Lecture 1
ELE 301: Signals and Systems

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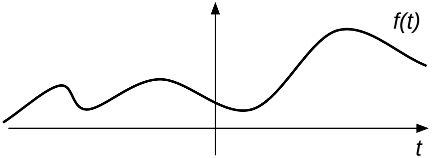
Course Overview

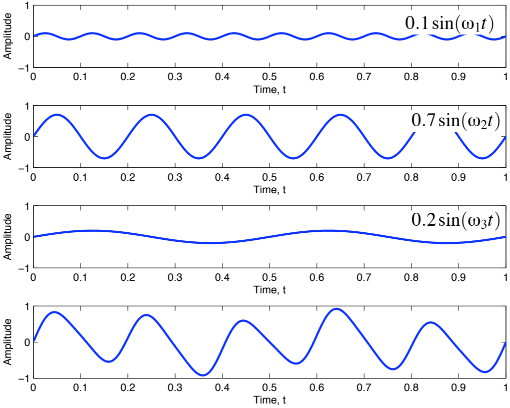
- Time-Series Representation of Signals
  Typically think of a signal as a "time series", or a sequence of values in time



Useful for saying what is happening at a particular time
Not so useful for capturing the overall characteristics of the signal.

Idea 1: Frequency Domain Representation of Signals

- Represent signal as a combination of sinusoids



$$f(t) = 0.1\sin(\omega_1 t) + 0.7\sin(\omega_2 t) + 0.2\sin(\omega_3 t)$$

- This example is mostly a sinusoid at frequency ω_2, with small contributions from sinusoids at frequencies ω_1 and ω_3.
    - Very simple representation (for this case).
    - Not immediately obvious what the value is at any particular time.

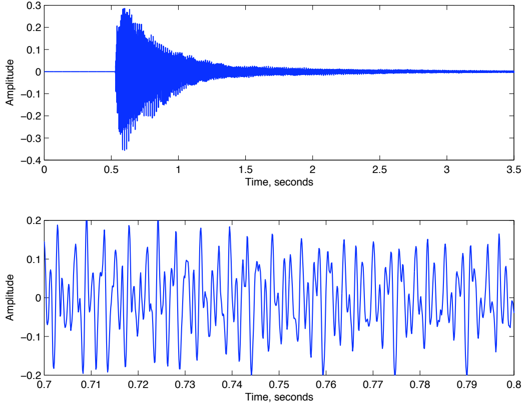
- Why use frequency domain representation?
    - Simpler for many types of signals (AM radio signal, for example)
    - Many systems are easier to analyze from this perspective (Linear Systems).
    - Reveals the fundamental characteristics of a system.

- *Rapidly becomes an alternate way of thinking about the world.*

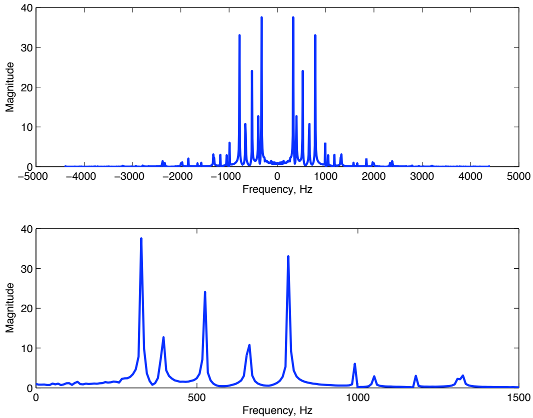
Demonstration: Piano Chord

- You are already a high sophisticated system for performing spectral analysis!

- Listen to the piano chord. You hear several notes being struck, and fading away. This is waveform is plotted below:

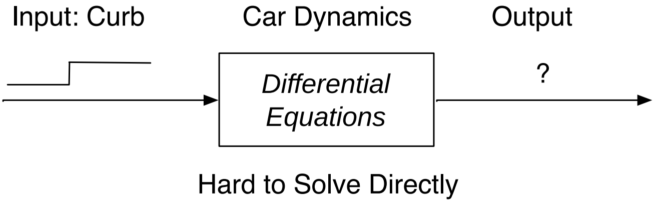


- The time series plot shows the time the chord starts, and its decay, but it is difficult tell what the notes are from the waveform.
- If we represent the waveform as a sum of sinusoids at different frequencies, and plot the amplitude at each frequency, the plot is much simpler to understand.

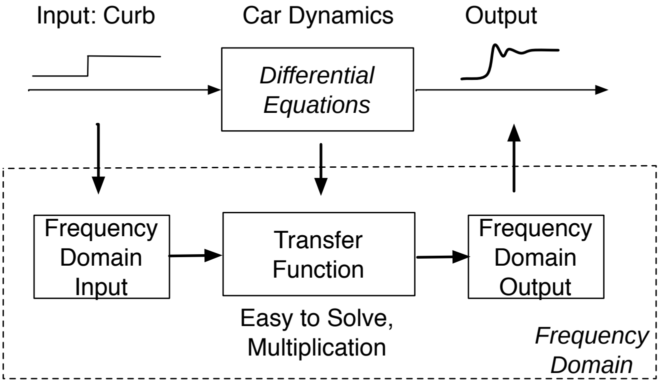


Idea 2: Linear Systems are Easy to Analyze for Sinusoids

Example: We want to predict what will happen when we drive a car over a curb. The curb can be modelled as a "step" input. The dynamics of the car are governed by a set of differential equations, which are hard to solve for an arbitrary input (this is a linear system).



After transforming the input and the differential equations into the frequency domain,



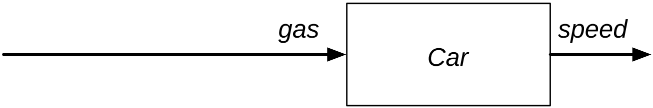
Solving for the frequency domain output is easy. The time domain output is found by the inverse transform. We can predict what happens to the system.

Idea 3: Frequency Domain Lets You Control Linear Systems

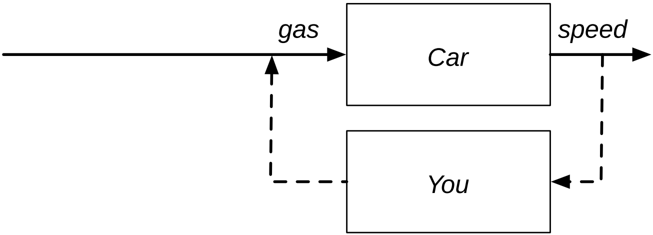
- Often we want a system to do something in particular automatically
  - Airplane to fly level
  - Car to go at constant speed
  - Room to remain at a constant temperature

- This is not as trivial as you might think!

Example: Controlling a car's speed. Applying more gas causes the car to speed up

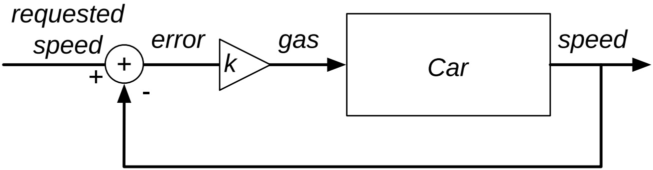


Normally you "close the loop"



How can you do this automatically?

Use feedback by comparing the measured speed to the requested speed:



This can easily do something you don't want or expect, and oscillate out of control.

Frequency domain analysis explains why, and tells you how to design the system to do what you want.

Course Outline

- It is useful to represent signals as sums of sinusoids (the frequency domain)
- This is the "correct" domain to analyze linear time-invariant systems
- Linear feedback control, sampling, modulation, etc.

What sort of signals and systems are we talking about?

Signals

- Typical think of signals in terms of communication and information
  - radio signal
  - broadcast or cable TV
  - audio
  - electric voltage or current in a circuit

- More generally, *any* physical or abstract quantity that can be measured, or influences one that can be measured, can be thought of as a signal.
  - tension on bike brake cable
  - roll rate of a spacecraft
  - concentration of an enzyme in a cell
  - the price of dollars in euros
  - the federal deficit

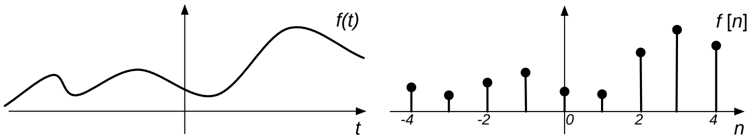
  Very general concept.

Systems

- Typical systems take a signal and convert it into another signal,
  - radio receiver
  - audio amplifier
  - modem
  - microphone
  - cell telephone
  - cellular metabolism
  - national and global economies
- Internally, a system may contain many different types of signals.
- The systems perspective allows you to consider all of these together.
- In general, a *system* transforms *input signals* into *output signals*.

Continuous and Discrete Time Signals

- Most of the signals we will talk about are functions of time.
- There are many ways to classify signals. This class is organized according to whether the signals are continuous in time, or discrete.
- A *continuous-time* signal has values for all points in time in some (possibly infinite) interval.
- A *discrete time* signal has values for only discrete points in time.



- Signals can also be a function of space (images) or of space and time (video), and may be continuous or discrete in each dimension.

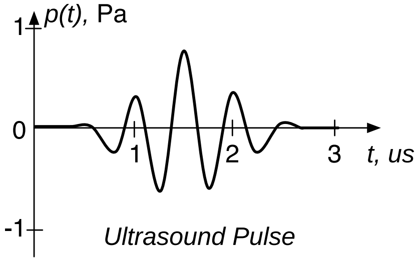
Types of Systems

Systems are classified according to the types of input and output signals

- *Continuous-time system* has continuous-time inputs and outputs.
  - AM or FM radio
  - Conventional (all mechanical) car
- *Discrete-time system* has discrete-time inputs and outputs.
  - PC computer game
  - Matlab
  - Your mortgage
- Hybrid systems are also very important (A/D, D/A converters).
  - You playing a game on a PC
  - Modern cars with ECU (electronic control units)
  - Most commercial and military aircraft

Continuous Time Signals

- Function of a time variable, something like t, τ, t_1.
- The entire signal is denoted as v, $v(.)$, or $v(t)$, where t is a dummy variable.
- The value of the signal at a particular time is $v(1.2)$, or $v(t), t = 2$.



Discrete Time Signals

- Fundamentally, a discrete-time signal is sequence of samples, written

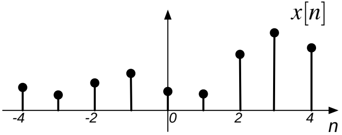
$$x[n]$$

where n is an integer over some (possibly infinite) interval.

- Often, at least conceptually, samples of a continuous time signal

$$x[n] = x(nT)$$

where n is an integer, and T is the *sampling period*.



- Discrete time signals may not represent uniform time samples (NYSE closes, for example)

Summary

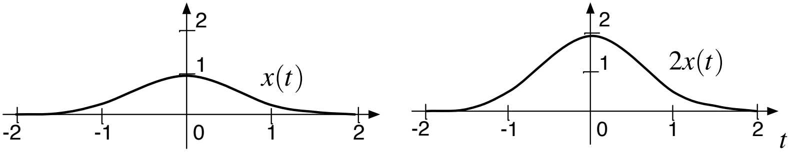
- A signal is a collection of data
- Systems act on signals (inputs and outputs)
- Mathematically, they are similar. A signal can be represented by a function. A system can be represented by a function (the domain is the space of input signals).
- We focus on 1-dimensional signals.
- Our systems are not random.

Signal Characteristics and Models

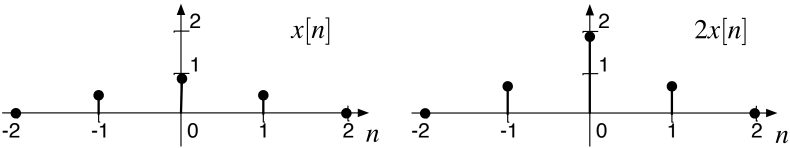
- Operations on the time dependence of a signal
  - Time scaling
  - Time reversal
  - Time shift
  - Combinations
- Signal characteristics
- Periodic signals
- Complex signals
- Signals sizes
- Signal Energy and Power

Amplitude Scaling

- The scaled signal $ax(t)$ is $x(t)$ multiplied by the constant a

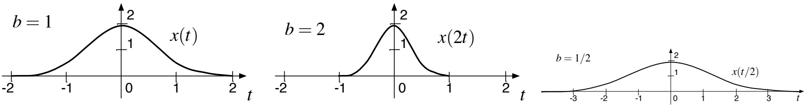


- The scaled signal $ax[n]$ is $x[n]$ multiplied by the constant a



Time Scaling, Continuous Time

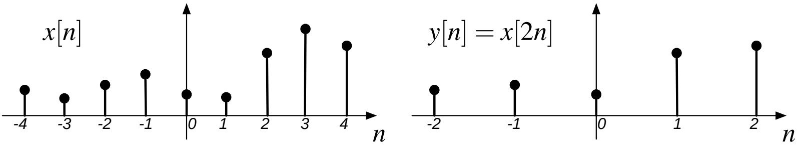
A signal $x(t)$ is scaled in time by multiplying the time variable by a positive constant b, to produce $x(bt)$. A positive factor of b either expands $(0 < b < 1)$ or compresses $(b > 1)$ the signal in time.



Time Scaling, Discrete Time

The discrete-time sequence $x[n]$ is *compressed* in time by multiplying the index n by an integer k, to produce the time-scaled sequence $x[nk]$.

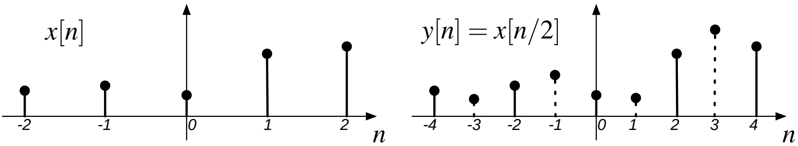
- This extracts every k^{th} sample of $x[n]$.
- Intermediate samples are lost.
- The sequence is shorter.



Called *downsampling*, or *decimation*.

The discrete-time sequence $x[n]$ is *expanded* in time by dividing the index n by an integer m, to produce the time-scaled sequence $x[n/m]$.

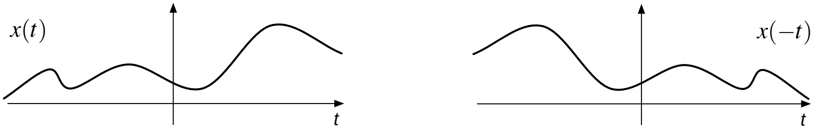
- This specifies every m^{th} sample.
- The intermediate samples must be synthesized (set to zero, or interpolated).
- The sequence is longer.



Called *upsampling*, or *interpolation*.

Time Reversal

- Continuous time: replace t with $-t$, time reversed signal is $x(-t)$



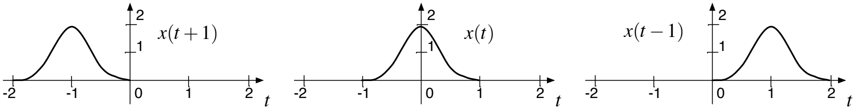
- Discrete time: replace n with $-n$, time reversed signal is $x[-n]$.

- Same as time scaling, but with $b = -1$.

Time Shift

For a continuous-time signal $x(t)$, and a time $t_1 > 0$,

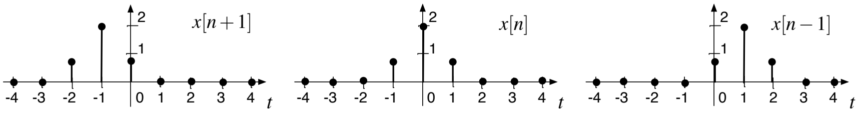
- Replacing t with $t - t_1$ gives a *delayed* signal $x(t - t_1)$
- Replacing t with $t + t_1$ gives an *advanced* signal $x(t + t_1)$



- May seem counterintuitive. Think about where $t - t_1$ is zero.

For a discrete time signal $x[n]$, and an integer $n_1 > 0$

- $x[n - n_1]$ is a delayed signal.
- $x[n + n_1]$ is an advanced signal.
- The delay or advance is an integer number of sample times.

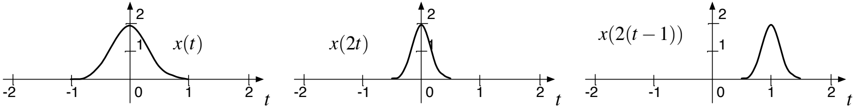


- Again, where is $n - n_1$ zero?

Combinations of Operations

- Time scaling, shifting, and reversal can all be combined.
- Operation can be performed in any order, but care is required.
- This *will* cause confusion.
- Example: $x(2(t-1))$

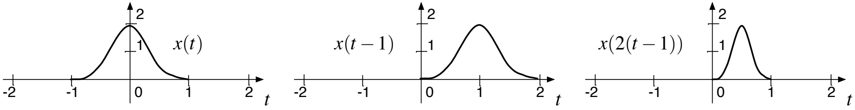
Scale first, then shift
Compress by 2, shift by 1



Example $x(2(t-1))$, continued

Shift first, then scale

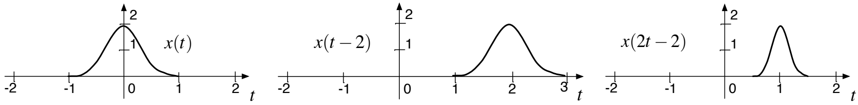
Shift by 1, compress by 2 *Incorrect*



Shift first, then scale

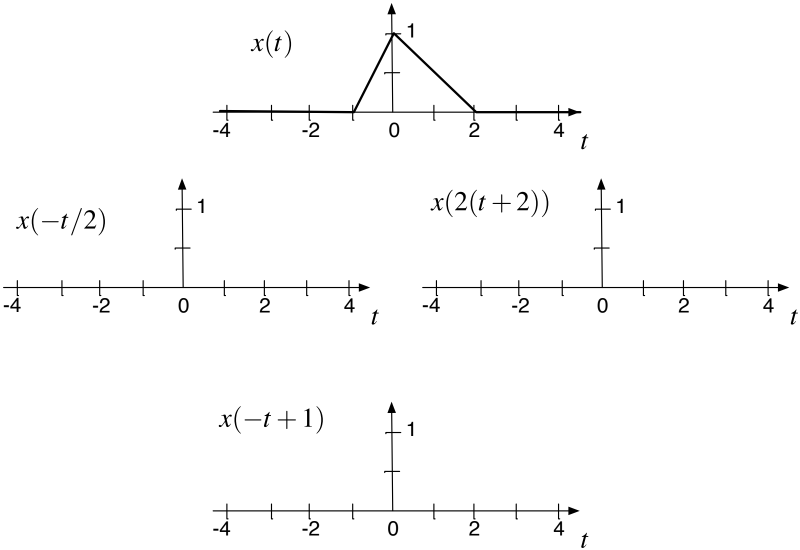
Rewrite $x(2(t-1)) = x(2t-2)$

Shift by 2, scale by 2 *Correct*



Where is $2(t-1)$ equal to zero?

Try these yourselves ....



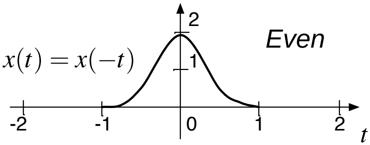
Even and Odd Symmetry

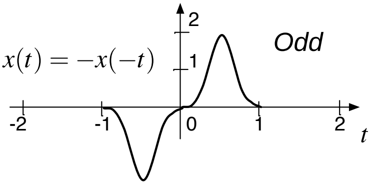
- An *even* signal is symmetric about the origin

$$x(t) = x(-t)$$

- An *odd* signal is antisymmetric about the origin

$$x(t) = -x(-t)$$





- Any signal can be decomposed into even and odd components

$$\begin{aligned} x_e(t) &= \frac{1}{2}\left[x(t) + x(-t)\right] \\ x_o(t) &= \frac{1}{2}\left[x(t) - x(-t)\right]. \end{aligned}$$

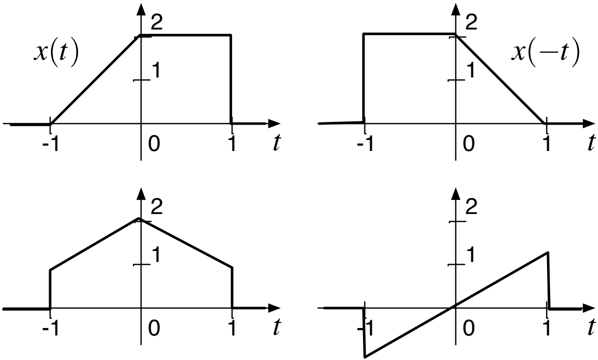
Check that

$$\begin{aligned} x_e(t) &= x_e(-t), \\ x_o(t) &= -x_o(-t), \end{aligned}$$

and that

$$x_e(t) + x_o(t) = x(t).$$

- Example

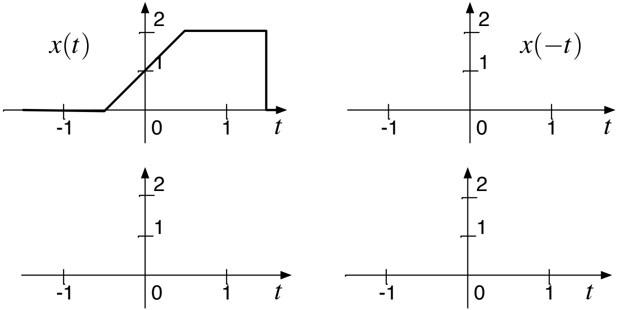


$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \qquad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

- Same type of decomposition applies for discrete-time signals.

The decomposition into even and odd components depends on the location of the origin. Shifting the signal changes the decomposition.

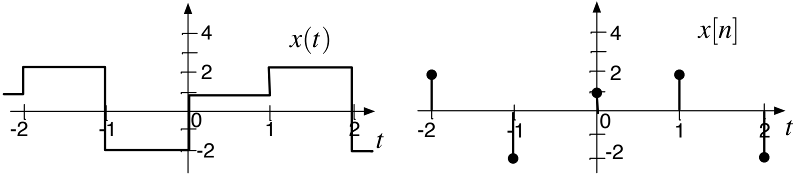
Plot the even and odd components of the previous example, after shifting $x(t)$ by $1/2$ to the right.



$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \qquad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Discrete Amplitude Signals

- Discrete amplitude signals take on only a countable set of values.
- Example: Quantized signal (binary, fixed point, floating point).
- A *digital signal* is a quantized discrete-time signal.
- Requires treatment as random process, not part of this course.



Periodic Signals

- Very important in this class.

- Continuous time signal is periodic if and only if there exists a $T_0 > 0$ such that

$$x(t + T_0) = x(t) \qquad \text{for all t}$$

  T_0 is the period of $x(t)$ in time.

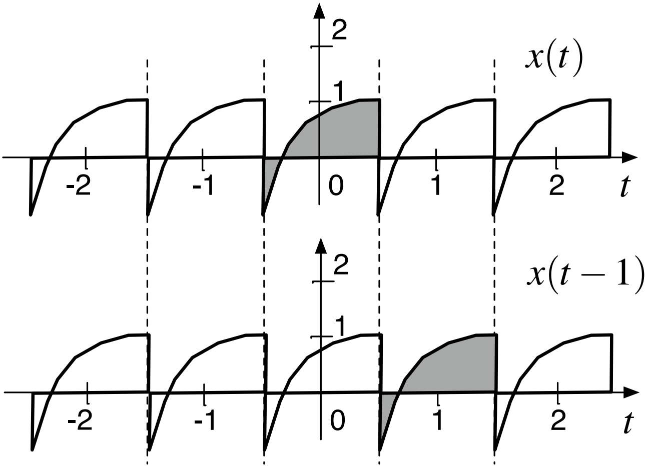
- A discrete-time signal is periodic if and only if there exists an integer $N_0 > 0$ such that

$$x[n + N_0] = x[n] \qquad \text{for all } n$$

  N_0 is the period of $x[n]$ in sample spacings.

- The smallest T_0 or N_0 is the *fundamental period* of the periodic signal.

Example:

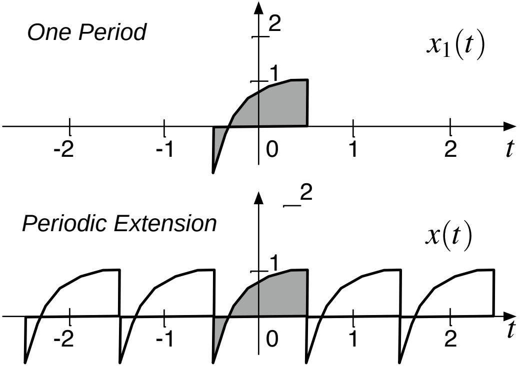


Shifting $x(t)$ by 1 time unit results in the same signal.

- Common periodic signals are sines and cosines

Periodic Extension

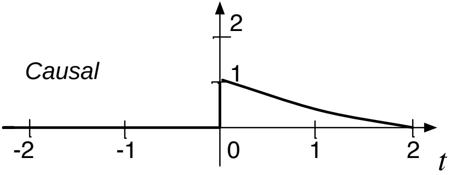
- Periodic signals can be generated by *periodic extension* by any segment of length one period T_0 (or a multiple of the period).



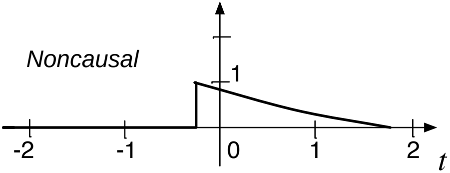
- We will often take a signal that is defined only over an interval T_0 and use periodic extension to make a periodic signal.

Causal Signals

- *Causal signals* are non-zero only for $t \geq 0$ (starts at $t = 0$, or later)



- *Noncausal signals* are non-zero for some $t < 0$ (starts before $t = 0$)



- *Anticausal signals* are non-zero only for $t \leq 0$ (goes backward in time from $t = 0$)

Complex Signals

- So far, we have only considered real (or integer) valued signals.

- Signals can also be complex

$$z(t) = x(t) + jy(t)$$

  where $x(t)$ and $y(t)$ are each real valued signals, and $j = \sqrt{-1}$.

- Arises naturally in many problems
  - Convenient representation for sinusoids
  - Communications
  - Radar, sonar, ultrasound

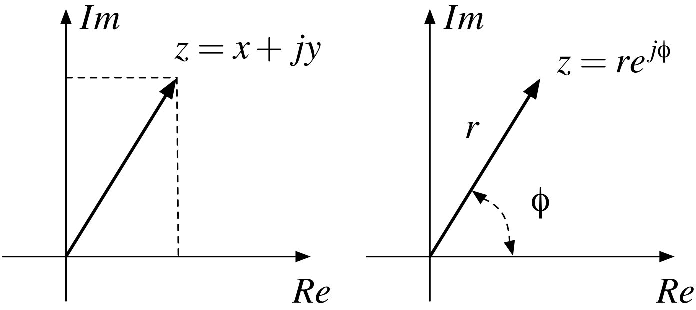
Review of Complex Numbers

Complex number in Cartesian form: $z = x + jy$

- $x = \Re z$, the *real part* of z
- $y = \Im z$, the *imaginary part* of z
- x and y are also often called the *in-phase* and *quadrature* components of z.
- $j = \sqrt{-1}$ (engineering notation)
- $i = \sqrt{-1}$ (physics, chemistry, mathematics)

Complex number in polar form: $z = re^{j\phi}$

- r is the *modulus* or *magnitude* of z
- ϕ is the *angle* or *phase* of z
- $\exp(j\phi) = \cos\phi + j\sin\phi$



- complex exponential of $z = x + jy$:

$$e^z = e^{x+jy} = e^x e^{jy} = e^x(\cos y + j\sin y)$$

Know how to add, multiply, and divide complex numbers, and be able to go between representations easily.

Signal Energy and Power

If $i(t)$ is the current through a resistor, then the energy dissipated in the resistor is

$$E_R = \lim_{T\to\infty} \int_{-T}^{T} i^2(t)\ R\ dt$$

This is energy in *Joules*.

The signal energy for $i(t)$ is defined as the energy dissipated in a 1 Ω resistor

$$E_i = \lim_{T\to\infty} \int_{-T}^{T} i^2(t) dt$$

The *signal energy* for a (possibly complex) signal $x(t)$ is

$$E_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt.$$

In most applications, this is not an actual energy (most signals aren't actually applied to 1Ω resistor).

The average of the signal energy over time is the *signal power*

Properties of Energy and Power Signals

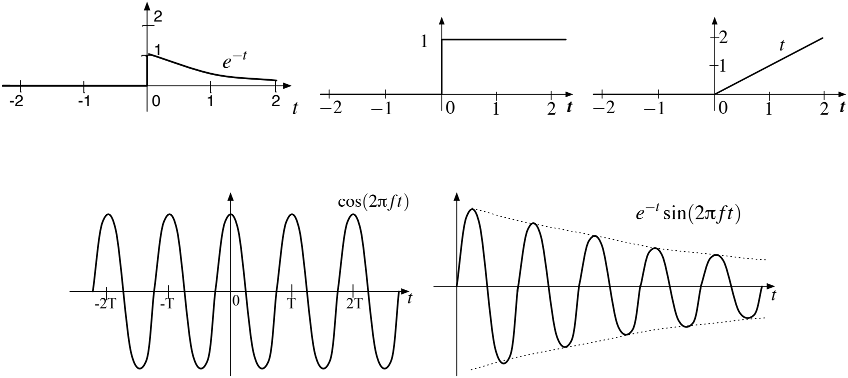
An energy signal $x(t)$ has zero power

$$\begin{aligned} P_x &= \lim_{T\to\infty} \frac{1}{2T} \underbrace{\int_{-T}^{T} |x(t)|^2 \, dt}_{\to E_x < \infty} \\ &= 0 \end{aligned}$$

A power signal has infinite energy

$$\begin{aligned} E_x &= \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt \\ &= \lim_{T\to\infty} 2T \underbrace{\frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt}_{\to P_x > 0} = \infty. \end{aligned}$$

Classify these signals as power or energy signals



A bounded periodic signal.
A bounded finite duration signal.