(T) Roll No. .....

PAPER ID—15710

B.Tech. EXAMINATION, 2025

(Third Semester)

CALCULUS AND ORDINARY DIFFERENTIAL EQUATIONS

Btech-2023-108

Time : 3 *Hours* *Maximum Marks :* 70

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (i) State the condition for continuity of a function $f(x, y)$ at a point (x_0, y_0).

(ii) What is the difference between partial derivative and total derivative ?

(iii) What do you mean by the change of order of integration ?

(iv) Write the transformation equations for converting Cartesian coordinates to polar coordinates.

(v) Write the standard form of Bernoulli's equation.

(vi) Define an exact differential equation.

(vii) Write the general form of a linear differential equation with constant coefficients. **7×2=14**

Unit I

2. (a) Explain the concept of limit and continuity of a function of two variables. Test the continuity of : 7

$$f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}; & (x,y)\neq(0,0)\\ 0; & (x,y)=(0,0)\end{cases}$$

(b) Prove Euler's theorem for homogeneous functions of degree n and show that for a homogeneous function $z = f(x, y)$: Verify it for $z = x^{2y} + y^3$. 7

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=nz$$

Verify it for $z = z^2y + y^3$.

3. (a) Find and classify the stationary points of $f(x, y) = x^3—3xy^2$; determine whether they are maxima, minima, or saddle points. 7

(b) Explain the method of Lagrange's undetermined multipliers for finding extrema of functions subject to constraints. 7

Unit II

4. (a) Convert the integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} f(x,y)\,dy\,dx$ into polar coordinates and simplify. 7

(b) Change the order of integration and evaluate : 7

$$\int_0^1 \int_x^1 (x+y)\,dy\,dx$$

5. (a) Derive the formula for finding the area of a region in the plane using a double integral. Apply it to find the area bounded by $y = x$ and $y = x^2$. 7

(b) Find the area bounded by the circle $x^2 + y^2 = a^2$ using polar coordinates. 7

Unit III

6. (a) Solve the Bernoulli's equation : 7

$$\frac{dy}{dx} + \frac{2y}{x} = x^2 y^3$$

(b) Find an integrating factor and solve : 7

$(y + 2x)dx + xdy = 0.$

7. (a) State and derive Newton's law of cooling in the form of a differential equation. 7

(b) Define orthogonal trajectories. Derive the condition that two families of curves are orthogonal trajectories of each other. 7

Unit IV

8. (a) Find the complementary function and particular integral of : 7

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^{2x}$$

(b) Solve : 7

$$(1+y^2)dx-(1+x^2)dy=0$$

9. (a) Find the particular integral of $\frac{d^2y}{dx^2}+4y=\tan(2x)$ using the method of variation of parameters. 7

(b) Derive the differential equation for the charge $q(t)$ in an L-R-C series circuit and discuss the cases of damped and undamped oscillations. 7

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