50-5}{50}\right)\left(\frac{50-20}{50}\right) = 21.6 \text{ L}$$

14.

Let the cost price of the second variety be Rs. C/kg.

Let the cost price of the mixture be Rs. x/kg.

Using the method allegation, we get

C +5   C

25

25 – C   C + 5 – 25

25 – C   C – 20

2 : 3

$$\frac{25-C}{C-20} = \frac{2}{3}$$

$C = 23 \qquad C + 5 = 28$

$$\frac{28\times3+23\times2}{5} = 26/\text{kg}$$

Hence, required cost price $= \frac{(28)(3)+23(2)}{5}$ = Rs. 26/kg.

15. The housewife added 250 ml of water to one litre mixture containing 750 ml of milk and 250 ml of water.

$\therefore$ 1.25 litre of mixture contains 750 ml of pure milk.

$\therefore$ Ratio of milk to water finally is 3 : 2.

Pure milk in the mixture after 250 ml of mixture is removed $= 750 - 250\times\frac{3}{5} = 600$ ml

CHAPTER

13

Time and Work, Pipes and Cisterns

- If A can do a piece of work in n days, then A's 1 day's work = $\frac{1}{n}$
- If A's 1 day's work = $\frac{1}{n}$, then A can finish the work in n days.
- If A is thrice as good a workman as B, then:

  Ratio of work done by A and B = 3 : 1

  Ratio of times taken by A and B to finish a work = 1 : 3

  Concept: Say A is working on tool machine and he can make 100 tools (work) in 10 days working for same hours each day with same efficiency.

  So A's 10 day work = 100 tools

  A's 1 day work = 100/10 = 10 tools = 1/10 of the total work

  So we can say that if A can do a piece of work in n (say 10) days, then A's 1 day's work = 1/n

  or

  If A's 1 day work = 1/n, then A can finish the work in n days

Example 1.

A takes 5 days to complete a piece of work and B takes 15 days to complete a piece of work. In how many days can A and B complete the work if they work together?

Sol. Let us consider work to be 1 unit. So if W = 1 Unit and A takes 5 days to complete the work then in 1 day A completes 1/5th of the work. Similarly B completes 1/15th of the work.

If they work together, in one day A and B can complete (1/5 + 1/15 = 4/15) of the work. So the complete 1 unit of work they will take 15/4 days.

New method:

Let us assume W = 15 units, which is the LCM of 5 and 15.

Given that total time taken for A to complete 15 units or work = 5 days

→ A's 1 day work = 15/5 = 3 units

Given that total time taken for B to complete 15 units of work = 15 days

→ B's 1 day work = 15/15 = 1 unit

→ (A + B)'s 1 day work = 3 + 1 = 4 units

→ 15 units of work can be done in 15/4 days.

Many solve Time and Work problems by assuming work as 1 unit (first method) but We feel it is faster to solve the problems by assuming work to be of multiple units (second method). This would be more evident when we solve problems which are little more complex than the above one.

Example 2.

X can do a work in 15 days. After working for 6 days he is joined by Y. If Y complete the remaining work in 3 more days, in how many days can Y alone complete the work?

Sol. Assume W = 15 units.

Note: You can assume work to be any number of units but it is better to take the LCM of all the numbers involved in the problem so that you can avoid fractions

X can do 15 units of work in 15 days

→ X can do 1 unit of work in 1 day

Note: If 1 had assumed work as 13 units for example then X's 1 day work would be 13/15, which is a fraction and hence I avoided it by taking work as 15 units which is easily divisible by 15 and 3

Since X worked for 6 days, total work done by X = 6 days × 1 unit/day = 6 units.

Units of work remaining = 15 – 6 = 9 units.

All the remaining units of work have been completed by Y in 3 days

→ Y's 1 day work = 9/3 = 3 units.

If Y can complete 3 units of work per day then it would take 5 days to complete 15 units of work. So Y takes 5 days to complete the work.

Example 3.

P can do a piece of work in 20 days and Q can do the same work in 30 days. They finished the work with the help of R in 8 days. If they earned a total of Rs. 6000, then what is the share of R?

Sol. Let total work = 120 units

P's 1 day's work= 6 units, also Q's 1 day's work = 4 units

(P+Q+R)'s 1 day's work = 120/8 = 15 units so R's 1 day's work = 15 – 4 – 6= 5 units

Given that R worked for 8 days' R's 8 day's work = 40 units= $1/3^{rd}$ of the total work

So R's share in total income would be 1/3 of the total amount = 1/3 6000 = 2000

Note

If A is thrice as good a workman as B, then ratio of work done by A and B = 3:1, Ratio of time taken by A & B to finish a work = 1:3.

Example 4.

P is thrice as good a workman as Q and together they finish a piece of work in 24 days. In how many days will P alone finish the work?

Sol. Assume Q's 1 day's work = 1 unit' P's 1 day's work = 3 units

(As it is given that P is thrice as good a workman as Q)

Given that together they can complete the work in 24 days

So in 24 days (P+Q)'s work = 24 × [(P+Q)'s 1 day's work] = 24 × (3+1) = 96 units= total work

⇒ *P alone will finish the work (96 units) in 96/3= 32 days*

Example 5.

P can dig a well in 5 hours. He invites Q and R who together can dig 5/4th as fast as he can, to join him. He also invites S and T who together can dig 8/5th as fast as he can, to join him. If the five person team digs the same well and they start together, how long will it take for them to finish the job?

Sol. Let the total work = 100 units

So P's 5 hour's work = complete work = 100 units, P's 1 hour's work = 20 units

Given that (Q+R)'s work $5/4^{th}$ as fast as P's, (Q+R)'s 1 hour's work = 5/4 × 20 = 25 units

Given that (S+T)'s work $8/5^{th}$ as fast as P's, (S+T)'s 1 hour's work = 8/5 × 20 = 32 units

Hence if all of them work together they can do 20+ 25+ 32 = 77 units of work in 1 hour so they can complete the total work (100 units) in 100/77 hours

Man-days Problems

If 1 man's 1 day's work = M *units and 1 man's* N *day's work* = M × N *units*

⇒ 10 *Men's* N *day's work* = 10 × M × N *units*

Also if 1 woman's 1 day's work = W *units so 10 women's* N *day's work* = 10×W×N *units*

Say if 10 men and 15 women can complete a piece of work in 5 days

So total work = (10 × M × 5 + 15 × W × 5) = (10M + 15W) 5 *units*

Example 6.

If 12 men are able to complete 240 units of work in one day, how many units of work is done by 5 men in the same time?

Sol. 12 men's 1 day's work = 240 units '! 1 men's 1 day's work = 240/12 = 20 units

So 5 men's 1 day work = 5 × 20 = 100 units

Example 7.

A piece of work can be done by 8 boys in 4 days working 6 hours a day. How many boys are needed to complete another work which is three times the first one in 24 days working 8 hours a day?

Sol. $\frac{8\times4\times6}{W} = \frac{B\times24\times8}{3W}$

$B = 3$

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PIPES & CISTERNS

Introduction

What comes as problem of the pipes & cistern is very similar to the problem of time and work involving the two or three people. The only difference is that in pipes and cisterns problems, we have an inlet pipe that is doing the positive work (filling up the tank or cistern) and there is the outlet pipe this is doing the negative work (leaking).

Concept

Inlet: A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an **inlet**

Outlet: A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an **outlet**

If an inlet pipe can fill a tank (100 L) in 10 hours, then part of the tank filled by the inlet pipe in 1 hour = 100/10 = 10 L= $1/10^{th}$ of the tank capacity.

Or we can say that if an inlet pipe fills $1/10^{th}$ (10 L) of the tank (100 L) in 1 hour, the inlet pipe will take 10 hours to fill the complete tank.

In a similar way if a outlet pipe (leak) can empty the fully filled tank (120 L) in 15 hours, then part of the tank empted by the outlet pipe (leak) in 1 hour = 120/15 = 8 L.

If both inlet and outlet pipe are connected to the same tank (120 liter) where inlet pipe is filling say 10 liter per hour and outlet pipe empting 8 liter per hour, then net filling of the tank in 1 hour = 10 – 8 = 2 liter per hour

Example 8.

There are three hoses, A, B and C, attached to a reservoir. A and B can fill the reservoir alone in 20 and 30 mins. respectively whereas C can empty the reservoir alone in 45 mins. The three hoses are kept opened alone for one minute each in the order A, B and C. The same order is followed subsequently. In how many minutes will the reservoir be full?

Sol. These kinds of problems can be solved in the same way as we solve problems where one or more men are involved. A, B and C are equivalent to three people trying to complete a piece of work.

The amount of work to be done would be the capacity of the reservoir. Let assume capacity of the reservoir = W = 180 (LCM of 20, 30, 45) litres.

A can fill the reservoir in 20 mins ⇒ In 1 min A can fill 180/20 = 9L.

B can fill 180/60 = 6 L in a minute.

In one minute C can empty 180/45 = 4 L from the reservoir.

1st Minute = > A is opened = > fills 9 L

2nd Minute = > B is opened = > fills another 6 L

3rd Minute = > C is opened = > empties 4 L

Hence every 3 minutes = > (9 + 6 – 4) = 11 L are filled into the reservoir.

So in 45(3 × 15) minutes (11 × 15) = 165 L are filled.

In the 46th minute A is opened and it fills 9 litres. In the 47th minute B is opened and it fills 6 L.

Hence the reservoir will be full in 47 minutes.

Example 9.

There is an empty reservoir whose capacity is 30 litres. There is an inlet pipe which fills at 5 L/min and there is an outlet pipe which empties at 4L/min. Both the pipes function alternatively for 1 minute. Assuming that the inlet pipe is the first one to function, how much time will it take for the reservoir to be filled up to its capacity?

Sol. The work ot be done = Capacity of reservoir = W = 30 litres

1st Minute = > inlet pipe opened = >5 L filled

2nd Minute = > inlet pipe closed; outlet pipe opened = > 4 L emptied

In 2 minutes (5 – 4) = 1 L is filled into the reservoir.

It takes 2 minutes to fill 1 L = > it takes 50 minutes to fill 25 L into the tank.

In the 51st minute inlet pipe is opened and the tank is filled.

Example 10.

Pipe A can fill a tank in 5 hours, pipe B in 10 hours and pipe C in 30 hours. If all the pipes are open simultaneously, in how many hours will the tank be filled?

Sol.

Similar to the work & time problem here we will assume the total capacity of the tank = LCM of 5, 10 and 30 = 60 L

So Pipe A can fill the tank (60 liters) in 5 hours, Pipe A fills in 1 hr= 60/5 = 12 L

Similarly Pipe B fills in 1 hr = 60/10 = 6 L and Pipe C fills in 1 hr = 60/30 = 2 L

Net amount filled by all the three pipes in 1 hour = 12 + 6+ 2 = 20 L , when all the three pipes will operate simultaneously the tank will get filled in 60/20= 3 hr.

Example 11.

A tank is filled by an inlet pipe in 5 hours. However because of a leak, it takes 30 minutes (0.5 hours) more to fill up the tank. In how much time will the leak empty the fully filled cistern independently?

Sol. Assume the total capacity of the tank = LCM of 5 and 5.5 = 55

Inlet pipe alone without leak can fill in 1 hr = 55/5 = 11 L

Inlet pipe with leak filling the tank (55 L) in 5.5 hr (adding extra 30 minutes) , so inlet pipe with leak fills in 1 hr = 55/5.5 = 10 L which means that in 1 hr leak is emptying 1 L of water out of the tank.

So if leak alone is emptying 1 L of water out of the tank in 1 hr so leak alone will empty the fully filled tank is 55/1= 55 hr.

Example 12.

A tank is fitted with 22 pipes. Some of these are fill pipes that fill the tank and others waste pipes that drain the tank. If each fills pipe can fill the tank in 24 hr and each waste pipe can drain the tank in 16 hr, and if all the pipes are kept open simultaneously when the tank is empty, the tank get fully filled in 12 hr. How many of these are waste pipes?

Sol.

Assume the total capacity of the tank = LCM of 12, 16 and 24 = 48

So each fill pipe fills in 1 hr = 2 and each waste pipe drain in 1 hr = 3

Say n fills pipes are there so number of waste pipes will be 22-n, now according to the questions all pipes operating simultaneously will fill the tank in 12 hr.

$[n(2)-(22-n)3)] \times 12 = 48$

$[2n - 66 + 3n] = 48/12$

$5n - 66 = 4$

$n = 14$

Hence there are 14 fills pipes and 22-14 = 8 waste pipes.

Objective Questions

1. P can finish a work in 18 days and Q can do the same work in half the time taken by A. Then, working together, what part of the same work they can finish in a day?

(a) $\frac{1}{9}$ (b) $\frac{1}{6}$

(c) $\frac{2}{5}$ (d) $\frac{2}{7}$

2. P, Q and R can complete a piece of work in 8, 12 and 16 days respectively. Working together, they will complete the same work in :

(a) 4 days (b) $\frac{45}{13}$ days

(c) $\frac{48}{13}$ days (d) $\frac{48}{15}$ days

3. A can do a piece of work in 16 days and B can do the same work in 12 days, with the help of C, they did the job in 4 days. Then C, alone can do the job in:

(a) $9\frac{1}{5}$ days (b) $9\frac{3}{5}$ days

(c) $9\frac{2}{5}$ days (d) $9\frac{4}{5}$ days

4. P and Q can do a piece of work in 6 days, Q and R can do it in 8 days, R and P can do it in 12 days. Who among these will take the least time if put to do it alone?

(a) P (b) Q

(c) R (d) Data inadequate

5. A can do a piece of work in 15 days. B is 25% more efficient A the number of days taken by B to do the same piece of work is:

(a) 10 days
(b) 12 days
(c) 18 days
(d) 13 days

6. P, Q and R can do a job in 15 days. After working with Q and R for 5 days, P leaves. Then Q and R finish it 20 days more. In now many days can P do the job along?

(a) 27 days
(b) 25 days
(c) 28 days
(d) 30 days

7. P does 75% of the work in 18 days. He then calls in Q and they together finish the remaining work in 2 days. How long Q alone would take to do the whole work?

(a) 15
(b) 20
(c) 12
(d) 24

8. A and B can complete a piece of work in 45 days and 40 days respectively. They began to work together but A leaves after some time and B completed the remaining work in 23 days. After how many days did A leave?

(a) 14 days
(b) 32 days
(c) 16 days
(d) 9 days

9. A can complete a piece of work in 15 days B in 20 days and C in 12 days. They began to work together but 2 days before the completion of work, both A and C left. How long it took overall to complete the work?

(a) 9.5 days
(b) 6.5 days
(c) 7.5 days
(d) 8.5 days

10. A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened, what is the total time taken to fill the tank completely?

(a) 4 hrs 15 min
(b) 4 hrs
(c) 3 hrs 45 min
(d) 3 hrs 15 min

11. Two pipes P and Q together can fill cistern in 4 hours. Had they been opened separately, then Q would have taken 6 hours more than P to fill the cistern. How much time will be taken by P to fill the cistern separately?

(a) 8 hrs
(b) 6 hrs
(c) 2 hrs
(d) 1 hrs

12. Two pipes P and Q can fill the tank in 15 hours and 20 hrs respectively while a third pipe R can empty the full tank in 25 hrs. All the three pipes are opened in the beginning. After 10 hours pipe R is closed. In how much time, will the tank be full?

(a) 14 hrs
(b) 12 hrs
(c) 16 hrs
(d) 15 hrs

13. A larger tanker can be filled by two pipes P and Q in 60 hours and 40 hours respectively. How many hours will it take to fill the tanker from empty state if Q is used for half the time and P and Q fill it together for other half?

(a) 20 hours
(b) 15 hours
(c) 30 hours
(d) 27.5 hours

14. A tank is filled by three pipes with uniform flow. The first two pipes opening simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. The time required by the second pipe alone to fill the thanks.

(a) 8 hrs (b) 10 hrs
(c) 12 hrs (d) 9 hrs

15. A person employs 45 men to complete a project in 45 days. However these 45 men while working together complete only one-third of the project in 30 days. How many extra men does the person need to employ so that the project is completed in 45 days?

(a) 90 (b) 60
(c) 45 (d) 50

16. 4 men and 6 women can complete a job in 8 days, while 3 men and 7 women can complete it in 10 days. How many days will 10 women working alone take to complete the same work?

(a) 36 days (b) 42 days
(c) 40 days (d) 24 days

17. Twenty-four women can complete a work in 16 days. 32 children can complete the same work in 24 days. 16 women and 16 children started working together and worked for 12 days. How many more children are to be added to complete the remaining work in 2 days?

(a) 32 (b) 48
(c) 60 (d) 40

18. If the output of 3 men is equal to that of 4 boys who can complete a job in 12 days working together, how much time would 5 boys and 3 men together take to complete the same job?

(a) $6\frac{1}{2}$ days (b) $10\frac{2}{3}$ days
(c) 18 days (d) 15.5 days

19. A, B and C individually can finish a work in 6, 8 and 15 hours respectively. They started the work together and after completing the work got Rs.94.60 in all. When they divide the money among themselves, A, B and C will respectively get (in Rs.)

(a) 44, 33, 17.60 (b) 43, 27.20, 24.40
(c) 45, 30, 19.60 (d) 42, 28, 24.60

20. Ramesh takes twice as much time as Mahesh and thrice as much time as Suresh to complete a job. They together complete the job in 4 days. Then the time taken by each of them separately to complete the work is

(a) 36, 24 and 16 days (b) 20, 16 and 12 days
(c) 24, 12 and 8 days (d) None of these

21. There is a leak in the bottom of the tank. This leak can empty a full tank in 8 hours. When the tank is full, a tap is opened into the tank which admits 6 litres per hour and the tank is now emptied in 12 hours. What is the capacity of the tank?

(a) 28.8 litres (b) 36 litres
(c) 144 litres (d) Cannot be determined

22. Two candles of the same height are lighted at the same time. The first is consumed in 4 hours and the second in 3 hours. Assuming that each candle burns at a constant rate, in how many hours after being lighted was the first candle twice the height of the second.

(a) $\frac{3}{4}hr$ (b) $1\frac{1}{2}hr$

(c) 2 hr (d) $2\frac{2}{5}hr$

23. Sixty men can do a work in 10 days for which the total wages are Rs. 700. The same work is now done in the following manner. On the first day one man starts the work and from the second day onwards one more man joins the group on each day. If the work is completed in this fashion in how many days will the work be completed?

(a) $34\frac{1}{7}$ (b) $35\frac{2}{7}$

(c) $34\frac{3}{7}$ (d) $33\frac{6}{7}$

24. The rates at which Alok and Bhaskar work are in the ratio 5 : 4. They work on alternate days to complete a job. Bhaskar started the job and he worked on the last day. The job was completed in 1 day more than the time that Alok alone would take to complete it. Find the time taken (in days) by them working together to complete the job.

(a) $4\frac{1}{2}$ (b) $3\frac{1}{3}$

(c) $4\frac{1}{3}$ (d) $4\frac{4}{9}$

25. There are ten taps that when working together can fill a certain empty tank in five hours. The capacities of the ten taps are in the ratio of 10 : 9 : 8 : 7 : 6 : 5 : 4 : 3 : 2 : 1. Initially all ten taps are opened (into the empty tank) and after every one hour the tap of the maximum capacity is closed. What is the time in which the tank will be filled if the taps are operated in this manner?

(a) $5\frac{1}{2}$ hours (b) $8\frac{1}{2}$ hours

(c) $9\frac{1}{2}$ hours (d) Tank is never filled

26. A alone can do a piece of work in 30 days and if he works alongwith B, he can do the same work in 20 days. Then again A alone works for 4 days after which both of them work together till the work is completed. How many days did it take from the beginning to complete the work?

(a) 20 (b) 17

(c) 18 (d) None of these

Answers
Objective Questions

1.	(b)	2.	(c)	3.	(b)	4.	(b)	5.	(b)
6.	(d)	7.	(c)	8.	(d)	9.	(d)	10.	(c)
11.	(b)	12.	(b)	13.	(c)	14.	(b)	15.	(c)
16.	(c)	17.	(b)	18.	(b)	19.	(a)	20.	(c)
21.	(c)	22.	(d)	23.	(a)	24.	(d)	25.	(d)
26.	(b)								

Solutions:—

1. By question Q can finish the work in a days assume total work = LCM of a and 18 = 18 units. So P's 1 days work = 1 unit and Q's 1 day's work = 2 units. (P + Q)'s 1 days work = 3 units which means in a day they together can finish 3 units of work as $\frac{1}{6}$ of the total work.

2. Assume total work = LCM of 8, 12 and 16 = 48 units , so P's 1 day's work = 6 units, Q's, 1 day's work = 4 units and R's 1 day's work = 3 units. (P + Q + R)'s 1 day's work = 6 + 4 + 3 = 13 units which means together they can finish the complete work in $\frac{48}{13}$ days.

3. Assume total work = LCM of 4, 12 and 16 = 48 units. A's 1 day's work = $\frac{48}{16}$ = 3 units , B's 1 day's work = 4 units
According to the question (A + B + C) can finish the work (48 units) in 4 days. So (A + B + C)'s 1 day's work = $\frac{48}{4}$ = 12 units.
C's 1 day's work = (A + B + C)'s 1 day work – sum of A's and B's 1 day work
C's 1 day's work = 12 – (4 + 3) = 5 units
C will finish the work in $\frac{48}{5}$ days = $9\frac{3}{5}$ days

4. Assume total = LCM of 6, 8 and 12 = 24 units
(P + Q)'s 1 day's work = $\frac{24}{6}$ = 4 units ———— I
(Q + R)'s 1 day's work = $\frac{24}{8}$ = 3 units ———— II
(P + R)'s 1 day's work = $\frac{24}{12}$ = 2 units ———— III

On adding I, II and III

(P + Q + R)'s 2 day's work = 9 units

(P + Q + R)'s 1 day's work = $\frac{9}{2}$ units ———— IV

Now P's 1' day's work = (P + Q + R)'s 1 day's work – (Q + R)'s 1 days work

$= \frac{9}{2} - 3 = \frac{3}{2}$ units.

Similarly Q's 1 day's work = $\frac{9}{2} - 2 = \frac{5}{2}$ units

R's 1 day's work = $\frac{9}{2} - 4 = \frac{1}{2}$ units

As Q's work per day is more than P's and R's work, so Q will finish the work in least time

5. B = $\frac{15}{1.25} = 12$ days

6. Assume total work = LCM of 5, 15 and 20 = 60 units

(P + Q + R)'s 1 day's work = $\frac{60}{15}$ = 4 units .... (I)

As it is given that together (P + Q + R) worked for 5 days

(P + Q + R)'s 5 day's work = 5 × 4 = 20 units

Now remaining work of 60 – 20 = 40 units is done by Q and R in 20 days, which means

(Q + R)'s 1 day's work = $\frac{40}{20}$ = 2 units ....(II)

From I and II

P's 1 day work = 4 – 2 = 2 units

P alone can finish the work in $\frac{60}{2} = 30$ days

7. According to the question P does $\frac{3}{4}$th of the work in 18 days so he will do the complete work in $18 \times \frac{4}{3} = 24$ days

Assume total work = 24 units

P's 1 day's work = 1 unit

P's 18 day's work = 18 units, Now remaining 6 units of work is done by P and Q in 2 days

(P + Q)'s 1 days work = $\frac{6}{2}$ = 3 units

8. Assume total work = LCM of 40 and 45 = 360 units

A's day's work $= \frac{360}{45} = 8$ units

B's 1 day's work $= \frac{360}{40} = 9$ units

Assume that they together worked for x days

So total work is x days = $x(9 + 8) = 17x$

Remaining work = $360 - 17x$

Given that remaining work is done by B in 23 days

B last 23 days work = remaining work

$23 \times 9 = 360 - 17x \Rightarrow x = 9$, so A left after 9 days as they worked together for 9 days.

9. Assume total work = LCM of 15, 20 and 12 = 60 units

Now A's, B's and C's 1 day's work are 4, 3 and 5 units respectively.

Assume that A, B and C work together fill x days.

So (A + B + C)'s x day's work = $x(4 + 3 + 5) = 12x$ units

Gives that after x days both A and C left and remaining work was completed by B in 2 days

$12x$ + B's 2 day's work = total work

$$12x + 2 \times 3 = 60 \Rightarrow x = 6.5$$

It means they took $(6.5 + 2) = 8.5$ days to complete the overall work.

10. Assume total capacity of the tank = any multiplied of 6 = say 60 liters.

$$\text{A's fills in 1 hr} = \frac{60}{6} = 10 \text{ liters}$$

Now tap A can fill half the tank in 3 hours ***i.e*** $3 \times 10 = 30$ liters

Given that remaining half is filled by A and 3 taps similar to A.

4 taps fills in 1 hour = $4 \times 10 = 40$ liters

But we have to fill half of he tank ***i.e*** 30 liters which will be filled by 4 taps in $\frac{3}{4}$ hours = 45 min

Tank will get filled in 3 hours 45 minute.

11. Assume total capacity of the cistern = any multiple of 4 say = 12 liters

(P + Q) fill in 4 hrs = full cistern

(P + Q) fill in 1 hr = $\frac{12}{4} = 3$ liters assume that P alone take t hour so fill the cistern

Q will take $(6 + t)$ hours to fill the cistern completely.

Now P's 1 hr fill + Q's 1 hr. fill = (P + Q)'s 1 hr fill

$$\frac{12}{t} + \frac{12}{6+t} = 3$$

$$12(6 + t) + 12t = 3t^2 + 18t$$

$$3t^2 - 6t - 72 = 0 \Rightarrow t^2 - 2t - 24 = 0$$

$$(t - 6)(t + 4) = 0$$

$t = 6$, so P alone take 6 hours to fill the cistern completely.

12. Assume total capacity of tank = LCM of 15, 20 and 25 = 600 litres

$$\text{P's fill in 1 hr} = \frac{600}{15} = 40 \text{ litres}$$

$$\text{Q's fill in 1 hr} = \frac{600}{20} = 30 \text{ litres}$$

$$\text{R's empty in 1 hr} = \frac{600}{25} = 24 \text{ litres}$$

For 10 hours P, Q and R opened together so (P + Q + R) fill in 10 hr. = 10 (40 + 30 - 24) = 460 litres (- 24, ∵ R is emptying the tank)

Remaining 600 – 460 = 140 litres is filled by P and Q as R is closed after 10 hours.

(P + Q) will take $\frac{140}{(40+30)}$ = 2 hrs to fill the remaining 140 liters.

Hence total time to fill the tank = 10 + 2 = 12 hrs.

13. Assume total capacity of the tank = LCM of 40 and 60 = 120 litres

$$\text{P's fill in 1 hr} = \frac{120}{60} = 2\text{L}$$

$$\text{Q's fill in 1 hr} = \frac{120}{40} = 3\text{L}$$

$$(\text{P + Q}) \text{ fill in 1 hr} = 5\text{L}$$

Assume total time to fill the tank = t hours

Now according to question

Q fill in $\frac{t}{2}$ hr and (P + Q) fill in $\frac{t}{2}$ hr = tank filled

$$\Rightarrow \quad \frac{3t}{2} + \frac{5t}{2} = 120 \Rightarrow t = 30 \text{ hours}$$

Hence it will take 30 hours to fill the tank.

14. Assume second pipe fill the tank in t hrs.

First pipe will fill in t + 5 hrs.

Third pipe will fill in t – 4 hrs.

According to question

$$\frac{C}{t+5} + \frac{C}{t} = \frac{C}{t-4}$$

(Where C is the total capacity of the tank)

$$\frac{1}{t+5} + \frac{1}{t} = \frac{1}{t-4}$$

$$(t + t + 5)(t - 4) = t(t + 5)$$

$$(2t + 5)(t - 4) = t^2 + 5t$$

$$2t^2 - 3t - 20 = t^2 + 5t$$

$$t^2 - 8t - 20 = 0$$

$$(t - 10)(t + 2) = 0 \Rightarrow t = 10$$

($t \neq -2$, as time can't be –ve so second pipe alone will fill the tank in 10 hours.

15. Assume 1 man's 1 day work = M units so 45 men's 45 day's work = total work = (45×45) M = 2025 M

Now given that these 45 men did only 1/3rd of the work in 30 days, so let x more men are needed to complete the work (remaining two-third) in time

Remaining $\frac{2}{3}$ rd work is done by $(x + 45)$ men in 15 days

$$(x + 45)\text{ M} \times 15 = \frac{2}{3}(\text{total work}) = \frac{2}{3}(45 \times 45\text{M})$$

$x = 45$, hence 45 extra men are needed .

16. Assume a man 1 day's work = M units

A women 1 day's work = W units

4 men and 6 women 8 day's work

$= (4M + 6W) \times 8 =$ total work —— I

Also $(3M + 7W) \times 10 =$ total wok ——II

From I and II

$$(4M + 6W) \times 8 = (3M + 7W) \times 10$$

$$1M = 11W$$

$$\text{Total work} = (4M + 6W) \times 8 = (4 \times 11W + 6W) \times 8 = 400\ W$$

Now 10 women's 1 day's work = 10W

Hence 10 women will take $\frac{400W}{10W} = 40$ days to complete the work.

17. Assume 1 women one day's work = W units

1 Child 1 day's work = C units

Given $(24\ W) \times 16 =$ work ———— I

Also $(32\ C) \times 24 =$ work ————II

From I and II

$$(24\ W) \times 16 = (32\ c) \times 24 \Rightarrow W = 2C$$

16 women and 16 children 12 day's work $= (16w + 16c)12 = [16\ (2c) + 16c]12 = 576$ C units

Remaining work = total work – 576 C

$(32C) \times 24 - 576 =$ 192C units

Given that these 16 women and 16 children will work for next 2 days so their work

$= (16w + 16c) \times 2$

$(32c + 16c) \times 2 =$ 96c units remaining work of $192c - 96c = 96c$

Units will be done by 48 children as each child does 2 units of work in 2 days.

Hence 48 more children are needed.

18. 1 man 1 day's work = M units

1 boy 1 day's work = B units

Given $3M = 4B$

Also $(3M + 4B) \times 12 = \Rightarrow (4B + 4B) \times 12 = 96$ B = total work

5 boys and 3 men 1 days work

$= (5B + 3M) = (5B + 4B) = 9B$

Now 5 boys and 3 men will take $= \frac{96}{9} = 10\frac{2}{3}$ days to complete the work.

19. If there are 120 units of the work to be done, A would finish 1/6th of it in 1 day i.e. 20 units, B will finish 1/8 of it in 1 day i.e. 15 units and C will finish 1/15th of it in 1 day i.e. 8 units. So, the amount of work done by A., B and C are in ratio 20 : 15 : 8. This should be ratio in which the total earning should be divided into. So, A. B and C would get Rs. 44, Rs. 33 and Rs. 17.60 respectively.

20. Let suresh, Mahesh and Ramesh take2, 3 and 6 days respectively to complete a job.

Since the job is completed in 4 days

Then, time taken by suresh = 2 × 4 = 8 days

Then, time taken by suresh = 3 × 4 = 12 days

Then, time taken by Ramesh = 6 × 4 = 24 days

21. Since the leak can empty the tank in 8 hours, the rate of leak = 1/8. And since the leak along with the tap can empty it in 12 hours, we can write the equation as : 1/x – 1/8 = –1/12 (where x is the time taken by the tap to fill the tank). Simplifying we get, 1/x = 1/24 or x = 24. This means that the tap can fill the tank in 24 hours. Since the tap admits 6 litres per hour, it will admit (6 × 24) = 144 litres in 24 hours, which should be the capacity of the tank.

22. Let h be the height of both candles.

1st burns: 1/4 per hr and 2nd burns: 1/3 per hr

after x hrs 1st candle $h-\frac{xh}{4}$ and 2nd candle $h-\frac{xh}{3}$

$h-\frac{xh}{4}=2\left(h-\frac{xh}{3}\right)$ or $h-\frac{xh}{4}=2h-\frac{2\times h}{3}$ or $\frac{2x}{3}-\frac{x}{4}=1$

or $\frac{5x}{12}=1$ or $x=\frac{12}{5}=2\frac{2}{5}$ hrs.

23. Given on the first day 1 man works.

Hence we get 1 man day.

Similarly on the second day we get 2 man days

Hence the total number of man days in n days will be

$1+2+3+\ldots\ldots+n$ i.e., $\frac{n(n+1)}{2}$

It is given that the total work requires 600 man days

$\frac{n(n+1)}{2}=600$ For n = 34, We get $\frac{n(n+1)}{2}=595$

In 34 days a total of 595 man days are contributed.

On the 35th day we get 35 man days but since we need only 5 man days, it is sufficient if they work for 1/7th of that day.

So, the total work will be completed in $34\frac{1}{7}$ day

24. Let the times taken by Alok and Bhaskar to complete the job be 4x days and 5x days respectively.

Part of the job done in the first 2 days = $\frac{1}{4x}+\frac{1}{5x}=\frac{9}{20x}$

In the first 4x days, part of the job done = $\frac{9}{20x}\left(\frac{4x}{2}\right)=\frac{9}{10}$

Remaining part of the job = $\frac{1}{10}$, This must have been done in 1 day by Bhaskar.

$\therefore$ $\frac{1}{10}$ of the job can be done by Bhaskar in a day

$\therefore$ 5x = 10 and 4x = 8

Time taken by them to complete the job working together = $\frac{(10)(8)}{18}$ or $4\frac{4}{9}$

25. Let the capacity of the first tap be 1.

$\Rightarrow$ The capacity of the other taps = 2, 3, ... till 10

$\Rightarrow$ Total capacity of tank = $\left(\sum_{t}^{10} n\right) \times 5 = 275$ hours

But the sum $\sum_{10} + n \sum_{9} n..........\sum_{1} n = 220$

i.e. 55 + 45 + 36 + 28 + 21 + 15 +10 + 6 + 3 + 1 = 220

(This is since the taps are stored in thespecified order)

$\therefore$ at the end of the length here all taps are closed though the taps is not yet tank.

The tank will never be filled.

26. When A and B work together, they take 10 days.

Hence B alone can do the work in 15 days

(because 1/10 – 1/30 = 1/15);

A works for 9 days and B for 3 days; the work completed in this time = 9/30 + 3/15 = 1/2;

Hence 1/2 the work is remaining which A and B together can complete in 5 days (because they can complete the work together in 10 days); so from the beginning it will take 9 + 3 +5 = 17 days.

CHAPTER

14

Simple and Compound Interest

CONCEPT

Principal	: Denoted by P, is the value at the outset.
Amount	: Denoted by A, is the final value of the principal after the applicable growth.
Rate of Interest	: Denoted by r, is the percentage growth of the principal for any single time period.
Interest	: Denoted by I, is the amount by which a principal increases.
Time Period	: Denoted by T or n, is the time interval after which any growth/interest is calculated.
Simple Interest (SI)	: The principal grows at a constant rate in absolute terms.
Compound Interest(CI)	: The principal grows at an increasing rate in absolute terms. The interest is calculated on the new principal at the end of every time period.

SIMPLE INTEREST

The Simple Interest is given by S.I. = $\frac{P \times r \times n}{100}$

One can also break this formula as

$$S.I. = \frac{P \times r}{100} + \frac{P \times r}{100} + \frac{P \times r}{100} + ...n \text{ times i.e.,}$$

Total S.I. in years = Interest in 1 year × number of years

Another understanding from the equation is

S.I. = $P \times \frac{r \times n}{100}$ = rn% of P i.e., if rate of interest is 5% and principal is invested for 6 years, the total S.I. will be 30% of the principal invested.

Please note that the formula gives the Simple Interest and to calculate the final amount one must add it to the principal.

$$\text{Amount} = P + \frac{P \times r \times n}{100} = P\left(1 + \frac{r \times n}{100}\right)$$

If p is the principal kept at CI @ r% p.a., amount after n years will be $P\left(1 + \frac{r}{100}\right)^n$

Example 1.

If r = 10%, n = 4 years, what is the simple interest charged on a loan of Rs. 2,000?

Sol.

$$S.I. = \frac{P \times n \times r}{100} = \frac{2000 \times 4 \times 10}{100}$$

Alternately, for 4 years the interest payable would be equal to 10 × 4 = 40% of the principal, i.e. 40% of 2000, which is Rs. 800.

Example 2.

In the above problem, if the case was one of compound interest, what is the CI?

Sol.

The amount at the end of 4 years, when interest is compounded annually, is

$$= P\left(1+\frac{r}{100}\right)^n$$

So $$A = 2000\left(1+\frac{10}{100}\right)^4$$

$$= 2000 \times (1.1)^4 = 2000 \times (1.4641) = \text{Rs. } 2928.2$$

So compound interest payable

$$= \text{Rs. } 2928 - \text{Rs. } 2000 = \text{Rs. } 928.2$$

	Simple Interest	Compound Interest		
		Annual Compounding	Semi-annual Compounding	Quarterly Compounding
Interest in n years	$\frac{P \times n \times r}{100}$	$P\left(1+\frac{r}{100}\right)^n - P$	$P\left(1+\frac{r}{200}\right)^{2n} - P$	$P\left(1+\frac{r}{400}\right)^{4n} - P$
Amount at the end	$P + \frac{P \times n \times r}{100}$	$P\left(1+\frac{r}{100}\right)^n$	$P\left(1+\frac{r}{200}\right)^{2n}$	$P\left(1+\frac{r}{400}\right)^{4n}$

Suppose that we invest the two amounts of Rs 100 each. both at the annual rate of 10%, one at simple Interest and the other at compound interest. Let's see their growth:

	Initial Quantity	1st Year	2nd Year	3rd Year	4th Year
Simple Growth	Rs 100	Rs 110	Rs 120	Rs 130	Rs 140
Yearly Interest		Rs 10	Rs 10	Rs 10	Rs 10
Compounded Growth	Rs 100	Rs 110	Rs 121	Rs 133.1	Rs 146.41
Yearly Interest		Rs 10	Rs 11	Rs 12.1	Rs 13.31

Example 3.

The simple interest on a certain sum of money for $2\frac{1}{2}$ years at 12% per annum is Rs. 40 less than the simple interest on the same sum for $3\frac{1}{2}$ years at 10% per annum. Find the sum.

Sol.

Let the sum be Rs. x. Then,

$$\left(\frac{x\times10\times7}{100\times2}\right)-\left(\frac{x\times12\times5}{100\times2}\right)=40 \qquad \left(\text{By Applying the formula SI}=\left(\frac{p\times r\times t}{100}\right)\right)\times$$

$$\frac{7x}{20}-\frac{3x}{10}=40$$

$$x=(40\times20)=800$$

Hence, the sum in Rs. 800.

Example 4.

A certain sum of money at simple interest amounts Rs.1008 in 2 years and to Rs. 1164 in $3\frac{1}{2}$ years. Find the sum and the rate of interest.

Sol.

SI for $1\frac{1}{2}$ years = (1164 – 1008) = Rs. 156

$$\left(\begin{array}{l}\text{SI for } \frac{3}{2}\text{year}=156\\ \text{SI for 1 year}=156\times\frac{2}{3}\end{array}\right)$$

SI for 2 years = $\left(156\times\frac{2}{3}\times2\right)$ = Rs. 208

∴ Principal = Rs. (1008 – 208) = Rs. 800.

Now, P = 800, T = 2 and SI = 208.

∴ Rate = $\left(\frac{100\times208}{800\times2}\right)\%$ = 13%

Example 5

Two customers borrowed the same-amount of money, one on a compounded interest and the other on a simple interest. If after 2 years, the interest payable by one was Rs. 200 and by the other was Rs. 220. What was the principal lent to each of them?

Sol. The extra Rs. 20 payable by the first person is the interest on the interest of the first year, which is Rs. 100, so the rate of interest 20%.

The interest payable at the end of the first year = Rs. 100, which is 20% of the principal.

Hence, princip al = Rs. 500.

Example 6.

At what simple rate of interest shall a sum of money double itself in 4 years?

Sol. Important point to be noted is that the amount received by the lender is double the amount given, which means Interest = Principal

So, if x is the Principal, then x is the Simple Interest.

Or, $$x = \frac{(x \times R \times 4)}{100}$$

Or, $$R = \frac{100}{4} = 25\%$$

Example 7.

If a sum of money at simple interest doubles in 6 years, it will become 4 times in

Sol. Let sum be x. Then S.I. = x, as when money doubles means interest is equal to the sum.

$$\therefore \quad \text{Rate} = \left(\frac{100 \times x}{x \times 6}\right)\% = \frac{50}{3}\%$$

Now sum is x and S.I. is 3x, Rate = $\frac{50}{3}\%$

$$\therefore \quad \text{Time} = \frac{100 \times 3x}{x \times \frac{50}{3}} = 18 \text{ years}$$

Example 8.

Two equal sums of money were invested at an annual rate of 10%, one sum at simple interest and the other at compound interest. If the difference between the interests after 2 years was Rs 200, what were the sums invested?

Sol. This question can be solved by taking both the amounts equal to Rs 100 at 10% and then using the unitary method.

At Rs 100 and at the rate of 10%, the amount at simple interest after two years is Rs 120 and the amount at compound interest after 2 years is 121. The difference between the amounts, of the difference between the interests is 1 rupee.

The difference in the interests in 1 rupee when the amounts invested are Rs 100.

→ difference between the interests is Rs 200 when the amounts invested are 200 × 100 = Rs 20,000.

Example 9.

A sum of money was invested in bank at compound interest. The interest in the second year was Rs 550 and the interest in the 3rd year was Rs 605. Find the rate of interest and the amount invested.

Sol. We known that compound interest is nothing but interest on interest. Therefore interest in the 3rd year would be interest added on the interest in the 2nd year, i.e., the interest in the 3rd year would be percentage growth on the interest in the 2nd year, Let the rate of interest be r

$$550\left(1+\frac{r}{100}\right) = 605$$

$$r = 10\%$$

Now for rate of interest equal to 10%, here is table when the amounts invested is Rs 100.

	Initial Quantity	1st Year	2nd Year	3rd Year	4th Year
Compounded Growth	Rs 100	Rs 110	Rs 121	Rs 133.1	Rs 146.41
Yearly Interest	Rs 0	Rs 10	Rs 11	Rs 12.1	Rs 13.31

Now we can solve it by unitary method.

At 10%, the interest is Rs11 in the second year when the amount invested is Rs 100.

$\rightarrow$ at 10% the interest will be Rs 550 in the second year when the amount invested is

$\frac{100}{11}\times 550$ = Rs. 5000.

Hence the original amount invested is Rs. 5000.

Example 10.

Find the amount for Rs. 80,000 at 20% per annum, compounded semi-annually for 2 years.

Sol. Here n = (2 years) × 2 = 4 periods.

Similarly, $R = \frac{20}{2} = 10\%$ per time period

(As interest compounded semi-annually)

P = 80000

$$A = 80000\left(1+\frac{10}{100}\right)^4 = 80000 \times 1.4641 = \text{Rs. } 11712.8$$

Example 11.

Find C.I. on Rs. 10,000 at 10% for 9 months compounded quaurterly.

Sol. n = 3 periods, R = 2.5% per period and P= Rs. 10,000

$$\text{Amount} = 10000\left(1+\frac{2.5}{100}\right)^3 = \text{Rs. } 10{,}769 \text{ (approx)}$$

C.I. = Amount – Principal= 10769 – 10000 = Rs. 769

Example 12.

If the C.I. on a certain sum for 3 years at 20% p.a. is Rs. 728, what is the sum invested?

Sol. $C.I. = 728 = P\left(1+\frac{20}{100}\right)^3 - P$

$(1.2)^3 - P = 728$

$P(1.728 - 1) = 728$

$P(0.728) = 728$

$P = Rs.\ 1000$

Example 13.

A sum of money amounts to Rs 6690 after 3 years and to Rs 10, 035 after 6 years on compound interest. Find the sum.

$$P\left(1+\frac{R}{100}\right)^3 = 6690 \qquad \text{..........(i)}$$

and
$$P\left(1+\frac{R}{100}\right)^6 = 10035 \qquad \text{.........(ii)}$$

On dividing, we get,

$$\left(1+\frac{R}{100}\right)^3 = \frac{10035}{6690} = \frac{3}{2}$$

Substituting this value in (i), we get:

$$P \times \frac{3}{2} = 6690$$

or
$$P = \left(6690 \times \frac{2}{3}\right) = 4460$$

Hence, the sum is Rs. 4460.

Example 14.

A sum of money doubles ifself at compound interest in 15 years. In how many years will it become eight times of itself?

Sol. $P\left(1+\frac{R}{100}\right)^{15} = 2P$

$$\left(1+\frac{R}{100}\right)^{15} = \frac{2P}{P} = P \qquad \text{............(i)}$$

Let after n years the money will become eight times of itself.

$$P\left(1+\frac{R}{100}\right)^n = 8P$$

Use (i) $\left(1+\frac{R}{100}\right)^n = \left(\left(1+\frac{R}{100}\right)^{15}\right)^3$

$$\left(1+\frac{R}{100}\right)^n = \left(1+\frac{R}{100}\right)^{45}$$

n = 45.

Objective Questions

1. Rs. 800 becomes Rs. 956 in 3 years at a certain rate of simple interest. If the rate of interest is increased by 4%, what amount will Rs. 800 become in 3 years?
   (a) Rs. 1020.80 (b) Rs. 1025
   (c) Rs. 1052 (d) Data inadequate
2. A sum invested at 5% simple interest per annum grows to Rs. 504 in 4 years. The same amount at 10% simple interest per annum in $2\frac{1}{2}$ years will grow to:
   (a) Rs. 420 (b) Rs.450
   (c) Rs. 525 (d) Rs. 550
3. A bank offers 5% compound interest caluclated on half-yearly basis. A customer deposits Rs. 1600 each on 1st January and 1st July of a year. At the end of the year, the amount he would have gained by way of interest is
   (a) Rs. 120 (b) Rs. 121
   (c) Rs. 122 (d) Rs. 123
4. What will be the difference between simple and compound interest @ 10% per annum on a sum of Rs. 1000 after 4 years?
   (a) Rs. 31 (b) Rs. 32.10
   (c) Rs. 40.00 (d) Rs. 64.10
5. The difference between compound interest and simple interest on a sum of 2 years at 10% per annum; when the interest is compounded annually is Rs. 16. If the interest were compounded half-yearly, the difference in two interests would be:
   (a) Rs. 24.81 (b) Rs.26.90
   (c) Rs. 31.61 (d) Rs. 32.40
6. At what rate percent of simple interest will a sum of money double itself in 12 years ?
   (a) $8\frac{1}{4}\%$ (b) $8\frac{1}{3}\%$
   (c) $8\frac{1}{2}\%$ (d) $9\frac{1}{2}\%$

7. Simple interest on a certain amount is $\frac{9}{16}$ of the principal. If the numbers representing the rate of interest in percent and time in years be equal, then time, for which the principal is lent out, is:

(a) $5\frac{1}{2}$ years
(b) $6\frac{1}{2}$ years
(c) $7\frac{1}{2}$ years
(d) 9 years

8. A sum of money placed at compounded interest doubles itself in 5 years. It will amount to eight times itself at the same rate of interest in.

(a) 7 years
(b) 10 years
(c) 15 years
(d) 20 years.

9. An amount of Rs. 12500 was deposited for a period of 3 years at a compound interest rate of 20%. what will be the amount of money, in rupees, at the end of 3 years?

(a) 21600
(b) 22600
(c) 23600
(d) None of these

10. The differece between compound interest and simple interest at the same rate Rs. 5,000 for 2 years is Rs. 72. The rate of interest per annum is:

(a) 6%
(b) 8%
(c) 10%
(d) 12%

11. Amit takes a loan from a bank at 18% CI for 2 years. At the end of the period, he pays back Rs. 6,962. What was the loan amount?

(a) Rs. 4,0000
(b) Rs. 6,000
(c) Rs. 5,000
(d) None of these

12. At a certain rate of compound interest, Rs. 15,320 becomes Rs. 30,640 in 6 years. What is the rate of interest?

(a) 12%
(b) 13%
(c) 14%
(d) 15%

13. The difference in CI and SI for 2 years on a certain sum at 10% rate of interest in Rs. 450. What is the principal?

(a) Rs. 30,000
(b) Rs. 40,000
(c) Rs. 45,000
(d) Rs. 50,000

14. The CI on a certain principal in second year is Rs. 420 and that in third year is Rs. 462. What is the rate of interest?

(a) 8%
(b) 10%
(c) 12%
(d) 15%

15. A sum of money increased by 56.25% in 2 years. When lent at a certain rate of compound interest. Interest being compounded annually if the same sum is lent at simple interest at the same rate of interest, in how many years would it become 11 times itself?

(a) 27
(b) 30
(c) 40
(d) 48

16. The difference between the compound interest and the simple interest on a certain sum at a certain rate of interest for 2 years is Rs. 125 and for 3 years is Rs. 380. Find the sum

(a) Rs. 68825 (b) Rs. 78125
(c) Rs. 81920 (d) Rs. 84585

17. A man invests a certain sum of money at 6% SI and another at 7% SI. His income from interset afer two year was Rs 354. One fourth of the first sum is equal to one fifth of the second sum. Then the total sum invested is

(a) Rs. 1200 (b) Rs. 2400
(c) Rs. 2500 (d) Rs. 2700

18. A sum of Rs. 32000 was deposited in a savings account in two parts. One part was deposited at 8% p.a. and the other part was deposited at 12% p.a. The sum earns an interest of 9% p.a. Find the difference (in Rs.) of the two parts.

(a) 12000 (b) 14000
(c) 16000 (d) 10000

Answers Objective Questions

1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(d)	**5.**	(a)
6.	(b)	**7.**	(d, c)	**8.**	(c)	**9.**	(a)	**10.**	(d)
11.	(c)	**12.**	(a)	**13.**	(dc)	**14.**	(b)	**15.**	(c)
16.	(b)	**17.**	(d)	**18.**	(c)				

Solutions :—

1. S.I. Rs (956 – 800) = Rs. 156.

$$\text{Rate} = \left(\frac{100 \times 156}{800 \times 3}\right)\% = 6\frac{1}{2}\%.$$

$$\text{New rate} = \left(6\frac{1}{2} + 4\right)\% = 10\frac{1}{2}\%.$$

$$\text{New S.I.} = \left(800 \times \frac{21}{2} \times \frac{3}{100}\right) = \text{Rs. } 252$$

$\therefore$ New amount =(800 + 252) = Rs. 1052.

2. Let the sum be Rs. x. Then, S.I. = Rs (504 – x).

$$\therefore \left(\frac{x \times 5 \times 4}{100}\right) = 504 - x$$

$$20x = 50400 - 100x$$
$$120x = 50400$$
$$x = 420$$

Now, P = Rs 420, R = 10%, T = $\frac{5}{2}$ years

$$\text{S.I.} = \left(\frac{420 \times 10}{100} \times \frac{5}{2}\right) = \text{Rs. } 105.$$

$\therefore$ Amount = Rs. (420 + 105) = Rs. 525.

3. Amount

$$= \left[1600 \times \left(1 + \frac{5}{2 \times 100}\right) + 1600 \times 1 + \left(\frac{5}{2 \times 100}\right)\right] = \left[1600 \times \frac{41}{40} \times \frac{41}{40} + 1600 \times \frac{41}{40}\right]$$

$$= \left[1600 \times \frac{41}{40}\left(\frac{41}{40} + 1\right)\right] = \left(\frac{1600 \times 41 \times 81}{40 \times 40}\right) = \text{Rs. } 3321$$

$\therefore$ C.I. = (3321 – 3200) = Rs. 121.

4. $\text{S.I.} = \left(\frac{1000 \times 10 \times 4}{100}\right) = \text{Rs. } 400$

$$\text{C.I.} = \left[1000 \times \left(1 + \frac{10}{100}\right)^4 - 1000\right] = \text{Rs.} 464.10$$

$\therefore$ Difference = (464.10 – 400) = Rs. 64.10.

5. For first year, S.I. = C.I.

Now, Rs. 16 is the S.I. on S.I. for 1 year.

$\therefore$ Rs. 16 is S.I. on $\left(\frac{100}{10} \times 16\right) = \text{Rs. } 160$

So. S.I. on principal for 1 year at 10% is Rs. 160.

$$\therefore \quad \text{Principal} = \left(\frac{100 \times 16}{10 \times 1}\right) = \text{Rs. } 160.$$

Amount for 2 years compounded half yearly

$$= \left[1600 \times \left(1 + \frac{5}{100}\right)^4\right] = \text{Rs.} 1944.81$$

$\therefore$ C.I. Rs. (1944.81 – 1600) = Rs. 344.81

$$S.I. = \left(\frac{1600 \times 10 \times 2}{100}\right) = Rs.\ 320$$

$\therefore$ $(C.I.) - (S.I.) = (344.81 - 320) = Rs.\ 24.81.$

6. Let sum = x.

Then, S.I. = x.

$$\therefore \text{Time} = \left(\frac{100 \times S.I.}{P \times R}\right) = \therefore \quad Time = \left(\frac{100 \times S.I.}{P \times R}\right) = \left(\frac{100 \times x}{x \times 12}\right) \text{years} = 8\frac{1}{3} = 8 \text{ years 4 months}$$

$$\therefore \text{Rate} = \left(\frac{100 \times S.I.}{P \times T}\right) = \left(\frac{100 \times x}{x \times 12}\right) = \frac{100}{12} = \frac{25}{3}\% = 8\frac{1}{3}\%$$

7. $\frac{9}{16}P = \frac{P \times x^2}{100}$

$x = \frac{9 \times 25}{4}$

$x = \frac{3 \times 5}{2} = 7\frac{1}{2}$

8. $P\left(1 + \frac{R}{100}\right)^n = 2P$

$$\left(1 + \frac{R}{100}\right)^5 = 2$$

Let $P\left(1 + \frac{R}{100}\right)^n = 8\ P$

$$\left(1 + \frac{R}{100}\right)^n = 8 = 2^3 = \left\{\left(1 + \frac{R}{100}\right)^5\right\}^3$$

$$\left(1 + \frac{R}{100}\right)^n = \left(1 + \frac{R}{100}\right)^{15}$$

$\therefore$ Required time = 15 years.

9. Let A be the amount at the end of 3 years.

Given : P = Rs. 12500, n (time) = 3 and

R (rate of interest) = 20%

We have, $A = P\left[1 + \frac{R}{100}\right]^n$

$$A = 12500\left[1 + \frac{20}{100}\right]^3 = 12500\left[\frac{6}{5}\right]^3 = 12500 \times \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5} = 21600$$

Hence, amount of money at the end of 3 years is Rs. 21600

10. Let R be the rate of interest per annum.

As we know,

$$CI - SI = \frac{R \times SI}{T \times 100}$$

where T is the time period

Given, $CI - SI = 72$ and $T = 2$

$$\therefore \quad 72 = \frac{R \times S.I}{200}$$

$$R \times S.I. = 14400 \quad \text{.........(i)}$$

Also, we have $S.I. = \frac{5000 \times R \times 2}{100}$

$$S.I. = R \times 100 \quad \text{.........(ii)}$$

From equation (i) and (ii), we have

$$R^2 = 144$$

$$R = 12$$

11. $A = P\left[1 + \frac{r}{100}\right]^n$

$$6962 = P\left[1 + \frac{18}{100}\right]^2$$

$$P = \text{Rs. } 5,000$$

Alternative method

The other 18% CI for 2 year is nothing but two sucessive changes of 18%.

Net Increase

$$18 + 18 + \frac{324}{100} = 39.247\% = 40\%$$

This means the loan amount × 1.4 = 6962

$$\text{Loan amount} = \frac{6962}{1.4} = \text{Rs. } 5,000$$

12. $A = P\left[1 + \frac{r}{100}\right]^n$

or $30640 = 15320\left(1 + \frac{r}{100}\right)^6$

$$\frac{30640}{15320} = \left(1 + \frac{r}{100}\right)^6$$

$$r = 12\% \text{ (approx.)}$$

Alternative method:

We see that the amount is double the principal

∴ By the formula $n = \frac{72}{r}$

We get $r = \frac{72}{6} = 12\%$

13. $P[(1.1)^2 - 1] - \frac{P \times 10 \times 5}{100} = 450$

P = Rs. 45,000

or Rs. 450 is interest

Rs. 450 = 10% of interest on principal

So, interest on principal = Rs. 4,500

∴ Principal = Rs. 45,000

Alternative Method: $\Delta I = R^2 P$

Where ΔI = Difference between the two interests for two years

P = Principal

R = Rate in fractions

∴ $450 = (0.1)^2 \times P$

P = Rs. 45,000

14.

Here 462 – 420 = 42

Rs. 42 is the interest on Rs. 420

i.e. 10% of 420

So, R = 10%

15. Let the rate of interest be R% p.a.

In 2 years, the sum becomes $\left(1+\frac{R}{100}\right)^2$ times.

$\therefore \left(1+\frac{R}{100}\right)^2 = 1.5625$

R = 25

If the sum is lent at simple interest it would become 11 times itself in $\frac{100(11-1)}{25} = 40$ years

16. Let Rs. P be the sum and r% be the rate of interest p.a.

Given, $P\left(\frac{r}{100}\right)^2 = \text{Rs. } 125$ ....(1)

$\left(\frac{r}{100}\right)^2 \left[3+\frac{r}{100}\right] = 380$ ....(2)

Divide equation (2) by (1)

$$3+\frac{r}{100} = \frac{380}{125}$$

$$\frac{r}{100} = \frac{5}{125}$$

$$r = 4$$

From (1), $P = 125\times\left(\frac{100}{4}\right)^2 = 125 \times 625 = 78125$

17. Let man invests Rs. $4x$ at 6% S.I.

Then he will invest Rs. $5x$ at 7% interest.

According to the question,

$$\frac{4x\times 6\times 2}{100}+\frac{5x\times 7\times 2}{100} = 354$$

$$118x = 35{,}400$$

$$\therefore \quad x = \frac{35{,}400}{118} = 300$$

Hence, total sum = $4x + 5x = 9 \times 300$ = Rs 2,700.

18. Let the parts deposited at 8% p.a. and 12% p.a. be Rs. x and Rs. 32000-x respectively.

$$\frac{8}{100}x+\frac{12}{100}(32000-x) = \frac{9}{100}(32000)$$

$x = 24000$ and $32000 - x = 8000$

Difference = Rs 16000

CHAPTER

15

Time Speed and Distance

Introduction

Speed is the rate at which distance is covered and the basic relation between Time, Speed and Distance is

$$\text{Speed} = \frac{\text{Distance covered}}{\text{Time taken}}$$

Thus the unit of speed will be either meter/second or kilometre/hour.

$$1\text{km/hr} = \frac{1\text{ km}}{1\text{ hr}} = \frac{1000\text{ m}}{3600\text{ s}} = \frac{5}{18}\text{m/s} \quad \text{OR} \quad 1\text{m/s} = \frac{18}{5}\text{km/hr}$$

Example 1.

How many minutes does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/hr?

Sol.

$$\text{Aditya's speed} = 20\text{ km/hr} = \left(20 \times \frac{5}{18}\right)\text{m/sec} = \frac{50}{9}\text{ m/sec}$$

$$\therefore \text{Time taken to cover 400 m} = \left(400 \times \frac{9}{50}\right)\text{sec} = 72\text{ sec} = 1\frac{12}{60}\text{min} = 1\frac{1}{5}\text{min}$$

Important

(a) Speed and distance are directly proportional. If speed is doubled, then distance travelled is also doubled in the same time. $(s \propto d)$

(b) Distance and time are directly proportional. If distance to be travelled is tripled, then time taken would also be tripled at the same speed. $(d \propto t)$

(c) Time is inversely related to speed. If the distance remains same, and speed doubles, then time taken to travel that distance becomes $\left(\frac{1}{2}\right)$ times the original time taken at the original speed. Similarly, if time taken to travel the same distance has become five times the original time, then we can conclude that the speed must have become $\left(\frac{1}{5}\right)$ of the original speed. $(s \propto 1/t)$

Example 2.

Walking at $\frac{5}{6}$ of its usual speed, a train is 10 minutes late. Find its usual time to cover the journey.

Sol.

New speed = $\frac{5}{6}$ of the usual speed

$\therefore$ New time taken = $\frac{6}{5}$ of the usual time ($t \propto 1/s$)

As the distance is constant.

So, $\frac{1}{5}$ of the usual time = 10 min $\Rightarrow$ usual time = 50 min.

Example 3.

Rakesh see Roma standing at a distance of 200 m from his position. He increases his speed by 50% and hence takes 20s now to reach her.

(a) If he travels at the original speed, how much time will he take?

(b) What was his original speed (in km/hr)?

(c) What is his new speed?

Sol. (a) Travelling at 150% of his original speed (i.e. $\frac{3}{2}$ times) his original speed, Rakesh should take $\frac{2}{3}$ of the original time, i.e. 20 sec. So his original time is 30 sec.

(b) Original speed = $\frac{\text{Distance}}{\text{Original time}} = \frac{200}{30}$ m/s or $\frac{20}{3} \times \frac{18}{5} = 24$ km/hr.

(c) His new speed is 50% more = 24 × 1.5 = 36 km/hr.

Example 4.

Mr. Rajesh arrives at his office 30 min late everyday. On a particular day, he reduces his speed by 25% and hence arrived 50 min late instead.

(a) Find how much time would he take to travel to his office if he decides to be on time on a particular day?

(b) If he has to arrive on time, by what percentage should he increase his speed?

(Assume on each of these days, he starts from his home at the same time.)

Sol. (a) Speed has become $\frac{3}{4}$ of the speed that he travelled with when he was 30 min late.

So time taken to travel should become $\frac{4}{3}$ of the earlier time or $\frac{1}{3}$ of the earlier time extra. So $\frac{1}{3}$ of the earlier time = (50 – 30) = 20 min.

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This implies when he was 30 min late, he was taking 3 × 20 = 60 min to travel the distance.

So if he decides to come on time, he would take 30 min to travel.

(b) If on time Mr. Rajesh takes $\frac{1}{2}$ hr, whereas if 30 min late, he takes 1 hr. If he has to arrive on time he needs to double his speed or, in other words, he must increase his speed by 100%.

Example 5.

If a person increases his usual speed by 15 km/h, he reaches his destination one hour earlier than his usual time. If the decreases his speed by 10km/h, he will be late by one hour. The distance traveled by him is equal to:

(a) 420 km (b) 360 km (c) 480 km (d) 300 km

Sol. Let the usual speed be v and the normal time taken be t, Distance is constant in each case.

$(v + 15) \times (t - 1) = vt \Rightarrow 15t - v = 15$ .... (1)

$(v - 10) \times (t + 1) = vt \Rightarrow v - 10t = 10$ ....(2)

Solving (1) and (2) we get v = 60 and t = 5. Therefore, distance = 300km.

Example 6.

If a person increases his usual speed by 20%, he reaches his office 15 minutes early. By how many minutes will he be late to his office, if he reduces his usual speed by 20%?

(a) 10 (b) 20 (c) 15.5 (d) 22.5

Sol. If the person increases his speed by $20\%\left(\frac{1}{5}\right)$, his speed becomes $\frac{6}{5}$ times of his original speed and therefore, his time for travel becomes $\frac{5}{6}$ times his original time. Therefore, $\frac{1}{6}$ th time is saved. This is equal to 15min, therefore, original time taken = 15 × 6 = 90 min.

If the person decreases his speed by $20\%\left(\frac{1}{5}\right)$, his speed becomes $\frac{4}{5}$ times his original speed and therefore his time for travel becomes $\frac{5}{4}$ times his original time i.e. $\frac{5}{4}\times 90 = 112.5$ min.

Therefore he will be late by 112.5 – 90 = 22.5 min. Average speed.

$$\textbf{Average Speed} = \frac{\textbf{Total distance travelled}}{\textbf{Total time taken}}$$

Example 7.

An aeroplane flies along the four sides of a square at the speeds of 200, 400, 600 and 800 km/hr. Find the average speed of the plane around the field.

Sol. Let each side of the square be x km and let the average speed of the plane around the field be y km/hr. Then,

$$\frac{x}{200}+\frac{x}{400}+\frac{x}{600}+\frac{x}{800} = \frac{4x}{y}$$

$$\frac{25x}{2400} = \frac{4x}{y}$$

$$y = \left(\frac{2400 \times 4}{25}\right) = 384$$

$\therefore$ Average speed = 384 km/hr.

Note

If you travel equal distance with speeds U and V, then the average speed over the entire journey is $\frac{2UV}{(U+V)}$.

Relative Speed

Relative speed is a phenomena that we observe daily whenever two objects are moving simultaneously.

Suppose you are travelling in a train and there is a second train coming in the opposite direction on a parallel track. It seems the second train is moving much faster than it is actually.

On the other hand if you are travelling on the Mumbai Pune expressway at 100 kmph in your car and there is another car moving besides you, also moving at 100 kmph in the same direction. The cars when seen from the other car seem stationary! All these are examples of relative speed.

If two objects are moving in opposite direction (either towards each other or away from each other), each with a speed of U and V respectively, the relative speed of either of them with respect to other is U + V

If two objects are moving in same direction, each with a speed of U and V respectively, the relative speed of either of them with respect to other is U – V or V – U.

Example 8.

A thief is spotted by a policeman from a distance of 100 metres. When the policeman starts the chase, the thief also starts running. If the speed of the thief be 8 km/hr and that of the policeman 10 km/hr, how far the thief will have run before he is overtaken?

Sol. Relative speed of the policeman = (10 – 8) km/hr = 2 km/hr.

Time taken by policeman to cover 100 m $= \left(\frac{100}{1000} \times \frac{1}{2}\right)$ hr $= \frac{1}{20}$ hr.

In $\frac{1}{20}$ hrs, the thief covers a distance of $\left(8 \times \frac{1}{20}\right)$ km $= \frac{2}{5}$ km = 400m

Example9.

A and B start from X and Y respectively at 9 am and travel towards Y and X respectively at uniform speed. If A reaches Y at 1 pm and B reaches X at 3pm on same day, then at what time do they meet.

Sol. Let the distance between X and Y be d. Thus A takes 4 hours (9 am to 1 pm) and B takes 6 hours (9 am to 3 pm) to travel this distance. Thus their speed is $\frac{d}{4}$ and $\frac{d}{6}$

After how much time will they meet.

And the answer is they would meet after

$$\frac{d}{\frac{d}{4}+\frac{d}{6}} = \frac{6\times4}{6+4} = 2.4$$ hours i.e. at 11.24 am

Note → not 11:40

Problems on Trains

1. Time taken by a train of length L metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover L metres.
2. Time taken by a train of length L metres to pass a stationary object of length B metres is the time taken by the train to cover L + B metres
3. If two trains (or bodies), start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then (A's speed): (B's speed) = $\left(\sqrt{b}:\sqrt{a}\right)$.

Example 10.

A train 100m long is running at the speed of 30 km/hr. Find the time taken by it to pass a man standing near the railway line.

Sol. Speed of the train = $\left(30\times\frac{5}{18}\right)$ m/sec = $\left(\frac{25}{3}\right)$ m/sec.

Distance moved in passing the standing man = 100 m

Required time taken = $\frac{100}{\left(\frac{25}{3}\right)} = \left(100\times\frac{3}{25}\right)$ sec = 12 sec.

Example 11.

A man is standing on a railway bridge which is 180 m long. He finds that a train crosses the bridge in 20 seconds but himself in 8 seconds. Find the length of the train and its speed.

Sol. Let the length of the train be x metres

Then, the train covers x metres in 8 seconds and $(x + 180)$ metres in 20 seconds.

$$\therefore \frac{x}{8} = \frac{x+180}{20} \Leftrightarrow 20x = 8(x+180) \Leftrightarrow x = 120$$

∴ Length of the train = 120 m.

Speed of the train = $\left(\frac{120}{8}\right)$ m/sec = 15 m/sec = $\left(15\times\frac{18}{5}\right)$ kmph = 54 kmph

Example 12.

A man starts from L to M, another from M to L at the same time. After passing each other, they complete their journey in $3\frac{1}{3}$ and $4\frac{4}{5}$ hr respectively. Find the speed of the second man if the speed of the first is 24 km/hr.

Sol. $S_1 : S_2 = \sqrt{\frac{b}{a}}$, where a and b are the time taken by 1st man and 2nd man respectively after meeting each other.

$$\sqrt{\left[\frac{24}{5}\times\frac{3}{10}\right]} = \frac{S_1}{S_2} = \sqrt{\left[\frac{36}{25}\right]} = \frac{6}{5}$$

Thus, 2nd man's speed = $\frac{5}{6}\times 24$ = 20 km/hr.

Example 13.

Two trains of lengths 400 m and 600 m have the speeds of 36 km/hr and 54 km/hr respectively. In what time will the trains be able to completely pass each other, given that the trains are moving in

(a) the same direction with the faster train approaching the slower,

(b) the opposite direction?

Sol. (a) When the trains are moving in the same direction, the faster crosses the slower the tail of the faster just passes the front end of the slower train.

Hence, the distance that the faster train has to cover before crossing the slower train

= Sum of the lengths of the trains

= (400 + 600) m = 1000 m.

Relative speed of the faster train with respect to the slower

= (54 – 36) km/hr = 18 km/hr = 5 m/s.

Therefore, time taken for the faster train to cross the slower = $\frac{1000}{5}$ = 200 s

(b) The distance travelled by the trains together

= Sum of the lengths of the two trains = 400 + 600 = 1000 m.

Relative speed of the trains = (54 + 36) km/hr = 25 m/s.

Therefore, time taken = $\frac{1000}{25}$ = 40 s.

Example 14.

Suppose you were sitting inside this train. The train is moving at a speed of, say, 60 km/hr. Another train on a parallel track is moving at a speed of 40 km/hr in the same direction as yours. There is a guy on the other train. Answer the following based on this data.

(a) What is your speed relative to the guy on the other train?

(b) The guy in the other train moves in the direction opposite to that of the trains themselves at a speed of 10 km/hr. What is the relative speed of the guy with respect to you?

Sol.

(a) Your relative speed is same as the relative speed of your train, i.e. 20 km/hr.

(b) The relative speed of the second person in this case is (60 – 30) = 30 km/hr. In this case, the effective speed of the guy in the other train = (40 – 10) = 30 km/hr, since he is moving in a direction oposite to the train.

Boats and Streams

1. In water, the direction along the stream is called **downstream.** And, the direction against the stream is called **upstream.**
2. If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr, then:

   Speed downstream = (u + v) km/hr

   Speed upstream = (u – v) km/hr.
3. If the speed downstream is a km/hr and the speed upstream is b km/hr, then

   Speed in still water = $\frac{1}{2}(a+b)$ km/hr

   Rate of stream = $\frac{1}{2}(a-b)$ km/hr

Example 15.

A man takes 3 hours 45 minutes to row a boat 15 km downstream of a river and 2 hours 30 minutes to cover a distance of 5 km upstream. Find the speed of the river current in km/hr.

Sol. Rate downstream = $\left(\frac{15}{3\frac{3}{4}}\right)$ km/hr = $\left(15 \times \frac{4}{15}\right)$ km/hr = 4 km/hr.

Rate upstream = $\left(\frac{5}{2\frac{1}{2}}\right)$ km/hr = $\left(5 \times \frac{2}{5}\right)$ km/hr = 2 km/hr.

$\therefore$ Speed of current = $\frac{1}{2}(4-2)$ km/hr = 1 km/hr.

Example 16.

In a stream running at 2 kmph, a motorboat goes 6 km upstream and back again to the starting point in 33 minutes. Find the speed of the motorboat in still water.

Sol. Let the speed of the motorboat in still water be x kmph. Then,

Speed downstream = $(x + 2)$ kmph; Speed upstream = $(x - 2)$ kmph.

$\therefore \frac{6}{x+2} + \frac{6}{x-2} = \frac{33}{60} \Leftrightarrow 11x^2 - 240x - 44 = 0$

$= 11x^2 - 242x + 2x - 44 = 0$

$= (x - 22)(11x + 2) = 0 \Leftrightarrow x = 22.$

Hence, speed of motorboat in still water = 22 kmph.

Example 17.

The speed of a boat in still water is 10m/s and the speed of the stream is 6 m/s. If the boat is moving upstream and again downstream, what is the ratio of the time taken to cover a particular stretch of distance in each direction?

Sol. Since the distances travelled in each direction is the same, and the effective speeds are in the ratio $(10 - 6) : (10 + 6) = 1 : 4$, the time taken to travel upstream and downstream would be in the ratio

$\frac{1}{1} : \frac{1}{4} = 4 : 1$ $(t \propto 1/s)$

Example 18.

A boatman rows for 3 hours downstream and then for 3 hours upstream. In this whole process he covers a total distance of 12 kms. If the speed of stream is 1 kmph, for how much more time will he have to row upstream to reach the starting point?

Sol. Knowing that speed downstream is B + R and speed upstream is B – R, and time spend is 3 hours both way, we have

$3(B + R) + 3(B - R) = 12$

$\Rightarrow 6B = 12 \Rightarrow B = 2$ kmph

Also distance left to be covered to reach starting point is $3(B + R) - 3(B - R) = 6R = 6$ km

Thus time taken to reach the top

Extra time $= \frac{6}{B-R} = \frac{6}{1} = 6$ hours,

Total time to reach the starting point while going upstream = 3 + 6 = 9 hours

Races and Games of Skill

Example 19.

A runs $1\frac{3}{4}$ times as fast as B. If A gives B a start of 84 m, how far must the winning post be so that A and B might reach it at the same time?

Sol. Ratio of the rates of A and B = $\frac{7}{4}:1 = 7:4$

So, in a race of 7 m, A gains 3m over B.

$\therefore$ 3 m are gained by A in a race of 7 m.

$\therefore$ 84 m are gained by A in a race of $\left(\frac{7}{3}\times 84\right)$ m = 196 m.

$\therefore$ Winning post must be 196 m away from the starting point.

Example 20.

A, B and C are three contestants in a km race. If A can give B a start of 40 m and A can give C a start of 64 m, how many metre's start can B give C?

Sol. While A covers 1000 m, B covers (1000 – 40) m = 960 m and

C covers (1000 – 64) m or 936 m.

When B covers 960 m, C covers 936 m.

When B covers 1000 m, C covers = $\left(\frac{936}{960}\times 1000\right)$ m = 975 m

$\therefore$ B can give C a start of (1000 – 975) or 25m.

Circular Motion

Problems in circular motion make use of both relative speed and LCM concepts

Example 22.

A and B, as a warm-up exercise, are jogging on a circular track. B is a better athlete and jogs at 18 km/hr while A jogs at 9 km/hr. The circumference of the track is 500 m (i.e. $\frac{1}{2}$ km). They start from the same point at the same time and in the same direction. When will they be together again for first time?

Sol. Since B is faster than A, he will take a lead and as they keep running, the gap between them will also keep widening. Unlike on a straight track, they would meet again even if B is faster than A.

The number of rounds the faster person makes is always one round more than the slower runner whenever and wherever they meet for the first time.

The same problem could be rephrased as: In what time would B take a lead of 500m over A?

Every second B is taking a lead of $\left[18\times\frac{5}{18}-9\times\frac{5}{18}\right]$ m = 2.5 m over A. Hence, he takes $\frac{500}{2.5}$ = 200 s to take a lead of 500 m over A. Hence, they would meet for the first time after 200 s.

In general, the first meeting if both are moving in the same direction and afterboth have started **simultaneously occurs after t =** $\frac{\textbf{Circumference of the circle}}{\textbf{Relative speed}}$

Or

For every round that A makes, B would have made 2 rounds because the ratio of their speeds is 1 : 2. Hence, when A has made one full round, B would have taken a lead of one round. Therefore, they would meet after $\frac{500}{2.5}$ s $\left[\frac{\text{1 round}}{\text{Sachin's speed}}=\frac{500}{2.5}\right]$

$\left[\text{Here } 9\times\frac{5}{18} \text{ m/s} = 2.5 \text{ m/s is A's speed}\right]$

Example 23.

Suppose, in the earlier problem, when would the two meet for the first time if they are moving in the opposite directions

Sol. If the two are moving in the opposite directions, then relative speed = 2.5 + 5 = 7.5 m/s.

Hence, time for the first meeting = $\frac{\text{Circumference}}{\text{Relative speed}}=\left(\frac{500}{7.5}\right)=\left(\frac{200}{3}\right)$ s

Example 24.

If the speeds of B and A were 8 km/hr and 5 km/hr, then after what time will the two meet for the first time at the starting point if they start simultaneously?

(If you are wondering in which direction... Do not bother. Read on.)

Sol. Let us first calculate the time B and A take to make one full circle.

Time taken by B = $\frac{500}{\left(8\times\frac{5}{18}\right)}$ = 225 s.

Time taken by A = $\frac{500}{\left(5\times\frac{5}{18}\right)}$ = 360 s.

Hence, after every 225 s, B would be at the starting point and after every 360 s, A would be at the starting point. The time, when they will be together again at the starting point simultaneously for the first time, would be the smallest multiple of both 225 and 360, which is the LCM of 225 and 360.

Hence, they would both be together at the starting point for the first time after LCM(225, 360) = 1800 s. Thus, every half an hour, they would meet at the starting point.

From the solution you could realise that it is immaterial whether they move in the same direction or in the opposite.

Let us now discuss all these cases of motion with three people.

Example 25.

C joins B and A, (in Example 1) and all of them run in the same direction from the same point simultaneously. C moves at 3 km/hr, A at 5 km/hr and B at 8 km/hr. When will they all be together again

(a) for the first time.

(b) for the first time at the starting point?

Sol. (a) Break the problem into two separate cases.

In the first case, B moves at a relative speed of (8 – 5) = 3 km/hr with respect to A.

At a relative speed of 3 km/hr, he would meet A after every $\frac{500}{\left(3\times\frac{5}{18}\right)} = 600$ s.

In the second case, B moves at (8 – 3) km/hr = 5 km/hr with respect to C.

At a relative speed of 5 km/hr, he would meet C after every $\frac{500}{\left(5\times\frac{5}{18}\right)} = 360$ s.

Therefore, if all the three have to meet, they would meet after every LCM (600, 360) = 1800 sec = 30 min or $\frac{1}{2}$ hr. Hence, they would all meet for the first time after 30 min

(b) If we need to find the time after which all of them would be at the starting point simultaneously for the first time, we should use the same method as in the case involving two people.

At a speed of 8 km/hr, B takes 225 s to complete one circle.

At a speed of 5 km/hr, A takes 360 s to complete one circle.

At a speed of 3 km/hr, C would take 600 s to complete one circle.

Hence, they would meet for the first time at the starting point after LCM (225, 360, 600) = 1800s.

Example 26.

In a 4000 meter race around a circular stadium having a circumference of 1000 meters, the fastest runner and slowest runner reach the starting point at the end of 5th minute, for the first time after the start of the race. All the runners have the same starting pt. and each runner maintains a uniform speed throughout the race. If the fastest runner runs at twice the speed of slowest runner, what is the time taken by the fastest runner to finish the race?

(a) 20 min (b) 15 min

(c) 10 min (d) 5 min

Sol. As the speed of fastest runner is twice the speed of the slowest runner, the fastest runner travels twice the distance as that traveled by the slowest runner in the same time. Also, to catch the slowest runner again, the fastest runner will have to travel one round whereas the fastest runner is taking two rounds in 5 min, therefore, to take 4 rounds, the fastest runner will take 10 min.

Objective Questions

1. An express train travelled at an average speed of 100 km/hr, stopping for 3 minutes after every 75 km. How long did it take to reach its destination 600 km from the starting point?
   (a) 6 hrs 21 min
   (b) 6 hrs 24 min
   (c) 6 hrs 27 min
   (d) 6 hrs 30 min

2. A farmer travelled a distance of 61 km in 9 hours. He travelled partly on foot @ 4 km/hr and partly on bicycle @ 9km/hr. The distance travelled on foot is:
   (a) 14 km
   (b) 15 km
   (c) 16 km
   (d) 17 km.

3. A person travels from P to Q at a speed of 40 kmph and returns by increasing his speed by 50%. what is his average speed for both the tips?
   (a) 36 kmph
   (b) 45 kmph
   (c) 48 kmph
   (d) 50 kmph

4. A man travels 600 km by train at 80 km/hr, 800 km by ship at 40 km/hr, 500 km by aeroplane at 400 km/hr and 100 km by car at 50 km /hr. What is the average speed for the entire distance?
   (a) 60 km/hr
   (b) $60\frac{5}{123}$ km/hr
   (c) 62 km/hr
   (d) $65\frac{5}{123}$ km/hr

5. A car travels the first one-third of a certain distance with a speed of 10 km/hr, the next one-third distance with a speed of 20 km/hr, and the last one-third distance with a speed of 60 km/hr. The average speed of the car for the whole journey is:
   (a) 18 km/hr
   (b) 24 km/hr
   (c) 30 km/hr
   (d) 36 km/hr

6. A train can travel 50% faster than a car, Both start from point A at the same time and reach point B 75 kms away from A at the same time. On the way, however, the train lost about 12.5 minutes while stopping at the stations. The speed of the car is:
   (a) 100 kmph
   (b) 110 kmph
   (c) 120 kmph
   (d) 130 kmph

7. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km /hr and the time of flight increased by 30 mintues. The duration of the flight is:
   (a) 1 hour
   (b) 2 hours
   (c) 3 hours
   (d) 4 hours

8. A train passes a station platform in 36 seconds and a man standing on the platform in 20 seconds. If the speed of the train is 54 km/hr, what is the length of the platfrom?
   (a) 120 m
   (b) 240 m
   (c) 300 m
   (d) None of these

9. A train 110 meters long is running with a speed of 60 kmph. In what time will it pass a man who is running at 6 kmph in the direction opposite to that in which the train is going?
   (a) 5 sec
   (b) 6 sec
   (c) 7 sec
   (d) 10 sec

10. Two trains of equal length are running on parallel lines in the same direction at 46 km/hr and 36 km/hr. The faster train passes the slower train in 36 seconds. The length of each train is:
(a) 50 m
(b) 72 m
(c) 80 m
(d) 82 m

11. Two trains are runnings in opposite directions with the same speed. If the length of each train is 120 metres and they cross each other in 12 seconds, then the speed of each train (in km/hr) is:
(a) 10
(b) 18
(c) 36
(d) 72

12. Two trains are runnings at 40km/hr and 20 km/hr respectively in the same direction. Fast train completely passes a man sitting in the slower train in 5 seconds. What is the length of the fast train?
(a) 23 m
(b) $23\frac{2}{9}$ m
(c) 27 m
(d) $27\frac{7}{9}$ m

13. Two trains running in opposite directions cross a man standing on the platfrom in 27 seconds and 17 seconds respectively and they cross each other in 23 seconds. The ratio of their speeds is:
(a) 1 : 3
(b) 3 : 2
(c) 3 :4
(d) none of these

14. Two trains, one from Howrah to Patna and the other from Patna to Howrah, start simultaneously After they meet, the trains reach their destinations after 9 hours and 16 hours respectively. the ratio of their speeds is:
(a) 2 : 3
(b) 4 : 3
(c) 6 : 7
(d) 9 : 16

15. A motorboat, whose speed is 15 km/hr in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes. The speed of the stream (in km/hr) is:
(a) 4
(b) 5
(c) 6
(d) 10

16. A boat takes 90 minutes less to travel 36 miles downstream than to travel the same distance upstream. If the speed of the boat in still water is 10 mph, the speed of the stream is:
(a) 2 mph
(b) 2.5 mph
(c) 3 mph
(d) 4 mph.

17. A man walks to his office at a speed of 5 km/hr and reaches office 10 min later than his normal time. The next day, he walks at a speed of 7 km/hr and reaches 2 min earlier than normal time. What is his normal time to reach the office?
(a) 30 min
(b) 40 min
(c) 32 min
(d) 42 min

18. A and B are 200 m apart on a straight road. They start moving towards each other, A at 10 m/min and B at 20 m/min. After A has covered 30 m he takes a left turn, moves 10 m; then takes a right turn, moves 20 m/min; and then turns right to meet the road. When A is back on the road again, how far are A and B?
(a) 0 m
(b) 10 m
(c) 60 m
(d) 20 m

19. A train is travelling at a speed of 96 km/hr. It takes 3s to enter a tunnel and 30 s more to pass through it completely. What is the length of the train?
(a) 60 m
(b) 80 m
(c) 100 m
(d) 120 m

20. Two runners run in the same direction along a circular track 4 km in length. The faster runner overtakes the slower every hour. Find the speed (in kmph) of the slower runner, If the faster runner completes each length of the track 2 minutes sooner than the other.

(a) 10 (b) 15
(c) 20 (d) 25

21. Two guns were fired from the same place at an interval of 12 min; but a person in the train approaching the place hears the second shot 10 min after the first. The speed of the train, if speed of sound is 330 m/s, is

(a) 65 m/s (b) 65 km/hr
(c) 66 m/s (d) none of these

22. A man travels 35 km partly at 4 km/hr and partly at 5 km/hr. If he covers the former distance at 5 km/hr, he could cover 2 km more in the same time, The time taken to cover the whole distance at the original rate is

(a) $4\frac{1}{2}$ (b) 7.5 hr
(c) 7.4 hr (d) 9 hr

23. A train starts from station P at 7 a.m. and moves at 60 km/hr towards Q. At 1 p.m. another train starts from Q towards P at 80 km/hr. When do the two trains meet, if the distance between P and Q is 1,200 km

(a) 6 p.m. (b) 7 p.m.
(c) 8 p.m. (d) none of these

24. A man can row at 5 kmph in still water. If the river is running at 1 kmph, it takes him 75 minutes to row to a place and back. How far is the place?

(a) 2.5 km (b) 3 km
(c) 4 km (d) 5 km

25. A person is standing on a stair case. He walks down 4 setps, up 3 steps, down 6 steps, up 2 steps, up 9 steps, and down 2 steps. Where is he standing now in relation to the step on which he stood when he started?

(a) 2 steps up (b) 1 step up
(c) At the same place (d) 1 step down

26. Prakash and Pramod started running simultaneously from a certain point in the same direction along a circular track. The radius of the track is 7m and the speeds of Prakash and Pramod are 22 m/sec and 11/m/sec respectively. When both met for the N^{th} time, Prakash had covered 484 m more than Pramod. Find N.

(a) 10 (b) 15
(c) 19 (d) 11

27. In a 500 m race. L beats M by 40 seconds and beats N by 125m. If M and N run a 500 m race, M beats N by 40 seconds. Find the time (in seconds) taken by L to run the race.

(a) 200 (b) 240
(c) 160 (d) 120

28. Along a road lie an odd number of stones placed at intervals of 10m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried out the job starting with the stone in the middle, carrying stones in succession, thereby covering a distance of 4.8 km. Then the number of stones is

(a) 35 (b) 15
(c) 29 (d) 31

29. The winning relay team in a high school sports competition clocked 48 minutes for a distance of 13.2 km. Its runners A, B, C and D maintained speeds of 15 kmph, 16 kmph, 17 kmph, and 18 kmph respectively. What is the ratio of the time taken by B to than taken by D?

(a) 5 : 16 (b) 5 : 17

(c) 9 : 8 (d) 8 : 9

30. Two boys A and B start at the same time to ride from Delhi to Meerut, 60 kilometers aways. A travels 4 kilometers an hour slower than B, B reaches Meerut and at once turns back meetings A 12 kilometers from Meerut. The rate of A was :

(a) 4 km/h (b) 8 km/h

(c) 12 km/h (d) 16 km/h

31. two cyclist, k kilometers apart, and starting at the same time, would be together in r hours if they traveled in the same direction, but would pass each other in t hours if they traveled in opposite direction. The ratio of the speed of the faster cyclist to that of the slower is :

(a) $\frac{r+t}{r-t}$ (b) $\frac{r}{r-t}$

(c) $\frac{r+t}{r}$ (d) r/t

32. Hari and Ravi started a race from opposite ends of the pool. After a minute and a half, they passed each other in the center of the pool. If they lost no time in turning and maintained their respective speeds, how many minutes after starting did they pass each other the second time.

(a) 3 (b) $4\frac{1}{2}$

(c) 6 (d) $7\frac{1}{2}$

33. In a ten-kilometers race first beats Second by 2 kilometres and First beats third by 4 kilometers. If the runners maintain constant speeds throughout the race, by how many kilometres does Second beat Third?

(a) $2\frac{1}{4}$ (b) $2\frac{1}{2}$

(c) $2\frac{3}{4}$ (d) 3.

34. A boy is running at a speed of p kmph to cover a distanceof 1 km. But, due the slippery ground, his speed is reduced by q kmph (p>q). If he takes r hour to cover the distance then.

(a) $\frac{1}{r}=\frac{1}{p}+\frac{1}{q}$ (b) $\frac{1}{r}=p-q$

(c) r = p + q (d) r = p − q

35. Two cyclists start from the same place in opposite direcion, One goes towards North at 18 kmph and the other goes towards south at 20 kmph. What time will they take to be 47.5 km apart?

(a) $2\frac{1}{4}$ hrs. (b) $1\frac{1}{4}$ hrs.

(c) 2hrs. 23 min. (d) $23\frac{1}{4}$ hrs.

36. In covering a distance of 30 km Amit takes 2 hours more than Suresh. If Amit doubles his speed, he would take 1 hour less than Suresh. Amit's speed is

(a) 5 km/hour (b) 7.5 km/hour

(c) 6 km/hour (d) 6.25 km/hour

Answers
Objective Questions

1.	(a)	**2.**	(c)	**3.**	(c)	**4.**	(d)	**5.**	(a)
6.	(c)	**7.**	(a)	**8.**	(b)	**9.**	(b)	**10.**	(a)
11.	(c)	**12.**	(a)	**13.**	(b)	**14.**	(b)	**15.**	(b)
16.	(a)	**17.**	(c)	**18.**	(b)	**19.**	(b)	**20.**	(c)
21.	(c)	**22.**	(c)	**23.**	(b)	**24.**	(b)	**25.**	(a)
26.	(d)	**27.**	(b)	**28.**	(b)	**29.**	(c)	**30.**	(b)
31.	(a)	**32.**	(a)	**33.**	(b)	**34.**	(b)	**35.**	(b)
36.	(a)								

Solutions :—

1. Time taken to cover 600 km = $\left(\frac{600}{100}\right)$ = 6 hrs.

Number of stoppages = $\frac{600}{75} - 1 = 7$ Total time of stoppages = (3 × 7) min = 21 min

hence, total time taken = 6 hrs 21 min.

2. Let the distance travelled on foot be x km.

Then, distance travelled on bicycle = (61 – x) km.

So, $\frac{x}{4} + \frac{(61-x)}{9} = 9 \Leftrightarrow 9x + 4(61 - x) = 9 \times 36$

$5x = 80 \Leftrightarrow x = 16$km

3. Speed on return trip = 150% of 40 = 60 kmph.

$\therefore$ Average speed = $\left(\frac{2 \times 40 \times 60}{40 + 60}\right) = \frac{4800}{100}$ = 48 km/hr.

4. Total distance travelled = (600 + 800 + 500 + 100) km = 2000 km.

Total time taken = $\left(\frac{600}{80} + \frac{800}{40} + \frac{500}{400} + \frac{100}{50}\right) = \frac{123}{4}$ hrs.

$\therefore$ Average speed = $\left(2000 \times \frac{4}{123}\right) = \frac{8000}{123} = 65\frac{5}{123}$ km/hr.

5. Let the whole distance travelled be x km and the average speed of the car for the whole journey be y km/hr.

Then, $\frac{(x/3)}{10}+\frac{(x/3)}{20}+\frac{(x/3)}{60}=\frac{x}{y}$

$\frac{x}{30}+\frac{x}{60}+\frac{x}{180}=\frac{x}{y}$

or y = 18 km/hr

6. Let speed of the car be x kmph.

Then, speed of the train = $\frac{150}{100}x=\left(\frac{3}{2}x\right)$ kmph

$\therefore \frac{75}{x}-\frac{75}{\frac{3}{2}x}=\frac{125}{10\times 60} \Leftrightarrow \frac{75}{x}-\frac{50}{x}=\frac{5}{24}$

$x=\left(\frac{25\times 24}{5}\right)$ = 120 kmph

7. Let the duration of the flight be x hours. Then,

$\frac{600}{x}-\frac{600}{x+\frac{1}{2}}=200 \Leftrightarrow \frac{600}{x}-\frac{1200}{2x+1}$

$=200 \Leftrightarrow x(2x+1)=3$

$2x^2+x-3=0$

$(2x+3)(x-1)=0$

$x=1$ hr.[neglecting the – ve value of x]

8. Speed = $\left(54\times\frac{5}{18}\right)$ = 15m/sec.

Length of the train = (15 × 20) m = 300m.

Let the length of the platfrm be x metres.

Then, $\frac{x+300}{36}=15 \Leftrightarrow x+300=540 \Leftrightarrow x=240$ m

9. Speed of train relative to man = (60 + 6)km /hr = 66 km/hr

$=\left(66\times\frac{5}{18}\right)=\left(\frac{55}{3}\right)$ m/sec.

$\therefore$ time taken to pass the man = $\left(110\times\frac{3}{55}\right)$s = 6 sec.

10. Let the length of each train be x meters. then, distance covered = 2x metres.

Releatives speed = (46 – 36) km/hr = $\left(10\times\frac{5}{18}\right)=\left(\frac{25}{9}\right)$ m/sec.

$\therefore \frac{2x}{36} = \frac{25}{9} \Leftrightarrow 2x = 100 \Leftrightarrow x = 50$

11. Let the speed of each train be x m/sec.

Then, relative speed ot the two trains = 2x m/sec.

So, $2x = \frac{(120+120)}{12} \Leftrightarrow 2x = 20 \Leftrightarrow x = 10$

$\therefore$ Speed of each train = 10 m/sec = $\left(10 \times \frac{18}{5}\right) = 36$ km/hr.

12. Relative speed = (40 – 20) km/hr

$= \left(20 \times \frac{5}{18}\right) = \left(\frac{50}{9}\right)$ m/sec.

Length of faster train = $\left(\frac{50}{9} \times 5\right) = \frac{250}{9} = 27\frac{7}{9}$ m

13. Let the speeds of the two trains be x m/sec and y m/sec respectively. Then, length of the first train = 27x metres, and length of the second train = 17y meters.

$\therefore \frac{27x+17y}{x+y} = 23 \Leftrightarrow 27x + 17y = 23x + 23y \Leftrightarrow 4x = 6y \Leftrightarrow \frac{x}{y} = \frac{3}{2}$

14. Let us name the trains as A and B. then,

(A's speed) : (B's speed) = $\sqrt{b} : \sqrt{a} = \sqrt{16} : \sqrt{9} = 4 : 3$

15. Let the speed of the stram be x km/hr. Then,

Speed downstream = (15 + x) km/hr, Speed upstream = (15 – x) km/hr.

$\therefore \frac{30}{(15+x)} + \frac{30}{(15-x)} = 4\frac{1}{2}$

$\frac{900}{225-x^2} = \frac{9}{2}$

$9x^2 = 225 \Leftrightarrow x^2 = 25 \Leftrightarrow x = 5$ km/hr.

16. Let the speed of the stream be x mph. then,

Speed downstream = (10 + x) mph. speed upstream = (10 – x) mph.

$\frac{36}{(10-x)} - \frac{36}{(10+x)} = \frac{90}{60}$

$7x \times 60 = 90(100 - x^2)$

$x^2 + 48x + 100 = 0$

$(x + 50)(x - 5) = 0$

$x = 2$ mph.

17. Let the distance be d. Then $\frac{d}{5} - \frac{d}{7} = \frac{12}{60}$ min

$d\left(\frac{2}{35}\right) = \frac{1}{5}$

$d = \frac{7}{2} = 3\frac{1}{2}$ km

when speed is 5 km/hr = $\frac{7}{2} \times \frac{1}{5} \times 60$ min = 42 min

Normal time = 42 – 10 = 32 min

18. Time taken by A to cover the 30 m on the straight road = $\frac{30}{10}$ = 3 min and Time taken by A to cover the 10 m after the left turn = $\frac{10}{10}$ = 1min and Time taken by A to cover the 20 m parallel to the straight road = $\frac{20}{10}$ = 2 min

Time taken by A to cover the 10 m to meet the straight road again = $\frac{10}{10}$ = 1min

Total time = 3+1+2+1 = 7 min

Distance covered by A on the straight road = 30+20 = 50

Distance covered by B on the straight road = Speed × Distance = 7 × 20 = 140

Thus, distance between them = 200 – 50 – 140 = 10 m

19. The speed of the train = 96 km/hr

$= 96 \times \frac{5}{18} = \frac{80}{3}$ m/s

If L is the length of the train, then we get $\frac{L}{\frac{80}{3}} = 3 \Rightarrow L = 80$

20. Let the speeds of the faster and slower runners be a kmph and b kmph respectively

$\frac{4}{a-b} = 1 \Rightarrow a - b = 4$ and $\frac{4}{a} = \frac{4}{b} - \frac{2}{60}$ or $\frac{4}{b+4} = \frac{4}{b} - \frac{2}{60}$

Substituting the choices in the equation above only b = 20 satisfies it.

21. If the person had not moved towards the source of the gunfire, he would have heard the second shot 12 min after the first shot, Since the person is actually moving towards the source, the shot is now heard after 10 min. It means the sound would have taken 2 min more to reach the initial position of the person; but this distance was travelled by the train in 10 min. It means it is the speed of the train in 10 min, It means speed of the train is $\frac{1}{5}$ of the speed of the sound, i.e., $\frac{330}{5}$ m/s = 66 m/s.

22. When he travels the entire journey at 5 km/hr, total distance travelled = 35 + 2 = 37 km.

$\therefore$ Time of travel = $\frac{37}{5}$ = 7.4 hr., which is same as the time taken at the original rate.

23. From 7 a.m. to 1 p.m., i.e. in 6 hr, the first train travels a distance of 60 × 6= 360 km.

$\therefore$ Distance between the trains at 1 p.m.

= 1200 – 360 = 840 km.

$\therefore$ Time taken to cover this relative distance

$= \frac{840}{140} = 6$hr. i.e. 7 p.m.

24. let the distance of place is x km.

Speed in still water = 5 km/hr.

River speed = 1 km.hr

∴ Speed in direction of flow = 6 km.hr

Speed in reverse direction of flow = 4 km/hr.

Now time taken to reach + Time taken to come back = 75 min.

$\therefore \frac{x}{6}+\frac{x}{4}=\frac{75}{60} \Rightarrow \frac{2x+3x}{12}=\frac{75}{60} \Rightarrow \frac{5x}{12}=\frac{5}{4} \Rightarrow x = 3$ km

25. Take (+) for the person walking down and (–) for walking up therefore, his relative position from the starting point = + 4 –3 + 6 – 2 – 9 + 2= –2.

Hence, the answer is 2 steps above the starting step.

26. Speed of Prakash and Pramod are in the ratio 2 : 1.

∴ They will meet at their starting point after every time Pramod completes the round.

∴ The additional distance travelled by Prakash would be the total distance travelled by Pramod. When both met for the Nthtime, total distance travelled by Pramod

= (N)(2)(22/7)(7)m

(N)(2)(22/7)(7) = 484

N = 11

27. Let the time taken by L to run the race be t seconds. Time taken by M to run the race = (t + 40) seconds. If M and N run a 500 m race, they would take the same time to run it as they do running along with L.

∴ Time taken by N to run the race = (t + 80) seconds

$$\frac{t+80}{t} = \frac{500}{375}$$

$$t = 240$$

28. Totally, a person covers 4.8km.That means he covers 2.4km on one side and 2.4km on other side. So, distances he will be covering will be 20+40+60+.......

∴ 2400 = (n/2)(2 × 20 + (n –1)20) = 10n(n + 1). After solving, we get n=15

∴ Total number of stones=15+15+1=31

29. Since it is a relay race, all the runners ran the same distance. Hence for a same distance, (ratio of times) = 1/(ratio of speeds). Hence ratio of times taken by B and D = 18 : 16 = 9 : 8.

30. Let the speed of B be x kmph and the speed of A be x – 4 kmph.

when they meet each other, the distance covered by A and B is 48 km and 72 km respectively.

Since the time required is constant.

$$t_1 = t_2$$

i.e., $$\frac{D_1}{S_1} = \frac{D_2}{S_2}$$

$$\frac{72}{x} = \frac{48}{x-4}$$

or

$$48x = 72x - 72 \times 4$$
$$24x = 72 \times 4$$

$\Rightarrow$ $x = 12$

i,e. speed of A = 8 kmph

31. Let the speed of faster and slower cyclist be x and y respectively.

By the concept of relative speed

$$\frac{k}{x-y} = r \quad \text{..........(i)}$$

$$\frac{k}{x+y} = t \quad \text{..........(ii)}$$

From (i) and (ii)

$$\frac{x+y}{x-y} = \frac{r}{t}$$
$$rx - yr = xt + yt$$
$$(r-t)x = (r+t)y$$

$$\therefore \quad \frac{x}{y} = \frac{r+t}{r-t}$$

32. In $1\frac{1}{2}$ minute, they passed each other in the center of the pool means they took same time to cover half of the pool. i.e., their speeds are constant.

Thus, they will take the same time ($1\frac{1}{2}$ times) to rach opposite ends and come back.

So time elapsed before 2nd meeting

$$1\frac{1}{2} + 1\frac{1}{2} = 3$$

Total time after starting

$$= 3 + 1\frac{1}{2} = 4\frac{1}{2}$$

33. Let speed of the first be v_1

Then speed of second, $v_2 = \frac{8}{10} = \frac{4}{5}v_1$

and speed of third, $v_3 = \frac{6}{10} = \frac{3}{5}v_1$

$$\therefore \quad \frac{v_2}{v_3} = \frac{4}{3} \Rightarrow v_3 = \frac{3}{4}v_2$$

$\therefore$ When v_2 completes 10 km, v_3 completes $\frac{3}{4} \times 10 = 7\frac{1}{2}$ km

So, the second beat the third by $2\frac{1}{2}$ km

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34. We have, speed = $\frac{\text{Distance}}{\text{times}}$

In the given question, distance = 1 km, speeds are p, q and time = r hr.

∴ According to the question

$$\frac{1}{r} = p - q \qquad (\because p>q)$$

35. Since, two cyclists are moving in opposite direction with speeds 18 kmph and 20 kmph respectively.

∴ Their relatives speed = 18 + 20 = 38 kmph.

$$\text{Now, Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\therefore \quad \text{Required Time} = \frac{47.5}{38} = 1.25 = 1\frac{1}{4} \text{ hrs.}$$

36. Let Suresh takes 't' hours to cover distance 30 km with speed S_s.

Let speed of Amit is S_A, he will take (t + 2) hours to cover the same distance.

$$\therefore \quad S_S = \frac{30}{t} \qquad .....(i)$$

$$S_A = \frac{30}{t+2} \qquad .....(ii)$$

If $S_{A'}$ represents the double speed of Amit then

$$S_A = \frac{30}{t-1} \qquad .....(iii)$$

Now $S_A = 2S_A$

$$\frac{30}{t-1} = \frac{2\times 30}{t+2} \Rightarrow t = 4$$

From (ii) $S_A = \frac{30}{4+2} = \frac{30}{2} = 5$ km/hr

CHAPTER

16

Clocks and Calenders

The face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called min. spaces.

A clock has two hands, the smaller one is called the hour hand or short hand while the larger one is called the min. hand or long hand. In 60 minutes or 1 hour, the minute hand gains 55 minutes on the hour hand.

In every hour, both the hands concide once.

The hands are in the same straight line when they are coincident or opposite to each other.

When the two hands are at right angles, they are 15 minute spaces apart.

When the hands are in opposite directions, they are 30 minute spaces apart.

Angle traced by hour hand in 12 hrs = 360°.

Angle traced by minute hand in 60 min = 360°.

Note

There are two hands in the clock: the hour and the minute hand. Both are moving in the same direction. The hour hand moves $\left(\frac{1}{2}\right)^{\circ}$ *per minute, whereas the minute hand moves 6° per minute. The minute hand is constantly chasing the hour hand. The relative speed of the minute hand with respect to the hour hand is* $\left(5\frac{1}{2}\right)^{\circ}$ *per minute.*

Example 1.

At 3:45, what is the (acute) angle between the hands of a clock?

Sol.

At 3 o'clock, the minute hand of a clock would be 90° behind the hour hand. In 45 min. the minute hand of a clock would move 45° × 6 = 270° forward.

The hour hand would move $\frac{1}{2} \times 45° = \left(22\frac{1}{2}\right)^{\circ}$ forward.

Hence, the angle between the hands would be

$270° - \left(90° + 22\frac{1}{2}\right) = \left(157\frac{1}{2}\right)$

Example 2.

At what time between 2 and 3 o'clock will the hands of a clock be together?

Sol.

At 2 o'clock, the hour hand is at 2 and the minute hand is at 12, i.e., they are 10 min. spaces apart.

To be together, the minute hand must gain 10 minute over the hour hand.

Min hand gains 55 min over hour hand in = 60 min.

Min hand gains 1 min over hour hand in = $\frac{60}{55}$ min

Min hand will gains 10 min over how hand in = $\frac{60}{55} \times 10 = \frac{120}{11} = 10\frac{10}{11}$ min

$\therefore$ The hand will coincide at $10\frac{10}{11}$ min. past 2.

Example 3.

At what time between 4 and 5 o'clock will the hands of a clock be at right angle?

Sol.

At 4 o'clock, the minute hand will be 20 min. spaces behind the hour hand. Now when the two hands are at right angles, they are 15 min. spaces apart.

So, they are at right angles in following two cases.

Case I

When minute hand is 15 min. spaces behind the hour hand:

In this case min. hand will have to gain (20 – 15) = 5 minute sapces.

55 min. spaces are gained by it in 60 min.

5 min. spaces will be gained by it in $\frac{60}{55} \times 5 = 5\frac{5}{11}$ min

$\therefore$ They are at right angles at $5\frac{5}{11}$ min. past 4.

Case II

When the minute hand is 15 min. spaces ahead of the hour hand:

To be in this position, the minute hand will have to gain (20 + 15) = 35 minute sapces. 35 min. Spaces are gained in $\frac{60}{55} \times 35 = 38\frac{2}{11}$ min.

$\therefore$ They are at right angles at $38\frac{2}{11}$ min. past 4.

Example 4.

A watch which gains uniformly, is 5 min. slow at 7 o'clock in the morning on Monday and it is 5 min. 48 sec. fast at 7 p.m. on following Monday. When was it correct?

Sol.

Time from 7 a.m. on Monday to 7 P.M. on following Monday = 7 days 12 hours = 180 hours

∴ The watch gains $\left(5+5\frac{4}{5}\right)$ min or $\frac{54}{5}$ min. in 180 hrs.

Now $\frac{54}{5}$ min. are gained in 180 hrs.

∴ 5 min. are gained in $\left(180\times\frac{5}{54}\times5\right)$ hrs.= 83 hrs 20 min. = 3 days 11 hrs. 20 min

∴ Watch is correct 3 days 11 hrs 20 min. after 7 a.m. of Monday.

∴ It will be correct at 20 min. post 6 p.m. on Thursday.

Example 5.

Two clocks show the same time at 5 p.m. the first clock loses 10 min every 2 hr and the second gains 10 min every hour. When will they both show the same time again?

Sol.

If they had not lost or gained any time, they would both show the same time always. But in this case, the first clock would be behind the second clock by 15 min at the end of 1 hr. (Since the first clock loses 5 min and the second gains 10 min in 1 hr). They would both show the same time again if they are separated by 12 hr = 12 × 60 min.

Number of hours the first clock takes to be behind the second by 12hr = $\frac{(12\times60)}{15}$ = 48 hr. so

∴ they would show the same time again after exactly 2 days.

Example 6.

A clock is set right at 9 a.m. The clock gains 10 minutes in 24 hours. What will be the true time when the clock indicates 2 p.m. on the following day?

Sol.

Time from 9 a.m. on a day to 2 p.m. on the following day = 29 hours.

24 hours 10 min. of this clock = 24 hours of the correct clock.

$\frac{145}{6}$ hrs.of this clock = 24 hrs of the correct clock

29 hrs of this clock = $\left(24\times\frac{6}{145}\times29\right)$ hrs. of the correct clock

= 28 hrs 48 min. of correct clock

∴ The correct time is 28 hrs 48 min. of correct clock i.e., 12.48 p.m. on the following day.

CALENDERS

In this topic we are supposed to find the day of the week on a given date.

For this, we use the concept of odd days.

I. Odd Days: In a given period, the number of days more than the complete weeks are called odd days.

II. Leap Year:

(i) Every year divisible by 4 is a leap year, if it is not a century.

(ii) Every 4th century is a leap year and no other century is a leap year.

Note

A leap year has 366 days.

Examples

(i) Each of the years 1948, 2004, 1676, etc. is a leap year.

(ii) Each of the century years 400, 800, 1200, 1600, 2000 etc. us a leap year.

(iii) None of the years 2001, 2002, 2003, 2005, 1800, 2100 is a leap year.

III. Ordary Year: The year which is not a leap year is called an ordinary year. An ordinary year has 365 days.

IV. Counting of Odd Days:

(i) 1 ordinary year = 365 days = (52 weeks = 1day).

$\therefore$ 1 ordinary year has 1 odd day.

(ii) 1 leap year = 366 days = (52 weeks + 2 days).

$\therefore$ 1 leap year has 2 odd days.

(iii) 100 year = 76 ordinary years + 24 leap years

= $(76 \times 1 + 24 \times 2)$ odd days = 124

= (17 weeks + 5 days) = 5 odd days.

$\therefore$ Number of odd days in 100 years = 5.

Number of odd days in 200 years = (5×2) = 3odd days.

Number of odd days in 300 years = (5×3) = 1 odd day.

Number of odd days in 400 years = $(5 \times 4 + 1)$ = 0 odd day.

Similarly, each one of 800 years, 1200 years, 1600 years, 2000 years etc. has 0 odd day.

Remember

Day of the Week Related to Odd Days:

No. of odd days	0	1	2	3	4	5	6
Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat

Example 7.

What was the day of the week on 16th July, 1776?

Sol.

16th July, 1776 = (1775 years + Period from 1.1.1776 to 16.7.1776)

Counting of odd days:

Number of odd days in 1600 years = 0

Number of odd days in 100 years = 5

75 years = 18 leap years + 57 ordinary years

= $(18 \times 2 + 57 \times 1)$ odd days = 93 days

= (13 weeks + 2days) = 2 odd days.

∴ 1775 years have = (0 + 5 + 2)odd days = 7 odd days = 0 odd day.

Jan	Feb	Mar	Apr	May	Jun	Jul	
(31 +	29 +	31 +	30 +	31 +	30 +	16)=	198 days

198 days = (28 weeks + 2 days) = 2 odd days.

∴ Total number of odd years = (0 + 2) = 2

Hence, the required day is Tuesday.

Example 8.

Jan 5, 1991was a Saturday. What day of the week was one March 3, 1992?

Sol.

Total no. of days between Jan 5, 1991 and March 3, 1992 = 360 days in 1991 + (31 + 29 + 3) days in 1992 = 3 odd days.

Therefore, March 3, 1992 is three days beyond Saturday, i.e., Tuesday.

Example 9.

Monday falls on 4th April, 1988. What was the day on 3rd Now 1987?

Sol.

No. of days between 3rd Nov. 1987 and 4th April 1988 = 27(Nov) + 31(Dec) + 31(Jan) + 29(Feb) + 31(Mar) + 4(Apr)= 153 days = 21 weeks + 6 days = 6 odd days.

Then, 3rd Nov 1987 was 7 – 6 = 1 day beyond the day on 4th April, 1988. So, the day was Tuesday.

Example 10.

Today is 21st August. The day of the week is Monday. This is a leap year. What will be the day of the week on this day after 3 years?

Sol. Since this is a leap year, non of the next 3 years is a leap year. So, the day of the week will be 3 days beyond Monday, i.e., it will be Thursday.

(As each normal year has 1 odd day so 3 normal years will have 3 odd days)

Example 11.

If was Thursday on 2nd Jan 1993. What day of the week will be on 15th March 1993. What day of the week will be on 15th March 1993?

Sol. Total no. of days = 29(Jan) + 28(Feb) + 15(Mar)

= 72 days = 10 weeks + 2days = 2 odd days.

Thus, the given date will fall on two days beyond Thursday, i.e., Saturday.

Objective Questions

1. An accurate clock shows 8 o'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon?
   (a) 144° (b) 150°
   (c) 168° (d) 180°
2. At what time between 7 and 8 o'clock will the hands of a clock be in the same straight line but, not together?
   (a) 5 min. past 7 (b) $5\frac{2}{11}$ min. past 7
   (c) $5\frac{3}{11}$ min. past 7 (d) $5\frac{5}{11}$ min. past7
3. A watch which gains uniformly is 2 minutes slow at noon on Monday and is 4 min. 48 sec fast at 2 p.m. on the following Monday. When was it correct?
   (a) 2 p.m. on Tuesday (b) 2p.m. on Wednesday
   (c) 3p.m. on thursday (d) 1p.m. on friday.
4. It takes the pendulum of a clock 7 seconds to strike 4 o'clock. How much time will it take to strike 11 o'clock?
   (a) 18 seconds (b) 20 seconds
   (c) 19.25 seconds (d) 23.33 seconds
5. The times between 7 and 8 o'clock, correct to the nearest minute, when the hands of a clock will form an angle of 84 degrees are:
   (a) 7 : 23 and 7 : 53 (b) 7 : 20 and 7 : 50
   (c) 7 : 22 and 7 : 53 (d) 7 : 23 and 7 : 52
6. A man on his way to dinner shortly after 6 : 00 p.m. observes that hands of his watch form an angle of 110°. Returning before 7 : 00 p.m. He notices that again the hands of his watch form an angle of 110°. The number of minutes that he has been away is :
   (a) $36\frac{2}{3}$ (b) 40
   (c) 42 (d) 42.4

Answers (Objective Questions)

1.	(d)	2.	(d)	3.	(b)	4.	(d)	5.	(a)
6.	(b)								

1. Angle traced by the hour hand in 6 hours $= \left(\frac{360}{12} \times 6\right)^{\circ} = 180^{\circ}$

2. Sol. When the hands of the clock are in the same straight line but not together, they are 30 minute spaces apart.

At 7 o'clock, they are 25 min. spaces a part.

$\therefore$ Minute hand will have to gain only 5 min. spaces.

55 min. spaces are gained in $\left(\frac{60}{55} \times 5\right)$ min $= 5\frac{5}{11}$ min

$\therefore$ Required time $5\frac{5}{11}$ min. past 7.

3. Time from 12 p.m. on Monday to 2 p.m. on the following Monday = 7 days 2 hours = 170 hours.

$\therefore$ The watch gains

$\left(2 + 4\frac{4}{5}\right)$ min $= \frac{34}{5}$ min in 170 hrs.

Now, $\frac{34}{5}$ min. are gained in 170 hrs.

$\therefore$ 2 min are gained in $\left(170 \times \frac{5}{34} \times 2\right)$ hrs. = 50 hrs.

$\therefore$ Watch is correct 2 days 2 hrs. after 12 p.m. on Monday i.e., it will be correct at 2 p.m. on Wednesday.

4. Students please note that the more relevant thing in this case is not the number of strikes but the number of time intervals. In other words, if a clock has to strike 4, there are 3 time intervals between the 4 strikes (this is so taken because the first strike happens at the zeroth second). So in 7 seconds the pendulum elapses 3 time intervals. To strike 11, there has to be 10 time intervals, which will take (10 × 7)/3 = 23.33 seconds.

5. Angle between the hands of a clock

$$\theta = 6\left(5H - \frac{11}{12}M\right) \quad \text{or}$$

$$\theta = 6\left(\frac{11}{12}M - 5H\right)$$

Where $\theta = 84$ and M = 7

$$84 = 6\left(5\times7 - \frac{11}{12}M\right) \quad \text{or}$$

$$84 = 6\left(\frac{11}{12}M - 5\times7\right)$$

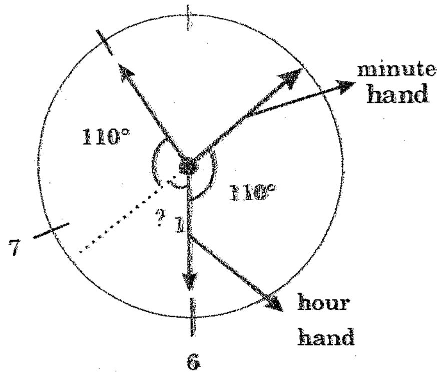
$$14 = 35 - \frac{11}{12}M \text{ or } 14 - \frac{11}{12}M - 35$$

$$\frac{11}{12}M = 12 \text{ or } \frac{11}{12}M = 49$$

Thus M = 22.9 ≈ 23 or M = 53.45 ≈ 53.

6. Let he was away for n minutes and for n minutes, the watch makes an angle θ_1. Let the complete angle θ_2 be made in between 6.00 to 7.00.

Thus, $\theta_1 = \frac{n}{2}$ and $\theta_2 = 6n$,



Thus, we have

$$\theta_2 = 110° + 110° + \theta_1$$

$$\theta_2 = 220° + \theta_1$$

$$\theta_2 - \theta_1 = 220°$$

$$6n - \frac{n}{2} = 220°$$

$$11n = 440°$$

$$n = 40.$$

Thus, he was away for 40 minutes.

CHAPTER

17

Data Interpretation

Orientation

By data we generlly mean quantities, figures and statics relating to any event.

Generally the massive data collected is in diserete (scattered) form. For any useful interpretation of data, it needs to be arranged (organised). Data in an organised form is generally referred to as information (quantitative). The primary objective of data organisation should be that the requfred/desired information can be drawn with minimum effort and in the least possible time.

Why do Entrance Examinations test the DI skills?

These tests check a student's ability to interpret data from graphs, tables charts etc., calculate fast and comparehend relevant information. As professionals of tomorrow, you will constantly come across data arranged in different tables, pie-graphs, line graphs etc. Hence it is imperative that you must be skilled in DI.

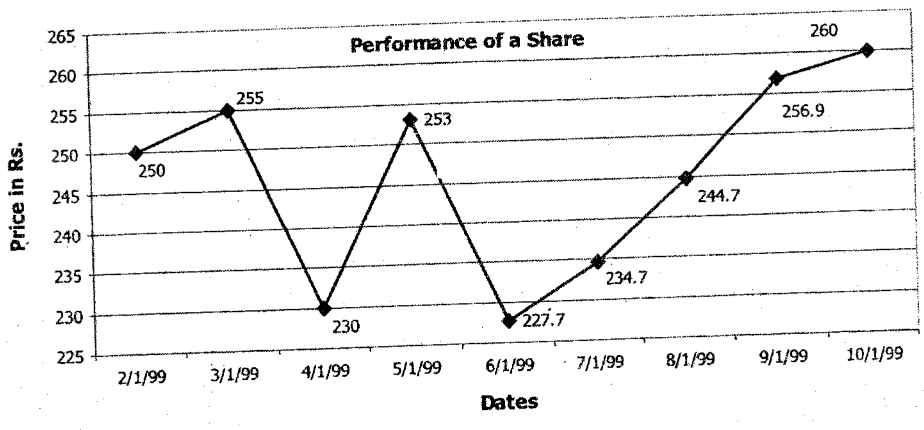
Why is a graphical representation required?

Let's take an example to answer the above question.

e.g. A detailed report of the performance of a share in a national daily.

Info: The share price of a company on January 2, 1999 was Rs. 250. On the next day, it rose by Rs. 5, followed by a fall of Rs. 25 the next day. The 5th of January saw a rise of 10%. Increase of Rs. 7 was seen in the share price on 7th January, followed by another increase of Rs. 10 the next day. A cut in import duties of the raw material of the company saw a further increase of 5% the next day. The share finally acquired a value of Rs. 260 on 10th january, 1999.

now, representing the same data through a Cartesian graph, we see that



Hope you do get an idea of how comparehensive a data becomes when represented in a graphical manner.

Are there some important formulae used often?

Yes! knowledge of some formulae always helps most of the questions in DI. The same are mantioned below.

1. Percentages

- Percentage Change = $\dfrac{\text{New value} - \text{Old Value}}{\text{Old Value}} \times 100$
- Percentage Increase = $\dfrac{\text{Increase}}{\text{Old Value}} \times 100$
- Percentage Decrease = $\dfrac{\text{Decease}}{\text{Old Value}} \times 100$
- Percentage Increase over a given period = $\dfrac{\text{Final Value} - \text{Intial Value}}{\text{Intial Value}} \times 100$
- Average Percentage Increase over a given period

$$= \frac{\text{Final Value} - \text{Intial value}}{\text{Initial Value}} \times \frac{100}{\text{Number of Periods}}$$

These are very useful in claculations. Direct calculations like -- if the share price changes from 250 to 275, then the percentage chage is 10% -- can be easily made with the help of these formulae. In fact, with some practice, you can internalise these formulae and perform a lot lf calculations mentally.

2. Averages

Average is the sum of quantities divided by the number of quantities.

- Average = $\dfrac{\Sigma X_n}{n}$. Here n is the number of quantities and X is the value of the different quantities.

e.g., the sum of temperatures from Monday to Saturday was 165°C. The average temperature for Monday to Saturady would be $\dfrac{165}{6} = 27.5^\circ C$

Another variation to Average is Weighted Average. In this, we take the quantities and their respective weights.

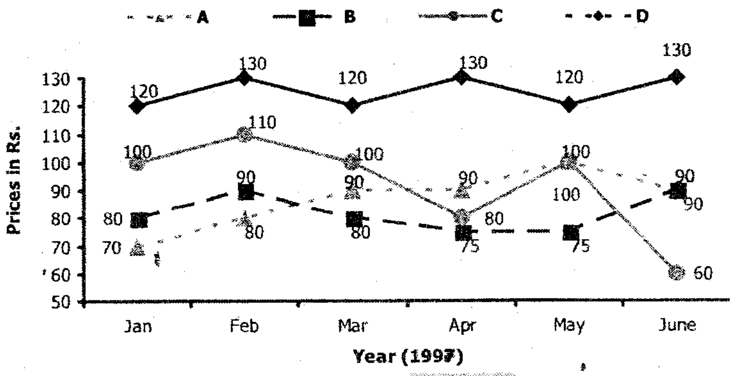
- Weighted Average = $\dfrac{\sum_{i=1}^{n} W_i X_i}{\sum_{i=1}^{n} W_i}$. Here W is the respective weight and X is the quantity.

e.g., If the temperatures from Monday to Saturday are 25°, 28°, 24°, 29°, 30° and 30° respectively and the weightage given to Monday is 2 and that given to Tuesday is 3 and all the others are given weightage 1, then the weighted average will be

$$= \frac{2 \times 25 + 28 \times 3 + 25 + 29 + 30}{2+3+1+1+1+1} = \frac{247}{9} = 27.44^\circ$$

Objective Questions — Part — I

Directions : *The following line graph shows the prices for different shares for different months in 1997. Refer to the graph to answer the questions that follow.*



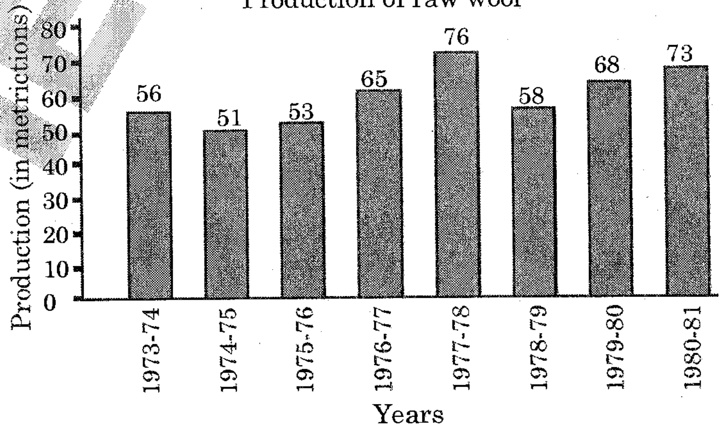
1. For which share is the avearge price for the six given months less than its price in January 1997?
   (a) A (b) B (c) C (d) D
2. The percentage increase in the price from January to February is least in the case of
   (a) A (b) B (c) D (d) All equal
3. If Stability Index for the given half year is defined as

$$\frac{\text{Price in June - pirce in January}}{\text{Price in January}} \times 100,$$ then stability Index is greatest for

   (a) A (b) B (c) C (d) D
4. A man buys 100 shares of A in January and sells them in February. with this money, he is able to buy X shares of C in June. The value of X is
   (a) 133 (b) 234 (c) 315 (d) 128
5. The average price of A over the six months
   (a) is greater than that of B (b) is less that of B
   (c) is equal to that of B (d) None of the abvoe

Directions : *Study the following bar graph and answer the questions that follow.*

Production of raw wool



6. The maximum absolute rise in the production of raw wool in the year
(a) 1977-78 (b) 1976-77 (c) 1980-81 (d) 1979-80

7. What was the percentage increase or decrease in the total production during the last four years compared to that during the first four years?
(a) 26.8% (b) 19.8% (c) 22.2% (d) 25%

8. The average production during the eight years was
(a) 67.2 MT (b) 50.0 MT (c) 62.5 MT (d) 52.8 MT

9. The maximum percentage decrease in production over the previous year was in the year
(a) 1978-79 (b) 1973-74 (c) 1977-78 (d) 1974-75

10. The production of raw wool in 1974-75 was ...... of the production in 1979-80.
(a) 1/3 (b) 1/2 (c) 2/3 (d) 3/4

Directions : *The table shows the production characteristics of four machines used by xyz ltd. Please complete the entires in the last two columns and answer the questions that follow*

Note : Total production = Accepted + Rejected

Machine Number	Production Accepted (units)	Percentage of Production Rejected	Total production (Accepted+Rejected)	Production Rejected (Units)
1	400	20%		
2	385	30%		
3	438	27%		
4	420	40%		

11. The total production (in units) of machine 3 is
(a) 500 (b) 550 (c) 600 (d) 700

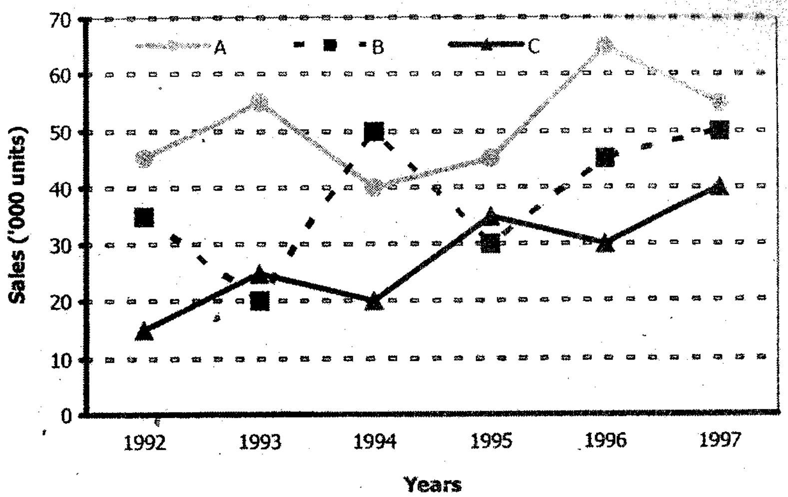
12. The rejected production (in units) of machine 4 is
(a) 700 (b) 280 (c) 300 (d) 180

13. The difference (in units) between accepted production and total production of machine 2 is
(a) 330 (b) 130 (c) 165 (d) 300

14. The total production of all four machines is
(a) 2600 (b) 2750 (c) 2350 (d) 2470

15. The total rejected production, considering all four machines, is
(a) 520 (b) 707 (c) 680 (d) 570

Directions : *The following graph shows the number of unit sold of three brands of TV (A,B and C) from **1992** to **1997**. Refer to the graph to answer the questions that follow.*



16. What is the ratio of the total sale of TVs in the year 1995 and 1997?

(a) 11 : 19 (b) 22 : 29 (c) 11 : 29 (d) 22 : 11

17. Three-fourths of the sale of brand C in 1996 in percent of two-third of the sale of brand B in the same year is

(a) 50% (b) 75% (c) 150% (d) 125%

18. What is the difference (in '000 units) between the average sale of brand A and Brand B over the years 1992 to 1997?

(a) 10.75 (b) 11.25 (c) 12.5 (d) 13.75

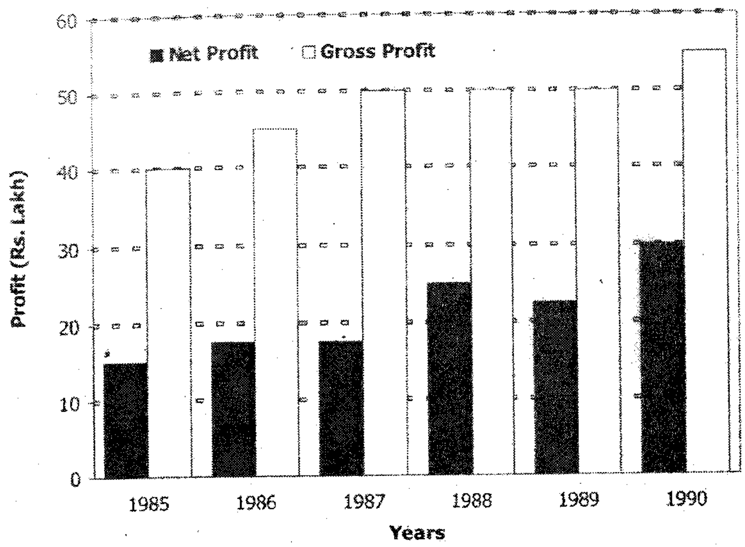
19. The sale of brand A in 1997 is by what percent more than the sale of brand C in 1993?

(a) 80% (b) 100% (c) 120% (d) 125%

20. In which of the following pair of year was the total sale of TVs the same?

(a) 1992, 1996 (b) 1993, 1996 (c) 1993, 1994 (d) 1994, 1995

Directions : *Study the graph carefully and answer the questions given below it.*



21. In which year was the ratio of gross profit (GP) to net profit (NP) equal to 2?
(a) 1986 (b) 1987 (c) 1988 (d)1989

22. In which year was the ratio NP to GP highest?
(a) 1987 (b) 1988 (c) 1989 (d)1990

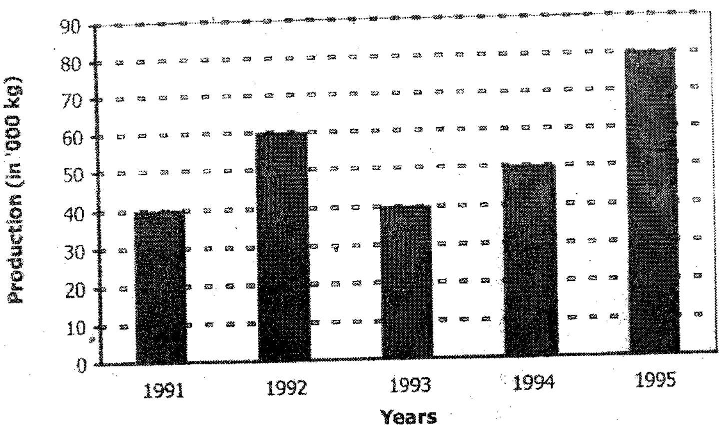
23. In which year did GP register maximum growth over the previous year?
(a) 1986 (b) 1987 (c) 1988 (d)1989

24. For how many years was there a paired rise in GP and NP?
(a) 0 (b) 1 (c) 2 (d) 3

25. The ratio of the largest difference between the GP and NP for a particular year to the smallest difference between them for a paticular year is
(a) > 2 (b) < 1 (c) > 1.5 but < 2 (d) > 1 but < 1.5

Directions : *Refer to the following bar graph and answer the questions that follow.*

PRODUCTION OF A FIRM FROM 1991 TO 1995



26. The average annual percentage increase in production in 1995 over 1991 is
(a) 25% (b) 30% (c) 50% (d) 40%

27. For how many of the given years is production greater than the average production for the given period?
(a) 2 (b) 3 (c) 4 (d) 1

28. If the cost of production is Rs. 10/kg in 1991 and increases @ 10% p.a., then what is the total cost of production in 1993? (in Rs. lakh)
(a) 4.84 (b) 5.63 (c) 3.68 (d) 4.12

29. If all that is produced is sold, what is the total value of sales (in Rs. lakh) in 1992 if the sale value per kg in 1992 is 150% of production cost per kg in 1992?
(a) 9 (b) 11 (c) 13 (d) 9.9

30. The showing maximum percentage change in production over any preceding year is
(a) 1992 (b) 1993 (c) 1994 (d) 1995

Directions : *A professor prepares a chart of his students on the basis of their performance and gender. The data is kept on a computer disk, but unfortunately some of it is lost because of a virus. Only the following could be recovered*

	Performance			Total
	Average	Good	Excellent	
Male			10	
Female				32
Total		30		

Panic buttons ware pressed but to no available. An expert committee was formed, which decided thdt the following facts were self-evident.

1. *Half the students ware either excellent or good.*
2. *40% of the students were females.*
3. *One-third of the male students were average.*

31. How many students are both female and excellent?

(a) 0 (b) 8 (c) 16 (d) 32

32. What proportion of good students are male?

(a) 0 (b)0.73 (c) 0.4 (d) 1.0

33. What proportion of female students are good?

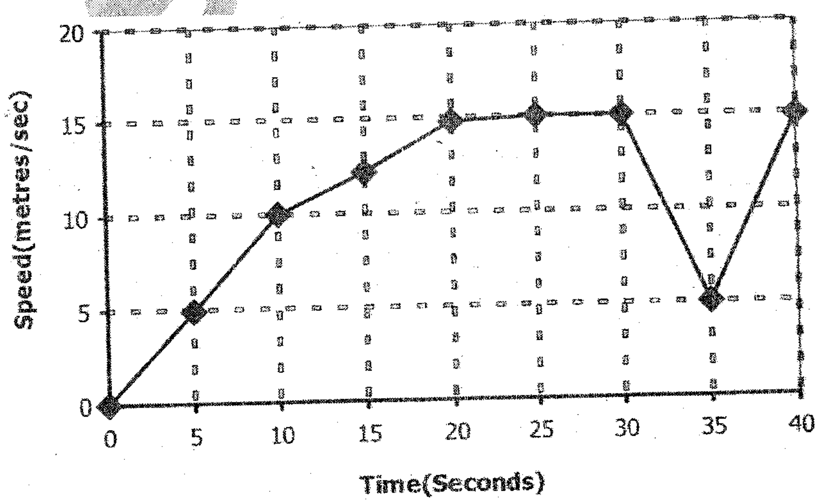
(a) 0 (b) 0.25 (c) 0.5 (d) 1.0

34. How many students are both male and good?

(a) 10 (b) 16 (c) 22 (d) 48

Objective Questions Part – II

Directions : *The following line graph shows the speed and time for a journey. Refer to the graph to answer the questions that follow.*



35. The average speed for entire journey is

(a) 12.325 m/sec (b) 15.625 m/sec. (c) 10.625 m/sec (d) 8.435 m/sec.

36. The total distance travelled at the end of 25 seconds is

(a) 250m (b) 175m (c) 290 m (d) 380m

37. The average speed from 0.20 seconds is

(a) 6 m/sec (b) 8.75 m/sec (c) 9.5 m/sec (d) 9 m/sec

38. Acceleration is zero in the time interval (sec.)

(a) 0-10 (b) 10-20 (c) 20-30 (d) 30-35

39. The average speed from 0:30 second is

(a) 13 m/sec (b) 10.0 m/sec (b) 12.25 m/sec (d) None of these

40. What is the total distance travelled at the end of 15 seconds?

(a) 110 m (b) 112.50m (c) 106.25 m (d) None of these

Directions : *The following table shows the data for few companies for the year 1994. Refer to the table to answer the questions that follow.*

Recruitment	Company							
	A		B		C		Total	
	E	T	E	T	E	T	E	T
Manager		4			5	5		
Employee	20		10	6				15
Total			25		5	10	60	30

41. How many Managers were recruted?

(a) 10 (b) 20 (c) 30 (d) 35

42. How many Employees were terminated in Type A companies?

(a) 2 (b) 3 (c) 4 (d) 5

43. What was the overall increase in recruitment (New employment – Terminations) in Type A companies?

(a) 30 (b) 19 (c) 20 (d) 22

44. If the total number of Managers at the beginning of 1994 ware 65, then what was the percentage increase in it during 1994?

(a) 21% (b) 23% (c) 6% (d) 8%

45. In which of the following categories did the maximum Net Recruitment take place?

(Net Recruitment = Employment - Termination)

(a) A, Employee (b) B, Manager (c) A, Manager (d) C, Employee

Objective Questions Part – III

Directions : *The following tables shows the educational background of 600 couples. Refer to the table to answer the questions that below.*

Husband \ Wife	Post Graduate	Graduate	10-12 class	Less than 10th class	Total
Post Graduate	30	65	35	20	150
Graduate	35	85	50	30	200
10-12 class	40	17	38	45	140
Less than 10th class	07	15	38	50	110
Total	112	182	161	145	600

46. For how many couples, the educational levels of both the spouses are equal?

(a) 195 (b) 203 (c) 200 (d) 207

47. The number of husbands who had lower qualifications than their wives is ..... % of the couples surveyed.

(a) 33.4 (b) 32.1 (c) 28.2 (d) 25.3

48. The average number of husbands in any category whose wives are post graduates is

(a) 27 (b) 30 (c) 28 (d) 31

49. Among the husbands with educational background of 10 – 12 class, in which category is the number of wives less than the average number of husbands in the given category?

(a) Post graduate (b) Graduate (c) 10-12 class (d) Less than 10th class

50. The number of husbands in a particular category which has more than the average number of husbands in any category, bears a ratio of .... to the total number of husbands.

(a) 1 : 3 (b) 1 : 4 (c) 1 : 5 (d) 2 : 5

Directions : *A market survey was conducted to assess the readership pattern of a sample population. The results are tabulated below. No person reads anything else except as given (entries in the table denote the number of readers). Refer to the table to answer the questions that follow.*

Age Group	Reader's Digest	India Today	Illustrated Weekly	Sportstar	None	Total
10-20	8	13	7	19	5	52
20-35	11	25	8	18	8	70
35-50	15	12	13	12	11	63
> 50	21	9	19	21	7	77
Total	55	59	47	70	31	262

51. What percent of the sample population reads either Reader's Digest or India Today?
(a) 43.5% (b) 62.5% (c) 53% (d) 34.5%

52. In the age group < 35 years, what percent reads Sportstar or nothing at all?
(a) 14% (b) 29% (c) 41% (d) 53%

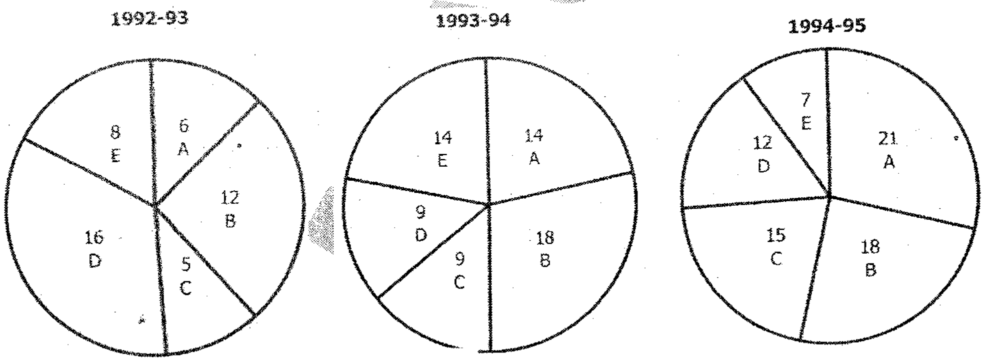
53. If 20% of all readers above 35 subscribe to magazines how many subscribers would there be in the sample?
(a) 32 (b) 33 (c) 35 (d) 24

54. What percent of readers in the age group 20-50 do not read the Illustrated Weekly?
(a) 81% (b) 16% (c) 84% (d) 70%

55. If the costs of Reader's digest, Illustrated Weekly, and Sportstar are Rs. 16, 20, and 28 respectively per month, what percent of sales does India Today command? (Assue the cost of India Today to be Rs. 30 per month and that everybody buys a copy)
(a) 13.6% (b) 31.9% (c) 69.6% (d) 22%

56. The number of people reading India Today expressed as a percent of Sportstar readers works out to
(a) 69% (b0 55% (c) 84% (d) 46%.

Directions : *The following pie charts show the number of cotton bales (in lakhs) pruduced by five states A, B, C, D and E during 3 years. Study the following pie charts carefully and answer the questions given below:*



57. The production of state D in 1993-94 was how many times its production in 1994-95?
(a) 1.33 (b) 0.75 (c) 0.56 (d) 1.77

58. In which of the states there is a steady increase in the production of cotton during the given period?
(a) A and B (b) A and C (c) B only (d) D and E

59. If each bale of cotton weights 100kg, how many tons of cotton was produced by stae E during the given period?
(a) 2900 (b) 290000 (c) 29000 (d) 2900000

60. How many states showing below average production i.e. (average of 1992-93, 1993-94, 1994-95) in 1992-93 showed above average production in 1993-94?
(a) 4 (b) 5 (c) 3 (d) 1

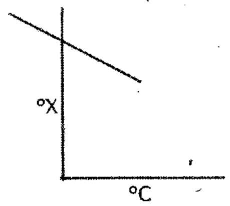
61. Which of the following statements is false?
(a) States A and E showed the same production in 1993-94.
(b) There was no improvment in the production of cotton in the state B during 1993-94, and 1994-95.

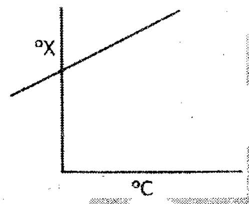
(c) State A had produced maximum cotton during the given period.

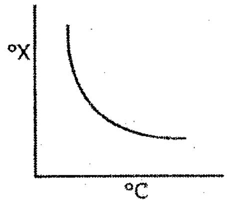
(d) Production of states C and D together was equal to that of state B during 1993-94.

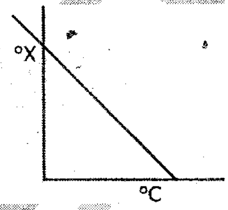
Directions: *For the following questions, four options are given. Choose the best option.*

62. If 0°C is given by 4° × and 100°C is given by 24° X, then which of the following gives roughly the relationship C and X?

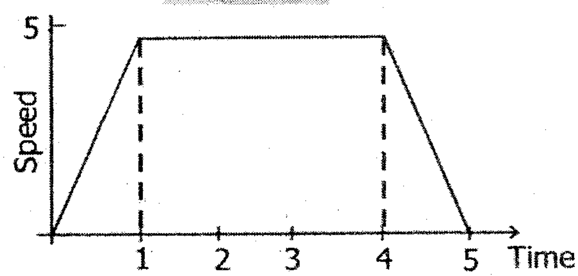
(a) 

(b) 

(c) 

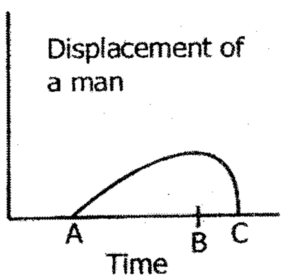
(d) 

63. If a body follows the motion as shown in the following figure, then what is the total distance covered by the body? (speed is in m/sec and the time in seconds)



(a) 10 (b) 50 (c) 80 (d) 20

64. According to the graph, which of the following conclusions is correct?



(a) A man starts from a point and comes back to the same place.

(b) He comes back to the same place at a faster rate.

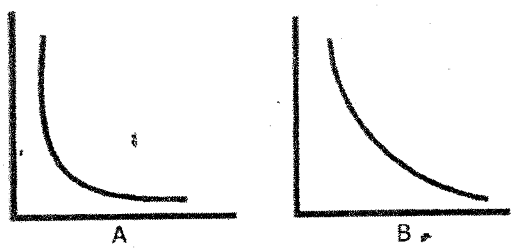
(c) He comes back to the same place at a slower rate

(d) The man's speed is constant throughout the movement.

65. Twon 'P' is located in a particular district. Town 'A' is west of 'P'. Town 'T' is east of 'P'. Town 'K' is east of 'B' but west of 'T' and 'A'. They are all in the same district. Which town is located farthest west?

(a) P (b) K (c) b (d) A

66. The two curves A and B show the variation of the variables A and B. Which of the following conclusions is correct?



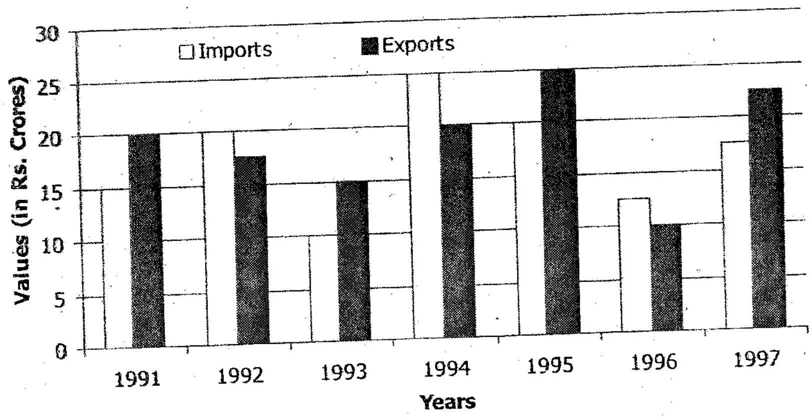
(a) Deviation of A is more as compared to B.
(b) Deviation of B is more as compared to A.
(c) Deviation of A and B is the same.
(d) Deviation of A and B can't be found out.

Objective Questions Part – IV

Directions : *Study the following graph carefully and answer the questions given below.*

Imports and exports of Nirsa Ltd. Over the Years 1991 — 1997

(Figure in Crore)



67. In which of the following pairs of years were the Imports the same?

(a) 1997, 1996 (b) 1992, 1996 (c) 1993, 1994 (d) 1992, 1995

68. What was the percentage increase in Exports from 1996 to 1997?

(a) 25% (b) 125% (c) 100% (d) 150%

69. In how many of the given years were Exports more than Imports?

(a) 2 (b) 3 (c) 4 (d) 5

Answers (Objective Questions)

Answer : (Data Interpretation) Part — I, II, III and IV.

1. (c)	2. (c)	3. (a)	4. (a)	5. (a)	6. (b)	7. (c)
8. (c)	9. (a)	10. (d)	11. (c)	12. (b)	13. (c)	14. (c)
15. (b)	16. (b)	17. (b)	18. (c)	19. (c)	20. (d)	21. (c)
22. (d)	23. (a)	24. (c)	25. (d)	26. (a)	27. (a)	28. (a)
29. (d)	30. (d)	31. (a)	32. (b)	33. (b)	34. (c)	35. (c)
36. (a)	37. (b)	38. (c)	39. (d)	40. (c)	41. (c)	42. (c)
43. (d)	44. (b)	45. (a)	46. (b)	47. (d)	48. (c)	49. (b)
50. (a)	51. (a)	52. (c)	53. (d)	54. (a)	55. (b)	56. (c)
57. (b)	58. (b)	59. (b)	60. (c)	61. (c)	62. (b)	63. (d)
64. (b)	65. (c)	66. (a)	67. (d)	68. (b)	69. (c)	

Solutions:—

1.

$$A = \frac{70+80+90+90+100+90}{6} = 86.66$$

$$B = \frac{80+90+80+75+75+90}{6} = 81.66$$

$$C = \frac{100+110+100+80+100+60}{6} = 91.66$$

$$D = \frac{120+130+120+130+120+130}{6} = 125$$

2.

$$A = \frac{80-70}{70}\times 100$$

$$= 14..28\%$$

$$B = \frac{90-80}{80}\times 100$$

$$= 12.5\%$$

$$C = \frac{130-120}{120}\times 100$$

$= 8.33\%$

3.

$$A = \frac{90-70}{70} \times 100$$

$= 28.57\ \%$

$$B = \frac{90-80}{80} \times 100$$

$= 12.5\%$

$$C = \frac{60-100}{100} \times 100$$

$= -40\%$

$$D = \frac{130-120}{120} \times 100$$

$= 8.33\%$

4. CP of shares = 100 × 70

= 7000

SP of shares = 80 × 100

= 8000

No. of shares of C in fine = $\frac{800}{60} = 133$

7.

First four years production = 225 MT

Last four years production = 275 MT

$$\frac{50}{225} \times 100 = 22.2\%$$

8.

$$\frac{500}{8} = 62.5 \text{ MT}$$

10.

$$\frac{51.}{68} = \frac{3}{4}$$

For Q. No. 11 to 15

Machine No.	Production Accepted	Percentage of Production Rejected	Total Production	Production Rejected
1.	400	20%	500	100
2	385	30%	550	165
3.	438	27%	600	162
4.	420	40%	700	280

16. Required ratio = $\frac{30+35+45}{40+50+55} = \frac{110}{145} = \frac{22}{29}$

17. Average % = $\frac{30 \times 3/4}{45 \times 2/3} \times 100 = 75\%$

18. Average difference = $\frac{305}{6} - \frac{230}{6} = \frac{75}{6} = 12.5$

19. Required % = $\frac{55-25}{25} \times 100 = 120\%$

20. Check directly from the options

21. In 1988, (Gross profit ÷ Net profit) = $50 \div 25) = 2$

22 Observe that for the years 1995, 1986, 1987 and 1989, the ratio would be less than 0.5 while for 1988, it would be equal to 0.5 and for 1990, it would be greater than 0.5.

23. The growth in gross profit in 1986, as compared to the previous year is "5 units in 40". In case of 1987, it is "5 units in 45" and in case of 1990, it is "5 units in 50". Hence the growth would be maximum for 1986 (wothout knowing the exact values).

24. The paired rise in gross profit and net profit took place for 2 years (1986 and 1990).

25. The larges difference is for the year 1987, ie.., 32.5 lakh and the smallest difference is for the year 1988, i.e., 25 lakh.

Hence, the required ratio = $\frac{23.5}{25} = 13$.

26. [(80 – 40) ÷ 40] × (100 ÷ 4) = 25%

27. Average = (40 + 60 + 40 + 50 + 80)/5 = 54 ⇒ 2 years

28. In 1993, production cost = 10 × 1.1 × 1.1 = Rs 12.1

⇒ Cost = 12.1 × 40 × 1000 = 484000 = 4.84 lakh.

29. The cost of production in 1992 will be Rs. 11 per kg.

Hence, sale value = 11 × 60000 × 1.5 = 9,90000 = 9.9 lakh

30 Percentage change for years 1992, 1993, 1994 and 1995 are 50% – 33.33%, 25% and 60, respectively.

For 31 to 34

Prepare the table as shown below on the basis of the data given. All questions can be answered with the help of his table.

	Performance			Total
	Average	Good	Excellent	
Male	16	22	10	48
Female	24	8	0	32
Total	40	30	10	80

Part — II

Distance travelled at the end of :

5 sec = ½ ×5 × 5 = 12.5 m

10 sec = ½ × 10 × 10 50 m

20 sec = ½ × 10 × 10 × ½ (10 + 15) × 10 = 175 m

30 sec = 175 + 15 × 10 = 325 m

40 sec = 325 + ½ × (15 + 5) 5 5 + ½ × (15 + 5) = 425 m

35. Average speed = 425/40 = 10.625 m/sec

36. 175 + 15 × 5 = 250 m.

37. 175/20 = 8.75 m/sec.

38. Acceleration = $\frac{\text{change in speed}}{\text{Time}}$

As speed is constant in interval (20 – 30) sec. So, acceleration,

Note that distance travelled is always given by the area under the speed time cuve.

39. 325/30 = 10.8 m/sec.

40. $\frac{1}{2}\times5\times5+\frac{1}{2}(5+10)\times5+\frac{1}{2}(10+12.5)$ = 12.5 + 37.5 + 56.25 = 106.25

For Q. 41 to 43

From the given data we can complete the table as follows.

Recruitment	Company							
	A		B		C		Total	
	E	T	E	T	E	T	E	T
Manager	10	4	15	6	5	5	30	15
Employee	20	4	10	6	0	5	30	15
Total	30	8	25	12	5	10	60	30

With the help of the table we can give the answer as follows.

44. 30 – 8 = 22

45. At the beginning, the number of Mangers = 65

During the years, the number increased by (30 – 15) = 15

∴ % increase = $\frac{15}{65}\times100=23\%$ (approx)

Part — III

46. The number of spouses having equal equational levels are given by number in the leading diagonal. their total number is

30 + 85 + 38 + 50 = 203.

47. The figure below the leading have to be taken

⇒ (35) + (40 + 17) + (7 + 15 + 38) = 152

⇒ Required percentage = (152/600) × 100 = 25.3%

48. The first column gives 112 wives with post graduate qualification. So the average number of husbands in any category = 112/4 = 28

49. The husbands of 10-12 class are 140 in number . Their average number is 140/4 = 35 i.e., the average number of wives in any category, whose husband have educational level of 10 – 12 class is 35. Among them, the number of gradute wives is 17 which is less than 35. So the number of wives < the average numbner of wife belongs to the gradutate category.

50. The average number of husbands in any category = 600/4 = 150. so among the husband the category of "graduates" has 200 husbands which is more than the average number. Hence the ratio = 200 : 600 = 1 : 3.

51. Total readers of Reader's Digest = 55. Total readers of India Today = 59.

Total sample population = 262.

⇒ Required percentage = (55 + 59)/262 × 100 = 43.5% (you must approximate)

52. Total strength of 10-12 group = 52. Total strength of 20-35 group = 70.

⇒ Required pecentage = {(19 + 5) (18+8)}/(52 + 70) × 100 = 41%. (Again, try to approximate & save time)

53. 20% of (63 + 77 – 7) = 24

54. Number of Illustrated weekly readers = 8 + 13 = 21.

Total number of readers in the age group (20-50) = 70 + 63 – (8 + 11)

= 114 ⇒ Required % = (114 – 21)/114 × 100 = 81% (approx).

55. Sales of India Today = 30 × 59 = 1770;

Total Sales = (55 × 16) + (30 × 59) + (20 × 47) + (28 × 70) = 5580

⇒ required percentage = $\left(\frac{1770}{5580}\right) \times 100 = 31.9\%$

56. (59/70) × 100 = 84%

57. Let (production of D in 1993-94) = k × (production of D in 1994-95). then, 900000 k × 1200000 or k = (9/12) = 0.75

58. Clearly, there is a steady increase in the production of cotton during the given period in case of states A and C.

59. Total number of bales produced by E = (8 + 14 + 7) lakhs = 29 lakhs

∴ Its weight = [2900000 × 100]/1000 tons = 290000 tons.

60. Average production of

A → (6 + 14 + 21)/3 = 13.66

B → (12 + 18 + 18)/3 = 16

C → (5 + 9 + 15)/3 = 9.66

D → (16 + 9 + 12)/3 = 12.3

E → (8 + 14 + 7)/3 = 9.66

States A, B and E showed below average production in 1992-93 and above average production in 1993-94.

61. During the given period, state B has produced 48 lakhs of bales while state A has produced only 41 lakhs

Hence, the statement (3) is false.

62. The graph should be increasing because with increase in C, the value of x° also increases.

63. Distance = velocity × time = total enclosed area

$$= \frac{1}{2}\times 5\times 1 + 5\times(4-1) + \frac{1}{2}\times 5\times 1 = 20 \text{ meters.}$$

64. At A, displacement is zero and also at C, the displacement is zero, thus we conclude that he comes back. Also, we see that the slope aftger B is steeper than it was before B. Thus we conclude that he returns at faster rate.

65. The towns are located from west to east in this order: B, K, A, P, T.

66. The smoother the curve, the greater the deviation. In other words, when a curve is steeper, it has more deviation.

Part — IV

67. imports in 1992 = 20 cr.

Imports in 1995 = 20 cr

68 Percentage increase in export from 1996 to 1997

$$= \frac{22.5-10}{10}\times 100 = 125\%$$

69 The exports point lies above the imports point in the years 1991, 1993, 1995 and 1997. So exports in these four years is more than the imports

Reasoning Aptitude

CHAPTER

18

Data Arrangement

Class Assignment Part - A

1. There are 10 books - two each of English, Hindi, Maths, Physics and Biology. The books are kept in bags of five colours - /red, blue, black, yellow and green. Each bag is allotted to one of the following students: Rajesh, Anita, Veena, Asha and Sophia. No two books of same subject are in the same colour bag. A Hindi book is not nkept with either a Biology book or a Maths book. Asha Carries the book from the subjects among Maths, Biology, Physics and Hindi only. Anita carries an English and a Hindi book. Rajesh does not like to carry books on Biology, English or Maths, Red, blue and black coloured bags are allotted to Anita, Rajesh and Sophia respectively; Red and green coloured bags have one book of the same subject. Black and yellow coloured bags have the same subject books. the books that Veena carries are of

   (a) physics and Maths (b) Maths and Biology

   (c) Physics and English (d) Physics and Hindi

2. Some packets are to be transported to a location 20 km away. Any number of workers can be used to transport the packets. The packets are transported in any number of 20, 40 or 80. Worker changes are Rs. 20 per hour. Workers travel at the speed of 20 km/hr if they are not carrying any load, at 10 km/hr if carrying 20 packets, at 4 km/hr if carrying 40 packets and at 2 km/hr if carrying 80 packets. A worker cannot carry more than 80 packets of load. Find the minimum cost at which 160 packets can be transported to its destination.

   (a) Rs. 340 (b) Rs. 320 (c) Rs. 300 (d) Rs. 280.

Direction : *Read the following information and answer the questions that follow.*

Six persons Mr. Ajay Deo, Ms. Veena Murali, Mrs. Suprabha Sen, Mr. Roshan singh, Mr. Dushyant Vaidya and Mrs. Vama Chitnis are working in 3 different branches of the same company. Two each working at Mumbai, Bangalore and Pune office. Three of them are Supervisors, one is Manager and remaining are Assistant Managers. One of them is in personnel Department, two other are in Accounts Department, *while the remaining three are in marketing department.*

- Mrs. Suprabha Sen is neither in Mumbai Branch nor in Bangalore Branch.
- Mr. Roshan singh holds the position of Assistant manager. However, he does not work at Bangalore Branch.

- Mr. Ajay Deo is Supervisor in Accounts Department at Mumbai Branch.
- Ms. Veena Murali is the assistant manager (Markting) at Bangalore Branch.
- One of the gentlemen working in the Accounts department is at Pune Branch. He is the Senior Supervisor.
- Mrs. Vama Chitnis works as a Manager. She is neither in Markting nor in Accounts department.

3. Which of the following groups consists of Supervisors?
   (a) Veena Murali, Roshan Singh and Vama Chitnis.
   (b) Ajay Deo, Suprabha Sen and Roshan Singh.
   (c) Ajay Deo, Dushyant Vaidya and Suprabha Sen.
   (d) Ajay Deo, Roshan Singh and Dushyant Vaidya.
4. Who among the following is the Personnel Manager?
   (a) Vama Chitnis (b) Ajay Deo
   (c) Veena Murali (d) Roshan Singh
5. From which branch does Mr. Dushyant Vaidya operate?
   (a) Mumbai (b) Pune (c) Bangalore (d) Either Pune or Mumbai
6. Which of the following statements is true?
   (a) Mr. Ajay Deo is Supervisor in Marketing Department.
   (b) Mr. Roshan Singh is Assistant Manager operating from Bangalore Branch.
   (c) One of the persons working from Bangalore Branch is in Accounts Department.
   (d) Both the Assistant Managers work in Marketing Department.
7. Which of the following combination is not ture?
   (a) Veena Murali, Assistant Manager, Bangalore
   (b) Vama Chitnis, Personnel Department, Bangalore
   (c) Ajay Deo, Supervisor, Mumbai
   (d) Roshan Singh, Manager, marketing

Directions : ***Read the information given below and answer the questions that follow.***

- Aurbindo, Bubblo, Chitranjan, Dwarka, Enjalina and Farooq are six students in a class.
- Bubblo and Chitranjan are shorter than Farooq but heavier than Aurbindo.
- Dwarka is heavier than Bubblo and taller than Chatranjan.
- Enjalina is shorter than Dwarka but tailler than Farooq.
- Farooq is heavier than Dwarka.
- Aurbindo is shorter than Enjalina but taller than Farooq.

8. Who among them is the tallest?
(a) Aurbindo (b) Bubblo (c) Dwarka (d) Enjalina

9. Who is third from the top when they are arranged in descending order of their heights?
(a) Aurbindo (b) Bubblo (c) Chitranjan (d) Enjalina

10. Which of the following groups of friends is shorter than Aurbindo?
(a) Bubblo, Chitranjan only (b) Dwarka, Bubblo, Chitranjan only
(c) Enjalina, Bubblo, Chitranjan (d) Farooq, Bubblo, Chitranjan only

11. Who among them is the lightest?
(a) Aurbindo (b) Bubblo or Chitranjan
(c) Chitranjan (d) Data inadequate

12. Which of the following statements is ture for Farooq as regards to height and weight?
(a) He is lighter than Enjalina and taller than Enjalina.
(b) He is heavier than Bubblo and taller than Enjalina.
(c) He is heavier than Bubblo and Chitranjan but shorter than Dwarka.
(d) He is Lighter than Enjalina and also shorter than Engalina.

Directions : Read the folloowing information carefully and answer the questions given below it.

Five professors Dr. Joshi, Dr. Davar, Dr. Natrajan, Dr. Choudhary and Dr. Zia teach five different subjects Zoology, Physics Botany, Geology and History in four Universities Delhi, Gujarat, Bombay and Osmania.

- *Dr. Choudhary teaches Zoology in Bombay Univiersity.*
- *Dr. Natrajan is neither in Osmania University nor in Delhi University and he neither teaches Geology nor History.*
- *Dr. Zia teaches Physics but neither in Bombay University nor in Osmania University.*
- *Dr. Joshi teaches History in Delhi University.*
- *Two professors are from Gujarat University.*

13. Who teaches Geology?
(a) Dr. Natrajan (b) Dr. Zia (c) Dr. Davar (d) Dr. Joshi

14. Dr. Zia is from which University?
(a) Gujarat (b) Bombay (c) Delhi (d) Osmania

15. Who teaches Botany?
(a) Dr. Zia (b) Dr. Davar (c) Dr. Joshi (d) Dr. natrajan

16. Who is from Osmania University?
(a) Dr. Natrajan (b) Dr. Davar (c) Dr. Joshi (d) Dr. Zia

17. Which of the following combinations is correct?
   (a) Dr. Zia – Delhi University
   (b) Dr. Choudhary – Geology
   (c) Dr. Davar – Bombay University
   (d) Dr. Natrajan – Gujarat University

Answers (Objective Questions)

Answer : (Data Arrangement)

1. (c)	**2. (b)**	**3. (c)**	**4. (a)**	**5. (b)**	**6. (d)**	**7. (d)**
8. (c)	**9. (a)**	**10. (d)**	**11. (d)**	**12. (d)**	**13. (c)**	**14. (a)**
15. (d)	**16. (b)**	**17. (d)**				

Solutions :—

1.

Name	**Book**	**Colour of the bags**
Anita	English and Hindi	Red
Rajesh	Hindi & Physics	Blue
Sophia		Blank
Veena	English/?	Green
Asha	Maths/Bio/Physics	Yellow

Veena will carry either English of Hindi and another book of different subjects red and green coloured bags have one book of the same subject. Hence, the options left out are (3) and (4). (4) is not the answer since then blue and green coloured bags will have same book which shouldn't be so since given that only black and yellow coloured bags have same books.

2. The minimum cost would be charged when carrying 20 packets are travelling at a speed of 10 km/hr. The charges are Rs. 20 per hour. Time taken to cover 20 km at a speed of 10 km/hr = 2 hrs. And the cost (worker) to transport 20 packets = 2 × 20 = 40.
Since any number of workers can be used to transport the packets to its distination, therfore 8 workers can be used. Hence, the total cost = Rs. (8× 40) = Rs. 320.

For Q. No. 3 to 7

The information given can be tabulated as follows:

Sr,	Name	Branch	Position	Department
1	*Mr. AjayDeo*	*Mumbai*	*Supervisor*	*Accounts*
2.	*Ms. Veena Murali*	*Bangalore*	*Asstt. Mgr.*	*Marketing*
3.	*Mrs.Suprabha Sen*	*Pune*	*Supervisor*	*Marketing*
4.	*Mr. Roshan Sing*	*Mumbai*	*Asstt. Mgr*	*Marketing*
5.	*Mr. Dushyant Vaidya*	*Pune*	*Supervisor*	*Account*
6.	*Mrs. VamaChitnis*	*Bangalore*	*Manager*	*Personnel*

For Q. No. 8 to 12

In terms of height, we have

B < F, C < F, C < D, E < D, F < E, A < E, F < A.

So , C < F < E < D, B < F, F < A < E.

Thus, the sequence becomes

B, C < F < A < E < D or B, C < F < A < E < D

In terms of weight, we have

A < B, A < C, B < D, D < F.

So, A < B < D < F, A < C

Thus the sequence becomes

A < C < B < D < F for A < B < C < D < F for A < B < D < C < F.

8. Clearly, Dwarka is the tallest.
9. The decending order of height is
   D > E > A > F > B > C or D > E > A > F > C > B.
10. Clearly, Farooq, Bubblo and Chitranjan are shorter than Aurbindo.
11. Data is inadequate as no clue regarding Engilina's weight is given.
12. Clearly Farooq is heavier than Bubblo and Chitranjan but shorter than Aurbindo.

For Q. No. 13 to 17

By the twin matrix procedure, we can prepare following table:

Names	Subjects					Universities			
	Zoo.	Phy.	Bot.	Geo.	Hist.	Del.	Guj.	Bom.	Osmn
Dr. Joshi	×	×	×	×	√	√	×	×	×
Dr. Davar	×	×	×	√	×	×	×	×	√
Dr. Natrajan	×	×	√	×	×	×	√	×	×
Dr. Choudhary	√	×	×	×	×	×	×	√	×
Dr. Zia	×	√	×	×	×	×	√	×	×

CHAPTER

19

Venn Diagram

Objective Questions

Directions for exercise 1 to 5 : These questions are based on the following data.

Out of a group of 245 pilgrims, 105 visited Badrinath, 95 visited Kedarnath and 95 visited Somnath. Fifteen of them visited all three shrines, while 190 visited exactly one of the three shrines. The number of pilgrims who visited exactly two out of the three shrines is three times as many as those who have not visited any one of the three shrines.

1. How many pilgrims have not visited any one of the three shrines ?

   (a) 20 (b) 10 (c) 15 (d) 25

2. How many pilgrims visited not more than one shrine ?

   (a) 50 (b) 100 (c) 150 (d) 200

3. If the number of pilgrims who have visited at least one of the two shrines Kedarnath and Somnath is 165, then how many pilgrims visited only Kedarnath and Somnath ?

   (a) 20 (b) 30 (c) 10 (d) 15

4. If 180 pilgrims visited at least one of the two shrines Kedarnath or Badrinath, then how many pilgrims visited only Somnath ?

   (a) 55 (b) 40 (c) 35 (d) 60

5. If there is nobody who visited only Badrinath and Somnath then how many people visited only Kedarnath ?

   (a) 90 (b) 80 (c) 70 (d) 50

Direction for question 6 : Select the correct alternative from the given choices.

6. Out of some members who have accounts in SBI or ICICI or HSBC banks, 10 have accounts in all the three banks, 54 have accounts in at least two banks, 26 have accounts in both SBI and ICICI and 30 have accounts in both SBI and HSBC. How many members have accounts in only ICICI and HSBC banks ?

   (a) 6 (b) 8 (c) 10 (d) 12 (e) 7

Direction for question 7 to 10 : These questions are based on the following information.

In a class 60% of the students failed in Physics and 50% of the students in Biology. The number of students who failed in Chemistry is 150% of the number of students passed in Physics. 40 students passed in both Physics and Chemistry and 40 students failed in Chemistry only. 20 students failed in all the three subjects and 10 students passed in all the three subjects.

7. How many students passed in Biology ?

   (a) 120 (b) 160 (c) 180 (d) 200 (e) 240

8. How many students passed in Chemistry only ?
(a) 50 (b) 60 (c) 70 (d) [illegible] (e) [illegible]

9. What percentage of the students passed in exactly one subject ?
(a) 60% (b) 62.5% (c) 66.67% (d) 75% (e) [illegible]

10. What percentage of the students failed in at least one subject ?
(a) 2.5% (b) 5% (c) 95% (d) 90% (e) 97.5%

Directions for questions 11 to 12 : Select the correct alternative from the given choices.

11. 25% students of a class failed in Chemistry and 40% failed in Physics where 10% did not pass in any of the two subjects. If 30 students passed in only Physics, how many of them passed in both the subjects.
(a) 10 (b) 15 (c) 90 (d) 30 (e) 45

12. The number of positive integers not greater than 100, which are not divisible by 2, 3 or 5 is
(a) 26 (b) 18(c) 31 (d) None

13. Out of 100 families in the neighbourhood, 45 own radios, 75 have TVs, 25 have VCRs. Only 10 families have all three and each VCR owner also has a TV. If 25 families have radio only, how many have only TV?
(a) 30 (b) 35(c) 40 (d) 45

Direction for Q14 –Q16 : Answer the Questions based on the following information:

Ghosh babu is staying at Ghosh Housing Society, Aghosh Colony, Dighospur , Calcutta. In Ghosh Housing Society 6 persons read daily Ganashakti and 4 read Anand Bazar Patrika; in his colony there is no person who reads both. Total number of persons who read these two newspapers in a Ghosh Colony and Dighospur is 52 and 200 respectively. Number of persons who read Ganashakti in a Ghosh Colony and Dighospur is 33 and 121 respectively; while the persons who read Anand Bazar Patrika in a Ghosh Colony and Dighospur are 32 and 117 respectively.

14. Number of persons in Dighospur who read only Ganashakti is
(a) 121 (b) 83 (c) 79 (d) 127

15. Number of persons in a Ghosh Colony who read both of these newspapers is
(a) 13 (b) 20 (c) 19 (d) 14

16. Number of persons in a Ghosh Colony who read only one paper
(a) 29 (b) 19 (c) 39 (d) 20

17. N the set of natural numbers is partitioned into subsets $S_1 = (1)$, $S_2 = (2, 3)$, $S_3 = \{4, 5, 6)$, $S_4 = \{7, 8, 9, 10\}$ and so on. The sum of the elements of the subset S_{50} is
(a) 61250 (b) 65525 (c) 42455 (d) 62525

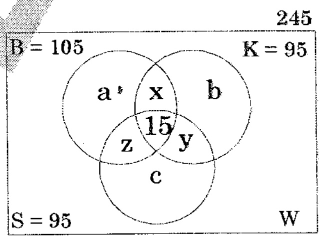
18. If A = { 1, 2, 3, 4} and B = {3, 5, 7}, then the set $(A-B)\cup(B-A)$ would be:
(a) {1,2,3,4,5,7} (b) {5,7} (c) {1,2,4,5,7} (d) {3}

19. In a classs of 60 students at a business school, 40 students like marketing, 36 like production management and 24 like both. The number of students who like only production management are:
(a) 16 (b) 52 (c) 8 (d) 12

Answers (Objective Questions)

1.	(b)	**2.**	(d)	**3**	(c)	**4.**	(a)	**5.**	(d)
6.	(b)	**7.**	(d)	**8.**	(c)	**9.**	(b)	**10.**	(e)
11.	(c)	**12.**	(a)	**13.**	(c)	**14.**	(b)	**15.**	(a)
16.	(c)	**17.**	(d)	**18.**	(c)	**19.**	(d)		

Solutions for 1 to 5:



$x + y + z = 3w,\ \ a + b + c = 190$

We have $105 + \{95 - (x + 15)\} + \{95 - (y + z + 15\} + w = 245$

i.e.,$x + y + z - w = 20$

$2w = 20 \ \Rightarrow\ w = 10$

$x + y + z = 30.$

1. It is w and hence 10.
2. $245 - 30 - 15 = 200$
3. $S + K - (S \cap K) = S \cup K$

   $95 + 95 - p = 165$

   where p = (only S & K) + (S, K & B) = y + 15

   $p = 25$

   Since S, K & B = 15, $y = 25 - 15 = 10.$
4. $105 + 95 - q = 180$

   where q = x + 15 = (only B & K) + (S, K & B) q = 20

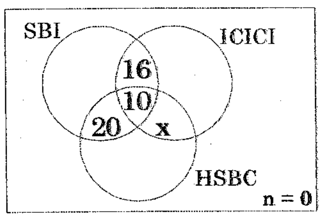
   Hence, $x = 20 - 15 = 5$; So, $y + z = 30 - 5 = 25$

   Only Somnath $= 95 - 15 - (y + z)$

   $= 95 - 15 - 25 = 55.$
5. Given z = 0; Only Kedarnath $= 95 - 15 - x - y$

   $= 95 - 15 - 30 = 50.$
6. If we draw the venn-diagram for the given information, we get the following figure.

   Given 54 have accounts in at least two banks

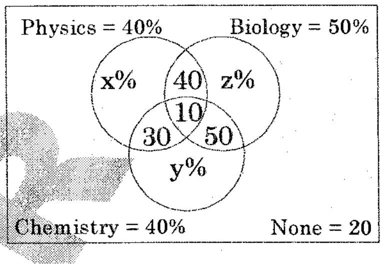
   $\therefore x + 16 + 20 + 10 = 54$

   $\therefore \qquad x = 8.$

Solutions for 7 to 10 :

60% of students failed in Physics

$\Rightarrow$ 405 of students passed in Physics. 50% of students failed in Biology



$\Rightarrow$ 50% of students passed in Biology = 150% of (40%) is 60%.

$\therefore$ 60% of students failed in Chemistry

$\Rightarrow$ 40% of students passed in Chemistry.

Also given that, 40 students passed in Physics and Chemistry, 60 students passed in Biology and Chemistry, 20 students failed in all the three and 10 students passed in all the three subjects.

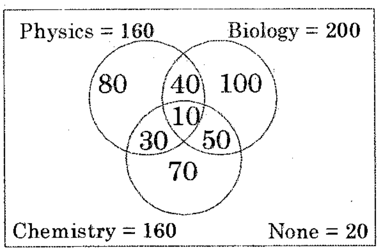
40 students failed in Chemistry only $\Rightarrow$ 40 students passed in Physics and Biology but not Chemistry.

Hence, if we draw a venn diagram on the above data. We get the following figure :

As the total sum of percentage equals to 100, assume that the total strength of the class is 100%.

$\therefore$ x% + y% + z% + 40 + 30 + 50 + 10 + 20 = 100% or x% + y% + z% = 100% – 150

Also, the number of students passed in Physics + Number of students passed in Chemistry + Number of students passed in Biology = Number of students passed in exactly one subject + 2 (number of students passed in exactly two subjects) + 3 (number of students passed in exactly three subjects)



$\Rightarrow$ 40% + 50% + 40%

$\Rightarrow$ (x + y + z)% + 2(30 + 40 + 50) + 3(10)

$\Rightarrow$ 130% = 100% – 150 + 270

$\Rightarrow$ 30% = 120 or 10% = 40

Hence 40% = 160 and 50% = 200

7. 200 students passed in Biology.

8. 70 students passed in Chemistry only.

9. 80 + 100 + 70 = 250 students passed in exactly one subject.

$$\frac{250}{400} \times 100 = 62.5\%$$

10. Apart from the 10 students who passed in all the three subjects, the remaining 390 students failed in at least one subject.

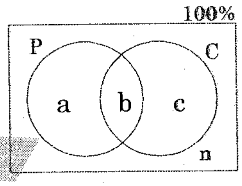
$$\therefore \quad \frac{390}{400} \times 97.5\%$$

Solutions for 11 and 12 :

11. P ⇒ students passed in Physics

C ⇒ students passed in Chemistry

n ⇒ students did not pass in any of the subjects



Given, $a + n = 25\%$; $a = 25\% - 10\% = 15\%$

Also, $c + n = 40\%$

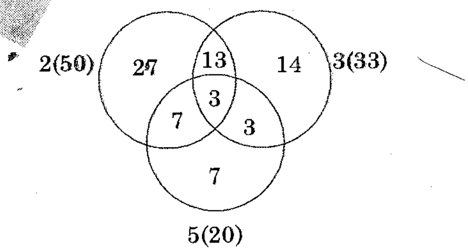
$\therefore c = 40\% - 10\% = 30\%$

$\therefore b = 100 - (a + c + n) = 100 - (15 + 30 + 10) = 45\%$

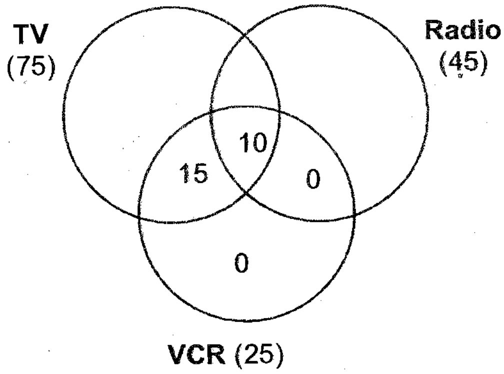
Students passed in only Physics = a = 15% = 30.

Students passed in both = b = 45% = 90.

12. The following Venn diagram shows the distribution on numbers between 1 & 100 that are divisible by 2, 3 or 5 or a combination or two or more of them, So we can see that there are 50 numbers that are divisible by 2, 33 numbers by 3 and 20 numbers by 5. There are 3 numbers are divisible by 2,3 & 5, while 7 are divisible by 2 & 5 (only), 13 are divisible with 2&3 (only) and 3 that are divisible by 5 &3 (only). that leaves with 27 of them divisible by 2 only, 14 by 3 only and 7 by 5 only. So, (27+13+14+7+3+3+7) = 74 numbers are divisible by one or more among 2, 3 & 5. So 26 numbers are not divisible by them.



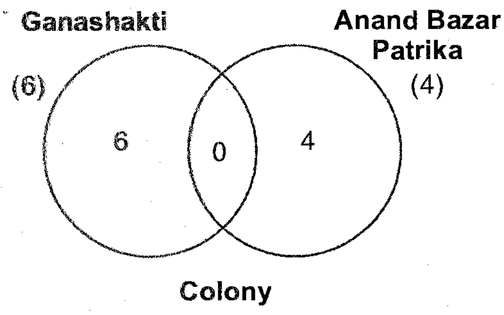
13. From the data given in the question, we can make the following Venn diagram. Since no VCR owner owns only a VCR or a Radio, we can say all 100 of them own at least on among a TV or a Radio. So if we are to look at the part of the diagram for TV and Radio we can say that : Total number of pepole who are in these two sets = 100, of which 75 own TV and 45 own radio. so number of pepole who own both TV & Radio = (75 + 45) – 100 = 20. Of these, 20, 10 own all three. Hence number of pepole having only TV & Radio = 10. Hence number of pepole ownings only TV = 75 – 10 – 10 – 15 = 40.

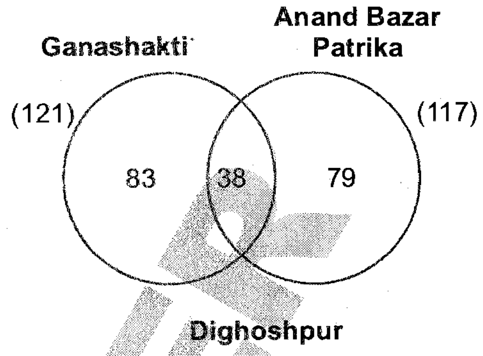


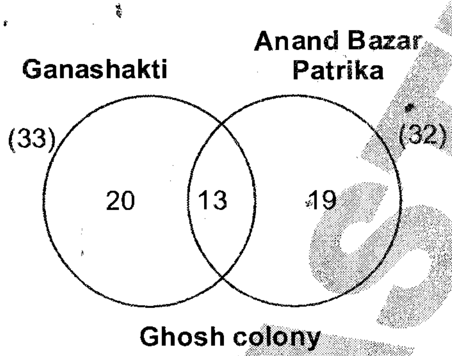
Direction for Q14 –Q16 : Answer the Questions based on the following information:

Ghosh babu is staying at Ghosh Housing Society, Aghosh Colony, Dighospur , Calcutta. In Ghosh Housing Society 6 persons read daily Ganashakti and 4 read Anand Bazar Patrika; in his colony there is no person who reads both. Total number of persons who read these two newspapers in a Ghosh Colony and Dighospur is 52 and 200 respectively. Number of persons who read Ganashakti in a Ghosh Colony and Dighospur is 33 and 121 respectively; while the persons who read Anand Bazar Patrika in a Ghosh Colony and Dighospur are 32 and 117 respectively.

Sol.







14. Number of persons in Dighoshpur who read only Ganashakti = 83.

15. Number of persons in a Ghosh Colony who read both the newspapers = 13.

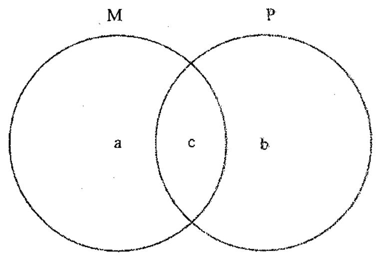
16. Number of persons in a Ghosh colony who read only 1 newspaper = 20+19 = 39.

17. If we are to take out the first elements of each set we find them as : 1, 2, 4, 7, 11, 16....

18. Given A = {1,2,3,4,}, B = { 3,5,7} $\therefore$ A – B = { 1,2,4} and B – A = { 5,7}

Hence, $(A-B)\cup(B-A)$ = {1, 2, 4, 5, 7}

19.



Let M denote the marketing management and P denote the producing management.

Given : a + c = 40, b + c = 36 and c = 24

To find : No. of people who like only production management. is b = 36 – c = 36 – 24 = 12

■ ■ ■

CHAPTER

20

Cubes and Cuboids

Introduction

A cubes is a three dimensional solid having 6 faces, 12 edges and 8 corners. All the edges of a cube are equal and hence all the faces are square in shape.

In competitive exams a few questions may be asked based on cubes.

The questions on cubes may belong to anyone of the following categories.

I. A cube is cut by making certain specified number of cuts. The directions in which the cuts are made may or may not given. We are to find the number of identical pieces resulting out of the given cuts.

II. The number of identical pieces, into which a cube is cut, is given and we need to find the number of cuts.

III. A cube could be painted on all or some of its faces with the same colour of different colours and then cut into a certain specified number of identical pieces. Then questions of the form – "How many small cubes have 2 faces painted?". "How many smaller cubes have only one face painted?" etc. could then be framed.

(Objective Questions)

Direction for examples 1 to 4

These questions are based on the following data.

125 small but identical cubes are put together to form a large cube. This larger cube is now painted on all six faces.

1. How many of the smaller cubes have no face painted at all ?
   (a) 27 (b) 64 (c) 8 (d) 36
2. How many of the smaller cubes have exactly one face painted ?
   (a) 49 (b) 54 (c) 64 (d) 72
3. How many of the smaller cubes have exactly two faces painted ?
   (a) 25 (b) 16 (c) 36 (d) 64

4. How many of the smaller cubes have exactly three faces painted ?
(a) 4 (b) 5 (c) 9 (d) 8

Direction for examples 5 to 10

Select the correct alternative from the given choices.

5. What is the least number of cuts required to cut a cube into 24 identical pieces ?
(a) 2 (b) 4 (c) 6 (d) 8

6. What is the maximum number of identical pieces in a cube can be cut into by 7 cuts ?
(a) 36 (b) 49 (c) 25 (d) 56

7. What is the least number of identical cuboids, each of dimentions 2 cm × 4 cm × 5 cm, that are required to form a cube ?
(a) 160 (b) 240 (c) 220 (d) 200

8. It was found that a cube can be cut into certain number of identical cuboids each measuring 1 cm × 2 cm × 5 cms. What is the side of the smallest such cube ? How many such cuboids can be formed from such a cube ?
(a) 10 cm, 100 (b) 5 cm, 50 (c) 20 cm, 800 (d) 15 cm, 100

9. A cube of side 6 cm, has been cut into 64 smaller but identical cubes. If it was estimated that it would take 4 ltrs. of paint to paint all the faces of the original cube, then how much paint is required to paint all the faces of all the smaller cubes ?
(a) 6 litres (b) 12 litres (c) 20 litres (d) 16 litres

10. Each face of a cube is painted either white or black. In how many different ways can the cube be painted ?
(a) 8 (b) 10 (c) 12 (d) 16

Answers (Objective Questions)

1.	(a)	2.	(b)	3.	(c)	4.	(d)	5.	(c)
6.	(a)	7.	(d)	8.	(a)	9.	(d)	10.	(b)

Solutions for examples 1 to 4 :

125 cubes will be put together as $5 \times 5 \times 5$ only.

1. (a) Out of $5 \times 5 \times 5$ cubes, if one layer on all sides is removed, the inside cubes, i.e. $3 \times 3 \times 3$ will have no paint at all.

2. (b) On each face if all the cubes on the outside are removed, the remaining 3×3 cube (i.e., nine cubes) has one face painted. Hence, on 6 faces, $6 \times 9 = 54$ cubes.

3. (c) On each edge, if the two corner cubes are removed the three cubes in the middle have two faces painted. Since there are 12 faces, $12 \times 3 = 36$ cubes with three faces painted.

4. (d) The 8 corner cubes have 3 faces painted.

Solutions for examples 5 to 10

5. (c) Three cuts parallel to one face, two cuts in a perpendicular plane and one cut in the third perpendicular plane. Hence, six cuts.

6. (a) Seven cuts to be distributed as 3 – 2 – 2 along the three perpendicular planes (all cuts parallel to the faces) and hence, we obtain $4 \times 3 \times 3 = 36$ pieces.

7. (d) LCM of 2, 4, 5 (which is 20) will be the side of the cube of minimum dimensions.

   Hence, number of cuboids $= \dfrac{20 \times 20 \times 20}{2 \times 4 \times 5} = 200$

8. (a) Side of the cube = LCM of 1, 2 and 5 = 10 cms.

   No. of cuboids $= \dfrac{10 \times 10 \times 10}{1 \times 2 \times 5} = 100$

9. (d) Since there are 64 small cubes, the original cube is cut into four pieces along each face, i.e., $4 \times 4 \times 4$. Hence, the paint also would be 4 times, as the total surface area increases by 4 times.

10. (b) Only distinct combinations have to be counted. Eg. if we say two adjacent sides are painted in one colour, it can be any two adjacent sides and we get the same combination always and not distinguishable from each other. Thus, the number of distinguishable combinations will be

All Black	– 1
1 Black + 5 White	– 1
[Two black can be two adjacent or two opposite faces - hence 2 ways]	– 2
3 Black + 3 White	– 2
[Three of one colour can be 3 adjacent or 3 faces in a row - hence 2 ways]	
4 Black + 2 White	– 2
5 Black + 1 White	– 1
All white	– 1
Total	10

CHAPTER

21

Coding and Decoding

Letter Coding

In these questions, the letters in a word are replaced by certain other letters according to a specific rule to form its code. The candidate is required to detect the coding pattern/rule and answer the questions accordingly.

(Objective Questions)

1. If ROAST is coded as PQYUR in a certain language, then how will SLOPPY be coded in that language?

(a) MRNAQN (b) NRMNQA

(c) QNMRNA (d) RANNMQ

2. If HEALTH is written as GSKZDG,then how will NORTH be written in that code?

(a) OPSUI (b) GSQNM

(c) FRPML (d) IUSPO

3. In a certain code language, BEAT is written as YVZG, then what will be the code of MILD?

(a) ONRW (b) NOWR

(c) ONWR (d) NROW

Direct letter coding

4. If the word EARTH be written as QPMZS in coded form, how can HEART be written following the same coding?

(a) SQPZM (b) SQMPZ

(c) SPQZM (d) SQPMZ

5. In a coding system, PEN is written as NZO and BARK as CTSL. How can we write PRANK in that coding system?

(a) CSTZN (b) NSTOL

(c) NTSLO (d) NZTOL

6. In a certain code, STOVE is written as FNBLK, then how will VOTES be written in the same code?

(a) FLKBN (b) LBNKF
(c) LKNBF (d) LNBKF

Number/Symbol Coding

7. If E = 5, PEN = 35, then PAGE = ?

(a) 27 (b) 28
(c) 29 (d) 36

8. If in a certain code, BAT = 23 and CAT = 24, then how will you code BALL ?

(a) 27 (b) 28
(c) 32 (d) 120

9. If O = 16, FOR = 42, then what is FRONT equal to?

(a) 61 (b) 65
(c) 73 (d) 78

10. In a certain code, EAT is written as 318 and CHAIR is written as 24156. What will TEACHER be written as?

(a) 8312346 (b) 8321436
(c) 8312436 (d) 8313426

Deciphering Message Word Codes

11. If in a certain language, '*oka peru*' means '*fine cloth*'; '*meta lisa*' means '*clear water*' and '*dona lisa peru*' means '*fine clear weather*', which word in that language means '*weather*'?

(a) Peru (b) oka
(c) meta (d) dona

12. In a certain coding system, '*rbm std bro pus*' means '*the cat is beautiful*', '*tnh pus dim std*' means '*the dog is brown*', '*pus dim bro pus cus*' means '*the dog has the cat*'. What is the code for '*has*'?

(a) std (b) dim
(c) bro (d) cus

Deciphering Number and Symbol codes for Messages

13. In a certain code language, '234' means '*spark and fire*', '456' means '*spark is cause*' and '258' means '*fire is effect*'. Which of the following numerals is used for '*cause*'?

(a) 3 (b) 4
(c) 5 (d) 6

14. In a certain code language, '253' means *'books are old'*, '546' means *'man is old'* and '378' means *'buy good books'*. What stands for *'are'* in that code?

(a) 2 (b) 4

(c) 5 (d) 6 (e) 9

Answers (Objective Questions)

1.	(c)	**2.**	(d)	**3**	(d)	**4.**	(d)	**5.**	(b)
6.	(b)	**7.**	(c)	**8.**	(a)	**9.**	(d)	**10.**	(c)
11.	(d)	**12.**	(d)	**13.**	(d)	**14.**	(a)		

1.

Sol.

Clearly, the letters in the word ROAST are moved alternately two steps backward and two steps forward to obtain the letters of the code. Thus, we have:

R	O	A	S	T	S	L	O	P	P	Y
−2↓	+2↓	−2↓	+2↓	−2↓	−2↓	+2↓	−2↓	+2↓	−2↓	+2↓
P	Q	Y	U	R	Q	N	M	R	N	A

So, the required code is QNMRNA.

2.

Sol. Clearly, the letters of the given word are written in a reverse order and then each letter is moved one step backward to obtain the code.

Reversing the order of letters in NORTH,we get HTRON.Thus, we have:

H	T	R	O	N
−1↓	−1↓	−1↓	−1↓	−1↓
G	S	Q	N	M

So, the required code is GSQNM.

3.

Sol. B, E, A, T are respectively the 2nd, 5th, 1st, 20th letters from the beginning of the English alphabet. The letters of the code Y, V, Z, G are respectively the 2nd, 5th, 1st and 20th letters from the end of the English alphabet.

Similarly, M, I, L, D are respectively 13th, 9th, 12th, 4th letters from the beginning of the English alphabet. And, the 13th, 9th, 12th, 4th letters from the end of the English alphabet are N, R, O, W respectively. So, the required code is NROW.

4.

Sol. Observing the above question, we may notice that HEART consists of the same letters as EARTH and the four possible codes given as alternatives also consist of the same letter codes as those in the code for EARTH. This indicates that this is a question on direct-coding.

Thus, we have:

Letter	E	A	R	T	H
Code	Q	P	M	Z	S

So, the code for HEART becomes SQPMZ

5.

Sol.

Letter	P	E	N	B	A	R	K
Code	N	Z	O	C	T	S	L

The code for PRANK is NSTOL.

6.

Sol.

Letter	S	T	O	V	E
Code	F	N	B	L	K

The code for VOTES is LBNKF.

7. If E = 5, PEN = 35, then PAGE = ?

Sol. clearly, putting A = 1, B = 2, C = 3, D = 4, E = 5, ..., M = 13, ..., X = 24, Y = 25, Z = 26, we have :

PEN = P + E + N = 16 + 5 + 14 = 35

So, PAGE = P + A + G + E = 16 + 1 + 7 + 5 = 29.

8.

Sol. C is one step ahead of B and the code for CAT is 1 more than that for BAT. Thus, the letters are coded by numerals denoting their positions in the English alphabet.

i.e. A = 1, B = 2, ..., Z = 26

So, BALL = B + A + L + L = 2 + 1 + 12 + 12 = 27.

9.

Sol. We have : A = 2, B = 3, ..., Z = 27. Then,

For = F + O + R = 7 + 1 6 + 19 = 42.

FRONT = F + R + O + N + T = 7 +19 + 16 + 15 + 21 = 78.

10.

Sol.

Letter	E	A	T	C	H	I	R
Code	3	1	8	2	4	5	6

The code for TEACHER is 8312436.

11.

Sol. In the first and third statements, the common code-word is '*peru*' and the common word is '*fine*'. So, '*peru*' means '*fine*'.

In the second and third statements, the common code-word is '*lisa*' and the common word is '*clear*'. So, '*lisa*' means '*clear*'.

Thus, in the third statement, '*lisa*' means '*clear*' and '*peru*' means '*fine*'. So, '*dona*' means '*weather*'.

12.

Sol. In the third statement, the code-word '*pus*' occurs twice and the word '***the***' also occurs twice. So, the code-word for '*the*' is '*pus*'.

Now, in the first and third statements, the common code-word '*pus*' stands for '*the*'. So, the other common code-word '*bro*' stands for the other common word *i.e.* '*cat*'. Similarly, in the second and third statements, the common code-word '*dim*' stands for the common word '*dog*'.

13.

Sol. In the first and second statements, the common code digit is '4' and the common word is 'spark'. So, '4' means 'spark'. In the second and third statements, the common code digit is '5' and the common word is 'is'. So, '5' means 'is'.

Thus, in the second statement, '6' means 'cause'.

14.

Sol. In the first and second statements, the common code digit is '5' and the common word is 'old'. So, '5' means 'old'.

In the first and third statements, the common code digit is '3' and the common word is 'books'. So, '3' means 'books'.

Thus, in the first statement, '2' means 'are'.

■ ■ ■

CHAPTER

22

Direction Sense

Directions:

The given questions are pertaining to movement of a person or of a vehicle in a given direction Using sense of direction, you are required to determine the location of the person or vehicle, after the person or vehicle has covered a certain distance, taking turns towards right or left.

1. Suchit travels 7 km North, then turns right and walks 3 km. He again turns to his right and moves 7 km forward. How many km is Suchit away from the place of his starting the journey?

(a) 7 km (b) 3 km (c) 6 km (d) 14 km

2. Riyaz drives to North of her place of stay at A and find after travelling 25 km that she has driven in the wrong direction. She then turns to the right and travels 2 km and then again turns right and drives straight another 25 km. How much distance she has now to cover to go back to the point from where she started?

(a) 25 km (b) 2 km (c) 4 km (d) 50

3. Rachit travels 10 km North turns left and travels 4 km and then again turns right and covers another 5 km. He then turns to right and travels another 4 km. How far is he from the point of starting his journey?

(a) 15 km (b) 4 km (c) 5 km (d) 10 km

4. Sparsh and Rabee both start from a point towards North. Sparsh turns to left after walking 10 km. Rabee turns to right after walking the same distance. Sparsh waits for some time and then walks another 5 km, whereas Rabee walks only 3 km. They both then return to the South and walk 15 km forward. How far is Sparsh from Rabee?

(a) 15 km (b) 10 km (c) 8 km (d) 12 km

5. A taxi driver commenced his journey from a point and drove 10 km towards North and turned to his left and drove another 5 km. After waiting to meet one of his friends, he turned to his right and continued to drive another 10 km. He has covered a distance of 25 km so far but in which direction is he now heading?

(a) North (b) East (c) West (d) South

6. There is a ring road connecting points A, B, C and D. The road is in a complete circular form but having several approach roads leading to the centre. Exactly in the

centre of the ring road there is a tree which is 20 km from point A on the circular road. You have taken a round of the circular road starting from point A and finish at the same point after touching points B, C and D. You then drive 20 km interior towards the tree from point A and from there reach somewhere in between B and C on the ring road. How much distance you have to travel from the tree to \reach the point between B and C on the ring road?

(a) 20 km (b) 15 km (c) 80 km (d) 40 km

7. A tourist drives 10 km towards East and turns to right hand side and takes a drive of another 3 km. He then drives towards West (turning to his right) another 3 km. He then turns to his left and walks another 2 km. Afterwards, he turns to his right and travels 7 km. How far is he from his starting point and in which direction?

(a) 10 km East (b) 9 km North (c) 8 km West (d) 5 km South

8. Rachit walks 30 m towards south. Then turns to his right and starts walking straight till he completes another 30 m. Then again turning to his left he walks for 20 m. He then turns to his left and walks for 30 m. How far is he from his initial position?

(a) 50 m (b) 30m (c) 10 m (d) 60m

9. Vishakha drove her car for 30 km due North. Then she turned left and drove for 40 km. She then turned left again and drove yet another 30 km. Again she turned left and drove her car 50 km. How far she actually drove her car from the initial position?

(a) 40km (b) 50km (c) 30km (d) None of these

10. Sankar ran 20 m to the east, then he turned left and walked for 15 m, then turned right and went 25 m and then turned right again and went 15 m. How far was Sankar from the starting point?

(a) 45m (b) 35m (c) 25m (d) 15m

Directions : *Read the given information and answer the questions that follow.*

If you start running from a point towards north and after covering 4 km you turn to your left and run 5 km and then again turn to left and run 5 km and then turn to left again and run another 6 km and before finishing you take another left turn and run 1 km.

11. How many km are you from the place you started?

(a) 1 km (b) 2 km (c) 3 km (d) 4 km

12. In which direction will you be running while finishing?

(a) East (b) West (c) North (d) South

13. After taking the second turn, in which direction will you be running?

(a) East (b) West (c) North (d) South

14. From the finishing point if you have to reach the point from where started, in which direction will you have to run?

(a) East (b) West (c) North (d) South

15. A child is looking for his father. He went 90 m in the East before turning to his right. He went 20 m before turning to his right again to look for his father at his uncle's

place 30 m from this point. His father was not there. From here he went 100 m to the North before meeting his father in a street. How for did the son meet his father from the starting point?

(a) 80 m (b) 100m (c) 140m (d) 260m

16. The door of Adil's house faces the East. From the back side of his house, he walks straight 50m, then turns to the right and walks 50 m again. Finally, he turns towards left and stops after walking 25m. Now Adil is in which direction from the starting point?

(a) South-East (b) North-East (c) South-West (d) North-West

17. Two buses start from the opposite points of a main road, 150 km apart. The first bus runs of 25 km and takes a'right and then runs for 15 km. It then turns left and runs for another 25 km and takes the direction back to reach themain road. In the mean time, due to a minor breakdown, the other bus has run only 35 km along the main road. What would be the distance between the two buses at this point?

(a) 65 km (b) 75 km (c) 80km (d) 85 km

18. Kushal walks 10 km towards North. From here he walks 6 km towards south. Then, he walks 3 km towards East. How far and in which direction is he with reference to his starting point?

(a) 5 km West (b) 7 km West (c) 7 km East (d) 5 km North-East

19. If A is to the south of B and C is to the East of B, in what direction is A with respect to C?

(a) North-East (b) North-West (c) South-East (d) South-West .

20. A is 40 m South-West of B. C is 40m south-East of B. Then, C is in which direction of A?

(a) East (b) West (c) North-East (d) South

Answers (Objective Questions)

1. (b)	2. (b)	3. (a)	4. (c)	5. (a)	6. (a)	7. (d)
8. (a)	9. (d)	10. (a)	11. (a)	12. (c)	13. (d)	14. (b)
15. (b)	16. (d)	17. (a)	18. (d)	19. (d)	20. (a)	

CHAPTER

23

Blood Relation

In the exams, the success of a candidate in the questions on blood relations depends upon his knowledge about various blood relations. Some of the relationships given below help in solving the problems.

The easiest and non-confusing way to solve these types of problems would be to draw a tree diagram and increase the levels in the hierarchy as shown below:

1st stage	Grand-Parents	→ (Grandfather, Grandmother, Granduncle, Grandaunt)
2nd stage	Parents & In-Laws	→ (Father, Mother, Uncle, Aunt, Father-in-law, Mother-in-law)
3rd stage	Siblings, & Spouse	→ (Brother, Sister, Cousin, Wife, Husband, Brother-in-law, In-Laws, sister-in-law)
4th stage	Children & In-Laws	→ (Son, Daughter, Niece, Nephew, Son-in-law, Daughter-in-law)
5th stage	Grand Children	→ (Grandson, Granddaughter)

Mother's or father's son	Brother	Mother's or father's daughter	Sister
Mother's or father's brother	Uncle	Mother's or father's sister	Aunt
Mother's or father's father	Grandfather	Mother's or father's mother	Grandmother
Son's wife	Daughter-in-law	Daughter's husband	Son-in-law
Husband's or wife's sister	Sister in law	Husband's or wife's brother	Brother-in-law
Brother's son	Nephew	Brother's daughter	Niece
Uncle or aunt's son or daughter	Cousin	Sister's husband	Brother-in-law
Brother's wife	Sister-in-law	Grandson's or Grand daughter's daughter	Great grand daughter

Objective Questions

1. Tina told Mani, "The girl I met yesterday at the beach was the youngest daughter of the brother-in-law of my friend's mother." How is the girl related to Tina's friend?
   (a) Cousin (b) Daughter (c) Niece (d) Friend
   (e) Anut

2. A woman going with a boy is asked by another woman about the relationship between them. The woman replied, "My maternal uncle and the uncle of his maternal uncle is the same." How is the lady related with that boy?
   (a) Grandmother and Grandson (b) Mother and Son
   (c) Aunt and Nephew (d) None of these

3. A's father's brother's father is D. How is D related to A?
   (a) Father (b) Grandfather (c) Uncle (d) Son

4. A's father's wife's mother is C, whose only child is D. How is D related to A's brother?
   (a) Grandmother (b) Aunt (c) Sister (d) Mother

5. Pointing to a photograph Ramesh said, "she is the sister of my father's mother's only child's son." How is the person in the photograph related to Ramesh ?
   (a) Sister (b) Aunt (c) Mother (d) Cousin

6. Introducing Reena, Monika said, "She is the only daughter of my father's only daughter." How is Monika related to Reena?
   (a) Aunt (b) Niece (c) Cousin
   (d) Data inadequate (e) None of these

7. B's father's father is the husband of C's mother's mother. How is B related to C?
   (a) Brother (b) Sister
   (c) Cousin (d) Cannot be determined

8. Pointing towards a girl in the picture, Sarita said. "She is the mother of Neha whose father is my son." How is Sarita related to the girl in the picture?
   (a) Mother (b) Aunt (c) Cousin
   (d) Data inadequate (e) None of these

9. Pointing to a woman, Naman said, "She is the daughter of the only child of my grandmother." How is the woman related to Naman?
   (a) Sister (b) Niece
   (c) Cousin (d) Data inadequate (e) None of these

10. A and B are brothers. C and D are sisters. A's son is D's brother. How is B related to C?

(a) Father (b) Brother

(c) Grandfather (d) Uncle (e) None of these

11. A, B and C are sisters. D is the brother of E and E is the daughter of B. How is A related to D?

(a) Sister (b) Cousin

(c) Niece (d) Aunt

Directions (12-14): Read the following information and answer the questions given below it:

A is the father of C. But C is not his son.

E is the daughter of C. F is the spouse of A.

B is the brother of C. D is the son of B.

G is the spouse of B. H is the father of G.

12. Who is the grandmother of D?

(a) A (b) C (c) F (d) H

13. Who is the son of F?

(a) B (b) C (c) D (d) E

14. C is A's father's nephew. D is A's cousin but not the brother of C. How is D related to C?

(a) Father (b) Sister (c) Mother (d) Aunt

Directions (15-18): Read the information given below and answer the questions that follow:

(i) In a family of six persons A, B, C, D, E and F, there are two married couples.

(ii) D is grandmother of A and mother of B.

(iii) C is wife of B and mother of F.

(iv) F is the grand daughter of E.

15. What is C to A?

(a) Daughter (b) Grandmother (c) Mother

(d) Cannot be determined (e) None of these

16. How many male members are there in the family?

(a) Two (b) Three (c) Four

(d) Cannot be determined (e) None of these

17. Which of the following is true?

(a) A is brother of F (b) A is sister of F. (c) D has two grandsons.

(d) B has two daughters. (e) None of these

18. Who among the following is one of the couples?

(a) CD (b) DE.

(c) EB (d) Cannot be determined (e) EB

Answers (Objective Questions)

1.	(a)	**2.**	(c)	**3**	(b)	**4.**	(d)	**5.**	(*)
6.	(e)	**7.**	(d)	**8.**	(e)	**9.**	(a)	**10.**	(d)
11.	(d)	**12.**	(c)	**13.**	(a)	**14.**	(b)	**15.**	(c)
16.	(d)	**17.**	(e)	**18.**	(b)				

1.

Sol.

The relations may be analysed as follows :

Daughter of brother-in-law – Niece; Mother's niece – Cousin.

So, the girl is the cousin of Tina's friend. Hence, the answer is (a).

2.

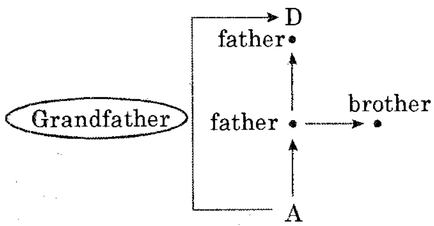
Sol. Clearly, the brother of woman's mother is the same as the brother of the father of boy's maternal uncle. So, the woman's mother's brother is the boy's maternal uncle's father. Thus, the woman's mother's brother's son is boy's maternal uncle, i.e. woman's mother's brother's daughter is boy's mother.

So, the woman and boy's mother are cousins. Thus, the woman is boy's aunt.

Hence, the answer is (c)

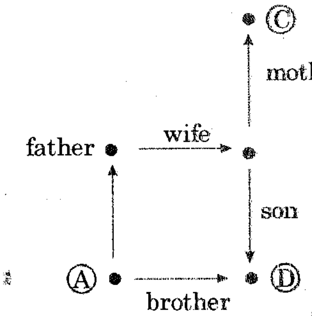
3.

Sol. A's father's brother is A's uncle. His uncle's father is A's father's father. D is the grandfather of A.



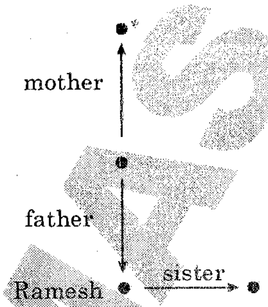
4.

Sol. A's father's wife is A's mother. A's mother's mother is C whose only child is D. Hence, D is the mother of A. Similarly, D is the mother of A's brother.



5.

Sol. My father's mother's only child is my father. My father's son's sister is in the photograph. Hence she is Ramesh's sister.



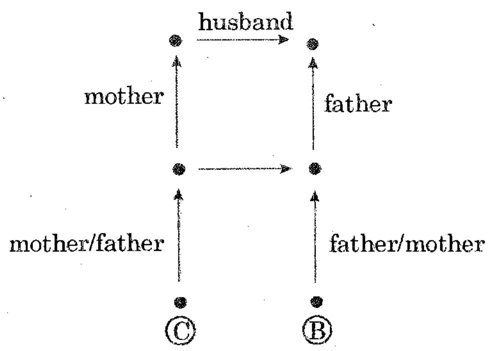
6.

Sol. Monika's father's only daughter → Monika.

So, Reena is Monika's daughter, i.e. Monika is Reena's mother.

7.

Sol. B's father's father is B's grandfather. B's grandfather is the husband of C's mother's mother i.e., grandmother. It is possible that B and C are sibling's and the persons mentioned are their paternal/maternal grand parents. It is also possible that B and C are cousins. Hence the relationship cannot be determined.



8.

Sol. Neha is the daughter of Sarita's son, and the girl is Neha's mother.

So, the girl is Sarita's son's wife i.e. Sarita is the girl's mother-in-law.

9.

Sol. Only child of Naman's grandmother → Naman's father/mother.

Daughter of Naman's father/mother → Naman's sister.

10.

Sol. Clearly, B is the brother of A; A's son is D's brother. This means D is the daughter of A. Since C and D are sisters, C is also the daughter of A.

So, B is the uncle of C.

11.

Sol. E is the daughter of B and D is the brother of E. So, D is the son of B. Also, A is the sister of B. Thus, A is D's aunt.

12.

Sol. D is the son of B, B is the brother of C and A is the father of C. This means that B is the father of D and A is the father of B. So, A is the grandfather of D. Since F is the spouse of A, so F is the grandmother of D.

13.

Sol. As explained above, B is the son of A and F is the spouse of A. So, B is the son of F.

14.

Sol. C is A's father's nephew means C is the son of A's father's brother i.e., C is the cousin of A. D is also A's cousin. So, D must be real brother or sister of C. But D is not brother of C. So, D must be sister of C.

15.

Sol. C is the wife of B and D is the mother of B. Also, D is grandmother of A. So, C is the mother of A.

16.

Sol. Clearly, the sex of A cannot be determined.

17. (e)

Sol. The sex of A is not known. So, neither (a) nor (b) is definitely true. Clearly, D is the grandmother of A and F.

18.

Sol. C is wife of B. So, one couple is BC. Now, D is grandmother of A. B is the son of D and his wife C is the mother of F. So, D is also the grandmother of F. But F is the grand daughter of E. So, E is the grandfather of F and the husband of D. Thus, DE is another couple.