UNIT 6

Signals and Systems

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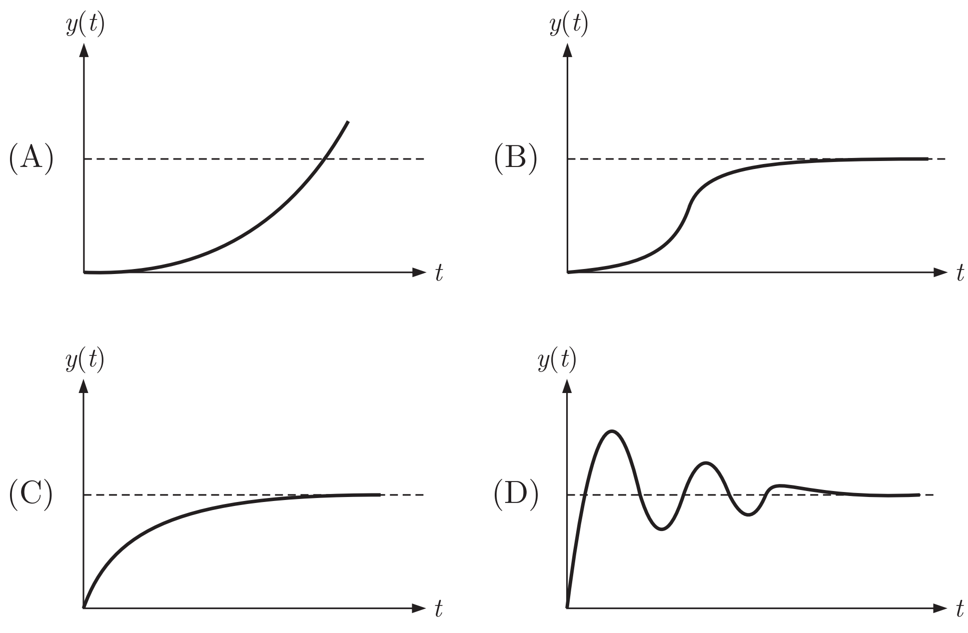
2011 **ONE MARK**

MCQ 6.1

The differential equation $100\frac{d^2y}{dt^2} - 20\frac{dy}{dt} + y = x(t)$ describes a system with an input $x(t)$ and an output $y(t)$. The system, which is initially relaxed, is excited by a unit step input. The output $y(t)$ can be represented by the waveform



MCQ 6.2

The trigonometric Fourier series of an even function does not have the

(A) dc term (B) cosine terms

(C) sine terms (D) odd harmonic terms

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MCQ 6.3

A system is defined by its impulse response $h(n) = 2^n u(n-2)$. The system is

(A) stable and causal
(B) causal but not stable
(C) stable but not causal
(D) unstable and non-causal

MCQ 6.4

If the unit step response of a network is $(1 - e^{-\alpha t})$, then its unit impulse response is

(A) $\alpha e^{-\alpha t}$
(B) $\alpha^{-1} e^{-\alpha t}$
(C) $(1 - \alpha^{-1}) e^{-\alpha t}$
(D) $(1 - \alpha) e^{-\alpha t}$

2011 **TWO MARKS**

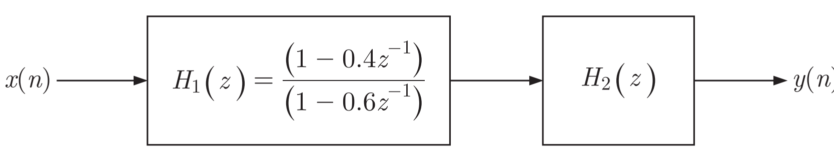
MCQ 6.5

An input $x(t) = \exp(-2t)\, u(t) + \delta(t-6)$ is applied to an LTI system with impulse response $h(t) = u(t)$. The output is

(A) $[1 - \exp(-2t)]\, u(t) + u(t+6)$
(B) $[1 - \exp(-2t)]\, u(t) + u(t-6)$
(C) $0.5[1 - \exp(-2t)]\, u(t) + u(t+6)$
(D) $0.5[1 - \exp(-2t)]\, u(t) + u(t-6)$

MCQ 6.6

Two systems $H_1(Z)$ and $H_2(Z)$ are connected in cascade as shown below. The overall output $y(n)$ is the same as the input $x(n)$ with a one unit delay. The transfer function of the second system $H_2(Z)$ is



(A) $\dfrac{1 - 0.6z^{-1}}{z^{-1}(1 - 0.4z^{-1})}$
(B) $\dfrac{z^{-1}(1 - 0.6z^{-1})}{(1 - 0.4z^{-1})}$
(C) $\dfrac{z^{-1}(1 - 0.4z^{-1})}{(1 - 0.6z^{-1})}$
(D) $\dfrac{1 - 0.4\, z^{-1}}{z^{-1}(1 - 0.6z^{-1})}$

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MCQ 6.7

The first six points of the 8-point DFT of a real valued sequence are $5, 1-j3, 0, 3-j4, 0$ and $3+j4$. The last two points of the DFT are respectively

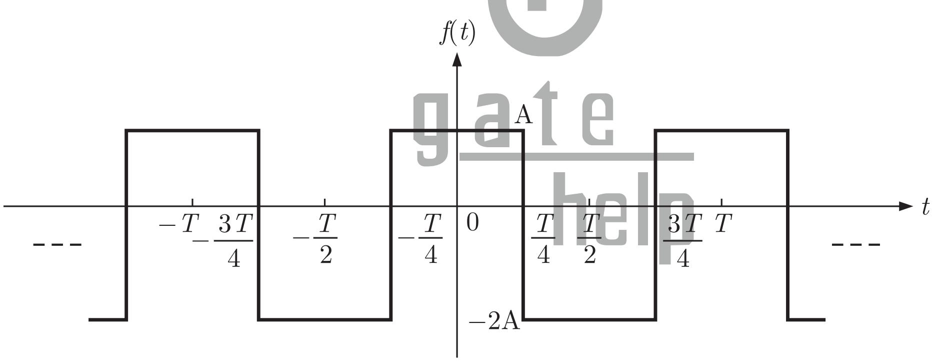
(A) $0, 1-j3$ (B) $0, 1+j3$

(C) $1+j3, 5$ (D) $1-j3, 5$

2010 **ONE MARK**

MCQ 6.8

The trigonometric Fourier series for the waveform $f(t)$ shown below contains



(A) only cosine terms and zero values for the dc components

(B) only cosine terms and a positive value for the dc components

(C) only cosine terms and a negative value for the dc components

(D) only sine terms and a negative value for the dc components

MCQ 6.9

Consider the z-transform $x(z) = 5z^2 + 4z^{-1} + 3;\ 0 < |z| < \infty$. The inverse z- transform $x[n]$ is

(A) $5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$

(B) $5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$

(C) $5u[n+2] + 3u[n] + 4u[n-1]$

(D) $5u[n-2] + 3u[n] + 4u[n+1]$

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MCQ 6.10

Two discrete time system with impulse response $h_1[n] = \delta[n-1]$ and $h_2[n] = \delta[n-2]$ are connected in cascade. The overall impulse response of the cascaded system is

(A) $\delta[n-1] + \delta[n-2]$
(B) $\delta[n-4]$
(C) $\delta[n-3]$
(D) $\delta[n-1]\,\delta[n-2]$

MCQ 6.11

For a N-point FET algorithm $N = 2^m$ which one of the following statements is TRUE ?

(A) It is not possible to construct a signal flow graph with both input and output in normal order
(B) The number of butterflies in the m^{th} stage in N/m
(C) In-place computation requires storage of only 2N data
(D) Computation of a butterfly requires only one complex multiplication.

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2010 TWO MARKS

MCQ 6.12

Given $f(t) = L^{-1}\left[\dfrac{3s+1}{s^3 + 4s^2 + (k-3)\,s}\right]$. If $\lim\limits_{t \to \infty} f(t) = 1$, then the value of k is

(A) 1
(B) 2
(C) 3
(D) 4

MCQ 6.13

A continuous time LTI system is described by

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$$

Assuming zero initial conditions, the response $y(t)$ of the above system for the input $x(t) = e^{-2t}u(t)$ is given by

(A) $(e^{t} - e^{3t})\,u(t)$
(B) $(e^{-t} - e^{-3t})\,u(t)$
(C) $(e^{-t} + e^{-3t})\,u(t)$
(D) $(e^{t} + e^{3t})\,u(t)$

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MCQ 6.14

The transfer function of a discrete time LTI system is given by

$$H(z) = \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Consider the following statements:

S1: The system is stable and causal for ROC: $|z| > 1/2$

S2: The system is stable but not causal for ROC: $|z| < 1/4$

S3: The system is neither stable nor causal for ROC: $1/4 < |z| < 1/2$

Which one of the following statements is valid ?

(A) Both S1 and S2 are true
(B) Both S2 and S3 are true
(C) Both S1 and S3 are true
(D) S1, S2 and S3 are all true

2009 **ONE MARK**

MCQ 6.15

The Fourier series of a real periodic function has only

(P) cosine terms if it is even
(Q) sine terms if it is even
(R) cosine terms if it is odd
(S) sine terms if it is odd

Which of the above statements are correct ?

(A) P and S
(B) P and R
(C) Q and S
(D) Q and R

MCQ 6.16

A function is given by $f(t) = \sin^2 t + \cos 2t$. Which of the following is true ?

(A) f has frequency components at 0 and $\frac{1}{2\pi}$ Hz

(B) f has frequency components at 0 and $\frac{1}{\pi}$ Hz

(C) f has frequency components at $\frac{1}{2\pi}$ and $\frac{1}{\pi}$ Hz

(D) f has frequency components at $\frac{0.1}{2\pi}$ and $\frac{1}{\pi}$ Hz

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MCQ 6.17

The ROC of z-transform of the discrete time sequence

$$x(n) = \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(-n-1) \text{ is}$$

(A) $\frac{1}{3} < |z| < \frac{1}{2}$

(B) $|z| > \frac{1}{2}$

(C) $|z| < \frac{1}{3}$

(D) $2 < |z| < 3$

2009 **TWO MARKS**

MCQ 6.18

Given that $F(s)$ is the one-side Laplace transform of $f(t)$, the Laplace transform of $\int_0^t f(\tau)\,d\tau$ is

(A) $sF(s) - f(0)$

(B) $\frac{1}{s}F(s)$

(C) $\int_0^s F(\tau)\,d\tau$

(D) $\frac{1}{s}[F(s) - f(0)]$

MCQ 6.19

A system with transfer function $H(z)$ has impulse response $h(.)$ defined as $h(2) = 1, h(3) = -1$ and $h(k) = 0$ otherwise. Consider the following statements.

S1 : $H(z)$ is a low-pass filter.

S2 : $H(z)$ is an FIR filter.

Which of the following is correct?

(A) Only S2 is true

(B) Both S1 and S2 are false

(C) Both S1 and S2 are true, and S2 is a reason for S1

(D) Both S1 and S2 are true, but S2 is not a reason for S1

MCQ 6.20

Consider a system whose input x and output y are related by the equation $y(t) = \int_{-\infty}^{\infty} x(t-\tau)\,g(2\tau)\,d\tau$ where $h(t)$ is shown in the graph.

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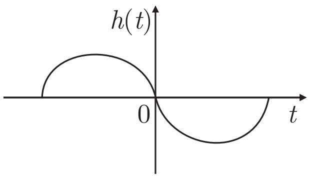

Which of the following four properties are possessed by the system ?

BIBO : Bounded input gives a bounded output.

Causal : The system is causal,

LP : The system is low pass.

LTI : The system is linear and time-invariant.

(A) Causal, LP
(B) BIBO, LTI
(C) BIBO, Causal, LTI
(D) LP, LTI

MCQ 6.21

The 4-point Discrete Fourier Transform (DFT) of a discrete time sequence {1,0,2,3} is

(A) $[0,\ -2+2j,\ 2,\ -2-2j]$
(B) $[2,\ 2+2j,\ 6,\ 2-2j]$
(C) $[6,\ 1-3j,\ 2,\ 1+3j]$
(D) $[6,\ -1+3j,\ 0,\ -1-3j]$

MCQ 6.22

An LTI system having transfer function $\frac{s^2+1}{s^2+2s+1}$ and input $x(t) = \sin(t+1)$ is in steady state. The output is sampled at a rate ω_s rad/s to obtain the final output $\{x(k)\}$. Which of the following is true ?

(A) $y(.)$ is zero for all sampling frequencies ω_s

(B) $y(.)$ is nonzero for all sampling frequencies ω_s

(C) $y(.)$ is nonzero for $\omega_s > 2$, but zero for $\omega_s < 2$

(D) $y(.)$ is zero for $\omega_s > 2$, but nonzero for $\omega_2 < 2$

2008 **ONE MARK**

MCQ 6.23

The input and output of a continuous time system are respectively denoted by $x(t)$ and $y(t)$. Which of the following descriptions corresponds to a causal system ?

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(A) $y(t) = x(t-2) + x(t+4)$ (B) $y(t) = (t-4)\,x(t+1)$

(C) $y(t) = (t+4)\,x(t-1)$ (D) $y(t) = (t+5)\,x(t+5)$

MCQ 6.24

The impulse response $h(t)$ of a linear time invariant continuous time system is described by $h(t) = \exp(\alpha t)\,u(t) + \exp(\beta t)\,u(-t)$ where $u(-t)$ denotes the unit step function, and α and β are real constants. This system is stable if

(A) α is positive and β is positive

(B) α is negative and β is negative

(C) α is negative and β is negative

(D) α is negative and β is positive

2008 **TWO MARKS**

MCQ 6.25

A linear, time - invariant, causal continuous time system has a rational transfer function with simple poles at $s = -2$ and $s = -4$ and one simple zero at $s = -1$. A unit step $u(t)$ is applied at the input of the system. At steady state, the output has constant value of 1. The impulse response of this system is

(A) $[\exp(-2t) + \exp(-4t)]\,u(t)$

(B) $[-4\exp(-2t) - 12\exp(-4t) - \exp(-t)]\,u(t)$

(C) $[-4\exp(-2t) + 12\exp(-4t)]\,u(t)$

(D) $[-0.5\exp(-2t) + 1.5\exp(-4t)]\,u(t)$

MCQ 6.26

The signal $x(t)$ is described by

$$x(t) = \begin{cases} 1 & \text{for} -1 \le t \le +1 \\ 0 & \text{otherwise} \end{cases}$$

Two of the angular frequencies at which its Fourier transform becomes zero are

(A) $\pi,\ 2\pi$ (B) $0.5\pi,\ \ 1.5\pi$

(C) $0,\ \ \pi$ (D) $2\pi,\ \ 2.5\pi$

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MCQ 6.27

A discrete time linear shift - invariant system has an impulse response $h[n]$ with $h[0]=1, h[1]=-1, h[2]=2$, and zero otherwise The system is given an input sequence $x[n]$ with $x[0]=x[2]=1$, and zero otherwise. The number of nonzero samples in the output sequence $y[n]$, and the value of $y[2]$ are respectively

(A) 5, 2 (B) 6, 2

(C) 6, 1 (D) 5, 3

MCQ 6.28

Let $x(t)$ be the input and $y(t)$ be the output of a continuous time system. Match the system properties P1, P2 and P3 with system relations R1, R2, R3, R4

Properties	Relations
P1 : Linear but NOT time - invariant	R1 : $y(t)=t^2 x(t)$
P2 : Time - invariant but NOT linear	R2 : $y(t)=t\lvert x(t)\rvert$
P3 : Linear and time - invariant	R3 : $y(t)=\lvert x(t)\rvert$
	R4 : $y(t)=x(t-5)$

(A) $(P1, R1), (P2, R3), (P3, R4)$

(B) $(P1, R2), (P2, R3), (P3, R4)$

(C) $(P1, R3), (P2, R1), (P3, R2)$

(D) $(P1, R1), (P2, R2), (P3, R3)$

MCQ 6.29

$\{x(n)\}$ is a real - valued periodic sequence with a period N. $x(n)$ and $X(k)$ form N-point Discrete Fourier Transform (DFT) pairs. The DFT $Y(k)$ of the sequence $y(n)=\frac{1}{N}\sum_{r=0}^{N-1}x(r)\,x(n+r)$ is

(A) $\lvert X(k)\rvert^2$ (B) $\frac{1}{N}\sum_{r=0}^{N-1}X(r)\,X(k+r)$

(C) $\frac{1}{N}\sum_{r=0}^{N-1}X(r)\,X(k+r)$ (D) 0

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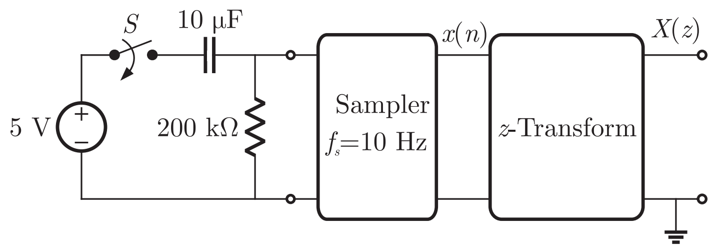
Statement for Linked Answer Question 6.31 and 6.32:

In the following network, the switch is closed at $t = 0^-$ and the sampling starts from $t = 0$. The sampling frequency is 10 Hz.



MCQ 6.30

The samples $x(n)$, $n = (0, 1, 2, ...)$ are given by

(A) $5(1 - e^{-0.05n})$ (B) $5e^{-0.05n}$

(C) $5(1 - e^{-5n})$ (D) $5e^{-5n}$

MCQ 6.31

The expression and the region of convergence of the $z-$transform of the sampled signal are

(A) $\frac{5z}{z - e^{5}}$, $|z| < e^{-5}$ (B) $\frac{5z}{z - e^{-0.05}}$, $|z| < e^{-0.05}$

(C) $\frac{5z}{z - e^{-0.05}}$, $|z| > e^{-0.05}$ (D) $\frac{5z}{z - e^{-5}}$, $|z| > e^{-5}$

Statement for Linked Answer Question 6.33 & 6.34:

The impulse response $h(t)$ of linear time - invariant continuous time system is given by $h(t) = \exp(-2t)\,u(t)$, where $u(t)$ denotes the unit step function.

MCQ 6.32

The frequency response $H(\omega)$ of this system in terms of angular frequency ω, is given by $H(\omega)$

(A) $\frac{1}{1 + j2\omega}$ (B) $\frac{\sin\omega}{\omega}$

(C) $\frac{1}{2 + j\omega}$ (D) $\frac{j\omega}{2 + j\omega}$

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MCQ 6.33

The output of this system, to the sinusoidal input $x(t) = 2\cos 2t$ for all time t, is

(A) 0
(B) $2^{-0.25}\cos(2t - 0.125\pi)$
(C) $2^{-0.5}\cos(2t - 0.125\pi)$
(D) $2^{-0.5}\cos(2t - 0.25\pi)$

2007 ONE MARK

MCQ 6.34

If the Laplace transform of a signal $Y(s) = \frac{1}{s(s-1)}$, then its final value is

(A) -1
(B) 0
(C) 1
(D) Unbounded

2007 TWO MARKS

MCQ 6.35

The 3-dB bandwidth of the low-pass signal $e^{-t}u(t)$, where $u(t)$ is the unit step function, is given by

(A) $\frac{1}{2\pi}$ Hz
(B) $\frac{1}{2\pi}\sqrt{\sqrt{2}-1}$ Hz
(C) ∞
(D) 1 Hz

MCQ 6.36

A 5-point sequence $x[n]$ is given as $x[-3] = 1$, $x[-2] = 1$, $x[-1] = 0$, $x[0] = 5$ and $x[1] = 1$. Let $X(e^{i\omega})$ denoted the discrete-time Fourier transform of $x[n]$. The value of $\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega$ is

(A) 5
(B) 10π
(C) 16π
(D) $5 + j10\pi$

MCQ 6.37

The $z-$transform $X(z)$ of a sequence $x[n]$ is given by $X[z] = \frac{0.5}{1-2z^{-1}}$. It is given that the region of convergence of $X(z)$ includes the unit circle. The value of $x[0]$ is

(A) -0.5
(B) 0
(C) 0.25
(D) 05

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MCQ 6.38

A Hilbert transformer is a

(A) non-linear system
(B) non-causal system
(C) time-varying system
(D) low-pass system

MCQ 6.39

The frequency response of a linear, time-invariant system is given by $H(f) = \frac{5}{1+j10\pi f}$. The step response of the system is

(A) $5(1-e^{-5t})\,u(t)$
(B) $5[1-e^{-\frac{t}{5}}]u(t)$
(C) $\frac{1}{2}(1-e^{-5t})\,u(t)$
(D) $\frac{1}{5}(1-e^{-\frac{t}{5}})u(t)$

2006 **ONE MARK**

MCQ 6.40

Let $x(t) \longleftrightarrow X(j\omega)$ be Fourier Transform pair. The Fourier Transform of the signal $x(5t-3)$ in terms of $X(j\omega)$ is given as

(A) $\frac{1}{5}e^{-\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$
(B) $\frac{1}{5}e^{\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$
(C) $\frac{1}{5}e^{-j3\omega}X\left(\frac{j\omega}{5}\right)$
(D) $\frac{1}{5}e^{j3\omega}X\left(\frac{j\omega}{5}\right)$

MCQ 6.41

The Dirac delta function $\delta(t)$ is defined as

(A) $\delta(t) = \begin{cases} 1 & t=0 \\ 0 & \text{otherwise} \end{cases}$

(B) $\delta(t) = \begin{cases} \infty & t=0 \\ 0 & \text{otherwise} \end{cases}$

(C) $\delta(t) = \begin{cases} 1 & t=0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty}\delta(t)\,dt = 1$

(D) $\delta(t) = \begin{cases} \infty & t=0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty}\delta(t)\,dt = 1$

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MCQ 6.42

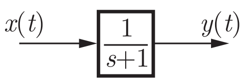
If the region of convergence of $x_1[n] + x_2[n]$ is $\frac{1}{3} < |z| < \frac{2}{3}$ then the region of convergence of $x_1[n] - x_2[n]$ includes

(A) $\frac{1}{3} < |z| < 3$ (B) $\frac{2}{3} < |z| < 3$

(C) $\frac{3}{2} < |z| < 3$ (D) $\frac{1}{3} < |z| < \frac{2}{3}$

MCQ 6.43

In the system shown below, $x(t) = (\sin t)\, u(t)$ In steady-state, the response $y(t)$ will be



(A) $\frac{1}{\sqrt{2}}\sin\left(t - \frac{\pi}{4}\right)$ (B) $\frac{1}{\sqrt{2}}\sin\left(t + \frac{\pi}{4}\right)$

(C) $\frac{1}{\sqrt{2}}e^{-t}\sin t$ (D) $\sin t - \cos t$

2006 TWO MARKS

MCQ 6.44

Consider the function $f(t)$ having Laplace transform

$$F(s) = \frac{\omega_0}{s^2 + \omega_0^2}\,\text{Re}[s] > 0$$

The final value of $f(t)$ would be

(A) 0 (B) 1

(C) $-1 \le f(\infty) \le 1$ (D) ∞

MCQ 6.45

A system with input $x[n]$ and output $y[n]$ is given as $y[n] = (\sin \frac{5}{6}\pi n)\, x[n]$. The system is

(A) linear, stable and invertible

(B) non-linear, stable and non-invertible

(C) linear, stable and non-invertible

(D) linear, unstable and invertible

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MCQ 6.46

The unit step response of a system starting from rest is given by $c(t) = 1 - e^{-2t}$ for $t \geq 0$. The transfer function of the system is

(A) $\frac{1}{1+2s}$

(B) $\frac{2}{2+s}$

(C) $\frac{1}{2+s}$

(D) $\frac{2s}{1+2s}$

MCQ 6.47

The unit impulse response of a system is $f(t) = e^{-t}, t \geq 0$. For this system the steady-state value of the output for unit step input is equal to

(A) -1

(B) 0

(C) 1

(D) ∞

2005 ONE MARK

MCQ 6.48

Choose the function $f(t);\ -\infty < t < \infty$ for which a Fourier series cannot be defined.

(A) $3\sin(25t)$

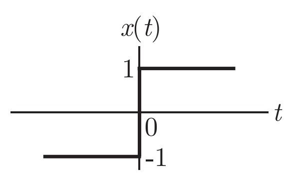
(B) $4\cos(20t+3) + 2\sin(710t)$

(C) $\exp(-|t|)\sin(25t)$

(D) 1

MCQ 6.49

The function $x(t)$ is shown in the figure. Even and odd parts of a unit step function $u(t)$ are respectively,



(A) $\frac{1}{2}, \frac{1}{2}x(t)$

(B) $-\frac{1}{2}, \frac{1}{2}x(t)$

(C) $\frac{1}{2}, -\frac{1}{2}x(t)$

(D) $-\frac{1}{2}, -\frac{1}{2}x(t)$

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MCQ 6.50

The region of convergence of $z-$ transform of the sequence $\left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1)$ must be

(A) $|z| < \frac{5}{6}$

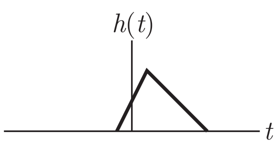
(B) $|z| > \frac{5}{6}$

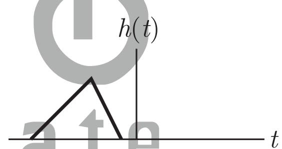
(C) $\frac{5}{6} < |z| < \frac{6}{5}$

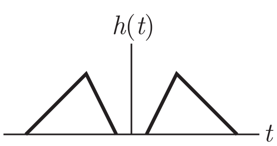
(D) $\frac{6}{5} < |z| < \infty$

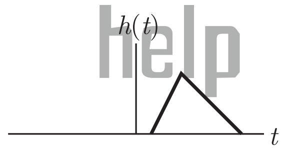
MCQ 6.51

Which of the following can be impulse response of a causal system ?

(A)


(B)


(C)


(D)


MCQ 6.52

Let $x(n) = (\frac{1}{2})^n u(n), y(n) = x^2(n)$ and $Y(e^{j\omega})$ be the Fourier transform of $y(n)$ then $Y(e^{j0})$

(A) $\frac{1}{4}$

(B) 2

(C) 4

(D) $\frac{4}{3}$

MCQ 6.53

The power in the signal $s(t) = 8\cos(20\pi - \frac{\pi}{2}) + 4\sin(15\pi t)$ is

(A) 40

(B) 41

(C) 42

(D) 82

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2005 **TWO MARKS**

MCQ 6.54

The output $y(t)$ of a linear time invariant system is related to its input $x(t)$ by the following equations

$$y(t) = 0.5x(t - t_d + T) + x(t - t_d) + 0.5x(t - t_d + T)$$

The filter transfer function $H(\omega)$ of such a system is given by

(A) $(1 + \cos\omega T)\, e^{-j\omega t_d}$
(B) $(1 + 0.5\cos\omega T)\, e^{-j\omega t_d}$
(C) $(1 - \cos\omega T)\, e^{-j\omega t_d}$
(D) $(1 - 0.5\cos\omega T)\, e^{-j\omega t_d}$

MCQ 6.55

Match the following and choose the correct combination.

Group 1

E. Continuous and aperiodic signal
F. Continuous and periodic signal
G. Discrete and aperiodic signal
H. Discrete and periodic signal

Group 2

1. Fourier representation is continuous and aperiodic
2. Fourier representation is discrete and aperiodic
3. Fourier representation is continuous and periodic
4. Fourier representation is discrete and periodic

(A) E – 3, F – 2, G – 4, H – 1
(B) E – 1, F – 3, G – 2, H – 4
(C) E – 1, F – 2, G – 3, H – 4
(D) E – 2, F – 1, G – 4, H – 3

MCQ 6.56

A signal $x(n) = \sin(\omega_0 n + \phi)$ is the input to a linear time- invariant system having a frequency response $H(e^{j\omega})$. If the output of the system $Ax(n - n_0)$ then the most general form of $\angle H(e^{j\omega})$ will be

(A) $-n_0\omega_0 + \beta$ for any arbitrary real
(B) $-n_0\omega_0 + 2\pi k$ for any arbitrary integer k
(C) $n_0\omega_0 + 2\pi k$ for any arbitrary integer k
(D) $-n_0\omega_0\phi$

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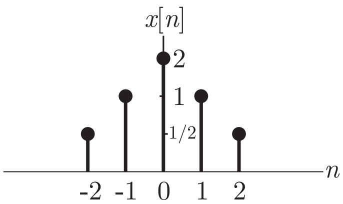
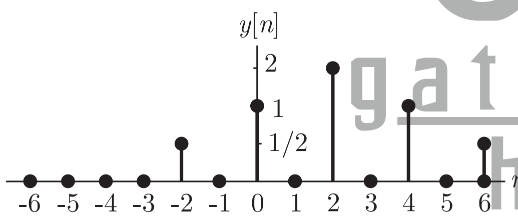
Statement of linked answer question 6.59 and 6.60 :

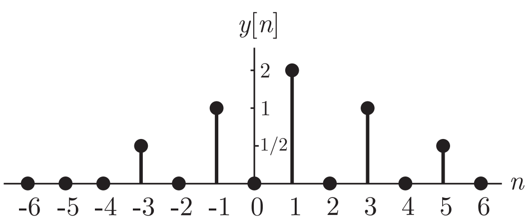
A sequence $x(n)$ has non-zero values as shown in the figure.

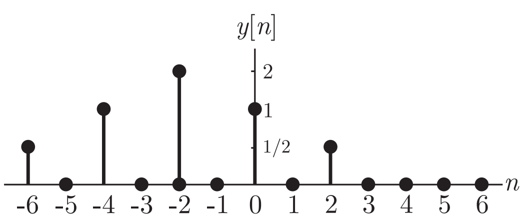


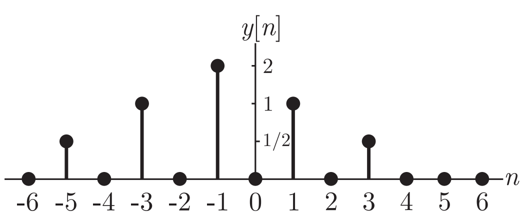
MCQ 6.57

The sequence $y(n) = \begin{cases} x\left(\frac{n}{2} - 1\right), & \text{For } n \text{ even} \\ 0, & \text{For } n \text{ odd} \end{cases}$ will be

(A)


(B)


(C)


(D)


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MCQ 6.58

The Fourier transform of $y(2n)$ will be

(A) $e^{-j2\omega}[\cos 4\omega + 2\cos 2\omega + 2]$ (B) $\cos 2\omega + 2\cos\omega + 2$

(C) $e^{-j\omega}[\cos 2\omega + 2\cos\omega + 2]$ (D) $e^{-j2\omega}[\cos 2\omega + 2\cos + 2]$

MCQ 6.59

For a signal $x(t)$ the Fourier transform is $X(f)$. Then the inverse Fourier transform of $X(3f+2)$ is given by

(A) $\frac{1}{2}x\left(\frac{t}{2}\right)e^{j3\pi t}$ (B) $\frac{1}{3}x\left(\frac{t}{3}\right)e^{-\frac{j4\pi t}{3}}$

(C) $3x(3t)\,e^{-j4\pi t}$ (D) $x(3t+2)$

2004 **ONE MARK**

MCQ 6.60

The impulse response $h[n]$ of a linear time-invariant system is given by $h[n] = u[n+3] + u[n-2) - 2n[n-7]$ where $u[n]$ is the unit step sequence. The above system is

(A) stable but not causal (B) stable and causal

(C) causal but unstable (D) unstable and not causal

MCQ 6.61

The z-transform of a system is $H(z) = \frac{z}{z-0.2}$. If the ROC is $|z| < 0.2$, then the impulse response of the system is

(A) $(0.2)^n u[n]$ (B) $(0.2)^n u[-n-1]$

(C) $-(0.2)^n u[n]$ (D) $-(0.2)^n u[-n-1]$

MCQ 6.62

The Fourier transform of a conjugate symmetric function is always

(A) imaginary (B) conjugate anti-symmetric

(C) real (D) conjugate symmetric

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2004 **TWO MARKS**

MCQ 6.63

Consider the sequence $x[n] = [-4 - j5 \; 1 + \underset{\uparrow}{j2} \; 5]$. The conjugate anti-symmetric part of the sequence is

(A) $[-4 - j2.5, \; j2, \; 4 - j2.5]$

(B) $[-j2.5, \; 1, \; j2.5]$

(C) $[-j2.5, \; j2, \; 0]$

(D) $[-4, \; 1, \; 4]$

MCQ 6.64

A causal LTI system is described by the difference equation

$$2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$$

The system is stable only if

(A) $|\alpha| = 2, |\beta| < 2$

(B) $|\alpha| > 2, |\beta| > 2$

(C) $|\alpha| < 2$, any value of β

(D) $|\beta| < 2$, any value of α

MCQ 6.65

The impulse response $h[n]$ of a linear time invariant system is given as

$$h[n] = \begin{cases} -2\sqrt{2} & n = 1, -1 \\ 4\sqrt{2} & n = 2, -2 \\ 0 & \text{otherwise} \end{cases}$$

If the input to the above system is the sequence $e^{j\pi n/4}$, then the output is

(A) $4\sqrt{2}\, e^{j\pi n/4}$

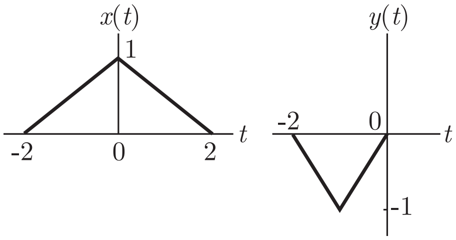
(B) $4\sqrt{2}\, e^{-j\pi n/4}$

(C) $4e^{j\pi n/4}$

(D) $-4e^{j\pi n/4}$

MCQ 6.66

Let $x(t)$ and $y(t)$ with Fourier transforms $F(f)$ and $Y(f)$ respectively be related as shown in Fig. Then $Y(f)$ is



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(A) $-\frac{1}{2}X(f/2)\,e^{-j\pi f}$ (B) $-\frac{1}{2}X(f/2)\,e^{j2\pi f}$

(C) $-X(f/2)\,e^{j2\pi f}$ (D) $-X(f/2)\,e^{-j2\pi f}$

2003 ONE MARK

MCQ 6.67

The Laplace transform of $i(t)$ is given by

$$I(s) = \frac{2}{s(1+s)}$$

At $t \to \infty$, The value of $i(t)$ tends to

(A) 0 (B) 1

(C) 2 (D) ∞

MCQ 6.68

The Fourier series expansion of a real periodic signal with fundamental frequency f_0 is given by $g_p(t) = \sum_{n=-\infty} c_n e^{j2\pi f_0 t}$. It is given that $c_3 = 3 + j5$. Then c_{-3} is

(A) $5 + j3$ (B) $-3 - j5$

(C) $-5 + j3$ (D) $3 - j5$

MCQ 6.69

Let $x(t)$ be the input to a linear, time-invariant system. The required output is $4\pi(t-2)$. The transfer function of the system should be

(A) $4e^{j4\pi f}$ (B) $2e^{-j8\pi f}$

(C) $4e^{-j4\pi f}$ (D) $2e^{j8\pi f}$

MCQ 6.70

A sequence $x(n)$ with the z-transform $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$ is applied as an input to a linear, time-invariant system with the impulse response $h(n) = 2\delta(n-3)$ where

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

The output at $n = 4$ is

(A) -6 (B) zero

(C) 2 (D) -4

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2003 **TWO MARKS**

MCQ 6.71

Let P be linearity, Q be time-invariance, R be causality and S be stability. A discrete time system has the input-output relationship,

$$y(n) = \begin{cases} x(n) & n \geq 1 \\ 0, & n = 0 \\ x(n+1) & n \leq -1 \end{cases}$$

where $x(n)$ is the input and $y(n)$ is the output. The above system has the properties

(A) P, S but not Q, R
(B) P, Q, S but not R
(C) P, Q, R, S
(D) Q, R, S but not P

Common data for Q 6.73 & 6.74 :

The system under consideration is an RC low-pass filter (RC-LPF) with $R = 1$ kΩ and $C = 1.0$ μF.

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MCQ 6.72

Let $H(f)$ denote the frequency response of the RC-LPF. Let f_1 be the highest frequency such that $0 \leq |f| \leq f_1 \frac{|H(f_1)|}{H(0)} \geq 0.95$. Then f_1 (in Hz) is

(A) 324.8
(B) 163.9
(C) 52.2
(D) 104.4

MCQ 6.73

Let $t_g(f)$ be the group delay function of the given RC-LPF and $f_2 = 100$ Hz. Then $t_g(f_2)$ in ms, is

(A) 0.717
(B) 7.17
(C) 71.7
(D) 4.505

2002 **ONE MARK**

MCQ 6.74

Convolution of $x(t+5)$ with impulse function $\delta(t-7)$ is equal to

(A) $x(t-12)$
(B) $x(t+12)$
(C) $x(t-2)$
(D) $x(t+2)$

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MCQ 6.75

Which of the following cannot be the Fourier series expansion of a periodic signal?

(A) $x(t) = 2\cos t + 3\cos 3t$ (B) $x(t) = 2\cos \pi t + 7\cos t$

(C) $x(t) = \cos t + 0.5$ (D) $x(t) = 2\cos 1.5\pi t + \sin 3.5\pi t$

MCQ 6.76

The Fourier transform $F\{e^{-1}u(t)\}$ is equal to $\dfrac{1}{1+j2\pi f}$. Therefore, $F\left\{\dfrac{1}{1+j2\pi t}\right\}$ is

(A) $e^{f}u(f)$ (B) $e^{-f}u(f)$

(C) $e^{f}u(-f)$ (D) $e^{-f}u(-f)$

MCQ 6.77

A linear phase channel with phase delay T_p and group delay T_g must have

(A) $T_p = T_g =$ constant

(B) $T_p \propto f$ and $T_g \propto f$

(C) $T_p =$ constant and $T_g \propto f$ (f denote frequency)

(D) $T_p \propto f$ and $T_p =$ constant

2002 TWO MARKS

MCQ 6.78

The Laplace transform of continuous - time signal $x(t)$ is $X(s) = \frac{5-s}{s^2-s-2}$. If the Fourier transform of this signal exists, the $x(t)$ is

(A) $e^{2t}u(t) - 2e^{-t}u(t)$ (B) $-e^{2t}u(-t) + 2e^{-t}u(t)$

(C) $-e^{2t}u(-t) - 2e^{-t}u(t)$ (D) $e^{2t}u(-t) - 2e^{-t}u(t)$

MCQ 6.79

If the impulse response of discrete - time system is

$$h[n] = -5^n u[-n-1],$$

then the system function $H(z)$ is equal to

(A) $\dfrac{-z}{z-5}$ and the system is stable

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(B) $\frac{z}{z-5}$ and the system is stable

(C) $\frac{-z}{z-5}$ and the system is unstable

(D) $\frac{z}{z-5}$ and the system is unstable

2001 ONE MARK

MCQ 6.80

The transfer function of a system is given by $H(s) = \frac{1}{s^2(s-2)}$. The impulse response of the system is

(A) $(t^2 * e^{-2t})\,u(t)$

(B) $(t * e^{2t})\,u(t)$

(C) $(te^{-2}t)\,u(t)$

(D) $(te^{-2t})\,u(t)$

MCQ 6.81

The region of convergence of the $z-$ transform of a unit step function is

(A) $|z| > 1$

(B) $|z| < 1$

(C) (Real part of z) > 0

(D) (Real part of z) < 0

MCQ 6.82

Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right)dt$ is

(A) 1

(B) -1

(C) 0

(D) $\frac{\pi}{2}$

MCQ 6.83

If a signal $f(t)$ has energy E, the energy of the signal $f(2t)$ is equal to

(A) 1

(B) $E/2$

(C) $2E$

(D) $4E$

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2001 **TWO MARKS**

MCQ 6.84

The impulse response functions of four linear systems S1, S2, S3, S4 are given respectively by

$$h_1(t) = 1,\ h_2(t) = u(t),$$

$$h_3(t) = \frac{u(t)}{t+1} \text{ and}$$

$$h_4(t) = e^{-3t}u(t)$$

where $u(t)$ is the unit step function. Which of these systems is time invariant, causal, and stable?

(A) S1

(B) S2

(C) S3

(D) S4

2000 **ONE MARK**

MCQ 6.85

Given that $L[f(t)] = \frac{s+2}{s^2+1}$, $L[g(t)] = \frac{s^2+1}{(s+3)(s+2)}$ and

$$h(t) = \int_0^t f(\tau)\,g(t-\tau)\,d\tau.$$

$L[h(t)]$ is

(A) $\frac{s^2+1}{s+3}$

(B) $\frac{1}{s+3}$

(C) $\frac{s^2+1}{(s+3)(s+2)} + \frac{s+2}{s^2+1}$

(D) None of the above

MCQ 6.86

The Fourier Transform of the signal $x(t) = e^{-3t^2}$ is of the following form, where A and B are constants :

(A) $Ae^{-B|f|}$

(B) Ae^{-Bf^2}

(C) $A + B|f|^2$

(D) Ae^{-Bf}

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MCQ 6.87

A system with an input $x(t)$ and output $y(t)$ is described by the relations : $y(t) = tx(t)$. This system is

(A) linear and time - invariant

(B) linear and time varying

(C) non - linear and time - invariant

(D) non - linear and time - varying

MCQ 6.88

A linear time invariant system has an impulse response $e^{2t}, t > 0$. If the initial conditions are zero and the input is e^{3t}, the output for $t > 0$ is

(A) $e^{3t} - e^{2t}$
(B) e^{5t}
(C) $e^{3t} + e^{2t}$
(D) None of these

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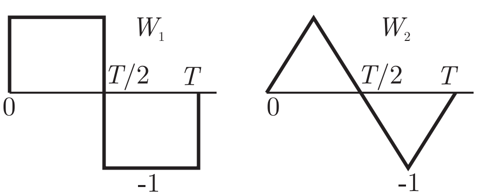
2000 **TWO MARKS**

MCQ 6.89

One period $(0,\ T)$ each of two periodic waveforms W_1 and W_2 are shown in the figure. The magnitudes of the n^{th} Fourier series coefficients of W_1 and W_2, for $n \geq 1, n$ odd, are respectively proportional to



(A) $|n^{-3}|$ and $|n^{-2}|$
(B) $|n^{-2}|$ and $|n^{-3}|$
(C) $|n^{-1}|$ and $|n^{-2}|$
(D) $|n^{-4}|$ and $|n^{-2}|$

MCQ 6.90

Let $u(t)$ be the step function. Which of the waveforms in the figure corresponds to the convolution of $u(t) - u(t-1)$ with $u(t) - u(t-2)$?

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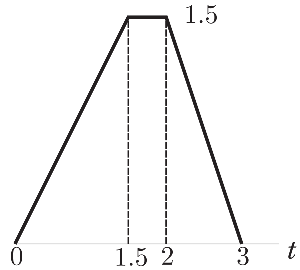
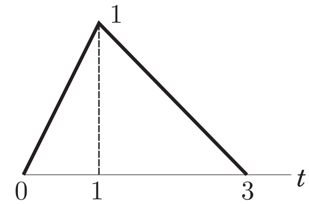
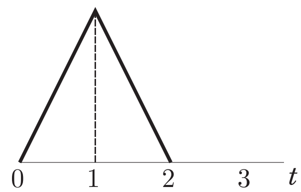
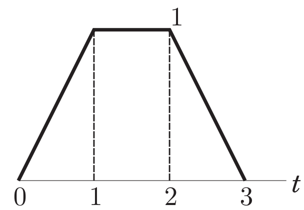
(A) 

(B) 

(C) 

(D) 

MCQ 6.91

A system has a phase response given by $\phi(\omega)$, where ω is the angular frequency. The phase delay and group delay at $\omega = \omega_0$ are respectively given by

(A) $-\dfrac{\phi(\omega_0)}{\omega_0}, \ -\dfrac{d\phi(\omega)}{d\omega}\Big|_{\omega=\omega_0}$

(B) $\phi(\omega_o), \ -\dfrac{d^2\phi(\omega_0)}{d\omega^2}\Big|_{\omega=\omega_o}$

(C) $\dfrac{\omega_o}{\phi(\omega_o)}, \ -\dfrac{d\phi(\omega)}{d(\omega)}\Big|_{\omega=\omega_o}$

(D) $\omega_o\phi(\omega_o), \ \int_{-\infty}^{\omega_o}\phi(\lambda)$

1999 ONE MARK

MCQ 6.92

The z-transform $F(z)$ of the function $f(nT) = a^{nT}$ is

(A) $\dfrac{z}{z-a^T}$

(B) $\dfrac{z}{z+a^T}$

(C) $\dfrac{z}{z-a^{-T}}$

(D) $\dfrac{z}{z+a^{-T}}$

MCQ 6.93

If $[f(t)] = F(s)$, then $[f(t-T)]$ is equal to

(A) $e^{sT}F(s)$

(B) $e^{-sT}F(s)$

(C) $\dfrac{F(s)}{1-e^{sT}}$

(D) $\dfrac{F(s)}{1-e^{-sT}}$

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MCQ 6.94

A signal $x(t)$ has a Fourier transform $X(\omega)$. If $x(t)$ is a real and odd function of t, then $X(\omega)$ is

(A) a real and even function of ω

(B) a imaginary and odd function of ω

(C) an imaginary and even function of ω

(D) a real and odd function of ω

1999 — TWO MARKS

MCQ 6.95

The Fourier series representation of an impulse train denoted by

$$s(t) = \sum_{n=-\infty}^{\infty} d(t - nT_0) \text{ is given by}$$

(A) $\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\exp - \frac{j2\pi nt}{T_0}$

(B) $\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\exp - \frac{j\pi nt}{T_0}$

(C) $\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\exp \frac{j\pi nt}{T_0}$

(D) $\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\exp \frac{j2\pi nt}{T_0}$

MCQ 6.96

The z-transform of a signal is given by

$$C(z) = \frac{1z^{-1}(1 - z^{-4})}{4(1 - z^{-1})^2}$$

Its final value is

(A) 1/4

(B) zero

(C) 1.0

(D) infinity

1998 — ONE MARK

MCQ 6.97

If $F(s) = \frac{\omega}{s^2 + \omega^2}$, then the value of $\lim\limits_{t \to \infty} f(t)$

(A) cannot be determined

(B) is zero

(C) is unity

(D) is infinite

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MCQ 6.98

The trigonometric Fourier series of a even time function can have only

(A) cosine terms
(B) sine terms
(C) cosine and sine terms
(D) d.c and cosine terms

MCQ 6.99

A periodic signal $x(t)$ of period T_0 is given by

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < \frac{T_0}{2} \end{cases}$$

The dc component of $x(t)$ is

(A) $\frac{T_1}{T_0}$
(B) $\frac{T_1}{2T_0}$
(C) $\frac{2T_1}{T_0}$
(D) $\frac{T_0}{T_1}$

MCQ 6.100

The unit impulse response of a linear time invariant system is the unit step function $u(t)$. For $t > 0$, the response of the system to an excitation $e^{-at}u(t), a > 0$ will be

(A) ae^{-at}
(B) $(1/a)(1 - e^{-at})$
(C) $a(1 - e^{-at})$
(D) $1 - e^{-at}$

MCQ 6.101

The z-transform of the time function $\sum_{k=0}^{\infty}\delta(n-k)$ is

(A) $\frac{z-1}{z}$
(B) $\frac{z}{z-1}$
(C) $\frac{z}{(z-1)^2}$
(D) $\frac{(z-1)^2}{z}$

MCQ 6.102

A distorted sinusoid has the amplitudes $A_1, A_2, A_3, ...$ of the fundamental, second harmonic, third harmonic,..... respectively. The total harmonic distortion is

(A) $\frac{A_2 + A_3 + ...}{A_1}$
(B) $\frac{\sqrt{A_2^2 + A_3^2 + ...}}{A_1}$

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(C) $\frac{\sqrt{A_2^2 + A_3^2 + ...}}{\sqrt{A_1^2 + A_2^2 + A_3^2 + ...}}$ (D) $\left(\frac{A_2^2 + A_3^2 + ...}{A_1}\right)$

MCQ 6.103

The Fourier transform of a function $x(t)$ is $X(f)$. The Fourier transform of $\frac{dX(t)}{df}$ will be

(A) $\frac{dX(f)}{df}$ (B) $j2\pi fX(f)$

(C) $jfX(f)$ (D) $\frac{X(f)}{jf}$

1997 ONE MARK

MCQ 6.104

The function $f(t)$ has the Fourier Transform $g(\omega)$. The Fourier Transform

$$ff(t)\, g(t)\left(= \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt\right) \text{ is}$$

(A) $\frac{1}{2\pi} f(\omega)$ (B) $\frac{1}{2\pi} f(-\omega)$

(C) $2\pi f(-\omega)$ (D) None of the above

MCQ 6.105

The Laplace Transform of $e^{\alpha t}\cos(\alpha t)$ is equal to

(A) $\frac{(s-\alpha)}{(s-\alpha)^2 + \alpha^2}$ (B) $\frac{(s+\alpha)}{(s-\alpha)^2 + \alpha^2}$

(C) $\frac{1}{(s-\alpha)^2}$ (D) None of the above

1996 ONE MARK

MCQ 6.106

The trigonometric Fourier series of an even function of time does not have the

(A) dc term (B) cosine terms

(C) sine terms (D) odd harmonic terms

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MCQ 6.107

The Fourier transform of a real valued time signal has

(A) odd symmetry
(B) even symmetry
(C) conjugate symmetry
(D) no symmetry

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SOLUTIONS

SOL 6.1

We have $$100\frac{d^2y}{dt^2} - 20\frac{dy}{dt} + y = x(t)$$

Applying Laplace transform we get

$$100s^2Y(s) - 20sY(s) + Y(s) = X(s)$$

or $$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{100s^2 - 20s + 1}$$

$$= \frac{1/100}{s^2 - (1/5)s + 1/100} = \frac{A}{s^2 + 2\xi\omega_n s + \omega^2}$$

Here $\omega_n = 1/10$ and $2\xi\omega_n = -1/5$ giving $\xi = -1$

Roots are $s = 1/10, 1/10$ which lie on Right side of s plane thus unstable.

Hence (A) is correct option.

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SOL 6.2

For an even function Fourier series contains dc term and cosine term (even and odd harmonics).

Hence (C) is correct option.

SOL 6.3

Function $h(n) = a^n u(n)$ stable if $|a| < 1$ and Unstable if $|a| \geqslant 1$We

We have $$h(n) = 2^n u(n-2);$$

Here $|a| = 2$ therefore $h(n)$ is unstable and since $h(n) = 0$ for $n < 0$

Therefore $h(n)$ will be causal. So $h(n)$ is causal and not stable.

Hence (B) is correct option.

SOL 6.4

$$\text{Impulse response} = \frac{d}{dt}(\text{step response})$$

$$= \frac{d}{dt}(1 - e^{-\alpha t})$$

$$= 0 + \alpha e^{-\alpha t} = \alpha e^{-\alpha t}$$

Hence (A) is correct option.

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SOL 6.5

We have $x(t) = \exp(-2t)\mu(t) + s(t-6)$ and $h(t) = u(t)$

Taking Laplace Transform we get

$$X(s) = \left(\frac{1}{s+2} + e^{-6s}\right) \text{ and } H(s) = \frac{1}{s}$$

Now $\quad Y(s) = H(s)\,X(s)$

$$= \frac{1}{s}\left[\frac{1}{s+2} + e^{-6s}\right] = \frac{1}{s(s+2)} + \frac{e^{-6s}}{s}$$

or $\quad Y(s) = \frac{1}{2s} - \frac{1}{2(s+2)} + \frac{e^{-6s}}{s}$

Thus $\quad y(t) = 0.5[1 - \exp(-2t)]\,u(t) + u(t-6)$

Hence (D) is correct option.

SOL 6.6

$$y(n) = x(n-1)$$

or $\quad Y(z) = z^{-1}X(z)$

or $\quad \frac{Y(z)}{X(z)} = H(z) = z^{-1}$

Now $\quad H_1(z)\,H_2(z) = z^{-1}$

$$\left(\frac{1-0.4z^{-1}}{1-0.6z^{-1}}\right)H_2(z) = z^{-1}$$

$$H_2(z) = \frac{z^{-1}(1-0.6z^{-1})}{(1-0.4z^{-1})}$$

Hence (B) is correct option.

SOL 6.7

For 8 point DFT, $x^*[1] = x[7]; x^*[2] = x[6]; x^*[3] = x[5]$ and it is conjugate symmetric about $x[4]$, $x[6] = 0$; $x[7] = 1 + j3$

Hence (B) is correct option.

SOL 6.8

For a function $x(t)$ trigonometric fourier series is

$$x(t) = A_o + \sum_{n=1}^{\infty}[A_n \cos n\omega t + B_n \sin n\omega t]$$

Where, $\quad A_o\ \frac{1}{T_0}\int_{T_0} x(t)\,dt \qquad T_0 \rightarrow$ fundamental period

and $\quad A_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega t\,dt$

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$$B_n = \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega t\,dt$$

For an even function $x(t), B_n = 0$

Since given function is even function so coefficient $B_n = 0$, only cosine and constant terms are present in its fourier series representation

Constant term $$A_0 = \frac{1}{T}\int_{-T/4}^{3T/4} x(t)\,dt$$

$$= \frac{1}{T}\left[\int_{-T/4}^{T/4} A\,dt + \int_{T/4}^{3T/4} -2A\,dt\right]$$

$$= \frac{1}{T}\left[\frac{TA}{2} - 2A\frac{T}{2}\right] = -\frac{A}{2}$$

Constant term is negative.

Hence (C) is correct option.

SOL 6.9

We know that $\alpha Z^{\pm a} \xleftrightarrow{\text{Inverse Z} - \text{transform}} \alpha\delta[n \pm a]$

Given that $X(z) = 5z^2 + 4z^{-1} + 3$

Inverse z-transform $x[n] = 5\delta[n+2] + 4\delta[n-1] + 3\delta[n]$

Hence (A) is correct option.

SOL 6.10

We have $h_1[n] = \delta[n-1]\, or\, H_1[Z] = Z^{-1}$

and $h_2[n] = \delta[n-2]\, or\, H_2(Z) = Z^{-2}$

Response of cascaded system

$$H(z) = H_1(z)\cdot H_2(z) = z^{-1}\cdot z^{-2} = z^{-3}$$

or, $h[n] = \delta[n-3]$

Hence (C) is correct option.

SOL 6.11

For an N-point FET algorithm butterfly operates on one pair of samples and involves two complex addition and one complex multiplication.

Hence (D) is correct option.

SOL 6.12

We have $$f(t) = \mathcal{L}^{-1}\left[\frac{3s+1}{s^3 + 4s^2 + (k-3)s}\right]$$

and $$\lim_{t\to\infty} f(t) = 1$$

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By final value theorem

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s) = 1$$

or $$\lim_{s \to 0} \frac{s.(3s+1)}{s^3 + 4s^2 + (k-3)s} = 1$$

or $$\lim_{s \to 0} \frac{s(3s+1)}{s[s^2 + 4s + (k-3)]} = 1$$

$$\frac{1}{k-3} = 1$$

or $$k = 4$$

Hence (D) is correct option.

SOL 6.13

System is described as

$$\frac{d^2 y(t)}{dt^2} + 4\frac{dt(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$$

Taking laplace transform on both side of given equation

$$s^2 Y(s) + 4sY(s) + 3Y(s) = 2sX(s) + 4X(s)$$

$$(s^2 + 4s + 3)Y(s) = 2(s+2)X(s)\,s$$

Transfer function of the system

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2(s+2)}{s^2 + 4s + 3} = \frac{2(s+2)}{(s+3)(s+1)}$$

Input $$x(t) = e^{-2t}u(t)$$

or, $$X(s) = \frac{1}{(s+2)}$$

Output $$Y(s) = H(s) \cdot X(s)$$

$$Y(s) = \frac{2(s+2)}{(s+3)(s+1)} \cdot \frac{1}{(s+2)}$$

By Partial fraction

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+3}$$

Taking inverse laplace transform

$$y(t) = (e^{-t} - e^{-3t})u(t)$$

Hence (B) is correct option.

SOL 6.14

We have

$$H(z) = \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

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By partial fraction $H(z)$ can be written as

$$H(z) = \frac{1}{(1-\frac{1}{2}z^{-1})} + \frac{1}{(1-\frac{1}{4}z^{-1})}$$

For ROC : $|z| > 1/2$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[n],\ n > 0 \qquad \frac{1}{1-z^{-1}} = a^n u[n], |z| > a$$

Thus system is causal. Since ROC of $H(z)$ includes unit circle, so it is stable also. Hence S_1 is True

For ROC : $|z| < \frac{1}{4}$

$$h[n] = -\left(\frac{1}{2}\right)^n u[-n-1] + \left(\frac{1}{4}\right)^n u(n), |z| > \frac{1}{4}, |z| < \frac{1}{2}$$

System is not causal. ROC of $H(z)$ does not include unity circle, so it is not stable and S_3 is True

Hence (C) is correct option.

SOL 6.15

The Fourier series of a real periodic function has only cosine terms if it is even and sine terms if it is odd.

Hence (A) is correct answer.

SOL 6.16

Given function is

$$f(t) = \sin^2 t + \cos 2t = \frac{1-\cos 2t}{2} + \cos 2t = \frac{1}{2} + \frac{1}{2}\cos 2t$$

The function has a DC term and a cosine function. The frequency of cosine terms is

$$\omega = 2 = 2\pi f \rightarrow f = \frac{1}{\pi} \text{ Hz}$$

The given function has frequency component at 0 and $\frac{1}{\pi}$ Hz.

Hence (B) is correct answer.

SOL 6.17

$$x[n] = \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(-n-1)$$

Taking z transform we have

$$X(z) = \sum_{n=0}^{n=\infty} \left(\frac{1}{3}\right)^n z^{-n} - \sum_{n=-\infty}^{n=-1} \left(\frac{1}{2}\right)^n z^{-n}$$

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$$= \sum_{n=0}^{n=\infty} \left(\frac{1}{3}z^{-1}\right)^n - \sum_{n=-\infty}^{n=-1} \left(\frac{1}{2}z^{-1}\right)^n$$

First term gives $\quad \frac{1}{3}z^{-1} < 1 \rightarrow \frac{1}{3} < |z|$

Second term gives $\quad \frac{1}{2}z^{-1} > 1 \rightarrow \frac{1}{2} > |z|$

Thus its ROC is the common ROC of both terms. that is

$$\frac{1}{3} < |z| < \frac{1}{2}$$

Hence (A) is correct answer.

SOL 6.18

By property of unilateral laplace transform

$$\int_{-\infty}^{t} f(\tau)\, d\tau \xleftrightarrow{L} \frac{F(s)}{s} + \frac{1}{s}\int_{-\infty}^{0^-} f(\tau)\, d\tau$$

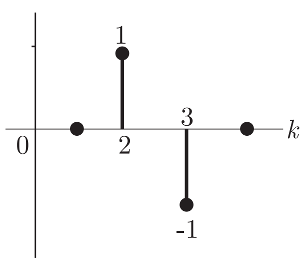
Here function is defined for $0 < \tau < t$, Thus

$$\int_0^t f(\tau) \xleftrightarrow{L} \frac{F(s)}{s}$$

Hence (B) is correct answer.

SOL 6.19

We have $h(2) = 1$, $h(3) = -1$ otherwise $h(k) = 0$. The diagram of response is as follows :



It has the finite magnitude values. So it is a finite impulse response filter. Thus S_2 is true but it is not a low pass filter. So S_1 is false.
Hence (A) is correct answer.

SOL 6.20

Here $h(t) \neq 0$ for $t < 0$. Thus system is non causal. Again any bounded input $x(t)$ gives bounded output $y(t)$. Thus it is BIBO stable.
Here we can conclude that option (B) is correct.
Hence (B) is correct answer.

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SOL 6.21

We have $x[n] = \{1,0,2,3) \text{ and } N = 4$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j2\pi nk/N} \quad k = 0,1\dots N-1$$

For $N = 4$, $X[k] = \sum_{n=0}^{3} x[n]\, e^{-j2\pi nk/4} \quad k = 0,1,\dots 3$

Now
$$X[0] = \sum_{n=0}^{3} x[n]$$
$$= x[0] + x[1] + x[2] + x[3] = 1 + 0 + 2 + 3 = 6$$
$$x[1] = \sum_{n=0}^{3} x[n]\, e^{-j\pi n/2}$$
$$= x[0] + x[1]\, e^{-j\pi/2} + x[2]\, e^{-j\pi} + x[3]\, e^{-j\pi 3/2}$$
$$= 1 + 0 - 2 + j3 = -1 + j3$$
$$X[2] = \sum_{n=0}^{3} x[n]\, e^{-j\pi n}$$
$$= x[0] + x[1]\, e^{-j\pi} + x[2]\, e^{-j2\pi} + x[3]\, e^{-j\pi 3}$$
$$= 1 + 0 + 2 - 3 = 0$$
$$X[3] = \sum_{n=0}^{3} x[n]\, e^{-j3\pi n/2}$$
$$= x[0] + x[1]\, e^{-j3\pi/2} + x[2]\, e^{-j3\pi} + x[3]\, e^{-j9\pi/2}$$
$$= 1 + 0 - 2 - j3 = -1 - j3$$

Thus $[6, -1 + j3, \quad 0, -1 - j3]$

Hence (D) is correct answer.

SOL 6.22

Hence (A) is correct answer.

SOL 6.23

The output of causal system depends only on present and past states only.

In option (A) $y(0)$ depends on $x(-2)$ and $x(4)$.

In option (B) $y(0)$ depends on $x(1)$.

In option (C) $y(0)$ depends on $x(-1)$.

In option (D) $y(0)$ depends on $x(5)$.

Thus only in option (C) the value of $y(t)$ at $t = 0$ depends on $x(-1)$ past value. In all other option present value depends on future value.

Hence (C) is correct answer.

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SOL 6.24

We have $\quad h(t) = e^{\alpha t}u(t) + e^{\beta t}u(-t)$

This system is stable only when bounded input has bounded output

For stability $\alpha t < 0$ for $t > 0$ that implies $\alpha < 0$ and $\beta t > 0$ for $t > 0$ that implies $\beta > 0$. Thus, α is negative and β is positive.

Hence (D) is correct answer.

SOL 6.25

$$G(s) = \frac{K(s+1)}{(s+2)(s+4)}, \text{ and } R(s) = \frac{1}{s}$$

$$C(s) = G(s)R(s) = \frac{K(s+1)}{s(s+2)(s+4)}$$

$$= \frac{K}{8s} + \frac{K}{4(s+2)} - \frac{3K}{8(s+4)}$$

Thus $\quad c(t) = K\left[\frac{1}{8} + \frac{1}{4}e^{-2t} - \frac{3}{8}e^{-4t}\right]u(t)$

At steady-state, $c(\infty) = 1$

Thus $\quad \frac{K}{8} = 1$ or $K = 8$

Then, $\quad G(s) = \frac{8(s+1)}{(s+2)(s+4)} = \frac{12}{(s+4)} - \frac{4}{(s+2)}$

$$h(t) = L^{-1}G(s) = (-4e^{-2t} + 12e^{-4t})u(t)$$

Hence (C) is correct answer.

SOL 6.26

We have $\quad x(t) = \begin{cases} 1 & \text{for } -1 \le t \le +1 \\ 0 & \text{otherwise} \end{cases}$

Fourier transform is

$$\int_{-\infty}^{\infty} e^{-j\omega t}x(t)\,dt = \int_{-1}^{1} e^{-j\omega t}1\,dt$$

$$= \frac{1}{-j\omega}[e^{-j\omega t}]_{-1}^{1}$$

$$= \frac{1}{-j\omega}(e^{-j\omega} - e^{j\omega}) = \frac{1}{-j\omega}(-2j\sin\omega)$$

$$= \frac{2\sin\omega}{\omega}$$

This is zero at $\omega = \pi$ and $\omega = 2\pi$

Hence (A) is correct answer.

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SOL 6.27

Given
$$h(n) = [1, -1, 2]$$
$$x(n) = [1, 0, 1]$$
$$y(n) = x(n) * h(n)$$
The length of $y[n]$ is $= L_1 + L_2 - 1 = 3 + 3 - 1 = 5$
$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$
$$\begin{aligned} y(2) &= \sum_{k=-\infty}^{\infty} x(k)\, h(2-k) \\ &= x(0)\, h(2-0) + x(1)\, h(2-1) + x(2)\, h(2-2) \\ &= h(2) + 0 + h(0) = 1 + 2 = 3 \end{aligned}$$
There are 5 non zero sample in output sequence and the value of $y[2]$ is 3.
Hence (D) is correct answer.

SOL 6.28

Mode function are not linear. Thus $y(t) = |x(t)|$ is not linear but this functions is time invariant. Option (A) and (B) may be correct.
The $y(t) = t|x(t)|$ is not linear, thus option (B) is wrong and (a) is correct. We can see that
R_1: $y(t) = t^2 x(t)$ Linear and time variant.
R_2: $y(t) = t|x(t)|$ Non linear and time variant.
R_3: $y(t) = x|(t)|$ Non linear and time invariant
R_4: $y(t) = x(t-5)$ Linear and time invariant
Hence (B) is correct answer.

SOL 6.29

Given :
$$y(n) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)\, x(n+r)$$
It is Auto correlation.
Hence
$$y(n) = r_{xx}(n) \xrightarrow{DFT} |X(k)|^2$$
Hence (A) is correct answer.

SOL 6.30

Current through resistor (i.e. capacitor) is
$$I = I(0^+)\, e^{-t/RC}$$
Here,
$$I(0^+) = \frac{V}{R} = \frac{5}{200k} = 25\mu\text{A}$$

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$$\begin{aligned} RC &= 200k \times 10\mu = 2\,\text{sec} \\ I &= 25e^{-\frac{t}{2}}\,\mu\text{A} \\ &= V_R \times R = 5e^{-\frac{t}{2}}\ \text{V} \end{aligned}$$

Here the voltages across the resistor is input to sampler at frequency of 10 Hz. Thus

$$x(n) = 5e^{\frac{-n}{2\times 10}} = 5e^{-0.05n}\ \text{For } t > 0$$

Hence (B) is correct answer.

SOL 6.31

Since $\quad x(n) = 5e^{-0.05n}u(n)$ is a causal signal

Its z transform is

$$X(z) = 5\left[\frac{1}{1 - e^{-0.05}z^{-1}}\right] = \frac{5z}{z - e^{-0.05}}$$

Its ROC is $|e^{-0.05}z^{-1}| > 1 \rightarrow |z| > e^{-0.05}$

Hence (C) is correct answer.

SOL 6.32

$$\begin{aligned} h(t) &= e^{-2t}u(t) \\ H(j\omega) &= \int_{-\infty}^{\infty} h(t)\,e^{-j\omega t}\,dt \\ &= \int_0^{\infty} e^{-2t}e^{-j\omega t}\,dt = \int_0^{\infty} e^{-(2+j\omega)t}\,dt = \frac{1}{(2+j\omega)} \end{aligned}$$

Hence (C) is correct answer.

SOL 6.33

$$H(j\omega) = \frac{1}{(2+j\omega)}$$

The phase response at $\omega = 2$ rad/sec is

$$\angle H(j\omega) = -\tan^{-1}\frac{\omega}{2} = -\tan^{-1}\frac{2}{2} = -\frac{\pi}{4} = -0.25\pi$$

Magnitude respone at $\omega = 2$ rad/sec is

$$|H(j\omega)| = \sqrt{\frac{1}{2^2 + w^2}} = \frac{1}{2\sqrt{2}}$$

Input is $\quad x(t) = 2\cos(2t)$

Output i $\quad = \frac{1}{2\sqrt{2}} \times 2\cos(2t - 0.25\pi)$

$$= \frac{1}{\sqrt{2}}\cos[2t - 0.25\pi]$$

Hence (D) is correct answer.

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SOL 6.34

$$Y(s) = \frac{1}{s(s-1)}$$

Final value theorem is applicable only when all poles of system lies in left half of S-plane. Here $s = 1$ is right $s-$plane pole. Thus it is unbounded.

Hence (D) is correct answer.

SOL 6.35

$$x(t) = e^{-t}u(t)$$

Taking Fourier transform

$$X(j\omega) = \frac{1}{1+j\omega}$$

$$|X(j\omega)| = \frac{1}{1+\omega^2}$$

Magnitude at 3dB frequency is $\frac{1}{\sqrt{2}}$

Thus $$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+\omega^2}}$$

or $$\omega = 1 \text{ rad}$$

or $$f = \frac{1}{2\pi} \text{ Hz}$$

Hence (A) is correct answer.

SOL 6.36

For discrete time Fourier transform (DTFT) when $N \to \infty$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

Putting $n = 0$ we get

$$x[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega 0}\, d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$$

or $$\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega = 2\pi x[0] = 2\pi \times 5 = 10\pi$$

Hence (B) is correct answer.

SOL 6.37

$$X(z) = \frac{0.5}{1-2z^{-1}}$$

Since ROC includes unit circle, it is left handed system

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$$x(n) = -(0.5)(2)^{-n}u(-n-1)$$
$$x(0) = 0$$

If we apply initial value theorem

$$x(0) = \lim_{z\to\infty} X(z) = \lim_{z\to\infty}\frac{0.5}{1-2z^{-1}} = 0.5$$

That is wrong because here initial value theorem is not applicable because signal $x(n)$ is defined for $n < 0$.
Hence (B) is correct answer.

SOL 6.38

A Hilbert transformer is a non-linear system.
Hence (A) is correct answer.

SOL 6.39

$$H(f) = \frac{5}{1+j10\pi f}$$
$$H(s) = \frac{5}{1+5s} = \frac{5}{5(s+\frac{1}{5})} = \frac{1}{s+\frac{1}{5}}$$

Step response $\quad Y(s) = \frac{1}{s}\frac{a}{(s+\frac{1}{5})}$

or $\quad Y(s) = \frac{1}{s}\frac{1}{(s+\frac{1}{5})} = \frac{5}{s} - \frac{5}{s+\frac{1}{5}}$

or $\quad y(t) = 5(1-e^{-t/5})u(t)$

Hence (B) is correct answer.

SOL 6.40

$$x(t) \xleftrightarrow{F} X(j\omega)$$

Using scaling we have

$$x(5t) \xleftrightarrow{F} \frac{1}{5}X\left(\frac{j\omega}{5}\right)$$

Using shifting property we get

$$x\left[5\left(t-\frac{3}{5}\right)\right] \xleftrightarrow{F} \frac{1}{5}X\left(\frac{j\omega}{5}\right)e^{-\frac{j3\omega}{5}}$$

Hence (A) is correct answer.

SOL 6.41

Dirac delta function $\delta(t)$ is defined at $t = 0$ and it has infinite value a $t = 0$. The area of dirac delta function is unity.
Hence (D) is correct option.

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SOL 6.42

The ROC of addition or subtraction of two functions $x_1(n)$ and $x_2(n)$ is $R_1 \cap R_2$. We have been given ROC of addition of two function and has been asked ROC of subtraction of two function. It will be same.
Hence (D) is correct option.

SOL 6.43

As we have $x(t) = \sin t,$ thus $\omega = 1$

Now $$H(s) = \frac{1}{s+1}$$

or $$H(j\omega) = \frac{1}{j\omega + 1} = \frac{1}{j+1}$$

or $$H(j\omega) = \frac{1}{\sqrt{2}}\angle -45^\circ$$

Thus $$y(t) = \frac{1}{\sqrt{2}}\sin\left(t - \frac{\pi}{4}\right)$$

Hence (A) is correct option.

SOL 6.44

$$F(s) = \frac{\omega_0}{s^2 + \omega^2}$$

$$L^{-1}F(s) = \sin\omega_o t$$

$$f(t) = \sin\omega_o t$$

Thus the final value is $-1 \leq f(\infty) \leq 1$

Hence (C) is correct answer.

SOL 6.45

$$y(n) = \left(\sin\frac{5}{6}\pi n\right)x(n)$$

Let $x(n) = \delta(n)$

Now $y(n) = \sin 0 = 0$ (bounded) BIBO stable

Hence (C) is correct answer.

SOL 6.46

$$c(t) = 1 - e^{-2t}$$

Taking laplace transform

$$C(s) = \frac{C(s)}{U(s)} = \frac{2}{s(s+2)} \times s = \frac{2}{s+2}$$

Hence (B) is correct answer.

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SOL 6.47

$$h(t) = e^{-t} \xrightarrow{L} H(s) = \frac{1}{s+1}$$

$$x(t) = u(t) \xrightarrow{L} X(s) = \frac{1}{s}$$

$$Y(s) = H(s)X(s) = \frac{1}{s+1} \times \frac{1}{s} = \frac{1}{s} - \frac{1}{s+1}$$

$$y(t) = u(t) - e^{-t}$$

In steady state i.e. $t \to \infty$, $y(\infty) = 1$
Hence (C) is correct answer.

SOL 6.48

Fourier series is defined for periodic function and constant.
$3\sin(25t)$ is a periodic function.
$4\cos(20t+3) + 2\sin(710t)$ is sum of two periodic function and also a periodic function.
$e^{-|t|}\sin(25t)$ is not a periodic function, so FS can't be defined for it.
1 is constant
Hence (C) is correct option.

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SOL 6.49

$$\text{Ev}\{g(t)\} = \frac{g(t) + g(-t)}{2}$$

$$\text{odd}\{g(t)\} = \frac{g(t) - g(-t)}{2}$$

Here $\quad g(t) = u(t)$

Thus $\quad u_e(t) = \frac{u(t) + u(-t)}{2} = \frac{1}{2}$

$$u_o(t) = \frac{u(t) - u(-t)}{2} = \frac{x(t)}{2}$$

Hence (A) is correct answer.

SOL 6.50

Here $\quad x_1(n) = \left(\frac{5}{6}\right)^n u(n)$

$$X_1(z) = \frac{1}{1 - \left(\frac{5}{6}z^{-1}\right)} \qquad \text{ROC} : R_1 \to |z| > \frac{5}{6}$$

$$x_2(n) = -\left(\frac{6}{5}\right)^n u(-n-1)$$

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$$X_1(z) = 1 - \frac{1}{1 - \left(\frac{6}{5}z^{-1}\right)} \qquad \text{ROC : } R_2 \to |z| < \frac{6}{5}$$

Thus ROC of $x_1(n) + x_2(n)$ is $R_1 \cap R_2$ which is $\frac{5}{6} < |z| < \frac{6}{5}$

Hence (C) is correct answer.

SOL 6.51

For causal system $h(t) = 0$ for $t \leq 0$. Only (D) satisfy this condition. Hence (D) is correct answer.

SOL 6.52

$$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$y(n) = x^2(n) = \left(\frac{1}{2}\right)^{2n} u^2(n)$$

or
$$y(n) = \left[\left(\frac{1}{2}\right)^2\right]^n u(n) = \left(\frac{1}{4}\right)^n u(n) \qquad ...(1)$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{n=\infty} y(n)\, e^{-j\omega n} = \sum_{n=0}^{n=\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}$$

or
$$Y(e^{j0}) = \sum_{n=0}^{n=\infty} \left(\frac{1}{4}\right)^n = 1 + \left(\frac{1}{4}\right)^1 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^4$$

or
$$Y(e^{j0}) = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

Alternative :

Taking z transform of (1) we get

$$Y(z) = \frac{1}{1 - \frac{1}{4}z^{-1}}$$

Substituting $z = e^{j\omega}$ we have

$$Y(e^{j\omega}) = \frac{1}{1 - \frac{1}{4}e^{-j\omega}}$$

$$Y(e^{j0}) = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

Hence (D) is correct answer.

SOL 6.53

$$s(t) = 8\cos\left(\frac{\pi}{2} - 20\pi t\right) + 4\sin 15\pi t$$

$$= 8\sin 20\pi t + 4\sin 15\pi t$$

Here $A_1 = 8$ and $A_2 = 4$. Thus power is

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$$P = \frac{A_1^2}{2} + \frac{A_2^2}{2} = \frac{8^2}{2} + \frac{4^2}{2} = 40$$

Hence (A) is correct answer.

SOL 6.54

$$y(t) = 0.5x(t - t_d + T) + x(t - t_d) + 0.5x(t - t_d - T)$$

Taking Fourier transform we have

$$Y(\omega) = 0.5e^{-j\omega(-t_d+T)}X(\omega) + e^{-j\omega t_d}X(\omega) + 0.5e^{-j\omega(-t_d-T)}X(\omega)$$

or
$$\frac{Y(\omega)}{X(\omega)} = e^{-j\omega t_d}[0.5e^{j\omega T} + 1 + 0.5e^{-j\omega T}]$$

$$= e^{-j\omega t_d}[0.5(e^{j\omega T} + e^{-j\omega T}) + 1] = e^{-j\omega t_d}[\cos\omega T + 1]$$

or
$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = e^{-j\omega t_d}(\cos\omega T + 1)$$

Hence (A) is correct answer.

SOL 6.55

For continuous and aperiodic signal Fourier representation is continuous and aperiodic.
For continuous and periodic signal Fourier representation is discrete and aperiodic.
For discrete and aperiodic signal Fourier representation is continuous and periodic.
For discrete and periodic signal Fourier representation is discrete and periodic.
Hence (C) is correct answer.

SOL 6.56

$$y(n) = Ax(n - n_o)$$

Taking Fourier transform

$$Y(e^{j\omega}) = Ae^{-j\omega_o n_o}X(e^{j\omega})$$

or
$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = Ae^{-j\omega_o n_o}$$

Thus
$$\angle H(e^{j\omega}) = -\omega_o n_o$$

For LTI discrete time system phase and frequency of $H(e^{j\omega})$ are periodic with period 2π. So in general form

$$\theta(\omega) = -n_o\omega_o + 2\pi k$$

Hence (B) is correct answer.

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SOL 6.57

From $\qquad x(n) = [\frac{1}{2}, 1, 2, 1, 1, \frac{1}{2}]$

$$y(n) = x(\tfrac{n}{2} - 1), n \text{ even}$$

$$= 0, \text{ for } n \text{ odd}$$

$n = -2, \qquad y(-2) = x(\frac{-2}{2} - 1) = x(-2) = \frac{1}{2}$

$n = -1, \qquad y(-1) = 0$

$n = 0, \qquad y(0) = x(\frac{0}{2} - 1) = x(-1) = 1$

$n = 1, \qquad y(1) = 0$

$n = 2 \qquad y(2) = x(\frac{2}{2} - 1) = x(0) = 2$

$n = 3, \qquad y(3) = 0$

$n = 4 \qquad y(4) = x(\frac{4}{2} - 1) = x(1) = 1$

$n = 5, \qquad y(5) = 0$

$n = 6 \qquad y(6) = x(\frac{6}{2} - 1) = x(2) = \frac{1}{2}$

Hence $\qquad y(n) = \frac{1}{2}\delta(n+2) + \delta(n) + 2\delta(n-2) + \delta(n-4) + \frac{1}{2}\delta(n-6)$

Thus (A) is correct option.

SOL 6.58

Here $y(n)$ is scaled and shifted version of $x(n)$ and again $y(2n)$ is scaled version of $y(n)$ giving

$$z(n) = y(2n) = x(n-1)$$

$$= \frac{1}{2}\delta(n+1) + \delta(n) + 2\delta(n-1) + \delta(n-2) + \frac{1}{2}\delta(n-3)$$

Taking Fourier transform.

$$Z(e^{j\omega}) = \frac{1}{2}e^{j\omega} + 1 + 2e^{-j\omega} + e^{-2j\omega} + \frac{1}{2}e^{-3j\omega}$$

$$= e^{-j\omega}\left(\frac{1}{2}e^{2j\omega} + e^{j\omega} + 2 + e^{-j\omega} + \frac{1}{2}e^{-2j\omega}\right)$$

$$= e^{-j\omega}\left(\frac{e^{2j\omega} + e^{-2j\omega}}{2} + e^{j\omega} + 2 + e^{-j\omega}\right)$$

or $\qquad Z(e^{j\omega}) = e^{-j\omega}[\cos 2\omega + 2\cos\omega + 2]$

Hence (C) is correct answer.

SOL 6.59

$$x(t) \xleftrightarrow{F} X(f)$$

Using scaling we have

$$x(at) \xleftrightarrow{F} \frac{1}{|a|}X\left(\frac{f}{a}\right)$$

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Thus $$x\left(\frac{1}{3}f\right) \xleftrightarrow{F} 3X(3f)$$

Using shifting property we get

$$e^{-j2\pi f_0 t}x(t) = X(f+f_0)$$

Thus $$\frac{1}{3}e^{-j\frac{4}{3}\pi t}x\left(\frac{1}{3}t\right) \xleftrightarrow{F} X(3f+2)$$

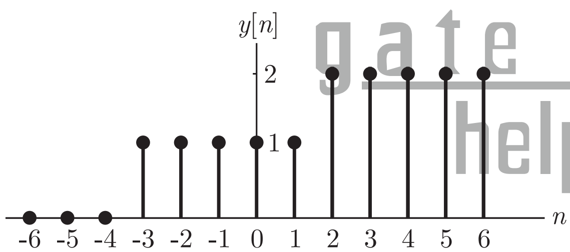
$$e^{-j2\pi\frac{2}{3}t}x\left(\frac{1}{3}t\right) \xleftrightarrow{F} 3X(3(f+\tfrac{2}{3}))$$

$$\frac{1}{3}e^{-j\pi\frac{4}{3}t}x\left(\frac{1}{3}t\right) \xleftrightarrow{F} X[3(f+\tfrac{2}{3})]$$

Hence (B) is correct answer.

SOL 6.60

A system is stable if $\sum_{n=-\infty}^{\infty}|h(n)| < \infty$. The plot of given $h(n)$ is



Thus $$\sum_{n=-\infty}^{\infty}|h(n)| = \sum_{n=-3}^{6}|h(n)|$$

$$= 1+1+1+1+2+2+2+2+2$$

$$= 15 < \infty$$

Hence system is stable but $h(n) \neq 0$ for $n < 0$. Thus it is not causal.
Hence (A) is correct answer.

SOL 6.61

$$H(z) = \frac{z}{z-0.2} \qquad |z| < 0.2$$

We know that

$$-a^n u[-n-1] \longleftrightarrow \frac{1}{1-az^{-1}} \qquad |z| < a$$

Thus $$h[n] = -(0.2)^n u[-n-1]$$

Hence (D) is correct answer.

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SOL 6.62

The Fourier transform of a conjugate symmetrical function is always real.
Hence (C) is correct answer.

SOL 6.63

We have
$$x(n) = [-4 - j5,\quad \underset{\uparrow}{1 + 2j},\quad 4]$$
$$x^*(n) = [-4 + j5,\quad \underset{\uparrow}{1 - 2j},\quad 4]$$
$$x^*(-n) = [4,\quad \underset{\uparrow}{1 - 2j},\quad -4 + j5]$$
$$x_{cas}(n) = \frac{x(n) - x^*(-n)}{2}$$
$$= [-4 - j\tfrac{5}{2},\quad \underset{\uparrow}{2j},\quad 4 - j\tfrac{5}{2}]$$

Hence (A) is correct answer.

SOL 6.64

We have $2y(n) = \alpha y(n-2) - 2x(n) + \beta x(n-1)$
Taking z transform we get
$$2Y(z) = \alpha Y(z) z^{-2} - 2X(z) + \beta X(z) z^{-1}$$
or
$$\frac{Y(z)}{X(z)} = \left(\frac{\beta z^{-1} - 2}{2 - \alpha z^{-2}}\right) \qquad \text{...(i)}$$
or
$$H(z) = \frac{z(\frac{\beta}{2} - z)}{(z^2 - \frac{\alpha}{2})}$$
It has poles at $\pm\sqrt{\alpha/2}$ and zero at 0 and $\beta/2$. For a stable system poles must lie inside the unit circle of z plane. Thus
$$\left|\sqrt{\frac{\alpha}{2}}\right| < 1$$
or
$$|\alpha| < 2$$
But zero can lie anywhere in plane. Thus, β can be of any value.
Hence (C) is correct answer.

SOL 6.65

We have $x(n) = e^{j\pi n/4}$
and $h(n) = 4\sqrt{2}\,\delta(n+2) - 2\sqrt{2}\,\delta(n+1) - 2\sqrt{2}\,\delta(n-1) + 4\sqrt{2}\,\delta(n-2)$

Now $y(n) = x(n) * h(n)$

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$$= \sum_{k=-\infty}^{\infty} x(n-k)\,h(k) = \sum_{k=-2}^{2} x(n-k)\,h(k)$$

or $$y(n) = x(n+2)\,h(-2) + x(n+1)\,h(-1) + x(n-1)\,h(1) + x(n-2)\,h(2)$$

$$= 4\sqrt{2}\,e^{j\frac{\pi}{4}(n+2)} - 2\sqrt{2}\,e^{j\frac{\pi}{4}(n+1)} - 2\sqrt{2}\,e^{j\frac{\pi}{4}(n-1)} + 4\sqrt{2}\,e^{j\frac{\pi}{4}(n-2)}$$

$$= 4\sqrt{2}\left[e^{j\frac{\pi}{4}(n+2)} + e^{j\frac{\pi}{4}(n-2)}\right] - 2\sqrt{2}\left[e^{j\frac{\pi}{4}(n+1)} + e^{j\frac{\pi}{4}(n-1)}\right]$$

$$= 4\sqrt{2}\,e^{j\frac{\pi}{4}n}\left[e^{j\frac{\pi}{2}} + e^{-j\frac{\pi}{2}}\right] - 2\sqrt{2}\,e^{j\frac{\pi}{2}n}\left[e^{j\frac{\pi}{4}} + e^{-j\frac{\pi}{4}}\right]$$

$$= 4\sqrt{2}\,e^{j\frac{\pi}{4}n}[0] - 2\sqrt{2}\,e^{j\frac{\pi}{4}n}\left[2\cos\tfrac{\pi}{4}\right]$$

or $$y(n) = -4e^{j\frac{\pi}{4}n}$$

Hence (D) is correct answer.

SOL 6.66

From given graph the relation in $x(t)$ and $y(t)$ is

$$y(t) = -x[2(t+1)]$$

$$x(t) \xleftrightarrow{F} X(f)$$

Using scaling we have

$$x(at) \xleftrightarrow{F} \frac{1}{|a|}X\left(\frac{f}{a}\right)$$

Thus $$x(2t) \xleftrightarrow{F} \frac{1}{2}X\left(\frac{f}{2}\right)$$

Using shifting property we ge

$$x(t-t_0) = e^{-j2\pi f t_0}X(f)$$

Thus $$x[2(t+1)] \xleftrightarrow{F} e^{-j2\pi f(-1)}\frac{1}{2}X\left(\frac{f}{2}\right) = \frac{e^{j2\pi f}}{2}X\left(\frac{f}{2}\right)$$

$$-x[2(t+1)] \xleftrightarrow{F} -\frac{e^{j2\pi f}}{2}X\left(\frac{f}{2}\right)$$

Hence (B) is correct answer.

SOL 6.67

From the Final value theorem we have

$$\lim_{t\to\infty} i(t) = \lim_{s\to 0} sI(s) = \lim_{s\to 0} s\frac{2}{s(1+s)} = \lim_{s\to 0}\frac{2}{(1+s)} = 2$$

Hence (C) is correct answer.

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SOL 6.68

Here $\quad C_3 = 3 + j5$

For real periodic signal

$$C_{-k} = C_k^*$$

Thus $\quad C_{-3} = C_k = 3 - j5$

Hence (D) is correct answer.

SOL 6.69

$$y(t) = 4x(t-2)$$

Taking Fourier transform we get

$$Y(e^{j2\pi f}) = 4e^{-j2\pi f2}X(e^{j2\pi f}) \qquad \text{Time Shifting property}$$

or $$\frac{Y(e^{j2\pi f})}{X(e^{j2\pi f})} = 4e^{-4j\pi f}$$

Thus $$H(e^{j2\pi f}) = 4e^{-4j\pi f}$$

Hence (C) is correct answer.

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SOL 6.70

We have $$h(n) = 3\delta(n-3)$$

or $$H(z) = 2z^{-3} \qquad \text{Taking } z \text{ transform}$$

$$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$$

Now $$\begin{aligned} Y(z) &= H(z)X(z) \\ &= 2z^{-3}(z^4 + z^2 - 2z + 2 - 3z^{-4}) \\ &= 2(z + z^{-1} - 2z^{-2} + 2z^{-3} - 3z^{-7}) \end{aligned}$$

Taking inverse z transform we have

$$\begin{aligned} y(n) = 2[&\delta(n+1) + \delta(n-1) - 2\delta(n-2) \\ &+ 2\delta(n-3) - 3\delta(n-7)] \end{aligned}$$

At $n = 4$, $\quad y(4) = 0$

Hence (B) is correct answer.

SOL 6.71

System is non causal because output depends on future value

For $n \le 1$ $\quad y(-1) = x(-1+1) = x(0)$

$$y(n-n_0) = x(n-n_0+1) \qquad \text{Time varying}$$

$$y(n) = x(n+1) \qquad \text{Depends on Future}$$

i.e. $$y(1) = x(2) \qquad \text{None causal}$$

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For bounded input, system has bounded output. So it is stable.

$$\begin{aligned} y(n) &= x(n) \text{ for } n \geq 1 \\ &= 0 \text{ for } n = 0 \\ &= x(x+1) \text{ for } n \leq -1 \end{aligned}$$

So system is linear.

Hence (A) is correct answer.

SOL 6.72

The frequency response of RC-LPF is

$$H(f) = \frac{1}{1 + j2\pi fRC}$$

Now $\qquad H(0) = 1$

$$\frac{|H(f_1)|}{H(0)} = \frac{1}{\sqrt{1 + 4\pi^2 f_1^2 R^2 C^2}} \geq 0.95$$

or $\qquad 1 + 4\pi^2 f_1^2 R^2 C^2 \leq 1.108$

or $\qquad 4\pi^2 f_1^2 R^2 C^2 \leq 0.108$

or $\qquad 2\pi f_1 RC \leq 0.329$

or $\qquad f_1 \leq \dfrac{0.329}{2\pi RC}$

or $\qquad f_1 \leq \dfrac{0.329}{2\pi RC}$

or $\qquad f_1 \leq \dfrac{0.329}{2\pi 1k \times 1\mu}$

or $\qquad f_1 \leq 52.2 \text{ Hz}$

Thus $\qquad f_{1\max} = 52.2 \text{ Hz}$

Hence (C) is correct answer.

SOL 6.73

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

$$\theta(\omega) = -\tan^{-1}\omega RC$$

$$t_g = -\frac{d\theta(\omega)}{d\omega} = \frac{RC}{1 + \omega^2 R^2 C^2}$$

$$= \frac{10^{-3}}{1 + 4\pi^2 \times 10^4 \times 10^{-6}} = 0.717 \text{ ms}$$

Hence (A) is correct answer.

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SOL 6.74

If $$x(t)^*\ h(t) = g(t)$$
Then $$x(t-\tau_1)^*\ h(t-\tau_2) = y(t-\tau_1-\tau_2)$$
Thus $$x(t+5)^*\ \delta(t-7) = x(t+5-7) = x(t-2)$$
Hence (C) is correct answer.

SOL 6.75

In option (B) the given function is not periodic and does not satisfy Dirichlet condition. So it cant be expansion in Fourier series.

$$x(t) = 2\cos\pi t + 7\cos t$$
$$T_1 = \frac{2\pi}{\omega} = 2$$
$$T_2 = \frac{2\pi}{1} = 2\pi$$
$$\frac{T_1}{T_2} = \frac{1}{\pi} = \text{irrational}$$

Hence (B) is correct answer.

SOL 6.76

From the duality property of fourier transform we have

If $$x(t) \xleftrightarrow{FT} X(f)$$
Then $$X(t) \xleftrightarrow{FT} x(-f)$$
Therefore if $$e^{-t}u(t) \xleftrightarrow{FT} \frac{1}{1+j2\pi f}$$
Then $$\frac{1}{1+j2\pi t} \xleftrightarrow{FT} e^{f}u(-f)$$

Hence (C) is correct answer.

SOL 6.77

$$\theta(\omega) = -\omega t_0$$
$$t_p = \frac{-\theta(\omega)}{\omega} = t_0$$
and $$t_g = -\frac{d\theta(\omega)}{d\omega} = t_0$$
Thus $$t_p = t_g = t_0 = \text{constant}$$
Hence (A) is correct answer.

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SOL 6.78

$$X(s)=\frac{5-s}{s^2-s-2}=\frac{5-s}{(s+1)(s-2)}=\frac{-2}{s+1}+\frac{1}{s-2}$$

Here three ROC may be possible.

$$\text{Re }(s)<-1$$
$$\text{Re }(s)>2$$
$$-1<\text{Re }(s)<2$$

Since its Fourier transform exits, only $-1<$ Re (s) <2 include imaginary axis. so this ROC is possible. For this ROC the inverse Laplace transform is

$$x(t)=[-2e^{-t}u(t)-2e^{2t}u(-t)]$$

Hence (*) is correct answer.

SOL 6.79

For left sided sequence we have

$$-a^n u(-n-1)\xleftrightarrow{z}\frac{1}{1-az^{-1}} \qquad \text{where } |z|<a$$

Thus $$-5^n u(-n-1)\xleftrightarrow{z}\frac{1}{1-5z^{-1}} \qquad \text{where } |z|<5$$

or $$-5^n u(-n-1)\xleftrightarrow{z}\frac{z}{z-5} \qquad \text{where } |z|<5$$

Since ROC is $|z|<5$ and it include unit circle, system is stable.

Alternative :

$$h(n)=-5^n u(-n-1)$$
$$H(z)=\sum_{n=-\infty}^{\infty}h(n)z^{-n}=\sum_{n=-\infty}^{-1}-5^n z^{-n}=-\sum_{n=-\infty}^{-1}(5z^{-1})^n$$

Let $n=-m$, then

$$H(z)=-\sum_{n=-1}^{-\infty}(5z^{-1})^{-m}=1-\sum_{m=0}^{\infty}(5^{-1}z)^{-m}$$
$$=1-\frac{1}{1-5^{-1}z}, \qquad |5^{-1}z|<1 \text{ or } |z|<5$$
$$=1-\frac{5}{5-z}=\frac{z}{z-5}$$

Hence (B) is correct answer.

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SOL 6.80

$$\frac{1}{s^2(s-2)} = \frac{1}{s^2} \times \frac{1}{s-2}$$

$$\frac{1}{s^2} \times \frac{1}{s-2} \xleftrightarrow{L} (t * e^{2t})\, u(t)$$

Here we have used property that convolution in time domain is multiplication in $s-$ domain

$$X_1(s)\, X_2(s) \xleftrightarrow{LT} x_1(t) * x_2(t)$$

Hence (B) is correct answer.

SOL 6.81

We have $\quad h(n) = u(n)$

$$H(z) = \sum_{n=-\infty}^{\infty} x(n)\,.z^{-n} = \sum_{n=0}^{\infty} 1.z^{-n} = \sum_{n=0}^{\infty} (z^{-1})^n$$

$H(z)$ is convergent if

$$\sum_{n=0}^{\infty} (z^{-1})^n < \infty$$

and this is possible when $|z^{-1}| < 1$. Thus ROC is $|z^{-1}| < 1$ or $|z| > 1$

Hence (A) is correct answer.

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SOL 6.82

We know that $\delta(t)\, x(t) = x(0)\, \delta(t)$ and $\int_{-\infty}^{\infty} \delta(t) = 1$

Let $x(t) = \cos(\frac{3}{2} t)$, then $x(0) = 1$

Now $\quad \int_{-\infty}^{\infty} \delta(t)\, x(t) = \int_{-\infty}^{\infty} x(0)\, \delta(t)\, dt = \int_{-\infty}^{\infty} \delta(t)\, dt = 1$

Hence (A) is correct answer.

SOL 6.83

Let E be the energy of $f(t)$ and E_1 be the energy of $f(2t)$, then

$$E = \int_{-\infty}^{\infty} [f(t)]^2\, dt$$

and

$$E_1 = \int_{-\infty}^{\infty} [f(2t)]^2\, dt$$

Substituting $2t = p$ we get

$$E_1 = \int_{-\infty}^{\infty} [f(p)]^2 \frac{dp}{2} = \frac{1}{2} \int_{-\infty}^{\infty} [f(p)]^2\, dp = \frac{E}{2}$$

Hence (B) is correct answer.

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SOL 6.84

Since $h_1(t) \neq 0$ for $t < 0$, thus $h_1(t)$ is not causal
$h_2(t) = u(t)$ which is always time invariant, causal and stable.
$h_3(t) = \frac{u(t)}{1+t}$ is time variant.
$h_4(t) = e^{-3t}u(t)$ is time variant.
Hence (B) is correct answer.

SOL 6.85

$$h(t) = f(t) * g(t)$$

We know that convolution in time domain is multiplication in $s-$ domain.

$$f(t) * g(t) = h(t) \overset{L}{\longleftrightarrow} H(s) = F(s) \times G(s)$$

Thus $$H(s) = \frac{s+2}{s^2+1} \times \frac{s^2+1}{(s+2)(s+3)} = \frac{1}{s+3}$$

Hence (B) is correct answer.

SOL 6.86

Since normalized Gaussion function have Gaussion FT

Thus $$e^{-at^2} \overset{FT}{\longleftrightarrow} \sqrt{\frac{\pi}{a}}\, e^{-\frac{\pi^2 f^2}{a}}$$

Hence (B) is correct answer.

SOL 6.87

Let $$x(t) = ax_1(t) + bx_2(t)$$
$$ay_1(t) = atx_1(t)$$
$$by_2(t) = btx_2(t)$$

Adding above both equation we have

$$ay_1(t) + by_2(t) = atx_1(t) + btx_2(t)$$
$$= t[ax_1(t) + bx_2(t)]$$
$$= tx(t)$$

or $$ay_1(t) + by_2(t) = y(t)$$ Thus system is linear

If input is delayed then we have

$$y_d(d) = tx(t - t_0)$$

If output is delayed then we have

$$y(t - t_0) = (t - t_0)\, x(t - t_0)$$

which is not equal. Thus system is time varying.
Hence (B) is correct answer.

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SOL 6.88

We have $h(t) = e^{2t} \xrightarrow{LS} H(s) = \frac{1}{s-2}$

and $x(t) = e^{3t} \xrightarrow{LS} X(s) = \frac{1}{s-3}$

Now output is
$$Y(s) = H(s)X(s)$$
$$= \frac{1}{s-2} \times \frac{1}{s-3} = \frac{1}{s-3} - \frac{1}{s-2}$$

Thus $y(t) = e^{3t} - e^{2t}$

Hence (A) is correct answer.

SOL 6.89

We know that for a square wave the Fourier series coefficient

$$C_{nsq} = \frac{A\tau}{T} \frac{\sin \frac{n\omega_0 \tau}{2}}{\frac{n\omega_0 \tau}{2}} \quad \text{...(i)}$$

Thus $C_{nsq} \propto \frac{1}{n}$

If we integrate square wave, triangular wave will be obtained,

Hence $C_{ntri} \propto \frac{1}{n^2}$

Hence (C) is correct answer.

SOL 6.90

$$u(t) - u(t-1) = f(t) \xleftrightarrow{L} F(s) = \frac{1}{s}[1 - e^{-s}]$$

$$u(t) - u(t-2) = g(t) \xleftrightarrow{L} G(s) = \frac{1}{s}[1 - e^{-2s}]$$

$$f(t)^* \ g(t) \xleftrightarrow{L} F(s)G(s)$$
$$= \frac{1}{s^2}[1 - e^{-s}][1 - e^{-2s}]$$
$$= \frac{1}{s^2}[1 - e^{-2s} - e^{-s} + e^{-3s}]$$

or $f(t)^* \ g(t) \xleftrightarrow{L} = \frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{e^{-s}}{s^2} + \frac{e^{-3s}}{s^2}$

Taking inverse laplace transform we have

$$f(t)^* \ g(t) = t - (t-2)u(t-2) - (t-1)u(t-1) + (t-3)u(t-3)$$

The graph of option (B) satisfy this equation.

Hence (B) is correct answer.

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SOL 6.91

Hence (A) is correct answer.

SOL 6.92

We have $f(nT) = a^{nT}$

Taking z-transform we get

$$F(z) = \sum_{n=-\infty}^{\infty} a^{nT} z^{-n} = \sum_{n=-\infty}^{\infty} (a^T)^n z^{-n} = \sum_{n=0}^{\infty} \left(\frac{a^T}{z}\right)^n = \frac{z}{z - a^T}$$

Hence (A) is correct answer.

SOL 6.93

If $\mathcal{L}[f(t)] = F(s)$

Applying time shifting property we can write

$$\mathcal{L}[f(t-T)] = e^{-sT} F(s)$$

Hence (B) is correct answer.

SOL 6.94

Hence (A) is correct answer.

SOL 6.95

Hence (A) is correct answer.

SOL 6.96

Given z transform

$$C(z) = \frac{z^{-1}(1 - z^{-4})}{4(1 - z^{-1})^2}$$

Applying final value theorem

$$\lim_{n \to \infty} f(n) = \lim_{z \to 1} (z-1) f(z)$$

$$\lim_{z \to 1} (z-1) F(z) = \lim_{z \to 1} (z-1) \frac{z^{-1}(1 - z^{-4})}{4(1 - z^{-1})^2}$$

$$= \lim_{z \to 1} \frac{z^{-1}(1 - z^{-4})(z-1)}{4(1 - z^{-1})^2}$$

$$= \lim_{z \to 1} \frac{z^{-1} z^{-4}(z^4 - 1)(z-1)}{4z^{-2}(z-1)^2}$$

$$= \lim_{z \to 1} \frac{z^{-3}}{4} \frac{(z-1)(z+1)(z^2+1)(z-1)}{(z-1)^2}$$

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$$= \lim_{z \to 1} \frac{z^{-3}}{4}(z+1)(z^2+1) = 1$$

Hence (C) is correct answer.

SOL 6.97

We have $\quad F(s) = \dfrac{\omega}{s^2 + \omega^2}$

$\lim\limits_{t \to \infty} f(t)$ final value theorem states that:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

It must be noted that final value theorem can be applied only if poles lies in –ve half of s-plane.

Here poles are on imaginary axis ($s_1, s_2 = \pm j\omega$) so can not apply final value theorem. so $\lim\limits_{t \to \infty} f(t)$ cannot be determined.

Hence (A) is correct answer.

SOL 6.98

Trigonometric Fourier series of a function $x(t)$ is expressed as :

$$x(t) = A_0 + \sum_{n=1}^{\infty} [A_n \cos n\omega t + B_n \sin n\omega t]$$

For even function $x(t)$, $B_n = 0$

So $\quad x(t) = A_0 + \sum\limits_{n=1}^{\infty} A_n \cos n\omega t$

Series will contain only DC & cosine terms.

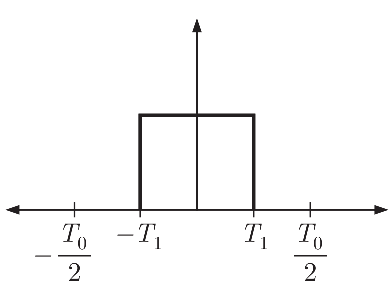
Hence (D) is correct answer.

SOL 6.99

Given periodic signal

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < \frac{T_0}{2} \end{cases}$$

The figure is as shown below.



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For $x(t)$ fourier series expression can be written as

$$x(t) = A_0 + \sum_{n=1}^{\infty}[A_n \cos n\omega t + B_n \sin n\omega t]$$

where dc term

$$A_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\,dt$$

$$= \frac{1}{T_0}\left[\int_{-T_0/2}^{-T_1} x(t)\,dt + \int_{-T_1}^{T_1} x(t)\,dt + \int_{T_1}^{T_0/2} x(t)\,dt\right]$$

$$= \frac{1}{T_0}[0 + 2T_1 + 0]$$

$$A_0 = \frac{2T_1}{T_0}$$

Hence (C) is correct answer.

SOL 6.100

The unit impulse response of a LTI system is $u(t)$

Let $h(t) = u(t)$

Taking LT we have $H(s) = \frac{1}{s}$

If the system excited with an input $x(t) = e^{-at}u(t)$, $a > 0$, the response

$$Y(s) = X(s)\,H(s)$$

$$X(s) = \mathcal{L}[x(t)] = \frac{1}{(s+a)}$$

so $$Y(s) = \frac{1}{(s+a)}\frac{1}{s} = \frac{1}{a}\left[\frac{1}{s} - \frac{1}{s+a}\right]$$

Taking inverse Laplace, the response will be

$$y(t) = \frac{1}{a}[1 - e^{-at}]$$

Hence (B) is correct answer.

SOL 6.101

We have $$x[n] = \sum_{k=0}^{\infty}\delta(n-k)$$

$$X(z) = \sum_{k=0}^{\infty}x[n]\,z^{-n} = \sum_{n=-\infty}^{\infty}\left[\sum_{k=0}^{\infty}\delta(n-k)\,z^{-n}\right]$$

Since $\delta(n-k)$ defined only for $n = k$ so

$$X(z) = \sum_{k=0}^{\infty}z^{-k} = \frac{1}{(1-1/z)} = \frac{z}{(z-1)}$$

Hence (B) is correct answer.

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SOL 6.102

Hence (B) is correct option.

SOL 6.103

$$x(t) \xleftrightarrow{\mathcal{F}} X(f)$$

by differentiation property;

$$\mathcal{F}\left[\frac{dx(t)}{dt}\right] = j\omega X(\omega)$$

or

$$\mathcal{F}\left[\frac{dx(t)}{dt}\right] = j2\pi f X(f)$$

Hence (B) is correct answer.

SOL 6.104

We have $\quad f(t) \xleftrightarrow{\mathcal{F}} g(\omega)$

by duality property of fourier transform we can write

$$g(t) \xleftrightarrow{\mathcal{F}} 2\pi f(-\omega)$$

so

$$\mathcal{F}[g(t)] = \int_{-\infty}^{\infty} g(t)\,e^{-j\omega t}\,dt = 2\pi f(-\omega)$$

Hence (C) is correct answer.

SOL 6.105

Given function

$$x(t) = e^{\alpha t}\cos(\alpha t)$$

Now $\quad \cos(\alpha t) \xleftrightarrow{\mathcal{L}} \dfrac{s}{s^2+\alpha^2}$

If $\quad x(t) \xleftrightarrow{\mathcal{L}} X(s)$

then $\quad e^{s_0 t}x(t) \xleftrightarrow{\mathcal{L}} X(s-s_0) \qquad$ shifting in s-domain

so $\quad e^{\alpha t}\cos(\alpha t) \xleftrightarrow{\mathcal{L}} \dfrac{(s-\alpha)}{(s-\alpha)^2+\alpha^2}$

Hence (B) is correct answer.

SOL 6.106

For a function $x(t)$, trigonometric fourier series is :

$$x(t) = A_0 + \sum_{n=1}^{\infty}[An\cos n\omega t + Bn\sin n\omega t]$$

where $\quad A_0 = \dfrac{1}{T_0}\displaystyle\int_{T_0} x(t)\,dt \qquad T_0$=Fundamental period

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$$A_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega t dt$$

$$B_n = \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega t dt$$

For an even function $x(t)$, coefficient $B_n = 0$
for an odd function $x(t), A_0 = 0$

$$A_n = 0$$

so if $x(t)$ is even function its fourier series will not contain sine terms.
Hence (C) is correct answer.

SOL 6.107

The conjugation property allows us to show if $x(t)$ is real, then $X(j\omega)$ has conjugate symmetry, that is

$$X(-j\omega) = X^*(j\omega) \qquad [x(t) \text{ real}]$$

Proof :

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

replace ω by $-\omega$ then

$$X(-j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{j\omega t}\, dt$$

$$X^*(j\omega) = \left[\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt\right]^* = \int_{-\infty}^{\infty} x^*(t)\, e^{j\omega t}\, dt$$

if $x(t)$ real $x^*(t) = x(t)$

then $$X^*(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{j\omega t}\, dt = X(-j\omega)$$

Hence (C) is correct answer.

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