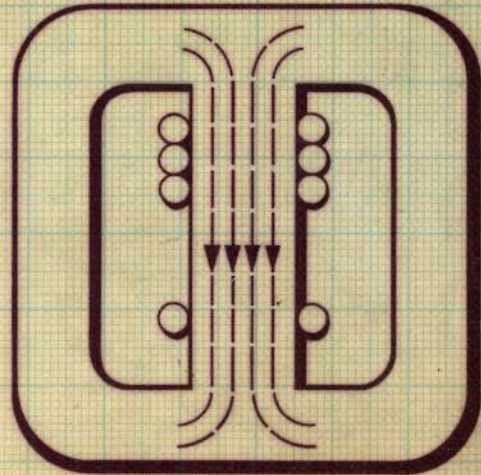
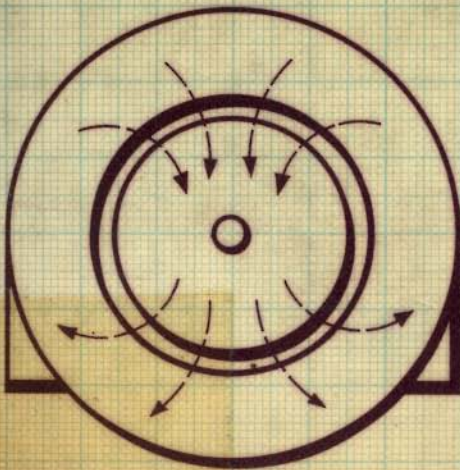


A PITMAN INTERNATIONAL TEXT

Alternating Current
Machines

Alternating Current Machines

M G Say



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Alternating Current Machines

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Preface

The forerunner of this book was *The Performance and Design of A. C. Machines*, first published in 1936 and subsequently revised and reprinted a dozen times. The combination of design and performance proved gratifyingly acceptable, and the book found its way around the English-reading world.

But times have changed. Design, revolutionized by the digital computer, has become the esoteric preoccupation of the few: performance interests have widened as a result of the advent of the big machine, the problem of machine and power-system interaction, the transient dynamics of starting and braking, and the influence on machine characteristics and control methods of power electronics. That few textbooks have explored this theme is due in part to the strong analytical counter-attractions of the 'generalization' concept, in which machine 'types' become variants of a common principle expressed in terms of electric-circuit equations of behaviour. Powerful though the method is, circuit equations can model a machine only in a restricted way. Real machines have real magnetic, mechanical and thermal attributes as well, and although their basic electromagnetic principles are the same, the several types do, in fact, differ markedly in performance.

The present text expresses my belief that machine performance can be grasped more readily through appreciation of flux-current interaction than by the manipulation of matrix equations; the latter give meaningful quantitative results only if the physical concepts are understood. The treatment therefore begins with the unified electro-magnetic principles of transformers and machines, followed by material common to both on magnetic circuits, windings and heat-dissipation. In dealing with practical transformers and three-phase induction and synchronous machines, the simple theory of the ideal case is followed progressively by departures from the ideal that affect the practical performance in both steady and transient states. Design is discussed in general terms, and some constructional details are illustrated. Machines having special features, and those to which normal two-axis methods are not so readily applied, are discussed in the final Chapter.

All formulae, whether concerned with electrical, magnetic, mechanical or thermal phenomena, are couched consistently in SI units. Only in numerical examples are decimal multiples or sub-multiples employed for convenience, in conformity with IEC recommendations. Guidance in the use of SI units,

if needed, can be gained from a booklet ('Symbols and Abbreviations') published by the Institution of Electrical Engineers.

My thanks are due to good friends in the industry and in the Universities, abroad as well as in the UK: their help has been warm and unstinting. I hope that my readers, too, will help by their critical comments and suggestions for improvement.

Edinburgh 1976

M. G. Say

1

Introductory

1.1 *Alternating-current machines*

Electric and magnetic fields both store energy, and useful mechanical forces can be derived from them. In air (or other gas at normal pressure) the dielectric strength of the medium restricts the working *electric* field intensity to about 3 MV/m and the energy density in consequence to about 40 J/m^3 ; this corresponds to a force between displaced charges of 40 N/m^2 . There is no comparable restriction on *magnetic* fields, but the saturation of ferromagnetic media required to complete the magnetic circuit limits the working flux density to about 1.6 T, for which the energy density in air is about 1 MJ/m^3 . As this is 25 000 times as much as for the electric field, almost all industrial electric machines are magnetic in principle; and they are inherently a.c. machines because of the occurrence of magnetic poles in pairs, leading to a natural N—S sequence. The movement of a conductor through such a sequence induces an e.m.f. which changes direction in accordance with the magnetic polarity: in brief an *alternating* e.m.f.

At present, the chief source of electrical energy is the mechanical form, direct (as in hydraulic storage reservoirs or rivers), or indirect from chemical or nuclear energy via heat. Conversion of mechanical to or from electrical energy is by use of the mechanical force manifested when a current-carrying conductor lies in a magnetic field.

The wide use of electrical energy has been fostered by the development of systems of transmission and distribution, so that interconnected networks enable generating plant to be concentrated, and a reliable, economic supply to be achieved. A high voltage is required to limit loss in the transmission of large powers; a much lower one is necessary for distribution for reasons connected with safety and convenience. The transformer makes this economically possible.

Thus both generation and transformation are inherently a.c. phenomena, and the collateral development of a.c. generators and motors has been the result. The a.c. system is not, however, a finality: in some applications the d.c. machine is superior, and d.c. transmission is technically possible. But a.c. machines and transformers seem likely to remain important components in the pattern of electrical energy generation and utilization.

1.2 Performance

Generators, motors and transformers form part of an electro-mechanical energy network. The various parts of such a system are interconnected and react upon each other, sometimes considerably and on occasion catastrophically. No machine can be studied in isolation, for its behaviour is influenced by the electrical supply at one end and the mechanical attachments at the other, between which it forms a dynamic link. The performance of a machine is therefore not just its steady-state characteristics, but more importantly its dynamic and transient behaviour, in which the rest of the system plays a possibly paramount role.

Consider an underground coal-cutter motor at the far end of a long mine cable. When the machine is started on load there will be a considerable voltage drop in the cable impedance, the motor terminal voltage will be abnormally low, and the starting torque will be seriously reduced. The 'normal' steady-state characteristic of the motor is not applicable: the performance must be taken in relation to the system to which the machine is connected. Again, although a large supply system is not likely to be significantly affected by a 50 W hand-tool motor, the starting of a 25 MW industrial machine may require previous notification to the supply authority. A machine that develops substantial harmonic voltages and currents can be an embarrassment to other consumers' plant.

The performance of a machine has a great many facets, among them the mechanical characteristics of prime-movers or fluctuating loads, frequent starting and stopping duty-cycles, stored kinetic energy, ambient conditions (particularly of cooling), and the like. The machine is a link in a *system*.

System

Systems engineering is concerned with the planning, design, construction and operation of complex interconnected systems. A *system* is regarded as an assembly of *sub-systems*: in an electric supply network these could be fuel-handling equipments, boiler plant, turbines, generators, transformers, transmission lines, distributors, motors and mechanical loads. The operation of the system as a whole is a function of the subsystem characteristics. The overall performance is studied in systems engineering in accordance with an objective such as optimized cost, overall reliability, or stability under conditions of fault.

Model

To evaluate system performance it is necessary to have a quantitative model of each subsystem in the form of a set of equations, a simulation or a compilation of graphical data. The subsystem models are then interconnected with due regard to their individual constraints. The model of an electrical machine can take many forms, from an elementary description in terms of current sheets and fluxes to sets of sophisticated equations in terms of voltage, current, speed, time, inertia, friction and electrical parameters. Much depends on the use to which a model is to be put. A transformer, for example, can for

normal operating conditions be represented by a simple T-network of impedances; but this will not serve for problems such as surge distribution, short-circuit force or temperature-rise. Performance must be interpreted in terms of the significant conditions of operation. The more comprehensive the study, the more complex must be the model. Eventually, the only possible model of a machine is the machine itself.

1.3 Design

Design, the creative translation of a concept into a reality, is (in the present context) the application of science, technology and invention to the realization of a machine to perform specified functions with optimum economy and efficiency. In a large-scale manufacturing process the design activities include a balanced consideration of the following.

Basics. Previous experience and new ideas; research into extension of technology of materials, stresses and limits; manufacturing and transport facilities; ergonomics, environmental effects, aesthetics, safety, reliability, maintenance; economics and cost analysis; optimization.

Communication. Transfer of design to factory floor (drawings, instructions, processes, numerical and automatic methods); job-flow, critical path, timing, delivery date.

Specification. Customer's requirements; standards, codes; interpretation of specification, subdivision into specialist design areas, co-ordination.

Information. Data sources (previous contracts, technical journals, research reports); *ad hoc* research and development.

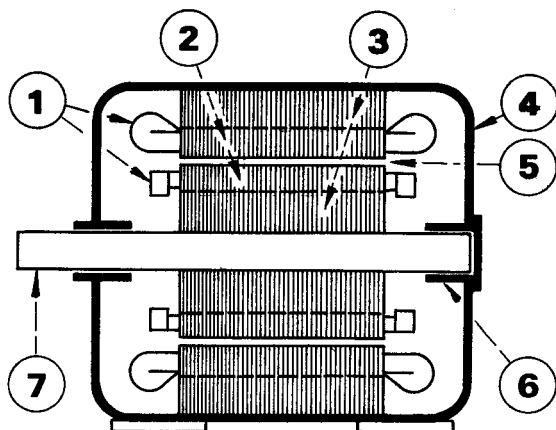
Design Team. Allocation of tasks; technical design analyses; group design, computation; synthesis of group designs, matching and compatibility, prediction of performance, achievement of specification and guarantees, optimization.

Optimization is not easy to define. An optimum solution can be so only in relation to specific objectives, but the choice of these depends on what is meant by optimization. It may not be merely 'cost reduction', but may involve a quantitative appreciation of all the criteria that identify the best design for a given duty. A single criterion, such as minimum mass or minimum manufacturing cost, might be readily definable; more often the weighted combination of several conflicting requirements is the determining factor. Such formulations are inherently less amenable to quantization than are analyses of technical performance.

Machines and Transformers

As an example, Fig. 1.1 lists the main areas concerned in the design of a motor. Many of the items are interrelated; the choice of slot dimensions affects leakage flux and therefore reactance, starting performance, torque/speed relation, heating, ventilation . . . , but the analytical relations between them are indirect and not easy to formulate.

Given the specification that a machine has to meet, the main design areas are: the magnetic circuit, the windings, the insulation, the heat dissipation and the mechanical construction. There is an economic conflict for space



1.1 Design and constructional features of a rotating machine.

Specification: rating, standards, guarantees, loadings, frame size, starting, efficiency, temperature-rise.

Performance: starting torque, transient behaviour, system considerations, short-circuit forces, load loss, optimization.

Manufacture: materials handling, works practice, works processes, available skills, transportation to site.

- (1) *Windings:* type, connection, conductors, turns, transposition, bracing, current density, resistance, eddy-currents, I^2R loss, cooling, leakage inductance, insulation, connections, commutator/slip-rings, mass.
- (2) *Slots and Teeth:* dimensions, insulation, flux density, core loss, mechanical forces, wedges.
- (3) *Cores:* material, stamping, clamping, insulation, mounting, dimensions, flux density, core loss, mass.
- (4) *Frame:* size, stiffness, vibration, fixing, enclosure, distortion, expansion, brushgear, mass.
- (5) *Airgap:* length, gap factor, flux distribution, space harmonics, clearance, magnetic pull, noise.
- (6) *Bearings:* type, loading, loss, cooling, lubrication.
- (7) *Shaft:* size, stiffness, deflection, critical speed.

between constructional steel, ferromagnetic material, copper (or aluminium), insulation and coolant. Limitations arise from the inherent material properties such as elastic limit, magnetic saturation, temperature-rise and efficiency — or, rather, loss as affecting the dissipation of heat. Many questions have to be answered: What form of insulation will have adequate life when subject to given electrical, thermal and mechanical stresses? How much coolant can be induced to flow along paths of given size, and how effective will it be in picking up and carrying away the heat? What size of shaft with a given space between bearings will give acceptable limits of deflection, with the consequent effects on out-of-balance magnetic pull and on critical speed?

Computer-aided design

Supposing that the functional relationships between many machine constructional variables can be established, the evolution of a design to meet some specified optimum criterion is a matter of long and tedious iteration. As such, it is clearly in the field of the digital computer. The fully *synthetic* design of a machine is possible in principle, but so far the great complexity of setting up the array of functional relationships has not been mastered. On the other hand, the 'slave' function of the computer has been widely developed to carry out difficult and complicated (but still routine) calculations. This is *computer-aided* design.

Programs have been devised for calculating the leakage inductance of induction machines, and refined by comparison with test results; for accepting details of machine rating, frame size and windings and evaluating performance; and for the prediction of transformer surge-voltage distribution. On occasion a program is written for rapid 'first-shot' design for size, mass and cost as an aid to tendering. Subsystem programs specifying performance and design parameters are then run, and subsequently matched to form the complete design. Some of the design parameters are continuously variable, others are discontinuous and can have only certain discrete values. There will also be absolute limits, such as temperature-rise or dimensions or mechanical stress, that must not be exceeded, and the subsystem programs must take account of them. After integrating the subsystems the overall behaviour is investigated for conformity with the master specification. In this process the computer is used as a decision-making device.

The computer accelerates the design process enormously, makes possible more 'trial' designs, and enables sophisticated calculations to be made without intolerable tedium and excessive time. It can reduce empiricism, readily handle non-linearities, and incorporate much of the designer's 'know-how'. A valuable feature is the man/computer symbiosis, in which the designer can 'talk' to the computer, ask questions and get quick informative answers, change his mind and ask again. The designer is released from numerical drudgery to grapple with physical and logical ideas.

In brief, computer-aided design gives the advantages of eliminating tedious hand-calculations, making data checks at every stage, facilitating trial designs preliminary to a final decision, permitting much more detailed and precise functional relations (provided that they can be formulated), and giving the possibility of developing new and more comprehensive design procedures.

This book does not deal specifically with computer design. It discusses the electromagnetic principles upon which the parameters depend, and the effect on performance.

1.4 Methods of approach

In design, a machine is considered a structure in which the interaction of magnetic fields and electric currents is the basic physical principle. It gives rise to calculable forces, torques, e.m.f.s, saturation effects, heat production, mechanical stresses, vibration, noise and several other phenomena of practical importance. In the assessment of performance, various secondary effects, both in-

ternal and external to the machine, have to be studied in terms of flux-current interactions, machine structure and system considerations. When the machine is analysed as a member connected into a complex electromechanical system, it is almost essential to treat it as a 'black box' possessing a set of terminal characteristics of behaviour.

The method of treatment depends upon the problem to be solved. The approach chosen is that which appears to 'model' the machine in the most appropriate way. It may be in terms of flux densities and current sheets, or multiple circuit equations, or voltage-current/torque-speed relations; it may be simplified by restriction to the steady state or by ignoring all purely mechanical considerations. The method of approach unavoidably shifts its ground to suit. Even a basic electromagnetic approach can be undertaken in several ways: we choose a flux-current interaction approach because it is most readily pictured in physical terms.

1.5 Basic principles

All electromagnetic machines can be related to three principles, namely those of *induction*, *interaction* and *alignment*.

Induction

The essentials for the production of an electromotive force by magnetic means are electric and magnetic circuits, mutually interlinked. Summation of all products of magnetic flux with complete turns of the electric circuit gives the total flux linkage ψ . If ψ is made to change, an e.m.f. is induced in the electric circuit. The e.m.f. exists only while the change is taking place, and the magnitude of its time-integral is equal to the change of linkage, so that

$$\int e \cdot dt = -\psi \quad \text{or} \quad e = -d\psi/dt$$

The direction of the e.m.f. is such as to oppose the change. If the electric circuit is closed and the linkage with an externally produced magnetic field is reduced, then the e.m.f. generates a current in the closed circuit, developing a self-flux the linkage of which tends to make up the deficiency. Comparable conditions obtain when the change of linkage is the result of a change of current in the electric circuit concerned: then the self linkage may be written as $\psi = Li$ where L is the inductance coefficient. The self-induced e.m.f. is

$$e = -d\psi/dt = -d(Li)/dt$$

acting in the direction of positive current. For reasons of simple convention, set out in Sect. 1.7, it is more convenient to consider the e.m.f. as directed in *opposition* to positive current, giving

$$e = +d\psi/dt = +d(Li)/dt \quad (1.1)$$

For engineering purposes this is used in the form

$$e = N(d\Phi/dt) \quad (1.2)$$

where all N turns link the flux Φ and result in a linkage $\psi = N\Phi$. The flux Φ

may be resolved into (i) a *useful* or working component, and (ii) a non-useful or *leakage* component.

Change of linkage in a coil may occur in three ways:

- (i) Supposing the flux constant, the coil may move through it.
- (ii) Supposing the coil stationary with respect to the flux, the flux may vary in magnitude.
- (iii) Both changes may occur together: i.e. the coil may move through a time-varying flux.

In (i) the flux-cutting rule can be applied, and the e.m.f. in a single conductor of length l calculated from the rate at which it cuts across a magnetic field of uniform density B when moving at speed u in a direction at right-angles to the direction of the flux: then

$$e = Blu \quad (1.3)$$

This is the *motional* e.m.f., always associated with conversion of energy between the mechanical and electrical forms. In (ii) the e.m.f. is found by applying eq. (1.1) to give the *pulsational* (or *transformer*) e.m.f.; no motion is involved and there is no energy conversion. In (iii) both e.m.f. components are concerned.

Interaction

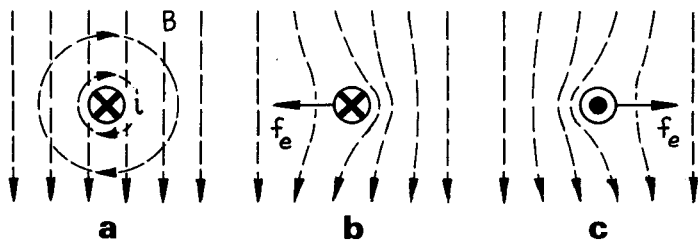
When a current i lies in and perpendicular to a magnetic field of density B over an active length l , a mechanical force

$$f_e = Bli \quad (1.4)$$

is developed on it in a direction perpendicular to both current and field. In (a) of Fig. 1.2, B represents the flux density of an undisturbed field. Introduction of a conductor carrying a current imposes a corresponding field component, developing the resultant in (b). In the neighbourhood of the conductor the resultant density is greater than B on one side and less than B on the other; provided that the increase on one side has the same magnitude as the reduction on the other, the force on the conductor is given by eq. (1.4).

Alignment

Pieces of high-permeability material (e.g. iron) in an ambient low-permeability

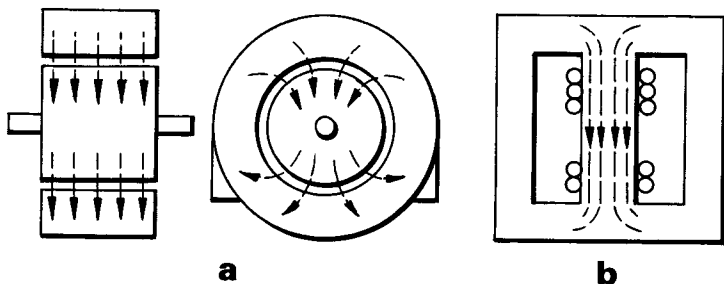


1.2 Interaction law.

medium (e.g. air) in which a magnetic field is established, experience mechanical forces tending to align them with the field direction in such a way as to minimize the reluctance of the system.

1.6 Basic forms

Electromagnetic transformers and machines all utilize the induction principle. Most machines exploit interaction, a few are based on alignment, and some utilize both principles. The devices can be built in a wide variety of forms, of which the most elementary are illustrated in Fig. 1.3.

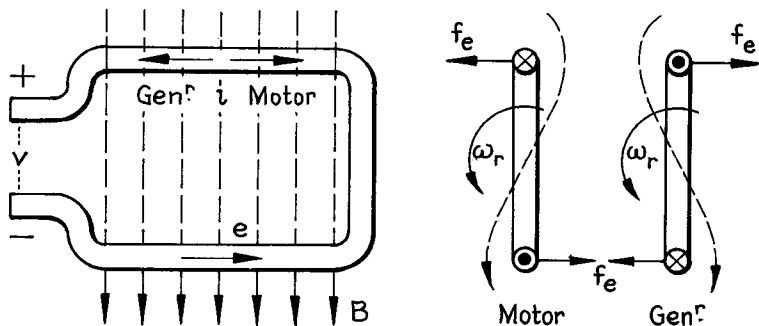


1.3 Basic forms

Rotary Machines

Two magnetic elements (*a*), one fixed (stator) and the other (rotor) capable of rotation, are separated by a narrow annular airgap. For mechanical convenience the stator is normally the outer member. Each element carries windings arranged to lead currents axially along the airgap surface, and a mutual magnetic flux crosses the gap to link them. Rotation of the rotor results in e.m.f. induction and in electromechanical energy conversion through interaction and alignment torques.

Because of the heteropolar nature of the magnetic field, it is convenient in a 2-pole machine to take pairs of gap-surface conductors and join them in series to make turns, as in Fig. 1.4. Then the two opposing forces combine to form



1.4 Motor and generator action.

a torque. Directions of flux, current and force are shown, together with the directions of current for *generator* and *motor* action. This reversibility of mode is a valuable feature of electromechanical devices.

Generator. An e.m.f. is produced by movement of a coil in a magnetic field. The current in an external circuit produced by the e.m.f. interacts with the field to develop a mechanical force opposing the movement, so that a mechanical driving torque must be provided. The generated electrical power is converted from the mechanical power input.

Motor. If current is supplied to the machine from an external source, interaction develops mechanical forces to move the rotor. The motion causes an e.m.f. to be induced in opposition to the current. Thus the motor requires electrical power to produce a corresponding mechanical power.

Despite considerable constructional variety, the electrical differences between machines are only secondary: they arise from (i) the kind of electrical system, a.c. or d.c., to which the windings of the machine are connected; and (ii) the kind of connection made between the windings and the electric system — i.e. tapped phase windings, or commutator windings with switched connections.

Transformers

Transformers do not convert energy to or from the mechanical form and in this sense are not 'machines'. For the *static* transformer, Fig. 1.3(b), no gap is needed as there is no movement, so that only pulsational e.m.f.s are concerned. Interaction forces are certainly developed, but the windings are braced against them. In the induction *regulator* transformer, in which the relative position of the two elements has to be adjustable, the constructional form is identical with that in (a) of Fig. 1.3. Again there is an interaction effect, and the torque produced makes it necessary to hold the rotor mechanically against rotation.

1.7 Conventions

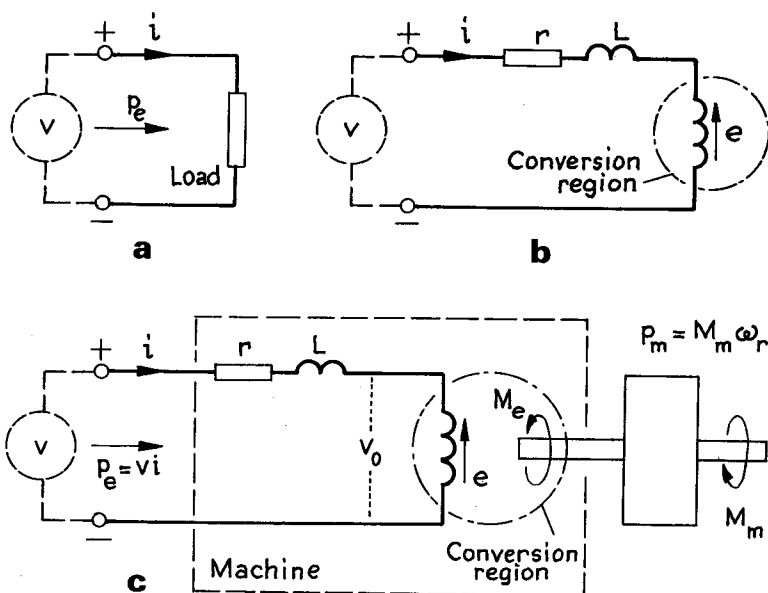
Voltage, current and power

An electrical device, connected to a source of voltage v of polarity as indicated in Fig. 1.5(a), takes a current i of positive direction (i.e., in at the positive terminal) and a positive input power $p_e = vi$. If the current reverses (to become $-i$ in the positive direction) then the power also reverses, becoming the negative input $-p_e$ corresponding to an output.

The terminal voltage is taken as the voltage *applied* and the current as that in the positive input direction, so permitting a direct distinction between the action of the load or machine as an absorber of electrical input power (*motor* mode) or as a producer of electrical output power (*generator* mode) corresponding to negative input.

Electromotive force

In that part of a machine where energy storage or conversion takes place magnetically, an e.m.f. given by eq. (1.1) appears. Its direction is in opposition to the direction of positive current. The active part of the machine, enclosed in



1.5 Machine conventions.

a circle in Fig. 1.5(b), is the seat of the e.m.f. $e = d\psi/dt$ which in general can be regarded as comprising two components:

Motional E.M.F.: This e.m.f., e_r , in a winding results from the movement at angular speed ω_r of its conductors through the flux densities in the gap.

Pulsational E.M.F.: The pulsational or 'transformer' e.m.f. e_p appears in the winding if the flux in which it lies has a rate of change with time.

The motional e.m.f. is concerned with electrical/mechanical energy conversion, while the pulsational e.m.f. provides a means of electrical energy transfer between magnetically coupled windings, as from primary to secondary winding in a transformer or from stator to rotor winding in a rotary machine. Then, if a winding carries a current i , the product $e_r i$ represents the rate of energy conversion, and $e_p i$ the rate of energy transfer.

General form: With its two components stated explicitly, the e.m.f. $e = d\psi/dt$ can be written

$$e = \frac{\partial \psi}{\partial \theta} \cdot \frac{d\theta}{dt} + \frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial \theta} \omega_r + \frac{\partial \psi}{\partial t} = e_r + e_p \quad (1.5)$$

The term $\partial \psi / \partial \theta$ is a measure of the distribution in the gap of the flux density B .

Machine Winding

The conventions for a machine winding are set out in Fig. 1.5(c). The effective resistance of the winding is r , and the loss in it is ri^2 . A further quality of

the winding is its leakage inductance L : not all of the flux linkage is usefully employed in linking the primary and secondary windings of a transformer or the stator and rotor windings of a machine. The leakage flux-linkage is considered as separated from the active region of the device.

The voltage and input power relations at any instant are

$$v = ri + L(di/dt) + e_r + e_p \quad \text{and} \quad p_e = ri^2 + L(di/dt)i + e_r i + e_p i$$

Energy-rate balance: A machine is an electro-mechanical device linking an electric source, of voltage v supplying current i , to a mechanical source of angular speed ω_r and torque M_m (or moving at linear speed u under a driving force f_m). Both vi and $M_m\omega_r$ are treated as *input* powers p_e and p_m . The total input to the machine is $p_e + p_m$, and within the machine there are three general rates of power absorption, namely

p : the rate of loss in I^2R , in the core, and in mechanical friction and windage;
 dw_f/dt : the rate of change of stored magnetic-field energy;
 dw_s/dt : the rate of change of mechanical energy stored in inertia, deformation, vibration, shaft strain, etc.

The energy-rate balance is consequently

$$p_e + p_m = dw_f/dt + dw_s/dt + p \quad (1.6)$$

For simplicity, let the machine run at constant speed in a steady-state condition and with no change of field or mechanical stored energy. Relative motion of stator and rotor generates a motional e.m.f. e_r in opposition to the applied voltage v . The behaviour can be summarized as follows. With the machine acting as a *motor*, the applied voltage v drives an input current $+i$ against e_r to give an electrical input power $p_e = v(+i)$, of which the part $e_r i$ is converted. The conversion process means the development of an electromagnetic torque M_e which drives the rotor against the mechanical input torque M_m at speed ω_r , to produce the negative mechanical input $p_m = (-M_m)\omega_r$, corresponding in effect to a positive mechanical output. With the machine as a *generator* driven at ω_r by a positive mechanical input torque M_m and so receiving the positive input $p_m = (+M_m)\omega_r$, the e.m.f. e_r now exceeds v . The current reverses to provide the negative electrical input (or positive output) $p_e = v(-i)$. The sum of the positive inputs is the rate of energy dissipation within the machine.

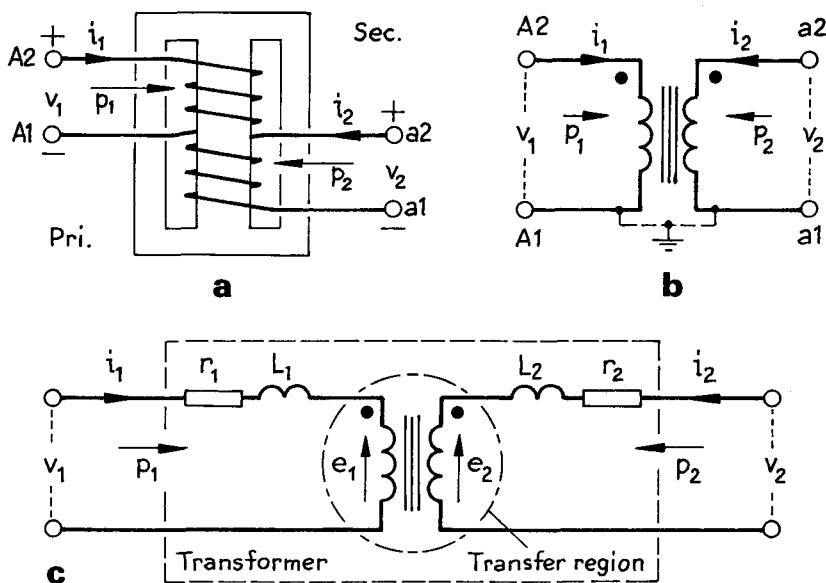
At the points of attachment of the machine to its associated electrical and mechanical system (i.e. at the terminals and the shaft), the convention is that input powers are taken as positive. In the motor mode, positive current enters the machine from the positive terminal and there is a positive electrical power input (but the mechanical input is negative). In the generator mode, the current direction is reversed and the electrical input is negative (but the mechanical input is positive).

Conversion Region: That region within the chain-dotted line in Fig. 1.5(c) contains only the essential electromagnetic/mechanical conversion quantities, respectively e and i , and ω_r and M_e . Losses, magnetic leakage and mechanical storage are excluded so that attention can be directed to the essential physical process of energy conversion. This process is intrinsically 'perfect', making it

possible to consider electromagnetic machines as ideal devices with respect to their conversion regions, with other effects taken into account externally.

Transformers

A two-winding transformer is basically a pair of coils embracing a common magnetic circuit of high-permeability steel, Fig. 1.6(a). It is both useful and



1.6 Transformer conventions.

convenient to adopt conventions for the two coils, primary and secondary, comparable with those for a machine. For this purpose it is necessary to define the *sense* of the windings. Both are seen in Fig. 1.6(a) to encircle the common flux in the same sense: then a positive polarity at terminal A2 of the primary pair A1A2 corresponds to positive polarity at terminal a2 of the secondary pair a1a2. The same information in a schematic diagram (b) is given by placing a dot at each of the corresponding positive terminals A2 and a2. If the ends A1 and a1 are tied together and earthed, then the polarities of the dotted ends swing together with common flux changes. A further consequence is that, if both primary and secondary circuit currents flow in the same direction, (e.g. both in at the dotted terminal) their m.m.f.s. are additive in the magnetic circuit.

The windings possess resistance, and not all of the flux due to one m.m.f. links the other winding. Leakage fluxes not contributing to the mutual coupling (by means of which energy can be transferred from one winding to the other) therefore exist. By treating the resistance and the leakage inductance as separate entities, it is possible to set up an elementary equivalent circuit, Fig. 1.6(c).

Transfer Region: The region within the chain-dotted line in (c) is that in which the essential energy transfer takes place. The primary and secondary inputs are $e_1 i_1$ and $e_2 i_2$ at any instant. Again the process is intrinsically 'perfect', making it is possible to consider a transformer as an ideal device with external defects.

1.8 Dynamic circuit theory

The study of electrical machines is devoted (a) to understanding how they work, and (b) to predicting their performance. The former is the concern of the applications engineer and the student: the latter chiefly of the designer.

Ideally, it ought to be possible to express the behaviour of electrical machines in terms of Maxwellian electromagnetic field equations. Such an approach is exceedingly difficult, and the normal method is through the simplified concepts of induction and interaction. Various techniques (e.g. complexor and locus diagrams, equivalent circuits), mostly of particular rather than of general application are employed. The disadvantage of this approach is its practical limitation to the steady state. But if one more step is taken from 'reality' into the abstract, machines can be dealt with in terms of *dynamic circuits*. A truly equivalent circuit applies to its prototype machine under all conditions, steady and transient, balanced and unbalanced. To such a circuit can be applied all the powerful techniques of circuit analysis — circuit laws, network theorems, operational and matrix methods, network analysers and computers. The circuit can be used to obtain the three primary characteristics (voltage, current and torque) in advanced studies of generator faults, changes of load, or automatic control, all so important in modern engineering.

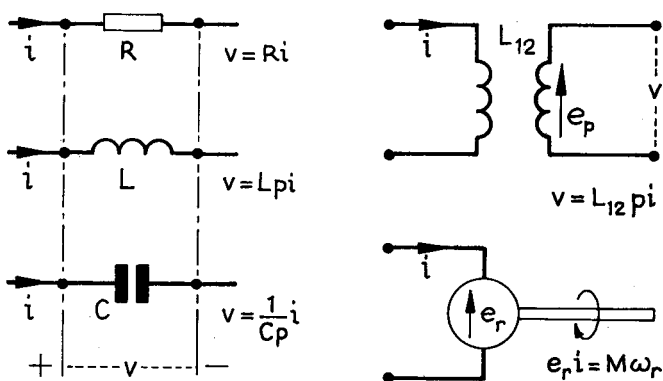
There are some sacrifices. Manipulation of the equations is only tractable on the assumption of linear parameters. Non-linear effects, such as saturation, must be the subject of subsequent adjustment in the light of experience by semi-empirical methods. Space-harmonics of flux, brush-contact phenomena and commutation effects must be ignored. These effects are far from negligible in practical machines; nevertheless to ignore them is to assume conditions differing only in degree from those of normal circuit theory. Finally, the circuits are still not truly equivalent because propagation phenomena are disregarded, so that surge-voltage effects need separate consideration.

Circuit Elements: Fig. 1.7 shows the basic elements of dynamic circuits with their equations of behaviour, in terms of the operators p (representing d/dt) and $1/p$ (representing time-integral).

1.9 Prototype ideal transformer

Power transformer action under steady-state sinusoidal conditions can be established in terms of an *ideal transformer* embodying the transfer region. The magnetic circuit of an ideal transformer has infinite permeability and zero reluctance, so that a vanishingly small m.m.f. is needed to excite the magnetic flux; all the flux completely links both primary and secondary windings (there is no magnetic leakage); there are no losses in the windings or in the core.

As the practical power transformer approximates to these conditions, it can be taken as equivalent to an ideal transformer with the following external 'defects': a small but finite m.m.f. to magnetize the core, small but finite leak-



1.7 Dynamic circuit elements.

age inductances to account for the slightly imperfect coupling, and resistances to simulate loss effects in the windings and the core.

Ideal Transformer under Sinusoidal Conditions

When a sine voltage v_1 of angular frequency $\omega = 2\pi f$ is applied to the primary winding with the secondary open-circuited, the voltage is balanced by an e.m.f. induced by the rate of change of flux linkage $\psi_1 = N_1 \Phi$, and the core flux must therefore vary with time. Let it be $\Phi = \Phi_m \cos \omega t$, a variation at angular frequency ω between oppositely directed peaks of Φ_m : then from eq. (1.1) the corresponding primary voltage is

$$v_1 = d\psi_1/dt = N_1 \omega \Phi_m \cos \omega t$$

which has the peak value $v_{1m} = N_1 \omega \Phi_m$ and the r.m.s. value

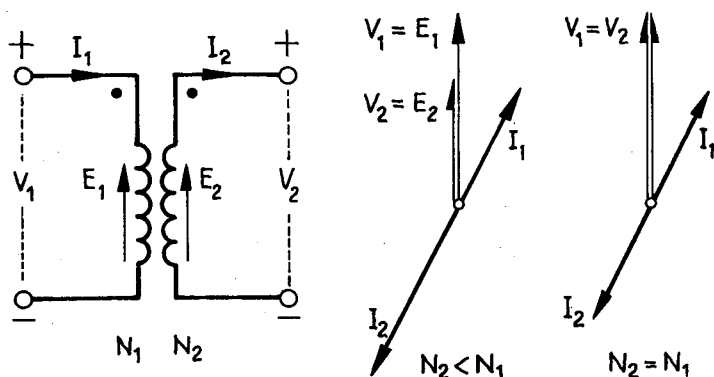
$$V_1 = (1/\sqrt{2}) \omega N_1 \Phi_m = 4.44 f N_1 \Phi_m$$

As the same flux links each turn, the voltage per turn is

$$V_t = (1/\sqrt{2}) \omega \Phi_m = 4.44 f \Phi_m \quad (1.7)$$

Thus the primary terminal voltage is $V_1 = N_1 V_t$ and that of the secondary is $V_2 = N_2 V_t$. The primary/secondary voltage ratio V_1/V_2 is equal to the primary/secondary turns ratio N_1/N_2 .

With the secondary an open circuit the primary input current is vanishingly small because the core is infinitely permeable and the flux is generated in it by negligible m.m.f. The primary winding therefore presents between terminals an infinite magnetizing inductance. But if the secondary terminals are connected to a load impedance such that the instantaneous current is i_2 , this develops an m.m.f. $N_2 i_2$ acting on the magnetic circuit. To preserve unaffected the flux Φ whose e.m.f. balances v_1 in the primary winding, the primary must take a balancing current i_1 such that $N_1 i_1 = N_2 i_2$ at any instant. In r.m.s. terms, $N_1 I_1 = -N_2 I_2$. The current ratio is thus the inverse of the turns ratio, with $I_1/I_2 = N_2/N_1$ and the m.m.f.s in opposition.



1.8 Ideal transformer.

Phasor Diagram. With the 'sense' of the windings as in Fig. 1.8, the primary and secondary voltage phasors are coincident in direction and of length proportional to the respective number of turns. The requirement of m.m.f. balance is satisfied by opposition of the current phasors. If I_1 enters the primary at the positive terminal, then I_2 leaves the secondary at the positive terminal, giving the condition $N_1 I_1 = -N_2 I_2$. Thus a clear distinction is made between primary input and secondary output conditions. In machine phraseology, the primary acts in a 'motor' mode, the secondary in a 'generator' mode. The primary input apparent power is $S_1 = V_1 I_1$, the secondary *input* is $V_2 (-I_2) = -S_2$. These are numerically equal:

$$S_1 = V_1 I_1 = V_2 (N_1/N_2) \cdot (-I_2) (N_2/N_1) = -V_2 I_2 = -S_2$$

The apparent power rating is therefore the volt-ampere product of either primary or secondary.

Impedance Transformation. If the secondary terminals are connected to a load of impedance Z , then $V_2/I_2 = Z$. The corresponding ratio V_1/I_1 is the effective input impedance of the primary:

$$V_1/I_1 = (N_1/N_2)^2 (V_2/I_2) = (N_1/N_2)^2 Z = k_n^2 Z$$

Thus Z across the secondary terminals appears in effect across primary terminals as Z multiplied by the square of the turns ratio $k_n = N_1/N_2$.

Summary. For sine conditions we have the following relations for the ideal transformer:

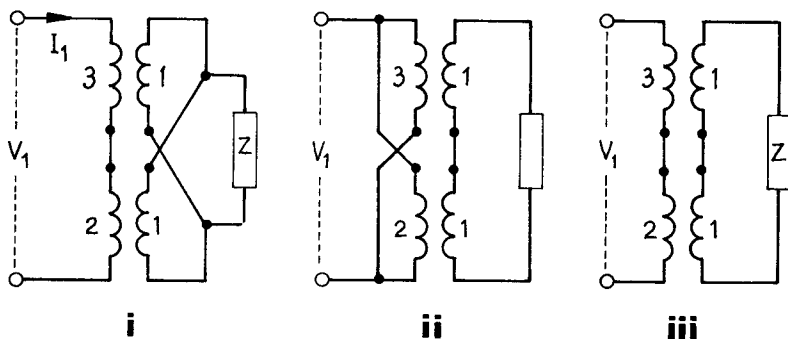
$$\begin{aligned} \text{Ratios: } V_1/V_2 &= N_1/N_2 \text{ and } I_1/I_2 = N_2/N_1 \\ \text{Rating: } S &= V_1 I_1 \text{ or } V_2 I_2 \quad \text{M.M.F. balance: } I_1 N_1 + I_2 N_2 = 0 \end{aligned} \quad (1.8)$$

EXAMPLE 1.1: The core of an ideal 1-ph 11 000/415V, 50 Hz, 300 kVA transformer has a net cross-section of area 0.032 m^2 . Find (i) the number of primary and secondary turns, (ii) the rated currents, (iii) the secondary load and primary input impedances, for a peak core flux density of about 1.5 T.

(i) For $B_m = 1.5$ T the voltage per turn, eq. (1.7), is $V_t = 10.7$ V, whence $N_2 = V_2/V_t = 38.8$. Fractions of a turn are not possible, and it may be convenient for N_2 to be even. Rounding to $N_2 = 40$ turns, for which $B_m = 1.46$ T and $V_t = 10.4$ V, the number of primary turns is $N_1 = 40(11\,000/415) = 1060$.

(ii) The rated currents are $I_1 = 300 \times 10^3/11\,000 = 27$ A, and $I_2 = 720$ A. The m.m.f.s are $N_1 I_1 = N_2 I_2 = 28\,800$ A-t.

(iii) The secondary load impedance is $Z = V_2/I_2 = 0.58\ \Omega$. The primary input impedance is $V_1/I_1 = 11\,000/27 = 415\ \Omega = Z(N_1/N_2)^2$.



1.9 Concerning m.m.f. balance.

M.M.F. balance

This is a concept useful where connections are not straightforward. Fig. 1.9 shows ideal transformers with ratios 3/1 and 2/1 interconnected to a common load Z . In (i) the primaries are in series and the secondaries in parallel. If the voltage across Z is V_0 , then the primary voltages must be $3V_0$ and $2V_0$, giving $V_1 = 5V_0$. As the current is I_1 in each primary, for m.m.f. balance the secondary currents must be $3I_1$ and $2I_1$. The current in Z is $5I_1$ (or I_1 depending on the 'sense' of the secondaries). As $V_0 = 5I_1 Z = V_1/5$, then $I_1 = V_1/25Z$ (or $I_1 = V_1/5Z$). The terminal input impedance V_1/I_1 is consequently $25Z$ (or $5Z$).

In (ii) the secondary voltages are respectively $V_1/3$ and $V_1/2$, and $V_0 = 5V_1/6$ (or $V_1/6$). From m.m.f. balance, the two primary currents are $5V_1/18Z$ and $5V_1/12Z$ summing to $I_1 = 25V_1/36Z$ (or $V_1/18Z$ and $V_1/12Z$, summing to $5V_1/36Z$).

In (iii) m.m.f. balance is impossible with any finite current. The network therefore draws no current and the input impedance is infinite (equivalent to an open circuit). With both primaries and secondaries in parallel, the input terminals are short-circuited.

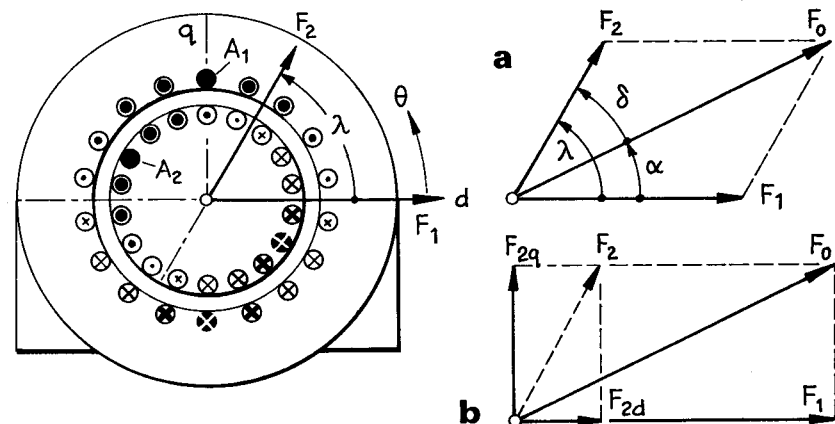
1.10 Prototype machines

If the ferromagnetic parts of the magnetic circuit of a machine can be considered as infinitely permeable, then the whole action is concerned with electromagnetic conditions in the airgap. The energy-rate balance of eq. (1.6) can

be expressed in terms of the action of windings on the gap surface, which are subject to e.m.f.s induced by the gap flux and give rise to distributed m.m.f.s resulting from the winding currents. By ignoring the ends of the machine, the problem becomes *two-dimensional*. In practical machines it is not, of course, justifiable to ignore the end regions, which present leakage-flux problems of some complexity.

1.11 Prototype two-pole uniform-gap machine

The 2-pole machine of Fig. 1.3(a) is shown in detail in Fig. 1.10. The stator



1.10 Prototype two-pole uniform-gap machine.

and rotor members have each a conductor arrangement so fed as to produce a *sine-distributed current-sheet*, the respective peak linear densities (in amperes per metre of gap periphery) being A_1 and A_2 . The magnetic axis of the stator, called the *direct-* or *d-axis*, is taken as horizontal; that of the rotor is displaced counterclockwise from the *d-axis* by an angle λ . The distribution around the gap of the stator current-sheet density is $A_1 \sin \theta$, whilst for the rotor it is $A_2 \sin (\theta - \lambda)$. Interaction of the two current-sheets develops a torque tending to reduce λ to zero and so to align the rotor axis with that of the stator.

In an ideal machine, attention is concentrated on the active gap region, the dimensions of which are given in Fig. 1.14. It is assumed that the iron parts of the magnetic circuit are highly permeable, and that in consequence the gap length l_g accounts for the whole of the magnetic circuit reluctance.

The total current in one-half of the stator current-sheet is $A_1(2/\pi)\frac{1}{2}\pi D = A_1 D$. This is the total stator m.m.f., acting horizontally across two gaps in series, so that the m.m.f. *per pole* is $F_1 = \frac{1}{2} A_1 D$. Across a gap of length l_g it produces the flux density of peak value $B_1 = F_1 (\mu_0/l_g) = \frac{1}{2} A_1 D (\mu_0/l_g)$ on the *d-axis*. At any angle θ the m.m.f. is $F_{1\theta} = F_1 \cos \theta$, and the corresponding gap flux density is $B_{1\theta} = B_1 \cos \theta$. In a similar way the rotor, acting alone, produces a total m.m.f. per pole $F_2 = \frac{1}{2} A_2 D$ and a peak density $B_2 = \frac{1}{2} A_2 D (\mu_0/l_g)$ on the

axis at the angle λ . For any angle θ the rotor m.m.f. is $F_2 \cos(\theta - \lambda)$ and the corresponding gap flux density is $B_2 \cos(\theta - \lambda)$.

With both stator and rotor excited, their combined m.m.f. at angle θ is $F_{1\theta} + F_{2\theta}$ producing $B_{1\theta} + B_{2\theta}$ in the gap. This is a new sine-distributed flux density of peak B_0 corresponding in magnitude and axis to the resultant F_0 of F_1 and F_2 , which can be found by treating F_1 and F_2 as if they were space vectors and using the relation

$$F_0^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \lambda \quad (1.9)$$

The peak gap energy density is $\frac{1}{2} B_0^2 / \mu_0 = \frac{1}{2} F_0^2 (\mu_0 / l_g^2)$, the mean is one-half of the peak, so that the total energy in the gap volume $\pi D l g$ is

$$w_f = \frac{1}{4} F_0^2 \pi D l (\mu_0 / l_g)$$

For given currents the torque is $M = dw_f / d\lambda$. The only term in eq.(1.9) dependent on λ is $2F_1F_2 \cos \lambda$, so that

$$\begin{aligned} M &= -\frac{1}{2} \pi D l (\mu_0 / l_g) F_1 F_2 \sin \lambda \\ &= -\frac{1}{2} \pi D l B_1 A_2 \sin \lambda = -\frac{1}{2} \pi D l B_2 A_1 \sin \lambda \end{aligned} \quad (1.10)$$

the minus sign indicating that the torque acts to decrease λ , i.e. to align the rotor and the stator magnetic axes. The torque depends on the main dimensions of the gap, the stator and rotor m.m.f.s, and the angle by which the two magnetic axes diverge.

Torque and load angles: The torque is proportional to sine of the angle between the magnetic axes of stator and rotor, and λ is defined as the *torque angle*. The m.m.f. diagram in Fig. 1.10(a) shows λ to be the sum of α and δ , the respective angular displacements between F_1 and F_2 and their resultant F_0 ; then α and δ are defined as respectively the stator and rotor *load angles*. In the analysis of certain machines, one or other of the load angles can be more readily identified from outside than can the torque angle λ . Suitably applying eq. (1.9) it is possible to write the torque in the alternative forms

$$M = -k F_1 F_2 \sin \lambda = -k F_1 F_0 \sin \alpha = -k F_2 F_0 \sin \delta \quad (1.11)$$

where $k = \frac{1}{2} \pi D l (\mu_0 / l_g)$.

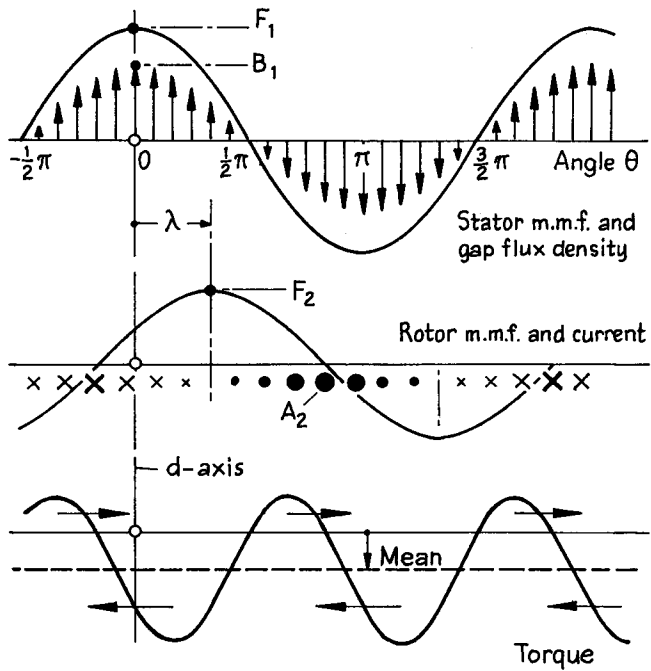
Torque distribution: Eq.(1.10) shows that we could consider the torque as arising from the interaction of a rotor current-sheet of peak density A_2 lying in the gap density B_1 of the stator acting alone; alternatively of A_1 lying in B_2 . Fig. 1.11 illustrates the former in a developed diagram. In an element of angle $d\theta$ there lies a rotor current element $A_2 \sin(\theta - \lambda) \cdot d\theta$ in a stator-produced flux density $B_1 \cos \theta$. From eq. (1.4) the torque on the current element is

$$dM = \frac{1}{2} D l B_1 \cos \theta \cdot A_2 \sin(\theta - \lambda) \cdot d\theta$$

Integrating over the range $0 < \theta < 2\pi$ gives for the total torque

$$M = -\frac{1}{2} \pi D l B_1 A_2 \sin \lambda$$

as in eq. (1.10). The curve of torque distribution over the periphery shows



1.11 Interaction torque distribution.

there to be regions of oppositely directed torque. These become equal and the total resultant torque vanishes when, for $\lambda = 0$ or π rad, the rotor and stator m.m.f. axes are in alignment. The distribution is completely unidirectional and the mean torque is greatest when the magnetic axes are at right-angles and $\lambda = \pm \frac{1}{2} \pi$ rad.

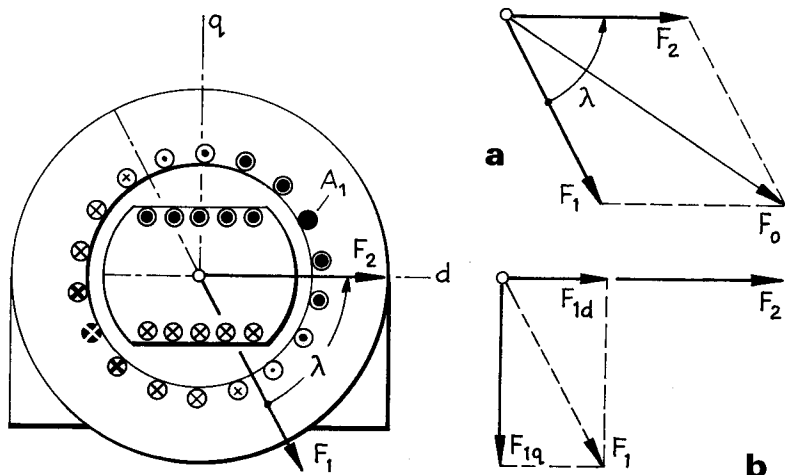
Two-axis concept: The first expression in eq.(1.11) can be written

$$M = -kF_1 (F_2 \sin \lambda) = -kF_1 F_{2q}$$

where $F_{2q} = F_2 \sin \lambda$ is the component of F_2 directed in the *quadrature* axis or q-axis, Fig. 1.10(b). The effect is as if the rotor had two separate current sheets, one of peak linear density $A_2 \sin \lambda$ magnetizing along the q-axis and producing torque, the other $A_2 \cos \lambda$ magnetizing along the d-axis and coupled with the stator (d-axis) winding. It would be equally valid to fix the d-axis on the rotor and resolve the stator m.m.f. into components $F_1 \sin \lambda$ and $F_1 \cos \lambda$; and the stator would now carry the two equivalent axis windings with their sine-distributed m.m.f.s.

1.12 Elementary two-pole salient machine

If one member, e.g. the rotor as in Fig. 1.12, is structurally polarized (salient), its m.m.f. F_2 cannot be taken as sine-distributed, nor does the resultant m.m.f. F_0 act everywhere across a uniform gap. The simple m.m.f. summation in (a) does not apply.



1.12 Elementary two-pole salient machine.

Two-axis Concept: This is particularly useful for the treatment of saliency effects. The d-axis is chosen to be the pole-axis of the *salient* member. In the case shown in Fig. 1.12 the stator m.m.f. F_1 is resolved along the rotor d- and q-axes to give $F_{1d} = F_1 \cos \lambda$ and $F_{1q} = F_1 \sin \lambda$ respectively. The d-axis m.m.f.s F_2 and F_{1d} act mainly across the short gap l_g , but the q-axis m.m.f. F_{1q} is presented with a much longer gap of variable length. It is still possible to write the torque as

$$M = -\frac{1}{2} \pi D l B_2 (A_1 \sin \lambda) = -\frac{1}{2} \pi D l B_2 A_{1q} \quad (1.12)$$

where B_2 is the peak of the *fundamental* component of the gap-flux density produced by F_2 , and A_{1q} is the peak of the equivalent sine-distributed stator q-axis current sheet.

1.13 Multipolar machines

A machine with p pole-pairs has p successive sets of N-S poles, each considered as spread over an electrical angle of 2π rad, so that although the physical (or mechanical) angle around the periphery is 2π (mech) rad, the total electrical angle is $2\pi p$ (elec) rad. An electrical torque angle λ therefore corresponds to a mechanical displacement angle $\lambda_m = \lambda/p$. The torque, being a mechanical quantity measured externally, is given now by $M = dw_f/d\lambda_m = p(dw_f/d\lambda)$. This does not mean that a machine of given gap dimensions produces more torque the greater the number of poles, because with given peak linear current densities A_1 and A_2 (which are determined by heat dissipation) the m.m.f.s per pole are reduced by the factor $(1/p)$.

EXAMPLE 1.2: The main dimensions of a uniform-gap machine are $D = 0.50$ m, $l = 0.20$ m and $l_g = 0.005$ m (5 mm). The stator has a winding excited to produce a sine-distributed current-sheet of peak density $A_1 = 12\,800$

A/m; the rotor has a corresponding density $A_2 = 9\,600$ A/m, with its magnetic axis displaced by a torque angle $\lambda = \pi/3$ (elec) rad from the stator d -axis.

Neglecting the m.m.f. required for the iron parts of the magnetic circuit, find the torque with the machine windings arranged to give (i) a 2-pole field, (ii) an 8-pole field.

(i) The m.m.f.s per pole are $F_1 = \frac{1}{2}A_1D = \frac{1}{2}(12\,800 \times 0.5) = 3\,200$ A-t, and $F_2 = 2\,400$ A-t. Applying eq. (1.7) with $\sin \lambda = 0.87$ and $\mu_0 = 4\pi \times 10^{-7} \cong 1/800\,000$ gives

$$M = -\frac{1}{2}\pi \times 0.50 \times 0.20(1/800\,000 \times 0.005)3\,200 \times 2\,400 \times 0.87 \\ = -260 \text{ N-m}$$

Alternatively, F_1 produces in the gap a peak flux density $B_1 = F_1\mu_0/l_g = 0.80$ T. From the appropriate expression in eq. (1.10),

$$M = -\frac{1}{2}\pi \times 0.50 \times 0.20 \times 0.80 \times 2\,400 \times 0.87 = -260 \text{ N-m}$$

(ii) Each 2-pole unit covers an arc $\pi D/p$, reducing the stator m.m.f. per pole to $F_1 = 800$ A-t and the corresponding peak gap flux density to $B_1 = 0.20$ T. Similarly $F_2 = 600$ A-t. The rate of change of gap energy with electrical angle has been reduced by the factor $1/p^2 = 1/16$; but as $\lambda_m = p\lambda = 4$, the reduction of torque is given by the factor $(1/p)$. By either of the forms of eq. (1.7) the torque is therefore

$$M = -260/4 = -65 \text{ N-m}$$

The average flux density $\bar{B}_1$ in the gap due to the stator excitation has been reduced by the factor $1/p = 1/4$, but the average rotor linear current density is unchanged.

1.14 Main dimensions

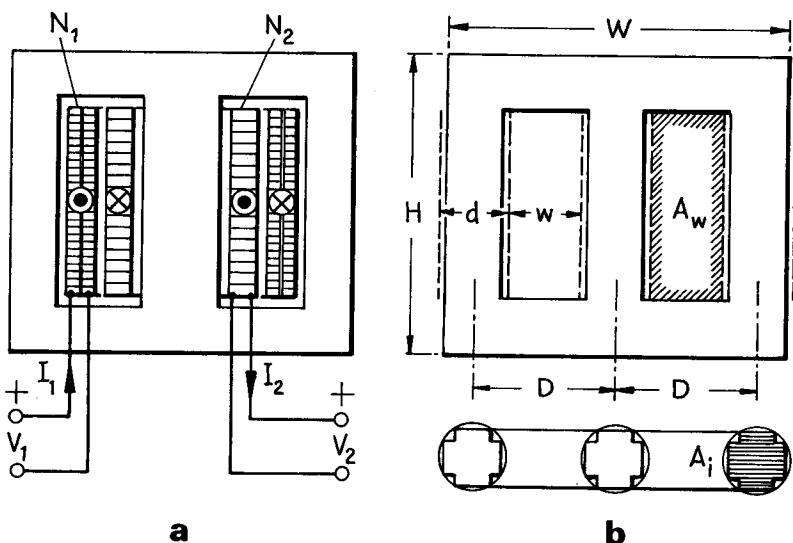
Although the thermal and mechanical design of transformers and machines is often complicated, there is a simple design relation between the electrical rating and the main dimensions.

Transformer

The electromagnetic quantities concerned are the mutual flux in the magnetic circuit and the current linkage in the electric circuit. The basic dimensions, Fig. 1.13(a), are the cross-section of the magnetic core and the 'window' area enclosed by it. The active ferromagnetic area A_i of the core (which is an assembly of steel plates thinly coated with insulation to reduce eddy-current loss) is a roughly constant fraction (about 0.9) of the gross core area. The peak mutual flux is $\Phi_m = B_m A_i$, the peak density B_m being chosen with due regard to core loss and saturation. The voltage per turn of the primary winding, from eq. (1.7), has the r.m.s. value

$$V_t = V_1/N_1 = 4.44 f B_m A_i \quad (1.13)$$

The primary and secondary windings must be accommodated in the window area A_w . Insulation and clearance reduce the available area for actual conduc-



1.13 Transformer: main dimensions.

tor cross-section to $k_w A_w$: the space factor k_w may be quite small if the working voltage is high. Because of the m.m.f. balance ($N_1 I_1 = N_2 I_2$) the primary and secondary windings each occupy about one-half of the available area so that the primary current linkage is $N_1 I_1 = \frac{1}{2} k_w A_w J$. The r.m.s. current density J is limited by $I^2 R$ loss and heat dissipation. The total rating $S = V_1 I_1$ of the 2-limbed 1-phase transformer in Fig. 1.13(a) is

$$S = (V_1/N_1) N_1 I_1 = 2.22 f B_m J A_i k_w A_w \quad (1.14)$$

This can be considered as $\frac{1}{2} S$ per limb, so that for the 3-phase transformer in (b), with the overall dimensions defining its 'frame size', the total rating is

$$S = 3.33 f B_m J A_i k_w A_w \quad (1.15)$$

directly relating the rating to the basic dimensions through B_m and J , the flux and current densities.

EXAMPLE 1.3: A 50 Hz 1-ph transformer has a core of net section $A_i = 0.032 \text{ m}^2$ and a window area $A_w = 0.07 \text{ m}^2$. The density loadings are $B_m = 1.5 \text{ T}$ and $J = 2.7 \text{ MA/m}^2 = 2.7 \text{ A/mm}^2$. Insulation and clearance take up 0.7 of the window area (i.e. $k_w = 0.3$). Find the rating of the transformer.

From eq. (1.14),

$$S = 2.22 \times 50 \times 1.5 \times 2.7 \times 10^6 \times 0.032 \times 0.3 \times 0.07 = 300\,000 \text{ VA}$$

so that the rating is 300 kVA.

Machines

The gap dimensions of a cylindrical machine, Fig. 1.14(a), are the diameter

D and the axial length l . Let the density of the flux crossing the gap be B_1 and the linear density of the axial rotor current sheet be A_2 . Their interaction develops on the rotor a tangential force $B_1 A_2$ per unit of gap surface, or $B_1 A_2 \pi D l$ for the whole machine. The torque with radius $\frac{1}{2}D$ is $M = B_1 A_2 \frac{1}{2} \pi D^2 l$; and at a rotor angular speed $\omega_r = 2\pi n$ the power is

$$M\omega_r = B_1 A_2 \frac{1}{2} \pi D^2 l \omega_r = B_1 A_2 \pi^2 D^2 l n$$

For mechanical reasons the current sheet is normally formed by discrete conductors, sunk into slots of pitch y_s and carrying current $I_c = bh_1 J_2$ as in Fig. 1.14(b), the current density being chosen on a heat-dissipation basis. Then $A_2 = I_c / y_s = (bh_1 / y_s) J_2 = c J_2$, and the power can be expressed as

$$M\omega_r = B_1 J_2 c \pi^2 D^2 l n$$

which directly relates developed power to the electric and magnetic densities J and B_1 , the speed n and the basic dimensions. In this idealized approach it is assumed that the interaction forces have, all at points on the rotor periphery, the same direction. A relation specific to a.c. machines requires further data.

Rating: Number of phases m , phase voltage V , phase current I , frequency f , rotational speed n , rating $S = mVI$.

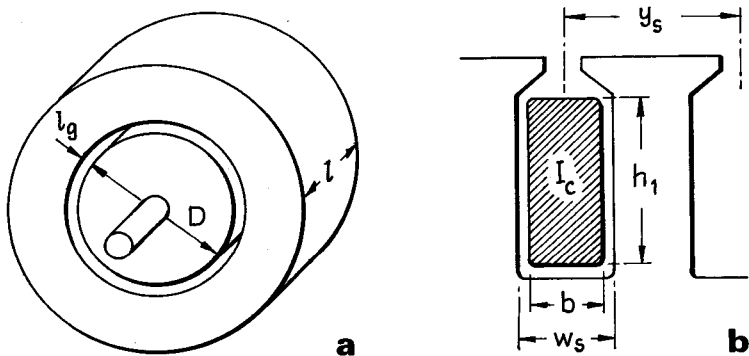
Magnetic Circuit: Flux per pole Φ , number of pole-pairs p , mean gap-flux density $\bar{B} = 2p\Phi / \pi D l$. The specific magnetic loading $\bar{B}$ is chosen in relation to excitation and saturation conditions, core loss and power factor.

Electric Circuit: Active conductors Z per phase, each carrying current I_c , and the specific electric loading (linear r.m.s. current density) $A = mZI_c / \pi D$, chosen with regard to m.m.f., $I^2 R$ loss and cooling.

The product of the *specific magnetic* and *electric loadings* is, as discussed above, the specific force $\bar{B}A = 2p\Phi mZI_c / \pi^2 D^2 l$. Now $2(pn)\Phi Z = 2f\Phi Z$ is proportional to the phase e.m.f. and so to the applied phase voltage V , while I_c is (or is a fraction of) the phase current I . Hence the rating of the machine is

$$S = mVI = k\bar{B}A \cdot D^2 l n \quad (1.16)$$

where k is a coefficient related to the technical details of the phase windings.



1.14 Machine: main dimensions.

EXAMPLE 1.4: In a typical rotating machine the mean gap flux density is $\bar{B} = 0.50$ T. The slots are spaced $y_s = 40$ mm apart and each contains a conductor of section 35 mm $\times$ 12 mm. The current density is $J = 3.3$ MA/mm² = 3.3 A/mm². Find the tangential force per unit length of gap periphery and per unit axial length of the machine.

The specific electric loading is $A = (35 \times 12)3.3 \times 1\,000/40 = 35\,000$ A/m, and consequently the specific force is

$$f_e = \bar{B}A = 0.5 \times 35\,000 = 17\,500 \text{ N/m}^2$$

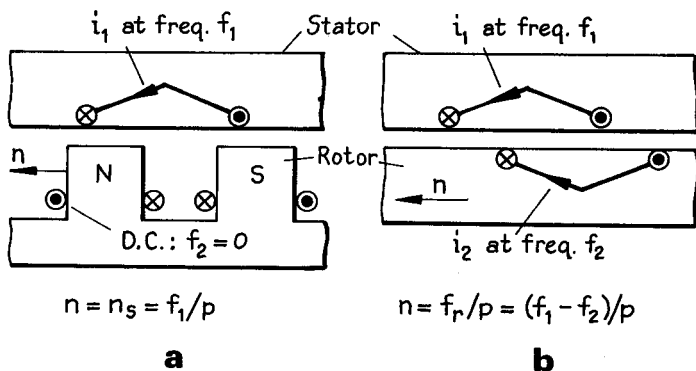
which is typical of a medium-rated machine. The torque depends on the diameter D and axial length l of the rotor member.

1.15 Torque maintenance

To maintain torque production when the machine rotates, the relative current-sheet patterns and the angle λ between their magnetic axes must be maintained. The manner in which this happens is dependent on the form (d.c. or a.c.) of the stator and rotor currents and on the windings in which they circulate. Only polyphase synchronous and induction machines are considered here.

Synchronous machine

Fig. 1.15(a). The stator phase windings carry currents of frequency f_1 and the rotor is d.c. excited, i.e. its poles are 'fixed' with respect to the rotor.



1.15 Synchronous and induction machines: torque maintenance.

Stator current reversal every half-period must be matched by rotor movement. In a 2-pole machine the rotor must complete one revolution in one period of the stator current, so that its speed is $n = f_1$, or $\omega_r = 2\pi n = 2\pi f_1 = \omega_s$. In a machine with p pole-pairs the argument applies to successive 2-pole units and the synchronous speed is f_1/p , or $\omega_s = 2\pi f_1/p$.

Induction machine

Fig. 1.15(b). The stator is the same, but the rotor is excited with currents of frequency f_2 so that its poles move with respect to its windings. In a 2-pole

machine the torque is maintained only if rotation at $n = \omega_r/2\pi = f_r$ together with current variation at f_2 combine to give $f_r + f_2 = f_1$. For let the speed be a little less than synchronous: the rotor slips back from the position it would occupy if running synchronously, but its poles move forward at f_2 with respect to its windings so that the field in effect moves synchronously with respect to the stator. In a machine with p pole-pairs the only difference is that $n = f_r/p$.

1.16 Classification

Machine windings are excited so as to develop particular current-sheet and m.m.f. patterns. A winding for the d.c. excitation of a working flux is often concentrated. For an a.c. flux it is generally distributed to reduce the leakage reactance. Polyphase windings can be arranged to give, to a close approximation, the sine-distributed current sheets of Fig. 1.10. A discussion of windings in general is given in Chapter 3.

Transformers and machines can be classified in accordance with (i) the kind of electrical supply, d.c. or a.c., to which each of its windings is connected; and (ii) the kind of connections made between the winding and its supply, i.e. by phase or tapped windings, or by switched connections (as in a commutator winding).

The Table lists a number of possible combinations, giving the names by which they are generally known.

Gap flux	Supply and Windings		Name	
	Member 1	Member 2		
<i>Fixed-axis</i>				
Constant	d.c. concentrated	d.c. commutator	D.C. commutator	A
Constant	a.c. phase	d.c. concentrated	1-ph synchronous	B
Alternating	a.c. distributed	a.c. commutator	1-ph commutator	C
Alternating	a.c. phase	a.c. phase	1-ph induction	D
Alternating	a.c. concentrated	a.c. concentrated	Transformer	E
<i>Travelling-wave</i>				
Constant	a.c. polyphase	d.c. concentrated or distributed	3-ph synchronous	F
Constant	a.c. polyphase	a.c. polyphase	3-ph induction	G

This book is devoted mainly to cases E, F and G. Types F and G, particularly the latter, can take a linear as well as a rotary form.

2

Magnetic Circuits

2.1 Magnetic circuit properties

The engineering approach to magnetic circuit design is to consider the magnetic flux Φ in a circuit of uniform cross-sectional area a , length l and absolute permeability μ established by the m.m.f. $F = Ni$ of a linked electric circuit of N turns carrying a current i . This relation is

$$\Phi = F\mu(a/l) = F\Lambda = F/S \quad (2.1)$$

where $\Lambda = \mu a/l$ is the permeance (in Wb per A-t) and $S = l/a\mu$ is the reluctance (in A-t per Wb). The flux Φ is assumed to be uniformly distributed over the area to give a flux density $\bar{B} = \Phi/a$. Practical magnetic circuits are usually divisible into component parts of uniform cross-section, and assignable length and permeability: for each the m.m.f. is $(B/\mu)l = HL$. For several such parts in series the total m.m.f. is

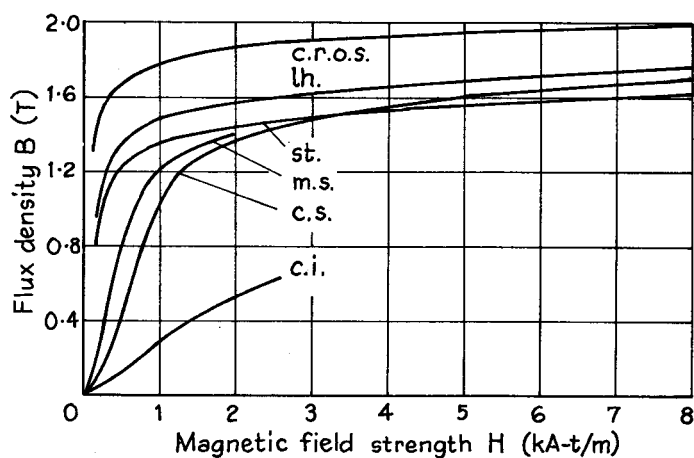
$$F = Ni = H_1 l_1 + H_2 l_2 + \dots \quad (2.2)$$

For air and for nonmagnetic materials in general, the absolute permeability is $\mu = \mu_0 = 4\pi \times 10^{-7}$ (in T per A-t/m, or H/m, approximating to $H = 800\,000\,B$). For ferromagnetic materials the absolute permeability is $\mu = \mu_r \mu_0$, where the relative permeability μ_r (a numeric) may lie between a few hundred and one-half million, and is highly dependent upon the level of B , i.e. upon the saturation condition. Thus H , the m.m.f. per metre length of path, may be between $2B$ and $2\,000\,B$, and it is necessary to employ graphical magnetization curves relating B to H . Strictly, each specimen has a unique B/H relation, but it is customary to employ 'average' curves. Typical comparative curves are given in Fig. 2.1, more detailed curves for structural, transformer and machine-core materials in Figs. 2.2–2.4.

For digital computation some analytic relation between B and H may be more convenient. Some typical expressions (from many) are

$$B = \frac{aH}{1 + bH} \quad \text{and} \quad B = \frac{a_0 + a_1 H + a_2 H^2 + \dots}{1 + b_1 H + b_2 H^2 + \dots} \quad (2.3)$$

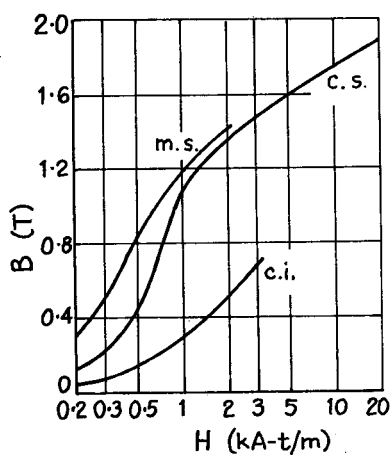
The former gives reasonable approximation for higher values of B , the latter is a better overall fit. If the flux is to reverse, only odd powers of H must be included.



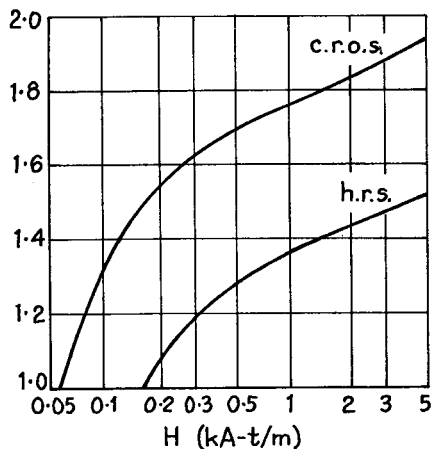
2.1 Magnetization curves: comparative.

Cold-rolled grain-oriented steel (c.r.o.s.) Lohys (lh.)

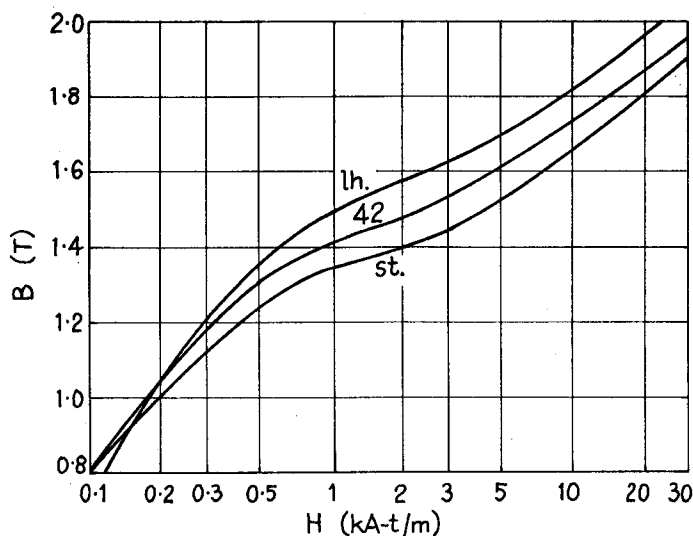
Stalloy (st.) Mild steel (m.s.) Cast steel (c.s.) Cast iron (c.i.)



2.2 Magnetization curves: structural materials.



2.3 Magnetization curves: transformer steel.



2.4 Magnetization curves: machine core steel.

Flux Distribution

The total flux in a ferromagnetic part of a magnetic circuit subject to a sine-varying excitation voltage varies sinusoidally with time. It might be expected that the flux density at every point in the cross-section would be Φ/a , directed parallel to the section and sine-varying. This is not so: practical magnetic materials lack magnetic, metallurgical and dimensional homogeneity, so that the density at a point is a function of the local permeability and structure. In laminated materials there are regions in which B has a component at right-angles to the magnetic axis as flux crosses from one plate to the next ('cross-fluxing'). In transformer and machine design a macroscopic view is taken of the density as a simple mean value, but the consequences of the non-homogeneity appear in the core loss.

Magnetostriction

Small changes in physical dimensions occur in a ferromagnetic material subjected to a magnetizing m.m.f. The lengthwise change is of the order of 10^{-5} and it is accompanied by corresponding transverse changes of opposite sign. These changes are due to magnetostriction, are of a hysteretic nature (i.e. they dissipate energy) and are intimately related to other physical conditions such as mechanical stress. A practical result of magnetostriction is the audible noise that emanates from a power transformer. With the high core flux densities commonly employed, the noise from distribution transformers may be the subject of limiting terms in the specification, and offending transformers may require sound-baffle structures to reduce the nuisance level.

Core loss

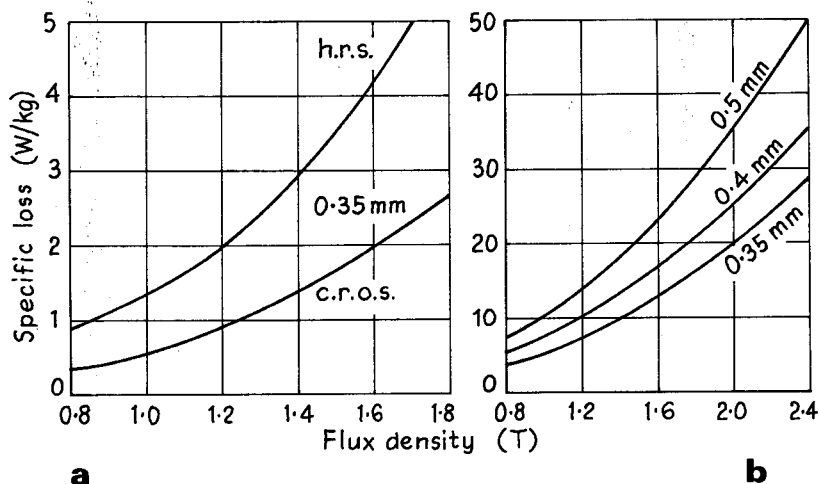
A ferromagnetic core carrying an alternating flux is the seat of magnetization loss that appears as heat. Traditionally the phenomenon is regarded as arising (i) from *hysteresis* loss due to the cycling of the material through its hysteresis loop, and (ii) from *eddy-current* loss resulting from the induction of currents circulating within the material. For sinusoidal flux-density variation of peak value B_m and r.m.s. value B , the specific loss (in W/kg) on a simple theoretical basis has the components

$$(i) p_h = k_h f B_m^x \quad (ii) p_e = k_e d^2 f^2 B^2 / \rho \quad (2.4)$$

The factor k_h depends on the molecular structure of the material; the exponent x lies between 0.8 and 2.3, and for the usual flux density levels may be about 2. The thickness d of the material takes account of the fact that cores carrying varying flux are sub-divided, usually into sheet form, and the appearance of ρ shows that to limit eddy-current loss the material should have a high electrical resistivity.

The idealized conditions on which eq.(2.4) is based do not obtain in machines and transformers, nor is it profitable in other than a metallurgical context to 'separate' the components. The loss is better regarded as a mass effect, distinguished only by the manner in which the flux variation occurs: (i) as a cyclic reversal on the same axis (i.e. a true alternating flux), or (ii) as a roughly constant magnitude but an axis that varies cyclically or rotates. In an actual core both processes occur. In a transformer, for example, there would seem to be only true alternating flux, but in fact rotational core-loss effects occur at the joints between limbs and yokes. The core loss may therefore be regarded as comprising components proportional respectively to the first and second powers of the frequency, and these indeed can be separated.

Attempts to optimize core loss must be based on tests, as some anomalies



2.5 Core loss at 50 Hz: (a) Transformers, (b) Machines.

occur. The actual loss in a built core is increased by contact between plate edges, imperfect insulation between sheets, hardening and burring by shearing and punching processes, and the non-uniformity of flux density distribution. Designers employ curves such as those in Fig. 2.5 for the core loss in constructed plant, using test results as a basis.

Core clamping

Cores must be kept tight by compression. A loose core vibrates under magnetic repulsion forces and is noisy, and narrow parts such as slot lips may break under fatigue; but excessive pressure may distort the core and impair its magnetic properties.

Magnetic sheet and strip material received from the steelmakers has small surface asperities, waviness, and variations of thickness of as much as ± 0.05 mm. Cores may shrink in service, due in part to progressive bedding of asperities, in part to the compression of intersheet insulation. Tests show that the average pressure in a machine core should not be less than about 300 kN/m^2 ; that stiffener bars should not be more than 25 mm apart; and that long through-bolted stacks should have high-tensile bolts stressed during core building to about 0.75 of the elastic limit to take up shrinkage in service. Such bolts develop eddy-current loss, which must be limited by insulating them from the surrounding core.

2.2 Magnetic materials

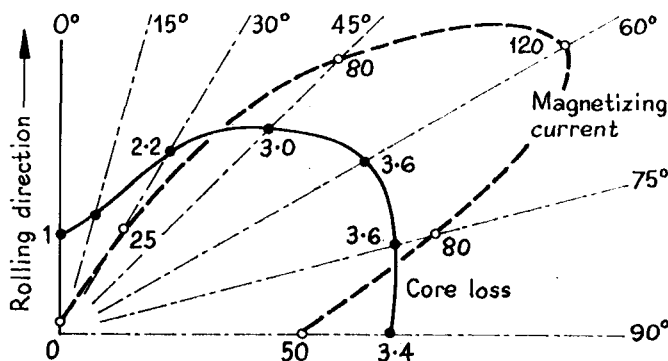
Materials may be classified in accordance with the flux, pulsating or steady, that they normally carry. The former must be laminated; the latter may be, if constructionally convenient. The mechanical properties of a built core are also of importance.

Laminated materials

The characteristics important in producing an alternating flux in minimum section with minimum loss are the permeability and the specific core loss. The designer is more interested in the m.m.f. than in the permeability as such, and uses 'static' B/H curves such as those in Figs. 2.1–2.4.

Magnetic sheet materials are alloy steels. The chief alloying constituent is silicon, which raises the permeability at low flux densities, reduces hysteresis, and, by augmenting the resistivity, eddy-current losses. The silicon content affects the mechanical properties, increasing tensile strength but impairing ductility. A 5% silicon steel is difficult to punch or shear, but addition of nickel makes *cold-rolling* possible with important resulting advantages. Where core loss is not a primary consideration, and in induction motors where the magnetizing current must be kept small, a low silicon content may be used. In large and totally-enclosed machines where cooling is difficult, a high-grade low-loss steel may be called for.

Transformer steels. Two kinds are available: (i) high-resistance steel (h.r.s.) of 4–5% silicon content and nominal thickness 0.35 mm; (ii) cold-rolled grain-oriented sheet and strip (c.r.o.s.) of 0.33 mm nominal, for which the magnetic properties in the rolling direction are far superior to those on any other axis.



2.6 Directional properties of cold-rolled grain-oriented transformer steel plate.

Typical directional variations are shown in Fig. 2.6, where the loss and magnetizing current in the rolling direction for a given peak flux density are each taken as unity. For example, the relative loss and m.m.f. are respectively 3 and 80 for magnetization on an axis at 45° to the rolling direction. If the directional properties can be exploited, c.r.o.s. is highly advantageous and can be worked to higher flux densities than h.r.s. It is therefore used in all large transformers and in most of the lower ratings. It may even be applied in large machines in spite of the absence of a fixed flux axis. Both c.r.o.s. and h.r.s. are available in several grades, with specific losses down to 1.0 and 2.4 W/kg respectively at 1.5 T, 50 Hz.

Solid materials

Where the flux is normally constant, the magnetic criterion is the B/H curve. Mechanical considerations may, however, entail alloying that also decreases the permeability, as in steel castings or forgings for high peripheral speeds.

Castings. Low-carbon steels are used for stationary parts not subject to large mechanical stressing. The carbon content is increased for rotating parts, with some sacrifice in permeability. A 10% nickel, 5% manganese cast iron is nearly non-magnetic, and may be employed when it is necessary to avoid shunting a magnetic field. For small frames the use of cast iron is less common as cast aluminium is light and convenient.

Forgings. Very large steel forgings are used for turbo-generator rotors where high mechanical strength is essential. The overhang windings are retained by forgings of non-magnetic alloy.

Working flux densities

The maximum flux densities (in T) normally employed are:

Core plates. Transformers (c.r.o.s.), 1.3–1.6 (up to 2.0 on 20% overvoltage).

Machines: teeth, 1.8–2.2; cores, 0.8–1.2.

Cast steel. Poles, 2.0; yokes, 1.5.

2.3 Magnetic circuit calculation

Having available the B/H curves for the materials employed, the magnetic circuit is divided into convenient parts in series, the flux densities estimated for given conditions, and the m.m.f. per unit length found and multiplied by the length of the part. Summation yields the total magnetic circuit m.m.f.

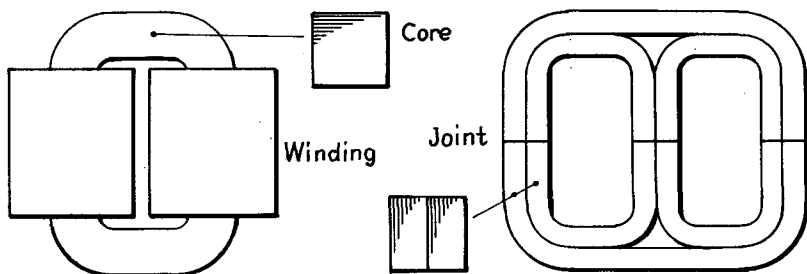
The estimation is basically simple for transformers except for the corners or joints of the closed magnetic circuit, and in the region of punched clamping or cooling vents. The design must be such as to limit cross-fluxing between adjacent plates, as this increases the loss and m.m.f. demand. In machines the shape of the magnetic circuit is much more complicated, and in particular the airgap, because of its low permeance relative to that of the ferromagnetic parts, requires detailed assessment.

2.4 Transformer magnetic circuits

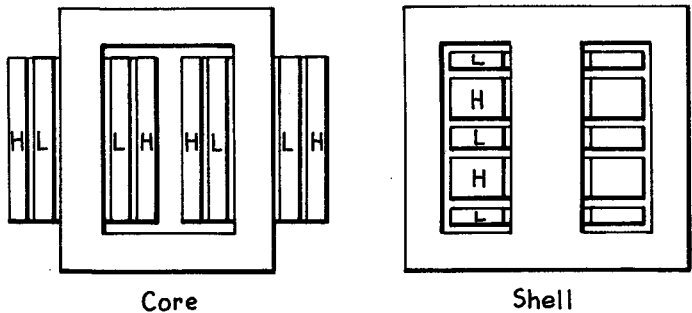
Rolls of c.r.o.s., up to 2 Mg in mass, are slit into widths by gang-operated slitters. The strips are then cut to length. With h.r.s. the material is sheared to size in power guillotines and punched in multiple presses. The working impairs the properties of c.r.o.s., so that sheets (or complete small cores) are given a stress-relief anneal at about 800 °C, in an inert atmosphere to avoid oxidation and carbon contamination. Surface insulation to reduce eddy currents is a thin coating of kaolin or varnish for h.r.s., but c.r.o.s. usually comes from the makers with a phosphate-base coating capable of withstanding the annealing process. Small cores need no further insulation, but kaolin or varnish is applied to laminations intended for transformers of 10 MVA upward.

Magnetic circuit design

Transformer cores up to about 5 kVA rating can be formed by winding continuous strip through pre-formed coils using special machinery, eliminating joints and using c.r.o.s. exclusively in its rolling direction. Larger sizes, Fig. 2.7, may have strip-wound cores, impregnated and then cut across to form 'C' cores which can be butt-jointed into pre-formed coils; core thicknesses up to 30 mm in widths up to 60 mm can be wound to give window dimensions of 100 mm square. By far the most common construction, however, is by stacking cut strips with interleaved joints to form core- or shell-type magnetic circuits, Fig. 2.8.



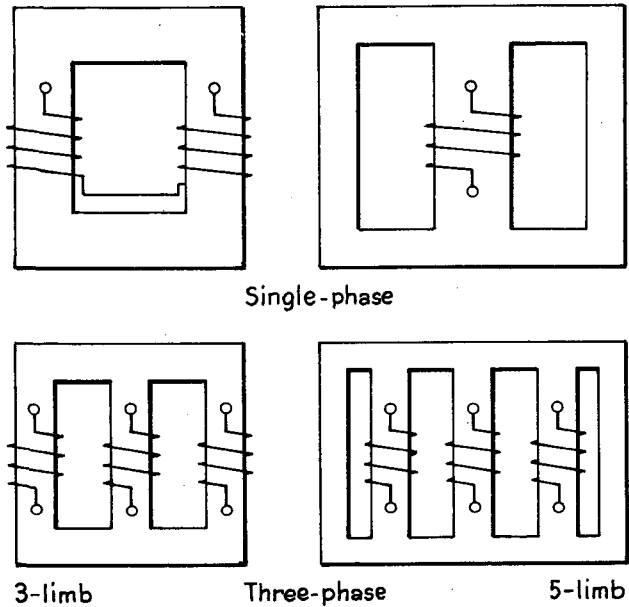
2.7 Small cores for grain-oriented steel.



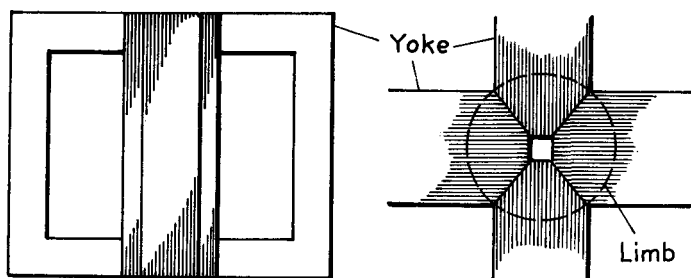
2.8 Basic 1-ph coil and core arrangements.

Most 1-ph and 3-ph transformers are of the core type, Fig. 2.9. If transport loading limits are likely to be reached, a 5-limbed core may be used to reduce the overall height. For very large units it may be necessary for transport to use three 1-ph transformers, or some form of demountable construction. Fig. 2.10 shows an example of four yokes fitted round a central limb for height limitation.

The basic parameters of a 3-ph core (Fig. 1.13b) are the core-circle diameter d , the limb spacing between centres D , and the window length l . From these the net core area A_i and the overall core width w give the available window area A_w . The overall height H and width W are of importance in tank design

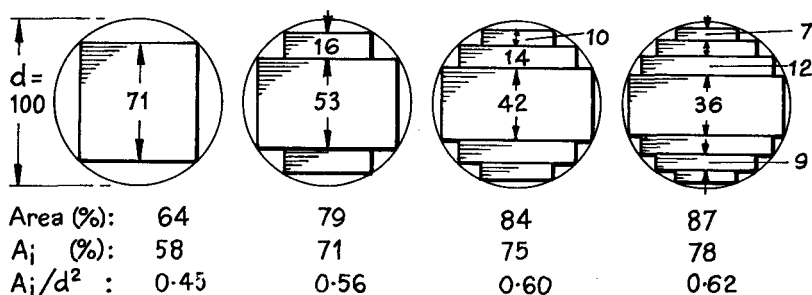


2.9 Core arrangements.



2.10 Four-yoke 1-ph core.

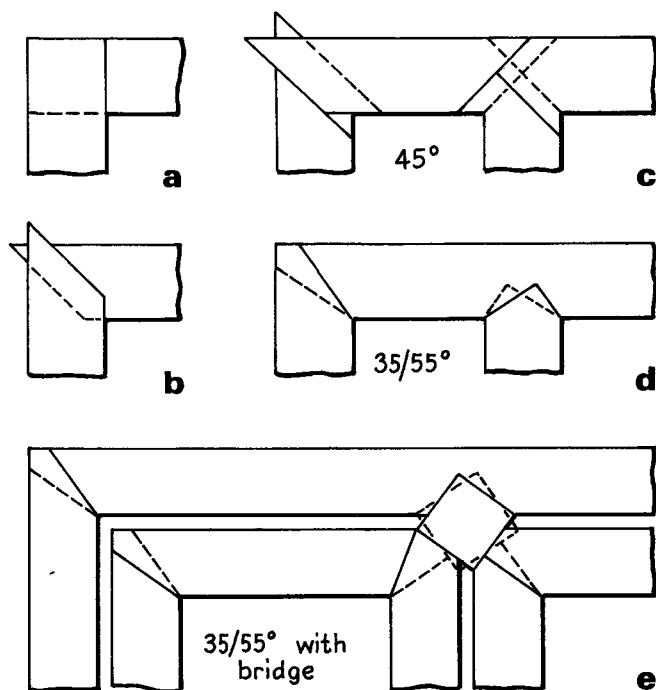
and transport. For a given type and shape there is a relation $d \propto \sqrt{S}$ between the core-circle diameter and the rating S . For economic design it is essential to restrict d by using as much as possible of the core-circle area. The shape of the core-packet assembly must therefore approximate to a circle. Typical optimum proportions are shown in Fig. 2.11 with the gross core area as a percentage of the circumscribing circle area, and with the net area A_i based on a stacking factor of 0.9 (which on large units can be raised). Seven or more steps may be used for larger sections, but if too massive to allow adequate cooling, ducts are arranged between sections of the core packets.



2.11 Sections of core-type transformer limbs.

Joints. Limb and yoke laminations are interleaved to reduce reluctance and loss. There are several methods, Fig. 2.12. Simple overlap (a) is suitable for h.r.s. plate material but not for c.r.o.s., for which mitred overlap (b) is preferred, either in the 45° form (c) or the more elaborate $35^\circ/55^\circ$ variant in (d). When the core-circle diameter exceeds about 0.8 m, commercially available strip is too narrow to span the full diameter and a core split becomes necessary. Bridging sections (e) are needed for the flux passing between the halves of the core at the central limb. The gap between the half-cores aids cooling because it exposes twice as much plate-edge area to the coolant.

Clamping. Limb sections may be clamped by plates and through-bolts; both may be subject to eddy-current loss, and the bolt-holes distort the flux path. A preferred method is to hold the limb plates together by bands e.g. of woven



2.12 Corner joints.

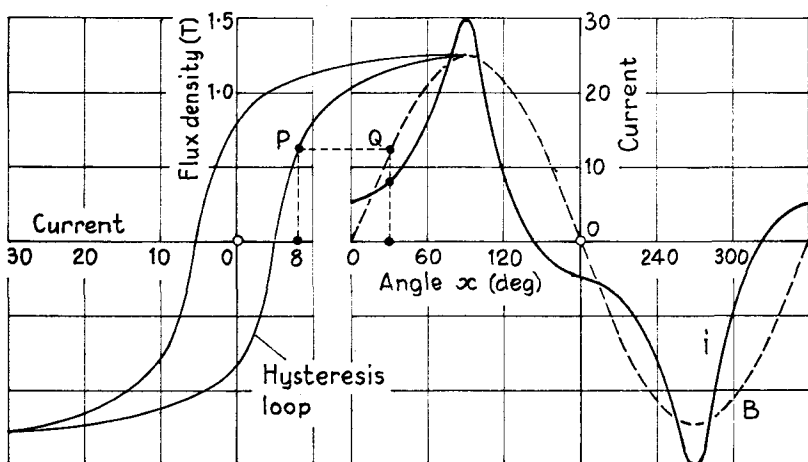
glass-fibre tape impregnated with a polyester resin. After banding with the core under pressure, the bands are heated to cure the resin. To spread the pressure more evenly, the outer packets of core plates may be bonded by a suitable adhesive such as Araldite.

Magnetizing current and power

The exciting m.m.f. is calculated with adjustment for gap and joint effects and for divergence of the flux path from the rolling direction, on the basis of the peak density B_m . The r.m.s. value of the m.m.f. is obtained by dividing the result by $\sqrt{2}$. This takes no account of saturation nonlinearity nor of the asymmetry introduced by hysteresis. A plot of the magnetizing current waveform for a particular case is shown in Fig. 2.13. An ascending flux density corresponding to point P on the hysteresis loop and to Q on the sinewave of time variation requires 8 units of magnetizing current. Proceeding in this way, the current waveform is seen to contain harmonics, which in this case (in terms of a fundamental of 100) analyse to

$$100 \sin(x + 18^\circ) - 39 \sin(3x + 7^\circ) + 18 \sin(5x + 9^\circ) - 8 \sin(7x + 10^\circ)$$

The displacement of the fundamental implies that it lags the voltage applied by $90 - 18 = 71^\circ$, and that there is consequently an active power component

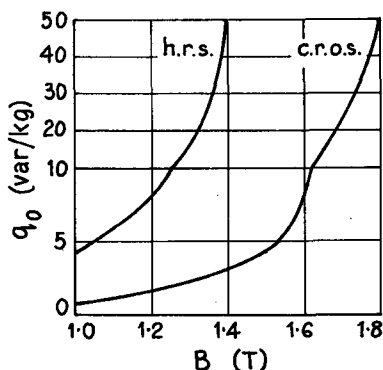
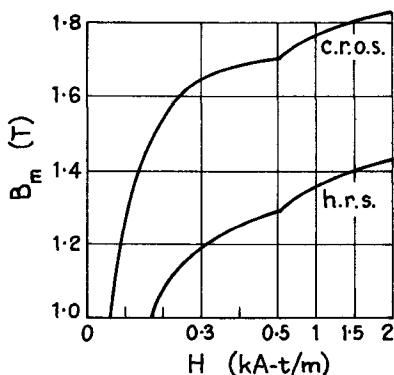


2.13 Magnetizing current waveform.

$100 \cos 71^\circ$ corresponding to the hysteresis loss. (The eddy-current loss is additional to this). The example is for a peak density of 1.25 T, and there is a 40% 3rd harmonic. For the higher densities in large transformers the 3rd, 5th and 7th harmonics are considerably intensified. However, on load the harmonics are negligible because the magnetizing current is only 0.05 p.u. of full-load current, or thereabouts,

Magnetizing reactive power. Writing the total m.m.f. for a peak density B_m as $H_m l_m$, where l_m is the mean length of the flux path around the magnetic circuit, the magnetizing current in the N_1 turns of the primary winding is $I_{or} = H_m l_m / \sqrt{2} N_1$, ignoring harmonics. Using eq.(1.7) for the primary voltage, the magnetizing reactive power (in var) is

$$Q_0 = V_1 I_{or} = 4.44 f B_m A_i (H_m l_m) / \sqrt{2}$$



2.14 Magnetizing m.m.f. and reactive power for 0.35 mm transformer steel cores.

As $A_l l_m$ is the core volume and the density of the steel is $7\,500\text{ kg/m}^3$, the specific magnetizing reactive power is

$$q_0 = 4.2fB_m H_m \times 10^{-4} \text{ var/kg} \quad (2.5)$$

Curves relating q_0 and B_m for given steels may be used for a rapid estimate of the requirements of a preliminary design. Fig. 2.14 gives curves for h.r.s. and c.r.o.s. materials.

No-load current. With a total core loss P_i , the active component of the no-load current I_0 is $I_{0a} = P_i / V_1$, and the total no-load current is the phasor sum of the active and reactive current components.

Asymmetry. In 3-ph core-type transformers the magnetizing current of the central limb is somewhat less than that of either outer limb. The unbalance is small because the limbs are relatively long in comparison with the yokes.

2.5 Machine magnetic circuits

In a machine, the intricacy of the shapes of the various parts, particularly of the airgap boundaries, makes a precise determination of the flux distribution very difficult. Fortunately it is possible to employ certain simplifications without undue loss of practical accuracy. It is usual to consider the end-winding as separate from the gap region; in fact, most of the individual parts are given an *ad hoc* treatment because, to be tractable, the boundaries must be redefined for each. The problem is aggravated by the several states in which the machine may be magnetized. The simplest is that for which only one member (stator or rotor) is excited. Less simply, but very important, is the estimate of leakage in the condition when both members have current-carrying windings. Most difficult of all is the transient condition, when induced currents in normally quiescent windings or in core masses affect the flux distribution and therefore the saturation condition from point to point. In this Section attention is confined to one of single excitation in the steady state.

Airgap

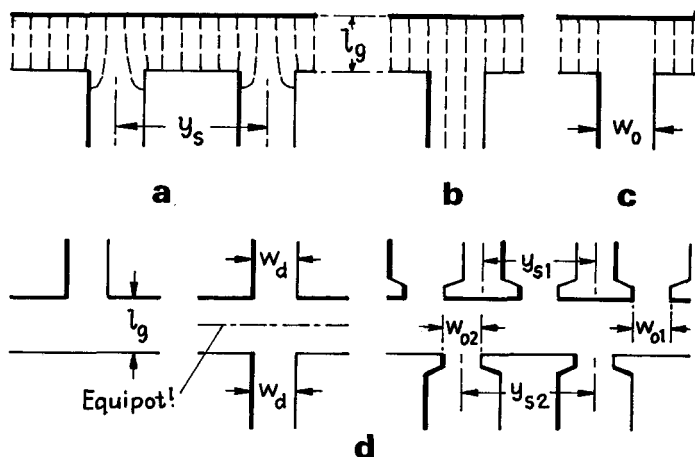
The gap is bounded on either side by iron surfaces, either or both slotted and sectionalized by cooling ducts. The principal quantities concerned are

l	total axial length of core	y_s	slot pitch
l_i	net iron length of core	w_o	slot opening
l_g	radial length of gap	w_d	duct width

The gap reluctance is solved using the analytical results given by Carter [2]. In Fig. 2.15, (a) shows a possible gap flux distribution in the presence of openings in the gap boundaries. The permeance of the gap is less than if the openings were ignored (b), but greater than if they were deemed to carry no flux at all (c). The effective 'contracted' width of the slot pitch is

$$y_s' = y_s - k_o w_o \quad (2.6)$$

where k_o is a function of the ratio (w_o/l_g) from Fig. 2.16, a distinction being made as to the nature, semi-closed or open, of the boundary opening. Radial



2.15 Gap surface openings.

ducts can be handled in a similar way by contracting the axial length l of the gap surface to l' where

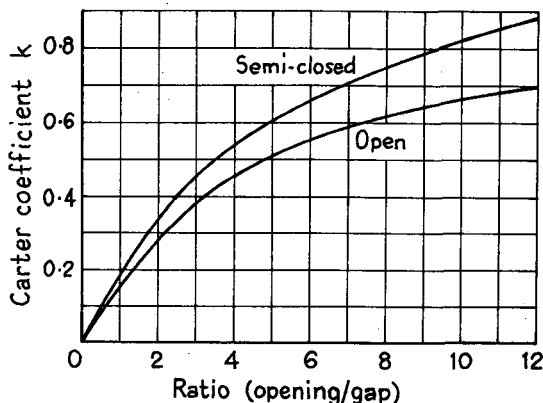
$$l' = l - k_d n_d w_d \quad (2.7)$$

for a total of n_d ducts, with k_d a function of (w_d/l_g) . If stator and rotor ducts coincide, their central plane is considered as an equipotential and eq.(2.7) applied, but with k_d now a function of $(w_d/\frac{1}{2}l_g)$.

When both gap surfaces are slotted, a more convenient method is to consider the gap length l_g to be extended to l'_g , i.e.

$$l'_g = k_g l_g \quad (2.8)$$

$$\text{where } k_g = k_{g1} \cdot k_{g2} = \frac{y_{s1}}{y_{s1} - k_{o1} w_{o1}} \cdot \frac{y_{s2}}{y_{s2} - k_{o2} w_{o2}}$$



2.16 Carter coefficients for airgaps.

The several quantities are shown in Fig. 2.15: the factors k_{o1} and k_{o2} are functions respectively of (w_{o1}/l_g) and (w_{o2}/l_g) from Fig. 2.16.

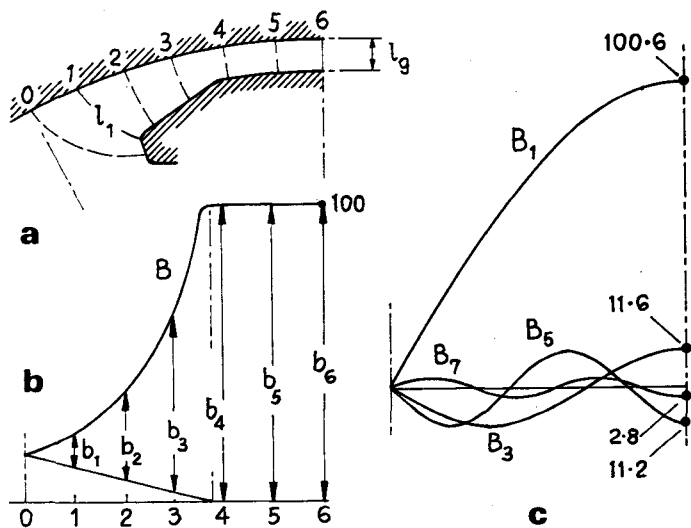
For an airgap between a dentated surface on one side and a salient pole on the other, the gap m.m.f. is

$$F_g = 800\,000 K_g B_g l_g \quad (2.9)$$

where l_g is the gap length at the pole centre, B_g is the peak gap-flux density (at the same point) and $K_g = (l/l')(y_s/y_s')$ takes account of slot-openings and ducts.

Gap-density distribution

A method for evaluating the gap-density waveform for odd-order harmonics up to the 7th is illustrated in Fig. 2.17. The outline of the half pole-arc is laid out and the half pole-pitch divided into six equal intervals. From each division mark is sketched a flux line, leaving and entering the iron surfaces in a normal direction, using as aid the technique for two-dimensional flux plotting by curvilinear squares. At the pole centre where the gap length is l_g , the flux density is assumed to be 100 units. At other points it is $100(l/l')$, where l is the length of the sketched flux line. A plot (b) of the result shows a positive value at position 0. This should in fact be zero, so a straight line is drawn from the ordinate of B at position 0 down to base at a point where the pole bevel begins. The actual flux density is taken as the intercept between the straight line and the B -plot, on the assumption that the omitted flux is leakage direct to the adjacent pole. The net curve, with ordinates $b_0 = 0, b_1, b_2 \dots b_5, b_6 = 100$ can



2.17 Salient-pole gap-density distribution.

now be analysed for its space harmonic content by use of the expression $B_n = k_1 b_1 + k_2 b_2 + \dots + k_6 b_6$ with the following factors k :

Harmonic density	k_1	k_2	k_3	k_4	k_5	k_6
B_1	0.086	0.167	0.236	0.289	0.323	0.167
B_3	0.236	0.333	0.236	0	-0.236	-0.167
B_5	0.323	0.167	-0.236	-0.289	0.086	0.167
B_7	0.323	-0.167	-0.236	0.289	0.086	-0.167

Then $B = B_1 \sin \theta + B_3 \sin 3\theta + B_5 \sin 5\theta + B_7 \sin 7\theta$.

EXAMPLE 2.1: A flux plot, Fig. 2.17, yields the values $b_1 = 10$, $b_2 = 31$, $b_3 = 68$, $b_4 = b_5 = b_6 = 100$ units. Derive the harmonic series for the flux density B , and find its mean and r.m.s. values.

The harmonic components, shown in Fig. 2.17(c), are:

$$B_1 = 0.86 + 5.16 + 16.72 + 28.9 + 32.3 + 16.7 = 100.6$$

$$B_3 = 2.36 + 10.32 + 16.04 + 0 - 23.6 - 16.7 = -11.6$$

$$B_5 = 3.23 + 5.16 - 16.04 - 28.9 + 8.6 + 16.7 = -11.2$$

$$B_7 = 3.23 - 5.16 - 16.04 + 28.9 + 8.6 - 16.7 = 2.8$$

The mean gap density and its r.m.s. value are

$$\bar{B} = (2/\pi) [100.6 - \frac{1}{3}11.6 - \frac{1}{5}11.2 + \frac{1}{7}2.8] = 60.4$$

$$B = \sqrt{[\frac{1}{2}(100.6^2 + 11.6^2 + 11.2^2 + 2.8^2)]} = 72.0$$

The amplitude of the fundamental density usually approximates to the peak gap density. The effect of the harmonics on the r.m.s. value is minimal.

Induction machines. These have very short airgaps, and tooth saturation flattens the peak density, the waveform of which can therefore be considered as a fundamental with a superposed positive 3rd harmonic. As the r.m.s. winding e.m.f. is almost unaffected by the harmonic, magnetic circuit calculation can be based on the flux density B_{30} at an electrical angle of 30° to the flux axis, a point where the 3rd harmonic is zero. If then a fundamental flux Φ_1 occupies a path of area a , its density at the 30° point is

$$B_{30} = B_{m1} \cos 30^\circ = \frac{1}{2}\pi(\Phi_1/a)0.87 = 1.36(\Phi_1/a)$$

The total m.m.f. on this basis is F_{30} , and its relation to the stator phase current I_1 , turns N_1 , winding factor K_{w1} and number of pole-pairs p is

$$F_{30} = 1.17 I_1 N_1 K_{w1}/p \quad (2.10)$$

Teeth

The slots of all but very small machines are parallel-sided so that the teeth are tapered. High saturation makes necessary a careful estimate of the m.m.f. The methods are as follows.

Graphical. From the known flux per tooth, the density is evaluated for a number of sections from tip to root, correcting for the true density B_t' if necessary

by the method described later. From a B/H curve the m.m.f. summation is made.

Three-ordinate. For simple trapezoidal teeth of moderate taper, Simpson's rule can be applied to the values H_1 , H_2 and H_3 relating to three equidistant sections. The mean value is then $H = (H_1 + 4H_2 + H_3)/6$.

One-third density. For slight taper and low densities it is sufficient to find the density $B_{t1/3}$ at a section one-third of the tooth length from the narrower end, and to assume that this obtains over the whole tooth length.

Integration. Accurate practical methods have been worked out for a number of tooth profiles and presented as curves for the usual core materials.

True and apparent tooth density. If the tooth flux density is high, the m.m.f. is sufficient to produce appreciable flux in the parallel paths in slots and ducts. Consequently an estimate of tooth density based on the tooth area alone is erroneous. Calling this the apparent tooth density B_t' and the true density B_t , the two are related by

$$B_t' = B_t + \mu_0 H(K - 1) \quad (2.11)$$

the second term being the flux density in the non-magnetic paths. The factor K is obtained from the ratio

$$K = (\text{gross area of iron and air})/(\text{net iron area})$$

which has to be found for each section taken. When reckoning the iron area the stacking factor (about 0.9) of the laminated core must be incorporated. K may have values between 1.5 for a machine without cooling ducts and 4 for the roots of turbo-generator teeth.

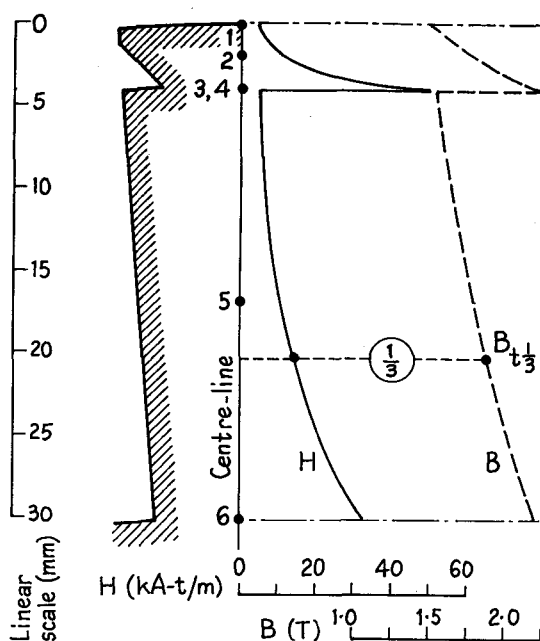
EXAMPLE 2.2: A machine with a gross axial length $l = 0.17$ m and active length $l_t = 0.153$ m has a flux $\Phi_t = 3.16$ mWb in one tooth pitch $y_s = 25.6$ mm. Data for points 1 ... 6 of the tooth profile, Fig. 2.18, are:

Section	Tooth width mm	Tooth area mm ²	B_t' T	H A-t/m	K	H A-t/m
1	13.6	2 080	1.52	2 300	—	—
2	11.4	1 740	1.82	11 000	—	—
3	9.2	1 410	2.24	80 000	3.0	50 000
4	13.2	2 020	1.56	2 800	—	—
5	11.5	1 760	1.79	9 500	—	—
6	9.8	1 500	2.10	40 000	2.5	32 000

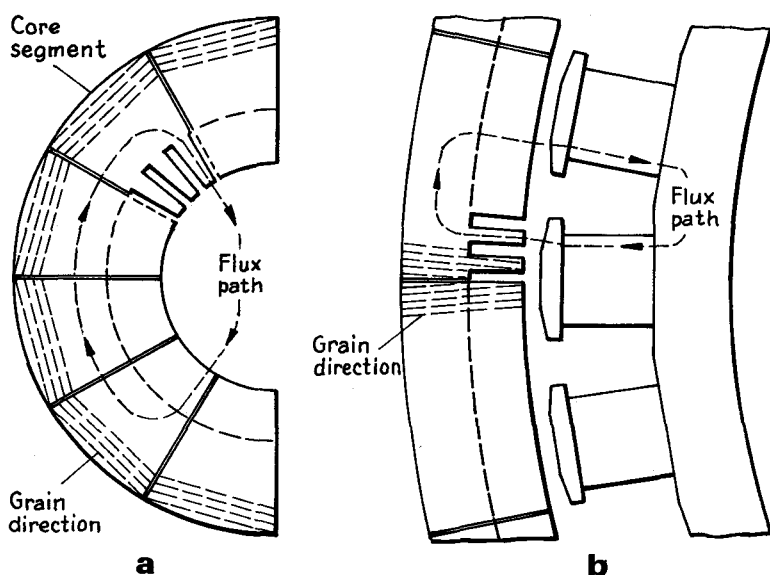
Find the tooth excitation F_t by (i) a graphical method, (ii) the density at one-third of the tooth length.

(i) Plots of B_t and H are given in Fig. 2.18, taking account of the factor K for sections 3 and 6. Summation gives $F_t = 390$ A-t.

(ii) At 10 mm from the root, the uncorrected density $B_{t1/3} = 1.86$ T requires $H = 13\,000$ and $F_t = 390$ A-t. The method is adequate for this moderate saturation level.



2.18 Tooth excitation.



2.19 Application of grain-oriented steel.

Cores

Excitation for poles, cores and similar sections is based on the choice of suitable regions for the calculation of area and the estimation of flux density. For salient poles it may be necessary to take account of the change of pole flux resulting from leakage, Sect. 2.6.

If it is feasible to choose a favourable magnetizing direction, the use of c.r.o.s. has possibilities in machines. In 2-pole turbo-generators the core-flux direction is mainly circumferential in the parts external to the slotting, Fig. 2.19(a). The core-plates are cut with the preferred direction at right-angles to a central radius. In large hydro-generators it is the teeth rather than the core that may benefit, and the arrangement (b) may give some reduction in both excitation and core loss.

Magnetizing current

Summation of the m.m.f.s required for the several series parts of the magnetic circuit gives the total to be provided by the exciting winding. The magnetizing current depends upon the number of turns and how they are distributed. For a concentrated coil of N turns such as that on a salient pole, the exciting current is $I = F/N$. For 3-ph uniformly distributed windings with the common 60° phase-spread and carrying sinusoidally varying currents, the m.m.f. distribution has a peak F_a related (as shown in Sect. 3.14) to the r.m.s. phase current by

$$F_a = 1.35 INK_w/p \quad (2.12)$$

where p is the number of pole-pairs and K_w is the winding factor of the N turns per phase in series.

2.6 Total flux

The total flux in a machine is a combination of the working, main or mutual flux crossing the gap, and the leakage flux in slots and end-windings. These may be summed by superposition on the assumption that the presence of one does not affect the other. The assumption is unwarranted because of the effects of saturation, but it has to be made for simple analysis. Saturation can be taken into account only by iterative digital computation, and even then the program is difficult to formulate.

On the total flux and its complicated distribution depends the estimation of the self and mutual inductance of windings, the conditions that give loss-generating flux pulsations, and the development of magnetic forces giving rise to the production of noise, the need for braced end-windings, harmonic torque effects and a number of other practical matters. Binns [2] has attacked the flux problem (particularly of airgaps with both iron surfaces slotted) by conformal transformation, the resulting equations being solved numerically by computation. Here we shall deal with leakage flux basically, in order to illustrate simple estimates which (though sometimes based on questionable assumptions) do give reasonable approximate results.

Leakage flux

The flux that traverses paths outside the active transfer or conversion regions (e.g. the core of a transformer or the airgap of a rotating machine) affects the excitation demands of salient poles, the leakage inductance of windings, forces in and between windings (especially under conditions of short circuit), load (stray) losses, circulating currents in transformer tank walls and metallic clamps, voltage regulation, and several more phenomena of practical import.

From eq.(2.1), the flux per unit current in a coil of N turns linked with a magnetic path of permeance Λ is $\Phi = N\Lambda$. The leakage inductance is therefore $L = N\Phi = N^2 \Lambda$. If the current in the coil alternates at angular frequency $\omega = 2\pi f$ its leakage reactance is

$$X = \omega L = 2\pi f N^2 \Lambda \quad (2.13)$$

Because of the complex geometry of leakage-flux paths, it is not easy to estimate Λ . Normally it is assumed that those parts of the path within iron require negligible m.m.f.; that is, they are infinitely permeable, so that the iron surfaces from which leakage flux emerges can be taken as equipotentials or to have some predetermined magnetic potential distribution. Attention is then concentrated on the non-magnetic ('air') parts of the flux path. Estimating the flux pattern by a flux plot or some similar method, the path can be divided into series or parallel sections for each of which a *permeance coefficient* λ is estimated from the ratio area/length to give $\lambda = a/l$. The permeance of a series of x sections is then written

$$\Lambda = \mu_0 \Sigma(a_x/l_x) = \mu_0 \Sigma(\lambda_x) \quad (2.14)$$

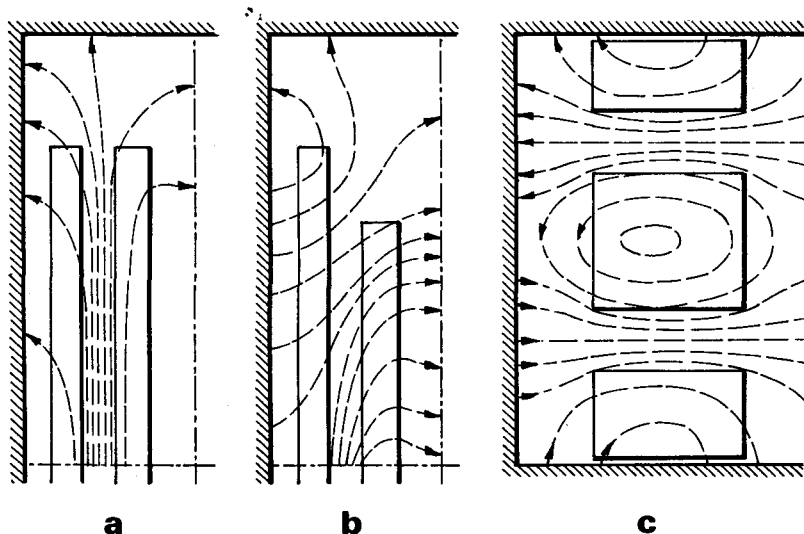
and the problem reduced to determination of λ . In practical cases the exciting circuits comprise conductors of finite area, and some of the leakage flux penetrates the area.

2.7 Leakage flux effects in transformers

The most common arrangements of the core and the windings are those of the core- and shell-types, Fig. 2.8. The flux plots in Fig. 2.20, based on calculations by Hague [3] show typical leakage field patterns for these. In (a), for concentric cylindrical primary and secondary windings of equal length, the leakage flux runs parallel with the axis for nearly the full winding length. Where, as in (b), the windings are of unequal lengths, the field is greatly modified. Plot (c) is for a shell-type transformer with sandwich coils. Detailed analyses of these field patterns have been given by Billig [4]. Here we attack the problem by simplifying the field geometry to obtain straightforward approximate estimates. The leakage flux is considered as a superposition of straight components in one or in two directions, Fig. 2.21(a).

Concentric cylindrical coils

Equal length. The leakage field is taken to comprise an axial flux (i) of uniform and constant value in the interspace between primary and secondary, and (ii) of value decreasing linearly to zero at the outer and inner surfaces of the windings and threading through them, as in Fig. 2.21(a). Further, the permeance of



2.20 Transformer leakage flux.

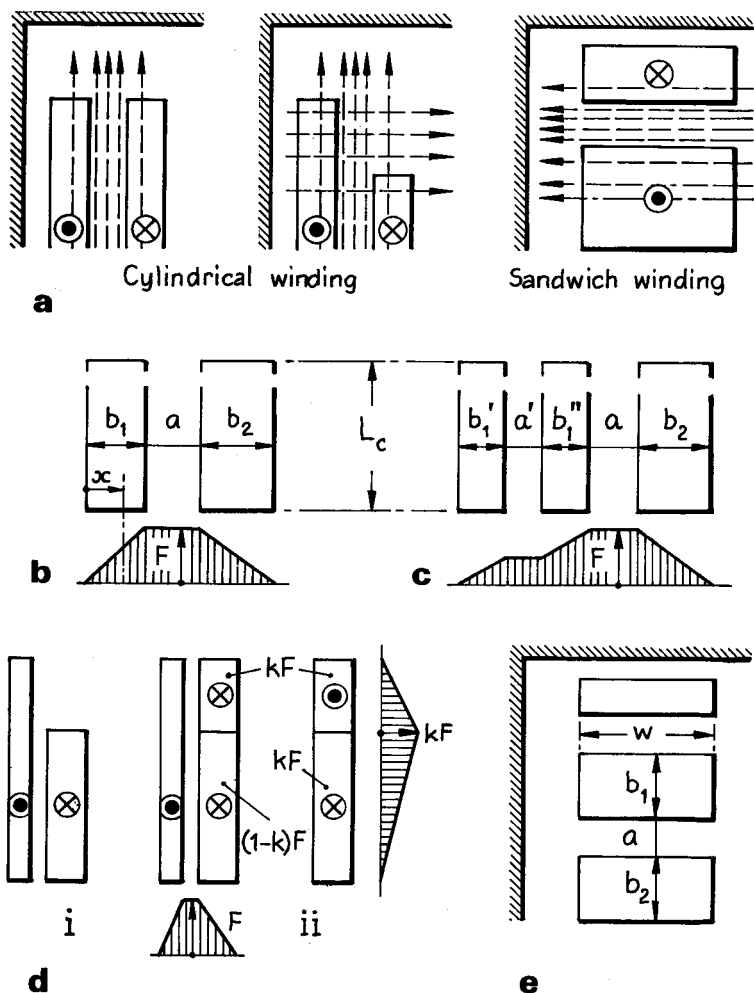
the leakage path external to the winding length L_c is taken to be so large as to require the expenditure of a negligible m.m.f.; i.e. the whole of the combined winding m.m.f. is expended on the path length L_c . It is further assumed that there is primary and secondary m.m.f. balance $N_1 I_1 = N_2 I_2 = F$. The flux density in the duct of width a is $B_a = \mu_0 F / L_c$, and the duct flux is $\Phi_a = B_a a L_{mt}$, where L_{mt} is the mean circumference of the duct, nearly the same as the length of a primary or secondary turn. One-half of the duct flux is taken as linking the N_1 primary turns to give a linkage $\frac{1}{2} N_1 \Phi_a$. Within the primary winding at radial distance x from the surface, Fig. 2.21(b), the flux density is $B_a(x/b_1)$ and the flux in an elemental annulus of width dx and (approximate) circumference L_{mt} is $d\Phi_b = B_a(x/b_1)L_{mt} \cdot dx$. Now $d\Phi_b$ links only the fraction (x/b_1) of the turns: hence integrating $N_1(x/b_1) \cdot d\Phi_b$ over the width b_1 gives the internal linkage as $B_a N_1 L_{mt}(b_1/3)$.

The primary leakage inductance is the linkage per ampere, for which $F = N_1 I_1 = N_1$. Summing the two components gives

$$L_1 = \mu_0 N_1^2 \frac{L_{mt}}{L_c} \left[\frac{a}{2} + \frac{b_1}{3} \right]$$

Substituting N_2 for N_1 and b_2 for b_1 gives the secondary leakage inductance. The total equivalent inductance in primary terms (from the impedance transformation in Sect. 1.9) is $L = L_1 + L_2(N_1/N_2)^2$. The corresponding leakage reactance for frequency f is therefore

$$X_1 = \mu_0 2\pi f N_1^2 \frac{L_{mt}}{L_c} \left[a + \frac{b_1 + b_2}{3} \right] \quad (2.15)$$



2.21 Calculation of leakage flux.

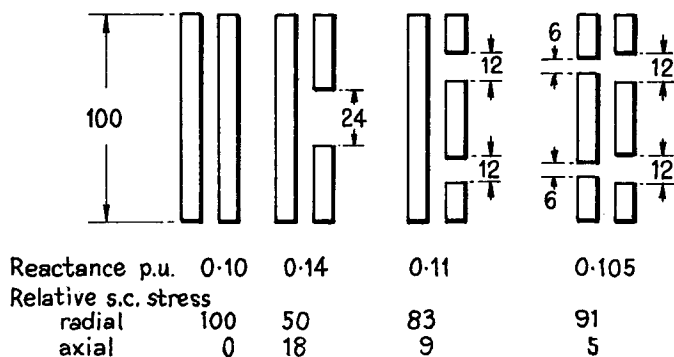
Alternatively, with F as the rated m.m.f. per limb of either primary or secondary, and with the e.m.f. per turn $E_t = V_1/N_1$, the *per-unit* leakage reactance of the transformer is

$$\epsilon_x = \mu_0 2\pi f \frac{F}{E_t} \frac{L_{mt}}{L_c} \left[a + \frac{b_1 + b_2}{3} \right] \quad (2.16)$$

If the h.v. winding is split, as in Fig. 2.21(c), into equal parts the square-bracketed terms in the two equations above are replaced by the expression $[a + \frac{1}{3}(b_1' + b_1'' + b_2) + \frac{1}{4}a']$.

Unequal length. This common case may be the result of the manufacturing process, non-uniform turn insulation, or the provision of tapped sections for voltage regulation. One approach is to resolve the leakage flux into axial and radial components, as in Fig. 2.21(a) for a difference of length at one end.

The concentric coils, each of m.m.f. F , are shown at (i) in Fig. 2.21(d). The arrangement is resolved as in (ii) into two superposed m.m.f. distributions



2.22 Cylindrical coil asymmetry.

in accordance with the lengthwise unbalance k . The first is dealt with as if it comprised balanced coils of equal length, the latter in a corresponding way but with different orientation and appropriate physical dimensions. Typical cases are shown in Fig. 2.22, with figures of percentage reactance compared with the case of symmetrical coils of equal length.

Sandwich coils

Fig. 2.21(a) shows the simplifying assumption made. It is usual to arrange the coils as in (e), where the end coils have one-half of the turns of the remainder. There are n h.v. coils, $(n-1)$ l.v. coils plus two half-coils. Taking two adjacent half-coils as the unit, the reactance is obtained by analogy with the cylindrical coil arrangement. For the n sections and $2n$ units the total leakage reactance referred to the primary is

$$X_1 = \mu_0 2\pi f \frac{N_1^2}{2n} \frac{L_{mt}}{w} \left[a + \frac{b_1 + b_2}{6} \right] \quad (2.17)$$

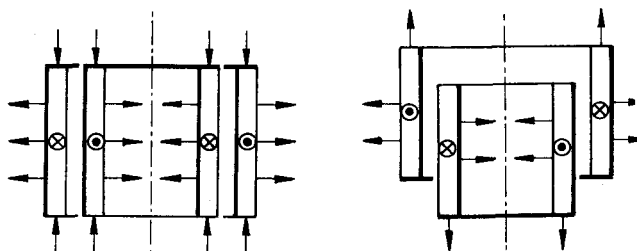
where N_1 is the total number of primary turns. The per-unit reactance is given by

$$\epsilon_x = \mu_0 2\pi f \frac{F}{E_t} \frac{L_{mt}}{2nw} \left[a + \frac{b_1 + b_2}{6} \right] \quad (2.18)$$

with F the rated phase m.m.f. of either primary or secondary.

Mechanical forces

The primary and secondary currents of a loaded transformer are in magnetic opposition with respect to the common core, but cumulative with respect to the space between them, so exciting leakage flux. One important consequence is the production of mechanical forces of mutual repulsion. Consideration of



2.23 Mechanical force directions.

the simplified diagrams in Fig. 2.23 shows that these forces can be analysed into the following components:

- (i) *radial*, tending to burst outer and crush inner coils;
- (ii) *axial*, tending to compress coils lengthwise; and
- (iii) *axial*, tending to force apart axially displaced coils.

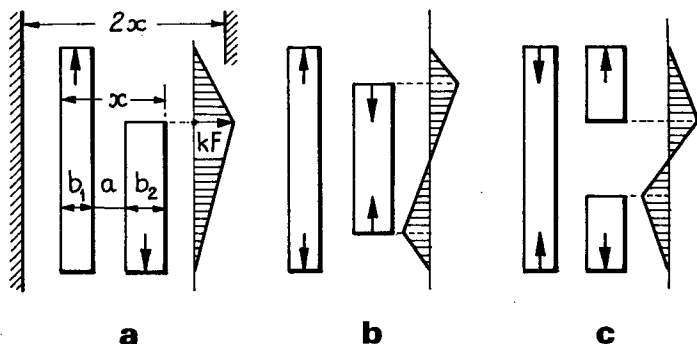
For rated currents the forces are comparatively small. Under the worst fault conditions, however, instantaneous currents may reach a level of nearly $2/\epsilon_x$ per-unit because of the doubling effect. In a 100 kVA transformer with $\epsilon_x = 0.04$ p.u. the radial force may reach 400 kN. In a 100 MVA transformer with the same per-unit reactance the force might be 180 MN; and even with 0.10 p.u. reactance the force is 30 MN.

Radial force. Circular coils have a shape best suited to withstand radial forces. For an instantaneous current i in the N turns of one winding the flux density in the annular duct is $B_a = \mu_0 F/L_c$ and the mean value in the windings is $\frac{1}{2}B_a$, where $F = Ni$. The mean force per unit length of perimeter and per unit axial length is therefore $\frac{1}{2}\mu_0(F/L_c)(F/L_c)$. For the whole surface area $L_{mt}L_c$ the force is

$$f_r = \frac{1}{2}\mu_0 F^2 (L_{mt}/L_c) \quad (2.19)$$

Axial force. Radial components of leakage flux originate axial compressive forces tending to squeeze the windings. When the coil arrangement is symmetrical the resulting stresses are unimportant even under fault conditions, and can normally be supported by the winding material. In shell types the outer coils experience repulsion forces which are resisted by the core in the buried length, but those parts of the coils external to the core must be braced.

Axial force of asymmetry. The radially directed leakage field produces coil forces illustrated by three typical cases in Fig. 2.24. Case (a) is the worst condition, and if as a result of a short circuit the coils are displaced by the forces, the forces themselves are further intensified. In (b) there is (ideally) less ten-



2.24 Axial forces of asymmetry.

dency to produce such an effect. Cases (a) and (c) are typical respectively of end-tapped and centre-tapped coils. The mechanical superiority of (c) over (a) is obvious. The forces of asymmetry can be estimated from the cross-flux developed by the out-of-balance m.m.f. kF , shown for example in case (a). The windings are deemed to be enclosed between iron surfaces separated by a distance $2x = 2(a + b_1 + b_2)$: the cross-flux density for a given instantaneous m.m.f. $F = Ni$ has the maximum value $\mu_0 kF/2x$. The axial force is that produced by the mean cross-flux density (one-half of the maximum) acting on N turns, i.e.

$$f_a = \frac{1}{2} \mu_0 kF^2 L_{mt} / 2(a + b_1 + b_2) \quad (2.20)$$

In the comparative cases in Fig. 2.22 the short-circuit currents reduce with subdivision because of increased leakage flux. The forces due to asymmetry depend upon how closely the primary and secondary discontinuities are 'matched'. **Bracing.** In so far as the stresses are inwards axially, they are taken by the windings themselves. Inward radial stresses are passed to the formers, packing pieces and cores. Outward axial stresses must be withstood by the end insulation. A well-constructed transformer will have a suitable choice of conductor dimensions and interturn insulation, and coils well supported and braced, with the compressive stresses kept in view. The end supports are not easily arranged to give good insulation and at the same time great mechanical strength, so that as far as possible the need for the latter quality must be avoided by maintaining symmetry, or by a suitable distribution of the unpreventable asymmetry caused by tappings, connections, etc.

EXAMPLE 2.3: The windings of a 600 kVA, 7.5/0.435 kV, 50 Hz, 1-ph, 2-limb core-type transformer have the following dimensions with reference to Fig. 2.21(c):

$$\begin{array}{llll} a = 15.75 \text{ mm} & b_1' = b_1'' = 9.0 \text{ mm} & L_{mt} = 1.26 \text{ m} & N_1 = 378 \\ a' = 8.5 \text{ mm} & b_2 = 23.0 \text{ mm} & L_c = 0.35 \text{ m} & N_2 = 22 \end{array}$$

Then $a + (b_1' + b_1'' + b_2)/3 + a'/4 = 31.5 \text{ mm} = 0.0315 \text{ m}$. Each limb has $L_c = 0.35 \text{ m}$ and $N_1 = 378/2 = 189$ primary turns, and an equivalent total

reactance (referred to the primary) given by eq.(2.15). The reactance of the two limbs forming the 1-ph winding is therefore

$$X_1 = 2[\mu_0 2\pi 50 \times 189^2 \times (1.26/0.35) \times 0.0315] = 3.2 \Omega$$

The full-load primary current is $600/7.5 = 80$ A, so that the per-unit reactance is

$$\epsilon_x = 3.2(80/7500) = 0.034 \text{ p.u.}$$

Neglecting winding resistance, the r.m.s. symmetrical short-circuit current is $1/0.034 = 29.4$ p.u., corresponding to an instantaneous peak of $\sqrt{2} \times 29.4 = 42$ p.u. With asymmetry this can be taken as, say, $1.8 \times 42 = 75$ p.u. or $75 \times 80 = 6000$ A. Applying eq.(2.19) with 189 turns per limb, the peak instantaneous radial outward force on the h.v. winding is

$$f_r = \frac{1}{2}\mu_0(6000 \times 189)^2 \times (1.26/0.35) = 3100 \text{ kN}$$

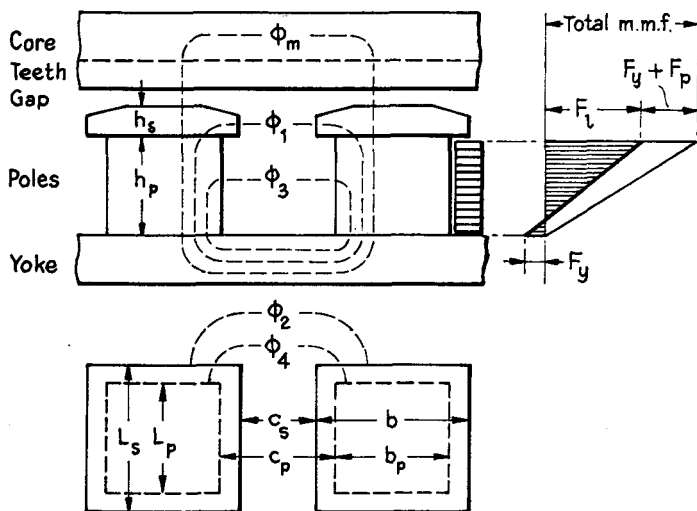
Suppose that the h.v. winding is 5% shorter than the l.v. winding and that the difference is all at one end. The result is an unbalance between the h.v. and l.v. distributions of m.m.f., with $k = 0.05$. From eq.(2.20), with $a + a' + b_1' + b_1'' + b_2 = 0.065$ m, the axial force of asymmetry is

$$f_a = \frac{1}{2}\mu_0 0.05 \times (6000 \times 189)^2 \times 1.26/2 \times 0.065 = 4200 \text{ kN}$$

If this force causes further divergence of the windings, a subsequent short-circuit current of the same value will produce an even greater force.

2.8 Leakage flux effects in machines

Estimation of leakage flux is more difficult than for transformers. Flux-plot



2.25 Leakage flux of salient poles.

methods include the fitting of a pattern of curvilinear squares by freehand drawing, electrolytic tanks, membranes, resistance networks, etc. With the advent of digital computers it has become possible to predetermine flux patterns accurately without having recourse to analogues. Mamak and Laithwaite [5] have described a method of deriving the relevant finite-difference equations for a two-dimensional problem using the concept of vector potential, enabling the flux pattern to be computed. A simpler approach through m.m.f. and permeance is given here, indicating the order of the results rather than their precise values.

Salient poles

Most low-speed salient-pole machines have many poles on a large diameter, so that the pole-axes can be assumed to be parallel. Components of the leakage flux are set out as in Fig. 2.25. The m.m.f. F_l producing leakage on no load comprises that for the airgap, teeth and core. Approximately, F_l produces Φ_1 and Φ_2 , while $\frac{1}{2}F_l$ produces Φ_3 and Φ_4 . Taking the leakage flux between facing sides to be rectilinear, and that between parallel faces (Φ_2 and Φ_4) to comprise straight lines plus two quarter-circles, then the estimations of leakage flux per pole are

- (i) between pole-shoes, $\Phi_{sl} = 2\Phi_1 + 4\Phi_2$:

$$\Phi_{sl} = \mu_0 F_l \left[2 \frac{L_s h_s}{c_s} + 2.9 h_s \log_{10} \left\{ 1 + \frac{\frac{1}{2} \pi b}{c_s} \right\} \right] \quad (2.21a)$$

- (ii) between poles, $\Phi_{pl} = 2\Phi_3 + 4\Phi_4$:

$$\Phi_{pl} = \mu_0 F_l \left[\frac{L_p h_p}{c_p} + 1.5 h_p \log_{10} \left\{ 1 + \frac{\frac{1}{2} \pi b_p}{c_p} \right\} \right] \quad (2.21b)$$

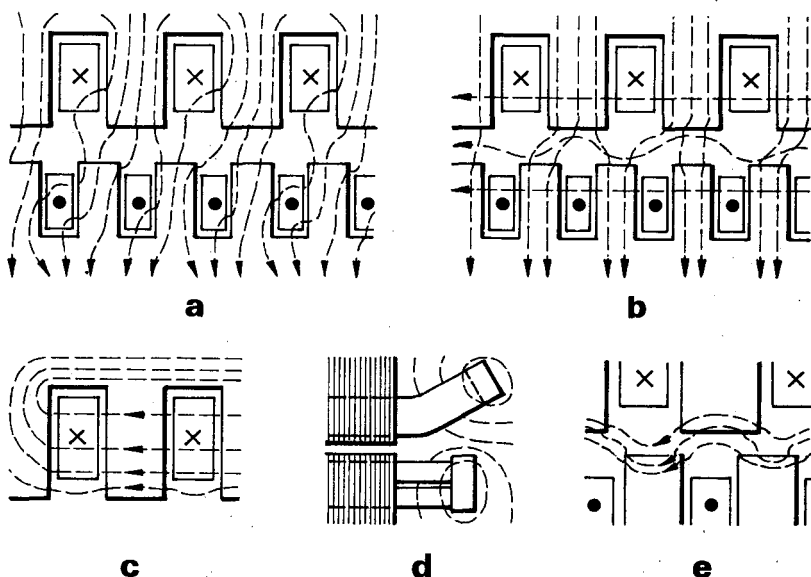
Φ_{sl} is generally of the order $20F/10^8$, while Φ_{pl} approximates to $80F/10^8$. The flux at the back of the pole-shoes is $\Phi + \Phi_{sl}$, and that at the root of the pole (where it joins the yoke) is $\Phi + \Phi_{sl} + \Phi_{pl}$, where Φ is the active gap flux.

Non-salient poles

The non-salient rotor has considerable leakage if the retaining rings clamping the overhang are not non-magnetic; the amount is dependent on the saturation flux density of the material, for the density will reach this level in all normal cases. A disadvantage of such leakage is that it raises the m.m.f. required for the core and teeth of the rotor. Non-magnetic steel is generally preferred for the retaining rings for this reason. Estimation of the leakage of non-salient rotors has to be taken step by step with that of the working flux, and cannot be treated separately.

Armature

By 'armature' is meant the member carrying distributed a.c. windings. In the gap region the leakage flux is a component of the total flux; only in the end



2.26 Leakage flux of slotted members.

regions can it be recognized as a separate entity. Fig. 2.26 shows at (a) the total flux in the gap region with both members excited, and (b) its arbitrary components. The working flux links both windings while the leakage flux either links one winding only, or is so directed as not to contribute to useful energy transfer between stator and rotor.

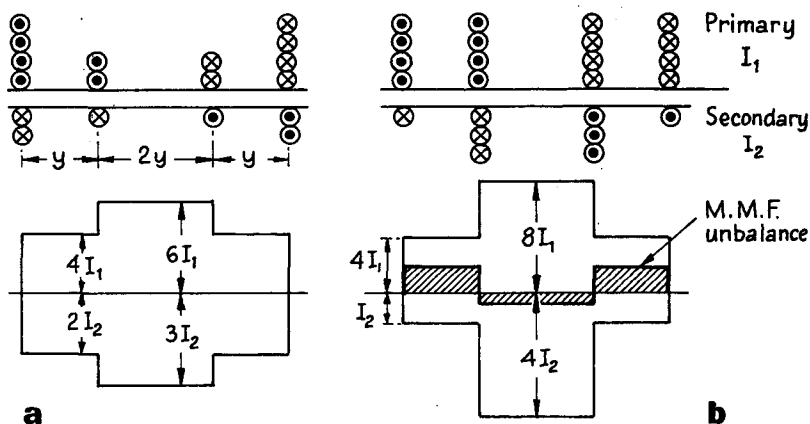
Summarizing the leakage flux components with reference to Fig. 2.26, there are:

Overhang leakage (d), produced by the end-windings and of value depending on the winding arrangement and on the proximity thereto of metal masses, such as core-stiffeners and end-covers, having conducting and magnetic properties.

Slot leakage (c), crossing the slot conductors from tooth to tooth.

Zig-zag leakage (e), which depends on the gap length and on the relative position of the stator and rotor tooth-tips; it tends to 'zig-zag' across and along the gap, utilizing the relatively high tooth-tip permeances.

Differential leakage, Fig. 2.27, the result of a difference between the current distributions on opposite sides of the gap. In (a), where two windings with identical distributions have a turn-ratio of 2/1, m.m.f. balance both in magnitude and in distribution is obtained if $I_2 = 2I_1$. But with the same turns-ratio and a dissimilar distribution as in (b), the condition for zero secondary linkage is that $(72yI_1 - 34yI_2) = 0$, giving $I_2 = (72/34)I_1$, a ratio differing from 2. However, the primary linkage is $(160yI_1 - 72yI_2) = 7\frac{1}{2}yI_1$. Thus the m.m.f.s cannot precisely balance. The net flux acts as leakage and is called the differential or harmonic leakage, the latter term because of an alternative approach by the

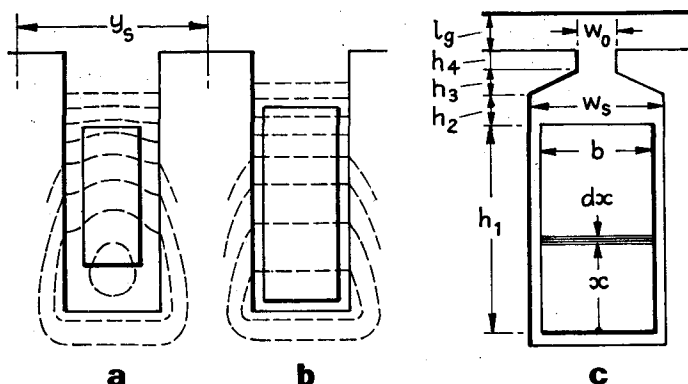


2.27 Differential or harmonic leakage.

harmonic resolution of the two m.m.f. distributions. The differential leakage is mainly the result of the higher space harmonics, and for normal windings may often be small enough to ignore. In a cage rotor, the currents can balance the stator currents at every point without restraint, so that differential leakage vanishes except for those space harmonics whose wavelength is comparable with the slot-pitch.

Slots

Fig. 2.28 shows the very common parallel-sided slot shape, accommodating one or more rectangular conductors. When the conductors carry current, they set up a slot-leakage flux pattern of complex shape. Typical patterns have been obtained for steady-state conditions by Hague [3], by applying the condition that the magnetic vector potential within the slot must satisfy the



2.28 Slot leakage.

Poisson equation within the section of the conductors, and the Laplace equation elsewhere. If there is appreciable space below the bottom conductor and its width is substantially less than that of the slot, the field pattern is that shown in (a). However, apart from the case of high-voltage machines requiring thick slot insulation, the proportions are more usually those in (b), for which the field pattern is very nearly in the form of straight lines across the slot. There is thus some justification for the classical assumption that an elemental leakage path is completed around the bottom of the slot as a consequence of the assumption that the ferromagnetic material has infinite permeability. The pattern is readily analysed, and is adequate for steady currents or those of low frequency, but fails (unless the conductors are subdivided and transposed) for frequencies approaching normal industrial supply values (e.g. 20 Hz upward). We adopt the straight-line assumption here, leaving more complicated cases for Chapter 3.

Semi-closed rectangular slot: The permeance coefficient per unit of axial slot length is the ratio (area/length) of the air path considered. A number of component parallel paths can be taken. Referring to the dimensions in Fig. 2.28(c) we have: (h_2/w_o) for the region immediately above the conductor; $2h_3/(w_s + w_o)$ for the wedge portion taken as an average width; and (h_4/w_o) for the lip.

The slot height h_1 occupied by the conductor has to be treated differently. All the leakage flux *above* the conductor links it wholly, but the flux within the height x links only the lower part of the conductor and contributes only partial linkage. Also, such flux has a lower density as only the fraction (x/h_1) of the total conductor current is available to provide the m.m.f. For 1 A in the conductor, uniformly distributed, the flux in the elemental path dx is $d\Phi = \mu_0(x/h_1 w_s) \cdot dx$ and it links the fraction (x/h_1) of the conductor. The permeance coefficient for this part is the summation from $x = 0$ to $x = h_1$ of $(x^2/h_1^2 w_s) \cdot dx$, giving $h_1/3w_s$. Adding the several parallel coefficients, the specific slot permeance is obtainable from

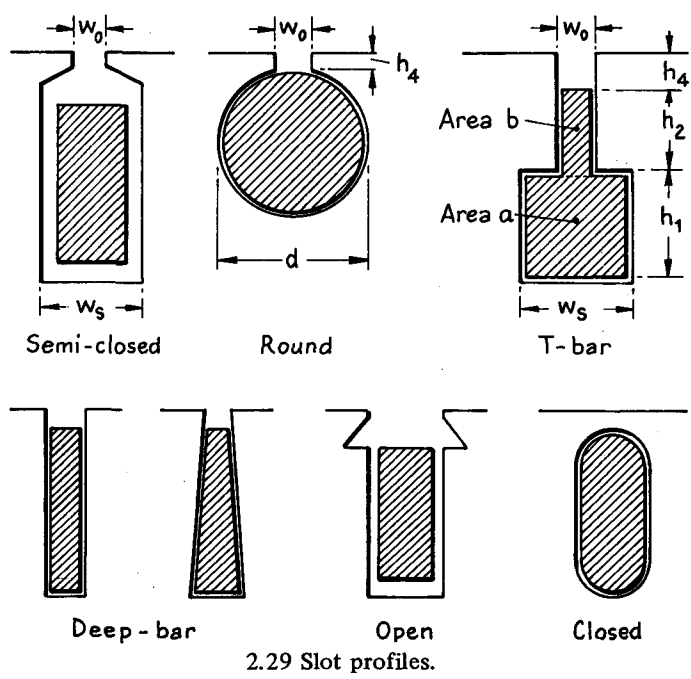
$$\lambda_s = \frac{h_1}{3w_s} + \frac{h_2}{w_s} + \frac{2h_3}{w_s + w_o} + \frac{h_4}{w_o} \quad (2.22)$$

The total slot permeance of a net core-length L_s is the product $\mu_0 \lambda_s L_s$.

Other slot shapes: Fig. 2.29 shows some of the several forms that slots can take. The *semi-closed rectangular* has already been discussed. The *circular* slot is often used for the cage rotors of induction machines, as is also the *T-bar* slot. Two *deep-bar* slots are shown, one trapezoidal and the other parallel-sided. The *open* slot shape permits of the easy insertion of pre-formed coils. The *closed* slot is sometimes used for induction machine rotors with cast-in aluminium cages, the 'bridge' mechanically retaining the molten metal during casting.

Round slot: With the assumption that flux crosses the slot in straight lines (an even less tenable concept than with rectangular slots) the result is independent of the slot diameter. It is found to be 0.62, usually raised to 0.66 to compensate for the actual flux pattern. The permeance coefficient is then

$$\lambda_s = 0.66 + (h_4/w_o) \quad (2.23)$$



T-bar slot: With uniform current distribution in the areas *a* and *b*, condition that applies for frequencies below about 5 Hz, the permeance coefficient is found as for a rectangular slot: it is

$$\lambda_s = \frac{1}{(a+b)^2} \left[\frac{h_1}{w_s} \cdot \frac{a^2}{3} + \frac{h_2}{w_o} \left\{ a^2 + ab + \frac{b^2}{3} \right\} \right] + \frac{h_4}{w_o} \tag{2.24a}$$

The total conductor section is $(a + b)$, the two parts sharing the current in proportion to their area. For currents of industrial frequency the current is substantially confined to *b*, and the permeance coefficient approximates to

$$\lambda_s = \frac{h_1}{2w_s} + \frac{h_2}{3w_o} + \frac{h_4}{w_o} \tag{2.24b}$$

Values intermediate between these two extremes may be of significance during starting and speed change. An elegant solution of the T-bar and similar shaped conductors is given by Bruges [6].

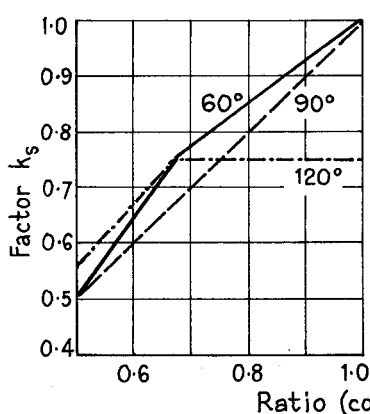
Deep slot: This is a variant of the foregoing, exploiting the effect of leakage flux in disturbing the current distribution by making the conductor deep and narrow. Eqs.(2.24) apply, with the obvious modifications.

Open slot. Facility in winding is counterbalanced by the aggravation of high-frequency no-load tooth loss and the increase of gap reluctance, particularly in induction machines whose gaps are very short. It is possible to ameliorate such effects by use of semi-magnetic slot-wedges.

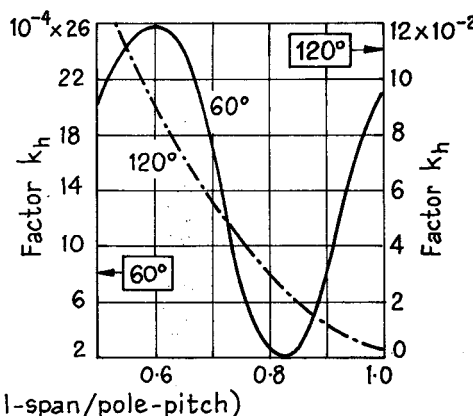
Closed slot: The 'bridge' reduces gap reluctance, but increases slot leakage flux by increasing the last term in eq.(2.22) by the factor μ_r , the effective relative permeability of the bridge. As μ_r is dependent on the degree of saturation its assessment is difficult: its value may change e.g. from 25 at low conductor currents down to 3 or 4 when the current is large, as at starting.

Modifications

In the calculation of the inductance of windings from their permeance coefficients, certain effects may require to be taken into account. The influence of slot *skewing* is discussed in Chapter 3. The *chording* of a winding affects its winding factor K_w ; and also the leakage flux because some slots hold conductors belonging to different phases. The effective slot-leakage permeance is reduced by the factor k_s in Fig. 2.30. Where the iron paths of leakage flux are



2.30 Slot leakage factor.



2.31 Differential leakage factor.

saturated, the flux per ampere is reduced. It is common to base the leakage on the assumption of infinite permeability and then to apply saturation factors. Agarwal and Alger [7] produced a simple method in which the iron is assumed to have a B/H relation in which the permeability is infinite ($H = 0$) for any flux density up to 2.0 T, and thereafter a fixed saturation flux density of 2.0 T regardless of the value of H . The method was developed to deal with combined tooth-tip and zigzag leakage components, and has been extended by Chalmers and Dodgson [8].

Zigzag

One expression (of many) gives the zigzag leakage reactance of a phase winding in terms of the reactance offered to the main flux by the winding, i.e. the magnetizing reactance x_m , in the form

$$x_z = \frac{5}{6} x_m \left[\frac{1}{g_1^2} + \frac{1}{g_2^2} \right] \quad (2.25)$$

where $g_1 = S_1/2p$ and $g_2 = S_2/2p$ are the numbers of slots per pole of the two members.

Differential

An expression for the differential or harmonic leakage reactance is

$$x_h = x_m(k_{h1} + k_{h2}) \quad (2.26)$$

where values of the coefficients k_h are obtained from Fig. 2.31 due to Alger [9]. This reactance is ignored for induction machines with cage rotor windings.

Overhang (end-winding)

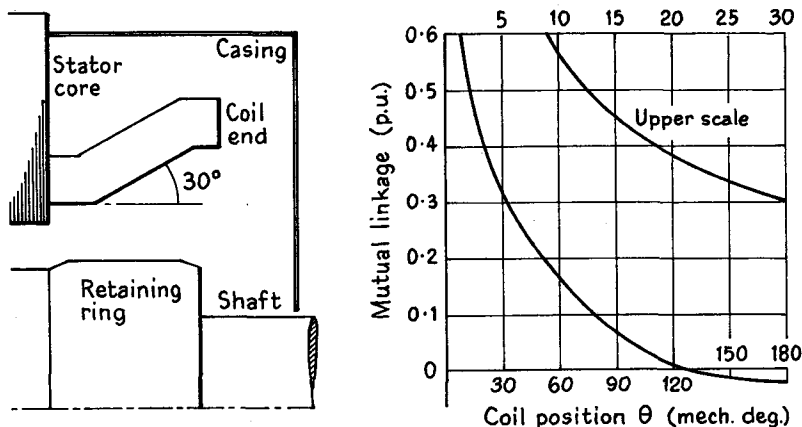
One simplified expression for small machines is in terms of the length L_o of an overhang conductor and the pole-pitch Y :

$$L_o \lambda_o = k_s Y^2 / \pi y_s \quad (2.27)$$

where y_s is the slot-pitch and k_s is given by Fig. 2.30.

Simple direct methods fail completely to deal with the overhang leakage problem in cases where its assessment is very important, as in turbo-generators. The leakage must be related to the length of the end-connectors and their shapes, the interleaving of phase conductors, the spacing between the stator and rotor overhangs, the proximity and configuration of neighbouring magnetic or conducting parts (cores, clamping and stiffening plates, housings, retaining rings, etc.) and the level of saturation in ferromagnetic parts. Not only is the leakage reactance required, but also the consequent 'load' loss and the magnetically generated mechanical forces.

Approximate analyses based on a solution of the Laplace equation for the scalar magnetic potential have been advanced by Reece and Pramanik [10]. A 2-dimensional field-plot method for estimating the leakage of a given machine has been described by Hawley, Edwards, Heaton and Stoll [11]. Carpenter [12]



2.32 Overhang leakage in turbo-generator.

has extended the principle of images to cases of currents partly free and partly embedded in one of the reflecting surfaces, and in particular to end-winding problems in which this surface is divided by an airgap. Lawrenson [13] has used the method of images for dealing with *magnetic* and *eddy-current-carrying* boundaries. Eddy-current boundaries are assumed to be non-permeable and to reduce leakage-flux linkage, while magnetic boundaries increase it. Lawrenson's method is (i) to obtain the basic linkage of a coil-end, (ii) to modify the value by factors introduced by the boundary surfaces, and (iii) to superpose the linkages of all the coils in a phase band in accordance with their relative position.

The shape of a single end-coil, and the significant boundary surfaces in the neighbourhood of the end winding, are shown in Fig. 2.32. The steps in the estimation of the linkage per ampere are as follows:

(i) Using the length L_o of the overhang of a coil of cross-section $b \times c$ and comprising N_c conductors all in series, the basic linkage of an isolated coil-end is obtained from

$$\psi = 1.5L_o(1 + 0.0075L_o)N_c^2 \left[\ln \frac{2L_o}{b+c} + 0.5 \right] 10^{-7}$$

(ii) Taking ψ as 1.0 p.u., the boundaries make proportional contributions 0.18 for the cylindrical casing, 0.07 for the end casing, 0.08 for the stator core-end, and 0.11 for a magnetic retaining ring on the rotor. These contributions must be made with the appropriate sign, positive for a magnetic boundary and negative for an eddy-current boundary. A non-magnetic retaining ring has no steady-state effect, but appears as an eddy-current boundary under transient conditions. These contributions result in the factors k_b by which ψ has to be multiplied to give the total linkage of an isolated end-coil:

VALUES OF FACTOR k_b

Type of rotor retaining ring: With or without core screen:	magnetic		non-magnetic	
	with	without	with	without
Operating condition: steady-state	1.30	1.44	1.19	1.35
transient	0.56	0.56	0.56	0.56

(iii) The graph in Fig. 2.32 shows the mutual inductance between one coil-end at position 0° and a second at some mechanical angle θ to it, in per-unit of the linkage of the first coil. The total linkage of a phase group is (a) the summation for each coil in turn of its own linkage and that due to other coils in the group, and (b) summation of all the totals in (a). The result is then doubled (in a 2-pole machine) for the opposite coil-end group of the same phase. The graph is for coils with a 150° span.

2.9 Phase reactance

The reactance of a phase winding of $g'z_s$ turns per pole-pair, with a conductor

length L_c and a permeance coefficient λ is, from eq.(2.13),

$$X = 2\pi f N^2 \lambda = 2\pi f (g' z_s)^2 \mu_0 2L_c (\lambda/g')$$

because the turn length includes two conductors and the flux has to traverse g' slots. If the turns in all pole-pairs are in series, the phase reactance is

$$X = 4\pi f \mu_0 N_{ph}^2 L_c (\lambda/pg') \quad (2.28)$$

Induction machine

The total phase reactance referred to the stator is

$$X_1 = x_1 + x_2' = x_{s1} + x_{s2}' + x_o + x_z + x_h \quad (2.29)$$

where x_{s1} is the stator slot reactance using the net core-length L_s in eq.(2.28) with the appropriate permeance coefficient for the slot shape; x_{s2}' for the rotor is calculated in a similar way and then multiplied by the square of the effective turns ratio, i.e. by $K_{w1}^2 S_1 / K_{w2}^2 S_2$; and x_o , x_z and x_h are the overhang, zig-zag and harmonic (differential) reactances.

For calculation of performance from an equivalent circuit it is sufficient to take $x_1 = x_2' = \frac{1}{2} X_1$. As (apart from slot leakage) the leakage flux is a result of the stator and rotor currents in combination, it is not strictly possible to separate it completely into individual stator and rotor components.

Synchronous machine

The leakage flux must be known to assess regulation under steady-state conditions; but it enters also into many complex problems such as short-circuit behaviour and damping. For the calculation of regulation, the leakage flux may be taken as the sum of the slot and overhang leakage fluxes for normal full-load r.m.s. current I_c per conductor. With eq.(2.1) as basis, the peak leakage flux associated with the slots for a coil of N_c turns is $\sqrt{2} I_c N_c \Lambda_s$, where Λ_s is $\mu_0 2L_s \lambda_s$. For a group of g' coils of the same phase, the turns producing the flux are $g' N_c$, but the slot flux now has to cross g' slots and its air path is g' times as long. This gives the total slot leakage flux per phase as

$$\Phi_s = 2\sqrt{2} \mu_0 I_c N_c (L_s \lambda_s) \quad (2.30)$$

An identical expression, except that $(L_o \lambda_o)$ is substituted for $(L_s \lambda_s)$, gives the overhang leakage flux. Zigzag leakage does not occur for a plain salient-pole machine, but may be present (though reduced by the comparatively long air-gap) if the pole shoes carry damper windings.

2.10 Unbalanced magnetic pull

In an excited rotary machine, a magnetic flux crosses the gap between stator and rotor, and these members exert on each other a strong magnetic attraction, considerably more intense than the useful force of interaction between the flux and the gap-surface currents. In a completely symmetrical machine the forces of attraction balance out with no resultant; but if there is asymmetry the balance is incomplete, leaving a resultant force called the *unbalanced magnetic pull* (u.m.p.).

A simplified introduction to the analysis of u.m.p. can be based on the well-known force of attraction between infinitely permeable ferromagnetic surfaces with a flux of density B crossing the gap between them. Per unit area this force is $\frac{1}{2}B^2/\mu_0$.

Symmetry. In the ideal 2-pole machine of Fig. 1.10 the rotor is set symmetrically within the stator bore, both gap surfaces are purely cylindrical, and the gap length is everywhere l_g . Taking the gap flux as distributed sinusoidally with its peak B_m on the x -axis, then at angle θ to the axis the radial density is $B = B_m \cos \theta$, and the radial attractive force over an elemental angle $d\theta$ is $[\frac{1}{2}(B_m \cos \theta)^2/\mu_0] \frac{1}{2}Dl \cdot d\theta$, where D is the diameter of the gap and l is the axial length. This quantity multiplied by $\cos \theta$ is the component of force along the x -axis, and integration over the whole periphery ($0 < \theta < 2\pi$) gives the resultant x -axis magnetic pull as zero, because the force $(Dl/3\mu_0)B_m^2$ for one pole is balanced by an equal and opposite force on the other.

Asymmetry. Let the rotor axis be displaced by a small distance e in the direction of the polar x -axis. At angle θ the radial gap length is now $l_g' = l_g(1 - \epsilon \cos \theta)$ where $\epsilon = e/l_g$ is the eccentricity expressed as a fraction of the symmetrical gap length. Assuming the flux density to be inversely proportional to gap length, then at θ , where with no eccentricity the flux density was B , it is now $B' = B/(1 - \epsilon \cos \theta) \approx B(1 + \epsilon \cos \theta)$. Integrating the x -axis force components around the whole rotor gives the resultant $(\pi Dl/4\mu_0)B_m^2\epsilon$, which is directly proportional to the eccentricity.

EXAMPLE 2.4: For the 2-pole machine of Example 1.2, find the unbalanced magnetic pull on the rotor when its axis is displaced by $e = 0.001$ m = 1 mm. The machine has $D = 0.50$ m, $l = 0.20$ m, $l_g = 0.005$ m = 5 mm; the stator and rotor peak current-sheet densities are $A_1 = 12\,800$ A/m and $A_2 = 9\,600$ A/m; and the torque angle is $\lambda = \pi/3$ rad. The m.m.f.s per pole are

$$F_1 = \frac{1}{2}A_1D = 3\,200 \text{ A-t} \quad \text{and} \quad F_2 = \frac{1}{2}A_2D = 2\,400 \text{ A-t}$$

and applying eq.(1.9), the resultant gap m.m.f./pole is $F_0 = 4\,450$ A-t, which produces in the gap a sine-distributed flux density of peak value $B_m = F_0\mu_0/l_g = 1.11$ T. Then the magnetic pull per pole is

$$(Dl/3\mu_0)B_m^2 = 33.0 \text{ kN}$$

which, balanced by the opposition of polar attractive forces, gives zero u.m.p. as resultant if the rotor is *symmetrically* centred within the stator bore. With the displacement e giving an *eccentricity* $\epsilon = 0.001/0.005 = 0.2$, then there is a resultant u.m.p. of

$$(\pi Dl/4\mu_0)B_m^2\epsilon = 15.5 \text{ kN}$$

The useful torque of the machine is $M = 260$ N-m, corresponding to a peripheral force of $260/0.25 = 1.04$ kN, so that the u.m.p. is fifteen times as great as the useful force.

The simplified analysis given above assumes that the stator and rotor axes, though displaced, remain parallel; this is not necessarily the case if the bearing alignments differ. It is also assumed that the peak flux density is unaltered

by the eccentricity, which may not be true. The u.m.p. is calculated for the worst condition, i.e. coincidence of eccentricity with the axis of maximum flux density, but this condition may change with speed and load. A form of eccentricity is produced by non-circularity: it is possible to grind a cage rotor, but much more difficult to machine the stator bore, so that asymmetry may arise from this cause.

Binns and Dye [110] have summarized much of the work on u.m.p. with reference to induction motors, in which the short airgap length intensifies the effect of small eccentricities. Experiment shows that the u.m.p. is proportional to ϵ for eccentricities not greater than 0.1, but that between 0.1 and 0.3 the increase is small as a result of saturation (especially in the tooth-tips) imposing a limit on the peak flux density. Leakage flux, which increases with load, plays a significant role in saturating tooth-tips so that u.m.p. effects are likely to be more prominent under light- or no-load conditions.

Practical effects

In manufacturing electrical machines it is necessary to allow dimensional tolerances on core-plate stampings, core assemblies, frames, bearings and endshields. Wide tolerances cheapen a machine, but may impair its performance; in particular, there may be variations in gap length which are significant in short-gap machines such as induction motors in giving rise to u.m.p. and vibration. Stiffer shafts and bigger bearings are made necessary, and the machines may be more noisy than is tolerable. Reckoned as an effective reduction in the stiffness of the shaft, u.m.p. reduces the critical speed, possibly into a region too close to the normal running speed.

In 2-pole machines appreciable homopolar flux is produced by the gap asymmetry, and the flux path may be completed through the shaft and frame. In effect there is a modulation of the gap length, inducing currents in cage-rotor bars and in parallel-connected parts of the stator winding.

The use of parallel paths in machine windings allows counteracting currents to flow and mitigates u.m.p. Cage windings provide multiple parallel paths which have maximum effect on no load, less on load and almost none at starting. Parallel paths in stator windings are most effective when equalizer connections are fitted.

Noise production in machines can be strongly supported by u.m.p., especially if its variation corresponds to a natural frequency of the stator structure. During the start of an induction machine, the u.m.p. has an alternating component at the rotor slot frequency and may set up vibration at speeds given by the natural resonance frequency divided by the number of rotor slots, possibly producing noise.

2.11 Open magnetic circuits

The rotary machines so far discussed have magnetic circuits that are substantially symmetrical, at least apart from constructional imperfections. The development of 'linear' machines (Sect. 12.11) has made practical use of

magnetic circuits of marked asymmetry and of an 'open' nature. The inter-linked electrical and magnetic circuits that are characteristic of electromagnetic machines have preferably minimum possible resistance and reluctance, the latter achieved by use of a short working airgap and high-permeability steel cores. Further, the symmetry leads (ideally) to a properly balanced magnetic pull. If the magnetic flux is made to pass over a long air path, some compensation for the high reluctance can be obtained by arranging for the path area to be large. But it is now a far more complex path in which the assessment of flux distribution must be based on three dimensions, and not on two as has been possible for the annular airgap of a conventional machine. Again, it is not possible to neutralize the unbalanced magnetic pull which is inherent in the structural form, with the result that mechanical means must be provided to sustain it.

The methods used in this Chapter are therefore unsuitable for the linear machine, and it is necessary to change radically the analytical processes for magnetic circuit design. The problem is discussed in detail by Laithwaite [82, 96].

3

Windings

3.1 Materials

The winding of a machine or transformer conveys electrical energy to or from the working region, and is concerned with e.m.f. induction and the development of magneto-mechanical force. Windings are formed from suitably insulated conductors.

Conductors

If a current of density J flows in a conductor of resistivity ρ and density δ , the loss per unit cube is $J^2\rho$ and the loss per unit mass (the specific loss) is $p = J^2\rho/\delta$. Copper is the most common conducting metal, but there is an increasing use of aluminium.

Copper: The International Annealed Copper Standard (IACS) has at 20°C a resistivity 0.017 241 $\mu\Omega\text{-m}$, a resistance-temperature coefficient 0.003 93 per °C, and a tensile strength 220–250 MN/m². Hot and cold working (such as wire-drawing) raises the mechanical strength at a small sacrifice in conductivity. Most machines employ windings of annealed high-conductivity copper, but it is necessary in large transformers to use a worked copper of controlled proof-strength, in turbo-generators a silver-bearing copper with resistance to thermal softening and creep. Cadmium-copper is adapted to cage windings because it can be flame-brazed without deterioration.

Aluminium. Aluminium-alloy conductors to BS 3242 have a resistivity at 20 °C of 0.0325 $\mu\Omega\text{-m}$ (i.e. a conductivity of 53% IACS). Sheet material is often used for the low-voltage windings of small- and medium-rated transformers. Cage windings can be fabricated by brazing slot bars to the end-rings, or by integral casting with silicon-aluminium alloy (6–12% Si) of resistivity 0.04–0.05 $\mu\Omega\text{-m}$.

Typical characteristics at 20 °C for resistivity ρ , temperature coefficient α per °C, and density δ , are:

	$\rho(\mu\Omega\text{-m})$	$\alpha(\text{per } ^\circ\text{C})$	$\delta(\text{kg/m}^3)$
Copper: annealed	0.0172	0.003 93	8 900
hard-drawn	0.0178	0.003 90	
Aluminium: cast	0.045	0.003 90	2 700
hard-drawn	0.0325	0.003 90	

I^2R Loss: This is the significant electrical property. As an example of magnitude, the loss in hard-drawn material at 75°C carrying a current of uniform density 5 MA/m^2 (or 5 A/mm^2) is

Copper: 540 kW/m^3 , 61 W/kg Aluminium: 990 kW/m^3 , 365 W/kg .

Insulants

Insulating materials, essentially non-metallic, are organic or inorganic, uniform or heterogeneous in composition, natural or synthetic. An ideal insulant would have: (i) high dielectric strength, sustained at elevated temperatures; (ii) good thermal conductivity; (iii) permanence, non-deteriorating at high temperatures; (iv) good mechanical properties such as ease of working and application, non-hygroscopic, and resistant to vibration, abrasion and bending. Almost all usable materials are subject to temperature limitation, and are classified in accordance with agreed limits of operating temperature:

Class Y. Cotton, silk, paper, wood, cellulose, fibre, etc., without impregnation or oil-immersion.

Class A. The materials of Class Y impregnated with natural resins, cellulose esters, insulating oils, etc.; also laminated wood, varnished paper, cellulose-acetate film, etc.

Class E. Synthetic-resin enamels, cotton and paper laminates with formaldehyde bonding, etc.

Class B. Mica, glass fibre, asbestos, etc. with suitable bonding substances; built-up mica, glass-fibre and asbestos laminates.

Class F. The materials of Class B with more thermally-resistant bonding material.

Class H. Glass-fibre and asbestos materials, and built-up mica, with appropriate silicone resins.

Class C. Mica, ceramics, glass, quartz and asbestos without binders or with silicone resins of superior thermal stability.

The classification is based on the following maximum permitted temperatures:

Insulation class:	Y	A	E	B	F	H	C
Maximum temp., $^\circ\text{C}$:	90	105	120	130	155	180	> 180

The figures are based on a 20-year working life under average conditions. The life of an insulating material is closely related to the "hot-spot" temperature within the winding it covers. Some notes follow on typical machine insulation.

Mica in the virgin or sheet state is difficult to work, and it is used in the form of sheets of splittings with shellac, bitumen or synthetic polyester bonding.

Micafolium is a wrapping composed of mica splittings bonded to paper and air dried. It may be wound on to conductors, then rolled and compressed between heated plates to solidify the material and exclude air.

Fibrous glass is made from material free from alkali metal oxides (soda or potash) that might form a surface coating that would attack the glass silicates. Glass absorbs no moisture volumetrically, but may attract it by capillary action between the fine filaments. Tapes and cloths woven from continuous-filament yarns have a high resistivity, thermal conductivity and

tensile strength, and form a good Class B insulation. The space factor is good, but the material is susceptible to abrasive damage. Thin glass-silk coverings are available for wires for field coils or mush windings: varnishing is necessary to resist abrasion.

Asbestos is mechanically weak, even when woven with cotton fibres, and can only occasionally compete with fibre glass. Laminates of asbestos with synthetic resins have good mechanical strength and thermal resistance. Asbestos, wire- and strip-coverings have resilience and abrasion resistance, but the space factor is low.

Cotton fibre tapes woven from acetylated cotton, recently developed, have remarkable resistance to heat "tendering", and are much less hygroscopic than ordinary cotton materials.

Polyamides such as nylon make tapes of high mechanical strength and effect a saving in space by their thinness. Nylon film is one of the few plastic films having adequate resistance to temperature and opposition to tearing.

Synthetic-resin enamels of the vinyl-acetate or nylon types give an excellently smooth finish and have been applied for mush windings, with considerable improvement in winding times and in length of mean turn. Varnishes of the same basic materials give good bonding of windings.

Slot-lining materials have in the past been various mica-composites, but the mica content is easily damaged in forming. With small motors a two-ply varnished cotton cloth bonded to pressboard has been found satisfactory; while three-ply material may serve for heavier windings.

Wood, in the form of synthetic-resin-impregnated compressed laminations, has proved a robust and accurate material for packing blocks, coil supports and spacers. If the electrical properties are not adequate, phenolic paper laminates will be preferred although their cost is greater.

Silicones are semi-inorganic materials with a basic structure of alternate silicon and oxygen atoms. They are remarkably resistant to heat, and as binders in Class H insulation permit of continuous operation at 180°C . Even when disintegrated by excessive temperatures, the residue is the insulator silica. Silicones are water-repellent and anti-corrosive: they have been successfully used in dry (oil-less) transformers, traction motors, mill motors and miniature aircraft machines operating over a winding temperature range of 200 to -40°C . An additional advantage is the superior thermal conductivity, improving heat-transfer from conductors and facilitating dissipation.

Epoxide thermosetting resins have become important in casting, potting, laminating-adhesive and varnishing applications, and in the encapsulation of small transformers.

Synthetic resin bonded paper, cotton and glass-fibre laminates have good electrical and mechanical properties as sheets, large cylinders and tubes.

Petroleum-based mineral oils are commonly employed in the cooling and insulation of immersed transformers. The characteristics of importance are chemical stability, expansion coefficient, resistance to sludging by oxidation, and viscosity. When clean and moisture-free their electric strength is good.

Askarels are synthetic non-flammable insulating liquids which, when decomposed by an electric arc, evolve only non-explosive gases. The commonest

askarel is a 60/40 mixture of hexachlorodiphenyl/trichlorobenzene giving a low pour point and a satisfactory viscosity/temperature characteristic.

3.2 Eddy currents

The main or working flux in a transformer or machine induces in a winding the useful e.m.f. concerned in energy transfer or conversion. The non-useful leakage flux also produces an e.m.f., which may introduce a loss arising from the disturbance of the current distribution in the conductors of the winding.

The parasitic eddy-currents in an *isolated* conductor due to its own field are called the *skin effect*. They arise on account of the inductance of the central parts of the conductor exceeding that of the outer parts. The reactance of the centre is therefore greater, and the current flows consequently more readily in the outer layers of the conductor. But any departure from uniform current density increases the I^2R loss over its d.c. value. The greater induced e.m.f. of self-induction in the middle parts of the conductor causes circulating currents which, superimposed on the main current, increases the I^2R loss.

In machines and transformers the conductors forming the windings are not, however, isolated, and the effects of alternating leakage fields are intensified by the proximity of ferromagnetic material. There are losses — usually undesirable — in the conductors themselves, and in neighbouring permeable or conducting masses (such as transformer tanks, and the stiffeners and housings of machines) resulting from currents induced therein by the conductors. The boundary shapes differ so radically that it is necessary to deal piecemeal with a variety of combinations of conductors and neighbouring masses.

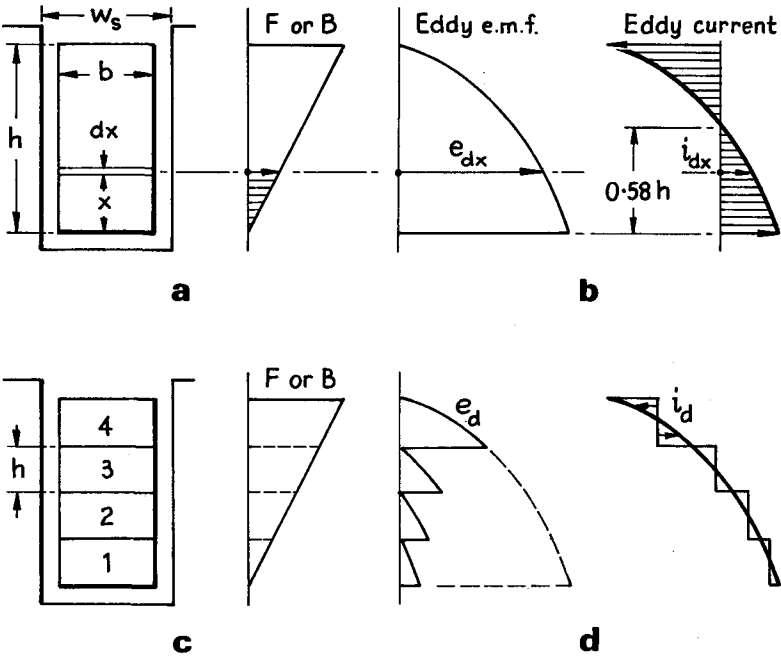
Slot conductors

As the core is laminated, the teeth flanking a slot can be taken as having high permeability and infinite resistance, so that the leakage flux path is the same as that assumed in Fig. 2.28. Consider the single rectangular conductor of Fig. 3.1(a), set into a parallel-sided open slot. Let the conductor carry a uniformly distributed current of peak i_m alternating at angular frequency ω . At the instant of peak current the m.m.f. F acting across the slot from wall to wall increases from zero at the bottom to i_m at the top.

At height h the cross-flux density is $\mu_0 i_m / w_s$, so that the total flux crossing the conductor per unit axial length of slot is $\Phi_h = \frac{1}{2} \mu_0 i_m (h/w_s)$. Up to the lower height x the corresponding flux is $\Phi_x = \Phi_h (x/h)^2$. The flux linking the lower part x is the flux $\Phi = \Phi_h - \Phi_x$ crossing the slot *above* it. The peak eddy e.m.f. induced in the elemental lamina dx is $e_{dx} = \omega \Phi$:

$$e_{dx} = \frac{1}{2} \mu_0 \omega i_m \frac{h}{w_s} \left[1 - \frac{x^2}{h^2} \right] = k i_m \left[h - \frac{x^2}{h} \right]$$

where $k = \frac{1}{2} \mu_0 \omega / w_s$. The eddy e.m.f. is plotted in Fig. 3.1(b) together with the eddy current that it produces. The average e.m.f. of $k i_m \frac{2}{3} h$ occurs at $x = h/\sqrt{3} = 0.58h$: at this height there is no circulating eddy current, but above it an eddy current i_d flows axially in one direction, to return in the lower part of



3.1 Eddy currents in slot conductors.

the conductor. As i_d lags i_m by $\frac{1}{2}\pi$ rad in time phase, the I^2R loss due to each can be calculated separately.

This *first-order* eddy current disturbs the originally uniform distribution of current density, superposing a *second-order* m.m.f., flux, e.m.f. and eddy current, which in turn develops a third-order component and so on. Not only do the eddy-current components cause further I^2R loss, but also they disturb the field distribution and affect the slot leakage flux and linkage. At this point it is necessary to distinguish between the eddy-current effects: A, in conductors in which the eddy currents are minimized; and B, in conductors in which the effect is encouraged for a useful purpose, such as the suppression of hunting in synchronous machines or the development of high values of starting torque in cage induction motors.

CASE A: This is the most common, and for it we can ignore all eddy-current components other than that of the first order, and neglect all disturbance to the effective leakage inductance.

The resistance r_x per unit length of an elemental lamina in a conductor of resistivity ρ is $\rho/b \cdot dx$. The effective eddy e.m.f. acting on it is $e_x = k i_m \times [h - (x^2/h) - \frac{2}{3}h]$. The laminar eddy current is therefore $i_{dx} = e_x/r_x$. In the same lamina the main current component is $i_m \cdot dx/h$. If the main and eddy components are squared and multiplied by the resistance they give the co-

corresponding I^2R losses. Integrating their ratio over the conductor height gives the result

$$\frac{\text{eddy-current } I^2R}{\text{main-current } I^2R} = \frac{4}{45} \frac{k^2 b^2}{\rho} h^4 = \frac{4}{45} (\alpha h)^4$$

where

$$\alpha = \sqrt{\frac{kb}{\rho}} = \sqrt{\frac{\frac{1}{2}\mu_0\omega}{\rho}} \cdot \sqrt{\frac{b}{w_s}} \quad (3.1)$$

The first term always occurs in the analysis of skin effect. The second is the square root of the ratio (conductor-width/slot-width).

It is now possible to define the *eddy-loss ratio*

$$K_d = \frac{\text{total loss}}{\text{loss due to main current}} = 1 + \frac{4}{45} (\alpha h)^4 \quad (3.2)$$

which makes it possible to deduce the actual loss from the I^2R based on a d.c. calculation. For a frequency of 50 Hz and the resistivities (hot) given in Sect. 3.1 the value of α becomes approximately

$$\text{Copper: } \alpha = 100\sqrt{(b/w_s)} \quad \text{Aluminium: } \alpha = 150\sqrt{(b/w_s)} \quad (3.3)$$

For a solid copper conductor of width 4 mm in a slot 5 mm wide, then $\alpha = 100\sqrt{(4/5)} = 90$. If the conductor height is $h = 5 \text{ mm} = 0.005 \text{ m}$, we have $\alpha h = 0.45$, and $K_d = 1 + (4/45)(0.45)^4 = 1.0035$; and the d.c. loss is increased by eddy currents only to the extent of under 1%. But if the conductor is 30 mm deep, with $\alpha h = 2.7$, then $K_d = 1 + 4.7 = 5.7$, and the I^2R loss is increased by a factor of nearly six. Such an excessive loss could not be tolerated. It is unusual to find $\alpha h > 0.7$ for a single bar.

Where the cross-section of a conductor has to be large, it is essential to sectionalize it as indicated in Fig. 3.1(c), with the sections lightly insulated from each other. The reduction in the eddy current is made clear in (d). With m sections or layers, the loss in the p th layer ($p = 1$ at the bottom of the slot) is

$$K_{dp} = 1 + \frac{p(p-1)}{3} (\alpha h)^4 \quad (3.4)$$

and the average loss ratio for the m layers, each of height h , is

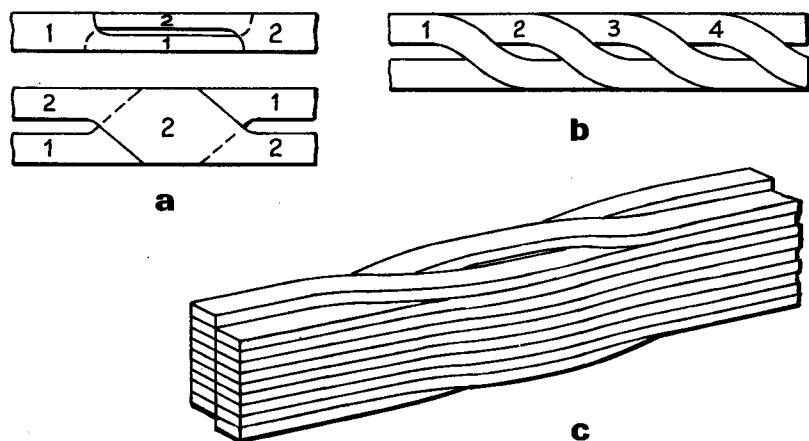
$$K_{dav} = 1 + \frac{m^2}{9} (\alpha h)^4 \quad (3.5)$$

The original solution for K_d was given by Field [14], who published his results in the form of curves.

The formulae for the loss factors given for sectioned conductors are valid so long as the subdivisions are not connected in parallel in such a way as to

permit a path between laminae for eddy currents. Otherwise the utility of sub-division is almost entirely lost. Of course, a parallel connection is essential for distributing the conductor current, so that the connection in parallel must be made so as to secure complete balance between the individual subdivisions. This may be obtained by *twisting* or *transposition*.

Twisted slot-conductors have their layers transposed or twisted in the slot so as to obtain symmetrical lengths such that each sub-division occupies all possible layer positions for the same length of slot. In a long-cored turbo-generator, the twisting may be carried out two or three times in a single slot. The effect is to equalize the eddy e.m.f.s in all laminae, and to allow the layers to be paralleled at the ends without producing eddy circulating currents



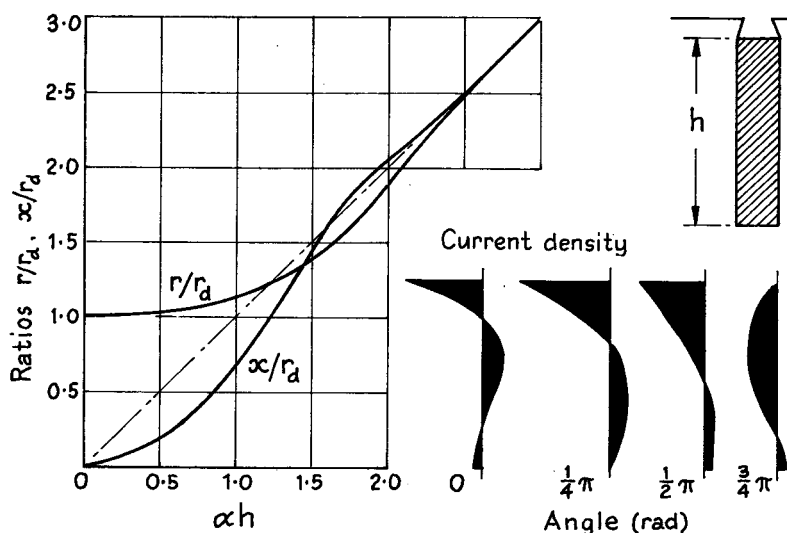
3.2 Slot conductor transposition.

between the layers. Considerable ingenuity is shown in the design of the twist, which must be accommodated in the slot space. Fig. 3.2 shows typical slot transpositions. For (a) and (b) the crossover is obtained by special shaping of the conductor. Method (c) is more compact but involves some reduction of the conductor area.

CASE B: The simplified expressions for Case A do not apply to deep conductors in which αh is large, because the eddy-currents of higher order are much too great. Fig. 3.3 shows a simple deep conductor with three instantaneous current distributions in a half-period of the alternating current carried. There is a distinct impression of downward *propagation*, and in fact the case is one of wave motion. The treatment, based on the Maxwell equations, gives the effective resistance and inductive reactance of a slot conductor in terms of its d.c. resistance r_d in the following form:

$$z = r + jx$$

$$= r_d(\alpha h) \left[\frac{(\sinh 2\alpha h + \sin 2\alpha h) + j(\sinh 2\alpha h - \sin 2\alpha h)}{\cosh 2\alpha h - \cos 2\alpha h} \right] \quad (3.6)$$

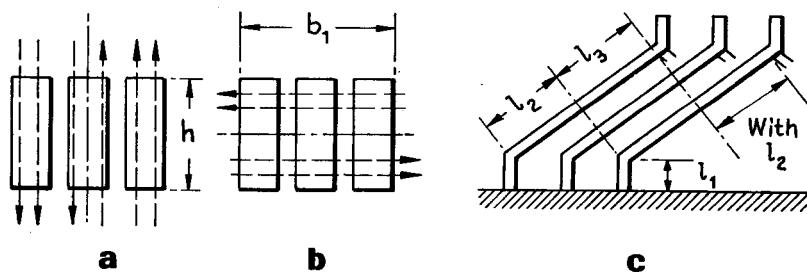


3.3 Eddy currents in deep bars.

This is plotted in Fig. 3.3. The case of a slot with a lip has been solved by Swann and Salmon [15], and an expression based on a ladder network (i.e., an equivalent transmission line) by Bruges [6]. All assume that the core has infinite permeability; the effects of saturation are evaluated by Chalmers and Dodgson [8].

Overhang conductors

The problem of the estimation of eddy current losses in overhang conductors is very difficult, as the leakage field is not readily estimated either in magnitude or direction. Consider Fig. 3.4, in which a typical case is illustrated. The actual field distribution may, for analysis, be resolved into approximate fields parallel to and across the conductors, (a) and (b). Since the eddy loss factor is proportional to h^4 , the parallel field may frequently be neglected. The length of path of the leakage flux is assumed to be $b_1 + h$, and the



3.4 Overhang leakage.

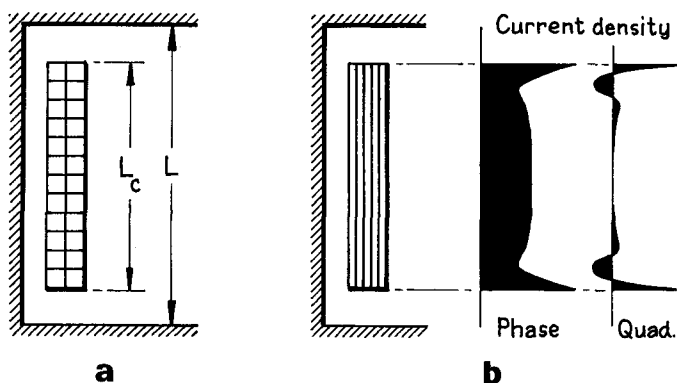
overhang considered as contained in an imaginary slot $b_1 + h$ wide and $h/2$ deep. With a basket winding (c), the overhang length might be taken in three parts l_1, l_2 and l_3 ; with $n = 1$ for l_1 , $n =$ two-thirds of the number in the group for l_2 , and $n =$ total number in a phase group for l_3 . Whence for the overhang

$$K_d = \frac{K_{d1}l_1 + K_{d2}l_2 + K_{d3}l_3}{l_1 + l_2 + l_3}$$

Transposition n in the overhang connectors is a method of making a layer occupy all possible slot positions by twisting. A relation between the number of layers and the number of slots per phase is essential. This transposition is obtained automatically in diamond and basket windings, in that the upper conductors of one coil side become the lower conductors of another. This may be sufficient for all but the largest machines, where twisting in slots has to be combined with overhang transposition to keep the eddy loss within bounds.

Transformer coils

For normal transformers, Fig. 3.5(a), the eddy-current loss factor may be found by considering the effective conductor length as the coil length L_c contained in a 'slot' of width L so that the ratio b/w_s is represented by L_c/L . The



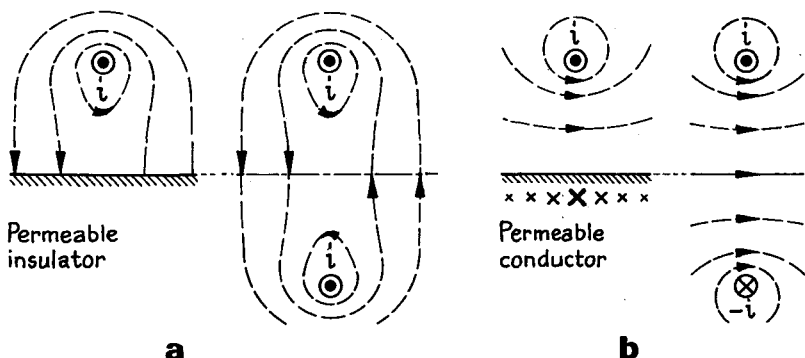
3.5 Eddy currents in transformer windings.

conductor height h is replaced by b , and eqs.(3.1) – (3.5) applied.

In sheet-wound transformers, where the l.v. winding is a spiral (usually of aluminium) of the full usable length of the limb, the current density is much higher at the ends than elsewhere. Fig. 3.5(b) shows typical distributions of the l.v. current, one for the component opposing the h.v. current, the other in quadrature. The loss factor is not likely to exceed 1.1, and will generally be less. This case is examined by Mullineux, Reed and Whyte [16].

Permeable and conducting masses

Conductors (apart from those in slots) carrying alternating currents may lie



3.6 Leakage near permeable and conducting masses.

near metal masses having permeable and/or conducting properties. The leakage field may penetrate such masses, inducing eddy currents which give rise to I^2R loss and which react to modify the leakage field pattern.

The conductor carrying current i in (a) of Fig. 3.6 lies parallel to a semi-infinite iron mass. If the iron is assumed to have *infinite* permeability and *zero* conductivity, its effect on the external field is as if it were replaced by an *image conductor* carrying an equal current i in the same direction. The mass in (b) is taken to be non-magnetic and of *infinite* conductivity, and its external effect is again that of an image current, but reversed, summing the contributions of the actual eddy-current distribution. In neither (a) nor (b) is there any eddy loss.

In practice both kinds of mass have finite conductivity, which impairs the simple image concept and introduces loss. Further, the permeable mass has saturation. If these 'defects' are to be included, an analysis of the leakage field and of the eddy-current loss in the mass becomes exceedingly complicated, and in any case the initiating leakage field is due to a distribution of current-carrying conductors and not to a single isolated current filament. Several attempts, e.g. by Stoll and Hammond [17] and by Mullineux and Reed [18], have been made to analyse various aspects of the problem. The solutions are complex, but can be computed without much difficulty for specific cases. As might be expected, the results depend on the relation between the permeability of the mass and the eddy-current skin depth (a function of the same variables as α above), and may approach closely to the two idealized extreme cases in Fig. 3.6.

The general conclusion is that the eddy loss in a permeable mass can be significantly reduced by facing it with a relatively thin layer of material of good conductivity, such as copper or aluminium.

3.3 Electromotive force

The useful e.m.f. in the windings of a transformer or machine are obtained by application of the Faraday–Lenz law, eq.(1.1). Because of the two ways in

which linkage can change, we can write

$$N \frac{d\Phi}{dt} = N \left[\frac{\partial \Phi}{\partial \alpha} \cdot \frac{d\alpha}{dt} + \frac{\partial \Phi}{\partial t} \right] = e_r + e_p \quad (3.7)$$

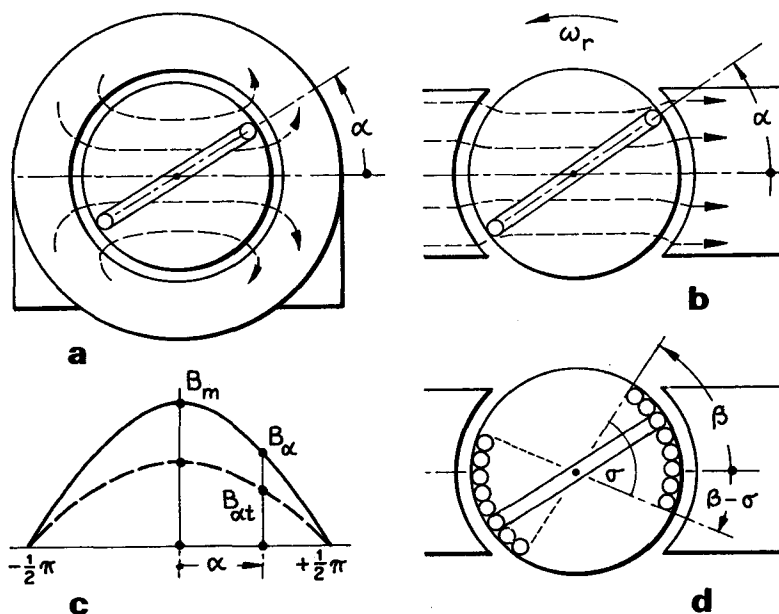
a form which allows for variation of linkage due to both causes occurring together. The first term refers to the change of flux linking a coil moving at angular speed $\omega_r = d\alpha/dt$ relative to the flux, the distribution of the flux at the instant considered being expressed as $\partial \Phi / \partial \alpha$. The second term refers to flux pulsation.

Although the turns of a winding are arranged in coils having a variety of shapes and (in machines) are located in slots in the airgap periphery, it is initially helpful to idealize the conditions. The assumptions made here are:

- (i) the structure is 2-polar;
- (ii) the distribution of the flux around the airgap is sinusoidal;
- (iii) the flux alternates sinusoidally in time;
- (iv) the turns of a winding, although finite in number, are considered to be uniformly distributed and of full diametral pitch;
- (v) there is complete magnetic and electric symmetry.

The effects of slotting and of non-sinusoidal flux distribution, which are matters of practical importance, are considered in detail in Sect. 3.12.

Flux. The stator can be cylindrical (a) or salient (b), Fig. 3.7. The maximum



3.7 Electromotive force.

instantaneous flux per pole is Φ_m , which corresponds to a peak gap flux density $B_m = \Phi_m/Dl$ at the pole-centre ($\alpha = 0$) and a density $B_\alpha = B_m \cos \alpha$ at an angle α to the pole centre. The flux alternates at angular frequency ω and is taken as $\Phi = \Phi_m \sin \omega t$. The instantaneous gap flux density at angle α is therefore

$$B_{\alpha t} = B_\alpha \sin \omega t = B_m \cos \alpha \cdot \sin \omega t$$

as shown in Fig. 3.7(c). The instantaneous flux linking a single-turn full-pitch coil in a plane making the angle α to the pole centre is

$$\Phi_\alpha = \int_{-\alpha}^{+\alpha} B_{\alpha t} \cdot \frac{1}{2} Dl \cdot d\alpha = Dl B_m \sin \omega t \cdot \sin \alpha = \Phi_m \sin \omega t \cdot \sin \alpha$$

Coil E.M.F. Eq.(3.7) can now be applied to obtain the e.m.f. in a concentrated coil of N_c turns rotating at angular velocity ω_r and momentarily situated at angle α : it is

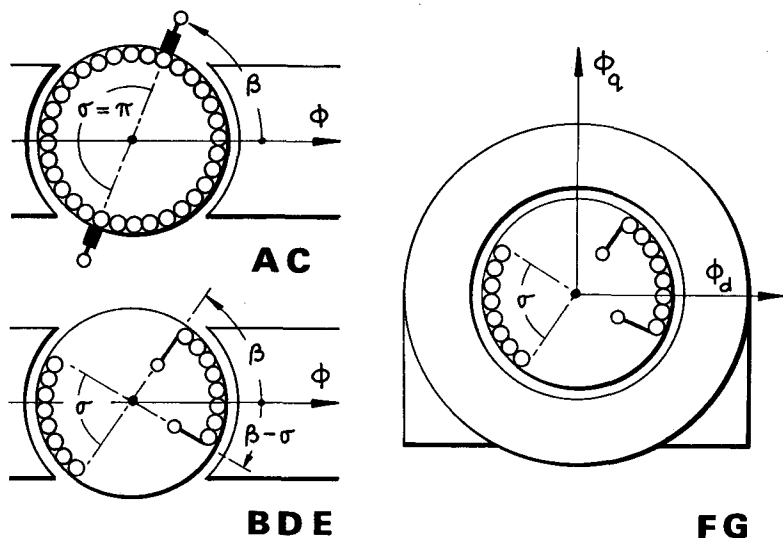
$$e_c = \omega_r N_c \Phi_m \sin \omega t \cdot \cos \alpha + \omega N_c \Phi_m \cos \omega t \cdot \sin \alpha \quad (3.8)$$

Winding E.M.F. A winding, Fig. 3.7(d), is made up of m coils in series, uniformly spread over an angle σ and making $mN_c = N$ turns in all. The winding e.m.f. is the sum of the e.m.f.s of coils making various angles α :

$$\begin{aligned} e = \Sigma(e_c) &= \left[\omega_r N \Phi_m \sin \omega t \cdot \frac{1}{\sigma} \int_{\beta-\sigma}^{\beta} \cos \alpha \cdot d\alpha \right. \\ &\quad \left. + \omega N \Phi_m \cos \omega t \cdot \frac{1}{\sigma} \int_{\beta-\sigma}^{\beta} \sin \alpha \cdot d\alpha \right] \quad (3.9) \\ &= e_r + e_p \end{aligned}$$

Here β is the angle between the position of the 'start' of the phase winding and the pole-centre. The coil e.m.f. has two components: e_r is the result of movement of the coil through the flux and is the *e.m.f. of rotation*, associated (when current flows in the coil) with interaction force and therefore with the conversion of energy between the electrical and mechanical forms; and e_p is the *e.m.f. of pulsation*, concerned with energy transfer between rotor and stator windings. Eq.(3.9) is general, in that it applies to any electrical machine subject to the stated idealizations. Clearly e_r is a maximum when the phase-centre lies in the flux axis, while e_p is a maximum when the phase-axis coincides with the flux axis.

A classification of common flux and winding arrangements is given in Sect. 1.17. As it is immaterial which of the members 1 and 2 is the stator and which the rotor, we take the more convenient form in considering Cases A to G, Fig. 3.8.



3.8 Basic winding arrangements in rotating machines.

Fixed-axis flux

CASE A: Constant stator flux, rotor with commutator winding. In a commutator winding there is, between brushes, a constant number of turns occupying always the same position with respect to the flux axis, making the brush e.m.f. have the same frequency as that of flux pulsation, in this case zero frequency. The e.m.f. is therefore unidirectional and due only to rotation. The N turns of the rotor are divided into two equal parallel sets each of $\frac{1}{2}N$ in series. The angle of spread is $\sigma = \pi$, so that eq.(3.9) reduces to

$$e = e_r = \omega_r \frac{1}{2} N \Phi \frac{1}{\pi} \int_{\beta-\pi}^{\beta} \cos \alpha \cdot d\alpha = \omega_r N \Phi \frac{1}{\pi} \sin \beta$$

With the rotor brush axis set at $\beta = \frac{1}{2}\pi$, i.e. at right-angles to the flux axis, then $\sin \beta = 1$. The e.m.f. can be written

$$e = E_r = 2 n N \Phi$$

where $n = \omega_r / 2\pi$ is the rotor speed (in rev/s) This case represents that of the common d.c. machine.

CASE B: Constant stator flux, rotor with phase winding. Again the flux is constant and there is no pulsational e.m.f. From eq.(3.9)

$$e = e_r = \omega_r N \Phi \frac{1}{\sigma} \int_{\beta-\sigma}^{\beta} \cos \alpha \cdot d\alpha$$

The integral is $[\sin \beta - \sin(\beta - \sigma)]$. It is a maximum for $\beta = \frac{1}{2}\sigma$ and the corresponding peak e.m.f. of the phase winding is

$$e_{rm} = \omega_r N \Phi \left[\frac{2}{\sigma} \sin \frac{\sigma}{2} \right] = \omega_r N \Phi k_d$$

where $k_d = (2/\sigma) \sin (\sigma/2)$ is the distribution factor. The turns, being distributed over the angle σ , cannot all link the same flux at a given instant, and the distribution factor is a measure of the reduction of the average linkage. Values of k_d for a number of common phase-spread angles are given in the Table:

Phase-spread, σ	$\pi/3 = 60^\circ$	$\pi/2 = 90^\circ$	$2\pi/3 = 120^\circ$	$\pi = 180^\circ$
Distribution factor k_d	0.955	0.905	0.827	0.636

The phase e.m.f. alternates at angular frequency $\omega_r = 2\pi f_r$. Its instantaneous value is $e = \omega_r k_d N \Phi \cos \omega_r t$ and its r.m.s. value

$$E = \sqrt{2} \pi f_r k_d N \Phi \quad (3.10)$$

This case is applicable to the synchronous machine.

CASE C: Alternating stator flux, rotor with commutator winding. This is the condition for the a.c. commutator machine. Both modes of e.m.f. induction are present and it is necessary to include both terms of eq.(3.9). Remembering that the number of series-connected turns is $\frac{1}{2}N$ and that the angular spread of the winding is $\sigma = \pi$, the e.m.f. for a flux $\Phi = \Phi_m \sin \omega t$ is

$$e = \omega_r \frac{1}{2} N \Phi_m \sin \omega t \cdot \sin \beta - \omega \frac{1}{2} N \Phi_m \cos \omega t \cdot \cos \beta = e_r + e_p$$

The e_r component has the flux frequency ω and is in phase with the flux; its magnitude is proportional to speed ω_r . The presence of e_p means that, in a closed rotor winding, energy could be transferred to the rotor from the stator through the common magnetic field by transformer action.

CASE D: Alternating stator flux, rotor with phase winding. The e.m.f. is given by eq.(3.9) as it stands. Using the distribution factor given for case B, the rotor-phase-winding e.m.f. can be written

$$e = k_d N \Phi_m [\omega_r \cdot \sin \omega t \cdot \cos \omega_r t + \omega \cdot \cos \omega t \cdot \sin \omega_r t]$$

We can express the difference of the angular frequencies of flux pulsation and of rotor rotation as $\omega - \omega_r = \omega_2$, and define $\omega_2/\omega = s$ as the fractional slip.

Thus $\omega_2 = s\omega$ and $\omega_r = (1-s)\omega$. With these substitutions, the e.m.f. of the rotor phase winding is

$$\begin{aligned} e &= \frac{1}{2}k_d N \Phi_m [(2\omega - \omega_2) \sin(2\omega - \omega_2)t - \omega_2 \sin \omega_2 t] \\ &= \frac{1}{2}k_d N \Phi_m \omega [(2-s) \sin(2-s)\omega t - s \sin s\omega t] \end{aligned} \quad (3.11)$$

The terms in the brackets involve sinusoidal variations at frequencies defined by $2\omega - \omega_2 = \omega + \omega_r$ and $\omega_2 = \omega - \omega_r$. The effect is as if the single *pulsating* flux Φ_m were divided into equal parts $\frac{1}{2}\Phi_m$ *rotating* in opposite directions at synchronous speed ω . The armature runs at an angular speed ω_r differing by ω_2 from one of these rotating fluxes, and therefore by $2\omega - \omega_2$ from the other. The induced e.m.f. can be considered as the combination of the e.m.f.s induced by the two fluxes acting separately. This equation forms the basis of one theory of the single-phase induction machine.

CASE E: Alternating primary flux, stationary secondary with phase winding.

The 1-ph induction regulator takes the form shown in Fig. 3.8E; as there is no sustained rotation the motional e.m.f. component is zero and only the pulsational term e_p of eq.(3.9) appears. Its magnitude is adjustable by changing the axis of the rotor phase winding. Writing $\gamma = \sigma - \frac{1}{2}\beta$ for the position of the phase centre, the instantaneous rotor e.m.f. is

$$e = e_p = \omega k_d N \Phi_m \cos \omega t \cdot \sin \gamma$$

and the r.m.s. value is

$$E = \sqrt{2\pi f} k_d N \Phi_m \sin \gamma$$

The static transformer belongs to this class of machine. The secondary coils ideally link the whole primary flux, the airgap is no longer necessary, and the e.m.f. is

$$e = e_p = \omega N \Phi_m \cos \omega t$$

with the r.m.s. value

$$E = \sqrt{2\pi f} N \Phi_m = 4.44 f N \Phi_m \quad (3.12)$$

Travelling-wave flux

CASES F and G: A travelling-wave field is produced by a balanced m -phase excitation, where m is normally 2, 3 or 6. Under steady-state ideal conditions such a field can be represented by direct- and quadrature-axis fluxes respectively given by

$$\Phi_d = \Phi_m \sin \omega t \quad \text{and} \quad \Phi_q = -\Phi_m \cos \omega t$$

The e.m.f. of a winding can then be obtained by summing the results of applying eq.(3.9) for Φ_d and a comparable equation for Φ_q , with due regard to the

relative orientations of winding and flux axis. The expressions obtained are naturally lengthy, but they all reduce to the r.m.s. form

$$E = (1/\sqrt{2})\omega' k_d N \Phi_m \quad (3.13)$$

where ω' depends on the angular speed of the phase considered relative to the angular speed of the travelling wave (Ref. [1]).

3.4 Transformer windings

The basic coil forms are shown in Fig. 2.8, where H and L refer to the high- and low-voltage sides respectively. The constructional details depend on rating and voltage, the forms in common use for the normal core-type transformer being given in the Table:

Transformer Windings					
Service	Rating MVA	H. V. winding kV	H. V. winding type	L. V. winding kV	L. V. winding type
Distribution	up to 1	11–33	foil, cross- over or multilayer	0.43	helix
System	1–30	33–66	disc	11	disc or helix
Trans- mission	30 and upward	132–500	disc or multilayer	11, 33, 66	disc or disc-helix
Generator	30 and upward	132–500	disc or multilayer	11–22	disc-helix

Various coil designs are illustrated in Fig. 3.9.

Helix. This simple coil consists of a single layer over the full axial winding length, the conductor being a single section or a number of strands in parallel. At each end of the coil a wedge-shaped packing is required.

Disc-helix. When the current rating demands many parallel conductors the individual strips can be assembled in a radial pack, either as a single column or as two columns in parallel.

Multilayer helix. This can be used for the h.v. windings of large transformers. The augmented capacitance between layers gives advantage in surge-voltage distribution, but it is difficult to provide the long, thin layers of conductors with adequate strength to sustain axial forces.

Cross-over. A cross-over or bobbin coil is wound in sections on a former with side cheeks. It is similar to a multi-layer winding except that each section is short. Separate sections are connected in series, with horizontal cooling ducts provided by spacers. Interconnection is more usually made back-to-back and front-to-front with successive sections reversed.

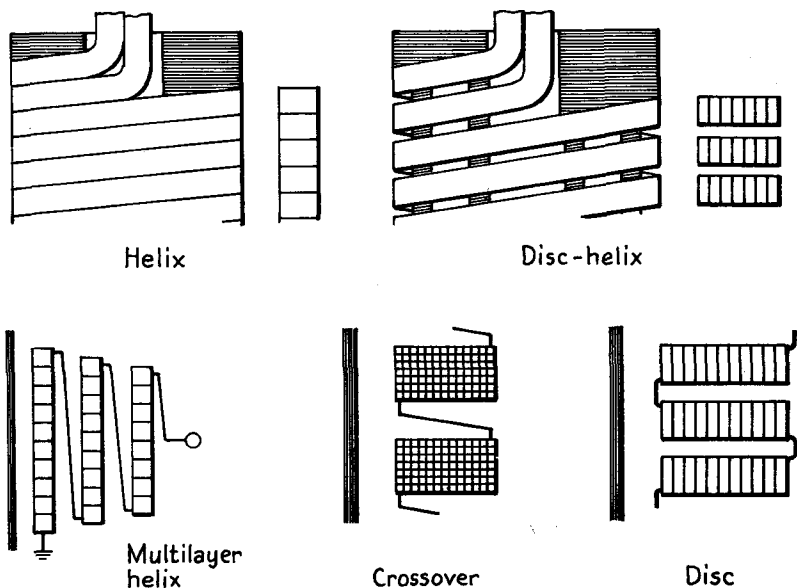
Disc. The disc coil differs from other forms in that adjacent turns, consisting of strip conductors, are wound in a spiral from the centre outward, rather like a cross-over coil with only one turn per layer. To achieve the required disposition of turns the coils are formed in pairs, the requisite turns for one disc being loosely wound so that the conductor finishes in a position to provide the start of the inside turn of the adjacent disc, which is then wound from inside outwards. The first disc is then rearranged in such a manner that the start is located as an outside turn: it can then be tightened.

Continuous disc. Following the formation of one disc, the procedure is repeated without cutting the conductor, saving jointing and joint space – an advantage with an inner winding. For large currents the coils can be wound with multiple conductors, or constructed as separate parallel-connected pairs. Alternatively two half-windings of reversed direction can be stacked, with the line connection taken from the centre of the stack.

Foil. Aluminium could be used for any of the forms mentioned above, but is uniquely employed in foil windings because it can be rolled to thinner and more flexible sheet than copper. The sheet is used in bobbin-type coils of one turn per layer.

Insulation

Classes A, B, C, F and H are all used in “dry” (i.e. non-oil-immersed) transformers, the silicone-treated materials being advantageous because of their water-repellent property. For oil-immersed transformers the coil insulation is



3.9 Types of transformer windings.

a Class A material, generally paper or a derivative thereof. Cotton is confined to small units, but the use of synthetic enamels for interturn insulation is increasing, and may be extended to the l.v. windings (up to about 17.5 kV) of larger units.

3.5 D.C. field windings

Synchronous machines require d.c.-excited field windings, which may be placed on salient-pole cores or in the distributed slots of a cylindrical rotor. In synchronous-induction motors a 3-ph winding on the rotor is arranged for d.c. excitation in the synchronous mode.

Salient-pole windings

The features of coil design are the voltage, required m.m.f., coil dimensions, conductor material, and thermal dissipation. For a given coil voltage V and current I the resistance of a winding of N turns each of mean length L_{mt} is given by $R = V/I = \rho L_{mt} N/a$; whence the cross-sectional area a of the conductor is

$$a = \frac{\rho L_{mt} IN}{V} = \frac{\rho L_{mt} F}{V} \quad (3.14)$$

for a given m.m.f. $F = NI$. The current cannot be settled until the cooling conditions are known. Assuming a current density J , the current is $I = Ja$ and the number of turns is $N = F/I$. The conductor area totals aN so that the gross cross-section of the coil is $A_w = aN/k_s$ where k_s is the space factor. Thus for a given coil voltage, mean length of turn and conductor resistivity the conductor area is directly proportional to the m.m.f. required.

If the thermally emitting surface S of the coil can dissipate a power c per unit area and per degree rise of temperature above ambient, the loss $P = VI$ of the coil under steady sustained conditions gives a surface temperature-rise $\theta_m = cP/S$. On intermittent or short-time rating the rise is a function of the time-constants (heating and cooling).

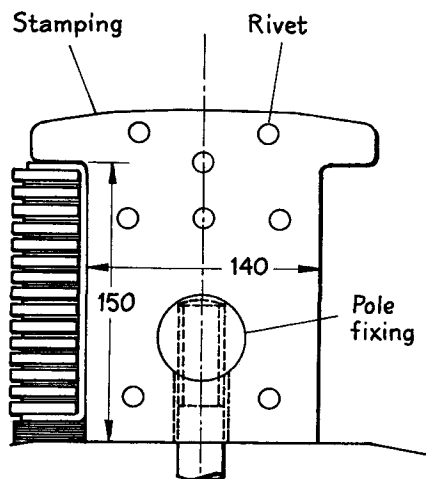
As the field voltage of a salient-pole a.c. machine can usually be chosen to suit the requirements, a value permitting strip-on-edge conductors, Fig. 3.10, is selected to give mechanical strength and effective cooling.

EXAMPLE 3.1: Each rotor pole of a 20-pole synchronous machine has the dimensions given in Fig. 3.10 and is required to develop an m.m.f. $F = 9\,000$ A-t. The field working voltage is 80 V (i.e. 4 V per pole); the mean length of turn is estimated to be $L_{mt} = 1.25$ m. For a copper winding at 75 °C the conductor area is

$$a = 0.021 \times 10^{-6} \times 1.25 \times 9000/4 = 60 \times 10^{-6} \text{ m}^2 = 60 \text{ mm}^2$$

The outside surface area is approximately $S = L_{mt} h_p = 1.25 \times 0.15 \text{ m}^2$; the cooling coefficient is estimated to be $c = 0.019 \text{ }^\circ\text{C m}^2/\text{W}$; whence for a temperature-rise of 65 °C the field coil can dissipate

$$p = \theta_m S/c = 65 \times 1.25 \times 0.15/0.019 = 620 \text{ W}$$



3.10 Salient pole.
Dimensions in mm.

The field current is therefore $I = 620/4 = 155$ A and the number of turns per pole is $N = 9\,000/155 \approx 60$. With 60 turns of 2 mm $\times$ 30 mm strip wound on edge, insulation of 0.3 mm between turns and 12 mm at the coil ends, the total coil depth is 150 mm. The current density is $J = 155/60 = 2.6$ A/mm².

Non-salient-pole windings

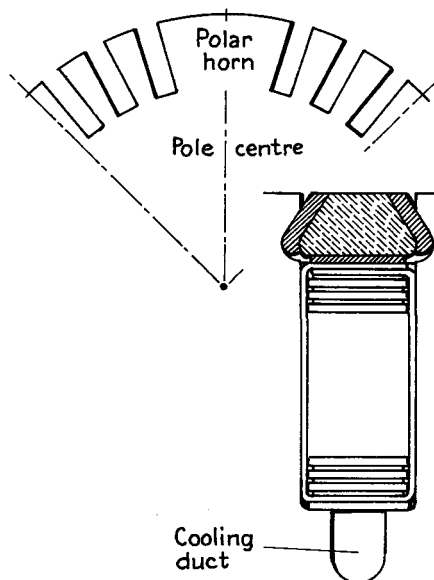
For high-speed rotors the centrifugal forces are too great to be sustained by a salient-pole construction, so that the field winding is distributed into slots. Fig. 3.11 shows one pole-pitch of a 4-pole rotor and a slot detail. The wedge is partly magnetic, and there is a cooling duct below the winding.

3.6 A.C. armature windings

The a.c. armature winding develops e.m.f.s in a number of *phases* when associated with a heteropolar magnetic field. The e.m.f.s are normally equal in magnitude and correctly displaced in time-phase relationship. The winding is composed of conductors in slots spaced around the periphery of the airgap, connected together at the ends, and grouped to form separate phase windings. When excited the phase windings in combination should produce an m.m.f. that is, as far as possible, sinusoidally distributed in the gap.

Conductors, turns and coils

Consider the armature conductors moving in the constant-flux field systems in Fig. 3.12. Each conductor, of length l , moving at peripheral speed u in a flux density B has an induced e.m.f. $e = Blu$. As the conductor passes N and



3.11 Non-salient pole.

As poles successively, its e.m.f. alternates. Diagram (a) shows a double pole-pitch of a machine in which the spatial distribution of the gap flux density over the circumference is sinusoidal. A conductor, 1, in the position shown has an e.m.f. proportional to the density B in which it is moving. A second conductor, 2, displaced a pole-pitch from 1, moves in a flux density of the same magnitude but opposite polarity. The instantaneous conductor e.m.f.s e_1 and e_2 are therefore equal but oppositely directed. Let the conductors be joined together at one end to form a single turn: then the e.m.f.s e_1 and e_2 are additive round the turn and sum to the turn-e.m.f. e_t plotted in (a) in accordance with the position of conductor 1. The flux linkage of the turn is also shown: comparison with e_t confirms eq.(1.1).

Fig. 3.12(b) is for a rectangular distribution of the airgap flux density. The turn-e.m.f. has a similar form. In an actual distributed phase winding the summation of turn-e.m.f.s gives a phase-e.m.f. more nearly sinusoidal.

Connections. The two conductors are series connected to form a turn. A coil may have more than one turn. The connections joining the conductors form the end-connectors or, in the mass, the *overhang* or *end-winding*. When, as in Fig. 3.12, the coil-sides are spaced one pole-pitch apart they are said to be of *full pitch*. The coil pitch may be less than a pole-pitch, in which case it is described as *short-pitched* or *chorded*.

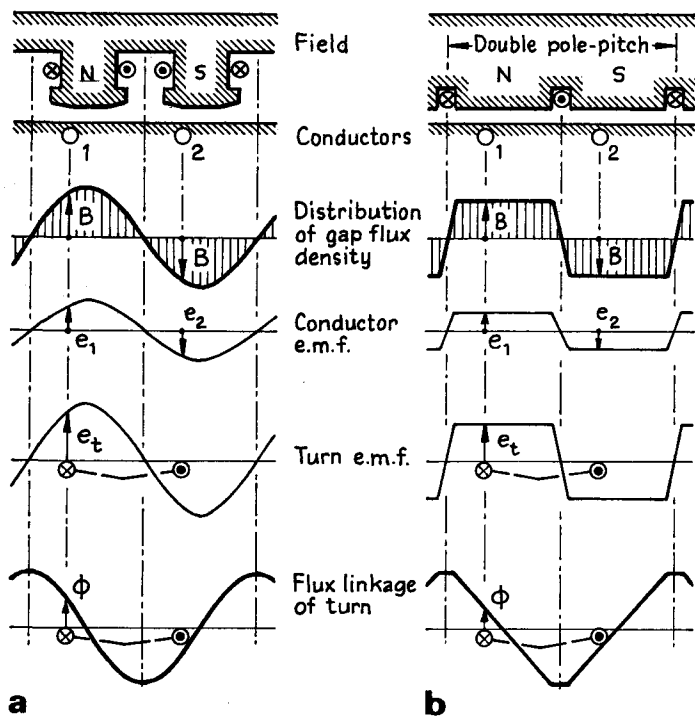
Windings

The slots in which the coils lie are normally punched uniformly around the gap surface. The coils are then arranged and connected to form phase groups.

In a *single-layer* winding each coil-side occupies a whole slot, whereas in a *double-layer* winding one coil-side lies in the upper position in a first slot with the other occupying the lower position in a second slot spaced one pole-pitch (or less) from the first.

Phases

The requirements of an a.c. winding are that it shall produce a symmetrical m -phase system of e.m.f.s of identical magnitude, frequency and waveform,

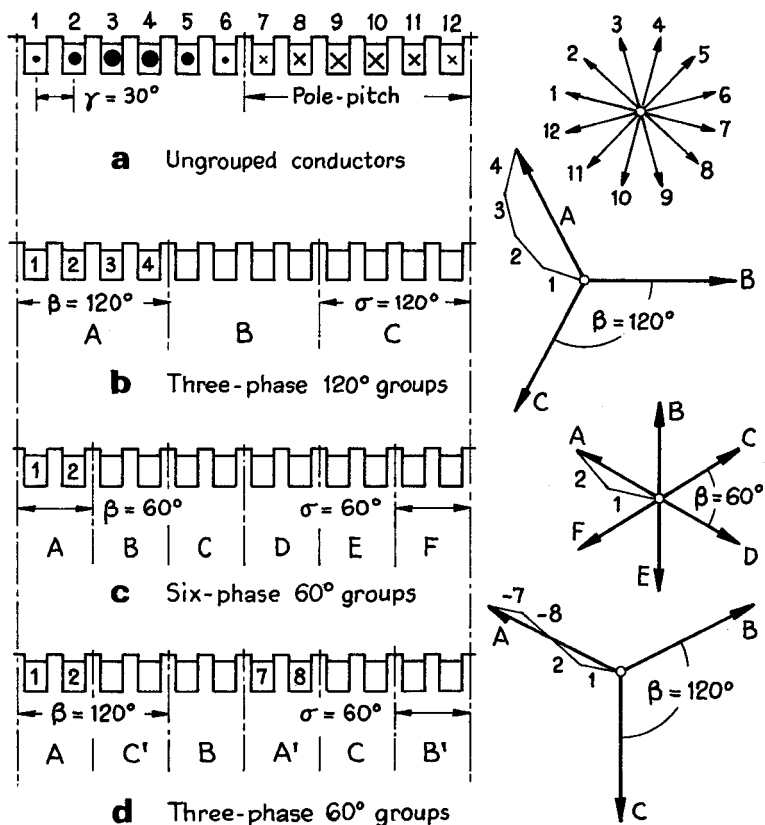


3.12 Rotational e.m.f. in a single turn.

displaced in time-phase by $\beta = 2\pi/m$ elec. rad. This is secured by providing identical phase groups arranged with their axes displaced by $2\pi/m$. The most usual arrangement is that in which the spread of each phase is $\sigma = \pi/m$, i.e. $1/m$ of a pole-pitch. The result is a sequence of phase bands A C B A C B in the double pole-pitch of a 3-ph machine.

In Fig. 3.13, an array of conductors in twelve slots is set out at (a) in a diagrammatic development of a double pole-pitch. The individual conductor e.m.f.s are shown on the right as a symmetrical group of phasors. At (b) a 3-ph grouping into phase-bands is made. Each band has a 120° spread, the summation of conductor e.m.f.s yielding three phase-e.m.f.s with a time-phase

displacement of $2\pi/3$ rad. It will be seen that the phase e.m.f.s are less than the arithmetic sum of the conductor e.m.f.s as a result of spreading the winding. In (c) is shown a grouping for six phases with 60° spread. Grouping (b) could not be used with a single-layer winding for three phases, nor could (c) be used for six, because the subdivision of the winding leaves no further free conductors a pole-pitch apart to form the return conductors. But (c) could be used for *three* phases if D, E and F were employed as the completion of

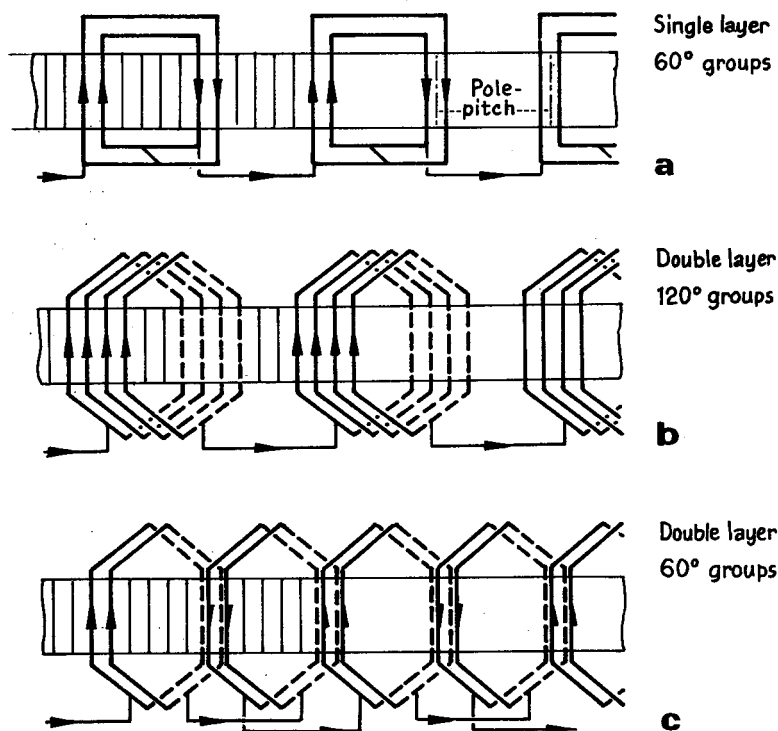


3.13 Phase grouping.

A, B and C. This is shown in (d), where the phase-bands are arranged in this way, and the phase sequence A, B, C obtained by re-lettering. The *phase-bands* follow in the order A, C', B, A', C, B'.

Connections between coil groups

The overhang comprises the end-connections which form the conductors into turns. In addition, the links made at the connection end of a winding comprise



3.14 One phase of 3-ph winding.

Full-pitch, 6 slots/pole, lower coil-sides of 2-layer winding shown dotted.

(a) coil-to-coil connectors, (b) phase-group connectors, and (c) paralleling connectors. When windings are required for large currents and low e.m.f.s, the groups will not generally be connected in series round the whole of a multipolar armature, but will be split up into sections each containing the same number of similar coil-groups: the sections are then paralleled by suitable connectors.

Examples of coil and group connectors are given in Fig. 3.14, for part of one phase of three typical 3-ph windings, viz. (a) single-layer, 60° groups; (b) double-layer, 120° groups; and (c) double-layer, 60° groups. A winding of type (c) in a sixteen-pole machine, for example, could have one, two, four, eight, or even sixteen parallel circuits per phase by making provision for the appropriate paralleling connectors. The number of circuits is equal to the number of poles divided by the number of groups per circuit.

Nomenclature

A double pole-pitch covers 2π electrical radians. A machine with p pole-pairs has $2\pi p$ electrical radians in its periphery. A distinction has therefore to be

made between mechanical (physical) and electrical angle. The following symbols are used in dealing with winding technology:

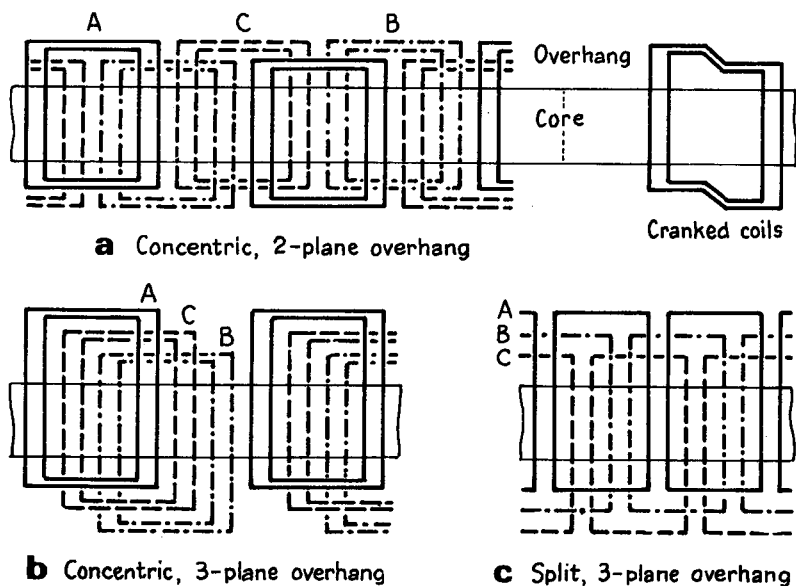
- S total number of slots on a uniformly-slotted armature;
- C number of coils;
- e e.m.f. of a conductor;
- e_t e.m.f. of a turn;
- p number of pole-pairs;
- $\gamma = 2\pi p/S$ slot pitch, electrical radians;
- m number of phases;
- $g = S/2p$ number of-slots per pole;
- $g' = S/2pm$ number of slots per pole and per phase;
- σ spread of a phase-group, electrical radians;
- $\beta = 2\pi/m$ angle between e.m.f.s of successive phases, electrical radians.

S_{ab}, S_{ac} numbers of slots between given points AB, AC.

3.7 Single-layer windings

For more than one phase, a single-layer winding must be designed to allow the end-connections to be accommodated in separate tiers or planes. For a 3-ph winding, Fig. 3.15, the overhang may be arranged in two or in three planes.

Single-layer windings fall into two main classes: (i) *unbifurcated* windings in which the coils comprising a pair of phase groups in adjacent pole-pitches are concentric, and (ii) *bifurcated* windings in which each group is split into



3.15 Single-layer winding.

Two slots per pole and phase, narrow spread

two sets of concentric coils, each set sharing its return coil-sides with those of another group in the same phase. The windings (a) and (b) in Fig. 3.15 are unbifurcated (*concentric*), the former being a continuous and the latter a broken chain arrangement. This affects the number of overhang planes needed to accommodate the end-windings. The winding (c) is bifurcated (*split concentric*) type, requiring a three-plane overhang.

Single-layer coils may be of "hairpin" shape, Fig. 3.18(a), pushed through semi-closed slots, but the subsequent jointing is difficult and expensive. A single-layer constant-span coil arrangement is sometimes employed for small induction motors, using circular-section insulated wire for the conductors. This is a *mush* winding. Each coil is first wound on a former making one coil side shorter than the other. In winding, the wires are dropped individually into the slots, first the short side (the wires emerging beneath the long sides of other coils in adjacent slots), then the long sides. Round the armature, long and short sides occupy alternate slots, and the coils are of constant pitch with a whole number of slots per pole and per phase.

The number of coils in a single-layer winding is one-half the number of slots available, because each coil side completely occupies one slot: thus $C = \frac{1}{2}S$.

3.8 Double-layer windings

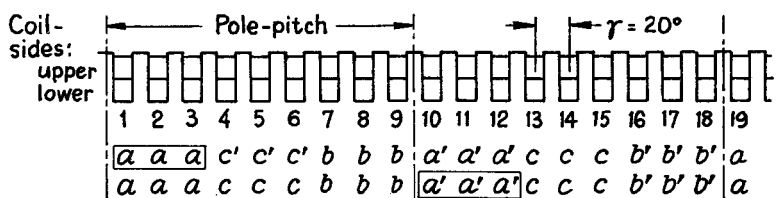
The armatures of nearly all synchronous generators and motors, and of most induction motors above a few kilowatts, are wound with double-layer windings. Windings with a whole number of slots per pole (g an integer) are called *integral-slot* windings. It is common, however, to find g fractional, and such arrangements (in which the number of slots S is not a multiple of the number of poles $2p$) are termed *fractional-slot* windings. These have a number of advantages. In all cases the number of coils is equal to the number of slots: i.e. $C = S$

Integral-slot windings

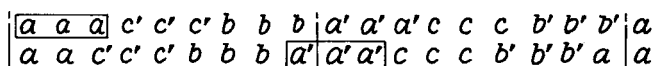
In Fig. 3.16(a, b) some basic arrangements are shown for integral-slot 3-ph windings. The letters indicate conductors to correspond with the phases ABC. Letters arranged vertically designate conductors in the same slot. The angular distance between successive slots is the angular slot-pitch γ in electrical radians. The coils are former-wound, and all have the same span.

Fig. 3.16(a) shows a winding with $g = 9$ slots per pole, $g' = 3$ slots per pole and phase, and full-pitch coils. The slot-pitch is $\gamma = 2\pi/18$ rad, or 20° . Each phase-band spreads over 3 slot-pitches so that the phase-spread is $\sigma = 60^\circ$. The conductors b are arranged 120° and conductors c 240° to the right of a , giving $\beta = 2\pi/3 = 120^\circ$ for the angle between successive phase positions and for their corresponding e.m.f.s. The coils being of full pitch of π rad or 180° , each slot contains coil-sides belonging to one phase only.

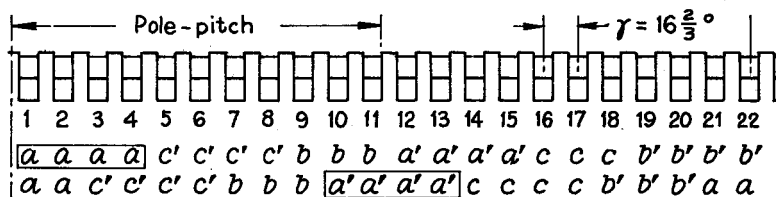
In order to reduce the end-connections and to reduce or suppress certain harmonics in the phase-e.m.f.s, the coils may be *chorded* or *short-pitched*. Such a winding is shown in Fig. 3.16(b). It has $g = 9$ and $g' = 3$, also $\beta = 120^\circ$ as before, but the coil-span is 8 slots instead of 9 as in (a), giving a coil span (or coil-pitch) of $8/9$ of a pole-pitch, i.e. 160° . Certain slots now hold coil-



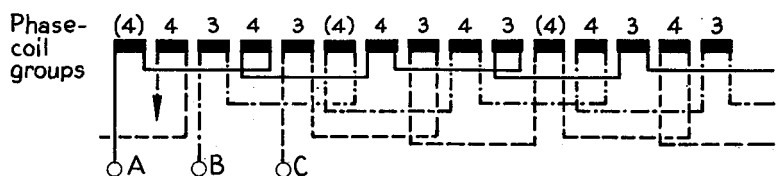
a Integral-slot full-pitch: $g = 9$, $g' = 3$



b Integral-slot chorded: $g = 9$, $g' = 3$, coil-span $8/9$



c Fractional-slot chorded: $g = 10\frac{4}{5}$, $g' = 3\frac{3}{5}$, span $9/10\frac{4}{5}$



d Connection of 5-pole unit of fractional-slot winding

3.16 Double-layer winding.

sides of different phases. Chording reduces the phase-e.m.f., and the coil-span is rarely made less than $\frac{2}{3}$ pole-pitch because additional turns become necessary which offset the saving of overhang material.

Fractional-slot windings

These have non-integral slots per pole g and slots per pole and phase g' : the value of g' is usually an improper fraction such as $\frac{7}{5}$, or in general $S'/2mp'$. In a unit of $2p'$ poles there are S' slots in all, and S'/m slots for each of the m phases, with S'/m integral. The fraction $\frac{7}{5}$ thus represents 7 slots per phase spread over 5 pole-pitches. In a 3-ph machine this means 21 slots for every unit of 5 poles, 42 for 10 poles and so on. All phase windings must be

arranged for electrical and magnetic balance, with an exact 120° angle between phase e.m.f.s.

A fractional-slot winding is shown diagrammatically in Fig. 3.16(c) for one double pole-pitch. It will be seen that the coils cannot have a full span of 180° , and that the phases are unbalanced. The balance is, of course, secured over the complete unit of poles ($2p' = 5$ in this case), after which the whole layout repeats for the next unit of poles, $2p'$, if any.

The coil-span that can be used has considerable freedom. That in Fig. 3.16(c) has a span of 9 slots in $g = 10\frac{1}{2}$ slots per pole; the equivalent $g' = 3\frac{3}{2}$ slots per pole and phase, although this number is not actually found, it representing only the average number in the 5 poles of the unit.

3.9 Fractional-slot windings

The apparent complication just described has theoretical advantages, reinforced by desirable simplifications in manufacture. These are: (a) the simple but effective reduction of slot harmonics, and (b) the use of a total number of armature slots that is not necessarily a multiple of the number of poles. Thus a particular number of slots for which notching gear exists may be used for a range of machines running at different speeds; this is of especial value where a wide range of numbers of poles may be called for, as with synchronous machines.

Fractional slotting is practicable only with double-layer windings. It limits the number of available parallel circuits because phase groups under several poles must be connected in series before a unit is formed and the winding repeats the pattern to give a second unit that can be paralleled with the first.

The simplest cases of fractional-slot windings are those for which g is an integer plus one-half, which employ alternate groups of coils differing by 1. Thus with $g = 7\frac{1}{2}$ slots per pole (or $g' = 2\frac{1}{2}$ slots per pole and phase) in a 3-ph winding with a 60° angular spread, alternate phase-groups will contain respectively 2 and 3 slots, and the two phase-groups, under two successive poles, connected in series will constitute the basic repeatable unit. The number of units is determined by the total number of poles $2p$.

As mentioned in Sect. 3.8, when g' is expressed as the fraction $S'/2mp'$ reduced to its lowest terms, S'/m represents the number of slots and coils per phase in the basic unit of $2p'$ poles. Thus for $g' = 2\frac{3}{4} = 11/4$, there are 11 coils per phase distributed among 4 successive poles, and the four groups may contain 3, 3, 3 and 2 coils in series. The basic unit of $2p'$ poles must naturally be a factor of the total number of poles $2p$, and if a is the number of parallel circuits required, the product $2ap'$ must also be a factor of $2p$.

EXAMPLE 3.2: (a) A 16-pole 3-ph machine with 108 slots and a 60° phase spread will have

$$g' = S/2pm = 108/16 \times 3 = 2\frac{1}{4} \text{ slots per pole and per phase.}$$

The phase groupings will be 3, 2, 2 and 2 coils in the basic unit of 4 poles. All 4-pole units may be series connected, or there may be 2 or 4 circuits in parallel.

(b) If a machine with 108 slots has 10 poles, then

$$g' = 108/10 \times 3 = 3\frac{3}{5} \text{ slots per pole and per phase}$$

the groupings being 4, 4, 4, 3 and 3 coils covering a basic 5-pole-pitches. Only one circuit is possible with both units in series, or one pair only of parallel circuits.

Symmetry

Fractional-slot windings must fulfil the requirements of e.m.f. balance. In each phase the group-sequence must be the same, and the groups must occupy slots so selected as to obtain the required phase-angle $\beta = 2\pi/m$.

Let $S_{ab}, S_{ac} \dots$ be the number of slot-pitches displacement between a given slot in phase A and similar slots in phases B, C. . . . Then, with an angular slot-pitch $\gamma = 2\pi p/S$ electrical radians (i.e. the total electrical angle in the p pole-pairs divided by the total number of slots)

$$S_{ab}\gamma = 2\pi/m \text{ or } (2\pi/m) + x\pi$$

gives the position of the required slot in phase B, with x having any positive integral value. For the former expression the "polarity", i.e. the positive direction of the e.m.f., of the group in which the phase B slot lies will be the same as that in phase A. The same is true of the second expression if $x = 0, 2, 4, 6 \dots$; but if $x = 1, 3, 5 \dots$ the polarities will be opposite. This must be allowed for in making phase connections. In a similar way

$$S_{ac}\gamma = 4\pi/m \text{ or } (4\pi/m) + x\pi$$

and so on for the remainder of the m phases.

As already indicated, a fractional-slot winding may divide itself naturally into basic units of $2p'$ poles containing S' slots each, so that the values $S_{ab}, S_{ac} \dots$ will be appropriate within each unit. Using for convenience the slots per pole and phase $g' = S/2pm = S'/2p'm = \pi/m\gamma$, the slot spacings are

$$S_{ab} = g'(mx + 2), \quad S_{ac} = g'(mx + 4) \dots$$

These always give whole numbers when an appropriate value of x is chosen except when $2p'$ is a multiple of m . Hence when $2p'$, in the improper fraction $g' = S'/2p'm$ reduced to its lowest terms, is a multiple of the number of phases, a balanced winding is not possible.

In 3-ph windings ($m = 3$) the grouping of phase B is S'/m slots from that of phase A when $2p' = 2, 5, 8, 11 \dots$, and the phase C grouping is $2(S'/m)$ slots from phase A. When $2p'$ is 4, 7, 10, 13 . . . these displacements are interchanged.

When the starts (or, depending on the polarity, the finishes) of the various phases have been determined, it remains to fill in a succession of similar groups in each phase from these points. All subsequent slots in each phase will then be correctly located. The sequence of slot groups, e.g. 4, 4, 4, 3, 3 slots in Example 3.2(b), is determined in such a way as to give the greatest symmetry in the winding. One 5-pole unit of the winding might be tabulated as follows—

Phase A	(+4)	-4	+4	-3	+3
Phase B	+3	(-4)	+4	-4	+3
Phase C	-4	+3	-3	(+4)	-4
Pole-pitch	1	2	3	4	5

The figures indicate the number of slots containing the *upper* coil-sides in each group with its appropriate polarity, those numbers in brackets being inserted first from the expressions given for S_{ab} and S_{ac} . A layout of the winding is drawn in Fig. 3.16(d). The diagram of two pole-pitches is shown-in (c), the lower coil-sides in the left-hand pole-pitch corresponding to the arrangement of the upper coil-sides of pole-pitch 5 in the Table.

Coil span

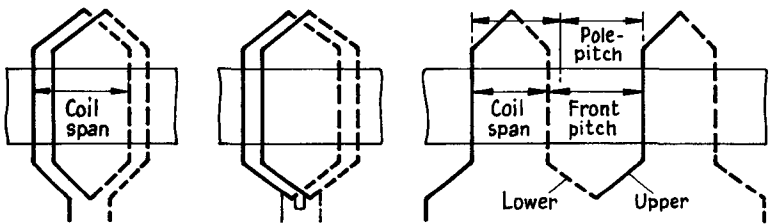
When the grouping of the upper layer of coil-sides has been decided, it follows that the groupings of the corresponding lower coil-sides are determined also, for these have the same group sequence but displaced from the upper groups by the *winding pitch* or *coil-span*. The coil-span must be the same for all coils in the winding, and because g is fractional the span cannot be a full pitch. Thus with $g = 7\frac{1}{2}$ slots per pole the coils might span from slot 1 to slot 9 (8 slot pitches), or 1 to 8 (7 slot pitches): the former is greater and the latter is less than the pole-pitch of $7\frac{1}{2}$ slot-pitches. The span chosen is almost invariably short rather than long. With $g = 7\frac{1}{2}$, a span of 7 slots is 89% of full pitch, while 6 slots gives 80% of full pitch. Chording has technical advantages in the reduction of overhang copper, and of e.m.f. and m.m.f. harmonics.

3.10 Types of double-layer winding

A coil in a double-layer winding represents the entire set of conductors in one slot layer in association with the similar set in the other layer of another slot. The number of coils is therefore the same as the number of slots: $C = S$.

The end connections of a double-layer winding may have a variety of shapes, depending on the method employed to bring about the transition from the upper to the lower slot position. Basically there are only two types, viz. *lap*- and *wave-connected*. The coils may have one turn or several. In the former case single-turn coils require only one conductor in each layer, but this will generally be laminated to facilitate bending in the end-windings and to reduce eddy-current losses. These coils form *bar-windings*, and are commonly found in low-voltage machines with large phase currents. The choice of coil depends on the number of turns per phase required to generate the e.m.f., the current rating, and the number of parallel circuits obtainable.

Fig. 3.17 shows (a) a pair of single-turn (bar) lap coils, (b) two multi-turn



a Bar lap **b** Multiturn lap **c** Single-turn (bar) wave

3.17 Double-layer chorded lap and wave coils.

(bar) lap coils, and (c) two bar wave coils. Multi-turn wave coils are not used, because lap coils are more easily connected.

The connection of lap coils in groups is as indicated in Fig. 3.14(b) and (c). The wave-connection of bars is not so straightforward, since after one complete tour round the armature (i.e. after series connection of p coils) the winding would naturally close on to the start of the first coil. To avoid this, a connection is made to continue the winding into the second coil-side of the first group, and so on until all coil-sides of *alternate* groups in each layer are fully wound. Another independent wave winding is made to utilize the remaining half of the phase. The two parts can then be connected in series or parallel to form the complete phase winding.

When fractional-slot wave windings are used, the problem is complicated by the fact that isolated coils are left after the full number of complete turns round the armature has been made, necessitating special connectors.

3.11 Choice of winding

In the following items specifying a 3-ph winding, those features italicized are the most common:

- (a) Type of coil: concentric, *lap*, wave.
- (b) Overhang: *diamond*, multiplane, involute, mush.
- (c) Layers: single, *double*.
- (d) Slots: *open*, closed, semi-closed.
- (e) Connection: *star*, *mesh*.
- (f) Phase-spread: 60° , 120° .
- (g) Slotting: integral, *fractional*.
- (h) Coil-span: full-pitch, *chorded*.
- (j) Circuits: series, or parallel.
- (k) Coils: single-turn, multiturn.

Multiturn coil windings. Machines with small numbers of poles and low values of flux per pole require a relatively large number of turns per phase: this is true also of high-voltage machines. Multiturn coils are needed, and the choice will lie between the double-layer lap winding and the single-layer winding. The former is dropped into open slots in all cases where good slot insulation is essential, although in small low-voltage machines the conductors may be fed through a narrowed slot-opening into a slot-liner which serves as the main insulation. Single-layer windings are rarely used in large machines except with partially-closed slots and push-through hairpin coils.

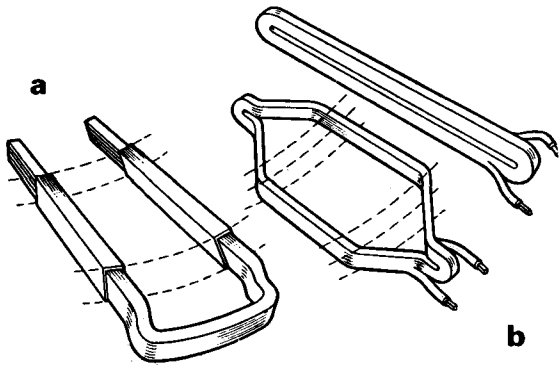
In general it may be said that modern practice favours the double-layer winding except where the slot-openings would be large compared with the length of the air-gap, as in high-voltage induction motors.

Bar windings. Where single-turn coils are necessary, as with turbo-generators or multipolar low-voltage machines, the choice lies between double-layer bar lap or wave windings. Both types may have the bars pushed through partially-closed slots and be bent to shape at the other end when the conductor section is moderate, but the heavy bars used in turbo-alternators must be completely formed before being inserted into open slots.

The chorded bar lap winding has the advantage of a shorter overhang at the connection end than the bar wave, and is more suited to cases where several parallel circuits are needed.

Coil making

Fig. 3.18(a) gives a view of a single-layer coil for a concentric winding, indicating its general shape before being pushed through a pair of slots. At (b) is shown stages in the production of a multiturn double-layer diamond coil of

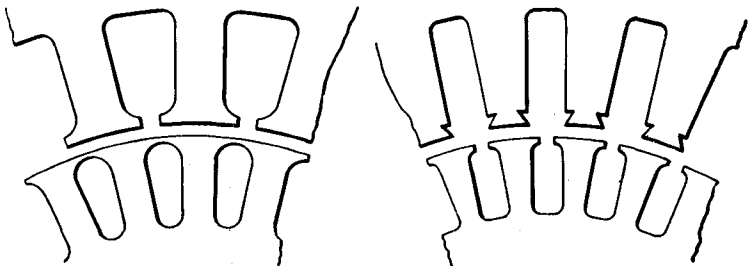


3.18 Single- and double-layer coils.

the *pulled* type. When such a coil is not too heavy it may be wound over two dowels as a flat loop, and the slot portion then gripped in a machine and pulled apart to the required coil span. More massive double-layer coils have to be built up on formers, and the overhang portion may be bent up to make an angle with the slot parts, to form an *involute* shape. When assembled on the stator the overhang presents a conical surface suitable for bracing.

Slotting

In very small machines, using round wires of the smaller gauges, the tooth shape is improved if tapered slots are used, Fig. 3.19. In all but these, parallel-sided slots are employed. The slot-openings serve to avoid excessive slot-leakage,



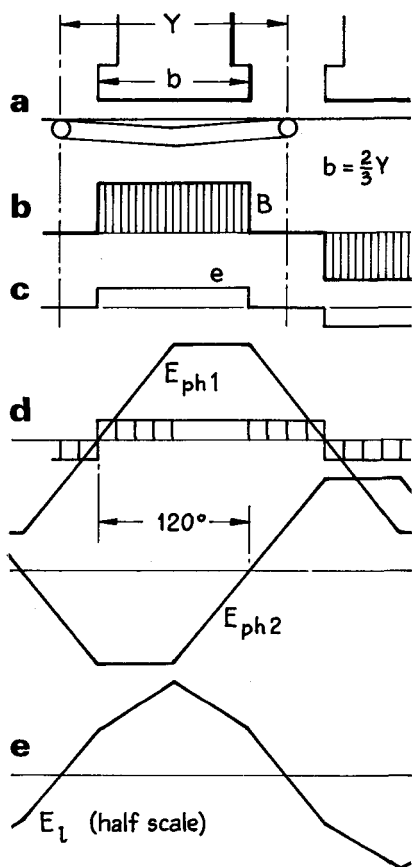
3.19 Types of slot.

but they cause an effective lengthening of the magnetic path across the air gap, and tunnel slots are occasionally used for small induction motors. Open slots (i.e. with the opening as wide as the slot) must be used where it is desired to complete the coils outside the armature and drop them into the slots.

3.12 E.M.F. of windings

The e.m.f. generated in basic windings under idealized conditions was considered in Sect. 3.3: these conditions were (i) a sinusoidal distribution of gap flux density, and (ii) a uniform spread of conductors over the gap periphery. The flux is never exactly distributed as in (i), particularly in salient-pole machines, and the windings (ii) are bedded in slots.

The e.m.f. of a phase is the summation of the e.m.f.s of slot conductors. Such a phase is normally associated with other phase windings in star to give a resultant line voltage, the effect being to make the line voltage more nearly



3.20 E.M.F. waveform due to rectangular gap-field distribution.

sinusoidal. The effects of e.m.f. summation and of phase interconnection are explained in Fig. 3.20, taking as an example a salient-pole machine with a pole-arc b two-thirds of the pole-pitch Y . The full-pitch coil in (a) is then associated with the rectangular gap-density distribution (b), and the e.m.f. in each coil-side has the waveform (c). A group of coils spread over 60° gives the phase e.m.f. E_{ph1} . (The stepping due to the location of coil-sides in separate slots is here ignored.) This phase e.m.f. subtracted from that of the associated phase, E_{ph2} , gives the line-to-line e.m.f. E_l in (e).

It was shown in Sect. 2.5 that a non-sinusoidal gap-density distribution could be expressed as a harmonic series

$$B = B_1 \sin \theta + B_3 \sin 3\theta + \dots + B_n \sin n\theta$$

A machine can thus be considered to have $2p$ "fundamental" poles, together with families of $6p, 10p \dots 2np$ "harmonic" poles, all individually sinusoidal and all generating e.m.f.s in an associated winding. The magnitude of the fundamental e.m.f. is determined by (i) the fundamental flux, (ii) the phase spread and (iii) the coil-span. The harmonic e.m.f.s are similarly determined by (iv) the harmonic fluxes, (v) and (vi) the effective electrical phase spreads and coil-spans, and (vii) the method of interphase connection. There is a further effect, however: the slotting imposes local changes in the gap reluctance, so that besides the harmonic content of B , there is a superposed *slot-ripple*. We deal first with the response of a uniformly distributed winding to fundamental and harmonics in the wave of gap flux density, and then with the slot-ripple.

Uniformly distributed winding

A full-pitch coil of one turn, moving through a fixed fundamental sine-distributed flux Φ_1 at a speed corresponding to frequency f (as in Case B of Sect. 3.3) develops an e.m.f. of r.m.s. value $E_t = \sqrt{2\pi f \Phi_1}$. In a phase winding of N such turns spread uniformly over an electrical angle σ , the e.m.f. in each turn is E_t but its time-phase depends on its position in the winding. The total phase-winding e.m.f. E_{ph} of the N turns in series is not NE_t but is the *phasor* sum of the individual turn e.m.f.s, approximating to the arc in Fig. 3.21(a). Then

$$\frac{E_{ph}}{NE_t} = \frac{\text{phasor sum}}{\text{arithmetic sum}} = \frac{\text{chord}}{\text{arc}} = \frac{\sin \frac{1}{2}\sigma}{\frac{1}{2}\sigma} = k_{d1}$$

where k_{d1} is the breadth coefficient or distribution factor for the fundamental of the gap flux.

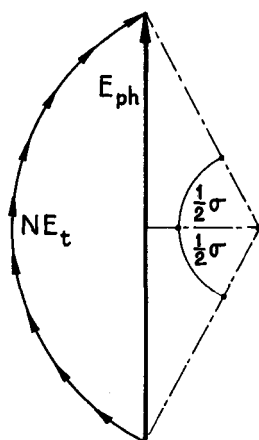
Distribution factor. For space harmonics in the wave of flux density the 'harmonic poles' are narrower and the phase spread σ covers more of the wavelength: in fact, for the n th harmonic the effective phase spread is $n\sigma$. Hence in general for a harmonic flux of order n the distribution factor is

$$k_{dn} = \frac{\sin \frac{1}{2}(n\sigma)}{\frac{1}{2}(n\sigma)} \quad (3.15)$$

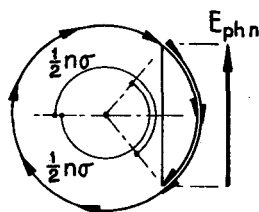
Compare (a) and (b) in Fig. 3.21 for the same phase spread: (a) is for the fundamental ($n = 1$) and (b) for the 5th harmonic ($n = 5$). The distribution factor for harmonics is generally different from that of the fundamental. If $\sigma = 2\pi/m$ where m is the number of phases in a multiphase winding, then although there may be a flux harmonic there will be no corresponding phase e.m.f. harmonic for the order $n = m$. The Table below gives values of fundamental and low-order harmonic distribution factors with the more usual phase spreads.

Uniformly distributed winding: values of k_{dn}

No. of phases m	Phase-spread σ	k_{d1}	k_{d3}	k_{d5}	k_{d7}	k_{d9}
3 or 6	60°	0.955	0.637	0.191	-0.136	-0.212
3 or 1	120°	0.827	0	-0.165	0.118	0
2	90°	0.900	0.300	-0.180	-0.129	0.100
1	180°	0.637	-0.212	0.127	-0.091	0.071

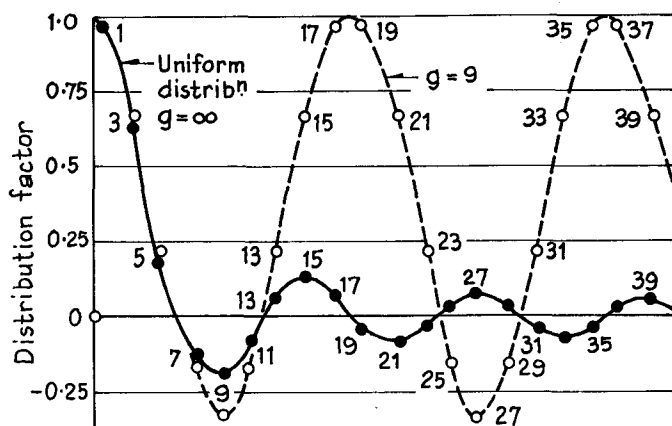


a Fundamental



b n^{th} harmonic

3.21 Distribution factor: uniformly-distributed winding.



3.22 Distribution factor.

The distribution factors for the common phase spread $\sigma = 60^\circ$ are plotted (full line and dots) in Fig. 3.22.

Coil-span or chording factor. If the span of a coil is not a full pole-pitch, but is chorded to cover $(\pi \pm \epsilon)$ electrical radians, the flux that it links is reduced by the factor $\cos \frac{1}{2}\epsilon$ for the fundamental, as shown by (a) and (b) in Fig. 3.23: its e.m.f. is reduced in proportion. In the case of harmonic flux components of order n , the effective chording angle is $n\epsilon$, so that the general coil-span or chording factor is

$$k_{en} = \cos \frac{1}{2}(n\epsilon) \quad (3.16)$$

Fig. 3.23(c) illustrates the effect on a 3rd harmonic of a chording angle of 60° , i.e. a coil-span of 120° . Such a coil covers two harmonic pole-pitches and links zero 3rd harmonic flux, so that 3rd harmonic e.m.f.s disappear from the phase e.m.f. In general, when a coil is pitched short or long by $\pi/n, 3\pi/n \dots$ no harmonic of order n survives in the coil or phase e.m.f.s.

Slotted winding

For an integral-slot winding there are g' slots/pole per phase, and individual coil e.m.f.s are represented by phasors displaced by the angle σ/g' , as in Fig. 3.24 for the fundamental. A little trigonometry shows that the general distribution factor is given by

$$k_{dn} = \frac{\sin \frac{1}{2}(n\sigma)}{g' \sin \frac{1}{2}(n\sigma/g')} \quad (3.17)$$

For a fractional-slot winding g' is given by $S'/2mp'$, and a layout of a $2p'$ -pole unit contains S'/m coils in S'/m slots per phase distributed evenly over a spread σ . The distribution factor is

$$k_{dn} = \frac{\sin \frac{1}{2}(n\sigma)}{(S'/m) \sin \frac{1}{2}(n\sigma)(m/S')} \quad (3.18)$$

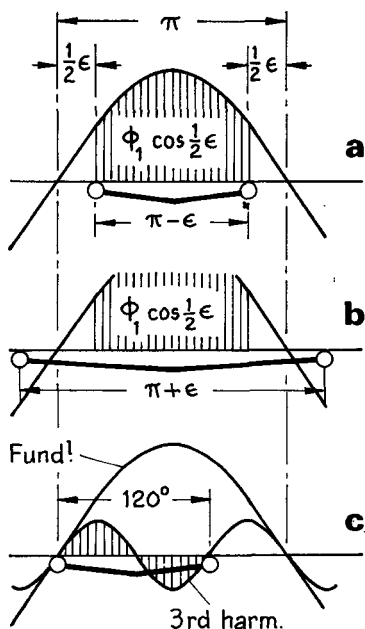
For example, a 3-ph winding ($m = 3$) with a spread of $\sigma = 60^\circ$ and $g' = 2\frac{1}{2}$ slots/pole per phase has

$$g' = S'/2mp' = 5/2$$

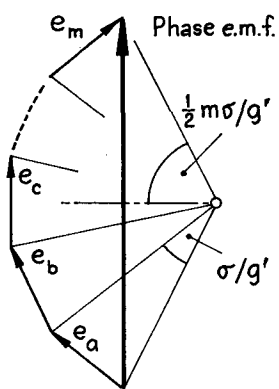
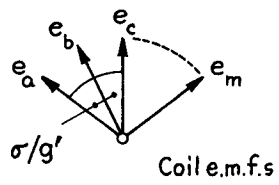
The unit comprises $2p' = 2$ pole-pitches in which there are $S'/m = 5$ coils or slots per phase and $S' = 15$ slots altogether. The effective angular displacement of the coils (and of their e.m.f.s) is $60^\circ/5 = 12^\circ$, and the distribution factor for the fundamental is $k_{d1} = \sin 30^\circ/5 \sin 6^\circ = 0.956$.

The variation of the permeance of the gap over a succession of slots and teeth may generate corresponding gap-flux harmonics in the wave of flux density, particularly if the number of slots per pole g is integral. Thus if there are 9 slots per pole ($g = 9$) and therefore 18 slots per pole-pair, there will be in the gap an 18th harmonic of flux density modulated in amplitude by the poles, giving the effect of constant-amplitude flux harmonics of orders 18 ± 1 , that is a 17th and a 19th. Now it can be shown that the distribution factors for harmonics of orders n are the same as those of orders $6Ag' \pm n$, where A is any integer 0, 1, 2 . . . Thus with $g = 9$ and $m = 3$, giving $g' = 3$, then for $A = 1$ we find that the distribution factors k_{d17} and k_{d19} are both equal to k_{d1} . Unless precautions are taken, such as skewing or the substitution of fractional for integral slotting, these harmonics will appear strongly in the phase e.m.f. The effect is shown by the circles (linked for clarity by the dotted line) in Fig. 3.22.

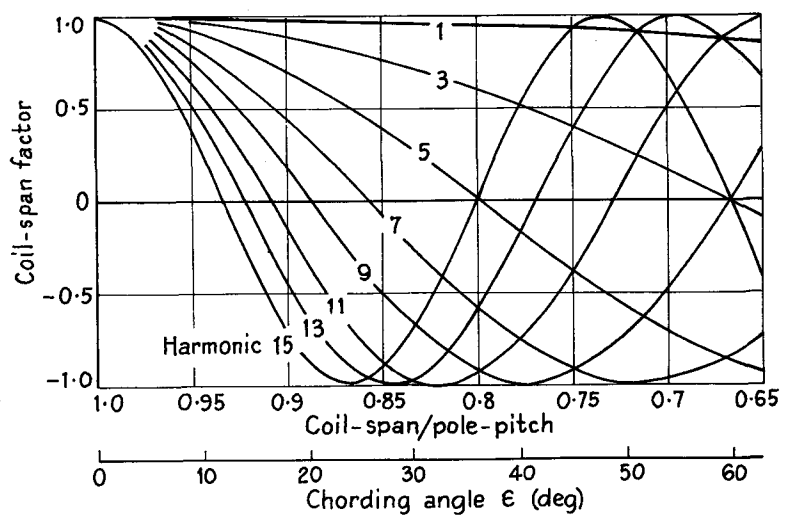
Coil-span factor. The appropriate factor is given by eq.(3.16). Fig. 3.25 gives



3.23 Chording.



3.24 Distribution factor.



3.25 Chording (coil-span) factors.

a plot of k_{en} for harmonics up to the 15th. For example, for a coil-span of two-thirds of a pole-pitch, $\epsilon = \pi/3 = 60^\circ$, the factor $k_{e1} = 0.866$. For the 3rd harmonic $k_{e3} = 0$; thus all 3rd (and *triplen*) harmonics are eliminated from the coil and phase e.m.f.s as we have seen already. The triplen harmonics in a 3-ph machine are normally eliminated by the phase connection, and it is usual to select the coil-span to reduce the 5th and 7th harmonics instead. A pitch of 150° corresponding to a chording angle $\epsilon = 30^\circ$ is useful in this respect, as it gives the following factors:

Harmonic order n	Fund.	3	5	7	9
Coil-span factor k_{en}	0.966	0.707	0.259	0.259	0.707

Phase E.M.F.

The e.m.f. of a phase due to the fundamental component of the flux per pole is

$$E_{ph1} = 4.44 f k_{d1} k_{e1} N_{ph} \Phi_1 = 4.44 f K_{w1} N_{ph} \Phi_1 \quad (3.19)$$

where $K_{w1} = k_{d1} \cdot k_{e1}$ is the winding factor. For the n th harmonic

$$E_{phn} = 4.44 n f K_{wn} \Phi_n \quad (3.20)$$

where $K_{wn} = k_{dn} \cdot k_{en}$. A more useful form is obtained from an analysis of the gap flux density distribution, Sect. 2.5; for $\Phi_1 = (2/\pi) B_1 Y_l$ and $\Phi_n = (2/\pi) B_n (Y/n) l = \Phi_1 (B_n/nB_1)$. The n th harmonic and fundamental e.m.f. components are now related by

$$\frac{E_{phn}}{E_{ph1}} = \frac{B_n K_{wn}}{B_1 K_{w1}}$$

The r.m.s. phase e.m.f. is

$$E_{ph} = \sqrt{(E_{ph1})^2 + E_{ph3}^2 + \dots + E_{phn}^2}$$

In all ordinary cases this is indistinguishable from E_{ph1} . The line e.m.f. in star connection is $\sqrt{3} E_{ph}$ calculated with all triplen harmonics omitted.

EXAMPLE 3.3: A 3.75 MVA 10 kV 3-ph 50 Hz 10-pole low-speed hydro-generator has 144 slots containing a two-layer diamond winding with 5 conductors per coil-side in each slot. The coil-span is 12 slot-pitches. The flux per pole is 0.116 Wb, distributed as in Example 2.1. A layout of the winding and a calculation of the phase e.m.f. is required.

Winding. The slot angle is $\gamma = 2\pi p/S = 10\pi/144 = 0.218 \text{ rad} = 12\frac{1}{2}^\circ$. The fractional value of slots per pole per phase is

$$g' = S/2mp = 144/3 \cdot 10 = 24/5 = 4\frac{4}{5}$$

There will consequently be 24 slots per phase, or 72 slots in all, in each of two 5-pole units. The spacing between the starts of A and B will be

$$S_{ab} = g'(3x + 2) = 4\frac{4}{5}(3 \cdot 1 + 2) = 24 \text{ slots}$$

and for C

$$S_{ac} = g'(3x + 4) = 4\frac{4}{3}(3 \cdot 2 + 4) = 48 \text{ slots}$$

To show the arrangement of coil groups, 5, 5, 5, 5 and 4 for each phase, the Table p. 102 can be drawn up. The slots are numbered 1, 2, 3 ... 72 in order, with their angles $0^\circ, 12\frac{1}{2}^\circ, 25^\circ \dots$, values above 180° being reduced by 180° or integral multiples thereof. Coil-sides for phase A are then allocated to slots with angles from 0° up to (but not including) 60° ; phase B from 120° ; and phase C from 60° . The letters *a*, *b* and *c* in square brackets are those located from the values of $S_{ab} = 24$ and $S_{ac} = 48$ already calculated; i.e. slots 1 for A, 25 for B and 49 for C.

The sequence of groups, from each starting point, is seen to be 5, 5, 5, 4, 5. The slots occupied by phase A cover the angle $0^\circ - 57\frac{1}{2}^\circ$ in steps of $2\frac{1}{2}^\circ$, so that the winding is uniformly spread with an angle of $2\frac{1}{2}^\circ$ between coil e.m.f.s. A winding Table, including polarities, is given below —

Phase A	[+5]	-5	+5	-4	+5
Phase B	+5	[-5]	+5	-5	+4
Phase C	-5	+4	-5	[+5]	-5
Pole-pitch	1	2	3	4	5

The lower coil-sides will lie in slots 12 slot-pitches to the right of their corresponding upper coil-sides.

E.M.F.: Example 2.1 shows that the total flux per pole of 0.116 Wb can be accepted as Φ_1 . The 3rd harmonic flux is ignored as it does not affect the line e.m.f. The coil-span is $12/(3 \times 4\frac{4}{3}) = 5/6$ pole-pitch or 150° , so that the chording angle is $\epsilon = 30^\circ$. From eqs.(3.18) and (3.16)

$$k_{dn} = \frac{\sin \frac{1}{2}(n60^\circ)}{24 \sin \frac{1}{2}(n2\frac{1}{2}^\circ)} \quad \text{and} \quad k_{en} = \cos \frac{1}{2}(n30^\circ)$$

giving the winding factors $K_{wn} = k_{dn} \times k_{en}$:

$$K_{w1} = 0.956 \times 0.966 = 0.925 \quad K_{w5} = 0.191 \times 0.259 = 0.049$$

$$\text{and} \quad K_{w7} = (-0.136)(-0.259) = 0.035$$

There is a material reduction in the harmonic factors. The e.m.f.s from eqs.(3.19) and (3.20) are

$$E_{ph1} = 4.44 \times 0.925 \times 50 \times 240 \times 0.116 = 5750 \text{ V}$$

$$E_{ph5} = 5750 (0.049/0.925)(11.2/100.6) = 34 \text{ V}$$

$$E_{ph7} = 5750 (0.035/0.925)(2.8/100.6) = 6 \text{ V}$$

The harmonic e.m.f.s are quite negligible, so that the phase e.m.f. is $E_{ph} = 5.75 \text{ kV}$, giving $E_l = 10 \text{ kV}$ with star connection.

Graded and interleaved windings

Excessive harmonics lead to vibration and noise, additional stator and rotor

Slot: 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Angle: 0	12½	25	37½	50	62½	75	87½	100	112½	125	137½	150	162½	175
Phase: [a]	a	a	a	a	c	c	c	c	c	b	b	b	b	b
Slot: 16	17	18	19	20	21	22	23	24	25	26	27	28	29	
Angle: 7½	20	32½	45	57½	70	82½	95	107½	120	132½	145	157½	170	
Phase: a	a	a	a	a	c	c	c	c	[b]	b	b	b	b	
Slot: 30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
Angle: 2½	15	27½	40	52½	65	77½	90	102½	115	127½	140	152½	165	177½
Phase: a	a	a	a	a	c	c	c	c	c	b	b	b	b	b
Slot: 45	46	47	48	49	50	51	52	53	54	55	56	57	58	
Angle: 10	22½	35	47½	60	72½	85	97½	110	122½	135	147½	160	172½	
Phase: a	a	a	a	[c]	c	c	c	c	b	b	b	b	b	
Slot: 59	60	61	62	63	64	65	66	67	68	69	70	71	72	
Angle: 5	17½	30	42½	55	67½	80	92½	105	117½	130	142½	155	167½	
Phase: a	a	a	a	a	c	c	c	c	c	b	b	b	b	

loss, and (in induction motors) to parasitic torques. If a winding had a sinusoidal distribution of conductors per slot there would be no harmonics other than those caused by slotting. In special cases such a *graded* winding, with more conductors per coil in the phase centre than at the ends, can be used for space-harmonic reduction.

Another way, not so difficult to build, gets something of the same effect by *interleaving*. The end coils of each phase group are interchanged with the corresponding end coils of adjacent groups. A 3-ph winding with 5 slots/pole per phase in normal and in interleaved (or interspersed) form would appear as below, the letters indicating coil-sides belonging respectively to phases A, B and C:

Normal:	b'	a	a	a	a	a	c'	c'	c'	c'	c'	b	b	b	b	b	b	a'	...	
Interleaved:	a	b'	a	a	a	c'	a	c'	c'	c'	b	c'	b	b	b	b	b	a'	b	...

The result is a distribution reducing the low-order harmonics, and for this example the distribution factors are

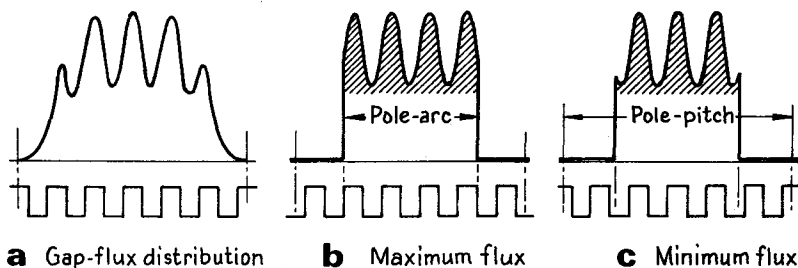
Harmonic order	Fund.	5	7	11	13	17
Normal	0.957	0.200	0.149	0.110	0.102	0.102
Interleaved	0.915	0.0	0.118	0.256	0.289	0.289

The fundamental is 5% reduced, but the 5th is zero. The 7th, somewhat reduced, could be chorded out. With a $2/3$ pitch the overall design could actually be advantageous, especially for large machines in which stray load losses are important.

3.13 Tooth harmonics

In addition to the e.m.f.s generated in the windings by those space harmonics of flux which move at the same velocity as the fundamental flux relative to the conductors, certain harmonic voltages of a particularly undesirable order may be produced by the effect of the openings of the slots in which the conductors themselves are located. This effect is aggravated when the slot-openings are large compared with the tooth width and gap length, because these affect the air-gap permeance, to variations of which the harmonics are due. The ripples in the flux wave caused by variation of permeance from point to point in the air-gap are of the form indicated in the oscillogram of Fig. 3.26(a): but it is important to note that, since the ripples are due to slotting, they do not move with respect to the conductors but glide over the flux-distribution curve, always opposite the slots and teeth which cause them.

Pulsation of Total Gap Permeance. If the main flux wave were assumed to be rectangular (as in a salient-pole machine without fringing) and the ripple flux a pure sine-wave superposed on it, the particular case in which the pole-arc covers an integral-plus-one-half slot-pitches would give a total flux varying between the two extremes shown in Fig. 3.26(b) and (c), totalling $\pm$ the flux



3.26 Tooth ripple

contained in one loop of the ripple. The frequency of flux pulsation would correspond to the rate at which the slots cross the pole-face, i.e. $2gf$. Now this stationary pulsation may be regarded as two waves of fundamental space-distribution rotating at angular velocity $2g\omega$, one forwards and one backwards, where $\omega = 2\pi f$ corresponds to synchronous speed. The component fields will have velocities of $(2g \pm 1)\omega$ relative to the armature winding and will generate therein harmonic e.m.f.s of frequencies $(2g \pm 1)f$. But this is only a secondary cause of tooth-ripple.

Rotor Current Pulsation. The prime cause of tooth-ripple is the induction in rotor circuits of currents of frequency $2gf$. These flow in suitably-pitched damper bars in the poles of a salient-pole machine, in the brass or bronze wedges of a turbo rotor, and (to a minor extent) in the main rotor field windings.

The gap permeance at the stator surface has space undulations, and the variations from the average may be represented approximately by a sine fluctuation $\lambda_1 \sin 2g\theta$. The fundamental rotor m.m.f. acting at the stator bore rotates with respect to the stator slots and may be written, for a given point on the stator, as $F_1 \sin(\theta - \omega t)$. The flux density at the stator surface will have a fundamental corresponding to the average permeance, together with a superposed ripple due to the fluctuation given by

$$\lambda_1 F_1 \sin 2g\theta \cdot \sin(\theta - \omega t)$$

Resolving this into its two oppositely-rotating components gives

$$\frac{1}{2} \lambda_1 F_1 \{ \cos[(2g-1)\theta + \omega t] - \cos[(2g+1)\theta - \omega t] \}$$

With respect to the stator these ripple fluxes vary with position in a slot-pitch, and with time at angular frequency ω : thus they cannot contribute any e.m.f. at tooth-ripple frequency.

The same two waves have, however, different velocities relative to the rotor. Any point designated by θ on the stator will require to be designated by $\theta + \omega t$ with reference to the rotor. Making this substitution gives

$$\frac{1}{2} \lambda_1 F_1 \{ \cos[(2g-1)\theta + 2g\omega t] - \cos[(2g+1)\theta + 2g\omega t] \}.$$

As these flux waves have a velocity $2g\omega$ relative to the rotor, they will generate currents of frequency $2gf$ in any closed rotor circuits of suitable pitch. Such

currents will superpose an m.m.f. variation of $2gf$ on the resultant pole m.m.f. Resolving these again into forward- and backward-moving components of $2g\omega$ relative to the *rotor*, and therefore $2g\omega \pm \omega = (2g \pm 1)\omega$ relative to the *stator*, then stator e.m.f.s at frequencies $(2g + 1)f$ and $(2g - 1)f$ will be generated. These constitute the principal tooth ripples.

In the above only the fundamentals of the slot-permeance variation λ_1 and of the rotor m.m.f. F_1 have been considered. Further ripple effect may occur due to the harmonics of each. Quite objectionable ripple may be generated by the second harmonic of the slot permeance.

The suppression of slot ripples is accomplished by: (1) skewing the stator core; (2) displacing the centre-line of the damper bars in successive pole-shoes; (3) offsetting the pole-shoes in successive *pairs* of poles: these refer to the mitigation of ripple directly due to slot-permeance fluctuation. To counter the effects of harmonics in the field form, the methods are: (4) shaping the pole-shoes so that the maximum gap length is about twice the minimum, in salient-pole machines; and (5) the use of composite steel-bronze wedges for the slots of turbo rotors.

Tooth-ripple Loss

Pole-entry Loss. As a slot, moving with respect to a salient pole, enters the region of the pole-tip, rapid changes of flux occur which can give rise to parasitic and transient losses both in the slot conductors and in the tooth steel. The sudden change of slot-field pattern generates eddy currents and develops tangential forces on the teeth that may contribute an acoustic noise effect.

Pole-face Loss. The tooth-ripple flux causes surface pulsation loss in the pole-faces. An appreciable amount of this flux penetrates deeply into laminated pole shoes along the boundary surfaces of the (thick) pole-shoe punchings. Several analytical solutions to the problem of the surface loss produced have been suggested. One, by Greig and Freeman [19], when reduced and written in the symbolism of this book, gives the specific pole-face loss (power per unit surface area) as

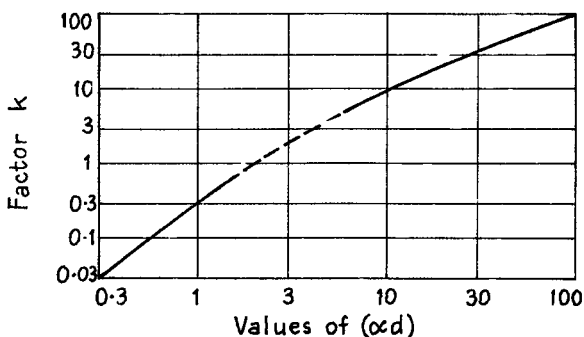
$$p_e = \frac{B_t^2 y_s \omega_s}{8\pi\mu} \cdot k \quad (3.21)$$

in which B_t is the peak value of the fundamental of the ripple flux density of angular frequency ω_s , and y_s is the slot-pitch and μ the permeability of the pole-shoe material. The factor k depends on the kind of pole-shoe, and has the following values:

Solid pole-shoes: $k = \alpha(y_s/\pi)$

Laminated pole-shoes: $k = \frac{1}{3}(\alpha d)^2$ for $0 < (\alpha d) < 1.5$
 $k = \frac{1}{4}(\alpha d) \cdot f(\alpha d)$ for $1.5 < (\alpha d) < 5$
 $k = (\alpha d)$ for $5 < (\alpha d) < 100$

where d is the thickness of a pole-shoe lamina, and α is from eq.(3.1) with



3.27 Tooth-ripple loss factor.

μ_0 replaced by μ and with $b/w_s = 1$. The function $f(\alpha d)$ is complicated, but can adequately be approximated by joining up the plots of k for the ranges 0–1.5 and 5–100, as indicated by the dotted line in Fig. 3.27. It is found that the second harmonic of the ripple flux density can add up to 25% loss: it can be evaluated from eq.(3.21) using appropriate values of ω_s and α .

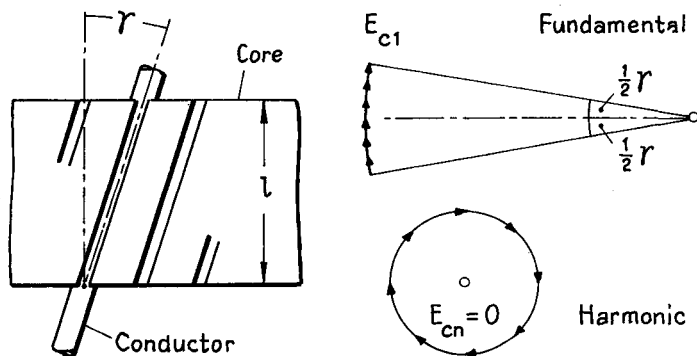
Induction Machines. Large tooth pulsations can occur. Chalmers and Richardson [20] have given experimental results and compared them with formulae devised by Alger [21].

Skewed slots

Skew, as indicated in Fig. 3.28, is an angular 'twist' of a slot away from the axial direction by an electrical angle γ which is typically in the range 0.1–0.5 rad (5°–30°). Most cage induction motors have their rotor slots skewed by one slot-pitch to reduce those space harmonics in the airgap flux density that are introduced by the slotting. There are several consequences.

Winding Factors. A skewed rotor bar is, in effect, 'spread' over the angle γ and its induced e.m.f. is reduced in accordance with the winding factors

$$k_{d1} = (\sin \frac{1}{2}\gamma) / \frac{1}{2}\gamma \quad \text{and} \quad k_{dn} = (\sin \frac{1}{2}n\gamma) / \frac{1}{2}n\gamma$$



3.28 Skewing.

for the fundamental and the n th harmonic respectively of the gap-flux density distribution. Ideally, slot harmonics are eliminated because the skew gives an effective phase-spread of 2π rad at the slot-harmonic wavelength.

Reactance. Reduction of the bar e.m.f. is effectively a drop in the main-flux rotor linkage and a rise in the rotor leakage flux. Thus the rotor leakage reactance is increased.

Flux Distribution. Skew produces at each end of the axial length a displacement of $\frac{1}{2}\gamma$ between the stator and rotor m.m.f. patterns, which therefore "match" only in the mid region. The mismatch elsewhere causes a non-uniform axial distribution of gap-flux density by as much as 40%.

Axial Force. The skewed rotor-bar current has a circumferential component proportional to $\sin \gamma$, developing an axially directed force which imposes an additional load on thrust bearings.

Losses. The non-uniform gap density increases core and rotor surface losses. Where (as is usual) the rotor bars are uninsulated, it is possible for currents to flow from bar to bar circumferentially, producing I^2R loss with no corresponding useful torque.

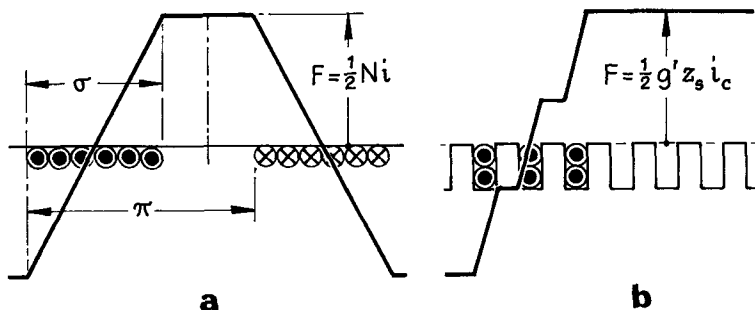
3.14 Magnetomotive force of windings

The airgap flux in a machine results from the combined m.m.f.s of all the windings, whether on the stator or on the rotor, that act on the gap region. We consider first the individual m.m.f.s of a number of typical windings.

Phase M.M.F.

The idealized m.m.f. of a uniformly distributed phase-type winding, considered as a uniform current sheet of spread σ , has the trapezoidal shape shown in Fig. 3.29(a). With N turns in the phase winding and a current i , the m.m.f. per pole is $\frac{1}{2}Ni$. A Fourier analysis of the trapezoidal waveform yields a series of harmonic sines of order $n = 1, 3, 5 \dots$ in terms of the electrical angle x , each term having the form

$$F_n = \frac{1}{2}Ni \frac{4}{n\pi} \frac{\sin \frac{1}{2}(n\sigma)}{\frac{1}{2}(n\sigma)} = i \frac{2}{n\pi} (k_{dn}N) \sin nx$$



3.29 Phase m.m.f.

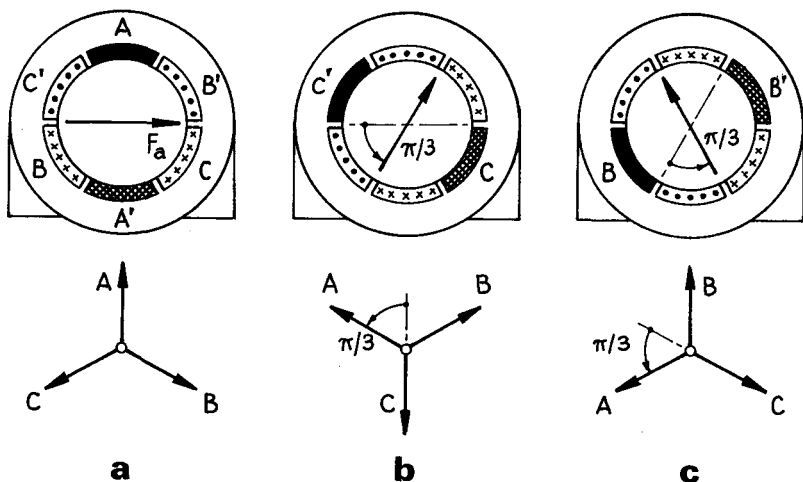
where k_{dn} is the distribution factor, eq.(3.15). If the winding is chorded the coil-span factor is introduced to give $k_{dn}k_{en}N = K_{wn}N$ for the effective turns of the phase winding.

With an alternating phase current $i = i_m \cos \omega t$, then each term of the Fourier series becomes

$$F_n = i_m \cos \omega t \frac{2}{n\pi} K_{wn} N \sin nx \quad (3.22)$$

The m.m.f. wave rises, falls and reverses with the change of the current, but its axis remains fixed with respect to the winding.

The subdivision of practical windings into slots results in the 'stepped' wave of Fig. 3.29(b). The fundamental and low-order harmonics are scarcely affected, but slot harmonics become prominent in integral-slot windings. With g' slots/pole per phase each containing z_s conductors carrying a current i_c , the instantaneous amplitude of the m.m.f. wave is $\frac{1}{2}Ni = \frac{1}{2}g'z_s i_c$.



3.30 Travelling-wave field.

Travelling-wave M.M.F.

A symmetrical m -phase winding, with $m = 2, 3, 4 \dots$ has its phase axes displaced successively around the airgap by an electrical angle $2\pi/m$, and is fed with a balanced set of m -phase currents. The resultant m.m.f. is nearly constant, and its axis moves over one double pole-pitch in the time of one period at almost constant speed. The very common 3-ph case ($m = 3$) is an example.

The stator in Fig. 3.30 carries a 3-ph narrow-spread 2-pole winding AA', BB', CC'. The current phasors for three successive instants are shown in (a), (b) and (c). Let the positive direction of current in ABC be outward (and in A'B'C' therefore inward). In (a) the current of phase A is a positive maximum, while in both B and C it is one-half negative maximum. The current pattern magnetizes along the axis of F_a . One-sixth of a period later, (b), phase C now has a maximum negative current, the whole current pattern shifting counter-

clockwise by 60° ; a similar shift occurs after a further one-sixth of a period, as in (c). For all three instants the current pattern is identical in magnitude, and changes only in direction. The result is a travelling wave of m.m.f.

It would be incorrect to assume that the m.m.f. is constant between the 60° intervals, nor even that its angular rotation is at a constant speed; but it can be shown that the *fundamental* component is constant and moves at uniform speed.

Three-phase M.M.F.

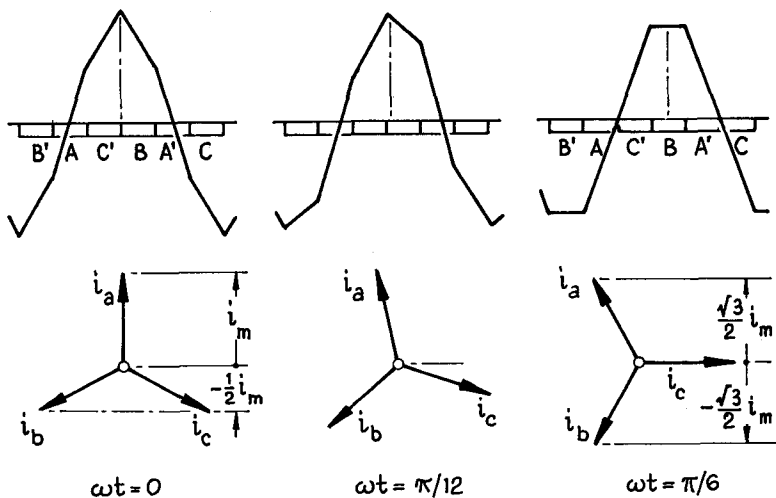
Using eq.(3.22), we write the m.m.f.s of phases A, B and C with due regard (i) to the time-phase displacement of the currents, and (ii) to the space displacement of the phase-m.m.f. axes. The currents and angles are

Phase A:	$i_m \cos \omega t$	nx
Phase B:	$i_m \cos(\omega t - \frac{2}{3}\pi)$	$n(x - \frac{2}{3}\pi)$
Phase C:	$i_m \cos(\omega t - \frac{4}{3}\pi)$	$n(x - \frac{4}{3}\pi)$

The resultant m.m.f. is the summation $F_a + F_b + F_c$ giving

$$F = \frac{3}{\pi} Ni_m \left[K_{w1} \sin(x - \omega t) + \frac{1}{5} K_{w5} \sin(5x - \omega t) - \frac{1}{7} K_{w7} \sin(7x - \omega t) + \frac{1}{11} K_{w11} \sin(11x - \omega t) \dots \right] \quad (3.23)$$

There is a constant fundamental, and harmonics of orders $5, 7, 11, 13$, or $6a \pm 1$, where a is any positive integer. All multiples of 3 are absent. The fundamental travelling wave moves at a speed corresponding to ω , while the harmonics move with speeds proportional to the reciprocal of their order, in the same direction as the fundamental for $n = 6a + 1$, and opposite for $n = 6a - 1$.



3.31 M.M.F. waveforms of travelling-wave field.

The m.m.f. wave naturally changes its shape from instant to instant, as explained by the three cases in Fig. 3.31 for a uniformly distributed 3-ph narrow-spread winding. For the extreme shapes we have

$$\omega t = 0: F = 2 \times \frac{1}{2} Ni_m \quad \text{and} \quad \omega t = \pi/6: F = \sqrt{3} \times \frac{1}{2} Ni_m$$

where $\frac{1}{2} Ni_m$ is the maximum m.m.f. of one phase.

Although the m.m.f. space harmonics are of practical importance it is sometimes permissible to consider the fundamental term alone. Its amplitude is $(6/\pi)^{\frac{1}{2}} Ni_m$. With an r.m.s. current I_c in each phase conductor, and with a total of N_{ph} turns/ph all in series in a machine with p pole-pairs, the fundamental armature m.m.f. per pole is

$$\begin{aligned} F_a &= (6/\pi) K_{w1} \frac{1}{2} g' z_s (\sqrt{2} I_c) \\ &= 1.35 K_{w1} N_{ph} I_c / p \end{aligned} \quad (3.24)$$

Equivalent M.M.F.

The behaviour of a machine depends on the interaction of the m.m.f.s of its stator and rotor members. It is therefore necessary to be able to equate m.m.f.s that are contributed by different kinds of winding.

Two Three-phase Windings. For two 3-ph windings 1 and 2 it is only necessary to write eq.(3.24) for each, giving

$$K_{w1} N_{ph1} I_{c1} = K_{w2} N_{ph2} I_{c2}$$

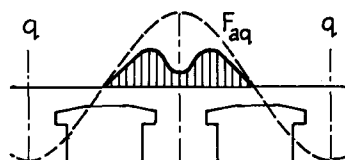
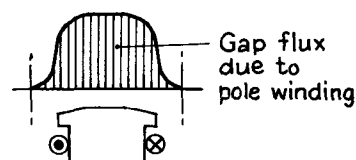
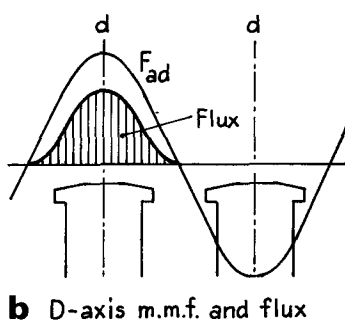
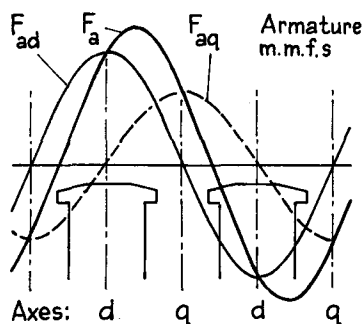
where K_{w1} and K_{w2} are the *fundamental* winding factors for the respective windings.

Three-phase Winding and Distributed D.C. Winding. In a non-salient-pole synchronous machine (such as a turbo-generator) the field winding is distributed in slots, Fig. 3.11, giving the m.m.f. distribution (b) in Fig. 3.29. Apart from m.m.f. harmonics arising from the trapezoidal waveform, it is sufficient to equate the fundamentals of the field and armature m.m.f.s.

Three-phase Winding and Salient Pole System For salient-pole machines, Fig. 3.32, the method of equating m.m.f.s is not valid because the gap is not uniform over the pole-pitch. Equivalence has to be based on the fundamental fluxes that the 3-ph and salient-pole windings would separately produce in the airgap. Analysis of a salient-pole flux distribution is given in Sect. 2.5. The results so obtained, with the ratio b/Y of pole-arc to pole pitch between 0.65 and 0.75, give approximately

$$F_{ad} = 1.07 K_{w1} N_{ph} I_c / p \quad (3.25)$$

for the pole excitation corresponding to the 3-ph winding current I_c which produces m.m.f. in the *direct axis*, i.e. in the direction of the pole-centre. The component of 3-ph winding m.m.f. wave that is directed along the interpolar axis has no equivalent in the field winding. The m.m.f. component in this, the *quadrature axis*, is applied to the interpolar gap which presents a widely varying permeance, so that the quadrature-axis flux distribution has a large spatial third-harmonic. The usual star connection of the 3-ph winding prevents any



3.32 M.M.F. reactions in salient-pole machine.

corresponding 3rd-harmonic e.m.f.s from appearing at the line terminals. The fundamental of the q-axis flux does however generate an e.m.f. of fundamental frequency in the 3-ph winding, giving the q-axis reaction the nature of a leakage reactance. Evaluation of the q-axis flux fundamental from the profile of the airgap is based on flux-plotting or the application of gap-permeance coefficients.

Phase-sequence Components

The foregoing discussion relates to m.m.f. distributions produced by normal, positive-sequence currents. Under fault conditions, or unbalanced loading, or the deliberate application of asymmetrical voltages, currents of negative and (in certain cases) zero sequence will flow. Each will produce a corresponding m.m.f.

When the negative-sequence currents are present in the windings of a 3-ph machine, an m.m.f. is developed which rotates at synchronous speed in a direction opposite to that of the pole-system. Double-frequency e.m.f.s are consequently induced in the field winding by the negative-sequence flux. In a salient-pole machine the flux will not be constant, because of the strongly variable gap permeance presented to it as it passes in succession the direct and quadrature axes. This is especially true if the machine has laminated pole-faces and no damper windings, for the latter have a compensating effect tending to make the machine equivalent to a cylindrical-rotor construction. The effect is naturally accompanied by considerable pole-face and damper losses.

Zero-sequence currents, being co-phased in time in all three phases, produce normally no flux in the air-gap.

Two-phase Winding

A symmetrical 2-ph winding, AA' and BB', with N_{ph} turns in series per phase and with the winding axes separated by $\frac{1}{2}\pi$ elec. rad will develop a travelling wave of gap m.m.f. in a way closely similar to that in the 3-ph case, if its phase currents have also a 90° phase displacement, e.g.,

$$i_a = i_m \cos \omega t \quad \text{and} \quad i_b = i_m \sin \omega t$$

The resultant m.m.f. has a fundamental of magnitude

$$F_a = 0.92 K_{w1} N_{ph} I_c / p \quad (3.26)$$

The harmonics are naturally rather more prominent than for a 3-ph winding.

Cage Winding

If the number of phases m in an m -phase winding is large, and if the phase currents are all of equal r.m.s. magnitude I_c with time-phase displacement $2\pi/m$ successively, the winding approximates to a sinusoidally distributed current sheet with the resultant m.m.f.

$$F_a = \sqrt{2}(m/\pi) K_{w1} N_{ph} I_c / p$$

A uniform cage winding with an adequate number of bars approximates closely to this condition. A cage in S slots (and therefore S/p slots per pole-pair) can be considered as a multi-phase winding with $m = S/2p$ phase windings each of a single turn. The winding factor $K_{w1} = k_{d1} k_{e1}$ is clearly unity. The fundamental of the m.m.f. becomes

$$F_a = \frac{1}{\sqrt{2\pi p}} S I_c \quad (3.27)$$

Harmonics of orders other than those close to $n = S/2p$ will normally be very small.

4

Loss Dissipation

4.1 Dissipation of heat

The processes of transfer in a transformer and interaction in a machine involve currents (and therefore I^2R loss) in conductors and fluxes (producing core loss) in ferromagnetic circuits. Stray leakage fluxes develop comparable losses in associated parts such as tanks, end-plates and covers, accounting for further 'load' loss. As all losses appear as heat, the temperature of each part so affected is raised above that of the ambient medium (normally the surrounding air). The temperature-rise above ambient is related (i) to the rate of heat production, (ii) the rate of cooling and (iii) the thermal capacity. The temperature-rise is paramount in determining the plant rating particularly through its effect on the life of the winding insulation, and is the subject of specified operating limits.

Heat is removed by a combination of *conduction* and *convection*, assisted by *radiation* from outer surfaces. Briefly, a cool fluid is passed through passages in a machine to conduct away the heat but a 'scrubbing' action of the coolant is relied upon for effective heat exchange. This is a kind of *forced convection*, whereas air-current dissipation from the outside surfaces is *natural convection* unless it is augmented by fans.

Convection by random air currents is mainly relied upon for a small machine, usually aided by rotor fans. For bigger machines the core volume is too large for purely surface cooling: radial and axial ducts must be provided to increase it, with means for directing the coolant through the labyrinth. The cooling system for the largest machines is complicated, but it is vital to the operating safety in view of the very large loss-rates.

4.2 Ideal temperature-rise/time relation

An ideal homogeneous body, internally heated and surface cooled, has a maximum surface temperature-rise proportional directly to the loss rate and inversely to that of the cooling. With simplifying assumptions and a constant rate of internal loss, the temperature-rise and the time are exponentially related. The symbols used are

p	energy-loss rate, or loss power
G	mass of body

c_p	specific heat
S	area of surface
λ	emissivity
c	cooling coefficient = $1/\lambda$
θ	temperature-rise
τ	time-constant

If the surface of the body is at some temperature θ above ambient, then in an element of time dt the heat energy $p \cdot dt$ is produced, a heat storage $Gc_p \cdot d\theta$ occurs due to an elemental rise $d\theta$ in temperature, and the heat $S\theta\lambda \cdot dt$ is dissipated. The energy balance is

$$Gc_p \cdot d\theta + S\theta\lambda \cdot dt = p \cdot dt$$

whence

$$\theta = \theta_m [1 - \exp(-t/\tau)] \quad (4.1)$$

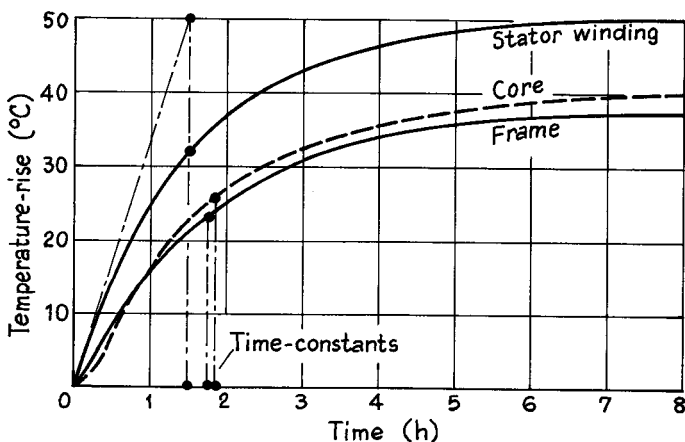
where $\theta_m = p/S\lambda$ is the maximum temperature-rise, and $\tau = Gc_p/S\lambda$ is the *heating time-constant*. The basic assumption is that the dissipation is proportional to the temperature-rise above ambient.

EXAMPLE 4.1: A small totally-enclosed induction motor, of mass $G = 41$ kg, has on continuous full load a total loss $p = 110$ W. Its frame has a cooling surface $S = 0.10$ m² with an emissivity $\lambda = 29$ W/(m²K). The specific heat of the machine averages $c_p = 420$ J/(kg K). Obtain the heating/time curve.

The final steady temperature-rise is $\theta_m = p/S\lambda = 110/0.10 \times 29 = 38^\circ\text{C}$ and the time-constant is $\tau = Gc_p/S\lambda = 41 \times 420/0.10 \times 29 = 6\,000$ s (about 1.7 h). The temperature-rise/time relation is

$$\theta = 38 [1 - \exp(-t/6\,000)]$$

The initial rate of rise is $\theta/\tau = 38/6\,000 = 0.0063^\circ\text{C/s}$, or about 23°C/h .



4.1 Temperature-rise/time relation.

The temperature-rise of the frame of the actual machine, taken by thermometer, is shown in Fig. 4.1, together with the core and winding temperatures. The frame temperature is seen to follow the expression above fairly closely, and comparable exponential relations apply to the core and windings, the temperature-rises of which are naturally greater.

It is an over-simplification to regard a machine or a transformer as a homogeneous body. There are many identifiable parts, each with a characteristic surface area, mass, heat capacity, thermal conductivity and loss. The temperature-rise of different parts (and of various points in the same part) may be very uneven, especially on intermittent load or with forced cooling, as discussed by Bates and Tustin [22]. Hot-spot temperatures are particularly important.

Rate of Temperature-rise

With the surface at θ above ambient, the rate of rise is

$$\frac{d\theta}{dt} = \frac{1}{Gc_p} (p - S\lambda\theta) = \frac{1}{\tau} (\theta_m - \theta) \quad (4.2)$$

At the start of a heating cycle ($\theta = 0$) the rate of temperature-rise depends on the heat capacity and the loss only, and $d\theta/dt = \theta_m/\tau = p/Gc_p$. The same condition holds approximately when there is a sudden rise in the loss rate. A typical case occurs when a transformer is subjected to considerable overload or to a short-circuit lasting a time Δt . If the conductor material has a density δ , and a resistivity ρ_1 at the initial temperature θ_1 , the specific loss (power per unit mass) for a current density J is $J^2\rho_1/\delta$: this may be augmented by the eddy-loss factor K_d as described in Sect. 3.2. Assuming that the heat is stored in the conductor material without dissipation, the temperature-rise is

$$\Delta\theta = \frac{K_d J^2 \rho_1}{\delta c_p} \Delta t \quad (4.3)$$

If $\Delta\theta$ is large, then ρ_1 should be replaced by the resistivity at the mean temperature, $\theta_1 + \frac{1}{2}\Delta\theta$.

EXAMPLE 4.2: A 1 000 kVA transformer with a rated I^2R loss of 4 times the core loss operates in an ambient temperature of 25 °C. From cold it has a winding temperature-rise of 24 °C after 1 h and 38.5 °C after 2 h on rated load. Estimate (a) the final temperature-rise on rated load and the heating time-constant of the windings; (b) for a maximum winding temperature of 97 °C (i) the continuous overload that can be sustained, and (ii) the time for which 2 p.u. rated load can be taken after continuous operation on no load.

(a) From eq. (4.1), with time for convenience in hours:

$$24 = \theta_m [1 - \exp(-1/\tau)] = \theta_m [1 - k] \quad \text{and} \quad 38.5 = \theta_m [1 - k^2]$$

whence $38.5/24 = 1 + k = 1.6$; thus $k = 0.6$, $\exp(1/\tau) = 1.67$ and $\tau = 1.96 \approx 2$ h. The final steady temperature-rise becomes $\theta_m = 60$ °C, corresponding to a scale temperature of 85 °C.

(b) The allowable temperature-rise is now $97 - 25 = 72^\circ\text{C}$, which is 1.2 times that for normal continuous rating.

(i) The rated losses are proportional to $(1 + 4) = 5$; they can now be $5 \times 1.2 = 6 = (1 + 5)$. The load is therefore $\sqrt{(5/4)} = 1.12 \text{ p.u.} = 1\,120 \text{ kVA}$.

(ii) On no load with only core loss, i.e. $1/5$ of rated loss, the steady temperature-rise is $60(1/5) = 12^\circ\text{C}$: this is the starting value. For 2 p.u. load the losses are proportional to $(1 + 4^2) = 17$, and a final steady temperature-rise of $60(17/5) = 204^\circ\text{C}$. But the rise is limited to a further $72 - 12 = 60^\circ\text{C}$: hence

$$60 = 204 [1 - \exp(-t/2)]$$

giving $t = 0.7 \text{ h} = 42 \text{ min}$.

These estimates are valid only insofar as the transformer can be regarded as a homogeneous body. The greater overload I^2R loss causes changes in the circulation of the coolant and the conditions of heat-transfer, so that the transformer does not operate thermally in the same way as on rated load. The calculations do, however, show practical trends.

4.3 Cooling of small units

Small transformers and machines use the ambient air as a coolant, the units being small enough to incur little danger of the build-up of excessive hot-spot temperatures.

Transformers

Most small transformers are oil-immersed, and the surface of the tank provides heat-transfer to the ambient air. Plain sheet-steel tanks can transfer about 13 W/m^2 per $^\circ\text{C}$ temperature-rise. Above about 25 kVA 3-ph rating the emitting surface must be increased, most commonly by setting vertical tubes of 40–50 mm diameter welded steel and 1.5 mm wall thicknesses into the sides of the tank. For ratings between 2 and 5 MVA, elliptical-section tubes are preferred as more can be accommodated.

Experiment shows that radiation accounts for about 6 W and convection for about 6.5 W per m^2 of plain tank wall, per $^\circ\text{C}$ difference between tank and ambient air or surroundings. For a rise of 35°C a plain tank will consequently dissipate about 440 W/m^2 . If this is insufficient, the total plain surface is increased x times by tubing. Then from 1 m^2 of plain tank there is a dissipation of $(6 + 6.5) \text{ W/}^\circ\text{C}$, and from the $(x - 1)$ corresponding area of tube wall a convection of $(x - 1) 6.5 \times 1.35 \text{ W/}^\circ\text{C}$, the factor of 1.35 accounting for the improved convection conditions obtained from tubes. Thus per unit of total area there is a dissipation

$$p = (8.8 + 3.7/x) \text{ W/(m}^2 \text{ }^\circ\text{C)} \quad (4.4)$$

The tank top and bottom are usually ignored.

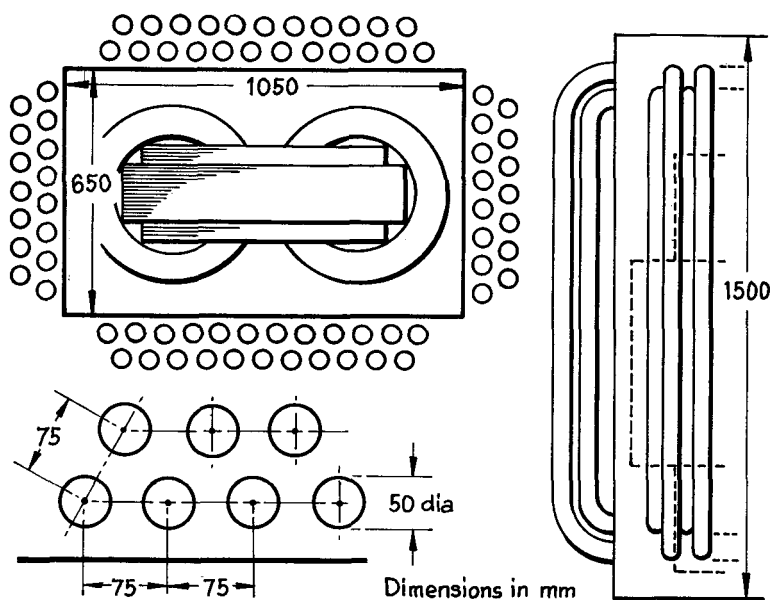
EXAMPLE 4.3: The tank of a 575 kVA 1-ph oil-immersed self-cooled transformer is 1.50 m in height and $0.65 \times 1.05 \text{ m}$ in plan. The rated loss is 6.5 kW. It is required to arrange 50 mm diameter tubes, spaced 75 mm between

centres and averaging 1.275 m in length for a mean surface temperature-rise of 35°C .

The vertical surface area of the plain tank is 5.1 m^2 . With the area increased x times by the provision of tubes, the dissipation from eq. (4.4) is

$$6.5 \times 10^3 / 35 \times 5.1 x = 36.5/x = (8.8 + 3.7/x) \text{ W}/(\text{m}^2\text{ }^{\circ}\text{C})$$

from which $x = 3.75$. The total tube area is therefore $2.75 \times 5.1 = 14.0\text{ m}^2$. The wall area of each tube averages 0.2 m^2 so that $14.0/0.2 = 70$ tubes are required.



4.2 Cooling from transformer tank.

These are set out as shown in Fig. 4.2 on 75 mm centres in two rows, the arrangement being;

$$\left. \begin{array}{l} \text{Long sides: } 12 \text{ and } 11 \text{ tubes, total } 46 \\ \text{Short sides: } 8 \text{ and } 7 \text{ tubes, total } 30 \end{array} \right\} 76 \text{ tubes}$$

Rotating Machines

The scheme of air ventilation is closely related to the construction, in particular the type of enclosure:

Open-pedestal, in which the stator and rotor ends are in free contact with the outside ambient air, the rotor being carried on pedestal bearings mounted on the bedplate.

Open-end-bracket, where the bearings form part of the end-shields which are fixed to the stator housing, a common construction for small and medium-sized machines. The air is in comparatively free contact with the stator and rotor through the openings.

Protected, or end-cover types with guarded openings: the protection may be by *screen* or by *fine-mesh covers*. In the latter case, the machine is regarded as totally enclosed.

Drip-, Splash- or Hose-proof, a protected machine with the ventilation opening in the end-shields designed to exclude falling water or dirt, or jets of liquid.

Pipe- or Duct-ventilated, with end-covers closed except for flanged apertures for connection to pipes along which the cooling air is drawn.

Totally-enclosed, where the enclosed air has no connection with the ambient air: the machine is not necessarily airtight. Total enclosure may be associated with an internal rotor fan, an external fan, water cooling, or closed-air-circuit ventilation, in which the air is circulated to a refrigerator and returned to the machine.

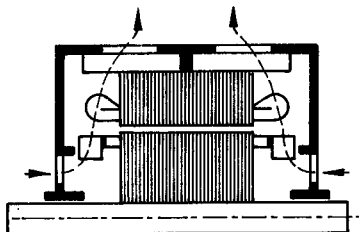
Weatherproof, or *watertight*, as specified.

Flame-proof, for use in mines.

As a general rule, the more difficult the ventilation the lower must a given machine be rated.

Cooling-air Circuit

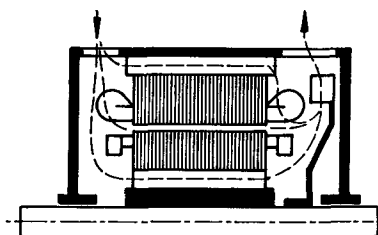
The arrangement of the path of the cooling air depends on the size of the machine (a larger core requiring more subdivision and more elaborate arrangements for gas distribution) and on its type.



4.3 Radial ventilation.

Radial ventilation is commonly employed, because the rotor induces natural centrifugal movement of the air, augmented as required by rotor fans. Fig. 4.3 shows a typical method suitable for machines below 20 kW. The end-brackets are shaped to guide air over the overhang and the back of the core, and baffles are fitted to improve the fan efficiency. For larger machines, subdivision of the core is necessary and the air paths through the radial ducts are in parallel with those across the overhang convectors. A high rate of heat dissipation is possible in the gap between smooth-bore stators and rotors — unless this gap is very short — on account of high air speeds accompanied by turbulence.

Axial ventilation is suitable for machines of moderate output and high speed. The practically solid rotor construction and restricted spider, necessary to avoid undue centrifugal forces, make it difficult to provide adequate air-paths to radial ventilating ducts. Further, the tendency in design is to increase core lengths and to restrict core diameters in order to get a cheaper machine.



4.4 Axial ventilation.

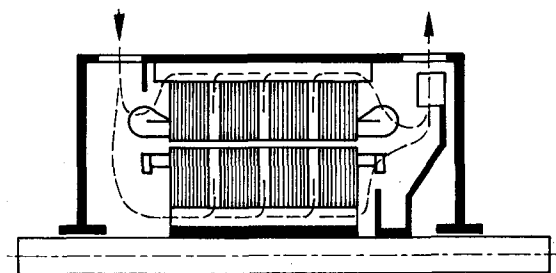
Fig. 4.4 shows a method of applying axial ventilation to a small machine with plain cores. To increase the cooling surface, holes may be punched in the core plates to form through-ducts where considerable heat-dissipation occurs. This greatly improves the cooling, but requires a larger core diameter for the increased core depth necessary.

Combined radial and axial ventilation is employed for large machines.

Fig. 4.5 shows the arrangement of an induction motor for mixed ventilation. The air is drawn in at one end, and encouraged to pass through the ducts by baffling the fan end of the rotor spider. The shaft-mounted fan ejects the air.

Multiple-inlet ventilation. With greater lengths of core, there is a tendency to starve the central radial ducts of air. With the multiple-inlet system it is possible to build machines with long cores and obtain effective centre cooling. The stator frame is divided into separate air circuits fed in parallel.

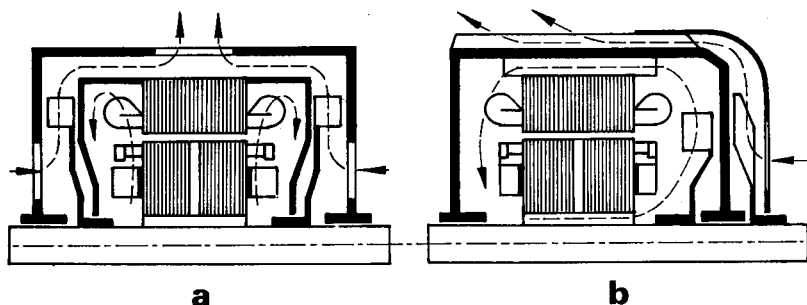
Totally-enclosed machines present a special problem in that the inside of the machine can have no air-connection with the outside. Two methods of fan cooling are shown in Fig. 4.6. At (a) a shaft-mounted fan external to the working parts of the machine blows air over the carcass through a space between the main housing and a thin cover plate. Internal air circulation is produced by an internal fan: this avoids the temperature gradient across the air gap. At (b) an internal fan circulates the heat to the carcass. Air is also blown over the outside of the carcass to improve the dissipation.



4.5 Combined axial and radial ventilation.

Cooling Coefficient

The maximum temperature-rise of a given part of a machine is given by $\theta_m = p/S\lambda = c(p/S)$, where c is the cooling coefficient. The Table gives typical



4.6 Ventilation of totally enclosed machine.

values of c in terms of a characteristic speed u (in m/s) and the dissipating surface area S (in m^2). The values can be applied only when justification is forthcoming from suitable tests.

Cooling Coefficient

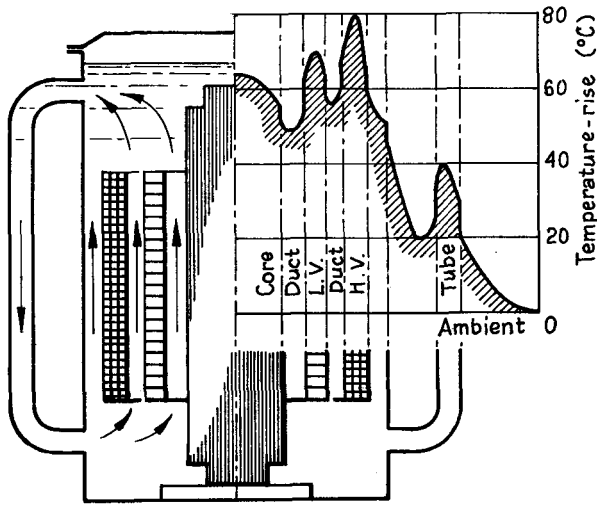
Part	c	u	Notes
Cylindrical surfaces of stator and rotor	$\frac{0.03 \text{ to } 0.05}{1 + 0.1u}$	Relative peripheral speed	Lower figures for forced cooling
Back of stator core	0.025 to 0.04	0	
Rotating field coils	$\frac{0.08 \text{ to } 0.12}{1 + 0.1u}$	Peripheral speed	Based on total coil surface
	$\frac{0.06 \text{ to } 0.08}{1 + 0.1u}$		Based on exposed coil surface only
Ventilating ducts in cores	$\frac{0.08 \text{ to } 0.2}{u}$	Air velocity in ducts	u taken as 0.1 of peripheral speed of core

4.4 Cooling of large units

Consider a transformer or a rotating machine with k times the linear dimensions of another smaller but otherwise similar unit. Its core and conductor areas are k^2 times greater and its rating (with the same flux and current densities) increase k^4 times. The losses increase by the factor k^3 but the surface area is multiplied only by the factor k^2 . Thus the loss per unit area to be dissipated is increased k times. Large units are therefore more difficult to cool than small ones, and require more elaborate methods. The cores are massive, the conductors are long, of large section and rated with high current densities. The coolant must therefore penetrate into the cores, and even into the conductors; for if not, interior heat must travel undue distance to the coolant, with the result that excessive temperature gradients occur and hot-spots develop. Forced-cooling systems are essential. Each part of the unit must be considered, both by itself and in relation to neighbouring parts, in respect of the conditions of thermal flow in the whole machine.

Transformers

The cooling of transformers differs from that of rotating machines in that no moving parts are available to assist the circulation of the coolant. If the natural thermal 'head' (resulting from the expansion and consequential upward flow of heated coolant) is not sufficient, forced circulation is necessary.



4.7 Oil-circulation and temperature distribution.

Natural Circulation. Fig. 4.7 is drawn to indicate on the left the thermal flow of oil in a transformer tank. The oil in the ducts, and at the surfaces of the coils and cores, takes up heat by conduction, and rises, cool oil from the bottom of the tank rising to take its place. A continuous circulation of oil is completed by the heated oil flowing to the tank sides (where cooling to the ambient air occurs) and falling again to the bottom of the tank. Oil has a large coefficient of volume expansion with increase of temperature, and a substantial circulation is readily obtained so long as the cooling ducts in the cores and coils are not unduly restricted. A typical temperature distribution is shown. On rated load the greatest temperature-rise will be in the coils. The maximum oil temperature may be about 10 °C less than the coil figure, and the mean oil temperature another 15 °C less.

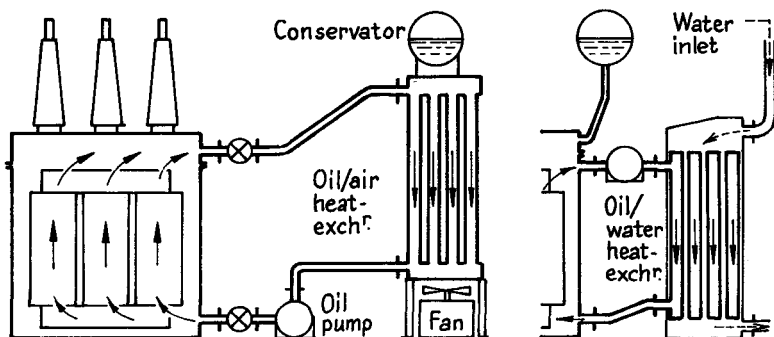
The tubed tank (Sect. 4.3) provides considerable additional cooling surface, but for transformer ratings exceeding about 5 MVA, or losses exceeding about 50 kW, cooling by natural oil circulation becomes inadequate.

Forced Circulation. The choice of the method of cooling will depend largely upon the conditions obtaining at the site. Air-blast cooling can be used, a hollow-walled tank being provided for the transformer and oil, the cooling air being blown through the hollow space. The heat removed from the inner walls of the tank can be raised to five or six times that dissipated naturally, so

that very large transformers can be cooled in this way, but it is more usual to employ radiator banks, of corrugated or elliptical tubes, separate from the transformer tank and cooled by fanned air circulation.

A cheap method of forced cooling where a natural head of water is obtainable is the use of a cooling coil, consisting of tubes through which cold water is circulated, inserted in the top of the tank. This method has, however, the disadvantage that it introduces into the tank a system containing water under a head greater than that of the oil. Any leakage will, therefore, be from the water to the oil, so that there is a risk of contaminating the oil and reducing its insulating value. Fins are placed on the copper cooling tubes to assist in the conduction of heat from the oil, since heat passes three times as rapidly from the copper to the cooling water as from the oil to the copper tubing. The inlet and outlet pipes are lagged to avoid water from the ambient air condensing on them and getting into the oil.

For large installations the best cooling system appears to be that in which the oil is circulated by pump from the top of the transformer tank to a cooling plant, returning when cold to the bottom of the tank. When the cooling medium is water, this has the advantage that the oil can be arranged to work at a higher static head than the water, so that any leakage will be in the direction of oil to water. The system is suitable for application to banks of transformers, but for reliability not more than, say, three tanks should be connected in one cooling pump circuit.

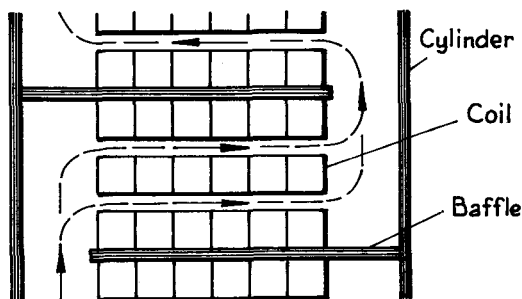


4.8 Transformer cooling methods.

Fig. 4.8 shows two methods of cooling with separate heat-exchanger banks. It is common to bring in the fan and oil-circulating pump at predetermined load levels or when the oil temperature in the transformer tank has reached a set limit; this improves the overall efficiency at small loads.

Cores and Coils. In small transformers the dissipation of core loss is simple, as the core surface/volume ratio is high. Very large cores, on the other hand have a relatively small surface/volume ratio, so that the cooling surface must in some way be augmented by additional ducts. There are two ways of arrang-

ing ducts — either parallel or perpendicular to the direction of the laminations. The first is easy, the second requires special punching. Unfortunately the first method does not present to the oil any additional plate *edges*. Heat flows twenty times more readily along the laminations to the edges than from plate to plate across the intervening insulation, which has a low thermal conductivity.



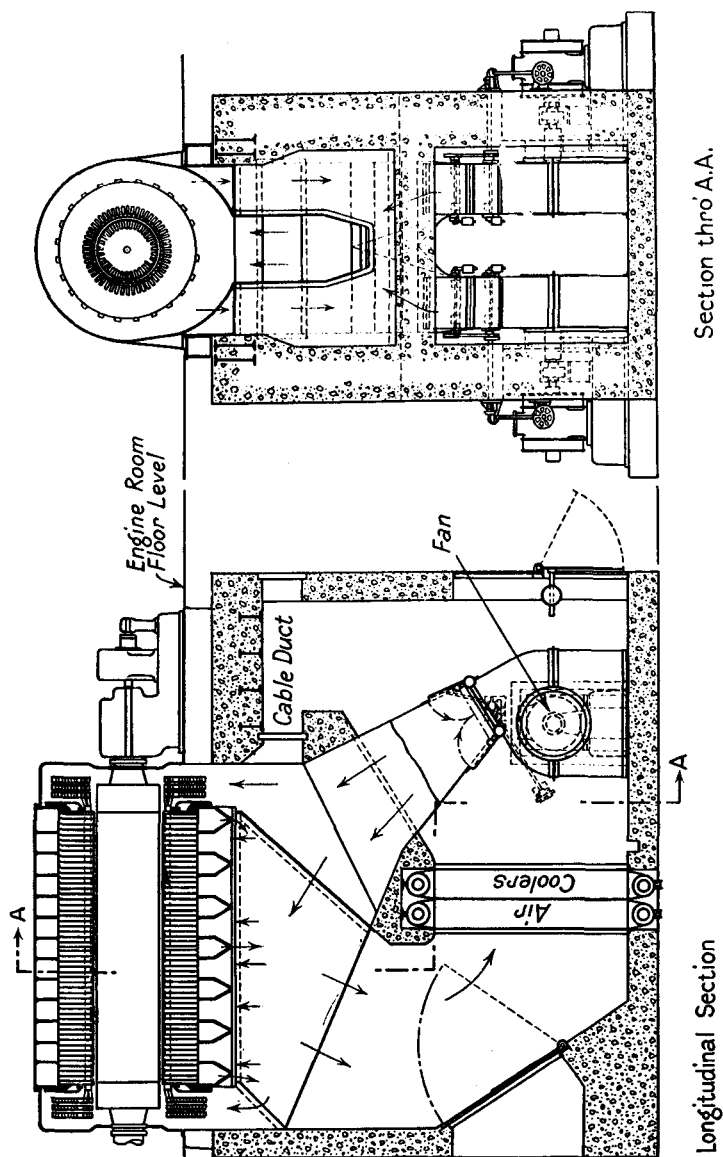
4.9 Directed oil-flow in transformer winding.

It is important that oil circulation is such that there is a flow over the main coil cooling surfaces. This is relatively simple when these surfaces are vertical, as with helix coils. But with disc coils in large transformers the major surfaces are horizontal. Oil-flow deflectors, Fig. 4.9, are necessary to reduce the conductor/oil temperature gradient.

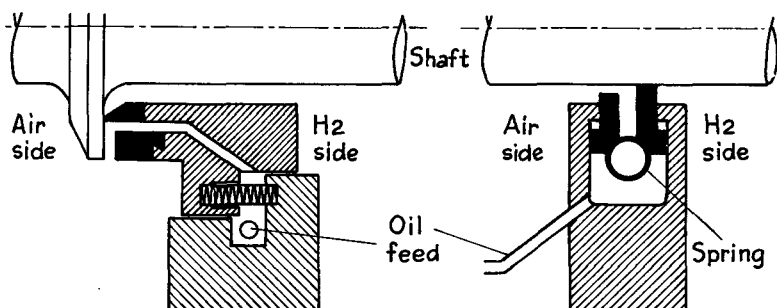
Rotating Machines

Forced Air Cooling. For large machines, which may require many tonnes of cooling air per hour, forced ventilation permits the cleaning of the air by suitable filters, avoiding the clogging of the ducts. In an arrangement for a large induction motor, the end covers are airtight, and air from an external fan is fed in from a duct. On high-speed machines rotor-mounted fans may be sufficient. The air is filtered or washed with a water spray, then baffled against flooded scrubbing surfaces to precipitate the dirt. It is then dried by passing over a series of dry scrubbing plates. A more complete means of securing clean cooling air is the *closed-circuit* system, as employed for the smaller ratings of turbo-generators. The hot air from the machine is passed through a water-cooled heat-exchanger and then returned to the machine by a centrifugal fan. The arrangement is shown in Fig. 4.10.

Hydrogen Cooling. Turbo-generators have forced ventilation and total enclosure to deal with the large losses and the high rating per unit volume. Certain advantages are gained by the use of hydrogen in place of air as a coolant: hydrogen has $\frac{1}{14}$ of the density, reducing windage loss and noise; 14 times the specific heat; $1\frac{1}{2}$ times the heat-transfer, so more readily taking up and giving up heat; 7 times the thermal conductivity, reducing temperature gradients; reduces insulation corona; and will not support combustion so long as the hydrogen/air mixture exceeds 3/1. As a result, hydrogen cooling at 1, 2 and 3 atm can raise the rating of a machine by 15, 30 and 40% respectively.



4.10 Forced air ventilation.



4.11 Hydrogen shaft seals.

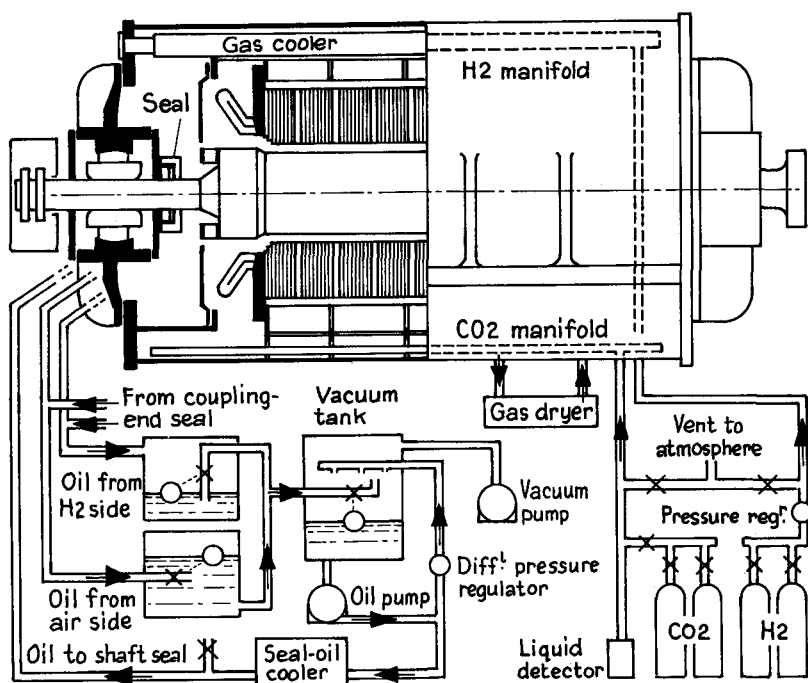
The stator frame must be gas-tight and explosion-proof. Oil-film gas-seals at the rotor shaft ends are necessary. Two forms are shown in Fig. 4.11: each must accommodate axial expansion of the rotor shaft and stator frame. Oil is fed to the shaft and the flow is split, part towards the interior (gas) side and part to the air side. The latter mingles with the bearing oil, while the former is collected and degassed.

Fans mounted on the rotor circulate hydrogen through the ventilating ducts and internally-mounted gas-coolers. The gas pressure is maintained above atmospheric by an automatic regulating and reducing valve controlling the supply from normal gas cylinders. When filling or emptying the casing of the machine, an explosive hydrogen-air mixture must be avoided, so air is first displaced by carbon dioxide gas before hydrogen is admitted: the process is reversed for emptying. It is usual to provide a drier to take up water vapour entering through seals. The hydrogen purity is monitored by measurement of its thermal conductivity.

Turbo-generators operating at hydrogen pressures just above atmospheric (so that leaks will be outwards) require about 0.03 m^3 per MW of rating per day. This rises to about 0.1 m^3 for hydrogen pressure of 2 atm. The gas consumption of synchronous capacitors, which do not need shaft seals, is very much less.

Hydrogen cooling results in substantial increase in rating for a given temperature-rise, and the reduction in windage may add 0.5–1.0% to the efficiency of a 100 MW machine. Fig. 4.12 gives a diagram of the auxiliary equipment required for a hydrogen-cooled machine.

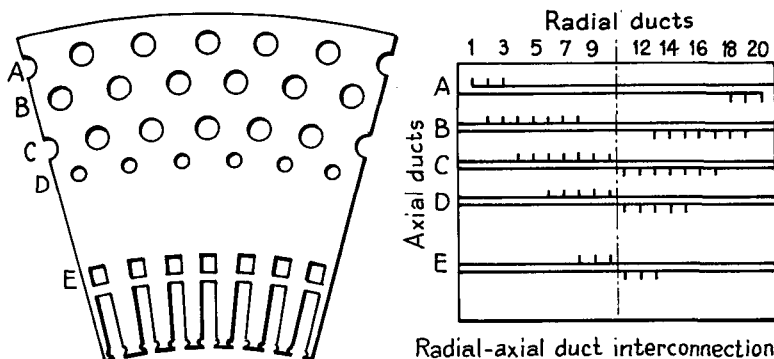
Direct Gas Cooling. Large stator cores for turbo-generators are provided with both axial and radial ducts, Fig. 4.13. Large rotors may have comparable ducting as indicated in Fig. 4.14(a), but for machines of 100 MW or more the temperature gradient over the conductor insulation is high enough to call for direct contact between the coolant and the material of the conductors themselves. In Fig. 4.14(b) the rotor conductors comprise rectangular tubes, ventilated by a cooling circuit separate from that of the stator, the hydrogen gas being admitted to the tubes through insulating flexible connections at the ends from a centrifugal impeller mounted on the outboard end of the rotor shaft. The slot tubes are electrically connected at the overhang by suitably



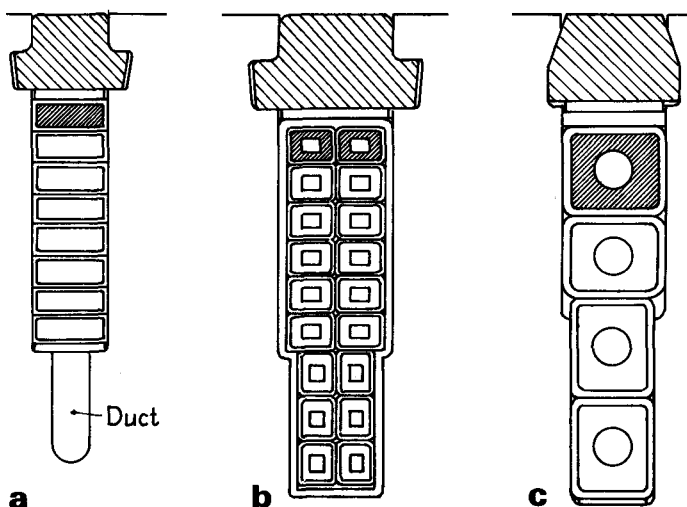
4.12 Hydrogen-cooling system.

shaped copper bars forming inlet or outlet ports. The hollow conductors are of hard-drawn silver-bearing copper, with synthetic-resin-bonded glass-cloth laminate insulation.

The direct gas cooling of stator and particularly of rotor windings permits of much higher electric loadings. With ratings of 1 000 MW even this is not enough to cope with the very large gas flow-rate and pressure, which might be $15 \text{ m}^3/\text{s}$ of hydrogen at 1.2 atm to absorb an excitation loss of 5 MW.



4.13 Cooling ducts in large core.



4.14 Turbo-rotor conductor cooling.

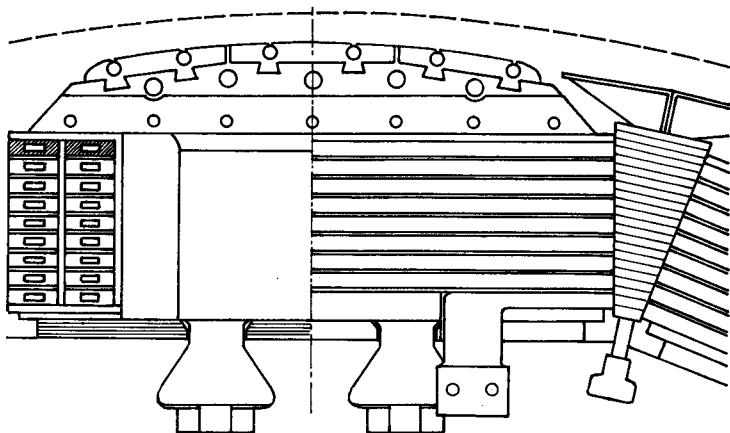
(a) Conventional; (b) Direct gas; (c) Direct water.

Direct Water Cooling. Turbo-generators of the highest ratings so far contemplated are likely to have hydrogen-cooled stator cores, and direct water-cooled stator and rotor windings. The thermal capacity of water is 750 times that of hydrogen at 500 kN/m^2 , but there are limits on the permissible speed of water flow in channels (e.g. 2.5 m/s) to avoid erosion and cavitation. Nevertheless, the conductor water-duct area may be small, allowing more space for copper.

In the direct water cooling of *stator* windings, one problem is to devise flexible water-tube connections with insulation against the high winding voltages and to preserve a low water conductivity (less than 0.5 mS/m). Water cooling of the *rotor* winding, even more desirable because of its high electric loading, offers more mechanical difficulty. Fig. 4.14(c) shows a rotor slot-conductor arrangement, the slot-width being reduced near the root to allow for greater tooth width to sustain the centrifugal force. To enable water to enter the bore of the shaft, an inlet seal is attached to the outboard (non-coupling) end. The coolant travels axially to a point in the plane of the end-winding, where its flow becomes radial into manifolds extending axially under the end-windings (which are reasonably accessible before the end-bells are fitted). The shaft passages are lined with stainless steel to prevent corrosion and water contamination. Flexible insulating connectors transfer the water from the stainless steel manifolds to the winding, entering at the coil ends and leaving at the mid point of each coil-side. On emerging from the conductors, the water flows radially inward to the rotor bore channel, then axially towards the coupling end. The water outlet is a simple catcher box surrounding the shaft. The external water system comprises an air-release tank, heat-exchangers, filters and circulating pumps. Reservoir tanks maintain the necessary water

levels. The purity is monitored, and can be adjusted by a demineralization unit connected across a bypass in the main circuit.

Water-cooled field windings have been used for the salient poles of large hydro-generators, a typical arrangement being shown in Fig. 4.15. The inter-polar space, normally open to the cooling air in an orthodox machine, can be closed to give more copper space and a lower excitation loss. The mass of cooling water required is only one-quarter of that of air at atmospheric pressure for the same cooling effect.



4.15 Direct water-cooling of salient-pole winding.

Superconducting Rotor Windings. Feasibility studies have been made on machines based on a substantially conventional water-cooled stator winding and a superconducting rotor winding. In one proposal by Lorch [106] for a 660 MW turbo-generator, the rotor has no magnetic steel but is a hollow cylinder of glass reinforced epoxy resin with steel ends. The superconducting windings are in slots and wrapped with glass epoxy resin. The windings are cooled to a temperature of about 5 K by a flow of helium entering and leaving the rotor on the axis at the outboard end. A copper cylinder shell rotates with the rotor and screens it from harmful a.c. magnetic fields; this shell is thermally insulated from the rotor by a sealed stainless steel vacuum enclosure and is maintained at a temperature between 80 and 135 K. Liquid nitrogen is passed through a stationary system of cylinders within the stator bore to form a cold screen at about 77 K. The gap between the fixed and rotating members is filled with hydrogen at 0.1 mmHg to reduce 'windage' but not unduly lower the thermal conductivity. In this way the complexity of a liquid nitrogen supply to the rotor is avoided. The exciter is conventional, but has to supply 2.5 MW only for a few seconds on initial excitation. The field is lowered by reversing the exciter or by means of a discharge resistor.

4.5 Heat transfer

Heat transfer is a complex phenomenon presenting formidable analytical dif-

ficulties. The loss heat is transferred by a combination of conduction, radiation and convection processes to the ambient air and surroundings which are considered in the ultimate to constitute a heat sink having the character of an 'infinite thermal busbar'. The symbols used in this section, and typical values of relevant physical quantities, are set out below:

<i>a</i>	area	δ	density
<i>b</i>	gas pressure ratio	ϵ	radiation factor
c_p	specific heat at constant pressure	θ	temperature-rise
<i>d</i>	tube diameter	θ	Celsius (scale) temperature
<i>k</i>	thermal conductivity	<i>Subscripts:</i>	
<i>k</i>	coefficient	<i>a</i>	absorption
<i>p</i>	power	<i>c</i>	conduction
<i>S</i>	surface	<i>i</i>	iron
<i>T</i>	Kelvin (abs.) temperature	<i>r</i>	radiation
<i>w</i>	width	<i>s</i>	space
<i>x</i>	dimension	<i>t</i>	tank
		<i>u</i>	tube
		<i>v</i>	convection

Material	Specific heat c_p J/(kg K)	Density δ kg/m ³	Thermal conductivity k W/(m K)	Thermal resistivity $1/k$ (m K)/W	Expansion coeff. $\times 10^{-6}/K$	
Aluminium	920	2 700	200	0.0050	25.5	†
Copper	390	8 900	380	0.0026	16.7	†
Steel	500	7 800	150	0.0067	11.0	†
Air*	1 000	1.3	0.025	40	3 660	‡
Hydrogen*	3 400	0.09	0.18	5.5	3 660	‡
Oil	1 900	850	0.16	6.3	1 000	‡
Water	4 200	1 000	0.63	1.6	—	
Mica	—	—	0.33	3	—	
Micanite	—	—	0.12	8	—	
Paper	—	800	0.12	8	—	

* At s.t.p.

† Linear expansion

‡ Volume expansion

Conduction

Internal flow of heat, from the region in which it originates to the surface at which it is transferred to the coolant, is important in determining hot-spot temperatures and the thermal conditions to which associated insulants are subjected. A 50 °C hot-spot temperature might be made up of a rise of 11 in the coolant, 25 at the heat-exchange surface, and 14 °C internally from the surface to the hot-spot.

Heat conduction is a function of the temperature gradient $d\theta/dx$, the thermal capacity of the path and the thermal conductivity of the material, the latter being the power transferred across a path of unit length and unit area per kelvin of temperature difference. For one-directional flow the basic equation is

$$\frac{\partial \theta}{\partial t} = \frac{k}{c_p \delta} \cdot \frac{\partial^2 \theta}{\partial x^2}$$

with corresponding forms for 2- and 3-dimensional flow. These govern transient heating. Under steady-state conditions $\partial \theta / \partial x$ is constant and there is no further thermal storage; the rate of energy flow through a path of length x between thermal equipotential planes of area a and having a temperature-difference $\Delta \theta$ is then

$$p_c = \Delta \theta \cdot k(a/x) \quad (4.5)$$

EXAMPLE 4.4: A transformer core, of plate width $w = 0.50$ m and with a stacking (space) factor $k_s = 0.94$, has a uniformly distributed core loss of $p_i = 3.0$ W/kg. The thermal conductivity of the steel is $k = 150$ W/(m K) and the surface temperature is 40°C . Estimate the maximum core temperature if the heat flow is (i) all to one end of the core, (ii) one-half to the surface at each end.

The heat flow is assumed to be along the laminae. The specific loss p_i corresponds to $p = p_i k_s \delta = 3.0 \times 0.94 \times 7\,800 = 22\,000$ W/m³.

(i) At a point distant x from one cooling surface, the power-flow density is $p(w-x)$ from the region of the core beyond x . The temperature gradient is $(d\theta/dx) = p(w-x)/k$, whence

$$\theta = px(w - \frac{1}{2}x)/k + 40$$

For $x = w$ the core temperature is

$$\theta_m = (pw^2/2k) + 40 = 18 + 40 = 58^\circ\text{C}$$

(ii) The power density at x is now $p(\frac{1}{2}w - x)$, and for $x = \frac{1}{2}w$

$$\theta_m = (pw^2/8k) + 40 = 5 + 40 = 45^\circ\text{C}$$

Perfect contact between a hot body and the coolant at the interface is rare. The temperature drop $\Delta \theta$ is strongly dependent on the fluid-flow conditions (stream-line or turbulent) and upon the condition of the surface. There may be interfering oxide films, gas bubbles in liquids, and a stagnant surface layer of fluid. In any case the thermal conductivity of coolants is much lower than that of metals. If the surface temperature is θ_s and the bulk temperature of the coolant is θ_f , the thermal conduction across the interface can be written

$$p_c = k_c (\theta_s - \theta_f) \text{ W/m}^2 \quad (4.6)$$

where the coefficient k_c is 50–1 500 for oil, and 10–50 for air.

Quantity of Coolant. Where heat is conducted away from a surface by a cool-

ant, a flow of 1 kg/s can absorb a power θc_p for a temperature-rise of θ . For a power p the required rate of flow in mass or in volume per unit time is

$$q = \frac{p}{c_p \theta} \text{ kg/s} \quad \text{or} \quad q = \frac{p}{c_p \delta \theta} \text{ m}^3/\text{s} \quad (4.7)$$

For *liquid* coolants these evaluate to

$$\text{Water: } \frac{p}{\theta} 24 \times 10^{-3} \text{ kg/s} \quad \frac{p}{\theta} 24 \times 10^{-8} \text{ m}^3/\text{s}$$

$$\text{Oil: } \frac{p}{\theta} 53 \times 10^{-3} \text{ kg/s} \quad \frac{p}{\theta} 62 \times 10^{-8} \text{ m}^3/\text{s}$$

For *gases* the volumes depend on absolute temperature and pressure so that

$$q = \frac{p}{c_p \theta} \text{ kg/s} \quad \text{or} \quad q = \frac{p}{c_p \delta \theta} \cdot \frac{T}{273} \cdot \frac{1}{b} \text{ m}^3/\text{s}$$

For both *air* and *hydrogen* the volume of coolant becomes

$$q = \frac{p}{\theta} \cdot \frac{T}{273} \cdot \frac{1}{b} 8 \times 10^{-4} \text{ m}^3/\text{s} \quad (4.8)$$

but hydrogen has important advantages in thermal conductivity.

Radiation

The radiant energy interchange between two surfaces involves the 'view' that they have of each other, their surface emissivity factors and their respective absolute temperatures. According to the Stefan-Boltzmann law, a body with a surface emissivity factor ϵ_1 and absolute temperature T_1 within a full radiator ('black body') of temperature T_2 radiates

$$p_r = 5.7 \times 10^{-8} \epsilon_1 (T_1^4 - T_2^4) \text{ W/m}^2 \quad (4.9)$$

For metal surfaces ϵ is low and dependent on the surface roughness; for non-metals (e.g. paints) ϵ approaches that of a full radiator, and generally exceeds 0.8. The net heat exchange between two plane surfaces, of temperatures T_1 and T_2 and emissivity factors ϵ_1 and ϵ_2 respectively, is

$$p_r = 5.7 \times 10^{-8} \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 + \epsilon_1 \epsilon_2} (T_1^4 - T_2^4) \text{ W/m}^2$$

Radiation does not normally occur by itself in machines and transformers, being accompanied in almost every case by convection. It is therefore often sufficient to use the expression

$$p_r = 2.9 \epsilon \theta^{1.17} \text{ W/m}^2 \quad (4.10)$$

for a body with the temperature θ above ambient in the range 0–100 °C.

Emissivity and Absorption Factors. The emissivity factor ϵ depends on the

surface and its treatment. It is higher for lead paints than for metallic paints, and varies with the colour. The Table gives typical values for ambient conditions between 0 and 100°C.

Emissivity and absorption factors

Factor	Al	Cu	Metal		Metal paint Al	Lead paints	
			rough	Steel sheet		white	grey
Emissivity ϵ	0.10	0.15	0.24	0.55	0.55	0.90	0.95
Absorption ϵ_a	0.15	—	—	—	0.55	0.25	0.75

Some plant, particularly transformers, may have to operate in the open, and may absorb radiant heat from the sun by *insolation*. The earth's outer atmosphere receives about 1.3 kW/m², and up to about two-thirds of this may reach its surface when climatic and air conditions are favourable. The effect depends on the absorption factor ϵ_a : if it approaches unity the insolation may raise the surface temperature by several degrees, but if it is low some of the sun's radiation is re-emitted. The factors ϵ and ϵ_a will normally differ because they refer to different heat wavelengths.

Ambient Temperature. For radiation, the ambient temperature is that of the sky and the ground to which the body radiates. For other forms of heating to the atmosphere, the ambient temperature is that of the bulk of the air at a distance remote enough to be unaffected by the thermal field of the body (a few metres). Strictly, the use of the air temperature as the ambient is justified only when convection is the major cooling process.

Convection

Convection is a transfer of heat from a surface to a coolant fluid, causing the fluid to expand and to create a thermal head and a consequent circulation. It is a complicated phenomenon involving many variables: the power density; the temperature difference between the surface and the bulk of coolant; the height, orientation, configuration and condition of the emitting surface; the thermal conductivity, density, specific heat, viscosity and coefficient of volume expansion of the fluid; and the value of the gravitational constant. It is the custom to deal with convection problems in terms of dimensionless parameters (Nusselt, Grashof and Prandtl), some of which are kept constant while others are varied, and formulae drawn up on an empirical basis for calculation. Even then the formulae have to be used with great caution.

For the temperature range and structure of machines and transformers, the following simplified formulae have been developed:

For vertical planes of height not less than 1 m and with a temperature difference θ between the surface and the ambient air of pressure b atm:

$$p_v = 2.0b^{1/2} \theta^{5/4} \text{ W/m}^2 \quad (4.11)$$

For vertical tubes of diameter d :

$$p_v = 1.3d^{-1/4}\theta^{5/4} \text{ W/m}^2 \quad (4.12)$$

EXAMPLE 4.5: A 7.5 MVA transformer has a rated core $+I^2R$ loss of $14 + 46 = 60$ kW. The tank is $1.4 \text{ m} \times 2.5 \text{ m}$ in plan, and 3.0 m in height, fitted with a total of 925 m of 0.05 m diameter tubes. Find the surface temperature-rise θ above 25°C (298 K) ambient.

Eqs. (4.9), (4.11) and (4.12) are summed and solved by iteration for θ , giving $\theta = 45^\circ\text{C}$ rise (corresponding to a surface temperature of 343 K). For this value the calculations check as follows:

Radiation. The radiating area is the tank walls and top, giving $23.4 + 3.5 = 29.6 \text{ m}^2$ (the tubes making no additional contribution). With $\epsilon = 0.9$

$$p_r = 5.7 \times 10^{-8} \times 0.9(343^4 - 298^4) = 303 \text{ W/m}^2$$

$$P_r = p_r S_t = 303 \times 29.6 = 9\,000 \text{ W} = 9.0 \text{ kW}$$

Convection: The tank surface (walls only) is 23.4 m^2 . Assuming normal atmospheric pressure, $b = 1$,

$$p_{vt} = 2.0 \times 45^{5/4} = 2.0 \times 117 = 234 \text{ W/m}^2$$

$$P_{vt} = p_{vt} S_t = 234 \times 23.4 = 5\,500 \text{ W} = 5.5 \text{ kW}$$

The tube area is $S_u = \pi 0.05 \times 925 = 145 \text{ m}^2$, so that

$$p_{vu} = 1.3 \times (0.05)^{-1/4} \times 45^{5/4} = 319 \text{ W/m}^2$$

$$P_{vu} = p_{vu} S_u = 319 \times 145 = 46\,200 = 46.2 \text{ kW}$$

The total dissipation is $9.0 + 5.5 + 46.2 = 60.7 \text{ kW}$, as required.

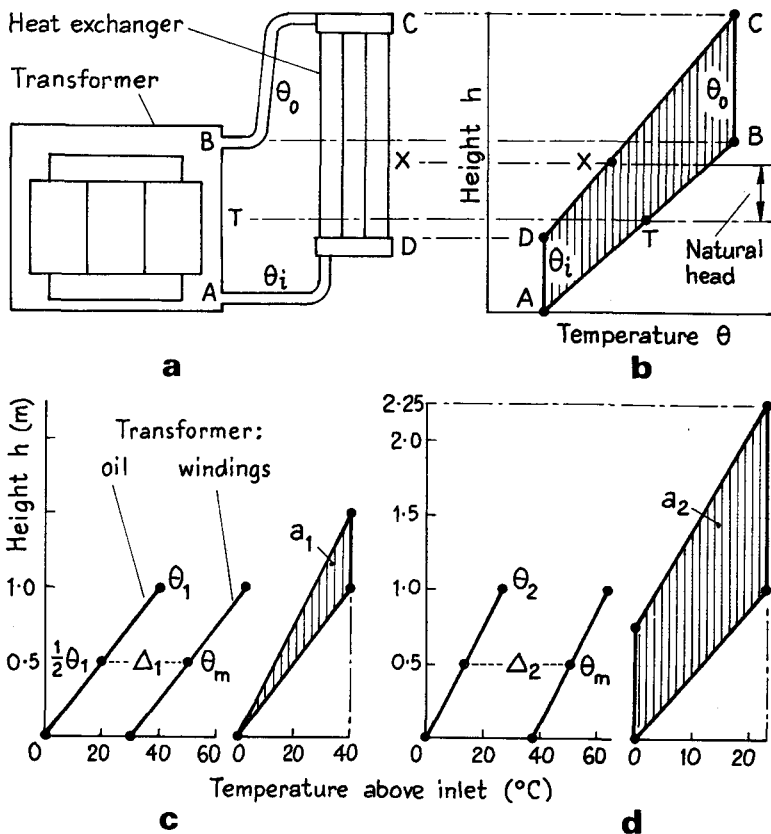
4.6 Flow of coolant in transformers

In the classical theory of isothermal steady flow of a fluid in a long straight tube, the flow can be (i) *streamline* (or laminar) with a parabolic velocity profile having a mean equal to 0.5 of the maximum, or (ii) *turbulent* in which the mean velocity is about 0.8 of the maximum and there remains a thin annular streamline flow at the tube-wall boundary. Actual conditions are rarely as straightforward, but turbulence has to be encouraged if effective heat-transfer is required.

Circulation of the coolant fluid — air or hydrogen — is relatively simple with rotating machines, which can be fitted with rotor-mounted fans. Although fan efficiencies are rather low (e.g. 0.2 – 0.4), rapid and directed air streams can be produced in the cooling ducts, the nature of which stimulates turbulence. In large machines, especially hydrogen-cooled turbo-generators, gas pressures up to 2 atm may be used to obtain the high-speed flow of large quantities of coolant.

In *natural* cooling by thermal head, the method employed in most oil-immersed transformers, the flow of the coolant is complex. The fluid 'drags' at

the walls of the channel, constituting a partial heat barrier, and the heat dissipation produces complicated velocity and temperature profiles. The temperature falls sharply across the walls of the cooling surface; and although there is a small bulk velocity in one direction, the 'core' of the column of oil may actually move in the opposite direction. While the details are thermally and hydrodynamically very intricate, the system as a whole may be looked upon as operating on a combination of thermal and gravitational 'heads'.



4.16 Temperatures in transformer and heat-exchanger.

Consider the transformer/exchanger system in Fig. 4.16(a). With the oil having the same temperature throughout the system the gravitational forces balance and there is no circulation. With the transformer on load, oil in the region AB absorbs the core and load losses; the temperature in AB rises and the oil density falls, unbalancing the columns and initiating circulation in the direction ABCD. At the base A the tank inlet oil temperature is θ_i . At B and C (ignoring dissipation from the pipework BC) the outlet temperature is θ_o . Assuming for simplicity that the height/temperature relation is linear, the h/θ diagram in Fig. 4.16(b) can be drawn. The shaded area is a measure of

the effective head causing oil circulation, for if multiplied by $g(d\delta/d\theta)$, where g is the acceleration due to gravity and δ is the oil density, the result is the pressure-difference or 'head'. For a given circuit the oil flow-rate can be taken as proportional to the head. Thus the circulation is determined (i) by the fall of oil density resulting from the temperature-rise ($\theta_o - \theta_i$), and (ii) by some function of the height of the exchanger with respect to the tank.

Clearly there can be no circulation without temperature-rise, and equally there can be no 'density head' unless the 'centre of cooling' (X) of the heat-exchanger is above that (T) of the transformer. In practice the designer is limited by specification, Sect. 4.6, in his choice of the temperature-rise ($\theta_o - \theta_i$), and he must therefore arrange that X is adequately above T. Truncating the heat-exchanger at its lower end may actually result in some improvement in oil circulation if it raises point X. Even in transformers carrying tubes on the tank walls (for which case the circulating 'head' is somewhat marginal) a small gain may be had by raising the tube entry level at the lower end.

EXAMPLE 4.6: In a 20 MVA transformer on steady full load, oil at the inlet enters the tank from the exchanger at 15°C above ambient, flows upward by a vertical distance of 1.0 m through the oil ducts and reaches a temperature of 55°C above ambient. It then flows to the header of a heat-exchanger bank 1.5 m high. The mean temperature-rise of the windings (estimated from a resistance/time curve taken after shut-down) is 65°C . Estimate the additional load obtainable if the heat-exchanger is raised bodily by 0.75 m for the same mean winding temperature as before, assuming that the oil inlet temperature remains at 15°C above ambient.

The original temperature condition is shown in Fig. 4.16(c). It is assumed that the temperature difference Δ_1 between the oil and the windings is constant. Taking the inlet temperature as reference, then the transformer top-oil temperature is θ_1 , the mean oil temperature is $\frac{1}{2}\theta_1$, the mean winding temperature is θ_m and the temperature-difference is $\Delta_1 = \theta_m - \frac{1}{2}\theta_1$. The rate of loss transfer from the transformer to the oil is proportional to Δ_1 . The area a_1 of the height/temperature diagram is a measure of the oil flow-rate. The numerical values are

$$\theta_1 = 40; \frac{1}{2}\theta_1 = 20; \theta_m = 50; \Delta_1 = 50 - 20 = 30^\circ\text{C};$$

$$a_1 = \frac{1}{2} \times 0.5 \times 40 = 10^\circ\text{C}\cdot\text{m}.$$

With the heat-exchanger raised by 0.75 m the shape of the height/temperature diagram changes to that in (d). The new values are

$$\theta_2, \text{ unknown}; \theta_m = 50; \Delta_2 = 50 - \frac{1}{2}\theta_2;$$

$$a_2 = \theta_2 \times \frac{1}{2}(0.75) + 1.25 = \theta_2.$$

The mean winding temperature is

$$\begin{aligned} \theta_m = 50 &= \frac{1}{2}\theta_2 + \Delta_2 = \frac{1}{2}\theta_2 + \Delta_1(a_2/a_1)(\theta_2/\theta_1) \\ &= \frac{1}{2}\theta_2 + 30 \frac{\theta_2}{10} \frac{\theta_2}{40} \end{aligned}$$

whence $\theta_2 = 23^\circ\text{C}$ and $\Delta_2 = 50 - 11\frac{1}{2} = 38\frac{1}{2}^\circ\text{C}$. The loss rate is increased by the factor $\Delta_2/\Delta_1 = 1.3$. The implication in terms of load rests on several factors, such as the ratio of core to I^2R loss on rated load, the relative effectiveness of heat transfer to the oil from the core and the windings respectively, the reduction of the oil fluidity as a result of the lowering of the oil temperature, and the reduction of heat dissipation from the tank itself. Nevertheless, within the validity of the assumptions, the advantage of raising the centre of cooling of the heat-exchanger is evident.

It is also clear that a further increase in loss dissipation could be obtained (i) by forced oil circulation to reduce Δ , and (ii) by placing fans below the heat-exchanger to increase thermal emission to the ambient air.

Multiflow Principle

The thermal rating of power transformers is based on the average temperature-rise of the windings as determined from the increase in resistance (Sect. 4.7), because this is the best practical measurement. The insulation life, however, depends on the temperature of the hottest region of the windings, which exceeds the average by some value dependent on the individual design characteristics; and it is not measurable directly.

Even for natural cooling the assumptions made in Fig. 4.16(a) are an oversimplification. With normal forced-oil cooling, the greater part of the flow will be outside the windings, between their perimeter and the inner wall of the tank. Only a small part of the oil will actually pass through the windings because of the greater hydraulic resistance. The top-oil temperatures of the streams will differ before they mix and leave the top of the transformer. On this *multiflow principle* it would be possible to identify, for example, 43°C rise in oil temperature at the top of the h.v. windings, 52°C for the l.v. winding, and 39°C for the oil flow in the tank external to the windings, with the combination of the three oil streams to give a 42°C rise of the oil outlet to the cooler. Carruthers and Norris [25] have shown that the hottest-spot temperature-rises can be more closely estimated on the multiflow principle from measurements made in a single heat run, and the estimate freed from the basis of temperature-rise of windings by resistance. As a result they show that it is feasible in forced-oil cooled transformers to allow an increase from 65 to 75°C in the average winding temperature-rise without any increase in either the hottest-spot or hottest-oil temperature. The multiflow principle has not created this increase; rather it has shown the possibility by a more sophisticated examination of the cooling conditions.

4.7 Limits of temperature-rise

The maximum working temperature of a transformer or a machine depends upon the thermal quality of its insulation. It is commonly remarked that a rise of 6°C in the operating temperature halves the life of insulation, and although the figures may be arguable, the basic truth is amply supported by experience. The classes into which insulating materials are divided are set out in Sect. 3.1 together with their maximum working temperatures. The temperature-rise

limits are such as to keep within these levels from arbitrary ambient or inlet coolant temperatures [25, 26].

Measurement of temperature

The temperature of parts of a transformer or electrical machine may be measured by

T: thermometer, applied to the surface of the part. This gives the temperature of the *surface* at one point only;

E: embedded temperature detector (thermo-couple or resistance coil), which gives the temperature at one *internal point*;

R: resistance, involving the measurement of the resistance both cold and hot, and estimating the *mean* rise by use of the resistance temperature coefficient: it is available for windings only.

The designations T, E and R are used in the following Tables to indicate the methods by which the limits of temperature-rise specified are to be determined.

These methods give results that do not refer to the same thing and furnish different bases for estimating hot-spot temperatures. If an embedded temperature detector can be placed, during construction, at a point at which experience shows the temperature to be highest, it will give an indication of the hot-spot temperature. In practice, however, the problem of insulation prevents the detectors from being placed too near the conductors.

Although methods are available for measuring the resistance of a winding in operation, it is more usual to make resistance (or thermometric) measurements immediately after shut-down. Extrapolation is then necessary to cover the period between actual shut-down and the start of temperature measurement. The temperature/time relation during cooling is plotted and extrapolated backward to the instant of shut-down by means of a simple application of the exponential cooling curve, Sect. 4.2.

Transformers

The temperature-rises given below are for transformers (other than miniature or other special types) operating at altitudes not exceeding 1 000 m above sea-level. For water cooling, the inlet water temperature must not be greater than 25 °C; for air cooling the limits of ambient temperature are -25 and +40 °C, with the proviso that the air temperature shall not exceed a mean of 30 °C in any one day, nor 20 °C over a year. Adjustments are made for conditions out-with those stated. The cooling nomenclature (i.e. the designations AN, AF, etc.) is detailed in Sect. 5.17.

Industrial rotating machines

The Table on page 139 refers to machines other than those intended for ship, aircraft or traction application, turbo-generators, and fractional-kilowatt machines. Temperature-rises are based on an inlet coolant temperature of 40 °C, and continuous rating at a voltage not exceeding 11 kV. Temperature-rises are reduced by 1 °C per kV for the excess of the working voltage above

Transformer temperature-rise, °C

Part		Cooling method	Oil circn.	Insulation class					
				A	E	B	F	H	C
DRY									
Windings	R	AN, AF	—	60	75	80	100	125	150
Cores near windings	T	any	—	60	75	80	100	125	150
not near windings	T	any	—	non-injurious value					
OIL-IMMERSED									
Windings	R	NA, FA W(int)	N	65					
	R	FA W(ext)	F	65					
Top oil	T	—	—	60 sealed or with conservator					
	T	—	—	65 unsealed and without conservator					
Cores	T	—	—	non-injurious value					

11 kV, and may be increased by agreement for short-time ratings. Adjustments are necessary for abnormal coolant temperatures and for altitudes above 1 000 m.

4.8 Thermal rating of transformers

The nominal rating of a transformer is a conventional value of apparent power conforming to the specification and test requirements, and establishing the 'rated' primary and secondary currents. But the performance and useful life of a transformer are functions of the temperature of its insulation, so that loads in excess of the nominal rating can be sustained for limited periods provided that the temperature limitations are not exceeded. This is a consequence of the thermal lag provided by the heat capacity of the transformer as expressed by eq.(4.1). However, the observations of numerous investigators have shown that, over the temperature range 80–140 °C, the rate of reduction of life-expectancy of transformer insulation is doubled for every increase of 6 °C in its hot-spot temperature. Overload conditions are therefore classified as (i) those that do not impair the 'normal' life-expectancy of some tens of years, and (ii) those in which a more rapid deterioration is acceptable. Estimation of the crucial hot-spot temperature θ_h is based on the simple thermal diagram of Fig. 4.16(c), using the top-oil temperature θ_1 and the temperature-difference Δ in the expression

$$\theta_h = \theta_1 + 1.1 \Delta$$

For (i) the hot-spot temperature must not exceed 98 °C. For a 20 °C ambient this corresponds to a hot-spot temperature-rise of 78 °C, viz. 13 °C above the mean winding temperature-rise of 65 °C. For (ii) the hot-spot temperature must not exceed 140 °C, and the current must not be greater than 1.5 p.u. of

rated value (except in emergency) in order to avoid difficulties that would otherwise arise from the limited thermal capacity of associated bushings, tap-changers, cable-ends and circuit-breakers.

Loading tables [107] appropriate to various forms of natural and forced cooling methods are available, from which acceptable over-loading conditions can be determined. They are based on a simplified daily load cycle of constant load K1 of duration $(24 - t)$ hours and a load K2 for t hours. For parti-

Industrial machine temperature-rise, °C

Part	Insulation class					
	A	E	B	F	H	
A.C. WINDINGS						
<i>Air-cooled:</i>						
Output 5 MW or more, or core-length 1 m or more	RE	60	70	80	100	125
Smaller than above	R	60	75	80	100	125
<i>Indirect hydrogen-cooled:</i>						
At absolute pressure (bar) 1–1.5	E	—	—	80	100	
2	E	—	—	75	95	
3	E	—	—	70	90	
4	E	—	—	65	85	
<i>Direct-cooled:</i>						
By hydrogen or liquid	E	—	—	80	100	
COMMUTATOR WINDINGS						
	T	50	65	70	85	105
	R	60	75	80	100	125
D.C. FIELD WINDINGS						
<i>Air- or hydrogen-cooled:</i>						
High-resistance, insulated conductors	T	50	65	70	85	105
	R	60	75	80	100	125
Low-resistance single-layer with insulated conductors	TR	60	75	80	100	125
Exposed bare conductors	TR	65	80	90	110	135
OTHER PARTS						
Short-circuited windings with insulated conductors; cores in contact with insula- ted windings	T	60	75	80	100	125
Commutators and slip-rings	T	60	70	80	90	100
Short-circuited windings with un- insulated conductors; cores not in con- tact with insulated windings	T	non-injurious value				

cular load cycles not tabulated it is possible to estimate the conditions by interpolation.

4.9 Thermal rating of machines

The Table of temperature-rise limits θ_m for industrial machines is interpreted in accordance with the duty that the machine is called upon to perform.

Although this is rarely predictable in precise terms, it may be comparable with one of a number of internationally agreed standard duty cycles (BS 2613), designated as follows:

- S1 Continuous running at rated load
- S2 Short-time operation
- S3 Intermittent periodic operation
- S4 As S3 but with starting
- S5 As S3 but with electric braking
- S6 Continuous cyclic operation
- S7 As S6 but with electric braking
- S8 As S6 but with related load/speed characteristic

The operating conditions are illustrated with associated temperature-rise/time curves in Fig. 4.17. The conditions are denoted

- N operation under rated conditions of load
- R machine at rest and de-energized
- D starting duty
- F braking duty
- V operation at no load, but rotating

Cyclic-duration Factor: For the more elaborate duty cycles a cyclic-duration factor is defined: it influences the class of rating. The definitions, referred to the diagrams in Fig. 4.17, are

$$\begin{array}{ll} \text{S3: } N/(N + R) & \text{S4: } (D + N)/(D + N + R) \\ \text{S5: } (D + N + F)/(D + N + F + R) & \text{S6: } N/(N + V) \end{array}$$

Rating Classes

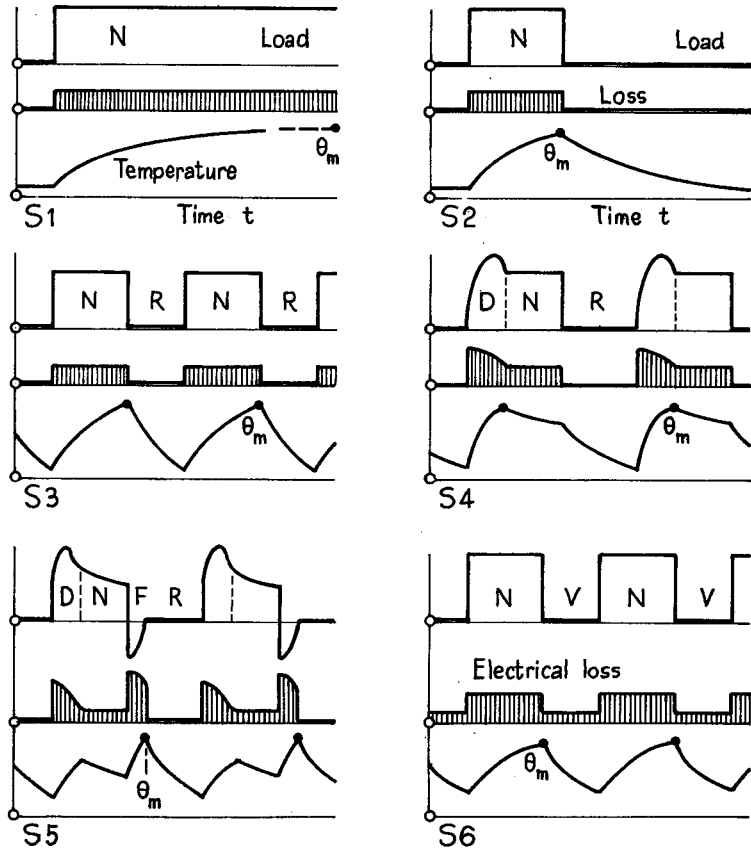
These are brief indications of the rating on which the design of the machine is based.

Maximum Continuous (MCR): continuous operation as in S1.

Short-time (STR): the load durations in S2 are usually 10, 30, 60 and 90 min, and the limits of temperature-rise may, by agreement, be increased by 10 °C.

Equivalent Continuous (ECR): this is for test purposes in the case of duties S1 and S2.

Duty Type (DTR): corresponding to S3–S8. The time for a cycle should be 10 min, and the cyclic duration factors should be 0.15, 0.25, 0.40 or 0.60. The information required for determining the d.t.r. includes the stored kinetic energy of the motor and of the load, the duration of the cycle and the cycle-duration factor.



4.17 Standard duty-cycles for rotating machines.

Under the stated conditions the maximum temperature-rise must not exceed the appropriate value in the Table for the part of the machine considered; except that for cage machines operating on a d.t.r., the cage temperature-rise in the *middle* of the final operating cycle following the attainment of equilibrium thermal conditions shall not exceed the value given in the Table.

5

Transformers: Theory and Performance

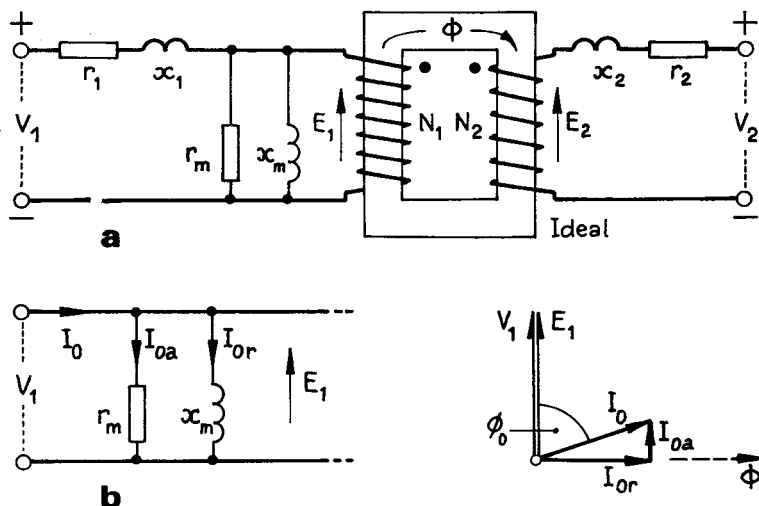
5.1 *Theory of the power transformer*

The essential behaviour of an *ideal* power transformer under conditions of sinusoidal excitation is discussed in Sect. 1.9. In brief, the behaviour is expressed in terms of the characteristic ratios set out in eq.(1.8), using the conventions in Fig. 1.6 in which terminal voltages are taken as applied, and the positive direction of power (whether active or reactive) is into the windings.

An actual transformer differs from the ideal in the following respects:

- (i) The core is not infinitely permeable, and to establish the working flux it requires a magnetizing m.m.f. F_0 . The m.m.f. balance is no longer $F_1 + F_2 = 0$, but now becomes $F_1 + F_2 = F_0$. As the core has a non-linear B/H relation, the magnetizing m.m.f. depends upon the degree of core saturation.
- (ii) The core is the seat of loss arising from alternating magnetization, so that the exciting (primary) winding must also supply the core-loss power.
- (iii) The windings have individual resistances between terminals, of values determined by the resistivity of the conductor material and augmented by eddy-current loss.
- (iv) The windings are not perfectly coupled, so that not all the flux developed by the primary links the secondary, and the latter produces further flux not linking the primary. The effect is taken into account by assigning to each winding the property of leakage inductance. As the leakage fluxes are established in paths partly external to the core, the leakage inductances can be taken as constant. At a given operating frequency they can be represented in an equivalent circuit by leakage reactances.

The actual transformer can now be taken as an ideal transformer with resistance r_1 and leakage reactance x_1 in series with the primary winding, and correspondingly r_2 and x_2 in series with the secondary, as in Fig. 5.1 (a), to take account of the defects (iii) and (iv). The m.m.f. F_0 is introduced by means of a magnetizing reactance x_m shunted across the primary terminals of the ideal transformer to take a magnetizing current I_{0r} such that $N_1 I_{0r} = F_0$. The magnetizing current is in time phase with the core flux, and is taken for simplicity to be sinusoidal, an assumption untrue for practical levels of core saturation but acceptable because I_{0r} is usually a very small fraction of the rated current.



5.1 Transformer on no load.

To represent core loss, a resistance r_m is connected in parallel with x_m ; again this is an approximation, for the loss is not strictly proportional to the square of the flux. The shunt resistance r_m takes an active current I_{0a} such that $r_m = P_0/I_{0a}^2$ where P_0 is the core loss.

We have in this way taken account of the defects (i)–(iv) by representing them in an equivalent circuit, Fig. 5.1 (a), containing an ideal transformer with r_1 and x_1 , r_2 and x_2 , r_m and x_m external to it. The combinations $z_1 = (r_1 + jx_1)$ and $z_2 = (r_2 + jx_2)$ are referred to as the *leakage impedances*.

Let the ideal-transformer part of the system carry a sinusoidally alternating mutual (or working) magnetic flux of peak value Φ_m and angular frequency $\omega = 2\pi f$. By definition this flux links every turn, inducing therein the e.m.f. $e_t = d\Phi/dt$ having the peak value $\omega\Phi_m$ and the r.m.s. value

$$E_t = \omega\Phi_m/\sqrt{2} = \sqrt{2}\pi f\Phi_m = 4.44f\Phi_m \quad (5.1)$$

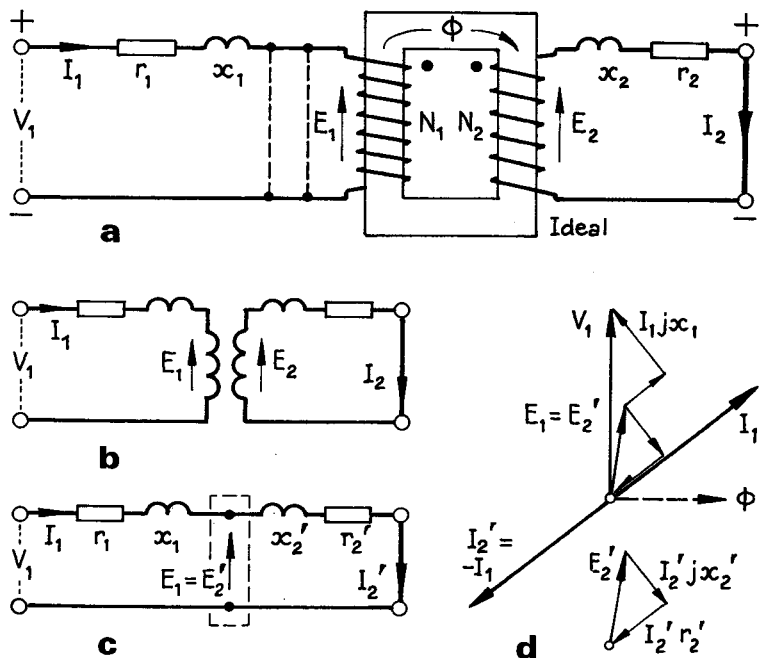
The total primary e.m.f. in N_1 turns in series is $E_1 = N_1E_t$, and for the secondary correspondingly $E_2 = N_2E_t$. We shall first consider the secondary terminals to be open-circuited: this is the *no-load* condition.

No-load Condition

The actual core requires the magnetizing m.m.f. F_0 , which in equivalent-circuit terms is the current in x_m , namely I_{0r} . Further, the core loss is provided in the equivalent circuit by the current I_{0a} in r_m . The e.m.f. E_1 balances the primary applied voltage V_1 (neglecting the very small volt drops due to the magnetizing and loss currents flowing through the primary leakage impedance z_1). Thus $E_1 = V_1$ and the no-load primary input current is

$$I_0 = \sqrt{(I_{0r}^2 + I_{0a}^2)}$$

at a phase angle to V_1 (or E_1) of ϕ_0 . Typically, I_0 is 0.05 p.u. of normal rated current. The phasor diagram corresponding to no-load conditions in Fig. 5.1(b) shows the active and reactive components of the primary no-load current, with the volt drops due to I_0 in r_1 and x_1 neglected. These drops could be typically 0.0005 and 0.003 p.u. respectively of normal primary voltage. The no-load power is therefore almost identical with $V_1 I_{0a} = P_0$, which is the core-loss power.



5.2 Transformer on short circuit.

Short-circuit Condition

Suppose the secondary terminals to be short-circuited, as in Fig. 5.2(a). The secondary e.m.f. E_2 is absorbed entirely in circulating I_2 through the leakage impedance $z_2 = r_2 + jx_2$, and the current is likely to be very large. To limit I_2 to the level of normal rated current it will be necessary to limit the primary voltage V_1 to about 0.1 or less of normal. The primary current of the ideal transformer is $I_1 = I_2 (N_2/N_1)$ and its e.m.f. $E_1 = E_2 (N_1/N_2)$. The primary voltage is employed in providing E_1 and the volt drop in the primary leakage impedance z_1 . As the primary and secondary per-unit leakage impedances are roughly equal, it follows that E_1 is about one-half of V_1 . With V_1 equal to 0.1 p.u. then E_1 is about 0.05 p.u., which means that the mutual flux Φ_m is of the same order. The core loss, which is proportional approximately to the square of the flux density, is consequently about 0.0025 of what it was on no load; further, the magnetizing current is greatly reduced, partly by reason of

the reduced flux and partly because of the low saturation level. In the equivalent circuit it is legitimate to ignore completely the shunt magnetization branches r_m and x_m , to give the simplified equivalent circuit of Fig. 5.2(b) and the corresponding phasor diagrams for primary and secondary. The turns-ratio can have any designed value: if it is large, $E_1 \gg E_2$ and $I_2 \gg I_1$, making the phasor diagrams somewhat awkward. We therefore adopt the unity turns-ratio equivalent.

Unity Turns-Ratio: Suppose that $N_1 = N_2$ and that consequently $E_1 = E_2$ in the ideal-transformer part of the equivalent circuit. Energy is now transferable from primary to secondary without any change of voltage or current, so that there is no need to have the ideal transformer at all. To retain such simplicity when $N_1 \neq N_2$ it is necessary to imagine the actual secondary winding of N_2 turns to be replaced by an equivalent winding of N_1 turns with the proviso that there is no change in the I^2R loss nor in the effective leakage reactance. For this the equivalent secondary must have a resistance r_2' and leakage reactance x_2' such that $I_1^2 r_2' = I_2^2 r_2$ (as the secondary current is now $I_2' = I_1$), and $I_1^2 x_2' = I_2^2 x_2$; hence the resistance and leakage reactance of the equivalent N_1 -turn secondary are

$$r_2' = k_n^2 r_2 \quad \text{and} \quad x_2' = k_n^2 x_2 \quad (5.2)$$

where $k_n = (N_1/N_2) = E_1/E_2 = I_2/I_1$. This result could be inferred directly from the impedance transformation of an ideal transformer, discussed in Sect. 1.9.

The equivalent circuit for a power transformer with secondary short-circuited can now be reduced to that in Fig. 5.2(c) and the phasor diagrams (d) for primary and secondary combined. Obviously, if the magnetizing branches are ignored, the condition becomes that of a primary voltage V_1 applied to a simple series circuit of total resistance R_1 and leakage reactance X_1 given by

$$R_1 = r_1 + r_2' \quad \text{and} \quad X_1 = x_1 + x_2' \quad (5.3)$$

As the terms 'primary' and 'secondary' describe merely which winding accepts electrical energy and which delivers it, the argument above can equally well be applied to a transformer in which the actual primary turns N_1 are assumed to be changed to N_2 and the primary current from I_1 to I_1' . It is immaterial which winding is used as the 'base'.

EXAMPLE 5.1: A 400 kVA 11/0.415 kV 3-ph delta/star-connected transformer has on rated load an h.v. I^2R loss of 2.46 kW and l.v. loss of 1.95 kW. The total leakage reactance is 0.055 p.u. Find the ohmic values of resistance and leakage reactance.

The phase voltages are 11 kV and 240 V. The phase currents are 12.1 A and 555 A, and the I^2R losses per phase are 0.82 kW for the h.v. side and 0.65 kW for the l.v. side. Hence $r_1 = 820/12.1^2 = 5.6 \Omega/\text{ph}$ and $r_2 = 650/555^2 = 2.1 \text{ m}\Omega/\text{ph}$. Referred to the h.v. winding, the total resistance is

$$R_1 = 5.6 + 0.0021(11/0.240)^2 = 5.6 + 4.4 = 10.0 \Omega/\text{ph}$$

giving $I_1^2 R_1 = 12.1^2 \times 10.0 = 1.47 \text{ kW/ph}$, which is of course the sum $(0.82 + 0.65) \text{ kW/ph}$. Referred to the l.v. side

$$R_2 = 5.60(0.240/11)^2 + 0.0021 = 0.0027 + 0.0021 = 4.8 \text{ m}\Omega/\text{ph}$$

The reactance referred to the h.v. side is such that $I_1^2 X_1/V_1 I_1 = I_1 X_1/V_1 = 0.055$, whence

$$X_1 = 0.055 \times 11\,000/12.1 = 50 \text{ }\Omega/\text{ph}$$

and referred to the l.v. side

$$X_2 = 50(240/11\,000)^2 = 24.0 \text{ m}\Omega/\text{ph}$$

or obtained directly, $X_2 = 0.055(240/555) = 24.0 \text{ m}\Omega/\text{ph}$. The division of X_1 or X_2 between the h.v. and l.v. windings is entirely arbitrary: a common way is to put one-half in each. Then

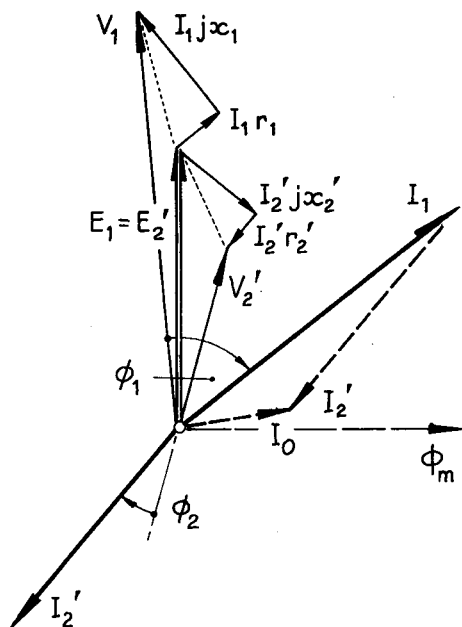
$$x_1 = 25 \text{ }\Omega, x_2' = 25 \text{ }\Omega \quad \text{or} \quad x_2 = 12 \text{ m}\Omega, x_1' = 12 \text{ m}\Omega.$$

5.2 Power transformer on load

If a load impedance Z is connected across the secondary terminals, the e.m.f. E_2 will drive an output current I_2 through it, and the secondary winding will impress the m.m.f. $F_2 = N_2 I_2$ on the magnetic circuit. To maintain the working flux Φ_m and the primary e.m.f. E_1 (which must always be in near balance with the applied voltage V_1) the primary must take an input current I_1 to develop an m.m.f. $F_1 = N_1 I_1$ to balance F_2 and at the same time to provide for the magnetizing m.m.f. F_0 . Thus a secondary current causes a counterpart to flow in the primary, and the combined primary and secondary m.m.f.s must result in a net magnetizing m.m.f. F_0 .

If we adopt the equal-turns-ratio condition with $N_2 = N_1$, primary and secondary currents can be used to represent also the winding m.m.f.s, making the phasor diagram more easily drawn. It is shown for a partially inductive secondary load in Fig. 5.3. The equivalent secondary current is I_2' . The primary current I_1 is the phasor sum of $-I_2'$ and I_0 , the latter corresponding very nearly to the no-load current as the mutual flux is only slightly less than on secondary open-circuit. For load conditions the phasor diagram incorporates the basic features of both the no-load and short-circuit conditions.

The applied primary voltage V_1 balances E_1 and provides for the primary resistance and leakage reactance components $I_1 r_1$ and $I_1 j x_1$. On the secondary side the e.m.f. E_2' circulates I_2' through the equivalent load impedance and provides for the secondary leakage impedance volt drops $I_2' r_2'$ and $I_2' j x_2'$. The secondary phase angle is ϕ_2 , the secondary operating in a 'generator' mode. The primary phase angle ϕ_1 differs from ϕ_2 by reason of the necessary provision of reactive (magnetizing) power in addition to that required by the secondary load. The current I_0 and the leakage impedance volt drops in the phasor diagram correspond to those that might be expected in a small transformer: for a large unit they are very much smaller.



5.3 Transformer on load: phasor diagram.

5.3 Losses and efficiency

The transformer has no moving parts, and its efficiency has far less significance than its losses, which are:

Core Loss resulting from the alternating core magnetization.

Dielectric Loss in insulating materials, particularly in the oil and solid insulation of high-voltage units.

I^2R Loss in the windings.

Load (Stray) Loss, largely the result of leakage magnetic fields inducing eddy currents in the tank walls and conductors.

A close approximation to the losses can be obtained from open- and short-circuit tests, without operating the transformer on load.

No-Load Loss

With open-circuited secondary, the primary current is I_0 . The I^2R loss due to it is in most cases quite negligible (e.g. 1/400 of the rated normal I^2R loss).

The no-load power input is consequently concerned with core and dielectric loss, the latter having significance only for transformers operating at very high voltages. The e.m.f. E_1 is substantially identical with V_1 , the conditions being represented by the equivalent circuit of Fig. 5.1 in respect of the core magnetization. If the dielectric effect is negligible the no-load power input $P_0 = V_1 I_{0a}$ is the loss P_i in the iron of the core. In large h.v. transformers, however, it will also include the loss P_d in the dielectric material. In the

latter case, too, the core magnetizing current I_{0m} is accompanied by a leading capacitive current I_{0c} , the effect of which is to reduce the lagging reactive component I_{0r} of the no-load current. Thus the core and dielectric losses are measurable by means of an *open-circuit test* at normal voltage and frequency. They are generally assumed to be *constant* and independent of the load.

Short-Circuit Loss

If rated secondary current is circulated on short circuit by means of a low value of primary voltage of normal frequency, the circuit in primary terms is effectively that in Fig. 5.2(d), from which the complexor relation is

$$V_1 = I_1 [(r_1 + r_2') + j(x_1 + x_2')] = I_1 (R_1 + jX_1) = I_1 Z_1$$

and the power input $I_1^2 R_1$, which is the total $I^2 R$ loss in the primary and secondary windings. In practice the active power input includes (stray) load loss, for with normal primary and secondary currents the leakage fluxes are normal (or nearly so), producing eddy-current loss in metallic parts of the transformer adjacent to the windings. Thus a *short-circuit test* at rated current and frequency can establish the load loss to a close approximation. It also gives information about the leakage impedance and effective resistance, from which the leakage reactance can be calculated.

Power Efficiency

The peak flux normally varies so little between no-load and full-load conditions that the core loss can be considered as constant at the no-load value P_i . If the short-circuit loss at rated current be P_c , the loss at any fraction k of the rated load S is $k^2 P_c$. The total loss at load kS of power factor $\cos \phi$ is $P_i + k^2 P_c$, and the output/input power ratio, i.e. the power efficiency, is

$$\eta = \frac{kS \cdot \cos \phi}{kS \cdot \cos \phi + P_i + k^2 P_c} = 1 - \frac{P_i + k^2 P_c}{kS \cdot \cos \phi + P_i + k^2 P_c} \quad (5.4)$$

Solving $d\eta/dk = 0$ gives $P_i = k^2 P_c$ for the maximum efficiency condition, i.e. a load of the fraction $k = \sqrt{(P_i/P_c)}$ of full-load rating for which the variable $I^2 R$ loss is equal to the constant core loss.

Eq.(5.4) shows that the efficiency is dependent upon the load power factor. Greatest efficiency is naturally obtained with a power factor of unity; for a purely reactive load the efficiency is zero, but the losses are the same. It is for this reason that the losses are more significant than the efficiency.

EXAMPLE 5.2: A 300 kVA transformer has a core loss of 1.5 kW and a full-load $I^2 R$ loss of 4.5 kW. Calculate its efficiency for 25, 58 (maximum efficiency load), 75, 100 and 125% of rated load at power factors respectively of unity, 0.8 and 0.6.

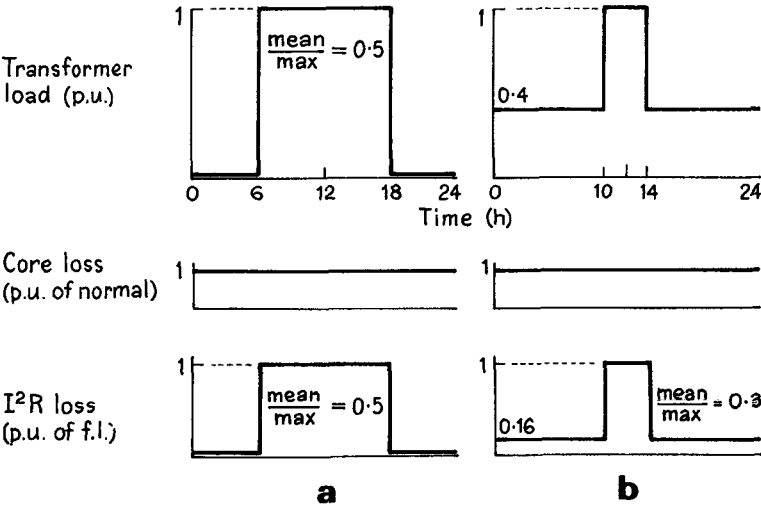
The core loss is $1.5/300 = 0.005$ p.u. and the full-load $I^2 R$ loss is 0.015 p.u. Calculations in per-unit values based on eq.(5.4) with the full-load rating $S = 1.0$ p.u. and the full-load active power $P = 1.0 \cos \phi$ p.u. are given in the Table:

Load, p.u.			0.25	0.58	0.75	1.0	1.25
Output power:	p.f.	1.0	0.25	0.58	0.75	1.00	1.25
		0.8	0.20	0.46	0.60	0.80	1.00
		0.6	0.15	0.35	0.45	0.60	0.75
Core loss			0.005	0.005	0.005	0.005	0.005
I ² R loss			0.0009	0.005	0.0084	0.015	0.023
Total loss			0.0059	0.010	0.0134	0.020	0.028
Efficiency:	p.f.	1.0	0.968	0.983	0.982	0.980	0.978
		0.8	0.971	0.979	0.978	0.976	0.972
		0.6	0.962	0.972	0.971	0.968	0.963

Energy Efficiency

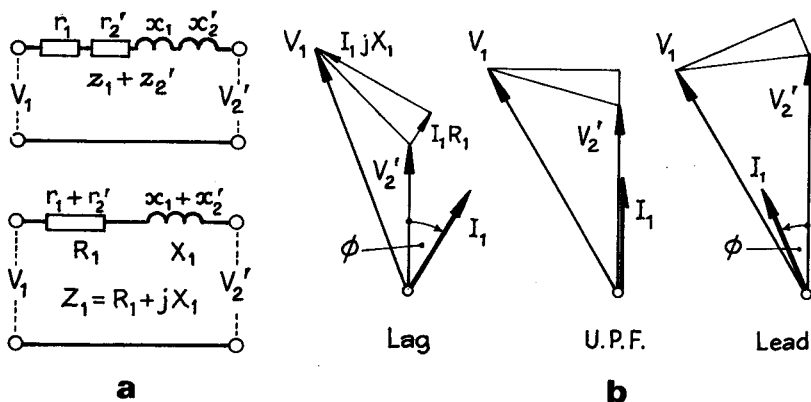
A transformer in circuit continuously has a steady core-loss rate. Its I²R energy loss depends on the square of the load. The annual cost of losses therefore depends on the hour-to-hour variation in the transformer loading. Fig. 5.4 shows two hypothetical daily load curves each of the same *peak* and the same *average* daily value, the load factor (mean/maximum) being 0.5 for both. But they produce different I²R losses, averaging 0.5 of the maximum for (a) but only 0.3 for (b). For minimum daily or yearly energy loss, the I²R/core loss-ratio should be chosen in accordance with the type of load variation. This is by no means easy to predict; a rough relation between the load-factor of the load and the corresponding load-factor of the I²R loss in typical supply network loadings is:

Load-factor of transformer load:	0.30	0.40	0.50	0.60	0.70	0.80
Load-factor of I ² R loss:	0.16	0.25	0.35	0.47	0.60	0.72



5.4 Load- and loss-energy relation.

The total loss in a transformer working on a load with a load-factor of 0.5 may be expected to consist of the normal constant core loss together with 0.35 p.u. of the rated I^2R loss. On the basis of energy-loss equality this suggests that a transformer with a full-load I^2R equal to $1.0/0.35 \approx 3$ times the core loss is likely to be the most economical on the assumption that the cost of wasted energy is the same per unit for each loss. This is not necessarily so, for the I^2R loss predominates when the system load is high and energy most valuable. Evidently the choice of a transformer on economic grounds is a complex matter. Briefly, it is necessary to consider capital cost and the charges thereon, life, depreciation, load factor, loss factor, loss ratio and energy cost: a formidable list. A computer solution is obtainable if all these can be analytically related.



5.5 Regulation.

5.4 Regulation

The regulation is the fall (or rise) of secondary terminal voltage between no-load and full-load conditions with the primary voltage at constant rated value. It is usually quoted as a percentage or per-unit value for a specified power factor.

On no load the secondary terminal voltage is very nearly $V_2' = V_1$. On full load corresponding to a primary current I_1 it is given by the phasor expression based on Fig. 5.5(a):

$$V_2' = V_1 - I_1(z_1 + z_2') = V_1 - I_1 Z_1$$

ignoring magnetizing current. The regulation is the *scalar* difference between V_1 and V_2' , and as shown in (b) it depends on the power factor of the secondary load.

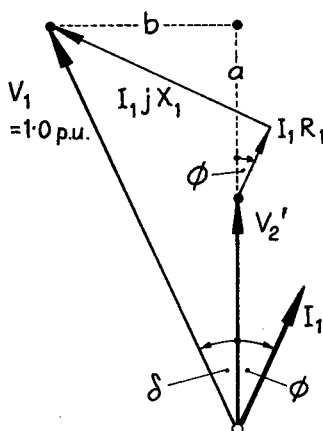
It is convenient to define the *per-unit resistance* (or per-unit full-load I^2R loss) and the *per-unit reactance* respectively in the form

$$\begin{aligned} \epsilon_r &= I_1^2 R_1 / V_1 I_1 = I_2^2 R_2 / V_2 I_2 = P_c / S \\ \epsilon_x &= I_1^2 X_1 / V_1 I_1 = I_1 X_1 / V_1 = I_2 X_2 / V_2 \end{aligned} \quad (5.5)$$

These can be combined to give the *per-unit leakage impedance*

$$\epsilon_z = \sqrt{(\epsilon_r^2 + \epsilon_x^2)} \quad (5.6)$$

Multiplied by 100 these give the percentage resistance, leakage reactance and leakage impedance respectively. The voltage phasor diagram in Fig. 5.6 uses



5.6 Assessment of regulation.

per-unit values to obtain the regulation. With $V_1 = 1.0 \text{ p.u.}$, then $a = \epsilon_r \cos \phi + \epsilon_x \sin \phi$ and $b = \epsilon_x \cos \phi - \epsilon_r \sin \phi$, where ϕ is the phase angle of the secondary load. The angle δ between V_1 and V_2' is defined by $\delta \cong b/V_1 = b$, and $\cos \delta = (V_2' + a)/V_1 = V_2' + a$. Expanding $\cos \delta = \cos b$ as a series, $(V_2' + a) = 1 - \frac{1}{2}b^2 + \dots$, whence the per-unit regulation is given by the *scalar* equation

$$\begin{aligned} \epsilon &= V_1 - V_2' = 1 - V_2' \\ &= \epsilon_r \cos \phi + \epsilon_x \sin \phi + \frac{1}{2}(\epsilon_x \cos \phi - \epsilon_r \sin \phi)^2 - \dots \end{aligned} \quad (5.7)$$

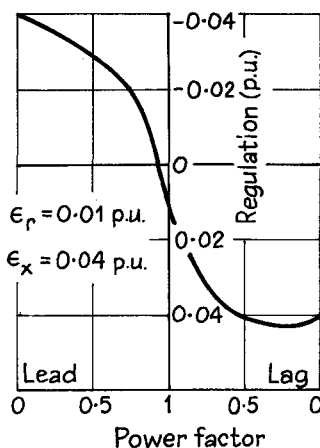
If ϵ_z is less than 0.1 (or 10%) the squared term can be ignored to give

$$\epsilon = \epsilon_r \cos \phi + \epsilon_x \sin \phi \quad (5.8)$$

The full-load regulation is a maximum when $\phi = \arctan(\epsilon_x/\epsilon_r) = \arctan(X_1/R_1)$; this is at a moderately low lagging power factor. It is zero for $\phi = -\arctan(\epsilon_r/\epsilon_x)$ corresponding to a high leading power factor, and for lower leading power factors the secondary voltage *rises* between no-load and full-load condition. A typical case for $\epsilon_r = 0.01 \text{ p.u.} = 1\%$ and $\epsilon_x = 0.04 = 4\%$ is shown in Fig. 5.7, the diagrams in Fig. 5.5(b) providing an explanation in phasor terms.

Regulation is a numeric, not a complex quantity, so that the regulation produced by two impedances connected in series can be combined algebraically. Thus we could write

$$\epsilon = \epsilon_{12} = \epsilon_1 + \epsilon_2$$



5.7 Regulation curve.

where ϵ_1 and ϵ_2 are the parts of the regulation associated with the individual impedances. In the present case the latter represent the primary and equivalent secondary leakage impedances in the equivalent circuit of a two-winding transformer. There is a clear distinction between r_1 and r_2' , but the division in $X_1 = x_1 + x_2'$ is arbitrary. However, if the windings are regarded as having distinct leakage reactances, each can be credited with individual leakage impedances and, consequently, per-unit regulations ϵ_1 and ϵ_2 . This takes care of all mutual effects and is extremely convenient where the transformer has more than two windings.

5.5 Operational equivalent circuits

The transformer is a device in which electromagnetic phenomena occur with electric and magnetic fields of considerable structural complexity interacting to transfer energy from primary to secondary circuit; in which intricate heat-exchanges take place between hot metals and fluid coolants; and in which the mechanical structure is subject to forces of thermal, magnetostrictive and magnetic origin, and responds by deflection, deformation and vibration.

Within normal limits of load and working conditions, the transformer behaves electromagnetically as a simple low-loss voltage-changer, but during faults or transient system conditions its reaction can be complicated. In consequence, a transformer may have to be considered from a viewpoint appropriate to particular conditions of operation. Much labour can be saved by including only those characteristics that are essential and omitting those whose influence is small. This is a matter of selecting a 'model' to suit the conditions of immediate interest. Typical models are listed below.

Mechanical. The laminated core, not a very rigid structure, has cyclic dimensional changes due to magnetostriction and magnetic forces of repulsion between laminae, in both cases at twice excitation frequency. Mechanical forces are developed on and between coils. Coils may deform, and cores vibrate in complex modes.

Thermal. The thermal transfer and the flow of coolant are vital in determining maximum working temperatures and the consequent life of the insulation. Generally the heat dissipation system can be considered as a problem almost on its own.

Electromagnetic. The behaviour of a transformer subjected to steep-fronted voltage surges is determined initially by the consequent distribution of the electric field, subsequently by the interchange of electric and magnetic field energies. The model can thus be represented in terms of a capacitor-inductor network with damping.

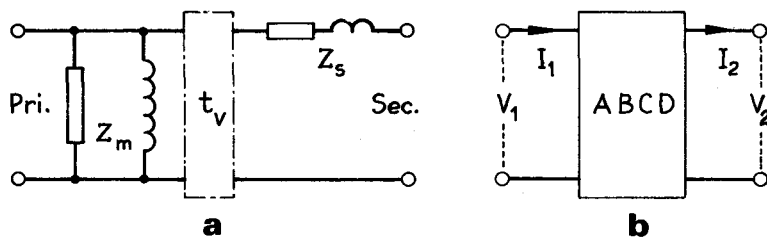
Magnetic. Switching-in phenomena are markedly affected by magnetic saturation, making it necessary to include some form of B/H relation in the appropriate model. Even in normal operation the flux and e.m.f. harmonics resulting from saturation may have to be taken into particular account, and not all harmonic fluxes have a path wholly within the core.

Normal Operation. This model, developed in Fig. 5.1, is the network equivalent using leakage impedance concepts in terms of 'lumped' parameters. As has already been shown, it may itself be simplified to the forms of Fig. 5.1(b) for no-load conditions (core loss and magnetizing current) and Fig. 5.2 for short-circuit conditions (I^2R loss and regulation).

System Network. In the analysis of supply-system operation, a transformer may be represented as a two-port, or even as a simple series impedance. In more precise studies account may be taken of the fact that the turns-ratio and the no-load voltage ratio are not quite the same, the divergence being greater with smaller ratings. Morris [23] has shown that *any* transformer can be accurately represented by the equivalent circuit in (a) of Fig. 5.8, constructed from direct measurements. The chain-dotted line represents a loss-free transformer which introduces the complex no-load voltage-ratio. The parameters are Z_m , a complex impedance found from the primary input voltage and current with the secondary open-circuited; Z_s , an impedance measured between the secondary terminals with the primary short-circuited; and t_v , the *complex* primary/secondary open-circuit voltage-ratio. The circuit (a) then follows from the Helmholtz-Thevenin theorem. An equivalent approach, shown in (b), is to invoke the theorem of passive quadripoles in which

$$V_1 = AV_2 + BI_2 \text{ and } I_1 = CV_2 + DI_2$$

where the quadripole parameters $ABCD$ can all be found from open-circuit and short-circuit tests.



5.8 Forms of practical equivalent circuit.

5.6 Vibration and noise

The rapid growth of domestic electrification and the ubiquity of large power transformers for local distribution, make noise produced by such apparatus a problem of social consequence. Under no-load conditions, the 'hum' developed by energized power transformers originates in the core, where the laminations tend to vibrate by magnetic forces. The noise is transmitted through the oil to the tank sides and thence to the surroundings. The essential factors in noise production are consequently: (a) magnetostriction (e.g., an extension of 0.000 12% for a flux density of 1 T); (b) mechanical vibration developed by the laminations, depending upon the tightness of clamping, size, gauge, associated structural parts, etc.; (c) mechanical vibration of the tank walls and (d) the damping.

This problem has been studied experimentally. It is established that magnetostriction is the first cause of transformer hum, but much of the noise depends on the natural frequency of vibration of the mechanical parts. Any constructional change which removes this frequency outside the audible range will be *ipso facto* an improvement. Transformer radiators and plain unreinforced tank walls are not usually troublesome as their natural frequencies lie as a rule below 100 Hz. Stiffening and clamping may do more harm than good if the effect is to raise the natural vibratory period into the audible range.

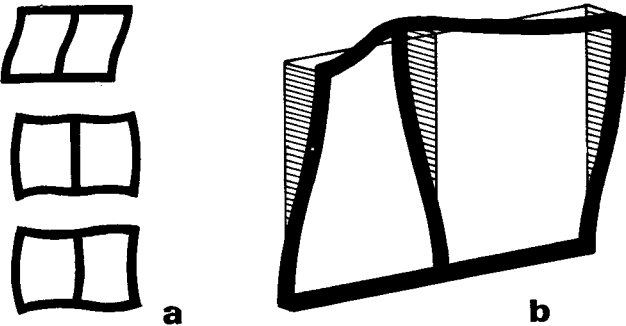
In general, the total noise emission may be reduced by (a) preventing core-plate vibration, which necessitates the use of a lower flux-density and attention to constructional features such as clamping bolts, proportions, and dimensions of the 'steps' in plate width, tightness of clamping and uniformity of plates; (b) sound-insulating the transformer from the tank by cushions, padding, or oil-barriers; (c) preventing vibration of the tank walls by suitable design of tank and stiffeners; and (d) sound-insulating the tank from the ground or surrounding air. There is no complete solution to the problem.

Core Vibration

If a core consisted of homogeneous steel, magnetostriction would cause vibration only in a plane parallel to that of the core. But as the laminations are not homogeneous, differential movements take place within the core, exciting vibrations in a direction perpendicular to the plane of the core. A core structure, having distributed mass and stiffness, possesses an infinite number of natural frequencies each associated with a particular mode. A few of these may lie in the range of magnetostrictive or magnetic-force harmonics, but the questionable rigidity of mitred corner joints makes the mechanical problem difficult to analyse. Some of the results computed and measured by Henshell and others [24] are illustrated in Fig. 5.9: (a) shows three modes of in-plane vibration, and (b) a typical out-of-plane mode, all modes having vibration frequencies in the range 40–300 Hz. Those modes of frequency near to a drive frequency (magnetic or magnetostrictive) can contribute considerably to the total transformer noise output.

5.7 Harmonics

The use of high flux densities in the cores of power transformers, imposed by

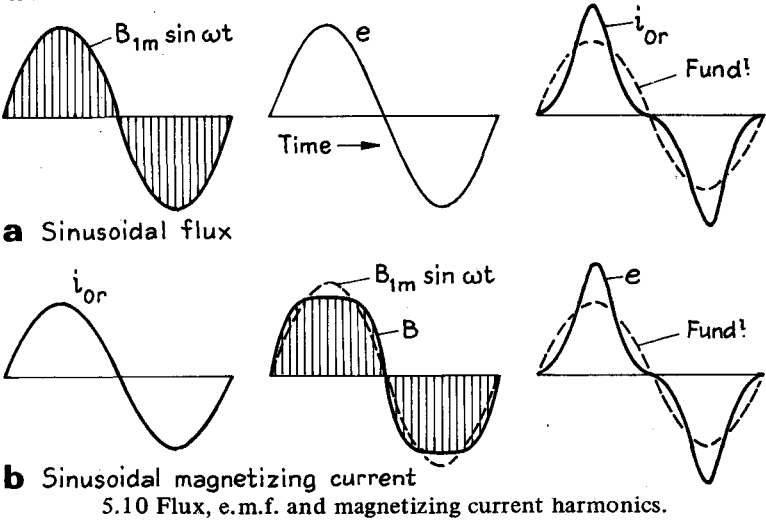


5.9 Vibration modes of 3-ph transformer core.

design requirements and the reduction of size, means that the saturation level is high. The typical magnetizing current waveform for a sinusoidally varying flux density, Fig. 2.14, clearly contains strong harmonics, chiefly those of order 3rd and 5th. Some of the implications can be seen by considering only the fundamental and the 3rd harmonic, as in Fig. 5.10(a), in which a flux density consisting only of the fundamental, of peak B_{1m} , induces the sinusoidal e.m.f. e but requires an exciting current i_{or} with a 3rd harmonic. If, as in (b), a purely sine magnetizing current were supplied, the flux density B would be flattened by saturation, introducing a 3rd harmonic flux density B_3 . Neglecting hysteretic phase-shift, the flux density can be taken as the sum of a fundamental $B_{1m} \sin \omega t$ and a 3rd harmonic $B_{3m} \sin 3\omega t$, whence the e.m.f. per turn embracing a net core area A_i is

$$e = \omega A_i [B_{1m} \cos \omega t + 3B_{3m} \cos 3\omega t]$$

a peaked waveform as indicated in Fig. 5.10(b). We examine the consequences of this.



5.10 Flux, e.m.f. and magnetizing current harmonics.

Single-phase Transformer

A sinusoidal supply voltage cannot give rise to harmonic magnetizing currents directly; it supplies fundamental reactive power, part of which is converted to harmonic power in the core by reason of its nonlinear B/H characteristic. The flow of a 3rd harmonic current implies the presence of a 3rd harmonic e.m.f. and therefore a corresponding harmonic in the flux waveform. The harmonic currents flow through the effective 3rd harmonic impedances of the primary winding and its supply network, and also (to a limited extent) in the secondary winding and its load. The core flux has therefore a waveform intermediate between the two shapes shown shaded in Fig. 5.10, being more flattened (and the magnetizing current less peaked) if the 3rd harmonic impedances are high. If the *network* impedance is negligible, the whole of the 3rd harmonic e.m.f. is expended in driving the harmonic current through the appropriate triple-frequency impedance of the transformer and no harmonic voltage appears across the terminals of the primary; but if not, the supply voltage will show a 3rd harmonic distortion. The higher harmonic effects due to core saturation are similar to those described for the 3rd harmonic.

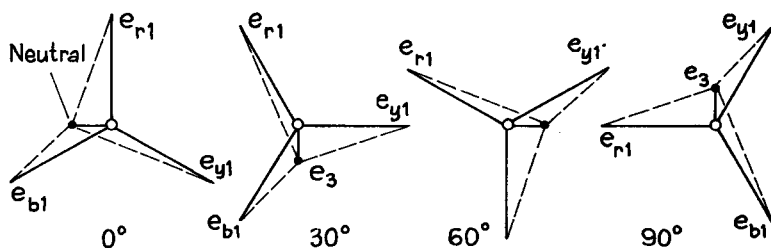
Three-phase Bank of Single-phase Transformers

Each of the three separate cores must carry the flux demanded by its operating conditions, but the effects are now modified by the phase interlinkage. In the following both supply and load are balanced and star-connected. The connection symbolism of the transformer bank is explained in Sect. 5.9.

(a) *Dd Connection.* In a symmetrical 3-ph voltage system, the fundamental and the harmonics of order $(3n \pm 1)$, where $n = 1, 2, 3 \dots$, are all representable by phasors with a 120° displacement. But those of order $3n$ (for brevity termed the *triplen* harmonics) are voltages co-phasal in all three phase windings and peak simultaneously. Delta connection provides a closed path to triplen e.m.f.s, which circulate triplen currents in the closed mesh and are absorbed by the triplen leakage impedance drop: there is no resultant triplen voltage at the line terminals. In the Dd case, therefore, the action is as follows. The supply voltage provides only a fundamental magnetizing current, excites a flattened flux waveform with a triplen content and therefore induced triplen e.m.f.s. The latter circulate triplen current in the delta, to add to the sinusoidal supply current as in Fig. 5.10(a) and to restore the flux waveform nearly to a pure sinewave. Thus the primary voltage and current are sinusoids, the flux is nearly so, and the necessary triplen magnetizing current is circulated around the closed mesh. This condition applies to the 3rd, 9th, 15th $\dots$ harmonics. Harmonics of other than triplen order (i.e. 5th, 7th $\dots$) behave like sets of balanced 3-ph quantities.

(b) *Yd and Dy Connection without Neutral.* Either connection operates as in (a), except that with a closed mesh on one side only, the triplen impedance is greater and the flux waveform diverges rather more from the sinusoidal.

(c) *Yy Connection without Neutrals.* The triplen e.m.f.s are all directed simultaneously away from or towards the star point and therefore cancel between any pair of lines. Consequently no triplen currents can flow, and (apart from the 5th, 7th $\dots$ harmonic components) the input magnetizing currents are



5.11 Oscillation of neutral point.

sinusoidal. The flux waveform is flattened, its triplen harmonics providing the triplen e.m.f.s. Balance between lines of the supply voltage and the e.m.f. is maintained, but the effect on the star-point voltage is to make it exhibit the *oscillating neutral* phenomenon, Fig. 5.11. The fundamental e.m.f.s, e_{r1} , e_{y1} and e_{b1} , are mutually displaced by $2\pi/3$ rad. The superposed 3rd harmonic e_3 is co-phased in all three legs, and the resultant phase voltages are shown by the dotted lines. The diagram shows four successive instants displaced by $1/12$ period, it being noted that the phasor e_3 rotates thrice as fast as the fundamental system. All the line-to-star-point voltages fluctuate, causing 'neutral oscillation'. The non-triplen harmonics form balanced systems with $2\pi/3$ displacement and appear across the lines.

(d) *Yy Connection with Neutrals*. The presence of the neutral connection effectively separates the three 1-ph transformers, the neutrals carrying the co-phased triplen compensating currents.

(e) *Other Yd Connections*. All variants of the possible connections will fall under one or other of the cases (a) to (d) above. A delta connection always provides a closed path for triplen currents; a neutral connection to the supply allows triplen currents to flow into the supply network.

(f) *Stabilizing D (Tertiary) Winding*. This is an auxiliary winding on each transformer, connected in delta and applied to Yy or Yz connected 1-ph banks to provide a closed path in which triplen currents can circulate. In effect, it decreases the negative-phase-sequence impedance.

Three-phase Unit Transformer

It is essential to distinguish between the cases where the phases are magnetically separate, and those where they are magnetically (as well as electrically) interlinked. In the 3-ph shell-type transformer the magnetic circuits are separate and do not interact, and the discussion above for the bank of 1-ph transformers will apply. In the more common 3-ph 3-limb core-type unit, however, this is not so. Any triplen flux harmonics are all upward or all downward at a given instant in the limbs, and their return path must be external (i.e. in the air or oil or the walls of the tank). These paths have a high reluctance and there is a strong tendency to suppress triplen harmonic core fluxes. The residuals may cause eddy-current loss in the tank, an effect mitigated if a copper band is fitted around the inside perimeter of the tank.

In five-limbed cores, the end limbs provide a magnetic path for triplen fluxes.

Harmonic Effects

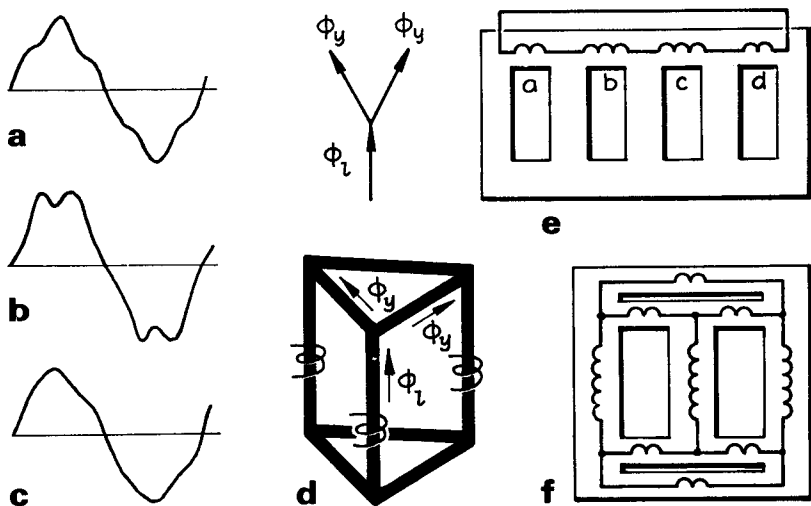
The effects of harmonic *currents* are: additional I^2R loss due to circulating currents; increased core loss; magnetic interference with protective gear and communication circuits.

Harmonic *voltages*, depending on their magnitude and frequency, may cause: increased dielectric stress; electric-field interference with communication circuits; resonance between the inductance of the transformer windings and the capacitance of a feeder to which it is connected.

Harmonic Current Compensation

For core densities exceeding 1.5 T, the harmonic current content markedly rises. It may in some cases be reduced by special compensating arrangements.

A saturated Yy isolated-star-point core-type transformer may have a magnetizing-current waveform like (a) in Fig. 5.12. The 3rd harmonic is precluded by the connection, the 5th is predominant; further, the high reluctance



5.12 Harmonic current compensation.

to 3rd harmonic fluxes forces the flux towards the sinusoidal. If the transformer has a 5-limb core (e), however, a 3rd harmonic flux path is provided by the end limbs and the flux waveform flattens; the magnetizing current waveform (b) shows that the 5th harmonic is reversed and the fundamental slightly reduced. Comparison of (a) and (b) suggests that if the end limbs had a suitable reluctance, there would be zero 5th harmonic in the magnetizing current, as in (c). The method applies only to isolated star/star and to star/zigzag connections.

The parallel connection of a star/star with a delta/delta transformer results in suppression of 5th and 7th harmonic currents since these are equal and opposite in phase, cancelling in the combined input. The same effect in a single transformer may be obtained by providing in one magnetic circuit a 'star' and

a 'delta' path. If a symmetrical 3-phase transformer, Fig. 5.12(d), is considered, it is seen that the limb flux Φ_l splits into two yoke fluxes Φ_y , differing in phase each by 30° with respect to the limb flux, and requiring 5th and 7th harmonic currents displaced respectively by $\pm 5 \times 30^\circ = 150^\circ$ and $\pm 7 \times 30^\circ = 210^\circ$ with respect to the current components in the limb. The two yokes together, by symmetry, take 5th and 7th harmonic excitation in phase opposition to that of the limb to which they are magnetically connected, and the combination may be devoid of harmonic magnetizing currents if the reluctances are suitably chosen unless high saturation densities are used, in which case harmonic yoke excitation can be provided by a yoke winding. This magnetic star-delta arrangement is inherent in the 5-limb core (e), and could also be obtained in a 3-limb core (f) by slitting the yokes. A more practical method would be to wind round the yokes a delta-connected winding to force the fluxes into the required relationship in spite of any difference in reluctance. In (f) there is in addition an auxiliary star-connected winding interlinked with the delta winding on the yokes to aid production between limbs and yokes of the 5th and 7th harmonic currents.

5.8 Transients

Apart from thermal transients, short-duration switching currents and line-surge voltages may occur in transformer operation, of such magnitude as to impose high mechanical stresses or insulation damage.

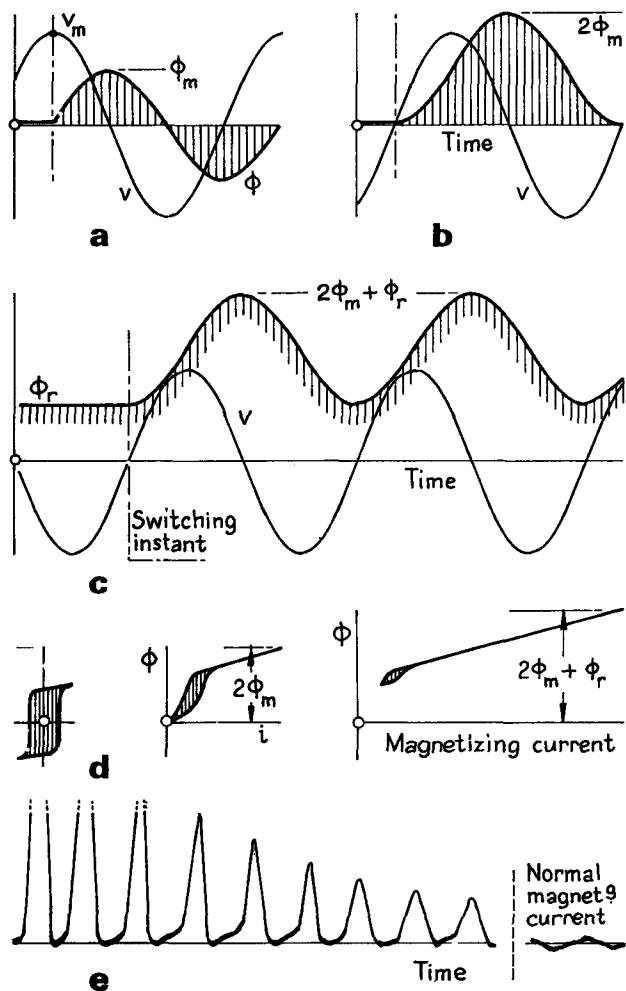
Switching

When the primary winding of an unloaded transformer is switched on to normal voltage supply, it acts as a nonlinear inductor. Neglecting resistance, the applied voltage must be balanced at every instant by the e.m.f. induced by the magnetizing current. The behaviour depends on the magnitude, polarity and rate of change of the applied voltage at the instant of switching. Assume first that the core is initially unmagnetized and that the switch is closed at a *voltage peak*. The balancing e.m.f. demands a flux having maximum rate of change, so that the flux starts to grow from zero in the required direction, as also does the magnetizing current. But such conditions are those of normal no-load operation, Fig. 5.13(a), and there is no transient. If, however, the switch closes at a *voltage zero* as in (b), the voltage is positive during the first half-period, throughout which the flux must therefore increase, with a total *change* corresponding to that of a 'normal' half-period. It therefore reaches a maximum of twice the normal peak, i.e. $2\Phi_m$, a phenomenon called the *doubling effect*. In subsequent half-periods the effect of losses rapidly reduces the flux waveform to symmetry about the time-axis.

Flux doubling implies a peak density of twice the normal steady-state value. With the high saturation levels employed for modern transformers, the magnetizing current required for twice normal flux density is very great. For a peak density of 1.4 T, for example, the excitation is about 3 kA-t/m; for twice this density it might be increased 100-fold. A normal magnetizing current of 0.05 p.u. is for such conditions raised to 5 p.u., producing electromagnetic forces 25 p.u. of normal. The doubled flux density will not in fact

be reached, for with such overcurrents the transformer resistance drop is significant, and further limitation is imposed by the source impedance of the supply network. The transformer switching transient is referred to as the *inrush current*. It causes difficulties in the application of current-balance protection.

Remanent flux in the core can aggravate the condition. If the primary voltage is applied at a zero instant and in such a direction that the rising field augments the existing remanent flux Φ_r , as in Fig. 5.13(c), the sinusoid of flux variation has still the amplitude $2\Phi_m$, but is entirely offset from zero and rises to a peak value $2\Phi_m + \Phi_r$.



5.13 Inrush current transients.

Hysteresis loops in terms of flux and current are shown in (d) for cases (a), (b) and (c). The first is normal and symmetrical, the second is unidirectional and the loop area is actually reduced. The third, for case (c), shows super-saturation and an almost negligible hysteresis loss.

An analytical method of predicting inrush currents using an exponential series to represent the magnetic nonlinearity (excluding hysteresis and eddy-current effects but including interphase coupling) has been devised by Macfadyen and co-workers [108]. Analysis is complicated by the drastic change in flux path. At low densities the flux is substantially confined to the core, but at high saturation the permeability of the core steel is so low that the core shares much of the flux with what are normally leakage paths. The peak magnetizing current approximates to that determined by the inductance L of the primary winding considered as an air-cored coil. The current peaks thereafter fall rapidly because the rate of decay is a function of the ratio R/L , where R is a large loss resistance and L is subnormal. As the flux density reduces towards its normal excursions the loss resistance reduces but the inductance increases, slowing the rate of decay and causing current distortion to persist for several seconds, as shown by the typical oscillogram in Fig. 5.13(e).

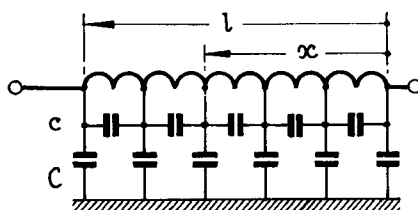
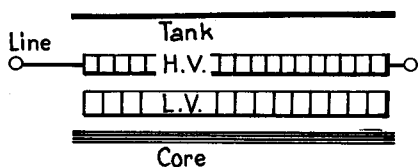
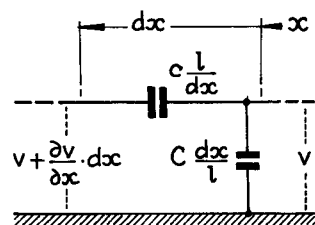
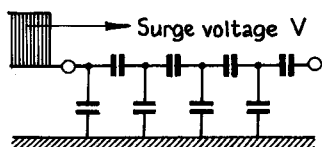
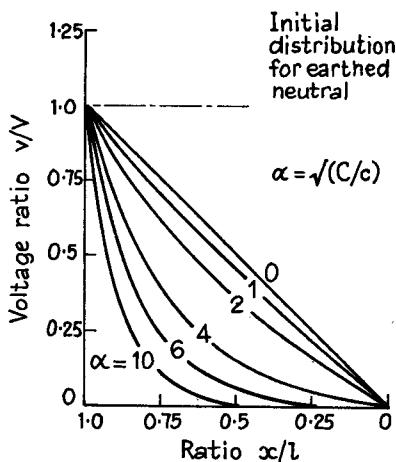
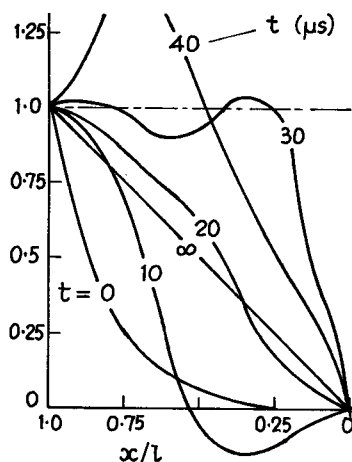
A condition akin to remanent flux can occur in a 3-limb transformer if one primary terminal is energized later than the other two, and in the neutral connection large inrush currents characterized by an exponentially decaying d.c. component may occur.

Surges

Surge overvoltages, initiated on overhead transmission lines by switching, faults or lightning discharges, may be imposed on the system transformers. The surges may have a very steep wavefront with a rate of rise 1 000 times as great as the peak rate of normal voltage at operating frequency, imposing intense and rapidly changing electric stresses within the transformer. For such a condition the appropriate model represents the transformer in terms of an equivalent capacitance network, the capacitance values being too small to affect normal-frequency behaviour. The problem has often been analysed, e.g. by Lewis [28], by Dent et al. [29] and by Adamson and Mansour [30].

Model. The network adopted is that of Fig. 5.14(a), in which c is the total distributed series capacitance through the winding of axial length l . If the winding has n evenly spaced turns (or coils), then c can be regarded as the series sum of n turn-to-turn (or coil-to-coil) capacitances each of nc . The total distributed shunt capacitance of the winding to the tank, core and l.v. winding (i.e. in effect to earth) is C . The winding has a distributed series inductance L and resistance r .

Response to Unit-function Surge Voltage. The winding inductance offers such a great opposition to rapid current changes that its presence can under these conditions be ignored. The model can then be simplified initially to the distributed series-parallel capacitance network (b). Consider an elemental length dx at a distance x from the far end: it consists of a series capacitance cl/dx and a shunt capacitance $C \cdot dx/l$. Then for the charge on the element of the winding

**a****b****c****d**

5.14 Surge voltage distribution.

and its voltage to earth we can write

$$\frac{\partial q}{\partial x} = \frac{C \cdot dx}{l} \cdot \frac{\partial v}{\partial t} \quad \text{and} \quad \frac{\partial v}{\partial x} = \frac{dx}{cl} \cdot \frac{\partial q}{\partial t}$$

Differentiating the former with respect to t and the latter with respect to x gives

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{l^2} \frac{C}{c} \frac{\partial^2 v}{\partial t^2} = \frac{\alpha^2}{l^2} \cdot \frac{\partial^2 v}{\partial t^2}$$

where $\alpha = \sqrt{C/c}$. The solution takes the form

$$v = A \exp [\alpha(x/l)] + B \exp [-\alpha(x/l)]$$

If the remote end of the winding is earthed, then $v = 0$ at $x = 0$ gives $A = -B$. At the line end $x = l$ and $v = V$, the magnitude of the unit-function surge voltage. With these boundary conditions the voltage at a point x in the winding is

$$v = V \frac{\sinh \alpha(x/l)}{\sinh \alpha} \quad (5.9)$$

A similar treatment for the case in which the remote end of the winding is isolated gives

$$v = V \frac{\cosh \beta(x/l)}{\cosh \beta}$$

where $\beta = \sqrt{(2C/c)}$. This case is less common.

Initial Voltage Distribution. Fig. 5.14(c) shows the *initial* distribution of a surge voltage over a uniform winding with the far end earthed (at $x = 0$). For $\alpha = 0$, which means negligible capacitance to earth, the distribution is uniform, but in the presence of large earth capacitance, e.g. $\alpha = 10$, most of the voltage is dropped across a small fraction of the line end of the winding, in which the voltage between turns becomes excessive. Because the surge voltage V may itself be several times the working voltage, the first few turns (or coils) may be subjected to an electric stress of several hundred times normal.

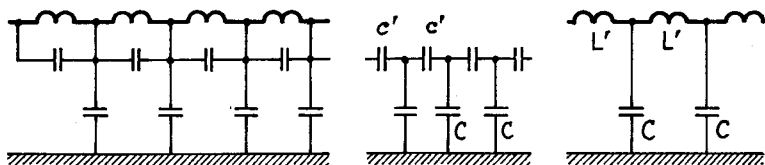
Final Voltage Distribution. After the transients have died away, the tail of the surge is the equivalent of a direct sustained voltage, and the voltage distribution obviously becomes

$$v = V(x/l) \quad (5.10)$$

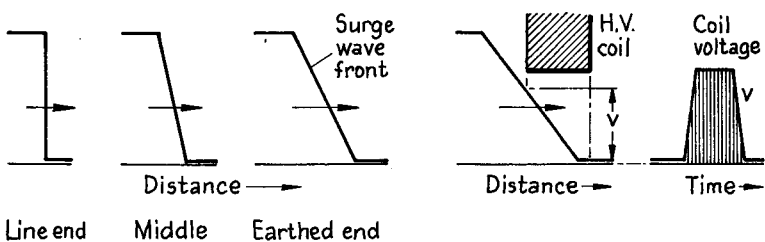
which is identical with that in (c) for $\alpha = 0$.

Intermediate Transient. Between the initial and final distributions, eqs.(5.9) and (5.10), an interchange of stored energy between capacitors through coil inductances generates complex oscillations at a variety of natural frequencies. As a result the voltage distribution changes characteristically with time. As an example, a transformer with $\alpha = 6$ might exhibit the distributions shown in Fig. 5.14(d) at intervals of $10 \mu s$ following the application of a unit-function voltage at $t = 0$, at which instant the initial distribution is that in (c) for this value of α . Strongly fluctuating electric stresses are thrown upon various parts of the winding insulation, until the oscillations die out by loss attenuation to give the uniform final distribution (marked $t = \infty$).

Travelling Waves. Another approach to the problem is to analyze the surge into a Fourier series of component sines impressed on one end of a ladder network, Fig. 5.15(a). At a critical frequency corresponding to the rejector resonance of the elements L' and c' , the ladder offers infinite impedance (neglecting loss). At *super-critical* frequencies the component sines are presented with the equivalent circuit (b), resulting in the initial surge-voltage distribution already discussed. At sub-critical frequencies the component sines encounter the equivalent circuit (c) and travel through the h.v. winding as through a low-pass filter. Both L' and c' are frequency dependent, and affect the velocity and attenuation of the travelling-wave components differently: this modifies



a Equivalent ladder netwk. **b** Supercrit. freq. **c** Subcrit. freq.



d Travelling wave in h.v. winding **e** Coil voltage due to travelling wave

5.15 Travelling waves in h.v. winding.

the wavefront as it passes through the winding, as in (d). At the end of the winding reflection takes place in accordance with the termination (e.g. earth or open circuit). The voltage to earth at any point and at a given instant is then the resultant of the initial and reflected voltage waves.

The voltage across an individual section or coil of the winding is seen from Fig. 5.15(e) to have a pulse form due to the passage of the travelling wave.

As the surge voltage distribution is affected by the several capacitances, and consequently by the insulation design, it is essential to co-ordinate the insulation with the effect that it will have on the voltage distribution.

Surge Protection. Before the effect of surge voltages was understood it was the practice to re-inforce the insulation of the line end of a winding to withstand the impulsive voltage gradients; but by reducing the series capacitance this actually intensified the trouble. External *surge-absorbers* may be connected between the transmission-line and transformer terminals to reduce the steepness of the wavefront and to dissipate some of the surge energy, but it is preferred to design the transformer in such a way that an approximation to the $\alpha = 0$ condition is achieved.

The provision of *static shields* aims to neutralize the coil-to-earth capacitance C by introducing coil-to-line capacitance, on the same principle as in grading a transmission-line insulator string. Suitable coil-to-line capacitance C_1 provides the coil-to-earth currents through C , leaving the inter-coil currents in c all equal and giving uniform voltage distribution over the winding. This may be done by mounting metal shields around the winding and connecting them to the line terminal, as in Fig. 5.16(a). The effect on the electric field is shown in (b), the figures indicating percentage equipotentials.

An alternative (or additional) method is the use of *interleaved coils*, (c).

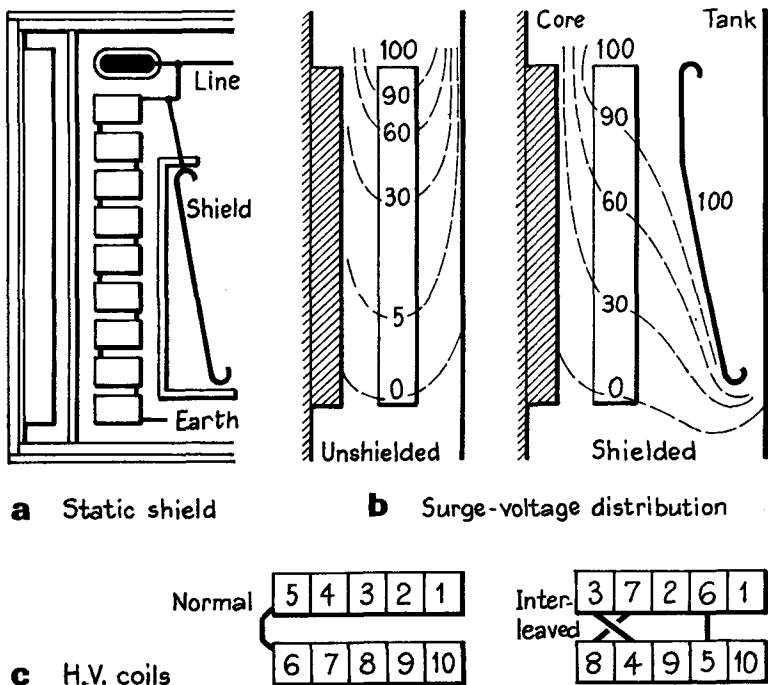
Two normal disc coils are shown: if the order of turn-connection is altered to increase the normal voltage between turns, the effect is to raise the inter-turn series capacitance c , and so reduce the ratio $\alpha = \sqrt{C/c}$. Windings of interleaved double-disc type are better able to withstand impulse voltages than non-interleaved double-disc coils; partially-interleaved windings combine the two, with the interleaved section normally at the h.v. end, but they introduce surge-voltage reflections. The problem of optimizing such windings is discussed by Schleich [31].

EXAMPLE 5.3: Compare the effective series capacitance of the interleaved disc-coil pair in Fig. 5.16(c) with that of the normal pair.

Let v be the voltage per coil-pair, and n the number of turns (in this case $n = 10$). The effective capacitance between the coil terminals (turns 1 and 10) can be found only by a field plot of the rather complicated electric-field pattern; but as the turns are much closer together than are the discs, we may as an approximation consider only the turn-to-turn capacitance C_t .

Normal Coil: The voltage between turns is $v_t = v/10$. The electric-field energy between successive and adjacent turns is

$$w_t = \frac{1}{2} C_t (v/10)^2$$



5.16 Surge voltage grading.

and the total for the complete pair of discs (8 interturn gaps) is

$$w_n = 8 \times \frac{1}{2} C_t (v/10)^2 = \frac{1}{2} C_n v^2$$

where C_n is the total effective series capacitance between turns 1 and 10, whence

$$C_n = C_t(8/100) = 0.08 C_t$$

Interleaved Coil: The geometry is unchanged, and so therefore is C_t ; but the effect of interleaving is to increase the voltage between adjacent (but no longer successive) turns to $5v_t$ (e.g. turn 1 to turn 6, 2 to 7 . . .) or to $4v_t$ (e.g. 6 to 2, 7 to 3 . . .). The total field energy is consequently

$$\begin{aligned} w_i &= \frac{1}{2} C_t [4(5v/10)^2 + 4(4v/10)^2] \\ &= 164 \times \frac{1}{2} C_t (v/10)^2 = \frac{1}{2} C_i v^2 \end{aligned}$$

and the effective series capacitance is

$$C_i = C_t (164/100) = 1.64 C_t$$

This markedly reduces the coefficient α and improves the surge-voltage distribution.

5.9 Three-phase connections

With windings that can be connected in star, delta, zigzag or vee, and with primary, secondary, tertiary and auto-connected windings, the variety of possibilities is considerable.

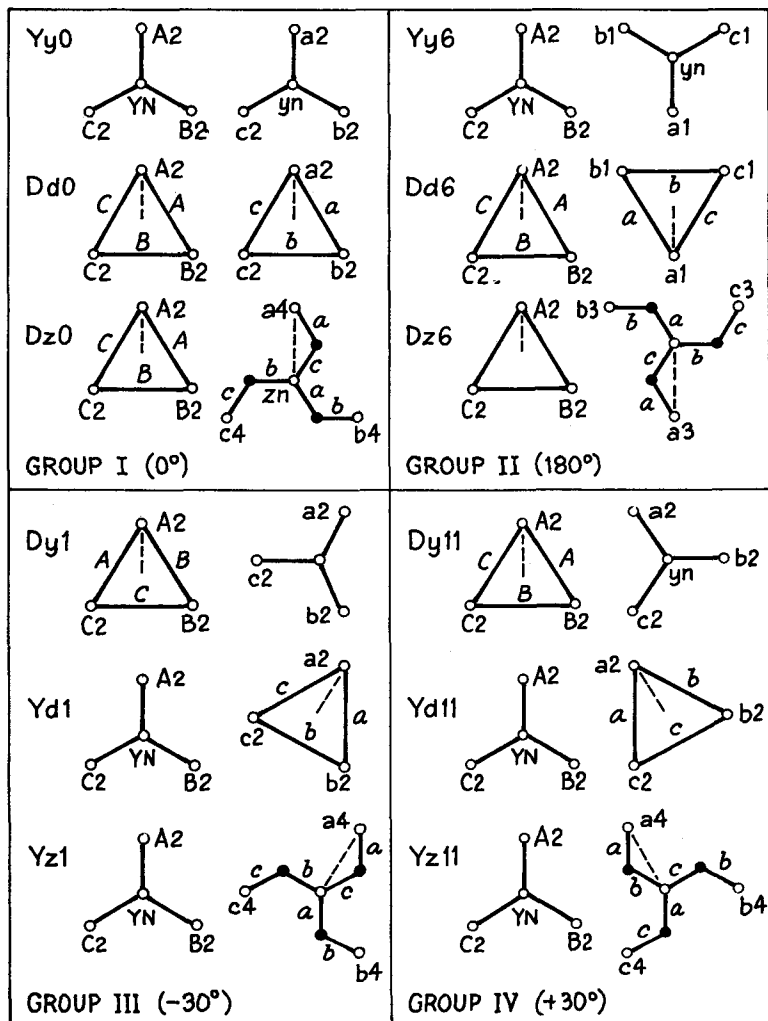
A choice is possible between a 3-ph unit and a bank of three 1-ph transformers. The unit may cost about 15% less than the bank and will occupy much less space. There is little difference in reliability, but it is cheaper to carry spare 1-ph than 3-ph transformers if only one installation is concerned. Single-phase units may be preferred in mines to ease underground transport. The possibility of operating two 1-ph units in open delta at 58% of nominal full load is a practice more favoured in the U.S.A. than in Europe.

The choice between star and mesh connection merits separate consideration in each case. Star connection is cheaper, since mesh connection needs more turns and more insulation. The difference is small, however, at voltages below 11 kV. With very high voltages a saving of 10% may be effected, mainly on account of the insulation. An advantage of the star-connected winding with earthed neutral is that the maximum voltage to the core (frame or earth) is limited to 58% of the line voltage, whereas with a delta-connected winding the earthing of one line (due to fault) increases the maximum voltage between windings and core to the full line voltage. Technically the mesh-connected primary is essential where the l.v. secondary is a star-connected four-wire supply to mixed 3-ph and 1-ph loads.

Nomenclature

Transformer terminals conforming to BS 171 are brought out in rows, h.v. on one side and l.v. on the other, and are lettered from left to right facing the

h.v. side. The h.v. terminals have capital letters (e.g. ABC); the l.v. terminals have small letters (e.g. *abc*); tertiary windings are lettered in capitals with the figure 3. Neutral terminals precede line terminals. Each winding has two ends designated with the subscripts 1, 2; or if there are intermediate tapplings these are numbered in order of their separation from end 1. Thus an h.v. winding with four tapplings is labelled $A_1, A_2 \dots A_6$, with A_1 and A_6 forming the ends of the phase winding. If the e.m.f. of an h.v. phase winding has, at a given instant, the direction A_1A_2 , then at the same instant the e.m.f. direction in



5.17 Standard connections for 3-ph transformers.

the associated l.v. winding is $a_1 a_2$. The diagrams in Fig. 5.17 show the connections and e.m.f. phasors for a number of common arrangements.

Polyphase transformers are all allotted symbols giving the type of phase connection and the angle of *advance* in passing from the phasor representing the e.m.f. of the h.v. winding to that of the l.v. phasor at the corresponding terminal. The angle is indicated in terms of a clock face, the h.v. phasor being at 12 o'clock (zero) and the corresponding l.v. phasor at the hour-hand number. Thus 'YzD3 11' represents a (h.v. star/l.v. zigzag/tertiary delta)-connected 3-ph transformer with the l.v. (secondary) e.m.f. in a given phase combination at 11 o'clock, i.e. 30° in advance of the 12 o'clock position of the h.v. phasor. The groups into which 3-ph transformers are classified are as follows:

Group	Phase displacement	Two-winding connections		
1	Zero	Yy0	Dd0	Dz0
2	180°	Yy6	Dd6	Dz6
3	30° lag	Dy1	Yd1	Yz1
4	30° lead	Dy11	Yd11	Yz11

Connections

The main features of the more common connections are noted below. References to harmonic behaviour are dealt with in Sect. 5.7.

Star/Star (Yy0 or Yy6)

This is economical for small h.v. transformers as it minimizes the turns/ph and the winding insulation. With star points available on both sides it is possible to provide a neutral connection. Triplen voltages are absent from the lines, and (unless there is a neutral connection) no triplen currents flow. The neutral voltage may oscillate, and triplen voltages may be high in shell-type units.

Delta/Delta (Dd0 or Dd6)

This suits large l.v. transformers as it requires more turns/ph of smaller section. Large load unbalance can be tolerated, and triplen voltages are damped out. The absence of a star point may be a disadvantage.

Star/Delta (Dy or Yd)

This is very common for supply networks. It has a star point for a neutral to serve mixed 1-ph and 3-ph loads, and a delta winding that can carry triplen currents and so stabilize the starpoint voltage.

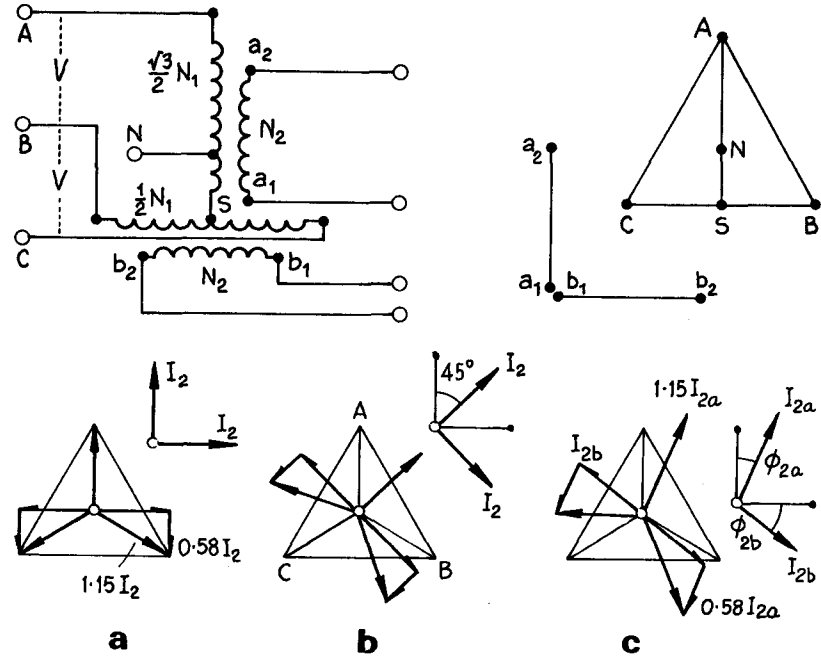
Interconnected-star/Star (Yz1 or Yz11)

The interconnected (or 'zigzag') star arrangement reduces triplen voltages and is not sensitive to conditions of unbalanced loading. But the zigzag is restricted to comparatively low-voltage windings, and as the phase voltages are composed of half-voltages with a 60° displacement, 15% more turns

are required for a given phase terminal voltage compared with a normal star. The connection in sometimes employed in rectifier supply.

5.10 Three/two- and three/one-phase connections

Phase conversion is needed in special cases, such as the supply to 2-ph electric furnaces.



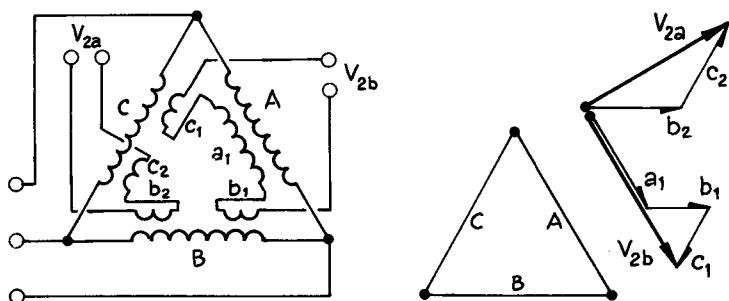
5.18 Scott connection.

Three/two-phase Connection

Scott Connection. This requires two transformers with different tapings and ratings, although they may be constructed for interchangeability. Consider in Fig. 5.18 the N_1 -turn primary of a 1-ph ('main') transformer connected between terminals B and C of a 3-ph supply of line voltage V . The voltage between line A and the mid point S of BC is $(\sqrt{3}/2) V = 0.87 V$. If now the primary of a second ('teaser') transformer with $0.87 N_1$ turns is connected across AS, the voltage per turn is the same for both main and teaser primaries. Then two identical secondaries with N_2 turns have terminal voltages of equal magnitude but phase quadrature. The 3-ph neutral point N is located on the teaser primary at one-third of the winding distance from S, where the voltage is $V/\sqrt{3} = 0.58 V$ in AN and $0.29 V$ in SN.

Let equal currents I_2 at unity p.f. be taken from the two secondary phases, Fig. 5.18(a), and let magnetizing current be neglected. There must, in each transformer, be an m.m.f. balance. In the main transformer the primary

balancing current is $2I_2N_2/\sqrt{3}N_1 = 1.15 I_2 (N_2/N_1)$. The teaser primary current is in phase with the star voltage AN. The total current in the main primary is the resultant of two components: the first is $I_2 (N_2/N_1)$ to balance the secondary current, the second is one-half of the teaser primary current in either direction from S (and having a zero net m.m.f. in consequence). The currents in lines B and C are shown in Fig. 5.18(a), taking $N_1 = N_2$ for clarity; their phasors, combining the rectangular components I_2 and $0.58I_2$, are co-phasal respectively with the star voltages NB and NC, and are equal to the teaser primary current. Thus for a balanced 2-ph load of unity p.f. the 3-ph side is balanced.



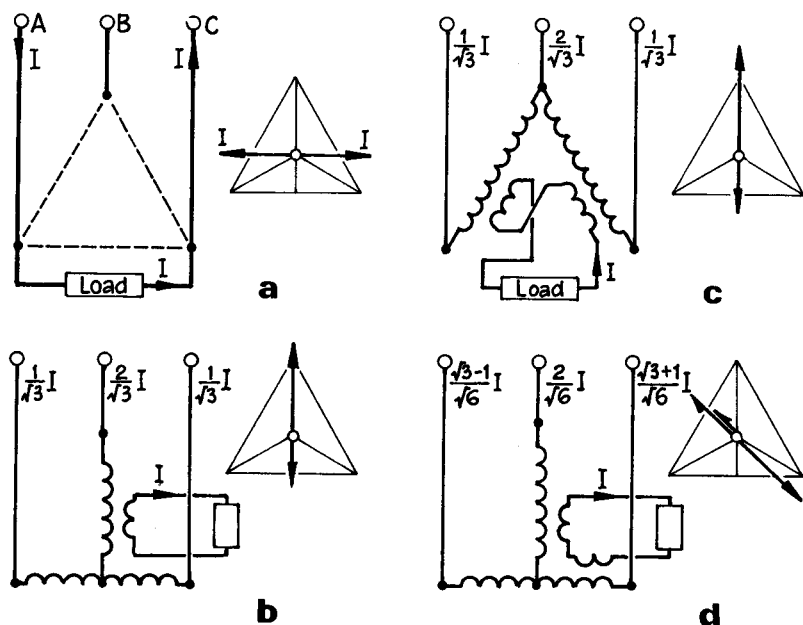
5.19 Le Blanc connection.

Case (b) is for a balanced 2-ph load of p.f. 0.71. Again the 3-ph load is balanced, and the main transformer loading is 15% greater than that of the teaser. Case (c) is for an unbalanced 2-ph load.

Le Blanc Connection. The 3-ph side consists of three windings in star or delta, wound on an ordinary 3-limbed core. If the 2-ph side is to have a phase voltage V , the limbs ABC in Fig. 5.19 are provided respectively with windings a_1 of voltage $\frac{2}{3}V$; b_1 of $\frac{1}{3}V$ with b_2 of $V/\sqrt{3}$; and c_1 of $\frac{1}{3}V$ with c_2 of $V/\sqrt{3}$. The windings when grouped give phase I of the 2-ph supply with a_1 , b_1 and c_1 in series; and phase II with b_2 and c_2 in series. The phasor sums show that the 2-ph side has equal voltages in phase quadrature. The use of b_1 and c_1 in phase I is to balance the m.m.f.s around the three interlinked magnetic circuits and to give 3-ph input balance for a balanced 2-ph load. The connection has the advantage of using a standard 3-ph core.

Three/one-phase connection

Single-phase power pulsates twice per period, but a balanced 3-ph load takes a power that, over the three phases together, is constant. It is therefore not possible to obtain 3-ph balance when supplying from it a 1-ph load unless there is some energy-storing device (such as rotating machine or a capacitor-inductor system) to give a 'flywheel' effect. Fig. 5.20 shows some simple direct methods not employing storage. The actual division of the 1-ph current can be made to assume any proportions between 1:1:0 and $1:\frac{1}{2}:\frac{1}{2}$. Connection (a) is the simplest; (b) is one phase of a Scott connection; (c) is an open delta; and (d) uses both units of the 2-ph Scott arrangement.



5.20 Three-phase/single-phase connection.

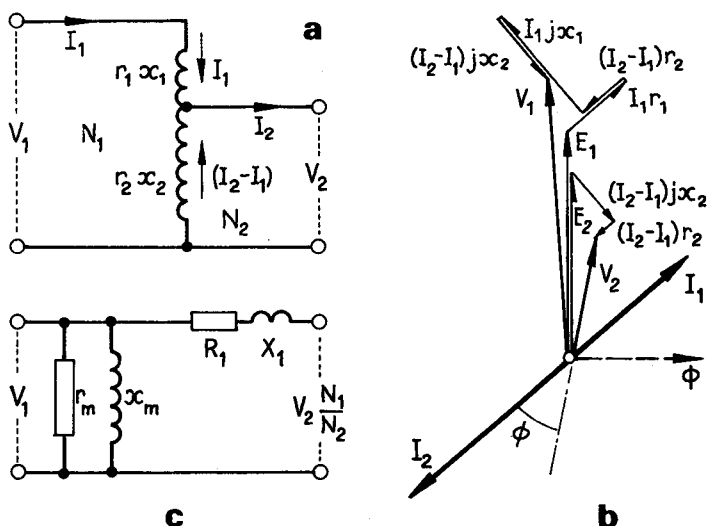
The 3-ph supply, because of its single-phase loading, carries equal positive- and negative-sequence currents. The p.p.s. currents supply the 1-ph load, while the n.p.s. currents, ineffective with the p.p.s. voltage, simply increase the line loss.

5.11 Auto connection

An auto transformer has windings common to primary and secondary, the input and output circuits being electrically connected as one continuous winding per phase, with input and output currents superposed in the common section. The principal application is to cases of voltage ratio not less than about 2:1 where electrical isolation of the primary from the secondary circuit is not essential; such applications include starters and the interconnection of h.v. transmission lines. The advantage gained is the saving in transformer size, rating and loss.

Neglecting magnetizing current, let the 1-ph auto-transformer in Fig. 5.21 have N_1 primary turns tapped at N_2 for a lower-voltage secondary. For an output of I_2 at voltage V_2 , the input is $I_1 = I_2(N_2/N_1)$ at voltage $V_1 = V_2(N_1/N_2)$ if the losses can be ignored. The current in the common part of the winding is $I_2 - I_1$ so that here the conductor section may be smaller. With the same conductor current density throughout, the ratio G_a/G_2 of conductor material in the auto and two-winding cases is

$$\frac{G_a}{G_2} = \frac{I_1(N_1 - N_2) + (I_2 - I_1)N_2}{I_1N_1 + I_2N_2} = 1 - \frac{V_2}{V_1}$$



5.21 Auto transformer.

The saving in material and cost is less than this, as the core is nearly as large as in the two-winding transformer. For a voltage ratio of 2:1, about 50% saving in winding material might be obtained, and the overall cost of the unit might be 60–70% of that of a normal transformer of the same input rating.

Equivalent Circuit

Fig. 5.21 shows at (a) the equivalent circuit ignoring magnetizing and core-loss current, and at (b) the corresponding phasor diagram. For a load phase angle ϕ the primary and secondary voltages are approximately

$$V_1 = E_1 + I_1(r_1 \cos \phi + x_1 \sin \phi) + (I_1 - I_2)(r_2 \cos \phi + x_2 \sin \phi)$$

$$V_2 = E_2 - (I_2 - I_1)(r_2 \cos \phi + x_2 \sin \phi)$$

Combining these and writing $E_2 = (N_2/N_1)E_1$ and $I_2 = (N_1/N_2)I_1$ yields the relation

$$\begin{aligned} V_1 &= (N_1/N_2)V_2 + I_1(r_1 + r_2') \cos \phi + I_1(x_1 + x_2') \sin \phi \\ &= (N_1/N_2)V_2 + I_1 R_1 \cos \phi + I_1 X_1 \sin \phi \end{aligned}$$

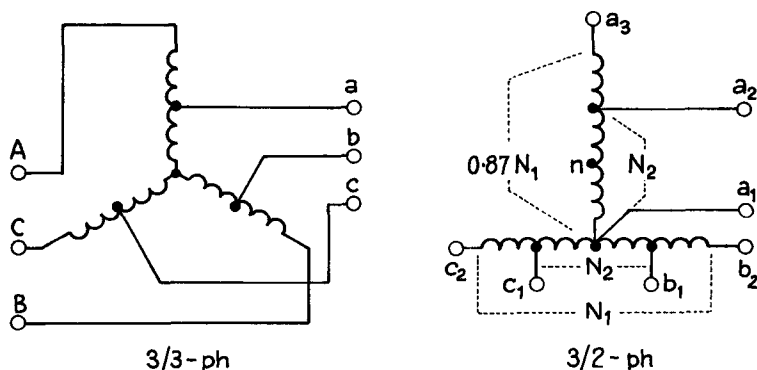
where

$$r_2' = \left(\frac{N_1}{N_2} - 1\right)^2 r_2 \quad \text{and} \quad x_2' = \left(\frac{N_1}{N_2} - 1\right)^2 x_2$$

and R_1 and X_1 are the equivalent resistance and leakage reactance referred to the primary. A test with the secondary short-circuited gives $Z_1 = R_1 + jX_1$ as the ratio of the applied primary voltage and input current, just as for a two-

winding transformer. The magnetizing branch included in Fig. 5.21 (c) is obtained from an open-circuit test.

Three-phase Auto Connection. This is readily obtained, as in Fig. 5.22. To avoid undue fault voltages it is usual to employ a star connection. The Scott connection is also possible, but it is not permissible to interconnect the 2-ph side as a 3-wire 2-ph system.



5.22 Three-phase auto transformers.

An auto-transformer starter for cage induction motors consist of a 3-ph star-connected unit with one or two (occasionally more) tapings per phase; it is designed for high current and flux densities for economy as the duty is required normally for only a few seconds at starting.

The interconnection of high-voltage supply networks (e.g. 275 and 132 kV) by auto-connected transformers results in a substantial saving of bulk and cost compared with two-winding transformers.

Comparison

Auto-transformers are not generally employed when the voltage-ratio exceeds 3:1, except for motor-starting duty, as the disadvantages preponderate. The latter are due to the direct electrical connection between the two sides, by reason of which disturbances on one system are liable to affect the other more seriously than when (as in the normal two-winding arrangement) there is only a magnetic link. The reactance is inherently low on account of (a) the 'coalescence' of the primary and secondary windings and (b) the smaller currents, so that the conditions on short circuit are more severe.

5.12 Three-winding connection

Transformers may be built with *tertiary* windings (i.e. additional to the primary and secondary) for any of the following reasons:

- (a) To supply a small additional load at a different voltage.
- (b) To supply phase-compensating devices (e.g. capacitors) requiring a different voltage or connection.
- (c) In star/star or star/zigzag transformers, a delta-connected tertiary winding

reduces the zero-phase-sequence impedance and allows adequate earth-fault current to flow for the operation of protective gear, and it limits voltage unbalance when the main load is asymmetric.

(d) To indicate voltage in a h.v. testing transformer.

(e) To load split-winding generators.

A tertiary winding for (a) is called *auxiliary*; for (c) it is termed a *stabilizing* winding.

Tertiary windings are frequently delta-connected: consequently, when faults and short-circuits occur on the primary or secondary sides (particularly between lines and earth), considerable unbalance of phase voltage may be produced, compensated by large tertiary circulating currents. The reactance of the winding must be such as to limit the circulating current to that which can be carried without overheating.

With given secondary and tertiary load currents I_2 and I_3 and with windings of N_1 , N_2 and N_3 turns/ph, the primary current I_1 is readily found by phasor summation of the secondary, tertiary and magnetizing m.m.f.s or equivalent currents.

Equivalent Circuit

Properly interpreted, a three-winding transformer can be represented by the equivalent 1-ph circuit of Fig. 5.23, with each winding assumed to have an equivalent resistance and separate leakage reactance. All values are reduced to a *common-voltage and common-rating* basis, and because the secondary output is not in general substantially the same as the primary input, the simple secondary equivalents used in the two-winding transformer are no longer valid: the load division between secondary and tertiary is, in fact, arbitrary.

Consider the primary and secondary operating as a two-winding transformer. The total per-unit resistance voltage $I r_{12}$ is the sum of the drops in the two windings separately. A similar statement is true for any pair of windings, and can be stated also in appropriate terms for the reactance drop. Thus we have

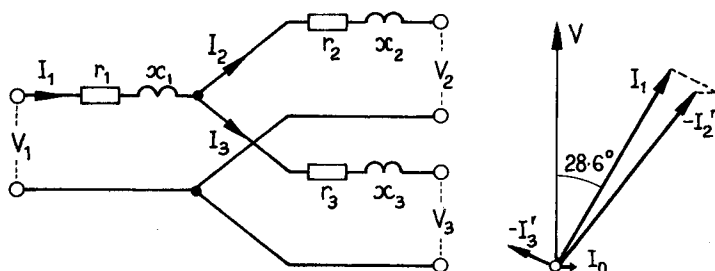
$$\begin{aligned} I r_{12} &= I r_1 + I r_2, & I x_{12} &= I x_1 + I x_2, \\ I r_{23} &= I r_2 + I r_3, & I x_{23} &= I x_2 + I x_3, \\ I r_{13} &= I r_1 + I r_3, & I x_{13} &= I x_1 + I x_3, \end{aligned}$$

for the possible combinations of pairs of circuits taken together as two-circuit transformers. Using these relations,

$$r_1 = \frac{1}{2} (r_{12} + r_{13} - r_{23}) = \frac{1}{2} \Sigma r - r_{23}.$$

The other corresponding relations are similar, whence

$$\begin{aligned} r_1 &= \frac{1}{2} \Sigma r - r_{23}, & x_1 &= \frac{1}{2} \Sigma x - x_{23}, \\ r_2 &= \frac{1}{2} \Sigma r - r_{13}, & x_2 &= \frac{1}{2} \Sigma x - x_{13}, \\ r_3 &= \frac{1}{2} \Sigma r - r_{12}, & x_3 &= \frac{1}{2} \Sigma x - x_{12}, \end{aligned}$$



5.23 Three-winding transformer.

Thus from a series of normal two-winding short-circuit tests, these expressions give, for the assigned voltage and rating, the ohmic values r_1, r_2, r_3 and x_1, x_2, x_3 in Fig. 5.23 taking completely into account all mutual effects.

Further, suppose the secondary to be short-circuited, the tertiary being left on open circuit. The total voltage applied to the primary will be the sum of the primary and secondary impedance voltages. A voltmeter across the tertiary output terminals is connected in effect across the junction of primary and secondary: it therefore indicates the secondary impedance voltage (after adjustment to the common voltage basis). In general, the voltage across one circuit when a second supplies short-circuit apparent power to the third, is the impedance voltage of the short-circuited winding.

Regulation. This is best considered from the individual contribution of each winding, the total for two windings in combination being the algebraic (not phasor) sum. For the primary alone, using eq.(5.8) adjusted for the ratio k = (load/base ratings),

$$\epsilon_1 = k(\epsilon_{r1} \cos \phi_1 + \epsilon_{x1} \sin \phi_1)$$

Summation of the part-regulations of two circuits together is an addition if power flows from one to the other, a subtraction if not. For a transformer with primary fed and both secondary and tertiary loaded, the regulations become

$$\epsilon_{12} = \epsilon_1 + \epsilon_2$$

$$\epsilon_{13} = \epsilon_1 + \epsilon_3$$

$$\epsilon_{23} = \epsilon_2 - \epsilon_3$$

$$\epsilon_{32} = \epsilon_3 - \epsilon_2$$

It is quite possible for these to be zero or negative.

EXAMPLE 5.4: A 3-ph 6 600/400/110 V star/star/delta transformer with a magnetizing current of 5.5 A has balanced 3-ph loads of 1 000 kVA at p.f. 0.8 lagging on the secondary, and 200 kVA at p.f. 0.5 leading on the tertiary winding. The per-unit resistance and reactance coefficients on a base of 6 600 V and 1 000 kVA are

$$\epsilon_{r1} = 0.0053$$

$$\epsilon_{r2} = 0.0063$$

$$\epsilon_{r3} = 0.0083$$

$$\epsilon_{x1} = 0.0265$$

$$\epsilon_{x2} = 0.0250$$

$$\epsilon_{x3} = 0.0332$$

Find the per-unit regulations.

Working in primary terms, we have

$$I_0 = -j 5.5 \text{ A}, \quad -I_2' = 70 - j 52.5 \text{ A}, \quad -I_3' = 8.8 + j 15.1 \text{ A}$$

which sum to $I_1 = 73.8 - j 42.9 = 89.7 \angle -28.6^\circ \text{ A}$. The primary load (neglecting losses) is therefore $S_1 = 1025 \text{ kVA}$ at p.f. $\cos 28.6^\circ = 0.88$ lagging. The phasor relations for the currents are shown in Fig. 5.23.

The primary regulation component with $k = 1025/1000 = 1.025$ is

$$\begin{aligned} \epsilon_1 &= k(\epsilon_{r1} \cos \phi_1 + \epsilon_{x1} \sin \phi_1) \\ &= 1.025(0.0053 \times 0.88 + 0.0265 \times 0.48) = 0.018 \text{ p.u.} \end{aligned}$$

For the secondary, $k = 1000/1000 = 1.0$, whence

$$\epsilon_2 = 1.0(0.0063 \times 0.80 + 0.0250 \times 0.60) = 0.020 \text{ p.u.}$$

For the tertiary, $k = 200/1000 = 0.2$, and thus

$$\epsilon_3 = 0.2(0.0083 \times 0.50 - 0.0332 \times 0.87) = -0.005 \text{ p.u.}$$

The primary-secondary and primary-tertiary regulations are therefore

$$\epsilon_{12} = \epsilon_1 + \epsilon_2 = 0.038 \text{ p.u.} \quad \epsilon_{13} = \epsilon_1 + \epsilon_3 = 0.013 \text{ p.u.}$$

The secondary-tertiary and tertiary-secondary regulations are

$$\epsilon_{23} = \epsilon_2 - \epsilon_3 = 0.025 \text{ p.u.} \quad \epsilon_{32} = -\epsilon_{23} = -0.025 \text{ p.u.}$$

Stabilization by Tertiary Windings

Star/star transformers comprising 1-ph units, or 3-ph *shell-* or *five-limb-core-* type units, have the disadvantages (a) that they cannot readily supply unbalanced loads between line and neutral, and (b) that their phase voltages may be distorted by triplen harmonic e.m.f.s. These arise because their phase magnetic circuits are not interlinked, and out-of-balance (zero-phase-sequence) currents have an iron path for the production of z.p.s. flux. The z.p.s. impedance is therefore high. The flow of earth-fault current is severely restricted, and may be insufficient for the satisfactory operation of protective gear. The zero-sequence impedance to out-of-balance or earth-fault currents must include the very high impedance (0.5–5 p.u.) between the secondary winding of the loaded or faulted phase and the primary windings of the other two phases. Abnormal voltages therefore occur when z.p.s. current flows. The output load or fault current concerns one phase only, whereas the corresponding input load current has, in the absence of a primary neutral, to be conducted through both of the other phases, which act as inductors. The voltage of the loaded phase is reduced, that of the others being raised. By use of a delta-connected tertiary winding, induced currents are caused to circulate in it, apportioning the load more evenly over the three phases. The action is, in brief, to interlink electrically the separate magnetic circuits.

The delta-connected tertiary provides a path for triplen magnetizing current. The disadvantages mentioned above are mitigated in 3-limb core-type transformers because, with z.p.s. flux forced out of the limbs, the z.p.s. im-

pedance is lower. Nevertheless it is usual to provide a delta-connected stabilizing tertiary winding, rated basically by its heat capacity under fault conditions.

5.13 Zero-phase-sequence impedance

The impedance of a transformer to positive- and to negative-sequence voltages is the same, as the phase order makes no difference to the operation. In determining the impedance to zero-phase-sequence currents, account must be taken of winding connections, earthing and, in some cases, the constructional form.

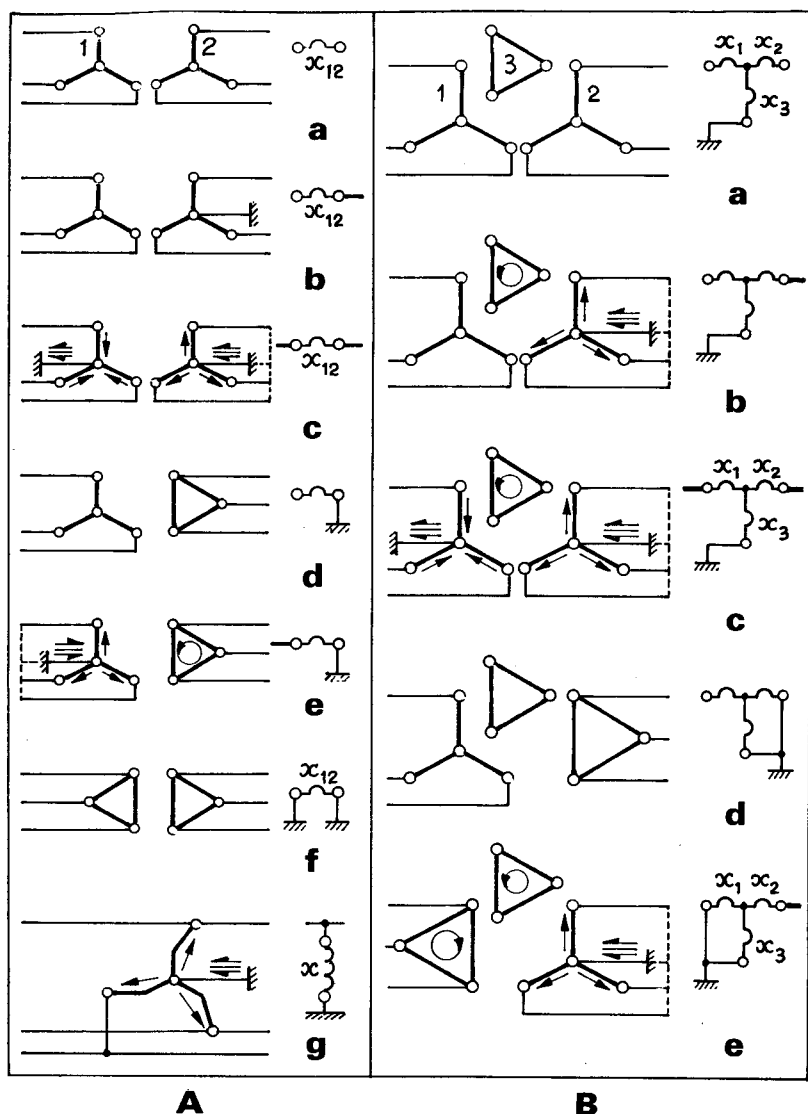
Fig. 5.24 sets out the connections and zero-sequence conditions, in A for two- and in B for three-winding transformers, neglecting resistance. In A the reactance x_{12} refers to $x_1 + x_2'$ or $x_1' + x_2$; in B the symbols x_{12} , x_{23} and x_{13} have the significance given to them in Sect. 5.12.

Two-winding Connections

- (a) *Yy (both star points isolated)*. There being no earth circuit, an infinite impedance is offered to z.p.s. currents.
- (b) *Yy (secondary neutral earthed)*. An earth fault on the primary side will not result in z.p.s. current. The same is nearly true for an earth fault on the secondary side if the transformer has no magnetic phase interlinkage (shell-type or three 1-ph units), but in a core-type unit the fluxes produced by z.p.s. currents find a high-reluctance path outside the core.
- (c) *Yy (both neutrals earthed)*. Provided that the primary circuit is complete for z.p.s. currents, a secondary earth fault will give rise to z.p.s. components whose counterparts flow in the primary circuit; and similarly for an earth fault on the primary. The phase reactance to z.p.s. currents is three times the reactance of the 3-ph windings in parallel, i.e. the reactance of one phase. It is therefore identical to the p.p.s. and n.p.s. values. With 3-ph core-type transformers the effect described in (b) may reduce the z.p.s. reactance value.
- (d) *Yd (star point isolated)*. No z.p.s. currents can flow to or from the primary or secondary, but they can circulate within the delta.
- (e) *Yd (neutral earthed)*. For a secondary earth fault the z.p.s. reactance is infinite. For a primary earth fault, compensating currents flow in the secondary delta. The z.p.s. reactance is as for (c).
- (f) *Dd*. No z.p.s. current to or from either side or in the delta.
- (g) *Z earthing transformer*. To p.p.s. and n.p.s. currents the reactance is infinite; to z.p.s. currents the reactance is due only to leakage as the two m.m.f.s on each limb are self-cancelling.

Three-winding Connections

- (a) *Yy (both star points isolated)*. The z.p.s. reactance is infinite.
- (b) *Yy (secondary neutral earthed)*. As for A(b). Compensating current flows in the tertiary delta, the reactance being x_{23} .
- (c) *Yy (both neutrals earthed)*. With a fault on either primary or secondary, z.p.s. currents can flow because of tertiary compensating currents. Currents can flow also in an unfaulted primary or secondary if the z.p.s. circuit is complete.



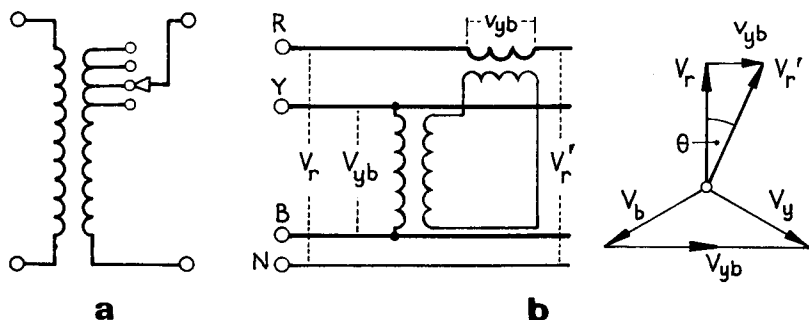
5.24 Zero-phase-sequence connections.

(d) *Yd (star point isolated)*. There is no path for z.p.s. currents.

(e) *Dy (neutral earthed)*. A secondary earth fault permits z.p.s. currents to flow, with compensating currents in primary and tertiary; the z.p.s. reactance is $x_2 + (x_1 x_3 / x_{13})$. For primary faults the z.p.s. reactance is infinite.

5.14 Tap-changing

Voltage control in a supply network is required for: (i) adjustment of consumers' terminal voltage within the statutory limits; (ii) control of active and reactive power flow in the network; (iii) seasonal (5–10%), daily (3–5%) and short-period (1–2%) adjustments in accordance with the corresponding variations of load. Much of the adjustment is done by altering the effective turns-ratio of the system transformers by tapplings on the windings. Occasional adjustments can be made by *off-circuit* tap-changing, the common range being $\pm 5\%$ in $2\frac{1}{2}\%$ steps. Daily and short-time adjustment is generally by means of *on-load* tap-changing gear.



5.25 Voltage control.

The essentials are indicated for one phase in Fig. 5.25. Simple up or down voltage regulation by tapplings is shown in (a), while (b) is for a control of phase angle in which a voltage v_{yb} derived from the line voltage V_{yb} gives an output voltage V_r' differing from V_r by the angle θ . It can be used for the control of reactive power flow on interconnector feeders.

Tapplings

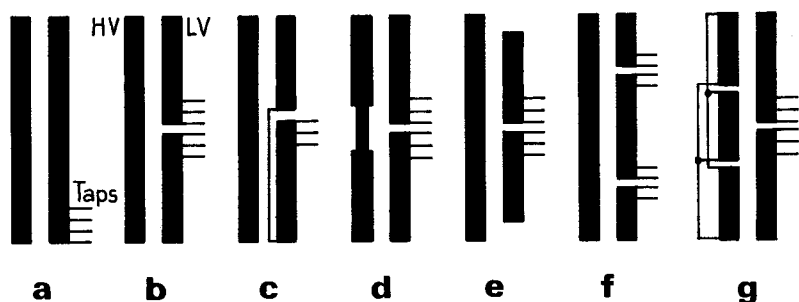
The *principal* tapping is that to which the rating of the winding is related. A *positive* tapping includes more, and a *negative* tapping less, turns than those of the principal tap. Tap-changing may be arranged in accordance with one of three conditions, namely (i) voltage variation with constant flux and constant turn-e.m.f.; (ii) with varying flux; and (iii) a mixture of (i) and (ii). In (i) the percentage tapping range is identical with that of voltage variation.

Location of the tapped part of a winding is partly a constructional question. With tapplings near the line ends the number of bushing insulators is reduced; with tapplings near the neutral ends the insulation conditions between phases are eased. Where a large voltage variation is required, tapplings should be near the centres of the phase windings to reduce magnetic asymmetry, but this arrangement cannot be used on l.v. windings placed next to the core.

It is not possible to tap other than an integral number of turns, and this may cause difficulty with l.v. windings. A 260 V phase winding with 15 V/turn cannot be tapped closer than 5%. It is consequently necessary to tap

the h.v. windings, a further advantage in a step-down transformer being that, on light load with lowest secondary voltage, the maximum number of turns is included on the h.v. side, reducing the e.m.f. per turn, the flux density and the core loss.

An effect of importance is that of unbalanced forces on short circuit when tapplings have produced magnetic asymmetry. Some methods of locating tapplings are shown in Fig. 5.26. Taps at one end (*a*) are permissible in small



5.26 Tapping and balancing.

transformers. In larger units they are placed centrally, both for (*b*) delta and (*c*) star-connected windings. Axial m.m.f. unbalance is mitigated by 'thinning' the l.v. winding (*d*), or by balancing parts of the winding more symmetrically (*e*) and (*f*). The untapped winding may be split into several parts (*g*) corresponding to the tapped side, and connected in parallel. For very large tapping ranges a special tapping coil may be used.

Tap-changing involves changes in leakage reactance, I^2R loss and core loss, and possibly some difficulty in operating dissimilar transformers in parallel. The response of the h.v. winding to a steep-fronted surge may also be impaired.

Off-circuit Tap-changing

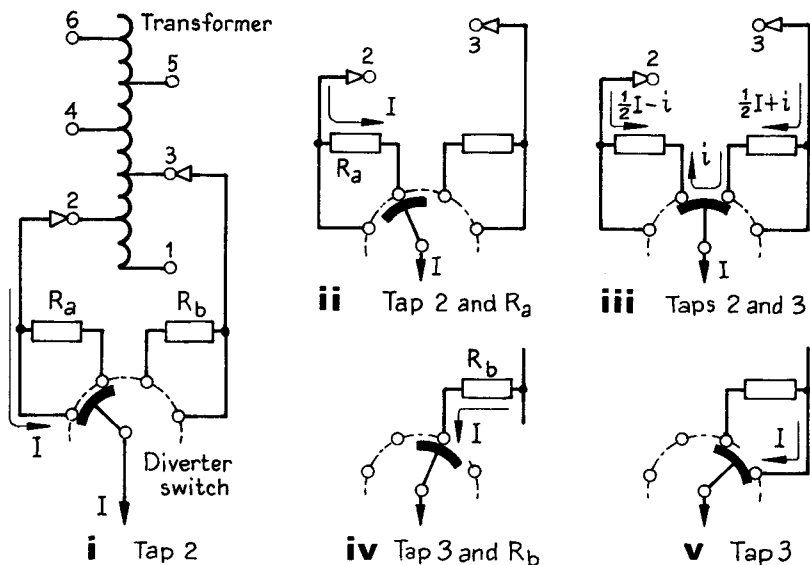
This adjustment is obtained by tapping the respective windings as required and taking connections therefrom to some position near the top of the transformer. Change of tapping is effected by means of hand holes in the cover. Alternatively the reconnection can be made by carrying the tapping leads through the cover for changing by hand or by manually operated switch. Two common types of switch are:

Vertical Tapping Switch. This comprises an insulated bar, one for each phase, carrying knife contacts, to be raised or lowered by handwheel and gearing to make connection to the tapping contacts.

Faceplate Switch. The spindles of movable contact arms are geared to a shaft with a handwheel external to the tank. The faceplate switches may be mounted on the upper yoke, or located nearer to the tapped positions on the windings.

On-load Tap-changing

The essential feature is the maintenance of circuit continuity throughout the tap-change operation. Momentary connection must be made simultaneously to two adjacent taps during the transition, and the short-circuit current between them must be limited by some form of impedance. Inductors have been used for this purpose, but in modern designs the current limiting is almost invariably obtained by means of a pair of resistors. Fig. 5.27 shows the method, which has a straightforward switching sequence. Back-up main contactors are provided which short-circuit the resistors for normal operation.



5.27 On-load tap changing.

Resistor transition requires one winding tap for each operating position. The resistors are short-time rated and it is essential to minimize their time of duty. Some form of energy storage must be incorporated in the driving mechanism to ensure that a tap change, once initiated, shall be completed irrespective of failure of the auxiliary control supply. All modern devices use springs, which operate to reduce the time that a resistor is in circuit to a few periods. Such a tap-changer is compact, and the high-speed break limits contact wear. The current break is eased by the fact that the short-circuit resistor current has unity p.f.

Irrespective of the form of transition, on-load tap-changers have one or other of the following switching arrangements:

1. A separate contactor for each winding tap, and camshaft operation to ensure the proper sequence.
2. Winding taps connected to a selector switch with an associated pair of contacts. Make and break occurs at the selector switch contacts, and some

degree of carbonization and wear is unavoidable. This 'single-compartment' tap-changer is common for transformers rated up to 20 MVA.

3. For very large transformers the tap selector switches do not move when carrying current: make and break is carried out by two separate diverter switches, usually in a separate compartment to minimize main-oil pollution by carbon. The diverters are mechanically interlocked with the selectors, which move (only when not current loaded) to provide the correct sequence of tap connection. Vacuum switches may be used as diverters, and as they do not contaminate oil they can be mounted in the same compartment as the selectors.

On-load tap-changer control gear can vary from simple push-button initiation to complex automatic control of as many as four transformers operating in parallel. The object of control is to maintain a set voltage level within a specified tolerance, or to raise it with load to compensate for line drop. The main components are an automatic voltage regulator, a time-delay relay, and compounding elements. The time-delay prevents initiation of a tap-change by a minor transient voltage fluctuation: it can be set for a delay up to 60 s. *Line-drop Compensation.* This is essentially an artificial transmission line consisting of adjustable resistor and inductor elements combined with current- and voltage transformers to provide a signal to the voltage relay coil. Variation of the load current then gives compensation for the line drop through the on-load tap-changing equipment.

5.15 Parallel operation

The satisfactory performance of transformers connected on both sides in parallel requires that they have the *essential* features of the same polarity, the same phase-sequence, and zero relative phase-displacement angle; a *near identity* of voltage ratio; and a *limited disparity* in per-unit impedance.

Polarity. This can be either right or wrong. If wrong it results in a dead short circuit.

Phase-sequence and relative phase-displacement. These questions arise with polyphase transformers. Only a few of the possible connections (Sect. 5.9) can be worked in parallel without excessive circulating current on small loads; for example, the secondary voltages of star/star and star/delta transformers have a phase difference of 30° , making parallel connection inadmissible. The various connections produce various magnitudes and phase displacements: magnitudes can be adjusted by tapplings, but phase divergence cannot be compensated.

The phase sequence must be identical for two paralleled transformers. If three secondary terminals $a_1b_1c_1$ of transformer 1 are to be paralleled with $a_2b_2c_2$ of transformer 2, the polarity and ratio being correct, then a_1 may be connected to a_2 . If the result is a potential difference across b_1b_2 or c_1c_2 , then the two phase-sequences differ.

The following are typical of the connections for which, from the viewpoint of sequence and phase divergence, transformers can be connected in parallel:

Transformer 1:	Yy	Yd	Yd
Transformer 2:	Dd	Dy	Yz

Thus all transformers in the same group can be paralleled. Further, transformers with a $+30^\circ$ and a -30° angle can be paralleled by reversing the primary and secondary phase-sequence of one.

Voltage Ratio

Equal voltage-ratio (not necessarily precisely the same as equal turns-ratio) is necessary to avoid no-load circulating current when transformers are in parallel on both primary and secondary sides. The leakage impedance being low, a small voltage difference may cause considerable no-load circulating current and additional I^2R loss. On load, the circulation is masked, but may cause over-current on one transformer when the paralleled group is loaded to the full combined rating.

Leakage Impedance

The leakage impedances of transformers required to operate in parallel may differ in magnitude and in quality (i.e. in reactance/resistance ratio). It is necessary also to distinguish between per-unit and ohmic impedance: the currents carried by two transformers are proportional to their ratings if their ohmic impedances are inversely proportional, and the per-unit impedances directly proportional, to those ratings.

A difference in the quality of the per-unit impedance results in a divergence of phase angle of the two currents, so that one transformer will be working with a higher, and the other with a lower, power factor than that of the combined output.

Load-sharing

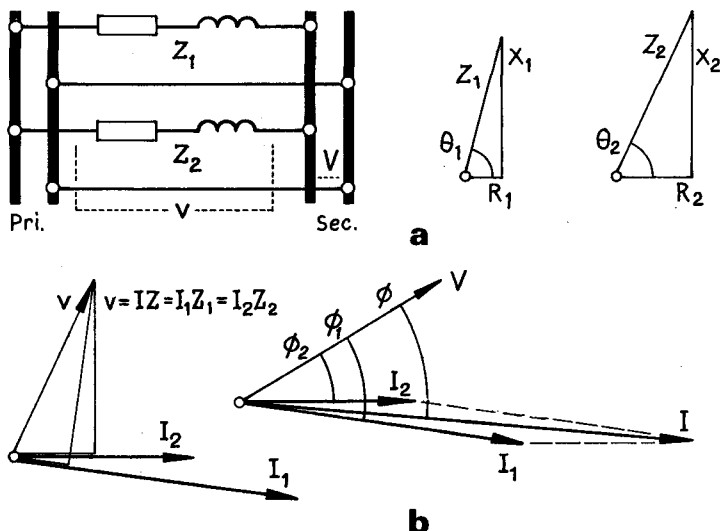
Equal Voltage Ratios. If the voltage ratios are equal and the voltages coincident in phase, both sides of a pair of transformers can be connected in parallel and no current will circulate between them on no load. Neglecting magnetizing admittance, the case can be represented by the equivalent circuit of Fig. 5.28(a), in which the leakage impedances (with respect to either primary or secondary side) are parallel connected. Let them be Z_1 and Z_2 , and let the two currents be I_1 and I_2 (at a common voltage V) which together provide the total load current I . The volt drop is

$$v = I_1 Z_1 = I_2 Z_2 = I Z_{12}$$

where $Z_{12} = Z_1 Z_2 / (Z_1 + Z_2)$ is the parallel equivalent of Z_1 and Z_2 . Then $I_1 = I(Z_{12}/Z_1) = I \cdot Z_2 / (Z_1 + Z_2)$; and similarly for I_2 . Multiplication by V gives the transformer loadings $S_1 = VI_1$ and $S_2 = VI_2$, and the combined load $S = VI$. Then

$$S_1 = S \frac{Z_2}{Z_1 + Z_2} \quad \text{and} \quad S_2 = S \frac{Z_1}{Z_1 + Z_2} \quad (5.11)$$

All these are complexor or phasor expressions, giving the loadings in magnitude and phase angle. Eq.(5.11) holds for per-unit loads and leakage impedances provided that all are expressed with reference to a common base.



5.28 Parallel connection.

Fig. 5.28(a) shows the leakage impedances $Z_1 = (R_1 + jX_1)$ with the angle $\theta_1 = \arctan(X_1/R_1)$, and Z_2 in similar terms. The currents (b) must be such as to make $I_1 Z_1$ equal in magnitude and phase to $I_2 Z_2$, and sum to the total load current I . The actual output phase angles ϕ_1 and ϕ_2 differ by $(\theta_1 - \theta_2)$. *Unequal voltage ratios.* Using the symbolism of the foregoing with the addition of Z , the load impedance across the secondary terminals, and E_1 and E_2 , the no-load secondary e.m.f.s, then

$$v = IZ_{12} = (I_1 + I_2)Z_{12}$$

as before. The transformer e.m.f.s are equal to the total voltages in their respective circuits, i.e.

$$E_1 = I_1 Z_1 + IZ \quad \text{and} \quad E_2 = I_2 Z_2 + IZ$$

whence $E_1 - E_2 = I_1 Z_1 - I_2 Z_2$. On no load with $I = 0$ this gives between the two transformers the circulating current

$$I_1 = -I_2 = \frac{E_1 - E_2}{Z_1 + Z_2}$$

On load we have

$$I_1 = \frac{(E_1 - E_2) + I_2 Z_2}{Z_1}$$

and by substitution the two currents become

$$I_1 = \frac{E_1 Z_2 + (E_1 - E_2)Z}{Z_1 Z_2 + (Z_1 + Z_2)Z}, \quad I_2 = \frac{E_2 Z_1 - (E_1 - E_2)Z}{Z_1 Z_2 + (Z_1 + Z_2)Z} \quad (5.12)$$

Normally E_1 and E_2 are co-phasal (or very nearly so); but eq.(5.12) could be used to show the severe results of incorrect parallel connection of 3-ph transformers with a phase-angle divergence.

The load-sharing of any number of transformers in parallel can more readily be found by applying the Millman (or parallel-generator) theorem: the common terminal voltage V of a number of transformers or generators, connected in parallel across a load of impedance Z , and having respective internal impedances $Z_1, Z_2, Z_3 \dots$ is $V = I_{sc} Z_0$, where I_{sc} is the sum of the generator short-circuit currents and Z_0 is the parallel sum of $Z, Z_1, Z_2, Z_3 \dots$. In symbols

$$V \left[\frac{1}{Z} + \frac{1}{Z_1} + \frac{1}{Z_2} + \dots \right] = \frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3} + \dots \quad (5.13)$$

The square bracket is $1/Z_0$ and the right-hand side is the sum of the individual short-circuit currents.

Reactive Power Absorption. Operating paralleled transformers on different ('staggered') tapings is a way of absorbing lagging reactive power at points in a supply network. In the case of two transformers each of leakage impedance $Z = R + jX$, with tapings staggered to give a voltage difference ΔV , the circulating apparent power is $S_c = (\Delta V)^2 / 2Z$. For example, two 125 MW units each with $R = 0.004$ and $X = 0.125$ p.u., operated on load with 5% voltage difference ($2\frac{1}{2}\%$ up and down), circulate $0.05^2 / 0.25 = 0.01$ p.u. apparent power, i.e. 1.25 Mvar. The additional I^2R loss is 0.04 MW, or 8% of the full-load I^2R .

EXAMPLE 5.5: A 500 kVA 1-ph transformer with 0.010 p.u. resistance and 0.05 p.u. leakage reactance is to share a load of 750 kVA at p.f. 0.80 lagging with a 250 kVA transformer with per-unit resistance and reactance of 0.015 and 0.04. Find the load on each transformer (i) when both secondary voltages are 400 V, and (ii) when the open-circuit secondary voltages are respectively 405 V and 415 V.

(i) The per-unit impedances expressed on a common base of 500 kVA are

$$Z_1 = 0.010 + j0.05 = 0.051 \angle 79^\circ$$

$$Z_2 = 2(0.015 + j0.04) = 0.085 \angle 69^\circ$$

$$Z_1 + Z_2 = 0.04 + j0.13 = 0.136 \angle 73^\circ$$

The load is $S = 750(0.8 - j0.6) = 750 \angle -37^\circ$ kVA. Applying eq.(5.11),

$$S_1 = 750 \angle -37^\circ \frac{0.085 \angle 69^\circ}{0.136 \angle 73^\circ} = 471 \angle -40^\circ = 359 - j305 \text{ kVA}$$

$$S_2 = 281 \angle -31^\circ = 241 - j305 \text{ kVA}$$

The total active power is $359 + 241 = 600$ kW ($= 750 \times 0.8$), and the total reactive power is 450 kvar ($= 750 \times 0.6$). The 250 kVA transformer operates with a $12\frac{1}{2}\%$ overload because of its smaller per-unit leakage impedance.

(ii) With the Millman theorem it is necessary to work in ohmic values of impedance

$$Z_1 = 0.0032 + j0.0160 = 0.0163 \angle 79^\circ \Omega$$

$$Z_2 = 0.0096 + j0.0256 = 0.0275 \angle 69^\circ \Omega$$

It is also necessary to estimate the load impedance Z : assuming an output voltage on load of 395 V, then $Z = 0.208 \angle 37^\circ \Omega$. From eq.(5.13)

$$\frac{E_1}{Z_1} + \frac{E_2}{Z_2} = \frac{405 \angle 0^\circ}{0.0163 \angle 79^\circ} + \frac{415 \angle 0^\circ}{0.0275 \angle 69^\circ} = 39\,700 \angle -75.2^\circ \text{ A}$$

$$\frac{1}{Z_0} = \frac{1}{0.208 \angle 37^\circ} + \frac{1}{0.0163 \angle 79^\circ} + \frac{1}{0.0275 \angle 69^\circ}$$

$$= \frac{1}{0.0099 \angle 73.5^\circ}$$

The secondary terminal voltage is

$$V = (39\,700 \angle -75.2^\circ)(0.0099 \angle 73.5^\circ) = 393 \angle -1.7^\circ = 393 - j12 \text{ V}$$

The internal volt drop in the first transformer is $E_1 - V = 405 - (393 - j12) = 17 \angle 45^\circ \text{ V}$, and in the second is $22 - j12 = 25 \angle 29^\circ \text{ V}$, whence

$$I_1 = \frac{17 \angle 45^\circ}{0.0163 \angle 79^\circ} = 1\,040 \angle -34^\circ \text{ A} \quad \text{and} \quad I_2 = 910 \angle -40^\circ \text{ A}$$

The loads are $S_1 = VI_1^*$ and $S_2 = VI_2^*$, giving

$$S_1 = 340 - j220 = 410 \text{ kVA} \quad \text{and} \quad S_2 = 270 - j220 = 350 \text{ kVA}$$

The combined load is $S = 610 - j445 \text{ kVA}$ ($\approx 600 - j450$) and the 250 kVA transformer is overloaded by 40%. The secondary circulating current on no load is

$$(E_1 - E_2)/(Z_1 + Z_2) = 230 \text{ A}$$

corresponding to about 95 kVA and a considerable waste in I^2R loss.

5.16 Cooling methods

The considerable variety of possible methods of heat dissipation make necessary a concise standard designation. The letter symbols employed are:

Medium: air A, gas G, synthetic oil L, mineral oil O, solid insulation S, water W.

Circulation: natural N, forced F.

Order: up to four letter symbols are used for each system for which the transformer is assigned a rating. The order is (1) the medium and (2) the circulation of the coolant in contact with the windings; and (3) the medium and (4) the circulation of the coolant in an external heat-exchanger system. The common forms are listed below. The limits of temperature-rise in each case can be obtained from the Table in Sect. 4.7.

Air Cooling

AN: This has the ambient air as coolant, with natural circulation by convection. The development of high-temperature insulating materials (glass and silicone resins) makes the method suitable for ratings up to 1.5 MVA, and for special conditions such as those in mines.

AF: With forced air circulation and improved heat dissipation, the specific loadings can be raised.

Oil-immersed, Oil Cooling

ONAN: This method of natural oil circulation and natural air flow over the tank (and tubes) is very common, and can be applied to transformers rated up to 5 MVA.

ONAF: Air is blown on to the tank surface, so that less surface is required for a given rating.

OFAN: This is uncommon, but can be useful where the coolers have to be well removed from the transformer. The oil is pumped round the system, and high current densities can be used in the transformer windings.

OFAF: The forced oil- and air-circulation method is the usual one for transformers of 30 MVA and upward. Mixed cooling may be used, with an ONAN condition up to 0.5 p.u. rating, temperature-sensing elements initiating oil pumps and air fans for the forced cooling at upper conditions of load.

Oil-immersed, Water Cooling

ONWF: Copper cooling coils are mounted in the tank above the level of the transformer core, but below the oil surface.

OFWF: Oil/water heat-exchangers are external to the transformer. The advantages over the ONWF system are that the transformer is smaller and the tank does not have to contain cooling coils; there are no condensation troubles; leakage of water into the oil is improbable if the oil pressure is greater than that of the water. Where the cooling water has considerable 'head', it is usual to employ cascaded heat-exchangers (oil/water and water/water), with the intermediate water circuit at low pressure. The use of water as a coolant is common in generating stations, particularly hydro stations, where an ample supply is available.

5.17 Protection

Transformers in service are often required to link extensive supply systems, and sometimes these are high-voltage systems with very large generation and load capability. The general nature of the hazards to which transformers may be subjected are listed in the Table on page 188.

Buchholz Protection

The Buchholz system is applicable to oil-immersed transformers — the great majority — and depends on the fact that transformer breakdowns are always preceded by the more or less violent generation of gas. A broken joint for example, produces a local arc, and vaporizes the oil in the vicinity. An earth fault has the same result. Sudden short-circuits rapidly increase the tem-

Transformer Protection

OVERCURRENTS: affecting insulation life, temperature-rise and mechanical stress.

Overloads exceeding rated value:

1. Winding temperature indication and alarm, allows full use of transformer overload capacity.
2. Oil temperature indication and alarm, winding temperature may be excessive.
3. Time-delay overcurrent relays, may isolate too rapidly on large and too slowly on small overcurrent.

External Faults in connected systems:

1. System protection to isolate faulty network, transformer impedance to limit throughput.

Internal Faults:

1. Buchholtz relay to give alarm or to initiate isolation.
2. Relays or fuses, high setting necessary for discrimination; restricted earth-fault relays give full discrimination but respond only to earth faults.
3. Balance methods, fully discriminative.

OVERVOLTAGES: affecting insulation stressing.

External, due to atmospheric causes and switching operations:

1. Insulation co-ordination and grading.
2. Protective gaps to lower surge crest voltage, disturb service continuity.
3. Cable connections to overhead lines, lower steepness of surge wave-front.

Internal, due to harmonics and/or resonance with system capacitance:

1. Delta-connected tertiary stabilizing winding.
2. System insulation co-ordination.

perature of the windings, particularly the inner layers, and pocketed oil is there vaporized. Discharges due to insulation weakness, e.g. by deterioration of the oil, will also cause oil dissociation accompanied by the generation of gas. Core faults, such as short circuits due to faulty core-clamp insulation, produce local heating and generate gas.

This generation of oil vapour or gas is utilized to actuate a relay. The relay is a hydraulic device, arranged in the pipe-line between the transformer tank and the separate oil conservator, Fig. 5.29(a): the relay is shown in greater detail at (b). The vessel is normally full of oil. It contains two floats, b_1 and b_2 , which are hinged so as to be pressed by their buoyancy against two stops. If gas bubbles are generated in the transformer due to a fault, they will rise and traverse the pipe-line towards the conservator, and will be trapped in the upper part of the relay chamber, thereby displacing the oil and lowering the float b_1 . This sinks and eventually closes an external contact which operates an alarm.

A small window in the wall of the vessel shows the amount of gas trapped and its colour. From the rate of increase of gas an estimate can be made of

the severity and continuance of the fault, while from the colour a diagnosis of the type of fault is possible.

If the rate of generation of gas is small, the lower float b_2 is unaffected. When the fault becomes dangerous and the gas production violent, the sudden displacement of oil along the pipe-line tips the float b_2 and causes a second contact to be closed, making the trip-coil circuit and operating the main switches on both h.v. and l.v. sides.

Gas is not produced until the local temperature exceeds about 150°C . Thus momentary overloads do not affect the relay unless the transformer is already hot. The normal to-and-fro movement of the oil produced by the cycles of heating and cooling in service is insufficient to cause relay operation.

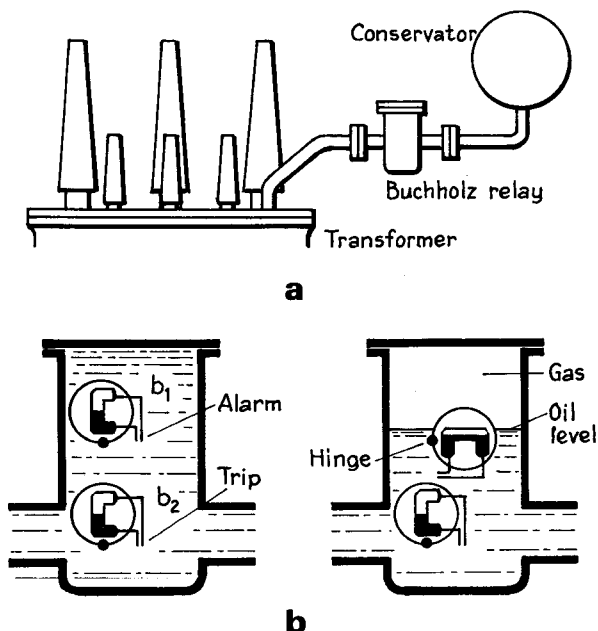
5.18 Testing

A transformer may be subjected to a range of tests for a variety of purposes, including—

- (a) Routine tests after manufacture;
- (b) Acceptance tests, heat runs, etc.;
- (c) Specialized investigations on particular details of design, performance or operation.

Phasing and Polarity

In manufacture, where all connections are traceable, the phases on primary and secondary sides may readily be grouped. If not, all phases are short-circuited



5.29 Buchholz protection.

except a primary and a supposedly corresponding secondary. A small direct current is circulated in the primary and a voltmeter is connected across the secondary. A momentary deflection of the voltmeter when the primary current is made and broken confirms that the two windings concerned belong to the same phase.

For this test all phase-ends should be separate. Difficulty may be experienced with internally connected zigzag (interconnected star) arrangements.

Polarity has to be in accordance with B.S. 171: see Fig. 5.17.

Voltage Ratio

Most manufacturers employ bridge methods, for checking coils before assembly.

Assembled transformers may be checked on a *ratimeter*, which is essentially a potential divider excited from the same supply as the transformer under test, and subdivided so as to read the l.v. voltage in terms of the h.v. Balance is obtained by connecting the ratimeter tapping to the l.v. winding through an ammeter and adjusting the former until the current is zero.

D.C. Resistance

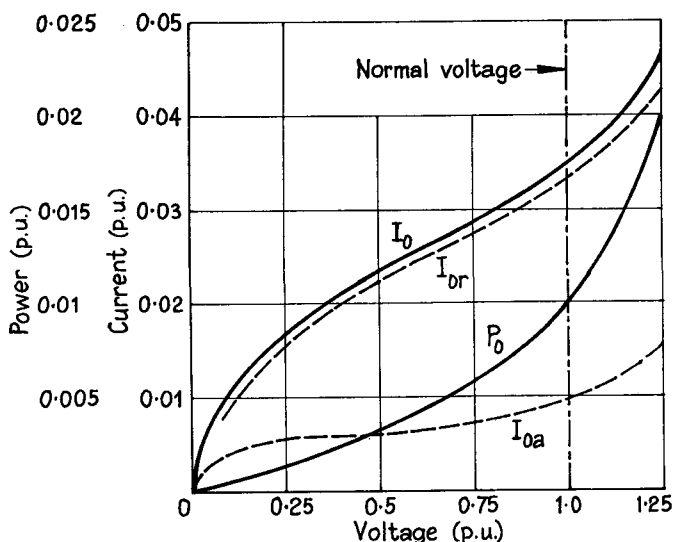
Any usual method may be used. For large low-voltage transformers a low resistance bridge method is necessary. The d.c. resistance must be made at known temperature, and may be used to check the design or to estimate the eddy-current loss ratio.

Magnetizing Current and Core Loss

This test is made on a transformer complete with tank and oil, as it is performed at normal rated voltage and electric stress. One winding (usually the h.v.) is left on open circuit and the other connected to a supply of normal frequency. Readings are taken of current and power at rated voltage. The power includes the core loss, the I^2R loss (often negligible) and the dielectric loss: the latter may be significant for transformers of extra-high voltage. The conditions are discussed in Sect. 5.1.

If a test is made at normal frequency and variable voltage V , the active and reactive components of the no-load current appear as in Fig. 5.30. As the peak flux density B_m is proportional to V and the core loss roughly to B_m^2 , i.e. to V^2 , the power/voltage curve is to a first approximation parabolic, while the active current I_{0a} is very roughly linear. The magnetizing component I_{0r} rises initially at low densities until the core attains its condition of maximum permeability, and thereafter rises steeply with the onset of saturation around normal voltage. The values in Fig. 5.30, which apply to transformers in the range 100–1 000 kVA, are expressed in per-unit terms.

Core-loss Voltmeter. Hysteresis loss (which accounts for 70–80% of the core loss) depends upon B_m , but the eddy-current loss depends upon the waveform of the flux density and therefore upon the r.m.s. density. The core loss should be measured with a sine applied voltage, but to avoid errors due to changes in the peak value, which can occur with negligible change in the r.m.s. voltage, the voltage is measured as a *mean* value, to which the peak flux density is pro-



5.30 No-load test values.

portional. The mean voltage is adjusted to $2\sqrt{2}/\pi = 0.90$ times the rated r.m.s. voltage. A rectifier type of voltmeter is suitable.

Magnetizing Current Unbalance. The magnetic path length associated with the central phase of a 3-ph 3-limbed core-type transformer is significantly shorter than that of either of the outer phases. The configuration  shows that the outer path lengths include two half-yokes additional to the limb. As a consequence, the magnetizing current and core loss values are asymmetric, to an extent depending on the path-length ratio. If the centre path length is one-half that of either outer, then its magnetizing current is likely to be about 30% less, and this is independent of the peak flux density level. The measurement of the total core loss by 2-wattmeter method is made difficult by the fact that it is the small difference between two large readings on account of the very low power factor: the measurement also depends upon which phase currents pass through the wattmeter current-circuits, and even on the phase sequence if the wattmeter volt-circuits have inductance errors. The most satisfactory way of measuring the core loss is to use one wattmeter for each phase.

Leakage Impedance and Load Loss

One winding is short-circuited, and the other (usually the h.v. to reduce the current and increase the voltage to be measured) is connected through voltmeters, ammeters and wattmeters to a supply of normal frequency but of a fraction (5–10%) of normal voltage. With rated full-load current circulated, the load loss is measured and adjusted to standard reference temperature, i.e. to 75 °C for insulation classes AEB, and to 115 °C for FHC. The power measurement includes a very small core loss (which is generally neglected) and the eddy and stray losses. The terminal short-circuit connections must be carefully

arranged, as l.v. winding resistances may be very low. In adjusting the measured loss to reference temperature it may be necessary to take account of the fact that while the ohmic I^2R loss increases with temperature, the eddy-loss ratio, eq.(3.1), actually falls.

The leakage impedance voltage is given by the voltage V_{sc} required to circulate rated current at reference temperature. The ratio V_{sc}/I_{sc} gives the leakage impedance Z_1 (if the measurements are made on the primary), from which the leakage reactance can be calculated.

Total Loss

This is the sum of the no-load loss at rated voltage, and the load loss at rated current adjusted to reference temperature. In the case of multiwinding transformers it is stated for one pair of windings.

EXAMPLE 5.6: An 11/0.44 kV 50 Hz 300 kVA 3-ph mesh/star transformer gave the following test results at rated voltage and current: No-load: 440 V 21.1 A, 1.30 kW (l.v. side). Short-circuit: 630 V, 15.7 A, 3.08 kW (h.v. side, winding temperature 30 °C). Evaluate the rated loss, impedance voltage, per-unit resistance and leakage reactance, and the efficiency and regulation on full load at p.f. 0.8 lagging.

The no-load loss is $P_0 = 1.30$ kW (0.043 p.u.); the no-load current is $I_0 = 21.1$ A (0.053 p.u.)

The s.c. test impedance voltage is $630/11\ 000 = 0.057$ p.u. and the resistance is $3.08/300 = 0.010$ p.u., giving the leakage impedance $\epsilon_x = \sqrt{(0.057^2 - 0.010^2)} = 0.057$ p.u. The adjusted load loss is $3.08 (235 + 75)/(235 + 30) = 3.60$ kW, whence $\epsilon_r = 0.012$ p.u. and $\epsilon_z = 0.057$ p.u.

The total adjusted loss is $1.30 + 3.60 = 4.90$ kW. At full load and p.f. 0.8 lagging the efficiency and regulation are

$$\eta = 1 - \frac{4.9}{300 \times 0.8 + 4.9} = 0.980 \text{ p.u.}$$

$$\epsilon = 0.012 \times 0.8 + 0.057 \times 0.6 = 0.043 \text{ p.u.}$$

With large transformers it is necessary to treat the ohmic and eddy-loss I^2R effects separately in determining the load loss.

Temperature-rise

This is measured during the performance of a heat-run intended to reproduce as far as is practicable the conditions of rated continuous load. The methods include (a) Back-to-back connection; (b) Delta/delta connection; (c) Equivalent open- or short-circuit run.

Back-to-back Connection. Two identical transformers A and B, Fig. 5.31 (a), are connected in parallel on one side to a supply of normal voltage and frequency. Disregarding the effect of the auxiliary transformer C, wattmeter W_1 reads the core loss of the two transformers together. The other sides are now properly paralleled, and a circulating voltage introduced by C to circulate full-load current in the primaries, and therefore also in the secondaries.

The circulating power is measured by wattmeter W_2 , which records the load loss. If the two transformers have suitable tapplings it may be possible to dispense with C by deliberate unbalance. The arrangement for 3-ph transformers is shown in (b). Because of the phasing of the magnetizing and circulating currents, there is a slight difference in the load loss of the pair; this may be averaged out if the auxiliary supply frequency is lower than normal.

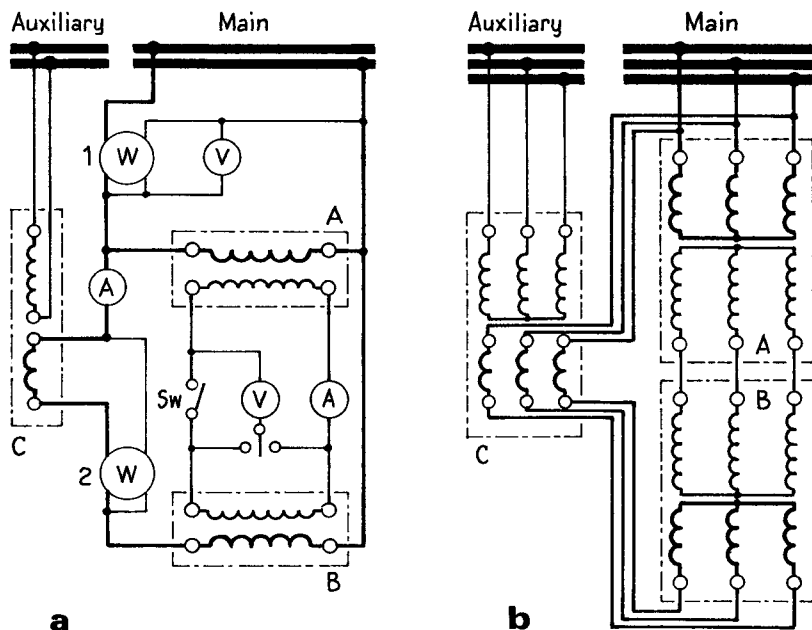
Delta-delta Connection. The primary side is excited normally: the secondaries, in open delta, are joined to the auxiliary supply to circulate full-load current, achieving the same result as the back-to-back connection.

Equivalent Run. Where the I^2R loss is several times the core loss, a heat-run may be made by a short-circuit test at a current sufficiently greater than full-load value to cover the (absent) core loss. With high-frequency transformers it may be possible to approximate to normal loss by an open-circuit test at overvoltage. A normal heat-run at low power factor may require a large synchronous compensator, possibly backed with a static capacitor bank. A 500 MVA transformer with 0.2 p.u. reactance would require a test equipment capable of 100 Mvar for the temperature-rise test.

Insulation

The integrity of the insulation to earth and between turns, coils, tapplings, windings and phases is tested (i) by power-frequency induced voltage, (ii) by power-frequency applied voltage, and (iii) by impulse applied voltage.

Induced voltages are generated by the transformer itself by open-circuit test

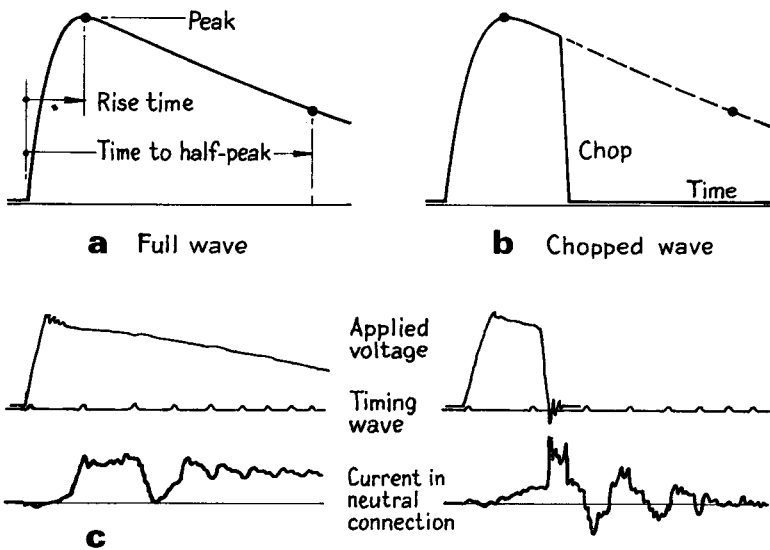


5.31 Back-to-back tests.

at a frequency up to twice normal in order to limit the level of core saturation. The peak induced voltage v_m of a given winding is measured, and $v_m/\sqrt{2}$ is taken as the r.m.s. voltage that must match the specified B.S. value for the system rated voltage level. Although the test voltage is greater than normal, its distribution is the same as in service.

Applied voltages are derived from test transformers. The whole of a tested winding is raised to terminal level, unlike the effect of induced voltage.

Impulse voltages are applied to the h.v. windings of a transformer by means of a surge or impulse generator. The test voltage wave takes the forms shown



5.32 Impulse tests.

in Fig. 5.32: (a) is a full wave, and (b) a chopped wave obtained by paralleling the impulse generator by a rod spark-gap, which has a delayed breakdown. The standard full wave is designated 1.2/50 μ s, indicating that the initial rise time is 1.2 μ s, and that it decays to one-half peak value in 50 μ s. A test sequence might be as follows: one full wave at reduced voltage; one full wave at test level; two chopped waves in succession; one final full wave at test level. For each test simultaneous oscillographic records are made of the applied impulse voltage and of the neutral current, the latter from the drop across a non-reactive resistor connected between the tank and earth. If an initial test is made at a voltage substantially below the expected impulse strength of the transformer, any failure that occurs during the normal test routine will show an anomaly in the current record. With modern techniques and considerable experience, failures can be detected with near certainty even when there are no other signs (e.g. noise or smoke). Fig. 5.32(c) shows typical oscillographic records.

Test Voltages

These are based on the highest r.m.s. system voltage V_s . Test voltage levels V_t (in r.m.s. for power-frequency tests and in peak for impulse tests) are given in BS 171. They approximate to the following:

Test Voltages, kV

- A: Air-insulated windings, and oil-immersed windings not subject to impulse testing.
 O: Oil-insulated windings designed for impulse testing.

System voltage V_s	Power-frequency test		Impulse test
	A	O	O
Less than 1.1	2.5	—	—
3–36	$2.1 V_s$	—	—
17.5–72.5	—	$7 + 1.8 V_s$	$20 + 4.2 V_s$
100–245 (Std. 1)*	—	$1.9 V_s$	$4.5 V_s$
100–420 (Std. 2)*	—	$1.5 V_s$	$3.5 V_s$

* Standard 1 for non-effectively-earthed windings; Standard 2 for effectively-earthed windings.

Overcurrent

System short-circuit currents have immediate mechanical effects, and rapidly developing thermal effects, on transformers. The most severe case is that of a terminal short circuit, for which the result for a transformer of leakage impedance z per-unit is a current comprising the components

- a symmetrical alternating current of r.m.s. value $1/z$ per-unit, and
- an exponentially decaying direct current with an initial value that may reach almost the peak of the a.c. component, as a result of the 'doubling effect'.

The overcurrent limits in BS 171 specify the symmetrical r.m.s. current I_{sc} in

- (i) for a range of ratings S :

S MVA :	0.63	1.25	3.15	6.3	12.5	25	100
I_{sc} p.u. :	25	20	16	14	12	10	8
z p.u. :	0.04	0.05	0.0625	0.0715	0.0835	0.10	0.125

A transformer will in general be designed to have leakage impedance corresponding to z .

Thermal Effects. If I_{sc} is not greater than 20 p.u. the transformer must withstand the heating effect for 3 sec without exceeding a specified temperature limit. For I_{sc} greater than 20 p.u. the time is reduced to 2 sec. Taking the initial winding temperature θ_0 to be the maximum coolant temperature (40°C) plus the permissible temperature-rise given in Sect. 4.7, then the maximum winding temperature θ_m reached must not exceed the following values (in $^\circ\text{C}$) corresponding to the insulation class and conductor material indicated:

Type and insulation class:	Dry	A	E	B	FHC	Oil A
Windings: copper		180	250	350	350	250
aluminium		180	200	200	—	200

The calculation is made on the basis of eq.(4.3) in the form

$$\theta_m = \theta_0 + aJ^2t \times 10^{-3}$$

for a short-circuit current of duration t (in sec) producing in the windings the current density J (in A/mm²). The factor a takes account of the resistivity, temperature coefficient, density and thermal capacity of the conductor material and is expressed as a function of the average temperature $\frac{1}{2}(\theta_m + \theta_0)$:

$\frac{1}{2}(\theta_m + \theta_0), ^\circ\text{C}$	140	160	180	200	220	240
Factor a , copper	7.41	7.80	8.20	8.59	8.99	9.38
aluminium	16.5	17.4	18.3	—	—	—

The conditions imposed settle the maximum short-circuit current densities, which range from 40 A/mm² for a 3-sec short circuit in a dry transformer with aluminium windings up to 100 A/mm² for an oil-immersed water-cooled transformer with copper windings and a 2-sec duration.

Mechanical Forces. The transformer must withstand the mechanical forces on short-circuit. The value of $F = Ni$ in eqs.(2.19) and (2.20) is found with $i = 1.8(\sqrt{2} I_{sc})$, the asymmetrical initial peak current assuming a small decrement.

5.19 Special types

Some types of transformer outwith those for normal electrical systems include the following.

Miniature Transformers

The open air-cooled construction has largely been superseded by hermetically sealed arrangements in air- or oil-filled containers, the connections being brought out through metal/glass or metal/ceramic seals. Transformers for mobile equipments and working at frequencies in the range 400–1 600 Hz employ cores of relatively costly materials and high loadings for reduction of mass. Very thin c.r.o.s. or nickel-iron cores are wound with coils insulated by high-temperature dielectrics such as fibreglass and silicones. A fully sealed 30 VA 1 600 Hz unit may have a mass as low as 0.1 kg.

Mining Transformers

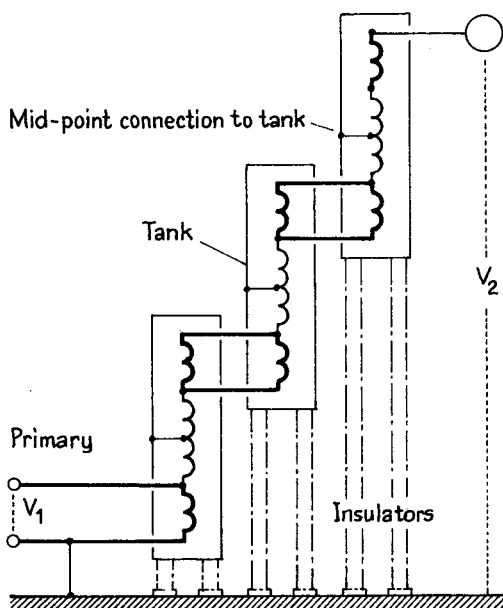
These work under particularly onerous conditions, with restricted headroom and a potentially explosive ambient atmosphere. The modern underground unit is 'dry', has a class C insulant, and is enclosed in a flameproof casing.

Welding Transformers

A.C. welding is simple, economical and efficient. The basic requirements are a secondary voltage between 70 and 100 V, and an adjustable series inductor to provide arc stability and to improve restriking after current zeros. Part of the necessary inductive reactance can be incorporated in the transformer itself. It is usual to employ capacitor correction of the inherently low power factor.

High-voltage Transformers

Industrial-frequency h.v. testing transformers have usually a duty comprising intermittent use with breakdown discharges amounting to partial short circuit. Although it is possible to produce 500 kV r.m.s. in a single unit, the problems of construction and transport may necessitate the cascade connection of two or more units. In the typical arrangement shown in Fig. 5.33, the mid-points



5.33 High-voltage testing transformer.

of the transformers are connected to the tanks, and each end is tapped and insulated to act as the primary winding or to provide load and magnetizing currents for the next higher-voltage unit. All tanks have to be insulated from earth, but the windings have to withstand only one-half of the voltage of the unit to core. The three units of Fig. 5.33, if each is designed for 333 kV and insulated for 167 kV to core, produce in cascade a voltage to earth of 1 MV. Considerable saving in floor space is achieved by mounting the cascaded units in a vertical column.

With its terminals open-circuited, the secondary of an h.v. transformer may take a leading self-capacitance current large enough to outweigh the

primary magnetizing current. On load, those sections of the primary carrying both load and magnetizing currents may have a large leakage reactance drop and require compensating windings.

Low-voltage Transformers

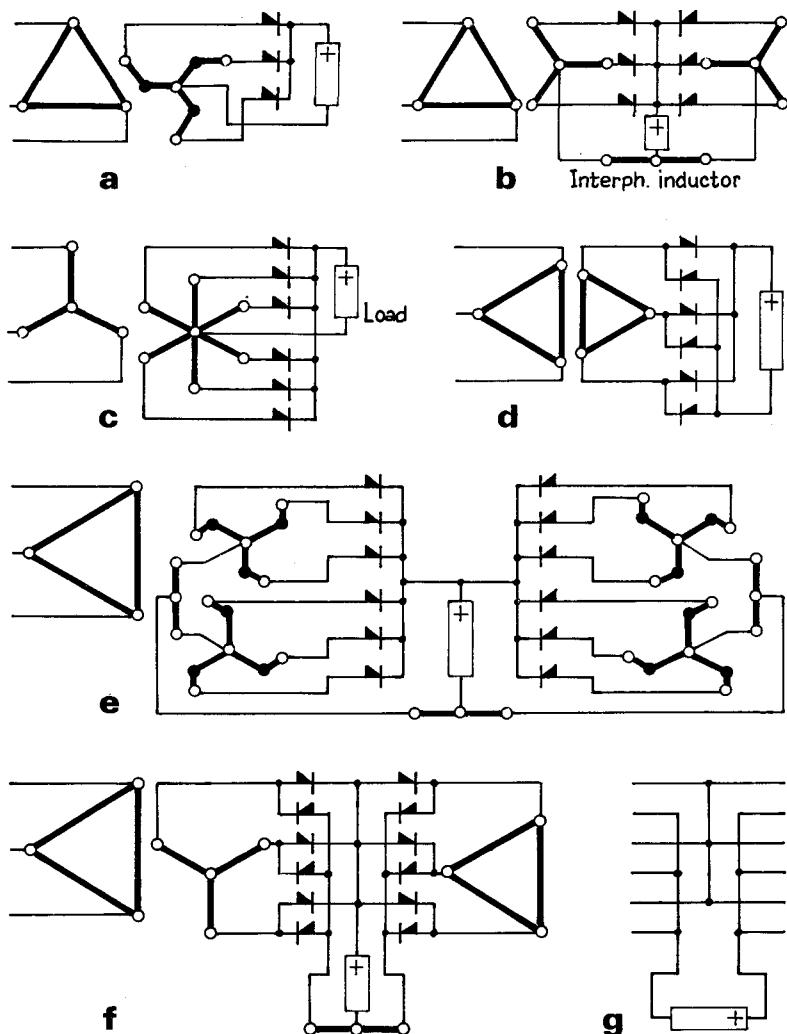
Electric arc furnaces and electrolytic processes need very large currents at variable low voltage. A shell design may be adopted so that the l.v. windings can be sectionalized to give a more uniform leakage impedance for parallel connection. A 3-ph furnace with a single turn per secondary phase may, however, be of core-type construction and employ sheet-type conductors. The phase terminal arrangements are designed for minimum magnetic leakage, as even a small inductance may cause a relatively large volt drop. Voltage regulation is provided by taps on the h.v. side in such a way as to minimize unbalanced mechanical forces in the event of a short circuit — a not infrequent occurrence.

Rectifier Transformers

Distortion of the current waveforms in transformers feeding power rectifiers increases the r.m.s. value for a given power and thus also the transformer apparent-power rating compared with that for a sinusoidal load current. Further, large rectifier transformers produce some voltage waveform distortion on supply systems, and for installations of 100 kW upward it may be desirable to limit distortion by increasing the number of pulses. The seven connections for 3-ph equipments in Fig. 5.34 are classified (i) in accordance with the number of *pulses* (i.e. commutations per period); (ii) as *single-* or *double-way* (i.e. transformer secondary currents either uni- or bi-directional); and (iii) whether or not a *bridge connection* or an *interphase inductor* is used:

- (a) Three-pulse; single-way; delta or star primary, zigzag secondary.
- (b) Six-pulse; single-way; delta or star primary, double-star secondary; interphase inductor.
- (c) Six-pulse; single-way; star primary with isolated star-point, diametral secondary.
- (d) Six-pulse; double-way; delta or star primary and secondary; bridge connection.
- (e) Twelve-pulse; single-way; delta or star primary, quadruple zigzag star secondary; three (or two) interphase inductors.
- (f) Twelve-pulse; double-way; delta or star primary, delta and star secondaries; two parallel 3-ph bridges, interphase inductor.
- (g) Twelve-pulse; double-way; windings as (f); two series 3-ph bridges.

Mercury-arc rectifiers are usually single-way connected (*a, b, c, e*) but all connections suit silicon diodes. Connection (*e*) is common for high-current low-voltage rectifier equipments. For higher voltages up to 2 kV, connection (*g*) can be used with two or more diodes in series in each path. In 6-pulse cases (*b, c, d*) the relative transformer ratings are 1.2:1.2:1, with an inductor of ratio 0.05 for (*b*). The relative ratings for 12-pulse arrangements (*e, f, g*) are 1.3:1:1 with (*e*) and (*f*) requiring inductors respectively of rating 0.06 and 0.01. For



5.34 Transformer-rectifier connections.

large outputs where harmonic limitation is essential, a 24-pulse system may be provided by two separate 12-pulse units with outputs in parallel, the 15° phase-shift being obtained by a $7\frac{1}{2}^\circ$ shift on each primary. Alternatively, four separate 6-pulse units may be used.

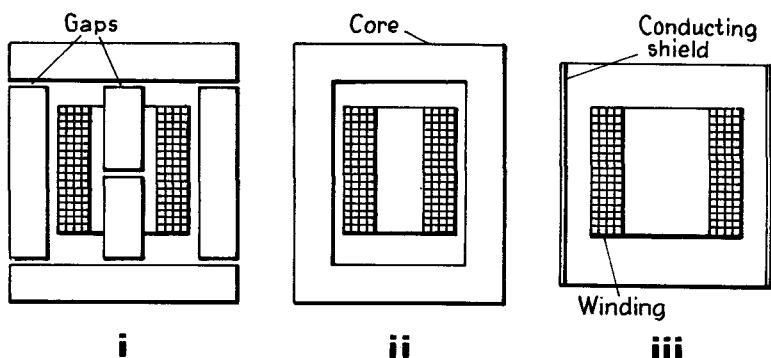
Traction-supply Transformers

High-voltage industrial-frequency railway traction systems (e.g. at 25 kV, 50 Hz) are fed by 1-ph track-side transformers spaced up to 40–50 km. Locomotive equipments, in view of the widely varying pick-up voltage result-

ing from supply-network, transformer and contact-wire impedances, are designed to operate satisfactorily over the range 0.65–1.1 p.u. of nominal voltage. Locomotives normally have rectifier-fed d.c. motors, so that the substation load current is likely to have a distorted waveform and to vary widely in amplitude (e.g. zero to 2.5 p.u.), giving temperature-rises and pulsating mechanical forces similar in many respects to those encountered in rectifier and arc-furnace transformers.

The 1-ph substation transformers have fully insulated primary windings connected across two lines of the main supply network, and are provided with off-load tap-changers that may be set to take account of local supply system regulation and traction conditions. When, as is usual, traction load is taken from a large interconnected supply network, the effects of 3-ph unbalance and harmonic distortion can generally be accepted without significant interference with other connected loads and generating plant; but occasionally it may be necessary to fit harmonic filters. Service conditions require short-circuit currents to be limited by adequate transformer leakage reactance (e.g. 0.1 p.u. for a unit of 10 MVA or above); and surge overvoltages up to 2.0 p.u. to be withstood. Typical design features include two- or three-limb 1-ph core-type constructions (Fig. 2.9) with a normal flux density up to 1.65 T; work-hardened copper-strip windings; and careful attention to mechanical resistance against short-circuit electromagnetic forces. Most transformers are natural oil cooled. Large units of 20 MVA and over may be provided with ONAN/OFAN/OFAF cooling by means of a separate heat-exchanger with airblast fans and a pump in the oil pipework, the changeover being controlled by switches in the winding hot-spot temperature detector.

The 1-ph shell-type construction is often considered where a high inherent mechanical strength is sought, as radial forces are thereby avoided, and the axial forces are more easily contained. But the interleaving of the h.v. and l.v. windings leads to a low inherent reactance, and the reduction of winding eddy-current loss is more complicated. Design studies show that the core type is to be preferred.



5.35 Inductive reactors.

Moving-Coil Regulators

These are described in Sect. 12.2.

Inductors

Series inductors ('reactors') are used in power-supply systems to limit fault levels, and shunt inductors for reactive-power compensation of long h.v. overhead transmission lines or cable networks. Series inductors may be of the cast-in-concrete form; both series and shunt inductors may, however, be built like transformers (i) with gapped cores, (ii) coreless but with magnetic shields, or (iii) coreless but with conductive shields, Fig. 5.35, and are normally oil-immersed. Form (i) follows transformer practice in that a coil embraces a laminated steel limb, the magnetic circuit being completed by yokes and the limb being sectionalized to control and stabilize the inductance. Form (ii) gives nearly perfect inductance linearity if the surrounding magnetic shield is given an adequate cross-section. The conducting screen in (iii) provides a path for currents that effectively suppress external flux that would otherwise penetrate the tank and cause eddy-current loss therein: there is, of course, some I^2R loss in the shield, and some reduction of the effective coil inductance. The isolated value of the coil inductance is L_1 , but in the presence of the shield the inductance is reduced to $L_1 - L_{12}^2/L_2$ by the effect of the mutual inductance L_{12} with the shield of self inductance L_2 . The design is a compromise between the loss in effective inductance and the increase of tank size and oil volume that would be demanded by a greater separation. Generally there is an inductance loss of about 10%. Under fault conditions the mechanical force between coil and screen may be considerable, and as a result the mechanical design is dominant.

Superconducting Transformer Windings

Wilkinson [27], in a study of the possibility of operating transformers under cryogenic conditions, assumed that a superconducting metal film could be found capable of the desired current density while immersed in a magnetic field. He concluded that the core must lie outside the refrigerated volume, for apart from the unknown performance characteristics of magnetic steels at very low temperatures, the removal of core loss at a temperature of a few kelvin above absolute zero might well require several hundred times the power actually extracted. Superconducting windings would have to conduct in the presence of a leakage field of 3.5 T and with very high linear current densities. In return there would be a prospective saving of the entire I^2R loss. In a 570 MVA design with a 50 kW core-loss reduction effected by lowering the core mass from 100 to 65 Mg, the conventional I^2R loss of 3 MW could be eliminated. Against this would have to be set a refrigeration power of about 0.5 MW, leaving a net loss reduction of about 2.7 MW on full load. The costs of development and of the refrigeration plant might absorb the capital equivalent of the saving in transformer loss, and Wilkinson concluded that the superconducting transformer was not yet economically feasible, even were the necessary conducting-film materials developed.

6

Transformers: Construction and Design

6.1 *Constructional features*

The main constructional elements are: cores, comprising limbs, yokes and clamping devices; primary, secondary and sometimes also tertiary phase windings, coil formers, spacers and conductor insulation; interwinding and winding/earth insulation and bracing; tanks, coolers, dryers, conservators and other auxiliaries; terminals and bushings, connections and tapping switches.

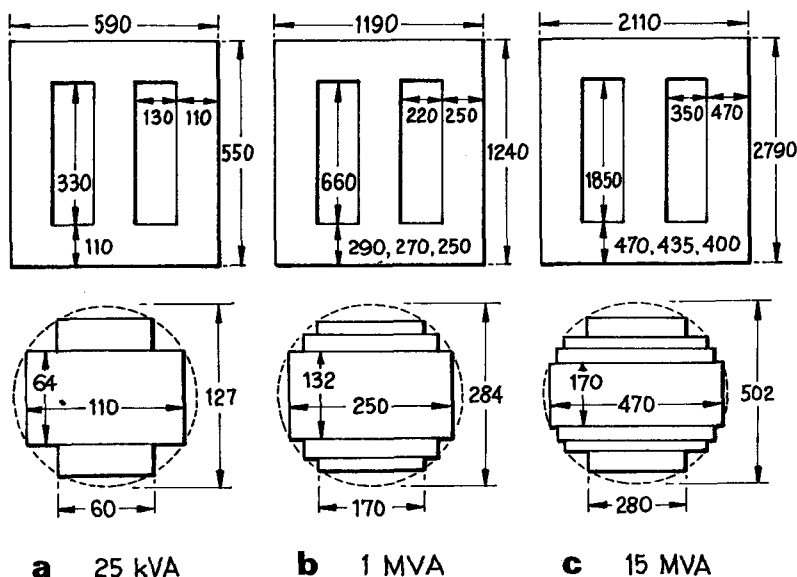
Dry Transformers. For indoor and well-protected sites, and for ratings up to about 1.5 MVA at 11 kV, 'dry' type transformers may be advantageous. They have class C insulation, and hot-spot temperatures up to 150 °C. They may have simple natural air cooling, or be sealed into cabinets with nitrogen filling. Mica-glass tape is employed for conductor insulation, while parts closely associated with the windings are of micalex or porcelain. These materials are stable at working temperatures. Dry units can be free-standing or be set within gated enclosures.

Oil-immersed Transformers. The great majority of industrial power transformers are immersed in oil for cooling, insulation and mechanical protection. Certain of the details below, in particular for tanks and oil, refer specifically to this type.

6.2 *Cores*

European practice is to standardize on core-type constructions. Some 1-ph shell types are built, but the 3-ph shell type is rare. General details of transformer magnetic circuit construction are given in Sect. 2.2 and 2.4. Core plates are slit and sheared from c.r.o.s. material and given a subsequent stress relief.

The limbs and lower yoke are assembled with the top yokes removable for admitting the windings. As many as 20 widths of steel strip may be used in a large core: Fig. 6.1 shows the dimensions of three typical cores. The advantage of multi-stepped cores in respect of active core cross-section has to be balanced against the intricacy of the building program. The insulated-lamina form of the core means a loss of 5–10% of the gross area, and to avoid further loss of active area requires clamping pressure to minimize gaps due to waviness of the core plates. With varnish as a plate insulation, a compressed core has little



6.1 Frame sizes.

Dimensions in mm; not to scale.

'spring' and remains substantially at its compressed dimensions. Core clamping may be effected by outside stiffeners and insulated non-magnetic through-bolts, but to retain the advantage of c.r.o.s. even very large cores may be bound under pressure by synthetic-resin-bonded glass or Terelene type which is subsequently heated to set the resin. Rigidity is aided by using varnish insulation for the outer packets of the core. Mechanical stresses imposed during assembly and in subsequent service are taken by a cradle comprising top and bottom yoke clamps, main tie-bolts, and substantial feet beneath the bottom yoke. Large bound cores have a high electrical resistance through the stack and must be provided with earth strips to prevent the static discharges that would otherwise occur during an impulse test.

Small cores wound with continuous varnished c.r.o.s. strip as in Fig. 2.7 do not need binding or clamping.

6.3 Tanks and oils

Small tanks are welded from steel plate, larger ones assembled from boiler-plate or cast-aluminium parts, usually mounted on a shallow fabricated steel base. A tank must withstand the stresses imposed by jacking and lifting, and should be no larger than is necessary to accommodate the core, windings and internal connections with appropriate electrical clearance. If the tank can support a partial vacuum, drying out on site (if required) is facilitated. The tanks of large transformers may be shaped to the outline plan of the core and windings to give a robust form not needing stiffener bars and minimizing the oil content. This 'form-fitting' shape is not possible for a 3-ph unit having the

tap-changer sited at one end because of the multiplicity of horizontal connectors needed.

Conservators. Transformer oil expands by about 7% over the temperature range from 0 to 100 °C. Conservator tanks are required to take up the cyclic expansion and contraction without allowing the oil to come in contact with the ambient air, from which it may absorb moisture. A typical conservator is an airtight cylindrical drum mounted on or near to the tank lid, or a flexible corrugated-disc container. The main tank is filled when cold, and expansion of the oil is taken up by the conservator. Displacement of the conservator air by oil takes place through a breather containing silica gel, which extracts moisture from ingoing air.

Oils

Transformer oil, which has to be both an insulator and a coolant, has to fulfil certain specifications, including the following.

Viscosity. This affects the oil-flow and therefore the effectiveness of cooling. A high viscosity causes sluggish flow through the many small orifices in the windings.

Electric Strength. Clean hydrocarbon oil normally has a fully adequate breakdown stress. More important is the severe reduction in electric strength due to the presence of water, which must be rigorously excluded. Further, if dust and small fibres are present in the oil, paths of low resistivity (and therefore of weakness) may be formed.

Flash Point. This is the temperature at which oil vapour ignites spontaneously. A flash point higher than 160 °C may be required.

Fire Point. The temperature at which an oil will ignite and burn is about 200 °C, and about 25% above the flash point.

Purity. The oil must not contain sulphur or its compounds, which cause corrosion of metal parts and accelerate the production of sludge.

Sludge. Sludging is the slow formation of semi-solid hydrocarbons as deposits on windings and tank walls; it is due to heat and oxidation, and is self-aggravating because the sludge deposits increase the thermal gradients and block the oil-circulation paths. Sludge seems to form more quickly in the presence of bright copper surfaces, and its acidic nature makes it corrosive.

Acidity. Among the products of oxidation are carbon dioxide, volatile water-soluble organic acids, and water. These in combination can attack and corrode steel and other metals. The rate of acidification is reduced by the use of breathers and conservators.

The deterioration of transformer oil during its working life can be retarded by anti-oxidants, usually of phenolic or amino type, which convert chain-forming molecules in the oil into inactive molecules, being gradually consumed in the process. Such inhibitors prolong the active life of *inhibited oil* by delaying the onset of deterioration by acid and sludge formation.

Fire- and Explosion-proofing. When it is essential to reduce the risks of fire and explosion, a synthetic oil (*askarel*) may be used. It is chemically stable, non-oxidizing, volatile, heavier than water and with greater dielectric strength than the normal petroleum-based mineral oil. Its relative permittivity is 4.5,

nearly twice that of normal oil, so that the distribution of electric stress between the liquid and solid insulants is affected. An askarel is a solvent of varnishes, binders and paints, but is resistant to the migration of moisture. When decomposed by an arc the chief product is hydrogen-chloride gas, which can form deleterious hydrochloric acid if water is present. However, with askarels a transformer may be economic in heavily loaded urban areas because fire-extinguishing equipment and fireproof masonry are not needed.

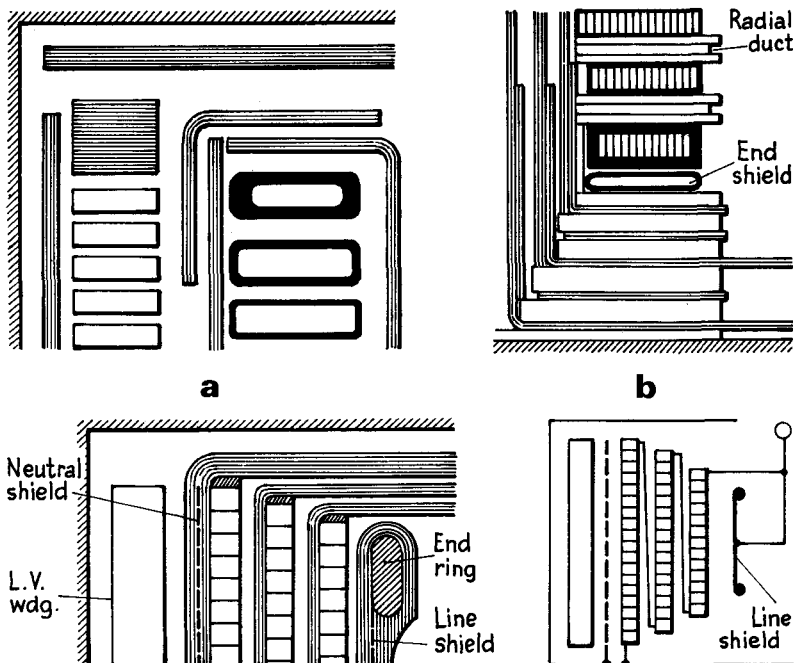
6.4 Windings and insulation

Types of coils and windings are described in Sect. 3.4. Their direct connection with electric power networks makes windings the most vulnerable parts of the transformer. The main design and constructional features are:

- (i) The provision of adequate *insulation* strength to withstand power-frequency applied or induced test voltages, both to earth and between coils, turns and phases; and full-wave and chopped impulse test voltages to prove capability against switching surges and lightning.
- (ii) The limitation of *load loss* (I^2R and stray) to the guaranteed performance figure.
- (iii) The provision of *cooling* to meet guaranteed limits of winding temperature-rise.
- (iv) The provision of the required *leakage inductance*.
- (v) The achievement of sufficient *mechanical strength* to withstand short-circuit forces.

For transformers of small and medium size, the inner (low-voltage) winding is insulated from the core by pressboard or by a synthetic-resin bonded paper cylinder, with axial bars arranged around it to form cooling ducts for the inside cylindrical surface of the winding. With disc or disc-helix coils, the bars have a wedge-shaped section on which intercoil or interturn dovetail-slotted spacers can be threaded. Similar bars are placed around the outer surface of the l.v. winding and (if necessary) between the layers of a helix winding. The main h.v./l.v. insulation is provided by another pressboard or s.r.b.p. cylinder, and the bars and spacers repeated for the h.v. winding. Insulation at the ends of the windings comprises blocks keyed to the axial bars in line with the spacers to form a series of columns by which the winding can be clamped.

For operating voltages demanding thick insulation it is customary to use concentric thin-walled cylinders spaced by axial bars, the material being carried round the ends of the outer (h.v.) winding by flanged collars as in Fig. 6.2. The 'solid' form (a), for 110 kV upward, has the collars as extensions of the cylindrical pressboard or paper winding insulation, the h.v. winding being assembled direct over the outer layer, with cooling ducts transferred from the inside turns at suitable points. In (b) the collars are separate. For multilayer h.v. helix windings the interlayer insulation may be a combination of paper or pressboard wraps, with bars arranged axially to form cooling ducts. These wraps may be formed in tapered halves when internal interlayer connections are required, providing maximum insulation thickness at the appropriate places.



6.2 Insulation details.

Conductor insulation is a tape, of paper for oil-immersed and glass for dry transformers. The tape is machine wrapped and lashed with cotton strands to keep it in place.

Surge-voltage Distribution. Methods of improving the distribution are given in Sect. 5.8. A multilayer winding has a higher series capacitance than a disc winding, but stress shielding may still be necessary at the ends of the layers. The choice between disc-coil and multilayer windings depends on the specified impulse level and the rating.

Load Loss. The conductor I^2R loss is calculable. The load (stray) loss arises from eddy currents in connectors, conductors (if not adequately transposed), tank walls and clamping structures. Where for a high reactance the leakage flux is large, the tank must be shielded from it.

Cooling. Most transformers are cooled by thermal circulation of oil over core and coil surfaces. The arrangements for heat dissipation are discussed in Sect. 4.2, 4.4 and 4.6.

Leakage Inductance. A given arrangement of core and windings has a 'natural' value of leakage inductance, Sect. 2.7. By dimensional adjustments this can be varied within limits without undue influence on cost and performance, but considerable changes involve expense and some reaction on load loss, surge-voltage distribution and other parameters subject to specification.

Short-circuit Mechanical Strength. The radial and axial forces to which transformer windings are subjected under short-circuit conditions are discussed in

Sect. 2.7. Thermal stresses can also be developed when the windings carry sustained overcurrents, and the temperature-rise in such cases is calculable from eq.(4.3), Sect. 4.2.

6.5 Terminals and fittings

Connections to the windings are copper straps, insulated wholly or in part, with the terminal connections taken to bushings on the tank top. The size and shape of the connectors may have to take account of electric stress, and sharp edges and corners avoided.

Bushings

Up to 33 kV, simple porcelain insulators can be used. Above this voltage, oil-filled or capacitor bushings are required. The *oil-filled* bushing consists of a hollow porcelain cylinder with a conductor, usually a cylinder, through its axis. The space between the conductor and the inner surface of the porcelain is filled with oil, and at the top there is a small expansion chamber to accommodate the variation of oil bulk due to changes in operating temperature. A capacitor bushing is constructed of thick layers of s.r.b.p. alternating with the graded layers of tinfoil to form between the outer layer and the inner conductor a series of capacitors of substantially equal capacitance, so giving an approximation to a uniform electric stress distribution over the radius.

For use outdoors, a bushing is covered by a porcelain rain-shed, corrugated circumferentially to provide a longer and partially protected leakage path.

Fittings

Among the devices normally provided for oil-immersed transformers are the following:

Cable Boxes for direct connection of paper- or plastic-insulated supply cables.
Oil Conservator, Oil Gauge and Oil-temperature Indicator.

Breather for transformers and conservators not fully sealed, provided with a chemical or refrigerant to remove moisture from the intake air.

Buchholz Relay. See Sect. 5.17.

Winding-temperature Indicator, essentially a thermometer with the bulb associated with a heater coil carrying a current proportional to the load current. The thermometer is immersed in the oil, and its indication, raised above the ambient oil by the heater, gives an analogue indication of the winding temperature.

Tapping Switch, either on- or off-load, Sect. 5.14.

Explosion Vent, relieving dangerous rises of internal pressure by a spring-loaded device or by a thin non-metallic diaphragm which breaks under excess pressure.

Base; Skids or Wheels.

6.6 Distribution transformers

The preferred ratings for steady loads are related approximately by the factor $2^{1/3} = 1.26$. They are

5	6.3	8	10	12.5	16	20	25	31.5	40
50	63	80	100	125	160	200	250	315	400
500	630	800	1000	...					

A range of distribution transformers for batch manufacture might comprise basic sizes of 63, 100, 160, 250, 400, 630 and 1 000 kVA rating. Laminations for the limbs and yokes are produced in the press shop from coils of grain-oriented sheet in one operation, heat treated in a stress-reducing tunnel oven, and then transported to the assembly shop. The h.v. and l.v. windings are produced on special-purpose machines, either as complete coils or as assemblies. Terminals and connections are prepared in advance, and the coils are aged and ready for installation.

The tank walls, bases and frames are welded together in the tank assembly shop, and pressure tested with hot oil for leaks. In the final production stage the tanks pass through steam cleaning and de-greasing plant, and are then painted.

The core and coil assembly line involves the following operations: placing the cores on trolleys; dismounting the top yoke; fit coils; re-fit yokes; assemble cover with bushings, switches and other equipment; fit cover on coil-core unit and secure electrical connections. The coils and cores are heated for some hours in an evacuated vessel, checked for tightness, fitted into the tank and bolted in position. Finally the tank is filled with dried and degassed oil.

Standard tests are carried out in the following sequence: resistance, voltage ratio, polarity; no-load tests; short-circuit test. The measurements are fed to a computer which processes them and prints out a test report.

The following Example traces the evaluation of the main features of a medium-rated distribution transformer. Normally all the calculations would be computed from suitable programs with the necessary specific data fed in.

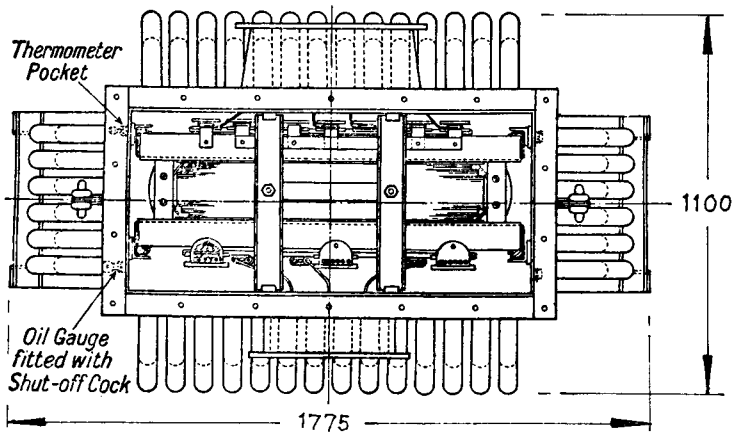
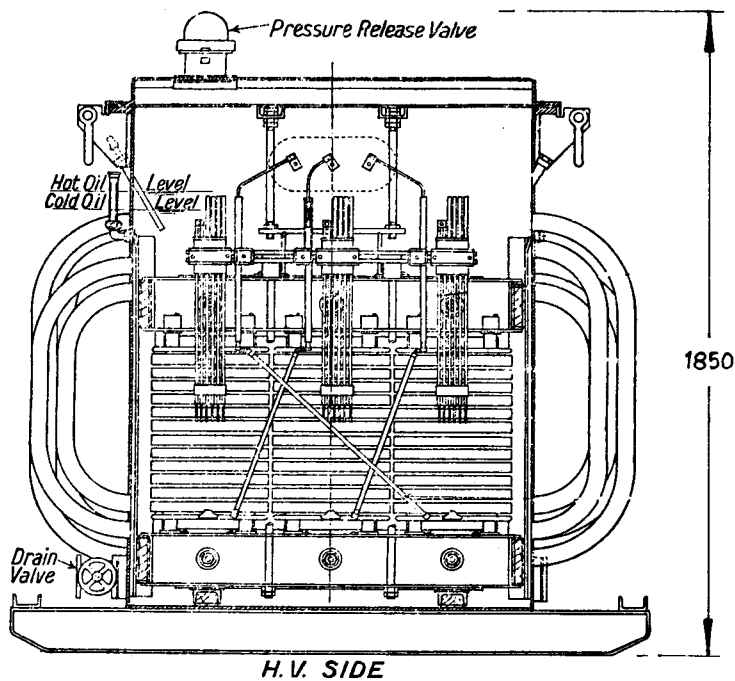
EXAMPLE 6.1: Three-phase 3-limbed core-type transformer to BS 171; rating 400 kVA, 6.6/0.41–0.45 kV in $2\frac{1}{2}\%$ steps, delta/star, oil-immersed, natural cooled. See Fig. 6.3.

Main Dimensions. The frame size (Sect. 1.14, Fig. 1.13(b)) has a stepped core of net area $A_i = 0.027 \text{ m}^2$ contained in an overall limb diameter $d = 0.21 \text{ m}$, with the limb centres spaced $D = 0.37 \text{ m}$. The yoke area is the same as that of a limb. With a peak flux density $B_m = 1.45 \text{ T}$, the voltage per turn from eq.(1.13) is $V_t = 8.7 \text{ V}$. The primary phase voltage is $V_1 = 6\,600 \text{ V}$; the secondary phase voltage at maximum flux density is $V_2 = 450/\sqrt{3} = 260 \text{ V}$. The turns per phase are therefore

$$N_2 = 260/8.7 = 30 \quad \text{and} \quad N_1 = 30 \times 6\,600/260 = 760$$

N_1 is increased by 76 turns to a total of 836 for the lower limit of secondary voltage. The rated primary current per phase is $I_1 = 133/6.6 = 20.0 \text{ A}$. With a current density $J = 3.0 \text{ MA/m}^2 = 3.0 \text{ A/mm}^2$, and a window space factor $k_w = 0.29$, the required window area from eq.(1.15) is $A_w = 0.071 \text{ m}^2$. The depth of the window is $l = A_w/(D - d) = 0.55 \text{ m}$. The main dimensions are thus

$$\begin{aligned} d &= 210 \text{ mm}; & D &= 340 \text{ mm}; & l &= 550 \text{ mm}; \\ W &= 2D + 0.9d = 870 \text{ mm}; & H &= 930 \text{ mm}. \end{aligned}$$



6.3 Core-type transformer: 400 kVA, 6.6/0.415 kV, 50 Hz, delta/star.

Magnetic Circuit. The material is c.r.o.s., and the yoke area is the same as the limb area. The total core volume is 0.093 m^3 and its mass is 700 kg. The specific loss (Fig. 2.5) is 1.50 W/kg , so that the total core loss (with a building factor of 1.3) is

$$P_i = 700 \times 1.50 \times 1.3 = 1.37 \text{ kW}$$

From the curves in Fig. 2.3, the total m.m.f. for the limbs is $3 \times 0.55 \times 150 = 250 \text{ A-t}$, and for the yokes is $2 \times 0.87 \times 150 = 260 \text{ A-t}$, a total of 510 A-t , raised to 600 A-t to allow for the greater reluctance of the joints. The magnetizing current per primary phase is

$$I_{0m} = 600/3 \times \sqrt{2} \times 760 = 0.19 \text{ A}$$

corresponding to a magnetizing reactive power per phase of $q_0 = 1 \text{ 260 var}$, or a total of 3 800 var . An estimate based on Fig. 2.14 gives 4.6 var/kg , i.e. 3 200 var .

Low-voltage Winding. With a current density $J_2 = 2.85 \text{ A/mm}^2$, a little lower than the mean to improve the inner cooling, the secondary phase current $I_2 = 133/0.26 = 512 \text{ A}$ requires a conductor area $a_2 = 512/2.85 = 180 \text{ mm}^2$. Taking an available winding depth of 465 mm , space is required to accommodate $30 + 1$ turns, giving 15 mm depth per turn for the helix winding. The conductor must be subdivided to reduce eddy currents. A suitable conductor comprises four parallel strips, each $13.5 \text{ mm} \times 3.5 \text{ mm}$ with corners rounded, each strip being wound with 0.25 mm paper tape and the whole conductor assembly taped over with 0.5 mm paper. Wound on an s.r.b.p. tube of inside diameter 215 mm and thickness 2.5 mm , the winding is 254 mm in overall diameter. The mean turn has a diameter of 237 mm and a length of 0.75 m . The phase resistance at 75°C is

$$r_2 = 0.021 \times 30 \times 0.75/180 = 0.0026 \Omega$$

and the I^2R loss is $512^2 \times 0.0026 = 0.69 \text{ kW/ph}$

High-voltage Winding. The conductor section $a_1 = I_1/J_1 = 20/3.1 = 6.45 \text{ mm}^2$ is suitable for circular-section wire of diameter 2.85 mm , insulated by paper wrap to 3.15 mm diameter. The h.v. winding is sectionalized into nine coils of 72 turns, two end coils of 55 turns with reinforced insulation, a central 76-turn coil tapped at each end and at three intermediate points, and two gunmetal support rings to form the outmost turn at each end; the total is $(9 \times 72) + (2 \times 55) + 76 + 2 = 760 + 76 = 836$ turns. The normal coils are arranged with 9 conductors axially and 8 radially. The axial length of each coil-section is 31.5 mm including taping. The coils are spaced 6 mm apart by pressboard U-pieces and the support rings are each 12 mm thick. The total depth of the h.v. winding is 480 mm , leaving 35 mm at each end for insulation and bracing.

The outside diameter of the l.v. winding is 254 mm . With an oil duct 12 mm wide to facilitate the cooling of the l.v. winding, the inner diameter of the s.r.b.p. cylinder carrying the h.v. winding is 278 mm . The h.v. winding is fitted over 5 mm axial spacer bars of pressboard, its diameters being 294 mm (inner),

348 mm (outer) and 320 mm (mean). The resistance of the h.v. winding at 75 °C per phase of 760 turns of mean length 1.00 m is

$$r_1 = 0.021 \times 760 \times 1.0/6.45 = 2.48 \, \Omega$$

and the I^2R loss is $20.0^2 \times 2.48 = 0.99$ kW/ph.

Cooling. The core loss is $P_i = 1.37$ kW, the I^2R loss is $P_c = 3(0.69 + 0.99) = 5.04$ kW and the total full-load loss is 6.41, say 6.5 kW. Allowing 10 mm between adjacent h.v. windings and adequate clearances to earth for connections and tappings, the transformer can be accommodated in a tank 1.15 m $\times$ 0.55 m in plan. The core height is 0.90 m; allowing 50 mm for the base and 250 mm of oil above, the oil depth is 1.2 m, and with a further 250 mm for connections to the tank top, the total tank-wall height is 1.45 m. The surface area of the tank walls is 5.0 m², making it necessary to fit cooling tubes. An array of 80 tubes each 50 mm diameter and mean length 1.05 m gives a total tube surface of 13.2 m². Using the method of Sect. 4.3, eq.(4.4), the ratio x is 3.6 and the specific dissipation rate is $p = (8.8 + 3.7/3.6) = 9.83$ W/(m² °C), whence the tank temperature-rise is

$$\theta_t = 6500/[9.83 \times (5.0 + 13.2)] = 36 \, ^\circ\text{C}$$

Impedance. From eq.(5.5), the per-unit resistance is $\epsilon_r = 6.5/400 = 0.016$ p.u. The terms for the per-unit leakage reactance in eq.(2.16) are:

$$\begin{array}{ll} f &= 50 \text{ Hz} & F &= N_2 I_2 = 30 \times 512 = 15\,360 \text{ A-t} \\ E_t &= 8.7 \text{ V} & L_{mt} &= \frac{1}{2}(0.75 + 1.00) = 0.88 \text{ m} \\ L_c &= 0.47 \text{ m} & a &= 0.020 \text{ m} & b_1 &= 0.026 \text{ m} & b_2 &= 0.019 \text{ m} \end{array}$$

and these give $\epsilon_x = 0.046$ p.u. The total leakage impedance is 0.0485 p.u.

Mass. The mass of active ferromagnetic material is $325 + 392 = 717$ kg. The copper in the windings has the mass $108 + 144 = 252$ kg. The ratio $G_i/G_c = 2.85$.

6.7 Large transformers

Rise in service voltage and increase in rating impose more stringent technical demands, making necessary more refined methods of assessing both electro-magnetic and mechanical problems. In transformers of 500 MVA rating and upward, the specific design problems are concerned with very large currents, powerful stray fields, the removal of heat in large quantities, and the short-circuit mechanical strength. Solutions must recognize the restraints imposed by transport facilities and the voltage levels.

The mass of a 1 000 MVA transformer core alone may exceed 200 t. Dimensional limits generally make a 5-limbed core essential, for loading-gauge reasons as well as those of the magnetic circuit. The 5-limbed core can be used to advantage for ratings above 100 MVA if the technical specification permits of optimum geometric proportions. The 5-limbed core is less massive than a 3-limbed core, but its mean specific core loss is higher because the yoke fluxes are no longer sinusoidal; however, the stray loss of a 5-limbed core is greatly reduced because of the magnetic path provided.

Winding design may demand double- or multiple-concentric windings to

carry very large currents (particularly, of course, on the l.v. side). The main insulation can, however, follow existing techniques based on paper, board and oil.

The stray magnetic field is responsible for the generation of eddy-current loss and of disruptive mechanical forces on short circuit. To determine and ameliorate the effects it is necessary to know the stray-field distribution pattern. Considerable advance has been made in field-pattern calculation by digital computer in the difficult cases of complex boundaries and of non-linearities. Screening is sometimes necessary to limit stray loss in tank walls by transferring the eddy-currents to aluminium or copper shields. Even more important than the absolute loss level is its concentration locally: critical sites in this respect are the ends of the coils, the stress rings, the tank sides, and the clamping beams and bolts.

The high power density and the necessarily compact construction of the windings necessitate intensive internal cooling, achieved only by forced oil-flow through the windings to reduce hot-spots and lower the temperature gradient between windings and oil. The basic construction is such that all concentrically arranged windings are hydraulically in parallel, and the flow in each path is determined by its hydrodynamic characteristics: these can vary widely, and may demand some baffle or valve control over the oil flow so that the individual paths may be assigned oil-flow rates proper to the heat-transfer demanded by each.

Avoidance of mechanical breakdown under short-circuit electromagnetic forces requires accurate calculation of the axial forces by digital methods, and mechanically rigid construction of each component winding with the end supports, spacers, cylinders and caps fixed to prevent them from working loose. Pressure elements pre-stress the windings to accommodate the effects of the drying-out and impregnation processes, and to ensure the maintenance of proper axial pressure in service. The windings must have carefully assembled supports to prevent the conductors from buckling due to inward radial forces. These means call for a high degree of dimensional accuracy, a demand not easy to satisfy because the thickness of the spacers varies during construction with the humidity in the winding shop. The use of forced oil circulation, necessary for proper cooling, enables coils to be sited more closely, but results in bigger radial forces: it is therefore necessary to make the inner coils more stable against inward force, and the outer coils resistant to expansion.

6.8 *Design*

Because it does not rotate, the transformer presents certain mechanical simplifications compared with rotating machinery, although it has a number of special features of its own. Considerable advances have been made in devising digital-computer design programs. Synthesis programs have been in regular use for several years. The analysis of surge-voltage distribution in windings, and the optimization of h.v. insulation were developed at an early stage, and later served as components of a full design program, in particular the basic dimensions directly from the customer's specification. Such programs can be run by computer staff having no detailed design knowledge, but minor adjustments

by designers are almost always made to improve details before the design is passed to production

The digital computation of particular features (such as leakage reactance) is normal. The more sophisticated the initial analysis in its use of design parameters, the more useful and accurate is the program.

Principles

The design clearly depends on the voltage and rating, less obviously on the type, service conditions, and the relative costs of active materials, labour, manufacturing equipment and works organization. The optimum design achieves the minimum total cost in relation to the capital cost, energy loss, thermal and mechanical depreciation (resulting from fault-generated stresses), the last few considerations not being readily expressible in money terms. Constraints are imposed by depreciation rates, transport to site, facilities for repair and labour expertise. The design process is an experienced manipulation of the many interrelated variables, and can be exceedingly complex. There are several possible starting points.

From eq.(5.4), a transformer has a *maximum power efficiency* for the fraction k of its rating S when the I^2R loss P_c and the core loss P_i are related by $k = \sqrt{(P_i/P_c)}$. Let G_i and G_c be the total active masses of core and conductor material, and p_i and p_c the specific losses (in power per unit mass); then for maximum efficiency the fractional load is such that $G_i P_i = k^2 G_c p_c$. It is possible to relate the specific losses explicitly to the main magnetic and electric loadings by writing

$$p_i = a + bB^n \quad \text{and} \quad p_c = J^2 \rho / \delta_c$$

where the exponent n is based on tests on core strips adjusted by the building factor (e.g. 1.07), and ρ and δ_c are respectively the resistivity and density of the conductor material. The ratio G_i/G_c can then be expressed in terms of k and the loadings.

Quite different criteria could be used, based on the *aggregate cost* of material (weighted to include processing costs) to secure the cheapest transformer.

Writing γ as the cost per unit mass of the processed and assembled parts, the total cost is the sum $G_i \gamma_i + G_c \gamma_c$. Although G_i and G_c are individually variable, it turns out that the cheapest transformer is one in which the ratio G_i/G_c is approximately the same as γ_c/γ_i .

If both maximum-efficiency and lowest-cost criteria are to be simultaneously satisfied, the requirement is

$$k^2 = \frac{\gamma_c}{\gamma_i} \cdot \frac{\delta_c(a + bB^n)}{J^2 \rho}$$

With k known, and no other unaccounted constraints can exert an influence, the optimum values of J and B can be computed; but if further conditions have to be imposed (such as transport dimensions) they must somehow be related to the specific magnetic and electric bondings, a process that may be rather complicated. For the largest transformers, the imposition of dimensional and

leakage-reactance constraints forces the designer to choose the maximum flux density and to let the current density emerge from the equation.

If k is unknown (the more common case), then B can be settled on saturation, harmonic or noise-limit conditions, and J on the cooling system necessary; but now k has little significance.

The specific loadings are continuously variable, but many data are variable only in steps (choice of coil type, conductor size, cooling method, core stepping, etc.). The current rating, impulse voltage test level and transport conditions dictate the type of winding, and if tapplings are necessary a decision must be made as to where they are to be placed and the consequent effect on leakage, asymmetry and short-circuit forces. Insulation and shielding are related in a complicated way to impulse strength. Flow of coolant affects temperature-rise. From such magnetic, dielectric, thermal and conduction considerations an array of significant parameters is selected: the type, form, section and construction of the core; the self and mutual arrangement of the windings; the dimensional allowances to meet the electrical and thermal boundary conditions.

Provided that analytical relationships can be worked out, the calculations can be made by means of a digital computer program. Many such programs have been devised and are in use in design offices. They are constantly refined in accordance with experience and become more comprehensive, and they are of great value in providing rapid appraisal of the consequences of changes in dimensions and in the cost and properties of new materials.

7

Poly phase Rotating Machines

7.1 *Synchronous and induction machines*

The basic concepts relating to rotating machines in general are discussed in Sect. 1.10–1.13, in which it is assumed that the distribution in the airgap of the flux produced by sine-distributed current sheets is itself sinusoidal. It is shown in Sect 3.14 that balanced currents in a 2- or 3-ph winding approximate to sine-distributed current sheets, and have in addition the further and important property of developing an airgap flux in the form of a travelling wave that moves one double pole-pitch in one period of the a.c. supply.

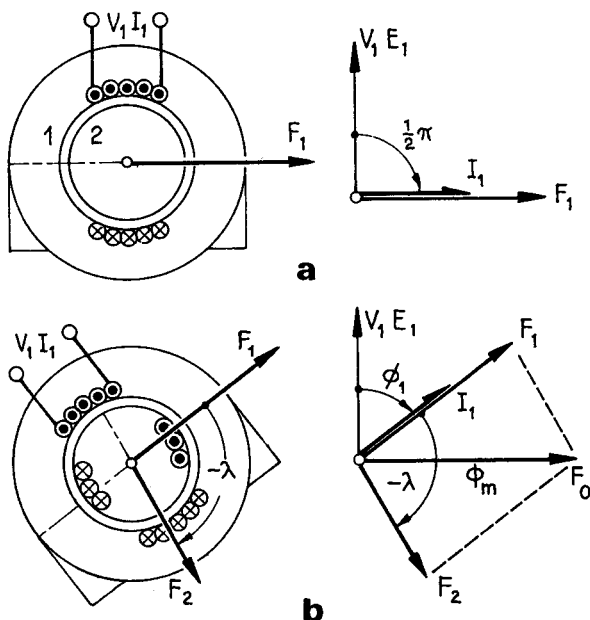
The torque set up in such an arrangement is a function of the stator and rotor m.m.f.s F_1 and F_2 , and of the torque angle λ between them. The condition that the torque shall be maintained unidirectionally under steady-state conditions is that for given stator and rotor m.m.f.s the torque angle shall remain constant. Thus both F_1 and F_2 must be waves travelling at the same speed. With the stator carrying a polyphase winding connected to a supply of angular frequency ω_1 then F_1 travels at the angular speed ω_1 in a 2-pole winding, or at the synchronous speed $\omega_s = \omega_1/p$ in a winding with p pole-pairs. The rotor m.m.f. F_2 must therefore travel at ω_s with respect to the stator. The speed of rotor rotation, ω_r , then depends upon the movement at speed ω_2 of F_2 with respect to its own rotor windings. If F_2 is 'fixed' by supplying d.c. rotor excitation, then the rotor must rotate at $\omega_r = \omega_s$ and can produce steady-state torque at no other speed: the arrangement gives a *synchronous* machine. If, however, the rotor m.m.f. is produced by a 2- or 3-ph winding carrying currents of frequency ω_2 , then the rotor rotation must be such as to give $\omega_r = \omega_1 \pm \omega_2$ in a 2-pole machine, or $\omega_r = (\omega_1 \pm \omega_2)/p$ in a $2p$ - pole machine. By far the commonest arrangement to give this effect is the *induction* machine.

In this Chapter the polyphase synchronous and induction machines are considered in general terms, with particular reference to ways in which they can be modelled for analysis. Such an analysis must deal not only with *steady-state* conditions but must also be able to take account of *transience*. In the former the machine can be taken as having a purely electromagnetic nature, but in the latter the model must be elaborated to include the mechanical effects of speed and load changes and the response of control and regulating systems. These result from the nature of the machine as a link between electrical and

mechanical networks, the responses of which vitally effect the behaviour of the machine.

Representative phase for steady-state conditions

Each stator phase of a balanced m -phase machine deals with $1/m$ of the total active and reactive power input to (or output from) the machine; and all have the same magnitudes respectively of voltage, current and phase angle, differing only in the time-phase displacement of $2\pi/m$ rad between the voltage or current of one phase and the next. It is useful to refer the machine as a whole to the measurable voltage V_1 and current I_1 of one *representative* phase winding. In Fig. 7.1(a) let only the stator be excited by connection to a polyphase



7.1 Polyphase machine: representative phase.

supply. At any instant that phase carrying *peak* current, $i_{1m} = \sqrt{2}I_1$, fixes the axis of the m.m.f. F_1 of the complete polyphase stator winding. In a time phasor diagram for this representative phase winding, its current I_1 can be drawn in the direction of the total-m.m.f. axis F_1 as an arbitrary, but pictorially advantageous, assumption. Then the stator applied voltage V_1 , and the e.m.f. E_1 generated in the phase winding, must be drawn in leading phase-quadrature with I_1 because the representative phase (as, indeed, the stator-winding as a whole) is simply an inductor taking only reactive magnetizing power (if loss can be ignored). Thus we can combine the phasor and m.m.f. diagrams into one that conventionally associates space conditions in the gap with time phasors of one of the contributing phase windings.

Now let both members be excited. The rotor is assumed to have a polyphase winding identical with that of the stator (i.e. it is 'referred' to the stator), and each rotor phase winding carries a current I_2 developing an m.m.f. F_2 . With F_2 aligned on the axis of that rotor winding carrying peak current at the instant considered, let F_2 make the torque angle $-\lambda$ to F_1 , corresponding to a motor mode. It is now the *resultant* m.m.f. F_0 that determines the gap flux Φ_m . The conditions are shown in Fig. 7.1(b), the stator having been tilted in the diagram in order to make the axis of F_0 and Φ_m horizontal. As Φ_m induces E_1 in the representative stator phase, both E_1 and V_1 lead Φ_m by $\frac{1}{2}\pi$ rad. But the phase angle ϕ_1 between V_1 and I_1 is now less than $\frac{1}{2}\pi$ and the representative stator phase is accepting a complex power $S_1 = V_1 I_1 (\cos \phi_1 + j \sin \phi_1) = P_1 + j Q_1$, where P_1 is $1/m$ of the total active input power, and Q_1 is $1/m$ of the total magnetizing reactive power of the stator.

Fig. 7.1 applies only to steady-state balanced conditions. Any transient or fault condition that disturbs the phase balance must be dealt with by considering the machine as an entity, with the individual phases making individual contributions to the operation in accordance with the appropriate constraints imposed by the circumstances that obtain.

Operating conditions

An electromagnetic machine, connected to an electrical supply system at its terminals and to a mechanical load or prime-mover at its shaft, is a device of great complexity. It is therefore customary to adopt conditions of operation which are simplified as far as possible while still retaining the essentials for a representation adequate to bring out the characteristics sought. The electrical supply system can, for some purposes, be taken as an *infinite busbar* system. If speeds can be considered as constant, then the mechanical system is reducible to a 'dead' load torque. If the behaviour can adequately be considered as consisting of small cyclic oscillations about a steady mean speed, the electrical side of the machine can be dealt with in terms of constant speed with appropriate electrical perturbations, and the mechanical side by cyclic fluctuations of stored kinetic energy together with any changes of load torque arising from the fluctuation of speed. But if the electrical machine is of large rating, and if substantial speed changes occur, the analysis must be comprehensive enough to cope with many interacting variables, electrical and mechanical, involving the supply network and the mechanical controls (such as governing) as well as the electromagnetic machine itself. In these circumstances an analytic solution is well-nigh impossible, and recourse must be had to the step-by-step solution of each specific problem by computer.

Infinite busbar. For steady-state conditions, and particularly for small machines that do not greatly affect the supply system, the machine can be assumed to run from infinite busbar providing a constant-voltage constant-frequency supply capable of supplying or absorbing electrical energy without limit and at any rate demanded of it.

7.2 Rotational effects

A rotating electromagnetic machine must be a 'machine' in the mechanical as

well as in the electrical sense, and is subject to mechanical effects that may greatly influence its design and behaviour.

Centrifugal force

The centrifugal force on a body of mass m revolving at angular speed ω_r is given by $m\omega_r^2 R$, where R is the radius of gyration.

Consider a cylindrical steel rotor, such as that of a turbo-generator, of diameter 1.0 m and rotating at 50 r/s: the centrifugal force on each 1 kg of mass at the surface is $(2\pi 50)^2 \times 0.5 = 50$ kN. Such a rotor is slotted to take copper windings, and the teeth have to withstand the centrifugal force of the windings as well as that arising from their own mass. Let the rotor be uniformly slotted with parallel-sided slots 35 mm wide and 200 mm deep, cut at an angular pitch of $1/30$ of the circumference. At its root, each tooth is consequently 28 mm wide, and has to contain the tooth and slot content above it against the centrifugal force. Taking the density of the slot content (copper and conductor insulation) as averaging the same as that of the steel ($7\,800$ kg/m³), the centrifugal force on an element of radius r and thickness dr for a 1 m axial length, is $\omega_r^2 r \cdot dm$ where $dm = 7\,800 (2\pi r/30)dr$. Over the tooth depth (i.e. from $r = 0.3$ m to $r = 0.5$ m) this integrates to 5.3 MN, a centrifugal force spread over a tooth-root area of 0.028 m² to give a tensile stress of 190 MN/m². If, as is usual, the rotor is designed to withstand a 25% over-speed, the stress is raised by the factor $(1.25)^2$ to 300 MN/m². As a result there are mechanical limitations imposed on the diameters of large high-speed machines.

Hoop stress

Turbo-generator rotor end-windings are retained by end-bells, and the rotor rims of hydro-generators carry the rotor poles. Rotating about their axis these develop hoop stress due to centrifugal force, both self-developed and imposed by the inner or outer masses that have to be retained.

An annular ring of radius R and unit cross-section develops a self centrifugal force of $\delta\omega_r^2 R$ per unit length of circumference which imposes a tensile hoop stress amounting to $2\delta\omega_r^2 R^2$ in the ring, where δ is the density of the material. For example, let the rotor windings of a turbo-generator to be retained at each end by rings 0.35 m in axial length and 45 mm in radial thickness, and of mean-radius 0.49 m; and the windings have a mass of 300 kg at a mean radius of 0.35 m. The self-produced hoop stress is $7\,800 (2\pi 50)^2 \times 0.49^2 = 190$ MN/m² and the copper adds a further 100 MN/m² to give a total of 290 MN/m² of which nearly two-thirds is self-produced. The rotor itself has a diameter of 1 m and runs at 50 rev/s; with a greater diameter or a higher speed, the limits of the retaining rings would be approached and they would be unable to retain any useful mass — perhaps not even their own.

Critical speed

As a structure with elastic mass, a rotor has a natural frequency of vibration. Consider a shaft running in two bearings without restraint, and carrying a disc

of mass m having its mass-centre displaced ϵ_0 from the axis of rotation. At a speed ω_r the centrifugal unbalanced force on the mass is $m\omega_r^2\epsilon_0$. This will produce a shaft deflection ϵ , increasing the unbalanced force to $m\omega_r^2(\epsilon_0 + \epsilon)$. The deflection is resisted by the elastic shaft, which develops a counter force equal, say, to $k\epsilon$. For equilibrium the two forces are equal, giving

$$\epsilon = m\omega_r^2\epsilon_0/(k - m\omega_r^2)$$

At some critical angular speed ω_c the denominator becomes zero and the deflection theoretically infinite. In practice ϵ is limited by friction and flexure loss, but at this *critical speed* will tend to be large. If there is no eccentricity ϵ_0 the expression becomes indeterminate, and a machine may indeed run through its critical speeds without vibration; but chance disturbance of centrality is probable and it is hazardous to operate a machine near to one of its critical speeds. The elementary case considered does not apply to practical systems. There may be more than one shaft span, bearing constraint, the development of nodes, and the complication of a rotor not equally stiff across all diameters (as in a slotted turbo-rotor). In small machines the critical speed is well above normal running speed, but the shaft must be stiff enough to withstand unbalanced magnetic pull arising from eccentricity.

Balance

Lack of static and dynamic balance may produce disastrous vibration. Large rotors are usually adjusted on a balancing machine at moderate speeds, at which they behave substantially as rigid bodies. In operation, however, they show a compound deflection curve between the first and second critical speeds through which they may have to pass. For this reason subsequent balancing on site at rated speed may be indispensable. The electrical sensing and compensating circuits of the balancing machine must then be imitated by some suitable mechanical artifice.

Bearings

Roller and ball bearings are common in small machines, and give satisfactory service with little attention. Conditions are very much more severe in large generators. In turbo-generators the journal bearing rubbing speeds may exceed 70 m/s, the pressures 2 MN/m² of the projected area, and the friction loss of the order of several hundred kilowatts. Advanced bearing design is necessary, as the oil-feed pressures are insignificant in comparisons with the bearing pressure, and serve only to keep the bearing flooded and the oil in circulation through a cooler. In hydro-generators the thrust bearing is of major importance, particularly in vertical-shaft machines where the bearing supports the weight of the rotor and turbine runner as well as a downward hydraulic thrust. A thrust bearing 2.5 m in diameter may carry a load of 1 000 Mg with a rubbing speed of 20 m/s normally or twice this value on overspeed. If the machine is shut down for some hours or days the oil film is squeezed out, in which condition a start would seriously damage the bearing. To restore the oil film, the rotor and and runner may be initially jacked up by about 5 mm, or raised by means of a

pressurized hydrostatic oil system by about 0.05 mm, using pressures up to 10 MN/m² which are reduced when the machine has been started and has reached about one-tenth of normal running speed.

Kinetic energy

Inertia, and the stored kinetic energy associated with it, are factors of importance in machine operation when starting and stopping, and in oscillatory and transient conditions. The necessary kinetic energy must be supplied (electrically and/or mechanically) to a machine to bring it from rest to running speed; and unless the machine is braked regeneratively the energy is eventually lost in friction. Speed fluctuations impose the release and absorption of amounts of kinetic energy, and in some cases strong oscillations can be sustained unless there is some way in which the oscillation energy can be dissipated by damping.

The inertia concerned is, of course, that of all the coupled rotating parts. In large modern turbo-generators the inertia of the steam-turbine rotor is dominant. In hydro-generators the inertia of the water-turbine runner is comparatively small, and additional inertia may have to be built into the electrical machine or incorporated in a flywheel, for a minimum inertia is necessary because of the sluggishness of hydraulic governing. If a dropped electric load is not to result in excessive rise in speed, endangering the structure by centrifugal forces, the inertia may have to be very great (especially if the normal speed is low) and may considerably influence the design.

A mass of inertia J , rotating at angular speed ω_r , stores a kinetic energy $w_s = \frac{1}{2}J\omega_r^2$, and its momentum is $m_0 = J\omega_r$. The 'flywheel effect' in electro-mechanical systems is often expressed in terms of H , the *inertia constant* with the dimensions of time, defined as the ratio of the stored energy (joules) at normal speed to the rating (volt-amperes) of the machine:

$$H = w_s/S = \frac{1}{2}J\omega_r^2/S = \frac{1}{2}m_0\omega_r/S \approx 2Qn^2/S \quad (7.1)$$

where n is the speed frequency (in r/s). The inertia constant is directly informative. For example, a generator on full load will, if its load is suddenly thrown off, accelerate at the rate (in rad/s²) of $\omega_r/2H$; and H represents one-half of the time taken to bring the rotating parts of a machine from rest up to rated speed with a torque corresponding to the rating, neglecting losses and damping.

The magnitude of H has a surprising small range: it lies commonly between 1 and 10, with the smaller figures for the larger ratings. With motors the inertia of the attached mechanical load can vary widely, but with synchronous generators and compensators the values of H can be expected to lie in the ranges following:

Steam turbo-generators: 7–2.8 (2-pole), 10–5 (4-pole)

Hydro generators: 4–2 (20-pole), 2.5–1.5 (60-pole)

Synchronous compensators: 1.25–1

7.3 Drive dynamics

The torque balance of a machine connected between an electric system and a

mechanical load or prime-mover is given at every instant by the equation

$$M_e = M_a + M_s + M_c + M_l \quad (7.2)$$

which relates the electrical input torque to the mechanical accelerating, elastic, damping and dead-load torques. The rate of change of angular speed of the rotating system of inertia J is

$$\alpha = d\omega_r/dt = M_a/J \quad (7.3)$$

whence the time for a small speed change $d\omega_r$ is

$$dt = (J/M_a) \cdot d\omega_r \quad (7.4)$$

If M_a is constant, the system accelerates from speed ω_p to a speed ω_q in a time $t = (\omega_q - \omega_p)/\alpha$.

Electrical torque, M_e . This is a function of time, voltage, current, speed, frequency, slip, load angle and the relevant circuit parameters of inductance and resistance. Sometimes, if the mechanical rotary system has a large inertia, it can be assumed that the electrical transients imposed by some sudden change of condition have vanished before the mechanical response begins to take effect; but this assumption, which greatly simplifies the analysis, is by no means always valid.

Elastic torque, M_s . This is associated with the elastic deformation of shaft and stator frame, and comes into play when an impulsive torque is suddenly applied to one end of a shaft which has an inertia load at the other, or with flexible mechanical couplings, or with the winding of mine cages at the end of long suspension cables. Except in the last-named case, the extent of the deformation is small and its effect on speed insignificant, but a substantial amount of strain energy may be imparted to shafts and structures by it, with such consequences as failure by shearing and the impact on frames and foundations.

Loss Torque, M_c . Losses in friction, windage, eddy-currents in damping windings or in solid parts of the magnetic circuit, develop heat energy and impose a corresponding torque on the system.

Load torque, M_l . Mechanical input or output torques, considered as distinct from inertia (accelerating) torque, are determined by the characteristics of the mechanical system. These are normally speed-dependent.

EXAMPLE 7.1: A synchronous-motor drive with a moment of inertia $J = 420 \text{ kg-m}^2$ and a friction torque $M_f = 135 \text{ N-m}$ (assumed to be independent of speed) runs on no load at 500 r/min. The drive is dynamically braked to standstill by connecting the stator windings to a bank of fixed resistors, the field current being held at a value that gives an initial electrical braking torque $M_e = 5.4 \text{ kN-m}$, the torque thereafter being proportional to speed. Estimate the time to brake the machine to rest, and the number of revolutions of the rotor in that time.

The synchronous speed is $\omega_s = 52 \text{ rad/s}$, so that the electric-braking torque can be expressed for any speed ω_r as $K\omega_r = 5.4 \times 10^3 (\omega_r/52) = 104\omega_r$. The total braking torque is $M_b = (M_f + K\omega_r)$, the acceleration is $d\omega_r/dt = -M_b/J$, and

therefore $t = -(J/K) \ln(M_f + K\omega_r) + \text{const.}$ As $\omega_r = \omega_s$ at $t = 0$, then

$$t = \frac{J}{K} \ln \left[\frac{M_f + K\omega_s}{M_f + K\omega_r} \right]$$

The time t_1 to reach zero speed, $\omega_r = 0$, is

$$t_1 = \frac{420}{104} \ln \left[\frac{135 + 5400}{135 + 0} \right] = 15 \text{ s.}$$

The total angular rotation θ is the time-integral of ω_r . The expression above for t gives

$$(M_f + K\omega_r) = (M_f + K\omega_s) \exp(-K/J)t,$$

and with $\theta = 0$ for $t = 0$, the total angular rotation θ_1 in time t_1 is

$$\begin{aligned} \theta_1 &= \frac{1}{K} \left[\frac{J}{K} (M_f + K\omega_s) \left\{ 1 - \exp \left(\frac{K}{J} \right) t_1 \right\} - M_f t_1 \right] \\ &= 192 \text{ rad} = 31 \text{ revolutions} \end{aligned}$$

Below $\omega_r = 1.3 \text{ rad/s}$ the electrical braking torque is less than the friction torque, and the machine is finally stopped by friction alone.

Electrical torque

Expressions for the electrical torque of a prototype machine in two-axis terms are given in Sect. 7.5. They are based on axis currents and inductances, but the currents are affected by a change of speed and also in practice the inductances themselves are current dependent as a result of saturation. All the terms in eq.(7.2) are in fact speed-dependent, and solutions of the equations of behaviour have to be obtained by an iterative process, a formidable task even with computer aid. Particular cases can, however, often be simplified by making appropriate assumptions or adopting special *ad-hoc* techniques.

In purely electrical problems it is common to select a chosen rating (not necessarily that of the machine under consideration) as the per-unit base of power. The technique is equally applicable to electromechanical cases. The inertia constant H of eq.(7.1) is expressed as a ratio. The torque corresponding to a rating S at a normal (synchronous) speed ω_s is $M_1 = S/\omega_s$, and per-unit torques can be stated as the ratio of the actual torque to M_1 . Similarly the per-unit speed is ω_r/ω_s for a given actual speed ω_r , and the per-unit acceleration is $\alpha_1 = \frac{1}{2}M_1/H$ (in per-unit speed-change per second). As this acceleration is normalized, conclusions from it are also normalized, making it possible to obtain normalized response curves for typical values of inertia and of torque/speed characteristics in per-unit terms. Such curves enable generalized performance to be derived, for application to specific problems after reduction to per-unit values.

Energy-rate balance

An electromechanical machine system in terms of instantaneous input powers

is described by eq.(1.6). With an inertia J , an elastic torque $(1/K)\theta$ arising from a displacement angle θ and a compliance K (which may be electrical as well as mechanical in origin), and specific forms of loss, some of the terms on the right-hand side of eq.(1.6) can be written

$$\text{Rate of change of stored energy:} \quad \frac{dw_s}{dt} = \left[J \frac{d^2\theta}{dt^2} + \frac{1}{K}\theta \right] \omega_r$$

$$\text{Rate of energy loss:} \quad p = C_1 \omega_r + C_2 + \Sigma I^2 R$$

The loss power comprises a term proportional to speed $\omega_r = d\theta/dt$, and typical of both mechanical viscous-friction and electrical eddy-current loss; a constant; and $I^2 R$ loss in the machine windings. Eq.(1.6) includes the term dw_f/dt for the rate of change of magnetic-field energy. The energy-rate balance equation for the system can then be written

$$p_e + p_m = \frac{dw_f}{dt} + \left[J \frac{d^2\theta}{dt^2} + \frac{1}{K}\theta \right] \omega_r + C_1 \omega_r + C_2 + p_c \quad (7.5)$$

where for convenience the $I^2 R$ loss is designated p_c . In the case of large machines it may be permissible to ignore p_c as being insignificant in comparison with other terms, and sometimes the change of magnetic energy can also be ignored.

Oscillations

The combination of an inertial and an elastic restoring torque makes possible the generation of oscillations by reason of the interchange of energy between the two. The basic idealized situation in which only the J and K terms are operative is one in which the system has a natural undamped oscillation angular frequency $\omega_n = 1/\sqrt{JK}$. Idealized damping proportional to the instantaneous angular velocity of the oscillation, and having the form $C(d\theta/dt)$, modifies the oscillation in accordance with its sign: if positive it reduces the amplitude with time according to an exponential law, and if it has the critical value $C_0 = 2\sqrt{J/K}$ its effect is just to suppress oscillation altogether; if the damping term is negative it increases the oscillation amplitude with time, again exponentially. Thus a torque M applied to a rotary inertial-elastic system yields the torque equation

$$M = Jp^2\theta + (1/K)\theta + Cp\theta = (p^2 + \omega_n^2)\theta + 2c\omega_n p\theta \quad (7.6)$$

where $c = C/C_0$ is the damping ratio.

The natural frequency of oscillation of an electromagnetic machine is usually low (one or two oscillations per second) and does not vary greatly with the size of the machine unless there is a shaft load of abnormal inertia. The compliance, however, may change with the excitation on the electrical side of the electro-mechanical system.

Oscillatory conditions may arise in several ways, especially where there are forced oscillations at a frequency near to that of the natural frequency. Oscillation amplitudes may be substantially increased by mechanical torques pulsating

at low frequency, as in a generator driven by a diesel engine, or a motor driving a reciprocating air-compressor. In ship propulsion with direct electric drive, fluctuating load torque may be impressed by the screws, and oscillation is possible in mill shafting connected to rolls some considerable distance apart. Rope elasticity in the vertical or inclined haulage of mine winders may resonate with the load mass at a frequency near to the natural frequency of the machine or of its closed-loop control devices.

Induction and synchronous machines respond in different ways to a change of load. The *induction* machine responds by a change of slip (and therefore of speed), and oscillations are of purely mechanical origin. The *synchronous* machine, when operating in parallel with other synchronous machines or on supply busbars, responds by adjusting its torque angle without change in *mean* speed: any divergence between the actual and equilibrium angles calls into play a synchronizing torque proportional to a function of the angle of divergence and so adding to the system an elastic torque of electrical origin. A case of interest is that for which the load change is cyclic.

Cyclic disturbing torque. Cyclic irregularity in the driving torque of a generator (as when it is driven by a diesel engine), or in the load torque of a motor (as for an air-compressor drive), or in a control system (e.g. governor 'hunting'), excite small fluctuations of speed. With induction motors the speed variations are constrained by the system inertia and the inherent damping by I^2R loss is large. With synchronous machines the synchronizing torque adds electrical elasticity. and provides a combination with a natural oscillation frequency. If this is close to the frequency of the cyclic disturbance, and if the damping is small, the conditions may be such as to result in considerable phase-swing. Suppose that the cyclic disturbing torque can be expressed as a simple sinusoidal variation $M_d \sin \omega_d t$ of angular frequency ω_d : the resulting acceleration α is proportional to the torque and in phase with it; the speed change ω is the time-integral of α ; and the angular divergence δ is the time integral of ω . Hence we have

disturbing torque:	$+ M_d \sin \omega_d t$
acceleration:	$+ \alpha_d \sin \omega_d t$
speed-change:	$- \omega_d \cos \omega_d t$
angular divergence:	$- \delta_d \sin \omega_d t$
synchronizing torque:	$+ (\delta_d/K) \sin \omega_d t$

the last expression being obtained from a synchronizing torque proportional to the angle of divergence and acting so as to reduce the displacement to zero. It is seen that the synchronizing torque is in phase with the disturbing torque, and reinforces it. Writing $p = j\omega_d$ and $\theta = \delta_d$ in eq.(7.6) for the sinusoidal conditions here assumed, the displacement due to the disturbance is

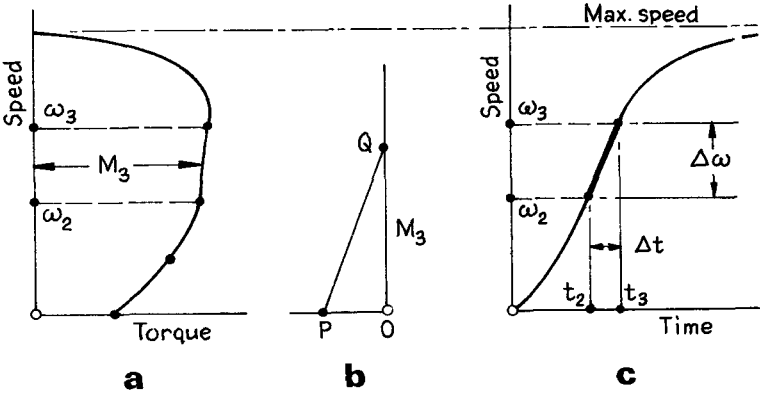
$$\delta = \frac{KM_d}{1 - \omega_d^2(JK) + j\omega_d(KC)} = \frac{KM_d}{(1 - k^2) + j2ck} \quad (7.7)$$

where $k = \omega_d/\omega_n$ is the ratio of the disturbing frequency to the natural undamped frequency of the system. Clearly, if the damping is small and ω_d approaches ω_n the divergence may grow large enough to swing the machine

out of step. It must also be noted that cyclic disturbing torques may be rich in harmonics, one of which may resonate. Low-frequency harmonics are as a rule more likely to be troublesome; in engine-driven generator sets, for example, there can be a torque harmonic at one-half engine speed, corresponding to the speed of the camshaft.

Graphical methods

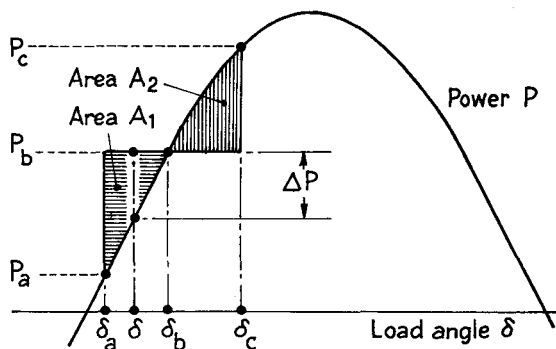
Motor starting. The speed/torque characteristics of motors and loads can be complicated, but they can be graphed and integrated in steps to give the run-up speed/time relation, it generally being necessary to ignore the short-lived electrical switching transient torques and to employ 'steady-state' motor characteristics. The construction shown in Fig. 7.2 shows at (a) the relation for a given



7.2 Graphical integration for speed/time relation.

case of the speed ω_r to the net accelerating torque M_a , derived as in eq.(7.2) by deducting $(M_s + M_c + M_l)$ from M_e . Then eq.(7.4) is solved by finite differences by finding $\Delta \omega_r / \Delta t = M_a / J$ for the slope of the speed/time curve (c) over selected speed steps ω_0 to ω_1 , ω_1 to $\omega_2 \dots$, using the auxiliary slope PQ in (b). Pole P is placed so that $OP = J$ to the same scale that $OQ = M_a$. The run-up time, theoretically infinite, can be taken as that for which the speed reaches 99% of the final steady-state value.

Swing curves. In a polyphase synchronous machine the electrical torque M_e is roughly a sine function of the load angle δ , as shown for the motor mode in Fig. 7.3. A change in load implies a corresponding change in the equilibrium load angle. Suppose that the machine is working at a load P_a and load angle δ_a , and that the mechanical power output is suddenly increased to P_b , the equilibrium angle for which is δ_b . The effect is an unbalanced power ΔP and a corresponding torque $\Delta M = \Delta P / \omega_s$ acting to retard the rotating parts of inertia J . (Although the result is a small change of speed, the *mean* speed must still be the synchronous value ω_s). The torque does work on the system in increasing the load angle from δ_a to δ_b , the retardation causes the rotating parts to lose a corresponding kinetic energy, and as a result the load



7.3 Equal-area criterion.

angle increases still further to δ_c by overshoot. During the overshoot the power-difference and the torque reverse, the electrical input power now exceeding the mechanical power demand; the speed begins to rise and the load angle to reduce again. Oscillation in this mode continues until damping has dissipated the cyclic component of the kinetic energy. The torque equation is

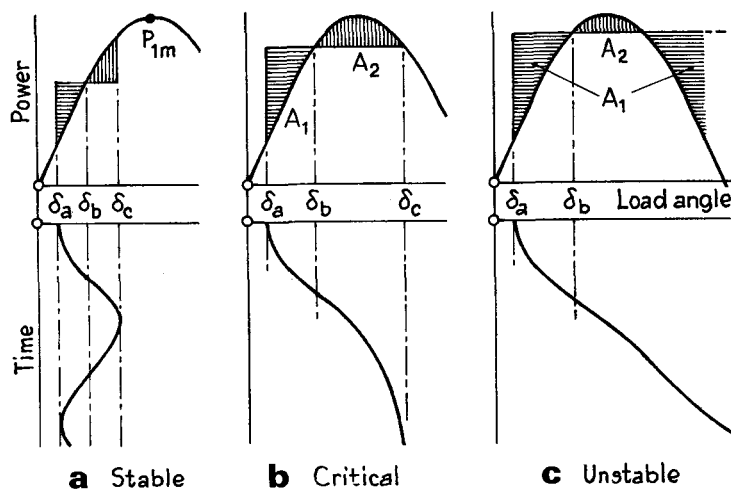
$$\Delta M = J(d^2\delta/dt^2) + C(d\delta/dt)$$

Alternatively the power equation can be written as $\Delta P = \omega_s \Delta M$. The difference torque depends on the load angle which in turn is a function of time, so that ΔM is a function of both δ and t .

The *equal-area criterion* can give a simple solution to the question of stability, i.e. of whether the rotating parts can swing through the angular excursion without falling out of step. The method ignores damping. If, in Fig. 7.3, the area A_2 is equal to the area A_1 , this condition defines the excursion of the rotor load angle for stable operation. The hatched areas are integrals of ΔM with respect to δ and are therefore proportional to kinetic swing energy. If $A_2 = A_1$ then energy lost in the change from δ_a to δ_b is fully regained in the change δ_b to δ_c .

Fig. 7.4(a) shows such a *stable* condition. In (b) the overswing energy represented by A_2 can just match that in A_1 of the initial swing and the condition is *critical*. In (c) there is insufficient torque during overshoot to provide the initial loss and the load angle passes into a region in which the rotor is actively retarded: the load angle rises as the machine falls out of step, an *unstable* condition.

The equal-area criterion, described for a motor, is equally valid for the generator mode. However, stability is most likely to be in danger under fault conditions, when the system voltage falls (changing the power angle relation), and both governor and voltage-regulator action further affect behaviour after a time delay. Again, the stabilizing property of damping is ignored. The criterion cannot give the angle/time relation which is important in considerations of control and protection; for this a step-by-step computation is necessary, yielding the swing curves in Fig. 7.4.



7.4 Stability.

7.4 Models

In the present context a model is a formulation of part or of the whole of a machine system in terms of appropriate parameters and on the basis of some acceptable theory of behaviour. The formulation may be (i) *mathematical*, as a set of integro-differential simultaneous equations or of transfer functions, or (ii) *analogue*, comprising hardware arranged to represent the prototype to some scale of size and time, and including such devices as analogue computers. In either case the aim is to observe the response of the model to a given stimulus, so that its behaviour can then be translated into the comparable performance of the prototype.

The model must have the appropriate *parameters*, and enough of them to give useful results. The choice of parameters is dictated by the stimulus to be applied and the response expected; and there must be some theory of behaviour into which the parameters will fit.

Some considerations in the choice of a model for an electrical machine are briefly discussed in Sect. 1.4. An ever-present difficulty is magnetic saturation, which produces physical effects that are not readily made analytic in the mathematical model, but can sometimes be incorporated into hardware by simulation.

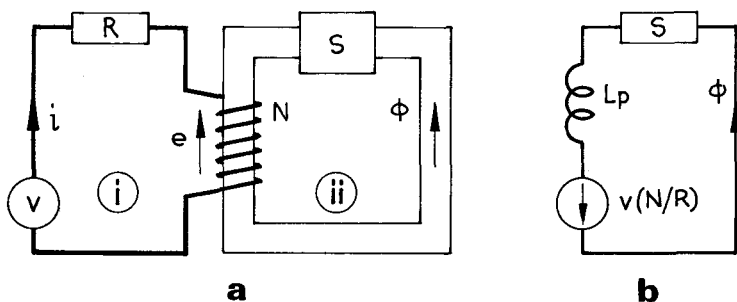
A model adequately representing a *complete* system of electrical supply, electromechanical machine and mechanical attachments is extremely complex. A usual method of simplification lies in the approximate representation of those parts of the system that are not concerned with a specific investigation, but modelling the parts of interest in more detail. Full-scale tests on a supply system under load, fault and transient conditions can sometimes be carried out (e.g. during holiday week-ends). Organization of such tests is elaborate and expensive, takes months of preparation and involves very comprehensive programming and instrumentation. When evaluated the results can be used to check

model parameters and to refine the modelling technique. But the fact remains: only the characteristics of a machine or system can be evaluated from the model by computation, not the actual system itself, and the answers given by a model have always to be treated with caution.

Equivalent circuits

An electric circuit associated with a magnetic circuit can, up to a point, be described in purely electrical terms by means of resistance and inductance parameters. Some considerations in the choice of parameters for the representation of a transformer are discussed in Sect. 5.5, from which the limitations of a simple resistance-inductance model can be inferred. Nevertheless, so long as such a model is employed only within its region of validity, it does provide a powerful concept in machine behaviour.

An electric circuit expressed in terms of parameters is an idealized model of a physically comparable circuit. An *electric equivalent circuit* in machine technology incorporates the effects of a magnetic circuit to which it is coupled. As Laithwaite [73] has pointed out, a magnetic circuit models a magnetic prototype, and a *magnetic equivalent circuit* could be set up to incorporate the effects of the coupled electric circuit with equal validity. Consider the coupled circuits in Fig. 7.5: the current i in the electric circuit is the source of the



7.5 Magnetic equivalent circuit.

m.m.f. Ni which develops the flux Φ in the magnetic circuit of reluctance S . The basic behaviour is expressed for the respective circuits (i) and (ii) in (a) by

$$(i) \quad e = N p\Phi \quad \text{and} \quad (ii) \quad Ni = S\Phi$$

The 'Kirchhoff' law in each case is

$$(i) \quad v = Ri + N p\Phi = (R + L p)i \quad \text{and} \quad (ii) \quad i = (S/N)\Phi$$

where $L = N^2/S$ is the inductance. Eliminating i ,

$$v(N/R) = S\Phi + (N^2/R)p\Phi = (S + L_p)\Phi$$

and $L = N^2/R$ is the *transference*, or magnetic equivalent of inductance. The result expresses a magnetic equivalent circuit (b) in which the coupled effects

of the associated electric circuit are included. The viewpoint has advantages in that a machine has only one main flux but several associated windings. Fiennes [97] works out a generalized machine theory using magnetic equivalent circuits. At present, however, analysis in electric-circuit terms is well established and more familiar.

Mathematical models for electrical machines

Comparatively simple models can be set up for specific parts of a machine, more elaborate ones for comprehensive analysis.

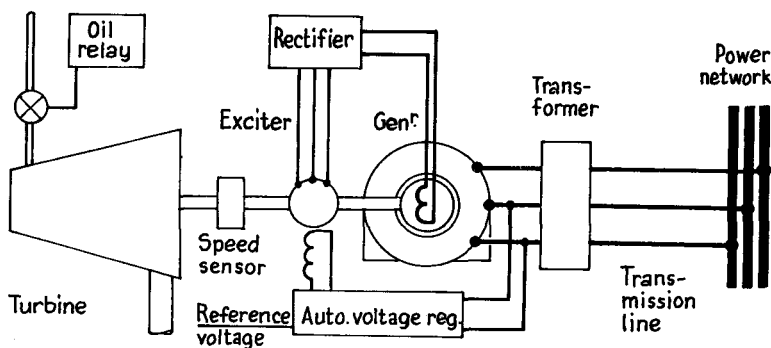
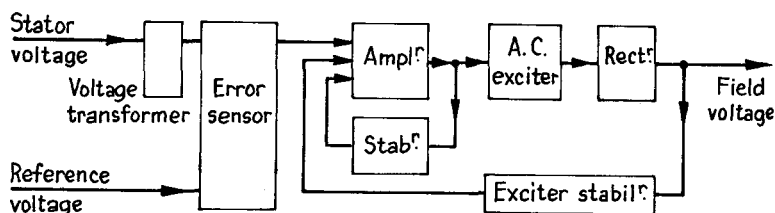
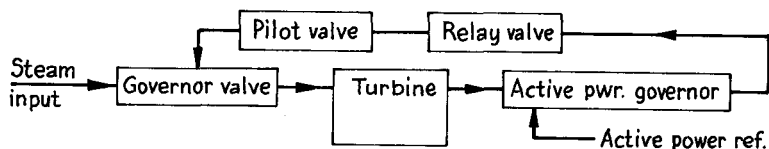
Electric circuit model. The representation of the electric circuits of a machine in two-axis terms is discussed in Sect. 7.5.

Electromechanical models. When both electrical and mechanical interactions are to be studied, the problem of modelling becomes more complicated. Particular examples concern generator stability, control-system design, the starting of synchronous motors, the behaviour of boiler-turbine-generator sets, the effect on supply systems of large induction machines during faults, and the optimization of generating-station performance.

The steam turbo-generator system has been analysed several times in the literature. In a classic paper [74] Shackshaft established a 'general-purpose' model for the system shown diagrammatically in Fig. 7.6(a), representing a real system which was subjected to full-scale field tests in order to verify the selected parameters or to derive them by such techniques as curve-fitting. The generator itself was modelled by the methods of Sect. 7.5. The voltage regulation and excitation network, (b), required the setting up of equations and transfer functions for the voltage sensor with its amplifiers and stabilizers, and for the exciter response; the governing system (c) required equations for the action of the centrifugal governor, relays and valves, and for the turbine power and torque; the transmission network was represented by lumped resistance and inductance. Humpage and Saha [75] gave a model suitable for digital computation of the same system as that field-tested by Shackshaft. Optimization of the running of a boiler-turbine-generator set by mathematical modelling, and for a multi-machine supply system, have been investigated by Nicholson [76]. These several references demonstrate the formidable task of modelling such multivariable systems, the justification being the great economic importance attached to an understanding of the performance of very large machines under abnormal conditions, as well as of their efficient operation on a system load cycle.

Micromachines

A micro-machine is specially designed to represent, on the basis of per-unit parameters, a large machine. A 2 kW low-voltage machine can be made (though with some difficulty and with auxiliary equipment to compensate for IR drop) to represent, say, a 1 000 MW synchronous generator. With alternative rotors the micro-machine can simulate many alternative large-machine designs. A driving motor can be controlled to represent a mechanical load, or a prime-mover with appropriate turbine and governor characteristics, and by the addition to the shaft of an appropriate flywheel can give the model the required in-

**a** System**b** Voltage regulation**c** Governing

7.6 Turbo-generator control system model.

ertia. The field resistance of a micro-machine is inevitably of higher per-unit resistance than that of a large machine, and for proper simulation must be provided with an adjustable negative resistance externally, operated by electronic feedback devices. Voltage regulators, either actual full-scale equipments or electronic simulators, can be connected to the field circuit. In such ways it is possible to represent very large machines by miniscule machines, greatly facilitating the prediction of large-machine behaviour under abnormal conditions with very detailed measurements. The investigations made possible are more comprehensive than can be achieved by computation based on machine parameters, for the direct effects of e.g. saturation and eddy-current induction can be included. The following are typical of the practical problems that can be investigated by micro-machine installations.

Generator short circuit. Unbalanced short-circuit currents, the analytical computation of which is difficult, can be investigated directly. The measurement of transient torques can readily be carried out if accelerometers and torque-meters are available.

Asynchronous operation. It is now recognized that a generator which pulls out of synchronism need not necessarily be disconnected provided that appropriate measures are taken to control the conditions and to restore synchronous running. Such an investigation with a real large generator is hazardous, but the small scale of the micro-machine arranged to simulate the generator conduces to exhaustive measurement of the transient currents and torques, and also to a study of the optimum control characteristics.

Solid rotors. The normal concepts of transient and subtransient reactance are based on the assumption that the damper system is equivalent to two discrete axis dampers. With a solid rotor the damping is by eddy-currents and the paths are complicated. A comparable problem arises in a salient-pole synchronous motor when the starting torque depends upon eddy-current effects in the solid pole-shoes, with the additional complexity of saliency and a full range of rotor frequency from standstill to synchronous speed. Both these cases are susceptible to direct measurement on micro-machines.

Induction-motor transients. The effect of large cage induction motors on power-system operation can be of importance in the prediction of system stability under conditions of fault. Analysis is particularly difficult by reason of the changing load angles of the synchronous plant and the rapid fall in speed of induction machines. A typical system can, although somewhat limited in extent, be set up with micromachines to represent, say, a synchronous generator, a transmission line and an infinite busbar. A fault on the line can be imposed, the phase-swung of the synchronous machine and the rise in slip of the induction motor directly observed, and the stability of the system resulting from the interaction of these effects deduced, with comprehensive instrumentation and under the advantageous conditions of a laboratory.

A description of the design of a micro-generator using negative-resistance devices, and dealing with its dimensional scaling, is given by Hammons and Parsons [79].

Power-system modelling

A quarter-century ago it was sufficient to model a power system electrically by 'network analysers', with transmission links represented by resistors, inductors and capacitors, and synchronous machines by voltage sources (variable in magnitude and phase) connected in series with 'internal' resistance and inductance. Later a more complex two-axis representation was introduced for the generators. Modern micromachines provide the most satisfactory solutions for qualitative problems, but it is not easy to obtain close adjustment of machine parameters and the number of machines that can be simulated is rather restricted.

Transient-stability studies at first required only more detailed versions of the two-axis equations. The modern digital computer can be programmed to provide numerical simulations of several hundred machines and several thousand busbars. The calculations cannot (as yet) be performed in real time,

but the digital machine is ideally suited to routine studies. The development of high-level languages, more comprehensive computers, multi-access systems and graphical output facilities will, however, lead almost certainly to optimization and the complete on-line control of the extensive power-supply system.

7.5 *Dynamic circuit theory*

It has for some time been recognized that rotating electrical machines differ not in their fundamental action, but in the arrangement and excitation of their windings. Early attempts to co-ordinate the piecemeal treatments of separate machine types led to what is called the *generalized* approach through dynamic circuit theory. The concept was founded by Park in a classic paper [50] on synchronous machines in 1929. Gabriel Kron developed Park's ideas to deal comprehensively with all rotating electrical machines by means of tensor analysis [75]. In the modern version of Park's method, all active circuits are described by resistance and inductance parameters. By writing the circuit equations, the required behaviour can be found in terms of voltages, currents and torques. As any given circuit in a machine may be magnetically coupled with any other, the circuit equations are complicated. Substantial simplification is achieved if each circuit is so transformed as to represent separately its effect on the direct and quadrature axes of the machine, for it can then be assumed that no equivalent circuit whose m.m.f. acts along one axis can have any magnetic coupling with any circuit acting along the other axis. Further, the two-axis concept can realize in a simple manner the normal travelling-wave field of a polyphase machine, and is therefore equally applicable to steady-state conditions.

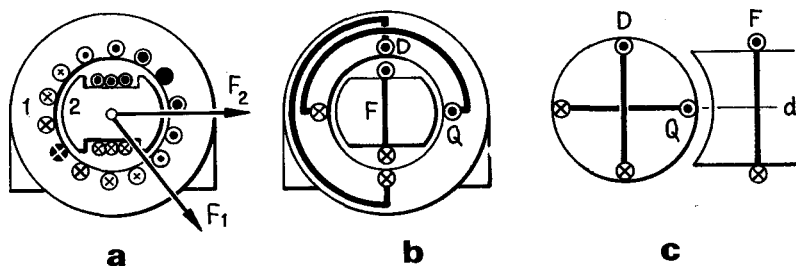
The process, in brief, is to *transform* a winding and its voltage and current into an equivalent coil on the d- or the q-axis (or on both) with the polyphase quantities thus converted into their axis equivalents; to set up the circuit equation for each such circuit; to impose the operating conditions; to solve the equations; and finally to convert the solutions back into terms of the currents and voltages of the actual machine windings.

There are some sacrifices. Manipulation of the equations is tractable only on the assumption of constant inductance and resistance parameters; saturation effects must usually be ignored; speed changes may render the equations non-linear in the mathematical sense; and secondary phenomena — such as brush-contact resistance or flux and m.m.f. harmonics — must be neglected. In spite of these drawbacks, the generalized two-axis dynamic circuit theory is very powerful, as can be appreciated from the work of Adkins [65] and Jones [66]. A simple introduction is given by Say [1].

Basic two-axis three-winding machine

This is a 2-pole unit with not more than one salient element, either stator or rotor. The d-axis is chosen as the axis of saliency. For this purpose the polarized field system on the cylindrical rotor of a turbo-type synchronous machine is treated as equivalent to a saliency. In a polyphase induction machine there is a less restricted choice of d-axis.

For convenience of reference framing, the d-axis is taken as horizontal, the q-axis as vertical and the rotation as counter-clockwise. As only relative motion is significant, the salient element is stationary, the moving element (or armature) rotating around or within it. The windings on each element are grouped or resolved so as to magnetize along one or other axis. Positive current in any axis winding then magnetizes in the positive axis-direction. With d- and q-axis coil groups thus in space quadrature there is no magnetic coupling between coils on different axes; but rotational e.m.f.s can be developed in coils on one axis that can be considered to move in the flux developed by coils on the other axis.



7.7 Basic two-axis three-winding machine.

The two-axis concept was briefly described in Sect. 1.11 and 12. The machine of Fig. 1.11 is shown schematically in Fig. 7.7(a), in which the m.m.f. axes of stator and rotor are emphasized. The m.m.f. of the polyphase stator current is resolved into two separate m.m.f.s in coils D and Q, as in (b), with the d-axis rotor m.m.f. produced by winding F. An equally valid (and more convenient) representation is that in (c). Magnetically, coils D, Q and F can be taken as single turns, but the fluxes due to them are nevertheless assumed to be distributed sinusoidally.

Quasi-stationary coils. The armature coils D and Q are fixed: but they represent the m.m.f. axes of a *real winding* that *rotates*. It is necessary to consider them to be the seat of rotational e.m.f.s in spite of the condition that they are stationary with respect to the d- and q-axes. The coils are termed *quasi-* (or *pseudo-*) *stationary*. Rotational e.m.f.s appear in Q due to d-axis flux, and in D due to q-axis flux.

Armature E.M.F.s. Consider coil D in Fig. 7.7(c). It is coaxial with the 'field' coil F, and any change of its linkage ψ_d will produce the pulsation e.m.f.

$$e_{pd} = d\psi_d/dt = p\psi_d$$

writing p for d/dt . Because of its orientation there can be no rotational e.m.f. in D due to ψ_d ; but if there were a q-axis linkage ψ_q a rotational e.m.f.

$$e_{rd} = \omega_r \psi_q$$

would result. In the case of coil Q, with its axis at right-angles to the d-axis of coil F, no pulsational e.m.f. can be induced by any change in ψ_d , but it will

have an e.m.f. of rotation in ψ_d when the winding it represents rotates at angular speed ω_r , of value

$$e_{rq} = -\omega_r \psi_q$$

The negative sign results from the fact that a q-axis coil with reference to a d-axis flux bears the opposite relation to that of a d-axis coil in a q-axis flux.

Armature applied voltages. Remembering that terminal voltages are considered as applied to the windings and that positive currents flow in a corresponding direction, then such voltages must account for the e.m.f.s resulting from axis linkages, and for volt drops in resistance and leakage inductance. Hence for armature coils D and Q the equations for the voltages applied are

$$v_d = r_d i_d + L_d p i_d + p \psi_d + \omega_r \psi_q \quad (7.8)$$

$$v_q = r_q i_q + L_q p i_q + p \psi_q - \omega_r \psi_d$$

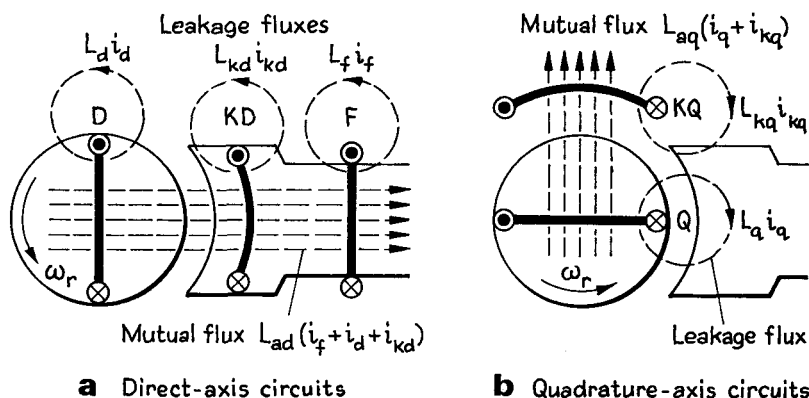
where r represents a winding resistance and L its leakage inductance.

Field applied voltage. The stationary coil F is not the seat of any rotational e.m.f., and it has no coupling with the q-axis: thus its applied voltage is given by

$$v_f = r_f i_f + L_f p i_f + p \psi_d \quad (7.9)$$

General multi-winding machines

While the D and Q coils are adequate to represent the armature, the field winding F is, for generality, augmented by at least one further coil KD on the d-axis as indicated in Fig. 7.8(a). KD represents, for example, a closed-circuit field collar or a damping winding in a synchronous machine, or the equivalent of the eddy-current path in a solid pole-shoe. For a like reason it is necessary to include a q-axis coil KQ, as in (b). Under transient conditions, currents induced in pole-shoes and dampers have an important effect on the flux of the machine, and



7.8 Axis coils: mutual and leakage fluxes.

the problem is to find the axis fluxes resulting from currents in all coils of both axes.

D-axis fluxes. There are three mutual inductances concerned, i.e. D with KD, KD with F and F with D. A useful simplification is to take these as all equal to a mutual inductance L_{ad} . For this purpose it is necessary to work with single-turn equivalent coils. Each coil has a leakage inductance of its own, and the total coil inductances will consequently be

$$D: (L_{ad} + L_d) \quad KD: (L_{ad} + L_{kd}) \quad F: (L_{ad} + L_f)$$

The self flux of F due to an instantaneous current i_f will thus be $(L_{ad} + L_f)i_f$, and similarly for the other d-axis coils. The mutual linkage with armature coil D when there are currents in all the axis coils is

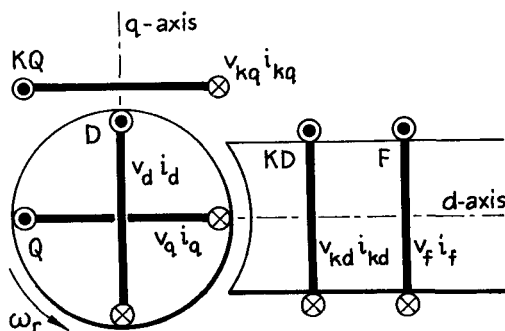
$$\psi_d = L_{ad}(i_f + i_{kd} + i_d) \quad (7.10)$$

Q-axis fluxes. Considering the group of q-axis coils in Fig. 7.8, and calling the common mutual inductance coefficient L_{aq} , the mutual linkage in coil Q is

$$\psi_q = L_{aq}(i_{kq} + i_q) \quad (7.11)$$

If the leakage inductances are respectively L_q and L_{kq} , the total inductances of Q and KQ will be $(L_{aq} + L_q)$ and $(L_{aq} + L_{kq})$ respectively.

Circuit equations. It is now possible to write down the voltage equations for all five circuits in the machine shown diagrammatically in Fig. 7.9. If for some



7.9 Basic two-axis five-winding machine.

purpose any further circuits are to be included, they are dealt with in precisely the same manner. Each circuit has an applied voltage v and an input current i .

F: This coil has a resistance r_f , a leakage inductance L_f and a total inductance $(L_{ad} + L_f)$. The current in any other d-axis coil will affect the linkage of F, but no q-axis coil can affect it, nor do any rotational e.m.f.s appear. Hence the applied voltage v_f has the components

$$v_f = r_f i_f + (L_{ad} + L_f) p i_f + L_{ad} p i_{kd} + L_{ad} p i_d \quad (7.12)$$

KD: This differs in no essential from F, so that

$$v_{kd} = r_{kd}i_{kd} + (L_{ad} + L_{kd})p i_{kd} + L_{ad}p i_f + L_{ad}p i_d \quad (7.13)$$

KQ: This has a resistance r_{kq} and a total inductance $(L_{aq} + L_{kq})$. It has no rotational e.m.f., but is affected magnetically by any current i_q in armature coil Q: thus any applied voltage must satisfy the condition

$$v_{kq} = r_{kq}i_{kq} + (L_{aq} + L_{kq})p i_{kq} + L_{aq}p i_q \quad (7.14)$$

In practice, coils KD and KQ usually represent short-circuited paths with an applied voltage of zero.

D and Q: The armature coils D and Q have the additional property of generating rotational e.m.f.s in the flux of the other axis. Their applied voltages are given in eq.(7.8), and using the expressions for ψ_d and ψ_q in eqs. (7.10) and (7.11)

$$v_d = r_d i_d + (L_{ad} + L_d)p i_d + L_{ad}p i_f + L_{ad}p i_{kd} + L_{aq}\omega_r i_{kq} + (L_{aq} + L_q)\omega_r i_q \quad (7.15)$$

$$v_q = r_q i_q + (L_{aq} + L_q)p i_q + L_{aq}p i_{kq} - L_{ad}\omega_r i_f - L_{ad}\omega_r i_{kd} - (L_{ad} + L_d)\omega_r i_d \quad (7.16)$$

We have now obtained a set of circuit equations for the machine represented by three d-axis and two q-axis coils. More coils can be added in a similar way.

Matrix form

The circuit equations can be written down accurately and rapidly in the matrix form $[V] = [Z] \cdot [I]$ by aid of a few simple rules. For the five coils F, KD, KQ, D and Q, a 5-row 5-column impedance matrix is set out as in Fig. 7.10.

	Fixed circuits			Moving circuits		
	i_f	i_{kd}	i_{kq}	i_d	i_q	
v_f	$r_f + (L_{ad} + L_f)p$	$L_{ad}p$		$L_{ad}p$		F
v_{kd}		$r_{kd} + (L_{ad} + L_{kd})p$				KD
v_{kq}			$r_{kq} + (L_{aq} + L_{kq})p$			KQ
v_d				$r_d + (L_{ad} + L_d)p$		D
v_q					$r_q + (L_{aq} + L_q)p$	Q

7.10 Construction of impedance matrix.

The rows are related to the voltages and the columns to the corresponding currents. Compartments in the main diagonal are emphasized: these contain the self-impedances. Thus r_f is entered only in row F and column F because only current i_f can produce a voltage drop in r_f , no resistance carrying more than one current. For the same reason the total self-inductances are also entered in the compartments of the main diagonal.

The remaining compartments contain the mutual impedances where any exist. Thus in row F, the second compartment contains $L_{ad}p$, the third zero, the fourth $L_{ad}p$ and the fifth again zero. There are no entries in row-F compartments lying in q-axis columns. Now, by inspection of the F-row, v_f is read as the sum of the products of the compartment impedances with their appropriate currents. This gives eq.(7.12).

The same procedure is employed to fill and read each row of the matrix. But in the case of the fourth and fifth rows (and only in these rows), rotational-e.m.f. terms will appear. Every coefficient of p in row Q appears again in row D in the same column, but with p replaced by $-\omega_r$. Similarly, every coefficient of p in row Q appears again in row D in the same column, but with p replaced by $+\omega_r$. The matrix is now complete, and if read off as already described it gives the circuit equations, eq.(7.12) to (7.16).

	F	KD	KQ	D	Q
F	$r_f + (L_{ad} + L_f)p$	$L_{ad}p$		$L_{ad}p$	
KD	$L_{ad}p$	$r_{kd} + (L_{ad} + L_{kd})p$		$L_{ad}p$	
KQ			$r_{kq} + (L_{aq} + L_{kq})p$		$L_{aq}p$
D	$L_{ad}p$	$L_{ad}p$	$L_{aq}\omega_r$	$r_d + (L_{ad} + L_d)p$	$(L_{aq} + L_q)\omega_r$
Q	$-L_{ad}\omega_r$	$-L_{ad}\omega_r$	$L_{aq}p$	$-(L_{ad} + L_d)\omega_r$	$r_q + (L_{aq} + L_q)p$

7.11 Impedance matrix of five-winding machine.

Fig. 7.11 gives the complete impedance matrix. For further coils, it is only necessary to add further rows and columns to correspond.

Torque

Torque is developed only in windings which, with the convention here adopted, move with respect to the fixed d- and q-axes. The torque produced by any winding is such that $M\omega_r = e_r i$, so that M is obtained from the rotational voltage multiplied by the current of the winding concerned, and divided by ω_r .

Consider row D of the matrix in Fig. 7.11. The rotational terms are $L_{aq}\omega_r$ associated with i_{kq} , and $(L_{aq} + L_q)\omega_r$ associated with i_q . The torque developed by armature coil D is therefore

$$M_d = [L_{aq}i_{kq} + (L_{aq} + L_q)i_q]i_d.$$

In the same way, for the armature coil Q,

$$M_q = [-L_{ad}i_f - L_{ad}i_{kd} - (L_{ad} + L_d)i_d]i_q.$$

The total armature torque is the sum of these two components giving

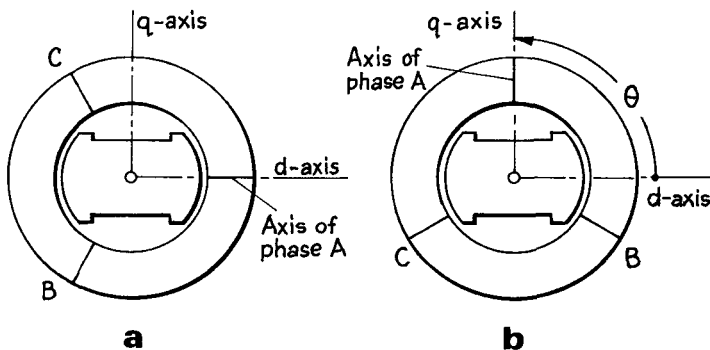
$$M = L_{aq}(i_{kq} + i_q)i_d - L_{ad}(i_f + i_{kd} + i_d)i_q - (L_d - L_q)i_d i_q \quad (7.17)$$

The last term is due to the saliency, which makes $L_d > L_q$.

Mechanical torque is defined as that *applied* to the shaft, so that the torque developed by an electrical input becomes a mechanical output and is therefore reckoned as negative. The total torque of a machine will, of course, include both electrically-developed and mechanically-applied torques. These normally balance (i.e. they sum to zero), but if they do not, an acceleration torque $Jp\omega_r$ is released to alter the speed of the rotating parts of inertia J . In this case the circuit equations become non-linear and a solution offers some difficulty.

Synchronous machine

A 3-phase winding rotates with reference to the d- and q-axes, and it is necessary to show (i) that it can be replaced by fixed D and Q armature coils, and (ii) the relation between the actual phase currents and their axis counterparts. **Armature phase inductance.** Consider phase A of a 3-phase winding. Its inductance is a combination of its own self inductance L_{aa} with its mutual inductance L_{ab} and L_{ac} with phase windings B and C. All three inductances vary with the relative position of stator and rotor because of saliency. When the axis of phase



7.12 Effect of position on phase inductance of 3-ph machine.

A coincides with the d-axis, Fig. 7.12(a), its self inductance is a maximum; and when coincident with the q-axis, (b), it is a minimum. Thus L_{aa} fluctuates twice per double pole-pitch. Fig. 7.12 also shows that the mutual inductance of phases B and C is a maximum at (a) and a minimum at (b). Both self and mutual inductances, being functions of the angle θ , can be expressed as Fourier series to take into account any 'shape' of inductance/angle relation. The simplest variations are

$$\begin{aligned} L_{aa} &= L_0 + L_2 \cos 2\theta \\ L_{bc} &= L_{cb} = -L_{m0} + L_{m2} \cos 2\theta \end{aligned} \quad (7.18)$$

From the latter we can, by symmetry, write

$$\begin{aligned} L_{ab} &= L_{ba} = -L_{m0} + L_{m2} \cos(2\theta - 2\pi/3) \\ L_{ac} &= L_{ca} = -L_{m0} + L_{m2} \cos(2\theta + 2\pi/3) \end{aligned} \quad (7.19)$$

We thus have expressions for all three components of the total inductance of phase A, and can write similar expressions for phases B and C.

Axis inductances. With balanced 3-phase currents and normal-speed rotation corresponding to the frequency, the phase currents can be written in terms of angles θ and γ . The position $\theta = 0$ corresponds to the coincidence of the axis of phase A with the rotor pole-centre, i.e. the d-axis; and γ is the phase angle of lag of the phase A current with respect to this instant:

$$\begin{aligned} i_a &= i_m \sin(\theta - \gamma) \\ i_b &= i_m \sin(\theta - \gamma - 2\pi/3) \\ i_c &= i_m \sin(\theta - \gamma + 2\pi/3) \end{aligned}$$

The total flux linkage of phase A is

$$\psi_a = L_{aa}i_a + L_{ab}i_b + L_{ac}i_c$$

Inserting the currents, together with the inductance coefficients from eqs.(7.18) and (7.19), gives the linkage ψ_a . If i_m is put equal to unity, the linkage per unit current is the effective inductance of phase A:

$$(L_0 + L_{m0}) \sin(\theta - \gamma) - (\frac{1}{2}L_2 + L_{m2}) \sin(\theta + \gamma)$$

A third-harmonic space variation has been ignored in this result.

Suppose that $\gamma = 0$; then phase A has peak current at the instant when $\theta = \frac{1}{2}\pi$, i.e. when its axis coincides with the q-axis. Its effective inductance is

$$L_0 + L_{m0} - \frac{1}{2}L_2 - L_{m2} = L_{aq}$$

If $\gamma = \frac{1}{2}\pi$, then phase A has peak current when on the d-axis with $\theta = 0$; for this the effective inductance is

$$L_0 + L_{m0} + \frac{1}{2}L_2 + L_{m2} = L_{ad}$$

Thus the flux per unit armature current (the 'armature reaction') is affected by saliency, and depends upon how the stator (or 'armature') m.m.f. is directed with respect to the d- and q-axes. The larger flux per unit phase current L_{ad} applies when the m.m.f. axis and the d-axis coincide; the smaller, L_{aq} , when the m.m.f. of phase A is along the q-axis. These differ by

$$L_{ad} - L_{aq} = L_2 + 2L_{m2}$$

that is, by the second-harmonic space-distribution coefficients. If there is no saliency, then $L_{ad} = L_{aq}$. It is to be noted that even with cylindrical-rotor machines there may be a magnetic saliency by reason of the distribution of the windings in slots and also, in practice, because of saturation effects.

Three-phase/two-axis transformation. Under *steady-state* conditions the m.m.f. set up by the currents in phases A, B and C is stationary with respect to the d-

and q-axes of the rotor. Consequently the fictitious coils D and Q, the quasi-stationary coils that reproduce the same m.m.f., must carry the *direct* currents i_d and i_q . The instantaneous axis currents are related to the instantaneous 3-phase stator currents and the instantaneous position θ of the axis of phase A by the expressions

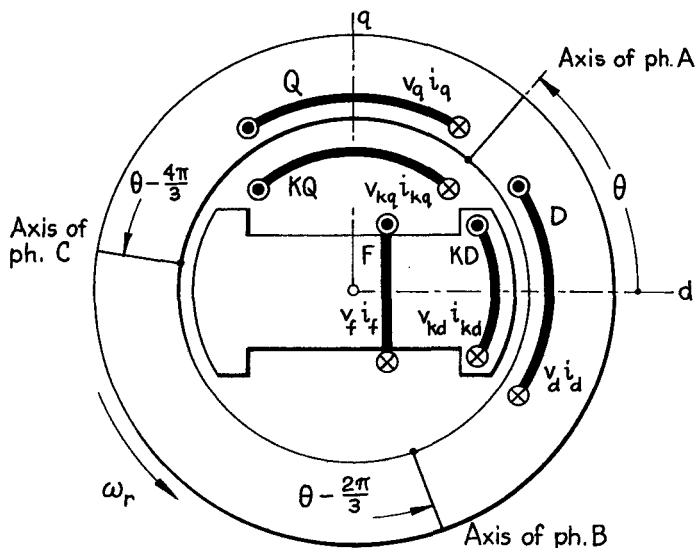
$$\begin{aligned} i_d &= \frac{2}{3} [i_a \cos \theta + i_b \cos(\theta - 2\pi/3) + i_c \cos(\theta - 4\pi/3)] \\ i_q &= \frac{2}{3} [i_a \sin \theta + i_b \sin(\theta - 2\pi/3) + i_c \sin(\theta - 4\pi/3)] \end{aligned} \quad (7.20)$$

The axis voltages v_d and v_q are related to the stator phase voltages v_a , v_b and v_c by expressions of identical form. The reverse transformations are

$$\begin{aligned} i_a &= i_d \cos \theta + i_q \sin \theta \\ i_b &= i_d \cos(\theta - \frac{2}{3}\pi) + i_q \sin(\theta - \frac{2}{3}\pi) \\ i_c &= i_d \cos(\theta - \frac{4}{3}\pi) + i_q \sin(\theta - \frac{4}{3}\pi) \end{aligned} \quad (7.21)$$

and again the expressions for the voltage have an identical form. If there are zero-sequence components, then the further equation $i_0 = \frac{1}{3}(i_a + i_b + i_c)$ and a corresponding expression for the voltage are added to eq.(7.21); the reverse transformations have simply the addition of i_0 (or v_0) to the right-hand side. **Power.** Taking unit power as that for the machine as a whole, then with per-unit voltages and currents in a 3-phase machine the per-unit input power is $p_1 = \frac{1}{3}(v_a i_a + v_b i_b + v_c i_c) = \frac{1}{3}(v_d i_d + v_q i_q)$.

Summary. The analysis in circuit form of a synchronous machine for any condition of operation requires first the reduction of all windings to d- and q-axis groups. Fig. 7.13 shows the basic 5-circuit machine. Coil F is always a d-axis



7.13 Five-winding synchronous machine.

winding. The armature (normally the stator) winding is transformed into the axis coils D and Q. The windings KD and KQ, associated with the presumed fixed field system, represent damping windings, eddy-current paths, single-turn equivalents of field collars and pole-shoes; they are normally closed and have no applied source voltage. If there are several distinguishable eddy-current paths it may be necessary to include additional coils, but here it is sufficient to assume one in each axis. The behaviour of the equivalent machine is determined from the conditions of voltage imposed, and the voltages and currents in the actual machine circuits derived therefrom, using transformations where phase windings are concerned.

Induction machine

With both stator and rotor cylindrical and symmetrical, the d-axis can be chosen arbitrarily as the axis of stator phase A. The 3-phase winding is transformed to a 2-phase 2-axis winding, so that the axis of the second equivalent phase becomes the q-axis. The stator winding then becomes two fixed-axis coils, 1D and 1Q.

The rotor carries a slip-ring winding or a short-circuited cage. In the former case it is possible to convert the winding to fixed d- and q-axis coils, just as for the armature of the synchronous machine.

A cage rotor is rather more complex; but insofar as it can, and does, produce an armature reaction m.m.f. substantially identical (apart from space harmonics) with that of a slip-ring winding, it can also be represented by d- and q-axis coils. These latter are called 2D and 2Q.

Disregarding zero-sequence voltages and currents, the conversion of a 3-phase stator winding to a 2-phase axis winding is by

$$v_{1d} = \frac{2}{3} [v_a - \frac{1}{2}v_b - \frac{1}{2}v_c] \quad (7.22)$$

$$\text{and } v_{1q} = \frac{2}{3} [-(\sqrt{3}/2)v_b + (\sqrt{3}/2)v_c].$$

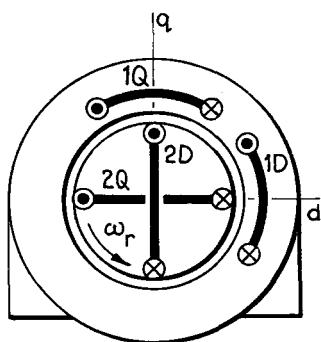
Identical coefficients relate the currents. For any zero-sequence voltage, $v_{10} = \frac{1}{3}(v_a + v_b + v_c)$. The reverse transformation is

$$\begin{aligned} v_a &= v_{1d}; & v_b &= -\frac{1}{2}v_{1d} - (\sqrt{3}/2)v_{1q}; \\ v_c &= -\frac{1}{2}v_{1d} + (\sqrt{3}/2)v_{1q}; \end{aligned} \quad (7.23)$$

and similarly for the currents. Any zero sequence voltage v_0 is added to each right-hand side.

For the rotor, the appropriate transformations are those given in eqs. (7.20) and (7.21).

The impedance matrix is set up as for the synchronous machine, appearing as in Fig. 7.14. The stator and rotor resistances and inductances are r_1 and r_2 , L_1 and L_2 for each axis. Coils 1D and 2D have the mutual inductance L_m , and similarly for 1Q and 2Q. Rotational e.m.f.s appear only in the rotor coils. Thus 2D has a quasi-rotation in the mutual flux of 1Q and also in the total flux of 2Q. The voltage equations are read directly from the impedance matrix in Fig. 7.14.



1D	1Q	2D	2Q
$r_1 + (L_m + L_1)p$		$L_m p$	
	$r_1 + (L_m + L_1)p$		$L_m p$
$L_m p$	$L_m \omega_r$	$r_2 + (L_m + L_2)p$	$(L_m + L_2)\omega_r$
$-L_m \omega_r$	$L_m p$	$-(L_m + L_2)\omega_r$	$r_2 + (L_m + L_2)p$

7.14 Four-winding induction machine.

Torque. This is obtained from the rotational-voltage terms, to give

$$M = L_m(i_{1q}i_{2d} - i_{1d}i_{2q}). \quad (7.24)$$

Speed changes

If a change of rotor speed is introduced, the matrix equations become mathematically nonlinear, making a step-by-step solution necessary. Where speed change is small, however, the calculus of variables can be applied. It is assumed that steady-state voltages v , currents i , electrical torque M and speed ω_r are changed by Δv , Δi , ΔM and $\Delta \omega_r$ respectively about constant mean values; and if these perturbations are small enough it is possible to enlarge the purely electrical matrix of Fig. 7.11 by one further row and column M , to represent torque and acceleration acting on the rotor mechanical axis (i.e. that of the shaft). The dynamic electromechanical matrix becomes that in Fig. 7.15, which permits of a solution to the behaviour of the machine when subjected to small impressed voltages or torques. Gibbs [77] discusses the application of the method to several cases.

	F	KD	KQ	D	Q	M
F						
KD						
KQ						
D						$L_{aq}i_{kq} + (L_{aq} + L_q)i_q$
Q						$-L_{ad}(i_f + i_{kd}) - (L_{ad} + L_d)i_d$
M	$L_{ad}i_q$	$L_{ad}i_q$	$-L_{aq}i_d$	$(L_{ad} - L_{aq})i_q - L_{aq}i_{kq}$	$L_{ad}(i_f + i_{kd}) + (L_{ad} - L_{aq})i_d$	Jp

7.15 Motional impedance matrix.

The blank compartments contain the corresponding terms in Fig. 7.11.

Applications

Dynamic circuit theory based on 2-axis transformations has a number of restrictions. It is not applicable to machines with saliency on both sides of the airgap because in such structures it is not possible to identify axes that present substantially constant reluctances to the axis m.m.f.s. It is a linear method based on sinusoidal distributions and is not capable without some elaboration of dealing directly with harmonics or with mechanical asymmetry. In particular, it cannot cope (except over restricted operating ranges) with magnetic saturation.

The problem of saturation needs special consideration. The generalized theory assumes linear relationships, not because saturation is unimportant but because there is no possible alternative. Any direct analytical treatment of nonlinear conditions, except in very simple cases, becomes impossibly complicated. It is an accepted method, used over the whole of engineering science, to develop a basic linear theory, which is modified where needed to allow for nonlinear effects. In the field of electrical machines the method gives good results if used with discretion. A full physical appreciation of all the ways in which the idealized machine differs from the real one is, however, essential if the method is to be applied successfully; moreover, repeated experimental verification of any deductions made must be available.

Exclusion from the theory of certain practical and important effects, such as mechanical soundness or the detailed distribution of flux and losses, means that design of a machine calls for skill and experience. However, provided that undesirable side effects are avoided by practical means, and that the most appropriate parameters can be selected, the generalized theory provides a reliable means of predicting machine performance. Naturally the design should not be subservient to a need for an amenable theoretical analysis; but in practice it turns out that the best design is often that one which conforms most nearly to the pattern of the idealized machine.

The method is very successful in analysing such problems as the short-circuit behaviour of large generators, the transient starting torques in induction motors and the run-up torques of synchronous motors. It forms an effective tool for the elucidation in particular of conditions of transience — conditions that are quite beyond the capability of simple steady-state theories to tackle.

Phase co-ordinate analysis

The two-reaction method described has the advantages that it makes evident that a machine has a single working flux produced by the action of all winding currents in combination, can be used for machines with any number of balanced phases, and is applicable directly (i.e. without transformations) to machines such as those with commutators in which the action depends on there being two distinct and recognizable physical d- and q-axes. But it has some disadvantages for a.c. machines, in that certain assumptions have to be made in regard to the distribution of gap-flux density resulting from the contributing windings. A salient-pole field winding or a damping winding, for example, is assumed to give a sinusoidal flux distribution, an assumption that is clearly untrue. The variations of self and mutual inductance expressed in eqs. (7.18) and (7.19) are

simplified by the neglect of all space-distribution harmonics except that of second order, whereas in fact there are higher-order harmonics actually present, some of which are of significant importance. Finally, it is necessary to transform stator axis-coil currents back into their phase equivalents after the basic solution because the only measureable currents are phase currents.

It was to avoid the use of position-dependent inductance coefficients that Park developed the two-axis dynamic circuit theory. With the advent of powerful digital computers, however, it is no longer essential to use phase-to-axis transformations, and by adequate iteration it is possible to calculate directly with eqs. (7.18) and (7.19) and to include, if necessary, higher space harmonics to express more exactly the variation of the inductance coefficients. The solutions for all three phase currents are then obtained. For practical purposes, apart from its heavy demands upon computer time, the 'phase co-ordinate' method has clear advantages. In effect, the matrix of Fig. 7.11 is changed by replacing the stator coils D and Q by phases A, B and C, and incorporating the appropriate angle-dependent inductance coefficients in place of the axis values. The method is particularly useful for cases of unbalance, whether of steady state or fault, conditions for which the two-axis method requires rather more computation.

7.6 *Industrial machines*

In specifying an electric motor for an industrial drive, many factors must be considered besides the voltage and frequency. The duty, starting performance, type of protection and ambient conditions may all have a direct influence on the machine to be selected. Sometimes a difficult decision is to settle which of the many alternatives is best: induction or synchronous, cage or slip-ring, pole-change or thyristor-controlled. By far the commonest industrial machine is the cage induction motor, which gives a cheap, robust and simple single-speed drive; but in its most elementary form its starting current is high and its starting torque low. With relatively minor rotor modifications using deep-bar, T-bar or double-cage designs, machines can offer significantly better starting performance, and two-speed operation is possible by suitable elaboration of the stator winding.

Where the drive demands an adjustable speed rather than one or two 'fixed' speeds, the choice of motor type widens considerably. Slip-ring induction motors with rotor resistance control are often adopted for intermittent use (e.g. for cranes and hoists) where the additional rotor-circuit losses at low speeds are acceptable. If they are not, and the load requirements justify the choice, use may be made of static frequency-converters to feed induction motors.

Small conventional synchronous motors are, compared with the simpler induction machines, expensive; but if motors of large output are required, the synchronous machine may be cheaper, is certainly more efficient, and is capable of operation at leading power factor. Modern synchronous-machine designs are generally of the 'brushless' type, devoid of slip-rings, with the d.c. exciter replaced by diodes carried on the rotor and fed from an a.c. excitation system. Brushless synchronous motors are somewhat more complicated than

brushless generators in that during run-up the diodes need protection from slip-frequency e.m.f.s induced into the field system, employing additional rotor resistors and solid-state switches.

Ratings and dimensions

Modern motors of small and medium rating are built to the standards of IEC 72, which lists a coherent range of main structural dimensions with centre heights between 56 and 1 000 mm. The outputs from standard motor frames are periodically reviewed to take account of technological advances. BS 3939 gives the standard ratings for the U.K. Some data are tabulated below.

Recommended Ratings and Dimensions of Rotating Machines
IEC Publications 72 and 72A

Preferred Ratings (kW):

0.06	0.55	5.5	30	110	220	355	500	710
0.09	0.75	7.5	37	132	250	375	530	750
0.12	1.1	11	45	150	280	400	560	800
0.18	1.5	15	55	160	300	425	600	850
0.25	2.2	18.5	75	185	315	450	630	900
0.37	3.7	22	90	200	335	475	670	950
								1000

Preferred Dimensions (mm):

FOOT-MOUNTED MACHINES, distance from shaft centre to mounting surface:

56	63	71	80	90	100	112	132	160	180	200	225	250
280	315	355	400	450	500	530	630	710	800	900	1000	

FLANGE-MOUNTED MACHINES, various flange pitch-circle diameters over the range 55 to 1080.

Cooling and enclosure

The simple construction of electric motors, more particularly of the cage induction machine, is readily cooled by straightforward circulation of the ambient air. Total enclosure has the merit that the windings are to a considerable degree protected from unfavourable environments; however, the cooling is restricted, and while there is little difference between the dimensions of open and totally-enclosed motors in ratings up to 15 kW, it becomes increasingly difficult to obtain adequate cooling-surface area for t.e. machines above 300 kW, and separate heat-exchangers may be necessary.

Modern systems of winding insulation are tolerant of dampness; nevertheless some minor refinements to the enclosure are generally worthwhile to suit specific operating conditions. Drip-proof, splashproof and hoseproof enclosures serve for the environmental conditions implied. Weatherproofing to the extent of preventing the ingress of moisture by spigots, flanges and bearing seals can be applied for motors to work in the open air without further protection. The

nomenclature for machine enclosures is embodied in the international IP and IC codes, indicating respectively the degree of protection afforded and the type of cooling employed. The basic IP code comprises two digits, the first indicating the method of protection against accidental contact with live or moving parts, the second that against the ingress of water or of foreign bodies. The IC code is similar, but there is a greater variety and complexity in cooling systems.

Some machines have to work in a corrosive atmosphere. Simple cage motors are relatively resistant to such an ambient medium, but machines with carbon brushes and sliding contacts rely on the maintenance of an oxide film on the sliding surfaces for satisfactory current collection. Total enclosure of the machine or brush-gear, or cooling by pressurized clean air, may be necessary. Operation in hazardous atmospheres of a flammable type may call for non-sparking cage motors with modifications to the enclosure aimed at minimizing risk. For mines, certificated flashproof motors are essential, the basis of acceptance being that if flammable gases should ignite within the machine, the explosion is contained in such a way that gases external to the machine are not ignited.

Noise

Much of the acoustic noise emitted by a machine is generated by the cooling air as it is passed through or over the machine by its fan. Some noise reduction may be possible by substituting unidirectional trailing-bladed fans for the normal radial-bladed form. Further reduction may require intake and outflow silencers or the adoption of closed-air-circuit ventilation.

The varying forces and pressures in the cooling medium responsible for the acoustic noise cannot be eliminated: they are inherent in the processes of circulation and heat-exchange. In very large machines the noise level may exceed 110 dB and be substantially unavoidable. The frequency spectrum of the noise has usually a broad band with a few superposed pure tones (which may be harmonics of the supply frequency or of the speed or rotation). The noise-power field around a machine is affected by the shape and dimensions of the outer surface of the carcass, and is subject to such phenomena as reverberation or reflection from hard surfaces.

A further cause of noise is magnetic in origin, and is dependent on the stator and rotor slotting. It often occurs in machines with fractional numbers of open slots per pole, producing during rotation a relatively large variation of airgap flux density. This may produce a penetrating note at approximately slot frequency (e.g. 0.5 – 1.5 kHz). The amplitude of the stator permeance variation appears to be the causative factor, but the amplitude also depends on high-order rotor m.m.f. space harmonics. An empirical rule, supported by experiment, suggests that the frequency of the noise is twice the product of the supply frequency with that integer nearest to the number of slots per pole.

Voltage levels

The user of a large machine may have a choice of more than one system voltage.

Motors can be wound for a wide range of supply voltages, but generally the higher the voltage the more costly is the motor, although the price differential may be swamped by the cost of switchgear and cabling in such a way that these exert a determining influence on the choice of voltage. It is naturally uneconomic to specify a high working voltage for a small machine, and the lower limits of rating might be taken typically as 150 kW for 3.3 kV, 300 kW for 6.6 kV and 1 MW for 11 kV.

Insulation

The temperature classification for machine insulation is given in Sect. 4.7. In order to maximize the rating for a given frame, most small- and medium-sized machines use class E insulation. Large machines almost always employ class B. The increasing availability of class F materials has led to a change from bitumastic bonded to synthetic-resin bonded insulants based on polyester or epoxy compounds for 6.6 and 11 kV machines.

Protection

The primary function of the commonly used current-operated protective devices is the recognition of abnormally large currents. The matching of these devices to the thermal characteristics of the machine may be difficult. As an alternative, *thermistors* (temperature-sensitive resistors) may be used to activate a protective switch when the winding temperature exceeds a predetermined value. The present internationally accepted method (where the machine is important enough to justify it) is to embed one thermistor in each phase section of the stator winding, i.e. three thermistors in a 3-phase machine for each speed range. The devices are normally connected in series into a bridge network which is balanced at temperatures not exceeding the assigned limit. For excessive temperatures the bridge unbalances and operates a control contactor. It is still advisable to use overcurrent relays to protect the connections and to act if necessary as a back-up.

All modern overcurrent relays are thermally operated devices that use the heating effect of phase currents to deflect bimetal elements. Tripping is caused by large deflection for overcurrent, and by differential deflection between phase elements to protect against single-phasing.

The protection afforded by thermistors is very effective when the rate of temperature-rise is slow, but for high rates (as when a motor is stalled) there may be too much thermal lag. In small machines, in which the stator temperature rises more rapidly than that of the rotor (a 'stator-critical' condition), thermistors in the stator winding may provide adequate protection. Large machines are more likely to be 'rotor-critical', and stator thermistors may not give proper protection.

Generators

Many of the points so far discussed apply also to generators, except that these machines are usually large, and in most cases are of 'one-off' design. (A turbo-generator of high rating takes some years to design, build, instal and commission).

Almost all a.c. generators, regardless of scale, are 3-ph star-connected synchronous machines, and are provided with elaborate equipments for control, synchronizing, voltage regulation, protection, thermal dissipation, excitation, governing, etc.

7.7 *Machines and power systems*

In the following Chapters the behaviour of machines is related in the first instance to operation on infinite busbars. This, though useful, is an oversimplification. Modern supply networks are very extensive and have enormous generator power; but there has been an increase in the size of electric motor loads. Attention must therefore be given to the effects on a supply network of the operating characteristics of motors connected to it and of the reaction of the network on the machines. To have practical validity, the machine performance must take account of starting, control, protection, load characteristics, fault consequences and system impedance. Particular aspects of performance are:

Normal: efficiency, temperature-rise, starting, control, noise.

Special: cyclic loading, pulsating torques, limitations on current level imposed by local system impedance.

Abnormal: operation and reaction during system faults, voltage depressions, excessive ambient temperatures, unexpected physical conditions (e.g. flooding).

The basic factors listed by Stephen [72] as concerned in the interaction of a large machine and its network are: (i) transient magnetizing-current inrush pulses caused by energizing an unmagnetized machine; (ii) transient currents produced, as in synchronizing, by voltage divergence between the machine and the supply; (iii) voltage surges produced by the interruption of machine currents; (iv) current pulsations produced by a synchronous machine when running asynchronously; (v) mechanical oscillations resulting from sudden voltage changes or transient load fluctuations; (vi) system voltage distortions due to large rectifying plant.

Voltage depressions

Engine-driven generators. These are increasingly installed for peak-logging service, the supply to construction sites, and the maintenance of essential services in the event of dislocation of the public supply by industrial conflict. Such installations must usually feed induction motors, some of which may be of rating large enough to impose a considerable load when starting. The engine-driven generator invariably has some form of automatic voltage regulator to hold the steady-state voltage variation within about 2%; but on the application of a large load increment the terminal voltage dips considerably, and the time-constants of the generator and regulator delay the recovery. The effect on induction-motor loads may be considerable.

Supply network. The Supply Authority is required by statute to limit voltage variation at the point of connection of a consumer's load to $\pm 6\%$ of the declared value. However, the starting of large motors can cause temporary voltage

dips of 10% or more, and greater but less frequent dips result from system faults.

Effects of voltage dips. A prolonged depression is significant more on account of its duration than of its magnitude. The effect on motors is governed initially by the transient characteristics of the electromechanical system: in some cases it is trivial, but the tripping of one machine in an interlocked continuous industrial process may trip the whole plant with serious effect.

No general induction motor characteristics can be specified to counter voltage dips, as the load and the supply system are also concerned, and the economic value of continuity of drive is an individual matter. The salient-pole synchronous motor is at some disadvantage if it pulls out of step on voltage depression, for apart from severe power fluctuations (reflected into the supply system at a time when they are particularly undesirable), the reaction on 'brushless' excitation circuits can be very damaging unless suitable protection is provided.

The frequency variation that accompanies voltage dip is usually small, but may on occasion be as much as 3%, adversely affecting both synchronous and induction machines.

Power-factor improvement

The power factor of an industrial load can be raised by the use of overexcited synchronous drives for certain loads, or of synchronous motor-generator sets which can compensate for standard induction motors driving pumps, fans, compressors and process auxiliaries. Apart from voltage regulation with load, the matter is basically economic, and p.f. improvement is justified only if there is a related tariff concession.

To be chosen in preference to a static capacitor, a synchronous motor must have several factors in its favour. Although the efficiency is less affected by starting and accelerating torque requirements than the cage induction motor (because the starting cage can be designed almost independently of the main-field running winding), the ideal condition is one of roughly constant load for long periods and not a high-inertia load requiring frequent starts. A disadvantage is that the synchronous motor will contribute to the current flowing into a short-circuit fault on the system and may call for additional expenditure in switchgear.

8

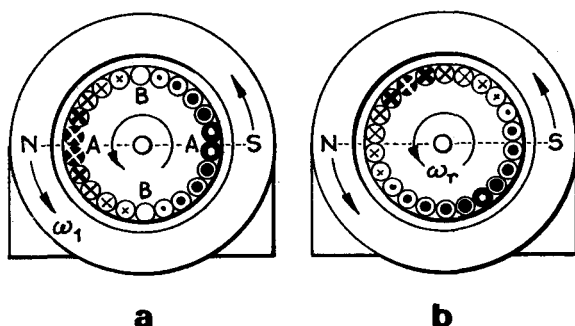
Induction Machines: Theory and Performance

8.1 *Development*

Nikola Tesla exhibited an elementary form of polyphase induction motor in 1891. Two years later, Dobrowolsky described a machine with a cage rotor and a distributed stator winding. The slip-ring machine was developed at the turn of the century. Since then, many detailed improvements to the 'plain' induction machine have been made. At the same time, attempts to circumvent the inherent speed and power-factor limitations have engaged the attention of inventors, but most of the ingenious devices developed have passed into history. The chief variants of the plain machine in modern machine technology are synchronous-induction machines, pole-change stator windings, double-cage rotors, and most recently the provision of variable-frequency drives by use of semiconductor rectifiers and inverters. Nevertheless, 80% of the world's industrial a.c. motors (apart from fractional-kilowatt machines) are plain cage motors working on constant-frequency supplies.

8.2 *Action of the ideal induction machine*

The essentials of a 2-pole induction machine are shown in Fig. 8.1. Poly-phase windings (not shown) on the stator produce a travelling sine-distributed m.m.f. which, acting on the short and uniform airgap, develops a correspond-



8.1 Elements of the induction machine.

ing travelling wave of flux of peak value Φ_m , as described in Sect. 3.14. The diagram indicates the axis of the gap flux at a given instant, and its direction of travel. The rotor carries a set of uniformly distributed full-pitch turns, each closed on itself.

In diagram (a), turns AA links no flux but has a maximum rate of change of flux linkage and so is the seat of the maximum induced e.m.f.; turn BB links maximum flux but the rate of change is momentarily zero and there is no e.m.f. The current that flows in each rotor turn at any instant depends upon the e.m.f. and the impedance: assuming this to be purely resistive, the current pattern, shown in (a), is identical with that of the e.m.f. But in fact each turn also has inductance, so that the alternating rotor currents are phase-delayed to give the pattern in (b). In each case the current distribution around the rotor gap surface is sinusoidal, and the condition equates to that in Fig. 1.10. As a consequence interaction mechanical forces are developed on the rotor conductors, urging them to move in the same direction as that of the stator field. The resultant torque in (b) is clearly less than in (a).

The behaviour of the machine is intimately related (i) to the *synchronous speed* and (ii) to the *slip*. The former is that of the travelling-wave airgap field, the latter is the fractional amount by which the angular speed of rotation of the rotor differs from synchronous speed.

Synchronous Speed: For a 2-pole machine the synchronous angular speed ω_s is the same as the angular frequency $\omega_1 = 2\pi f_1$ of the stator supply. In a machine with p pole-pairs, $\omega_s = \omega_1/p$.

Slip: If the angular speed of rotation of the rotor is ω_r , then the per-unit slip is $s = (\omega_s - \omega_r)/\omega_s$.

The rotor winding is isolated: it has no connected supply, and its power is transferred to it by means of the mutual gap flux. The amount of this power is primarily a function of the slip.

Let the rotor be at rest, a condition of *standstill*, with $\omega_r = 0$ and therefore $s = 1$. Both stator and rotor are at rest, and the angular frequency ω_2 of the rotor e.m.f.s and currents is the same as ω_1 , the frequency of the stator e.m.f.s and currents. Taking the resistance and leakage reactance of a representative rotor phase to be r_2 and x_2 , and its e.m.f. to be E_2 , then the rotor current is $I_2 = E_2/(r_2 + jx_2)$, lagging E_2 by the phase angle $\arctan(x_2/r_2)$. The rotor m.m.f. pattern travels around the gap in synchronism with that of the travelling stator field. Interaction torque is developed in the direction of the stator field. The stator takes a current to balance the rotor m.m.f., just as does the primary of a static transformer to balance the secondary m.m.f., for the application of a supply voltage to the stator windings demands the presence of an airgap magnetic field and an approximate m.m.f. balance between stator and rotor. If we could ignore the stator leakage impedance, then this balance would be exact, as in the ideal transformer.

Now let the rotor rotate under torque at a speed ω_r in the direction of the travelling-wave field, with a *slip* between 1 and 0. The rate of change of linkage in the rotor winding falls, and the e.m.f. is reduced to sE_2 at the lower frequency $s\omega_1 = \omega_2 = \omega_1 - \omega_r$. The leakage reactance is also reduced, and the leakage impedance is now $r_2 + jsx_2$, so that the rotor current may not

be much less than it was at standstill; moreover, its angle of lag is smaller and in consequence the torque is enhanced. The rotor m.m.f. pattern remains in synchronism with that of the stator, for it moves at angular speed ω_2 with respect to the rotor surface and is carried round bodily at ω_r to give a total angular rate of travel $\omega_2 + \omega_r = \omega_1$. As before, any current in the rotor calls for a balancing current in the stator.

If the rotor runs at *synchronous speed*, with $\omega_r = \omega_1$ and $s = 0$, the rotor windings move in an unchanging flux. Thus the rotor e.m.f. is $sE_2 = 0$; there is no current nor torque. The machine can develop torque only if there is a rotor e.m.f. and current, i.e. if there is a finite slip.

Operating Conditions

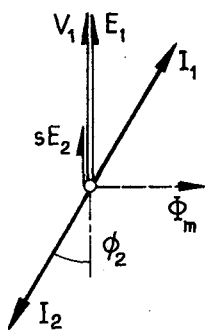
When a phase voltage V_1 is applied to the stator windings it is balanced (ignoring resistance) by the e.m.f. E_1 induced by a magnetic flux. Only part of this flux is mutual with the rotor and able to transfer energy thereto across the gap: the remainder is leakage flux proportional to the stator current. The greater the stator current, the smaller that proportion of the total flux which is mutual, so that Φ_m decreases with the load. In the ideal machine we shall disregard the stator leakage impedance, and assume that the mutual flux Φ_m is constant. For further simplicity, the machine is taken as having a basic 2-pole structure.

We shall consider the action of this ideal 2-pole machine in three operating modes, namely as a *motor* (with s between 1 and 0), as a *generator* (with s negative, i.e. with the rotor running at a speed higher than the synchronous), and as a *brake* (with s greater than unity, the rotor running backwards against the direction of the stator field). The assumptions, closely comparable to those adopted for the ideal transformer, are:

- (i) The stator and rotor windings are similar in distribution and have the same number of turns per phase.
- (ii) The stator applied phase voltage V_1 has a constant magnitude and frequency.
- (iii) The gap flux Φ_m is constant and travels at synchronous angular speed ω_1 .
- (iv) The only loss is in the rotor resistance r_2 .
- (v) The magnetizing m.m.f. is vanishingly small and the stator winding has negligible leakage impedance.

Although the assumptions imply a rather considerable simplification, the idealized machine can still present some significant characteristics.

Magnetization: The rotor winding is short-circuited on itself and has no external connection, the current being induced in it by the gap flux. As a result, the stator is always responsible for maintaining the magnetization by accepting from the supply the appropriate reactive power (here neglected). The stator applied voltage V_1 is balanced by an equal e.m.f. E_1 induced by the flux Φ_m , which also induces into the rotor winding the e.m.f. $E_2 = E_1$ at standstill and $sE_2 = sE_1$ at slip s . Any rotor current I_2 must be counterbalanced by an equal and opposite stator current $I_1 = -I_2$ to preserve m.m.f. equality



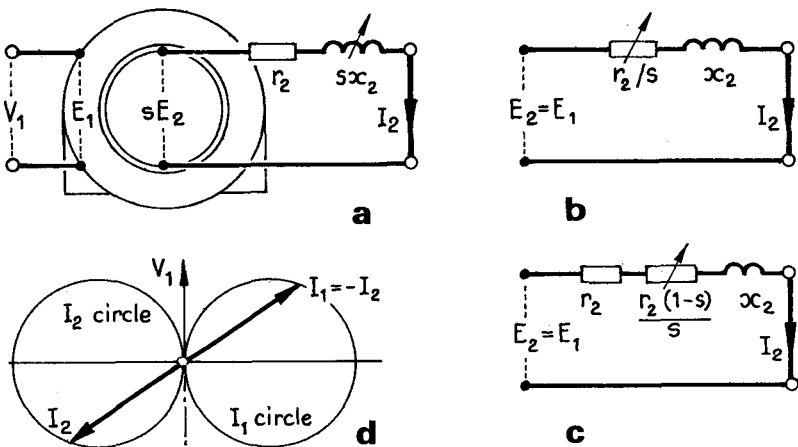
8.2 Ideal induction machine: phasor diagram.

and so maintain Φ_m . These conditions are represented in the phasor diagram of Fig. 8.2. The stator current, with reference to V_1 , represents an input to the machine; in a sense, the rotor current is an 'output' with reference to sE_2 .

Rotor Current: For any slip s , the rotor current is determined by the e.m.f. sE_2 and the appropriate leakage impedance $z_{2s} = r_2 + jsx_2$, whence the stator current balancing I_2 is

$$I_1 = -I_2 = \frac{sE_2}{z_{2s}} = \frac{sE_2}{\sqrt{[r_2^2 + (sx_2)^2]}} = \frac{E_2}{\sqrt{[(r_2/s)^2 + x_2^2]}} \tag{8.1}$$

which has a lagging phase angle $\phi_2 = \arctan (sx_2/r_2)$. The equivalent circuit for the penultimate expression is shown in (a) of Fig. 8.3, and for the last in (b). The currents in (a) and (b) are, of course, the same; but in (b) the voltage $E_2 = E_1$ is, with the stated assumptions, a constant. Further, the active power conditions in the two equivalent circuits differ, for in (a) it is $I_2^2 r_2$ and in



8.3 Ideal induction machine: rotor circuit.

(b) it is $I_2^2 r_2/s$. This difference is significant, and can be clarified by writing r_2/s as the series combination of r_2 and $r_2(1-s)/s$, as in (c). With I_2^2 the former component gives the rotor I^2R loss, the latter gives a power that must correspond to that *converted* into mechanical power.

Current Locus: Suppose that $E_2 = E_1 = V_1 = 100$ V, and that the rotor circuit leakage impedance is made up of $r_2 = 0.1 \Omega$ and $x_2 = 0.5 \Omega$. Working with the primary balancing current, then for a slip $s = 0.05$ p.u. we have $r_2/s = 2.0 \Omega$, and $I_1 = 48$ A lagging E_1 by $\phi_1 = \arctan(0.5/2.0) = 14^\circ$. Similarly for $s = 0.2$ p.u., $I_1 = 141$ A lagging by 79° . For a negative slip of $s = -0.05$ p.u. the current is again 48 A but the phase angle is $\phi_2 = 166^\circ$: this means that the active component is reversed, and the machine is delivering active power to the input terminals as a generator. For synchronous speed $s = 0$ the current is zero, while for infinite slip, forward or backward, the current is $E_1/x_2 = 200$ A at a phase angle of 90° . If all these current phasors are plotted, the locus of their extremities is found to be a circle, Fig. 8.3(d).

Power: From Fig. 8.2, the stator active power input is given by

$$P_1 = V_1 I_1 \cos \phi_2 = E_1 (-I_2) \cos \phi_2 = E_2 (-I_2) \cos \phi_2 = P_2 \quad (8.2)$$

for as the ideal stator has no loss, the whole of its input power P_1 is transferred to the rotor by transformer action. Of the rotor input P_2 , part is I^2R loss in the rotor circuit,

$$I_2^2 r_2 = (sE_2) I_2 \cos \phi_2 = sP_2$$

and the remainder is the power converted into the mechanical form:

$$P_m = (1-s)P_2$$

The important conclusion (general, and not dependent upon the assumption of constant flux) is that, of the power P_2 delivered across the gap to the rotor the fraction s becomes I^2R loss and the fraction $(1-s)$ appears as mechanical power (including windage and friction). Thus

$$P_2 : P_m : I^2R = 1 : (1-s) : s \quad (8.3)$$

A small slip, i.e. a running speed close to synchronous speed, is clearly advantageous. The rotor efficiency is $P_m/P_2 = (1-s)$.

Torque: As for a slip s the rotor speed is $\omega_r = \omega_1(1-s)$, the mechanical torque is

$$M = P_m/\omega_r = P_2(1-s)/\omega_1(1-s) = P_2/\omega_1 \quad (8.4)$$

The torque is thus proportional to the rotor power input regardless of slip. The power P_2 may be referred to as 'the torque in synchronous power' because it is the power that the actual torque would develop were it associated with synchronous speed ω_1 rather than the actual speed. The torque is obtained in terms of rotor quantities and the slip by using eqs.(8.1) and (8.2) and writing $\cos \phi_2$ as $r_2/\sqrt{[r_2^2 + (sx_2)^2]}$ to give

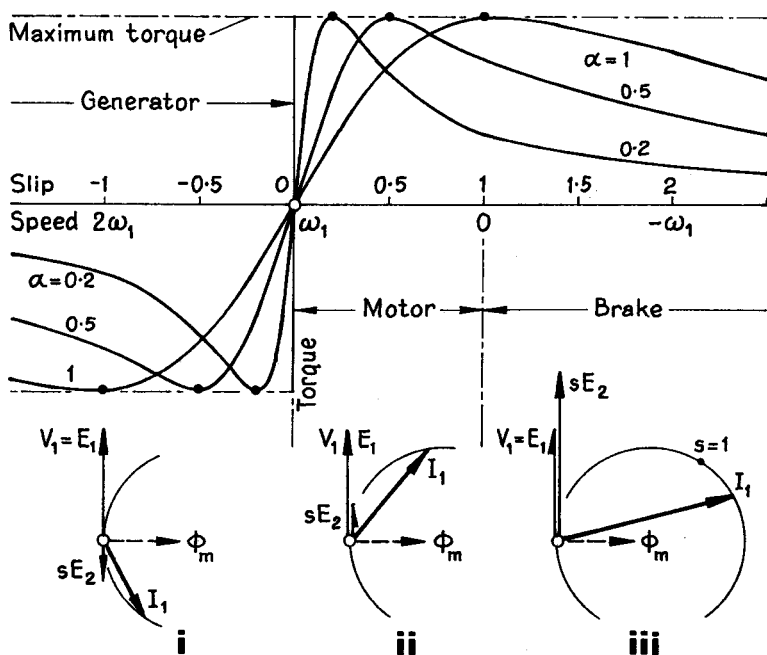
$$M = \frac{1}{\omega_1} \frac{sE_2^2 r_2}{[r_2^2 + (sx_2)^2]} = \frac{s\alpha}{s^2 + \alpha^2} K \quad (8.5)$$

where $K = E_2^2 / \omega_1 x_2$ and $\alpha = r_2 / x_2$. For a given ideal machine the torque therefore varies with slip and with the ratio of the rotor resistance to its standstill leakage reactance.

Eq.(8.5) gives $s^2 = \alpha^2$ for the condition of *maximum torque*, so that $s = \pm \alpha$ gives the information that there is a peak torque $M_{\max} = K \alpha^2 / 2\alpha^2 = \frac{1}{2} K$ at each of two speeds, one below and one above synchronous speed.

The factor K includes the term $1/x_2$, so that the magnitude of the maximum torque is raised if the standstill rotor reactance is reduced: this is obvious because the rotor current, on which the torque depends, will be larger if the rotor circuit impedance is smaller, the effect being to enlarge the diameter of the current locus in Fig. 8.3(d).

The torque/slip and torque/speed relations for three values of $\alpha = r_2/x_2$ are drawn out in Fig. 8.4 for a machine in which the standstill leakage reactance



8.4 Ideal induction machine: torque/slip relations and phasor diagrams.

x_2 is fixed, but the resistance r_2 can be varied. In all cases the maximum torque is the same, the variation of resistance only changing the slip at which this torque is developed.

For most normal operating conditions the machine works near synchronous speed with a small slip (up to about 0.05 p.u.). In eq.(8.5) we can then legitimately neglect s^2 and express the torque as $M \approx s(K/\alpha)$. For a given machine (K/α) is fixed, and the torque is directly proportional to the slip,

giving a speed/torque characteristic in which the speed falls only slightly with increase of the torque. This is generally referred to as a 'constant-speed characteristic' resembling that of a d.c. shunt machine.

Generator Mode

The phasor diagram for stator quantities is shown at (i) in Fig. 8.4. The speed is hypersynchronous, the slip is negative (and usually small), the rotor e.m.f.s and currents have such direction as to demand active power *output* from the stator terminals. But magnetization is still dependent on the stator winding accepting reactive power for this purpose from the electrical source, so that the induction generator can only operate when connected to a live and synchronous a.c. system. If a *lagging* reactive power *input* is equated with a *leading* reactive power *output*, then the generator can be described as operating with a leading power factor. The torque acts in a direction opposite to that of the travelling field, requiring a mechanical drive at the shaft.

Motor Mode

The rotor e.m.f. conditions are those in Fig. 8.1 (a) and the rotor currents as in (b). The machine operates at speeds between standstill and synchronous speed, with positive slips between 1 and 0. Let the machine run on no load, with a slip typically less than 0.01 p.u. The rotor e.m.f. is very small, the rotor circuit impedance is almost purely resistive, and a current sufficient to develop a torque to maintain rotation is developed. Suppose that now a mechanical load is put on the shaft: the rotor slows, increasing the slip. The rotor e.m.f. rises in magnitude and frequency, producing more current and torque so long as the effect of the leakage reactance permits. Greater load results in more torque up to a maximum pull-out value and then the motor stalls.

The limit to torque production at low slips is the rotor resistance, which should therefore be small. At starting, however, the current is limited largely by the leakage reactance. The torque may nevertheless be improved by increasing the resistance, for although this augments the total impedance it reduces the angle of lag and results in more rotor active power. As eq.(8.4) shows, greater rotor active power means greater torque.

Fig. 8.4 shows at (ii) the phasor diagram of stator quantities. With change of slip the stator current follows the circular locus.

Brake Mode

If the induction machine is driven mechanically in a backward direction (i.e. opposite to the travelling-wave field) it still produces a forward torque and therefore acts as a brake, absorbing the mechanical power in rotor loss. The effect can be very useful in bringing a motor rapidly to rest. Suppose a motor is running normally at a small slip s , and the direction of the field is suddenly reversed. The rotor now has in effect a slip of $(2 - s)$ and enters the brake mode. The speed falls towards zero and the machine can be brought to standstill. It will not run up thereafter in the reversed direction if the supply to the stator is interrupted. This method of stopping a motor is termed 'plugging'.

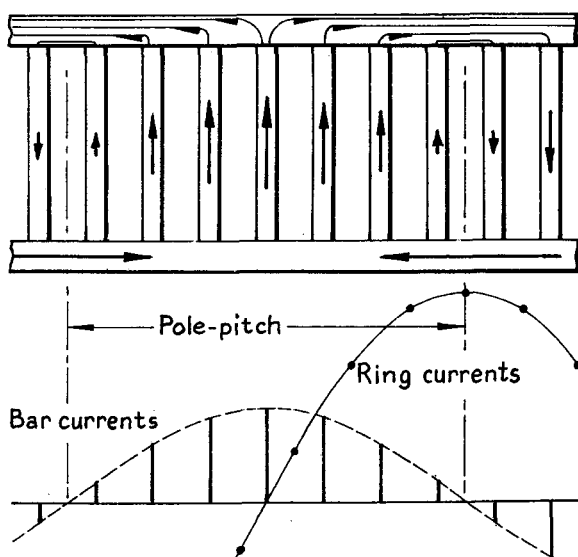
The brake-mode conditions are indicated by the phasor diagram (iii) in Fig. 8.4.

8.3 Cage and slip-ring rotor windings

The two types of rotor winding in common use for induction machines are the cage and the slip-ring windings.

Cage Rotor

The essential features of the multiphase cage winding were discussed in Sect. 3.14. In practice it consists of a set of uniformly spaced bars accommodated in the rotor slots and connected at each end to conducting rings. The winding, which is electrically closed on itself as is required for induction machine operation, can be considered to be a series of full-pitch 'turns' formed by pairs of conductor bars a pole-pitch apart, joined together in a closed loop by the end-conductors. The assumed sinusoidal space distribution of bar currents in Fig. 8.5 shows that the end-rings carry the summation of the bar currents and



8.5 Currents in cage rotor.

affect the resistance of each 'turn'. Generally the effect is small, and the currents in the bars are assumed to retain a sine distribution.

Slip-ring Rotor

A slip-ring rotor carries a normal 3- or 6-ph winding, connected in star or delta and terminated on three slip-rings, which are short-circuited when the machine is in normal operation. A 2-ph winding is an acceptable but unusual alternative.

Comparison

The cage and slip-ring rotors are thus electrically equivalent as long as attention is confined to the fundamental sine-waves of voltage, current, flux, etc. We may proceed on the conclusion that the operation of a machine will be the same whichever type of rotor winding it has.

The practical reasons for the choice between cage and slip-ring windings are based in most cases on the following considerations:

(a) the cage is permanently closed and its electrical characteristics are fixed, whereas the slip-ring winding permits of the variation of the electrical characteristics by the inclusion of external circuits via the slip-rings; and

(b) the cage is adaptable to any number of poles, whereas the slip-ring winding has to be made for one (or possibly two) definite numbers of poles.

Consideration (a) is most cogent from the viewpoint of starting the motor against large load torques, where it is advantageous to be able to increase the resistance of the rotor circuit by the inclusion of a rheostat connected across the slip-rings; while (b) is of importance in connection with speed control.

The cage is always preferred, as it is a more robust and much cheaper winding. Some of its starting-torque disadvantages can be overcome by use of the double-cage or the deep-bar cage construction.

EXAMPLE 8.1: A 2-pole 50 Hz induction machine has a rotor of diameter 0.20 m and core length 0.12 m. The polyphase stator winding maintains in the gap a sine-distributed travelling wave of flux of peak density 0.54 T. The rotor winding comprises a cage of 33 bars, each of resistance $120 \mu\Omega$ and leakage inductance $2.50 \mu\text{H}$, including the effect of the end-rings. At a given load the rotor runs with a slip of 0.064 corresponding to an angular speed of 294 rad/s (46.8 rev/s). Find (i) the peak current per rotor bar and the rotor I^2R loss; (ii) the rotor m.m.f. per pole; (iii) the load angle between the gap-flux axis and the axis of the rotor m.m.f.; (iv) the electromagnetic torque developed; and (v) the mechanical power developed.

(i) *Rotor bar current:* The slip velocity is $314 - 294 = 20$ rad/s or $20 \times 0.1 = 2.0$ m/s peripherally. The maximum e.m.f. induced in a bar is $e_{bm} = Blu = 0.54 \times 0.12 \times 2.0 = 0.13$ V. The bar impedance is $z_b = (120 + j20 \times 2.5) = (120 + j50) = 130 \angle 23^\circ \mu\Omega$. The maximum bar current is therefore

$$i_{bm} = e_{bm}/z_b = 0.13/130 \times 10^{-6} = 1\,000 \text{ A}$$

and the total rotor I^2R loss is $\frac{1}{2}(1\,000^2 \times 120 \times 10^{-6} \times 33) = 1\,980 \text{ W}$

(ii) *Rotor m.m.f.:* The peak linear current density is $A_2 = 1\,000 \times 33/\pi \times 0.2 = 52\,500 \text{ A/m}$. The discussion in Sect. 1.11 shows that the rotor m.m.f. is

$$F_2 = \frac{1}{2} A_2 D = \frac{1}{2} \times 52\,500 \times 0.2 = 5\,250 \text{ A-t/pole}$$

(iii) *Load angle:* If the rotor winding has only resistance, the peak current would occur in the bars at the positions of maximum gap flux density, and the rotor m.m.f. axis would be at right-angles to the gap flux axis. With the

actual phase-lag of 23° , the load angle is increased to $90^\circ + 23^\circ = 113^\circ$: compare Fig. 8.1(b).

(iv) *Torque*: Using eq. (1.10),

$$M = -\frac{1}{2} \pi \times 0.20 \times 0.12 \times 0.54 \times 52\,500 \times \sin 113^\circ = 98.5 \text{ N-m}$$

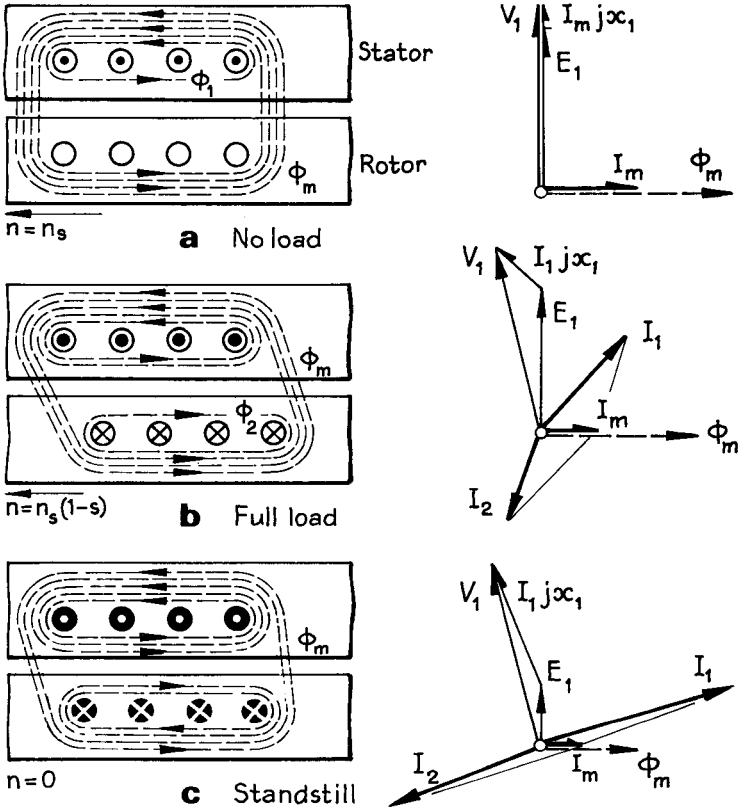
(v) *Power*: This is

$$P_m = M\omega_r = 98.5 \times 294 = 29\,000 \text{ W}$$

which can be checked from the rotor I^2R loss, using $P_m [s/(1-s)] = 29\,000 (0.064/0.936) = 1\,980 \text{ W}$. The total rotor input is $P_2 = 29\,000 + 1\,980 = 30\,980 \text{ W}$ and the efficiency (neglecting stator and mechanical losses) is 0.936 p.u.

8.4 *Practical induction machine*

In a practical induction machine the gap reluctance demands a magnetizing m.m.f., so that the stator current has to balance the rotor current and in ad-



8.6 Magnetic flux patterns.

dition provide for magnetizing current and core loss. Again, the presence of stator winding inductance and leakage reactance makes the assumption of constant gap flux untenable.

Flux Changes

A simple idea of the flux variation can be gained from the three conditions of motor operation in Fig. 8.6. In (a), for no load with negligible rotor current, most of the flux is mutual (Φ_m) but a small proportion (Φ_1), perhaps 2–3% of the total, does not link the rotor. The total stator flux is $\Phi_m + \Phi_1$, corresponding to the two components E_1 and $I_m x_1$ of the applied voltage V_1 , where I_m is the magnetizing current and the volt drop in stator-winding resistance is ignored. At full load (b) the mutual flux, now smaller, induces e.m.f.s and currents in the rotor, producing rotor leakage flux Φ_2 and requiring a balancing current component in the stator which increases the stator leakage. As V_1 has to provide for an increased stator leakage reactance drop $I_1 x_1$, the mutual flux component E_1 is less and Φ_m falls appreciably from its no-load value. At standstill (c) the rotor e.m.f. and current are large, and so must be the stator current. Much of the stator flux is now leakage, and the mutual flux has dropped to about one-half of the no-load level.

These effects, together with resistance, are taken into account by including in the equivalent circuit the stator phase resistance and leakage reactance. The magnetization requires that the stator and rotor m.m.f.s combine to produce the magnetizing and core-loss requirements.

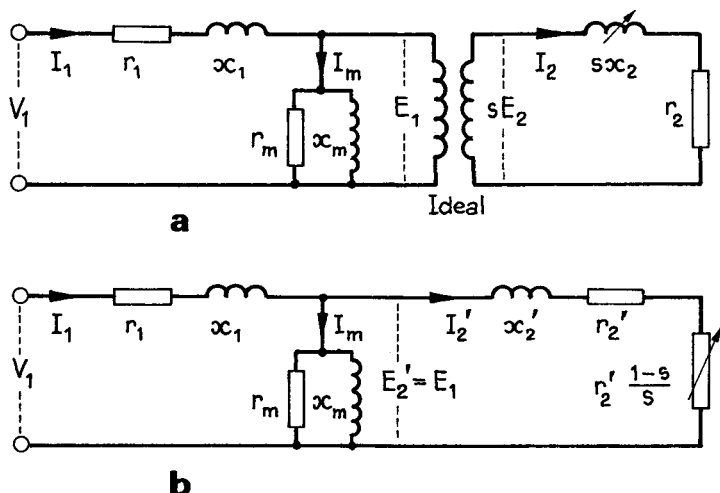
Equivalent Circuit

This is constructed for one representative stator and rotor phase, and is developed from the elementary circuits in Fig. 8.3. The stator resistance r_1 and leakage reactance x_1 are separated from the common magnetic circuit, and the magnetizing and loss currents are considered as flowing through the shunt branches r_m and x_m . The equivalent circuit then appears as in Fig. 8.7(a), the ideal transformer developing in the rotor the e.m.f. sE_2 , which circulates I_2 through the resistance r_2 and the leakage reactance sx_2 .

The rotor power is represented by the $I^2 R$ loss in r_2 . A very useful modification is to use the rotor circuit (c) of Fig. 8.3 in order to include the mechanical power conversion: then the rotor e.m.f. is changed to E_2 and the rotor impedance parameters to r_2/s and x_2 . Dividing r_2/s into r_2 and $r_2(1-s)/s$ then includes the converted power $I_2^2 r_2(1-s)/s = P_m$ explicitly. The final simplifying step is to assume the actual representative rotor phase winding to be replaced as in Fig. 8.7(b) by one having the same number of effective turns as the stator: this is the *equivalent rotor referred to the stator*, with the referred quantities such that the power, both active and reactive, in the referred rotor remain unaltered. Indicating referred quantities by a prime, then

$$E_2' = E_1, \quad I_2'^2 r_2' = I_2^2 r_2 \quad \text{and} \quad I_2'^2 x_2' = I_2^2 x_2$$

The conversions are evaluated in a manner analogous to that used for the secondary of a transformer, as follows.



8.7 Equivalent circuits.

Slip-ring Rotor Winding: Here it is necessary only to write

$$E_2' = k_t E_2, \quad r_2' = k_t^2 r_2 \quad \text{and} \quad x_2' = k_t^2 x_2 \quad (8.6)$$

where $k_t = (K_{w1}N_1/K_{w2}N_2)$ using the appropriate numbers of turns per phase in series (N) and winding factors (K_w).

Cage Rotor Winding: Eqs.(3.24) and (3.27) give the fundamental m.m.f.s respectively of a 3-ph winding carrying a current I_2' in N_1 turns per phase in series, and of a $2p$ -pole cage winding in S_2 slots with an r.m.s. current I_b per bar: the expressions are

$$F_1 = 1.35K_{w1}N_1I_2'/p \quad \text{and} \quad F_2 = S_2I_b/\sqrt{2\pi p}$$

For m.m.f. equivalence, $F_1 = F_2$ whence

$$I_b = I_2'(6K_{w1}N_1/S_2) \quad (8.7)$$

To derive the equivalent resistance r_2' we must first find the I^2R loss in the cage winding. If r_b is the resistance per bar, the total loss in the S_2 bars is $S_2I_b^2r_b$. It can be seen from the current distribution diagram in Fig. 8.5 that the end-ring current has a maximum made up of the sum of the instantaneous bar currents in a half pole-pitch, and therefore an r.m.s. value $I_c = I_b(S_2/2p\pi)$. The total I^2R loss for the S_2 bars each of resistance r_b and the two end-rings each of resistance r_c (taken around the periphery) can be equated to the total I^2R loss in a 3-ph winding carrying I_2' :

$$I_b^2 [S_2r_b + 2(S_2/2p\pi)^2r_c] = 3I_2'^2r_2'$$

for a 3-ph stator, from which, using eq.(8.7), the stator equivalent of a cage-rotor resistance is

$$r_2' = \frac{12K_{w1}^2 N_1^2}{S_2} \left[r_b + \frac{0.1 S_2}{2p^2} r_c \right] \quad (8.8)$$

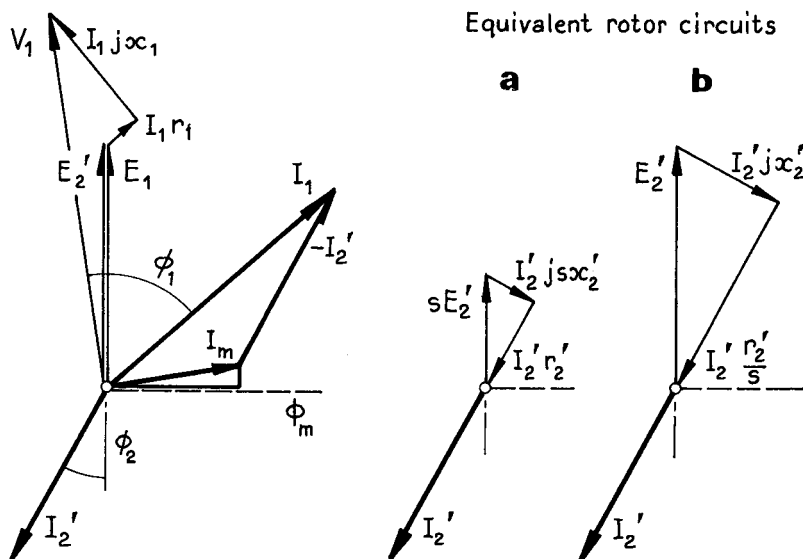
In the same way, the equivalent reactance is

$$x_2' = \frac{12K_{w1}^2 N_1^2}{S_2} x_2 \quad (8.9)$$

where x_2 is the reactance of a rotor phase. It is usual to calculate only the slot leakage separately; see Sect. 2.9.

Phasor Diagram

Either of the equivalent circuits in Fig. 8.7 can be used to construct a phasor diagram for steady-state operation at some chosen value of slip. The two forms are given in Fig. 8.8. The diagram in (a) is for the electrical quantities



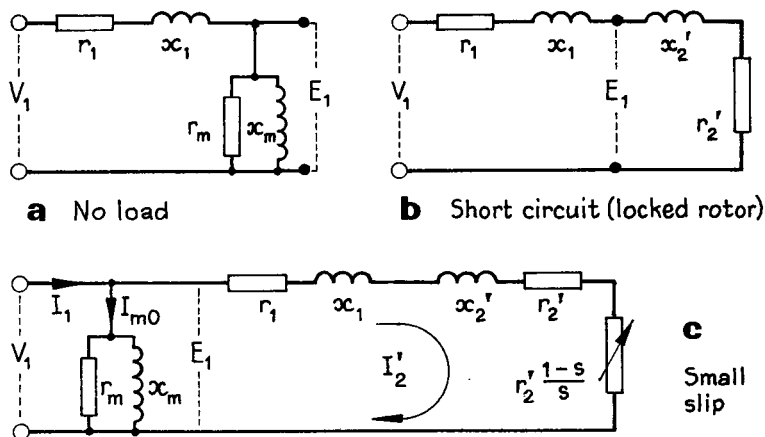
8.8 Phasor diagrams.

only, while (b) includes the total rotor power. The stator current comprises the components $-I_2'$ to balance the rotor m.m.f., and the magnetizing and core-loss components that make up the current I_m needed to magnetize the machine. The stator voltage V_1 accounts for the stator leakage impedance volt drops $I_1 r_1$ and $I_1 j\omega_1$, leaving the e.m.f. E_1 associated with the gap flux. In the rotor, I_2' is derived in (a) from sE_2' divided by the true rotor leakage impedance $(r_2' + jx_2')$, whereas in (b) it is E_2' divided by $[(r_2'/s) + jx_2']$.

Approximate Equivalent Circuit

The equivalent circuits in Fig. 8.7 enable the characteristics of the induction machine to be evaluated for steady-state conditions by the ordinary processes of a.c. network solution. Some useful approximations can be made for normal operating conditions with small positive or negative slip, and for no-load and short-circuit conditions.

No Load: On no load the slip is so small that r_2'/s becomes very high, making the rotor in effect almost open-circuited, reducing the equivalent circuit to that in Fig. 8.9(a). The magnetizing impedance preponderates, being so much



8.9 Equivalent-circuit approximations.

greater than r_1 and x_1 that $E_1 \approx V_1$. The loss is V_1^2/r_m very nearly. However, there is a further small loss, in friction and windage, that demands a torque and therefore a rotor current and a balancing stator current so that the actual no-load current is I_0 . A test of the machine on no load therefore gives, as stator input, the core and mechanical loss together with an almost negligible I^2R loss.

Short Circuit: In this condition the stator is excited (at normal, or for large machines, at reduced voltage) and the *rotor held at rest* ('locked'). The conditions in Fig. 8.9(b) show that, as E_1 is only about one-half of its no-load value and the rotor current is large, the magnetizing current can be neglected. The stator current is limited by the combined resistance $R_1 = r_1 + r_2'$ and the combined reactance $X_1 = x_1 + x_2'$. The active power input with the rotor locked is therefore the effective I^2R loss with its associated stray loss.

Operation with small slip: So long as the slip is small, the machine can be regarded as running with an almost constant gap flux, and the magnetizing branches r_m and x_m can be transferred to the primary terminals as in Fig. 8.9(c). From the network viewpoint this is a considerable simplification, and it is common where operating conditions permit to use this 'approximate' equivalent circuit for the purposes of easy calculation.

Limitations

There are numerous defects in the equivalent circuit. The impedance parameters are not constant. Apart from variation of ohmic resistance with temperature, the parameters may be expected to change with the loading conditions as follows:

r_1 and r_2 : tooth saturation alters the eddy-loss coefficient (Sect. 3.2) and consequently the effective resistance.

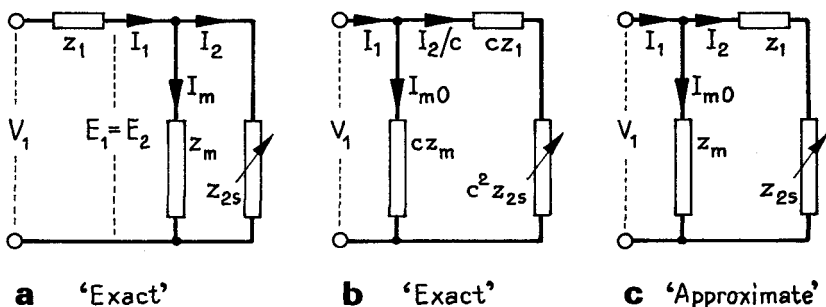
x_1 and x_2 : tooth and core saturation affect the leakage flux per unit current and therefore the effective inductive reactance.

r_m and x_m : the magnetizing reactance is dependent in magnitude on the saturation of the magnetic circuit; the core loss is not a simple function of the gap flux, as it is a summation of the loss in the ferromagnetic materials, in which the flux density varies in intensity from point to point, so that only at one fixed e.m.f. condition can r_m give the proper loss.

It is seen that the major cause of uncertainty is the saturation. The effect of saturation is to limit the flux in the iron of the flux paths concerned. The usual process in design is to calculate reactances on the assumption of infinite permeability, and then to apply a saturation factor. It is a formidable task, for an accurate solution for any given conditions of current and slip would require a knowledge of the path dimensions and the magnetic characteristics of the material at every point therein. The usual arbitrary division of the leakage flux into convenient regions sometimes involves two such components occupying in part the same iron path; but superposition of the components is not valid. A number of attempts have been made to deal with this problem, such as that devised by Chalmers and Dodgson [8].

8.5 Steady-state theory

The behaviour of the machine is investigated by use of the equivalent circuit, it being assumed that proper values can be assigned to the parameters to suit the operating conditions. Operation is considered in the steady state with constant stator applied voltage and frequency, and all values are per phase.



8.10 Exact and approximate equivalent circuits.

In order to include the converted power explicitly, the equivalent circuit is based on Fig. 8.7(b). To simplify the analysis, the network of Fig. 8.10(a) is taken with the assumption that the stator/rotor effective turns ratio is 1/1; in using the results for general calculation it is only necessary to remember that z_2 , r_2 and x_2 have to be read as z_2' , r_2' and x_2' . We seek the currents I_1 , I_2 and I_m , and the torque, for any given slip s .

Stator

The stator current is $I_1 = I_m + I_2 = (E_1/z_m) + (E_1/z_{2s})$, and the applied voltage is

$$\begin{aligned} V_1 &= E_1 + I_1 z_1 = E_1 + (I_m + I_2) z_1 \\ &= E_1 [1 + (z_1/z_m) + (z_1/z_{2s})] \end{aligned}$$

Now z_1 is small compared with z_m , but both are inductive and have impedance angles of the same order. It is convenient to write

$$c = 1 + (z_1/z_m) = c \angle \gamma \quad (8.10)$$

a complex number slightly greater than unity (e.g., 1.1) with a small angle γ , usually negative. Then

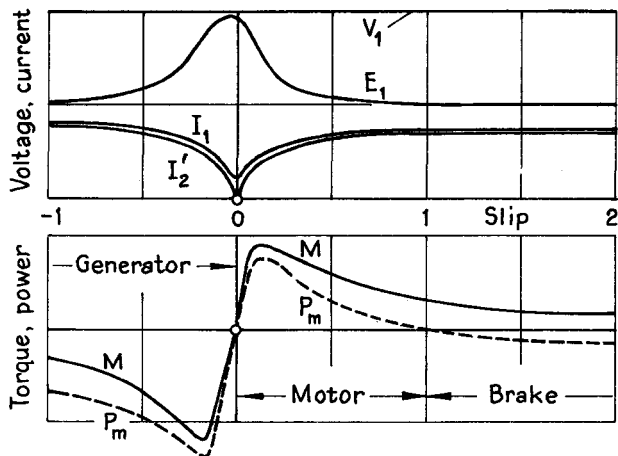
$$V_1 = E_1 [c + (z_1/z_{2s})]$$

Curves typical of the variation of E_1 and I_1 with slip are shown in Fig. 8.11.

Rotor

It is now possible to obtain I_2 in terms of V_1 :

$$\begin{aligned} I_2 &= \frac{E_1}{z_{2s}} = \frac{V_1}{z_1 + cz_{2s}} = \frac{V_1}{(r_1 + jx_1) + c[(r_2/s) + jx_2]} \\ &= \frac{V_1}{(r_1 + cr_2/s) + j(x_1 + cx_2)} \end{aligned} \quad (8.11)$$



8.11 Induction machine characteristics.

Any circuit of impedance $r + jx$ in which one component (here the reactance) is constant while the other (resistance) is varied has a straight-line impedance locus, and in consequence the locus of its phasor current at constant voltage is a circular arc [Ref. 1]. The scalar magnitude of the rotor current (taking c as a *pure numeric*) is

$$I_2 = \frac{V_1}{\sqrt{[(r_1 + cr_2/s)^2 + (x_1 + cx_2)^2]}} \quad (8.12)$$

Torque

The total rotor input power per phase is $P_2 = I_2^2 r_2/s$ and the torque is P_2/ω_s . It therefore follows from eq.(8.3) that the torque is

$$M = \frac{1}{\omega_s} \cdot \frac{V_1^2 (r_2/s)}{(r_1 + cr_2/s)^2 + (x_1 + cx_2)^2} \quad (8.13)$$

Curves typical of the torque (and rotor input power) are plotted in Fig. 8.11. Solving $dM/ds = 0$ for maximum torque gives the slip for this torque as

$$s_m = \pm \frac{cr_2}{\sqrt{r_1^2 + (x_1 + cx_2)^2}} \quad (8.14)$$

The value of the maximum torque is therefore

$$M_m = \frac{1}{\omega_s} \cdot \frac{V_1^2/2c}{\pm r_1 + \sqrt{r_1^2 + (x_1 + cx_2)^2}} \quad (8.15)$$

Magnetizing Current

At zero slip the rotor current vanishes and the magnetizing current is $I_{m0} = V_1/(z_1 + z_m) = V_1/cz_m$. When, for finite slips, rotor current flows and the e.m.f. falls to $E_1 = V_1 - I_1 z_1$, the magnetizing current reduces to I_m . Let the stator current $I_1 = I_m + I_2$ be written instead as $I_1 = I_{m0} + I_a$: then

$$I_a = I_2 + I_m - I_{m0} = I_2 + (E_1/z_m) - (V_1/cz_m)$$

Solving for I_a with $E_1 = V_1 - (I_{m0} + I_a)z_1$, we obtain (after some algebra) the result that $I_a = I_2/c$, whence

$$I_m = I_{m0} + I_a - I_2 = I_{m0}(c - 1) = I_{m0} - I_a(z_1/z_m)$$

As I_a has the same circular locus as I_2 , the magnetizing current I_m for a slip s is the value I_{m0} at slip zero less a phasor that itself has a circular locus. In terms of the rotor current

$$I_m = I_{m0} - I_2 [(c - 1)/c] = I_{m0} - I_2 [z_1/(z_1 + z_m)] \quad (8.16)$$

Stator Current

This is $I_1 = I_m + I_2$, and using eq.(8.16)

$$I_1 = \frac{V_1}{cz_m} - I_2 \frac{c - 1}{c} + I_2 = \frac{1}{c} \left[\frac{V_1}{z_m} + I_2 \right] \quad (8.17)$$

where $c = 1 + (z_1/z_m)$ from eq.(8.10), and I_2 is from eq.(8.11). Eq.(8.17) can be expressed by the alternative equivalent circuit in Fig. 8.10(b), which may offer some advantage in computing performance from known parameters.

A comparison of these results with those in Sect. 8.2 show that the stator impedance has an important influence on the behaviour of the machine. Generally the stator and rotor equivalent resistance and reactance values are comparable and the stator quantities cannot be neglected without serious error.

Within the limitations imposed by the variation of resistance and leakage reactance with saturation and frequency, eqs.(8.11)–(8.17) can be used to calculate machine performance.

EXAMPLE 8.2: A 36 kW, 415 V, 50 Hz, 3-ph, 10-pole, delta-connected cage machine has the following impedance parameters referred to a stator phase at normal operating temperature:

$$\begin{array}{lll} r_1 = 0.19 \, \Omega & r_2 = 0.29 \, \Omega & r_m = 143 \, \Omega \\ x_1 = 1.12 \, \Omega & x_2 = 1.12 \, \Omega & x_m = 16.8 \, \Omega \end{array}$$

The performance of the machine is required for the following operating conditions:

- (i) as a generator with a slip $s = -0.026$ p.u.;
- (ii) at synchronous speed, $s = 0$;
- (iii) as a motor at rated output, for which $s = 0.026$ p.u.;
- (iv) as a motor at a slip corresponding to pull-out torque;
- (v) at standstill, $s = 1.0$ p.u.;
- (vi) as a brake with a slip of $s = 2.0$ p.u.;
- (vii) at infinite slip, $s = \infty$.

For condition (iv), eq.(8.14) gives $s_m = 0.133$ p.u. The calculations are set out in the Table, using the 'exact' equivalent circuit, Fig. 8.10(a), the phase voltage $V_1 = 415 \angle 0^\circ$ V as reference phasor, and the following data:

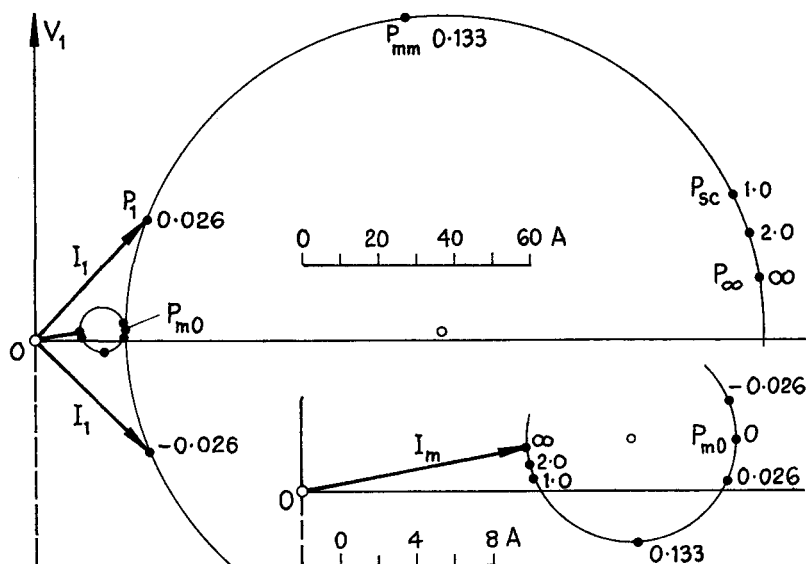
$$\begin{aligned} z_1 &= 0.19 + j1.2 = 1.14 \angle 80.4^\circ \, \Omega \\ z_{2s} &= 0.29/s + j1.12 \, \Omega \\ z_m &= 1/[(1/r_m) - j(1/x_m)] = 1.95 + j16.6 = 16.7 \angle 83.3^\circ \, \Omega \\ c &= 1 + (z_1/z_m) = 1.07 \angle -0.2^\circ \approx 1.07 \\ \omega_s &= 2\pi 50/5 = 62.8 \text{ rad/s} \end{aligned}$$

The three-phase values in the Table include an estimated mechanical loss in windage and friction: this has to be supplied to the machine by the mechanical drive when the operating mode is generation or braking, but is deducted from the converted power P_m for motor-mode conditions.

The seven values of s are marked on the stator current locus in Fig. 8.12. The locus of $I_1 = I_m + I_2$ is seen to be circular, as also is that of the magnetizing current I_m , the details of which are more clearly shown in the enlarged part of the diagram, drawn to 5 times the scale of the main diagram. In com-

EXAMPLE 8.2: Performance of 36 kW 415 V 50 Hz 10-pole Induction Machine

Condition	Generator	Syn. speed	Motor	Max. torque	Standstill	Brake	Inf. slip
[1] Slip s	p.u.	0.0	0.026	0.133	1.0	2.0	∞
[2] r_2/s	Ω	∞	11.2	2.18	0.29	0.145	0
[3] z_{2s}	Ω	∞	11.2 + j1.12	2.18 + j1.12	0.29 + j1.12	0.145 + j1.12	j1.12
[4] cz_{2s}	Ω	∞	12.0 + j1.2	2.34 + j1.2	0.31 + j1.2	0.16 + j1.2	j1.2
[5] $z_1 + cz_{2s}$	Ω	∞	12.2 + j2.3	2.53 + j2.3	0.50 + j2.3	0.35 + j2.3	0.19 + j2.3
[6] $z_{2s}/(z_1 + cz_{2s})$	-	0.93 $\angle$ 0°	0.91 $\angle$ -5.3°	0.71 $\angle$ -15°	0.49 $\angle$ -2.3°	0.48 $\angle$ 1.2°	0.48 $\angle$ 4.7°
[7] $E_1 = V_1 \times [6]$	V	386 $\angle$ 0°	378 $\angle$ -5.3°	296 $\angle$ -15°	203 $\angle$ -2.3°	199 $\angle$ 1.2°	199 $\angle$ 4.7°
[8] $I_m = E_1 \div z_m$	A	2.7 - j23.0	0.6 - j22.6	-2.8 - j17.6	0.9 - j12.1	1.6 - j11.8	2.3 - j11.7
[9] $I_2 = E_1 \div [3]$	A	0 - j0	32.6 - j6.2	88.2 - j81.2	37 - j171	26.4 - j175	14.6 - j177
[10] $I_2^2 r_2$	kW	0.0	0.32	4.20	8.86	9.00	9.20
[11] P_2	kW	0.0	12.52	31.50	8.86	4.52	0.0
[12] P_m	kW	0.0	12.20	27.30	0.0	-4.48	-9.20
[13] $M = P_2/\omega_s$	N-m	0.0	199	500	141	72	0.0
[14] $I_1 = I_2 + I_m$	A	2.7 - j23.0 = 23.2 $\angle$ -83°	33.2 - j28.9 = 44 $\angle$ -41°	85 - j99 = 131 $\angle$ -49°	38 - j183 = 186 $\angle$ -78°	28 - j187 = 188 $\angle$ -82°	17 - j189 = 190 $\angle$ -85°
[15] $\cos \phi_1$	p.u.	0.12	0.76	0.66	0.20	0.15	0.09
[16] P_1	kW	1.1	13.8	35.5	16.7	11.6	7.0
<i>Three-phase values:</i>							
Gross mechanical power $3P_m$	kW	0.0	36.60	81.90	0.0	-13.44	-27.6
Mechanical loss	kW	0.62	0.60	0.40	0.0	0.63	-
Net mech. power	kW	0.0	36.0	81.5	0.0	-14.1	-
Efficiency	p.u.	0.0	0.87	0.76	0.0	-	-



8.12 Circle diagram: 36 kW machine.

putting the tabulated results it is easily possible to introduce current-dependent variations in the resistance and reactance parameters; the locus diagrams will then not be precise circles.

8.6 Approximate theory

Use of the equivalent circuit (c) of Fig. 8.10 yields simpler expressions from which generally informative – but approximate – inferences can be made. In effect, the magnetizing current is taken as constant, and the conversion branch of the network becomes simply z_1 and z_{2s} in series with a constant phase voltage V_1 applied. The complex number c is now unity, and the following expressions can readily be derived.

Rotor Current

This is

$$I_2 = \frac{E_1}{z_{2s}} = \frac{V_1}{z_1 + z_{2s}} = \frac{V_1}{(r_1 + r_2/s) + jX_1} \quad (8.18)$$

where $X_1 = x_1 + x_2$ is the total combined stator and rotor leakage reactance at supply frequency referred to the stator winding. The scalar value of the current is

$$I_2 = \frac{V_1}{\sqrt{[(r_1 + r_2/s)^2 + X_1^2]}} \quad (8.19)$$

This has a circular locus.

Torque

Eq.(8.13) simplifies to

$$M = \frac{1}{\omega_s} \cdot \frac{V_1^2 (r_2/s)}{(r_1 + r_2/s)^2 + X_1^2} \quad (8.20)$$

For the slip

$$s_m = \pm \frac{r_2}{\sqrt{r_1^2 + X_1^2}}$$

the torque has its maximum value

$$M_m = \frac{1}{\omega_s} \cdot \frac{V_1^2/2}{\pm r_1 + \sqrt{r_1^2 + X_1^2}} \quad (8.21)$$

The negative sign refers to negative slip conditions (i.e. the generator mode). The *rotor* resistance affects the speed for maximum torque, but not the torque itself. To obtain maximum torque at starting ($s = 1$) the rotor resistance must be $r_2 = \sqrt{r_1^2 + X_1^2}$. It is notable that an increase of *stator* resistance affects the maximum torque, increasing it for the generator and reducing it for the motor mode:

$$\frac{M_m(\text{gen})}{M_m(\text{motor})} = \frac{r_1 + \sqrt{r_1^2 + X_1^2}}{-r_1 + \sqrt{r_1^2 + X_1^2}} \approx \frac{X_1 + r_1}{X_1 - r_1}$$

and the approximation holds if r_1 is small compared with X_1 as is normally the case. If $X_1 = 7r_1$, the torque ratio becomes 1.33. An attempt to control an overspeeding motor by connecting resistance into the stator lines may produce very high peak torques.

Losses and Efficiency

The obvious I^2R loss is $I_1^2 r_1 + I_2^2 r_2$ per phase, and is load-dependent. In addition there are various 'stray' losses (Sect. 8.12) which may also be taken as load-dependent. Certain losses are taken to sum to a constant, at least in the normal working range of speed: these are friction, windage and core losses. Per phase, the efficiency, or ratio [output/output + losses], is given approximately for an electromagnetically converted power P_m and mechanical loss p by

$$\eta = \frac{P_m - p}{(P_m - p) + \{I_2^2 (r_1 + r_2) + p_s\} + [I_{m0}^2 (r_1 + r_m) + p]}$$

where p_s is the stray loss, and the core loss is represented by $I_{m0}^2 r_m$. The term in square brackets is taken as roughly constant so that maximum efficiency can be assessed as that for which the two sets of bracketed terms are equal. The variable and 'constant' losses of a machine on test can be inferred from its input on no load and with locked rotor.

EXAMPLE 8.3: Evaluate the performance of the machine of Example 8.2,

using the 'approximate' equivalent circuit, for the same seven operating conditions.

The primary data are the same as in Example 8.3, but the magnetizing current is now

$$I_{m0} = V_1 [(1/r_m) - j(1/x_m)] = 2.9 - j24.7 \text{ A}$$

and from eq.(8.21) the slip for maximum torque is $s_m = 0.13$ p.u. The computed results are tabulated below: the items are numbered where possible in accordance with the Table for Example 8.2.

EXAMPLE 8.3: Performance of 36 kW 415 V 50 Hz Induction Machine

Condition		Generator	Syn. speed	Motor
[1] Slip s	p.u.	-0.026	0.0	0.026
[3] z_{2s}	Ω	-11.2 + j1.12	∞	11.2 + j1.12
[5] $z_1 + z_{2s}$	Ω	-11.0 + j2.24	∞	11.4 + j2.24
[9] $I_2 = V_1 \div [5]$	A	-36.3 + j7.4	0	35.0 - j6.9
[10] $I_2^2 r_2$	kW	0.40	0	0.37
[11] P_2	kW	-15.3	0	14.2
[12] P_m	kW	-15.7	0	13.8
[13] M	N-m	-244	0	225
[14] $I_1 = I_2 + I_m$	A	-33.4 - j32.1	2.9 - j24.7	37.9 - j31.6
[15] $\cos \phi_1$	p.u.	0.72	0.12	0.77
[16] P_1	kW	-13.9	1.2	15.7
Output	kW	-16.0	0.0	13.6
Efficiency	p.u.	0.87	0	0.87

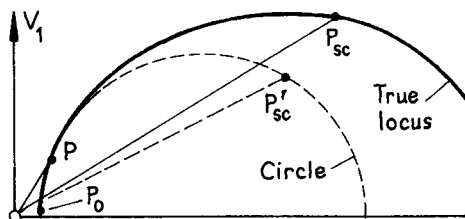
Condition		Max. torque	Standstill	Brake	Inf. slip
[1] p.u.	0.130	1.0	2.0	∞	
[3] Ω	2.24 + j1.12	0.29 + j1.12	0.15 + j1.12	j1.12	
[5] Ω	2.43 + j2.24	0.48 + j2.24	0.34 + j2.24	0.19 + j2.24	
[9] A	92 - j184	38 - j178	33 - j182	16 - j185	
[10] kW	4.50	9.60	9.95	10.55	
[11] kW	35.1	9.60	5.00	0.0	
[12] kW	30.6	0.0	-4.95	-10.55	
[13] N-m	560	153	79	0	
[14] A	95 - j109	41 - j202	36 - j207	19 - j210	
[15] p.u.	0.66	0.20	0.17	0.09	
[16] kW	39.4	17.0	15.0	7.9	
Output kW	30.5	0.0	0.0	0.0	
Efficiency	0.78	0.0	-	-	

There are divergences between the calculated figures in Examples 8.2 and 8.3 as a result of their different circuit bases. The stator and rotor current phasors again have a circular locus, but the magnetizing-current circle has shrunk to a single point.

8.7 Current diagrams

Current-locus diagrams for the induction machine are less common than they were before the advent of digital computation, but they are still useful for clarifying test results and for making comparisons of performance. Graphical niceties are out of place, however, because of the variability of impedance parameters, so that often a simplified diagram can give all that is necessary.

In general, r_1 , x_1 , r_2 and x_2 will be less for large than for small currents, with the reactances decreasing probably faster than the resistances. It may therefore be expected that the short-circuit current will be greater and of (probably) higher power factor than as estimated from values of r_1 , x_1 , r_2 and x_2 that are quite suitable for currents within the load range. It is evident that no single circle will be accurate over the whole range, and the diagram should be drawn with reference to the use to which it is put. If the full-load conditions are required, and maximum torque, the diagram should be drawn for characteristics applicable to the full-load current. It is not generally possible to do more than estimate the changes at very large short-circuit currents.



8.13 Practical stator-current locus.

That the true locus tends to an elliptical shape is shown by Fig. 8.13, drawn from test figures on a 415 V, 3-ph, 50 Hz, 6-pole, 1.2 kW machine with a short-circuit current small enough to be readily measured at normal voltage. The short-circuit test figures were:

Measured			Calculated			
Line voltage	Line current	Total power	Z_1	R_1	X_1	$\cos \phi$
V	A	kW	Ω	Ω	Ω	p.u.
63	2.5*	0.124	44	19.8	39.5	0.45
95	3.8	0.28	44	19.8	39.4	0.46
240	10.6	2.32	39	20.5	33.2	0.53
415	20.0	8.79	36	21.9	28.7	0.61

* Full-load current

From Fig. 8.13 it is clear that for finding currents in the normal load range the dotted circle, drawn from the no-load test and the short-circuit test at nor-

mal rated current, is adequate. But this circle will give erroneous results for starting current and torque.

‘Exact’ Current Locus Diagram

Using the theory developed in Sect. 8.5, the current locus can be drawn from a few known points, e.g. those marked in Fig. 8.12:

$$OP_{mo}: I_1 = I_{mo} = \frac{1}{c} \cdot \frac{V_1}{z_m} \text{ for the current at } s = 0;$$

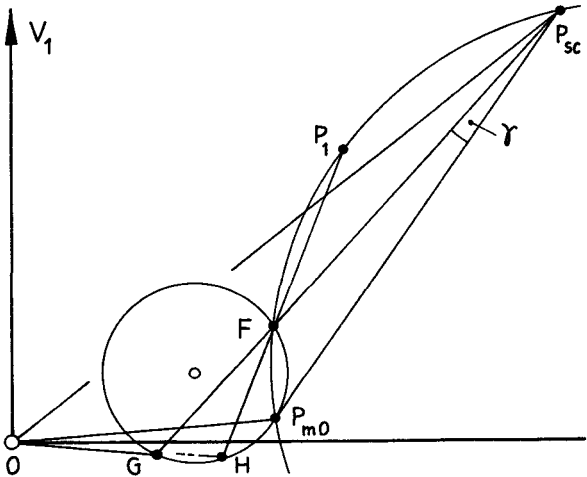
$$OP_1, OP_{mm}, OP_{sc}, OP_{\infty}: I_1 = \frac{1}{c} \left[\frac{V_1}{z_m} + I_2 \right]$$

where I_2 is given by eq.(8.11) with values of s corresponding respectively to some load (preferably full load), maximum torque, short-circuit $s = 1$, and infinite slip $s = \infty$. As all these points lie on a circle the centre is readily found by drawing a few chords.

The magnetizing current circle is very small. It can be drawn from the stator current circle by applying the analysis in Sect. 8.5, whence from eqs. (8.17) and (8.10)

$$I_1 = I_m + I_2 = I_{mo} + I_a = I_{mo} + I_2/c$$

For the phasor $OP_{sc} = I_{sc}$ of the stator current on short circuit. Fig. 8.14, $OP_{mo} = I_{mo}$ and $P_{mo}P_{sc} = I_a$. Draw a line $P_{sc}G$ of length cI_a and displaced in direction by the angle γ in c . Then $P_{sc}G = I_2$ and consequently OG must be I_m for the short-circuit condition. The same kind of construction can be used for any load point such as P_1 , and as the angle γ applies in each case, all lines $P_{sc}G, P_1H \dots$ must pass through the same focal point F on the stator current circle. Completing chords $FP_{mo}, P_{mo}H, HG \dots$ enables the centre of the magnetizing current circle to be found.



8.14 Geometrical construction for magnetizing current.

A short approximate method is to draw the line $P_{mo}P_{sc}$; bisect the angle $OP_{sc}P_{mo}$; and project the bisector to G, where the length $P_{sc}G = \sqrt{[(OP_{sc}) \cdot (P_{mo}P_{sc})]}$.

Diagram from Test Data

Although it is possible to obtain graphically a range of data from the current diagram, such information is limited in accuracy. A practical diagram therefore omits slip lines, loss lines and efficiency scales and aims to give a straightforward indication of full-load stator and rotor currents, full-load power factor, and pull-out torque, starting from test data or design figures. One of several such diagrams is given in Fig. 8.15, drawn to scale for one phase and normal rated voltage:

- $OP_0 = I_0$, the no-load current at phase angle ϕ_0 ;
- $OA = I_{0r}$, the reactive (pure magnetizing) component of I_0 ;
- $AP_0 = I_{0a}$, the active component of I_0 , associated with both stator core and mechanical losses per phase;
- $OB = I_{sci} \approx V_1/(x_1 + x_2')$, the 'ideal' short-circuit current;
- AB = diameter of current-locus circle, of centre M;
- $OP_{sc} = I_{sc}$, the stator short-circuit (standstill) current at phase angle $\phi_{sc} \approx \arctan[(x_1 + x_2')/(r_1 + r_2')]$;
- $P_{sc}D = I_{sc} \cos \phi_{sc}$, the active component of I_{sc} , proportional to the short-circuit $I^2 R$ loss and divided at C in the proportion $P_{sc}C/CD = I_2'^2 r_2'/I_1'^2 r_1$;
- P_0C = torque line, extended to give the infinite-slip point P_∞ ;
- P_0P_{sc} = output-power line.

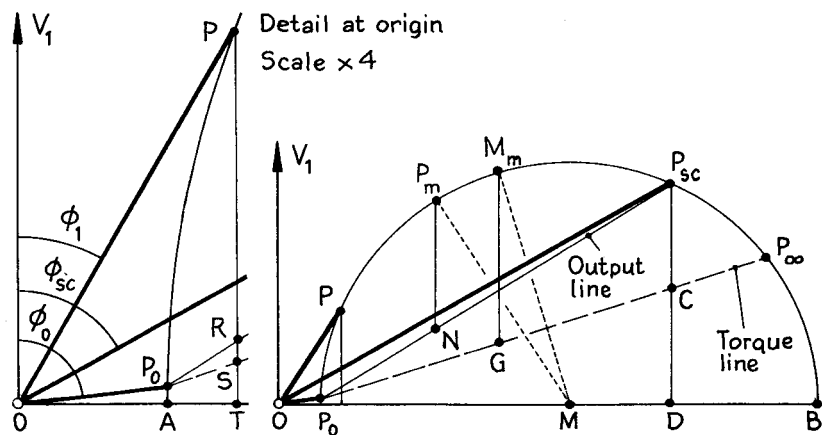
Radii from M, perpendicular respectively to the torque and power lines, locate the points M_m and P_m ; then the maximum torque is M_mG and the maximum output power is P_mN . Except for small machines the centre M of the circle can be taken as lying on the base line. A justification for the location of the output-power and torque lines is given below. For any given stator current $I_1 = OP$ as shown in the enlarged detail of Fig. 8.15, the net power output is proportional to PR and the torque to PS , while PT represents the input power.

'Approximate' Current Locus Diagram

A current locus diagram based on the 'approximate' equivalent circuit, with further simplifying assumptions, is of value not for its accuracy (which is doubtful) but for its clarity in comparing the behaviour of induction machines in respect of change of parameters and operation in various modes.

The main circle, Fig. 8.16, is constructed on the diameter P_0Y given by $V_1/(x_1 + x_2')$, the 'ideal' short-circuit current.

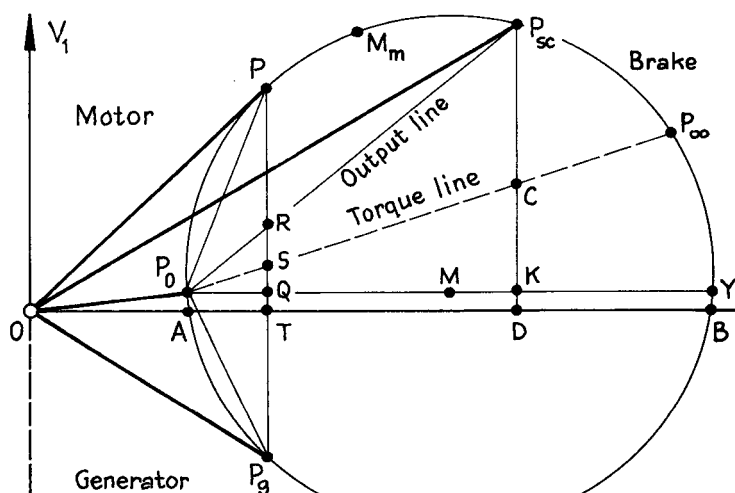
No-load: The stator active power input per phase is $P_0 = V_1 I_0 \cos \phi_0 = I_{0a} = P_0A$ to scale, and it comprises stator core loss, mechanical loss, and a small rotor power to maintain rotation (taken as negligible). The reactive component



8.15 Practical current diagram.

of the stator no-load current is $I_{or} = OA$ to scale, taken as constant. On load, the rotor speed falls and with it the mechanical loss; at the same time there is a rising rotor core loss. It is assumed that the sum of the mechanical and core losses is constant and that $P_0 = P_0A = QT = KD = YB$ to scale.

Short-circuit: The stator current is $I_{sc} = OP_{sc}$ and the stator active power is $P_{sc} = V_1 I_{sc} \cos \phi_{sc} = P_{sc}D$ to scale, of which KD has been taken as the 'constant' loss. The remainder, $P_{sc}K$, must be the total stator and rotor I^2R loss per phase. Straight lines P_0P_{sc} and P_0K then contain the appropriate I^2R loss



8.16 Approximate current diagram.

for any load, such as that represented by point P in the motor mode. The line PT, perpendicular to AB, has the intercept RQ: the ratio

$$\frac{RQ}{P_{sc}K} = \frac{P_0Q}{P_0K} = \frac{P_0P(P_0P/P_0Y)}{P_0P_{sc}(P_0P_{sc}/P_0Y)} = \frac{(P_0P)^2}{(P_0P_{sc})^2} = \frac{I_2^2}{I_{2sc}^2}$$

which, taking the equivalent rotor and the stator phase currents as very roughly equal for load and short-circuit conditions respectively, is the ratio of total I^2R losses for these cases. As active currents are the vertical components, and PT therefore represents the stator input active power, RQ the I^2R and QT the constant loss, then the output power is PR. The line P_0P_{sc} can then be labelled the *output-power line*.

Torque: The net output torque can be regarded as the net power output plus the rotor I^2R loss. The total I^2R loss, RQ for a load P and $P_{sc}K$ for short-circuit conditions, can be divided respectively in the same proportions RS/SQ and $P_{sc}C/CK$ corresponding to the rotor/stator I^2R loss ratio (or, approximately, the rotor/stator phase resistance ratio r_2'/r_1) by the line P_0C . Then the intercepts PS and $P_{sc}C$ represent torque (in synchronous power to the power scale), and the line P_0C can be called the *torque line*.

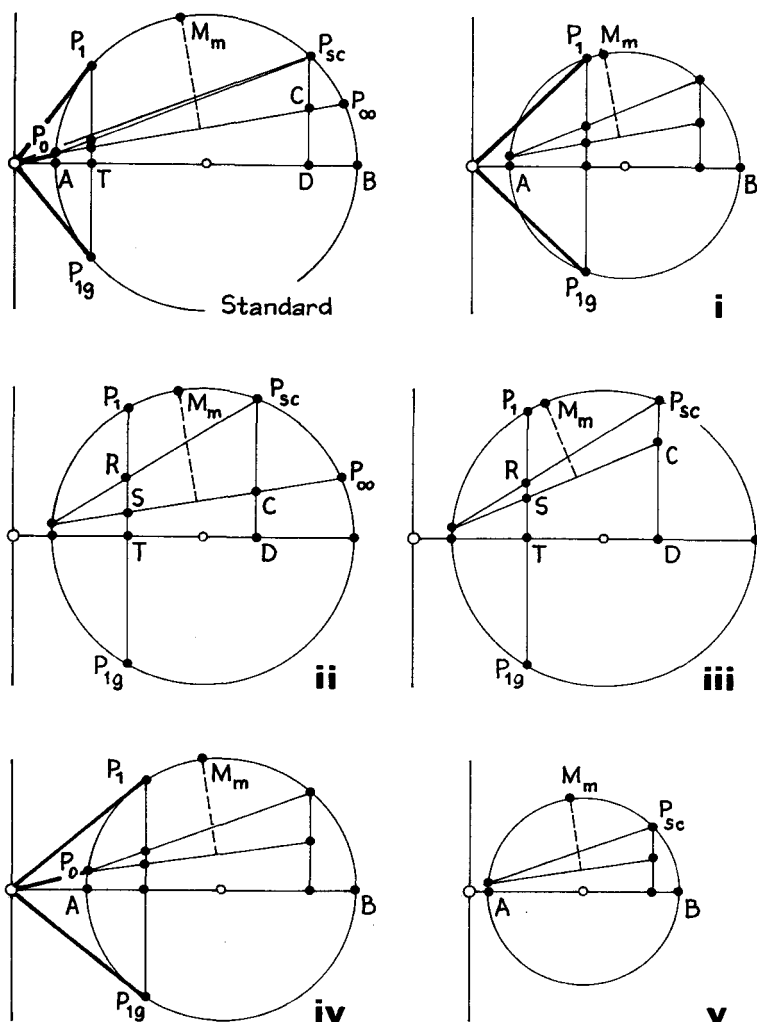
Infinite Slip: At $s = \infty$ there is no rotor current and consequently no rotor I^2R loss. Projecting P_0C to P_∞ therefore gives the corresponding condition. The part of the circle between P_{sc} and P_∞ then gives the brake mode.

Motor Mode: The circular locus between P_0 through P to P_{sc} is for operation of the machine as a motor. The stator current $I_1 = OP$ for a required output power PR or torque PS can then be located, and its power factor $\cos \phi_1$ found. The efficiency is PR/PT and the slip is RS/PS (but it would be idle to measure these from the diagram). The pull-out torque (or maximum output power) can be obtained from the point M_m (or P_m) where a radius from the centre M, drawn perpendicular to the torque line (or the power line), cuts the circumference.

Generator Mode: This is the lower part of the circle from P_0 to P_∞ . For a load point P_g the stator power output as a generator is P_gT , the mechanical input to drive the machine is P_gR , and the I^2R and 'constant' losses are, as for the motor mode, given by the intercepts RS, SQ and QT. Regarded as an output, the machine works at a leading power factor: in other terms, it has an active power *output* but still requires a lagging reactive *input* for magnetization.

8.8 Effects of machine parameters

Variations of the parameters have the effects on performance that can be illustrated with reference to a set of comparative approximate circle diagrams. The 'standard' machine in Fig. 8.17 has, in terms of 1.0 p.u. full-load rating as a motor (corresponding to the intercept P_1R) and a 1.0 p.u. stator current OP_1 , a magnetizing current of 0.3 p.u., a short-circuit current of 2.5 p.u., and equal stator and rotor I^2R loss. The per-unit currents are not to be taken as typical, as they are chosen rather to clarify the diagrams (i)–(iv) in Fig. 8.17. The lettering on all diagrams corresponds to that of Fig. 8.16.



8.17 Effect of varying machine parameters.

Leakage Reactance

It is not necessary to consider the leakage reactances of stator and rotor separately. In diagram (i) the total reactance is about 1.3 times that of the 'standard' machine, and it becomes difficult to accommodate the required rated output because the diameter of the circle, AB, is reduced to about 0.75 of its original size. The pull-out torque is of the same order as the full-load torque so that the machine as a motor has little reserve for overload. Comparable restrictions are placed on the performance of the machine as a generator.

The leakage reactance plays a dominant role in the capability of the machine, but not very much can be done in design to reduce it.

Resistance

Increase of the *rotor* resistance is the most important practical variation, as it has a direct effect on the torque. Diagram (ii), for a rotor phase resistance twice that of the 'standard', shows a substantial increase in the torque at starting as a motor, but the full-load efficiency ($P_1 R/P_1 T$ for the motor mode, and $P_{1g} T/P_{1g} R$ for the generator mode) is naturally reduced. The brake mode is extended and the braking torque is increased. Use is made of rotor resistance in the torque control of slip-ring machines by connecting a rheostat across the slip-rings (which for normal running are short-circuited). If, however, the higher rotor resistance is built into the machine (as when deep-bar rotor conductors are used) there may be cooling difficulties.

Doubling the *stator* resistance, as in (iii), lowers the efficiency without improving the low-speed or brake torque. In the generator mode there is a substantial increase in the driving torque demand. The additional stator $I^2 R$ loss may cause difficulties in cooling unless the added resistance is external, an unusual arrangement producing low-speed current limiting as the only advantage.

Magnetizing Reactance

Although the teeth and cores of the magnetic structure are semi-saturated to economize in material, the airgap between the stator and rotor presents the major portion of the magnetic-circuit reluctance. As the machine has to be magnetized from the a.c. supply system, the gap length is kept short (of the order of 1 mm) compatible with the limitations imposed by mechanical clearance, shaft deflection and out-of-balance magnetic pull. The shaft must be very stiff to preserve centrality of the rotor with the stator.

A long airgap yields a diagram like that in (iv). The full-load stator current is increased and its power factor lowered. The approximate equivalent circuit obscures the fact that excessive magnetizing current restricts the diameter AB of the current circle.

Voltage

Voltage reduction seriously reduces performance. A common case is that of starting a motor by star-delta switch. Diagram (v) is drawn for this case, the working voltage per phase in star connection being $1/\sqrt{3}$ of normal, reducing currents by this factor and powers by the factor squared, i.e. to one-third of normal.

8.9 Motor performance in the steady state

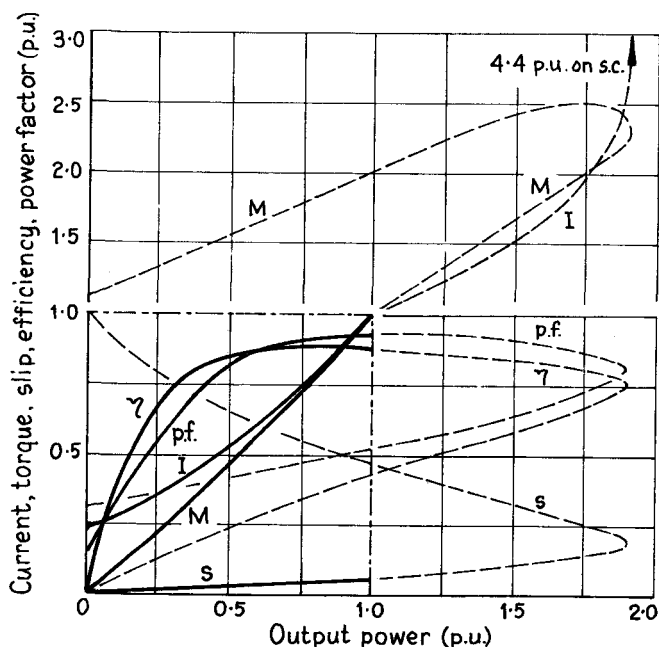
On no load with the rotor windings short-circuited, the machine runs at a speed very close to the synchronous speed at which the stator travelling-wave field revolves. The rotor current is very small, because only the loss torque (for windage and friction) has to be developed. The stator takes a correspond-

ing active current which, with the core-loss current, is the active no-load current component. The magnetizing current is much larger than the active current, so that the no-load power factor is low: e.g. 0.1 or (for small motors) 0.25.

If a load torque is applied to the shaft, the rotor tends to slow down. The rotor frequency is very low, the inductive reactance has little effect, and the rotor current is nearly in phase with the rotor e.m.f.: i.e. it is substantially a torque-producing current. The slip increases so as to produce enough rotor current and torque; and, very nearly, the load torque and slip are proportional. Thus both the slip/torque and speed/torque relations are rectilinear. The increase in rotor current necessitates a rise in the corresponding (active) component of stator current, and the power factor rises.

On full load the slip may be about 0.05 p.u. in a 2 kW motor down to around 0.01 p.u. in a 1 MW machine, and the full-load stator current is about three times the no-load value. The power factor rises to a value between 0.8 and 0.9. The efficiency, which is of course zero at no load, rises to a maximum at or (usually) below full load. This maximum occurs roughly where the I^2R losses in stator and rotor sum to an amount equal to the no-load 'constant' losses. At higher loads, the I^2R losses increase more rapidly than the output, so that the efficiency slowly falls.

For moderate overloads the stator current and slip increase roughly in proportion to the load, while the power factor, as the circle diagram indicates, remains sensibly unchanged. For larger overloads both current and slip increase



8.18 Motor performance curves.

more rapidly, until at maximum load (about 100% overload) the current may be about three times its full-load figure, at a power factor in the neighbourhood of 0.7. At this stage a small increase in overload will cause the motor to stop suddenly. The current will rise to e.g. five times rated full-load value, the torque will be small, and the power factor lower, 0.2–0.4. The machine will not restart until the load torque has been reduced below that available at starting. The rating of a motor must be reckoned in relation to its pull-out or stalling (maximum) torque, a usual relation of maximum to full-load torque being 2:1.

A typical set of characteristic curves over the complete range is given in Fig. 8.18.

The efficiency and power factor at rated load depend on the size and application, and particularly on the nominal speed $n_1 = f/p$ (r/s) or $60 f/p$ (r/min) for a machine with p pole-pairs and a supply of frequency f . Because of less effective use of material and greater leakage inductance a low-speed machine is heavier, less efficient and more reactive than a fast machine. The figures below compare the data for a 5 kW machine at various speeds, and a range of cage and slip-ring machines.

5 kW 50 Hz 3-ph Cage Motor

$2p =$	2	4	6	8	12	16
Efficiency p.u.	0.84	0.85	0.84	0.83	0.81	0.79
Power factor p.u.	0.90	0.86	0.81	0.74	0.72	0.70
Relative mass	0.85	1.0	1.3	1.6	2.4	4.4
Relative cost	0.98	1.0	1.25	1.45	2.0	3.1

Cage and Slip-ring 50 Hz Motors

Efficiency/power factor, p.u.						
Rating	Cage motors			Slip-ring motors		
kW	$2p = 4$	8	12	4	8	12
1	0.72/0.75	—	—	—	—	—
5	0.84/0.86	0.83/0.74	0.81/0.70	0.82/0.83	0.81/0.67	0.79/0.66
10	0.86/0.87	0.84/0.78	0.82/0.71	0.84/0.84	0.83/0.74	0.80/0.73
20	0.88/0.89	0.85/0.83	0.83/0.75	0.87/0.88	0.85/0.80	0.82/0.77
50	0.90/0.90	0.89/0.85	0.87/0.82	0.89/0.90	0.88/0.83	0.86/0.81
100	0.91/0.92	0.90/0.89	0.89/0.84	0.91/0.92	0.89/0.87	0.88/0.83
1 000	0.93/0.94	0.93/0.93	0.92/0.91	0.93/0.94	0.92/0.91	0.91/0.90

Types and Applications

Cage Motors: The four main classes are:

1. Motors with normal starting torque and current, having cages with low resistance and reactance, low full-load slip, good efficiency and power factor, and high pull-out torque.

2. Motors with normal starting torque and low starting current, having large rotor reactance but the same slip and efficiency as (1). The power factor and pull-out torque are less.

3. Motors with high starting torque and low starting current, using deep-bar or double-cage rotors. A 2 p.u. starting torque with moderate current is obtained on full voltage. The efficiency, power factor and pull-out torque are lower than for (1). The motors are useful for starting against load (e.g. reciprocating compressors).

4. Motors with high full-load slip, using a comparatively high-resistance cage, with large starting torque, low starting current and low efficiency. Used for drives with heavy starting but light running duty. If employed for loads with rapidly fluctuating torques (e.g. punch presses) they may be attached to flywheels for load-peak equalization.

Slip-ring Motors. Suitable for heavy, frequent starting and accelerating duty-cycles. A high starting torque is obtained with a low starting current at high power factor. The rotor losses are mainly in the external resistance, easing the problem of rotor cooling.

Multispeed Motors. These are usually cage motors with single or double pole-changing windings. They may be classified as:

(a) variable-torque, power output proportional to square of speed (fans, centrifugal pumps);

(b) constant-torque, power output proportional to speed (conveyors, stokers, reciprocating compressors, printing presses);

(c) inverse-torque, power output rating constant (machine tools, lathes, boring mills, drills, planers).

Less frequently multispeed motors are of the *slip-ring* type, as for hoists, conveyors and elevators.

Gear-motors are high-speed motors with integral unit gear construction giving output shaft speeds of 10–500 r/min. The motor is generally a 4-pole 1 500 r/min machine. If the service is heavy, the gear must be very robustly designed. Gear-motors may be applied to conveyors, agitators, low-speed fans, blowers and screens.

Special machines include: submersible motors, usually 2-pole machines with long small-diameter vertical rotors, the liquid passing through the rotor itself; motors with solid steel rotors, employed for intermittent starting of other machinery; 2-ph motors for servomechanisms in which one phase is supplied at constant voltage with the other phase provided with a quadrature voltage of varying magnitude and polarity; and selsyns for servo error-detection or control.

Linear motors utilize the induction principle in linear in place of rotary form, for short-throw actuators, crane traversing, and high-speed railway traction.

8.10 Motor starting

To accelerate a motor from rest to operating speed requires an input of energy to supply (i) the losses and (ii) the kinetic energy of the motor and its load.

The I^2R loss alone is equal to the stored kinetic energy, an important consideration in the frequent starting of high-inertia loads. In the design and choice of a drive motor it is necessary to take account of the frequency of the starting duty, the duration of each start, and the starting torque demanded. For large machines there may be limitations imposed by the supply authority on the magnitude of starting-current peaks.

The torque per phase is given by $M = I_2^2 r_2 / s \approx k I_1^2 / s$. Comparing the torques M_n for rated current I_n and slip s_n and M_s for a starting current I_s at slip unity, the ratio is

$$M_s / M_n = (I_s / I_n)^2 s_n \quad (8.22)$$

If a motor is started 'direct-on-line', i.e. switched on to a normal voltage supply, the starting current is $I_s = I_{sc}$. For a machine with $I_{sc} = 5 I_n$ and $s_n = 0.04$ p.u., the starting torque is $M_s = 5^2 \times 0.04 = 1.0 M_n$, the machine developing full-load torque at starting with five times full-load current.

It has already been seen that the starting torque of the induction motor can be increased by raising the rotor resistance. This is readily feasible with slip-ring motors, but not with cage motors. The inclusion of a high-resistance rotor certainly increases the starting torque of the latter, but decreases the efficiency. A starter may therefore either increase the rotor resistance or decrease the starting current. In the former case the starting torque may be increased and the current simultaneously limited, in the latter the torque must inevitably be reduced.

The methods available for the starting of induction motors are:

Slip-ring: Rotor rheostat.

Cage: Direct-on-line; star-delta switch; auto-transformer; clutch; external stator impedance.

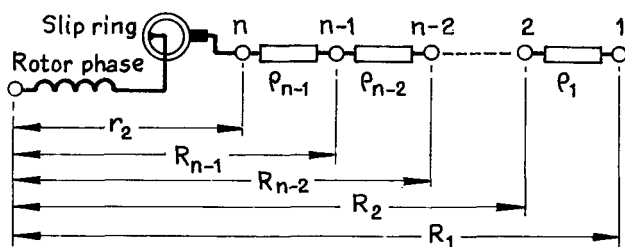
Slip-Ring Motors

The effect of increasing the rotor resistance is shown in Fig. 8.17 (ii). Maximum torque at starting can be obtained if the rotor resistance per phase (including an external resistance R) is made approximately equal to the total leakage reactance, as can be inferred from eq.(8.21) neglecting r_1 in comparison with X_1 . A consequent and useful effect is the reduction of the stator starting current and a raising of its power factor.

In rotor-rheostat starting, the phase ends of a star-connected rotor winding (or the phase junctions of a delta-connected winding) are taken to slip-rings, three in number, which are in turn connected through brushes to a star-connected rheostat. The latter may take the form of a stud- or contactor-type arrangement of metallic or carbon resistors, or for large currents it may comprise an electrolytic liquid. The main stator switch together with undervoltage and overcurrent protective devices may be incorporated.

The starting current is chosen on a basis of the starting torque required and the losses that can be allowed during the starting period. The losses can be reduced by using a larger starting current provided that the motor is operating on a stable torque/slip characteristic.

Where the number of resistance steps is small, the starting rheostats may be mounted on the rotor shaft, and arranged to be cut out by a centrifugal switch. The use of slip-rings and external switchgear is avoided, but the device is complicated if frequent starting is required (and consequently much heat developed), and if more than two resistance steps are needed. A motor fitted with a centrifugal switch will start itself automatically if stalled.



8.19 Rotor resistance starter.

Fig. 8.19 shows a rotor phase of resistance r_2 and one branch of the starter rheostat with sections of resistance $\rho_1, \rho_2 \dots$ such that the total resistance on steps 1, 2 ... is $R_1, R_2 \dots$. As each step is completed, the current becomes $I_{\max}$, the speed rises reducing the current, and when a lower limit $I_{\min}$ is reached a rheostat section ρ is cut out. Consequently, taking the upper and lower current limits to apply throughout the start, and the phase resistance at the start of a step to be R_p when the slip is s_p , falling to s_q just before the next step is made, we have

$$I_{\max} = \frac{V_1'}{\sqrt{[(R_p/s_p)^2 + X^2]}} \quad \text{and} \quad I_{\min} = \frac{V_1'}{\sqrt{[(R_p/s_q)^2 + X^2]}}$$

where X is the total motor leakage reactance and V_1' is the stator applied voltage, both referred to the rotor. These expressions give

$$R_1/s_1 = R_2/s_2 = \dots = r_2/s_n \quad \text{and} \quad R_1/s_2 = R_2/s_3 = \dots = R_{n-1}/s_n$$

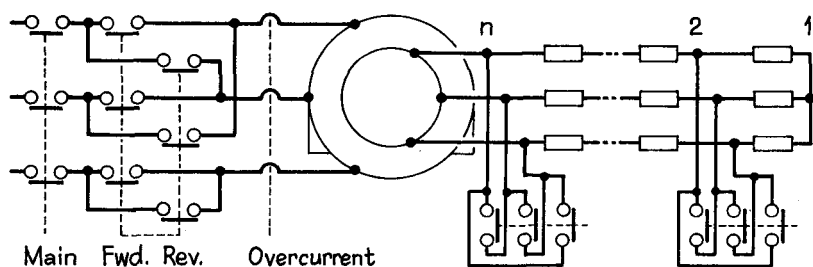
taking the slip for normal running on the given load to be s_n . Combining the two sets of equalities and writing $s_1 = 1$ yields

$$s_2/s_1 = s_3/s_2 = \dots = R_2/R_1 = R_3/R_2 = \dots = r_2/R_{n-1} = \gamma$$

whence $R_1 = r_2/s_n$, $R_2 = \gamma R_1$, $R_3 = \gamma R_2$, etc. Further, $\rho_1 = R_1 - R_2$, $\rho_2 = R_2 - R_3 = \gamma \rho_1$ etc.

Fig. 8.20 shows a typical arrangement. Stator contactors are provided for main switching (with overcurrent protection) and for forward and reverse rotation by interchange of a pair of stator lines. Rotor rheostat sections are short-circuited by standard 3-pole contactors in a controlled sequence.

Limitation of the starting current to constant peaks is rarely applied in practice as it requires too many contactors. The number can be reduced by cutting sections from the rotor phases in sequence instead of simultaneously, provided that the current unbalances are not prolonged by a slow start.



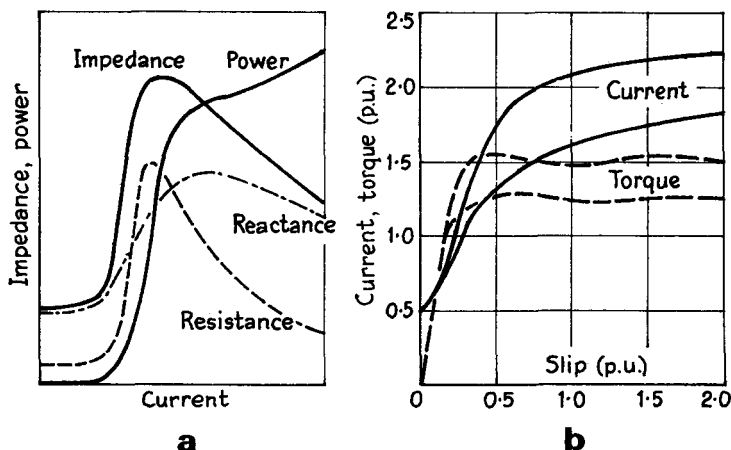
8.20 Connections of slip-ring machine.

The rotors of machines of like rating, but driving separate shafts, can be connected to a common rheostat to maintain equal speeds. If one rotor lags, its e.m.f. is phase-advanced and it takes a greater current. The action may fail for low slips because the circulating current, though large, has a low p.f. and may not develop an adequate alignment torque.

Rotor Voltage. The choice is arbitrary. For small machines there is little difficulty in keeping it below 500 V, an upper limit if starters and switchgear are hand-operated. With large machines it is necessary to limit the current to avoid difficulties with slip-rings, brushes and short-circuiting gear, and rotor voltages of 1–2 kV may be chosen.

Saturistor Control. If a 'hard' magnetic material such as Alnico is used as part of the core of an inductor excited by a coil carrying an alternating current, the effective resistance and inductance of the coil are low provided that the excursions of a.c. magnetic field intensity do not exceed the coercive force of the material. At higher excitations the hysteresis causes a marked rise in loss, and therefore in effective coil resistance and impedance. With a fully developed hysteresis loss, which for Alnico is typically 16 W/cm³, further rise in the power dissipation is due mainly to the coil resistance, so that the overall effective resistance declines with increasing current. The impedance and power of such an inductor vary with the exciting current in a way indicated in Fig. 8.21(a). At maximum resistance the power factor of the exciting coil is about 0.75, and is independent of frequency because the hysteresis loss is itself proportional to the frequency. The construction of the inductor usually takes the form of Alnico blocks between normal laminated-steel yokes, as the hard material is most readily produced in simple shapes. The operating characteristics can (if needed in a closed-loop control system) be adjusted by superposing a d.c. excitation in an additional control winding.

Such an inductor, called a *saturistor* (or saturable resistor), can replace each of the star-connected branches of a starting rheostat across the slip-rings of an induction motor to provide automatically an almost constant starting torque of, say, 1.5 times full-load torque over the starting range of speed, with a current of less than twice normal, as shown in Fig. 8.21(b). The characteristics are almost ideal for several classes of mechanical load. A suitable quantity of Alnico, or other similar hard magnetic material, is included in each inductor in accordance with the effective resistance/current relation required. It is usually desirable to short-circuit the saturistor bank when the machine attains



8.21 Saturistor control.

full working speed in order to limit the heating of the inductor windings and to preserve the full pull-out torque level. The control must be arranged to re-insert the saturistors during speed change and in the event of undervoltage conditions, in order to limit the subsequent current inrush. If starting is likely to be delayed, a heat sink must be provided to avoid excessive temperature-rise of the Alnico; and if the machine is to run as a brake it may be necessary to shunt the saturistors by a rheostat. If the saturistors can be mounted on the rotor itself, elimination of slip-rings and brushes is made possible.

Cage Motors

As a cage rotor winding is electrically isolated, the starting current can (apart from special devices) be reduced only by lowering the voltage applied to the stator. If the voltage be reduced from normal to the fraction x , the no-load and short-circuit currents will be changed roughly by the same proportion; but the torque and output for a given slip become the fraction x^2 of normal, as discussed in connection with diagram (v) of Fig. 8.17.

Direct Switching. A cage motor may be switched 'direct-on-line', i.e. on to normal voltage, momentarily taking several times full-load current at a low power factor. The motor may be damaged by overheating if the start is delayed by excessive load torque or supply-voltage depression. Direct switching may be the subject of Supply Authority regulations, which may be expressed as the permissible ratio of starting kVA to rated kW; the following are typical:

Range of rating, kW:	1-6	6-40	40-250	250-500	500-1 500	1 500-4 000
Maximum ratio kVA/kW:	10	9	8	7.7	7.4	7.2

From eq.(8.22) the ratio of starting to full-load torque (at which the slip is s_n) is $M_s/M_n = (I_{sc}/I_n)^2 s_n$.

EXAMPLE 8.4: A 3-phase, 3.3 kV, 50 Hz, 10-pole, star-connected induction motor has a no-load magnetizing current of 40 A and a core loss of 30 kW. The stator and referred rotor standstill leakage impedances are respectively $(0.18 + j1.6)$ and $(0.40 + j1.6) \Omega/\text{ph}$. With the motor supplied from 3.3 kV busbars having a short-circuit level of 27 MVA, estimate (i) the gross torque, stator current and power factor when the machine runs with a slip of 0.03 p.u., and (ii) the starting torque with direct-on-line switching.

The rated phase voltage is 1.90 kV. The busbar short-circuit current level is $27 \times 10^3/3 \times 1.90 = 4.75$ kA, and the effective system impedance (assumed to be purely reactive) is therefore $1.90/4.75 = j0.40 \Omega$.

(i) *Running.* At the slip $s = 0.03$ p.u., the total impedance is $Z_1 = [(0.18 + 0.40/0.03) + j(1.6 + 1.6)] = (13.5 + j3.2) \Omega/\text{ph}$. This is nearly 35 times as great as the system impedance so that the motor terminal voltage can be taken as 3.3 kV. Using the approximate equivalent circuit, Fig. 8.10(c):

$$I_2 = 1900/(13.5 + j3.2) = (134 - j32) = 138 \text{ A}$$

$$P_2 = 3 \times 138^2 \times 0.40/0.03 = 762 \text{ kW}$$

$$P_m = 762(1 - 0.03) = 740 \text{ kW}$$

$$M_e = 762/62.8 = 740/60.8 = 12.1 \text{ kN-m}$$

$$I_0 = (10/1.90) - j40 = (5 - j40) \text{ A}$$

$$I_1 = (5 - j40) + (134 - j32) = (139 - j70) = 156 \angle -27^\circ \text{ A}$$

$$\text{p.f.} = \cos(-27^\circ) = 0.89$$

(ii) *Starting.* The rotor current is large enough for the magnetizing branch current to be neglected. The series impedance of the motor is $Z_1 = [(0.18 + 0.40) + j(1.6 + 1.6)] = (0.58 + j3.2) \Omega/\text{ph}$; this is increased to $(0.58 + j3.6) = 3.65 \Omega/\text{ph}$ because of the system reactance. Then

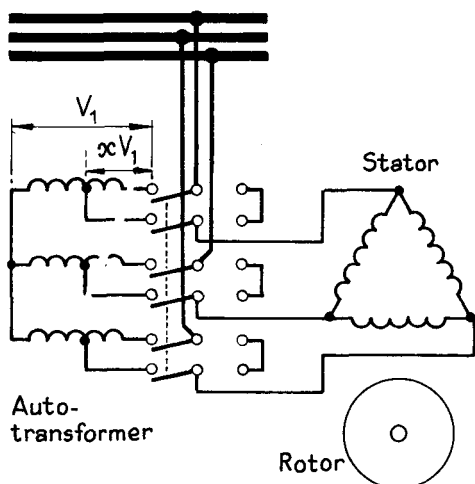
$$I_2 = 1900/3.65 = 520 \text{ A}$$

$$M_s = 3 \times 520^2 \times 0.40/62.8 = 5.2 \text{ kN-m}$$

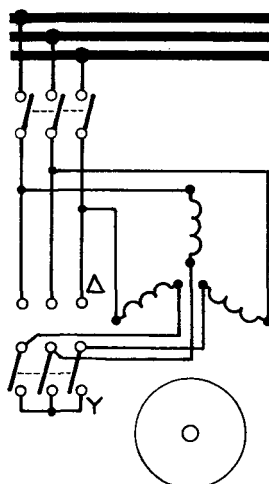
With zero system reactance the current would be 585 A and the torque 6.5 kN-m, an increase of 25%.

Stator Impedance Starting. The inclusion of a resistor or an inductive reactor in each of the stator input lines reduces the stator terminal voltage to a fraction x of normal, the fraction approaching unity as the speed of the motor approaches normal. The initial starting current is $I_s = xI_{sc}$ and the starting torque is $M_s = x^2 M_{sc}$. The ratio of starting to full-load torque is $M_s/M_n = x^2 (I_{sc}/I_n)^2 s_n$, or the fraction x^2 of that obtainable with direct switching. The method may be used for the smooth starting of small machines such as those driving centrifugal oil purifiers; but the star-delta switch may be cheaper and may give a better starting torque.

Auto-transformer Starting. Suppose that the auto-transformer in Fig. 8.22 is used to reduce the stator applied voltage to the fraction x of normal. Then the ratio of starting to full-load torque is $M_s/M_n = x^2 (I_{sc}/I_n)^2 s_n$ for a normal-voltage short-circuit current I_{sc} , the same as for the stator-impedance method.



8.22 Autotransformer starter.



8.23 Star/delta starter.

But here the transformer primary current is x times the secondary current and therefore $x^2 I_{sc}$. For the same line current, the starting torque is $1/x$ times as great as that for the stator-impedance method. Further tapings can provide voltage steps during starting, the auto-transformer being switched out after the start is completed. The transformer needs only a short-time current rating but its leakage impedance should be small to avoid undue limitation of the starting current.

Star-delta Switching. For this method, Fig. 8.23, a motor must be built to run normally with a delta-connected stator winding. For starting the windings are connected in star, reducing the phase voltages to $1/\sqrt{3} = 0.58$ of normal. The starting current per phase is $I_s = 0.58 I_{sc}$, the line current is $0.33 I_{sc}$, the starting torque is 0.33 of its short-circuit value, and the ratio

$$M_s/M_n = \frac{1}{3} (I_{sc}/I_n)^2 s_n$$

The method is cheap and effective so long as the starting torque is not required to exceed about 0.5 p.u. of full-load torque and the line voltage is less than about 3 kV.

Where motors may run for long periods on small loads, a star-delta switch enables the machine to be connected in star to reduce magnetizing current and increase efficiency.

Clutch Starting. Even with direct-on-line switching, the starting torque of a cage motor is low. A clutch is a device that allows the machine to start on light load and to take up the drive when its developed torque is near the maximum. A centrifugal clutch is a pulley or coupling with an inner part attached to the motor shaft and an outer part connected to the load. Spring-loaded blocks on the inner member come into contact with the outer member under centrifugal force to lock the driving and driven parts together. The blocks may be coated with friction-brake material, or may be metal running in oil. A

comparable method utilizes a hydraulic torque-converter or 'fluid coupling'. Both methods prevent stalling and permit the motor to be rated on running rather than starting conditions, but involve additional cost and maintenance.

EXAMPLE 8.5: A small cage motor has a short-circuit current of 6 times full-load current, and a full-load slip of 0.05 p.u. Find in terms of normal values the motor and line starting currents and torque for (a) direct switching; (b) stator resistance and (c) auto-transformer starting with the motor current limited to 2.0 p.u.; and (d) star-delta starting. (e) What auto-transformer ratio gives full-load torque at starting?

The results are tabulated, using the various expressions already found:

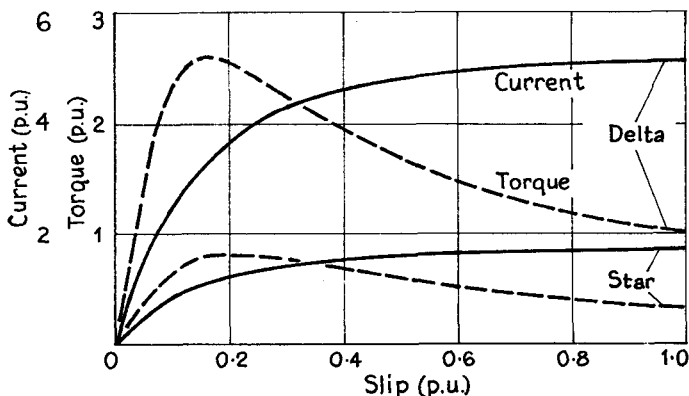
Per-unit of normal value	(a)	(b)	(c)	(d)
Motor phase voltage	1.0	0.33	0.33	0.58
current	6.0	2.0	2.0	3.5
Line current	6.0	2.0	0.67	2.0
Torque	1.8	0.2	0.2	0.6

The torque is the same in (b) and (c) for the imposed condition of twice full-load motor current, but the *line* current for the auto-transformer start is only one-third of that for the rheostat.

(e) For full-load torque at starting, $M_s/M_n = (I_s/I_n)^2 s_n = 1$, from which $I_s = I_n/\sqrt{s_n} = 4.46 I_n = 0.75 I_{sc}$. Thus a 75% tap is required. The line current is $(0.75)^2 = 0.56$ times full-load current.

Starting-current peaks

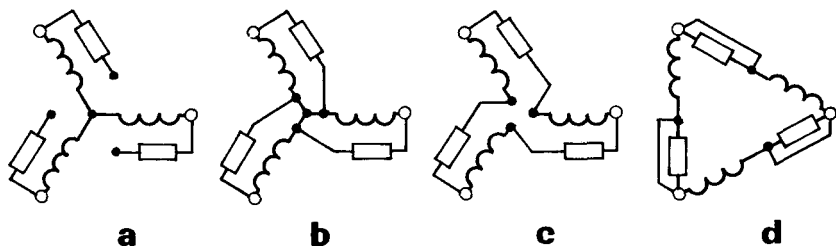
In the foregoing the short-duration switching transients are ignored: they are discussed in Sect. 8.21. The 'steady-state' starting current does not fall appreciably until the motor has attained over two-thirds of normal speed, and it is desirable to delay the sequence of transitions to the normal running connection. Thus in Fig. 8.24, for star-delta starting, the transition should not be



8.24 Torque and current in star/delta starting.

made until the speed is within 20% of synchronous; otherwise there will be a peak line current considerably greater than full load – greater in fact than the standstill current in star.

Any starting method that involves a momentary disconnection of the main supply may cause severe current peaks. Stator excitation of the magnetic field is withdrawn, but the short-circuited rotor develops induced currents that hold the original field at the position on the rotor surface that it had at the moment of interruption. This field, 'frozen' to the rotor and carried round by it, induces e.m.f.s of reducing magnitude in the open-circuited stator winding, and (because of the slip) of time phase anything between coincidence and opposition to the supply voltages when they are re-applied in delta. In the former condition there will be large current surges in all phases. In fast switching, with interruption between 0.1 and 0.3 s, the star to delta transition should preferably be such that the phase winding connected across terminals RN of a supply of sequence RYB is switched in delta to RB; similarly YN to YR and BN to BY. Then the applied line phasor voltages are retarded by 30° , compensating to some extent the effects of rotor retardation. The modified star-delta transition of Fig. 8.25 avoids current surges by retaining



8.25 Modified star/delta switching.

the line connections without interruption: (a) the motor is started in star; (b) resistors are paralleled with the phase windings leaving the motor unaffected; (c) the star point is opened putting the windings in delta with series resistance; and (d) the resistors are short-circuited.

Starters and Contactors

The functions of a 'starter' include stopping and possibly heavy-current braking duty, and protection of the motor from overcurrent and undervoltage conditions. Overcurrent relays are thermally operated by fixed heaters, associated with the individual phase currents, that deflect bimetal strips. Large deflections, or those differing between the respective phases, operate to trip the contactor for protection against sustained overcurrent or single-phasing, and may initiate an alarm. Protection from excessive winding temperatures is secured by thermistors embedded in the windings, one for each phase section. The thermistors have a nonlinear resistance/temperature characteristic which is used to unbalance a bridge network if the winding temperature rises excessively.

Contactors. Three standardized categories of a.c. contactor are concerned with motors: their designations (BS 775) are

AC4: for cage motors where starting, reversing, plugging and 'inching' duty is required and where very high peak currents may occur.

AC3: for cage motors involving starting and switching off with the motor still running, and so not normally required to break the starting or the stalling current.

AC2: for slip-ring motors in which current levels are limited.

In addition to its category designation, a contactor starter is given a class of intermittent duty which defines the number of operations per hour that it can deal with while still complying with the thermal, electrical and mechanical specifications.

8.11 Harmonic effects

With certain numbers of poles and of stator and rotor slots in cage motors, peculiar and deleterious behaviour may be observed when the machine is started. For example, with the number S_1 of stator slots equal to the number S_2 of rotor slots, the machine may refuse to start at all, a phenomenon termed *cogging*. With other ratios of S_2/S_1 the motor may exhibit a tendency to run stably at a sub-normal speed (e.g. one-seventh): this is called *crawling*. In some cases excessive *vibration* may be set up at sub-synchronous speeds, due to pulsations of mechanical force on the teeth, generating *noise* (e.g. a medium-pitched howl). The efficiency of a machine is inferior to that predicted by simple theory because of *load* (stray) *loss*. All these phenomena originate in the very complicated nature of the instantaneous gap flux distribution, which is strongly affected by the slot openings as these are usually wider than the length of the radial airgap. Slip-ring machines are similarly affected, but the essential difference in the present context is that a cage winding, being in effect a multiphase winding, can circulate a current under any harmonic e.m.f. produced by the gap flux, except that for which the wavelength is equal to the pitch of the bars.

In Sects. 8.4–8.8 the performance of an induction machine is discussed in terms of simple 'main' and 'leakage' fluxes, assuming the machine to be mechanically, magnetically and electrically balanced, with both time-varying and space-distributed quantities strictly sinusoidal. This is basically valid, but the second-order effects of present concern, which are of importance in design and performance, depend on the precise gap-flux distribution, i.e. on its harmonic content.

Gap-flux Harmonics

The m.m.f. of a uniformly distributed stator winding (Sect. 3.14) contains a sine-distributed fundamental and a family of space harmonic components of order $n = 6m \pm 1$, where m is a positive integer. The harmonics form waves of m.m.f. travelling at speeds $1/n$ of that of the fundamental, in the same direction if $n = 6m + 1$ (i.e. 7th, 13th), in reverse if $n = 6m - 1$ (e.g. 5th, 11th).

For a similar 3-ph rotor winding (also carrying sinusoidal phase currents) there is a comparable fundamental and harmonic family of m.m.f.s. Now the gap flux results from the stator and rotor m.m.f.s in combination, and as rotor movement continually alters the relative orientations of the stator and rotor phase windings, the resultant harmonic m.m.f.s, and the gap flux harmonics produced by them, change cyclically. A further effect is a cyclic change in the stator-rotor mutual inductance.

Such harmonics alone would have minor effect on a slip-ring machine, and even less with a cage machine. But they are greatly intensified by slotting, which not only introduces 'steps' in the m.m.f. wave, Fig. 3.29(b), and further harmonics, but also modulates the gap flux as, at a given radial plane, the rotor and stator teeth move in and out of register. High-frequency components, responsible for load loss and noise, are generated.

Binns and Schmid [111] list the following harmonic components developed in a cage machine:

Stator: (i) stator m.m.f.; (ii) rotor m.m.f.; (iii) slot-ripple modulated by (i); (iv) saturation, giving slot-ripple sidebands.

Rotor: (v) slip-frequency currents representing the main-flux fundamental; (vi) components associated with (i); slot-ripple due to stator teeth; (vii) saturation, as in (iv); (viii) currents induced by harmonic fluxes.

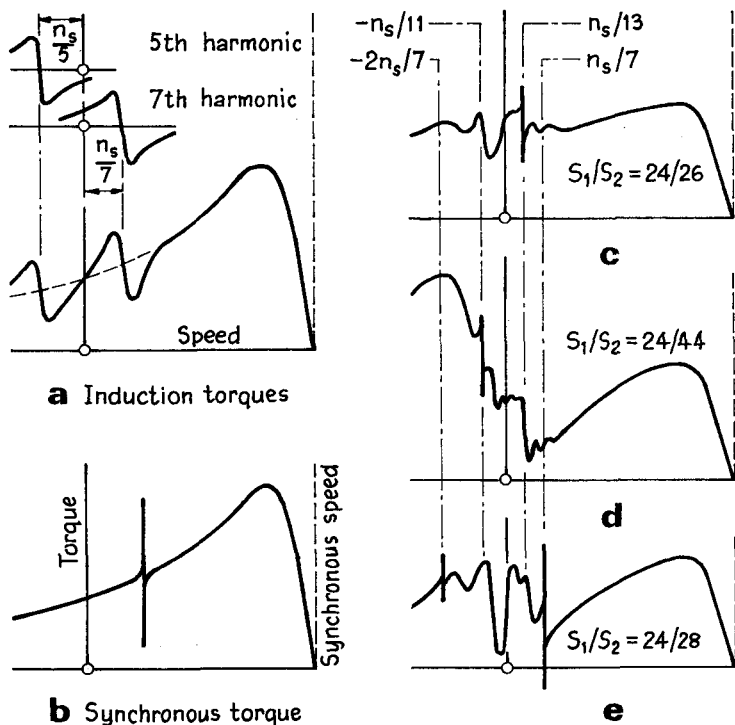
The chief loss-producing harmonics are those in (iii), but serious aggravation can be produced by such asymmetries as unbalanced windings and gap nonuniformity. In general, a harmonic effect in the rotor is reflected in the stator and re-reflected therefrom into the rotor in a cumulative armature reaction effect.

Cogging and Crawling

If the stator and rotor slot numbers, S_1 and S_2 , are equal or have an integral ratio, the slot-harmonic fluxes give rise to strong alignment forces when the machine is at rest, and these may exceed the tangential accelerating torque to cause the machine to *cog*.

Fifth- and seventh-order harmonic fluxes may be deemed as produced by sets of additional poles superimposed on the fundamental poles. They generate rotor e.m.f.s, currents and torques of the same general torque/slip shape as that of the fundamental, but related to 'synchronous' speeds respectively $1/5$ backwards and $1/7$ forwards, as indicated in Fig. 8.26(a). The resultant torque/speed curve, combined with the fundamental, shows a marked 'saddle'. If large, the 7th harmonic torque may prevent the speed from standstill rising above about $1/7$ of normal. The motor therefore *crawls*. With certain slot ratios it is possible for a stator and a rotor m.m.f. harmonic to rotate together at some subsynchronous speed to develop a synchronizing torque at that speed, as in Fig. 8.26(b), again with a tendency to crawl.

Harmonic Induction Torques. In a slotted m -phase stator winding, the e.m.f. distribution factors of harmonics of order $n = 6Ag' \pm 1$ (with $A = 0, 1, 2 \dots$ and g' slots per pole and phase) are the same as those for the fundamental. A 4-pole motor with 24 stator slots ($S_1 = 24, g_1' = 2$) will thus encourage 11th and 13th harmonics, and torque 'saddles' may be observable at $+1/13$ and



8.26 Harmonic torque effects.

(a) Harmonic asynchronous (induction) torques.

(b) Harmonic synchronous torques.

(c, d, e) Torque/speed curves of 50 Hz 4-pole motor with various slottings.

$-1/11$ of the synchronous speed. If the half-wave-length of a stator-produced harmonic is equal to the rotor bar pitch, the harmonic torque is intensified: e.g. with $S_1 = 24$, $S_2 = 44$ the 11th harmonic has 44 half-waves in the gap each similarly located with respect to a rotor bar. The result is a strong 11th harmonic torque.

Harmonic Synchronous Torques. The rotor slotting produces space harmonics of order $(S_2/p) \pm 1$, the plus sign referring to travel in the direction of rotor rotation. Consider a 4-pole motor with $S_1 = 24$, $S_2 = 18$: the stator m.m.f. has a reversed 11th and a forward 13th, the rotor a reversed 13th and a forward 15th harmonic. For a fundamental synchronous speed n_s the 'synchronous' speed of the 13th harmonic is $+n_s/13$ and $-(n_s - n)/13$ relative to stator and rotor respectively when the speed of rotation is n . The two 13th harmonics are in step when $+n_s/13 = n - (n_s - n)/13$, i.e. when $n = n_s/7$. Thus there is a torque discontinuity at $n_s/7$ produced not by the 7th but by the 13th harmonics. The torque/speed curves in (c), (d) and (e) of Fig. 8.26 exhibit both induction and synchronous harmonic torques. The two are distinguished in that

the operating speed changes slightly for the former but is constant for the latter, for a small variation of the shaft load.

Slotting. Alger [32] has analyzed the effects of slotting on the cogging and crawling phenomena in terms of gap-permeance waves, the permeance at any point in the gap being multiplied by the resultant m.m.f. there and the harmonic content extracted. The most important term is a function of rotor position angle θ and angular speed ω_r given by $\cos [(S_2 - S_1)\theta - S_2\omega_r t]$. Binns [33] criticizes the permeance-wave method as giving a poor approximation where m.m.f. and permeance harmonics of comparable order are concerned, because the flux paths across the gap, assumed by Alger to be radial, are not actually so. Binns uses an energy method based on zigzag leakage as well as gap permeance variation.

Many rules for slotting have been suggested, the recommendations depending to some extent on the analytical assumptions made. For example, cogging torques can be eliminated by making $(S_1 - S_2)/2p$ integral, S_1/S_2 fractional, and S_2 not widely different from S_1 ; asynchronous harmonic torques are limited if S_2 does not exceed $1.25 S_1$; to limit synchronous harmonic torques, S_2 should not be $6px$ or $6px \pm 2p$, where x is a positive integer; slot harmonics are reduced if S_2 is not made equal to $S_1 \pm 2p$ or $S_1 \pm p$ or $\frac{1}{2}S_1 \pm p$. The cases illustrated in Fig. 8.26 abrogate one or other of these rules.

Noise

Noise results from mechanical vibration in a machine and its supporting structure, and directly from the coolant, especially if it stimulates natural structural frequencies.

Mechanical Forces. Dynamic out-of-balance of the rotor generates vibration at a frequency determined by the speed of rotation, and in 50 Hz machines with p pole-pairs will have a frequency of $50/p$, to which the ear is relatively insensitive. Bearings, particularly of the ball or roller types, produce noise by the rolling or sliding motion between the fixed and moving surfaces.

Magnetic Forces. These are important. The magnetic flux crossing the gap varies rapidly with time and relative position owing to the fluctuation of slot currents and tooth configurations, with the result that forces of magnetic attraction across the gap are subject to change and so generate vibration, especially in machines with few poles in which the variations are more concentrated and the shafts less stiff. Eccentricity of the airgap surfaces, whether of the stator or the rotor, can cause strong vibratory forces of magnetic origin. Of the wide frequency band of forces, those of low frequency are the more important in their ability to transfer vibration to the structure, while those in the range 100–10 000 Hz produce directly radiated noise. The teeth respond to varying forces by a 'cantilever' vibration. The core responds as a rather complex thick cylinder; a long core may develop vibrations as a cylinder with travelling waves of mechanical displacement around it.

Aerodynamic Noise. This results from rapid changes in pressure of the coolant gas which radiate directly into the gas stream or excite structural vibrations. The causes of pressure fluctuation are: (i) Variation in the resistance to gas-flow brought about by the movement of rotor slots and ducts past the stator

gap surface. (ii) 'Air beating' by fan blades. (iii) Vortices in the wake of objects in the gas stream, which break up against other objects to develop aeolian tones. (iv) The flow of coolant through ducts having abrupt changes of cross-section, generating turbulence and random pressure fluctuations.

The methods of reducing noise, recommended by Ellison and Moore [34], are by reducing the known causes and by modifying coolant transmission paths. Noise of magnetic origin can be reduced only by lowering the gap flux density (which is uneconomic), but some mitigation is achieved by elimination of flux space harmonics. It is desirable to avoid any asymmetry, to keep the structural members stiff, to give careful attention to fan design, and to improve stream-line flow of the cooling gas (but this conflicts with the efficacy of thermal pick-up from the cooling surfaces). In some cases it may be helpful to employ anti-vibration mountings.

All parasitic fields are natural noise-producers. High tooth saturations, and zigzag leakage with unsuitable slot numbers, generate magnetic forces which move around the rotor as the slot openings fall in and out of register. It is usually considered that skewing of the rotor (or, occasionally, of the stator) slots can reduce magnetic force fluctuations and so reduce noise.

Skewed Slots

Some effects of skewing are discussed in Sect. 3.13, and Fig. 3.28 indicates how the aim of skewing — the reduction of slot harmonics — is achieved. However, there are some undesirable effects. The direction of the current in a rotor bar is no longer axial, but has a peripheral component proportional to the sine of the angle of skew which magnetizes the stator and rotor cores axially, superimposing a flux that disturbs the symmetry of the gap flux-density distribution and introducing some degree of further saturation. There are also the effects of increase in leakage reactance and stray loss.

It is shown by Binns and Dye [35] that the cogging torque of any cage machine can be made zero by a skew of a rotor slot-pitch or a multiple thereof, although this may not be the optimum value on other grounds. For example, the disturbance to the magnetic symmetry gives rise to a gap energy sensitive to rotor axial position, so that in consequence there is an axial force on the rotor of magnitude proportional to the fractional difference between the gap densities at the two ends of the machine. If the two densities are respectively B' and B'' , and the mean density is B (with all densities in r.m.s.) the axial force is

$$f_a = \frac{1}{2} \pi D l_g (B' - B'') B / \mu_0$$

for a machine of rotor diameter D and radial gap length l_g . In a typical case B' and B'' may differ by 20% and the force may be some score newtons.

The beneficial effect of skewing can be seriously impaired as a consequence of imperfect insulation between rotor bars and the slots in which they lie, a problem of particular difficulty with cast-aluminium cages.

Load Losses

Friction, windage, core and I^2R losses can be estimated from no-load and

short-circuit tests (Sect. 8.22). The measured loss on load is invariably greater, by the amount of what is called the 'stray' load loss, made up chiefly of high-frequency tooth loss and harmonic rotor I^2R . Varying fluxes in any conducting and/or magnetic parts of the teeth, cores, windings, shaft and enclosing structure will generate loss, but the chief locations are the following:

Fundamental-frequency: (i) Eddy-currents in stator conductors. (ii) Eddy currents in the end regions. (iii) Displacement of stator and rotor fundamental m.m.f. waves produced by skewing.

High-frequency: (iv) Harmonics in the rotor due to stator m.m.f harmonics, and (v) in the stator due to rotor m.m.f. harmonics.

It is necessary to distinguish between *no-load* and *load* stray losses. The former occur as a result of permeance variations producing harmonic fluxes and tooth-pulsation effects due to relative tooth positions on no load; the latter are the additional (and more important) effects arising from leakage fluxes on load, which may reduce the full-load efficiency by 1% or considerably more. The stray load-loss is particularly evident if the slip is large; it may double the torque developed in the braking mode and lead to serious errors in the estimation of accelerating and braking performance.

Alger [32] and Christofides [36] have given analytic treatments of several forms of stray loss. The physical concepts are, however, far from simple, and the superposition of arbitrary 'components' of leakage flux is highly complicated by saturation.

Losses associated with self-leakage fluxes (i.e. not mutual to stator and rotor windings) contribute nothing to primary energy transfer and fall logically under the heading of stray loss. But gap leakage flux (in contrast to slot and overhang leakage) is partly mutual as it crosses the gap: components of gap leakage can and do produce torques, and cause saddles in the torque/speed relation. In this connection, the tooth-pulsation fluxes induce harmonic currents in a cage winding leading to I^2R losses dependent on the winding impedance, and these losses rise with increasing difference between stator and rotor slot numbers. Moreover, the impedance of the cage to inducing e.m.f.s depends on its impedance to the core in view of the phase relation of some of the e.m.f.s, especially with skewed slots. Consequently the use of a suitable slot ratio and the insulation of the bars in their slots can substantially reduce the loss. But cage bars are rarely insulated because of structural difficulties.

Several tests [37] have been devised for measuring stray loss, in particular the so-called *reversed-rotation* test (Sect. 8.22). The tests are not suitable for large machines, and in any case cannot precisely measure the losses sought. In respect of the declaration of efficiency in motor specifications, Schwarz [39] concludes that making a nominal allowance for stray load loss is at present the only reasonable and practicable method.

8.12 High-torque cage motors

Compared with a slip-ring machine of like rating, the cage motor has the following advantages:

- (i) About one-half of the rotor conductor material and a smaller inherent I^2R loss.
- (ii) Smaller rotor leakage reactance, and in consequence a better power factor and a higher pull-out torque (the diameter of the current circle depends on the reactance).
- (iii) Higher efficiency, so long as the cage resistance does not have to be augmented to obtain adequate starting torque.
- (iv) No slip-rings, brushgear, short-circuiting devices, rotor terminals or starting rheostats, and consequently a lower cost.

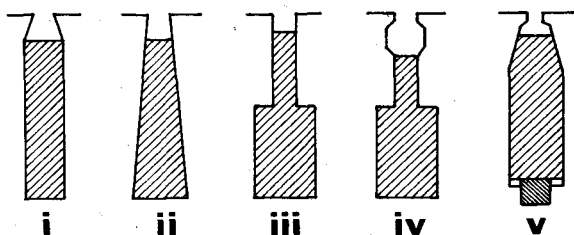
The disadvantages are almost entirely connected with starting. The most usual standard low-resistance cage machine gives 0.3–0.45 p.u. torque with 2.5–3.5 p.u. current on a start at 60% voltage. If the starting conditions are onerous, some form of high-torque cage machine may be suitable. The methods are: Deep-bar cages, using skin effect; or Double-cage construction.

Deep-bar Cage Machines

The behaviour of a deep bar of rectangular section is discussed in Sect. 3.2, giving eq.(3.6) and the characteristics in Fig. 3.3. But these are based on the assumption of infinite core and tooth permeability, a condition that does not exist at starting on normal voltage. Chalmers and Dodgson [8] discuss various attempts to take account of saturation (generally by taking a rectilinear B/H relation or even a fixed saturation density such as 2 T), and give a refined method of their own that can be programmed for computation. If saturation is neglected, the predicted short-circuit current, power factor and torque are likely to be in considerable error, with the current underestimated and the torque overestimated e.g. by 20%. With strong saturation in the region of the tooth tips, the leakage flux is less concentrated there, the bar current is consequently less constricted, and the effective bar resistance falls, producing the difference between predicted and actual results.

Some typical bar shapes are shown in Fig. 8.27: (i) is the simple rectangular bar requiring some means of retaining it, (ii) is the wedge bar, and (iii) and (iv) are versions of the T-bar which is preferred by some makers. The behaviour of the full-depth T-bar (iii) is less predictable than the recessed form (iv), largely because of the difficulty in assessing tooth-tip saturation. The bar (v) is fixed tightly by a steel key to provide rigidity.

The starting current being confined chiefly to the outer part of the bar,

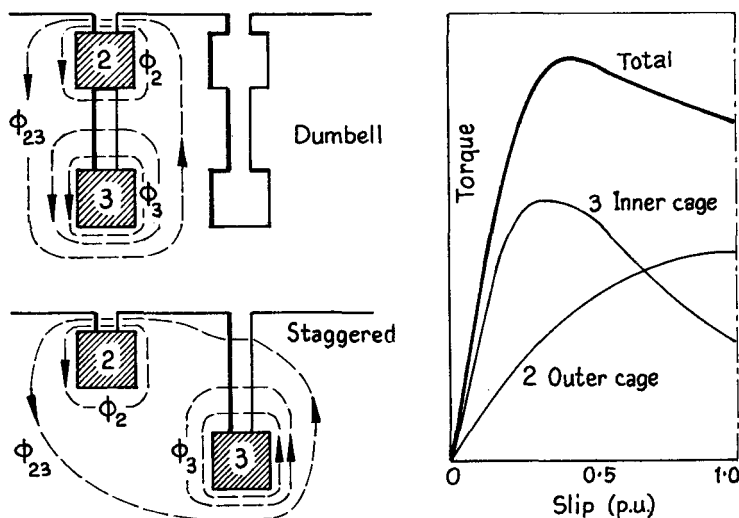


8.27 Deep-bar profiles.

this part rapidly heats, but the thermal conductivity spreads the heat to the lower part. The T-bar has the mechanical advantage that the shoulders keep the bar more tightly in the slot: vibration and thermal cycling (leading to expansion and contraction) can be very damaging to a slack bar and its insulation (if any).

Double-cage Machines

In these the rotor carries two concentric and separate cage windings, outer and inner, arranged in a slot arrangement that may take a variety of forms, Fig. 8.28. The outer cage has a high resistance together with the low reactance



8.28 Double-cage machine.

of a normal single cage, while the inner cage has a low resistance and the high reactance obtained as a result of the long, narrow, slitted lip. The slits are necessary to prevent the inner cage from being 'missed' by the main flux, for if the inner bars were buried in the rotor iron, the gap flux would link only the outer cage.

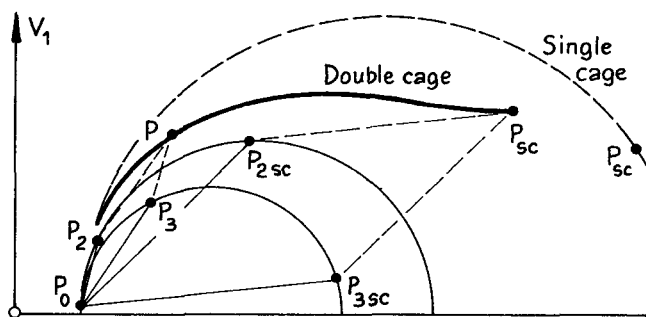
By suitable choice of the cage resistances, the number and breadth of the slot openings and the depth of the inner cage, it is possible to control the shape of the torque/speed curve so as to obtain, within certain limits, any desired torque characteristic. To a first (and rather rough) approximation it is possible to consider the cages as producing quite separate torques, their sum giving the total torque of the motor, as in Fig. 8.28. A wide range of operational characteristics is obtained by suitably modifying the individual cage resistances and leakage reactances. This flexibility is achieved at some cost. In comparison with a plain cage motor, the pull-out torque is smaller because one cage produces its maximum torque at a speed different from that of the other cage. The

high-resistance cage bears the burden of most of the loss during starting; with too frequent starts it may burn out. The additional inner-cage leakage lowers the full-load power factor, and the high outer-cage resistance increases the full-load loss and impairs the efficiency. However, as a normal single-cage machine with a small rotor I^2R loss could not develop the same starting torque, the energy efficiency of the double-cage motor may be greater for the given duty.

Diecast Rotors. The double-cage construction is sometimes used with diecast aluminium rotor windings. During the casting process the upper and lower parts 2 and 3 of the 'dumbbell' are connected by an aluminium fillet which provides electrical contact between cages along their length. However, analysis and experiment both show that the assumption of separate cages is a reasonable approximation.

Performance

Taking the two cages as magnetically as well as electrically separate, the performance of a machine can be estimated by drawing two circle diagrams and adding corresponding rotor current phasors to obtain the effective total. If in Fig. 8.29 points P_2 and P_3 correspond to outer- and inner-cage currents for

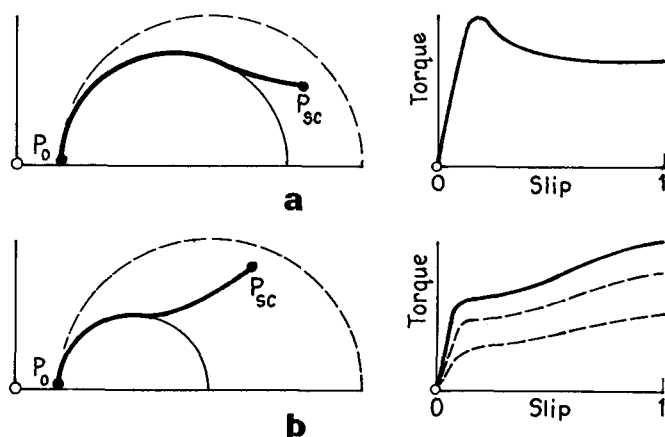


8.29 Current locus of double-cage motor.

a given slip, their phasor sum gives P , a point on the resultant current locus from P_0 . As the outer cage has the lesser reactance its circle has the greater diameter; and the short-circuit point P_{2sc} is near to the crest of the circle since the high resistance contributes the major part of the starting torque. The inner-cage circle is smaller and its short-circuit point P_{3sc} is lower. As both the resistance and leakage reactance of the double cage is greater, the combined rotor current must always lie within the circle of the corresponding plain motor. The torque of the double-cage machine can be obtained by drawing torque lines for each circle separately, and combining the two values for a given slip.

The allocation of resistance and leakage reactance to the two cages is intricate, and depends on the torque/speed characteristic required. The more nearly the inner cage approaches a normal single-cage construction, the bigger

is its circular locus, as in (a) of Fig. 8.30, and the more nearly will the pull-out torque approach that of the plain single-cage motor; but at the same time the starting current will increase and the starting torque reduce as a result of the greater parallelling effect of the two cage resistances. At the other extreme is the use of a large inner-cage reactance and an adjustment of the outer-cage resistance to achieve a high starting torque, Fig. 8.30(b); but the pull-out



8.30 Current loci and torque/slip curves of double-cage motor.

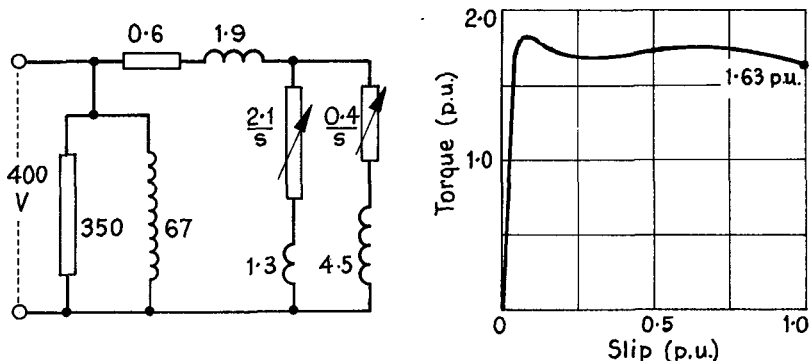
torque is now but little greater than the full-load value. This characteristic, combined with stator rheostat control as indicated by the dotted torque/slip relations, is applicable to cranes. It gives a controllable hoisting torque together with inherent overspeed limitation for lowering.

The Table below gives a comparison between typical slip-ring and cage motors of comparable rating.

Per-unit values	Slip-ring	Single-cage	Double-cage*	
			D	S
Efficiency, full-load	0.88	0.90	0.90	0.90
half-load	0.86	0.89	0.87	0.89
Power factor, full-load	0.85	0.92	0.86	0.88
half-load	0.67	0.82	0.76	0.82
Slip, full-load	0.04	0.028	0.021	0.022
Starting current, d.o.l.	—	6.0	5.6	5.5
star-delta	—	2.0	1.9	1.8
Starting torque, d.o.l.	—	1.3	2.1	2.1
star-delta	—	0.43	0.7	0.7

* Slotting: D, dumbell; S, staggered

EXAMPLE 8.6: A 22.5 kW, 400 V, 50 Hz, delta-connected double-cage motor with staggered slotting has the equivalent circuit per phase in Fig. 8.31 with



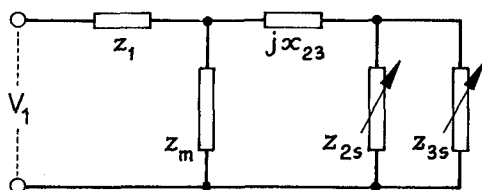
8.31 Equivalent circuit and torque/slip curves.

impedances in ohms and mutual cage reactance disregarded. Obtain the current locus and torque line, and plot the gross torque/slip curve.

Referring to Fig. 8.32, co-ordinate axes are drawn with origin at point O_{23} , with resistance values vertical and reactance values horizontal. The locus of the outer-cage impedance z_2' is obtained by drawing to scale from the point for $s = 1$ (that is, $r_2' = 2.1 \Omega$, $x_2' = 1.3 \Omega$) the vertical line z_2' marked off in terms of slip. The impedance locus z_3' is drawn in a similar manner. Each impedance locus is now inverted about O_{23} to give the circular loci y_2' and y_3' , with centres respectively at M_2 and M_3 , a suitable common admittance scale being chosen to give a convenient size of circle. On the admittance loci, the s values are marked in by inversion from the impedance lines. Now, for each chosen slip, y_2' is added to y_3' to give the dotted locus y_{23}' which, re-inverted about O_{23} , results in the locus of z_{23}' , the combined impedance of the inner and outer cages in parallel. The stator impedance is now added by shifting the origin by 0.6Ω to scale downwards, and 1.9Ω horizontally to the left, locating the new origin O_{123} . Impedances measured from O_{123} are for the whole machine excluding the magnetizing impedance. The corresponding admittance y_{123} is the inversion of z_{23}' with respect to O_{123} . The final origin O is located from O_{123} by shifting to scale by $1/r_m$ and $1/x_m$. Then O is the origin of the terminal admittance, and the locus y_{123} becomes also the locus of the total stator current $I_1 = V_1 y_{123}$, where V_1 is the phase voltage. The *torque line* is found by calculating the stator $I^2 R$ loss at a number of points, and setting up an intercept from the horizontal base-line equal to scale to the active component of current corresponding thereto. Then, very nearly, the remainder of the active current (and the corresponding active power) is the rotor input and (to another scale) torque. Thus at standstill the stator current is 114 A, the loss per phase is $114^2 \times 0.6 = 7\,800$ W, the active current component is $7\,800/400 = 19.5$ A; and this, set off vertically beneath the unity-slip point, locates the point on the torque line. The torque/slip curve, Fig. 8.31, can be drawn by measurement from the stator current locus marked in slip values; it shows a comparatively uniform torque over the speed range during the starting operation averaging about 1.7 p.u. of full-load torque. At standstill,

1.63 p.u. torque is obtained with 4.75 p.u. current. The pull-out torque is about 1.8 p.u.

'Exact' Equivalent Circuit. From the leakage-flux patterns in Fig. 8.28 it is clear that the outer cage has a small flux Φ_2 and the inner cage a much greater flux Φ_3 ; and there is a common flux Φ_{23} endowing the two cages with mutual inductance. It may therefore not be valid to assume the cages to form separate rotor circuits, particularly if the rotor has dumbbell slotting.



8.33 Double-cage motor with cage mutual reactance.

Modifying the equivalent circuit of Fig. 8.10(a) to accommodate two cages with mutual reactance gives the network in Fig. 8.33. The parallel branches z_{2s} and z_{3s} together give, at any slip s , a combined impedance $r_e/s + jx_e$ where

$$r_e = \frac{r_2 r_3 (r_2 + r_3) + s^2 (r_2 x_3^2 + r_3 x_2^2)}{(r_2 + r_3)^2 + s^2 (x_2 + x_3)^2} \quad (8.23)$$

$$x_e = \frac{r_2^2 x_3 + r_3^2 x_2 + s^2 x_2 x_3 (x_2 + x_3)}{(r_2 + r_3)^2 + s^2 (x_2 + x_3)^2} + x_{23} \quad (8.24)$$

These are rather complicated functions of slip, but if we take r_{e0} and x_{e0} as their values at $s = 0$ and define a ratio m for the increase in overall resistance of r_e divided by the decrease of x_e compared with r_{e0} and x_{e0} , for a given slip s , then

$$m = \frac{\Delta r}{\Delta x} = \frac{r_e - r_{e0}}{x_{e0} - x_e} = \frac{r_2 + r_3}{x_2 + x_3}$$

which is independent of slip and serves as a useful parameter for dealing with the allocation of cage impedances for securing the operating characteristics required. Alger [32] deals with several approximate analyses in terms of m , variations of which permit adjustment of the torque/slip curve, and of r_{e0} and x_{e0} on which the small-slip characteristics depend. Chalmers and Mulki [40] use the same parameters in a design-synthesis procedure in which the cage resistances and reactances are related to the shape of the torque/slip curve, the upper limits of short-circuit current and rotor $I^2 R$ (or rated slip), and the lower limits of starting and pullout torques and full-load power factor.

Eqs.(8.23) and (8.24) show that, for small slips, the double cage can be represented by a single cage with the approximate parameters

$$\frac{r_e}{s} = \frac{1}{s} \cdot \frac{r_2 r_3}{r_2 + r_3} \quad \text{and} \quad x_e = \frac{r_2^2 x_3}{(r_2 + r_3)^2} + x_{23}$$

in which x_2 has been neglected. The former is simply the parallel combination of r_2/s and r_3/s : these determine the full-speed characteristics. Calculation of the short-circuit conditions at $s = 1$ gives the starting torque, and inspection will show roughly the type of torque/slip curve to be expected.

8.13 Motor speed control

The plain induction motor in normal operation has inherently a nearly constant speed. Certain applications, however, call for two or more speeds, and in others a continuously variable speed range is a necessary feature. Two-speed motors, for example, are commonly required for boiler forced-draught fan drives; ships with a.c. propulsion need reversing and low-speed manoeuvring facilities; and cranes must have speeds adjusted to suit the loads to be lifted and lowered. In process plants such as paper-making and steel-rolling it may be vital to achieve closely accurate speed control of a number of associated motors to 0.01% or better, a requirement obtainable only by closed-loop control.

Basic Methods

The speed n of a motor with p pole-pairs, connected to a supply of frequency f and running with a slip s with respect to its synchronous speed $n_s = f/p$, is

$$n = (f/p)(1 - s)$$

showing that the three basic methods by which the speed can be varied are:

Slip: The slip s for a given torque can be varied in cage motors by changing the supply voltage, and in slip-ring motors by including external rheostats into the rotor circuits, or by applying to them slip-frequency voltages from auxiliary plant in order to convert the slip power into a useful form.

Poles. The synchronous speed f/p can be changed by altering the number of pole-pairs. Change-pole motors are specially wound for this purpose, usually for two speeds but occasionally for three or more.

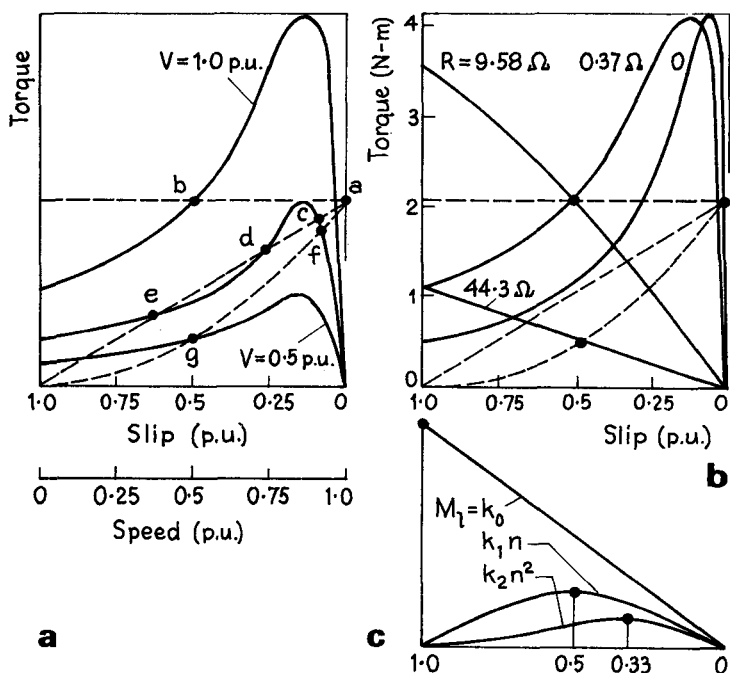
Frequency. The speed can be varied over a wide range by stepless control of the supply frequency f . Solid-state frequency-changers are generally used to obtain the necessary supply.

Alternative Methods

There are several special methods of occasional usefulness. A 3-ph motor with a 1-ph rotor winding can be made to run stably at one-half of normal synchronous speed. A motor with stator and rotor fed at different frequencies can operate at synchronous speeds corresponding to the sum and difference frequencies. If the essential simplicity of the induction machine is given up, a wide speed range can be obtained by providing the rotor with a commutator.

8.14 Slip control

The efficacy of slip control by voltage reduction is affected by the torque/speed characteristic of the load. For a given slip, the motor torque is proportional to the square of the applied stator voltage. Fig. 8.34(a) shows typical motor characteristics for applied voltages of 1.0, 0.71 and 0.5 p.u., and three



8.34 Slip control.

load torques M_l , viz (i) constant ($M_l = k_0$), (ii) proportional to speed ($M_l = k_1 n$) and (iii) proportional to speed-squared ($M_l = k_2 n^2$). Combining motor and load torque characteristics shows that for (i) there will usually be a single stable speed a and an unstable speed b , and that the motor cannot operate at all in either of the two low-voltage conditions. For (ii) there may be two stable speeds c and e , separated by an unstable speed d . For (iii) there is a stable speed f at 0.71 p.u. voltage, and an unstable one g for 0.5 p.u. Thus if the slip is to be controlled by voltage reduction, the motor must be suitably designed for its load and the voltage reduction properly chosen. Incidentally, it is possible to stabilize speeds b , d and g by providing a feedback loop control.

Slip Power. With substantial speed changes, high levels of slip power are concerned. Eqs.(8.3) and (8.4) show that a given torque M implies a corresponding active power input P_2 to the rotor regardless of speed, and that the fraction s of P_2 appears in the rotor circuits as slip power. With cage rotors the slip power is the rotor $I^2 R$ loss, but with slip-ring rotors most of the slip power is transferred to external devices. Let the torque demand at speed n be proportional to n^x ; then the corresponding power is $P_m = kn^{x+1}$. The rotor input power is $P_2 = P_m/(1-s)$ and the slip power is sP_2 . In terms of the power demand $P_{mm} = kn_s^{x+1}$ at synchronous speed, the slip-power coefficient

$$g_s = sP_2/P_{mm} = (n/n_s)^{x+1} [s/(1-s)] = s(1-s)^x$$

expresses the relative power (including rotor internal I^2R) that has to be dealt with for a given slip. Putting $x = 0, 1$ and 2 gives respectively the three representative torque/speed characteristics (i), (ii) and (iii) in Fig. 8.34(a), and the corresponding slip powers are shown in (c). For (i) the slip power is proportional to slip, and comprises the whole rotor input at standstill. For (ii) the slip power is a maximum of $0.25P_2$ at $s = 0.5$; and for (iii), typical of a fan drive, the peak slip power is $0.15P_2$ at $s = 0.33$.

Voltage Control

Stator voltage reduction by series resistors or inductors, though wasteful and current-dependent, can be applied to cage motors of rating 1 kW or less. Machines with star-delta starters may be run economically by switching to the star connection (reducing the phase voltage to 0.58 p.u. and the pull-out torque to one-third) provided that the load characteristic is suitable. For large machines a more sophisticated method is to adjust the stator voltage by interposing inverse parallel diodes with thyristors, or by bidirectional thyristors. Erlicki, Ben Uri and Wallach [41] describe the *burst-firing* method, i.e. the application of normal voltage for a few periods on and off in continuous succession, using thyristors for the switching function. Applied to the stator, burst-firing induces transients and increases loss, and is suitable only for small cage motors. It is more successful when applied to a slip-ring rotor, as the gap-field does not then collapse during the periods of quench.

Rotor Resistance Control

In the normal operating region the torque and slip are related to an approximation by $M = ks/r_2$, so that the slip for a given torque is proportional to the rotor-circuit resistance. Connection of external rheostats across the slip-rings (as for starting) therefore lowers the speed at the cost of a greater dissipation of slip power as heat, so that the overall efficiency must always be less than $(1 - s)$. Further, there is a greater change of speed with load. In practice the method is used for occasional minor speed reductions as in mine-ventilating fan drives, and also for 'load equalization'.

A simple analysis can be based on the approximate theory of Sect. 8.6 with the stator resistance r_1 neglected and the rotor-winding resistance r_2 augmented by an external rheostat R such that $R_2 = r_2 + R$. Then with all phase quantities referred to the stator, eq.(8.20) gives

$$M\omega_s = P_2 = V_1^2 (R_2/s) / [(R_2/s)^2 + X_1^2]$$

for the active power input to the rotor. The maxima M_m and P_{2m} occur for a slip $s = s_m = R_2/X_1$ for which $M_m\omega_s = P_{2m} = V_1^2/2X_1$, whence for a given fraction q of maximum torque

$$q = \frac{M}{M_m} = \frac{P_2}{P_{2m}} = \frac{2X_1(R_2/s)}{(R_2/s)^2 + X_1^2}$$

Rearrangement yields a quadratic equation which solves to

$$R_2 = s(X_1/q)[1 \pm \sqrt{(1 - q^2)}]$$

Of the two values of R_2 only one is realizable. The other gives an unstable operating point on the negative-slope region of the torque/speed curve, or is less than r_2 implying that R is negative. Fig. 8.34(b) shows the method applied to the following Example.

EXAMPLE 8.7: A 375 kW 3.0 kV 50 Hz 3-phase 10-pole slip-ring induction motor with star/star connection has a rotor resistance $r_2 = 0.39 \Omega/\text{ph}$ and a total leakage reactance $X_1 = 5.75 \Omega/\text{ph}$ (both referred to the stator). The stator/rotor effective turns ratio is 4.65 and the full-load slip is 0.022 p.u. Ignoring stator resistance and mechanical loss, estimate the external rotor resistance, the slip power and the efficiency for a speed of one-half synchronous when the load has (i) constant full-load torque, (ii) a torque of full-load value at normal speed but is proportional to speed-squared.

Synchronous speed is 62.8 rad/s. Per phase, the full-load output is 125 kW, the full-load torque is $125/(62.8 \times 0.978) = 2.04 \text{ kN-m}$ and the pull-out torque is $1730^2/(2 \times 5.75 \times 62.8) = 4.15 \text{ kN-m}$.

(i) *Constant Torque.* Here $q = 2.04/4.15 = 0.49$ and $q^2 = 0.24$. Then

$$R_2 = 0.50(5.75/0.49) [1 \pm \sqrt{(1 - 0.24)}] = 10.97 \text{ (or } 0.76) \Omega$$

Thus $R = R_2 - r_2 = 9.58 \text{ (or } 0.37) \Omega$ of which only the former gives a stable operating point. The slip power $sP_2 = 0.5(125/0.978) = 63.9 \text{ kW/ph}$, of which 55.8 kW is dissipated in R . The mechanical output is $125(0.5/0.978) = 63.9 \text{ kW}$, equal to the rotor slip power, and with other losses ignored the efficiency is 0.5 p.u. The actual resistance in rotor terms is found by dividing R by the square of the turns ratio, giving $9.58/4.65^2 = 0.44 \Omega$.

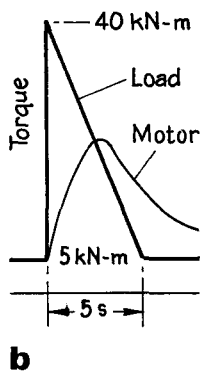
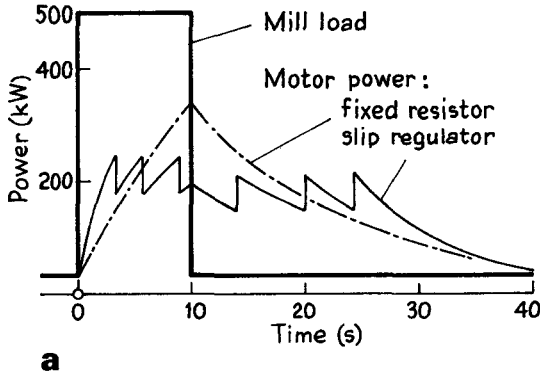
(ii) *Torque proportional to speed-squared.* At one-half synchronous speed the torque is $2.04(0.50/0.978)^2 = 0.53 \text{ kN-m}$; whence $q = 0.53/4.15 = 0.128$, and $q^2 = 0.016$. Then

$$R_2 = 44.7 \text{ (or } 0.18) \Omega \quad \text{and} \quad R = 44.3 \text{ (or } -0.21) \Omega$$

showing that the alternative solution is not possible. The slip power is 16.6 kW, nearly all dissipated in R , and the efficiency is again 0.5 p.u. In rotor terms $R = 2.05 \Omega$.

The motor characteristics in Fig. 8.34(b) are marked with the stator equivalents of the external rotor resistance R , that marked $R = 0$ being for a short-circuited slip-ring condition.

Slip Regulator. A motor driving a load (such as a rolling mill) subject to sudden load peaks may be fitted with rotor rheostats and a shaft-mounted flywheel. On sudden load application the speed falls, part of the torque demand being met by release of kinetic energy from the flywheel. With fall of load the motor accelerates, restoring energy to the flywheel. The equalization obtained by a contactor-controlled variable rotor rheostat is shown in Fig. 8.35(a), which also shows the less effective result when the rheostat is fixed. A liquid rheostat, Fig. 8.36, may be used, the electrode position being set by a 'torque motor' which exerts a static torque proportional to the main motor current to increase the resistance (and so the slip) with rise of load.

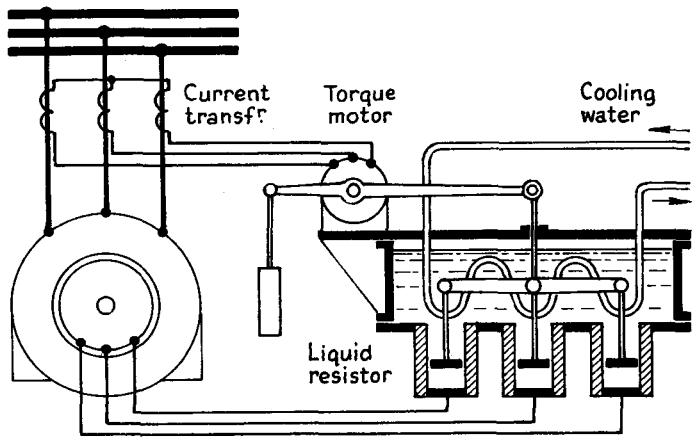


8.35 Load equalization

More elaborate methods employ a servo-motor with sensors responding to both torque and speed.

EXAMPLE 8.8: A 1 000 kW 50 Hz 10-pole induction motor developing full-load torque at 573 rev/min (60 rad/s) is provided with a flywheel, the total stored kinetic energy at 60 rad/s being 9.0 MJ. The fixed rotor external resistance is such that the slip for full-load torque is increased to 0.10 p.u. The load cycle, Fig. 8.35(b), comprises an idling torque $M_0 = 5.0$ kN-m, an instantaneous peak $M_1 = 40.0$ kN-m, and thereafter a uniform decrease to M_0 in 5.0 s. Find the motor torque/time relation.

Rated full-load torque is 16.7 kN-m. If this is developed at slip 0.10 p.u., and if the motor torque is assumed proportional to slip, then $M = ks$ where $k = 167$ kN-m per unit slip. With the load pulse applied at $t = 0$, the load torque is represented by $M_l = (40 - \alpha t)$ where $\alpha = 7.0$ kN-m/s. The system



8.36 Slip regulator.

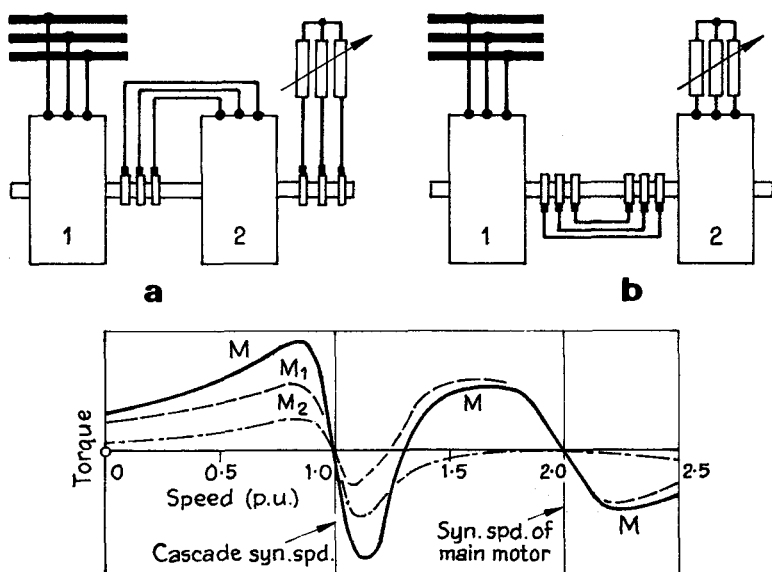
inertia is $J = 9.0 \times 10^6 / \frac{1}{2} \times 60^2 = 5\,000 \text{ kg-m}^2$ and the synchronous speed is $\omega_s = 62.8 \text{ rad/s}$. The motor torque M balances the mechanical load and inertia torques M_1 and $J(d\omega_r/dt)$ at any angular speed ω_r . Writing $\omega_r = \omega_s(1-s)$, then $d\omega_r/dt = -\omega_s(ds/dt) = -(\omega_s/k)(dM/dt)$, and the balance equation is

$$(dM/dt) + (k/J\omega_s)M = (k/J\omega_s)(M_1 - \alpha t)$$

which solves to

$$\begin{aligned} M &= [M_1 - M_0 + \alpha(J\omega_s/k)] [1 - \exp(-k/J\omega_s)t] + M_0 - \alpha t \\ &= 52 - 49 \exp(-t/1.88) - 7t \end{aligned}$$

over the interval $t = 0$ to $t = 5 \text{ s}$. The peak motor torque occurs at $t = 2.45 \text{ s}$, and the motor torque/time curve is given in Fig. 8.35(b).



8.37 Cascade connection.

Cascading

This arrangement, Fig. 8.37, is one of the earliest developed to make effective use of *slip power*, which is supplied by the main motor to an auxiliary induction machine mechanically coupled, direct or through gearing, to the main-motor shaft. The slip power may be fed to the auxiliary stator as in (a), its rotor having slip-rings and starting rheostats or a plain cage winding. Alternatively the two rotors may be interconnected as in (b).

Let p_1, p_2 be the respective pole-pair numbers of the main and auxiliary motors, and f_1 the supply frequency: then the synchronous speed of the main motor is $n_1 = f_1/p_1$. Suppose n to be operating speed of the set on no load. In connection (a), the frequency in the main rotor and auxiliary stator is $f_2 =$

$(n_1 - n)p_1$ and the synchronous speed of the auxiliary machine is $n_2 = f_2/p_2$. Taking the no-load slip of the auxiliary rotor to be zero, then the set runs at $n = n_2 = n_1[p_1/(p_1 + p_2)]$. This is for unidirectional torques, but if the main and auxiliary torques oppose, the no-load speed is $n = n_1[p_1/(p_1 - p_2)]$. When the set is loaded, the speed drops fractionally. Electrical power to the main stator is passed (except for stator loss) to its rotor, where the power divides into (i) a part proportional to n converted into mechanical power, and (ii) the slip power proportional to $(n_1 - n)$ transferred to the auxiliary machine which employs it in mechanical power output and I^2R loss. The mechanical outputs are in the ratio $p_1 : p_2$. Fig. 8.37 shows a group of typical torque, power and current characteristics for a set with $p_1 = p_2$. Cascade operation has inherently low power factor, efficiency and pull-out torque, but it has been applied in a.c. ship-propulsion systems, where low manoeuvring speeds are important.

Variable-speed Slip-power-recovery Schemes

These enable slip-ring induction motors to operate at continuously variable speeds efficiently and economically. There are two groups:

Mechanical Recovery. Slip power from the slip rings is converted into mechanical power to supplement the output of the main motor.

Electrical Recovery. Slip power is frequency-converted and returned to the main supply.

Early examples of the former are those due to Kramer (1906), and of the latter the Scherbius (1906) and Clymer (1942) systems. All were based on the use of auxiliary machines (d.c. and a.c. commutator motors, synchronous motors and converters, variable-ratio transformers, etc.). In modern variable-speed sets, solid-state rectifiers and inverters can replace most of these.

Simplified examples of mechanical and electrical slip-power recovery are shown in Fig. 8.38. In both the control of slip is by field control of the d.c. machine. Decreasing the field reduces the armature e.m.f., allowing increased currents to flow in the slip-ring rotor, which accelerates to a speed determined by the mechanical load. A small difference between the slip-ring and d.c. armature voltages (referred back through the rectifier) then circulates the load current. In a comparable manner, the speed of the main motor falls if the d.c. excitation is raised. In the control characteristics shown, the operating speeds correspond at no load approximately to the intersections of the slip-ring and referred d.c. armature voltages: on load there is a speed drop of 0.05 p.u. or less. The minimum operating (or *base*) speed is shown in each case as about 0.5 p.u. of synchronous speed.

Electrical recovery applied to a constant-torque load implies that the power input to the induction machine is constant regardless of its speed (but most of the slip power is returned to the supply). With mechanical recovery, the torque contribution of the d.c. motor reduces the mechanical load taken by the induction motor: the set then takes a constant power input for a drive with torque inversely proportional to speed (i.e. constant mechanical power). In each case the efficiency is reasonably good, being lowest at base speed. The mechanical recovery is simpler, while the electrical method is well adapted

unidirectional output current is smoothed and passed on to a 3-ph line-commutated inverter at a rate depending on the supply voltage, the rectified direct voltage and the thyristor firing angle. The inverted current has a fixed wave-form and a constant conduction angle of $2\pi/3$ rad. The onset of conduction with respect to the phase-voltage zero is controlled by the firing angle θ . The harmonic content of the inverted current is substantially invariable: in terms of a peak current i_m of unity, the magnitudes of the fundamental, 5th, 7th, 11th and 13th harmonics are 1.05, 0.24, 0.12, 0.10 and 0.07. As power flow through the rectifier is unidirectional, only subsynchronous speeds are obtainable.

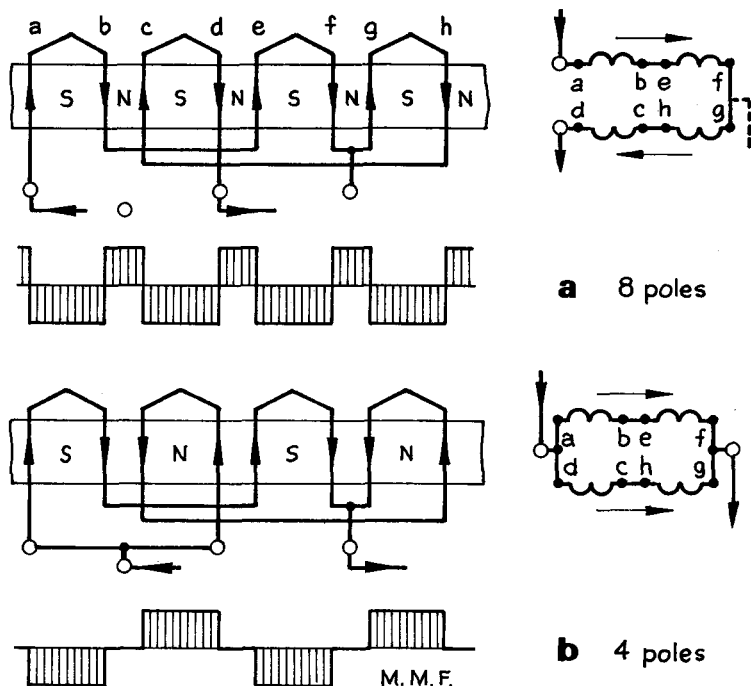
In rectifier/inverter recovery, part of the active power input to the motor passes through the rotor back to the supply, but the lagging reactive power is absorbed within the motor and rectifier. The bridge rectifier is, in effect, a loss-free lagging reactive power device, a frequency changer and a harmonic-current generator, feeding the rotor with reactive power to magnetize the gap at each harmonic frequency, while drawing from the rotor the equivalent in fundamental reactive power. The fundamental of the inverted current leads the phase voltage by an angle equal to the supplement of the firing angle and is therefore a source of leading reactive power (or a sink of lagging reactive power), and the harmonics cause 'distortion' power loss. As regards compensation, the provision of stator capacitors intensifies the harmonics, but capacitors across the rotor terminals can markedly decrease the leakage reactance of the motor and raise the peak torque.

The usual range of inverter firing angle for wide speed regulation down to standstill is 90° – 160° , allowing a safe interval for commutation. The ratio (active/reactive) power of the inverter varies from zero at a 90° firing angle corresponding to maximum speed up to 0.9 lagging for 160° , the low-speed condition.

8.15 Pole-changing

A cage motor with two (or more) separate stator windings of different pole numbers can run at two (or more) integrally related speeds. Certain change-pole motors have a single winding, the connections of which can be changed to give different pole numbers; but separate windings, appropriately matched to the load conditions, may be preferred for such loads as lifts and hoists where a speed change must be made without losing control of the motor. Separate windings are, however, difficult to accommodate.

The coil pitch of a stator winding is naturally fixed, but its electrical span depends on the number of poles. A coil pitch one-eighth of the circumference provides full-pitch coils for an 8-pole connection, two-thirds for a 6-pole and one-half for a 4-pole connection. Too narrow a span is to be avoided, and for a pole-change winding for a 2/1 speed ratio a possible arrangement is a coil span of $1\frac{1}{2}$ pole-pitch for the larger and $\frac{2}{3}$ for the smaller pole number, in each case with a coil-span factor of 0.87. The spans may in practice be nearer to 1 and $\frac{1}{2}$ (coil-span factors 1.0 and 0.71) in order to avoid excessive leakage reactance in the lower-speed connection.



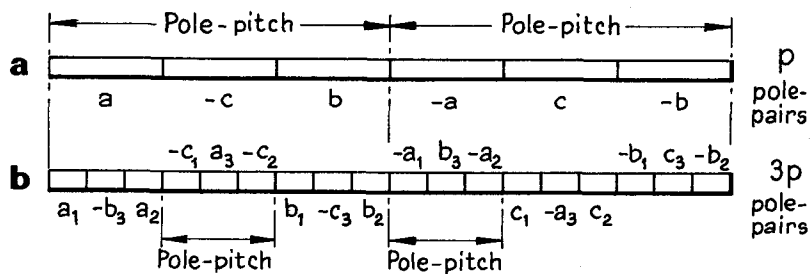
8.40 Pole-changing in ratio 2/1.

Pole Ratio 2/1

The essential method is given in Fig. 8.40, only one phase being shown. For the higher pole number the coils are in series (*a*). For the lower number (*b*) they are in series-parallel, giving the m.m.f. patterns corresponding respectively to 8 and 4 poles. It is common to use a double-layer winding, with a 120° spread for the higher number. An important design consideration is to limit space harmonics, which may produce a tendency to 'crawl'.

Pole Ratio 3/1

A method due to Rawcliffe and Jayawant [42] consists in making three subgroups of each phase winding. Fig. 8.41 shows (*a*) two pole-pitches of a normal 3-ph winding, and (*b*) its subdivision into what amounts to nine phases. If subgroups $a_1, -b_3, a_2$ are connected in series they form a normal phase of a 2-pole machine (the other phases are $b_1, -c_3, b_2$ and $c_1, -a_3, c_2$). If now the subgroup a_1, a_2, a_3 is series connected it forms one phase of a 6-pole winding, the *b* and *c* subgroups being similarly series connected. In each case the windings have an electrical spread of 60° for all pole numbers. For the same supply voltage the 6-pole arrangement gives a rather low gap flux density, and to increase it one-third of each phase may be omitted in the 2-pole connection.



8.41 Pole-changing in ratio 3/1.

Operating Conditions

Referring throughout to the stator winding, let subscripts 1 and 2 imply conditions with the smaller and larger pole-numbers p_1 and p_2 . The ratio of gap flux densities B and that of the pull-out torques M are

$$\frac{B_1}{B_2} = \frac{N_2 K_{w2} p_1}{N_1 K_{w1} p_2} \cdot \frac{V_1}{V_2} \quad \text{and} \quad \frac{M_1}{M_2} = \frac{(B_1 K_{w1})^2}{(B_2 K_{w2})^2} \cdot \frac{p_2}{p_1}$$

where N , K_w and V refer to the turns in series per phase, the winding factor and the phase voltage. The relative maximum output powers are $P_1/P_2 = (M_1/M_2)(p_2/p_1)$. Neglecting saturation, the magnetizing and short-circuit current ratios are

$$\frac{I_{m1}}{I_{m2}} = \frac{B_1 p_1}{B_2 p_2} \cdot \frac{N_2 K_{w2}}{N_1 K_{w1}} \quad \text{and} \quad \frac{I_{sc1}}{I_{sc2}} = \frac{N_2^2}{N_1^2} \cdot \frac{V_1}{V_2}$$

for the latter ignoring change in overhang leakage. If the same phase voltage is applied for each connection, then $V_2 = V_1$. The ratios are only approximate, the magnetizing-current ratio being 10–20% greater and the short-circuit current ratio 10–15% smaller than as given, because of the changed m.m.f. pattern and the different conditions of saturation.

The choice of winding arrangement depends on the load. Two cases are considered; others (e.g. for fan ventilation drives) may sometimes be more apposite.

Constant Torque: If the same torque range is required for both speeds (i.e. power proportional to speed), the windings are best utilized if

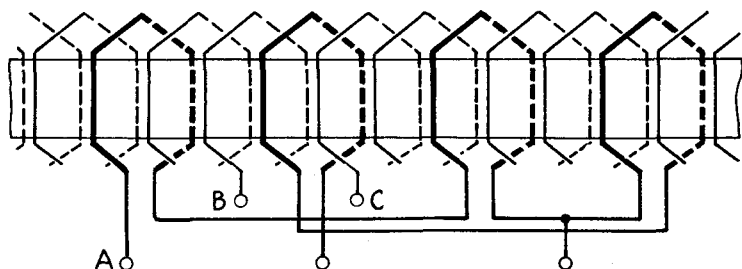
$$B_2 \approx B_1 \quad \text{or} \quad N_2 K_{w2} V_1 / N_1 K_{w1} V_2 \approx p_2 / p_1$$

and the ratio of the pull-out torques is then

$$M_1/M_2 \approx (K_{w1}/K_{w2})^2 (p_2/p_1)$$

Constant Power: For torque inversely proportional to speed, the ratio $B_1/B_2 = p_1/p_2$ gives the same overload capability on each speed and the same current density in the windings.

Fig. 8.42 shows one phase of a double-layer winding arranged for 2:1 pole-changing. The coil-span is $2/3$ of the larger and $1\frac{1}{3}$ of the smaller pole-pitch,



8.42 Pole-changing in ratio 2/1 with double-layer winding.

and for the latter the phase-spread is 120° . The method of pole change is that in Fig. 8.40. The winding factors are $K_{w1} = 0.955 \times 0.866 = 0.827$, and $K_{w2} = 0.827 \times 0.866 = 0.715$. If the coil groups are in series and the phase-connections star for the larger pole number, and series and delta for the smaller, then

$$N_1/N_2 = 1 \quad V_1/V_2 = \sqrt{3} \quad B_1/B_2 = 0.8 \quad M_1/M_2 = 1.5$$

$$I_{m1}/I_{m2} = 0.4 \quad I_{sc1}/I_{sc2} = 1.7 \quad P_1/P_2 = 3.0$$

so that 'constant torque' conditions are roughly approximated. For 'constant power' the greater pole number can have coils in series and delta connection, the smaller parallel groups and star, giving

$$N_1/N_2 = 1 \quad V_1/V_2 = 1/\sqrt{3} \quad B_1/B_2 = 0.5 \quad M_1/M_2 = 0.65$$

$$I_{m1}/I_{m2} = 0.5 \quad I_{sc1}/I_{sc2} = 2.3 \quad P_1/P_2 = 1.3$$

Common connections for a 2/1 pole-change are delta/parallel-star and star/parallel-star, because the gap flux densities do not then differ by more than 40 and 20% respectively. On pole-changing it is necessary to reverse the stator phase sequence to preserve the same direction of rotation.

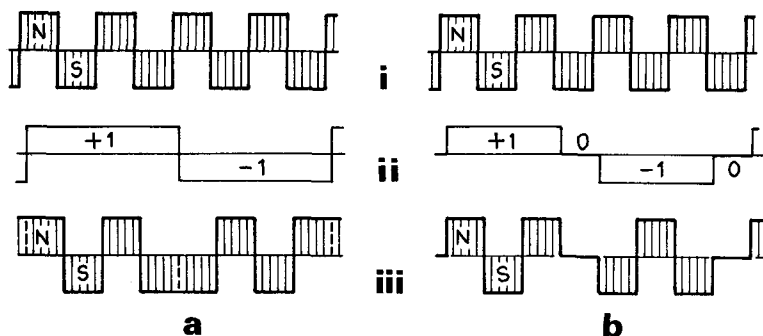
Voltage Control. Combining this with pole-changing can give more speed availability in certain cases. It can readily be inferred from Fig. 8.34(a) that if the number of poles could be doubled (and the synchronous speed halved) there may be several more running speeds.

Slip-ring Rotors. Cage rotors can respond to any stator pole number if properly designed, but slip-ring windings must be pole-changed to correspond to the stator polarity. Windings can be devised to act as conventional windings for one pole number (and so available for rheostat control) and as a permanent short-circuit to the other.

Pole-amplitude modulation

To have separate stator windings for pole-changing is wasteful of conductor material, requires deeper slots and a larger frame size, and involves greater I^2R loss and leakage reactance. The method of *pole-amplitude modulation* (so called by analogy with the balanced modulator in telecommunications in which the carrier wave is suppressed and there are two side-frequencies)

enables several pole-combinations to be economically obtained with a single winding and without increase in frame size. Modulation of the m.m.f. of a p -pole-pair gap field yields two superposed fields of pole-pairs $p_1 = p - k$ and $p_2 = p + k$, where k is an integer. Then by suitable 3-ph connections one of these is eliminated, leaving the other. The term 'modulation' refers to reversal or suppression of the current in parts of the winding. In Fig. 8.43, the m.m.f.



8.43 Pole-amplitude modulation.

in rectangular form of one phase of a 3-ph 8-pole winding is shown in (i), while (ii) represents two kinds of modulation. In (a) the modulation is in fact a reversal of current in the right-hand half of the winding; in (b) there is the additional feature of the suppression of the m.m.f. in certain poles (i.e. the relevant conductors are disconnected). The result is shown in (iii), and in each case the m.m.f. is a combination of 6-pole and 10-pole fields. Thus with $k = 1$ and the basic $p = 4$ for the unmodulated winding we obtain $p_1 = p - k = 3$ and $p_2 = p + k = 5$.

When these 1-ph patterns are allied to those of the other two phases, the resultant can be a 3-ph field of one or other pole-number; a given mechanical spacing has different electrical spacings for different pole-numbers, and can be made such as to favour one of the pole-numbers and suppress the other.

Let the angular phase spacing for the basic $2p$ -pole winding be $r(2\pi/3)$ rad where r is any integer other than 3 or a multiple of 3. After modulation to $p \pm k$ the electrical spacing becomes $[(p \pm k)/p]r(2\pi/3)$. One of the modulation products is suppressed if its phase spacing is a multiple of 2π rad, say $2m\pi$; that is

$$\frac{p \pm k}{p} r \frac{2\pi}{3} = 2m\pi \quad \text{whence} \quad \frac{m}{r} = \frac{1}{3} \left(1 \pm \frac{k}{p} \right)$$

To suppress $p_1 = p - k$ it is necessary for $m/r = \frac{1}{3}(1 - k/p)$; the angular phase spacing for p_2 is derived from $(1 + k/p) = 2 - 3m/r$ so that

$$\frac{p + k}{p} r \frac{2\pi}{3} = \left(2 - \frac{3m}{r} \right) r \frac{2\pi}{3} = (2r - 3m) \frac{2\pi}{3}$$

So long as r is not a multiple of 3, the spacing is $\pm 2\pi/3, \pm 4\pi/3, \pm 6\pi/3 \dots$ and so correct for developing p_2 . Similarly, suppressing p_2 develops p_1 . With $r = 1, 2, 4, 5 \dots$ and $m = 0, 1, 2 \dots$ the numbers of pole-pairs obtainable for pole-changing are

$$k = 1: 1/2, 4/5, 7/8, \dots (3m+1)/(3m+2)$$

$$k = 2: 2/4, 5/7, 8/10 \dots (3m+2)/(3m+4)$$

$$k = 3: 3/6, 12/15, 21/24 \dots (9m+3)/(9m+6)$$

Harmonics. A 3-ph m.m.f. has only an approximately sinusoidal space distribution, and when modulated by a rectangle (or shortened rectangle), itself equivalent to a fundamental and a series of odd-order harmonics, the result is rich in harmonic m.m.f.s. Each produces a component modulation and a corresponding number of space harmonics (some forward, others backward) capable of developing 'spurious' torques. Such undesirables could be reduced if sinusoidal modulation were practicable, but only an approximation in the form of a stepped wave is feasible.

Overall Modulation. In the original paper by Rawcliffe et al. [43] simple reversal was the modulation method, considered in respect of the m.m.f. or current distributions of individual phase windings. Later developments [44] dealt with total modulation, i.e. the effect of applying a space modulation wave to the current distribution of a complete winding. The modulation of a p -pole-pair basic winding by a $(p+q)$ -pole-pair wave is equivalent to modulation of the phase windings individually by separate waves each of $(p-q)$ pole-pairs, and conversely. The outcome was the development of a wide-ratio two-speed single-winding machine as described by Fong [45]. Considerable ingenuity has been expended on the technology of p.a.m. machines, which are commercially available for close or wide ratios of two or of three speeds, all from a single basic 3-ph winding.

Generally, p.a.m. windings have coil groups comprising different numbers of coils, or even different numbers of coil groups per phase. The design technique lies in the arrangement of the individual coils, the choice of the phase to which each coil should belong, the relative current directions, and whether these have to be reversed for pole-changing. The three component phase windings may each be unbalanced, but by the method of *symmetrization*, described by Fong and Broadway [112], they may in combination be equivalent to a balanced 3-ph winding.

8.16 Frequency control

Straightforward frequency control of motor speed is sometimes employed in the electrical propulsion of ships as the a.c. generator speeds are readily variable. In industry, portable tools such as drills, grinders and woodworking appliances may be operated by small induction motors fed from high-frequency rotary or static equipment; typical supplies are 110/220 V at 180 Hz and 125/250 V at 200 Hz. These drive 2-pole motors at speeds approaching 10 800 and 12 000 r/min respectively and give a ratio of active mass to output as low

as 1 kg/kW. The inherent pull-out torque limitation is a protection against overload.

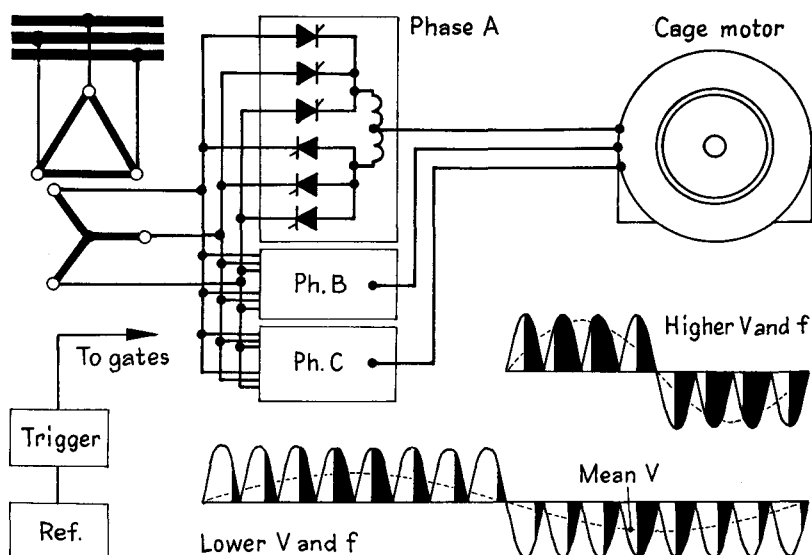
Many motor speed controls are based on d.c. machines fed from 3-ph industrial-frequency supplies through thyristors. However, with the introduction of fast turn-off capability and of special commutating capacitors, efficient frequency conversion plant is available to enable the more robust cage induction motor to be utilized instead. In modern practice, the frequency-changer for developing a variable secondary frequency supply from a fixed-frequency primary network is almost invariably a solid-state device based on the thyristor.

Static Frequency Conversion

Thyristors for frequency conversion operate in a switching mode in which they either conduct or 'block'. The thyristor trigger electrode (or 'gate') can be pulsed with a suitable voltage to initiate conduction, but thereafter loses control: it cannot cut off the current. Restoration to the blocking mode, a process of *commutation*, requires the thyristor current to be reduced to zero by some circuit property and a reverse bias to be applied between cathode and anode for several microseconds. If the thyristor is fed from an a.c. supply, the natural reversal of the alternating voltage each half-period can be exploited to secure both of these requirements, giving *natural* (or phase) *commutation*. This technique is applied in the cycloconverter. In the more important d.c.-link converter, a primary a.c. supply is first rectified to d.c., which is then inverted to give an a.c. output of controllable frequency. As the inverter thyristors are d.c. fed, natural commutation is not feasible. Auxiliary circuits for *forced commutation*, usually employing charged capacitors, must therefore be provided. For triggering the conduction mode, a single pulse of 150–300 μ s duration or a train of 25 μ s pulses with a mark/space ratio of 1/5 may be applied to the gate.

Cycloconverter

The simplest 3-ph/3-ph cycloconverter requires 18 thyristors to generate an output frequency necessarily lower than that of the primary supply, for if it exceeds about one-third of the input frequency the output waveform becomes intolerably distorted. The essential connections are given in Fig. 8.44. The thyristors are sequentially switched to 'fabricate' the output waveform from segments of the supply voltage, the switching rate and sequence being determined by the control circuitry (not shown in detail) which develops and distributes the low-power firing pulses. Two typical waveforms are illustrated for one phase bank, one for a frequency near the upper limit and the other for a low frequency. The control of voltage is obtained by suitable advance or delay of the firing pulses. Centre-tapped interphase inductors are required to smooth transfer from one thyristor to the next. The cycloconverter is able to change the frequency in either direction so that it can be applied to the regenerative braking of induction motors; but generally its limited frequency range restricts its use to traction (e.g. for developing $16\frac{2}{3}$ Hz from a 50 Hz primary supply for railways with low-frequency 1-ph series-motor drives),

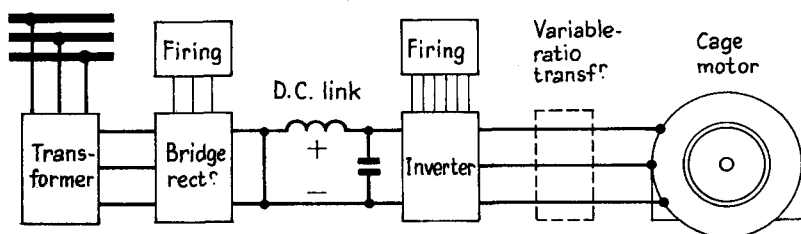


8.44 Cycloconverter.

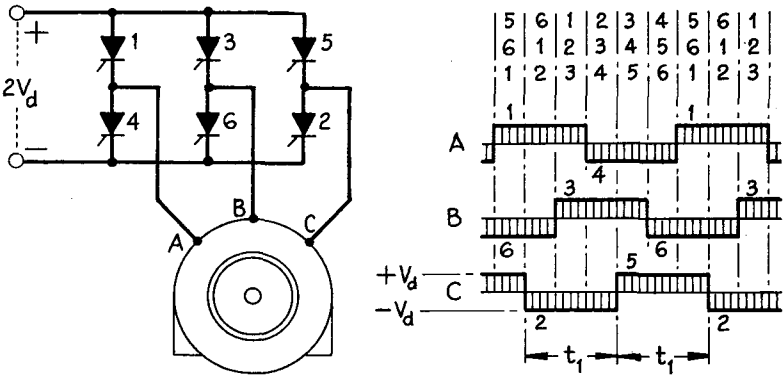
and to steel-rolling and comparable drives with many motors that must have group speed control.

D.C.-link Converter

If the primary a.c. supply is first rectified in a diode or thyristor bridge and the resulting direct voltage filtered, the d.c. supply can be fed to a static inverter to change it into a.c. (usually 3-ph) power at any other frequency within a wide range (e.g. 5–150 Hz). The inverter power thyristors are triggered in sequence so that a rectangular or stepped output waveform is generated. The control circuitry comprises an adjustable-frequency reference oscillator with logic elements to form and distribute firing pulses in the proper sequence and at the required rate, and as the inverter has a d.c. input its output frequency limit is determined by the turn-on and forced-commutation turn-off times of its constituent thyristors. For small equipments rated up to a few kilovolt-



8.45 D.C. link inverter.

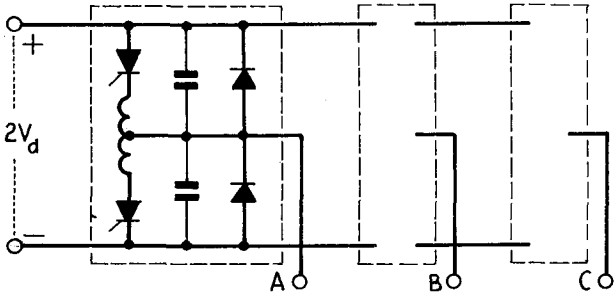


8.46 Action of inverter.

amperes, power transistors may be substituted for thyristors, with the advantages of higher switching rates, simpler commutation, and sinusoidal outputs at frequencies up to 400 Hz.

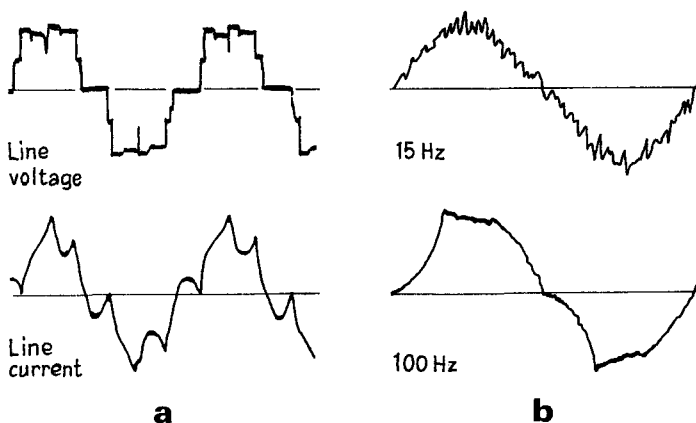
The frequency control of the speed of an induction motor generally involves also a concomitant control of its voltage, for in order to preserve a constant peak flux density in the gap and an invariable condition of core saturation, the voltage must be roughly proportional to the frequency. The d.c.-link arrangements in Fig. 8.45 differ in that in (i) the link voltage is variable while in (ii) it is constant. In (i) the link voltage is controlled by using thyristors in the bridge rectifier; in (ii) the rectifier is a diode bridge, and control of the output voltage is by inclusion of a variable-ratio transformer, or by a d.c. chopper interposed between the link and the inverter. In each case the output frequency is determined by the firing pattern of the inverter thyristors. More elaborate methods may be used if the speed accuracy is such as to demand closed-loop control.

The basic inverter action can be seen from the simplified diagram in Fig. 8.46. Let thyristor 1 be on and 4 be off: this condition is maintained for a time interval t_1 , at the end of which 1 is turned off and 4 on for a further time t_1 . The two rectangular half-waves form a complete period of frequency.



8.47 Three-phase inverter with forced commutation.

$1/2 t_1$ supplied to phase A of the motor. Similar waves with appropriate timing are applied to phases B and C. The motor inductance delays current decay, so that the thyristors must be paralleled by diodes as in Fig. 8.47, which also shows the forced-commutation capacitors. When accelerating the motor from rest, the inverter operates at minimum frequency (e.g. 5 Hz) initially, the frequency being raised at a rate appropriate to the inertial and load torque demand by feedback control of the reference oscillator frequency, which in turn controls the firing pulse timing. Fig. 8.48(a) shows typical oscillograms of the motor line voltage and current.



8.48 Inverter output waveforms.

(a) Thyristor inverter

(b) Chopped d.c.

For small motors it is possible to obtain a near-sinusoidal inverter output from a d.c. link by commutating each phase of the machine rapidly across the positive and negative terminals, the ratio of the durations of the opposing-polarity pulses determining the voltage level. Fig. 8.48(b) gives typical waveforms for lower and higher speeds. In the latter case the limit of sine-voltage forming by the chopping process is exceeded, and the supply unit then operates with the thyristors switched only once per period.

Motor Performance

Harmonics. The switching function of a diode or thyristor rectifier distorts the current waveform from the primary a.c. supply system, particularly if the system impedance is appreciable. The effect is to develop disturbing voltage dips and to impose on the current a strong harmonic content. With installations of large rating the supply authority may require supply to be taken at a higher voltage (e.g. 33 kV), or the installation of harmonic filters, or the use of a 24-pulse system of rectification.

The harmonic content of the output of a frequency-changer adds to motor

losses. Compared with sinusoidal operation, the losses in a cage motor fed from an inverter are augmented by: (i) I^2R arising from harmonic currents and by increased skin effect in windings; (ii) high-frequency core loss; (iii) loss due to skew leakage flux, which may be large at harmonic frequencies; (iv) loss in the end windings; (v) load (stray) loss arising from space-harmonic m.m.f.s excited by time-harmonic currents. Investigations of the additional losses have been undertaken by Chalmers and Sarkar [46] and by McLean [70].

Harmonic voltages, and the currents produced by them in the frequency-dependent motor leakage impedance, develop forward- or backward-directed harmonic torques due to the interaction at various speeds of pairs of gap-flux and current components. With k harmonic currents forming travelling-wave fields, there are k^2 harmonic torques, comprising one fundamental oscillating torque, $(k-1)$ forward or backward torques, and (k^2-k) hunting torques. According to Largiader [47] the last-named may be an appreciable fraction (e.g. 7½%) of the main driving torque. Hunting occurs when gap-flux harmonics and rotor currents, moving at different peripheral speeds, interact to generate a torque oscillation that makes no *mean* contribution to the useful torque. At the low-speed end of the control range the motor may exhibit a hunting torque modulation at six times working frequency.

Speed. Inverter drives gives a wide range of operating speed, continuously variable from standstill to 9 000 r/min for a 2-pole motor. Maximum speeds of 4 500 r/min or less are more economically provided if the motor is a 4- or 6-pole machine. The speeds quoted are for a maximum inverter output frequency of 150 Hz; higher values can be obtained if the inverter is equipped with fast-switching thyristors. The parameters of interest in control are the rotor resistance and the load inertia. Lawrenson and Stephenson [48] show that when a machine is reversed by dropping the inverter frequency, the speed falls more slowly than the instantaneous synchronous speed: the slip is therefore negative and the machine regenerates. Inverter-fed motors running at constant speed are analysed by Ward [49] using 2-reaction methods.

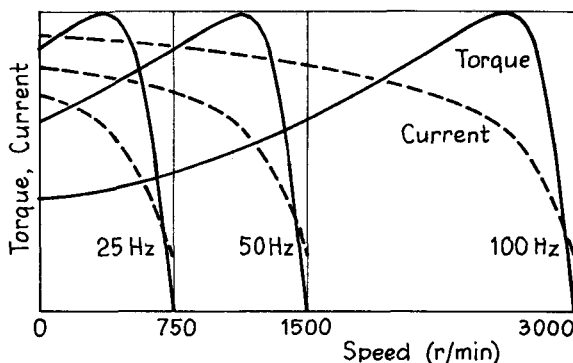
Constant-flux Operation. With voltage related to frequency, a drive can provide rated torque over the whole speed range. Consider the gross torque of a motor supplied (a) at a reference angular frequency ω_0 and running at slip s_0 for which the e.m.f. of the rotor is E_0 and its resistance and leakage reactance are r and x_0 ; and (b) at a different frequency ω , slip s , and rotor quantities E , r and x . From eq.(8.5) the torques per phase are

$$\frac{1}{\omega_0} \cdot \frac{s_0 E_0^2 r}{r^2 + s_0^2 x_0^2} \quad \text{and} \quad \frac{1}{\omega} \cdot \frac{s E^2 r}{r^2 + s^2 x^2}$$

where for ω the rotor reactance is $x = x_0(\omega/\omega_0)$. If $E_0/\omega_0 = E/\omega$ to preserve a constant gap flux, the torques are then

$$K \frac{\omega_0 s_0 r}{r^2 + s_0^2 x_0^2} \quad \text{and} \quad K \frac{\omega s r}{r^2 + s^2 [x_0(\omega/\omega_0)]^2}$$

These torques are the same, and take the same rotor current, if $s\omega = s_0\omega_0$.



8.49 Torque/speed characteristics for variable-frequency control.

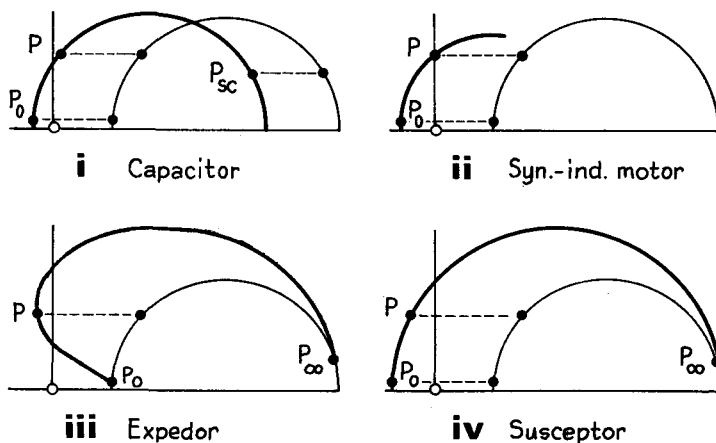
The torque/speed and current/speed relations for any operating speed are therefore closely related, as indicated in Fig. 8.49 for a motor operated on 25, 50 and 100 Hz. In this simple analysis no account is taken of changes in mechanical and core loss, or in effective resistance, or in magnetizing current or saturation, resulting from the change in frequency. To obtain a constant E/ω it is necessary to relate the motor voltage V to the operating frequency f by $V = a + bf$.

8.17 Power-factor adjustment

Because of its a.c. excitation, the full-load p.f. of an induction motor rarely exceeds 0.9, and to avoid tariff penalties it may pay to raise the p.f., by (i) connecting capacitors across the stator terminals or (ii) using the synchronous-induction machine. Formerly (iii) phase-advancers or (iv) special phase-compensated machines were employed. The broad effects of these methods are shown in Fig. 8.50 in terms of the modifications that they impose on the normal circle diagram. In (i) the circle is shifted bodily to the left by the amount of the constant leading current of the capacitor; in (ii) the normal operation is synchronous, the machine having a d.c. rotor excitation derived from a rotating exciter or a static rectifier; in (iii) and (iv) the operation is modified by injecting a capacitance effect into the rotor circuit or into the magnetizing branch of the equivalent circuit.

Static Capacitors

If the motor operates at full load with input active, reactive and apparent powers P , Q and S , and therefore at a power factor $\cos \phi = P/S$, the raising of the power factor to $\cos \phi'$ (for which $Q' = P \tan \phi'$) requires a leading reactive power $Q - Q' = P(\tan \phi - \tan \phi')$ and the capacitance required is $C = (Q - Q')/\omega V_1^2$. The capacitor may be made smaller by raising its operating voltage, e.g. by an autotransformer, if the motor supply voltage is low. Where adequate compensation is provided for full-load power-factor improvement, the system is overcompensated (though not, as a rule, significantly) at low levels of load. The switching requires care to avoid the possibility of momen-



8.50 Power-factor adjustment methods.

- (i) Capacitor. (ii) Synchronous-induction motor.
(iii) Expedor. (iv) Susceptor.

tary self-excitation of the motor as an induction generator, a condition that might produce a stator overvoltage condition. The capacitance of the phase-advancing capacitor bank may have to be limited to a value that makes self-excitation impossible.

Synchronous-induction motor

This machine is described in Chap. 10. A noteworthy feature is the large leading current taken at small loads, and the machine can be useful for compensating the power factor of a group of uncompensated motors.

Phase-advancers

A phase-advancer is a device connected into the rotor circuit of the induction motor, but mechanically external to it. The two basic devices can be classified as: (i) *Expedor*, developing a voltage that is a function of the rotor current and has some phase relation thereto; it generates or absorbs an e.m.f., equivalent to an impedance rise or fall. (ii) *Susceptor*, developing a voltage that is a function of the rotor open-circuit e.m.f. and has some phase relation thereto; it affects the magnetizing current of the motor, and consequently its effective magnetic-circuit susceptance.

In either case, the basic method consists in so adding to the rotor induced e.m.f. that the rotor current is advanced in phase, and the consequent m.m.f. reaction causes the stator current to be correspondingly advanced. The method requires the use of a slip-ring rotor and the injection thereby of voltages of suitable magnitude, phase-displacement and slip frequency from a separate phase-advancer machine. The principal types of advancer are Expedor: Kapp vibrator, Leblanc advancer, Scherbius advancer. Susceptor: frequency converter.

Kapp Vibrator. A small d.c. armature is connected in series with each slip-ring connection to carry the slip-frequency rotor phase current. The d.c. armature develops a slip-frequency vibratory torque with a permanent or separately-excited field system, and an e.m.f. corresponding to the instantaneous armature speed. On account of the inertia, the e.m.f. has a 'capacitive' effect. The vibrator must be cut out and replaced by a rheostat for starting.

Leblanc and Scherbius Advancers. Slip-frequency currents are supplied to a 3-phase commutator armature. If this armature is at rest, a rotating field of slip frequency is developed and a counter-e.m.f. of slip frequency appears at the commutator brushes. If now the armature is rotated by an auxiliary motor at a speed above that corresponding to slip frequency it develops (still at slip frequency) e.m.f.s proportional to the main rotor currents and of leading phase-displacement. The advancer is not applied to the slip-rings until the main motor has been started and is up to its operating speed.

Frequency Converter. This takes the form of an armature with d.c. type winding, commutator and slip-rings, connected mechanically to the main motor. Three-phase brushgear on the commutator is connected to the main-motor rotor circuit, and an e.m.f. of slip frequency is injected by energizing the advancer armature through its slip-rings from the main a.c. supply. Rotation of the armature at the speed of the main motor results in slip-frequency e.m.f.s being tapped off the commutator, of phase dependent on the brush-rockers position.

Successful attempts have been made to incorporate phase-advancing devices into small induction motors to avoid the complication of the additional machine. These, however, as well as the types briefly described above, are no longer in common use.

8.18 Braking

Simple friction brakes may serve for hoists, lifts and cranes. Electrical methods are adopted for sophisticated drives, especially where precision in braking time or load positioning is called for, but they may impose severe duty cycles and involve both thermal and mechanical stresses.

Regenerative Braking

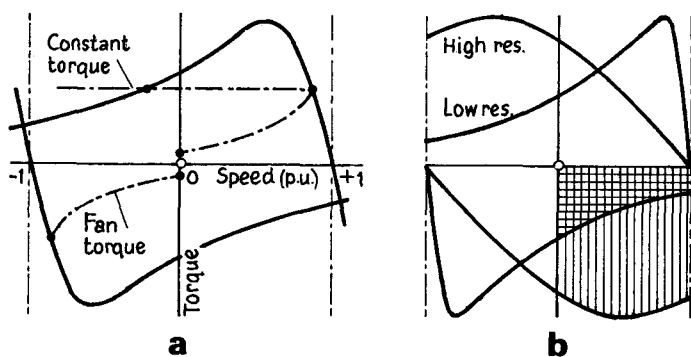
The motor, overdriven by the load into a negative-slip hypersynchronous speed, becomes an induction generator. In a slip-ring machine, the braking can be maintained by a moderate *rotor* external resistance and still return useful energy to the supply. Braking fails when the speed falls to synchronous speed. The braking range is, however, extended if the stator has a pole-change winding, in which case the rotor winding is normally of the cage type.

Plugging

Braking is obtained by reversal of the stator connections while the motor is running, so reversing the direction of the travelling-wave airgap field. The slip is then greater than unity and the machine develops a braking (i.e. a reversed)

torque, corresponding to the region between P_{sc} and P_{∞} in the circle diagram of Fig. 8.16. The stator and rotor currents are large. Cage motors of about 20 kW are plugged direct, using the star connection if a star-delta switch is provided; larger motors require stator resistors. Slip-ring motors employ rotor resistance for current limitation. Problems of thermal rating may arise for frequent braking duty, and contactors may need frequent servicing.

Insofar as torque transients (Sect. 8.20) can be ignored, or have a duration small compared with the braking time, the essentials of plugging can be based on the steady-state torque/speed characteristics of the motor and its load, Fig. 8.51, together with the inertia of the drive. In the diagram, (a) shows two



8.51 Torques during plugging.

typical load torques, one for a fan and the other for a hoist or elevator; (b) shows that the power available for plugging can be increased by augmenting the *rotor* resistance. Simultaneous reversal of the stator connections and inclusion of rotor resistance in a slip-ring machine is fully practicable, but the same effect can be achieved by including saturistors in the rotor winding. Alternatively, a deep-bar rotor cage may give a convenient rise in effective rotor resistance following the plug-switching operation as the slip frequency is suddenly increased from s to $(2-s)$ times ω_1 . Shepherd [54] examines the characteristics and requirements for rapid plugging (including the effect of control by saturistors) and shows that there is an optimum effective resistance.

EXAMPLE 8.7: A 420 V 50 Hz 3-phase star-connected 6-pole induction motor having stator and referred-rotor leakage impedances both given by $(1.0 + j2.0) \Omega/\text{ph}$, drives a pure-inertia load at 960 r/min. The total inertia of the drive is $J = 3.0 \text{ kg}\cdot\text{m}^2$. Estimate the time to bring the drive to standstill by plugging. Neglect friction, windage and load torques other than inertial, and the effect of transient switching torques.

Immediately after switching the speed is given by $\omega_r = \omega_s(1-s)$ and the angular acceleration by $\alpha = d\omega_r/dt = -\omega_s(ds/dt) = M/J$, where M is the motor torque over the range $s_1 = 1.96 \text{ p.u.}$ initially to $s_2 = 1.0$ at rest. Then $dt =$

$-\omega_s(J/M)ds$. Taking the torque per phase as the approximation in eq.(8.20), then

$$dt = -\frac{J\omega_s^2}{3V_1^2} \left[\frac{(r_1 + r_2/s)^2 + X_1^2}{r_2/s} \right] ds$$

Integrating the term in square brackets gives

$$\begin{aligned} & [(r_1^2 + X_1^2)/2r_2] (s_2^2 - s_1^2) + 2r_1(s_2 - s_1) + r_2 \ln(s_2/s_1) \\ &= -24.1 - 1.9 - 0.7 = -26.7 \end{aligned}$$

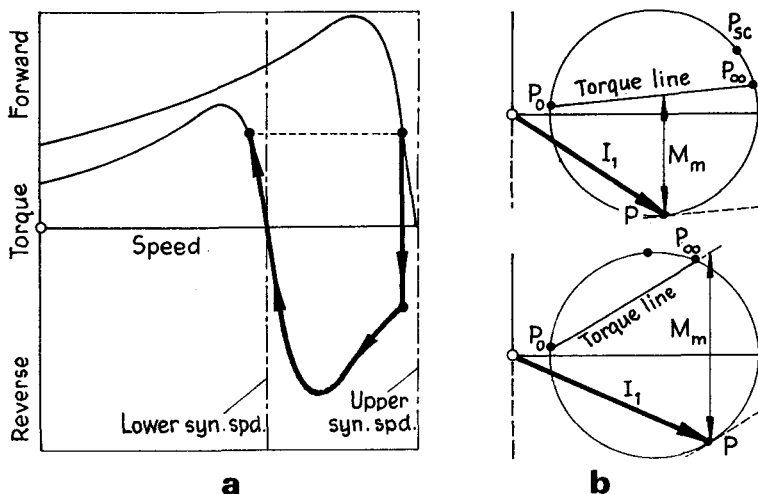
with $r_1 = r_2 = 1.0$ and $X_1 = (x_1 + x_2) = 4.0 \Omega$; then the total time is

$$t = 3.0 \times 105^2 \times 26.7/420^2 = 5.0 \text{ s}$$

in which $\omega_s = 2\pi \times 1000/60 = 105 \text{ rad/s}$.

Two-speed Plugging

In some circumstances, loads driven by pole-change motors may be retarded by switching from the high-speed to the low-speed connection. A braking torque is developed in this range. Fig. 8.52(a) indicates that during run-down the torque changes sign twice in rapid succession, passing through a strong reverse peak. Such a duty calls for a robust design, with taper-shaft couplings, welded instead of keyed cores, well-braced stator windings and reinforced cage-rotor bars and end-rings. It is to be noted that inclusion of resistance in series with the *stator* windings is deleterious, for it actually increases the peak reverse torque: this is shown by the circle diagrams (b) in Fig. 8.52.

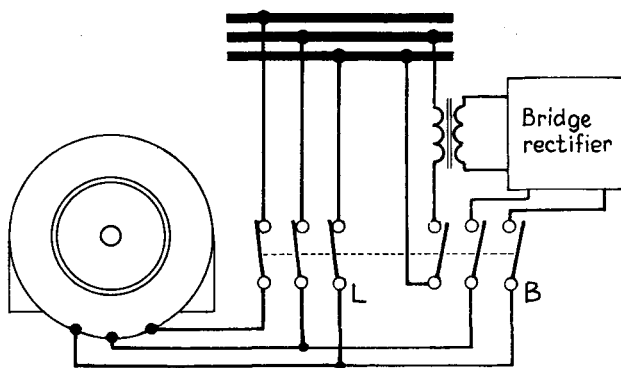


8.52 Two-speed plugging.

Motors required to plug high-inertia loads may have their own inertia minimized by low speed and small diameter, a combination which gives a high inherent leakage reactance. The magnetizing current tends to be large because the facility of an easily repaired stator winding necessitates the use of open slots.

D.C. Injection Braking

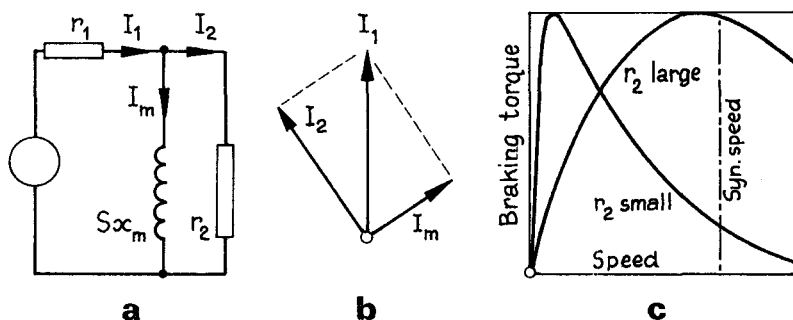
In this method the stator windings, immediately after their disconnection from the 3-phase supply, are excited with direct current from a rectifier and (in the case of slip-ring machines) resistance is introduced into the rotor circuits. The d.c. excitation establishes a *stationary* gap flux, the magnitude and position of which depends on the resultant combined m.m.f. of the d.c. stator and induced rotor currents. A rotor e.m.f. proportional to the flux and to the speed is generated in the rotor, the braking effect being the result of the I^2R loss. A typical connection diagram is given in Fig. 8.53; the



8.53 D.C. injection braking.

machine operates as a motor with contactors L closed, while with L open and B closed a direct current is fed through two stator phases, the third being left on open circuit. Alternative connections (such as those in Fig. 10.44) are possible.

Approximate Analysis. Rotation of the rotor at synchronous speed in the stationary airgap field gives conditions similar to those in an a.c.-fed motor at standstill; at rest, with no induced rotor e.m.f., the condition is like that of an a.c. machine at synchronous speed. The operation is therefore described by an equivalent circuit whose parameters have the factor $S = (1 - s)$, where s is the slip, and fed with a constant applied current, I_1 in Fig. 8.54(a), instead of a constant applied voltage. Ignoring saturation and rotor leakage, the rotor impedance is simply r_2 and the equivalent circuit has the phasor relation $I_m = I_1 - I_2$ and closely the scalar relation $I_m^2 = (I_1^2 - I_2^2)$. The flux due to I_m generates a rotor e.m.f. $E_2 = SI_m x_m$, where x_m is the magnetizing reactance chosen to relate the magnetizing current to the rotor e.m.f.: this leads to the



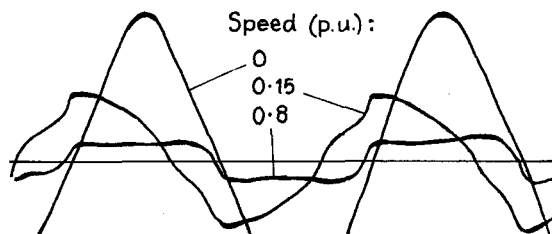
8.54 D.C. injection braking effects.

equivalent circuit in (a). Resolving Sx_m and r_2 into their series equivalent gives the resistance term

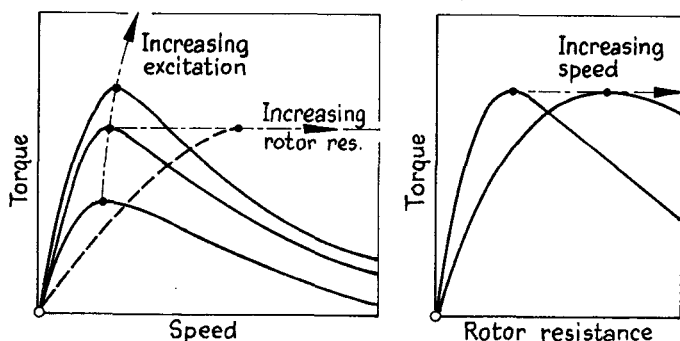
$$r_{2s} = r_2 (Sx_m)^2 / [r_2^2 + (Sx_m)^2]$$

from which the rotor I^2R loss per phase for a speed having the fraction S of synchronous speed is $I_1^2 r_{2s}$. The braking torque corresponds to a synchronous power per phase of $I_1^2 r_{2s} / S$. Maximum torque occurs for $S = r_2 / x_m$ and depends only on the stator current and the magnetizing reactance. The braking-torque/speed relation resembles that of the torque/slip for a normal induction motor, as shown in Fig. 8.54(c). For dynamic braking the average torque is of importance, and it can be shown to be greatest for $r_2 = \frac{1}{2} x_m$.

Effect of Saturation. The net magnetizing current is greatest at zero speed, for then there is no opposing rotor current, so that the airgap flux is a maximum and saturation conditions are inevitable. The value of x_m will be low at standstill and will rise with increase of speed. The difficulty in applying the approximate analysis is consequently the strong effect of saturation on x_m . Krishnamurthy [52] has investigated the gap-flux distribution for a machine excited as in Fig. 8.53 for various speeds, and some of these patterns are given in Fig. 8.55 to bring out the variations in flux level, position and waveform. Clearly the speed and braking effort is a complex function of the saturation. Butler [53], using the equivalent circuit of Fig. 8.54(a) but with rotor leakage reactance included, has derived operating characteristics with



8.55 Gap-flux distribution for constant exciting currents.



8.56 D.C. injection braking characteristics.

saturation taken into account. He shows that a severe torque limitation results from saturation but that for most practical applications it is permissible to ignore rotor leakage.

Characteristics. Typical performance curves are given in Fig. 8.56. Injection braking takes little power from the supply and gives smooth braking torque, useful for mine winders and high-inertia loads. Its advantage over plugging is the absence of a reverse-rotation airgap field (and therefore no tendency for the machine to run backwards) and a lower rotor I^2R loss. Dubey and De [55] give methods of evaluating the performance of a d.c. braking system with saturistors on the rotor, and show that the braking performance can be optimized by a suitable combination on the rotor of resistors and saturistors.

Capacitor Braking

A motor on load, disconnected from the supply and reconnected to a bank of capacitors, will self-excite as an induction generator (Sect. 8.20). An alternating voltage is built up in the stator windings until balance between the capacitive and saturating magnetizing reactance obtains. The load inertia drives the machine with a slip negative with respect to the 'synchronous' speed of self-excitation, and the speed falls. At the same time the magnetizing current also falls, reducing the saturation level and increasing the magnetizing reactance, so lowering the self-excitation frequency. By proper choice of capacitance, the 'synchronous' speed can be maintained below the rotor speed (i.e. with negative slip) down to low speeds. The method is of use where positive braking to standstill is not required, the load stopping the machine in the time allowable. Resistors may be used in the stator circuit in series with the capacitor bank to control the torque. Where braking to rest is a requirement, the magnetic field built up by self excitation is exploited by short-circuiting the stator windings through resistors for the final stage. If the load inertia is high, however, a d.c. injection scheme may be used instead.

Capacitor braking, alone or combined with d.c. or magnetic braking, has the advantages of high braking torque, lower loss, small energy demand from the supply, and quiet operation.

Braking Conditions

In terms of synchronous power, the motor torque is the rotor input P_2 , while the rotor I^2R loss is sP_2 . The ratio of I^2R loss to torque is therefore simply the slip s . The average loss between any two speeds corresponding to slips s_1 and s_2 is therefore $P_2(s_1 + s_2)/2$. The time t for the speed change depends on the inertia of the drive system, and the energy expended in the change is $P_2 t(s_1 + s_2)/2$ which, for given conditions, is proportional to $(s_1 + s_2)$. In acceleration from standstill ($s_1 = 1$) to normal running ($s_2 \approx 0$), that part of the rotor input dissipated electrically is proportional to 1. During retardation by plugging the initial condition is approximately $s_1 = 2$, and at standstill $s_2 = 1$: the loss is thus proportional to 3. If acceleration or braking is by variation of the stator supply frequency, as in the inverter or capacitor methods, the synchronous speed is itself changed and the slip remains small so that the loss is proportional to, say, 0.2. The motor duty therefore depends not only on the inertia but also on the method of braking. Because of the time taken to stop them, high-inertia loads may be unsuitable for plugging.

Precision methods of braking can be very effective. Conveyors and multi-motor drills stopped every 8 s, machine-tools brought to rest with a positional accuracy of ± 0.025 mm, and a 600 Hz motor running near 36 000 r/min brought to rest in $3\frac{1}{2}$ s are impressive examples.

8.19 Unbalanced operation

Unbalanced conditions of operation may arise accidentally, or may be deliberately introduced, by asymmetrical supply voltages or inequalities of phase impedance.

Voltage Asymmetry

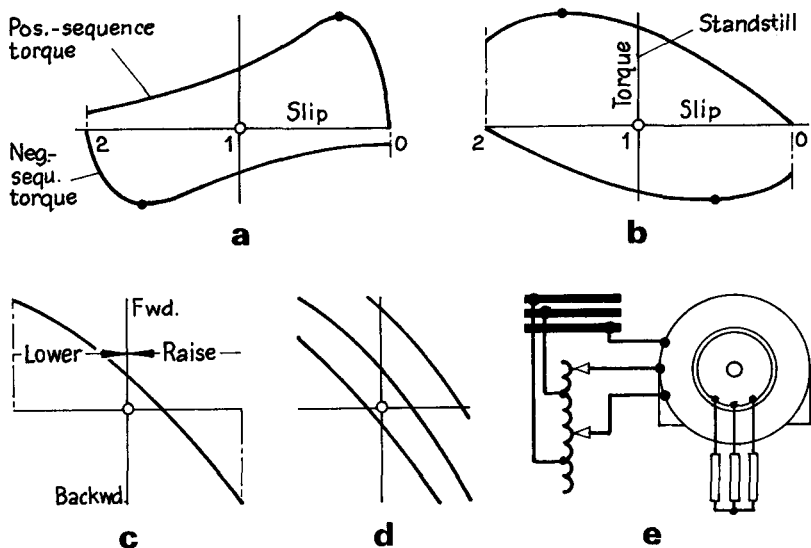
The performance of a 3-ph induction motor is modified if its stator phase voltages differ in magnitude or phase-displacement, or both. Such voltages can be considered as the superposition of symmetrical positive-, negative- and zero-sequence components, each of which (disregarding saturation nonlinearities) produces corresponding currents in the windings. The p.p.s. components develop a 'normal' travelling-wave m.m.f. in accordance with the phase sequence and direction of rotation; the n.p.s. components develop an m.m.f. travelling in the reverse direction. The effect of z.p.s. components, usually small, resembles 1-ph excitation of the machine with three times the number of poles.

As regards p.p.s. and n.p.s. components, the motor behaves as the combination of two machines, one with a phase voltage V_+ and slip s , the other with V_- and $(2 - s)$. The resultant mechanical power per phase is

$$P_m = I_{2+}^2 r_2 [(1 - s)/s] - I_{2-}^2 r_2 [(1 - s)/(2 - s)]$$

where the currents are determined by the respective sequence voltages and impedances. The resultant torque is $P_m/\omega_s(1 - s)$. The torque/slip curves in (a) of Fig. 8.57 are for a large degree of asymmetry. Minor asymmetries do not greatly affect the torque, but *any* asymmetry increases the I^2R loss.

Speed Control and Braking. Asymmetric-voltage control has been applied to



8.57 Negative-sequence control.

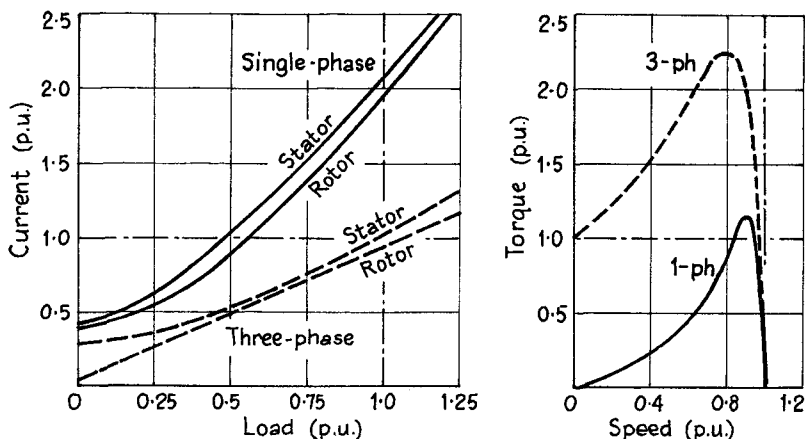
cranes. If a slip-ring motor is heavily loaded on rotor resistance, the torque/slip relations (a) in Fig. 8.57 are changed to those in (b), and the net torque approaches a linear relation with slip (c). The control enables a crane to develop a stable *upward* torque to restrain a load while it is being lowered. One method of achieving the family of characteristics in (d) is to connect an auto-transformer across two supply lines as in (e). If the tap positions can be crossed, it is possible to control from full forward symmetry to full backward symmetry through a single-phase condition when the two tappings coincide.

Single-phasing. This term refers to operation with one stator terminal accidentally disconnected (as by a blown fuse or a faulty contact on the 'run' side of a star-delta switch), a common cause of winding burnout. Fig. 8.58 compares the performance of a 3-ph motor under normal and single-phasing conditions. Clearly, if single-phasing occurs, a loaded motor may stall, because the disconnection of a stator terminal introduces a n.p.s. torque.

Single-phase Braking. If the stator windings of a 3-ph motor with a synchronous speed n_s are connected in series or parallel to a 1-ph supply so that all three currents are co-phasal, the distribution of the gap m.m.f. produces three times the normal number of poles, all pulsating at supply frequency. A rotor running at a speed higher than $n_s/3$ is therefore in a generating mode and is retarded. A machine with a high-resistance rotor can be braked down to standstill, a method sometimes used for the light-duty braking of machine-tools.

Winding Unbalance

A 3-ph motor with a 1-ph rotor winding has two stable operating speeds,



8.58 Normal and single-phasing performance.

respectively approaching synchronous and one-half synchronous speed. A comparable effect is observed when a 3-ph rotor has a winding unbalance, deliberate or resulting from a winding fault or a difference in the external impedances connected to the slip-rings.

The p.p.s. currents set up in the stator by a symmetrical p.p.s. 3-ph voltage form a normal travelling wave of m.m.f. of synchronous speed n_s with respect to the windings, and the corresponding gap flux induces p.p.s. stator and rotor e.m.f.s. The rotor e.m.f. circulates currents of slip frequency, but in consequence of phase unbalance these can themselves be resolved into p.p.s. and n.p.s. components. The latter travel relative to the rotor at a speed $-sn_s$, and therefore at $(-n_s + 2n)$ relative to the stator. The stator reacts to the rotor n.p.s. gap flux by induced currents of frequency $(1 - 2s)f$. The result is a reversed interaction torque which imposes a dip in the net torque/speed characteristic at one-half of synchronous speed. If the rotor phase impedance unbalance is large, the dip is deep enough to permit of stable half-speed operation. Several further phenomena occur. Negative-sequence currents are injected into the supply system, and pairs of p.p.s. and n.p.s. flux-current combinations develop pulsating torques and consequent vibration (except at half speed); the p.p.s. and n.p.s. flux axes move into and out of coincidence, producing strong cyclic magnetic saturation.

If half-speed running is to be an intended operating condition, the rotor unbalance should include a short-circuited phase winding, and saturation should be mitigated by use of an over-size frame. Transfer of the operating speed from one-half to full is difficult, and so is reduction from full to half speed unless the load torque is sufficient.

8.20 Induction generator

Operation of an induction machine in the generator mode, with negative slip, has been discussed in Sects. 8.1–8.8. There are two excitation conditions of interest.

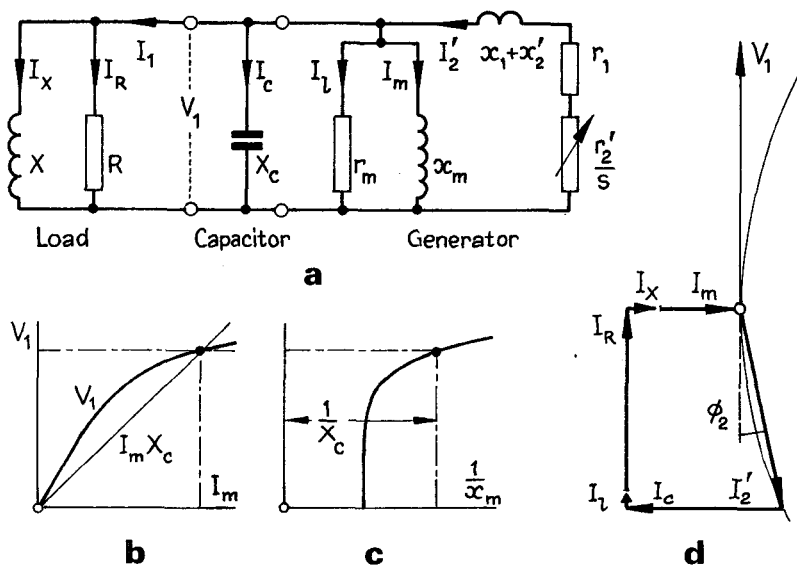
External Excitation

To determine its voltage and frequency, and to supply its lagging reactive magnetizing power, an induction generator is connected to an energized power network with synchronous machinery. The action can then be inferred in simple terms from Fig. 8.17: the active power generated is typically TP_{1g} , the mechanical drive power is $P_{1g}R$, and the network supplies the reactive power OT . The torque/slip characteristic lies in the negative-slip region of Fig. 8.11. If the prime-mover driving torque exceeds the peak electrical torque, the speed rises and at some high value of negative slip the generating effect vanishes.

The machine can utilize waste heat of process steam in chemical works and the power from small remote hydro plants. To minimize transients and heavy motoring currents, the set is brought up to synchronous speed before the stator windings are connected to a 3-phase network, remote starting and connection being readily possible. The induction generator needs little auxiliary equipment, it can run without hunting and its rotor can carry a robust cage winding or even be a solid cylinder. On external short circuit the excitation collapses, giving some automatic protection. The prime-mover need not be governed, although the generator must then be able to sustain overspeeds up to 2 or 3 p.u. The drawbacks are the need for a.c. magnetization, the moderate efficiency and (for mechanical reasons) the short air-gap.

Self-excitation

With the magnetizing reactive power provided by a capacitor bank, an external a.c. supply is not needed; but the operating frequency and the generated



8.59 Induction generator.

voltage are affected by the speed, load and capacitor rating. The capacitor must also supply any lagging reactive power demand.

Provided that the rotor has an adequate remanent field, the machine will self-excite. The operating conditions can then be described in terms of the approximate equivalent circuit (a) of Fig. 8.59. On *no load*, $I_c = V_1 \omega C = V_1/X_c$ must be equal the magnetizing current $I_m = V_1/x_m$. But V_1 is a function of I_m so that stable operation requires the line $I_m X_c$ in (b) to cut the voltage/excitation characteristic. If, as in (c), the terminal voltage V_1 is plotted as a function of the magnetizing susceptance $1/x_m$, the operating point is fixed by the corresponding capacitive susceptance $\omega C = 1/X_c$, and this settles the frequency. Under *load* conditions, the generated power $V_1 I_2 \cos \phi_2$ provides for the load power in R and the loss in r_m . The reactive currents must sum to zero, i.e.

$$V_1/X + V_1/x_m + I_2 \sin \phi_2 = V_1/X_c$$

which determines the capacitance for a given load and voltage. The several currents in network (a) are shown in phasor diagram (d).

EXAMPLE 8.9: The ideal short-circuit current of a 3 kV 50 Hz induction machine is 750 A/ph and the core-loss current is 10 A. The magnetizing current at 50 Hz is

Line voltage	kV:	1.45	2.5	3.0	3.4
Magnetizing current	A:	20	40	60	90

What star-connected capacitance is required for the machine (i) just to self-excite, (ii) to generate 3.0 kV? (iii) Determine the operating conditions for a load of $(125 - j20)$ A at 3.0 kV, 50 Hz.

Working per phase, the magnetizing susceptance is found:

Phase voltage V_1	kV:	0.84	1.44	1.73	1.96
Magnetizing current I_m	A:	20	40	60	90
Susceptance I_m/V_1	mS:	24	28	35	46

(i) The minimum capacitive susceptance for self-excitation is 24 mS, so that $C = 24 \times 10^{-3}/314 = 76 \mu\text{F/ph}$. (ii) Using the method of Fig. 8.59, interpolation gives 43 mS for 1.73 kV/ph (3.0 kV line), whence $C = 137 \mu\text{F/ph}$. (iii) The rotor current circle is drawn with diameter 750 A to scale; then $I_m = 60$ A, $I_l = 10$ A, $I_R = 125$ A, $I_X = 20$ A. The active current is $125 + 10 = 135$ A, and with $I_X + I_m = 80$ A the capacitor current is 104 A, requiring $C = 190 \mu\text{F/ph}$.

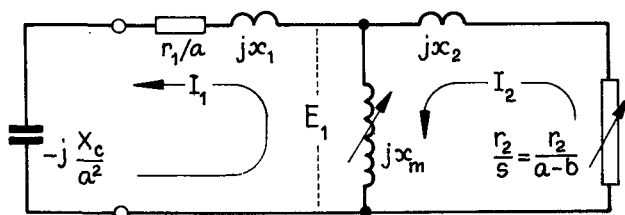
Performance. The voltage regulation of a self-excited induction generator may be large, and the terminal voltage waveform is likely to be distorted because of the need for a saturated condition to stabilize the excitation. The machine has been used in aircraft as a self-starting motor and brushless generator: for this application the driving speed for constant frequency must rise with increase of load, and because of the inevitably wide variations in magnetization characteristic it may be necessary, in machines to run in parallel, for star-connected saturable inductors to be fitted.

Accidental Self-excitation. Dangerous voltages may occur with induction generators working over long transmission lines if synchronous machines at the far end become disconnected and the line capacitance excites the induction machine. Motors with power-factor correction capacitors may generate if the capacitors remain connected to the running machine when for some reason the supply is cut off.

Braking. Self-excitation can be used for braking, as described in Sect. 8.18. On disconnection of the stator from the supply and its subsequent connection to a capacitor bank, the stator voltages are self-excited to develop a travelling-wave field, and for a generating mode the 'synchronous' speed of the field must be lower than the speed of rotor rotation. The drive power is drawn from the inertia energy and the speed consequently falls. The slip (i.e. the difference between the momentary field and shaft speeds) remains negative because, with reducing current, the effective magnetizing inductance rises — perhaps to 10 times its saturated value. But eventually the inductance stabilizes and further speed drop brings the machine into a motor mode.

Equivalent Circuit

The equivalent circuit for any given self-excited frequency ω_e can be expressed in terms of the parameters r_1, r_2, x_1, x_2, x_m and X_c all referred to the normal angular frequency ω_1 by using a factor $a = \omega_e/\omega_1$. If the shaft speed is expressed as the factor b of normal synchronous speed, then $a - b = s$, the slip (which is negative). The equivalent circuit for frequency ω_e can then be drawn as in Fig. 8.60, in which normal-frequency values of reactance are



8.60 Self-excited induction generator.

retained while the stator and rotor resistance and the capacitive reactance are suitably modified. As there is no externally applied voltage, the mesh equations for currents I_1 and I_2 are

$$0 = I_1 [(r_1/a) + j(x_1 - X_c/a^2)] + (I_1 - I_2) jx_m$$

$$0 = I_2 [(r_2/a) + jx_2] + (I_2 - I_1) jx_m$$

From these, eliminating I_1 and I_2 and separately equating the 'real' and 'imaginary' parts to zero, we have

$$\frac{x_m}{x_1} = \frac{(r_1 x_2 / r_2 x_1) [(b - a)/a] + (X_c / a^2 x_1) - 1}{1 - (r_1 / r_2) [(b - a)/a]}$$

Knowing x_m/x_1 and therefore x_m , for chosen values of a , b and X_c , the value of E_1 can be obtained from the magnetization characteristic and the rotor current derived. The braking torque per phase is then $I_2^2 r_2 / (b - a)$.

8.21 Transients

The steady-state stator and rotor currents and the travelling-wave gap flux that exist in an induction machine running at a fixed speed with a constant unidirectional torque have to be established from the instant when the stator of the inert machine is switched on to the supply. Magnetic energy has to be supplied, as has the kinetic energy of the rotating system and the various losses, as well as the energy absorbed by the load in raising it to the final steady-state condition. The energizing process is complicated by the reaction of cores to the rapid rise of flux density, the skin effect in conductors, and the mechanical deformations of motor shaft and structure as quickly varying torques are developed. The difference between the actual and the steady-state conditions for given speed and load are bridged by transient currents and fluxes, which appear instantaneously and thereafter decay exponentially at rates determined by the parameters and time-constants of the several electric circuits and the instantaneous speed of the machine. The term *switching* is applied to transients arising as a result of the energizing of an inert machine.

Transients are also generated when one steady state is changed to another, as when *re-switching*. If a running machine has its load suddenly altered, or if it is disconnected from the supply for a brief period and then reconnected, transient currents and torques appear. These transients are sometimes of great severity, as when a motor is plugged or dynamically braked, or subjected to supply-voltage interruptions especially where there are capacitor banks across the stator terminals. Considerable torque and current transients may occur even when motor terminal connections are changed during run-up by a star-delta switch. Very large induction machines, when switched, re-switched or subject to the voltage fluctuation resulting from a power-system fault, may react strongly on the system by drawing or supplying transient power.

Constant-linkage theorem. Physical appreciation of the transient condition is afforded by the *theorem of constant linkage*, which states that in a closed circuit of zero resistance, the algebraic sum of its magnetic linkages remains constant. No circuit (superconduction apart) is devoid of resistance, but the linkage nevertheless remains instantaneously constant immediately after a sudden change because it takes time for stored magnetic energy to be altered. a process approximating to a time-exponential rise or decay. When, for example, a 3-phase voltage is suddenly applied to the stator terminals of an inert induction machine, phase fluxes begin to build up at rates proportional to the instantaneous voltages (which are inevitably unequal). The stationary rotor develops corresponding opposition currents that initially maintain the inert condition of zero linkage. If the rotor moves, these currents may be carried round, but rapidly decay. On disconnecting a running motor, the stator currents are quenched; but as the short-circuited motor maintains its linkage by a unidirectional current, a trapped or 'frozen' flux is carried round with the rotor. The trapped flux decays, but induces rotational em.f.s in the stator windings. The

transient currents and torques that occur on re-switching depend on the stator e.m.f.s so induced, and if these have phase-opposition to the supply voltages at the instant of reconnection, the transient currents may be severe and the transient torque may have a negative (retarding) peak.

Analysis

Prediction of transient performance is difficult. The differential equations that must be used for adequate modelling of an induction machine are complicated, and if there is a change of speed during the process of a transient, the equations are (in the mathematical sense) nonlinear, quite apart from inevitable physical nonlinearity resulting from deep-bar effects and magnetic saturation.

For transient analysis, Wood and his co-workers [56] use the 2-axis model (Sect. 7.5). Using the impedance matrix of Fig. 7.14, the voltage equations for the stator 2-axis windings 1D and 1Q, and the corresponding rotor windings 2D and 2Q, are:

$$1D: v_{1d} = r_1 i_{1d} + (L_m + L_1) p i_{1d} + L_m p i_{2d}$$

$$1Q: v_{1q} = r_1 i_{1q} + (L_m + L_1) p i_{1q} + L_m p i_{2q}$$

$$2D: v_{2d} = r_2 i_{2d} + (L_m + L_2) p i_{2d} + L_m p i_{1d} \\ + (L_m + L_2) \omega_r i_{2q} + L_m \omega_r i_{1q}$$

$$2Q: v_{2q} = r_2 i_{2q} + (L_m + L_2) p i_{2q} + L_m p i_{1q} \\ - (L_m + L_2) \omega_r i_{2d} - L_m \omega_r i_{1d}$$

The 2-axis transient equivalent voltages for the case to be considered are then imposed and the equations solved, by iteration if there is a change in the rotor speed ω_r . In many cases the rotor circuits are short-circuited and have no externally applied voltage, so that $v_{2d} = v_{2q} = 0$. The validity of the method can be shown by presuming steady-state conditions with a constant slip s . Then p is replaced by $j\omega_1$ and ω_r by $\omega_s(1-s)$ where ω_s is related to ω_1 by the number of stator pole-pairs. Combining ω_1 with the inductances to give reactances, and inserting the condition of short-circuited rotor windings, then there is no difference between the d- and q-axis windings except one of time-phase. Consequently it is possible to write $v_{1d} = v_1$, $v_{1q} = -jv_1$, $i_{1d} = i_1$, $i_{1q} = -ji_1$, $i_{2d} = i_2$ and $i_{2q} = -ji_2$. Then

$$1D, 1Q: v_1 = [r_1 + j(x_m + x_1)] i_1 + jx_m i_2$$

$$2D, 2Q: 0 = [r_2 + j(x_m + x_2)] i_2 + jx_m i_1 \\ - (1-s) [(x_m + x_2) i_2 + x_m i_1]$$

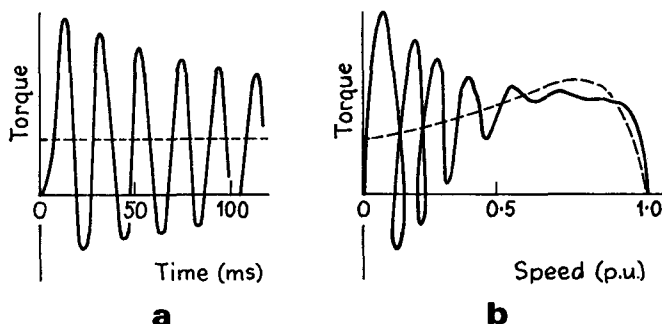
These express the behaviour of the steady-state equivalent circuit, and the d- and q-axis contributions to the torque are equal.

There are alternative analytical methods. Smith and Sriharan [57] apply instantaneous symmetrical components; Enslin, Kaplan and Davies [58] apply the differential equations of coupled circuits using parameters derived from

indirect tests and show that the transient stator and rotor current components each consist of a pair of terms with complex angular frequencies. Torque components arise from the consequent flux and m.m.f. distribution waves defined by amplitude, decrement and angular speed of rotation. Unidirectional synchronous torques are developed by the interaction of flux and m.m.f. waves of the same angular speed, pulsating torques by those of different speeds, with all stator-flux/rotor-m.m.f. and rotor-flux/stator-m.m.f. products accounted. There exist nine decaying transient torques superimposed on the steady-state torque.

Switching

Starting-torque transients are affected by the instant of connection of the first phase to close, and the delay (if any) in closure of the other two phases. The initial peak torque may also be affected by residual flux left after a previous duty cycle. Assuming the machine to remain at rest, Slater and Wood [56] give Fig. 8.61 (a) as a typical torque/time pattern with simultaneous switching



8.61 Starting transients.

on all phases, the main component being a damped alternating torque. An approximate expression for the total peak torque at starting in terms of the steady-state locked-rotor torque is

$$M_p \approx 1 + \sqrt{[k(1 + \omega_1^2 \tau^2)]}$$

where $\tau = (L_1 + L_2)/(r_1 + r_2)$ is a time-constant derived from the stator and rotor leakage inductance and resistance, and k is a factor to account for delay between the application of the first and last line voltages; $k = 1$ for simultaneous switching.

Transient torques persist for several periods of the supply frequency. In the *slow* start of a motor with a large inertia load, the effects may die away before the machine has gathered speed. The *fast* run-up of an unloaded machine may, in contrast, exhibit strong torque pulsations, Fig. 8.61 (b), the torque reversals producing the characteristic 'loops'. The steady-state torque/speed curve is given for comparison. Both curves in Fig. 8.61 are for sinusoidal applied volt-

age; additional and complicated effects are likely if the supply voltages are derived from thyristor inverters.

Re-switching

When a running motor is disconnected from the supply and then reconnected, a transient torque is developed, primarily as a result of the e.m.f.s induced into the stator phase windings by the gap flux of the unidirectional decaying rotor currents, as already described. The phase angle of these e.m.f.s with respect to the re-applied supply voltage is significant, and the amplitude and sign of the first peak of the transient torque are closely dependent on the rotor speed and the duration of the interruption. In the worst case the first peak may reach 15 p.u. of full-load torque. The corresponding transient peaks of stator current are typically 5–7 p.u. of full-load current.

Plugging, dynamic braking and busbar transfer are examples of fast re-switching in which re-connection may be made with appreciable e.m.f.s existing in the stator winding. In *plugging* the re-connected supply has the reversed phase-sequence; even at reduced voltage the transient torques and currents are severe, e.g. with 15 p.u. peak torque and 20 p.u. current in terms of rated values. Plugging is a drastic operation, complicated by the effects of eddy currents and saturation.

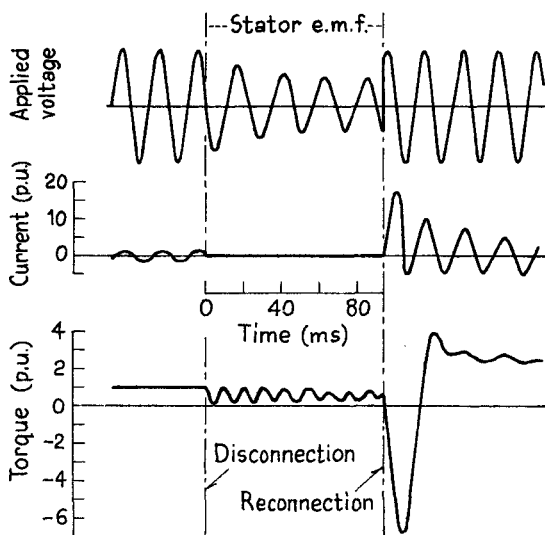
Marked transient effects occur when *terminal capacitors* are used for power-factor correction, self-excitation of an induction generator or capacitance braking. When a motor supply is interrupted with the capacitors left across the stator terminals, the capacitors tend to maintain the gap flux and the stator may build up an overvoltage in spite of the drop in rotor speed. On re-connection with unfavourable conditions of e.m.f. and supply voltage phasing, the resulting current and torque transients can be severe. In the case of an induction generator, the effects of reconnection are intensified by the rise in speed due to the prime-mover.

Transients during *d.c. injection braking* are normally small, and are more likely to be caused by short-circuit of the d.c. supply than by the injected voltage. Consequently they become less significant the longer the duration of disconnection.

An example of a re-switching transient is given in Fig. 8.62. The machine under full-load conditions is disconnected from rated voltage for 93 ms (4.65 periods of a 50 Hz supply) and then re-connected. The torque wave is a computed electromagnetic gross torque.

Electromechanical Transients

Run-up and Overshoot. During the direct-on-line start and run-up of an unloaded cage motor, the measured torque/speed characteristic may exhibit a depressed peak-torque, and be observed to reach a supersynchronous speed overshoot before settling to the small steady-state positive slip. Both effects are due to the action of the short-circuited rotor in opposing rapid changes of flux linkage. As the rotor approaches synchronous speed, the rate of decay



8.62 Reconnection transients

of its currents may be slow enough to give the temporary effect of d.c.-excited poles, which consequently develop a synchronising torque, like that of a synchronous motor with a retarded load angle. The machine therefore 'hunts' about synchronous speed for one or two oscillations before the rotor torque adjusts to the steady-state condition.

Change of Load. Reduced low-slip damping and transient synchronizing torque are also exhibited when a running motor is subjected to sudden changes of load. During the rapid retardation following a large load application, the trapped rotor flux combines with the main synchronously rotating field to produce synchronizing torque which reverses cyclically as the speed drops and develops wide resultant fluctuations of motor torque, the peaks decaying as the rotor currents settle to the new condition.

Fluctuating Load. If a load, such as a reciprocating compressor, imposes a slip and torque oscillation, cyclically generated rotor fluxes are trapped, of magnitude dependent on the load-oscillation frequency and the short-circuit time-constant. They produce synchronizing torques, the elastic nature of which in combination with the load inertia of the set gives a natural oscillation frequency. If this is close to the fluctuation frequency of the load (or to one of its harmonics) the slip-swing may be augmented by resonance, causing a strong pulsation of the stator-current waveform envelope. Analysis of such cases, with examples, is given by Middlemiss [109].

Faults

System Faults. Where these reduce the terminal voltage of a loaded induction motor the speed of the machine will fall. In the post-fault voltage-recovery period, large motors make heavy reactive power demands that may depress

the system voltage still further and inhibit re-acceleration. The nature of the torque/speed characteristic of the load is important; if, for example, load torque is proportional to speed, recovery of normal operating conditions is likely to be more quickly attained.

Short Circuit. When a system short-circuit fault occurs at or near the terminals of a large induction motor, the machine makes a significant contribution to the system fault current, particularly if it is running on light load. The short-circuit current is always more severe for a 1-phase than for a 3-phase fault. The most important quantity is the first peak current as it determines the 'make' rating of the protective circuit-breaker. In general the short-circuit current has a d.c. component of slow decay, and an a.c. component. The latter is larger than the direct-on-line starting current and may reach 10–20 times full-load current; it can be represented by two exponential terms related to the d-axis transient and subtransient reactances and the corresponding time-constants, viz

$$x = x_1 + \frac{x_m x_2}{x_m + x_2} \quad \text{and} \quad \tau = \frac{1}{\omega r_2} \left[x_2 + \frac{x_m x_1}{x_m + x_1} \right]$$

where x_d'' and x_d' are evaluated from x respectively for $s = 1$ and $s = 0$, and similarly τ_d'' and τ_d' from τ , using appropriate values of resistance and inductance if they vary over the range of slip. Then for a pre-fault terminal voltage V_1 the short-circuit current is

$$I = V_1 \left[\left(\frac{1}{x_d''} - \frac{1}{x_d'} \right) \exp \left(-\frac{t}{\tau_d''} \right) + \frac{1}{x_d'} \exp \left(-\frac{t}{\tau_d'} \right) \right]$$

To a reasonable approximation the current can be estimated on a basis of the subtransient reactance alone.

8.22 Testing

Few stock motors undergo a comprehensive test, but new types call for more detailed testing to confirm design figures. On large machines the stator is magnetically checked for imperfections, the rotor for balance and (for slip-ring machines required to run for long periods at high slip) an underspeed load test with slip-power recovery.

D.C. Resistance

Accessible windings are checked by a d.c. method, the conductor temperature being recorded for calculating the resistance at specified operating temperature.

Open-circuit (Voltage Ratio)

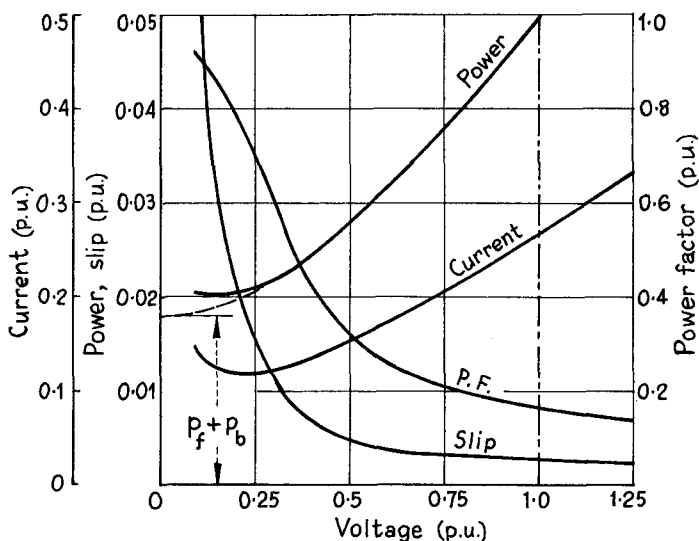
This, for slip-ring machines, consists in applying rated voltage and frequency to the stator windings and measuring the voltage between slip-rings as a check for balance. The measured phase voltages V_1 and V_2 do not give the effective turns-ratio because of stator leakage; but if V_2 is applied to the rotor and the stator voltage is then v_1 , the effective turns-ratio is close to $\frac{1}{2}(V_1 + v_1)/V_2$.

Run-up

A starting test may be applied to cage motors to confirm that they are capable of starting against a specified load torque and inertia without crawling or developing noise.

No-Load

This gives core, pulsation, friction and windage losses, magnetizing current and no-load power factor. It reveals mechanical out-of-balance and faulty connections. The stator windings are connected to a supply of rated voltage and frequency, with instruments to measure voltage, current, power and slip. After starting, the rotor (if it is of the slip-ring type) is short-circuited as for normal operation. Power and current are read at, say 1.2 p.u. of normal voltage and the test repeated at lower voltages down to the value at which the stator



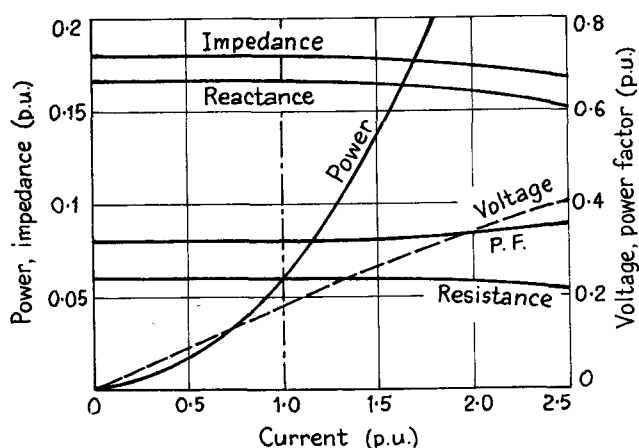
8.63 No-load test.

current begins to rise. Fig. 8.63 gives a plot of typical results. At normal voltage the no-load current I_0 is between 0.2 and 0.4 p.u. of full-load current and the power factor is low as the reactive (magnetizing) component I_{0r} of I_0 predominates. With reduction of voltage, the flux falls almost in proportion and the power curve is roughly parabolic as the core loss is proportional to the square of the flux density; the power factor rises as the active current component I_{0a} increases to satisfy the mechanical loss. At about 0.2 p.u. voltage the magnetizing current and core loss are small, but the mechanical loss calls for a larger active current component and the slip rises to permit its circulation. The power is therefore almost entirely for mechanical loss, and if the power P_0 is extrapolated to zero voltage (if necessary by plotting the straight-line relation of P_0 to V_1^2) the intercept is the mechanical loss p_b for brush friction

(if there are slip-rings with no brush-raising device) and p_f for windage and bearings. The power actually includes a small I^2R loss, for which, in small machines, a correction can be made.

Locked-rotor (Short-circuit)

The rotor, short-circuited as for a running condition, is held stationary, and the stator is supplied at a low voltage of normal frequency. The position in which the rotor is clamped may affect the stator current: if so, a mean position is found, or the rotor permitted to revolve very slowly during the test. The applied stator voltage is raised in steps, with readings of current and power, until the current reaches not more than twice normal. The temperature must be taken for each reading so that the conditions can be normalized. Typical results are shown in Fig. 8.64, plotted to a base of stator current. The



8.64 Short-circuit (locked-rotor) test.

power curve is nearly a parabola, as the power input is almost entirely I^2R loss. For, say, 0.2 p.u. voltage, the flux density is about one-tenth of its normal value and the core loss is consequently very small.

The leakage impedance is seen to fall with higher currents as a result of saturation, and the resistance may drop a little for the same reason. For deep-bar and double-cage machines, the effective rotor standstill leakage reactance increases at normal speed above its actual standstill value due to the redistribution of the rotor current. For accurate performance calculations the motor reactance may therefore be re-determined at e.g. 5 Hz for a 50 Hz machine.

Performance

In the assessment of efficiency by loss-summation, BS 269, the losses to be taken into account are:

Fixed loss: (a) core loss; (b) bearing friction; (c) total windage; (d) brush friction.

Direct load loss: I^2R loss (e) in stator winding, (f) in rotor winding, (g) brush loss.

Load loss: stray loss (h) in core, (j) in conductors.

Losses (a), (b), (c) and (d) are found by deducting the small no-load I^2R loss from the total measured loss in the no-load test. Losses (e) and (f) for slip-ring motors are calculated from the currents (obtained from the circle diagram) and resistances of the windings. For cage rotors, loss (f) with the small part of (h) and (j) appertaining to the rotor, is determined from the expression —

$$(\text{output} + \text{windage} + \text{friction}) \times s/(1 - s)$$

Loss (g) for all brushes together is taken as the slip-ring current $\times 1$ V. The stray load loss is covered by deducting $\frac{1}{2}\%$ from the full-load efficiency, and *pro rata*; or it may be estimated on the basis of a reverse-rotation test.

The circle diagram is an essential feature of the method, and it gives also the full-load current, power factor and slip, and an indication of the pull-out torque.

EXAMPLE 8.10: The following test data refer to a 22.5 kW, 500 V, 3-phase, delta-connected, 50 Hz, 4-pole cage motor:

No Load

Line voltage	V:	564	515	480	320	160	75
Line current	A:	9.8	8.6	7.8	5.4	4.5	7.9
Power, W1	kW:	-2.05	-1.55	-1.22	-0.25	0.28	0.36
W2	kW:	3.67	3.05	2.62	1.40	0.69	0.60
W1 + W2	kW:	1.62	1.50	1.40	1.15	0.97	0.96

Locked Rotor

Line voltage	V:	60	100	140
Line current	A:	18.6	32.0	44.8
Power, W1	kW:	-0.27	-0.75	-1.50
W2	kW:	0.83	2.35	4.65
W1 + W2	kW:	0.55	1.60	3.15

The figures above are corrected to 75 °C. The stator resistance (hot) is 0.68 Ω /ph. Separate the principal losses, and find the full-load current, power factor, efficiency and slip, the pull-out torque, and the maximum power output.

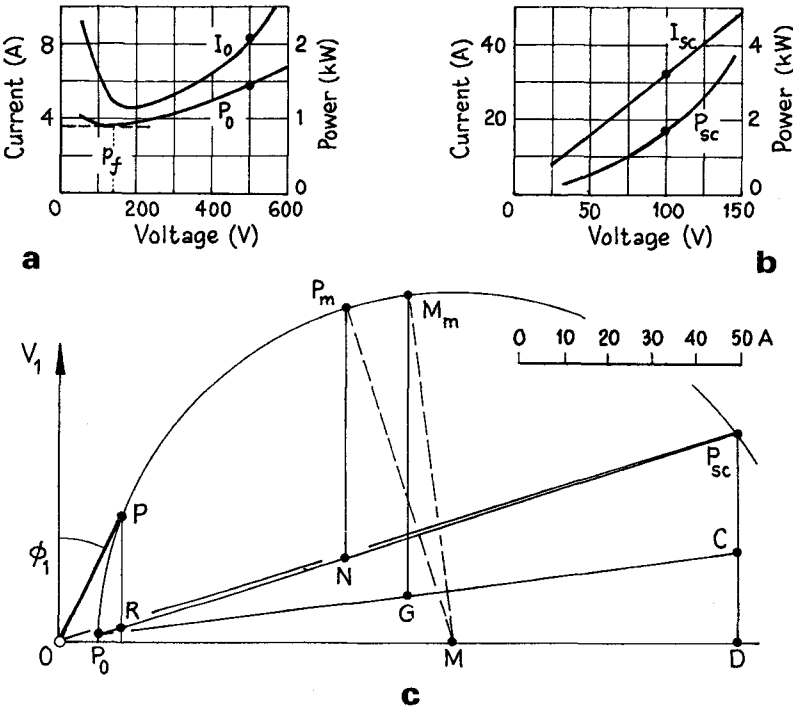
From the plot (a) of no-load readings in Fig. 8.65, the no-load values at normal voltage are $V_1 = 500$ V, $I_0 = 8.2$ A, $P_0 = 1.45$ kW, whence $\cos \phi_0 = 0.202$ and $\phi_0 = -78.4^\circ$. By extrapolation, the friction and windage loss is $p_f = 0.90$ kW. The no-load I^2R loss for the three phases, calculated from the stator phase resistance, is 0.05 kW. The core loss (using the nomenclature given in a later paragraph on loss-separation) is $p_{e1} + p_{h1} + p_p = 1.45 - 0.90 - 0.05 = 0.50$ kW.

Plotting the locked-rotor readings, (b), and taking values for full-load cur-

rent current (estimated at 32 A), we have $V_1 = 100\text{ V}$, $I_{sc} = 32\text{ A}$, $P_{sc} = 1.60\text{ kW}$, and therefore $\cos \phi_{sc} = 0.29$, $\phi_{sc} = -73.1^\circ$. The corresponding values *pro rata* for normal voltage are $V_1 = 500\text{ V}$, $I_{sc} = 160\text{ A}$, $P_{sc} = 40\text{ kW}$, $\cos \phi_{sc} = 0.29$; the stator I^2R loss is then 17.4 kW , so that the rotor I^2R loss is $40.0 - 17.4 = 22.6\text{ kW}$.

The circle diagram, Fig. 8.65(c), is drawn for convenience with line values and a current scale of 10 A per unit length. One current unit vertically therefore represents $\sqrt{3} \times 500 \times 10 \times 10^{-3} = 8.66\text{ kW}$. For a full load of 22.5 kW a vertical intercept of $22.5/8.66 = 2.60$ units is necessary between the full-load point P and the out-put line P_0P_{sc} . This fixes P and gives the stator current $OP = 32\text{ A}$ to scale at a power factor of 0.906. The full-load losses and input are therefore

	kW
Friction and windage	0.90
Core	0.50
Stator I^2R	0.70
Rotor I^2R	0.90
Output	22.5
Input	25.5



8.65 Test results and circle diagram.

The efficiency is $22.5/25.5 = 0.88$ p.u. (less $\frac{1}{2}\%$) and the slip is $s = 0.90/(22.5 + 0.9 + 0.9) = 0.037$ p.u. The maximum power is given by the vertical intercept between P_{max} and the $P_0 P_{sc}$ line: it measures 5.75 units, i.e. $5.75 \times 8.66 = 49.8$ kW, or 2.2 p.u. of rated full load.

The torque line is obtained by dividing $P_{sc}D$ at C in the proportion $22.6/17.4$ of the short-circuit rotor/stator I^2R loss. Raising the perpendicular from the centre M to the P_0C line and extending it to cut the circle at M_{max} gives the pull-out torque as 2.5 times full-load torque. The starting torque on full voltage is $P_{sc}C$ to scale, approximately equal to the full-load torque; but it is not a reliable guide.

Temperature-rise

In general a true test of the temperature-rise can be obtained only by directly loading the machine, either mechanically or through a d.c. or a.c. generator. The latter is economical if the generator can itself be loaded on to the supply, but otherwise a direct load test offers difficulty and expense.

When two identical induction machines are available, an informative output-input test may be made by coupling the machines mechanically, supplying one at normal voltage and frequency and the other at a slightly lower frequency so that it operates as a generator. A repeat with the roles of the machines interchanged gives a check and reduces instrument error. In a test department it may be possible to use the supply mains for the motor and a test generator for the lower-frequency supply.

Insulation

Insulation is tested by high-voltage ohmmeter and by the application of a specified overvoltage (e.g. from a test transformer) between open-circuited phase windings, and between windings and earth.

Loss-separation

Apart from the I^2R loss in accessible windings, which can be calculated, the losses in an induction machine include: p_b , the brush-friction loss; p_e , the eddy-current loss in the core steel; p_f , the mechanical losses in bearing friction and windage; p_h , the hysteresis loss; and p_p , the pulsation loss. The friction losses require no comment. Using the subscripts 1 and 2 for stator and rotor respectively, the stator core loss is $p_{e1} + p_{h1}$ at normal flux. At standstill the rotor core loss is $p_{e2} + p_{h2}$, when the rotor is open-circuited. At a slip s , the loss becomes $s^2 p_{e2} + s p_{h2}$, since the former is proportional to the square and the latter to the first power of the frequency. The eddy-current loss corresponds to a torque proportional to the slip, but the hysteresis loss corresponds to a constant torque, since $s p_{h2}$ is the product of the slip s and a constant power, or the slip-speed $s n_1$ and a constant torque. The rotating field produced by the stator may be conceived as magnetizing the rotor, producing corresponding rotor poles. The hysteretic effect is, however, to retard the rotor poles behind the stator ones, so that a continuous attraction is set up between the stator and induced rotor poles. This *hysteretic torque* urges the machine in the direction of the stator rotating field. The power supply for the loss in rotor hystere-

sis must naturally be given to the stator in association with the main rotating field: but the loss is employed usefully in producing hysteretic torque which tends to raise the speed.

The pulsation loss is a high-frequency tooth loss in stator and rotor, having the frequency S_2n in the former and S_1n in the latter, and produced by variations of gap reluctance as the tooth tips pass each other. The loss depends on the relative slot-pitches in a manner not easily susceptible to calculation.

No-load Test with Rotor Driven: Suppose a *slip-ring* machine to be driven with open-circuited rotor and stator excited from a normal-voltage, normal-frequency supply. As the rotor speed is varied, all the losses will vary with the exception of the stator loss ($p_{e1} + p_{h1}$). Since the friction losses are supplied by the drive, the stator electrical input corrected for I^2R loss consists of

$$(p_{e1} + p_{h1}) + (p_{e2} + p_{h2})$$

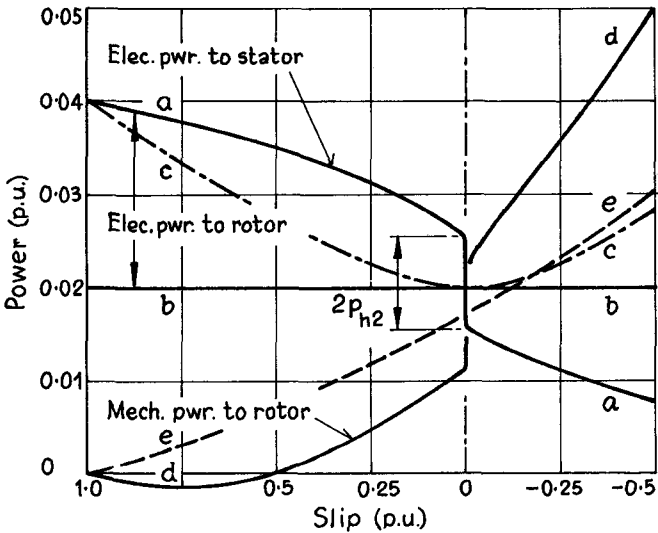
at standstill, and

$$(p_{e1} + p_{h1}) + s(sp_{e2} + p_{h2})$$

at slip s , for stator and rotor core losses. The rotor losses produce a forward torque, and therefore power, to drive the machine, and this must come also from the stator. From eq.(8.3) the stator takes in addition

$$(1 - s) (sp_{e2} + p_{h2}).$$

At synchronous speed the torque due to rotor eddy-currents is zero, but that due to rotor hysteresis may have any value between the limits $\pm p_{h2}$, depending on the position of the rotor with respect to the stator rotating poles. The electrical power supplied to the stator therefore changes discontinuously by $2p_{h2}$ between speeds just below and just above synchronism.



8.66 Loss estimation.

Fig. 8.66 shows loss/slip curves typical of the conditions described, viz. the motor driven by external means with its rotor open-circuited and the stator energized at normal voltage and frequency. Curve *a* represents the stator input; and curve *b* the constant stator loss ($p_{e1} + p_{h1}$). The intercept between *a* and *b* is the loss given electrically to the rotor ($sp_{e2} + p_{h2}$). The part $s(sp_{e2} + p_{h2})$ which is lost in the rotor is the intercept between *b* and *c*; the remainder between *c* and *a* is the part developed by the rotor as a forward driving power. The net mechanical power supplied to or by the rotor at various speeds is given by curve *d*. This is obtained by comparing the mechanical power needed to drive the rotor with the stator first excited then unexcited. The difference indicates what mechanical power the rotor develops from the electrical power it receives from the stator. The intercept between *d* and the base-line is the mechanical power that must be externally supplied to turn the rotor. Curve *e* is the sum of *d* and the intercept between *c* and *a*. Since *d* does not include purely mechanical losses, then *e* (which is the total mechanical power supplied from drive and stator together) is the pulsation loss which causes an effect resembling friction.

Subdivision: The following seven tests can be used to separate the several losses discussed above.

1. Measure the power to the stator at normal voltage and frequency with the rotor open-circuited and at rest. The stator input P_1 corrected for I^2R loss is the standstill core loss corresponding to curve *a* in Fig. 8.66.
2. Measure the power to the stator with the rotor short-circuited and running on no load: corrected for I^2R loss this gives the stator core and pulsation losses and the friction losses, P_2 . The rotor eddy-current loss is very small; its hysteresis loss is almost wholly returned as a driving torque, providing part of the friction loss.
3. Measure as in 2 but with the brushes raised, P_3 .
4. With the machine running as in 2, the rotor circuit is suddenly opened and the stator input measured immediately thereafter. The pulsation and friction losses cause the rotor to slow down.
5. Apply to the rotor the voltage that produces $\frac{1}{2}(V_1 + v_1)$ in the stator, as described above under 'Open-circuit (Voltage-ratio)'. Measure the rotor input with the machine running on no load with the stator short-circuited, to give the conditions in 2 but inverted. The corrected rotor input is P_5 .
6. With the machine running as in 5, open-circuit the stator. The rotor input falls to P_6 , corresponding to test 4.
7. Obtain the friction losses from a no-load test or a retardation test, to give P_7 .

From these power observations (corrected in each case for I^2R loss)

$$p_b = P_2 - P_3$$

$$p_{h1} = P_6 + \frac{1}{2}(P_2 - P_5 - P_1)$$

$$p_f = P_7 - p_b$$

$$p_{h2} = P_4 + \frac{1}{2}(P_5 - P_1 - P_2)$$

$$p_{e1} = P_1 - P_6$$

$$p_p = \frac{1}{2}(P_5 + P_2 - P_1) - P_7$$

$$p_{e2} = P_1 - P_4$$

The method, based on small differences, is not easy, but with care yields useful results.

Load Loss

The American Standard method [59] for assessing the 'stray' load loss of a single large cage machine comprises the two following tests.

Rotor-removed Test: A measurement of stator eddy-current and end-region fundamental-frequency loss components is obtained by circulating full-load current in the stator windings with the rotor removed from the machine. It is assumed that the end-region flux conditions are unaffected by the absence of the rotor, but this may not in fact be so especially with 2- and 4-pole machines with high electric loadings. The corrected fundamental-frequency stray loss is

p_{1s}

Reversed-rotation Test: This consists in driving the machine mechanically backwards at synchronous speed ($s = 2$) with the stator supplied at a voltage sufficient to circulate full-load current I_1 ; the rotor current then has full-load value I_2 , and its equivalent I^2R loss for m representative phases is $mI_2^2 r_2$. One-half of this, $mI_2^2 r_2/s = mI_2^2 r_2/2$, is provided by the stator and the other half by the mechanical drive. The total input to the stator is therefore

$$p_1 = mI_1^2 r_1 + \frac{1}{2}mI_2^2 r_2 + p_{1s}$$

while on the assumption that the high-frequency stray load loss p_{2s} is entirely supplied mechanically the total drive power is

$$p_m = p_f + \frac{1}{2}mI_2^2 r_2 + p_{2s}$$

From these two measurable powers is derived the expression

$$p_{2s} = p_m - p_f - (p_1 - mI_1^2 r_1 - p_{1s})$$

which is the mechanical power less the stator input corrected for its I^2R loss and fundamental-frequency load loss.

Chalmers and Williamson [38] discuss the validity of the method. They conclude that reasonable accuracy is obtainable in some cases in spite of the three assumptions made, namely that the tooth-frequency flux pulsations (i) are the main source of h.f. load loss, (ii) are supplied mechanically, and (iii) are the same for both directions of full-speed rotation. The errors can, however, be large where stator-winding pitch leads to high belt-leakage flux and loss, and where there are large sub-harmonic fluxes due to irregular stator windings.

9

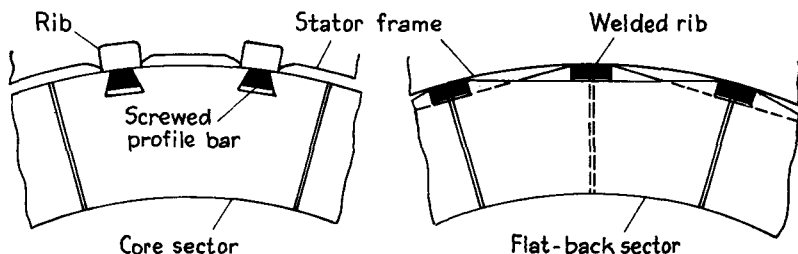
Induction Machines: Construction and Design

9.1 *Constructional features*

The essential features are: a laminated stator core carrying a poly-phase winding; a laminated rotor core carrying either a cage or a polyphase winding, the latter with shaft-mounted slip-rings; a stiff shaft to preserve the very short air-gap; a frame to form the stator housing and carry the bearings; and a cooling system.

Cores

The stator and rotor cores are built up of thin sheets of special core steel with good magnetic properties. The laminae are insulated from one another by varnish, paper or sprayed china clay films. In the smaller sizes the stator plates form complete annular rings, but for larger machines they are assemblies of segmental stampings. Typical methods of securing segmented stator cores are shown in Fig. 9.1. The gap surfaces of stator and rotor have suitable slots



9.1 Sector stampings for stator.

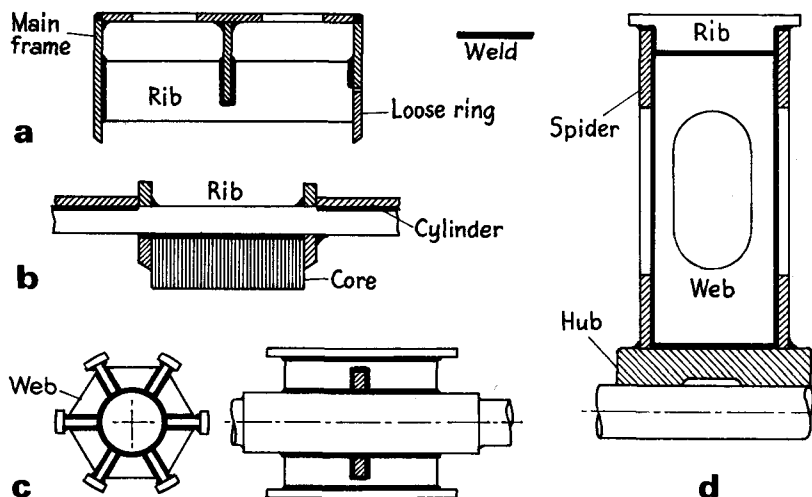
punched out, of shape dependent upon the kind of winding chosen. Long cores are provided with spacer plates at intervals to provide radial cooling ducts.

In small motors the rotor core is mounted directly on the shaft, to which it is keyed, and clamped between a shoulder on the shaft and a shrink ring, with the slots skewed. For large machines the rotor is carried on a fabricated spider.

Frame

The frame may be die-cast or fabricated. Machines up to about 50 kW rating may have frames die-cast in a strong silicon-aluminium alloy, sometimes with the stator core cast in. The end-covers carry the bearing housings, and the method allows of ready thickening of the wall in regions of higher mechanical stress. The surface finish requires no machining.

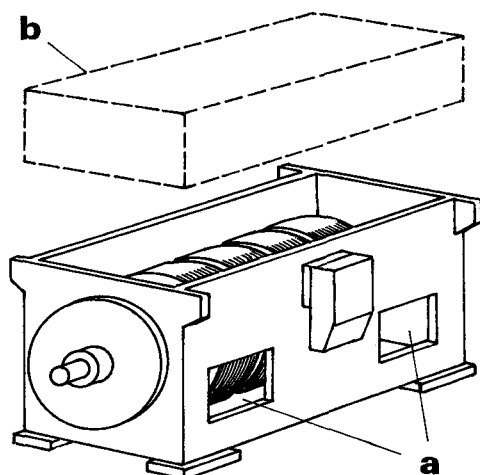
The advantage of fabrication for large machines is its adaptability to new designs and modifications. The main constructional difficulty is the avoidance of distortion when the component steel plates are welded together. Essentially the frame is a short cylinder, or a box, with end plates and axial ribs. When the inner rib surfaces have been machined and the ends turned where necessary for the end covers, the frame is ready to receive the stator core, assembled and wound externally. Some fabricated elements are shown in Fig. 9.2. At (a) the



9.2 Fabricated constructions.

core length has an additional centre plate for rigidity; at (b) the core is compressed between end-plates; the rotor spider in (c) comprises a shaft with arms and stiffeners; the large diameter rotor spider (d) is built up from a hub, arms and ribs, the latter machined to take the rotor core.

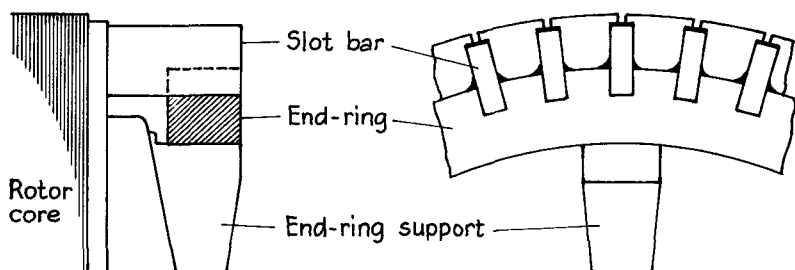
Machines in the range of 250 – 10 000 kW are often built on a modular basis to fit a variety of specifications. The motor frame is a fabricated rectangular enclosure, Fig. 9.3, containing the stator core and provided with end plates with bearings to carry the rotor. Openings *a* can be sealed by plates, fitted with louvres or connected to air ducting. The top is covered by a structure *b* adapted to requirements of drip- or flame-proofing, or of total enclosure with a heat-exchanger.



9.3 Modular construction.

Windings

Motors up to 250 kW operate usually at voltages in the range 380–440 V. For 3.3 kV the minimum economical rating is 250–300 kW, and for 11 kV about 750 kW. General details of windings are given in Chapter 3; the double-layer lattice winding is most common for stators. Small motors with few slots and requiring many turns per phase may be mush wound. Modern insulants for diamond coils are in classes E, B and F (Sect. 3.1). Polyester foil coated with compressed fibre may be used for slot and phase insulation to class E, and plastic foil backed with polyamide fibres for class F, in both cases impregnated with a class F varnish.



9.4 Connection of deep rotor bars to end-ring.

Cage rotor windings comprise copper or aluminium bars in slots (usually uninsulated), welded or brazed to the end-rings. Fig. 9.4 shows a method of connecting deep bars to end-rings: the jointing, as the most likely site of failure, requires care. It is also necessary to keep the bars tight in the slots, for with mechanical vibration and thermal cycling a loose bar is quickly damaged.

Much of the difficulty can be avoided in small cage machines by integral casting of aluminium bars and end-rings. The stacked rotor core is used to form part of the die in a gravity-fed process, or by a centrifugal or pressure technique. The end moulds form the rings with fins as simple fan blading. A silicon alloy (6–12% Si) is employed, as pure aluminium does not cast well.

Where slip-rings are required, the rotor winding is of the conventional 3-ph type, a wave winding being common for its convenient adaptability. The choice of voltage is usually free; its standstill open-circuit slip-ring level is typically 100 V in small machines, and up to about 1 kV for large machines.

Enclosure and cooling

Besides the usual types of enclosure (Sect. 7.6) a flameproof construction is available for Division 1 Areas (mines, petroleum plants, etc., with hazardous atmospheres). This is a totally-enclosed construction with all joints flanged so that any flame generated by an internal explosion will be cooled in its passage through the flange so as to be externally innocuous. Machines without such flame-proofing are used in Division 2 Areas where flammable gas is not normally present.

Surface-cooled 3-phase motors for low and medium outputs are commonly made with finned housings. With higher outputs this simple method is inadequate, and the machines may be self-ventilated with two separate air circuits and tubular heat-exchanger. The inner circuit takes up the heat from the windings and cores, and thence to the cooler where the outer air circuit passes the heat to ambient; the two circuits are not in direct air contact. There is a fan for each circuit. The construction is suitable up to about 5 MW. At higher ratings the outer circuit may be water-cooled if there is a convenient supply.

Shafts and bearings

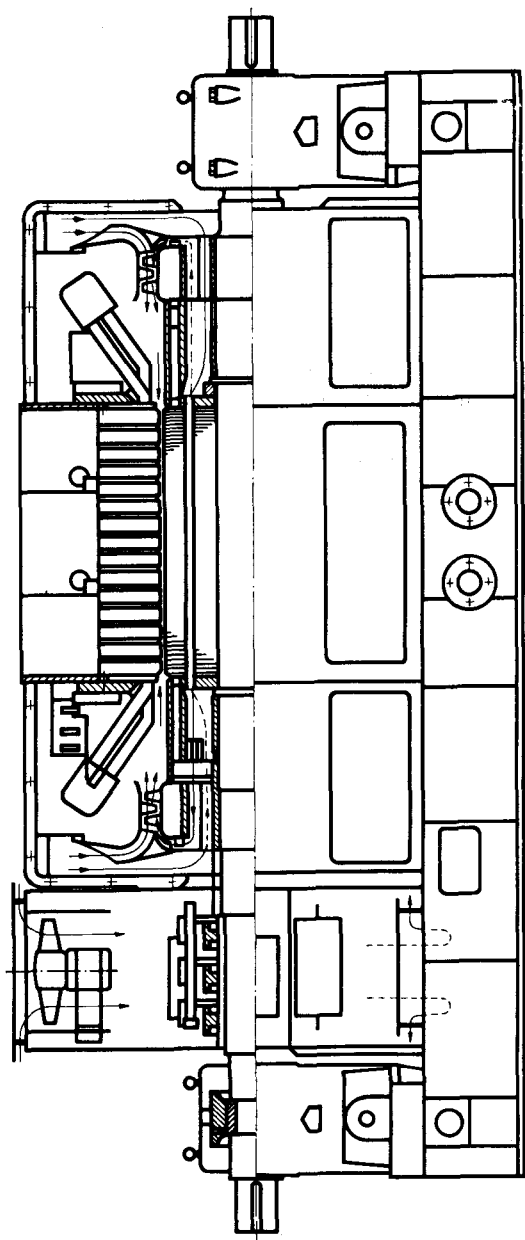
The shaft of an induction machine is short and stiff to maintain the necessarily short airgap. The use of ball or roller bearings makes accurate centring of the rotor readily possible. Where massive rotors demand journal bearings, the self-aligning spherical-seated type is useful.

Slip-rings may be pressed together on a body of reinforced thermo-setting resin carried on a mild-steel hub. Brush-lifting and short-circuiting gear is not generally fitted to modern motors. In small machines the rings may be located on the shaft between the rotor core and the bearing, or outboard. In the latter case the shaft has to be bored to take the slip-ring leads through the bearing.

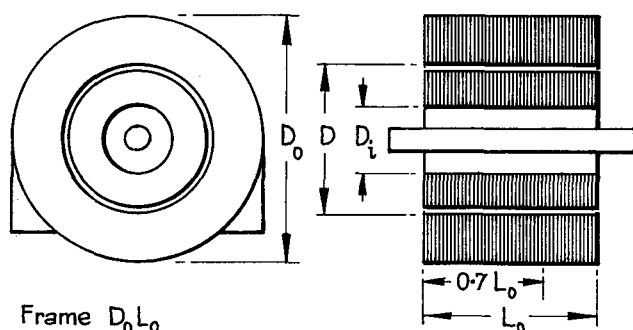
A drawing in Fig. 9.5 shows the general arrangement of a large slip-ring motor. A small cage motor is shown in Fig. 9.7.

9.2 Main dimensions

Apart from some special machines, the manufacture of small and medium-rated induction motors is concentrated into a series of standard frames to cover a wide range by adjustment of rotor diameter and length, as indicated in Sect. 7.6. The synchronous speeds available with 50 Hz machines are 3 000, 1 500, 750, 600 . . . r/min, and 3 600, 1 800, 900 . . . r/min for 60 Hz supply. All these can be produced with a limited number of frames. The frame houses a



9.5 Slip-ring induction motor: 3 700 kW, 11 kV, 50 Hz, variable speed.



9.6 Frame size.

stator of given outside diameter D_o , together with its bearings, end-covers etc., and a maximum active length L_o , Fig. 9.6. A variation of rating is possible by alternative core lengths down (say) to $0.7 L_o$, and the stator bore can be increased for multipolar designs.

The rating of a frame depends not only on its dimensions and the shaft speed, but also on the specific magnetic and electric loadings, the type of enclosure, load characteristics, and the starting-stopping duty. A demand for a low speed, such as 600 r/min, can at normal frequency be met only by a multipolar construction, which is uneconomic for machines of small rating.

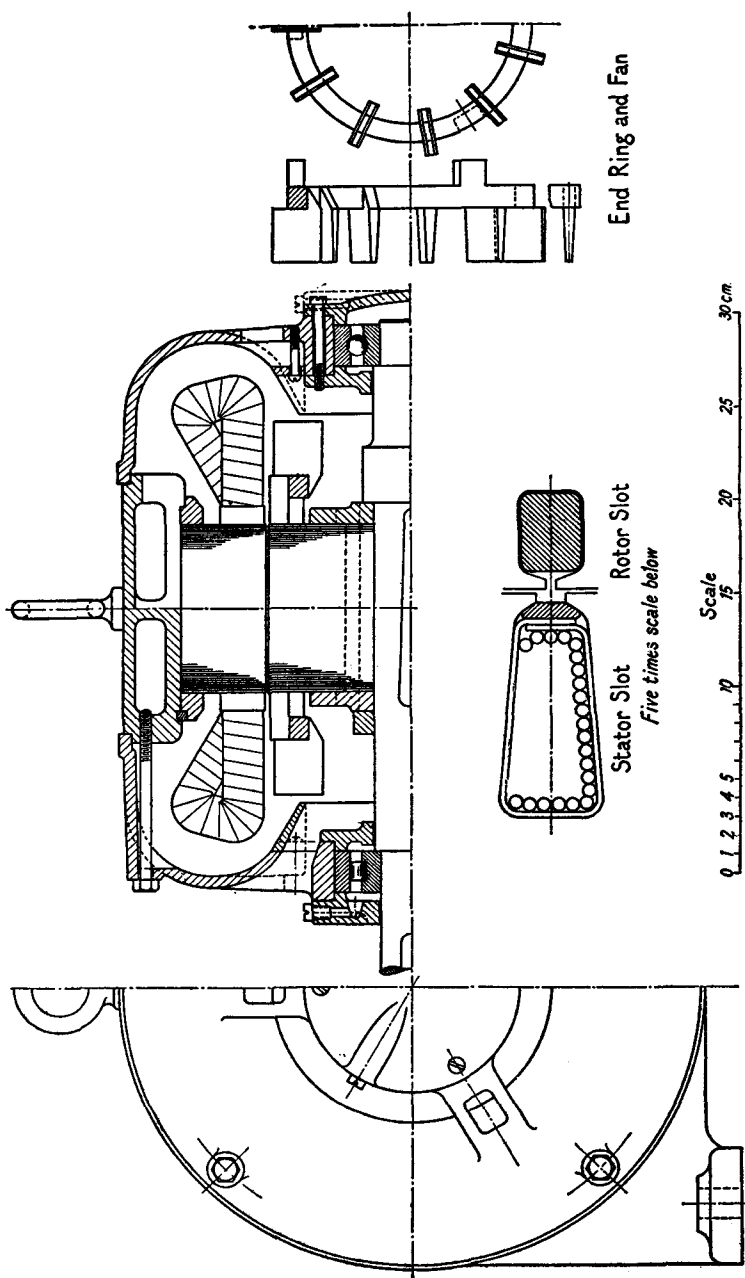
The manufacture of small motors is highly competitive. The aim is a range of standardized cores, frames, cooling systems and insulation with the facility for economically meeting minor variations in specifications. A basic choice is that of the ratio of the stator and rotor outside diameters and the slotting, on which the saturation levels and I^2R loss depend. A further requirement is that the stalling torque should be about twice the full-load torque, a ratio that depends on the total leakage reactance: the latter is proportional to the leakage permeance and the square of the number of stator turns per phase, and it cannot be changed without some sacrifice elsewhere, such as in a variation of slot depth.

Economy demands that as much as possible of a range design be standardized, and 4-pole motors up to 250 kW may have completely standardized core punching dimensions, consequent upon which the design process is virtually reduced to specifying an appropriate winding from a standard list.

The following Examples give typical design data. The first is set out as a 'hand' calculation to indicate the application of design formulae, and represents the end product of a series of trials.

EXAMPLE 9.1: 5 kW, 415 V, 3-ph, 50 Hz, 4-pole cage motor for star-delta starting, standard duty cycle S1 (Sect. 4.8), to BS 3939. See Fig. 9.7.

Main dimensions. The appropriate frame has $D_o = 240$ mm, $L_o = 90$ mm, foot-mounted with a shaft-centre height of 132 mm. The power factor and efficiency on rated load are estimated each to be 0.83 p.u. The rated input is therefore $S = 5\,000 / (0.83 \times 0.83) = 7\,260$ VA. The value of k in eq.(1.16) is $1.11\pi^2 K_w =$



9.7 Cage induction motor: 5 kW, 415 V, 50 Hz.

11 K_w , and with full-pitch stator winding $K_w = 0.955$. The synchronous speed is $n = 25$ r/s. The specific magnetic and electric loadings are taken as $B = 0.48$ T and $A = 27\,000$ A/m. The main dimensions are obtained from

$$D^2 l = 7\,260 / (11 \times 0.955 \times 0.48 \times 27\,000 \times 25) = 0.0021 \text{ m}^3$$

Taking the full available core length, then $l = L_o = 0.90$ m and hence $D = 0.15$ m. As the core length is too short to require cooling ducts, the net iron length is $l_i = 0.9l = 0.081$ m. The main airgap dimensions are thus

$$D = 150 \text{ mm}, \quad Y = 118 \text{ mm}, \quad l = 90 \text{ mm}, \quad l_i = 81 \text{ mm}, \quad l_g = 0.40 \text{ mm}$$

The radial gap is the minimum economically obtainable without surface grinding or special care.

Stator winding. The no-load flux is based on an e.m.f. equal to the supply voltage of 415 V/ph in delta connection. Then $\Phi_m = 0.48 \times 0.118 \times 0.09 = 5.1$ mWb. The number of turns per phase is

$$N_1 = 415 / (4.44 \times 0.955 \times 50 \times 5.1 \times 10^{-3}) = 384$$

With $S_1 = 36$ slots giving $g_1' = 3$ slots per pole and phase, then there are $z_1 = 64$ conductors per slot. The estimated full-load current is $I_1 = 7\,260 / (3 \times 415) = 5.8$ A. Taking a current density $J_1 = 4.9$ A/mm² the required conductor area is $a_1 = 1.18$ mm², obtained by enamelled round wire of diameter 1.22 mm (bare), 1.40 mm (insulated). The stator slots are shaped to permit of parallel-sided teeth each 6 mm wide. Allowing 0.5 mm slot liner and 1 mm slack, the slot depth including a 3 mm slot opening and a 1 mm lip, is 24 mm. The mush-type winding has a mean-turn length of 345 mm, and a total length per phase of 265 m. The phase resistance at 75 °C is

$$r_1 = 0.021 \times 265 / 1.18 = 4.7 \, \Omega$$

The total stator $I^2 R$ loss is $3 \times 5.8^2 \times 4.7 = 475$ W.

Rotor winding. A suitable cage is contained in $S_2 = 30$ slots, $g_2 = 7\frac{1}{2}$ slots/ph, slot-pitch $y_{s2} = 15.6$ mm, and a skew of one slot-pitch. An estimate of the rotor equivalent phase current is $I_1 \cos \phi_1 = 4.8$ A. From eq.(8.7) the r.m.s. bar and end-ring currents are

$$I_b = 4.8(6 \times 0.955 \times 384 / 30) = 350 \text{ A}$$

$$I_c = 350 \times 30 / 4\pi = 835 \text{ A.}$$

The rotor cooling is good, and the current density may be $J_2 = 7.6$ A/mm². The required bar area is $a_2 = 350 / 7.6 = 46$ mm². The bar dimensions are chosen with regard to the mechanical strength and saturation of the teeth. Slots 10.5 mm deep and 6.5 mm wide can house conductors 8.75 mm $\times$ 5.75 mm with rounded corners and light insulation. The bar length is 120 mm and its resistance at 75 °C is 0.055 m Ω ; the $I^2 R$ loss for 30 bars is 200 W. The end-rings are cast with fan blades, and have an area of about 120 mm² and a mean diameter of 117 mm. The resistance around each is 0.064 m Ω (hot), and for the two

rings the I^2R loss is 90 W. The total I^2R loss in the cage is 290 W. The rotor resistance expressed as a stator phase equivalent is

$$r_2' = 290/(3 \times 4.8^2) = 4.3 \, \Omega/\text{ph.}$$

No-load current. The effective gap length $l_g' = k_g l_g$ is found by applying eq. (2.8) in Sect. 2.5:

$$k_g = \frac{13.1}{13.1 - (0.74 \times 3.0)} \cdot \frac{15.6}{15.6 - (0.40 \times 1.0)} = 1.24$$

so that $l_g' = 1.24 \times 0.40 = 0.50$ mm. The effective gap area per pole is $Yl = 0.0106 \text{ m}^2$. The cores are constructed from No. 42 grade 0.4 mm plates. The Table shows the estimation of the magnetizing m.m.f., based on the procedure given in Sect. 2.5. The magnetizing component of the no-load current from eq.(2.10) is

$$I_{0m} = 530 \times 2/(1.17 \times 384 \times 0.955) = 2.47 \text{ A/ph} = 4.3 \text{ A/line}$$

The core loss, taken as confined to the stator core and teeth, is estimated from the curves in Fig. 2.5(b):

Core: peak density, 1.50 T; specific loss, 15 W/kg; mass, 8.75 kg; loss, 132 W.
Teeth: estimated peak density, 1.75 T; specific loss, 20 W/kg; mass, 3.15 kg; loss, 63 W.

The total core loss is 200 W. Including 60 W for mechanical loss, the total no-load loss is 260 W, requiring an active component $I_{0a} = 260/3 \times 415 = 0.21$ A/ph. The no-load current is therefore

$$I_0 = \sqrt{(2.47^2 + 0.21^2)} \approx 2.5 \text{ A/ph} = 4.3 \text{ A/line}$$

at a power factor $\cos \phi_0 = 0.085$. The magnetizing reactance per phase is $x_m = 415/2.47 = 168 \, \Omega$.

Magnetization: $\Phi_m = 5.1 \text{ mWb}$

a , area (m^2); l , length (m); $B_{30} = 1.36 \Phi_m/a$; $B_c = \Phi_m/2a$

Part	a	l	B_{30}	B_c	H	F
Stator core	0.0017	0.060	—	1.50	2 400	144
teeth	0.0044	0.024	1.59	—	4 200	100
Airgap	0.0106	$0.50/10^3$	0.65	—	520 000	260
Rotor teeth($\frac{1}{3}$)	0.0047	0.0105	1.49	—	2 100	20
core	0.0024	0.030	—	1.06	210	6

Total: 530 A-t

Short-circuit current. The specific slot permeances are

$$\lambda_{s1} = \frac{18}{3 \times 9} + \frac{2}{8} + \frac{2 \times 2}{8 + 3} + \frac{1}{3} = 1.61 \quad \text{and} \quad \lambda_{s2} = 1.65$$

Referring the rotor value to stator terms requires multiplication by $(K_{w1}^2 S_1)/(K_{w2}^2 S_2) = 0.955^2 \times 36/30 = 1.094$, giving 1.81. The total slot permeance coefficient is $1.61 + 1.81 = 3.42$. Then eq.(2.28) gives the *slot leakage reactance* in stator terms:

$$x_s = [4\pi f \mu_0 N_1^2 / pg_1'] L_s \lambda_s = 19.4 \times 0.09 \times 3.42 = 6.0 \Omega$$

For the overhang leakage, eq.(2.27) gives $L_0 \lambda_0 = 0.34$ and the *overhang leakage reactance* in stator terms:

$$x_o = 19.4 \times 0.34 = 6.6 \Omega$$

From eq.(2.25) the *zigzag leakage reactance* is

$$x_z = \frac{5}{6} 168 \left[\frac{1}{9^2} + \frac{1}{7.5^2} \right] = 4.2 \Omega$$

With a cage rotor there is negligible differential leakage. The total leakage impedance per phase referred to the stator winding is given by

$$R_1 = r_1 + r_2' = 9.0 \Omega; \quad X_1 = x_s + x_o + x_z = 16.8 \Omega; \quad Z_1 = 19.0 \Omega$$

The short-circuit current is $I_{sc} = 415/19.0 = 21.8 \text{ A/ph} = 38.0 \text{ A/line}$ at a power factor $\cos \phi_{sc} = 0.47$.

Performance. The current diagram or the equivalent circuit gives

$$I_1 = 5.7 \text{ A}, \quad I_2' = 4.6 \text{ A}, \quad \cos \phi_1 = 0.83$$

for the full-load phase values, confirming the design figures. The full-load efficiency is calculated as follows

	W
Stator $I^2 R$: $3 \times 5.7^2 \times 4.7$	458
Rotor $I^2 R$: $3 \times 4.5^2 \times 4.3$	262
Core loss:	200
Mechanical loss:	60
Total loss:	980
Output:	5 000
Input:	5 980

The efficiency is $5\,000/5\,980 = 0.835$, reduced by $\frac{1}{2}\%$ for stray load loss to 0.83 p.u.

The full-load *slip*, from the ratio of rotor $I^2 R$ loss to total rotor input, is

$$s = 262/(262 + 60 + 5\,000) = 262/5\,322 = 0.049 \text{ p.u.}$$

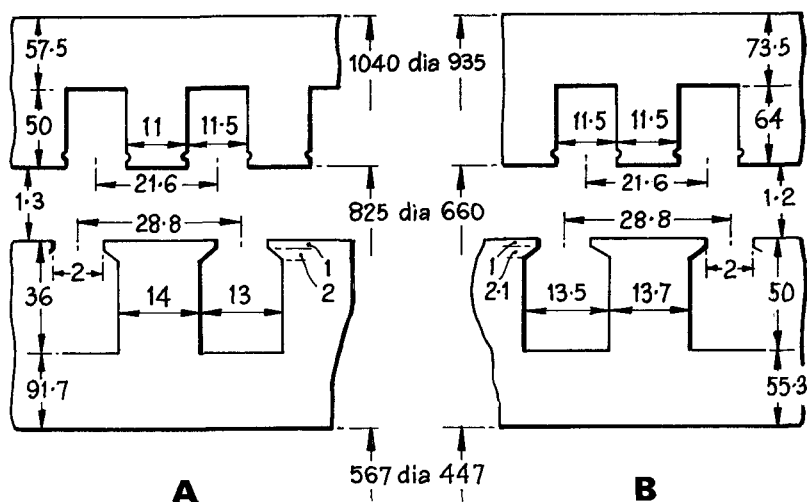
The *starting torque* for direct switching in delta is given by the current diagram or by the equivalent circuit as 1.16 p.u., which is reduced by star-delta switching to 0.39 p.u. It is in practice somewhat greater because saturation reduces the leakage reactance and eddy currents raise the rotor effective resistance at stand-still. The *pull-out torque* is 2.0 p.u.

Cooling. The rotor end-rings are bare and carry fans, and the stator overhang is well ventilated. From the drawing in Fig. 9.7 the cooling surface across which air is fanned is approximately a cylinder formed by the inner faces of the overhang and core, and the annular faces formed by the stator coil ends. The back of the stator also dissipates. The inner area is estimated as 0.17 m^2 . On account of the rotor peripheral speed ($u = 11.75 \text{ m/s}$) the inner surfaces are more effective than the back of the core by the factor $(1 + 0.1 u) = 2.2$, making the equivalent surface $0.17 \times 2.2 = 0.375 \text{ m}^2$. The back of the stator has a surface of area 0.068 m^2 , so that the total is 0.44 m^2 : it has to dissipate about 660 W of I^2R and core loss. Assuming from Sect. 4.3 a coefficient $c = 0.03$, the temperature-rise is

$$\theta_m = 0.03 \times 660/0.44 = 45^\circ \text{C}$$

The rotor loss is 260 W , and it has ample cooling surface: its temperature-rise is about 35°C , well within the specified limits and with a margin for the rapid heating during a start.

EXAMPLE 9.2: Outline data are given below for two slip-ring motors differing from the normal industrial range: the slotting and main dimensions are set out in Fig. 9.8.



9.8 Slot and core details.

Motor A: 270 kW, 3.0 kV, 3-ph, 50 Hz, 10-pole slip-ring motor to drive a centrifugal water pump, rated with reduced specific magnetic and electric loadings for a 30°C temperature-rise in ambient air at 55°C and 100% humidity.

Motor B: 75 kW, 3.0 kV, 3-ph, 50 Hz, 8-pole slip-ring motor, flame-proof for driving mine-haulage machinery, rated for a 50°C temperature-rise; the restricted ventilation enforces low I^2R loss, especially in the rotor.

		A	B
<i>Rating</i>			
Output	kW	270	75
Power factor	p.u.	0.84	0.86
Efficiency	p.u.	0.93	0.94
Input	kVA	346	93
Line/phase current	A	66.6	17.9

Main Dimensions

Magnetic loading	T	0.39	0.225
Electric loading	kA/m	30.9	14.9
Stator bore	m	0.825	0.66
Gross bore length	m	0.40	0.50
Ducts, no./width	mm	5/10	nil
Net core length	m	0.35	0.50
Iron length	m	0.32	0.45
Radial gap length	mm	1.30	1.20
Pole pitch	m	0.26	0.26

Stator

Winding/connection		2-layer diamond/star	
No-load flux per pole	mWb	40.0	29.1
Turns per phase	—	200	286
Number of slots	—	120	96
Distribution factor	—	0.958	0.958
Coil-span factor	—	1.0	0.991
Winding factor	—	0.958	0.950
Slot pitch	mm	21.6	21.6
Conductors per slot	—	2 × 5	2 × 9
Conductor size	mm	6.5 × 3	6.5 × 2.2
area	mm ²	19.0	14.0
Length of mean cond.	m	0.963	1.032
Resistance	Ω/ph	0.426	0.886
Current density	A/mm ²	3.50	1.27
Total f.l. I ² R loss	kW	5.70	0.84
Copper mass	kg	220	250

Rotor

Winding/connection	—	2-layer bar wave/star	
Slip-ring o.c. voltage	V	650	490
Turns per phase	—	45	48
Number of slots	—	90	72
Winding factor	—	0.960	0.960
Slot pitch	mm	28.8	28.8
Conductors per slot	—	2(2 × 3)	2 × 2
Conductor size	mm	13 × 3	20 × 5
area	mm ²	77	98.5
Length of mean cond.	m	0.862	0.963
Resistance	mΩ/ph	21.2	19.7

Table continued . . .

		A	B
Current density	A/mm ²	3.32	2.42
Total f.l. I ² R loss	kW	4.00	0.49
Copper mass	kg	175	265
<i>No-load current</i>			
M.M.F. per pole	A-t	1 100	500
Magnetizing current	A	24.6	6.3
reactance	Ω	70.5	275
Core loss	kW	5.7	2.2
No-load loss	kW	8.6	3.1
current	A	24.6	6.3
power factor	p.u.	0.067	0.095
<i>Short-circuit current</i>			
Slot reactance	Ω	2.26	9.85
Overhang reactance	Ω	1.56	3.75
Zigzag reactance	Ω	1.13	4.40
Differential reactance	Ω/ph	0.30	0.90
Total reactance	Ω/ph	5.25	18.9
Total resistance	Ω/ph	0.85	1.57
Total impedance	Ω/ph	5.3	19.0
Short-circuit current		327	91
power factor	p.u.	0.16	0.083
<i>Performance</i>			
Full-load losses:			
stator I ² R	kW	5.7	0.84
rotor I ² R	kW	4.0	0.49
brush	kW	0	0.14
core	kW	5.7	2.2
mechanical	kW	2.9	0.9
load (stray)	kW	0.9	0.33
total	kW	19.2	4.9
Output	kW	270.0	75.0
Input	kW	289.2	79.9
Efficiency	p.u.	0.93	0.94
Full-load rotor input	kW	276.9	76.5
slip	p.u.	0.0145	0.0061
speed	r/min	591	745
Pull-out torque	p.u.	2.70	2.87

9.3 Design

Traditional formulae and methods are inadequate for modern economic design to extended limits and greater loadings, and for the use of digital computers.

The earliest application of digital computation was simply as a slide-rule. A later step (taken first in induction machine design in 1956) was a form of progressive synthesis, the designer making initial estimates of the independent variables and intervening to modify them in the light of the computed performance. This method is still valuable for the investigation and refinement of specific details; but with fast modern computers it may be expensive to interrupt calculations.

In the design of small induction motors, which have completely standardized coreplates, synthesis is considerably simplified, for many independent variables become constants. The chief independent variables are the stator turns per phase and the core-length, and their determination can be optimized on a cost basis. This means an organized procedure for scanning the area of possible designs until a minimum of the cost function is found.

Only in the case of transformer design (Sect. 6.9) have full synthetic-design programs been so far possible, as a consequence of the relatively few variables compared with those essential in a rotating machine. Here the synthesis must include analysis and subsequent iterative processes, i.e. design analysis (the calculation of performance for any current set of independent variables) and direct-search or gradient iterations to find maxima or minima. Gradient methods require a knowledge of the derivatives of the appropriate function, which carry considerable information regarding the function but are often very difficult to formulate. If these derivatives cannot be stated, direct-search methods have to be used: such a condition frequently arises because some variables (such as slot numbers) are discrete, not continuous.

Formulation

For digital computation, the many variables in the traditional 'hand' design (Sect. 9.2) must be reduced by changing them into fewer new ones with suitable interrelations. Many such formulations are in use: one is the following.

Given the core length, outside diameter, current and flux densities and the specification, the method is (i) to cast the basic equations for starting and pull-out torque, starting apparent power and full-load slip into functions of three variables, the total leakage permeance Λ and the ratios G_1 and G_2 of the stator and referred rotor phase resistances to the total leakage reactance, so that $G_1 = r_1/X_1$ and $G_2 = r_2'/X_1$; (ii) to eliminate slot dimensions from the equations for Λ , G_1 and G_2 by using expressions for slot shape and employing tooth and core flux densities in terms of their levels compared with the gap density. For example, eq.(8.15) for the maximum torque can be expressed as

$$M_m = 0.075 \frac{[1.05 B_g(D - l_g)lK_w]^2}{p\Lambda [G_1 + \sqrt{(1 + G_1)^2}]}$$

G_1 is obtained from $r_1 = \rho_1 L_{mt} N_1/a_1$ and $X_1 = 2\pi f N_1^2 \Lambda$. The mean length of stator turn is based on the core-length l and pole-pitch $\pi D/2p$ in a form such as $L_{mt} = (2l + 1.6\pi D/p)$. The conductor area per slot $a_1 = k_{s1} a_{s1} S_1/6N_1$ uses

the gross slot area a_{s1} , the space factor k_{s1} and the number S_1 of stator slots. Then

$$G_1 = \frac{0.95 \rho_1 (2l + 5D/p)}{k_{s1} a_{s1} S_1 f \Lambda}$$

A rather more complicated expression gives G_2 . Often only a few slot combinations are acceptable for reasons of harmonic torques and noise, and the choice is further limited if the same core plates have to serve for several pole-numbers. It may be possible to fix S_1 and S_2 for a given frame. The calculation of Λ is in any case expressed in terms of a series of slot dimensions, based on expressions such as those in eqs.(2.22 – 27). The slot dimensions also settle the tooth and core areas for a given frame. The effects of changes in various parameters can be computed in the search for an optimum design. The Table shows typical directions of change in slot area, permeance, G -values and the ratio J_1/J_2 of stator/rotor current density that result from an increase in the stator bore D , core length l , and the ratios b_1 and b_2 of the stator and rotor core or tooth flux density to the gap density:

Increase in:		b_1	b_2	D	l
Effect on:					
Gross slot area	a_{s1}	up	0	down	0
	a_{s2}	0	up	up	0
Permeance	Λ	0	0	0	up
Ratio	G_1	down	0	up	0
	G_2	0	down	down	0
Current density	J_1/J_2	down	up	up	0

A design synthesis on this basis gives main dimensions to satisfy specified torque and starting apparent-power requirements. The program is run to arrive at a minimum number of punching forms to cover each frame, and some economic compromise struck for each required rating. It may be found that three sets of punchings per frame are needed: one for a 2-pole machine, in which the core behind the slots has to carry a large flux; one for 4- and 6-pole machines; and one for motors with 8 or more poles. For each set and frame the program is run with D and l independently varied to enable the compromise dimensioning to be selected. A motor can then be designed using the pull-out torque as a criterion. For a given pair of D and l values only one gap flux density gives the required pull-out torque and the core and tooth densities follow, as also does the stator current density. The cooling has, of course, always to be checked.

Programs

Design synthesis programs are not easy to devise. The problem is discussed by Chalmers and Bennington [60], who outline a method for large non-standard-

ized cage motors that co-ordinates a number of suitable techniques. The program routines are:

Passive: Routines that release data (e.g. tool sizes) to the program or which carry out certain design analyses.

Performance: Routines associated with the calculation of equivalent-circuit parameters and calculate performance therefrom.

Active: Program organizing the search for solutions.

Function: Holds a series of function subroutines such as those for calculating reactance, those stating magnetic B/H relations and those for dealing with saturation, slot-openings and end-winding dimensions.

Input and Output: Reads in data, checks its validity and transfers control to a 'steering' routine; each time control is referred back, more data are read in. An update routine records the state of all output parameters at the end of a total cycle. An output routine organizes the specific output routines, and prints out the design and performance information.

Monitor: This consists of two sets of orders, (i) printing information concerning the course of calculation of a particular design to indicate the manner in which the design is obtained, to facilitate detailed improvements; and (ii) trapping errors.

Such a program may take 80 s of computer time and cost about £20 per data case, but produces several alternative designs in the process.

10

Synchronous Machines: Theory and Performance

10.1 *Type and construction*

Large a.c. power networks operating at constant frequency (e.g. 50 or 60 Hz) rely almost exclusively on synchronous *generators* for the provision of electrical energy, and may have synchronous *compensators* at key points for reactive-power control. Private, stand-by and peak-load plant with diesel or gas-turbine prime-movers also have synchronous generators. Non-land-based synchronous plant is found on oil rigs (with a generating capacity of 50 MVA upward), on large aircraft (with hydraulically driven generators working at 400 Hz), and on ships for variable-frequency supply to synchronous propeller motors. Synchronous *motors* provide constant-speed industrial drives with the possibility of power-factor correction, but are not often built in small ratings for which the induction motor is cheaper.

The speed n_s (in r/s) of a synchronous machine is related to the frequency f_1 and the number of pole-pairs p by $n_s = f_1/p$. With a 2-pole construction this means 3 000 and 3 600 r/min for 50 and 60 Hz respectively. The use of a rotating d.c. field system is almost universal, as it permits the a.c. windings to be placed on the stator where they are more readily braced against electromagnetic force and insulated for high voltage. The rotor field construction may be with *salient poles*, or be *cylindrical* with no polar protuberances.

Generators. The prime-mover speed has a profound influence on the constructional form, and in all large units the limiting feature is the centrifugal force on the rotor. Steam-turbine-driven machines (*turbo-generators*) must run at high speed, and have 2-pole rotors of solid forged steel, of diameter limited to about 1.2 m and of axial length several metres. Generators driven by water power (*hydro-generators*) are built for a wide range of comparatively low turbine speeds, and are axially short, but have large diameters to accommodate the many salient poles.

Compensators. Except at start-up, these are not called upon to develop or absorb mechanical power, and are built with cylindrical or salient-pole rotors and a minimum of mechanical loss.

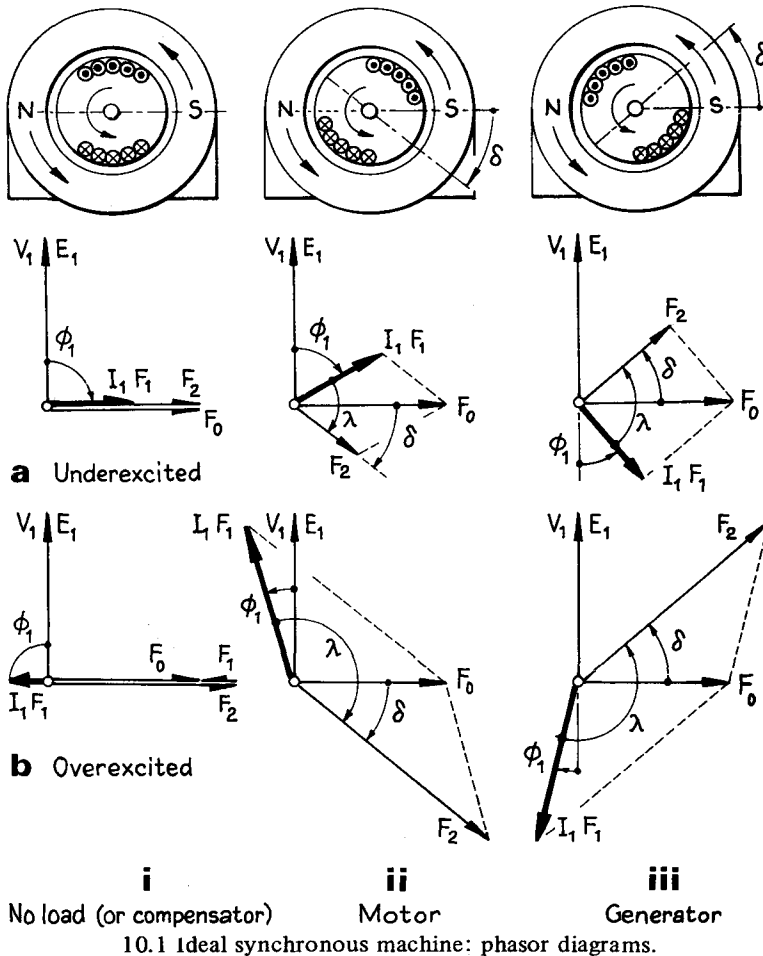
Motors. Both constructional forms are used, in accordance with the synchronous speed required. As a plain synchronous motor develops a torque only at

synchronous speed, provision must be made for it to start in some other way, usually by fitting the rotor with a cage winding to develop an induction torque.

10.2 Action of the ideal synchronous machine

The essential action of a polyphase synchronous machine is most simply introduced in terms of an *ideal* cylindrical-rotor machine connected to *infinite bus-bars*. The ideal machine has stator windings of negligible resistance and leakage inductance, a uniform airgap, a magnetic circuit of high permeability and devoid of saturation, and balanced loading. The infinite busbar system to which the machine is connected produces a balanced 3-phase voltage of constant magnitude and frequency, and can deliver or absorb both active and reactive power without limit.

The machine in Fig. 10.1 has stator windings (not shown) connected to the



infinite busbar, and a rotor carrying a d.c. exciting winding. To work at all the rotor must rotate at synchronous speed. No torque can be developed if the rotor is unexcited, but the stator must take in lagging reactive power to magnetize the machine to a gap flux per pole of Φ_m in order that a stator e.m.f. E_1 can be induced to balance the applied voltage V_1 . If the rotor is now given a small d.c. excitation, it takes over part of the task of exciting the magnetic circuit, reducing the demand of the stator for magnetizing power. This is the condition of *under-excitation*, (a) in Fig. 10.1

With the machine on *no load* (zero mechanical load, and therefore also zero active electrical power), the conditions in (i) apply, and the phasor diagram can be drawn using the representative-phase concept of Sect. 7.1 and incorporating the travelling-wave m.m.f. F_1 of the stator and the rotating m.m.f. F_2 of the rotor. As there is no torque, the m.m.f. axes remain in alignment with a zero torque angle, and the two m.m.f.s combine to give the resultant F_0 necessary to produce Φ_m . If, as in (b), the rotor m.m.f. is considerably raised into the condition of *overexcitation*, the stator must take a leading, demagnetizing current in order that the resultant F_0 shall remain unchanged and still generate Φ_m and $E_1 = V_1$. (For clarity in the phasor diagrams the currents correspond in length to the m.m.f.s that they produce.)

The *no-load* conditions, then, are these: the rotor runs with its axis aligned with that of the stator; the gap flux is constant so that always $E_1 = V_1$; the magnetization is shared between stator and rotor; the stator takes a reactive current, lagging to aid, or leading to oppose, the rotor m.m.f. in accordance with whether the rotor is under- or over-excited. As no torque is produced, both the torque angle and the load angle are zero.

Motor mode

Suppose the machine to be running on no load, and a mechanical retarding torque to be applied to its shaft. The rotor begins to fall back from its coaxial position. The axis of its m.m.f. F_2 is retarded, and to maintain normal gap flux and resultant m.m.f. F_0 , the stator m.m.f. must make the necessary adjustments. Depending on whether the rotor is (a) underexcited or (b) overexcited, the phasor/space diagrams (ii) in Fig. 10.1 describe the conditions of stability that must be reached. In either case the stator takes an active current component, thus accepting power from the supply and developing a forward torque on the rotor to balance the load torque. The reactive component acts, as on no load, to compensate for under- or over-excitation.

Torque and load angles. The torque developed between two cylindrical members with sinusoidally distributed m.m.f.s of peak values F_1 and F_2 per pole is proportional to the sine of the angle λ between their axes (Sect. 1.11). The torque can also be found from the sine of the load angle δ between F_2 and the resultant F_0 . As F_2 has the direction of the magnetic axis of the rotor, and F_0 is directly related to E_1 and therefore to the stator terminal voltage V_1 , it is more convenient here to deal with the *load angle* δ . This angle is indicated in the phasor diagrams of Fig. 10.1. It will be observed that δ is negative for the motor mode.

The synchronous machine thus acts as a motor by a backward adjustment of its load angle. Greater mechanical load calls for an increase in negative load angle. But this cannot go on without limit, for if δ exceeds $-\frac{1}{2}\pi$ rad (elec.) the torque falls below its maximum and the rotor (by 'falling out of step') will desynchronize and stop.

Diagrams (a) and (b) for the motor are drawn with the same load angle. Overexcitation results in greater active current and therefore torque, and the maximum power capability is markedly raised.

Generator mode

Suppose again that the machine is running on no load, but that now a prime mover connected to its shaft provides a driving torque in the direction of the stator travelling-wave field. The rotor moves forward, developing a load angle δ of sign opposite to that for the motor mode. The relative position of the axis of F_2 makes necessary a change in F_1 to provide the constant resultant F_0 . The conditions are shown in Fig. 10.1 (iii), with (a) for underexcitation and (b) for overexcitation. The active component of the stator current reverses, so delivering power into the supply and developing a counter torque to balance the driving torque. The machine thus generates. Again the reactive component of I_1 must provide proper compensation, and this affects the power factor at which the machine works. With respect to output, underexcitation causes ϕ_1 to be a leading angle (meaning that there is an input of lagging reactive magnetizing power from the supply); overexcitation has the opposite effect. For the same load angle δ , greater excitation means greater power, as it does for the motor mode.

Compensator (phase-modifier) mode

A synchronous machine designed to run unloaded, with its shaft connected neither to a mechanical load nor to a prime mover, can operate only in accordance with the no-load conditions (i) of Fig. 10.1. Variation of the rotor excitation causes the machine to take a purely reactive stator current, lagging if underexcited and leading if overexcited. It therefore behaves at its stator terminals as if it were a polyphase inductor or capacitor, changing from one to the other in accordance solely with the rotor excitation. Such a machine, called a *synchronous compensator*, can be used to control the voltage regulation of transmission-line systems by modifying the resultant receiving-end load. The limit to the capacitive effect is the level of rotor excitation, while the inductive effect, as it demands underexcitation, cannot be extended beyond the excitation level below which the machine becomes unstable (tends to lose synchronism).

Operating conditions

The same machine, without change in its stator connections or its direction of rotation, can operate as motor or generator solely by accommodating its load angle to the mechanical conditions imposed on its shaft. Varying the rotor excitation then controls the power factor of the stator current, but not its

power component, except that the *limit* of active power and torque is determined by the excitation. With mechanical load or drive beyond this limit the machine is unable to develop a balancing electromagnetic torque, and falls or rises out of synchronism

As synchronous speed is the only stable operating condition in which sustained torque can be developed, the machine has no torque when it is at rest. It has to be started and brought up to synchronous speed by some auxiliary means.

E.M.F diagrams

The gap flux Φ_m which generates E_1 is excited by F_1 and F_2 in combination. However, in the ideal cylindrical machine, the absence of saturation makes it valid to imagine that F_1 and F_2 , acting across the uniform gap length, excite *separate* sinusoidally distributed gap fluxes Φ_a and Φ_f which superpose to give Φ_m as resultant. The two flux components can then be considered to induce distinguishable e.m.f.s E_a and E_f in the stator windings, having the phasor sum E_1 . Consider the diagram (b)(ii) for the over-excited motor in Fig. 10.1: if this be drawn with e.m.f.s instead of m.m.f.s, then $E_1 = V_1$ is drawn leading $\pi/2$ rad on F_0 , and therefore E_a and E_f bear the same relation respectively to F_1 and F_2 . The e.m.f. E_a due to the stator's own m.m.f. and flux is in quadrature with the phasor of current I_1 and is proportional to I_1 . The e.m.f. diagram then appears as in Fig. 10.2(a), and it can be related to the equivalent circuit (b), in which E_a can be taken as produced by the stator current in an 'armature-reaction reactance' external to a 'machine' in which only E_f due to rotor excitation is generated.

Reaction reactance. The stator m.m.f. and its reaction on the rotor m.m.f. to give the resultant m.m.f. is real enough: its representation by an 'external' inductive reactance is fictitious, but convenient. Although it somewhat obscures the physical action and applies only to steady-state conditions, by its use we can derive direct expressions for power and torque in terms of measurable quantities — the stator terminal voltage, the rotor exciting current, and the load angle. The (armature-) *reaction reactance* $x_a = E_a/I_1$, a measure of the stator m.m.f. per unit current, is readily calculable from the stator winding details and the gap dimensions, or it can be obtained directly from the open- and short-circuit characteristics of the machine.

Open-circuit characteristic. The o.c.c. relates the phase e.m.f. of the stator on open circuit to the rotor excitation when the machine is driven at synchronous speed. The measured phase e.m.f. E_{oc} then directly determines E_f for any given rotor excitation.

Short-circuit characteristic. The s.c.c. is found by driving the machine (preferably at synchronous speed, but in an ideal machine the actual speed — provided that it is not zero — does not matter) with the stator phases short-circuited. The conditions are then that $V_1 = E_1 = 0$, so that there is no resultant gap excitation, and in consequence the stator and rotor m.m.f.s just balance out with $F_1 = F_2$.

Open- and short-circuit characteristics are shown in Fig. 10.2(c). On the

basis of the equivalent circuit (b), the argument is that E_{oc} for a given excitation is E_t , and that therefore I_{sc} for the same rotor excitation is such that $E_a = E_t$; whence the reaction reactance must be $x_a = E_{oc}/I_{sc}$.

Operating characteristics

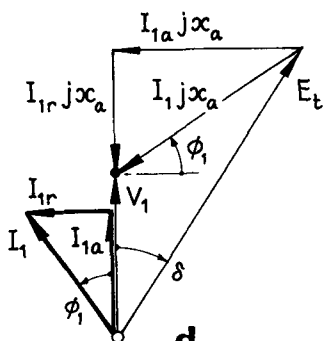
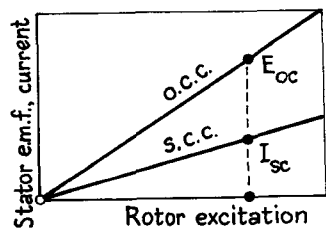
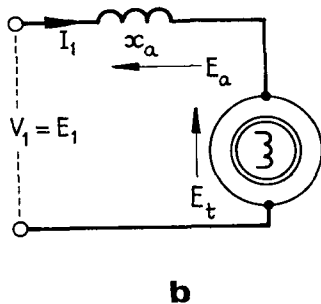
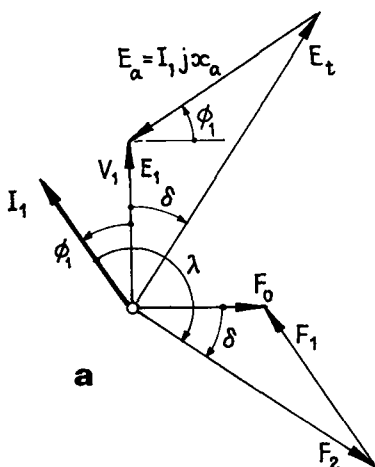
The phasor diagram in Fig. 10.2(d) is for the same conditions as in (a), namely an overexcited motor mode. The input current is shown resolved into the active component $I_{1a} = I_1 \cos \phi_1$ and the leading reactive component $I_{1r} = I_1 \sin \phi_1$. The corresponding components of reaction reactance voltage can be seen by inspection to have the scalar magnitudes

$$I_{1a}x_a = -E_t \sin \delta \quad \text{and} \quad I_{1r}x_a = V_1 - E_t \cos \delta$$

Multiplying through by V_1 gives the input active and reactive electrical powers per phase:

$$P_1 = -(V_1/x_a) E_t \sin \delta \quad (10.1)$$

$$Q_1 = (V_1/x_a) [E_t \cos \delta - V_1] \quad (10.2)$$



10.2 Ideal synchronous machine: equivalent circuit.

These relate the powers per phase to the rotor excitation level and the load angle for a machine of given reaction reactance connected to an infinite busbar of voltage V_1 . As the ideal machine has no losses, P_1 is also the mechanical power per phase, and if divided by the synchronous speed ω_s gives the torque. For a motor the load angle δ is negative, giving P_1 as a positive input; for a generator with a positive load angle, the input P_1 is negative, meaning an electrical output. A positive value for Q_1 means that the reactive power input is leading.

The relations in eqs.(10.1) and (10.2) are illustrated in Fig. 10.3 for a machine with a reaction reactance $x_a = 1.25$ p.u. The curves of active and reactive input powers are drawn for excitations E_t expressed in per-unit values, where 1.0 p.u. excitation is that which gives $E_t = V_1$. For any given power P_1 and excitation E_t there is a particular load angle δ and an associated leading or lagging reactive power Q_1 .

Synchronizing power and torque. A synchronous machine tends to maintain itself in synchronism with the supply. Suppose a machine, running on a steady load P_1 with a load angle δ , to begin to rise or fall in speed: then the load angle begins to diverge from the balance condition and the input power also changes, so that a power differential between input and load occurs. The difference acts against the angular disturbance to return the rotor axis towards its balance condition; for example, if a motor driving a constant-torque mechanical load begins to slow down, its load angle becomes more negative and it takes in more electrical power, the excess restoring the rotor axis to the original state. The machine will remain synchronized provided that an angular divergence brings about an appropriate change in power, called the *synchronizing power* and defined in terms of power per electrical radian of displacement: hence

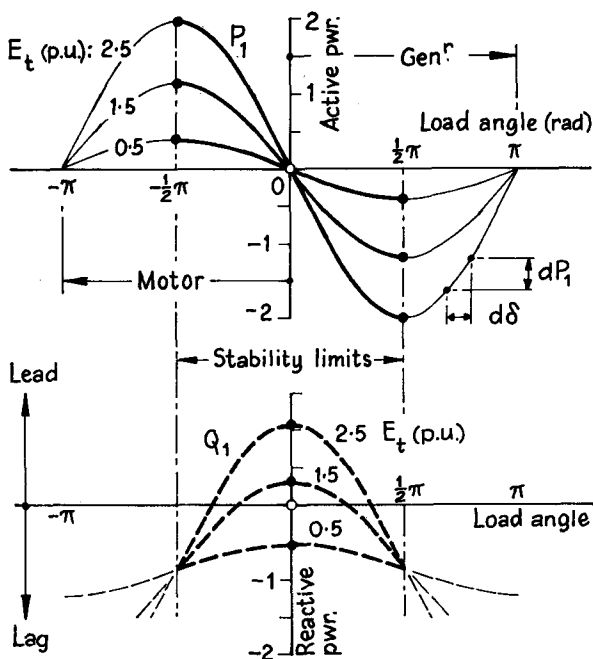
$$p_s = dP_1/d\delta = (V_1/x_a)E_t \cos \delta \quad (10.3)$$

is the synchronizing power per radian. The corresponding synchronizing torque is $M_s = p_s/\omega_s$. More practically, both p_s and M_s can be specified in terms of power or torque per mechanical degree of displacement, using the multiplier $p(\pi/180)$ for a machine with p pole-pairs.

The synchronizing power is a maximum for no load ($\delta = 0$) and reduces as the machine is more heavily loaded. For a load angle of $\pm\frac{1}{2}\pi$ rad (elec.) it vanishes altogether, because angular divergence no longer changes the power. Up to this limit, the larger the load angle, the less stably does the machine operate.

Synchronizing power can circulate between two machines in parallel (but not connected to an infinite busbar) to keep them in step. Eq.(10.3) holds if V_1 is replaced by the E_t value of the second machine, and x_a by the sum of the two individual reaction reactances. The angular divergence which brings the synchronizing power and torque into play is that between the axes of the two rotors. The 'faster' machine is slowed by the synchronizing power that it circulates to the 'slower' machine, as a result of which the latter is accelerated to reduce the divergence.

Hunting. Acceleration or retardation of a machine by synchronizing torque imparts to the rotating system an angular acceleration and a change of kinetic



10.3 Active and reactive power/load-angle relations.

energy. A machine will therefore over-swing beyond the equilibrium angle, build up a synchronizing torque in reverse, and continue to hunt (i.e. to support variations of speed about the mean synchronous speed) unless the kinetic swing energy can be dissipated, as it must be in a practical machine. As the machine has a synchronizing torque proportional to divergence, the analogue of an 'elastic' property, its natural undamped frequency of hunting is given by

$$\omega_0 = \sqrt{(M_s/J)} \quad (10.4)$$

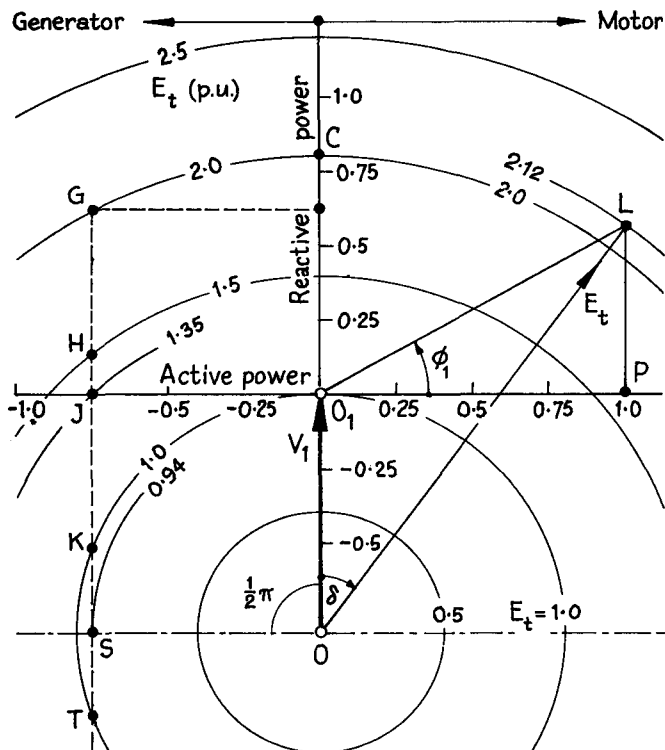
when the moment of inertia of the rotating parts is J .

Voltage regulation. For a generator, the *regulation* is the voltage rise at the terminals when a given load is thrown off, the machine being disconnected from the infinite busbar but the speed and excitation remaining unchanged. The rise is the scalar difference between E_t and V_1 and the regulation is defined as

$$\epsilon = (E_t - V_1)/V_1 \text{ p.u.} \quad (10.5)$$

The regulation depends on the load and power factor. It may be zero for a leading power factor load, or even be negative. Generally synchronous generators operate on lagging loads and have a large positive regulation.

Electrical load diagram. This shows all possible operating conditions for a



10.4 Ideal synchronous machine: electrical load diagram. The scales are in per-unit values for a reaction reactance of 1.25 p.u.

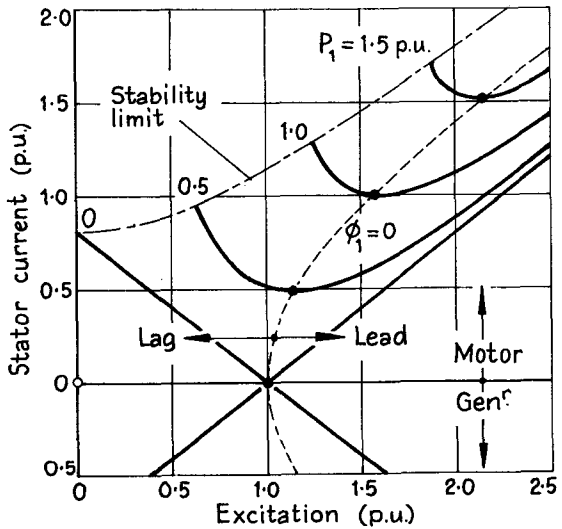
machine on infinite busbars. It is based on the phasor diagram (*d*) of Fig. 10.2: let this represent full-load conditions. Then the electrical load diagram, Fig. 10.4, is built round it with the constant busbar voltage V_1 represented by OO_1 , E_t by OL and $I_1 x_a$ by LO_1 , with PO_1 and LP as its active and reactive components. As PO_1 is for full-load active current and so also for 1.0 p.u. input active power P_1 , this line can be scaled in per-unit active power with positive values for motor and negative for generator action. The same scaling can be used for the vertical axis through OO_1 to give the per-unit reactive power Q_1 , with positive for leading and negative for lagging inputs. Load point L then gives direct scale values of P_1 and Q_1 for the operating condition, which requires $E_t = OL$ to scale. Constant values of E_t appear as circles struck from the origin O . The angle PO_1L is the terminal phase angle between current and voltage (in this case leading, because the machine is operating in the motor mode with overexcitation). The angle O_1OL is the load angle, in this case negative.

The diagram can give the essential operating features for any specified steady-state condition. Point G , for example, corresponds to inputs of $P_1 = -0.75$ p.u.

and $Q_1 = +0.60$ p.u.: it represents working in the generator mode with outputs $-P_1 = +0.75$ p.u. and $-Q_1 = -0.60$ p.u., obtained with an excitation $E_t = OG = 2.0$ p.u. (i.e. twice that corresponding to $E_t = V_1$). If the excitation is reduced, the prime-mover power remaining unchanged, the operating point travels along the dotted line GHJ . . . , the reactive power changing to compensate. At point S the load angle is $\frac{1}{2}\pi$ rad, corresponding to maximum power with an excitation of $E_t = 0.94$ p.u. With higher excitations the operating point can lie in a region where $\delta > \frac{1}{2}\pi$ rad, but it is not stable. The horizontal line OS, the maximum-power line, can also be called the *steady-state stability limit*.

The electrical load diagram gives solutions of eqs.(10.1) and (10.2). It makes evident that a synchronous machine working on infinite busbars can operate as a motor or compensator or generator, respectively at a negative, zero or positive load angle, depending upon the mechanical power given to it or demanded from it. The level of the excitation determines the operating power factor (and to a lesser extent the load angle), and the greater the excitation the greater is the maximum power capability of the machine.

V-curves. These, called so because of their shape, relate input current I_1 to excitation E_t for specified constant active powers. They can be constructed direct from the electrical load diagram. For example, the operating conditions for a generator, with its per-unit reaction reactance the same as that in Fig. 10.4 and delivering 0.75 p.u. active power, are given by the dotted line GHJKS. The stator phase current is proportional to $O_1G, O_1H \dots$ for excitations of 2.0, 1.5 . . . p.u., so that plots of the current/excitation relations can readily be drawn. Fig. 10.5 shows V-curves obtained in this way from Fig. 10.4. As the



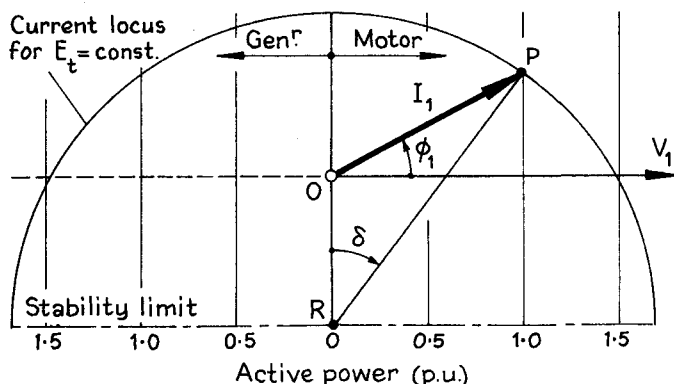
10.5 V-curves.

The scales are in per-unit values for a reaction reactance of 1.25 p.u.

the electrical load diagram is symmetrical with respect to generator and motor operation, the V-curves apply to both.

Constant active load, variable excitation. This condition implies a constant shaft torque (driving or driven) and a constant active current component I_{1a} . Change in excitation affects only the reactive current component I_{1r} , which acts to make up the deficiency in rotor excitation for underexcited conditions, or to oppose the excess for overexcitation. As a result, the terminal power factor is changed, and can be made leading or lagging at will. Excitation also affects the load angle, which reduces as the excitation is raised. Further, there are maximum and minimum excitations for a given active power; synchronous running is lost if the excitation is too low, while the maximum is not normally reached in a normal machine because of the very large stator and rotor currents involved. A typical characteristic is given by the dotted line GHJKS in Fig. 10.4

Constant excitation, variable active load. Triangle OLO_1 in Fig. 10.4 is composed by the scalars E_t , $I_1 x_a$ and V_1 . If all three are divided by x_a they yield the triangle RPO in Fig. 10.6 in which $OP = I_1$. The circular locus of P gives



10.6 Current circle diagram for constant excitation.

the current in magnitude and phase for any condition corresponding to a given fixed value of the excitation E_t . Angle ORP is the load angle δ . As the active load is increased, the terminal phase angle ϕ_1 changes from lead to lag through zero, and δ increases to the stability limit of $\pm \frac{1}{2}\pi$ rad. For underexcitation the input current may lag for all loads, and the maximum power will obviously be reduced.

Summary

The steady-state behaviour of an ideal synchronous machine can be inferred from its electrical load diagram. The important features are (i) the load angle, which changes from negative to positive values as the machine changes from the motor to the generator mode; (ii) the power factor of the input current, which varies in accordance with the requirement that the combined m.m.f.s F_1 and F_2 of stator and rotor shall yield across the gap a resultant F_0 to mag-

netize the machine to a flux that generates the e.m.f. E_1 balancing the applied voltage V_1 .

EXAMPLE 10.1: An ideal 1.5 MW, 50 Hz, 3-ph, 8-pole synchronous machine with a reaction reactance of 1.25 p.u. runs on 6.6 kV infinite busbars. The moment of inertia of the rotor and its mechanical attachments is 3 100 kg-m². The machine operates

- (a) on no load with 1.0 p.u. excitation;
- (b) as a compensator with 2.0 p.u. excitation;
- (c) as a motor on full load (1.0 p.u.) at power factor 0.87 leading;
- (d) as a generator on 0.75 p.u. active power at an output power factor of 0.78 lagging.

For each case estimate the load angle, synchronizing power and torque, and the natural frequency of hunting oscillation; for case (d) find the voltage regulation.

The electrical load diagram in Fig. 10.4 is drawn for $x_a = 1.25$ p.u.; the power/load-angle curves in Fig. 10.3 and the V-curves in Fig. 10.5 also apply here. Using per-unit values based on the full rating (1.5 MW) enables calculations to be made for the whole machine direct instead of per phase.

(a) *No Load:* The no-load operating point with $E_t = 1.0$ p.u. is O_1 in Fig. 10.4. The stator current and the load angle are both zero. From eq.(10.3) the synchronizing power is

$$p_s = (1.0/1.25)1.0 = 0.80 \text{ p.u./rad(e.)} = 1.20 \text{ MW/rad(e.)}$$

As 1 rad(m.) = 4 rad(e.), then also

$$p_s = 0.80 \times 4 = 3.2 \text{ p.u./rad(m.)} = 0.056 \text{ p.u./deg(m.)}$$

The synchronous speed is $\omega_s = 2\pi 50/4 = 78.6 \text{ rad(m)/s}$ so that the corresponding synchronizing torque is

$$M_s = 3.2 \times 1.5 \times 10^3/78.6 = 61 \text{ kN-m/rad(m.)}$$

Applying eq.(10.4), the natural angular frequency of hunting oscillation is

$$\omega_0 = \sqrt{(61\ 000/3\ 100)} = 4.44 \text{ rad(m.)/s}$$

that is, an oscillation frequency $f_0 = 0.71 \text{ Hz}$, or one complete swing in 1.4 s.

(b) *Compensator:* Raising E_t to 2.0 p.u. shifts the operating point to C. The load angle is still zero (there being no active power concerned) but the machine takes a reactive input of 0.80 p.u. = 1.2 Mvar leading as a synchronous capacitor. The values required are

$$p_s = (1.0/1.25)2.0 = 1.60 \text{ p.u./rad(e.)} = 6.4 \text{ p.u./rad(m.)}$$

$$M_s = 6.4 \times 1.5 \times 10^3/78.6 = 122 \text{ kN-m/rad(m.)}$$

$$\omega_0 = \sqrt{(122\ 000/3\ 100)} = 6.28 \text{ rad/s, } f_0 = 1.0 \text{ Hz.}$$

(c) *Motor:* The operating point is L, with 1.0 p.u. active power (1.5 MW), 0.57 p.u. leading reactive power (0.85 Mvar) and 1.15 p.u. apparent power (1.72 MVA). The necessary excitation is readily calculated, or measured from the

load diagram: it is $E_t = 2.12$ p.u. The load angle is $-36^\circ = -0.63$ rad(e.). The synchronizing power and torque are

$$p_s = (1.0/1.25) \times 2.12 \times \cos(-36^\circ) \times 4 = 5.49 \text{ p.u./rad(m.)}$$

$$M_s = 5.49 \times 1.5 \times 10^3 / 78.5 = 105 \text{ kN-m/rad(m.)}$$

and the natural frequency is $\omega_0 = 5.8$ rad(m.)/s, $f_0 = 0.92$ Hz.

(d) *Generator*: The conditions are given by G, with $OG = E_t = 2.0$ p.u., and inputs of -0.75 p.u. active and $+0.60$ p.u. reactive. The output of the machine as a generator is therefore $+0.75$ p.u. = 1.12 MW and $-0.60 = 0.9$ Mvar lagging. By calculation or measurement, $\delta = +28^\circ = +0.49$ rad(e.). Then

$$p_s = 5.64 \text{ p.u./rad(m.)}, \quad M_s = 108 \text{ kN-m/rad(m.)},$$

$$\omega_0 = 5.9 \text{ rad(m.)/s}, \quad f_0 = 0.94 \text{ Hz}$$

The regulation is $\epsilon = (E_t - V_1)/V_1 = 1.0$ p.u., because obviously the open-circuit terminal voltage is double the busbar voltage.

10.3 Practical synchronous machines

The assumptions, explicit and implicit, made in dealing with the ideal machine in Sect. 10.2 are the following:

- (i) a symmetrical cylindrical construction in which the stator and rotor m.m.f. and their resultant are sinusoidally distributed;
- (ii) an unsaturated magnetic circuit of high permeability, and a uniform radial gap providing the whole reluctance and producing a sine-distributed gap flux under all conditions;
- (iii) negligible electrical, magnetic and mechanical losses, and no magnetic leakage;
- (iv) purely steady-state operating conditions.

A practical machine may have salient poles, and even if its construction is cylindrical it still has some degree of effective polarization; it therefore has a non-sinusoidal distribution of gap flux. The windings are distributed in slots, and give rise to space harmonics of gap-flux density so that the induced e.m.f. contains time harmonics besides a fundamental. The machine has conduction, core and mechanical losses producing heat, and it has a significant level of leakage flux. Its operating conditions include not only the steady state, but also sudden load change, hunting, starting, and the effects of faults both within the machine structure and in the electrical network into which it is connected. Finally, saturation of the magnetic circuit varies widely with load and excitation and markedly influences the behaviour of the machine.

Losses

Core losses occur in the yoke, core, tooth and end-winding regions of the magnetic circuit, the stator (and, in cylindrical machines, the rotor) teeth being normally worked at high levels of saturation. I^2R and eddy-current losses occur in the stator windings, and the rotor excitation power requirement may be very large in machines of high rating. Load (stray) losses are caused by alternating

leakage fluxes that appear, when the machine is loaded, in pole-faces and end regions. Mechanical windage and friction loss may be considerable with machines of large diameter or high speed; and in heavy generators the bearing friction may demand forced cooling to avoid excessive temperature-rise in the lubricant.

Electrical and magnetic losses have to be dissipated by a cooling system, sometimes elaborate, in order to limit the thermal stress imposed on the winding insulation. In turbo-generators the structural temperature-rise may cause significant dimensional changes that have to be accommodated by permitting cyclic expansion and contraction.

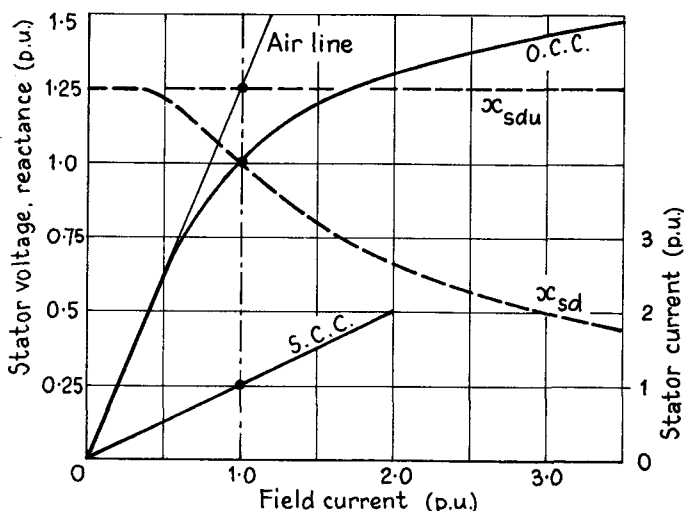
Leakage flux

Rotor leakage flux adds to the saturation conditions of the rotating part of the magnetic circuit. Self-flux linking the stator slot and overhang conductors, besides contributing to eddy-current and stray losses, endows the stator phase windings with leakage reactance x and a corresponding volt-drop $I_1 x$. It is particularly important during transients as it forms the chief limitation on current growth.

Saturation

Magnetic saturation considerably affects the performance of a synchronous machine in respect of its excitation, regulation and circuit impedances. By introducing a strong non-linearity it makes a linear analysis, like that in Sect. 10.2, almost impossible. The saturation phenomenon is a function of the magnetic material concerned and its working flux density, and is therefore a local effect that varies from point to point in space and (for varying flux densities) in time. For a machine its mass overall effect is normally taken as represented by the open-circuit characteristic.

Open-circuit and short-circuit characteristics. The *open-circuit characteristic* or *o.c.c.* relates the terminal voltage on open circuit at normal speed to the amount of field excitation. Since under these conditions the terminal voltage measured is the induced e.m.f., which depends on the total flux Φ_m linking the stator winding, the *o.c.c.* is a measure of the saturation in the magnetic circuit. At low values of field excitation the *o.c.c.* is rectilinear: the iron parts of the magnetic circuit have only a small magnetic loading and are highly permeable, so that the chief reluctance in the circuit is that of the gap. For a density of, say, 0.8 T a laminated ferromagnetic material requires only about 100 A-t/m, but an air path requires 640 000 A-t/m. The part of the excitation devoted to the gap is defined by the air-line. Fig. 10.7 shows an *o.c.c.* typical of a large turbo-generator: comparison with the *o.c.c.* in Fig. 10.2(c) indicates the effect of saturation. For high flux densities the ferromagnetic parts suffer a considerable decline in permeability, and the *o.c.c.* for the upper range of field excitation has a comparatively small slope. At normal terminal voltage, corresponding to 1.0 p.u. field excitation, about 20% of the exciting m.m.f. is expended on the gap, whereas at 1.25 p.u. voltage the gap m.m.f. has risen to 1.0 p.u. and the iron m.m.f. to 0.7 p.u. giving a total of 1.7 p.u. for the whole

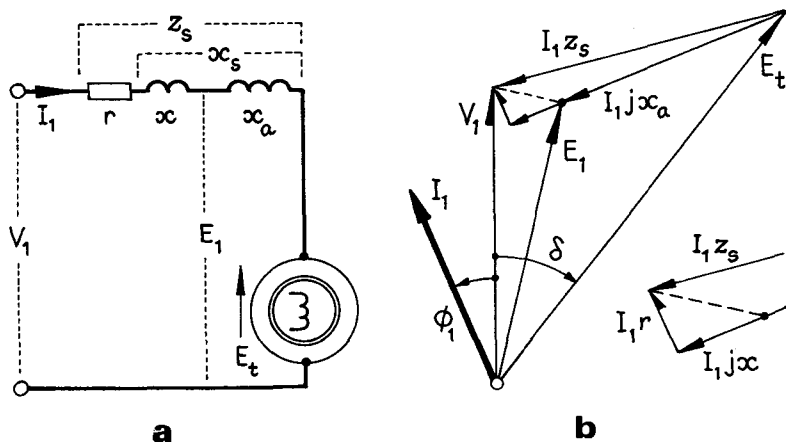


10.7 Open- and short-circuit characteristics.

magnetic circuit. At higher saturations the excitation expended on the iron paths exceeds that on the gap.

The *short-circuit characteristics* (s.c.c.) relates principally the 'armature reaction' to the field excitation. If there were no leakage reactance nor resistance, the short-circuit current in the stator would furnish the a.c. equivalent of the d.c. rotor excitation. The field excitation has to exceed this equivalent value by the amount necessary to produce the small flux that generates the e.m.f. for circulating the short-circuit current through the resistance and leakage reactance. So long as this flux is small enough the s.c.c. remains a rectilinear relation between armature and field currents. At high values of short-circuit current, however, the field current is high and the consequent leakage produces saturation in the poles after which a given increase in the exciting current produces less increase in the stator current. In most machines the curvature of the s.c.c. is only slight, and starts well above full-load current; thus a single point together with the origin suffices to locate the s.c.c.

Synchronous reactance. As just observed, in an actual machine the excitation on short circuit has not only to balance the stator current, but must also generate the voltage drop in resistance and stator leakage reactance. Thus the equivalent circuit for the ideal machine, Fig. 10.2(b), must be replaced by that in Fig. 10.8(a), and the ideal phasor diagram by Fig. 10.8(b). The winding resistance r and the true leakage reactance x are additions to the ideal machine, and now E_1 differs from V_1 by the volt-drops due to I_1 in r and x . Alternatively, V_1 and E_1 can be related more directly by combining x and the reaction reactance x_a to give an effective total reactance $x_s = x_a + x$ called the *synchronous reactance*. If further we include r , the total impedance is the *synchronous impedance*, z_s .



10.8 Practical synchronous machine: equivalent circuit and phasor diagram.

In a short-circuit test, the current for a given excitation is effectively determined by the synchronous impedance, and the ratio of the open-circuit terminal e.m.f. to the short-circuit current for the same field excitation is therefore $E_{oc}/I_{sc} = z_s$. The ratio is naturally a function of the saturation conditions, and its variation is shown in Fig. 10.7.

In large machines the leakage reactance is of the order of 0.10 p.u., and the reaction reactance x_a is typically 1.25 p.u. The phase resistance, however, may be only about 0.01 p.u., so that z_s scarcely differs at all numerically from x_s : in diagrams based on phasors the resistance drop can normally be ignored as regards its effect on the geometry of the diagram, especially in view of the saturation dependence of x_s .

Load angle. For strict consistency with the concept of load angle employed for the ideal machine, the angle between E_t and E_1 should be taken. However, it is far more convenient to adopt as a definition the angle between E_t and V_1 for a practical machine, as indicated in Fig. 10.8(b). In effect, this is to consider the synchronous reactance rather than the reaction reactance as the operative quantity.

Saliency

In a salient-pole machine, and also to a minor extent in a cylindrical-rotor machine, the gap flux produced by a distributed m.m.f. depends upon the orientation of the m.m.f. axis with respect to that of the saliency, and the gap reluctance presented to it. The very different effects of orientation have been discussed before, in Sect. 3.14 and Fig. 3.32. The component F_{1d} of the (assumed) sinusoidally distributed stator m.m.f. directed in the pole or direct axis operates over a magnetic circuit identical with that of the rotor field system; but the quadrature-axis component F_{1q} is applied across the interpolar space, producing a smaller flux per ampere and a markedly different distribution.

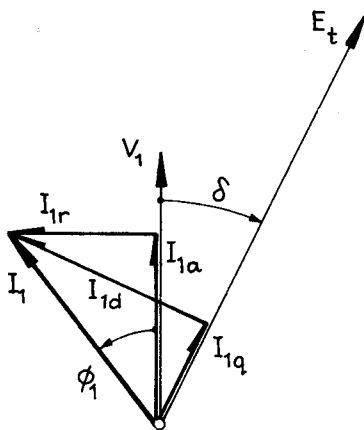
The two-reaction theory considers direct- and quadrature-axis reactions

separately, assigning different reaction reactances to each so that the appropriate axis synchronous reactances are

$$x_{sd} = x + x_{ad} \quad \text{and} \quad x_{sq} = x + x_{aq}$$

for the direct and quadrature axes respectively. The q-axis reactance is smaller than the direct-axis reactance because a given m.m.f. gives rise to a smaller gap flux.

It is necessary to distinguish the active and reactive components of I_1 from its d- and q-axis components. The former are related to the applied voltage V_1 through the terminal phase angle ϕ_1 , the latter to the magnetic axis of the rotor. In Fig. 10.2(a) it is seen that E_t is drawn at right-angles to the rotor magnetic axis given by F_2 ; hence the d-axis component is at right-angles to E_t . The two



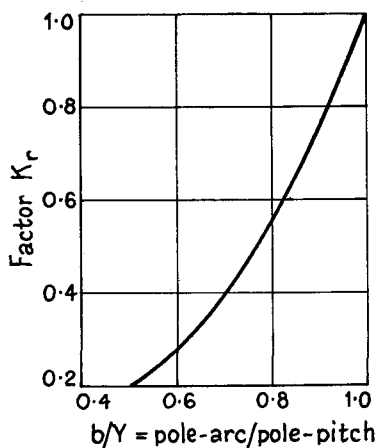
10.9 Current components.

sets of components are shown in Fig. 10.9. When δ is the load angle and ϕ_1 is the terminal phase angle, then

$$\begin{aligned} I_{1a} &= I_1 \cos \phi_1 & I_{1r} &= I_1 \sin \phi_1 & I_1 &= \sqrt{I_{1a}^2 + I_{1r}^2} \\ I_{1q} &= I_1 \cos(\phi_1 - \delta) & I_{1d} &= I_1 \sin(\phi_1 - \delta) & I_1 &= \sqrt{I_{1q}^2 + I_{1d}^2} \end{aligned}$$

On a short-circuit test $\delta = 0$ and ϕ_1 is very nearly $-\frac{1}{2}\pi$ rad, so that $I_1 = I_{1r} = I_{1d}$ and the reactance obtained from the o.c.c. and s.c.c. ratio is the direct-axis synchronous reactance x_{sd} (neglecting the very small effect of resistance). The q-axis synchronous reactance is obtainable in design from flux plots, and experimentally by a 'slip test', Sect. 10.15. As the relation between x_{sq} and x_{sd} depends upon the constructional geometry of the machine, particularly the ratio b/Y of the pole-arc to the pole-pitch, it is possible to infer x_{sq} from the expression $x_{sq} = K_r x_{sd}$, where the factor K_r is given approximately by the graph in Fig. 10.10.

Values of direct- and quadrature-axis synchronous reactances lie in the



10.10 Cross-reaction coefficient.

range 0.8–1.6 p.u. and 0.65–1.0 p.u. respectively, the higher figures applying to synchronous compensators.

Other practical features

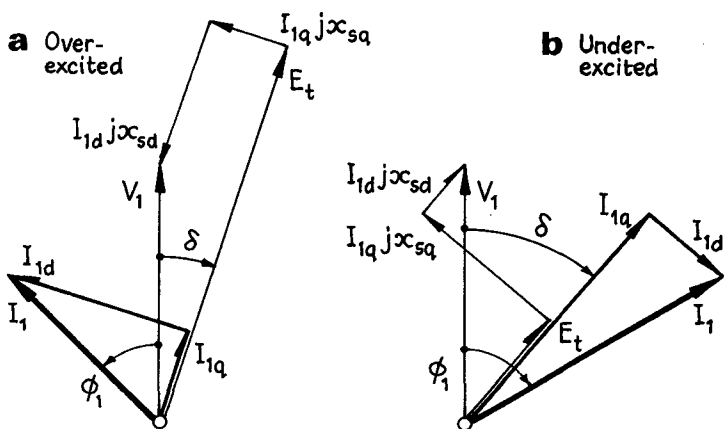
Harmonics are discussed in Chapter 3, and mechanical considerations in Chapter 7. Electrical features of generators and motors are dealt with in subsequent Sections of this Chapter. First, the steady-state action of the machine with saliency is taken up to show certain modifications that must be made to the theory of the ideal machine. The analysis of transient behaviour cannot be undertaken on a basis of steady-state theory: it requires the much more elaborate circuit model described in Chapter 7.

10.4 Steady-state theory

The theory of the practical synchronous machine differs from that of the ideal in respect (i) of the use of the synchronous reactance in place of the reaction reactance, (ii) of the inclusion, where its effect is significant, of stator-winding resistance, and (iii) of saliency. Further, the synchronous reactance is not a constant but varies with the conditions of magnetic-circuit saturation.

Phasor diagram

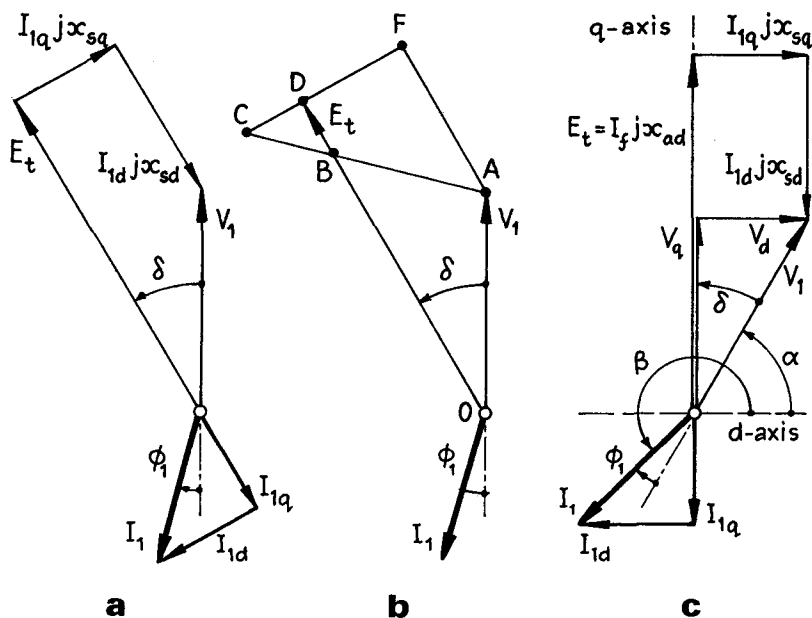
Motor mode. Phasor diagrams for a motor with a given active input power are given in Fig. 10.11, with (a) for overexcited and (b) for underexcited conditions and for known d- and q-axis synchronous reactances. (The volt drop $I_1 r$ in resistance has been omitted for clarity: it would be inserted as in Fig. 10.8(b) as a phasor cophasal with the current I_1 .) Each reactance voltage is drawn in quadrature with its respective axis current: $I_1 q x_{sq}$ is smaller than it would be in a machine without saliency (for which $x_{sq} = x_{sd} = x_s$) and as a result the load angle is also less. The salient-pole machine is magnetically 'stiffer' as the structural polarization contributes reluctance power by alignment effect.



10.11 Phasor diagram of salient-pole motor.

Generator mode. A phasor diagram for an overexcited generator is given by Fig. 10.12(a).

In drawing these phasor diagrams, the d- and q-axis currents and reactance voltages are not known until the load angle between V_1 and E_t is fixed, but δ in turn depends upon the axis reactance voltages. A graphical construction to resolve the difficulty is given in Fig. 10.12(b) for the case of the overexcited



10.12 Phasor diagram of salient-pole generator.

generator. It is assumed that the terminal conditions (voltage, current and phase angle) and the two axis reactances are known: then V_1 and I_1 can be drawn to locate the point A. Draw AC in a direction perpendicular to I_1 and of length equal to $I_1 x_{sd}$. Locate point B such that BA/CA is the ratio x_{sq}/x_{sd} . Then the line OB has the required direction of E_t , and the magnitude of E_t is found by dropping the perpendicular CDF on OB, where CFA is a right-angle. Then OD is the phasor E_t , DF is $I_1 x_{sq}$ and FA is $I_1 x_{sd}$. The ratio BA/CA is the same as K_r in Fig. 10.10. If the saliency is negligible, then $CA = I_1 x_s$ and $OC = E_t$, the load angle being increased as a result.

Circuit theory. The dynamic circuit theory of Sect. 7.5 can give the steady-state operation as a special case. For a 2-pole generator on steady balanced load and connected to a symmetrical 3-phase supply of angular frequency ω_1 , the speed is $\omega_r = \omega_1$, the angle θ in Fig. 7.13 is $\omega_1 t$ reckoned from the instant that the axis of phase A coincides with the d-axis, and the field current i_f is constant. Let the applied voltage and input current of phase A be $v_a = v_m \cos(\omega_1 t + \alpha)$ and $i_a = i_m \cos(\omega_1 t + \beta)$; corresponding values for phase B and C are written by adding $(-2\pi/3)$ and $(-4\pi/3)$ respectively to the cosine terms. Applying eq. (7.20) gives

$$\begin{aligned} v_d &= v_m \cos \alpha & i_d &= i_m \cos \beta \\ v_q &= -v_m \sin \alpha & i_q &= -i_m \sin \beta \end{aligned}$$

These, being independent of time, are *direct* currents and voltages.

The circuit equations are now written direct from the matrix. All terms in $p = d/dt$ are zero and there are no currents in KD or KQ. We can write $r_d = r_q = r$, and $L_d = L_q = L$ from symmetry, whence $\omega_1 L = x$, the leakage reactance. Further the reaction reactances x_{ad} and x_{aq} can be written for $\omega_1 L_{ad}$ and $\omega_1 L_{aq}$ respectively, and the axis synchronous reactances as $\omega_1 (L_{ad} + L) = x_{sd}$ and $\omega_1 (L_{aq} + L) = x_{sq}$. In per-unit terms, the equations for the three active circuits F, D and Q reduce to

$$\text{and} \quad \begin{aligned} v_f &= r_f i_f, & v_d &= r i_d + x_{sq} i_q \\ v_q &= r i_q - x_{sd} i_d - x_{ad} i_f \end{aligned}$$

The term $-x_{ad} i_f$ is a measure of the gap flux due to the field excitation and is consequently represented by e_t .

The results can be used to construct the complexor diagram, Fig. 10.12(c), in terms of r.m.s. values. The phasor applied voltage and input current for phase A are

$$\begin{aligned} V_1 &= V_d + V_q = (\cos \alpha + j \sin \alpha) (v_m / \sqrt{2}) = (v_d - j v_q) / \sqrt{2} \\ I_1 &= I_d + I_q = (\cos \beta + j \sin \beta) (i_m / \sqrt{2}) = (i_d - j i_q) / \sqrt{2} \end{aligned}$$

There is no essential difference between (c) and (b) of Fig. 10.12.

In per-unit terms, the inductance coefficients in eq. (7.17) can be expressed as their corresponding reactances. Only currents i_f , i_d and i_q are concerned, so that the torque is

$$M = -x_{ad} i_f i_q - (x_{sd} - x_{sq}) i_d i_q$$

The last term is the reluctance torque, which vanishes in a non-salient machine, leaving only the torque due to the main flux x_{ad} if.

Operating characteristics

Because of the dependence of synchronous reactance on the degree of saturation and its high per-unit value in a normal machine compared with the stator winding resistance, the latter is neglected in dealing with performance characteristics except, of course, where losses and their thermal implications are concerned. Analytical expressions are greatly simplified in this way, and are quite adequate for the purpose.

Active power. The active power input to a *non-salient* machine is given by eq.(10.1) modified to include the leakage reactance x , i.e. with x_s substituted for x_a to give

$$P_1 = -\frac{V_1}{x_s} E_t \sin \delta \quad (10.6)$$

which is positive for the motor mode (δ negative). In a *salient* machine it is necessary to employ the axis synchronous reactances. The active power is $P_1 = V_1 I_1 \cos \phi_1$: Fig. 10.9 shows that we may write

$$I_1 \cos \phi_1 = I_{1q} \cos \delta - I_{1d} \sin \delta$$

while from the phasor diagram in Fig. 10.11(a) we have

$$I_{1q} x_{sq} = -V_1 \sin \delta \quad \text{and} \quad I_{1d} x_{sd} = E_t - V_1 \cos \delta$$

Making the substitutions then gives

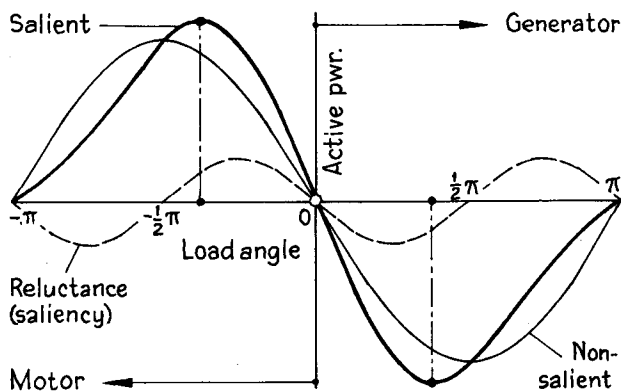
$$\begin{aligned} P_1 &= V_1 \left[-\frac{E_t}{x_{sd}} \sin \delta - \left(\frac{V_1}{x_{sq}} - \frac{V_1}{x_{sd}} \right) \sin \delta \cdot \cos \delta \right] \\ &= -\frac{V_1}{x_{sd}} E_t \sin \delta - \frac{V_1^2 (x_{sd} - x_{sq})}{2x_{sd} x_{sq}} \sin 2\delta \end{aligned} \quad (10.7)$$

The first term has the form of eq.(10.6). The double-angle term, due to saliency expresses the difference in the axis gap reluctances through the corresponding synchronous reactances; and if, as in a non-salient machine, there is no difference the term vanishes. It is of interest to observe that the saliency term is independent of E_t and therefore exists even when the machine is unexcited, for the presence of structural polarization adds an alignment component to the torque. Taking x_{sq} as typically equal to $0.6x_{sd}$, then eq.(10.7) becomes

$$P_1 = -(V_1/x_{sd}) E_t \sin \delta - \frac{1}{3} (V_1^2/x_{sd}) \sin 2\delta$$

If, for example, $E_t = 2V_1$ and the load angle is small enough to equate the sine of the angle to the angle itself, then the active power is $P_1 = -(V_1^2/x_{sd})(2 + \frac{2}{3})$, showing a 25% contribution made by saliency; and there is still an active power even if the excitation is zero.

Fig. 10.13 shows the non-saliency and saliency power components and their sum for a machine working on infinite busbars with $x_{ad} = 1.25$, $x = 0.15$ and



10.13 Salient-pole machine: active-power/load-angle relation.

therefore $x_{sd} = 1.40$ p.u. The q-axis synchronous reactance is taken to be $x_{sq} = 0.70$ p.u. For clarity the power/load-angle relation is graphed for the single excitation $E_t = 2.0$ p.u., for which eq.(10.7) gives

$$P_1 = -1.43 \sin \delta - 0.36 \sin 2\delta$$

for the per-unit active power input per phase. Maximum power occurs for $\delta = \pm 1.21$ rad (69°), compared with the $\frac{1}{2}\pi$ rad (90°) for a non-salient machine. **Synchronizing power and torque.** An additional term now appears in eq.(10.3) to give

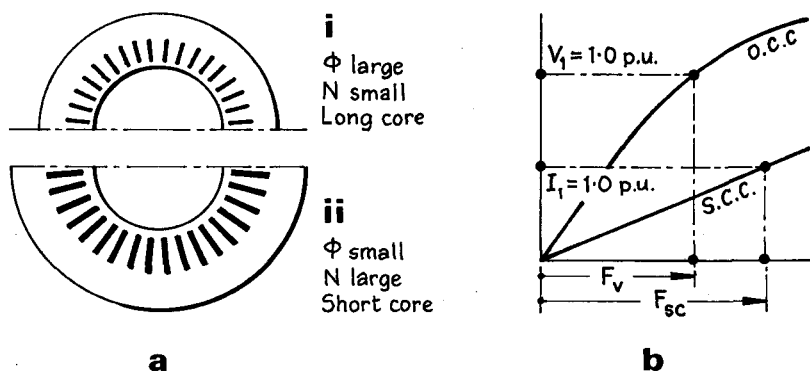
$$p_s = \frac{dP_1}{d\delta} = \frac{V_1}{x_{sd}} E_t \cos \delta + \frac{V_1^2 (x_{sd} - x_{sq})}{x_{sd} x_{sq}} \cos 2\delta \quad (10.8)$$

for the synchronizing power per phase and per electrical radian of displacement. Expressed as per mechanical radian and divided by the synchronous angular speed, the result is the synchronizing torque.

Steady-state reactance

Power, stability and regulation problems of a synchronous machine can be solved only if the reactances are known, but these depend on the flux conditions and the degree of saturation.

Short-circuit ratio. The stator phase voltage is proportional to the product of the effective gap flux Φ_m and the turns per phase N_1 . A large flux requires a bigger core area (which in turbo-type machines means a longer core as the diameter is limited by peripheral speed) but less conductor material. By contrast, with more turns a smaller core area is needed but the conductors must be deeper and have more cross-sectional area. The two shapes are indicated at (a) in Fig. 10.14. Now a fundamental thermal limit is the m.m.f. rating F_2 of the rotor. Part of F_2 opposes F_1 of the stator, part provides the gap-flux magnetization, F_0 . With fewer stator turns, F_1 is reduced and the gap length may be ex-



10.14 Short-circuit ratio.

tended: with more stator turns the gap length must be reduced, and for a given load the load angle is greater and the machine is less 'stiff' and therefore less stable.

The relative flux/turns proportionality is embodied in the *short-circuit ratio* shown in Fig. 10.14(b):

$$r_{sc} = \frac{F_v}{F_{sc}} = \frac{\text{p.u. excitation for normal voltage on o.c.}}{\text{p.u. excitation for rated stator current on s.c.}}$$

Modern turbo-generators normally have a s.c.r. between 0.5 and 0.6, but this must be raised to 1.0–1.5 if the loading is likely to be capacitive (as with connection to long unloaded overhead transmission lines or extensive h.v. cable runs). The range 1.0–1.5 is also common for low-speed generators. If a generator works on a leading zero-power-factor load, its stator m.m.f. F_1 is direct magnetizing, causing self-excitation. To retain voltage control the machine must have positive rotor excitation: the gap must therefore be lengthened to increase F_v and the stator turns reduced in number to limit F_{sc} .

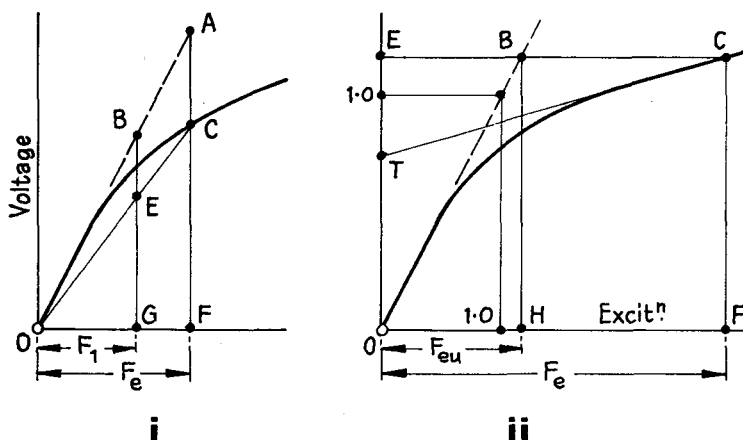
The short-circuit ratio is the reciprocal of the d-axis per-unit synchronous reactance if the latter is defined for normal voltage and rated current. But x_{sd} for a given load is affected by the saturation conditions, whereas r_{sc} is specific and uni-valued for a given machine.

Synchronous reactance. Defined from the o.c.c. and s.c.c. for any given excitation, x_{sd} is not a constant, as Fig. 10.7 clearly shows. To avoid ambiguity the *unsaturated synchronous reactance*, related to the gap alone, is defined as

$$x_{sdu} = \frac{\text{p.u. voltage on o.c.c. air-line}}{\text{p.u. stator current on s.c.c.}}$$

for any excitation, and it is a constant rather greater than the reciprocal of r_{sc} . It is the value normally quoted for modern machines.

Adjusted synchronous reactance. The choice of x_{sd} for a given machine and



loading condition requires judgement. Two methods of graphical estimation are given in Fig. 10.15.

(i) Let $OF = F_e$ be the excitation on the o.c.c. for the internal e.m.f. E_1 for a given load (i.e. the phasor addition of the terminal voltage and the internal voltage drops in resistance and in leakage reactance): then CF is the open-circuit voltage E_1 , while AF is the value E_{1u} that would be generated were there no saturation. Join OC. Let $OG = F_1$ be the stator m.m.f.: then $BG = E_{au}$ is the air-line reaction e.m.f. corresponding. The saturated value is taken to be $EG = E_a$, and the appropriate synchronous reactance is

$$x_{sd} = (E_g/I) + x = x_g + x$$

(ii) The o.c.c. is drawn with 1.0 p.u. excitation corresponding to 1.0 p.u. voltage on the air line. The internal e.m.f. E_1 is obtained as in (i). From it the o.c.c. excitation $OF = F_e$ and the air-line excitation $OH = F_{eu}$ are found. Then $OF/OH = F_e/F_{eu} = k_1$, a saturation factor. A tangent to the o.c.c. is drawn through C and produced to cut the ordinate axis in T, dividing EO in the ratio $TO/ET = k_2$. Then the synchronous reactance appropriate to the load condition is

$$x_{sd} = \frac{x_{sdu} - x}{\sqrt{[k_1(1 + k_2)]}} + x$$

Potier reactance. The Potier reactance is the leakage reactance obtained in a particular way from a test on the machine at full-load current of power factor zero lagging. Such a test, requiring but little power, furnishes the excitation for short circuit and for normal rated voltage, in each case with full-load z.p.f. current.

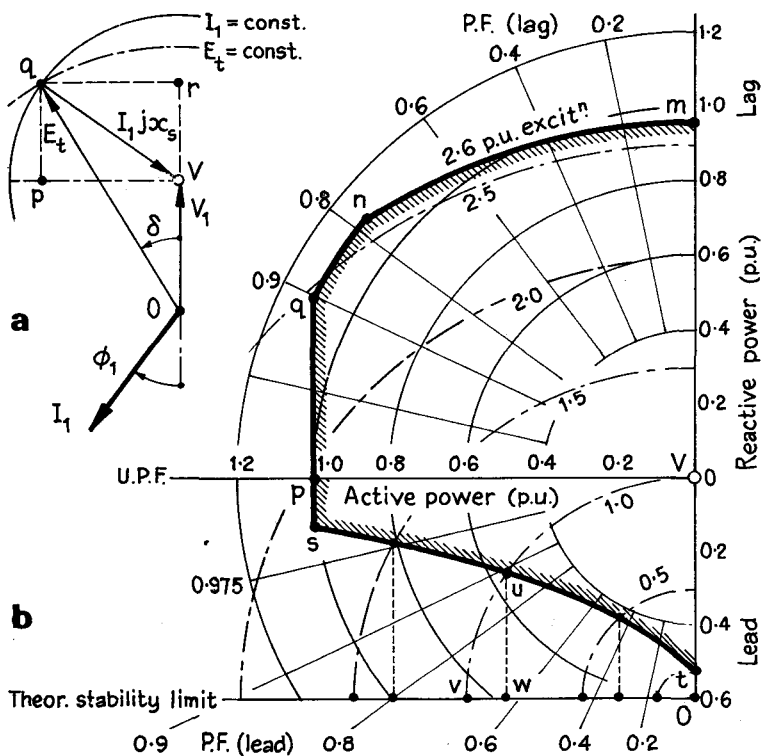
Consider an unsaturated machine with negligible resistance. Its o.c.c. is the straight line in Fig. 10.16 (a). Point S on the abscissa axis is such that $OS = F_{sc}$ is the field excitation producing full-load current on short circuit (which is

known, a z.p.f. characteristic is needed. Two points suffice: S for short circuit, and S' for normal voltage. Then from S' the horizontal line S'O' = SO is drawn, and O'A' parallel to OA to cut the o.c.c.

Electrical load diagram

Non-salient machine. With resistance neglected the electrical load diagram is identical with that in Fig. 10.4 for the ideal machine. Its practical use is in the operation of large synchronous generators. In such machines the basic parameters are the active power (which is settled by the output of the prime mover) and the power factor, chosen to suit the network loading conditions. The maximum allowable stator and rotor currents must also be considered, as influencing temperature-rise limits. Other factors include operation at leading power factors, and the general problem of synchronous stability.

To avoid undue complexity, saturation and resistance effects on the load diagram are ignored, and an unsaturated value of direct-axis synchronous reactance selected. The machine is assumed to be connected to infinite busbars. With the various operating limits included, the diagram is called an *operating chart*, Fig. 10.17. Its basis is shown in (a). For a given terminal voltage V_1 and



10.17 Operating chart for cylindrical-rotor generator.

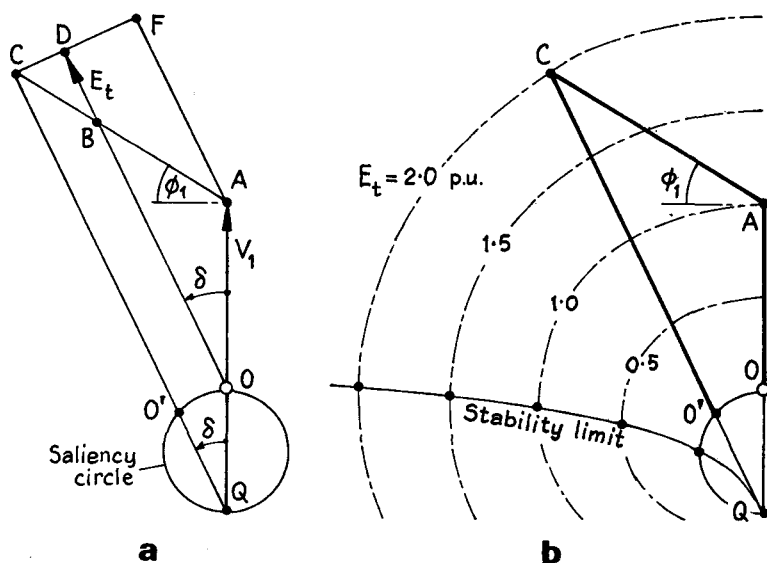
current I_1 at a phase angle ϕ_1 (i.e. for a given output power) the e.m.f. E_t is obtained by adding the phasors V_1 and $I_1 jx_s$. For constant current (i.e. constant apparent power), $I_1 jx_s$ is constant and its locus is a circle centred on the point V. Constant excitation circles are struck from O. The load angle is that between E_t and V_1 , the phase angle is contained between $I_1 jx_s$ and the horizontal line through V. Other features are based on Fig. 10.4, except that the diagram is drawn in terms of generator *output* and not a negative input. The operating chart in Fig. 10.17 is for a synchronous reactance of 1.67 p.u.: for zero excitation the current is $1.0/1.67 = 0.60$ p.u., so that the length OV corresponds to $Q_1 = 0.6$ p.u. reactive power, fixing both active and reactive power scales.

It remains to put in the working area such that any point therein lies inside the specified operating limits. Taking 1.0 p.u. active power as the limit set by the prime mover (given to scale by the length Vp), the vertical limit line spq is inserted through point p at 1.0 p.u. active power. Suppose the machine to be so rated as to deliver 1.0 p.u. active power at a power factor not less than 0.9: then this determines point q . Limitation of the stator current to such a value requires the limit-line to become the circular arc qn about point V. At n , a new limit, that of the rotor current, becomes operative, it being assumed here that the rotor current must not exceed that corresponding to $E_t = 2.6$ p.u. (i.e. 2.6 times V_1). The circular arc nm is therefore struck from O. This completes the upper limit pqn . The line qp cannot be continued downwards to the theoretical stability limit because the latter represents a condition for which the smallest increment of load will cause the machine to fall out of step. A more satisfactory condition is one for which the loading is, say, 0.1 p.u. less for any given excitation than the theoretical maximum. Consider point v for 1.0 p.u. excitation: reduce Ov to Ow by the amount $vw = 0.1$ p.u. power. Then wu , cutting the excitation circle in u , fixes the point for which there is 0.1 p.u. power in hand.

The completed working area, shown with a shaded outline, is $mnqpsut$. A load point within this area defines the active, reactive and apparent powers, the current, power factor and excitation. The load angle, if required, is found by direct measurement.

Salient machine. The difficulty here is the differing values of d- and q-axis synchronous reactance. The method described by Walker [61] represents a simple modification of the non-salient operating chart, given known (unsaturated) values of the two synchronous reactances. The basis of the method is shown in Fig. 10.18(a) which (in terms of generator *output*) restates Fig. 10.12(b) where $AC = I_1 jx_{sd}$ and $OBD = E_t$. Draw $O'C$ equal and parallel to OD : then O' is a point on a saliency circle of diameter $OQ = V_1 [(x_{sd}/x_{sq}) - 1]$. Further, CO' produced meets point Q.

Diagram (a) can now be used as a salient-pole generator operating chart, similar to that for the non-salient machine except that E_t is the intercept $O'C$ on the line $QO'C$. For constant excitation the appropriate E_t locus is found by drawing rays from Q and marking on them constant length $O'C$ external to the saliency circle. The loci are not circles but limacons, their shape becoming more evident for smaller excitations. Each such E_t locus has a point correspond



10.18 Basis of chart for salient-pole generator.

ing to maximum active power, and a curve drawn through these points forms the theoretical limit of stability, as indicated in Fig. 10.18(b).

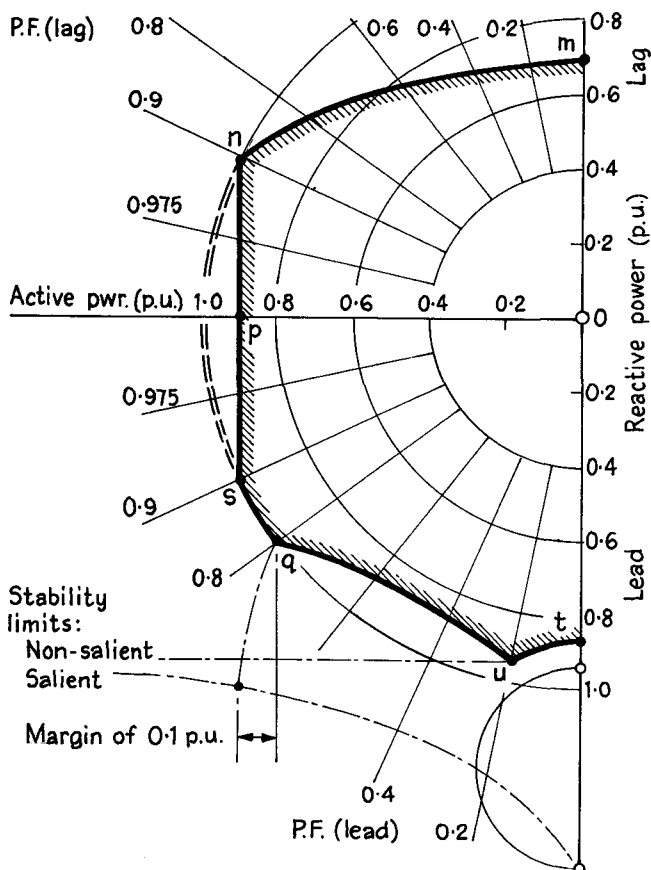
The generator operating chart obtainable is given in Fig. 10.19. It is substantially the same as for the non-salient machine except in the low-excitation region. The machine is presumed to have a current rating limited by 1.0 p.u. active power at a power factor not lower than 0.9. Then *mn* represents the excitation limit imposed by rotor heating; *nps* is the prime-mover mechanical power limit; *sq* is imposed by stator heating; *qu* is the practical stability limit; and *ut* is a restriction imposed by the requirement that there shall always be a positive field excitation, for it is possible for a salient-pole machine to run without excitation — or even with a small reversed excitation — though under such conditions it is susceptible to de-synchronizing should small disturbances occur. **Motors and compensators.** The motor action only requires the chart to be reflected across the vertical axis of V_1 to give an operating chart having features in every way comparable with those for generation. In the case of a compensator, the chart reduces simply to the vertical axis.

10.5 Steady-state performance

Where a machine operates on an infinite busbar, the general features of its performance can readily be inferred from the operating chart. Some generators may, however, work individually or in parallel on isolated loads (e.g. on ship-board or in large aircraft).

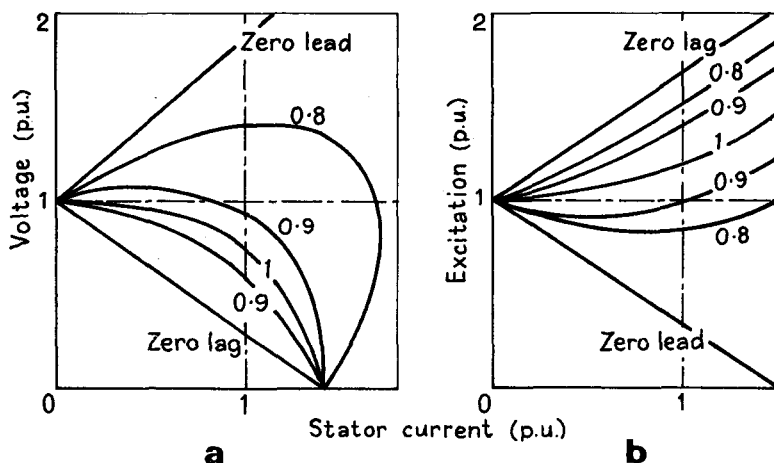
Isolated generator

Consider an isolated synchronous generator, driven at constant speed and pro-



10.19 Operating chart for salient-pole generator.

vided with constant excitation. Disregarding stator resistance and saliency it can be inferred from the phasor diagram of the ideal machine that for loads of *unity power factor*, the terminal voltage is given by $V_1^2 + (I_1 x_s)^2 = E_t^2 = \text{constant}$, and the V_1/I_1 relation, Fig. 10.20(a), is substantially an ellipse with semi-axes E_t and I_{sc} . The current I_{sc} is that which flows when the load resistance is reduced to zero, a condition giving $V_1 = 0$ and $I_{sc} = E_t/x_s$. For *zero power factor* loads, we have the scalar relation $V_1 = E_t \pm I_1 x_s$, giving the two straight lines in (a) for zero power factor leading and lagging. Loads of intermediate power factor give V_1/I_1 relations between these limits. If the terminal voltage V_1 is to be maintained constant, then the excitation will have to be adjusted to suit the load and power factor, as shown in Fig. 10.20(b); all unity and lagging-p.f. loads require increase of excitation with load current, but low leading-p.f. loads, for which the stator m.m.f. is direct-magnetizing, require the excitation to be reduced. Although variation in the conditions of satura-



10.20 Generator load characteristics.

(a) Constant excitation. (b) Constant voltage.

tion modify x_s , the effect on the V_1/I_1 relations is not enough to invalidate the load characteristics in Fig. 10.20.

Voltage regulation. This, defined above in eq.(10.5), is an important characteristic affecting protection, voltage control and exciter rating. Apart from a direct test, the regulation can be estimated from design or low-load test data. The regulation of a salient-pole machine is not likely to differ much from that of a cylindrical-rotor machine with the same value of x_{sd} , and as no simple method of prediction is precise the same method can be applied to each form of structure. The imprecision results from the complex changes in saturation that occur in the machine from full load to no load. Four methods of prediction of the regulation are given in the following example. The general data needed are the o.c.c., the s.c.c., the leakage impedance, and the z.p.f. full-load current test to obtain the Potier reactance.

EXAMPLE 10.2: Find the regulation for full-load current (a) at p.f. 0.8 lagging and (b) at p.f. 0.8 leading for a 500 kVA, 3.3 kV, 3-phase, star-connected, 50 Hz salient-pole generator. Resistance, 0.02 p.u.; leakage reactance, 0.08 p.u.; ratio (pole-arc/pole-pitch), 0.67; reaction coefficient $K_r = 0.34$; o.c.c. and s.c.c., Fig. 10.7 with 1.0 p.u. voltage corresponding to 3.3 kV line, 1.0 p.u. excitation to 5 kA-t/pole, and full-load current on short-circuit circulated by a field excitation of 5 kA-t/pole. The short-circuit ratio is $r_{sc} = 1.0$, and the unsaturated synchronous reactance is $x_{sdu} = 1.25$ p.u.

1. **Simple M.M.F. method:** This identifies E_1 with V_1 an open-circuit as in an ideal machine, and builds up the m.m.f. triangle F_1, F_2, F_0 as in Fig. 10.1 (iii) for a given terminal voltage V_1 and load phase angle ϕ_1 . The regulation so obtained is generally less than the true value

Taking $F_0 = 1.0$ p.u. and $F_1 = 1.0$ p.u., then for (a) $F_2 = 1.78$ p.u. giving

$E_t = 1.26$ p.u. from the o.c.c.; similarly for (b) $F_2 = 0.89$ and $E_t = 0.94$ p.u. The regulations are therefore

$$\begin{aligned}(a) \epsilon &= (1.26 - 1.0)/1.0 = 0.26 \text{ p.u.} \\(b) \epsilon &= (0.94 - 1.0)/1.0 = -0.06 \text{ p.u.}\end{aligned}$$

The negative sign indicates that the open-circuit voltage after throwing off the load is less than the terminal voltage on load.

2. *Synchronous impedance method.* This is based on the phasor diagram, Fig. 10.8(b), modified for a generator (i.e. with negative input current). There is considerable doubt as to the choice of the synchronous impedance. The value from the o.c.c./s.c.c. tests at maximum current may be taken, but even so the saturation conditions may be lower than for normal load. Generally, regulations greater than those actually occurring are given by the method.

At full-load current the synchronous impedance is almost identical with the synchronous reactance, i.e. $1.0/1.0 = 1.0$ p.u. Then for (a) $E_t = 1.80$ and for (b) $E_t = 0.90$ p.u., whence

$$\begin{aligned}(a) \epsilon &= 1.80 - 1.0 = 0.80 \text{ p.u.} \\(b) \epsilon &= 0.90 - 1.0 = -0.10 \text{ p.u.}\end{aligned}$$

3. *Adjusted synchronous impedance method.* The construction in Fig. 10.15(ii) is used. For full-load current at p.f. 0.8, taking into account the resistance and leakage reactance,

$$E_1 = 1.0 + (0.8 \pm j0.6)(0.02 + j0.08) = 1.06 \text{ or } 0.97 \text{ p.u.}$$

Then for (a): OF = 1.40 p.u., OH = 1.06 p.u., OF/FH = $k_1 = 1.32$, TO/ET = 1.51 $x_{sdu} = 1.25$; thence

$$x_{sd} = 0.10 + (1.25 - 0.10)/\sqrt{[1.21(1 + 0.76)]} = 0.87$$

and $E_t = 1.55$ p.u. For (b) $E_1 = 0.97$, OF = 1.18, OH = 0.97, $k_1 = 1.18/0.97 = 1.21$, TO/ET = 0.76, $x_{sd} = 0.87$, so that $E_t = 0.85$ p.u. The regulations are

$$\begin{aligned}(a) \epsilon &= 1.55 - 1.0 = 0.55 \text{ p.u.} \\(b) \epsilon &= 0.85 - 1.0 = -0.15 \text{ p.u.}\end{aligned}$$

4. *Reaction method:* In effect this is based on the phasor diagrams in Fig. 10.12 with the effect of resistance and leakage reactance included. The internal e.m.f. E_1 for the two cases, as already found in Method 3, is respectively 1.06 and 0.97 p.u. In (a), E_1 requires an excitation of 1.18 p.u., and the leakage impedance drop of 0.08 p.u. requires 0.07 p.u. excitation, both obtained from the o.c.c., so that the reaction m.m.f. is $1.0 - 0.07 = 0.93$ p.u.; AB = 0.34 AC, and F_2 is found to be 1.87 p.u. giving $E_t = 1.28$ p.u. from the o.c.c. In (b), $E_1 = 0.97$ and in a similar way $E_t = 0.94$ p.u. The regulations are

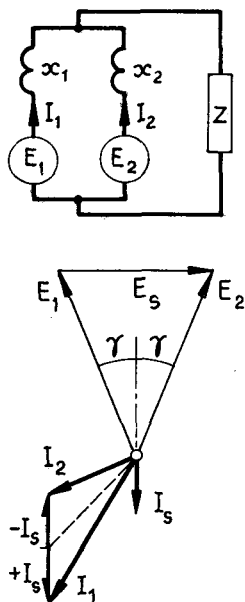
$$\begin{aligned}(a) \epsilon &= 1.28 - 1.0 = 0.28 \text{ p.u.} \\(b) \epsilon &= 0.94 - 1.0 = -0.06 \text{ p.u.}\end{aligned}$$

Summarizing the results: for (a) we obtain 0.26, 0.80, 0.55 and 0.28 p.u., while for (b) the values are -0.06, -0.10, -0.15 and -0.06. The very wide

differences give emphasis to the inaccuracy of regulation prediction in practical machines by simple methods.

Two generators in parallel

When two synchronous generators are connected in parallel to a local load, they have an inherent tendency to remain in step by exchange of synchronizing power, the development of which depends on the fact that the stator impedance is predominantly reactive. If it were not, the machines could not operate stably in parallel. Consider identical machines, 1 and 2 in Fig. 10.21, in parallel



10.21 Two generators in parallel.

and with a common load: if the internal e.m.f.s, E_1 and E_2 , diverge because of the acceleration of machine 1, the resulting phase-difference 2γ develops the voltage difference E_s that circulates a current I_s in the local circuit of the two stators through the two impedances x_1 and x_2 (neglecting resistance). The reactance causes I_s to have the phase of an active-power component: consequently machine 1 produces $P_s = E_1 I_s$ as a generator and supplies it to machine 2 as a motor. Within the limits of maximum power, therefore, the machines develop a mutual synchronizing power.

The argument does not, of course, require the machines to be identical, and in general they have different e.m.f.s, different impedances and different load/speed regulations. The latter are imposed by the prime-mover governors, which are arranged so that a speed reduction is necessary for an increase in mechanical power. Both machines must run in synchronism, and their output powers are

therefore determined by the respective governor setting and load/speed characteristic.

EXAMPLE 10.3: Two parallel-running synchronous generators have e.m.f.s of 1 000 V/ph, and their phase impedances are $z_1 = (0.1 + j2.0)$ and $z_2 = (0.2 + j3.2) \Omega$. They supply a common load of impedance $(2.0 + j1.0) \Omega/\text{ph}$. Find their terminal voltage, load currents, active power outputs and no-load circulating current for a phase divergence of 10° (e.). The governor characteristics are identical.

Taking $E_1 = (1\,000 + j0) \text{ V}$, then $E_2 = 1\,000(\cos 10^\circ - j\sin 10^\circ) = (985 - j174) \text{ V}$. Applying the Millman (parallel-generator) theorem with a common load impedance Z , then

$$V \left[\frac{1}{Z} + \frac{1}{z_1} + \frac{1}{z_2} \right] = \frac{V}{Z_0} = \frac{E_1}{z_1} + \frac{E_2}{z_2}$$

gives the common terminal voltage V . The admittance summation yields $Z_0 = 0.905 \angle 66.3^\circ$, and the right-hand side gives the total short-circuit current $I_{sc} = 808 \angle -90.7^\circ$. Hence

$$V = I_{sc} Z_0 = 730 \angle -24.4^\circ = (667 - j303) \text{ V}$$

is the common terminal voltage. For the individual load currents,

$$I_1 = (E_1 - V)/z_1 = 224 \angle -44.9^\circ = (160 - j159) \text{ A}$$

$$I_2 = (E_2 - V)/z_2 = 106 \angle -64.3^\circ = (46 - j96) \text{ A}$$

The corresponding active powers per phase are

$$P_1 = 155 \text{ kW} \quad \text{and} \quad P_2 = 60 \text{ kW}$$

so that machine 1 is retarded and machine 2 advanced. With the machines operating in parallel on zero load, the circulating current is

$$I_s = (E_1 - E_2)/(z_1 + z_2) = (34 - j1) \text{ A}$$

which is substantially an active-power current, corresponding to a generated power of 34 kW/ph in machine 1.

Unless the machines have an exceptionally large inertia, and respond only slowly to synchronizing power, the impedances z_1 and z_2 are not the synchronous impedances, but have lower (dynamic) values.

Generator on system busbars

If the system can, at the generator terminals, be considered as an infinite busbar, the generator characteristics are those derived from the operating chart of Fig. 10.17 or 10.19. If, however, the infinite busbar (or system reference point) is remote from the machine, it is necessary to include with the generator impedance an addition to take account of the transmission link.

Motor

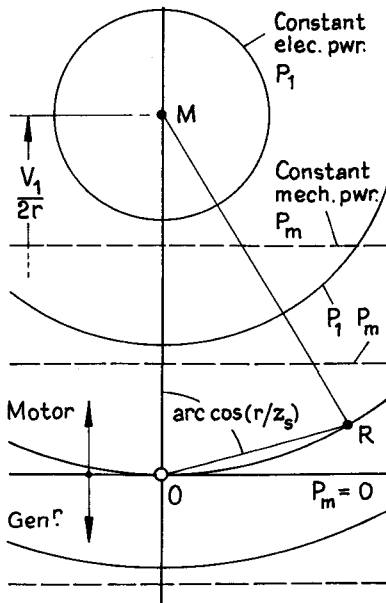
The performance of large motors is subject to system effects, but small machines can generally be considered to operate on infinite busbars with the characteristics already discussed.

V-curves. The variation of stator current with field excitation on constant power form curves of V shape. Although V-curves apply equally to generators, their interest lies mainly with motor and compensator operation.

O-curves: The electrical active-power input per phase to a motor is $P_1 = V_1 I_1 \cos \phi_1$. The converted power (i.e. the mechanical power plus constant rotational loss) is $P_m = P_1 - I_1^2 r$, whence

$$I_1^2 - (V_1/r)I_1 \cos \phi_1 + P_m/r = 0$$

a quadratic showing that there are two possible current values for a given converted power, one with a lagging and the other with a leading power factor. The active current I_{1a} taken to develop a given P_m is greater for power factors less than unity because of the additional $I^2 R$ loss that then obtains. The phasor locus of I_1 is consequently a circle, for if I_1^2 is written as $(I_1 \cos \phi)^2 + (I_1 \sin \phi)^2$ in the expression above, then I_1 must lie on a circle struck from a centre M displaced by a distance $V_1/2r$ from the origin O of the phasor V_1 , as shown in Fig. 10.22. The radius of the circle is $\sqrt{[(V_1/2r)^2 - (P_m/r)]}$. The construction of the mechanical load diagram (or O-curve diagram) is as follows. Let $OM = V_1/2r$ to scale: draw with M as centre a circle of radius OM. This circle represents the current locus for $P_m = 0$. Any smaller circle on centre M represents the current locus

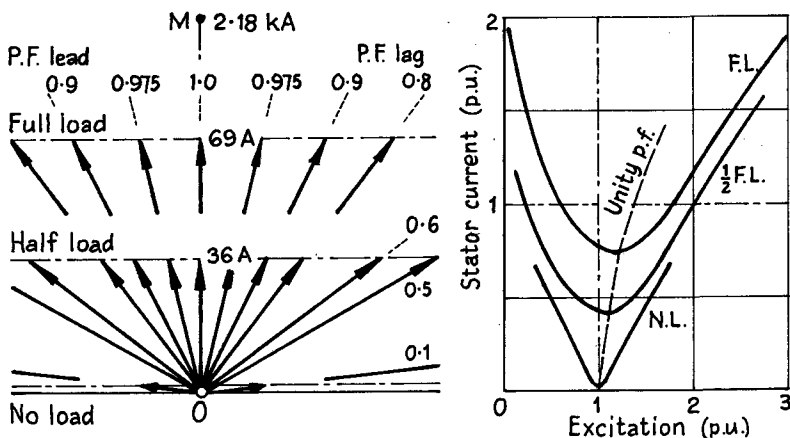


10.22 O-curves.

for some constant mechanical developed power P_m . For a load such that $P_m/r = (V_1/2r)^2$ there is a single value of current $I_1 = V_1/2r$ corresponding to the maximum possible power: the power circle has in fact shrunk to zero radius and become the point M. Such a condition, impracticable because of excessive heating and critical stability, is in fact the *maximum-power-transfer* condition commonly exploited in telecommunication circuits.

Fig. 10.22 is completed by the addition of the line OR, of length V_1/z_s and drawn at angle $\arccos(r/z_s)$ to MO. Circles with R as centre give the loci of operating points with constant excitation. Current circles from M with radii greater than $V_1/2r$ serve for operation in the generator mode. The O-curves are based on constant resistance and synchronous reactance and are therefore of academic interest only.

EXAMPLE 10.4: The 500 kVA machine of Example 10.2 is rated as a motor with a full-load output of 375 kW. Plot O- and V-curves for full, half and zero mechanical power output. Resistance, 0.02 p.u. = 0.435 Ω /ph; leakage reactance, 0.08 p.u. = 1.74 Ω /ph; core loss, 8 kW, and friction and windage loss, 7 kW, assumed invariant.



10.23 O-curves and V-curves.

The curves are shown in Fig. 10.23. Point M is located at a distance $V_1/2r = 2.18$ kA/ph from the origin O. As this is about 20 times full-load current, the curvature of the O-curves in the actual working range is so small that the circular arcs could justifiably be replaced by straight lines.

Full load: The output is 375 kW. The total converted power is $375 + 15 = 390$ kW which, at unity power factor, would demand an active input current of 68 A. This in turn produces an I^2R loss of 6 kW, so that the actual input is about 396 kW and 69 A/ph. A similar iteration for a power factor of 0.50 gives an input of $375 + 15 + 28 = 418$ kW, 836 kVA and 147 A, this current

producing 28 kW of I^2R loss. The result is obtainable directly from the full-load O-curve.

Half load: The converted power is $\frac{1}{2}(375) + 15 = 202.5$ kW. At unity p.f. the input is 204 kW, 36 A/ph.

No load: The power input consists of losses only, so that the I^2R loss is important, particularly at low power factors.

The phase currents for various power factors are as follows:

Power factor	1.0	0.95	0.8	0.5	0.1	0.05
Current, A/ph: full-load	69	73	88	147	—	—
half-load	36	38	46	74	—	—
no-load	2.6	2.7	3.3	5.3	28	85

Using the o.c.c. and s.c.c. and the phasor diagram for a salient-pole motor, the excitation for each load can be obtained and the V-curves constructed to give the result shown in Fig. 10.23.

10.6 Transient performance

The behaviour of a synchronous machine under conditions of transience may have a dominant effect on its application and may demand special attention in its design. For example, the possibility of a generator being suddenly short-circuited when running fully excited necessitates robust design of its end-windings to withstand the disruptive forces generated electromagnetically by the currents, which in a 500 MW turbo-generator may exceed 100 kN axially and 50 kN peripherally. These forces depend on the square of the current.

The phase-swinging of a machine resulting from switching operations, faults, or sudden changes of load is a typically occurring transient condition of an unpredictable nature, while the induction starting of a synchronous motor is a predictable transient important to the fulfilment of the motor's duty cycle.

The parameters that determine the performance of a machine in a transient condition are quite different from those employed in discussing its steady-state operation. As these differences are most evident under conditions of sudden short-circuit, this fault occurrence will be discussed first.

10.7 Short circuit

The events within a generator subjected to a sudden short circuit depend on several factors, including (i) the instant in the cycle at which the short circuit is initiated, (ii) the load and excitation of the machine at the time, (iii) the extent of the fault (i.e. the number of phase windings involved and the distance of the fault from the machine terminals), and (iv) the constructional features of the machine that affect flux leakage and damping.

The time immediately following a short circuit may be divided into three successive intervals:

1. A very short time (e.g. one or two periods of the supply frequency) during

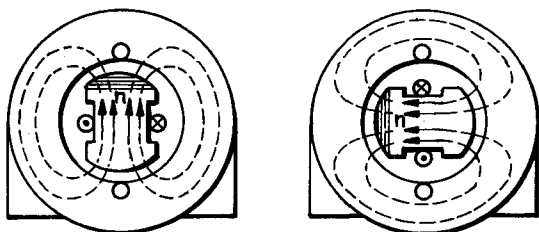
which the conditions are largely dependent upon the flux linking the stator and rotor windings at the instant of fault initiation.

2. A subsequent interval of transient decay of current amplitudes consequent upon damping and the rise of armature reaction.

3. A final period during which steady-short-circuit conditions obtain: a machine will normally be open-circuited before this period is reached.

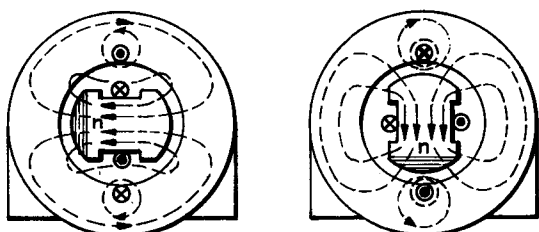
i

Instant of short circuit



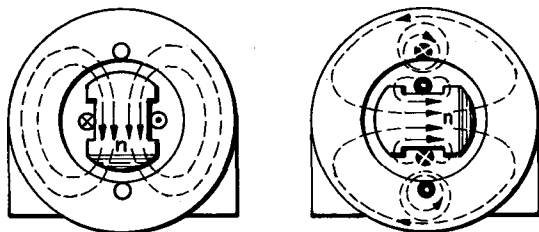
ii

Quarter - rev. later



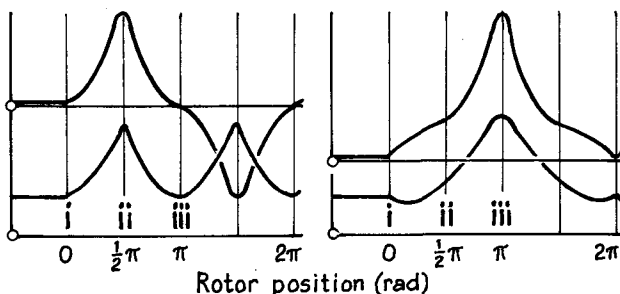
iii

Half - rev. later



Stator current

Rotor (field) current



a Zero linkage

b Maximum linkage

10.24 Constant linkage: single-phase short circuit.

Initial conditions

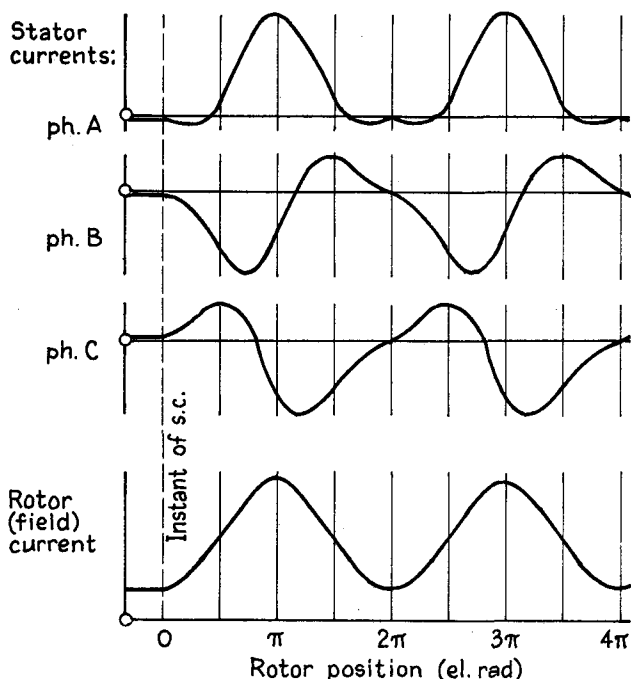
A physical insight into the behaviour of a generator during time-interval (1) above can be based on the *theorem of constant linkage*, which states that the linkage of a loss-free inductive circuit, closed on itself, cannot be altered. On stator short circuit, the stator windings are closed by the fault, while the rotor exciting winding is closed through its exciter; and if resistance can initially be neglected, the linkage of each winding must remain at the level it had at the instant of short circuit.

Consider an elementary *1-phase* machine, Fig. 10.24, comprising a 2-pole field system and a concentrated stator winding. Let the stator, initially on open circuit, be suddenly short-circuited as at (a) when in a condition of zero linkage, instant (i). As the rotor rotates to a new position (ii) at right-angles to the first, it tends to establish full normal linkage in the stator winding: this the stator must oppose by a current in the direction shown, forcing the rotor field into a leakage path. Simultaneously the rotor current must increase to maintain its own flux constant despite the now increased reluctance of its magnetic circuit. At position (iii) the stator current is again zero and the rotor current sinks to normal. A similar argument for subsequent instants gives the current/position relation shown.

Suppose now that the position of the rotor at the instant of stator short circuit is that at (i) in Fig. 10.24(b). The stator has maximum linkage. Rotation of the rotor into position (ii) removes its linkage from the stator, which consequently assumes a current to produce its own, in leakage paths. In position (iii) the rotor attempts to produce full reversed flux in the stator, and is counteracted by the stator current rising to a high value to maintain unchanged its original linkage. The peak current is determined by the reluctance of the leakage paths. The rotor in position (iii) has its flux forced into a leakage path mainly inside the stator bore, so that the rotor current rises too. The current changes are shown in (b). In actuality, the rotor current variations take place partly in the rotor winding, partly in other closed paths available for current flow, such as the rotor body (if solid), wedges and end-covers, or damper windings.

The short-circuit current is seen to be a function of relative position of stator and rotor, and immediately after short circuit the currents can be assessed in terms of rotor angle and appropriate leakage inductances.

At the moment of short circuit in a *3-phase* machine, the flux linking the stator from the rotor is caught and 'frozen' to the stator, giving a stationary replica of the main-pole flux. For this purpose each phase will in general carry a d.c. component. The rotor maintains its own poles, and accordingly there are two systems of similar poles, one rotating with the rotor and the other fixed with the stator. Each time the rotor poles are in the position they occupied at the moment of short circuit the rotor (field) current is normal and all the stator phase currents zero (if the machine were previously unloaded). One half-period later the stator and rotor poles are again axially coincident, but are opposite in sense: yet constant linkage must be maintained in each, and current peaks result. Further, the stationary field held by the stator induces a normal-frequency e.m.f. in the rotor. The resultant a.c. component in the effective rotor field



10.25 Constant linkage: three-phase short circuit.

circuit develops a fundamental-frequency flux, which due to rotation produces in the stator windings a *double-frequency* or *second-harmonic* current. Thus the general form of the transient phase current is a wave comprising fundamental, second-harmonic and d.c. components. The rotor current has a large fundamental-frequency fluctuation, part of which will flow in eddy-current paths. Fig. 10.25 gives curves of stator-phase and rotor-circuit currents exhibiting these effects.

Analysis of the currents in a simple ideal 2-pole machine when short-circuited can be based on the treatment in Sect. 7.5. The machine is reduced to the three circuits F, D and Q, with resistance neglected in accordance with the constant-linkage theorem. It is convenient to make $t = 0$ the instant of short circuit, at which the axis of phase A make the initial angle θ_0 with the d-axis: then $\theta = \omega_1 t + \theta_0$. Prior to zero time the stator axis voltages are $v_d = 0$ and $v_q = -\omega_1 \lambda_{ad} i_{f0}$ for an exciting current i_{f0} . The effect of the short circuit is to reduce v_q instantaneously to zero, represented by the sudden application of a step-function voltage $-v_q$ to the q-axis terminals. The currents thereafter are superimposed on those that existed prior to $t = 0$. In fact there is a current only in F, so that the currents in D and Q will be the short-circuit currents, while that in F is the component superimposed on i_{f0} . The speed is taken as invariable, with ω_r equal throughout to ω_1 . The only equations concerned are consequently (7.12), (7.15) and (7.16), omitting the resistances and all KD

and KQ effects. Then writing inductance coefficients for convenience as reactances, we have

$$\text{F: } 0 = (1/\omega_1)[x_{ad} + x_f] p i_f + x_{ad} p i_d \quad (\text{i})$$

$$\text{D: } 0 = (1/\omega_1)[(x_{ad} + x) p i_d + x_{ad} p i_f + \omega_1(x_{aq} + x) i_q] \quad (\text{ii})$$

$$\text{Q: } -v_q 1 = (1/\omega_1)[(x_{aq} + x) p i_q - \omega_1 x_{ad} i_f - \omega_1(x_{ad} + x) i_d] \quad (\text{iii})$$

From (i), $i_f = -i_d[x_{ad}/(x_{ad} + x_f)]$ can be eliminated from (ii) and (iii). Then from (ii)

$$i_q = -\frac{1}{\omega_1} \cdot \frac{1}{x_{aq} + x} \left[\frac{x_{ad} x_f}{x_{ad} + x_f} + x \right] p i_d = -\frac{1}{\omega_1} \cdot \frac{x_d'}{x_{sq}} \cdot p i_d$$

where x_d' , the term in square brackets, is called the *d-axis transient reactance*. Substituting for i_f and i_q in (iii) and simplifying then gives

$$i_d = \frac{\omega_1^2}{x_d'} \cdot \frac{1}{p^2 + \omega_1^2} \cdot v_q 1 \quad (10.9)$$

This solves by standard methods to give i_d as a function of time, and from it i_q :

$$i_d = (v_q/x_d')(1 - \cos \omega_1 t) \quad \text{and} \quad i_q = -(v_q/x_{sq}) \sin \omega_1 t$$

It remains to transform the axis currents into phase equivalents by eq.(7.19) with $\theta = \omega_1 t + \theta_0$. For phase A

$$i_a = v_q \left[\frac{1}{x_d'} \cos(\omega_1 t + \theta_0) - \frac{x_{sq} + x_d'}{2x_d' x_{sq}} \cos \theta_0 - \frac{x_{sq} - x_d'}{2x_d' x_{sq}} \cos(2\omega_1 t + \theta_0) \right]$$

normal frequency a.c. component	asymmetric (d.c.) component	double-frequency a.c. component	(10.10)

And similarly for phases B and C. The transient field current is

$$i_f = v_q \frac{x_{ad}}{x_{ad}(x_f + x) + x_f x} [1 - \cos \omega_1 t]$$

These results resemble the curves in Fig. 10.25. Neglect of resistance implies that all current components persist without decay.

EXAMPLE 10.5: A 2-pole generator running on no load and excited to normal busbar voltage is dead-short-circuited on all three phases at the instant that the axes of the field and phase A are in coincidence. Find the current in phase A and in the field winding. Per-unit reactances: $x_{ad} = 1.50$, $x_{aq} = 0.60$, $x = 0.10$, $x_f = 0.13$ p.u. Resistances: all negligible.

Here $v_q = 1.0$ p.u. and $\theta_0 = 0$. The d-axis transient reactance is

$$x_d' = \frac{x_{ad}x_f}{x_{ad} + x_f} + x = 0.22 \text{ p.u.}$$

and $x_{sq} = x_{aq} + x = 0.70$ p.u. Using the solutions above we get

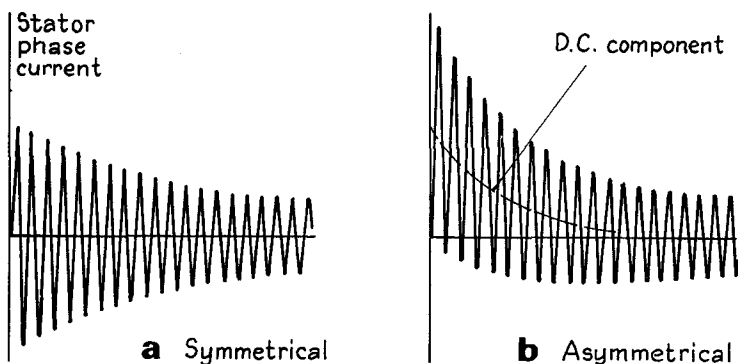
$$i_a = 4.5 \cos \omega_1 t - 3.0 - 1.5 \cos 2\omega_1 t$$

$$i_f = 4.2(1 - \cos \omega_1 t) + 1.0 = 5.2 - 4.2 \cos \omega_1 t$$

the unity being added to i_f for the initial value $i_{f0} = 1.0$ p.u. The values correspond to those plotted in Fig. 10.25.

Subsequent conditions

Eq.(10.9) above shows that, neglecting resistance, the normal-frequency component of the *initial* short-circuit current is limited by the transient reactance x_d' . The *steady-state* short-circuit current, however, is limited by $x_{sd} = x_{ad} + x$. In Example 10.5 we found $x_d' = 0.22$ p.u., whereas x_{sd} is given as 1.60 p.u., a much higher value. The high initial short-circuit current is determined by a true leakage inductance, and as armature reaction takes a finite time to build up the quite different flux pattern threading the various closed windings of the machine, there is a time interval of several seconds between the initial high-valued currents and the attainment of the much lower level of steady-short-circuit conditions.



10.26 Symmetrical and asymmetrical short-circuit phase currents.

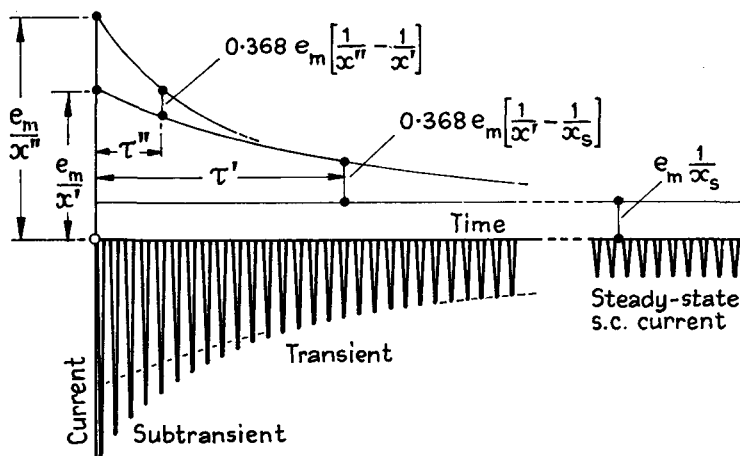
Fig. 10.26 shows two limiting patterns of phase short-circuit current. In (a) the instant of short-circuit is such that there is no asymmetry — it corresponds roughly to the conditions in Fig. 10.24(a) — while (b) is a case in which there is a maximum d.c. component. The rapid attainment of symmetry and the slower shrinking of the current envelope to the steady-short-circuit level each depend upon an effective resistance/inductance ratio related to the type of construction. Thus in the solid rotor body of a turbo-generator the eddy-

currents in rather ill-defined paths shunt the field winding resistance and contribute to the rate of energy loss, so damping both the asymmetry and the short-circuit current amplitude. In salient-pole machines the damping windings in the pole faces serve a similar purpose.

In brief, 3-phase short-circuit currents are characterized by (i) a high initial value determined by an effective leakage inductance; (ii) some phase-current asymmetry depending upon the instant in the cycle at which the short circuit occurs; (iii) rapid attenuation of the d.c. components; (iv) slower attenuation of the a.c. component to a value determined by the synchronous reactance.

If a short circuit occurs between one phase terminal and earth with the other two phases unaffected, the 1-phase short-circuit current develops a pulsating stator m.m.f. with an a.c. component that can be regarded as resolved into two equal and oppositely rotating component m.m.f.s. That component moving with the rotor gives rise to a demagnetizing effect, but the other reacts on the field winding with a double-frequency pulsation. The d.c. component of the phase current will, as in the 3-phase case, produce stationary magnetic poles reacting on the field circuit at normal frequency. The rotor current then exhibits (i) a general increase to balance the forward-rotating m.m.f. of the a.c. reaction, (ii) a double-frequency current due to the backward-rotating m.m.f., and (iii) a normal-frequency current resulting from the d.c. component, which vanishes as symmetry is approached (or is absent if there is no asymmetry).

Short-circuit current envelope. On a 3-phase dead-short-circuit, the a.c. components of the phase currents — that is, the currents that remain after the d.c. transient components have been extracted — occupy identical envelope shapes, of which Fig. 10.27 is typical. The envelope is substantially exponential in its variation with time, with a time-constant τ' of the order of 1–2 s. At the beginning, however, there is a much more rapid rate of decay with a time-constant τ'' of 0.2 s or less. Extrapolation of the main part of the envelope back to zero



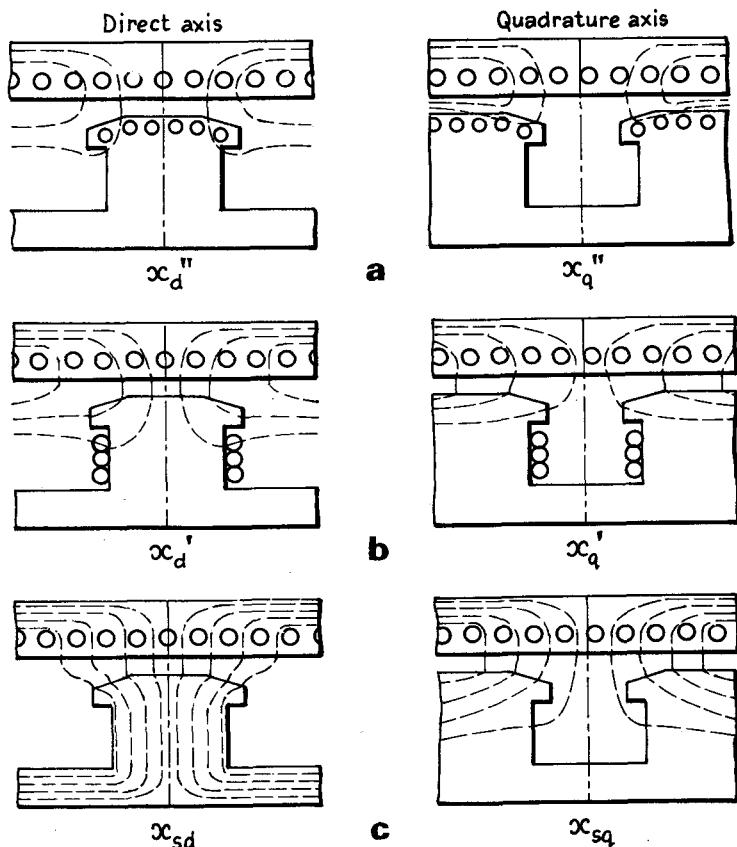
10.27 Analysis of symmetrical short-circuit current.

time would give a peak current of e_m/x' : the actual current peak at zero time, given by e_m/x'' , is in fact considerably greater. It is necessary to discuss the reason for the existence of the *sub-transient* reactance x'' and time-constant τ'' , as well as of the *transient* values x' and τ' . The explanation is based on the flux paths enforced at various times during the run of the sudden short circuit. *Axis reactances.* For a terminal short circuit, all the effective reactances — sub-transient, transient and synchronous — are d-axis quantities. If the short circuit occurs at a point remote from the generator terminals into the connected network, the phase angle may be less and quadrature-reaction effects may appear together with some increase in the resistance and inductance parameters concerned in the short-circuit paths. The general problem then requires for solution a knowledge of the subtransient, transient and synchronous reactances and time-constants for both d- and q-axes. Typical values are given in the Table, together with data for negative- and zero-sequence reactance. The n.p.s. reactance is a rough mean between x_d'' and x_q'' ; the z.p.s. reactance is small and is considerably affected by the stator winding coil-pitch and phase-spread.

Synchronous Machine Reactances (p.u.) and Time-constants (sec.)

Quantity		Turbo-generator	Hydro-generator with dampers		Compen- sator
Synchronous:	d-axis	x_{sd}	1.0–2.5	0.5–1.5	0.5–1.5
	q-axis	x_{sq}	1.0–2.5	0.35–1.0	0.35–1.0
Armature leakage:	x	0.1–0.25	0.1–0.2	0.1–0.2	0.1–0.2
Subtransient:	d-axis	x_d''	0.1–0.25	0.1–0.2	0.15–0.3
	q-axis	x_q''	0.1–0.25	0.2–0.8	0.35–1.0
Transient:	d-axis	x_d'	0.2–0.35	0.2–0.3	0.2–0.35
	q-axis	x_q'	0.2–0.35	0.25–0.8	0.35–1.0
Sequence:	n.p.s.	x_2	0.1–0.35	0.15–0.6	0.25–0.6
	z.p.s.	x_0	0.01–0.1	0.04–0.2	0.04–0.2
Time-constant: d.c.		τ_a	0.1–0.2	0.1–0.2	0.1–0.2
	subtransient	τ''	0.03–0.1	0.03–0.1	0.03–0.1
	transient	τ'	1.0–1.5	1.5–2.0	1.5–2.0

A physical appreciation of the terms subtransient and transient can be had by consideration of the diagrams in Fig. 10.28, drawn for a salient-pole machine in which the damping circuits are more explicit than they are in a turbo-type machine. For the d-axis, (c) shows the *steady-state* condition in which the stator d-axis m.m.f. contributes flux that traverses the 'normal' magnetic circuit. But if the stator current appears suddenly as in the case of a short circuit, the closed circuits prevent a corresponding sudden change of linkage: an opposing current is induced into the damper winding, forcing the stator flux into the gap as in (a). With the exponential decay of the damper-winding current, the stator m.m.f. is able to force its flux more deeply into the pole (although against the opposi-



10.28 Flux paths for salient-pole machine reactances.

tion of induced field current) as indicated in (b). Eventually the steady-state short-circuit condition (c) is reached. As the inductance of a winding is measured by the linkage per unit current, which is in turn dependent on the magnetic reluctance of the flux path, the effective stator inductance is a minimum in condition (a), and smaller in (b) than in (c). The inductance coefficients multiplied by ω_1 give the corresponding reactances x_d'' , x_d' and x_{sd} . A similar but less pronounced effect occurs for q-axis stator currents. Because the flux path in (b) differs but little from that in (c), the q-axis transient reactance x_q' has a magnitude of the same order as that of the synchronous reactance x_{sq} in a machine with saliency.

In a turbo-generator there are no specific damper windings in the pole faces, but the solid rotor body and, to a limited extent, the metallic slot wedges provide eddy-current paths and so contribute to the damping. Bharali and Adkins [67] have pointed out that the complicated eddy-current distribution causes the subtransient reactance to diverge from that predicted on simple theory, and

give a modification of the KD and KQ parameters for improved accuracy.

While the subtransient and transient inductances are well established in the literature, they do not have an identifiable existence. They are simplified concepts formulated to deal analytically with the exceedingly complex phenomena that occur in an electro-magnetic machine.

Time-constants. The subtransient time-constant is that of the damper or equivalent circuits operative immediately following the incidence of a sudden change of condition. Generally, the circuits concerned have a large resistance/inductance ratio and consequently the time-constants τ_d'' and τ_q'' are very short. The transient time-constants are up to 20 times as long. The effective resistance controlling a time-constant is a parameter derived from the loss due to the subtransient or transient current in a given part of the machine, and as the current path is not readily defined, its estimation is often rather difficult.

EXAMPLE 10.6: A 50 MVA, 13.5 kV, 50 Hz synchronous generator was run on open circuit and excited to give a terminal e.m.f. of 13.5 kV. A 3-phase terminal short circuit was applied at $t = 0$. The positive and negative amplitudes of the short-circuit current envelope, scaled from an oscillogram for one phase, are tabulated in columns p and n in the Table, for various times t . Estimate (i) the per-unit subtransient, transient and synchronous d-axis reactances, and (ii) the subtransient, transient and d.c. time-constants.

The current envelope has an asymmetry, like that in Fig. 10.26(b). If the d.c. component is extracted it can be plotted separately to a base of time and the time-constant obtained from the duration between the initial value and 0.368 of this value. The remaining symmetrical envelope is plotted as in the upper part of Fig. 10.27 and the rest of the constants obtained. For this purpose columns are added to the Table: $d = \frac{1}{2}(p + n)$ gives the d.c. component, and $a = \frac{1}{2}(p - n)$ yields the amplitude (above and below current zero) of the a.c. component. The results are:

t s	p kA	n kA	d kA	a kA	t s	p kA	n kA	d kA	a kA
0	50.8	-14.3	18.3	32.6	0.30	18.4	-14.1	2.2	16.2
0.02	43.7	-11.9	15.9	27.8	0.40	16.4	-14.3	1.1	15.4
0.04	38.6	-10.8	13.9	24.7	0.50	15.4	-13.8	0.8	14.6
0.06	34.3	-10.3	12.0	22.3	0.80	12.6	-12.6	0	12.6
0.10	29.1	-10.8	9.1	20.0	1.00	11.5	-11.5	0	11.5
0.14	25.3	-11.5	6.9	19.4	1.50	9.6	- 9.6	0	9.6
0.16	24.0	-11.9	6.1	18.0	...	...	...	...	...
0.20	21.9	-12.8	4.6	17.3	30.0	2.75	- 2.75	0	2.75

The rated current is 2.13 kA r.m.s. or 3.0 kA peak = 1.0 p.u. The initial peak of the a.c. component is 32.6 kA, whence $x_d'' = 3.0/32.6 = 0.09$ p.u. Extrapolating the plot gives the transient component peak as 22.4 kA, whence $x_d' = 3.0/22.4 = 0.13$ p.u. The steady-state short-circuit current is 2.75 kA peak, whence $x_{sd} = 3.0/2.75 = 1.1$ p.u.

The difference $32.6 - 22.4 = 10.2$ kA between the subtransient and transient currents is shown to fall to $10.2 \times 0.368 = 3.7$ kA in $\tau'' = 0.04$ s; similarly the difference $22.4 - 2.75 = 19.7$ kA between the transient and steady-state currents falls to 7.2 kA in $\tau' = 1.40$ s. A separate plot of the d.c. component, treated in a like manner, gives $\tau_a = 0.14$ s. The values required are thus:

- (i) $x_d'' = 0.09$ p.u.; $x_d' = 0.13$ p.u.; $x_{sd} = 1.1$ p.u.
 (ii) $\tau'' = 0.04$ s; $\tau' = 1.4$ s; $\tau_a = 0.14$ s.

Circuit theory

The short-circuit currents in Example 10.5 were evaluated on the assumption of zero resistance. If the field winding has a resistance r_f (all other circuits remaining resistance-free), the circuit analysis results in the appearance of two time-constants and corresponding exponential decays. Eq.(10.9) now becomes

$$i_d = \frac{\omega_1^2}{x_{sd}} \cdot \frac{1}{p^2 + \omega_1^2} : \frac{1 + p \tau_{d0}'}{1 + p \tau_d'} \cdot v_q \quad (10.11)$$

$$\text{where } \tau_d' = \frac{11}{\omega_1 r_f} \cdot \frac{x_{ad} x}{x_{ad} + x} + x_f \quad \text{and} \quad \tau_{d0}' = \frac{1}{\omega_1 r_f} (x_{ad} + x_f)$$

are the d-axis transient and open-circuit time-constants. As a result of the field resistance, i_d decays to v_q/x_{sd} , the steady-state short-circuit current limited by the d-axis synchronous reactance x_{sd} .

The more general case of five circuits (F, KD, KQ, D and Q), all with resistance, involves the solution of five simultaneous equations. As Adkins [65] shows, the task is formidable unless approximations are made and the speed is assumed invariant. The solution for phase A for a 3-phase short circuit on a machine initially excited, but unloaded, is

$$\begin{aligned} i_a = v_q \left[\frac{1}{x_{sd}} \cos(\omega_1 t + \theta_0) \right. & \quad \text{steady-state s.c. component} \\ + \left(\frac{1}{x_d'} - \frac{1}{x_d} \right) \exp \left(-\frac{t}{\tau_d'} \right) \cos(\omega_1 t + \theta_0) & \quad \text{normal-freq. transient} \\ + \left(\frac{1}{x_d''} - \frac{1}{x_d'} \right) \exp \left(-\frac{t}{\tau_d''} \right) \cos(\omega_1 t + \theta_0) & \quad \text{normal-freq. subtransient} \\ - \frac{x_d'' - x_q''}{2x_d'' x_q''} \exp \left(-\frac{t}{\tau_a} \right) \cos(2\omega_1 t + \theta_0) & \quad \text{double-freq. transient} \\ \left. - \frac{x_d'' + x_q''}{2x_d'' x_q''} \exp \left(-\frac{t}{\tau_a} \right) \cos \theta_0 \right] & \quad \text{asymmetric (d.c.) transient} \end{aligned} \quad (10.12)$$

The normal-frequency transient can be identified in Fig. 10.27 and the d.c. component in Fig. 10.26(b). The reactances and time-constants concerned are summarized below. They are based on the resistances $r_d = r_q = r$, r_f , r_{kd} and r_{kq} ; the leakage inductances $L_d = L_q = L$, L_f , L_{kd} and L_{kq} ; and the axis mutual inductances L_{ad} and L_{aq} , for the five circuits D, Q, F, KD and KQ. The reactances are formed from the inductances by multiplying each by ω_1 . The various combinations that give the operative reactances and time-constants are as follows:

Reactances	
Synchronous: d-axis	$x_{sd} = x_{ad} + x$
q-axis	$x_{sq} = x_{aq} + x$
Transient: d-axis	$x_d' = \frac{x_{ad}x_f}{x_{ad} + x_f} + x$
Subtransient: d-axis	$x_d'' = \frac{x_{ad}x_fx_{kd}}{x_{ad}x_f + x_fx_{kd} + x_{kd}x_{ad}} + x$
q-axis	$x_q'' = \frac{x_{aq}x_{kq}}{x_{aq} + x_{kq}} + x$
Time-constants	
O.C. transient: d-axis	$\tau_{d0}' = \frac{1}{\omega_1 r_f} [x_{ad} + x_f]$
O.C. subtransient: d-axis	$\tau_{d0}'' = \frac{1}{\omega_1 r_{kd}} \left[\frac{x_{ad}x_f}{x_{ad} + x_f} + x_{kd} \right]$
q-axis	$\tau_q'' = \frac{1}{\omega_1 r_{kq}} [x_{aq} + x_{kq}]$
S.C. transient: d-axis	$\tau_d' = \frac{1}{\omega_1 r_f} \left[\frac{x_{ad}x}{x_{ad} + x} + x_f \right]$
S.C. subtransient: d-axis	$\tau_d'' = \frac{1}{\omega_1 r_{kd}} \left[\frac{x_{ad}x_fx}{x_{ad}x_f + x_fx + x_{ad}x} + x_{kd} \right]$
q-axis	$\tau_q'' = \frac{1}{\omega_1 r_{kq}} \left[\frac{x_{aq}x}{x_{aq} + x} + x_{kq} \right]$
S.C. armature (d.c.)	$\tau_a = \frac{1}{\omega_1 r} \left[\frac{2x_d''x_q''}{x_d'' + x_q''} \right]$

It is important to note that the two-axis analysis is necessary for cylindrical-rotor as well as for salient-pole machines, because the field winding is a d-axis circuit and introduces effective saliency during the flow of transient currents, and because the solid metal of the rotor body provides a path for eddy currents, giving the effect of a winding KQ. It is not, in practice, valid to assume that such an eddy-current path can be represented by fixed resistance and inductance parameters, for the paths are ill-defined and the parameters are affected by the degree of saturation. Further, the assumption of an invariant speed is made to keep the equations mathematically 'linear': strictly, the considerable torque fluctuations must result in some changes of instantaneous speed. To take them into account makes the solution possible only by computer iteration.

Asymmetrical loading

Fault analysis is made more complicated if the conditions are not symmetrical. Graphed results for such cases are given by Saha and Basu [68] for line-line and line-earth faults, and analytical treatment by Ching and Adkins [78] based on constant speed and the Park two-axis theory. The problem is tractable only with the aid of important simplifications.

Asymmetry involves a condition that, for the steady state, is normally represented by positive-, negative- and zero-sequence components. The same concept can also be applied to fault conditions. If the negative-sequence system alone is considered to be present, a reversed travelling-wave field exists in the machine, having double synchronous speed relative to the rotor. The axis of the travelling-wave field therefore alternates between the d- and q-axes at double the system frequency. The mean value of reactance presented to negative-sequence currents is therefore the mean of the appropriate d- and q-axis reactances, approximately:

$$x_2 = \frac{1}{2}(x_d' + x_q') \quad \text{or} \quad \frac{1}{2}(x_d'' + x_q'')$$

the former for machines without and the latter for those with damper windings.

If the asymmetrical loading or fault includes zero-sequence currents, these components produce no travelling-wave field because they are co-phased in time in each stator phase-winding. Only leakage flux is produced. The magnitude of the leakage field is dependent upon the winding structure (in particular the slot-pitch and the arrangement of the overhang), so that as a consequence the zero-sequence reactance can be very different for otherwise similar machines (e.g. 0.03–0.15 p.u. for salient-pole machines and 0.01–0.10 for turbo-generators). If several generators with earthed neutrals work in parallel, quite small asymmetries can cause large neutral currents, unequally divided between the machines. These can be limited by substituting 0.1–0.2 p.u. inductors for the solid earthing connections.

Reactance levels

The various reactance ranges in a preceding Table affect the performance of the machine when working on a supply system. The influences on behaviour are briefly set out below.

Synchronous reactance. The static stability limit, discussed in Sect. 10.8, is given by the power $E_t V_1 / x_e$, where x_e is the total reactance of the machine, its transformer and the network up to the point having the constant voltage V_1 . If the highest steady-state pull-out load or torque is wanted, the machine constituent x_{sd} of the total x_e should be small. However, as the same result can be obtained by use of a voltage regulator to raise E_t rapidly, and as dynamic stability sets a more stringent constraint, a reduction of x_{sd} for this reason is rarely sought. There is, however, the further consideration of capacitive loading by cables and long overhead lines at the time of low consumer load. With positive excitation the synchronous reactance x_{sd} must not be greater than the equivalent capacitive reactance x_c , and this may provide a limiting level for x_{sd} in a generator, though there are means for avoiding the limitation. In the range $x_{sq} < x_c < x_{sd}$ the required excitation is negative.

Transient reactance. When sudden disturbances occur, the rotor flux is maintained by a current induced in the field winding. The same power-limit expression as before holds, except that E_t is replaced by the effective e.m.f. behind the transient reactance and that x_e is now the transient reactance of the machine and system. It is desirable to keep the transient reactance small in those cases in which dynamic stability is critical. A cage form of damping winding may considerably raise the stability by countering the rotor swing by its induction torque. A low transient reactance tends to reduce voltage dip when faults occur.

Subtransient reactance. Let a generator be subjected to a phase-phase short circuit. If the third phase winding is open, the highest overvoltage in it is given by $[2(x_q''/x_d'') - 1]$ p.u., with a large harmonic content. If the third phase is capacitively loaded by cables or lines, resonance may raise the overvoltage considerably. It is thus important to keep the ratio of q-axis to d-axis subtransient reactance as near as possible to unity. Complete cage dampers and solid pole-shoes give a ratio x_q''/x_d'' typically between 1.3 and 1.5 and are always sufficient in this respect. The separate pole-grid type of damper has a ratio between 2 and 3 and is wholly unsuitable where resonating conditions are foreseen. In general the absolute value of the subtransient reactance is less important than the torque that the damper can exert.

10.8 Stability

The capability of a power system to remain in synchronism when disturbed by load-changes, faults and similar contingencies is called its *stability*.

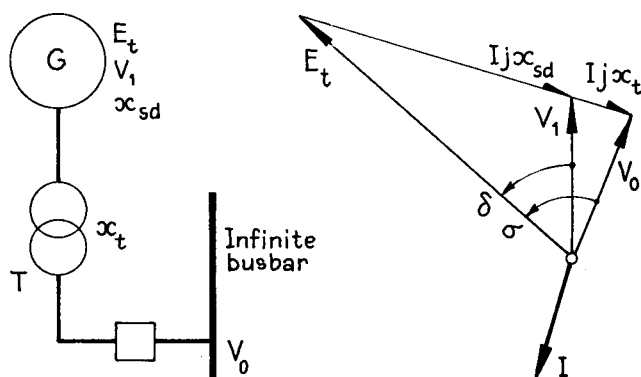
Power systems normally comprise synchronous generators sited in large power stations and connected to loads that are dynamic (i.e. comprise motors and rotating mechanical attachments) or static (e.g., heating, lighting and certain industrial processes). The load has an important influence on stability, as it may affect a generator that has become unstable to increase its pole-slip or to allow it to re-synchronize after slipping one or more pole-pairs. Under steady load it is economically necessary that full use be made of active and reactive power capability, and at the same time there must be an adequate margin of transient stability against faults and outages for reliable, good-quality supply

to the connected loads. Thus both *steady-state* and *transient* stability are of concern.

The consequences of instability can be severe, even disastrous, to system operation. Assessment of system behaviour is extremely complicated, for it involves the contribution of each item of generating plant including boilers, governors, exciters, voltage controllers, protective equipment and switchgear, all having interacting responses complicated by various degrees of nonlinearity.

Steady-state stability

A condition of steady-state stability exists when, after a small disturbance, the system returns to the initial operating state. For generators operating on an infinite busbar (strictly, an unrealized condition) it is shown in Fig. 10.13 that the maximum-power limit is reached for a load angle of 90 deg(e.) for a cylindrical machine, and somewhat less, for example 80 deg, for a salient machine. These load angles correspond to the maximum steady-state power that can be transferred through the synchronous impedance to a fixed system voltage.

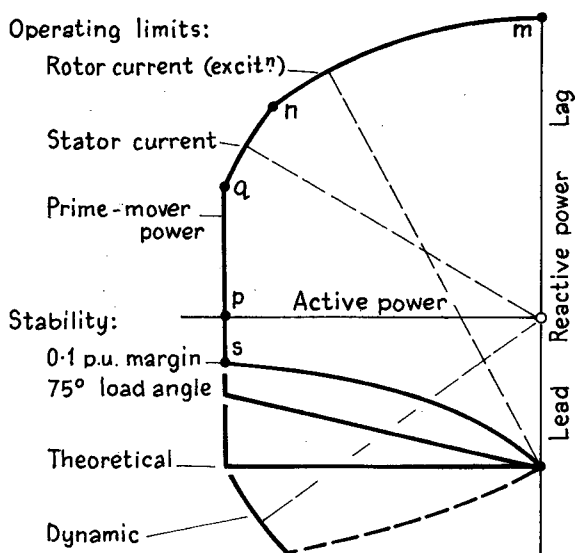


10.29 Stability.

Consider now the system of Fig. 10.29, in which a generator-transformer is connected between the machine and the infinite busbar. The power/angle and voltage/angle curves now relate to the active power into the busbar through the impedance of the machine and its transformer, the terminal voltage V_1 of the machine, and the angle σ between the e.m.f. E_t and the busbar voltage V_0 . So long as σ is less than the maximum-power angular divergence, the machine is stable to small disturbances; but if σ exceeds the maximum-power value, an increase in angle reduces the power, the mechanical drive torque exceeds the electromagnetic torque and the angle must increase still further. Thus regardless of damping the system is unstable. However, if the generator were provided with a continuously acting automatic voltage regulator, the tendency of the voltage V_1 to fall with increasing angle would invoke an increase in excitation and therefore in the active power developed. Provided that the excitation can be raised quickly enough, a positive synchronizing torque can be established

to stabilize the machine. This is clarified by the electrical load diagram in Fig. 10.4, where the maximum power corresponding to a generator working at point S can still be developed after an increase of load angle at point T, provided that the excitation is raised (in the case considered) from 0.94 to 1.0 p.u.

The working region of operation at angles exceeding 90 deg is the region of *dynamic stability*. The factors of importance in determining the dynamic stability limit are the inertia of the machine and its prime mover, the transient reactance, damping and exciter response rate. Methods of determining limits are given by Gove [51], and a typical generator capability chart, comparable with that in Fig. 10.17, is shown in Fig. 10.30. The theoretical stability limit



10.30 Generator capability chart.

line is based on a 90 deg load angle; the 0.1 p.u. margin is inserted; another limiting criterion is the 75 deg load angle; and the considerable addition to reactive power capability resulting from automatic excitation control shows the advantage to be gained by operation in the dynamic stability region.

Transient stability

The transient stability is the capability of a system to return to a steady-state operating condition following a large disturbance, such as a loss of a major load or of a large generating unit, or a severe fault. In systems loaded with heavy rotating plant, the mechanical reactions have caused generators to become unstable even after two or three safe swings. Instability may also occur at times of light system load, for with an extensive transmission network the demand for leading reactive power may require generators to be run at low

excitation to provide it. Operation at leading power factor is critical in several ways: the permissible angular swing of the generator is less than in normal lagging operation for which the excitation level is high; the system voltage collapses more rapidly on fault and consequently reduces the stabilizing effect of connected loads; and the design of the generator excitation and governing has to be highly sophisticated if reliable equipment is to be achieved.

Typically, the fault condition against which transient stability is assessed is a 3-phase fault which, when cleared in 140 ms, results in the outage of a double transmission circuit. Trials have shown that to achieve adequate stability it may be necessary to provide protection equipment and switchgear capable of clearing such faults in less than 100 ms.

Swing curve. The equation of motion for a synchronous generator, based on eq.(7.5) of Sect. 7.3, can be expressed by equating the difference between the electromagnetic and prime-mover torques to the torque required to accelerate the inertia J and provide a pair of damping torques, one proportional to the velocity and one a constant, i.e.

$$\Delta M = J(d^2\delta/dt^2) + C_1(d\delta/dt) + C_2$$

It is more convenient to work in terms of power at the mean synchronous speed. Then $\omega_s \circ \Delta M$ becomes Δp , the difference between the electrical power p_e and the prime-mover input power p_m . The damping torques multiplied by ω_s become damping powers, and the product $J\omega_s$ can be written as the angular momentum m_o . Then

$$\Delta p = m_o (d^2\delta/dt^2) + k_d(d\delta/dt) + p_c \quad (10.15)$$

The equation is not easy to solve for δ as a function of time. The power-difference term Δp is itself time-dependent. The electrical power varies not only with δ but also with voltage-regulator action, and the prime-mover power is affected by governor action. Again, during the swing the conditions may change rapidly because of the operation of circuit-breakers. The damping coefficient k_d can depend quite critically on the rate of change of load angle, particularly where significant damping is obtained by means of pole-face windings or induced eddy-current effects.

The momentum for a given machine is related by eq.(7.1) to the inertia constant H . This has a direct bearing on the frequency and amplitude of the rotor swing, for the larger the inertia of the generator-turbine combination, the more excess kinetic energy it can absorb for the same change in speed. Large modern turbo machines tend to have a rather low specific inertia because the physical dimensions do not increase in proportion to the rating, a consequence of the limitation on rotor diameter imposed by centrifugal force and the higher specific electric loadings that are obtained with advanced cooling methods. In the case of hydro machines a much larger inertia effect is essential because of the sluggish action of hydraulic governors made necessary by the great moving masses of water that they must control.

Various solutions for the dynamic stability equations have been given, such as that by Humpage and Saha [69], using various degrees of simplification in

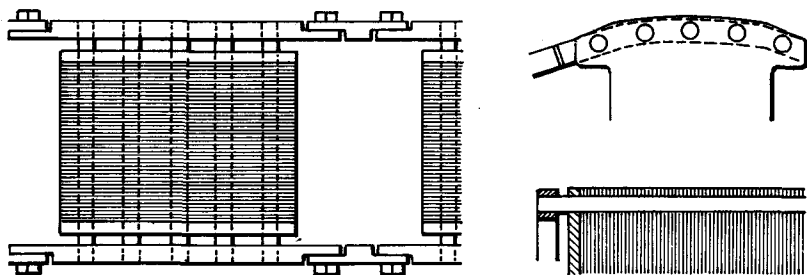
the boiler-turbine-generator system. The simplification acceptable depends largely on the number of rotor swings, and this in turn on the load. For static loads the first-swing stability criterion is adequate, but for highly dynamic loads a multi-swing investigation covering up to 20 s may be essential. The problem has some ramifications: for example, the auxiliary motors in a generating station must remain in step even if the main generators produce violent fluctuations of voltage and frequency at the motor terminals. In a computer study by Ramsden Zorbas and Booth [71] the dynamic performance of induction motors is considered in relation to system stability. They show that the large cage induction motors connected to a supply system can be important in determining the behaviour of the system under fault conditions. Such motors generate large initial currents immediately subsequent to the fault occurrence, and these are additional to those contributed by the synchronous generators. Thereafter the latter machines swing to an increasing load angle, while the induction machines suffer a fall in speed. After the fault is cleared, the stability of the system depends upon whether all the synchronous machines remain in synchronism, and whether all the induction motors recover to normal slip.

Initial currents can be calculated on the assumption that the speed is constant; but for subsequent behaviour the effect of the appreciable speed-change is an essential factor, and the only method available to determine the variations in load angle or speed is a numerical step-by-step computation. The analysis is complicated by the difficulty in establishing a suitable reference frame. Kalsi and Adkins [81] discuss some analytical methods and compare the computed results with tests on micromachines; they conclude that the effect on the system of induction machines ought to be included in system-fault studies, but that simplified methods must be justified by comparison with measured values.

Damping

If the rotor speed departs from a steady synchronous value, any short-circuited path linking the gap flux or part thereof behaves with reference to the travelling wave stator field as does the cage winding of an induction machine: slip-frequency currents are induced and the consequent I^2R loss dissipates some of the swing energy, developing a torque roughly proportional to the slip. At negative slip the damping torque acts to reduce the speed, and at positive slips to increase it. The estimation of the damping power $k_d(d\delta/dt)$ in eq.(10.15) is complicated, especially if the damping effect results from eddy-current loss in a solid rotor.

Damping circuit. In salient-pole machines, damping windings consist usually of solid copper bars through the pole-shoes. The ends of the bars can be connected together by short-circuiting straps to form a damping 'cage', as in Fig. 10.31. Sometimes interpolar connectors are omitted. An alternative method is to employ thick steel plate for the pole and shoe, or to use solid pole-shoes, in which the eddy-current damping effect is produced. It is possible theoretically to improve the damping by connecting the solid pole-shoes together by interpolar conducting strips, but the advantage gained is usually small. However, without interpolar connections, q-axis damping currents must flow in the pole-body and rim, and the ratio x_q''/x_d'' of the subtransient reactances is inher-



10.31 Damper windings for salient-pole machine.

ently greater than unity. The design of interpolar connections offers some difficulty in that they have to withstand tangential vibration between adjacent poles, and may require reinforcement against centrifugal force if the machine is subject to appreciable over-speed.

The rotors of turbo-generators are always of solid steel, and in addition there may be an effective damping cage effect provided by metallic slot wedges, with the end-caps acting as short-circuiting rings.

Negative-sequence damping. The constant-power term p_c in eq.(10.15) includes friction, which is normally comparatively small, and negative-sequence damping. Most stability studies involve fault conditions in which negative-sequence current components are concerned. These have a slip of approximately $s = 2$, setting up an asynchronous (induction) torque in a direction opposite to that of the rotor rotation; the effect is comparable with a friction torque.

Equation of motion

If eq.(10.15) can be solved, the motion of the rotor can be determined. Now Δp is a function of load angle δ , governor action and voltage-regulator response; damping powers vary with slip and fault conditions; and after some delay in the action of protective gear, circuit-breaker operation occurs to change the operating conditions. Only electronic computation methods with step-by-step calculation can properly cope with the evaluation of the equation of motion and make evident whether or not the machine concerned will remain in step.

The essential features of the calculation can be illustrated by use of the equal-area criterion (Sect. 7.3) and the following simplifying assumptions: (i) the prime-mover power remains constant, (ii) the generator power/angle relation remains unchanged for a given circuit condition, and (iii) damping is neglected so that the equation of motion reduces to $(d^2\delta/dt^2) = \Delta p/m_0$. These are employed in the following Example.

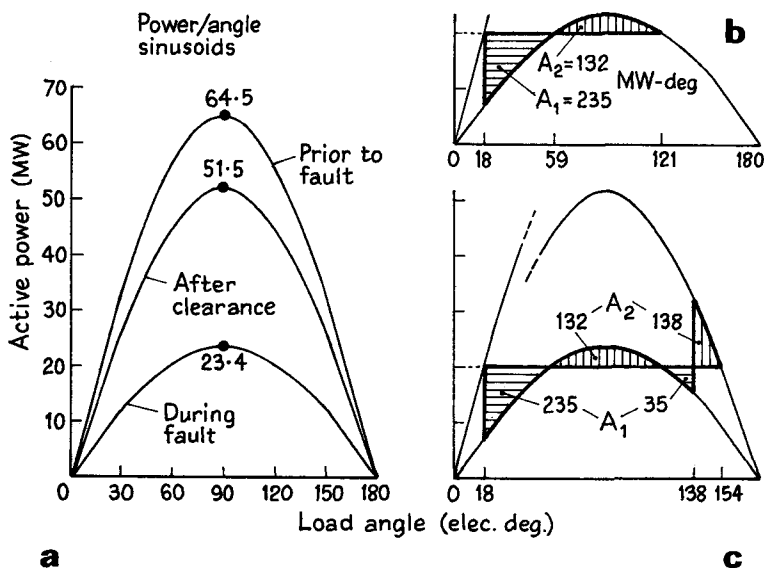
EXAMPLE 10.7: A 25 MVA, 33 kV, 50 Hz, 2-pole, 3-ph turbo-generator delivers a steady load of 20 MW over a double-circuit transmission line to an infinite-busbar system. A 3-ph fault occurs on one of the line circuits, and is cleared simultaneously at both ends (i) after 400 ms, (ii) after 650 ms. Obtain

the swing curves and the critical switching time. The required data are:

Inertia constant	$H = 2.3 \text{ MJ/MVA}$
Generator e.m.f. (line)	$E = 34.0 \text{ kV}$
transient reactance	$x_d' = 13.0 \Omega/\text{ph}$
Infinite busbar voltage	$V_0 = 33.0 \text{ kV}$
Reactances between generator and infinite busbar:	
prior to fault	$x_1 = 17.4 \Omega/\text{ph}$
during fault	$x_2 = 48.0 \Omega/\text{ph}$
after fault clearance	$x_3 = 21.8 \Omega/\text{ph}$

The power/load-angle relations are assumed to be sinusoids. From eq.(10.6), suitably modified for the three operating conditions and with output power taken for convenience as positive, the peaks of the three sinusoids are

$$P_1 = 33.0 \times 34.0 / 17.4 = 64.5 \text{ MW}; \quad P_2 = 23.4 \text{ MW}; \quad P_3 = 51.5 \text{ MW}$$



10.32 Transient stability: equal-area criterion.

Equal-area criterion. The three sinusoids are plotted in Fig. 10.32(a). The diagram (based on the treatment in Sect. 7.3) shows that the fault, if allowed to persist, is bound to lead to asynchronous running, for area A_2 is less than A_1 . But if the fault is cleared before $\delta = 154^\circ$ some additional compensating area is included. By trial it is found that clearance must take place by the time that δ reaches 138° . For this *critical switching angle* the acceleration energy, proportional to the total area $A_1' + A_1''$, is just matched by the energy absorption proportional to $A_2' + A_2''$. The equal-area diagram does not give the *critical switch-*

ing time, to determine which it is necessary to find a solution to the equation of motion.

Equation of motion. The momentum is $m_0 = 2HS/\omega_s = 0.37$ MJ per rad/s = 0.0064 MJ per deg(e.)/s. The equation of motion with the simplifying assumptions made is given by $(d^2\delta/dt^2) = \Delta p/m_0$, whence the swing velocity and load angle are approximately

$$\omega = \Delta\delta/\Delta t = (\Delta p/m_0) \Delta t + \omega'$$

$$\delta = \delta' + \omega \Delta t$$

for a short-time-interval Δt , where ω' and δ' are the initial values of swing velocity and load angle respectively. Inserting numerical values with the time-interval $\Delta t = 50$ ms = 0.05 s gives

$$\omega = \omega' + 7.82 \Delta p \text{ deg/s} \quad \text{and} \quad \delta = \delta' + 0.05 \omega \text{ deg}$$

with Δp in megawatts. Prior to the fault ($t = 0^-$) the load angle is such that $64.5 \sin \delta = 20$ MW so that $\delta = 18.1^\circ$. Immediately subsequent to the fault initiation ($t = 0^+$) the angle is the same but the power sinusoid changes to a peak of 23.4 MW for which the generator power is 7.25 MW: hence $\Delta p = 20 - 7.25 = 12.75$ MW available to accelerate the rotor. For $t = 0$ the mean $\Delta p = 12.7/2 = 6.37$ MW is taken, and the velocity at the end of the first interval Δt is

$$\omega = \omega' + 7.82 \Delta p = 0 + 7.82 \times 6.37 = 50 \text{ deg/s}$$

The load angle at the end of the interval is

$$\delta = \delta' + 0.05 \omega = 18.1 + 2.5 = 20.6 \text{ deg}$$

These become initial values for the second interval: $23.4 \sin 20.6^\circ = 8.25$ MW so that $\Delta p = 20 - 8.25 = 11.75$ MW. The velocity and load angle are now

$$\omega = 50 + 7.82 \times 11.75 = 142 \text{ deg/s}, \quad \delta = 20.6 + 142 \times 0.05 = 27.7 \text{ deg}$$

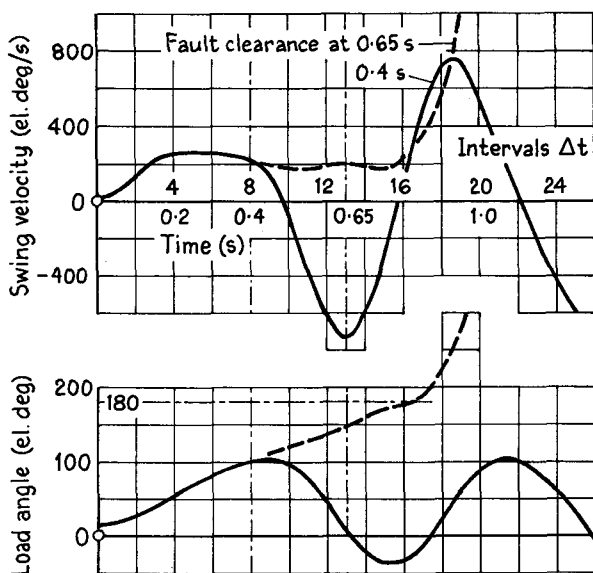
(i) Clearance at $t = 400$ ms: Tabulated values for 19 time-intervals are given below. Fault clearance occurs at the end of interval 8, where the power sinusoid changes to a peak of 51.5 MW, and the mean Δp is again taken as for $t = 0$. Graphs of swing velocity and load angle are given in Fig. 10.33. The machine swings over a wide angle but remains synchronized.

(ii) Clearance at $t = 650$ ms: The values up to interval 8 are the same as in (i), and continue under the same conditions up to interval 13, where a second transition occurs. The values are graphed in Fig. 10.33 (dotted line), showing that circuit-breaker operation is too late to avoid loss of synchronism.

Critical switching time. It is seen that the critical switching time exceeds 400 ms but is less than 650 ms. A guide can be obtained by a fairly simple adjustment of the switching angle in Fig. 10.32(c) to equalize the areas, giving the angle 138° as the critical angle. The dotted graph in Fig. 10.33 shows that this angle is reached in about 12 time-intervals, i.e. 600 ms, and this gives approximately the critical switching time.

Fault-clearance time, 400 ms

Interval No.	δ'	$\rho \sin \delta'$	Δp	ω'	$\Delta \omega$	ω	$\Delta \delta$	δ
0-	18.1	20.0	0	Initial, $\Delta p = 6.37$				
0+	18.1	7.25	12.75					
0	18.1	7.25	6.37	0	50	50	2.5	20.6
1	20.6	8.25	11.75	50	92	142	7.1	27.7
2	27.7	10.9	9.1	142	71	213	10.6	38.3
3	38.3	14.5	5.5	213	43	256	12.8	51.1
4	51.1	18.2	1.8	256	14	270	13.5	64.6
5	64.6	21.1	-1.2	270	-9	261	13.0	77.6
6	77.6	22.9	-2.9	261	-23	238	11.9	89.5
7	89.8	23.4	-3.4	238	-27	211	10.6	100.1
8-	100.1	23.0	-3.0	Transition, $\Delta p = -16.9$				
8+	100.1	50.8	-30.8					
8	100.1	50.8	-16.9	211	-132	79	4.0	104.1
9	104.1	50.0	-30.0	79	-234	-155	-7.7	96.4
10	96.4	51.3	-31.3	-155	-245	-400	-20.0	76.4
11	76.4	50.1	-30.1	-400	-235	-635	-31.8	44.6
12	44.6	36.3	-16.3	-635	-127	-762	-38.1	6.5
15	-42.1	-34.6	54.6	-319	427	108	5.4	-36.7
20	88.1	51.5	-31.6	524	-247	277	13.9	102.0
25	34.1	28.5	-8.5	-665	-66	-731	-36.5	-2.4



10.33 Transient stability: swing curves.

Improvement of stability

Governors contribute little to stability unless they have very fast responses to small changes of speed, e.g. full-throttle valve closure within 500 ms. Such a response demands special oil-pressure hydraulic servos. Automatic voltage regulators can act through the excitation system to change the generator power, but only by as much as the system voltage and rotor angle will allow. The most effective aid to stability is the fast-acting circuit-breaker. With rapid fault detection within 10 ms and air-blast breakers it is possible to attain full fault clearance in less than 100 ms. In this time-interval a large generator will not acquire much additional kinetic energy.

The complexities of system modelling have been discussed in Sect. 7.4. A generating set in particular is a mechanical-electrical conversion link between thermal or hydro plant at one end and an electrical supply network at the other. No single control element can be considered in isolation, for such elements interact and are usually nonlinear. However, the elements most immediately concerned in operation are the governing and the excitation, which must for stability be closely co-ordinated. Summarizing, the improvement of stability may be achieved as follows.

Steady state. Electrically, this is a matter of synchronizing and damping torques. Consider the system in Fig. 10.29. Let x_t be the reactance between the generator terminals and a point on the network that can be assumed to have infinite-busbar properties; and replace x_{sd} by x , the value most appropriate to the swing condition. Then adapting eq.(10.3), the synchronizing power coefficient is

$$p_s = [E_t V_0 / (x + x_t)] \cos \sigma = [V_1 V_0 / x_t] \cos (\sigma - \delta)$$

A strong synchronizing power depends on small reactances and divergence angle and on the maintenance of system voltage and generator excitation (conditions not favourable if faults occur). The natural damping of the generator is raised if control secures that the machine supplies more active power when accelerating than when retarding during a swing.

Transient state. Improved steady-state stability benefits transient stability too, but there are further requirements. Network faults must be rapidly detected and cleared, and fast governor and valve control provided to reduce the mechanical drive. It may be possible to augment the inertia of a hydro-generator at the design stage, but the natural inertia of turbo-generator sets tends to be low.

Dynamic state. Control methods that cause torques to be produced in phase with the rate of change of load angle δ improve dynamic stability, in particular excitation control to increase the electrical output in phase with $+d\delta/dt$, and the prime-mover power in phase with $-d\delta/dt$, the former by excitation and the latter by governing. Many control quantities have been suggested, including $d\delta/dt$ and its higher derivatives, or the output power P_1 and its time-derivative and time-integral.

10.9 Excitation and voltage regulation

Small synchronous machines are sometimes built with permanent-magnet field excitation, but normally a d.c. excitation supply is necessary. Industrial motors may operate with a constant excitation. With generators the aims of the exciting system are: (i) to control the voltage for operation (if needed) near the steady-state stability limit; (ii) to regulate the voltage under fault conditions; and (iii) to facilitate reactive-power load-sharing between generators operating in parallel.

To obtain the greatest possible rating from a given mass of active material, turbo-generator design with forced cooling and high current densities gives unavoidably a high synchronous reactance compared with that normal in a low-speed machine. The synchronous reactance x_{sd} has as its main component the stator (or 'armature') reaction reactance x_a which depends on the electric loading. Thus typically with a leakage reactance $x = 0.15$ p.u. and a reaction reactance $x_a = 1.6$ p.u., the synchronous reactance has the high value $x_{sd} = 1.75$ p.u.

With reactances of this order, the rotor field excitation at full load and p.f. 0.8 lagging must be more than $2\frac{1}{2}$ times that for no load at the same terminal voltage. The exciter must therefore have a voltage and current rating to correspond. Allowing for temperature-rise and voltage margin, the exciter must have a voltage range of the order of 4/1 over which it must possess a stable characteristic. Some tendency to instability occurs with weak fields when the reaction of the stator currents on the airgap flux is stronger than that of the exciting current. It is generally sufficient to have a gap length that makes strong excitation necessary and enables the exciter to achieve the wide range of voltage without undue saturation.

In large generators the excitation power is of the order of 0.005 p.u., approaching 0.01 p.u. during 'field forcing'. The power (which is little more than the field circuit $I^2 R$) for the largest machines may be about 4 kW/MVA.

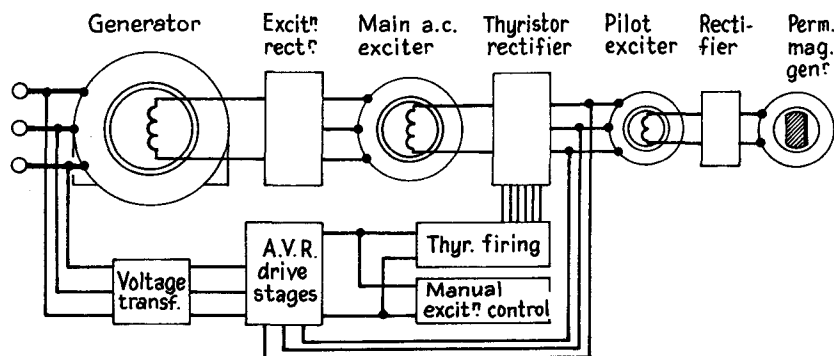
A compromise between large exciting currents at low voltage and thicker insulation for high voltage leads to a choice normally between 200 and 800 V.

Excitation systems

D.C. exciters. This is the traditional method. A d.c. generator is mounted on the main shaft and shunt- or separately excited, the output being fed to the rotor of the main generator through slip-rings and brushgear. Exciter construction follows normal d.c. generator practice, except that all or part of its field excitation may be provided by an auxiliary pilot exciter to improve the rate of response. Commutation difficulties at high speeds have sometimes led to geared or separately-driven exciters; low-speed hydro-generators may require exciters driven at some suitably higher speed by motors or by small water turbines.

Static excitation. Direct-current excitation can be obtained by means of a rectifier and a suitable supply of alternating voltage, and by this method the commutation limits inherent in the rotary d.c. exciter generator are avoided. The rectifier unit has no rotating parts, requires little maintenance and is immune to hazardous or dust-laden atmospheres. With associated control equipment it is accurate, reliable, compact, and fast-acting. There are several arrangements.

With an *a.c. exciter* machine mounted on the main shaft, its field fed from a pilot exciter whose field in turn is derived from a permanent-magnet generator, the exciter output is fed to the floor-standing rectifier, and the rectified current supplied to the main rotor winding through conventional slip-rings. The essential arrangement is shown in Fig. 10.34.



10.34 A.C. exciter/rectifier excitation for turbo-generator.

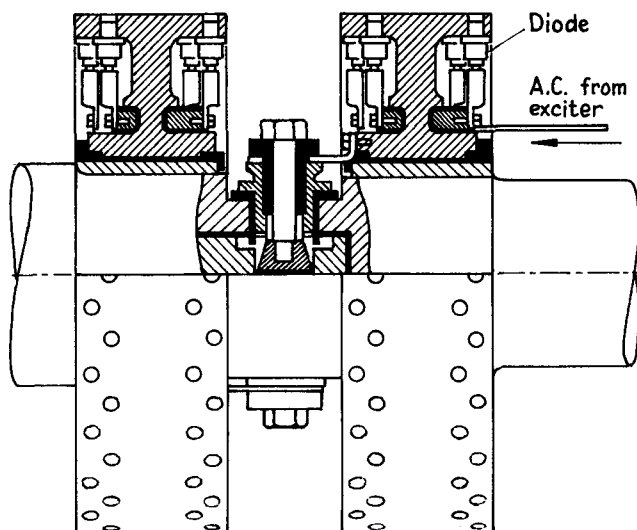
With *thyristors* in place of diodes a reliable excitation system can be achieved. A fully-controlled 3-phase bridge connection is most usual because of its advantages in securing lower voltage stresses on the semiconductors and greater utilization of the transformer capacity. With the high peak reverse voltage capability of thyristors, two elements in series per branch are sufficient. Advances in thyristor technology, particularly in the on-state characteristics and in improved triggering, now permit the direct series connection without cascading.

Design of the rectifiers must be conservative, so that they can withstand the severe conditions imposed by system faults, pole-slip, asynchronous running and faulty synchronizing. Typical diode rectifier elements, rated at 300 A and 3 kV reverse peak voltage, can be connected in parallel banks, and are readily maintained.

Brushless excitation. The term 'brushless' is applied to a machine in which the conventional brushgear is eliminated. An exciter with a fixed field and rotating phase windings is mounted on the main shaft, the a.c. output being converted to d.c. by means of shaft-mounted rectifiers and fed directly to the main rotor winding, no slip-rings or brushgear being needed.

Most modern synchronous motors are equipped with brushless excitation. The requirements differ from those that obtain in large turbo-generators, for while the brushless system for generators is excellent from the viewpoint of general maintenance, to replace a faulty rectifier diode it is necessary to shut down the machine, involving a long and very costly outage. Further, it is not possible to use a field switch or to reduce the natural time-constant of the rotor field winding by externally connected resistors, so that damage of a more extensive nature may result in the event of an internal fault. Remote-indicating instrumentation without re-introducing slip-rings is more complicated

The rotating rectifier should have the simplest mechanical arrangement to provide maximum reliability. Ideally the bridge arms should consist only of diodes in parallel since, with forward- and reverse-polarity diodes, this permits of the firm support of each unit in the metal housing of the rotating carrier. A construction suitable for a 500 MW generator is illustrated in Fig. 10.35: two T-section steel discs are provided, one for each polarity and insulated from



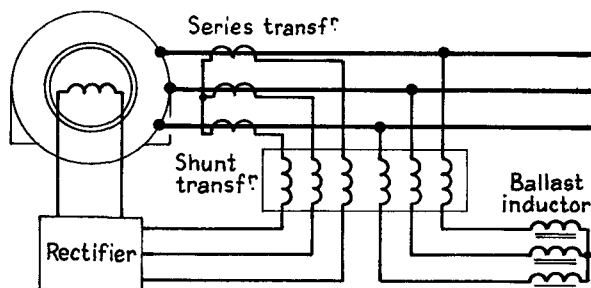
10.35 Rotating rectifier.

the shaft. The diodes are mounted on the rims with the inner flexible conductors joined to one of the a.c. phase terminals, each of which covers circumferentially somewhat less than 120° and is insulated from the T-section discs. Such a rectifier unit has an outside diameter of less than 1 m, and an axial length of about the same as would be needed for normal excitation slip-rings.

There are certain other ways of exciting a brushless machine, one being a combination of induction and rectification. In a machine operating on this principle, it is arranged that an auxiliary field is set up so as to induce an alternating e.m.f. in a winding carried on the rotor, and this is used with a rectifier to supply d.c. excitation to the main-field winding of the rotor. The auxiliary field may be stationary, pulsating or rotating relative to the stator, the only essential requirement being that it should alternate with respect to the rotor. For example, this field could be produced by space harmonics due to winding arrangement or distribution, or due to magnetic circuit geometry. It may have the same pole-number as the main field of the machine, in which case alternating voltages are induced into a single rotor winding which is closed through rectifiers. Alternatively the auxiliary field may have a different pole-number and induce e.m.f.s in a special pick-up winding connected through rectifiers to the main rotor exciting winding.

Self-excitation. Small generators, such as those for marine service, can be self-excited by means of a bridge-connected rectifier deriving its alternating voltage drive from a transformer with two primary windings per phase; one primary is excited from the generator terminal voltage, the other from the load current, as indicated in Fig. 10.36. As a result the excitation is responsive to both output voltage and load current, and is said to be compounded, or self-regulating. The advantages are rapid transient voltage response to suddenly applied loads together with the reliability of static equipment. As most generators have a residual rotor field too small to enable self-excitation to occur from a standstill start, a permanent-magnet pilot exciter is provided to establish an adequate generator terminal voltage for initiating self-excitation.

The method is applicable to synchronous motors, although here the initiation equipment is not needed. By proper design of the transformer and of the star-connected inductor used to terminate its 'voltage' primary windings, it is pos-



10.36 Compounded self-excitation.

sible to secure either constant power factor or constant reactive power for all motor loading conditions.

Double excitation

A major problem in the operating of generators connected to an extensive supply network is the supply of reactive power. A conventionally wound turbo-generator rotor can be made to furnish leading reactive power by underexcitation, but if stability is to be retained the limit of reactive power on zero active load is about 0.5 p.u. A fast-acting voltage regulator can make operation possible at load angles exceeding 90° , but dynamic stability is then endangered. Thus the turbo-generator with a conventionally-wound rotor (c.w.r.) operating at a leading power factor on an unloaded system works in its most unfavourable condition when the system voltage is most difficult to control.

To improve the conditions the double- or divided-winding rotor (d.w.r.) can be used. The d.w.r. makes it possible to shift the rotor m.m.f. axis with respect to the rotor body: it has the following features:

The d.w.r. has the same slotting as a c.w.r., but the slot conductors are separated by special end-connectors into two parts, the m.m.f. axes of which are mutually displaced (e.g. by an angle of 60°) to give *torque-winding* and *reactive-winding* m.m.f.s, individually controllable.

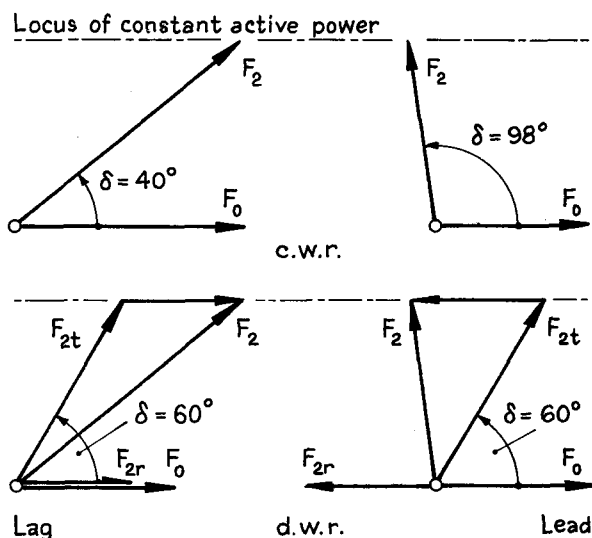
The m.m.f. axis of the *torque winding* is varied in proportion to the displacement of the rotor from a predetermined load angle, irrespective of whether that displacement results from a change of prime-mover power or from a system disturbance. For this purpose a control sensitive to load angle is required.

The m.m.f. of the *reactive winding* can, by the action of an automatic voltage regulator, be varied over a range of values from positive to negative as required by the divergence between the system voltage and that of a predetermined level of reference voltage.

The angle regulator controls the load angle to a value approximately equal to that between the torque-winding and reactive-winding m.m.f.s, although this condition is not essential and may, in fact, not be best for stability.

The essential concept can be conveyed by the m.m.f. diagrams in Fig. 10.37, drawn for constant generator full-load active power respectively at rated lagging and leading power factors. The rotor m.m.f. for the c.w.r. machine is F_2 and its resultant gap m.m.f. is F_0 . The m.m.f. diagram for lagging power factor is based on Fig. 10.1 for the over-excited generator condition (b)(iii). The load angle δ is 40° , the machine being taken to have a short-circuit ratio $r_{sc} = 0.4$ and negligible saturation. If F_2 is reduced, the power factor rises and then becomes leading; and m.m.f. F_2 reduced by 33% results in a load angle of 98° , which lies in an unstable region.

Consider now the action in the d.w.r. machine, in which the reactive winding produces the m.m.f. F_{2r} along the gap-flux axis F_0 , with the torque winding m.m.f. F_{2t} displaced from it by 60° . The lagging and leading power factor conditions are obtained by adjustment and reversal of F_{2r} , with the axis of F_{2t} unchanged.



10.37 M.M.F.s of conventional and double-wound rotors.

There are several advantageous effects. The d.w.r. can control active and reactive outputs without altering the rotor position with respect to the gap flux, so increasing steady-state stability. The transient stability is improved, largely because of an increase in the q-axis time-constant. When angle- and voltage-sensing regulators are used independently, with the reactive winding held in line with the flux axis, it is possible to employ very high voltage-regulator gains, again improving the transient stability limits. The d.w.r. system has been analysed by several writers, including Harley and Adkins [83]. The success of the d.w.r. machine is dependent upon an advanced and sophisticated system of field control.

Excitation response

When a sudden load change on a generator occurs, demanding a change in excitation, the automatic regulating equipment must first operate on the exciter field; the exciter output voltage changes, and then the main rotor field current. Magnetic fluxes cannot build up instantaneously, so that the rapidity of response to a load change depends on a sequence of events each with a time delay: the voltage regulator, the exciter and the main field, together with closed circuits and eddy-current paths which tend to sustain the fluxes linked by them, all contribute some delay.

An indication of the rapidity of response of an exciter is the average rate of rise of the rotor terminal voltage following the opening of the exciter armature circuit. For turbo-generators a response of 0.5 may be adequate (i.e. a rise of 200 V/s for a nominal 400 V). The rate required for hydro-generators may be much higher as a consequence of the overspeed to which these machines may be subjected.

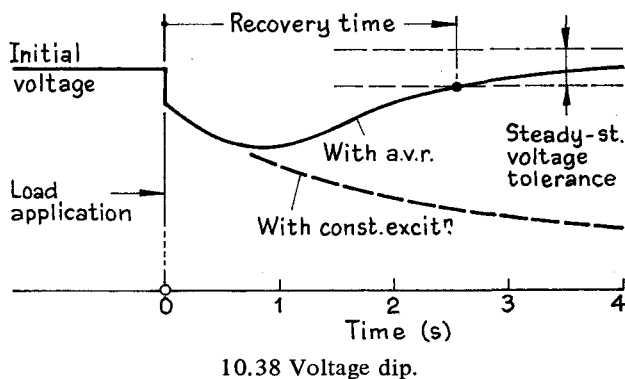
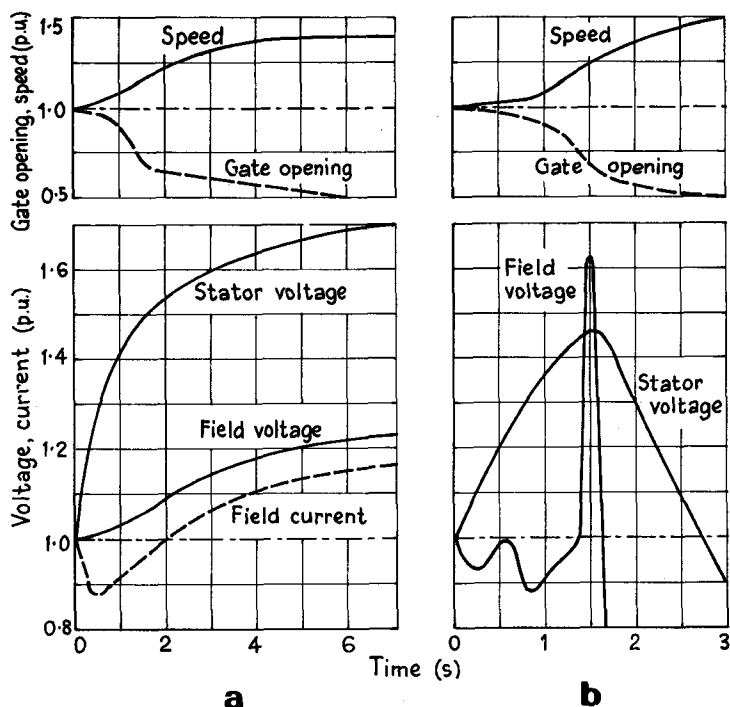


Fig. 10.38 shows stator terminal voltage variations for a generator on 0.4 p.u. load at p.f. 0.8 lagging, and undergoing at $t = 0$ a sudden increase of load to 0.8 p.u. The initial drop of about 4% is due to increased leakage reactance drop. Thereafter the voltage falls slowly as the gap flux alters to a value determined by the new armature reaction. After about 1 s the rising exciter current begins to restore the generator terminal voltage. A shunt-connected exciter gives the slowest restoration, separate excitation is more rapid, and if *field forcing* is employed (the application of e.g. twice normal field voltage) the delay may be reduced to little more than 2 s. Where the dropping of load is liable to cause overspeed, considerable voltage rises may initially occur. Fig. 10.39(a) shows conditions produced by suddenly unloading a 20 MVA hydro-generator working at a p.f. of 0.75 lagging and with a synchronous speed of 100 r/min. The water gate begins to close about 0.6 s after the load has been dropped. Meanwhile the speed rises, reaching 1.35 p.u. in 5 s. The voltage regulator reduces the field current but the rotor flux and overspeed combine to raise the generator voltage to 1.7 p.u. The curves in (b) are for the same conditions as in (a) except that an overvoltage overfrequency relay opens the field circuit; but there is still a voltage rise.

Field suppression. When a short circuit occurs in a machine in operation, not only should it be disconnected from the busbars but also its generated e.m.f. should be rapidly extinguished in order to avoid the aggravation of internal damage. The excitation must be interrupted by an automatic switch, by opening the main field or the exciter field circuit, or by reversing the field voltage, or by short-circuiting the main field winding and exciter.

The e.m.f. induced in a field winding disconnected from its exciter and closed on a resistor of resistance matching that of the field winding is twice normal voltage; but the field current may persist and cause the generator to feed into an internal fault, unless (as in turbo-generator) there are eddy-current damping paths in the rotor body to shorten the field persistence time.

Exciter reversal has little advantage over short-circuiting, but it reduces the field current more rapidly in the later stages. If the exciter is itself separately excited from a constant-voltage source, the generator field current can be



10.39 Hydro-generator: response to throwing off full load at p.f. 0.8 lagging.

rapidly reduced to zero. The main field switch should then be opened as soon as the stator fault current has been quenched, in order to prevent the build-up of a reversed main flux.

Automatic voltage regulators

Excitation control is required to maintain normal operating voltage, to vary the generation of reactive power, to maintain the voltage during external faults, and to increase the steady-state, dynamic and transient stability. Manual control of an exciter-field rheostat suffices for very small machines, but otherwise the use of automatic voltage regulators (a.v.r.) is universal.

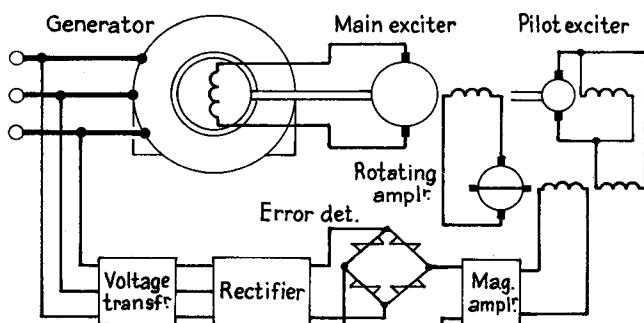
Carbon-pile regulator. For small machines a carbon-pile form of regulator is common. It makes use of the resistance variation of a pile of carbon plates with variation of compressive force, the mechanical pressure being provided by an electromagnet energized from the supply to be controlled. The pile may operate directly on the main field circuit, but usually controls the exciter field.

Torque-motor regulator. This is a sensitive and quick-acting device comprising multi-tapped resistors connected into the exciter field circuit and brought out to closely spaced rows of silver contacts. Short-circuiting sectors roll over the contacts in accordance with the torque of a small split-phase disc motor against

spring loading and eddy-current damping. Voltage setting is simple, and droop and compounding effects can be adjusted independently. The regulator has no parts subjected to wear and requires very little attention.

Vibrator regulator. The exciter field rheostat is rapidly switched in and out of circuit to correct deviations from nominal voltage setting. A control magnet is energized from the voltage to be regulated and its main contacts are closed by a spring. If the voltage rises, the pull of the spring is overcome and the contacts open, inserting the field rheostat into the field circuit. The resulting fall in voltage causes the contacts to reclose. The cyclic process takes place rapidly and repeatedly, holding the voltage within the limits prescribed.

High-speed regulator. The electromechanical regulators mentioned above are unsuitable for machines above about 75 MW owing to the large field currents required for the main exciter. High-speed regulators are required that can give rapid response rates with the largest generators. Such an automatic voltage regulator comprises error-detecting, follow-up, under-excitation limiter and under-voltage protection units, with appropriate amplifiers. Fig. 10.40 shows



10.40 Automatic voltage-regulating system.

the essentials of a typical regulator. The error unit is a semiconductor bridge of elements with a voltage/current characteristic of the form $v = k i^\beta$, the exponent β being different in each pair of elements; the output voltage from this bridge is zero for the specified reference input voltage. A power of a few milliwatts from the bridge is amplified by magnetic and rotary amplifiers up to 40 kW or more for the field of the main exciter. The pilot exciter acts through the rotating amplifier, with a 'bucking' winding fed through the error-detecting circuit. The operating frequency of the magnetic amplifier is usually 50 Hz, but it may be given a faster response by use of a higher frequency, e.g. 400 Hz. Manual control of the system is provided for emergency, with automatic follow-up. The limitation of the level of under-excitation can be achieved by monitoring the generator load angle, using a permanent-magnet generator on the main shaft to indicate the rotor position.

Continuous-acting a.v.r. are available for permitting stable operation in the region of dynamic stability, with load angles up to 120 deg(e.). In case of an

a.v.r. failure the undervoltage protection boosts the excitation to a safe limit to preserve synchronous running.

10.10 *Generator control*

A generator/prime-mover set is a very complicated and expensive plant. Its starting and running require sophisticated operating and control equipments.

Starting

In starting a hydro-generator, the main considerations are those outlined in Sect. 7.2. The operation is more complicated for a steam turbo-generator.

Much of the process of starting a turbo-generator 'from cold' concerns the thermal problems of the turbine expansion and stressing, more particularly of its rotor. Thermal bends in rotors may develop during run-up and cause damaging vibration, through which the set must be 'nursed'. It may take several hours for a turbo-generator set to become hot enough for loading, so that a common practice is to maintain some plant as 'spinning reserve' with the turbines taking steam largely for temperature maintenance, so that more rapid loading can be achieved. In large modern stations the starting and stopping processes are automated. The control engineer can preset the required active and reactive load and the rate of loading, and then initiate boiler starting, turbine, run-up, synchronization and loading by one switch operation.

Synchronizing

Almost all synchronous generators operate in parallel with others, and must be switched into operation by a synchronizing process. The conditions to be satisfied are these: (i) the speed of the machine to be connected to live busbars must be such that its frequency is nearly the same as that of the busbars and preferably 0.2–0.4 Hz high; (ii) the terminal voltage should be within 5% of that of the busbars, and co-phasal with it within about 5 deg(e.). The usual method of synchronizing a generator on to the system busbars is to apply rotor excitation with the coupling circuit-breaker open and to adjust the open-circuit voltage until it is in magnitude equal to that of the system. The phase displacement of the generator and system voltages is observed by synchroscope or lamps, and its rate of angular change adjusted by control of the generator speed. When the rate of change is slow, and the displacement within about 5 deg(e.) the circuit-breaker is closed. With manual synchronizing, allowance must be made by the operator for the delay, 200–400 ms, between initiating circuit-breaker action and the actual closure of the contacts. On large generators use is made of automatic synchronizing equipment with the appropriate voltage and angle sensors.

Another method ('rough synchronizing') consists in closing the coupling circuit-breaker with the rotor field unexcited, then raising the excitation and simultaneously controlling the speed until the generator self-synchronizes. There are three successive processes concerned: (i) the magnetizing time following closure of the circuit-breaker, with a peak stator current occurring within the first two a.c. periods and rapidly decaying asymmetric and subtransient

components; (ii) a duration of asynchronous running during which induction torque and governor action bring the speed close to normal synchronous speed; and (iii) the synchronizing time following closure of the d.c. field switch. Generally, the current peaks in (i) are smaller than those due to short-circuit faults, and unless a generator is operated on a daily two-shift duty the method has attractive simplifications, especially for automatic control.

Synchronizing currents. If at the instant of closure of the main circuit-breaker there are divergences of angle, frequency or voltage between the incoming machine and the busbars, there will be a subtransient synchronizing-current surge in the stator windings, with its effects reflected into the rotor. For a phase divergence α an active-power-producing current $i_\alpha = (v_1/x_d'') \sin \alpha$ flows, and if there is also a frequency divergence $s\omega_1$ due to a slip s , a further current of similar form (with $s\omega_1 t$ replacing α) occurs; in the limit, these in combination give a maximum of $2v_1/x_d''$, which may be very large. For a voltage difference Δv a switching current $i_v = \Delta v/x_d''$ will occur, demanding reactive power.

Resynchronizing. Restoring synchronous operation to a generator after it has lost synchronism following a system disturbance can be achieved without disconnecting it from the busbars, provided that certain limitations are observed. There is a maximum slip beyond which it is practically impossible to resynchronize, this maximum being determined by the ceiling voltage of the excitation system. The second most important requirement is that the excitation should be applied at a definite point in the slip cycle before the limiting swing angle is reached and with due regard to the field-circuit time-constant and the operating delay of the circuit-breaker. The conditions necessary can be established with the aid of a comparatively straightforward control loop. The conditions of asynchronous running are referred to later.

Steady-state operation

After synchronization, a generator is loaded by adjustment of the prime-mover governor to give the required output power, and of the excitation to cause the machine to take its assigned share of the reactive load. Normal operation of a generator involves maintaining a balance between the inputs to the various items of plant and their outputs. The required conditions are set by the operator, and thereafter control devices (such as the turbine governor and the boiler control loops) automatically maintain the condition. More elaborately, the sensing of the variables that determine the plant efficiency can be fed to a computer programmed to print out efficiencies and performance details on which the control engineer may take action. On-line computer control is a further step.

A factor of importance in the operation of large power-system generators is the maintenance of full loading for economic reasons. This involves steady-state stability in both the static and dynamic regions. Stable operations can be obtained by prime-mover governors and automatic voltage regulators in combination. A conventional speed governor by itself has little potential in maintaining stability, but substantial advantages can be obtained by the inclusion of acceleration sensing in governor-actuating mechanisms. The governor-a.v.r. combination forms a complex control system that can be expressed by an inter-

connection of transfer functions with feedback and feedforward, delay, dead-zone, differentiator and integrator elements.

In the chain modelling a steam turbo-generator, elements are required to represent the time response of the boiler, steam-flow control, turbine, generator, voltage regulator and system load:

Boiler. The transient response is slow, with a time-constant typically of 100 s, and for short-duration changes it may be regarded as an 'infinite steam busbar' of constant pressure and temperature.

Steam-flow control. The main element is usually a mechanical flyball governor backed by a two-stage hydraulic amplifier acting on the steam valve; it has characteristically a dead-zone of about 0.06% speed change, a fixed 'droop' which is higher for full-load high-efficiency sets and smaller for low-load sets, and a somewhat inflexible performance, but is reliable. The disadvantages are overcome by electrical governing, and a number of further controls can be added. It can be made to act on both high- and intermediate-pressure steam valves to give a faster mechanical response, provide acceleration feedback to aid system damping, and incorporate a load-control loop with integral action to zero the steady-state error for accurate setting. The most important control signals for dealing with load disturbances are the speed error and its derivative (i.e. rotor acceleration). For large disturbances the inherent mechanical limits on the rate of valve opening or closure mean that the acceleration signal has a dominant control function.

Turbine. This is an extremely complicated unit to model. It may be represented by a system of chambers interconnected by ducts. A chamber has a significant volume, in which a change in boundary conditions (such as mass-flow) produces time-dependent changes in the fluid conditions. A duct introduces modifications to the steam flow by friction and pressure-drop. Chamber conditions are found in large-diameter piping as well as in the main turbine cylinders and in the condenser, while pipes have also a partly ductlike nature, and valves can be modelled as pipes with 'nozzle' restrictions.

Generator. The generator model is based on the Park concept. For small-perturbation studies in which the current can be expressed as $i = i_0 + \Delta i$, the equations (Sect. 7.5) can be linearized to the following, stated in per-unit terms of voltage, power and flux.

$$\text{Axis voltages: } \Delta v_d = \Delta v_d'' + x_q'' \Delta i_q$$

$$\Delta v_q = \Delta v_q'' - x_d'' \Delta i_d$$

$$p(\Delta v_d'') = [(x_q' - x_q'') \Delta i_q - \Delta v_d''] / \tau_{q0}''$$

$$p(\Delta v_q'') = [\Delta \psi - (x_d' - x_d'') \Delta i_d - \Delta v_q''] / \tau_{d0}''$$

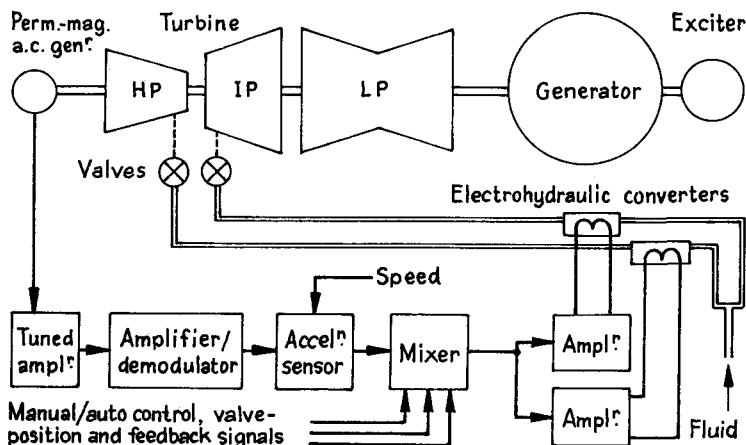
$$p(\Delta \psi) = [\Delta v_f - (x_d - x_d') \Delta i_d - \Delta \psi] / \tau_{d0}'$$

$$\text{Power: } \Delta P_e = v_{d0} \Delta i_d + i_{d0} \Delta v_d + v_{q0} \Delta i_q + i_{q0} \Delta v_q$$

$$\text{Terminal voltage: } \Delta v_1 = [v_{d0} \Delta v_d + v_{q0} \Delta v_q] / v_0$$

Voltage regulator. The field voltage v_f from the exciter system is controlled by the automatic voltage regulator, which gives a control signal to an amplifier to provide the excitation required. Commonly the equipment comprises an a.c. exciter, rectifier, magnetic and/or rotary amplifiers and sensors. The terminal voltage of the turbo-generator provides the main control signal, which may include line-drop compensation and rotor-angle elements. The difference between the terminal and the reference voltage can be taken as acting through a single time-lag network with an exciter-stability feedback.

Load. The network influences the loading conditions in a complex manner. For the simplified case of a generator connected through a tie-line of reactance x_e to an infinite busbar of voltage V then the power flow is dependent upon the overall load angle, and changes of d- and q-axis generator current can be related to the d- and q-axis generator voltages, the load angle and the voltage of the infinite busbar.



10.41 Electronic governing system for turbo-generator.

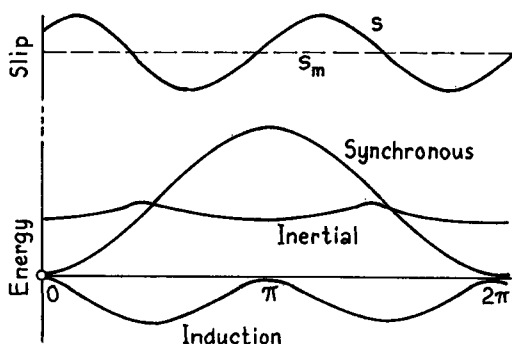
Fig. 10.41 shows a schematic diagram of an electronic governing system. A speed-sensing unit is driven from the turbine to give a frequency signal, which is then processed by means of a tuned circuit to provide a speed-error signal indicating the magnitude and sense of the deviation of the turbine from rated speed. To this signal is added a reference voltage indicating the load required from the set at nominal system frequency (e.g. 50.00 Hz) and the summed signal positions of the four electrohydraulically controlled high-pressure governor valves. Each valve-position servo includes a signal and a power amplifier. The output from the latter energizes the coil of an electrohydraulic valve to admit oil to the relay stage to position its governor or interceptor valve. Position transducers provide feedback signals to close the position-servo loops. Transient feedback terms provide appropriate damping of the fast response obtained. Acceleration sensors are included to improve control action and to limit speed rise resulting from load rejection.

Before synchronizing, the governor acts as a speed controller to stabilize the speed of the set. After synchronization, the system frequency is dependent upon the mean speed gain of all the working turbine governors. Variation of load with frequency depends on the gain of the speed-loop control (i.e. the 'droop'). With the electronic governing the speed change between no load and full load can be as low as 1%, compared with 4% for a mechanical governor. In purely mechanical governing the time-lags in the hydraulic relays are long enough to have a major effect on the generator speed which may swing to 5% above and then 20% below normal in the 50 s following a large disturbance. Such corresponding frequency swings could adversely affect the behaviour of station auxiliaries motor-driven from the generator transformer.

Asynchronous running

Turbo-generators may occasionally be required to operate asynchronously, as for example in the process of self-synchronizing. Further, with the rise in rating it becomes increasingly desirable, in the event of a machine losing excitation or desynchronizing for some other reason, that it should be able to run asynchronously at reduced output until normal running can be restored, in order to avoid the lengthy outage entailed by disconnection and re-loading. A basic problem in assessing the asynchronous performance is that of analysing the eddy-current phenomena in the saturable solid steel of the rotor body, for the effective torque depends on the total rotor I^2R loss, only part of which occurs in the field winding. Tests have shown that as the slip increases there is an increase of generated power (as an induction generator) up to a peak at typically 0.01 p.u. slip, but the peak torque depends on the stator voltage and the rotor-field connections, as might be expected. If, when operating at full load, the machine loses its d.c. excitation, it runs asynchronously with a slightly lower load but with a stator current much higher than rated value; however, the stator current level can be reduced to about normal if the load is quickly reduced to one-half normal value. Such operation should be possible for quite prolonged periods unless it gives rise to unacceptable voltage reduction on the system, as it may do if the rating of the machine is an appreciable fraction of the system rating.

The mean point of *sustained* asynchronous operation is determined by the trend of the mean torque/slip characteristics and the combined turbine-governor torque/speed characteristics. The point of intersection of these represents the mean operating point about which the speed and torque pulsate. With a rise in speed, a quantity Δw of kinetic energy is absorbed by the rotor which varies over the slip cycle and is given by the energies contributed by (i) the net induction torque, (ii) the synchronizing torque present if the rotor is d.c. excited, and a quantity dependent on the inertia J and given by $J(\frac{1}{2}s_m^2 - s_m)$ where s_m is the mean slip. A typical relation between these is given to a base of load angle δ in Fig. 10.42. If, at any instant, the change Δw in kinetic energy is zero, the rotor speed becomes equal to the synchronous speed and the machine attains a condition in which it may pull in to a synchronous condition. Malik and Cory [80] point out that the torque contribution (i) and the inertia are design parameters, but the synchronizing torque (ii) can be manipulated by means of excitation and voltage-regulator control. By applying a



10.42 Energy and slip during asynchronous operation.

sufficiently large excitation at a suitable instant in the slip cycle, it is possible to synchronize the machine successfully. Account must, however, be taken of the presence of nonlinear factors of importance, in particular the response of the excitation current to the exciter voltage, and the effect on the induction and synchronous torques of the very rapid rate of change of speed over the slip cycle.

Protection

A generator requires to be protected against the consequences of overspeed, overvoltage, overcurrent, unbalanced load, insulation failure, bearing failure, prime-mover failure, excessive coolant temperature, internal hot-spots, surges, and fire. The Table summarizes the protection means commonly employed.

Equipment for the indication of shaft distortion and rotor expansion in large steam turbo-generators has been developed. A distortion that caused the mass-centre of a 100 Mg rotor to deviate only 0.1 mm from the axis of rotation would, at 3 000 r/min, produce a centrifugal force equal to the weight of the rotor. Uneven heating or temporary misalignment may cause such a displacement; and the length of a turbine-generator set may increase by 40–50 mm between the cold-start and running-temperature conditions. Rotor eccentricity is detected by methods in which the amplitude or the frequency of a basic carrier signal is modulated by pick-up from inductors located close to the rotor shaft, and the modulated signal fed to recorders, indicators and alarms.

Rectifiers employed in excitation systems are subjected to the most onerous conditions by a short circuit across the d.c. side, and fuse protection must be provided. During a short circuit on the synchronous generator, the rotor current is positive during the transient so that the rectifier is not called upon to perform voltage blocking: but it carries a large (though decaying) current surge. Rotor slot wedges, damper bars and other metal parts of the rotor in the neighbourhood of the airgap magnetically screen the field winding. High-speed d.c. circuit-breakers for the field current, operating in little more than 30 ms, have been

Generator Protection

Fault	Method
Overspeed	Mechanical or electrical sensor
Overvoltage	Overvoltage relay (H)
Overcurrent	Overcurrent relay or differential protection
Load unbalance	Unbalance-detecting network with relay or alarm
Stator insulation failure	Differential protection
Rotor insulation failure	Alarm from earth-fault detector
Loss of bearing oil-pressure	(T) Automatic transfer to emergency oil feed (H) Standby oil pump
Loss of vacuum	(T) Vacuum load-suppression sensor
Prime-mover failure	(T) Reverse-power relay
Bearing over-temperature	Temperature sensor
Surges	Supply network protection sufficient
(T) Turbo-generator (H) Hydro-generator	

used in generators up to a 500 MW rating, with series high-rupturing-capacity fuses in the diode circuits as backup. For machines of larger rating and currents of the order of 5 kA to be broken, circuit-breakers are impracticable, and reliance for protection has to be placed on the diode fuses.

Pumped-storage plant

Where the terrain is suitable, spare generating capability on power systems at times of light load may be employed in driving hydraulic pumps to raise water to a high level and store it in a reservoir. At times of peak system load the water is used to drive hydro-generators to assist the base-load stations. Operational flexibility is an important feature. The plant can be started and fully loaded in the generating mode in a few minutes, and the output is readily adjusted to system demands. Similarly, a sudden reduction of system load releases surplus energy from steam and nuclear stations on the system which can be absorbed by pumping. The pumped-storage plant can also be employed for synchronous compensation.

In high-head installations it is possible to combine the hydro pump and turbine into a single unit, to which a synchronous machine can be attached to work in either generator or motor modes as required. There are special problems associated with starting the motor/pump mode. It is hydraulically necessary for the hydro runner to be below the outflow level to provide a suction head of 15–30 m. With the main inlet valve closed, the braking torque developed by a runner is high, but when dewatered it falls to less than 0.01 p.u. Hence it is necessary for starting to depress the water level temporarily, using compressed air. The static friction torque at breakaway is between 0.05

and 0.15 p.u. of normal, but this can be reduced by pumped high-pressure bearing lubrication. With the starting-torque demand so reduced, various starting methods (Sect. 10.11) such as asynchronous, synchronous, pony motor or pony turbine, or low-frequency rectifier/inverter systems are feasible, as discussed by Hammons and Loughran [84].

For most reversible hydro pump/turbine machines the maximum pumping head is greater than the head for turbine operation, and the optimum pumping speed is higher than for driving. Further, the utilization of pumped-storage water in some schemes involves a wide range of head, and the turbine/pump units would have an unacceptably low efficiency if they were connected to normal fixed-speed motor/generator machines. It is therefore desirable to adopt pole-change synchronous machines. There are several solutions to this problem. One is to use for the salient-pole rotor say 16 poles, uniformly spaced but with shoes arranged asymmetrically. When all are excited in successively NSN . . . polarity, a 16-pole field is produced in the gap with strong harmonics; for a 12-pole system certain pole excitations are reversed and others are left dead to produce the required pole-number; again the gap field has strong harmonics. The stator winding is pole-changed in ways similar to those used in induction machines, with due regard to harmonic suppression.

Where a reversible hydro machine is impracticable, and the pump and turbine are separate, the turbine can be used to start the pump motor, and the electrical design is not affected. With a reversible hydro units and a single synchronous machine, the latter must be self-starting.

10.11 Motor control

The inherent advantages of a synchronous motor are its high efficiency, high or leading power factor, and ability to retain synchronous running during voltage depressions. The motor can be built for speeds from the highest (e.g. 3 000 r/min at 50 Hz) down to the lowest likely to be required in practice and for which the poor performance of induction motors makes them less attractive, quite apart from their uneconomically low power factor. Constructionally, synchronous motors may have cylindrical rotors with embedded exciting windings as in a turbo-generator, a form suitable for high-speed 2-pole machines; solid poles (4 or more) also suitable for the higher range of speeds; salient poles with pole-face bars inserted for starting and damping and either confined to the pole-shoes or extended between the poles to form a cage winding; or may be built as cylindrical- or salient-pole-rotor synchronous-induction machines. As a *purely* synchronous machine develops a torque only at synchronous speed, a chief development has been in methods of introducing means for self-starting.

The methods of excitation described in Sect. 10.9 are equally applicable to synchronous motors, but almost all modern motors are provided with brushless excitation. The application is, however, not so simple as it first appears, because of the behaviour of the field circuit during the self-starting process.

Starting

Although a synchronous motor may be run up by use of a convenient 'pony'

motor attached to its shaft, self-starting is by far the most usual method: it employs the asynchronous (induction) torque developed in a cage winding, which may have deep bars, or sometimes only in solid salient-pole-shoes if the starting duty is light. The d.c. field winding could be closed to provide an effect comparable to that in a double-cage induction machine, but the stator current is increased thereby without much gain in starting torque; it is therefore usual to keep the field circuit open until the motor approaches synchronous speed. A characteristic feature of the starting torque/speed relation is a pronounced 'dip' at one-half synchronous speed, the severity of which is a function of the stator- and cage-winding resistances and of the saliency. The following brief descriptions list the most common starting methods.

Direct-switching. Motors with direct-on-line starting and with a limit imposed on the starting current must be designed to have an appropriate impedance at standstill as well as suitable acceleration and synchronizing characteristics. The limiting impedance may, however, be incorporated in the supply system, as when a motor is supplied from a step-down transformer. In generating stations, motors driving auxiliaries may be allowed starting peaks up to 6 or 7 times full-load current as, in spite of the negligible system impedance in such cases, the power level of the generating plant is very high.

Star-delta switching. This method is the same as for induction motors. It is suitable only for low-torque applications.

Special-connection switching. Various methods are available for reducing the starting current (at the expense of also reducing the starting torque). Machines with parallel stator windings may have these series-connected for starting; or the stator winding may be tapped so as to raise its impedance.

Reduced-voltage switching. This is effectively an increase of the impedance at starting by the inclusion of inductors or auto-transformers. The reactance of an auto-transformer has an important influence on synchronizing. Where a large machine is concerned and has a main transformer allocated to it, a standard tap-changer may be employed for voltage reduction at starting.

Rotor resistance. This method can be used with synchronous-induction motors. The rheostats are balanced to avoid a partial single-phasing effect which might cause the machine to 'lock' at one-half synchronous speed.

Synchronous. In particular installations, such as electrical ship-propulsion and pumped-storage stations, a motor may be connected to a stationary generator. If both generator and motor fields are excited and the prime-mover is started, the two machines pull into step at a low frequency and accelerate together up to normal running speed. Another method of achieving synchronous starting is to supply the motor from a static frequency converter, such as has been developed for variable-speed induction motors. The operating features of interest are the effects on the supply network introduced by harmonics and reactive power demand. The harmonics can be reduced by adopting a 12-pulse converter.

Pony-motor starting is suitable for easy duty against little or no mechanical load, and for cases where disturbance to the supply network must be strictly limited. The direct-switching method is simple and minimizes switchgear, but the starting current is large (e.g. 6 times full-load current) and of very low

power factor. The increasing capability of supply networks and the usual tolerance of occasional voltage dips have made current limitation of less significance, with the result that synchronous motors are commonly switched direct-on-line.

Run-up

A salient-pole synchronous motor runs up in a manner basically similar to that in an induction motor, with the cage and field windings developing an induction torque, but the conditions are complicated by the magnetic and electric asymmetry introduced by the field winding. The 3-phase stator winding, when energized, produces a synchronous travelling-wave field. The rotor, running with a slip s varying from $s = 1$ at standstill to a value approaching zero immediately before it synchronizes, presents to the stator field a fluctuating gap reluctance and consequently a pulsating torque is developed. In deriving the torque/speed relation only the mean torque is of significance (apart from the possibility of interaction with some load-torque fluctuation as might occur in a reciprocating compressor).

Dynamic circuit analysis. Consider a 2-pole motor with a salient-pole member (rotor) carrying a field winding F defining the d -axis, and also an actual or equivalent cage forming separate short-circuited axis coils KD and KQ . The other member (stator) has a symmetrical 3-phase winding ABC energized by balanced voltages of angular frequency ω_1 . In accordance with two-axis representation the voltages and currents of the phase windings are transformed into equivalents in axis coils D and Q , as discussed in Sect. 7.5.

In a synchronous machine in the synchronous steady-state condition the windings D and Q have direct applied voltages and carry constant direct currents. During run-up, however, the machine runs with a slip s varying from $s = 1$ to $s = 0$. It can be inferred that the voltages and currents in D and Q , and also those induced in KD , KQ and F (if it is closed) must alternate at slip frequency $s\omega_1$. These conditions can be inserted into the impedance matrix of Fig. 7.11. But we have the difficulty that the angular speed $\omega_r = \omega_1(1 - s)$ is a variable. It is usual to assume the motor to operate in the steady state at a series of constant slips and to assemble the results to give the torque/slip relation. Initial switching transients are ignored as too brief to have any significant effect. The instantaneous D and Q applied voltages, which are equal in magnitude and phase displaced by $\frac{1}{2}\pi$ rad in time, are therefore

$$v_d = v_m \cos \omega t \quad \text{and} \quad v_q = -v_m \sin \omega t$$

and ignoring stator resistance they can be equated to the d - and q -axis linkages by

$$v_d = p \psi_d + (1 - s) \omega_1 \psi_q \quad \text{and} \quad v_q = p \psi_q - (1 - s) \omega_1 \psi_d$$

and the instantaneous electromagnetic torque is given by

$$M = \frac{1}{2} \omega_1 [\psi_q i_d - \psi_d i_q]$$

The two voltage equations taken simultaneously lead to the result that $\omega_1 \psi_d = v_m \sin \omega_1 t$ and $\omega_1 \psi_q = v_m \cos \omega_1 t$. As for the axis currents i_d and i_q , these

are functions of the stator reactance and the reflected effects of rotor resistance and reactance. Let the resulting effective impedances of the D and Q coils be z_d and z_q having respective phase angles α and β . Then the two currents are

$$i_d = (v_m/z_d) \cos(s\omega_1 t - \alpha) \quad \text{and} \quad i_q = -(v_m/z_q) \sin(s\omega_1 t - \beta)$$

The torque can now be found: its instantaneous value is

$$M = \frac{1}{2} v_m^2 \left[(1/z_d) \{ \cos \alpha + \cos(2s\omega_1 t - \alpha) \} \right. \\ \left. + (1/z_q) \{ \cos \beta - \cos(2s\omega_1 t - \beta) \} \right]$$

indicating that the d- and q-axis torque contributions are separate, and that each has a pulsation at twice slip frequency. The physical explanation in simple terms is as follows. The stator develops a travelling-wave field with its N-S axis rotating at angular speed ω_1 , which overtakes the more slowly rotating rotor at a relative speed $s\omega_1$. As a stator pole approaches a rotor salient pole 'from behind', it exerts a retarding reluctance torque; and as it overtakes and passes the rotor pole it exerts an accelerating reluctance torque. As this action takes place for each stator pole, the total torque fluctuates at frequency $2s\omega_1$.

For most purposes it suffices to take the mean torque, for which the double-slip-frequency terms vanish to give

$$M = \frac{1}{2} V_1^2 \operatorname{Re} [1/z_d + (1/z_q)]$$

where the two axis impedances are here expressed in complex operator terms. The equation states, as might be expected, that the torque in terms of synchronous power is the same as the rotor power input, the machine operating as an asymmetrical induction machine. The complex impedances are expressed as functions of the axis synchronous reactances and time-constants:

$$z_d = jx_{sd} \frac{(1 + js\omega_1 \tau_d') (1 + js\omega_1 \tau_d'')}{(1 + js\omega_1 \tau_{d0}') (1 + js\omega_1 \tau_{d0}'')}$$

$$z_q = jx_{sq} \frac{(1 + js\omega_1 \tau_q'')}{(1 + js\omega_1 \tau_{q0}'')}$$

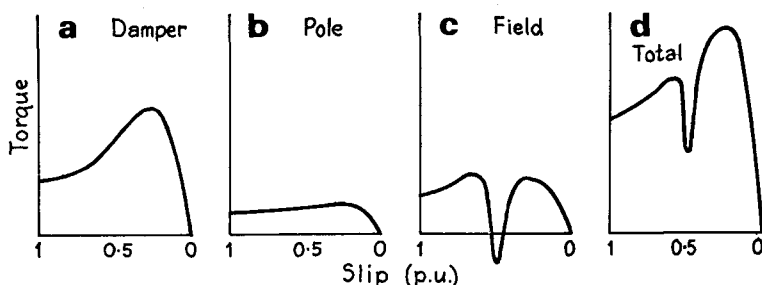
The d-axis time-constants must include the effects of the field circuit connection and external resistance. The stator resistance has been neglected for simplicity.

As in the case of the polyphase induction motor, the resistances and reactances of the rotor circuits play a major part in determining the asynchronous performance of a salient-pole synchronous motor. But the problem is more complicated in that there are three main rotor circuits concerned, namely the q-axis damping winding and the d-axis field and damping windings. Each of these may be represented to a reasonable approximation by resistance and inductance.

The two most common rotor constructional forms have (i) solid-steel poles (or pole-shoes), (ii) fully laminated poles with a cage winding in the pole-face.

In (i) the torque during starting results from the substantial eddy-current loss in the pole-shoes at slip frequencies, while in (ii) these losses are confined to the damper bars. In both cases the heat produced has to be effectively dissipated to avoid undue temperature rise. The pole-shoes or damper bars may be connected from pole to pole by interpolar links or end-rings, and this may have a substantial effect on the q-axis impedance and on the asynchronous performance.

Fig. 10.43 shows a typical set of component *mean* torques in a synchronous



10.43 Motor starting torque components.

motor during run-up, with the field winding closed through a resistor. The damper-winding torque (*a*) is characteristic of a cage induction motor; the contribution of the solid pole-shoes and poles in (*b*) has a similar form, but the eddy-current paths are much more complex and are affected by local saturation; and the torque (*c*) due to the field winding shows a saliency effect strong enough to produce the characteristic torque-reversal at one-half speed. The total torque (*d*) shows that there is still a dip at half speed. The relative magnitude of the torque components is naturally dependent on the design of the machine, an intricate process. A method of determining starting performance from design data is given in detail by Widger and Adkins [86].

Brushless excitation. The rectifiers in the field circuit prevent bi-directional current flow. The pulsating unidirectional field current may produce sufficient 'generating' torque to make run-up impossible, and high inverse voltages appear across the rectifier bridge. These effects are alleviated by shunting a resistor across the field winding, but can be prevented only by switching out the rectifier during the starting process. Various methods have been devised for this purpose. A permanently shunted field winding may be disconnected from the rectifier by a mechanically operated switch, a reliable method but one which involves random synchronizing when the machine reaches a low slip. Alternatively the switch may take the form of a thyristor, retained in the off position until excitation is applied, in which case 'point-on-wave' synchronizing is possible. In more elaborate methods the rectifier bridge is short-circuited during run-up by reversed thyristors fired from the induced field e.m.f. through zener diodes. The behaviour of motors with field rectifiers has been investigated by Chalmers and Richardson [85].

Synchronizing

A large synchronous motor or compensator, started against inertia by a separate pony motor, can be synchronized as for a generator. Alternatively the pony- and main-motor stator windings may be connected in series for starting, the initial voltage division between them being typically 0.8 to 0.2 p.u.; when the speed of the set approaches the synchronous, the main motor excitation is applied, it absorbs most of the applied voltage, and pulls into step. Subsequently the pony motor (an induction motor with a very low rotor resistance, or with one pole-pair fewer than the main machine, or even with a solid unwound rotor) is short-circuited to neutral.

Generally a synchronous motor is self-starting and direct-switched. The induction torque for acceleration falls to zero at a small value of slip. If excitation is now applied to the rotor d.c. winding, a synchronizing torque is developed. The machine has to be designed so that the limit of induction torque is at a slip small enough for the synchronizing torque to pull the machine into step against the load torque; in some cases it may be necessary to reduce the slip by supplying the motor at overvoltage derived from an autotransformer to prevent pole-slipping and the consequent fluctuations of stator current.

The synchronizing conditions involve several torque components. The synchronous torque is a function of the load angle δ , and its growth following initiation of the d.c. excitation is exponential. The induction torque is proportional to the slip $s = (d\delta/dt)$, the inertial torque for a machine with p pole pairs and inertia J is $(J/p)(d^2\delta/dt^2)$, and the dead-load torque can be considered as constant. The equation of motion during the synchronizing process therefore has the form

$$K_1(d^2\delta/dt^2) + K_2(d\delta/dt) + K_3 \cdot f(t) \sin \delta = M_l$$

where M_l is the load torque and $f(t)$ expresses the growth of the field excitation. The solution is analytically difficult, but can be computed in numerical cases.

When the d.c. excitation is switched on, the pulsating synchronous torque is superimposed on the steady induction torque. The motor may pull into step almost immediately: it may continue to run as an induction motor but with speed and current fluctuations: it may revolve several times and then pull into synchronism. The required condition is to have the motor synchronize on its first swing through the steady-state operating position. The inertia of the rotor and its connected load is a very important factor. The angle δ obtaining at the instant of switching in the d.c. excitation also affects the pulling into step. If the angle is such that the machine generates (i.e. if δ is an angle corresponding to a forward position of the rotor) the speed decreases because initially the generated energy is abstracted from the kinetic energy of the rotating masses. The rotor swings into the motoring region, upon which an additional motoring torque is produced to speed it up again.

The field excitation is an important factor as determining the maximum synchronizing torque; the larger the field current, the more certain is the pulling into step.

Operation

Normal conditions. A synchronous motor is often run with constant excitation, in which case the power factor varies with the mechanical load and becomes low and leading on no load. Excitation compounding may be provided by the method shown in Fig. 10.36 if the uncontrolled power-factor condition has to be avoided.

Asynchronous running. Apart from loss of synchronous operation on fault or excessive load, synchronous motors are sometimes required to run asynchronously as part of the normal duty-cycle. In electric ship-propulsion, for example they may be required to cover the speed range between full-ahead and full-astern, and deliver full-load torque to the screws during the reversal. Special design of the damping is necessary to yield a roughly constant ratio of torque to stator current. The design is intricate, as it is necessary to investigate the detailed linkage in symmetrical pairs of damper bars and to assess the heating of the bars and pole-shoes. A 'brushless' form of excitation is not likely to be practicable in these circumstances.

Unexcited motor. A synchronous motor for loads below 5 kW and for which the power factor is unimportant can be constructed with rotors resembling cage induction machines but with part of the slotting cut away to form a suitable degree of saliency. They can run up as induction motors and self-synchronize on reluctance torque, with the advantage that no d.c. field excitation is needed.

Two-speed motor. A method of pole-changing was briefly described for a pumped-storage generator/motor in Sect. 10.10. Rawcliffe and his co-workers [87] have devised a scheme for two-speed salient-pole motors in which the field pole-change is similar, but the stator winding is arranged for pole-amplitude modulation as discussed in Sect. 8.15. Only one stator winding is used, and the change of connections required is simple.

Protection

The standard protective requirements for a synchronous motor are:

- (i) A thermal relay giving overcurrent and phase-unbalance protection, with high-set instantaneous overcurrent and earth-fault devices.
- (ii) Undervoltage protection, which because of voltage maintenance by the machine itself if it has a large inertia may require reverse-power back-up or a suitable electronic sensor.
- (iii) Overall differential (circulating-current) protection for phase-phase and phase-earth faults.
- (iv) Field-failure protection.
- (v) Overpower relay for protection against mechanical overload or failure, in the case of some kinds of load.

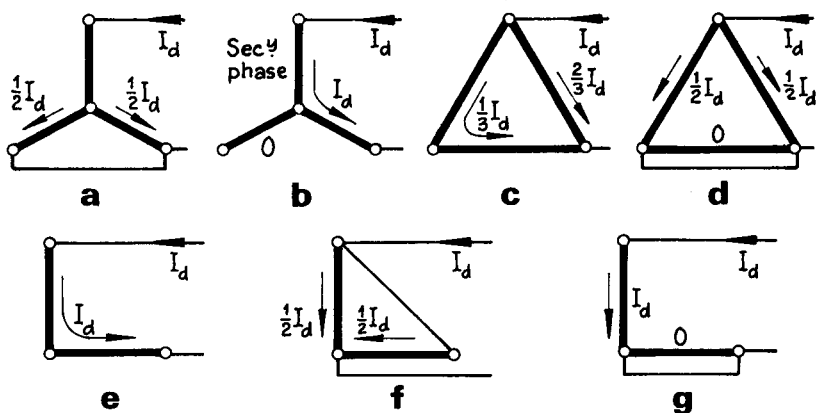
Protection of large machines may be required to operate in case of pole-slipping by tripping the motor or opening the field circuit and initiating re-synchronization, using for example a power-factor relay based on electronic logic devices. Overheating of the field system may be detected by a thermal sensor; this may have a parallel-connected inductor to increase the current in the thermal

element for protection during asynchronous running.

Where brushless excitation systems are employed, more elaborate rotor protective devices are necessary, particularly for the solid-state rectifier elements.

10.12 Synchronous-induction motor

If the rotor windings of a slip-ring induction motor are excited by a direct current, the m.m.f. distribution produces alternate N and S poles in the same way as does a 3-phase current, but the pole axes are fixed with respect to the rotor. In Fig. 10.44(a), the direct current I_d from an excitation system flows through



10.44 Synchronous-induction motor: connections for excitation.

one phase winding and then through the other two in parallel, so that the m.m.f. pattern is identical with that of a normal 3-phase excitation at the instant that the current in one phase is at its peak value. Other possible connections are also shown in Fig. 10.44. A cylindrical-rotor slip-ring induction motor can thus be readily synchronized. It is started on rotor rheostats through a change-over switch, and when a small slip has been reached, the switch is moved to provide d.c. excitation. The machine has then a starting torque that may be twice full-load torque or more, and a reasonable pull-out synchronous torque with the possibility of power-factor correction as an additional feature. If necessary the machine can be *inverted*, i.e. have the 3-phase supply connected by slip-rings to the rotor windings, with the starting rheostat and excitation system now readily connected to the stator. To cover both 'normal' and 'inverted' cases, the terms 'primary' and 'secondary' are used respectively for the 3-phase supply windings and the d.c.-excited windings.

The designer's object is to obtain the greatest d.c. m.m.f. with minimum excitation power, to distribute the I^2R loss evenly over the secondary windings, to obtain adequate damping for synchronous running, and to secure specified starting torque. The excitation m.m.f. depends mainly on the length of the air-gap: a long gap gives a stiffer machine with a larger overload capability, but the

starting is impaired and more excitation power is required. A short gap gives opposite effects, and to limit power-factor variation it may be necessary to apply compounding.

As can be inferred from the current distributions in Fig. 10.44, unbalanced secondary heating is unavoidable with a 3-phase secondary, but a 2-phase arrangement, as in (e) or (f), equalizes the heating. However there is, in (e), no closed secondary loop apart from that through the exciter, so that the damping is not as good as in (f). A reduction in exciting current can be achieved by use of a multi-turn secondary, but as a consequence the induced secondary e.m.f. at standstill is high, introducing insulation problems.

Equivalent secondary current. Connection (a), with phase currents in the ratio $1:\frac{1}{2}:\frac{1}{2}$, has I_d corresponding to the peak i_m of the equivalent alternating current, so that in r.m.s. terms $I_d = I_a/\sqrt{2}$. The Table below compares in like manner the features of the seven connections. All the 3-ph cases required the same exciter power for a given m.m.f., resistance and turns per phase. All the 2-ph connections take $12\frac{1}{2}\%$ more power and the gap flux has a somewhat greater harmonic content. Because they include a closed circuit methods (a), (f), (d) and (g) — particularly the two latter — give good damping.

Secondary connections for synchronous-induction motors

- (1) Relative d.c. excitation current for a given m.m.f.
- (2) Secondary input resistance in terms of resistance per phase;
- (3) Relative exciter voltage, (1) $\times$ (2).
- (4) Relative exciter power, (1) $\times$ (3).
- (5) Distribution of (4) between the secondary phases.
- (6) Starting qualities.
- (7) Damping qualities.
- (8) Asynchronous operation on overload after pull-out.
- (9) Notes on applications.

E = excellent; G = good; N = normal; P = poor; S = single-phase

Case	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(a)	1.41	1.50	2.12	3.0	2/0.5/0.5	N	G	G	Small machines
(b)	1.23	2.00	2.46	3.0	1.5/1.5/0	P	P	S	Steady drives
(c)	2.12	0.67	1.41	3.0	2/0.5/0.5	N	P	S	Seldom used
(d)	2.45	0.50	1.22	3.0	1.5/1.5/0	N	E	G	Modified
(e)	1.06	3.00	3.18	3.4	1.7/1.7	N	P	S	Seldom used
(f)	2.12	0.75	1.59	3.4	1.7/1.7	N	G	G	Commonly used
(g)	1.50	1.50	2.25	3.4	3.4/0	N	E	N	Patented

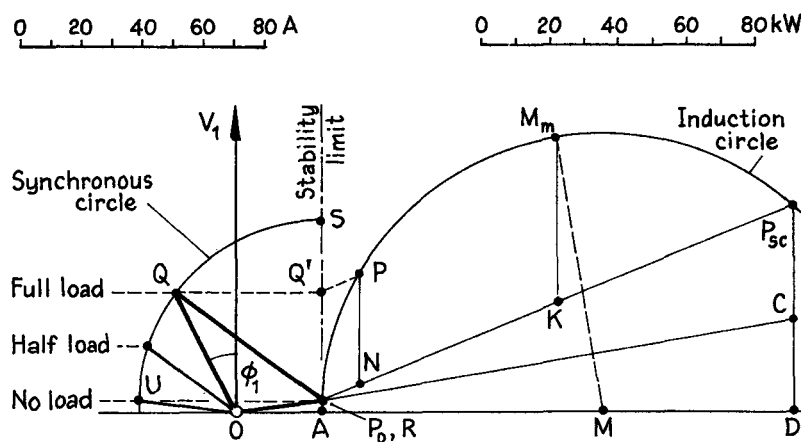
Salient-pole motor

The disadvantage of the motor described above is that if the secondary winding has a moderate standstill induced e.m.f., the number of turns is such that a large d.c. excitation at low voltage is necessary. The restriction on slot conductor area is lifted by use instead of a salient-pole construction, so combining the high

efficiency and low excitation loss of the salient-pole synchronous motor with the starting characteristics of the induction motor. The d.c. field winding is normal; and independent starting winding connected to slip-rings occupies slots in the pole-shoes. Each winding can be designed almost independently to fulfil its allotted function, with consequent improvement in synchronous running and exciter-circuit conditions.

Current diagram

When the machine operates synchronously, the locus of the stator current for constant rotor excitation is a circle, Fig. 10.6. In Fig. 10.45 the only difference



10.45 Synchronous-induction motor: current diagram.

is that OR is V_1/z_{sd} instead of V_1/x_{sd} as it is necessary to include primary resistance, and the angle V_1OR is $\arctan(x_{sd}/r)$. The impedance of an induction motor at synchronous speed is closely that of the magnetizing branches, z_m in Fig. 8.10, so that the no-load current phasor OP_0 fixes P_0 as the centre of the current circles for synchronous working. The intercept PN represents full-load induction-motor power output, and $Q'P_0$ is therefore a corresponding power for the synchronous condition. The theoretical limit of synchronous stability is the axis P_0S , which meets the voltage axis at a point remote $V_1/2r$ from O ; but as this distance for normal machines is very great, it is sufficient to make P_0S parallel to the voltage axis. The O -curves of constant power become the straight lines through Q' for full load and through P_0 for no load.

The secondary excitation is set off from P_0 as a circular locus with a radius corresponding to the a.c. equivalent of the exciting current. The primary currents are measured from O . The maximum load for synchronous operation is P_0S to scale, the associated primary current being OS . On synchronous no-load operation the primary current is OU , at a low leading power factor.

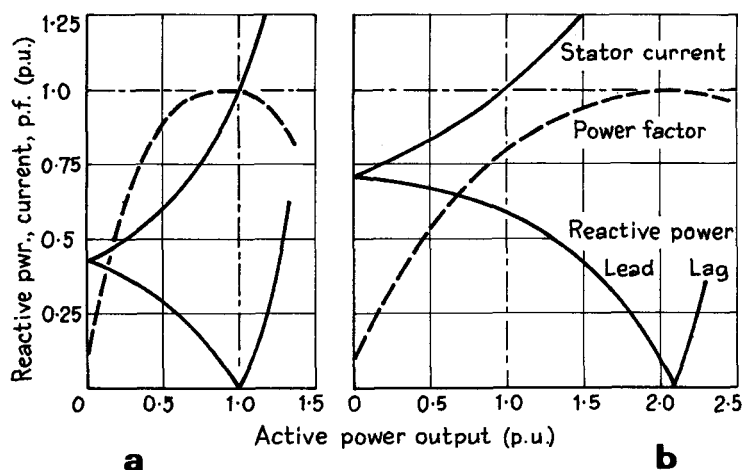
EXAMPLE 10.8: A 30 kW, 500 V, 3-phase, 50 Hz, star-connected synchronous-induction motor has, as an induction machine, a short-circuit current of 200 A at p.f. 0.35 and a no-load current of 30 A at p.f. 0.15. The ratio of effective stator/rotor turns per phase is 1/1.3, the phase resistance ratio is 1/1.2, and the rotor excitation connection is (a) in Fig. 10.44. Find the starting, stalling and pull-out torques, the rotor excitation for full load (synchronous condition) at a p.f. of 0.90 leading, and the current and power factor at one half and no load. Fig. 10.45 is drawn to scale for the machine. The induction-motor circle is drawn as described in Sect. 8.7. To obtain the synchronous motor circle, draw OQ leading the voltage axis by $\arccos 0.90 = 26^\circ$ and a horizontal no-load line P_0U ; then $Q'Q$ parallel to P_0U is displaced vertically by a scale distance of 30 kW to fix Q. Then P_0Q is the radius of the required synchronous-motor circle. The full-load torque is $Q'P_0$ to scale; the synchronous pull-out torque is P_0S ; the induction stalling torque is M_mK ; and the starting torque with direct switching is $P_{sc}C$. In per-unit terms of full-load torque, the synchronous pull-out torque is 1.74 p.u. and the stalling torque is 2.17 p.u. The starting torque, inherently 1.11 p.u., can be increased by use of a starter resistance up to a limit of 2.17 p.u. The a.c. equivalent of the rotor d.c. excitation is $P_0Q = 60$ A, so that the exciting current is $I_d = 60/\sqrt{2} = 85$ A for unity turns-ratio, and $85/1.3 = 65$ A for the turns-ratio given. Stator currents and p.f.s. (all leading) from the diagram are: full-load, 43 A at 0.90; half-load, 35 A at 0.62; no-load, 30 A at 0.15. Closer estimates are obtained if the exciter loss and the stator and rotor I^2R losses are included in the no-load loss.

Performance

Three important characteristic torques are (i) starting in the induction mode, (ii) pull-in, i.e. synchronizing from the induction to the synchronous modes, and (iii) pull-out, corresponding to maximum torque in the synchronous mode. There is naturally a close relation between (iii) and the full-load p.f.; thus a motor excited to run at a p.f. of 0.9 leading when developing full-load torque might have a pull-out torque of 1.5 p.u., but if it were designed for unity p.f. on full load, the pull-out torque and the necessary exciter rating would both be considerably reduced. Fig. 10.46 shows current, reactive power and p.f. curves typical of a machine excited to unity p.f. on rated active-power output in (a), while (b) shows the same quantities for a motor with a strong excitation. In case (b) the machine has a large overload capability, but over the range of normal load its p.f. is reduced and the I^2R loss is higher. To accommodate a load that varies widely, the exciter may be compounded by means of a current transformer and rectifier to optimize the p.f. and loss automatically.

The synchronous-induction motor is rarely built for ratings below 25 kW, as the exciter system is an additional expense and the p.f. adjustment of small units is comparatively unimportant. However, if a given drive calls for a low speed and therefore a multi-pole machine, and for a high starting torque, a synchronous-induction motor offers advantage over both plain induction and plain synchronous machines.

A 'good' induction motor makes a 'bad' synchronous motor. The former has a short gap length and a high magnetizing reactance, typically 3.0 p.u. This in



10.46 Synchronous-induction motor: performance characteristics.

(a) Full load, p.f. unity.

(b) Full load, p.f. 0.8 leading.

a synchronous machine corresponds to the synchronous reactance, and a value of this order would entail excessive change of p.f. with load and a limited pull-out torque. Reduction of the synchronous reactance implies a longer gap, and with this the induction machine is subject to pull-out torque limitation. The synchronous-induction motor in this respect demands compromise design.

10.13 Synchronous compensator

A synchronous compensator is designed to run without mechanical attachment but with its excitation so controlled as to provide a load taking a purely reactive power. Located at strategic points in a supply network, compensators can act as 3-ph inductors or capacitors. With over-excitation the machine is stable and capable of accepting leading reactive powers at almost zero p.f. When under-excited the input lags but the stability is reduced. Full rating at zero p.f. lagging can be obtained with excitation reversed, although such a mode is unusual. Compensators in large networks are equipped with automatic excitation control to maintain stable operation; the important requirements are related to the rapidity of response, which determines how quickly corrective measures can be applied to restore stability if lost. The control-loop time-lags and the reactances and time-constants of the compensator are crucial. The transient reactance should be lower and the subtransient reactance higher than those of other large synchronous machines on the system in order to mitigate voltage fluctuations caused by erratic lagging loads.

Hunting phenomena require the compensator to have a natural frequency appropriately related to its system, in respect of the natural frequency of other large machines, the position of the compensator in the network, and the voltage range over which it must work. The compensator is often arranged to main-

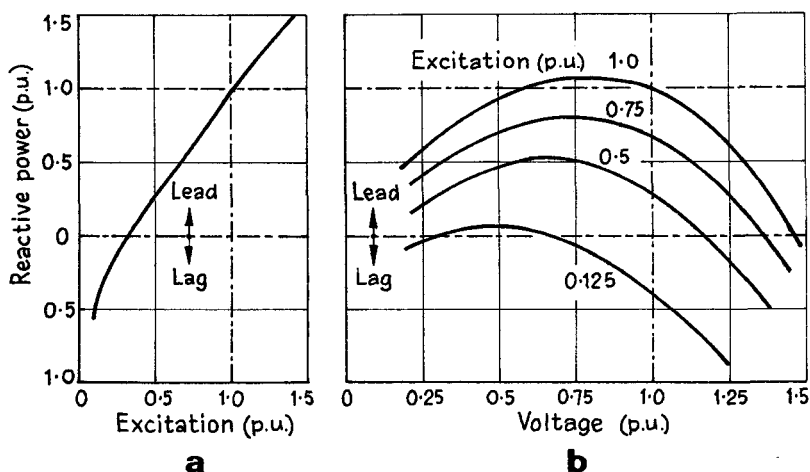
tain approximately constant the voltage at its own terminals, both at high-load times when the power factor is lagging and on light load when the system power factor is leading by reason of line-charging effects. The former requires the compensator to be over-excited, the latter under-excited.

Starting can be achieved by the same methods as those for motors. The conditions, however, are very different. The starting and synchronizing torques are low; but as a compensator, rated typically at 50 MVA, may represent a large fraction of the system rating, even small per-unit demands may be significant. Again, the machine may be required to start when the system is heavily loaded with lagging reactive power, or when the voltage is above or below normal, or even when the line is 'dead'.

The speed of a compensator is selected to give the cheapest machine that will fulfil the specification, which is influenced by the frequency of the system and the possible overspeeds and frequencies that may occur with hydro-generators. Cylindrical-rotor 2- and 4-pole designs are costly, and 6- or 8-pole designs may be preferred on this count. The former pair give a long-core small-diameter shape to the rotor, while with the latter the diameter is larger and the length smaller. Variation of dimensions, particularly of core length, affects the values obtainable of leakage, transient and subtransient reactance, and it also has an influence on the losses, more particularly those in windage and friction.

The service duty of a compensator is fairly easy as there is no torque output demanded. Machines can be built for mobile and out-door situation. Machines on a permanent site can be hydrogen-cooled, the absence of a shaft extension simplifying the achievement of gas-tightness. The external heat-exchangers can be integral with the frame.

Fig. 10.47 shows typical compensator characteristics. At (a) is the single V-curve for constant voltage operation. In this case 1.0 p.u. excitation gives 1.0



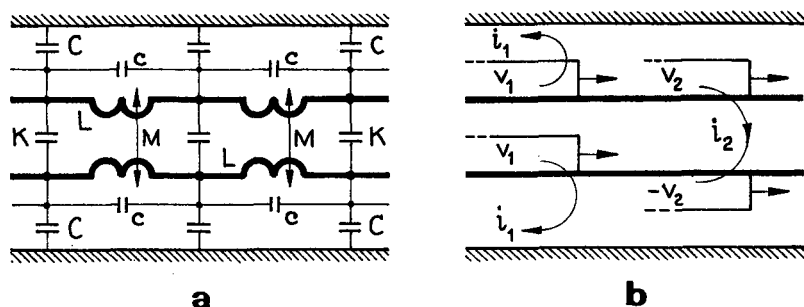
10.47 Synchronous compensator: characteristics.

p.u. of full rated reactive leading power. At about 0.3 p.u. excitation the current falls to a very small active value corresponding to losses other than those due to I^2R . At lower excitations the machine acts as a synchronous inductor. The maximum lagging reactive power may be limited to about 0.3 p.u. to avoid instability on low (or reversed) excitation. The curves (b) refer to performance at constant excitation but varying terminal voltage.

10.14 Surge voltages

The impact of a high-voltage surge on a synchronous machine has an effect comparable with that on a transformer; but a stator winding is structurally more complex, being an elaborate arrangement of coils insulated with material of high permittivity and embedded for about one-half of their length in iron-bounded slots. With the usual two-layer winding there will be at least two conductors, conductively separate but coupled both capacitively through their insulation and inductively by their mutual slot and overhang magnetic flux.

Consider a bar-type stator winding constructed with single-turn coils so that each slot has two conductors. Ignoring the discontinuity introduced by the end-windings the contents of one slot may be approximately represented by the equivalent circuit of Fig. 10.48(a). The self-inductances L of the bars have



10.48 Surge voltage in stator winding.

mutual coupling M , and there are mutual capacitances K between conductors, C to earth, and c between coils in adjacent slots due to their proximity in the overhang. These parameters are expressed per unit length of conductor. From the differential equations relating voltages and currents in this network, Robinson [62] found that the necessary conditions are satisfied by two sets of traveling waves, Fig. 10.48(b). The first set comprises a pair of equal voltages v_1 accompanied by currents i_1 associated with the 'transmission-line' system formed by a conductor and the earthed slot-wall. The second set consists of a pair of equal but oppositely-polarized voltages v_2 with currents i_2 . The respective currents and voltages are related by the surge impedances presented to them

$$Z_{01} = v_1/i_1 = \sqrt{[(L + M)/C]} \quad \text{and} \quad Z_{02} = v_2/i_2 = \sqrt{[(L - M)/(C + 2K)]}.$$

The velocities of propagation also differ. They are

$$u_1 = 1/\sqrt{[(L + M)C]} \quad \text{and} \quad u_2 = 1/\sqrt{[(L - M)(C + 2K)]}.$$

There is an upper limit of the propagation of travelling waves, however, due to the series capacitance c which resonates with $(L + M)$ to form a rejector network. If, therefore, a step-function surge is analysed into a constant term plus a frequency spectrum, only those components below the frequency of rejector resonance can pass through the winding as travelling waves. (The effect of this limitation is substantially to slope the wavefront to a finite initial steepness, and to make the steepness progressively less as the waves advance into the winding by slowing down the higher-frequency components.) The upper-frequency components in the spectrum, which are rejected, form an additional standing voltage component distributed substantially in accordance with the capacitances of the equivalent network: this leads to a high voltage-gradient in the turns of the stator winding near the line terminal.

Where the winding has two or more turns per coil, high transient voltages occur across the comparatively weak insulation of the adjacent turns, intensified by the comparatively large ratio C/K .

With considerable simplification a synchronous machine may, from the surge viewpoint, be represented as a cable with the appropriate surge impedance and propagation velocity. The surge impedance varies from about 600 Ω down to 50 Ω for large turbo- and hydro-generators, and the surge velocity is of the order of 20 m/ μ s.

Generator-transformer units

Surge voltages may be impressed on the terminals of a generator by transfer through the associated transformer from the h.v. transmission system, though few generator insulation failures can be attributed to transferred surges, the magnitude of which is small compared with the line surge voltage for generators of rating below 500 MW. The transfer mechanism is considered by White [63] to be partly inductive and partly capacitive, and he gives analyses with the generator represented by an LC network on the assumption that the steepness of the h.v. line surge has been considerably reduced by its passage through the transformer. The *inductive* transfer is based on the concept that, when loaded by terminal LC circuits, the transformer l.v. winding may be regarded as a voltage source with internal leakage inductance, and the transferred surge voltage sets up an oscillation of amplitude shared by division between the transformer and generator inductances. The *capacitive* transfer is initially between the h.v. and l.v. sides of the transformer, appearing as a voltage pulse of the same polarity as the line-surge voltage and transferring to the generator terminals through the inductance of the l.v. winding. Two factors contribute to suppressing the capacitive transfer. First, with normal h.v./l.v. capacitance (typically 1–2 nF) feeding into a low generator surge impedance, the decay time of the initial transferred voltage pulse is of the order of 0.1 μ s and therefore short compared with the line-surge rise time: thus the full theoretical maximum capacitive voltage (limited in any case by the ratio of h.v.—l.v. and l.v.—earth

capacitance) can never be fully developed. Second, the l.v. winding inductance and resistance combine to delay the build-up of transferred current by a time-constant of about $10\ \mu\text{s}$, so that again the development of the theoretically possible voltage at the generator terminals is inhibited. Hickling and Winder [64] give the results of practical surge transfer tests, which demonstrate that the mechanism of surge-voltage transfer is predominantly inductive.

10.15 Testing

Small and medium-rated synchronous motors offer no difficulty in testing, but large generators (up to 1 000 MW) and compensators cannot be direct-loaded, and indirect methods are accepted. The primary use of works testing equipment is to prove that each machine tested has the required performance in respect of load characteristics, loss and temperature-rise, and excitation. Works testing never involves the supply and dissipation of the rated power of a large machine, as this is quite impracticable: even the losses alone may be several megawatts.

In a *back-to-back* test, two identical machines are coupled both mechanically and electrically, and by suitable angular displacement at the mechanical coupling it can be arranged that both machines work at rated voltage and current to provide a thermal test under load conditions. The opportunity for such a test is rare, but it may be set up for research purposes to confirm the reliability of the simpler routine methods.

In a *reactive power* test two machines are connected together electrically only; they may differ in every way except in working voltage. By adjustment of the field excitations, rated current at a power factor near zero can be circulated between the machines. Driving power can be supplied electrically or by a motor coupled to either machine. Inductive reactors can be used to make up any significant difference in the ratings of the tested machines.

Separate *open-* and *short-circuit* tests are made in most cases. In each, only part of the full-load loss occurs: on o.c. mainly the core loss, on s.c. mainly the I^2R loss. As these losses occur in different parts of the machine it becomes a matter of careful estimate backed by test experience to deduce the heating that will result on service load. Determination of excitation characteristics follows from the methods adopted for heating tests. For accuracy a zero-power-factor point for normal voltage and current is needed besides the data from the o.c.c. and s.c.c.

Turbo-generators

These machines do not readily develop adequate torque from an a.c. supply to start against bearing friction, and they are usually coupled to a driving motor (of rating up to 5 MW) and gear-box or other method of speed control to deal with overspeed. For tests within the voltage range of the loading machine, the turbo-generator is connected to a group of inductors and brought up to speed (unexcited). On applying field excitation, the terminal voltage and reactive current are built up, and the turbo-generator synchronized with a motor-generator for a.c. driving. The starting motor is then disconnected. The motor-generator makes up any deficiency in the reactive power taken by the inductors,

and its power input provides the basis for the calculation of the turbo-generator loss.

Hydro-generators

Salient-pole machines can usually be started from a low-frequency supply. In horizontal-shaft machines the damper winding provides enough torque to overcome friction, but in vertical-shaft generators it is usually essential to jack the rotor to allow the bearings to flood. The generator is accelerated by increasing the frequency of the test-bed motor-generator, then synchronized on to a suitable normal-frequency supply, if necessary after an overspeed run. The driving set is then disconnected. As the generator now is driven electrically, its total loss is given by a terminal input power measurement, but care has to be taken in view of the very low power factor (which may typically be of the order of 0.02).

Synchronous motors

These are tested in the same way as hydro-generators. However, when rated at high power factors the reactive-load condition is different from the rated condition in such a way as to produce a deficiency in core loss and an excess in excitation loss. These divergences are calculable and the temperature-rise can be corrected accordingly.

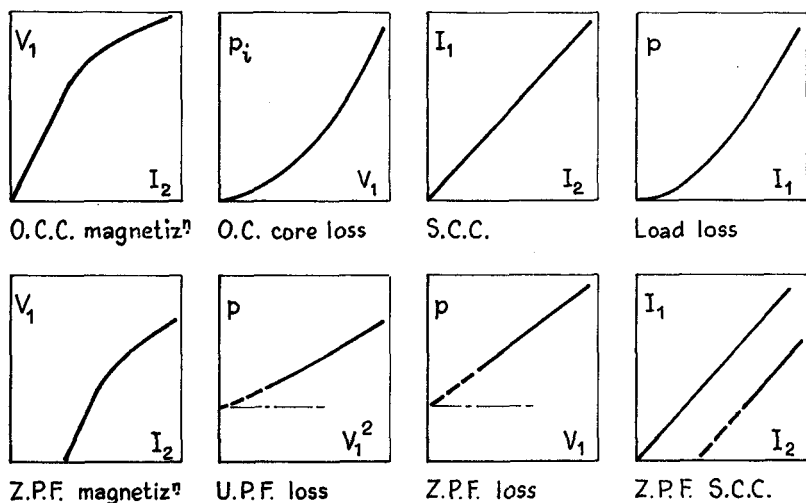
Synchronous compensators

The reactive-power test is the normal working condition, with the loss rather less than the convention summation of o.c. and s.c. losses. A feature is the test on underexcitation for lagging reactive output, usually at some under-voltage level, to demonstrate the range and stability of the excitation equipment.

Test-schedule

The following characteristics and losses may be measured:

- (a) Open-circuit characteristic and loss.
- (b) Short-circuit characteristic and loss.
- (c) Zero-power-factor characteristic and loss.
- (d) Temperature-rise (i) by full-load z.p.f. over-excited run, or (ii) by equivalent heat run. The latter comprises a rise measurement on the stator with excitation but no stator current, followed by a test with rated stator current but minimum excitation. The total temperature-rise is then obtained by combining the results.
- (e) Overspeed test, e.g. 0.25 p.u. for turbo- and 1.0 p.u. for hydro-generators, the latter in an enclosure; as the windage loss varies as the cube of the speed, considerable drive power may be demanded.
- (f) High-voltage tests.
- (g) Insulation-resistance tests, made before and after (f).
- (h) Waveforms, interference, gap length, balance, vibration, bearing currents, magnetic symmetry.



10.49 Test results.

Not all of these are necessarily made on each machine. The curves obtained from the major tests are shown in Fig. 10.49. For small machines the testing schedule is much simpler, and may be confined to (a), (b), (c), (d) and (f), apart from general mechanical constructional verification. Excitation and control systems are subject to full testing schedules before association with the machine itself.

The specified temperature-rises allowed are given in Sect. 4.7. Embedded temperature-detectors are called for in large machines. High-voltage tests are specified in BS 2613. Winding resistances are measured by a d.c. method, so that eddy-current and load-loss additions can be estimated.

Efficiency. The estimation by loss-summation is recognized in BS 269. The losses to be included are:

Excitation: field I^2R , rheostat, brushes, exciter system.

Fixed: core, bearings, brushes, windage and friction.

Direct Load: I^2R loss in stator windings.

Load (stray): conductors, iron parts, end windings, etc.

The load loss is due to (i) stray fields in solid parts near the stator, particularly coil-supports and end-covers, tooth-stiffeners and the outer packets of stator core laminations; (ii) field distortion increasing tooth loss; (iii) field pulsation arising from slotting and stator m.m.f. harmonics, setting up pole-face loss; (iv) eddy currents in stator slot conductors and end-connectors, increased by saturation in the teeth.

EXAMPLE 10.9: Evaluate the full-load efficiency of a synchronous machine of rating 23.4 MVA at p.f. 0.8 lagging from the following losses: Bearing friction 68 kW, measured by volume of lubricating oil circulation and tempera-

ture-rise; Windage 220 kW, from o.c. test unexcited, measuring cooling-air volume and temperature-rise; Core loss 165 kW, from o.c. test with excitation to give calculated e.m.f. at rated load, corrected for windage; Winding loss 200 kW, from full-load z.p.f. test; Stator d.c. I^2R loss 62 kW; Exciter loss 14 kW.

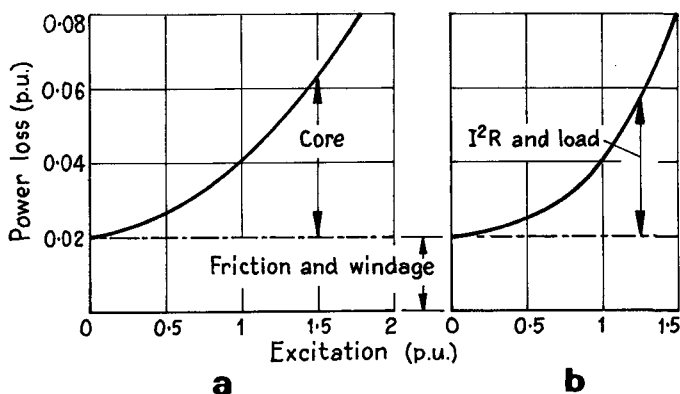
The load (stray) loss is the difference, 138 kW, between the winding loss and that calculated from the d.c. resistance. Then the losses sum to 763 kW. The output is $23\,400 \times 0.8 = 18\,750$ kW and the input is $18\,750 + 736 = 19\,513$ kW. The efficiency is

$$18\,750/19\,513 = 0.961 \text{ p.u.}$$

Specific tests

Some details of the more important tests are given.

Open-circuit test. The machine, whether motor or generator or compensator, is driven at rated speed with the stator windings on open circuit and the field current varied, to give the o.c.c., Fig. 10.7. With the machine driven by a calibrated motor the input to the synchronous machine, representing the core, friction and windage losses, can be calculated and the core loss and mechanical loss separated, Fig. 10.50(a).



10.50 Power/excitation curves.

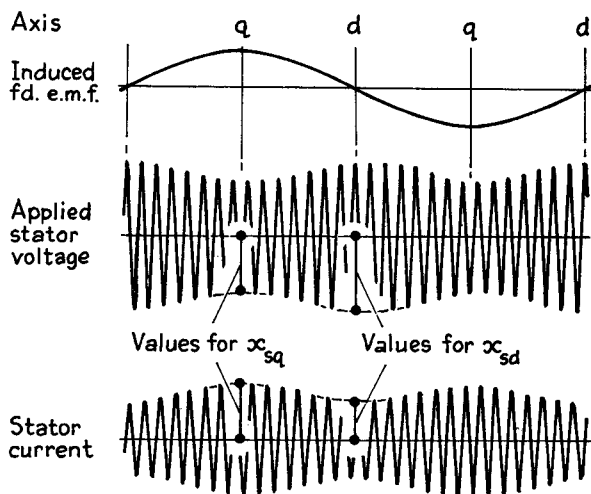
(a) Open circuit. (b) Short circuit.

Short-circuit test. The machine is driven at rated speed with the stator windings short-circuited. The field current is varied to give stator currents up to (or slightly over) full-load value. The relation between stator and field currents is substantially rectilinear, Fig. 10.7. Again the calculated input gives the loss, in this case the mechanical, stator I^2R and load losses summated, the core loss normally being negligible. The input power for the range of field currents is shown in Fig. 10.50(b), and the mechanical loss is readily separated.

Zero-power-factor test. A pseudo full-load test is carried out by loading the machine on static or synchronous inductors (or a combination thereof) so that

it operates at normal voltage and current. The gap flux is 4–5% above normal full-load level and the field current is 20–30% high. The test serves as a heat-run, with temperature-rises a little above normal: but appropriate correction factors can be applied. The I^2R and load (stray) losses are close to the normal full-load values.

Axis reactances. The d- and q-axis synchronous reactances, x_{sd} and x_{sq} , may be found from a slip test. The machine is coupled to a driving motor, the field circuit is opened, and a normal positive-sequence voltage applied to the stator. The drive torque is adjusted so that the machine loses synchronism and runs at a low value of slip. Oscillograms, Fig. 10.51, are taken of (i) the e.m.f. induced



10.51 Measurement of axis synchronous reactances.

in the field winding, (ii) the stator applied voltage, and (iii) the stator current: then x_{sd} is the maximum and x_{sq} is the minimum ratio of stator voltage to stator current. The method has the advantage of direct measurement, but the field saturation conditions are not normal. The slip should be small enough to avoid undue eddy-current induction in damper windings and pole-shoes. The test can be made with the machine self-driven, in which case a closely controllable supply voltage, of magnitude 0.3 p.u. or less and of normal frequency, must be available. The slip speed tends to fluctuate because of the variation of torque with load angle, with the result that x_{sd} tends to be under-estimated.

Ventilation

Machines of medium rating are often installed with cooling gas entering and leaving by ducts. The pressure-drop has to be reproduced on test by wire screens, so that the gas intake shall not exceed that in service. Larger machines with closed-circuit ventilation are tested with or without coolers, in the latter case again with provision for reproducing service conditions. Compensators with

air coolers mounted on the sides of the stator frame can usually be tested under conditions approximating adequately to those in the intended service. With vertical-shaft hydro-generators the enclosure of the air circuit on site is formed by the concrete foundations, and for works test enclosure is omitted because the pressure-drop in it is negligible. The cooling-water circuit is provided, particularly if it is concerned with direct-cooled field coils.

Temperature-rise

The following assumptions may be made for calculating the temperature-rise from the result of heat-runs in cases for which close accuracy is not sought or necessary:

- (i) The airgap is a thermal insulator, so that the rises of stator and rotor are not interdependent.
- (ii) The stator rise is the sum of three individual rises produced respectively by windage, core, and I^2R plus load loss.
- (iii) The stator core loss is a function of the terminal voltage as determined by the o.c.c. test.
- (iv) The load (stray) loss is the same on short-circuit, zero-power-factor and rated power-factor conditions, and is independent of temperature.
- (v) The rotor rise is the sum of two individual rises, respectively due to windage and to field I^2R .
- (vi) Heat re-distribution immediately following individual heat-runs is negligible.

11 Synchronous Machines: Construction and Design

11.1 *Types of synchronous machine*

Synchronous machines for general power-supply networks (at 50, 60 and occasionally 25 Hz) may be classified as follows:

Turbo-generators. Driven by steam turbines at high speeds (up to 3 600 r/min) and in ratings up to 1 000 MW.

Hydro-generators. Driven by water turbines at speeds between 90 and 1 000 r/min and in ratings up to 750 MW.

Engine-driven generators. Various forms of prime-mover of the internal-combustion type, with speeds up to 1 500 r/min and ratings up to about 20 MW. Gas-turbine drives may be of higher speed and rating.

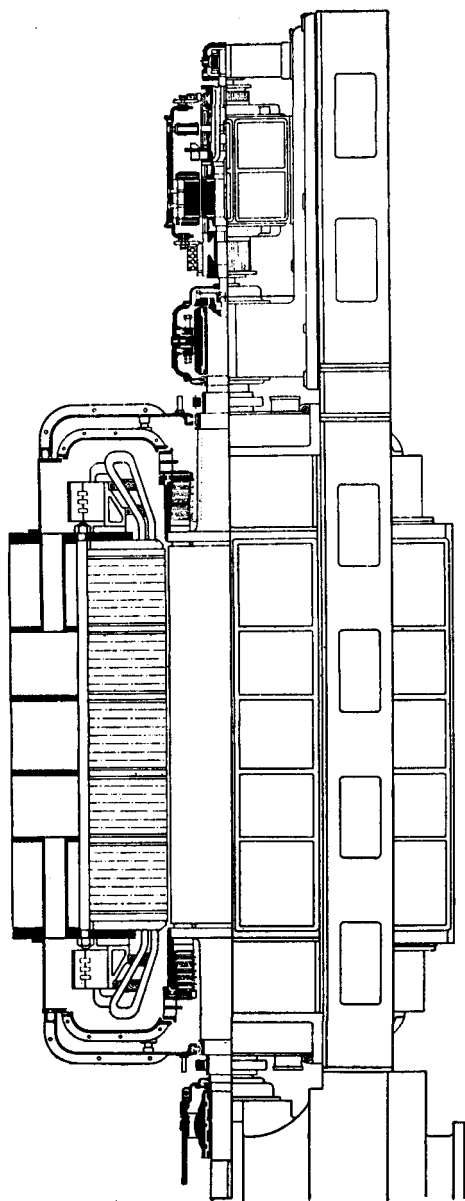
Compensators. Self-driven machines operating at speeds up to 3 000 or 3 600 r/min and in ratings up to about 100 Mvar.

Motors. The four main types are salient-pole, cylindrical, and salient or cylindrical synchronous-induction machines. The plain salient-pole synchronous machine has the highest efficiency and can be used for constant-speed industrial drives such as compressors, blowers, pumps and fans, in particular where the starting conditions are suitable or power-factor improvement is economically desirable.

Almost all synchronous generators are 3-phase star-connected machines, with the field windings carried on cylindrical or salient-pole rotors. The stator windings are of double-layer form so that harmonics can be reduced by chording. The phase-spread is almost invariably 60 deg(e). With large turbo-generators the armature (stator) m.m.f. may reach 300 kA-t/pole, and to avoid excessive demagnetization effects the field (rotor) m.m.f. must be of comparable magnitude as defined by the short-circuit ratio. As it is undesirable to have high core and tooth saturation levels, a long airgap is necessary, from 5 mm in a 1 MVA machine to 100 mm or more in a 500 MVA machine. Typical short-circuit ratios are 0.4 for machines with quick-acting excitation control, 0.7–1.0 for medium-speed machines, and 1.0–1.5 for hydro-generators and compensators.

11.2 *Turbo-generators*

Steam turbines run efficiently only at high speeds, so that 2-pole generator



11.1 Turbo-generator: 60 MW, 50 Hz, 2-pole.

construction is common even at high ratings. The electric and magnetic loadings are exceptionally high. The rotor diameter is limited to about 1.2 m, and in 50 Hz machines the active core length must be of the order of 10 mm per MVA. A 500 MW generator may thus have a core length of 5 m, and a shaft length of 12 m. The outside diameter of the stator core may be about 3 m and of the outer casing about 4 m. Fig. 11.1 shows the general arrangement of a 60 MW air-cooled machine. Larger ratings make more sophisticated cooling essential.

Rotor

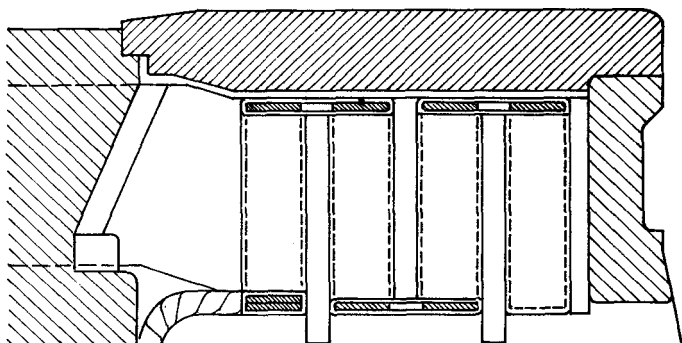
The I^2R loss per unit mass of conductor material in a 750 MW generator is of the order of 150 W/kg in the stator and 500 W/kg in the rotor. The rotor is therefore the limiting member. Within the diameter imposed by considerations of centrifugal force, deflection and critical speed, an excitation must be provided in accordance with the stator electric loading and the short-circuit ratio; and the exciting winding has to be contained in slots of such width as to leave teeth of adequate tensile and bending strength, able to carry the magnetic flux without excessive saturation. A cylindrical construction is necessary. Rotor bodies are normally machined from single-ingot forgings, and in the largest sizes may have a finished mass of 100 Mg. It may be necessary to use nickel-chrome-vanadium-molybdenum steel of ultimate strength up to 800 MN/m². Slots are milled out axially to receive the windings, typical profiles being shown in Figs. 3.11 and 4.14. The first critical speed is typically 1 200 r/min, the higher-order speeds occurring at 2.7, 3.0 . . . times this figure. One of these may be near to the normal running speed or the specified overspeed.

Windings. Excitation currents may reach 2.5 kA at 1 kV for a 750 MW generator. For lower ratings with conventional cooling the winding comprises continuous coils of silver-bearing copper strap. With tapered slots and direct cooling the coil-sides are constructed separately, and jointed at the ends after location in the slots. The end-windings, not being retained by slot wedges, must be contained within end-bells. To reduce rotor flux leakage it is the European practice to use for these a non-magnetic austenitic steel (e.g. 18% Mn, 3% Cr, 0.5% C) of ultimate strength reaching 1 150 MN/m². A typical end-bell and end-winding arrangement for an air-cooled machine is illustrated in Fig. 11.2.

Stator

The core is built up from sectors such as that shown in Fig. 4.13. Although grain-oriented steels, arranged as in Fig. 2.19(a), give lower core loss, the orientation is not everywhere favourable, and cold-rolled non-oriented steels containing 3% Si are more usual. Mechanical problems arise from the double-frequency vibration set up by the rotation of the axis of the magnetic field, and from the internal pressure of the hydrogen coolant. Core vibration is prevented from being transmitted to the stator casing by a form of flexible mounting.

Windings. For small machines the terminal voltage is usually fixed at some standard system level. For very large generators permanently connected to a transformer, the designer's choice is less restricted. A high voltage gives reduced



11.2 Retaining ring and rotor overhang.

current but entails more slot insulation; a typical compromise is 15 kV for generators of 100–200 MW rating, and 20–25 kV for large machines. Even so, the currents per phase may reach 20 kA, making essential a winding with several parallel circuits. The large currents per slot develop high pulsational forces between conductors, reaching e.g. 80 kN/m in a 500 MW machine. The winding is generally of conventional double-layer short-pitched form. Each conductor is elaborately subdivided to limit eddy-current loss (Sect. 3.2). The conductors are insulated by mica paper and asbestos or glass-fibre tape, with polyester or epoxy bonding to give mechanical strength and permanence. The end-windings must be very securely braced against movement under the impact of short-circuit electromagnetic forces. Hardwood blocks with glass-fibre cord or tape may be used, with metallic (but non-magnetic) brackets as anchorage.

Airgap windings. Modern turbo-generators have long radial gaps for reasons of operational stability, and it has been suggested that the stator winding could with some advantage occupy some of the gap space. In a conventional toothed stator, the slotting concentrates the flux into the teeth, saturation and core-loss levels are high, and pole-face losses occur. The deep slots make conductor transposition essential and introduce some difficulties in direct water-cooling; high-voltage insulation in the usual double-layer winding results in a low space-factor (e.g. 0.3–0.4); end-windings are bulky and need elaborate support.

The *slotless winding*, described by Davies [103], comprises a substantial cylindrical insulating tube fixed to the stator bore surface to form the main insulation to frame, with phase windings distributed within it and provision for cooling. An inner insulating tube insulates the windings from the mechanical air-gap. The advantages are a reduction in the overall stator-core diameter, saving mass; the elimination of tooth punching; simpler coil design and bracing; the possible reduction of eddy loss; lower transient reactance; and the suppression of tooth vibration and noise. The stator winding can be constructed complete externally to the core, and then inserted into the stator bore. There are, however, certain important differences, as the winding seals the active stator region from the rotor to prevent normal radial ventilation so that the rotor must be

water-cooled. The whole of the torque acts directly on the stator conductors, and it must be transferred through the tubular winding structure to the stator frame. For a 750 MW, 50 Hz, 2-pole machine the total torque is about 2.5 MN-m; but spread over the total surface of the stator bore this may amount to a concentration of only about $100 \text{ kN/m}^2 = 10 \text{ N/cm}^2$. The figure might be multiplied by 10 for short-circuit forces. As regards the magnetic conditions of the airgap winding, it faces the main flux and will have eddy-loss effects proportional to the main flux (instead of the slot-leakage flux as in a conventional winding). But conductor stranding can be minimal. The winding is almost inevitably concentric if the method is to be exploited to the full.

Cooling

Although in large machines the full-load loss may total no more than one or two percent, this may amount to 10 MW or more. The removal of the heat demands a comprehensive cooling system using appropriate fluid coolants. For machines up to 60 MW rating the coolant may be air in a closed circuit with a heat exchanger, as shown in Fig. 4.10. For larger ratings, hydrogen has been employed for its superior qualities, the arrangements being illustrated in Figs. 4.11 and 4.12. For ratings above 200 MW the concentration of winding loss calls for direct-cooling methods, using hydrogen or water as coolant. An analysis of the viability of water cooling for turbo-generator rotors is given by Noser and Kranz [105].

Large turbo-generators

The need for large generating capacity raises the question of the design, construction and transport to site of machines rated at 1 000 MW or more. Ratings of this order were considered by some Russian engineers [104] in 1963, and a number of developments since then have made these ratings practicable. With the speed fixed by the frequency, the disposable dimensions are represented by the rotor diameter D and the active core-length l . The mechanical limits on D are the body and end-bell centrifugal forces, while those on l concern the rigidity of the rotor and the need to attain normal running through the first critical speed. The present limit on D for 50 Hz machines is about 1.2 m, and the lengthening of the rotor beyond $l = 5D$ gives no improvement; but even a small increase in D could give a substantial rise in capability.

For a rotational speed n , the rotor stress is proportional to $D^2 n^2$. It is not easy to establish a stress criterion, but practice suggests that the tangential stress at the rotor surface should not exceed 50–60% of the yield stress for normal speed and 75–85% at 20% overspeed, the latter condition involving a factor of safety of only 1.15. At these high stresses the elastic/plastic behaviour of the material is complex.

Titanium alloy may be required for the end bells, the stress in which is largely self-produced. The yield point of the alloy is no greater than that of nickel steel, but its density is only $4\,500 \text{ kg/m}^3$ (compared with $7\,800 \text{ kg/m}^3$ for steel) and the lowering of its self-stress permits it to cope more safely with the centrifugal force of the windings that it has to retain.

The DI product affects the critical speed. For very large machines the most refined calculations are essential, taking account even of the differing rigidities of the direct and quadrature axes of the rotor, the effect of the rotor winding, the bearing deformation and the characteristics of the generator-turbine shaft coupling. The complete unit has several critical speeds related to the various turbine sections and the generator rotor.

Vibration problems can be serious. A 2-pole stator is deformed into an ellipse by the intense magnetic attraction, with the minor axis directed along the axis of the airgap flux; and as the rotor rotates, so does the deformation. The strain is not large — it may be of the order of $30\text{ }\mu\text{m}$ on the radius — but it initiates strong vibration at twice the rotational frequency, which may be too near to the natural vibrational frequency of the stator and its frame as an elastic mass. Increase of D lowers the natural frequency, although at the same time it acts to raise the hazardous second critical speed further from the normal operating speed. Vibration affects the windings, both in the slots and in the overhang, and in both regions there is an additional cause of double-frequency vibration in the electro-magnetic attraction and repulsion forces between conductors carrying large currents. Conductors insufficiently wedged in their slots, or attached to loose bracing blocks, may have their insulation damaged by abrasion or fatigue.

With the mechanical dimensions so great, an 80 Mg rotor in a 300 Mg stator attached to a multistage turbine in an overall length of, say, 25 m, there are considerable dimensional changes resulting from thermal expansion and contraction. In the generator rotor it is necessary to employ windings of silver-bearing copper.

Mechanical limits on D and I imply that increase in output can be achieved only by raising the magnetic and electric loadings. The magnetic flux density is limited by tooth saturation: it does not at present seem possible to find materials for the rotor body that combine mechanical strength with better magnetic properties. Raising the electric loading is the only possibility remaining. It requires the use of elevated current densities, especially in the rotor. Conductor heating itself no longer sets a problem, as direct water-cooling methods have been developed. The heat-transfer capability of water is 50 times that of air; but its density is 1 000 times as great, and this adds substantially to the complexity of the rotor cooling system, besides increasing the centrifugal force on the conductors.

Load (stray) loss in a large machine is always considerable. It arises mainly as a consequence of space harmonics of m.m.f. at the stator and rotor gap surfaces, of end-winding leakage flux and of end-bell leakage. Nonmagnetic end-bells are essential.

A 1 000 MW generator might have a rotor 1.25 m in diameter, 7 m long and 80 Mg in mass, giving an excitation m.m.f. of 1 MA-t/pole; a 300 Mg stator core; an airgap 150 mm in radial length; a total full-load loss of 14 MW; and a 12-ph stator winding connected one-half star/star and the other half star/delta to the generator transformer to obtain a higher winding factor and to reduce harmonics. The cooling would require special consideration.

Cooling. The 14 MW of loss in a 1 000 MW turbo-generator could be dissipated

by hydrogen cooling, but the fan power would be of the order of 1 400 kW. With water as the coolant the same loss could be removed by use of a pump rated at 100 kW. There are two schools of thought on the solution of the cooling problem. One accepts hydrogen cooling in general, with water applied only to those parts with a high loss concentration. The other takes the view that if water is essential for the high-loss regions, it is not justifiable to use hydrogen at all. Hydrogen cooling demands a leak-, pressure- and explosion-proof housing, shaft seals, rotor-mounted circulating fans, heat-exchangers within the machine and hydrogen plant externally. Water cooling permits of a lighter casing, eliminates fans and shaft seals, and allows the stator and rotor to be separated by an insulating cylinder in the gap.

With water as the major coolant, the stator windings have hollow conductors, or are associated with separate cooling-water tubes. The stator core has axial ducts in which stainless-steel cooling-water tubes are contained, fed from headers at each end. Such tubes would have to be insulated to a thickness depending on their radial position (i.e. on the magnitude of the e.m.f. induced in them) and be non-magnetic. Alternatively, the stator core could be oil-cooled, making tubes unnecessary and bringing the coolant into direct contact with the edges of the holes in the core plates.

The rotor cooling system could be that described in Sect. 4.4. The water could also be circulated through holes in the slot wedges, which serve in the turbo machine as a damping winding.

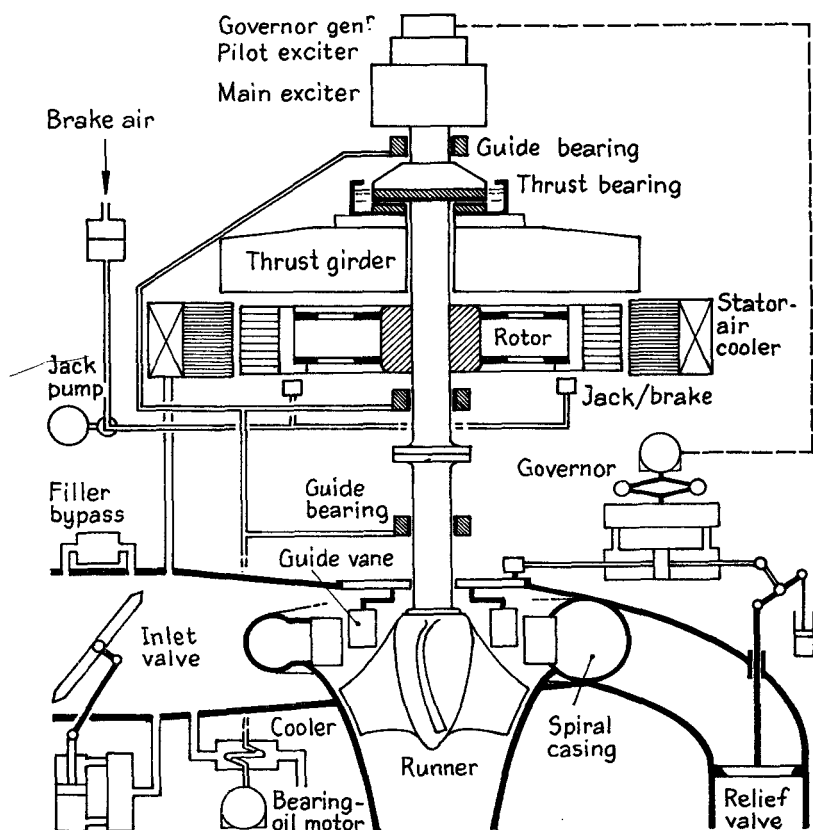
An insulating cylinder in the gap could serve several useful purposes. By separating the stator hermetically from the rotor, it would be possible to fill the stator housing with an inert gas such as nitrogen, reducing the risk of fire and explosion and suppressing corona discharges in the h.v. windings. A similar gas in the rotor region (but at lower pressure and without rotor fans) would substantially reduce the windage loss. The latter could be removed by a helical water-tube arrangement associated with the separate cylinder.

It seems probable that, with a radical change in cooling techniques such as those described, it would be possible to achieve a 2-pole turbo-generator with a rating approaching 2 000 MW.

Superconducting windings. The feasibility of loss reduction by the use of superconductivity has naturally been investigated for large turbo-generators. The stator windings carry alternating currents and cannot therefore be made superconducting, but the rotor — with its conventional loss of several megawatts in excitation — offers the possibility. In a design described by Lorch [106] the rotor body is a glass/epoxy-resin cylinder with steel ends; a normal steel forging would conduct too much heat from the bearings and would offer no magnetic advantage at the very high flux densities that superconductivity makes possible. The rotor cylinder is slotted to take coils cooled to about 5 K by a flow of helium entering and leaving the rotor on the axis at the outboard end. A copper cylindrical shell rotates with the rotor to screen it from harmful alternating magnetic fields, the shell being thermally insulated from the rotor by a sealed stainless-steel vacuum enclosure maintained at a temperature of about 100 K. Liquid nitrogen is passed through a system of stationary cylinders within the stator bore to form a cold screen at 77 K. The

gap between the fixed and rotating structures is filled with hydrogen at 0.1 mmHg to reduce windage but leaving adequate thermal conductivity. In this way the complexity of a liquid nitrogen supply to the rotor is avoided. The stator is conventional, and so is the exciter apart from its much lower rating: it has to supply 2.5 MW for only a few seconds on initial excitation to establish the magnetic field energy.

The problems are formidable, and much ingenious design is called for. The use of high gap flux densities is clearly possible, but for a peak density of 2 T it would be necessary to remove the stator teeth. As much as 5 T has been suggested, for which the stator core and teeth would be redundant but which would require a massive magnetic shield around the stator to localize the intense field. The problems of securing the exciting windings to the nonmetallic rotor body and the stator windings on a nonmagnetic stator against very large short-circuit forces are likely to be great. Some of the familiar machine parameters would be markedly changed, with a low synchronous reactance and very



11.3 Hydro-generator and turbine control scheme.

long transient and subtransient time-constants. Finally, the advantage of superconductivity must be balanced against the capital, operating and energy-loss costs of providing it.

11.3 *Hydro-generators*

Mechanical considerations dominate the constructional form of generators to be driven by water-turbines. The low speed, determined by the available head and type of turbine, demands a multipolar form and consequently large diameter, involving in turn a design such that the machine can be transported to site in sections. The turbine governing, or the transient stability of the associated power network, decides the total inertia of the set, most of which must be provided by the generator. It is usual to design the machine to rotate safely on overspeed of the order of twice normal operating speed; only rarely is the overspeed near to the first critical speed.

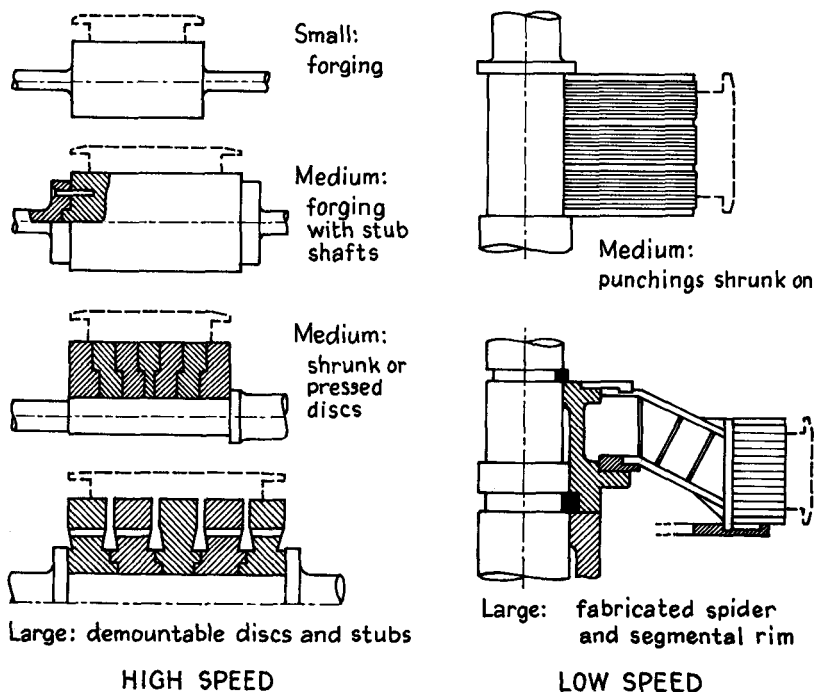
Both horizontal- and vertical-shaft designs are employed, the former for impulse-turbine drive and the latter for large reaction and Kaplan turbines. The bearings for horizontal-shaft machines must include a thrust bearing to resist the axial component of force produced by the turbine. In low-speed vertical-shaft machines, the hydraulic thrust may exceed the dead weight of the rotating masses, and there are a few variants possible in the mechanical arrangement of the unit.

Oil is supplied to the bearings by motor-driven pumps, and cooled externally. Alternatively the bearing itself may be direct-cooled by water through tubes embedded in the bearing metal. Vertical-shaft machines are provided with pneumatically operated brakes to bring the set to rest without protracted low-speed running. The same brakes, operated hydraulically, may be used as jacks when the set is at rest to allow oil to flood the thrust bearing or to permit maintenance of the thrust-bearing pads. The diagram in Fig. 11.3 shows the mechanical assembly with the operating and control system for a vertical shaft Francis turbine and generator.

Rotor

The salient poles are carried on a rotor body. For high-speed machines the body is (a) machined with its shaft from a forging; or (b) machined separately, with stub shafts attached thereafter; or (c) built up from discs shrunk on to a shaft. These methods are illustrated in Fig. 11.4. In the case of medium- and low-speed machines, method (c) may be used for diameters up to about 4 m, or (d) the body may be fabricated from a cast-steel spider mounted on the shaft and carrying a laminar ring of overlapping segmental plates tightly bolted to resist slipping under hoop stress. The rim rests on the spider arms and is driven from them by floating keys, thus relieving the spider of centrifugal force other than that due to its own mass. Up to a peripheral speed of about 80 m/s it is feasible to use a segmented spider. At higher peripheral speeds (up to 110 m/s normal and 220 m/s overspeed) a solid disc becomes essential.

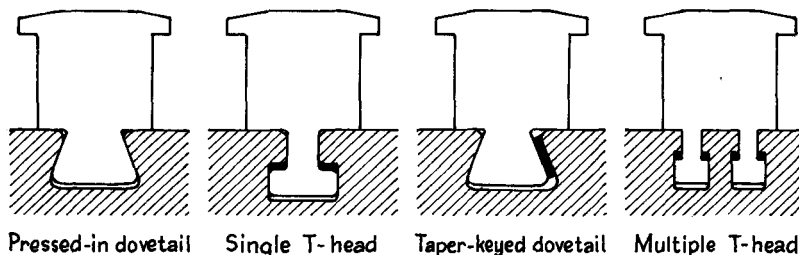
Laminated poles are used on all medium- and low-speed constructions. They comprise thin steel plates clamped between heavy end-plates and secured



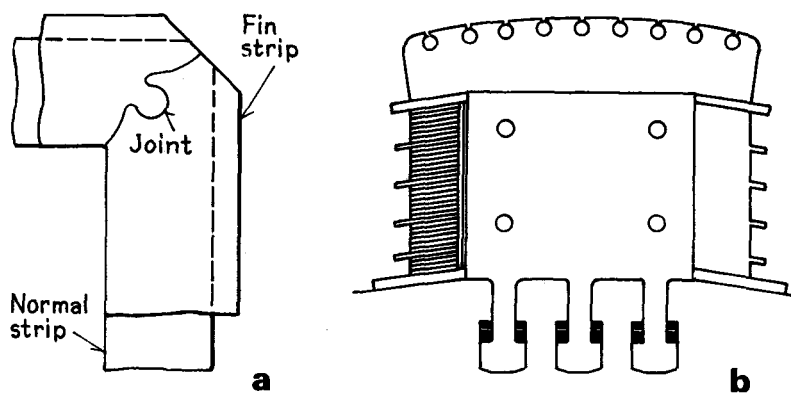
11.4 Salient-pole machines: rotor body construction.

by studs or rivets. The poles may be bolted to the rotor body if the peripheral speed does not exceed about 25 m/s. For higher speed the poles are dovetailed, as shown in Fig. 11.5.

Windings. The field coils are formed from rectangular-section copper (or aluminium) strap, wound on edge, with inter-turn insulation cured and consolidated under a pressure exceeding that due to the centrifugal force on the coil at overspeed. The coils of machines with axially long poles are fabricated from punched straight lengths of copper, Fig. 11.6(a). Certain turns can be made of wider strap to produce additional cooling fins. The method also



11.5 Salient-pole fixings.



11.6 Salient-pole field windings.

facilitates the construction of 'canted' field windings, (b), which better withstand centrifugal force.

Damper windings. With solid poles the eddy-current damping may be sufficient, but with laminated poles a damper winding is employed, comprising copper bars through the pole-shoes with end connections attached to the pole end-plates. Interpole connection between the damper windings is not normally necessary.

Stator

The stator of a large machine is fabricated in 3, 4 or 6 parts, sometimes completely wound except for coil joints, so that the transport problem is relieved. It is possible to employ grain-oriented magnetic sheet steel, Fig. 2.19(b), but the advantage is not great.

Windings. The two-layer diamond-coil form is normal, with single-turn bar or multiturn coils. Bar windings have Roebel or similar transposition arrangements. The insulation is generally of class B mica, with phenolic, epoxy or polyester bonding varnish.

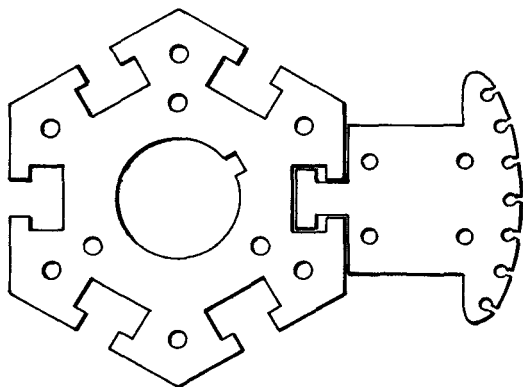
Cooling

Closed-circuit air cooling is frequently employed, with air/water heat-exchangers mounted outside the stator frame. In very large machines the liquid cooling of both rotor and stator windings is considered to be feasible. Fig. 4.15 shows a water-cooled field winding.

11.4 Industrial generators

Special-purpose and standby synchronous generators are sometimes useful in industry. They are driven from diesel engines or gas turbines, sometimes as a transportable unit, in sizes typically between 50 and 1 000 kVA. The most usual design has a 4-pole salient rotor with brushless excitation, and a conventional stator winding. The performance guarantees include automatic voltage regulation, voltage dip and recovery time, voltage build-up when starting,

and low harmonic content. A typical gas-turbine/generator set for 500 kW is fully enclosed on a single base, complete with turbine starting, cooling, fuel and oil systems, excitation control, switchgear and main cable connectors, all in a total mass of 6 Mg. With a top-mounted exhaust outlet, the plan dimensions are 4.5 m x 1.5 m, and the height is 2.75 m. Typical spider and pole punchings for a 6-pole generator are shown in Fig. 11.7.



11.7 Spider and pole punchings.

11.5 Compensators

Synchronous compensators for the control of reactive-power demand in supply system networks may be permanently sited, or may require to be transported at intervals in accordance with the growth and changing pattern of system conditions. They are designed to be compact and almost self-contained, with the minimum of external connections. Machines of 50 Mvar upward are often built with hydrogen cooling. Four- or six-pole construction is usual, with salient rotor poles and a conventional stator winding. Apart from excitation, the losses are low, and a power factor of 0.02–0.03 p.u. can be achieved.

11.6 Motors

The constructional features of motors do not differ from those of generators of comparable rating and speed, except that the salient-pole design is the most common, and that (unlike the generator) the machine is almost always required to be self-starting, using the pole-face cage as a starting winding. There is no essential difference between the stators of polyphase synchronous and induction motors of comparable rating.

11.7 Main dimensions

The output S of a synchronous machine is a function of the air-gap diameter D , core length l , speed n , and the specific magnetic and electric loadings $\bar{B}$ and A . In machines of the highest rating the limiting feature is the peripheral speed $u = \pi Dn$: then from eq.(1.16)

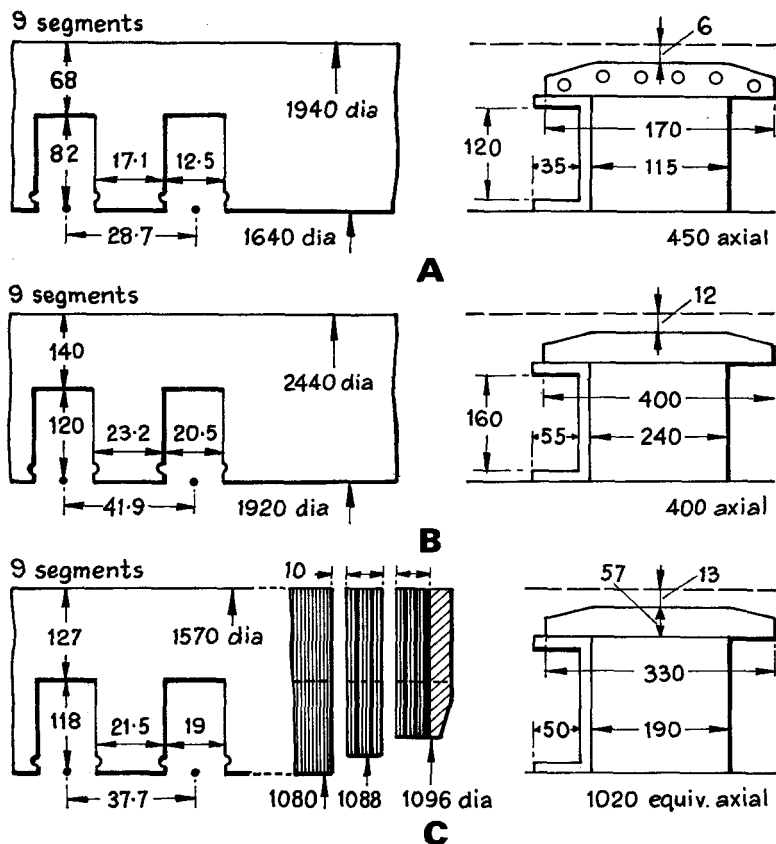
$$S = 11K_w \bar{B} A D^2 l n = 1.1 K_w \bar{B} A l u^2 / n$$

where the winding factor of the stator is of the order $K_w = 0.96$.

Salient-pole machine. With a mean gap density $\bar{B} = 0.65$ T and an electrical loading $A = 60$ kA/m, and with dovetailed solid-pole construction to permit a peripheral speed $u = 100$ m/s, the output available per unit length of core is

$$S/l = (400/n) \times 10^6 \text{ VA/m}$$

A 20-pole 50 Hz machine has $n = 5$ r/s and $D = 6.3$ m, so that with the loadings stated it can develop $S/l = 80$ MW/m of core length. The main dimensions may be subject to several adjustments to meet specified values of short-circuit ratio, synchronous and transient reactance, overspeed, transport facilities and inertia constant. The per-unit transient reactance varies directly as the product $\bar{B}A$, and inversely as a function of the pole-pitch and the shape of the interpolar space. Hydro-generators located at the end of long transmission lines may have particularly low synchronous reactance (e.g. 0.55 p.u.) and transient reactance (e.g. 0.2 p.u.); exceptional demands for supply-system charging current may be met by raising the short-circuit ratio or even by the use of nega-



11.8 Slot and pole details.

Dimensions in mm; not to scale.

tive excitation. Large-diameter low-speed generators of the punched-segment rim construction can be given higher inertia by increasing the depth of the rim. In higher-speed units with disc rotors, a demand for higher inertia may require an increase in diameter (involving more elaborate pole fixings), or the provision of a flywheel, or the application of turbine runaway speed limiters. *Cylindrical machine.* Considering 2-pole, 50 Hz turbo-generators with a peripheral-speed limit of $u = 195$ m/s and therefore a diameter $D = 1.25$ m, the magnetic loading may be 0.7 T. Liquid cooling enables the electric loading to be of the order of 220 kA/m, using stator and rotor current densities that respectively approach 10 and 18 A/mm². The output per unit of core length is then

$$S/l = 125 \times 10^6 \text{ VA/m}$$

The rotor of a 750 MW generator with these loadings must have an active length of 6 m.

EXAMPLE 11.1: Outline data are given below for three salient-pole machines: the essential stator and rotor dimensions are set out in Fig. 11.8.

Generator A. 1 250 kVA, p.f. 0.8 lagging, 6.6 kV, 3-ph, 50 Hz, 20-pole, 300 r/min for engine drive.

Generator B. 3 750 kVA, p.f. 0.8 lagging, 10 kV, 3-ph, 50 Hz, 10-pole, 600 r/min for hydraulic drive requiring an inertia of 3 400 kg-m².

Compensator C. 5 MVA leading/2.5 MVA lagging, 11 kV, 3-ph, 50 Hz, 6-pole, 1 000 r/min machine with a full-load loss of 125 kW (0.025 p.u.). To reduce load (stray) loss the stator core ends are stepped and the tooth stiffeners insulated from the core plates.

		A	B	C
<i>Rating</i> Output	kVA	1 250	3 750	5 000
	kW	1 000	3 000	0
Line/phase voltage	kV	6.6/3.8	10/5.8	11/6.35
Line/phase current	A	109	217	263
<i>Main Dimensions</i>				
Magnetic loading	T	0.51	0.48	0.46
Electric loading	kA/m	38.2	51.5	55
Stator bore	m	1.64	1.92	1.08
Gross core length	m	0.45	0.40	1.00
Ducts, no./width	mm	6/10	5/10	14/10
Net core length	m	0.39	0.35	0.86
Iron length	m	0.351	0.315	0.775
Radial gap length	mm	6	12	13
Pole pitch	m	0.26	0.60	0.57
<i>Stator</i>				
Winding/connection	—	double-layer diamond/star		
No-load flux per pole	mWb	59	116	261
Turns per phase	—	300	240	120
Number of slots	—	180	144	90

Conductors per slot	—	2 x 5	2 x 5	2 x 4
Coil pitch	—	9	12	12
Conductor size	mm	5 x 5	2(10 x 4)	2(8.5 x 4.5)
area	mm ²	25	80	77
Length of mean cond.	m	1.11	1.38	2.07
Resistance	mΩ/ph	560	174	136
Current density	A/mm ²	4.4	2.7	3.4
Total f.l. I ² R loss	kW	20	24.5	28
Copper mass	kg	475	1 520	1 060

Rotor

Type of pole	—	laminated	laminated	solid
fixing	—	bolted	dove-tailed	screwed
Damper bars/pole	—	8	0	0
Turns/pole	—	71½	99½	99½
Conductor size	mm	35 x 1.4	55 x 1.3	50 x 1.75
area	mm ²	49	71	87
Full-load current	A	135	207	270
Current density	A/mm ²	2.75	2.9	3.1
Length of mean turn	m	1.49	1.41	2.52
Resistance	Ω	0.89	0.42	0.37
Volt drop	V	120	86.5	100
I ² R loss	kW	16.1	18	27
Copper mass	kg	950	630	1 300
Peripheral speed	m/s	25.5	60.5	11.2
Overspeed	p.u.	0	0.8	0

Magnetization

Flux density, core	T	1.24	1.31	1.33
pole	T	1.20	1.23	1.40
teeth (¼)	T	1.61	1.67	1.74
gap	T	0.77	0.725	0.79
M.M.F./pole, no-load	kA-t	7.5	8.95	10.35
full-load	kA-t	9.65	20.6	26.9
Armature m.m.f./pole	kA-t	4.91	15.1	16.0

Performance

Full-load losses:				
stator I ² R	kW	20.0	24.5	28.0
rotor I ² R	kW	16.1	18.0	27.0
core	kW	20.0	35.0	38.0
mechanical	kW	3.5	14.5	23.0
load (stray)	kW	7.4	12.0	5.0
total	kW	67.0	104.0	121.0
Output	kW	1 000	3 000	0
Input	kW	1 067	3 104	121
Efficiency	p.u.	0.937	0.967	0

11.8 Design

The general remarks in Sect. 1.3, and those specific to induction machines in Sect. 9.3, may be taken as applying to the computer design of synchronous machines: but the process, particularly for very large machines, is considerably more complicated making synthetic design at present impossible.

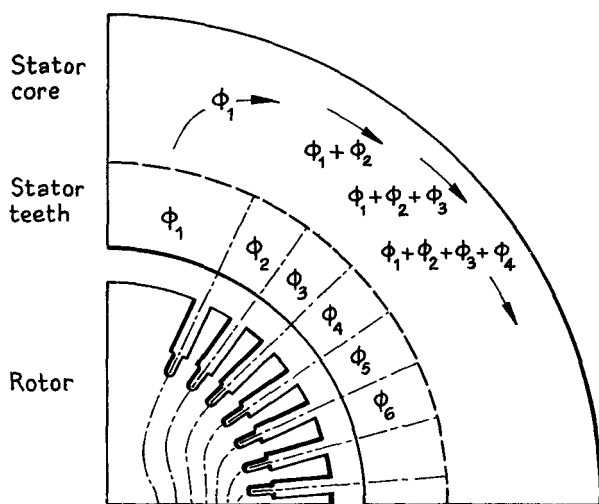
The approach to design programming is greatly affected by the type and size of the machine. In the case of small standard or semi-standard machines, many of the dimensions and performance criteria are imposed by international and manufacturers' standards, greatly reducing the number of variables; synthesis of the remainder of the design becomes practicable. The optimum design is generally based on minimized production cost (occasionally mass), and marginal improvements in performance are less important.

For large machines the number of design variables is so great that optimizing may vary with the application. The capitalized value of an optimized performance item (such as subtransient reactance) may outweigh the increase in price that the customer is prepared to pay for it. Again, the machine may be designed as part of an extensive system and its individual cost be of less consequence than its performance in the system.

A comprehensive synthesis program might take several man-years to develop, and could be both restrictive and obsolescent when completed. An analysis-synthesis combination may be more practicable, with the synthesis routines dealing e.g. with the adjustment of a few interrelated dimensions to meet a particular item in the specification, and the analysis routines coping with many of the design calculations (unbalanced magnetic pull, magnetization, inertia, stresses, resistance, leakage reactance and losses). Here the 'conversational mode' is valuable.

In the course of a design, coefficients may be used that are empirical functions of, say, two variables expressible as polynomials: techniques are available for interrelating them so that they can be stored in a main program concerned with the required coefficients. In all computer-aided design, the task of forming interrelations between variables is vital.

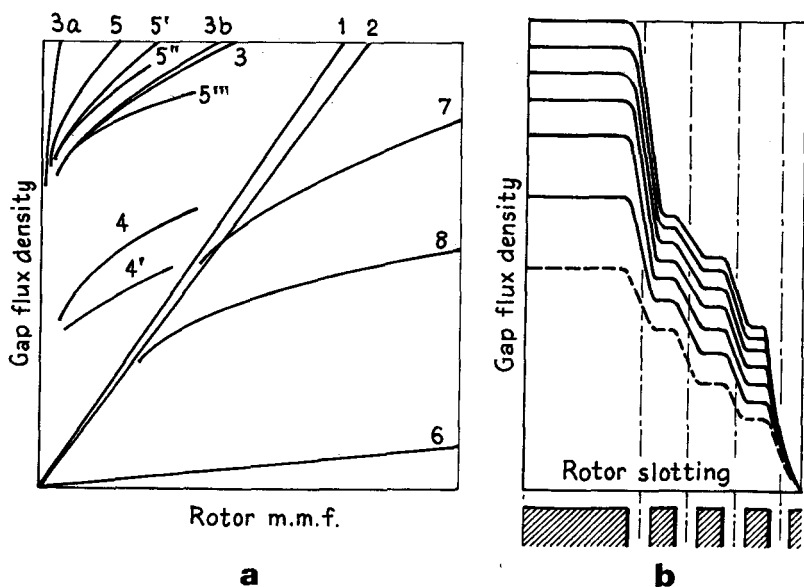
No-load magnetization of large turbo-generators. An example of the problems that arise in turbo-generators is the predetermination of the no-load magnetization characteristic. This is discussed by K. J. Binns. The magnetic circuit concerned is shown in Fig. 11.9. The flux distribution at the centre of the airgap is first assumed (it is approximately trapezoidal), the flux per pole being estimated from the required stator e.m.f. The useful flux is divided into a number of paths, bounded by the centre lines of the rotor slots and continuing in the rotor core to cross the q-axis at right-angles. For simplicity the stator slotting is considered as replaced by an equivalent slot system having angular alignment with the rotor slots but the same reluctance as the actual slots. The flux quantity passing through each flux path is determined. The rotor leakage flux is added to the rotor flux and then, because the cross-sectional area of ferromagnetic material at each point on a path can be found, the variation of the flux density along each path can be estimated. From each value of B the corresponding H is obtained from the magnetization curve of the steel. By taking a considerable number (e.g. 100) of points along



11.9 Calculation of magnetic circuit of turbo-generator.

each flux path it is possible to integrate H to obtain the drop of magnetic potential along the path. The m.m.f.s are used to determine the rotor slot current. If the assumed gap flux distribution is correct, each slot will require the same current; but if there is a discrepancy, the assumed distribution is modified until the error is minimized. The final flux distribution is employed in calculating the open-circuit e.m.f. of the machine. To program the calculation the independent variable is taken as B , and the H/B relation fitted by a number of linear equations each valid for a part of the H/B curve. The variation of B with length x along a path is expressed as a polynomial in x , and the magnetic potential difference along a path integrated by application of Simpson's cubic rule. Iteration enables the necessary equality of the rotor slot currents to be approximated, and finally a harmonic analysis of the gap-flux distribution is used to derive the r.m.s. value of stator e.m.f.

The saving of time compared with 'hand' calculation is important, and the accuracy is markedly improved because of the point-by-point rather than the region-by-region assessment. The 'hand' calculation of this problem requires a saturation curve to be determined for each part of the magnetic circuit on the assumption of no leakage. The curves are then corrected and combined to give a saturation curve for the whole circuit per pole with the exception of the rotor core. The core m.m.f. must be calculated from a knowledge of the field form, and as this m.m.f. does not primarily affect the rest of the circuit, it is added to the total m.m.f. at the end of the o.c.c. calculations. In dealing with the regions, the stator and rotor teeth are assessed graphically (Fig. 2.18), the polar horn as a large tooth and the stator core in terms of its average flux density. End-bell leakage is graphed, and the rotor slot leakage based on slot permeance. The saturation curves that emerge are shown in Fig. 11.10: they are



11.10 Magnetization curves and field form.

1. Air line for unwound part of the rotor.
2. Air line for wound part of the rotor.
3. Stator, combination of core (3a) and teeth (3b).
4. Wound part of rotor; 4' corrected for end-bell leakage.
5. Unwound part of rotor; 5' corrected for end-bell leakage; 5'' partly corrected for slot leakage; 5''' corrected for end-bell and slot leakage.
6. Slot leakage.
7. Magnetization curve for unwound part (1 + 3 + 5''').
8. Magnetization curve for wound part (2 + 4')

The distribution of gap flux density is now found for a series of rotor excitations, the results being shown in (b) of Fig. 11.10. These are *field-form* curves: the steps above the slots tend to slope more steeply at points in the gap more remote from the rotor surface, and the field-form curve tends toward a trapezoidal shape, which is the starting point for Binns' method.

12

Special Machines

12.1 *Forms of special machines*

The principle of the a.c. electromagnetic machine is not, of course, confined in its application to standard 3-phase synchronous and induction machines with rotary motion, although these are of major industrial importance. Linear forms of these machines have been developed for travelling cranes, railway traction and several other fields involving locomotion or restricted rectilinear movement. The prevalence of 1-phase supplies for domestic and small workshop loads has resulted in the demand for very great numbers of 1-phase machines, especially in the fractional-kilowatt range, for driving refrigerators, mixers, cleaners and hand tools. A useful range of machines for control-system use has been developed for position sensing, speed indication and several other control functions.

The performance of industrial synchronous and induction machines is closely related to the supply frequency, which determines their normal operating speed. If an a.c. machine is provided with a commutator, this speed restraint can be lifted. The associated technological difficulties were successfully overcome in the first decades of this century when large 1-phase series motors were built for several European railways, using special low-frequency supplies; and at the same time a number of 3-phase commutator motors for normal-frequency operation were invented to provide variable-speed industrial motors. Interest in a.c. commutator motors has declined with the later development of more sophisticated solutions to the problem of variable speed, notably those in which semiconductor devices are applied. The exception is the 1-phase series commutator motor for ratings of a few score watts and formerly called 'universal' because it is capable of giving comparable performance on both d.c. and a.c. supply.

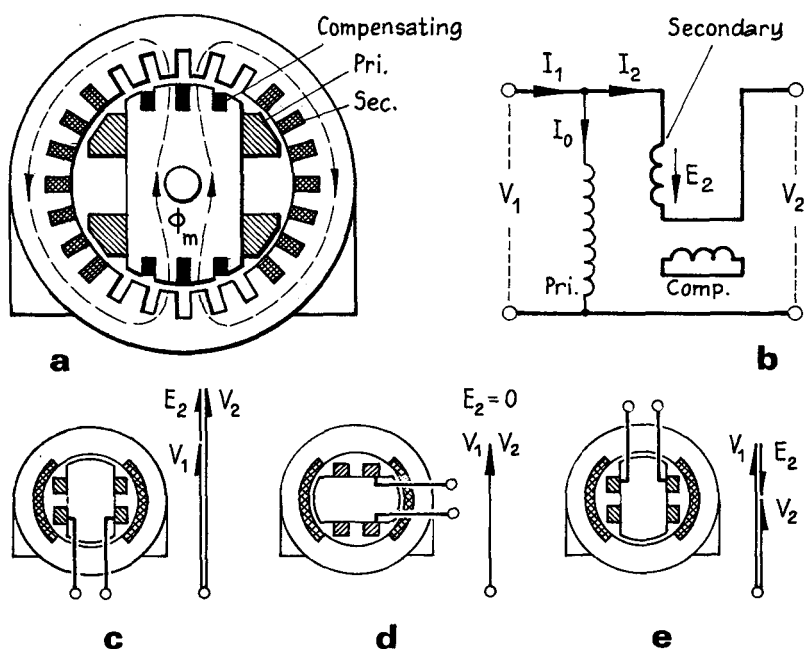
Certain devices for voltage adjustment and regulation are based on the induction machine and the transformer. A special class of high-frequency a.c. generator, called an inductor alternator, has been widely used for h.f. power supplies, e.g. for furnaces, and although such powers can be developed by static electronic means, the inductor generator has still some industrial utility. It is essentially the generator counterpart of the reluctance motor, the development of which has been the subject of much recent research.

12.2 Voltage regulators

These provide a variable voltage from a constant-voltage a.c. supply.

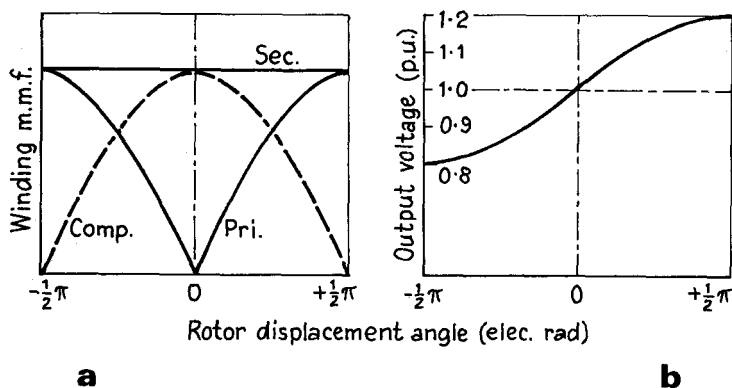
Induction regulator

The constructional features resemble those of a slip-ring induction motor. The movable member, however, requires to rotate only about one pole-pitch of arc, and is often arranged vertically with its shaft connected to a worm/pinion device for adjusting position and withstanding the torque.



12.1 Single-phase induction regulator.

Single-phase regulator. Fig. 12.1 shows the cross-section of a 1-ph 2-pole regulator and its connections. For convenience the primary constant-voltage supply is fed by flexible connectors to the inner (movable) member: its flux Φ_m induces in the stator (secondary) winding an e.m.f. E_2 which varies in magnitude in accordance with the angle between the axes of the two windings. In the position indicated in (c), with the connections as in (b), E_2 provides maximum *boost* to raise the output voltage from V_1 to $V_1 + E_2 = V_2$. A 90° rotation nullifies the mutual inductance between primary and secondary so that $E_2 = 0$, as in (d). A further 90° displacement induces again a maximum E_2 , but in such a direction as to *buck* the output voltage to $V_2 = V_1 - E_2$. The input current I_1 differs from the output current I_2 by the magnetizing current I_0 . The 1-ph regulator is always provided with a q-axis short-circuited winding, shown black in Fig. 12.1(a), to reduce the reactance in the midway position

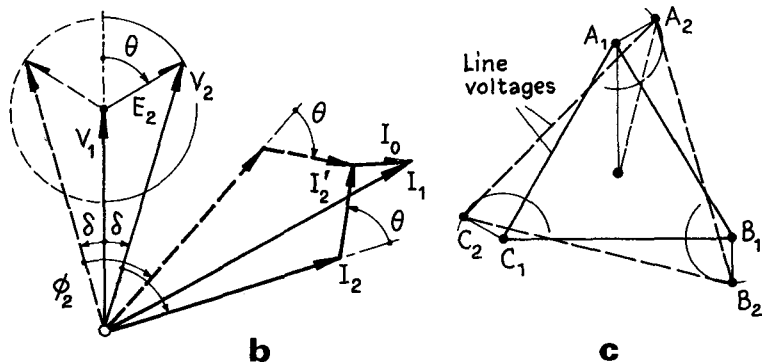
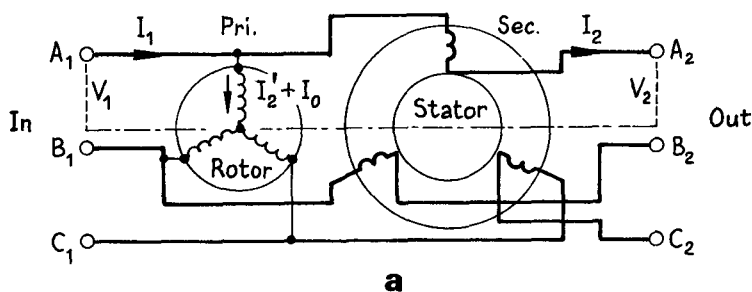


12.2 M.M.F.s and voltages in 1-ph regulator.

(d) and to stabilize the I^2R loss for a given output current over the whole boost-buck range. Variation of primary and compensating m.m.f.s with rotor displacement is shown at (a) in Fig. 12.2; and if the two windings have the same equivalent resistance the combined I^2R loss in the two together is substantially independent of rotor position. Fig. 12.2(b) gives an output-voltage/rotor-angle relation for a regulator with a $\pm 20\%$ variation.

Three-phase regulator. The electric and magnetic circuits are substantially similar to those of the 3-ph slip-ring induction motor, but with the primary supply to the rotor through flexible connectors, Fig. 12.3(a). The gap flux is a nearly constant travelling wave, so that the secondary phase e.m.f. E_2 is also constant, only its phase angle with respect to the primary voltage changing with the rotor position angle θ . The output voltage V_2 is the phasor sum of V_1 and E_2 , yielding the circular locus in (b). Regulation may be obtained along either half of the locus (full-line or dotted-line) depending upon the sense of θ .

Let the output per phase be I_2 at V_2 and phase angle ϕ_2 . The primary current is the phasor sum of the magnetizing current I_0 and the equivalent secondary current $I_2' = I_2(E_2/V_1)$. Inspection of the phasor diagram (b) shows that I_1 is smaller in magnitude for the full-line locus, and it is usual to arrange the rotor displacement sense to secure the lesser primary current. The phasor diagram (c) for all three phases indicates that the output voltage at $A_2B_2C_2$ is co-phasal with the input voltage at $A_1B_1C_1$ only at maximum boost and buck. For intermediate rotor angles there is a divergence δ reaching $\arctan(E_2/V_1)$. This is unimportant except where the regulator has to link two supply networks, for the displacement could then lead to large circulating currents. If necessary the displacement can be avoided by using a bank of 1-ph regulators or a *double regulator*, which comprises two mechanically coupled 3-ph regulators with their secondary windings in series and a common movement in opposing senses. An advantage is that the torque of one is to a large extent neutralized by the other.



12.3 Three-phase induction regulator.

The *torque* depends on the rotor position angle and the load, and is given by

$$M = kV_1I_2' \cos(\theta \pm \phi_2)$$

For a unity p.f. load this is greatest for maximum boost and buck, and zero for $\theta = 90^\circ$; for a zero p.f. load it is zero at maximum boost and buck. For a double regulator the torque has a resultant proportional to $\cos(\theta + \phi_2) - \cos(\theta - \phi_2)$, which vanishes for a load of unity p.f.

The *rating* is proportional to the maximum voltage boost. A 3-ph regulator for a 6.6 kV primary supply, to deliver 200 A/ph with a $\pm 10\%$ voltage variation, has the rating $\sqrt{3} \times 6.6 \times 0.1 \times 200 = 230$ kVA. Where the voltage exceeds 11 kV it is customary to supply the primary through a step-down transformer, and to reduce noise the regulator may be oil-immersed in a tank with cooling tubes.

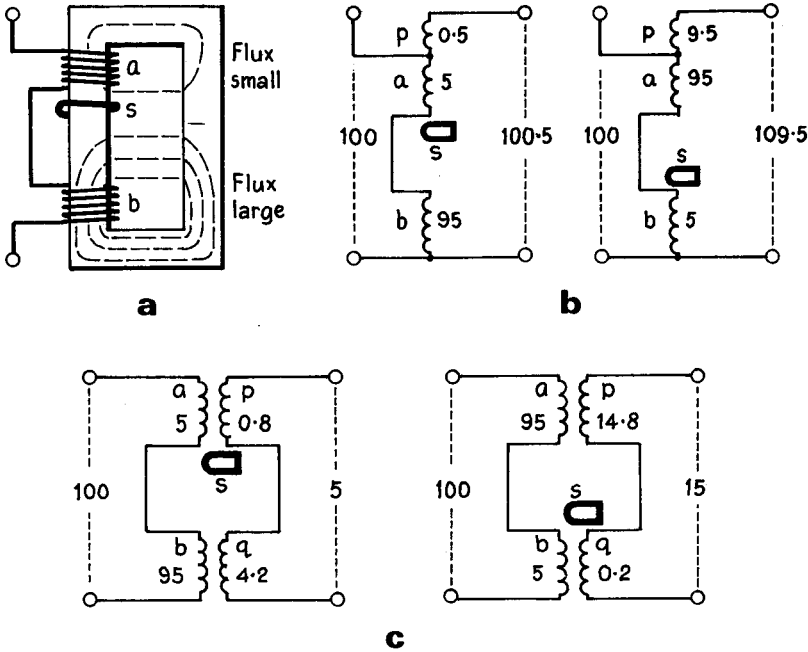
A general circuit theory of the double regulator is given by Jha [88], including a treatment of the biased form in which the locus of E_2 is elliptical.

Moving-coil regulator

Wide and stepless voltage control can be obtained by varying the coupling of coils on a common magnetic circuit. Fig. 12.4 shows the essentials of a 1-ph regulator: the 3-ph version comprises three 1-ph units, with the right-hand limb in common, but as this limb carries no resultant flux it may be omitted.

The two-limbed core in Fig. 12.4(a) has windings *a* and *b* in opposition at top and bottom respectively of one limb. (In practice each coil occupies nearly one-half of the limb space). An isolated short-circuited coil *s* can be moved up and down the limb. The closer *s* is to *a*, the greater is their mutual inductance and the lower is the input impedance of *a*. If a voltage be applied to *a* and *b* in series, the voltage distribution between them depends on their respective impedances, and the closer *s* is to one of them, the lower is the proportion of the applied voltage taken by it. The voltages across *a* and *b* can, in fact, be varied between 5 and 95% of the terminal voltage by positioning *s*, which by its induced current varies the leakage flux and therefore the reactance.

There are several useful arrangements. In the auto connection (b) an auxiliary winding *p* is used, with one-tenth of the turns of *a* or *b*. Coils *a* and *p* act together as a normal transformer, so that with *s* at the top and 5% voltage across *a* there is 0.5% voltage in *p* and the output voltage is 100.5%. With *s* at the bottom, the voltage of *a* is 95% and of *p* therefore 9.5%, giving an output of 109.5%. A smooth variation between these extremes is obtained by adjustment of the position of *s*. In the 2-winding arrangement (c) the turns-ratio of *a* and *p* is 6.43, and of *b* and *q* is 22.5. The output voltage is now variable between 5 and 15%. This simple regulator introduces no phase-angle displacement. It is discussed with further variants by Rawcliffe [102].



12.4 Moving-coil regulator.

Sliding-contact regulator

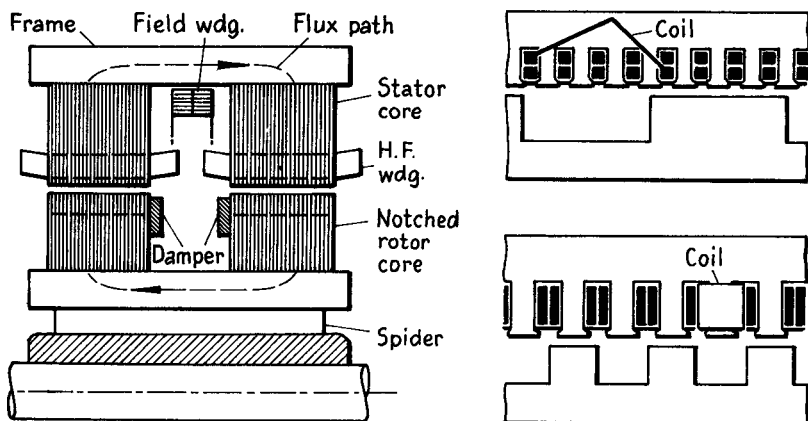
A two-winding or auto transformer with a toroidal core has the outermost winding bared over a circular track swept by a contact brush or roller, so picking off an output voltage in steps of the voltage per turn. The device ('variac') is essentially 1-ph and has a rating limited by the current that can be satisfactorily collected by the sliding connection.

12.3 Inductor generators

Unlike normal machines, these utilize the pulsation of a flux imposed by cyclic variation of the reluctance of the magnetic circuit. The constructional variants have the following terminology:

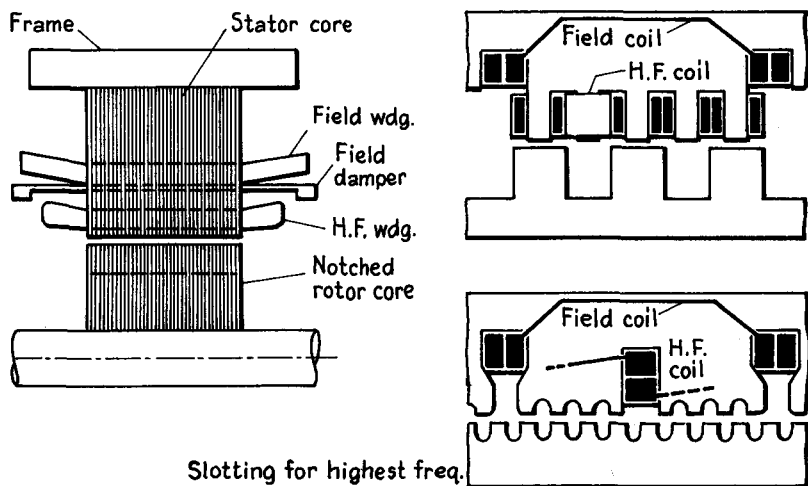
Field System. A working flux having no peripheral change in direction at the gap is termed *homopolar*: it has a field-coil axis parallel to the axis of rotation of the machine. A *heteropolar* system has alternate polarity around the gap periphery, with field coils wound in the plane of the armature coils.

Slotting. The *Guy* form has unwound slots ('inductors') on both sides of the airgap, with field and armature coils of equal pitch. The *Lorenz* form has semiclosed armature slots and an otherwise smooth stator airgap surface between the field slots; there are several armature coils within the pitch of one field coil. If the Lorenz machine has a dentated stator surface it is called a *Guy-Lorenz* generator.



12.5 Homopolar inductor generator.

The essential features are illustrated in Figs. 12.5 and 12.6. The homopolar machine has two stator and two rotor cores forming a single magnetic circuit, excited by an annular d.c. winding. The heteropolar machine has single stator and rotor cores and a short gap-length, an important advantage in raising the output. Both forms operate on cyclic changes of gap reluctance that occur when the rotor moves, optimized by a short gap and rectangular tooth shaping. In the *homopolar* machine, rotor movement through one tooth-pitch produces one complete cyclic variation of reluctance. With S_2 rotor slots (or



12.6 Heteropolar inductor generator.

teeth) the frequency for a rotor speed n is nS_2 , independent of the stator tooth arrangement. For best use of the flux variation the stator coil-pitch should be one-half of the rotor pitch.

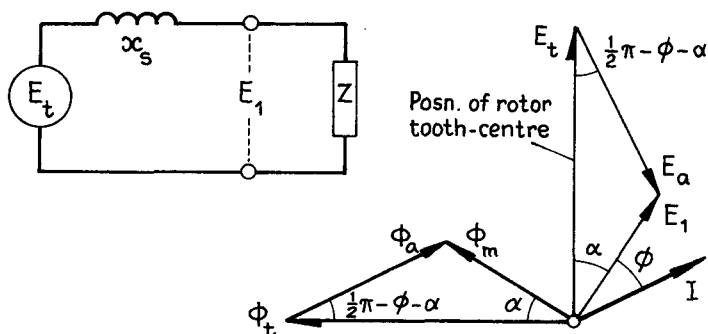
In the *heteropolar* machine, Fig. 12.6, the working flux is almost constant so that no alternating e.m.f. is induced into the d.c. exciting winding. To generate a frequency f with a rotor peripheral speed u the slot pitch is u/f , which may be too small for h.f. generation. It may be necessary then to adopt stator and rotor teeth of equal pitch with the stator coils lodged in larger slots of span equal to any suitable number of the smaller teeth.

Action

Analysis of the action in detail, concerned as it is with angular variations of reluctance or permeance, the effects of saturation and of h.f. currents, is complex. Bunea [89] has dealt with Guy machines, Davies and Kay [90] with Lorenz machines; they give *ad hoc* treatments of the flux distributions at the stator and rotor surfaces in analytic form. An elementary treatment is the following.

The gap permeance fluctuates as the stator and rotor teeth move in and out of register. Let it be represented by $\Lambda_g = \Lambda_0 + \Lambda_1 \cos \theta$, with θ an electrical angle of rotor displacement from the position of maximum gap permeance. Neglecting the iron-circuit effect, the no-load flux is $\Phi_0 = F_t \Lambda_g$ for a d.c. excitation m.m.f. F_t . On load the stator current is $i_1 \sin(\theta - \phi - \alpha)$ giving a corresponding reaction m.m.f. F_1 , where ϕ is the (lagging) load phase angle and α is an additional angle of lag caused by a shift in the position of the flux axis consequent upon the reaction F_1 . On load the flux is

$$\Phi = [F_t + F_1 \sin(\theta - \phi - \alpha)] (\Lambda_0 + \Lambda_1 \cos \theta)$$



12.7 Inductor generator equivalent circuit.

giving a constant (average) flux, a second-harmonic term here ignored, and a fundamental-frequency alternating flux

$$\Phi_m = \Phi_t + \Phi_a = F_t \Lambda_1 \cos \theta + F_1 \Lambda_0 \sin (\theta - \phi - \alpha)$$

The phasor diagram in Fig. 12.7 shows the e.m.f.s E_t and E_a corresponding to these flux components, and their resultant E_1 . The machine may be represented by a source e.m.f. separated from the terminals by a synchronous reactance $x_s = E_a/I_1$, and loaded on to an impedance Z . The apparent power output is a maximum for $Z = x_s$, and for this condition the output is

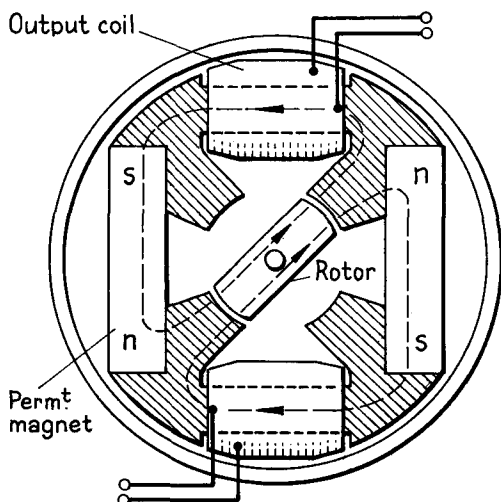
$$P = \frac{E_t^2}{4x_s} \cdot \frac{\cos \phi}{1 + \sin \phi}$$

In this analysis saturation and losses have been neglected. The core and load (stray) losses are large but difficult to assess with any precision.

Inductor generators have been built to generate 400 Hz supplies for aircraft, and to provide power for industrial h.f. heating in the range 0.25–10 kHz with ratings up to about 2 000 kVA for the lower frequencies. The generator is usually connected to its load through a series capacitor to improve the power factor e.g. of induction-melting equipment. For outputs between 0.1 and 5 kVA at frequencies from 1 to 10 kHz, *flux-switch* machines have been made, in which flux reversal in a pair of stator coils is achieved as indicated in Fig. 12.8 by the rotation of a very simple shaped rotor, the excitation being normally by means of a pair of permanent magnets. Such machines are designed to run at speeds in the range 150–850 r/s to give output frequencies from 1 to 10 kHz. The main disadvantages arise from the high centrifugal forces on the rotor and the high internal impedance.

12.4 Doubly-fed induction motors

A polyphase induction motor with balanced windings on both stator and rotor can operate as a synchronous motor at a specified speed if each member is connected to symmetrical polyphase supply. It can retain this speed over a range of mechanical load torque, exhibiting a synchronizing torque. Such a



12.8 Flux-switch machine.

machine could be used as a high-speed motor, or as a frequency-error detector with one member connected to a supply of reference frequency and the other to the source whose frequency deviation is to be detected.

Consider a 2-pole machine with stator and rotor fed respectively at angular frequencies ω_1 and ω_2 . Each member produces a travelling wave of m.m.f. around the gap with a direction corresponding to the phase sequence. Interaction produces a pulsating torque at all speeds ω_r except that for which $\omega_r + \omega_2 = \omega_1$, for only when the stator and rotor m.m.f. patterns rotate in synchronism can a sustained unidirectional torque be established. For this condition, $\omega_r = \omega_1 \pm \omega_2$, the sign depending upon the rotor phase sequence with respect to that of the stator. If these are the same, then $\omega_r = \omega_1 - \omega_2$, and a machine with both stator and rotor fed from the same supply remains at rest. But if the phase sequence of the rotor is reversed, then $\omega_r = 2\omega_1$ and the new synchronous speed is twice that given by ω_1/p .

The peak torque of the doubly-fed machine is always greater than that of a normal induction motor, and in consequence the peak output power in the double-speed condition approaches four times that of an induction motor with the same pole number $2p$. Unfortunately the doubly-fed motor suffers from (i) an inherent lack of starting torque, and (ii) the presence of damping torques tending to amplify a momentary divergence from synchronous speed and therefore to give rise to instability. Neither disadvantage has as yet been satisfactorily overcome. If a practical version of the machine could be devised, it might well prove to have considerable industrial significance.

12.5 Polyphase reluctance motors

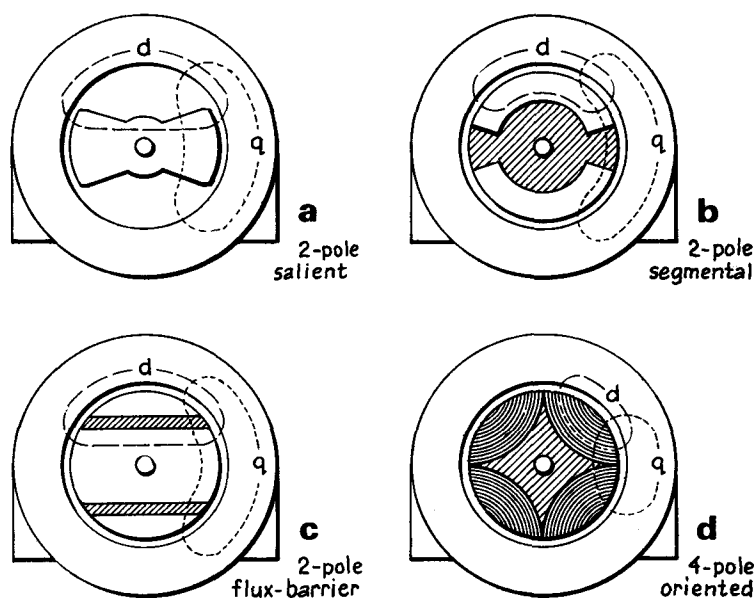
Reluctance machines have normal 3-ph primary (stator) windings producing

a travelling-wave field. The rotors are basically induction-motor equivalent cage-type structures, modified to produce saliency. The operation of the machine depends on the 'locking' of the rotor saliency with the travelling poles of the primary; the equivalent cage acts as a damper and (for fixed-frequency operation) to provide the asynchronous torque to start the machine and raise its speed to a value from which reluctance torque can provide synchronous pull-in.

Reluctance machines are cheap, robust and reliable; and despite their low power factor and specific output they have found application in several fields, such as positioning of control rods in nuclear reactors and providing textile-machine drives which are completely synchronized. In these drives the machine is adaptable to position- or speed-control or a combination thereof, using variable-frequency supplies such as those obtained from controlled switching of rectifier-inverter units. Where variable-frequency supply is at hand, the limitations imposed by pull-in requirements are avoided, the machine starting from rest with a frequency of zero and accelerating synchronously as the frequency is raised, permitting designs capable of maximum pull-out torque and relatively high power factor. Even for a simple form of motor, the maximum pull-out torque is typically five times the pull-in torque, and with permanently synchronous operation a power factor of at least 0.8 can be reached. An additional advantage of the machine is that for steady synchronous load there is no rotor I^2R loss.

Two-speed operation is possible. A conventional rotor in a 2-pole field rotates synchronously at an angular speed $\omega_r = \omega_1$ corresponding to the supply angular frequency ω_1 . However, the same rotor in a 4-pole field has still effective saliency, i.e. there are markedly preferred 4-pole axes of magnetization, and a torque is produced at angular speed $\omega_r = \frac{1}{2}\omega_1$. Provided that the rotor shape and proportions are properly chosen, it is possible to obtain good performance at each speed.

For variable-frequency operation the basic requirement of the rotor is an optimum saliency, a maximum difference in gap reluctance between the d-axis of the saliency and the q-axis in electrical quadrature therewith. For fixed-frequency operation the further essential is an adequate starting, run-up and pull-in torque. Various rotor constructions have been devised to fulfil these requirements: some typical designs are shown for 2-pole rotors in Fig. 12.9. The simple saliency in (a) may be improved on a basis of careful flux-pattern studies to increase the difference between the d- and q-axis reluctances. In (b) the shaped segments are obtained by interchanging the air and iron parts of rotor (a), and the preferred flux paths are now essentially circumferential instead of radial; by contrast the non-preferred or q-axis paths are of high reluctance on account of the large non-magnetic spaces between the segment ends. As a result a high reluctance ratio can be obtained without cutting away large sections of the rotor periphery and of the cage (which would be undesirable for asynchronous and pull-in performance). A further improvement is achieved by slotting the centre of each segment, the resulting form being then regarded as a combination of (a) and (b). A very simple rotor (c) is made by assembling laminations of the appropriate salient form (or machining a solid



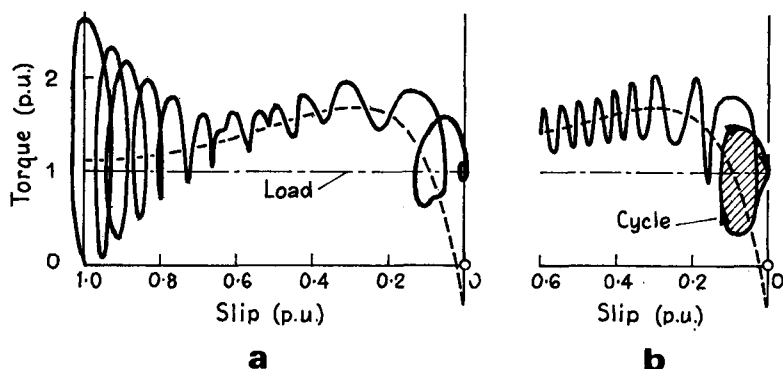
12.9 Reluctance motors.

steel rotor body) and flanking it with copper. Form (d) is made by winding a grain-oriented steel ribbon into an annular shape, deforming it into a rough square, and cutting it across into four pieces which are re-assembled to give a 4-pole rotor design.

Action

Basically the polyphase reluctance machine can be analysed in its various modes by applying the impedance matrix, Fig. 7.11, without the field winding F . However, the treatment in Sect. 7.5 is not readily employed where the flux patterns diverge considerably from the sinusoidal, which is the present case. The *general* conclusions hold, but the details of reluctance motor design present too great essential asymmetry for a valid analysis on these lines. The work of Lawrenson [91] has greatly advanced the technology of reluctance-motor design: he points out that the output torque is directly proportional to the difference between the d - and q -axis reactances, and analyses the gap permeance distribution on which this difference depends, deriving permeance and 'fringe' factors for machines in terms of the rotor geometry.

Starting. A machine working on a constant-frequency supply has to start by means of the asynchronous (induction) torque. Fig. 12.10 shows at (a) a typical torque/slip relation for a motor started from rest against a constant full-load torque, and synchronizing successfully. The saliency introduces considerable fluctuations, particularly near synchronous speed. The dotted line gives the 'steady-state' characteristic of the induction-torque component, which has



12.10 Reluctance motor: run-up torque/slip relations.

a small negative net value at zero slip: this makes the synchronizing process more difficult. The graph (b) shows the small-slip region corresponding to (a) but with an increased inertial load. In this case the motor just reaches synchronous speed but is unable to hold it, with the result that it runs asynchronously in a fixed cycle with large pulsations of both torque and speed. If either the dead load torque or the inertia (or both) were reduced, the machine could synchronize as in (a). Thus for any given machine the combination of load torque and inertia determines whether or not the machine can pull into synchronism.

Performance. The factors in achieving good performance are partly conflicting. A high reluctance ratio results in a high rated output and synchronous pull-out torque, but impairs the pull-in torque and stability. The design has therefore to be a compromise influenced by the duty and in particular the load inertia. For a 5 kW motor the following details are typical:

Reluctance ratio, 3–6; efficiency, 0.7–0.8; p.f. 0.6–0.8;

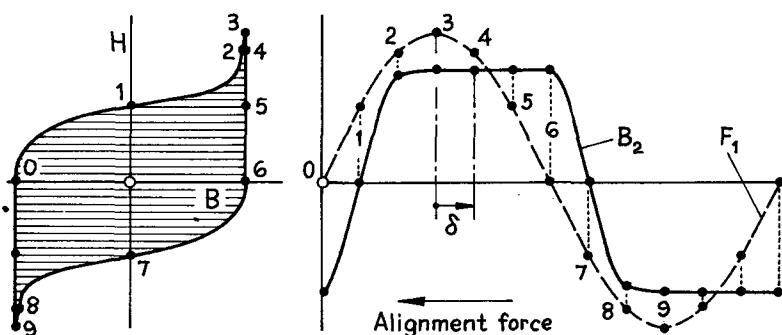
Pull-out torque, 2–2.5 p.u.; pull-in torque, 0.9–1.2 p.u.

The torques are expressed in terms of full-load torque.

Hysteresis motors

These machines have some resemblance to those operating on reluctance, but the polarization is produced in a different way. As a hysteresis motor can give an output no more than about one-quarter of that of an induction machine of the same dimensions, it is made only in very small ratings such as those suitable for driving gyros. The machine has a 3-ph (or 2-ph) conventional stator winding in semi- or fully-closed slots to reduce minor parasitic hysteresis loops. The rotor is an unwound cylinder of magnetically hard chrome or cobalt alloy steel with the characteristics of high hysteresis loss independent of the axis of magnetization. The material must have also a high resistivity to reduce eddy-current loss, so that for high-frequency supply the rotor may be laminated, sintered or built up from alloy powder set in a synthetic resin.

The travelling-wave of stator m.m.f. F_1 magnetizes the rotor, developing a distributed rotor flux of density B_2 . Hysteresis effects cause the rotor flux density to be displaced in angular position from the stator m.m.f., as shown in Fig. 12.11. At rotor speeds below synchronous the sine-distributed travelling wave of stator m.m.f. moves past a given elemental volume of the rotor,



12.11 Hysteresis motor action.

subjecting it to a slowly changing excitation H , and the consequent value of B is a function of the hysteresis loop of the core material. The excitation changes successively through the values 0, 1, 2 . . . to produce corresponding values of B . It is seen that there is an angle δ between F_1 and B_2 for the elemental volume, and therefore for the rotor as a whole. As a result a torque is developed in the direction of travel of F_1 . This torque is produced without rotor current, and as it depends only on δ it is independent of rotor speed up to synchronous speed, provided that the hysteresis loop does not change with the variation of the slip frequency. The motor thus has good starting properties and its ability to synchronize is not limited by the load inertia. It can be shown that the torque is proportional to the volume of the rotor and the hysteresis loss per cycle, i.e. the area of the BH loop.

At synchronous speed the relation of F_1 and B_2 around the rotor is unchanged in all respects except that the rotor flux is now fixed. Hence the torque remains constant. If the load torque reduces the rotor speed momentarily rises, with the result that magnetic changes begin in the rotor in a sense opposite to those that occurred during run-up. B_2 is reduced and the torque falls until the driving and load torques balance, possibly after a minor oscillation, and the final steady-state speed is synchronous.

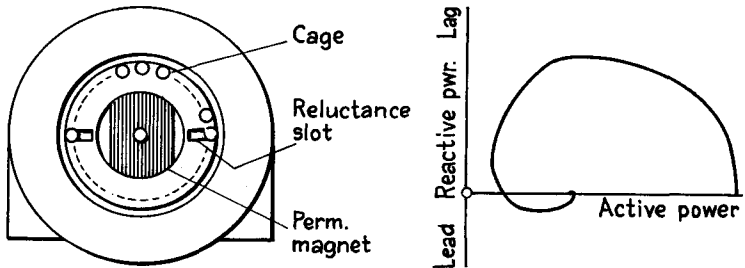
Hysteresis/reluctance motor

If two diametrically opposite slots are machined in the annular ring rotor of a 2-pole hysteresis motor an effective saliency is produced so that, although the hysteresis torque is somewhat impaired, a reluctance torque is introduced in compensation. If the two slots are used to accommodate a short-circuited copper winding, a third (induction) torque is added to improve run-up.

12.6 Permanent-magnet machines

A small synchronous machine, motor or generator, may have its excitation provided by a permanent magnet. A major problem, however, is to avoid demagnetization under service conditions. A generator may have its flux reduced to zero if a short-circuit occurs, and a motor is subjected to a reversal of flux every time it is started from standstill.

A permanent-magnet synchronous *motor* is constructed by fitting the magnet inside the rotor cage (which is necessary for induction starting) as in Fig. 12.12. The magnetic path through the annular core carrying the cage winding



12.12 Permanent-magnet synchronous generator.
Rotor construction and stator-current locus.

serves to protect the magnet, but it must be slotted to prevent the permanent-magnet flux from being 'short-circuited' in normal operation.

There are two periods during starting when large currents flow in the 3-phase stator winding: (i) at standstill on direct-switching, when the magnetic effect on the permanent magnet is almost entirely neutralized by the cage currents; (ii) at synchronous pull-in, when the cage is largely inoperative, and when in consequence demagnetization is most likely to occur if there is insufficient depth of annular core and if the magnet has too low a coercive force.

A permanent magnet *generator* does not require a starting cage on its rotor. Mainer [92] shows that the machine can be regarded as a self-regulating conventional synchronous generator in which the rotor m.m.f. F_2 is represented by the terminal m.m.f. of the magnet, with comparable expressions for power and a very similar phasor diagram.

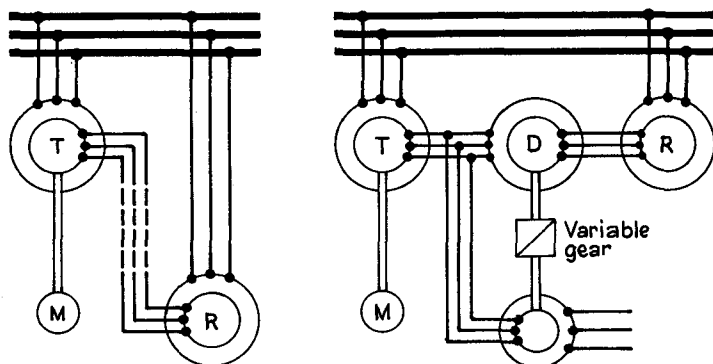
12.7 Power control machines

A travelling crane of wide span has little rigidity in the plane of travel. With drive from a single induction motor, a shaft must cross the span to link the motor with the gearing at each end. The cross-shaft cannot be regarded as a guarantee of alignment, as it is subject to flexure and twisting stresses. The use of two drive motors, one at each end of the span, is preferable to secure uniform motion, avoid skewing and improve adhesion; but it calls for an equivalent *electric shaft* or *electric tie*. Similar requirements occur in process industries (paper, steel, textile) with driving stages requiring specifically

related speeds, although these are more likely to be met by automatic speed control of each drive motor.

Power selsyns

Let two identical 3-ph slip-ring motors 1 and 2 have their stator windings connected to the same supply. When running at the same speed, their rotor phase e.m.f.s have the same magnitude but are displaced if the rotor positions with respect to the travelling gap field are not identical. If the slip-rings are interconnected in proper sequence, an advanced position of rotor 1 causes a circulating current which retards 1 and advances 2, so that the two rotors will tend to remain aligned. Machines so connected may be called 'power selsyns' if they are large enough to provide all the necessary load torque, and may be applied to crane drive. The electric shaft action tends to be lost if the machines run near synchronous speed, and there are various hazards in the system especially if the torque demand on the two machines is sensibly different. Under some operating conditions the speeds may oscillate.



12.13 Selsyns.

Pilot selsyns

When two 3-ph induction machines are interconnected as in Fig. 12.13 and their stator windings are energized, the two shafts retain a fixed relative angular position. Variation of the speed of the 'transmitter' T (by means of a small auxiliary motor or, in the case of a crane, by one of the rail wheels) causes the speed of the 'receiver' R to be similarly varied. If R is loaded, a small angular divergence is produced, the counterpart of the twist of a mechanical shaft. To change the speed of a receiver without changing that of the transmitter requires a frequency conversion, which can be achieved with the aid of a separate motor and variable gear to drive machine D, which acts as an *electrical differential*.

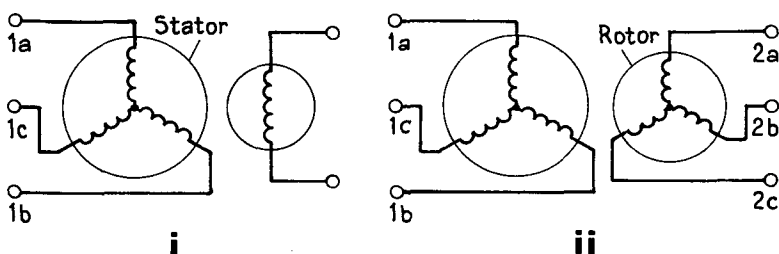
12.8 Servo control machines

A.C. machines developed for servo control systems include those for sensing,

position indication, actuation and speed tachometry. They resemble industrial machines in miniature, and are normally operated at frequencies such as 400 Hz, well above that of the standard power supply.

Synchros

The term 'synchro' is generic for a series of control machines, built with a 3-ph stator and a 3-ph or 1-ph rotor, Fig. 12.14 (i) and (ii); the two members



12.14 Synchros.

can be excited in a variety of ways. In construction great care is taken to preserve both mechanical and magnetic symmetry, as the accuracy of the synchro in service depends intimately on these qualities. Typical forms of synchro are listed below, with reference to the connections (i) and (ii) and the method of excitation.

Transmitter (i): The rotor is salient (structurally polarized) and a.c. excited; the three stator windings develop co-phased alternating voltages, of individual magnitudes depending on the angular position θ of the stationary rotor. Thus

$$v_{1a} = k \sin \theta \quad v_{1b} = k \sin (\theta - \frac{2}{3}\pi) \quad v_{1c} = k \sin (\theta - \frac{4}{3}\pi)$$

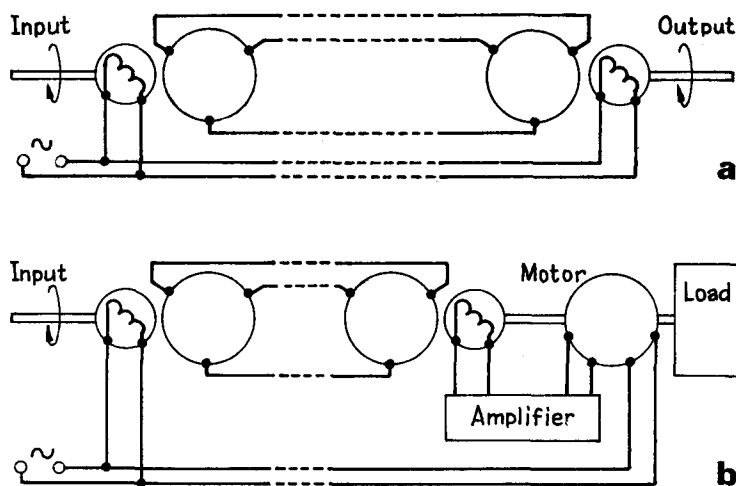
These voltages uniquely define the angle θ .

Control transformer (i). Appropriate input voltages are applied to the stator windings to define an angle θ ; the rotor output voltage is proportional to $\sin (\theta - \alpha)$, where α is the angular position of the rotor shaft. In this device the rotor is cylindrical.

Receiver (i). The rotor input is an alternating voltage, the stator input is a set of three alternating voltages defining an angle θ . With a rotor position-angle α , a torque proportional to $\sin (\theta - \alpha)$ is developed and appears at the rotor shaft.

Differential transmitter (ii). Both stator and rotor are cylindrical and carry 3-ph windings. When one member is excited with voltages defining an angle θ , the output voltages from the other member define a second angle $\beta = (\theta - \alpha)$, where α is the angular position of the rotor.

Differential receiver (ii). When three input terminals are connected to voltages defining an angle θ and the other three to voltages defining an angle α , a torque is produced on the rotor of magnitude proportional to $\sin (\theta - \alpha)$.



12.15 Synchro systems.

Typical synchro systems are illustrated in Fig. 12.15. System (a) employs direct transmission of position data without feedback. The output synchro must be supplied with an operating torque sufficient to position the load and overcome friction, and the unavoidable position-error is proportional to the load torque. In (b) the loop is closed, operation is now on the angular error, and the load torque is supplied by the amplifier-motor combination; the electrical torques on the synchros are minimized, and in consequence the overall angular error can be reduced to a very low level.

The expressions for the voltages, given above in terms only of angular position, represent the steady-state condition in which the rotor is stationary. When the rotor moves, rotational e.m.f.s as well as the useful pulsational e.m.f.s are developed, as discussed in Sect. 3.3.

Inherent asymmetries in synchros have to be minimized in order that position-angle accuracy shall be achieved. Manufacturing tolerances are imposed on the ellipsing of the stator and rotor members, eccentricity of rotor axis, difference in coils and in their resistance and leakage inductance, and variations in the pole-shaping of salient members. The presence of undue deformations leads to space harmonics. The chief such harmonic is the second, caused by ellipsing. Slotting produces higher-order harmonics. Residual errors, indicated by a voltage output at the null position for which it should be zero, result from nonlinearity and asymmetry in the ferromagnetic cores in spite of care in the assembly. It is usually necessary to 'pair' synchros together in order to select two machines in which the errors correspond and are so minimized. The items are compared with a synchro of known error and very high quality to avoid the very lengthy process of determining 'absolute' error.

Synchros are available in models that have maximum position-angle static errors from a few degrees down to less than 0.1 deg. Accuracy is greatly im-

proved by operation at a high frequency such as 400 Hz, and by using geared pairs at both transmitter and receiver ends. In the latter system each pair is geared e.g. by a ratio of 36/1 so that the motion of one is amplified at the second by the gearing. The arrangement allows the precision of synchro systems to approach that of the gear trains.

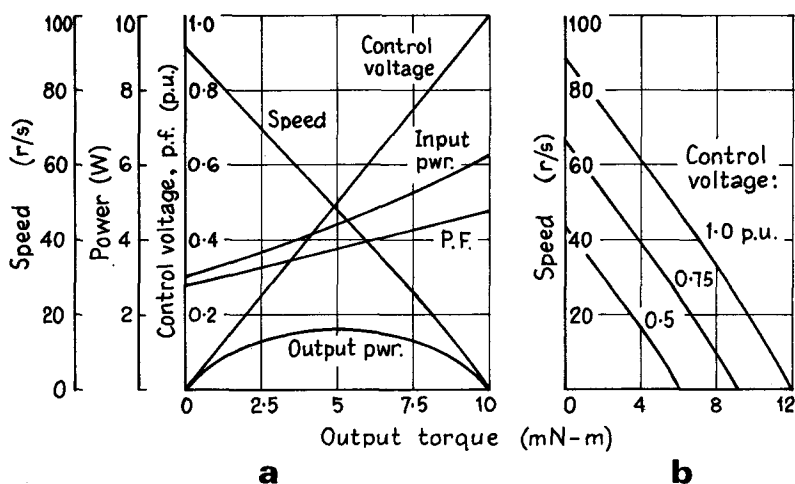
Control motors

Actuator requirements for servo systems are readily met by the 2-ph induction motor having a cage (or equivalent) rotor and two stator windings in space quadrature. The main stator winding is continuously excited to give a high torque per unit of power in the other (control) winding. The two windings are normally identical. The characteristics are generally designed to give a stalling torque proportional to the control-winding applied voltage, and a driving torque falling uniformly with rising speed for any given control voltage. Considering the machine as balanced (i.e. with main and control voltages equal and in phase quadrature), the rotor effective resistance is arranged so that maximum torque occurs at a slip between 1.5 and 2, about ten times that for a normal industrial polyphase machine: in the working range the torque/speed relation is then almost rectilinear. Although this limits the load and reduces the stalling torque, it gives better linearity and yields a torque approaching the maximum when reversing. The machine is inherently a high-speed low-torque device and must be geared down to drive the load, with a ratio to give the optimum stalling-torque/rotor-inertia figure. The length of the airgap must be very short, an achievable minimum being about 0.025 mm. High-quality steel is used for the cores, and the windings are encapsulated. Ratings above 1 kW are rare, for above this demand a d.c. machine is more satisfactory.

The 2-ph machine has the advantage that reversal of phase-sequence by means of the control winding gives a very simple reversal of torque. It is usual to base the torque and direction control on the amplitude of the control signal.

Design criteria differ basically from those for industrial power drive. Efficiency and power factor are unimportant; they are overridden by the requirement that the 2-phase machine give optimum performance in the servo system of which it forms a part. An important factor is that the torque should be zero when there is no signal applied to the control winding, a condition obtained by careful building and the provision of a high rotor resistance. A further factor is the inertia, because the duties of these small motors include rapid changes of speed and of direction of rotation.

Analysis. The electromagnetic structure of the servomotor is so complex that simplifying assumptions have to be made, with magnetic-circuit reluctance confined to the airgap, ideal stator and rotor current sheets, and slotting effects neglected except for skew. Hesmondhalgh [93] deals with the performance and design of these 2-ph machines: the basic design problem is that of producing maximum stall torque with required linearity for a given power input and temperature-rise. The small size of instrument 2-ph motors (e.g. rotor diameter 12.5 mm; stator outside diameter 40 mm; 8, 12, or 16 stator slots; 2-, 4-, 8- or 16-pole structure) and the relatively strong second-order effects make it necessary to relate design to measurements on experimental prototypes.



12.16 Two-phase servo motor characteristics.

Fig. 12.16 shows typical characteristics for steady-state conditions, to a base of torque. In (a), the *speed* curve is for full control-winding excitation and variable load torque; it is nearly linear, the effective rotor resistance being large. The speed drop with increase in load torque is the equivalent of viscous friction, providing damping in the servo system. The *power output* curve has naturally a roughly parabolic shape, peaking at about one-half of no-load speed. The *power input* and *power factor* are plotted with rated voltage on both windings and over the range no-load to stall. The *control* curve is the variation in stalling torque with control voltage; the curve is linear and gives the 'stiffness' $K = M_m/V_c$, i.e. the ratio of stalling torque to control voltage. The efficiency could be derived from the ratio of output to input power, but its peak value is quite low and, in an instrument motor, not significant. The curves in (b) show the relation between speed and torque for a range of three control voltages.

Tachometers

An induction tachometer is a small induction machine with a cage rotor and a stator with a pair of phase windings set in space quadrature. One stator phase is supplied at a constant voltage of constant frequency; an e.m.f. is generated in the other stator winding, of line frequency and of magnitude proportional to the rotor speed. The main errors arise from rotor asymmetry and divergence of the stator phase axes from strict space quadrature. Non-linearity is caused by resistance and leakage reactance, which reduce the primary flux as the speed is raised. Much of the error can be reduced by replacing the normal cage rotor by a 'drag-cup'.

Drag-cup tachometer. The drive shaft carries a copper cup, the cylindrical surface of which serves as a uniformly distributed cage. The cup shape is called

for to enable the copper to be rotated in the gap between the stator and a fixed internal core. The precision depends on the stability of the cup: uniformity is maintained by careful control of the material with respect to grain structure, stress-relief after annealing and during assembly, and by boring out of solid stock where tolerances must be minimized. To retain dimensional stability, the materials of all mating parts have matched expansion/temperature coefficients, and corrosion has to be prevented so that the drag-cup resistance is not increased. For very high precision it may be necessary to employ cup material (such as manganin) of low resistance/temperature coefficient, but the higher resistivity lowers the output. Low-accuracy cups may be made from drawn copper tube, or even aluminium if the inertia has to be limited. One of the errors likely to appear is the production because of asymmetry of an e.m.f. in the stator pick-up winding when the cup is at rest, giving the effect of a spurious rotational e.m.f.

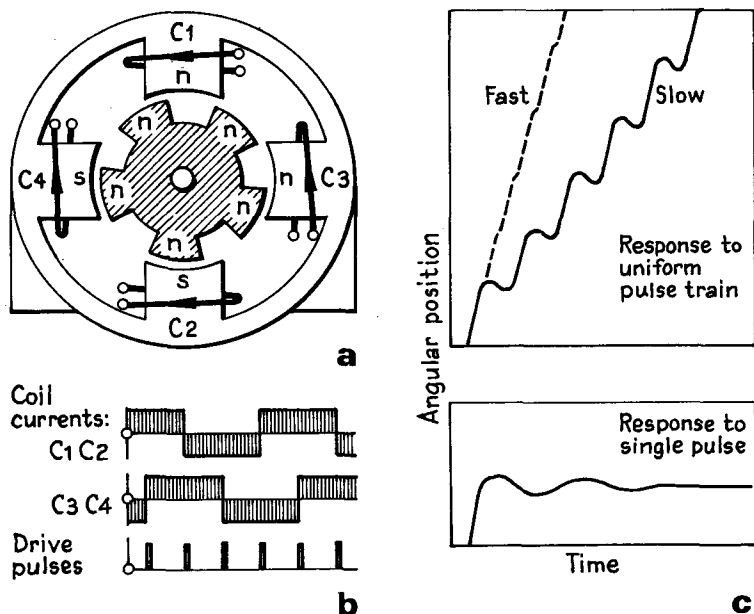
Permanent-magnet tachometer. This is essentially a 1-phase generator operating at variable and reversible speed. At low speeds the frequency of the pick-up e.m.f. is very low. The output may be rectified and smoothed, using a direction sensor on the shaft if indication of reversal is needed.

Stepper motor

A stepper motor is employed for the direct control of position. Its shaft is made to stop, with a high degree of angular accuracy, in any one of a number of positions in a complete revolution. A single step may, according to the design, correspond to an angular movement of the rotor between 1 and 180 deg, and is obtained by energizing the stator windings with constant amplitude of magnetizing current according to the dictates of a train of control pulses. By suitably timing the pulse sequence the effective motor output can be in the form of a constant or variable torque, or speed, or acceleration.

The essential operating feature is the movement of the rotor into a 'preferred' magnetic position. A typical arrangement is shown in Fig. 12.17. With the pairs of stator coils C1C2 and C3C4 energized as in (a) to give the indicated polarity, the 5-pole homopolar permanent-magnet rotor is in the position of minimum reluctance. The angle between rotor poles is 72° ; hence if either of the stator field pairs has its magnetizing current reversed, the least angle through which the rotor must move to regain a minimum-reluctance position is 18° , corresponding to 20 steps per revolution. A 50-pole rotor with a suitably excited stator would have 200 steps per revolution, with a step position error of $\pm 3\%$, non-accumulative. Thus if the stator is connected to a switching system which reverses the stator coil-pairs sequentially with the arrival of each pulse in a pulse train, as in (b), the motor rotates in a series of defined steps. The repetition rate of pulsing can be varied to give changes of speed. The stator supply has to provide corresponding pulses of constant current. Modern digital logic circuits with transistor switching can meet such demands economically.

The response of the rotor to the application of a step may be oscillatory with a frequency and damping depending on the inertial and frictional pro-

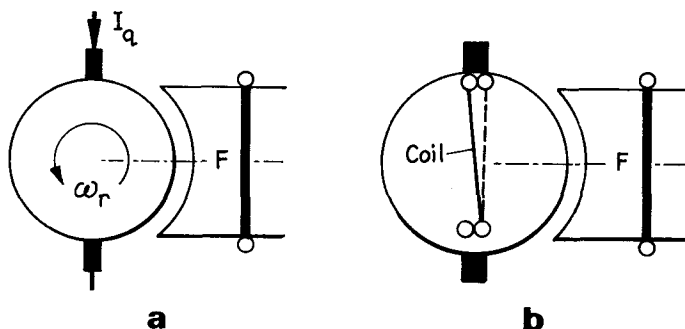


12.17 Stepper motor.

properties of the motor-load combination. Such a response is shown in Fig. 12.19(c), together with that for a rapid succession of pulses. The repetition frequency, if unduly high, may pulse the rotor when it has under- or overshoot, introducing position error or stalling: the precise instants of pulsing may therefore be critical. For applications such as machine-tool control, it is necessary to achieve a high slewing rate to start successive machining cycles, and the tendency to error or stalling can be eliminated by incorporating the stepper motor into a closed-loop control system. [Ref. 113]

12.9 Commutator machines

A synchronous machine can run stably only at synchronous speed; an induction machine runs at a speed related to the synchronous by the slip. Such restrictions are lifted by use of a rotor carrying a commutator winding and fed through a fixed brush system, because such an arrangement has a *frequency changing* effect, and wide speed ranges are possible in commutator machines. Consider, for example, a 2-pole d.c. (i.e. zero-frequency) machine, Fig. 12.18(a), with constant currents in its d-axis field winding F and in the connections to its diametral rotor brushes, which enable the rotor currents to set up a q-axis m.m.f. in the equivalent quasi-stationary coil Q . The direct current in a particular coil of the *actual* rotor winding reverses as the commutator sectors to which it is connected pass under the brushes, i.e. as they are conveyed by rotation from one side of the rotor to the other. The rotor-conductor currents thus alternate at the speed frequency ω_r : yet at the



12.18 Commutation conditions.

brushes the frequency is $\omega_1 = 0$. The commutator and brushes act as a frequency changer.

A similar conclusion holds for the e.m.f. between the rotor brushes. Individual coils have speed-proportional e.m.f.s of frequency ω_r which are rectified or 'commutated' by the commutator/brush system. The action is independent of rotor speed, accounting for the fact that stator and commutator-rotor windings can be connected in series to develop a unidirectional torque. As the direction of rotation and of torque are unaffected by reversal of the terminal polarity of a d.c. series machine, it can work satisfactorily if the supply voltage alternates at angular frequency ω_1 . The torque will, of course, fluctuate at frequency $2\omega_1$ (because the current goes through zero twice per period) but will remain unidirectional. Then across the rotor brushes the e.m.f. is proportional in magnitude to the rotor speed ω_r , while its frequency is ω_1 , i.e. that of the field in which the rotor rotates. The combination of a commutator rotor and a fixed-axis main-field system is thus applicable to a.c. supply and has the speed freedom associated with the d.c. machine.

Consider now a 2-pole rotor carrying a closed winding with its coils connected to a commutator, and also symmetrically tapped to slip-rings into which balanced polyphase currents are fed. The two basic conditions are: *Rotor stationary*. The polyphase currents set up a travelling-wave field of angular speed ω_1 corresponding to their frequency, and induce e.m.f.s of frequency ω_1 in the winding. The commutator brushes pick off an e.m.f. of the same frequency. Thus slip-ring, winding-conductor and commutator-brush e.m.f.s and currents all have the same frequency ω_1 .

Rotor rotating. If the rotor spins at angular speed ω_r , the speed of the travelling-wave field with respect to the winding conductors is still ω_1 because the tappings rotate with the rotor; the frequency of the conductor currents and e.m.f.s remains at ω_1 . But the field travels relative to the fixed brush axis at $(\omega_1 \pm \omega_r)$, and this is now the frequency in the external connections to the brushes.

Thus the commutator changes the frequency ω_1 in the conductors into $(\omega_1 \pm \omega_r)$ at the brushes. The sign depends upon whether the rotor rotation is in the same direction as, or opposite to, that of the travelling-wave field,

and the resultant frequency is in fact that of the *field* with respect to the *fixed brush axis*. This effect of frequency changing is basic to the winding arrangement of polyphase commutator motors.

Commutation

When a rotor coil undergoes commutation, i.e. while it is short-circuited by a brush as in Fig. 12.18(b), the current reversal is opposed by an e.m.f. of self inductance arising from the leakage flux in slots and overhang. This 'reactance e.m.f.' lengthens the commutation time and may lead to sparking. It appears in any commutator machine but is augmented in a.c. machines by further rotational and pulsational e.m.f.s, making the conditions of commutation much more severe. The e.m.f.s. concerned are:

Reactance e.m.f. This is directly proportional to the current, the turns per coil and the number of coils being commutated simultaneously. It is proportional also to the speed, as a higher speed shortens the available time for current reversal. It is an alternating e.m.f., the peak value of which occurs at peak current, and is therefore in phase with the current. The reactance e.m.f. in r.m.s. takes the form

$$E_{xc} = k_x I \omega_r$$

where k_x takes account of coil turns and brush width.

Rotational e.m.f. This is generated by rotation of the commutating coils in any flux Φ_c that may exist in the commutating zone, and is therefore given by

$$E_{rc} = k_r \omega_r \Phi_c$$

and has the frequency ω_1 of the alternating flux Φ_c . In poly-phase machines with a travelling-wave field Φ_1 of angular speed ω_1 the rotational e.m.f. is proportional to the relative speed of wave and rotor, so that

$$E_{rc} = k_r (\omega_1 \pm \omega_r) \Phi_1$$

Pulsational e.m.f. Fig. 12.18(b) shows that in a machine with a fixed main-flux axis, the short-circuited coils are coupled to the field winding F and so have a 'transformer e.m.f.' induced of

$$E_{pc} = k_p \omega_1 \Phi_1$$

where $k_p = k_r \cos \alpha$, with α the angle by which the brush axis diverges from the neutral axis position.

The resultant e.m.f. E_c is the phasor sum of the three components. Acting between brush heel and toe, it results in a circulating current that loads the commutating coils, heats the brush and degrades the quality of commutation.

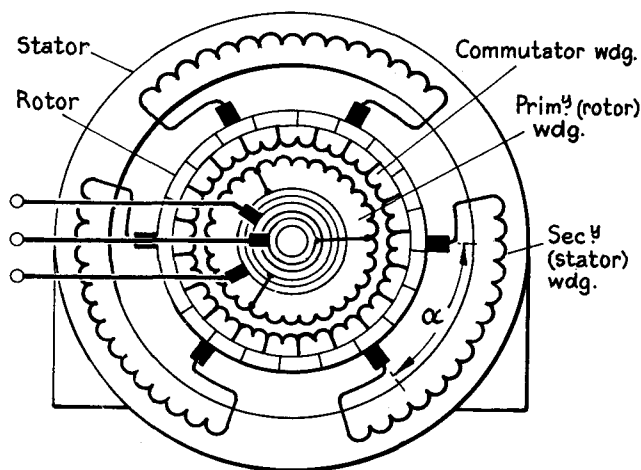
12.10 Polyphase commutator motors

The recovery of slip power (Sect. 8.14) can be achieved in a single machine if its rotor is provided with a commutator winding. Two such machines, the *Schrage* and the *doubly-fed* types, have some industrial application. Both

have a 'shunt' speed/torque characteristic and can operate over ranges both above and below synchronous speed.

Schrage motor

The primary 3-phase winding is carried by the rotor and is connected to the supply through slip-rings and brushes. The rotor also has a low-voltage commutator winding in the same slots as the primary with conductors set nearer to the slot-openings. The stator has a 3-phase distributed winding, each phase being terminated on a pair of commutator brushes. The brushgear comprises two movable rockers, geared together and fitted with three brush-spindles per pole-pair. The gearing is so arranged that the movement of a handwheel or pilot motor shifts the rockers in opposite directions. With the brush rockers aligned, the two ends of a stator phase winding are connected together through the brushes and the common commutator sector on which they stand. The opposing movement of the rockers enables any desired number of sectors to be included between these brushes. The arrangement of a 2-pole machine is shown in Fig. 12.19.



12.19 Schrage motor.

Consider the 2-pole machine with its primary fed at angular frequency ω_1 and rotating at angular speed ω_r . The travelling-wave rotor field moves at ω_1 with respect to the rotor body and at $\omega_1 - \omega_r = s\omega_1$ with respect to the stator. The stator (secondary) e.m.f. is therefore of slip frequency, and this is also the frequency of commutator-winding e.m.f.s at the brushes. Moreover, the coil-e.m.f. of the commutator winding is directly proportional to the secondary flux, which is very roughly a constant. Thus the e.m.f.s injected from the commutator into the stator phases are proportional to the number of commutator sector pitches between brushes connected to the

phase ends. With the brushes 'in line', the stator phases are in effect short-circuited and the machine runs at its natural slip as an inverted induction motor. When the brushes are moved so that the injected e.m.f. opposes the secondary current, the speed is reduced (positive slip); for the opposite movement the speed is increased (slip negative). For an electrical brush-separation angle α , the no-load speed is given approximately by

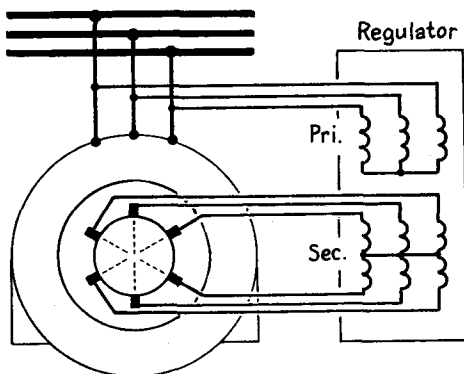
$$\omega_r = \omega_1 (1 - k \sin \frac{1}{2} \alpha)$$

where k is a constant depending on the turns-ratio of the stator secondary and rotor commutator windings. The limits of lower and higher speed are respectively about $0.4\omega_1$ and $1.6\omega_1$, a ratio of 4/1. Because of the additional winding, and in particular the commutator and brushes, the efficiency of the machine on load is a few percent lower than that of a corresponding induction machine. The power factor approaches unity at hypersynchronous speeds, but tends to be low at subsynchronous speeds. In non-reversing motors the power factor for all running speeds can be improved by giving the brush rockers an unequal divergence from the 'in line' position to change the phase-angle of the injected e.m.f.

Starting can be by direct switching with the brushes set in the lowest-speed position. The starting torque is typically 1.5 times full-load value with 1.5–2 times full-load current. Commutation difficulties limit the rated output to about 20 kW/pole, and the siting of the primary winding on the rotor restricts the main supply voltage to about 600 V.

Doubly-fed motor

The stator windings are identical with those of a conventional induction machine, while the rotor has a commutator winding with six brushes per pole-pair. The two members are connected to the main supply, the stator directly, the rotor (which has necessarily a low operating voltage) through a double induction regulator. The gap flux travels at synchronous speed, and



12.20 Doubly-fed motor.

as a result of the commutating function the brush e.m.f.s. have the frequency of the supply at all speeds of rotation. The induction regulator supplies the rotor windings with a speed-adjusting injected voltage, so that the speed can be varied from standstill to 1.5–2 times synchronous speed. At the zero-voltage position of the regulator, the brushes are in effect short-circuited through the regulator winding, and the motor operates as an induction machine with a rather high rotor impedance. Moving the regulator introduces the e.m.f. to give speed/torque relations similar to those of the Schrage motor. Unlike the latter, however, the doubly-fed motor is not self-contained; but it can be built for higher ratings and supply voltages, and the simpler brush gear makes the machine economic in ratings down to 2–3 kW. A schematic diagram of the connections is given in Fig. 12.20.

12.11 *Single-phase motors*

Apart from series traction motors still in use on low-frequency rail systems in Europe, 1-ph machines are uncommon in ratings over 5 kW. Fractional-kilowatt motors for industrial-frequency 1-ph supplies, however, account for four-fifths of the total numerical annual manufacture of electromagnetic machines. The range includes commutator motors with direct or inductively fed rotors, synchronous (mostly reluctance) motors and induction machines.

Commutator motors

Series motor. This resembles a d.c. series motor except that all ferromagnetic cores are laminated to limit core loss. Small motors for suction cleaners, drills and food mixers can run on d.c. and 1-ph supplies: the term 'universal' is no longer appropriate as few d.c. supplies are now extant.

Repulsion motor. This is a form of series motor with the rotor energized inductively. The rotor winding is designed for a low working voltage, and its brushes are joined by a short-circuiting connector to provide a closed current path. The brush axis is displaced from quadrature with the stator d-axis so that the induced rotor current can develop interaction torque. When the motor spins the d- and q-axis fluxes have a phase displacement, developing a crude travelling-wave resultant of 'elliptical' form. At a rotor speed corresponding to the synchronous, a nearly uniform travelling flux wave is attained, so that the commutation conditions are relatively good.

Repulsion-induction motor. If, in addition to an outer commutator winding the rotor carries an inner cage, the torques developed are additive. At starting the cage current is limited by its comparatively high reactance and the commutator winding develops a major part of the torque; but during run-up the cage torque rises, compensating for the natural drop in the torque/speed characteristic of the commutator winding. At normal operating speeds the low impedance of the cage allows it to develop most of the load torque. On no load the repulsion torque drives the rotor slightly above synchronous speed, balancing the counter-torque of the cage resulting from its negative slip. In the *repulsion-start* form of the machine, a mechanical device based normally on centrifugal force acts to short-circuit all the commutator sectors

together at about 75% of synchronous speed, converting the winding effectively into a cage for normal-speed running.

Synchronous motors

The accurate frequency stability of mains a.c. supplies has encouraged the use of small 1-ph synchronous motors for clocks, timers and turntables. The stators have laminated cores excited by a mains-fed winding, often multi-polar, with shading rings to develop starting torque. The rotors can be simple toothed steel discs with a strong reluctance-torque capability enabling the motor to synchronize rapidly after switch-on.

Induction motors

The 1-ph induction motor is applied to many low-power drives requiring constant speed where a 3-ph supply is not to hand and the output does not exceed a few kilowatts. Most motors have ratings between 100 and 500 W for domestic refrigerators, fans and small tools. The rotor is always a simple cage-wound structure.

Plain motor. The single pulsating field produced by the stator induces a co-axial current and m.m.f. distribution in the rotor, with the result that the machine at standstill has no starting torque. In the absence of some external means for giving the rotor an initial spin, there must be some internal method for developing a travelling-wave field in the gap.

Shaded-pole motor. The stator has salient poles with about one-quarter of each pole-face embraced by a conducting ring. The flux through this shading ring is phase-delayed with respect to the remainder of the pole flux, resulting in the development of a 'shifting field' and a starting torque. The efficiency is low, the p.f. is between 0.5 and 0.6, the pull-out (peak) torque about $1\frac{1}{2}$ times full-load torque, and the starting torque rather limited. Applications are to loads such as fans with low starting demands and small rating, e.g. 100 W.

Split-phase motor. The stator carries a main (running) and an auxiliary (starting) winding, connected in parallel and arranged in space quadrature. The starting winding has a high resistance, either inherent or provided by a series resistor. At starting the main current I_1 lags the applied voltage by 70–80 deg, the auxiliary current I_s by 30–40 deg, so that a non-uniform travelling-wave field results, with a rotor torque proportional to $I_1 I_s \sin \alpha$, where α is the difference between the respective angles of lag. A starting torque of $1\frac{1}{2}$ –2 times full-load value can be achieved. After run-up the starting winding may be open-circuited, and the machine then operates with a p.f. of 0.6–0.7 and an efficiency of 0.55–0.65.

Capacitor split-phase motor. A greater phase-difference α can be obtained if a capacitor (typically between 30 and 100 μF) is substituted for the external resistor of the starting winding. In the *capacitor-start* machine the starting winding is open-circuited when the motor is up to operating speed; for *capacitor-run* it is left in circuit to improve the p.f. Ideally, however, the capacitance for running should be less than one-half of that for starting if both conditions are to be optimized.

Two-speed motor. The principle of pole-amplitude modulation (Sect. 8.15) can be applied to a split-phase motor to obtain two operating speeds. It is shown by Broadway and Weatherley [94] that grading the number of turns in successive coils of the two stator windings provides the flexibility needed to design satisfactory p.a.m. machines that will give at each speed a performance comparable to that of the conventional single-speed motor. There are strong harmonics in the gap-flux distribution, but these are not so objectionable in a small 1-ph motor as they would be in a larger 3-ph machine.

Asymmetric-cage motor. Consider a 2-pole machine with a single stator winding and consequently no inherent starting torque. Let one pair of cage bars (at opposite ends of a diameter in a 2-pole case) be removed: then the rotor acquires a small degree of magnetic saliency. Provided that the axis of saliency is out of alignment with the axis of the stator winding when the machine is at rest, an initial alignment torque occurs at switch-on, the resulting movement being the necessary starting impulse. The start depends primarily on the initial rotor position, there being a range for which the motor will make a unidirectional start. Only light starting duty can be satisfied, but the machine has the advantage of needing no auxiliary stator winding. The running condition is similar to that of a conventional induction motor.

12.12 Single-phase commutator motors

Universal motor

This machine has a wide and easily controllable speed range, typically 10 000–3 000 r/min, a rating up to about 500 W and a similar performance on 1-ph and d.c. supplies giving an inverse-speed (i.e. 'series') characteristic. To reduce winding reactance it is preferably a 2- or 4-pole motor without commutating or neutralizing windings, reliance being placed on brush selection to ameliorate commutation conditions, for in cheap machines (and cost is the competitive factor) with few rotor slots and sectors the reactance and transformer voltages in the commutating coils may sum to 12–14 V. Brush pressures are high to counteract 'bounce', and their positioning is crucial to satisfactory commutation.

The rotor winding, in slots skewed by one pitch, is conventional. The stator laminations have salient poles with shanks shaped to accept machine-wound coils. To avoid saturation, large core sections and short airgaps are employed, reducing the requisite number of field turns and limiting the exciting m.m.f. Production tolerances on rotor eccentricity within the stator bore must be such as to keep down unbalanced magnetic pull which may cause bearing 'hammer'.

The behaviour equations of the machine can be set up from the discussion in Sect. 7.5, or directly from the matrix in Fig. 7.11. Reducing the motor to its simplest terms, it has a d-axis stator winding F and a q-axis rotor winding Q ; the respective voltages v_f and v_q sum in series connection to the applied terminal voltage v_1 , and the input current is $i_1 = i_f = i_q$. Then

$$v_1 = [r_f + r_q + (L_f + L_q)p + (L_{ad} + L_{aq})p - L_{ad}\omega_r] i_1$$

For steady-state sinusoidal supply ($p = j\omega_1$), putting $r_f + r_q = r$ and $\omega_1(L_f + L_q) = x$ for the total resistance and leakage reactance, then in r.m.s. terms

$$V_1 = [r + jx + j(x_{ad} + x_{aq}) - x_{ad}(\omega_r/\omega_1)] I_1$$

The components of the applied voltage are the volt drops in resistance and leakage reactance, the magnetizing reactance for the d-axis flux (useful) and q-axis flux (non-useful), and a speed-dependent term which is the e.m.f. of rotation $E_r = L_{ad}I_1\omega_r$. The torque has a double-frequency pulsation with the r.m.s. value

$$M = -x_{ad}(\omega_r/\omega_1)I_1^2$$

giving an inverse-speed ('series') torque/speed relation.

In the practical machine there is a third winding that, particularly in small machines, has an important influence: this is the effectively fixed q-axis winding KD formed by the short-circuited coils undergoing commutation. Its voltage equation is

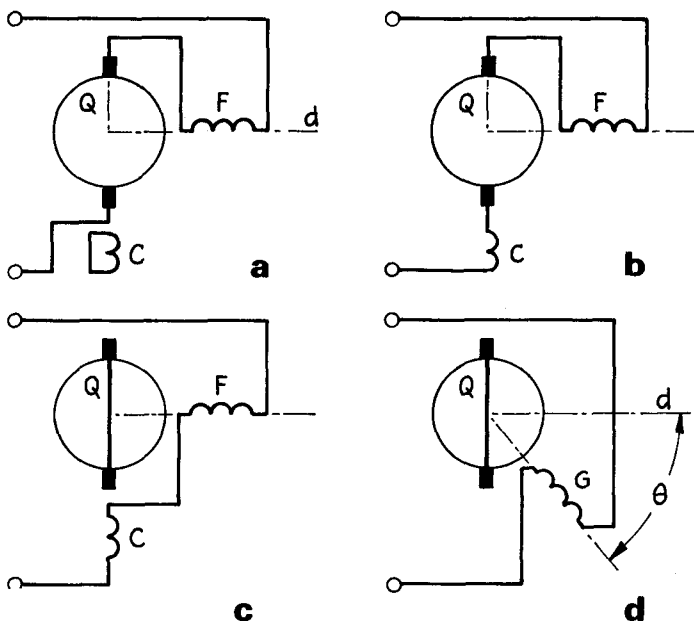
$$0 = L_{ad}p i_1 + [r_{kd} + (L_{ad} + L_{kd})p] i_{kd}$$

which relates i_{kd} to i_1 . Even this is not enough, for the commutating coils have also a magnetic effect due to reversal of i_1 . Again, a major component of the closed circuit is the nonlinear resistance arising from the brush/commutator contact. The problem would be treated by a designer in a different way, using perhaps the itemized analysis of commutation e.m.f.s as in Sect. 12.9, and with careful verification of his conclusions by test.

Neutralized series motor

The q-axis flux produced by the rotor has no useful torque production; it serves only to increase the series reactance of the machine and lower its p.f. Two methods of *neutralizing* (or 'compensating') the rotor flux is by adding to the stator a q-axis winding, distributed over the bore to simulate the rotor conductors, and capable of fully opposing its m.m.f. The neutralizing winding C may be isolated and closed on itself, acting then as the short-circuited secondary of a transformer and reducing the rotor flux to a low leakage value. The arrangement is shown diagrammatically in Fig. 12.21 (a). The alternative method (b) is to connect the neutralizing winding in series with the main stator and rotor windings. Method (a) has the advantage that the number of neutralizing turns is more freely chosen, for in (b) it is necessary for the neutralizing and rotor windings to be closely matched.

In large motors of several hundred kilowatts rating, commutation dominates the design. The transformer e.m.f. in the coils undergoing commutation (Sect. 12.9) is proportional to the coil turns, main flux and supply frequency. It is limited in magnitude and effect by (i) use of a multipolar structure, by which the flux per pole and the field reactance are reduced; (ii) narrow high-resistance brushes and single-turn rotor coils, making necessary a commutator of large diameter and axial length; (iii) current-limiting resistive connectors



12.21 Single-phase commutator motors.

between the coil-ends and the commutator sectors; (iv) the provision of commutating poles in the brush zone, shunted by resistors to secure a proper phasing of the commutating-pole flux; and (v) the use of a low supply frequency, such as $16\frac{2}{3}$ Hz.

Repulsion motor

Inductive neutralization of the q-axis rotor flux by means of a coupled short-circuited stator winding C is shown in Fig. 12.21 (a). The functions of the neutralizing and rotor windings can be interchanged, as in (c), with the former a series-fed winding and the latter having a short-circuit connection across the brushes. The two stator windings, F and C, can further be combined into a single winding G with its axis displaced from the d-axis by an electrical angle θ as in (d). With $\theta = 0$, the stator and rotor winding axes are in quadrature and have no mutual inductance: no e.m.f. can be induced in the rotor at rest, no rotor current flows (except non-usefully in the coils short-circuited by the brushes) and there is no torque. The stator takes mainly a magnetizing current and its input impedance is high. With $\theta = 90^\circ$ the stator and rotor axes coincide and there is a large induced rotor current: but there is no d-axis flux with which it can interact so again there is no torque. In this condition the stator input impedance is low, as the machine is acting like a transformer with a short-circuited secondary. At an intermediate value of θ , typically an optimum between 50° and 70° , the stator winding can be considered as resolved into the two windings F and C in Fig. 12.21 (c): there is a q-axis

flux to induce rotor current, and a d-axis flux for interaction torque production.

A simple basic assessment of the action of a repulsion motor is obtainable by considering an *ideal* 2-pole machine with separated stator winding (c), neglecting resistance and leakage reactance in windings F, G and Q. The stator current I_1 of angular frequency ω_1 develops in F the d-axis flux Φ_d in phase with I_1 ; but the q-axis flux Φ_q in G and Q results from the combination of their m.m.f.s, so that Φ_q lags 90° on I_1 . Let the rotor spin at angular speed ω_r . Across its brushes there is the e.m.f. $E_r = k\omega_r\Phi_d$ due to rotation in Φ_d and $E_p = k\omega_1\Phi_q$ induced in it by Φ_q . Now E_r is in phase with Φ_d while E_p is in quadrature with Φ_q , and with zero rotor leakage reactance they balance to give $E_r = E_p$ so that

$$\omega_r\Phi_d = \omega_1\Phi_q \quad \text{giving} \quad \Phi_q/\Phi_d = \omega_r/\omega_1$$

If $\omega_r = \omega_1$, therefore, the two axis fluxes are equal in magnitude and as they are also in both space and time quadrature, they produce together a constant synchronously travelling-wave field. Below synchronous speed $\Phi_d > \Phi_q$, above it $\Phi_d < \Phi_q$ and the travelling-wave field is 'elliptical'.

Commutation can be considered with respect to E_{rc} and E_{pc} in Sect. 12.9. These with appropriate change of subscripts are $E_{rc} = k_r\omega_r\Phi_q$ and $E_{pc} = k_r\omega_1\Phi_d$, whence with the flux/speed relation above and due regard to phasing, the net e.m.f. in the commutating coils (apart from reactance voltage) is $E_c = E_{pc}[1 - (\omega_r/\omega_1)^2]$ which is zero for synchronous speed, at which the commutating conditions are the least unfavourable.

Repulsion motors are built in ratings up to about 5 kW for normal industrial frequencies. The rotor winding is short-pitched to aid commutation and, being isolated, can be designed for optimum turns and sectors. The fully wound stator has a winding layout chosen to give an m.m.f. distribution roughly matching that of the rotor. The brushes are usually fixed, but adjustment of the torque/speed characteristic is possible if the brush axis can be shifted. This facility also enables the direction of rotation to be reversed, though reversal of the stator winding connections gives the same effect and is cheaper.

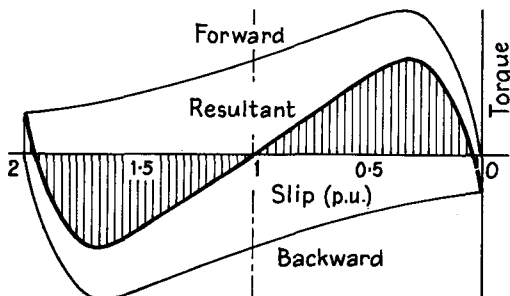
12.13 Single-phase induction motors

The several forms are briefly described in Sect. 12.12. Here we deal with the basic cage and shaded-pole machines.

Plain cage motor

The rotor has a simple skewed cage winding. The single stator winding can, ignoring harmonics, be considered to provide a sinusoidally distributed gap m.m.f. of peak value F_1 pulsating at supply frequency ω_1 . The behaviour of the machine can be based on (i) the *rotating-field* theory, or (ii) the *cross-field* theory. The latter is an application of the 2-axis field concept, Sect. 7.5. *Travelling-wave theory*. The fixed-axis alternating stator m.m.f. is regarded as the combination of a forward- and a backward-moving m.m.f. component,

travelling in opposite directions around the stator and exciting corresponding travelling-wave gap fluxes. With the rotor at standstill, each component m.m.f. has the constant value $\frac{1}{2}F_1$; each component flux acts as in a polyphase machine to develop torque, but the torques oppose and have no resultant and in consequence the motor is not self-starting. When the rotor spins in the forward direction, the forward component develops a torque/slip relation as indicated in Fig. 12.22. The backward component gives a similar relation



12.22 Single-phase induction motor: torque/slip relation.

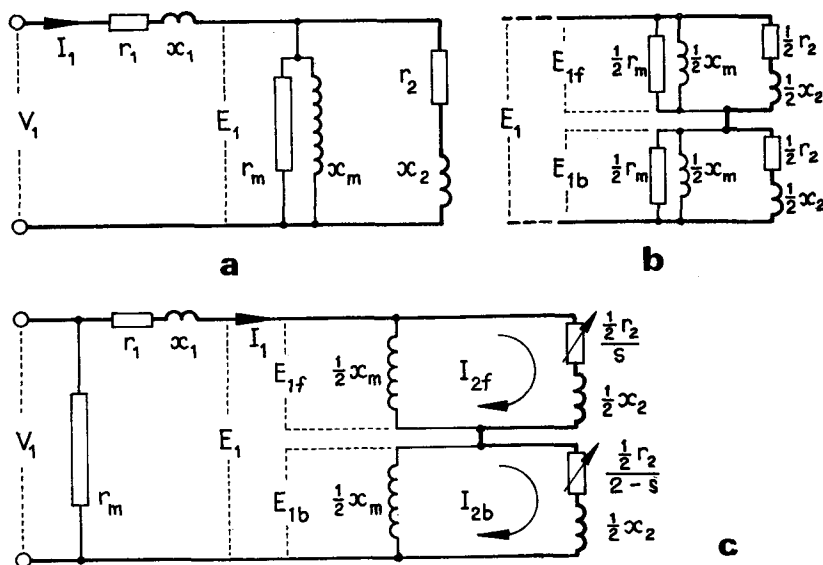
but with reference to a slip not of s but of $(2 - s)$. The resultant torque is therefore the difference, showing a zero at standstill for which $s = (2 - s) = 1$, and another zero close to synchronous speed. These basic features of the resultant torque/slip curve are shown in Fig. 12.22, but the intermediate values are only approximate, for while the stator m.m.f. components are equal, the gap fluxes depend also upon the corresponding rotor m.m.f. components, which differ by reason of dissimilar impedances at all slips other than unity.

The motor runs in either direction given an externally applied torque to start it against friction and load. The normal slip is greater than that of a comparable 3-ph machine, and the backward-moving field introduces additional rotor core loss. As in all 1-ph motors, the resultant torque pulsates at frequency $2\omega_1$. The efficiency and p.f. are typically 0.4 and 0.45 for a 40 W motor and 0.7 and 0.65 for a 400 W motor.

With the nomenclature of Sect. 8.2, the equivalent circuit (a) in Fig. 12.23 can be set up for the machine at standstill. The two flux components induce in the stator winding the respective e.m.f.s E_{1f} and E_{1b} . These sum to the applied voltage V_1 (less volt drops in the stator resistance r_1 and leakage reactance x_1), and so can be represented as in (b) with one-half of the magnetizing and rotor impedances attached to each in series. A simplifying approximation is to take the core, friction and windage loss as represented by r_m across the stator terminals: then for a slip s in the forward direction the equivalent circuit for running conditions is that in (c). In terms of synchronous power the net torque is

$$M = I_{2f}^2 \left[\frac{1}{2} r_2 / s \right] - I_{2b}^2 \left[\frac{1}{2} r_2 / (1 - s) \right]$$

giving the torque/slip relation in Fig. 12.22.



12.23 Single-phase induction motor: equivalent circuit.

EXAMPLE 12.1: A 200 W, 240 V, 50 Hz, 4-pole, 1-ph induction motor runs on rated load with a slip of 0.05 p.u. The parameters are: $r_1 = 11.4 \Omega$, $x_1 = 14.5 \Omega$, $\frac{1}{2}r_2 = 6.9 \Omega$, $\frac{1}{2}x_2 = 7.2 \Omega$, $\frac{1}{2}x_m = 135 \Omega$; core and mechanical loss, 32 W. Estimate the full-load performance.

Forward circuit. With $s = 0.05$, the rotor impedance is $(\frac{1}{2}r_2/s) + j\frac{1}{2}x_2 = 138 + j7.2 \Omega$: in parallel with $j\frac{1}{2}x_m = j135 \Omega$ this gives

$$z_{2f} = 64 + j69 = 94 \angle 47^\circ \Omega$$

Backward Circuit. Here $\frac{1}{2}r_2/(2-s) = 3.5 \Omega$ and $\frac{1}{2}jx_2 = j7.2 \Omega$ in series. The result of parallel combination with $j\frac{1}{2}x_m$ is

$$z_{2b} = 3 + j7 = 7.6 \angle 67^\circ \Omega$$

Output. The total series input impedance is $z_1 + z_{2f} + z_{2b} = 78.4 + j90.5 = 120 \angle 49^\circ$, the p.f. being $\cos 49^\circ = 0.66$. The forward and backward active rotor currents and torques are found as follows:

$$\begin{array}{ll} E_{1f} = 2.0 \times 94 = 188 \text{ V} & E_{1b} = 2.0 \times 7.6 = 15.2 \text{ V} \\ I_{2f} = 188/138 = 1.36 \text{ A} & I_{2b} = 15.2/8.0 = 1.9 \text{ A} \\ M_f = 1.36^2 \times 138 = 255 \text{ W} & M_b = 1.9^2 \times 3.5 = 13 \text{ W} \end{array}$$

The net torque (as a synchronous power) is $M = 255 - 13 - 32 = 210 \text{ W}$ and the shaft power is $210 \times 0.95 = 200 \text{ W}$. The input power is $240 \times 2.0 \times 0.66 = 316 \text{ W}$ and the efficiency is $200/316 = 0.63$ p.u.

Cross-field theory. The treatment is that in Sect. 8.21 with the one stator winding 1D. The circuit equations are then written with an applied voltage

v_1 and the stator current i_1 . For steady-state conditions with slip s , rotor speed $\omega_1(1-s)$, and with the supply frequency ω_1 combined with the inductances to form reactances, the equations in r.m.s. terms are

$$1D: V_1 = [r_1 + j(x_m + x_1)] I_1 + jx_m I_{2d}$$

$$2D: 0 = [r_2 + j(x_m + x_2)] I_{2d} + jx_m I_1 + (1-s)(x_m + x_2) I_{2q}$$

$$2Q: 0 = [r_2 + j(x_m + x_2)] I_{2q} - (1-s)x_m I_1 - (1-s)(x_m + x_2) I_{2d}$$

From these, I_1 , I_{2d} and I_{2q} can be expressed in terms of V_1 , the slip s and the circuit parameters, and the torque derived. The evaluation is tedious, but it is possible to relate I_{2d} and I_{2q} to the forward and 'backward' currents of the rotating-field theory by the phasor expressions

$$I_{2d} = [I_{2f} + I_{2b}]/\sqrt{2} \quad \text{and} \quad I_{2q} = -j[I_{2f} - I_{2b}]/\sqrt{2}$$

Making the substitutions leads to equations in I_1 , I_{2f} and I_{2b} for the equivalent circuit of Fig. 12.23(c).

Single-phase cage motors are always built with some means for starting, usually an auxiliary stator winding. If this winding is in space quadrature with the main winding, an additional equation for 1Q is required. Often, however, the angle between the windings differs from $\frac{1}{2}\pi$ elec. rad, in which case it may be represented by d- and q-axis equivalent windings in series. Leung and Szeto [95] set up the impedance matrix for such a case, and show that it can be applied to most forms of 1-ph motor provided that the stator core is cylindrical. Performance can readily be computed if the winding parameters are known with adequate accuracy.

Performance. Typical values of efficiency and p.f. for small 1-ph 50 Hz cage motors are:

Rating	W:	40	100	200	400	750	1 000
Efficiency	p.u.:	0.38	0.50	0.60	0.68	0.72	0.75
Power factor	p.u.:	0.45	0.55	0.60	0.65	0.67	0.70

The starting methods outlined in Sect. 12.11 are applied to suit the duty required. The following Table gives typical starting currents and torques in per cent of full-load values:

Machine	Rating kW	Current p.u.	Torque p.u.
Split-phase: resistance	0.01–0.04	6.0	1.5
capacitance	0.1–0.8	5.0	3.0
Repulsion-start	0.1–4.0	2.5	3.0
Repulsion-induction	0.4–4.0	3.5	3.0
Capacitor-run: one capacitor	0.1–0.4	2.5	0.5
two capacitors	0.1–4.0	5.0	3.0

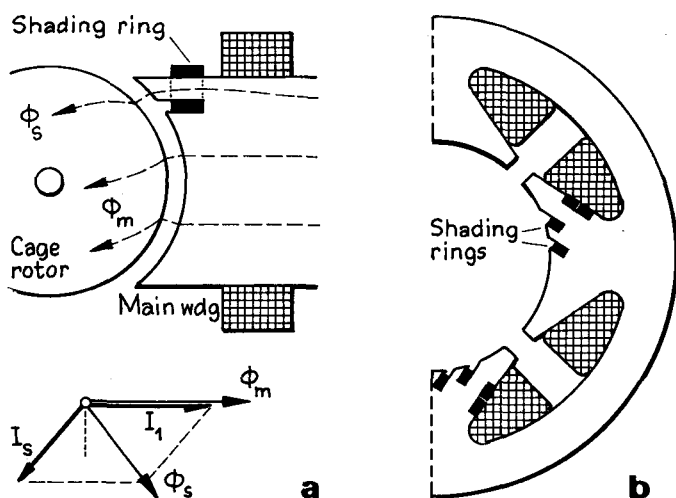
For capacitor-start, the capacitors vary in size from 20–30 μF for a 100 W motor up to 60–100 μF for a 750 W motor. Electrolytic capacitors are

economic, but their losses may lead to thermal breakdown if starts are too frequent.

Single-phase induction motors have a mean airgap flux density in the range 0.35–0.55 T, and to raise the magnetizing reactance the gap length may be 0.2–0.25 mm.

Shaded-pole motor

The split-phase motor is one of a class in which stator asymmetry is introduced to secure self-starting, using two (or more) windings that differ in magnetic axis and winding details (spread, slot size, conductors and number of turns). The shaded-pole motor may have all such asymmetries. The essential method is that in Fig. 12.24(a). A part of each salient stator pole is embraced by a



12.24 Shaded-pole motor.

short-circuited winding, normally of a single turn and called a shading ring. That part of the flux Φ_m produced by the main winding is modified in the ring by induced current I_s as if it were the short-circuited secondary of a transformer. As a result there is a flux Φ_s in the shaded region, delayed in time-phase with respect to the main flux Φ_m . The two polar fluxes are thus displaced in both space and time to give the effect of a crude shifting or travelling wave in the gap, capable of developing torque in a cage rotor at rest.

One-half of a typical 4-pole stator (b) shows a pair of shading rings on one horn of each pole, with the main winding embracing the whole pole shank. The magnetic conditions are complex. The motor performance is a function of the number of main-winding turns, the pole-arc/pole-pitch ratio, the proportion of the pole-arc in the shaded region, the shading-ring resistance, and the rotor gap resistance and slot skew. Analyses have been attempted by

Butler and Wallace [114], and by Eastham and Williamson [115], but design must rely heavily on tests to assess the appropriate parameters.

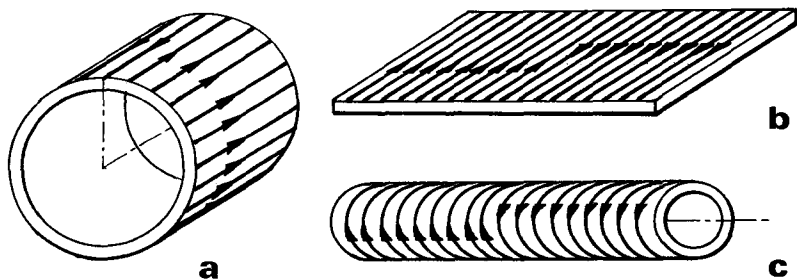
The important design features are the starting torque and the temperature-rise. The torque/speed characteristic resembles that of a 3-ph induction motor, apart from a 'dip' in torque at about 0.4 of synchronous speed, for which the strong harmonic content of the gap m.m.f. is responsible. This is a function of the pole-arc/pole-pitch ratio: for a ratio above 0.9 a 3rd harmonic torque assists the fundamental torque at starting, but the two oppose if the ratio is between 0.6 and 0.9. Some improvement in torque may be obtained by slightly lengthening the airgap in the unshaded end of the stator pole-arc. The I^2R loss in the shading ring on load (when it is of little service) lowers the efficiency to the range 0.2–0.5 p.u., although this is not a serious drawback in the ratings 15–250 W in which the machine is easily and cheaply manufactured. Such sizes have naturally considerable dimensional and operating tolerances.

The rotor is a simple cage with a 60° skew for optimum starting torque and for limiting the torque dip during run-up. A low cage resistance impairs both the starting torque and the dip, and usually the resistance is greater than for any other 1-ph motor cage, giving a starting and a pull-out torque in the respective ranges of 0.4–0.8 and 1.1–1.3 p.u. of full-load torque. The gap length is of the order of 0.25 mm. Too short a gap may result in starting-torque variations due to rotor slotting.

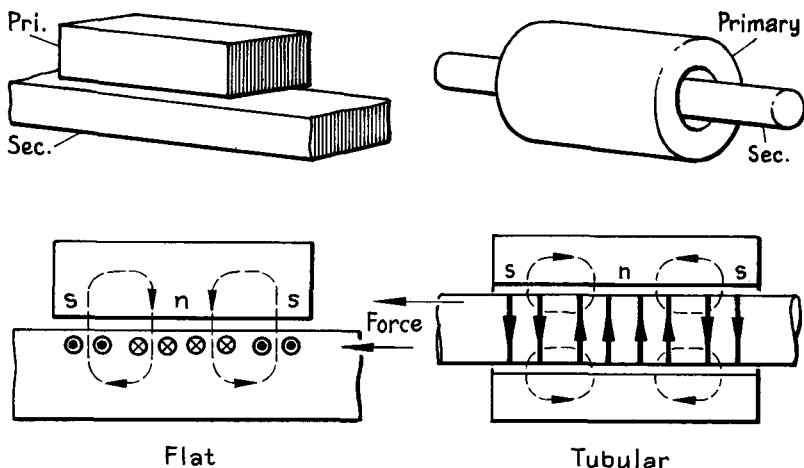
12.14 Linear machines

Although a drive producing linear motion can be mechanically derived from a rotating machine, it may not always be convenient to do so, especially where the linear motion must (as in a liquid-metal pump) be sustained locally, or (as in high-speed trains) be extended over several hundred metres or kilometres. In such cases the linear form of driving motor has advantage. A linear machine can be based on a change of shape of a conventional rotary machine.

The active gap region of a 2-pole cylindrical uniform-gap rotary machine is shown diagrammatically at (a) in Fig. 12.25. If the cylinder is cut axially and 'opened out', the gap region (b) is now typical of a *flat* linear machine;



12.25 Linear machine development.



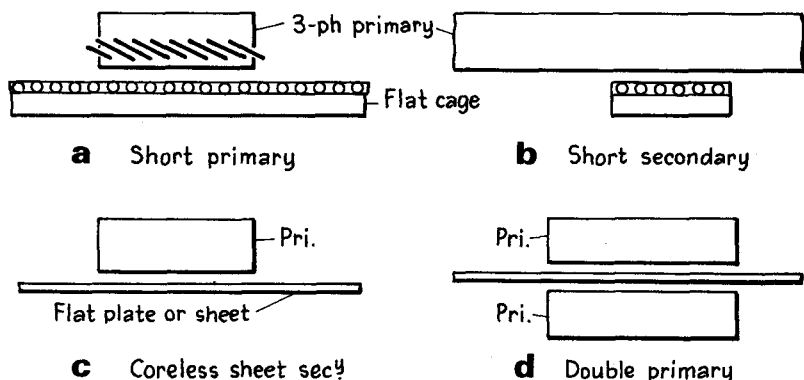
12.26 Flat and tubular linear machines.

and if, as in (c), the flat form is re-rolled on a lengthwise axis it becomes *tubular*. In (a) and (b) the end-connections remain, but in (c) each originally straight conductor is formed into a circular coil, eliminating the end-connectors entirely. Elementary realizations of flat and tubular structures are shown in Fig. 12.26. Of the two necessary members, one is the primary which produces a magnetic field in the flat or annular gap, the other is the secondary which carries currents so disposed as to develop interaction force with the primary flux.

A basically different form of linear motor, not derived directly from the rotary machine, is obtained by adopting a transverse flux path. This is discussed later.

Virtually any rotary machine has a linear counterpart, but the linear form has distinctive features: (i) the problem of dealing with a linear movement between primary and secondary that, if continued, must cause one member to part company with the other; (ii) marked magnetic asymmetry; and (iii) unbalanced magnetic attraction between members when both have ferromagnetic cores. To cope with (i), one of the members must span the full operating distance, which may be a few millimetres up to several hundred kilometres. The finite length of the machine means that it lacks the magnetic symmetry implicit in a normal rotary machine in which any and every S pole is flanked on both sides by N poles: a linear machine has 'ends'. Magnetic attraction between members can be balanced out in a rotary machine (and also in a tubular machine if the centrality of the secondary is maintained) but cannot be so balanced in a flat machine unless a double-sided construction is adopted or the secondary is non-magnetic. Mechanical arrangements must therefore be provided in some cases to prevent primary and secondary from sticking together.

Both d.c. and synchronous machines can be constructed in the linear form,



12.27 Flat linear 3-ph induction motors.

but have the disadvantage that power must be fed to both primary and secondary. A more convenient working principle is that of the polyphase induction machine with the secondary (in the form of an actual or equivalent cage winding) energized from the primary. Some of the arrangements then possible are indicated in Fig. 12.27. The short-primary type (a) is suitable where the operating distance is great, for it is uneconomic to wind a very long 3-ph primary. The short-secondary form (b) is useful with limited operating distance. In both (a) and (b) the secondary winding is a cage set into slots in a core. The coreless sheet or plate secondary (c) is often as effective, being cheaper. However, the airgap is now 'open' and ill-defined, and the secondary currents are no longer constrained to flow in paths of optimum direction with respect to the primary flux. The double-primary construction (d) re-establishes the definite gap, although for reasons of clearance the gap length will generally be much greater than that of a standard rotary induction motor. If the two facing primaries are fed so as to form the sides of a common magnetic circuit there will be a strong magnetic attraction between them, but maintenance of the separation offers little difficulty.

Linear induction motor

The *primary* of a linear induction motor is that member which is connected to the 3-phase supply, and the *secondary* is the wound-core or coreless-sheet member in which interaction currents are induced. Either member can be the *stator*, the other being the *runner* in accordance with the particular requirements imposed by the duty for which the machine is intended. In this respect three classes of linear motor can be distinguished:

Force (actuator) machine. This is concerned with producing thrust at rest or at low speed over a restricted length of travel. It is normally built in small sizes in which it can compete with hydraulic or pneumatic thrusters. The design criterion is the ratio of force to primary power (i.e., the $I^2 R$ demand).
Energy (accelerator) machine. Here the duty is to impart kinetic energy to a mass by accelerating the mass from rest to a high speed within a specified dis-

tance. The criterion is the ratio of imparted energy to the useful mass accelerated.

Power (drive) machine. The 'power' machine is concerned with the transport of masses, usually at high speed and with a reasonably high power efficiency. The application dictates the allocation of stator and runner functions to the primary and secondary members.

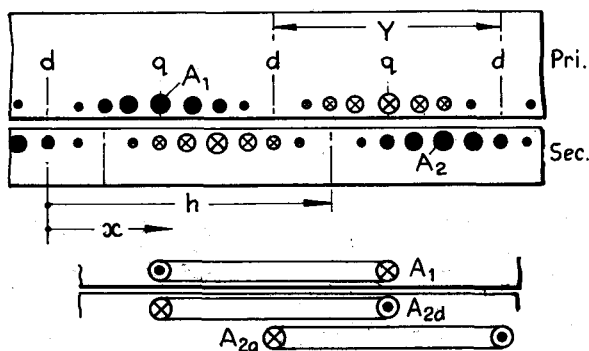
A linear motor may have any number of poles, odd or even. Let the pole-pitch be Y . The double pole-pitch is the wavelength, $\lambda = 2Y$. For a supply of angular frequency ω_1 the travelling-wave field has a linear synchronous speed given by $u_s = \omega_1(Y/\pi) = \omega_1(\lambda/2\pi)$. The synchronous speed for a 50 Hz supply and a wave-length of 1 m is $u_s = 50 \text{ m/s} = 3 \text{ km/min} = 180 \text{ km/h}$, so that high translational speeds are possible with 'power' machines. There is, of course, a slip s as in the rotary machine, and the speed of the runner with respect to the stator is

$$u = u_s(1 - s) = \omega_1(Y/\pi)(1 - s)$$

The division of the power P_2 to the secondary between mechanical output P_m and I^2R loss is the same as for the rotating induction machine, as given by eq.(8.3).

Ideal machine

One wavelength of the idealized arrangement in Fig. 12.28 is remote from the ends so that magnetic symmetry can be assumed. Currents in the primary and secondary slot conductors are represented by sine-distributed current-sheets of peak value A_1 and A_2 (in A/m); these respectively produce peak m.m.f.s $F_1 = A_1(2/\pi)^{1/2}Y = A_1(Y/\pi)$ and $F_2 = A_2(Y/\pi)$ per pole. With ideal cores and a uniform gap of length l_g , the primary m.m.f. sets up a gap flux of peak density $B_1 = \mu_0 F_1/l_g$. The successive parallel axes marked d and q in Fig. 12.28 correspond to the d - and q -axis respectively in a rotary machine. A force displacement h is the linear counterpart of the torque angle. The action is taken as confined within the gap region, the width of which is l perpendicular to the direction x .



12.28 Conditions in flat linear induction motor.

Force. Apart from the now linear geometry, the conditions differ in no respect from those discussed in Sect. 1.11 and illustrated in Fig. 1.10. In place of the general angle we have the distance x , so that in trigonometric functions θ is replaced by $x(2\pi/\lambda) = x(\pi/Y) = \beta x$, where $\beta = 2\pi/\lambda = \pi/Y$. The force displacement appears as βh , and the peak m.m.f.s per pole can be written as $F_1 = A_1/\beta$ and $F_2 = A_2/\beta$. Let Fig. 12.28 represent the conditions at any instant: then at a point x we have

$$\text{Primary: } A_{1x} = A_1 \sin \beta x \quad \text{and} \quad B_{1x} = B_1 \cos \beta x$$

$$\text{Secondary: } A_{2x} = A_2 \sin \beta(x - h)$$

Applying the interaction rule, eq.(1.4), or modifying eq.(1.10), gives for a double pole-pitch the translation force

$$f_e = -\frac{1}{2} Y B_1 A_2 \sin \beta h$$

The current sheet A_2 could be resolved into superposed sheets on the d - and q -axes with peaks $A_{2d} = A_2 \cos \beta h$ and $A_{2q} = A_2 \sin \beta h$. The latter is responsible for the interaction force, the former is a direct reaction on the primary accounting for the characteristic inductive energy transfer from primary to secondary.

Power. For a synchronous speed u_s and slip s , the runner speed is $(1-s)u_s$ and the slip speed is su_s . At a point x where the primary gap flux density is B_{1x} , the e.m.f. induced in the secondary is $e_{2x} = B_{1x}(su_s)$ and it produces therein the surface current density $A_{2x} = e_{2x}/\rho l$ where ρ is the surface resistivity and magnetic leakage inductance is ignored. The force on an element dx is then $df_e = B_{1x} l (A_{2x} \cdot dx)$. This force multiplied by the speed u is the mechanical power developed by the element:

$$dP_m = [lu_s^2 s/\rho] (1-s) B_{1x}^2 \cdot dx = k(1-s) B_{1x}^2 \cdot dx$$

The electrical power dissipated in the element is $e_{2x} A_{2x} \cdot dx = ks B_{1x}^2 \cdot dx$, and the total input to the element is the summation of the mechanical power and the dissipation, giving $dP_2 = k B_{1x}^2 \cdot dx$. Integrated over a wavelength the term $B_{1x}^2 \cdot dx$ becomes $\frac{1}{2} \lambda B_1^2$, whence the powers concerned in one double pole-pitch of the secondary are

$$\text{Input: } P_2 = (\frac{1}{2} \lambda l u_s^2 s/\rho) B_1^2$$

$$\text{Output: } P_m = P_2(1-s)$$

$$\text{Loss: } I^2 R = P_2 s$$

just as with the rotary machine. The mechanical output is a maximum for a slip $s = 0.5$ p.u., and the efficiency is $(1-s)$ p.u.

Practical machine

The conditions in a practical realization of the linear induction machine differ significantly from the ideal, especially where coreless sheet secondaries are employed.

In a *short-primary* machine, Fig. 12.27(a), secondary conducting material outside the influence of the primary is electrically 'dead', and as relative

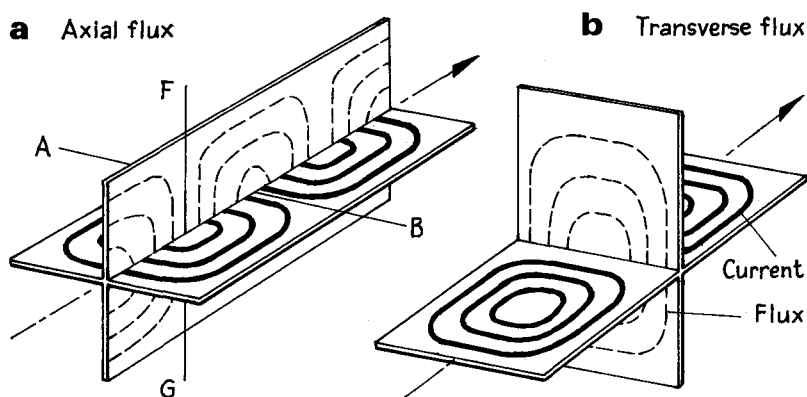
movement occurs, such material continually enters the energized region. Secondary currents are induced in opposition to the primary m.m.f., the flux in a secondary element building up as it passes into the working gap. When the element leaves at the further end, its current dies away. As a result, *entry* and *exit* energy losses occur. With operation at a slip s , the rotor entry currents move with the secondary into and out of phase with the primary travelling-wave m.m.f., giving rise to complicated resultant flux patterns which are not uniform and vary with slip. Every part of a short-primary machine works under transient conditions so that the conventional equivalent circuit is not strictly applicable. If the number of pole-pairs is p , then for slips between $s = 1/(2p + 1)$ and $s = 2/(2p + 2)$ the secondary entry loss may in fact be a gain, and may compensate for the always positive exit loss. At $s = 2/(2p + 2)$ the operation approaches that of a conventional rotary machine of the same pole-pitch and power output. At $s = 1/(2p + 1)$ the excess secondary loss is zero and the secondary efficiency is given by $(1 - s) = 2p/(2p + 1)$: thus p should be large. All the primary coils of a phase should be connected in series to maintain a uniform current; for if they were in parallel and connected to a constant-voltage source, the flux in all pole-pitches would be constrained to be the same, and the turns at the entry end of the primary would draw very large currents as a consequence of the current reaction of the secondary in that region.

In a *short-secondary* machine the roles of current and flux densities may be interchanged. With series connection the flux density is high just outside the secondary ends, while in a parallel-connected case the secondary induced e.m.f.s are substantially the same as in a slice of a conventional machine. However, as the secondary currents must flow in closed paths, there is a greater I^2R loss.

Current in sheet secondary. The linear induction motor produces thrust by interaction of the main working flux with the eddy-currents that it induces in the secondary sheet or slab. Such a secondary has no distinct 'bars' and 'end-rings' so that the current pattern is less precise. The axial end-currents flow mainly, but not entirely, in the edges of the secondary sheet, and their effect is strongly dependent upon the width of the sheet relative to the width of the primary core. If the width were infinite, the secondary currents would flow parallel to the primary slot currents as they do in an unskewed rotary machine. But in a practical linear motor the width of the secondary may be greater or less than that of the primary, and further may be quite small in comparison with the pole-pitch. The currents flow in elliptical paths of which the major axis lies in the direction of motion, so that much of the secondary I^2R loss is useless for force production. Accurate prediction of performance is possible only if the gap-flux density and secondary current patterns can be properly analyzed, a very difficult matter when the active conductors of the primary are straight but face secondary currents in elliptical paths. The secondary reaction is then greatest on the longitudinal centre-line of the machine, the flux density distribution across the width is non-uniform: it is greatest at the edges (where the primary and secondary current directions are at right-angles instead of running parallel), giving a *transverse-edge* effect.

Some typical distributions are derived by Preston and Reece [98]. In a double-primary machine designed for use with a very long secondary (as for long-distance rail traction) the secondary sheet may, to reduce its cost, be made narrower than the width of the primary. Its effectiveness is thereby reduced by the restriction on the width over which the secondary currents are substantially parallel to the primary slot-conductors and by the much greater length of the non-useful current paths along the sheet edges, particularly with the extensive pole-pitch dimensions usually required to obtain high speeds. The economic balance between the two conflicting requirements is an intricate one. Cross-slitting the secondary to constrain the current directions to the optimum is effective in improving force production, but adds heavily to the cost.

Axial-flux and transverse-flux machines. The forms so far discussed have gap flux and secondary current orientations approximately as indicated in Fig. 12.29(a). The feature that prevents the pole-pitch (and therefore the speed



12.29 Axial- and transverse-flux relations.

for a given supply frequency) from being increased indefinitely is the non-useful components of flux and secondary current parallel to the direction of motion: the useful components of flux are those in direction FG, which interact with currents parallel to AB, as for example in the drive of the disc of a polyphase electric energy meter. The non-useful flux path is *axial*; it contrasts with the *transverse* flux direction in (b), which gives a considerable magnetic advantage. The magnetic circuit is shortened, reducing the mass of core steel needed for a narrow machine of long pole-pitch. The novel conditions and problems that face the designer of a transverse-flux machine are discussed by Laithwaite [99].

Performance

The linear induction motor differs in electrical characteristics from its rotary counterpart by reason (i) of the much greater magnetizing current consequent

upon the longer and less well defined airgap, especially in single-sided machines with coreless secondaries; and (ii) of its discontinuous nature resulting in flux distortion and edge losses. Nevertheless, the performance can be predicted approximately on the basis of the equivalent circuit in Fig. 8.10. The secondary leakage inductance in a sheet member can be ignored; and the friction, windage, primary-core and secondary-edge losses deducted from the gross output. The parameters can be estimated by the methods proposed by Laithwaite [82], or for an actual machine evaluated from standstill and no-load tests. In the latter, the runner is driven with respect to the stator at synchronous speed. Let the measured impedance of one primary phase with the secondary locked to it be

$$Z_1 = r_1 + R_1 + jX_1$$

where r_1 is the measurable ohmic resistance of the primary. The other terms represent the effective series resistance and reactance of the parallel connection of r_2 and x_m for $s = 1$. The reactance $X_0 = x_1 + x_m$ can be obtained from the no-load test, whence the parameters for the equivalent circuit are

$$r_1, \quad r_2 = R_1 [1 + (R_1/X)^2] \quad \text{and} \quad x_m = X [1 + (R_1/X)^2]$$

where $X = (X_0 - X_1)$. Care must be taken with the application of the equivalent circuit because of the significant asymmetry of the machine. The translational force derived from the equivalent circuit calculation includes the frequency- and speed-dependent losses, which are rather intricate to assess. For each phase of a 3-ph linear 'power' motor the force/slip relation is given by

$$f_e = \frac{I_1^2 (r_2/s) x_m^2}{u_s [(r_2/s)^2 + x_m^2]}$$

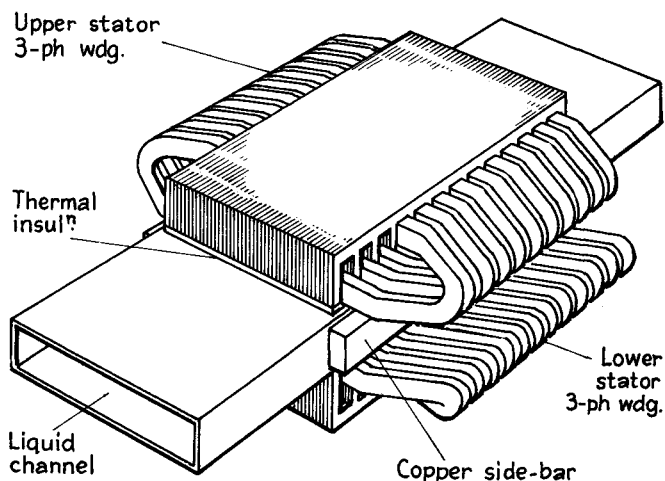
where u_s is the synchronous linear speed.

Applications

Power (drive) machines have been applied to conveyors, haulers, electromagnetic pumps, travelling cranes and high-speed rail traction. **Accelerator** machines have been developed for rope and car-crash test rigs, and for the launching of aircraft and small missiles. **Force** (actuator) machines serve to operate impact metal forming and blanking devices, stirrers and stop-valves. The *ad hoc* designs for particular applications have little characteristic similarity in appearance.

Power drives

Electromagnetic pump. This has been developed specifically for handling high-temperature liquid metals (such as sodium, sodium-potassium alloy or bismuth) used as heat-transfer media in nuclear reactor plants. A flat linear pump, Fig. 12.30, has a double-primary core, slotted and wound with 3-ph coils, with the 'airgap' comprising a rectangular-section tube which carries the liquid metal. End-bars are required to encourage an appropriate direction of current flow in the liquid; these may be of copper, cast or brazed to the

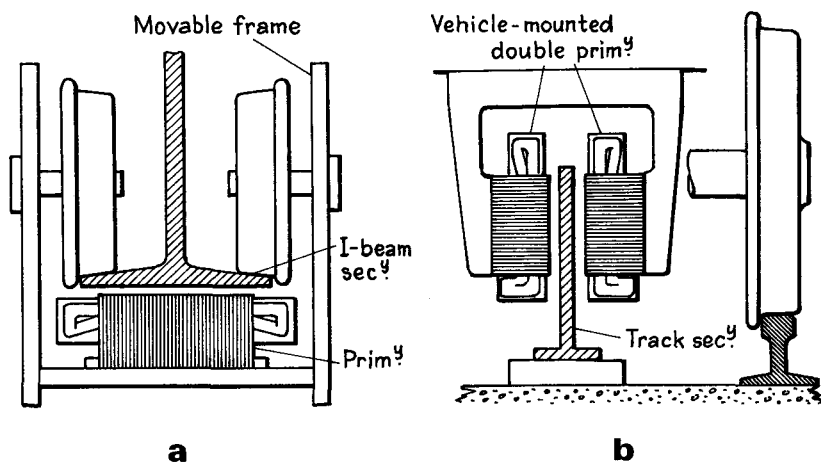


12.30 Linear induction pump.

outside of the tube walls. The liquid channel is formed from a tube, or welded from sheet. The latter shape is inherently weak and may have to be stiffened. Undue pulsation of the working flux can be avoided if the winding has an odd number of fully wound pole-pitches, one half-wound pole at each end, and series connection. Standard formers can then be used to produce the diamond coils.

The force developed on the liquid produces a pressure differential which, multiplied by the speed of flow, gives the mechanical power output. The efficiency is low. Its ideal value is $(1 - s)$, but this is lowered by a number of secondary effects. The liquid-metal channel, about 2 mm thick, shunts both working flux and secondary current; there is a contact drop at the sides; fluid friction drag absorbs some of the force developed; and the large gap between the primaries demands large magnetizing-current components and lowers the power factor. The high-temperature working conditions are onerous and necessitate thermal as well as electrical insulation between primary and secondary. Depending upon the liquid pressure and the flow-rate, higher working efficiencies may be obtained by a tubular instead of a flat construction, or by a combination of rotary and axial motion. In the latter case the primary strongly resembles the stator of a rotary machine, with a large 'airgap' and a stationary rotor. The annular space is occupied by an annular tube which leads the metal into the gap at one end and out at the other. A helical barrier then causes the liquid metal to have a component velocity in the axial direction.

Overhead travelling crane. The travelling gantry carries single-sided primaries at each end of the span, while the tracks or joists on which the crane travels act as secondaries. The value of the horizontal thrust is varied by voltage control. The advantages are the simplicity of construction avoiding rotating parts, elimination of gears, clutches and coupling shafts, and ease of control. The essential arrangement is shown in Fig. 12.31(a). Speeds up to about



12.31 Linear induction drive for traction.
(a) Crane (b) Railway

2 m/s are possible. At full speed the slip may be about 0.2, depending upon the resistivity of the secondary material and effective length of the airgap. The optimum operating frequency is usually appreciably lower than that of normal supply, but for convenience some sacrifice is acceptable to gain the advantage of a standard supply. Inputs of 5–10 VA are required per newton of thrust.

Stirrer. Stirring the molten contents of a metal furnace is possible by means of short-primary linear structures clamped to the outside of the furnace, provided that the shell is non-magnetic and the operating frequency is low. The molten metal in the vicinity of the primary field acts as the secondary. The efficiency is rather low, but the stirring action is readily controlled, and is said to improve the metallurgical properties of the metal.

Mine haulage. One system of moving loaded trucks along a track is to link them by flexible couplings to form trains of a specified minimum length (e.g. 250 m). Underneath each truck is fitted a horizontal flat metal plate, and short primaries are fixed between the rails at intervals rather shorter than the length of a train. As each truck passes over a primary it is subject to a forward thrust, maintaining the whole train in forward motion.

High-speed trains. Conventional railways achieve train speeds of 150–200 km/h, and might reach 300 km/h or more if the track were straight and level. The tractive effort, however, is limited by adhesion between wheel and rail. The linear motor is attractive as an alternative method of propulsion: it is independent of adhesion, has excellent braking capability and a high power/mass ratio. A high-speed train without wheels or normal rails, but employing a tracked hovercraft principle with linear-motor drive, could be capable of reaching a speed of, perhaps, 400 km/h. Economic considerations dictate that the vehicle should carry the primary member and that the secondary (which must extend the full length of the route) should

be reduced to the simplest and cheapest form. The moving primary demands either that power be supplied to the moving vehicle, or that fuel and a prime-mover/generator be carried on board.

An assessment of the problems of high-speed railways has been made by Laithwaite [100]. One system that has found favour is illustrated in Fig. 12.31(b), which shows a double-sided primary flanking the track member, a T-section aluminium plate. The mechanical clearances necessary together with the secondary plate thickness result in a considerable airgap length. However, the high-speed motor inevitably has a long pole-pitch as an essential feature and this reduces the magnetic disadvantage of the long gap. The tractive force is, as with all induction machines, a function of the secondary I^2R loss: when the vehicle is in motion a fresh supply of cool secondary plate is continually entering the gap, but at rest and at starting the rapid temperature-rise of the secondary may require some mechanical modification to avoid buckling.

The design procedure for a high-speed vehicle, its power supply and control gear, is complex and strongly affected by several features not encountered in normal industrial rotary machines. The vehicle might collect electrical power from some conductive or inductive track system as alternating current at a constant frequency, with the linear motor operated at variable slip over its whole speed range and controlled by voltage variation. A more elegant system would be to control speed by varying the frequency, using solid-state inverters on board, and it would permit of re-generative braking. This might be a valuable feature of a pneumatically suspended (hovercraft) vehicle, as the regenerated power could be employed in maintaining the fan drive so that the vehicle does not 'sit down' on its track in the event of a power failure until the speed has been reduced to a safe level.

Accelerator and actuator drives

The accelerator type of linear induction motor is concerned with imparting kinetic energy. An example is a test rig in which cars or lorries (of mass in the range 500–2 500 kg) are crashed into a concrete block, their speed being raised from standstill to 50 km/h by a short double-primary 3-ph 50 Hz 3.3 kV 400 kW motor. The total accelerating period is about 3 s and the energy absorbed is about 4 MJ (1 kWh).

Various realizations of the accelerator machine are possible to optimize the interrelation between the synchronous speed, the slip and the accelerating force at any point on the travel of the runner. With a primary covering the full length of the travel, the pole-pitch could be made progressively longer so that at any point the local synchronous speed could be matched to the runner speed with the supply at constant frequency. Alternatively the primary might have a constant pole-pitch but be supplied at progressively rising frequency. The latter method could also be used with a short-primary construction. The design problem is exceedingly complex, as it involves a combination of sustained mechanical and electrical transients that persist without ever reaching a steady state. Onuki and Laithwaite [101] suggest that a machine with a few different constant synchronous speeds is likely to be more practicable

than one attempting to give a continuous rise throughout the launching process.

Short-stroke tubular motors, comprising a fixed primary with circular 3-ph windings in a laminated core and a mild steel axial reaction-rod secondary, are commercially available for operating steam, hydraulic or pneumatic valves, hopper gates, sluice gates and sliding doors. They have the facility of being able to move slowly or rapidly, or move in timed steps, in either direction. If the stator coils are skewed, the path of the travelling-wave field is changed from the axial to the partly rotational so that a helical path is obtained. The runners can then be given a corkscrew motion, suitable for screw-down valve operation.

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List of Symbols

All quantities are expressed in SI units. Unit symbols are those listed in 'Symbols and Abbreviations for Electrical and Electronic Engineering' (Institution of Electrical Engineers, 1976).

CAPITALS

<i>A</i> area	<i>L</i> inductance
integer	overall length
linear current density	<i>L</i> transference
specific electric loading	<i>M</i> torque
<i>B</i> magnetic flux density	<i>N</i> number of turns
$\bar{B}$ specific magnetic loading	<i>P</i> active power
<i>C</i> capacitance	<i>Q</i> charge
constant	reactive power
damping coefficient	<i>R</i> radius
number of coils	resistance
<i>D</i> diameter	<i>S</i> apparent power
limb spacing	number of slots
<i>E</i> electromotive force	rating
<i>F</i> magnetomotive force	reluctance
<i>G</i> mass	surface area
resistance/reactance ratio	<i>T</i> absolute temperature
<i>H</i> inertia constant	<i>V</i> voltage
magnetic field intensity	<i>W</i> energy
overall height	overall width
<i>I</i> current	<i>X</i> reactance
<i>J</i> current density	<i>Y</i> admittance
moment of inertia	pole-pitch
<i>K</i> coefficient, factor	<i>Z</i> impedance
compliance	number of conductors

SMALL LETTERS

<i>a</i> acceleration	<i>a</i> constant
conductor area	dimension

a	number of paths	m	number of phases
b	breadth	m_0	momentum
	component flux density	n	harmonic order
	dimension		number
	gas pressure		speed of rotation
	pole-arc	p	active power
c	capacitance		number of pole-pairs
	cooling coefficient		specific loss
	damping ratio	p	d/dt
	dimension	q	charge
d	diameter		reactive power
	thickness		volume flow-rate
d	differential	r	integer
e	electromotive force		radius
f	force		ratio
	frequency		resistance
f	function	s	slip
g	gravitational constant	s	complex frequency
	slots per pole	t	time
g'	slots per pole and phase	u	coil-sides per slot
h	height, depth		linear speed
	load displacement	v	voltage
i	current	w	energy
j	90° operator		width
k	coefficient	x	displacement
	compliance		fraction
	constant		length variable
	factor		reactance
	fractional load	x'	transient reactance
	thermal conductivity	x''	subtransient reactance
l	gross core length	y	admittance
	length		dimension
m	integer		slot-pitch
	mass	z	impedance
	numbers of layers		conductors per slot

GREEK LETTERS

α	angle	β	root of capacitance ratio
	angular acceleration	γ	angle
	eddy-loss parameter		angular divergence
	load angle		slot-pitch angle
	resistance/reactance ratio		specific cost
	root of capacitance ratio	Δ	finite difference
	temperature coefficient	δ	density
β	angle		load angle
	phase coefficient	ϵ	chording angle
	phase-displacement angle		eccentricity

δ	per-unit impedance	μ_0	magnetic space constant
	radiation factor	ρ	resistance section
	regulation		resistivity
	shaft deflection	σ	angle
η	efficiency		angular phase-spread
θ	position angle	τ	time-constant
	temperature	τ'	transient time-constant
	temperature-rise	τ''	subtransient time-constant
Λ	permeance	Φ	magnetic flux
λ	permeance coefficient	ϕ	phase angle
	thermal emissivity	ψ	magnetic linkage
	torque angle	ω	angular frequency
	wavelength		angular speed
μ	absolute permeability		

SUBSCRIPTS

<i>a</i>	absorption	<i>d</i>	eddy-current
	acceleration	<i>e</i>	chording, coil-span
	active		effective
	air		electrical
	alternating-current		excitation
	armature-reaction	<i>er</i>	end-ring
	duct-width	<i>f</i>	field
	phase A		fluid
<i>ad</i>	d-axis mutual		friction
<i>aq</i>	q-axis mutual	<i>g</i>	gap
<i>au</i>	armature, unsaturated	<i>h</i>	harmonic
<i>av</i>	average		hot-spot
<i>b</i>	boundary		hysteresis
	braking	<i>i</i>	inlet
	duct-width		iron
	phase B		net-iron
<i>c</i>	cage	<i>kd</i>	d-axis winding
	capacitive	<i>kq</i>	q-axis winding
	coil	<i>l</i>	leakage
	conduction		line
	conductor		load
	control		loss
	critical	<i>m</i>	magnetizing
	damping		maximum
	phase C		mean
<i>d</i>	direct-axis		mechanical
	direct-current	<i>mc</i>	mean-conductor
	distribution	<i>mt</i>	mean-turn
	disturbing	<i>n</i>	harmonic-order
	duct		winding

<i>o</i>	opening outlet outside overall overhang	<i>t</i>	test tooth torque total transient turn
<i>oc</i>	open-circuit	$t\frac{1}{3}$	at one-third tooth level
<i>p</i>	pole pulsation transient-peak	<i>u</i>	tube unsaturated
<i>ph</i>	phase	<i>v</i>	convection voltage
<i>pl</i>	pole-leakage	<i>w</i>	width winding window
<i>q</i>	quadrature-axis	<i>x</i>	at position <i>x</i> reactive
<i>r</i>	radial radiation reactive relative remanent resistance remanent ring rotational, motional	<i>y</i>	yoke
<i>s</i>	mechanical-storage pole-shoe slot space starting stray-loss synchronising synchronous system	<i>z</i>	impedance zigzag
<i>sc</i>	short-circuit	0	characteristic free-space natural no-load normal resultant zero-phase-sequence
<i>sci</i>	ideal short-circuit	1	fundamental input primary stator supply
<i>sd</i>	d-axis synchronous	2	outer-cage rotor secondary
<i>sdu</i>	d-axis synchronous, unsaturated	3	inner-cage tertiary
<i>sl</i>	shoe-leakage		
<i>sq</i>	q-axis synchronous		
<i>t</i>	at time <i>t</i> tank		

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Alternating Current Machines

Alternating Current Machines is the successor to the same author's *Performance and Design of Alternating Current Machines*, and is designed to achieve the balance, which first degree and diploma students require, between theoretical mathematical models and technological realities. It is an in-depth coverage which recognizes that there is more to a real machine than a set of differential equations and that machines of various types do in fact have marked technological differences.

The book starts with a unification in terms of flux-current interaction to give a physical picture of how a machine works. The text leads from simple, idealized theory progressively to practical steady-state and transient performance, with generalized analyses introduced where appropriate. Reference is made to phenomena of engineering importance, such as starting and braking, harmonics and skewing, inrush current and surge distribution, disruptive forces and magnetic pull, noise and vibration, speed control and slip-power recovery.

The major part of the text is devoted to transformers, three-phase induction and synchronous machines, but special machines are also covered, including those to which generalized methods are not readily applicable.

Attention is given to modelling for particular operating conditions. A brief outline is provided of modern design methods, there are several worked examples, and an extensive bibliography is appended.

Alternating Current Machines is suitable for technical college, polytechnic and university students of electrical plant and machinery.

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