(T) Roll No. .....

PAPER ID—15710

B.Tech. EXAMINATION, 2024

(Third Semester)

COMPUTER SCIENCE AND ENGINEERING

Calculus and Ordinary Differential Equations

Time : 3 Hours *Maximum Marks : 70*

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit I to V. Q. No. 1 is compulsory. All questions carry equal marks.

Unit I

1. (a) Prove that $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ does not exist.

(b) If $Z = \log(x^2 + xy + y^2)$, find the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$.

(c) Evaluate $\int_0^1 \int_0^x e^{y/x} dy\, dx$.

(d) If $u = x^3 + y^3 + z^3$ and $x = e^{2t}$, $y = e^{2t} \cos 3t$, $z = e^{2t} \sin 3t$ then find $\frac{du}{dt}$.

(e) Solve $(x^2 + y^2 + e^x)dx + 2xy\, dy = 0$

(f) Find the particular integral of differential equation $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$.

(g) Evaluate $\int_0^a \int_0^a \int_0^a (yz + zx + xy)dx\, dy\, dz$.

2×7=14

Unit II

2. (a) If $u = \sin^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, then find the value

of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$. 7

(b) If $\theta = t^n e^{\frac{-r^2}{ut}}$, find the value of n which will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)=\frac{\partial\theta}{\partial t}$. 7

3. (a) Examine for minimum and maximum values of $\sin x + \sin y + \sin(x+y)$. 7

(b) Find the minimum value of $x^2 + y^2 + z^2$, given that $ax + by + cz = p$. 7

Unit III

4. (a) Solve $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$ by changing the order of integration. 7

(b) Show by double integration that area between the parabola $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^3$. 7

5. (a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz\, dy\, dx$. 7

(b) Find the smaller of area bounded by the ellipse $4x^2 + 9y^2 = 36$ and the straight line $2x + 3y = 6$. 7

Unit IV

6. (a) Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$. 7

(b) Solve the differential equation : $(xy^2 - x^2)dx + (3x^2y^2 + x^2y - 2x^3 + y^2)dy = 0$. 7

7. (a) Find the orthogonal trajectories of the family of curves $r^2 = a^2 \cos 2\theta$. 7

(b) The equation of electromotive force in terms of current i for an electrical circuit having resistance R and condenser of capacity C in series, is $E = Ri + \int \frac{i}{C} dt$, find the current i at any time t when $E = E_m \sin \omega t$. 7

Unit V

8. (a) Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=e^{2x}-\cos^2 x$. 7

(b) Solve :

$$(2x+3)^2\frac{d^2y}{dx^2}-(2x+3)\frac{dy}{dx}-12y=6x.$$ 7

9. (a) Solve $\frac{d^2y}{dx^2}+y=\operatorname{cosec} x$ by method of variation of parameters. 7

(b) Solve the simultaneous equations :

$$\frac{dx}{dt}-\frac{dy}{dt}+2y=\cos 2t$$

$$\frac{dx}{dt}+\frac{dy}{dt}-2x=\sin 2t.$$ 7