PAPER ID—15710

B. Tech. EXAMINATION, 2024

(Third Semester)

COMPUTER SCIENCES ENGINEERING

Calculus and Ordinary Differential Equations

Time : 3 Hours *Maximum Marks : 70*

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Solve $\lim_{\substack{x\to 1\\ y\to 2}} \frac{2x^2y}{x^2+y^2+1}$.

(b) Find the second order partial derivatives of $z = x^3 + y^3 - 3axy$.

(c) Evaluate :

$$\int_0^5 \int_0^{x^2} x\left(x^2 + y^2\right) dy\, dx .$$

(d) Solve the following equation :

$$\left(x^2 - ay\right) dx - \left(ax - y^2\right) dy = 0 .$$

(e) Find the orthogonal trajectory of the family of parabolas $y^2 = 4ax$.

(f) Solve :

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0 .$$

(g) Define Cauchy's homogeneous linear differential equation. **7×2=14**

Unit I

2. (a) If $z = \tan\left(y + ax\right) - \left(y - ax\right)^{3/2}$, show that :

$$\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2} .$$ 7

(b) Find the value of n so that the equation :

$$V = r^n \left(3\cos^2\theta - 1\right)$$

satisfies the equation :

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) = 0. \quad 7$$

3. (a) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u \text{ and}$$

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u. \quad 7$$

(b) Divide 24 into three parts such that the continued part of the first square of the second and cube of the third may be maximum. 7

9. (a) Solve the following simultaneous equations :

$$\frac{dx}{dt} = 5x + y, \frac{dy}{dt} = y - 4x . \qquad 7$$

(b) Solve the differential equation :

$$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x . \qquad 7$$