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# PAPER ID—16147

## B.Tech. EXAMINATION, 2024

(First Semester)

COMPUTER SCIENCE AND ENGINEERING

(Cyber Security)

**Code : BSM-101**

Mathematics-I

*Time : 3 Hours* *Maximum Marks : 70*

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

**Note :** Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (i) Define the rank of a matrix.

(ii) Write the general form of a system of linear equations in three variables.

(iii) Find the inverse of $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, if it exists.

(iv) Write all types of the indeterminate form.

(v) State the Rank-Nullity theorem.

(vi) What is the difference between Taylor and Maclaurin series ?

(vii) Define improper integrals and give one example. **14**

## Unit I

2. (a) Solve the following system of equations using Cramer's Rule :

$2x + y - z = 3$; $3x - 2y + 2z = -4$; $x + y + z = 2$

Verify your solution by substituting the values into the equations. 7

(b) Determine whether the following set of vectors is linearly independent : 7

$v_1 = (1, 2, 3)$, $v_2 = [2, 4, 6]$, $v_3 = [3, 6, 9]$.

3. (a) Reduce the following matrix to row echelon form using Gauss Elimination and find its rank : $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$. 7

(b) Find the inverse of the following matrix using the Gauss-Jordan elimination method : $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$. 7

## Unit II

4. (a) Find the basis and dimension of the vector space spanned by the following set of vectors : $v_1 = [1, 0, 2]$, $v_2 = [2, 1, 4]$, $v_3 = [3, 1, 6]$. 7

(b) Find the value of $k$ so that the set S of vectors is linearly dependent in $R^3$, where $S = \{(1, 2, 1); (k, 3, 1); (2, k, 0)\}$ 7

5. (a) For the linear transformation $T : R^3 \to R^3$ defined by : 7

$$T(x) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

find the range, null space, rank and nullity of T. Verify the Rank-Nullity Theorem.

(b) Prove that the set of vectors { [1, 0, 0], [0, 1, 0], [0, 0, 1] } is a basis for $R^3$. 7

## Unit III

6. Determine whether the following matrix is diagonalizable : $A = \begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$; if yes, then diagonalize it. 14

7. Orthogonalize the following set of vectors by using the Gram-Schmidt Process : 14

$v_1 = [1, 1, 0]$, $v_2 = [1, 0, 1]$, $v_3 = [0, 1, 1]$.

## Unit IV

8. (a) Show that $\lim_{x \to 0} \frac{(1+x)^{1/x} - e + \frac{ex}{2}}{x^2} = \frac{11e}{24}$. 7

(b) Evaluate the following improper integral

$$\int_1^\infty \frac{1}{x^2} dx.$$ 7

9. (a) Find the Taylor series expansion of $f(x) = e^x$ about $x = 0$ up to the $4^{th}$ degree term. 7

(b) Establish the relation between Beta and Gamma function. 7