(T) Roll No. ....

PAPER ID—10701

Bachelor of Technology (Electrical Engineering), Bachelor of Technology in Computer Science and Engineering (Cyber Security), Bachelor of Technology in Electronics Engineering (VLSI Design Technology), Bachelor of Technology (Civil Engineering) EXAMINATION, 2025

(Second Semester)

MATHEMATICS-II

Code : BSM-104

Time : 3 Hours *Maximum Marks* : 70

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Define exact differential equations. 2

(b) Define auxiliary equation. 2

(c) Define Laplace transform. 2

(d) State Heaviside's shifting theorem for Laplace transform. 2

(e) Define partial differential equations. 2

(f) Write P.I. for :

$$\left(D^2 - D'^2 + 3D' - 3D\right)z = e^{x+2y} + xy.$$ 2

(g) Write a short note on kurtosis. 2

Unit I

2. (a) Solve :

$$\left(\sin x \cos y + e^{2x}\right)dx + \left(\cos x \sin y + \tan y\right)dy = 0.$$ 7

(b) Find the orthogonal trajectory of the cardoids $r = a\left(1 - \cos\theta\right)$. 7

3. (a) Solve :

$$(D-2)^2 y = 8\left(e^{2x} + \sin 2x + x^2\right). \qquad 7$$

(b) Solve :

$$(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1. \qquad 7$$

Unit II

4. (a) Find the Laplace transform of :

$$\frac{e^{at} - \cos bt}{t}. \qquad 7$$

(b) Evaluate :

$$\int_0^{\infty} t^3 e^{-t} \sin t \, dt. \qquad 7$$

5. (a) Sketch the function and express it in terms of unit step function. Hence obtain the Laplace transform :

$$f(r) = \begin{cases} t^2, & 0 < t \le 2 \\ 4t, & t > 2 \end{cases}. \qquad 7$$

(b) Find the inverse Laplace transform of

$$\frac{6}{2p-3}-\frac{3+4p}{9p^2-16}+\frac{8-6p}{16p^2+9}.$$ 7

Unit III

6. (a) Form a partial differential equation by eliminating the arbitrary functions from $z = xf_1(x+t)+f_2(x+t)$. 7

(b) Solve the differential equation :

$$\left(x^2-y^2-z^2\right)p+2xyq=2xz.$$ 7

7. (a) Solve :

$$2zx-px^2-2qxy+pq=0$$

by Charpit's method. 7

(b) Use the method of separation of variables to solve the equation :

$$\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial y}+u; \quad u(x,0)=3e^{-5x}-2e^{-3x}.$$

7

Unit IV

8. (a) Ten students got the following percentage of marks in Mathematics and English :

Roll No.	Mathematics	English
1	78	84
2	36	51
3	98	91
4	25	60
5	75	68
6	82	62
7	90	86
8	62	58
9	65	53
10	39	47

Calculate the rank correlation coefficient.

10

(b) Derive the angle between two lines of regression. 4

9. (a) Fit a straight line to the following data : 7

X	Y
0	1
1	1.8
2	3.3
3	4.5
4	6.3

(b) Goals scored by two teams A and B in a football season were as follows :

No. of goals scored in a match	**No. of matches A**	**B**
0	27	17
1	9	9

2	8	6
3	5	5
4	4	3

Find out which team is more consistent. 7

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(T) Roll No. ....

PAPER ID—10707

B.Tech. Computer Science and Engineering (Internet of things and Cyber Technology including block chain technology), Bachelor of Technology (Computer Science and Engineering), Bachelor of Technology in Computer Science and Engineering (Artificial Intelligence), Bachelor of Technology in Computer Science and Engineering (Internet of things), Bachelor of Technology (Computer Science and Engineering (Artificial Intelligence and date science)), etc. (Common for All) EXAMINATION, 2025

(Second Semester)

MATHEMATICS-II

Time : 3 *Hours* *Maximum Marks* : 70

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Define random variable and state its types. 2

(b) Define Moments. 2

(c) Name a discrete distribution with equal mean and variance. Also state the mean and variance. 2

(d) State the 'lack memory' property of exponential distribution. 2

(e) Write the relation between central and non-central moments. 2

(f) Write a short note on skewness. 2

(g) Define level of significance. 2

Unit I

2. (a) State and prove Chebyshev's inequality. 7

(b) Derive Poisson approximation as a limiting case of binomial distribution. 7

3. For a discrete random variable, prove the following : **14**

(a) $E(aX_1 + bX_2) = aE(X_1) + bE(X_2)$

(b) $V(a_1X_1 + a_2X_2) = a_1^2V(X_1) + a_2^2V(X_2) + 2a_1a_2Cov(X_1, X_2)$

(c) $E(aX + b) = aE(X) + b$

Unit II

4. The amount of bread (in hundreds of pounds) X that a certain bakery is able to sell in a day is found to be a numerical valued random phenomenon, with a probability density function $f(x)$, given by :

$$f(x) = \begin{cases} A.x & \text{for} \quad 0 \le x \le 5 \\ A(10-x) & \text{for} \quad 5 \le x \le 10 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of A such that $f(x)$ is a probability density function. 2

(b) What is the probability that the number of pounds of bread that will be sold tomorrow is : 6

(i) D = more than 500 pounds,

(ii) E = less than 500 pounds,

(iii) F = between 250 and 750 pounds ?

(c) For D, E, F defined above : **6**

(i) Find P(D | E), P(D | F).

(ii) Are D and E independent events ?

(iii) Are D and F independent events ?

5. If the distribution of two independent random variables X and Y is given by $f_X(x) = 1$; $0 \leq x \leq 1$ and $f_Y(y) = 1$; $0 \leq y \leq 1$, then find the pdf of X + Y. **14**

Unit III

6. (a) Derive the angle between two lines of regression. **7**

(b) Write a note on Normal Distribution. Write its properties. 7

7. Fit an exponential curve of the form $Y = ab^X$ to the following data : 14

X	Y
1	1.0
2	1.2
3	1.8
4	2.5
5	3.6
6	4.7
7	6.6
8	9.1

Unit IV

8. (a) A random sample of 900 members has a mean 3.2 cm. Can it be reasonably regarded as a sample from a large population of mean 3.2 cm and standard deviation 2.3 cm ? Also find 95% confidence interval. Consider tabulated value 1.96 at 5% level of significance. 7

(b) Random samples of 400 men and 600 women were asked whether they would like to have a school near their residence. 200 men and 325 women were in favour of proposal. Test the hypothesis that the proportion of men and women in favour of the proposal are same at 1% level of significance. Consider tabulated value 2.58 at 1% level of significance. 7

9. The following table gives for a sample of married women, the level of education and marriage adjustment score :

		Marriage Adjustment Score			
		Very Low	Low	High	Very High
	College	24	97	62	58
Level of Education	High School	22	28	30	41
	Middle School	32	10	11	20

Can you conclude from the above, 'the higher the level of education, the greater is the degree of adjustment in marriage' ? Consider tabulated value 12.592 at 5% level of significance. **14**

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(220-CU-211611) Roll No. ....

PAPER ID—10707

B.Tech. EXAMINATION, 2024

(Second Semester)

ARTIFICIAL INTELLIGENCE AND DATA SCIENCE

Code : BSM-102

Mathematics-II

Time : 3 *Hours* *Maximum Marks* : 70

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all. Q. No. **1** (Unit I) is compulsory. Attempt any *four* questions from Unit II.

Unit I

1. (a) Find the probability of drawing a king and an ace in this order from a pack of cards in two successive draws assuming that first card drawn is not replaced.

   (b) Explain Bayes' theorem.

   (c) Probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men now 60, at least 7 would live to be 70 ?

   (d) Define Binomial probability distribution.

   (e) Rank differences for 5 pairs of ranks are 5, – 3, –1, 1, K. Find Spearman's rank correlation coefficient.

   (f) What is Skewness ? Define the various measures of skewness.

   (g) Describe Chi-square test. $2\times7=14$

Unit II

2. (a) Five defective bulbs are accidentally mixed with twenty good ones. It is not possible to just look at a bulb and tell whether or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot. 7

(b) A car hire firm has two cars which it hires out daily. The number of demand for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the number of days out of 100 days on which (i) neither car is used (ii) some demand is refused. 7

3. State and prove Chebyshev's Inequality. 14

4. (a) In a bolt factory, there are four machines A, B, C and D manufacturing 20%, 15%, 25% and 40% of the total output respectively. Of their outputs 5%, 4%, 3% and 2% in the same order are defective bolts. A bolt is chosen at random from the factory's production and is found defective. By using Bays' theorem find the probability that the bolt was manufactured by machine A or machine D. 7

(b) Explain continuous random variables and their properties. 7

5. (a) Fit the second degree parabolic trend to the following data : 7

Year	Production (in crore Rs.)
1983	5
1984	7

1985	4
1986	9
1987	12

(b) Calculate Karl Pearson's coefficient of correlation from the following data : 7

X	Y
24	18
27	20
28	22
28	25
29	22
30	28
32	28
33	30
35	27
35	30
40	22

4	59
5	57
6	59

By using Chi-square test whether the die is biased or not. Given that $\chi^2_{0.05}$ at 5 d.f. is 11.09.

(T) Roll No. ....

PAPER ID—15796

B.Tech. EXAMINATION, 2024

(First Semester)

ELECTRONICS ENGINEERING

(VLSI Design Technology)

Code : BSM-103

Mathematics-I

Time : 3 Hours *Maximum Marks : 70*

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Define skew-symmetric matrix with example. 2

(b) Define Orthogonal Transformation. 2

(c) Explain Euler's formula. 2

(d) Define the limit of a function. 2

(e) Write the half-range cosine series. 2

(f) Define a homogeneous function. 2

(g) Write the duplication formula for Gamma function. 2

Unit I

2. (a) Verify Cayley-Hamilton theorem, find the inverse of the matrix : 7

$$\begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

(b) Using the matrix method, show that the equations $x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$ are consistent and hence obtain the solutions for x, y and z. 7

3. (a) Find the eigen values and eigen vectors of the matrix : 7

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Find the value of k so that the equations $x + ky + 3z = 0$, $4x + 3y + k = 0$ and $2x + y + 2z = 0$ have a non-trivial solution. 7

Unit II

4. (a) Examine the convergence or otherwise of the series : 7

$$\frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + ... + \frac{1}{n(n+1)} + ... \text{ to } \infty.$$

(b) Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \leq x \leq \pi$. 7

5. (a) Show that the series of real numbers $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p < 1$. 7

(b) Obtain the half-range sine series for e^x in $0 < x \leq 1$. 7

Unit III

6. (a) If $y = (x^2 - 1)^n$, then prove that : 7

$$(x^2 - 1) y_{n+2} + 2x y_{n+1} - n(n+1) y_n = 0$$

(b) Show that the Maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{\frac{1}{e}}$. 7

7. (a) Examine the function $x^3 + y^3 - 3axy$ for maxima and minima. 7

(b) Find $\frac{\partial^3 u}{\partial x \partial y \partial z}$ If $u = e^{xyz}$. 7

Unit IV

8. (a) Evaluate the triple integral $\iiint_V (x + y + z)\, dV$ where V is the region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$. 7

(b) Evaluate the integral : 7

$$I = \int_1^{\ln 8} \int_0^{\ln y} e^{x+y}\, dx\, dy.$$

9. (a) Evaluate $\iint y\, dx\, dy$ over the region R bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. 7

(b) Show that : 7

$$\beta(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

350

(T) Roll No. ....

PAPER ID—16147

B.Tech. EXAMINATION, 2024

(First Semester)

COMPUTER SCIENCE AND ENGINEERING

(Cyber Security)

Code : BSM-101

Mathematics-I

Time : 3 Hours *Maximum Marks : 70*

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (i) Define the rank of a matrix.

(ii) Write the general form of a system of linear equations in three variables.

(iii) Find the inverse of $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, if it exists.

(iv) Write all types of the indeterminate form.

(v) State the Rank-Nullity theorem.

(vi) What is the difference between Taylor and Maclaurin series ?

(vii) Define improper integrals and give one example. **14**

Unit I

2. (a) Solve the following system of equations using Cramer's Rule :

$2x + y - z = 3; \quad 3x - 2y + 2z = -4;$
$x + y + z = 2$

Verify your solution by substituting the values into the equations. 7

(b) Determine whether the following set of vectors is linearly independent : 7

$v_1 = (1, 2, 3)$, $v_2 = [2, 4, 6]$, $v_3 = [3, 6, 9]$.

3. (a) Reduce the following matrix to row echelon form using Gauss Elimination and find its rank : $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$. 7

(b) Find the inverse of the following matrix using the Gauss-Jordan elimination method : $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$. 7

Unit II

4. (a) Find the basis and dimension of the vector space spanned by the following set of vectors : $v_1 = [1, 0, 2]$, $v_2 = [2, 1, 4]$, $v_3 = [3, 1, 6]$. 7

(b) Find the value of k so that the set S of vectors is linearly dependent in R^3, where $S = \{(1, 2, 1); (k, 3, 1); (2, k, 0)\}$ **7**

5. (a) For the linear transformation $T : R^3 \to R^3$ defined by : **7**

$$T(x) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

find the range, null space, rank and nullity of T. Verify the Rank-Nullity Theorem.

(b) Prove that the set of vectors { [1, 0, 0], [0, 1, 0], [0, 0, 1] } is a basis for R^3. **7**

Unit III

6. Determine whether the following matrix is diagonalizable : $A = \begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$; if yes, then diagonalize it. **14**

7. Orthogonalize the following set of vectors by using the Gram-Schmidt Process : **14**

$v_1 = [1, 1, 0]$, $v_2 = [1, 0, 1]$, $v_3 = [0, 1, 1]$.

Unit IV

8. (a) Show that $\lim_{x \to 0} \frac{(1+x)^{1/x} - e + \frac{ex}{2}}{x^2} = \frac{11e}{24}$. 7

(b) Evaluate the following improper integral $\oint_1^{\infty} \frac{1}{x^2} dx$. 7

9. (a) Find the Taylor series expansion of $f(x) = e^x$ about $x = 0$ up to the 4^{th} degree term. 7

(b) Establish the relation between Beta and Gamma function. 7

PAPER ID—10707

B. Tech. (CSE/AI/IOT/CS All Computer) EXAMINATION, 2023

(Second Semester)

MATHEMATICS-II

Time : 3 *Hours* *Maximum Marks* : 70

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory. Attempt any *four* questions from Unit II.

Unit I

1. (a) A bag contains 10 white and 5 black balls. Two balls are drawn at random one after the other without replacement. Find the probability that both balls are black.

(b) Define Bayes' theorem.

(c) Ten coins are tossed simulataneously. Find the probability of getting at least seven heads.

(d) Define binomial probability distribution.

(e) Calculate coefficient of rank correlation :

x :	8	7	6	3	2	1	5	4
y :	7	5	4	1	3	2	6	8

(f) What is Kurtosis ? How is it measured ?

(g) What is Chi-square test of independence under what condition it is applicable ?

2×7=14

Unit II

2\. (a) A random variable x has the following probability distribution :

x :	0	1	2	3	4	5	6	7	8
$p(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(i) Determine the value of a

(ii) Find $P(X \geq 3)$

(iii) What is the smallest value of x for which $P(X \leq x) > 0.5$? 7

(b) Between the hours of 2 pm and 4 pm the average number of phone calls per minute comming into a switch board of a company is 2.5. Find the probability that during one particular minute there will be :

(i) exactly 3 calls

(ii) at least 2 calls. 7

3. State and prove Chebyshev's inequality. 14

4. (a) In a bulb factory, machine A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B ? 7

(b) Explain continuous random variables and their properties. 7

5. (a) Price of commodity during 1981-1986 are given below. Fit a second degree parabola to the following data and calculate the trend, values and estimate the price of the commodity in 1987 : 7

Year :	1981	1982	1983	1984	1985	1986
Price :	110	114	120	138	152	218

(b) Find the coefficient of Correlation from the following data : 7

X :	10	12	18	16	15	19	18	17
Y :	30	35	45	44	42	48	47	46

6. (a) Calculate first four moments about mean for the following distribution : 7

x :	2.0	2.5	3.0	3.5	4.0	4.5	5.0
y :	5	38	65	92	70	40	10

(b) A manufacturer claims that only 4% of his products supplied by him are defective. A random sample of 600 products contained 36 defectives. Test the claim of the manufacturer. 7

7. (a) Random samples drawn from two countries gave the following data relating to the heights of adult males : 7

	Country A	Country B
Mean height (in inches)	67.42	67.25
Standard deviation	2.58	2.50
Number of Samples	1000	1200

(i) Is the difference between the means significant ?

(ii) Is the difference between the standard deviations significant ?

(b) Verify whether Poisson distribution can be assumed from the data given below and test the goodness of fit :

No. of defects :	0	1	2	3	4	5
Frequency :	6	13	13	8	4	3

Given that χ^2 at 5% level of significance for 4 d.f. is 9.49. 7

PAPER ID—10707

B. Tech. (CSE/AI/IOT/CS All Computer)

EXAMINATION, 2023

(Second Semester)

MATHEMATICS-II

Time : 3 *Hours* *Maximum Marks* : 70

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Note : Attempt *Five* questions in all. Q. No. **1** is compulsory. Attempt any *four* questions from Unit II.

Unit I

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(e) Calculate coefficient of rank correlation :

x :	8	7	6	3	2	1	5	4
y :	7	5	4	1	3	2	6	8

(f) What is Kurtosis ? How is it measured ?

(g) What is Chi-square test of independence under what condition it is applicable ?

2×7=14

Unit II

2\. (a) A random variable x has the following probability distribution :

x :	0	1	2	3	4	5	6	7	8
$p(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(i) Determine the value of a

(ii) Find $P(X \geq 3)$

(iii) What is the smallest value of x for which $P(X \leq x) > 0.5$? 7

(b) Between the hours of 2 pm and 4 pm the average number of phone calls per minute comming into a switch board of a company is 2.5. Find the probability that during one particular minute there will be :

(i) exactly 3 calls

(ii) at least 2 calls. 7

3. State and prove Chebyshev's inequality. **14**

4. (a) In a bulb factory, machine A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B ? 7

(b) Explain continuous random variables and their properties. 7

5. (a) Price of commodity during 1981-1986 are given below. Fit a second degree parabola to the following data and calculate the trend, values and estimate the price of the commodity in 1987 : 7

Year :	1981	1982	1983	1984	1985	1986
Price :	110	114	120	138	152	218

(b) Find the coefficient of Correlation from the following data : 7

X :	10	12	18	16	15	19	18	17
Y :	30	35	45	44	42	48	47	46

6. (a) Calculate first four moments about mean for the following distribution : 7

x :	2.0	2.5	3.0	3.5	4.0	4.5	5.0
y :	5	38	65	92	70	40	10

(b) A manufacturer claims that only 4% of his products supplied by him are defective. A random sample of 600 products contained 36 defectives. Test the claim of the manufacturer. 7

7. (a) Random samples drawn from two countries gave the following data relating to the heights of adult males : 7

	Country A	Country B
Mean height (in inches)	67.42	67.25
Standard deviation	2.58	2.50
Number of Samples	1000	1200

(i) Is the difference between the means significant ?

(ii) Is the difference between the standard deviations significant ?

(b) Verify whether Poisson distribution can be assumed from the data given below and test the goodness of fit :

No. of defects :	0	1	2	3	4	5
Frequency :	6	13	13	8	4	3

Given that χ^2 at 5% level of significance for 4 d.f. is 9.49. 7

PAPER ID—16147

B. Tech. EXAMINATION, 2024

(First Semester)

COMPUTER SCIENCE AND ENGINEERING
(CYBER SECURITY)

Code : BSM-101

Mathematics–I

Time : 3 Hours *Maximum Marks* : 70

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all. Unit I is compulsory. Select any *four* questions from Unit II. All questions carry equal marks.

Unit I

1. (a) If $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2\\ -1 & 0\\ 2 & -1\end{bmatrix}$,

then find the product AB. 2

(b) What is the rank of a non-singular matrix of order 4 ? 2

(c) State rank nullity theorem. 2

(d) Find the matrix representing the transformation $T:\mathbb{R}^3 \to \mathbb{R}^2$ given by $T(x, y, z)=(|x|, y-z)$ is not a linear transformation. 2

(e) If 1, 2, 3 are the eigen values of a matrix A then find the eigen values of A^{-1}. 2

(f) State Taylor's theorem with remainder. 2

(g) Evaluate the integral $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^3 x\, dx$.

2

Unit II

2. (a) Find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$ 7

(b) Solve the system of equations :

$$x + y + z = 6,$$
$$x - y + 2z = 5,$$
$$3x + y + z = 8$$

using Cramer's rule. 7

3. (a) Are the following vectors linearly dependent ? If so, find the relation between them :

$$x_1 = (1, 3, 4, 2),$$
$$x_2 = (3, -5, 2, 2),$$
$$x_3 = (2, -1, 3, 2).$$ 7

(b) Investigate the values of λ and μ for which the system of equations :

$$2x + 3y + 5z = 9,$$

$$7x+3y-2z=8,$$

$$2x+3y+\lambda z=\mu,$$

have (i) no solution (ii) unique solution (iii) infinite number of solutions. 7

4. (a) Verify that the mapping $T:\mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (2x - 3y, 7y + 2z)$ is a linear transformation. 7

(b) Show that the set $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ is a basis for the vector space $\mathbb{R}^3(\mathbb{R})$. 7

5. (a) Find the values of a, b, c, if the matrix

$$A=\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$ is orthogonal. 7

(b) Determine the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. 7

6. (a) Find the values of a and b such that :

$$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1.$$ 7

(b) Verify Lagrange's mean value theorem for the function :

$$f(x)=(x-1)(x-2)(x-3)$$

in the interval (0, 4). 7

7. (a) The curve $x=a\cos\theta$, $y=b\sin\theta$ revolves about the x-axis find the volume of solid generated. 7

(b) State and prove Legendre's duplication formula. 7

ID—220-CU-111611

B. Tech. EXAMINATION, 2023

(First Semester)

COMPUTER SCIENCE AND ENGINEERING

(CYBER SECURITY)

Code : BSM-101

Mathematics—I

Time : 3 *Hours* *Maximum Marks* : 70

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, Select *one* question from each Section I-IV. Section V is compulsory.

Section I

1. (a) Find the inverse of the matrix : 7

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(b) Show that the system of equations :

$$x + y + z = 6$$

$$x - y + 2z = 5$$

$$3x + y + z = 8$$

is consistent. If so, find the solution. 7

2. (a) Are the following vectors linearly dependent ? If so, find the relation between them : 7

$x_1 = (1, 2, 4)$, $x_2 = (2, -1, 3)$, $x_3 = (0, 1, 2)$, $x_4 = (-3, 7, 2)$

(b) Find the rank of the matrix : 7

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$$

Section II

3. (a) Define basis. Show that the set {(2, 1, 4), (1, –1, 2), (3, 1, –2)} form a basis in $\mathbb{R}^3$. 7

(b) State and prove rank-nullity theorem. 7

4. (a) Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (x, 0)$, where $x, y \in \mathbb{R}^2$. Show that T is a linear transformation. Also find range and null space. 7

(b) Let T : U → V is an invertible linear transformation and T^{-1} : V → U be its inverse. Show that T^{-1} is also a linear transformation. 7

Section III

5. (a) If A and B are two orthogonal matrices, then show that AB is also orthogonal. Further, show that the determinant value $|A| = \pm 1$. 7

(b) Using Gram-Schmidt orthogonalization process, find an orthogonal basis of $V_3(R)$ with standard inner product defined on it, given the basis : 7

$$\{(1, 0, 1), (1, -2, 1), (1, 2, 3)\}$$

6. Diagonalize the matrix : 14

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Section IV

7. (a) Evaluate : 7

$$\lim_{x \to \frac{\pi}{2}} (\sec x)^{\cot x}$$

(b) State and prove Rolle's theorem. 7

8. (a) Find the volume of the reel-shaped solid formed by the revolution about y-axis, of the part of the parabola $y^2 = 4ax$, cutoff by the latusrectum. 7

(b) Prove that : 7

$$B(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

Section V

9. (a) If $A=\begin{bmatrix}1 & 4 & 6\\ 5 & 0 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-1 & -2\\ 6 & 4\\ 9 & -3\end{bmatrix}$,

then find the product AB. 2

(b) If the value of a determinant is 5, then find the value of its transpose. 2

(c) Define inner product space. 2

(d) Find the matrix representing the transformation $T:\mathbb{R}^3\rightarrow\mathbb{R}^2$ given by $T(x,y,z)=(|x|,y-z)$ is not a linear transformation. 2

(e) What is the product of eigen values of singular matrix ? 2

(f) Show that $\lim_{x \to \infty} \left(1 + \frac{1}{n}\right)^n = e$. 2

(g) Evaluate the integral $\int_0^{\pi/2} \sin^2 x \cos^4 x \, dx$. 2

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B. Tech. EXAMINATION, 2023

(First Semester)

ELECTRONICS ENGINEERING

(VLSI DESIGN TECHNOLOGY)

Code : BSM-103

Mathematics–I

Time : 3 Hours *Maximum Marks : 70*

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all. Select *one* question each from Section I to Section IV. Section V is compulsory.

Section I

1. (a) Find the inverse of the matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$ 7

(b) Investigate the values of λ and μ so that the system of equations :

$$2x + 3y + 5z = 9,$$

$$7x + 3y - 2z = 8,$$

$$2x + 3y + \lambda z = \mu$$

Have (i) no solution (ii) unique solution (iii) infinite no. of solutions. 7

2. Diagonalize the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. **14**

Section II

3. (a) State and prove the necessary condition for convergence of an infinite series. 7

(b) Test the convergence of an infinite series :

$$1 = \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \ldots\ldots$$ 7

4. (a) Expand sin x in powers of $\left(x - \frac{\pi}{6}\right)$. Hence find the value of sin 32°. 7

(b) Obtain half range cosine series for the function :

$$f(x) = \begin{cases} kx, & 0 \le x \le \frac{l}{2} \\ k(l - x) & \frac{l}{2} \le x \le l \end{cases}$$ 7

Section III

5. (a) Find the derivative of the function $f(x) = \sec x$. 7

(b) If $y = (\sin^{-1} x)^2$, show that :

$$(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - n^2 y_n = 0.$$ 7

6. (a) State Euler's theorem. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u\,.$$ 7

(b) A rectangular box, open at the top, is to have a volume of 32 cc. Find the dimensions of the box requiring least material for its construction. 7

Section IV

7. (a) Find the volume of the reel-shaped solid formed by the revolution about y-axis, of the part of the parabola $y^2 = 4ax$ cutoff by the latusrectum. 7

(b) Change of the order of integration $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\,dy\,dx$. Hence evaluate it. 7

8. (a) Evaluate the integral :

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx\,.$$ 7

(b) Prove that :

$$B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$ 7

Section V

9\. (a) Define elementary matrices. 2

(b) What is the product of eigen values of singular matrix ? 2

(c) State test for convergence of an infinite series. 2

(d) Test the behaviour of an infinite series :

$$1+4-7+1+4-7+\ldots\ldots$$ 2

(e) If $x = r\cos\theta$, $y = r\sin\theta$, find the Jacobian $\frac{\partial(x, y)}{\partial(r, \theta)}$. 2

(f) Show that $\lim_{x \to \infty} \frac{\sin x}{x} = 0$. 2

(g) Evaluate the integral :

$$\int_0^{\frac{\pi}{2}} \sin^2 x \cos^4 x \, dx.$$ 2