(T-1268) Roll No. .....

ID—220-CU-111611

B. Tech. EXAMINATION, 2023

(First Semester)

COMPUTER SCIENCE AND ENGINEERING
(CYBER SECURITY)

Code : BSM-101

Mathematics—I

Time : 3 *Hours* *Maximum Marks :* 70

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, Select *one* question from each Section I-IV. Section V is compulsory.

Section I

1. (a) Find the inverse of the matrix : 7

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(b) Show that the system of equations :

$$x + y + z = 6$$

$$x - y + 2z = 5$$

$$3x + y + z = 8$$

is consistent. If so, find the solution. 7

2. (a) Are the following vectors linearly dependent ? If so, find the relation between them : 7

$x_1 = (1, 2, 4)$, $x_2 = (2, -1, 3)$, $x_3 = (0, 1, 2)$, $x_4 = (-3, 7, 2)$

(b) Find the rank of the matrix : 7

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$$

Section II

3. (a) Define basis. Show that the set $\{(2, 1, 4), (1, -1, 2), (3, 1, -2)\}$ form a basis in $\mathbb{R}^3$. 7

   (b) State and prove rank-nullity theorem. 7

4. (a) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (x, 0)$, where $x, y \in \mathbb{R}^2$. Show that T is a linear transformation. Also find range and null space. 7

   (b) Let $T : U \to V$ is an invertible linear transformation and $T^{-1} : V \to U$ be its inverse. Show that T^{-1} is also a linear transformation. 7

Section III

5. (a) If A and B are two orthogonal matrices, then show that AB is also orthogonal. Further, show that the determinant value $|A| = \pm 1$. 7

(b) Using Gram-Schmidt orthogonalization process, find an orthogonal basis of $V_3(R)$ with standard inner product defined on it, given the basis : 7

$$\{(1, 0, 1), (1, -2, 1), (1, 2, 3)\}$$

6. Diagonalize the matrix : 14

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Section IV

7. (a) Evaluate : 7

$$\lim_{x \to \frac{\pi}{2}} (\sec x)^{\cot x}$$

(b) State and prove Rolle's theorem. 7

8. (a) Find the volume of the reel-shaped solid formed by the revolution about y-axis, of the part of the parabola $y^2 = 4ax$ cutoff by the latusrectum. 7

(b) Prove that : 7

$$B(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

Section V

9. (a) If $A=\begin{bmatrix}1 & 4 & 6\\ 5 & 0 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-1 & -2\\ 6 & 4\\ 9 & -3\end{bmatrix}$, then find the product AB. 2

(b) If the value of a determinant is 5, then find the value of its transpose. 2

(c) Define inner product space. 2

(d) Find the matrix representing the transformation $T:\mathbb{R}^3\to\mathbb{R}^2$ given by $T(x,y,z)=(|x|,y-z)$ is not a linear transformation. 2

(e) What is the product of eigen values of singular matrix ? 2

(f) Show that $\lim_{x\to\infty}\left(1+\frac{1}{n}\right)^n = e$. 2

(g) Evaluate the integral $\int_0^{\pi/2} \sin^2 x \cos^4 x \, dx$. 2