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PAPER ID—16147

B. Tech. EXAMINATION, 2024

(First Semester)

COMPUTER SCIENCE AND ENGINEERING

(CYBER SECURITY)

Code : BSM-101

Mathematics–I

Time : 3 Hours *Maximum Marks : 70*

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all. Unit I is compulsory. Select any *four* questions from Unit II. All questions carry equal marks.

Unit I

1. (a) If $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2\\ -1 & 0\\ 2 & -1\end{bmatrix}$,

then find the product AB. 2

(b) What is the rank of a non-singular matrix of order 4 ? 2

(c) State rank nullity theorem. 2

(d) Find the matrix representing the transformation $T:\mathbb{R}^3 \to \mathbb{R}^2$ given by $T(x, y, z)=(|x|, y-z)$ is not a linear transformation. 2

(e) If 1, 2, 3 are the eigen values of a matrix A then find the eigen values of A^{-1}. 2

(f) State Taylor's theorem with remainder. 2

(g) Evaluate the integral $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^3 x \, dx$. 2

Unit II

2. (a) Find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}. \qquad 7$$

(b) Solve the system of equations :

$$x + y + z = 6,$$
$$x - y + 2z = 5,$$
$$3x + y + z = 8$$

using Cramer's rule. 7

3. (a) Are the following vectors linearly dependent ? If so, find the relation between them :

$$x_1 = (1, 3, 4, 2),$$
$$x_2 = (3, -5, 2, 2),$$
$$x_3 = (2, -1, 3, 2). \qquad 7$$

(b) Investigate the values of λ and μ for which the system of equations :

$$2x + 3y + 5z = 9,$$

$$7x+3y-2z=8,$$

$$2x+3y+\lambda z=\mu,$$

have (i) no solution (ii) unique solution (iii) infinite number of solutions. 7

4. (a) Verify that the mapping $T:\mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (2x - 3y, 7y + 2z)$ is a linear transformation. 7

(b) Show that the set $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ is a basis for the vector space $\mathbb{R}^3(\mathbb{R})$. 7

5. (a) Find the values of a, b, c, if the matrix

$$A=\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$ is orthogonal. 7

(b) Determine the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. 7

6. (a) Find the values of a and b such that :

$$\lim_{x \to 0} \frac{x(1+a\cos x)-b\sin x}{x^3}=1. \qquad 7$$

(b) Verify Lagrange's mean value theorem for the function :

$$f(x)=(x-1)(x-2)(x-3)$$

in the interval (0, 4). 7

7. (a) The curve $x=a\cos\theta$, $y=b\sin\theta$ revolves about the x-axis find the volume of solid generated. 7

(b) State and prove Legendre's duplication formula. 7