(T) Roll No. .....

PAPER ID—16018

B. Tech. EXAMINATION, 2025

(Third Semester)

DISCRETE MATHEMATICS

(Btech-2023-587)

Time : 3 *Hours* *Maximum Marks :* 70

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) What is a Hasse diagram ? Draw the Hasse diagram for the set {1, 2, 4, 8} under the divisibility relation. **2**

(b) Write the converse, inverse and contrapositive of the statement :

"If it rains, then the ground becomes wet." **2**

(c) Find the number of ways of selecting a committee of 3 men and 2 women from 6 men and 5 women. **2**

(d) Obtain the generating function of $S(n)\frac{1}{n!}$. **2**

(e) Find identity for the operation $a*b=a+b+1$ on $\mathbb{Z}$. **2**

(f) Determine whether $(\mathbb{Z}, +, x)$ is a ring, integral domain, or field, giving reasons. **2**

(g) State Euler's Theorem for connected planar graphs and verify it for K_4. **2**

Unit I

2. (a) Let A = {1, 2, 3, 4} and define a relation R on A as :

$$R=\{(x, y)|x \text{ divides } y\}$$

(i) Represent R using a matrix and digraph.

(ii) Determine whether R is reflexive, symmetric and transitive.

(iii) Draw the Hasse diagram for the corresponding partial order. **8**

(b) (i) Define injective, surjective and bijective functions with examples.

(ii) Determine whether the following function is bijective :

$f : \mathbb{R} \to \mathbb{R}, f(x) = 3x + 2$

Also, find its inverse function. **6**

3. (a) Prove by using set identities that :

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

and write its dual. **6**

(b) Using truth tables, determine whether the following statements are tautologies, contradictions, or contingencies :

(i) $\sim (p \vee q) \vee (\sim p \vee \sim q)$

(ii) $(p \to q) \vee (q \to p)$.

Also find all possible truth values for each case. **8**

Unit II

4. (a) How many 4-digit numbers can be formed using the digits 1, 2, 3, 4, 5, 6, 7 if :

(i) digits are not repeated

(ii) digits are repeated ? **4**

(b) How many arrangements of the word PROBABILITY can be made if all letters are used ? **3**

(c) Solve the recurrence relation :

$a_n - a_{n-1} = 3^n, a_0 = 2$. **7**

5. (a) Using the Euclidean Algorithm, find the GCD of 414 and 662 and express the GCD as a linear combination of 414 and 662. **7**

(b) Solve the recurrence relation :

$$a_n = 5a_{n-1} - 6a_{n-2}$$

given that $a_0 = 1$ and $a_1 = 4$. Find the explicit formula for a_n and hence compute a_5. **7**

Unit III

6. (a) Verify whether S_3 forms a group under composition of mappings and state whether it is Abelian or Non-Abelian. **8**

(b) State and prove Lagrange's theorem. **6**

7. Let B = {1, 2, 3, 6} be the set of positive factors of 6. Let the binary operations '+' and '.' on B are defined as follows :

$$a + b = l \text{ cm } (a, b)$$

and $a \cdot b = \gcd(a, b) \forall\ a, b \in B$.

Let the unary operation on B is defined by $a' = \frac{6}{a} \forall\ a \in B$. Show that $\{B, +, ;'\}$ is a Boolean algebra. **14**

Unit IV

8. State and prove Euler's theorem for connected planar graph. **14**

9. (a) Determine whether the graph with edges :

E = {AB, BC, CD, DA, AC, BD}

is Eulerian and/or Hamiltonian. If it is Hamiltonian, write one Hamiltonian circuit. Also Explain the degree conditions for Eulerian and Hamiltonian graphs and check them for this case. **8**

(b) Draw a bipartite graph $K_{3,2}$. Find the degree of each vertex and the total number of edges. Is the graph regular ? Explain why or why not. **6**

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