GATE 2022
EC | Electronics Engineering

SECTION - A GENERAL APTITUDE

Q.1 Mr. X speaks ________ Japanese ________ Chinese.

(a) neither / or (b) either / nor

(c) neither / nor (d) also / but

Ans. (c)

End of Solution

Q.2 A sum of money is to be distributed among *P*, *Q*, *R*, and *S* in the proportion 5 : 2 : 4 : 3, respectively.

If *R* gets Rs. 1000 more than S, what is the share of Q (in Rs.)?

(a) 500 (b) 1000

(c) 1500 (d) 2000

Ans. (d)

$P = 5x,\ Q = 2x,\ R = 4x,\ S = 3x$

Given: *R* gets Rs. 1000 more than *S*

$$4x - 3x = 1000$$

$$x = 1000$$

Share of $Q = 2x = 2 \times 1000 =$ Rs. 2000

End of Solution

Q.3 A trapezium has vertices marked as *P*, *Q*, *R* and *S* (in that order anticlockwise). The side *PQ* is parallel to side *SR*.

Further, it is given that, *PQ* = 11 cm, *QR* = 4 cm, *RS* = 6 cm and *SP* = 3 cm.

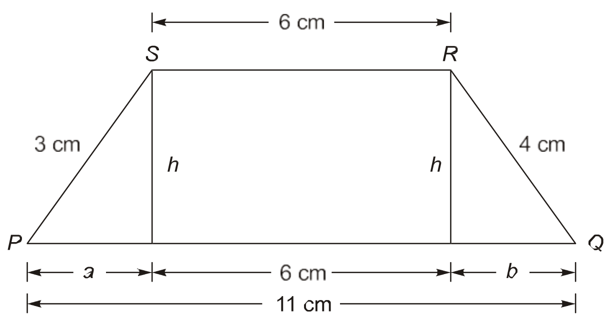
What is the shortest distance between PQ and SR (in cm)?

(a) 1.80 (b) 2.40

(c) 4.20 (d) 5.76

Ans. (b)

Given :



Trapezium PQRS

h is distance between parallel sides,

$$a + b = 11 - 6 = 5$$

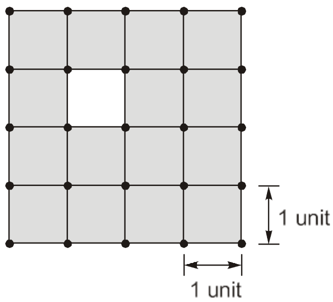
From its diagram,

$$h^2 + b^2 = 16 \quad \text{...(i)}$$
$$h^2 + a^2 = 9 \quad \text{...(ii)}$$
$$b^2 - a^2 = 7$$
$$(b + a)(b - a) = 7$$
$$5(b - a) = 7$$
$$(b - a) = \frac{7}{5}$$
$$(b + a) = 5 \quad \text{...(iii)}$$
$$2b = \frac{7}{5} + 5 = \frac{32}{5}$$
$$b = \frac{16}{5}$$

From equation (i),
$$h^2 = 16 - \left(\frac{16}{5}\right)^2 = 16 - \frac{256}{25} = \frac{400 - 256}{25} = \frac{144}{25}$$
$$h = \sqrt{\frac{144}{25}} = \frac{12}{5} = 2.4$$

End of Solution

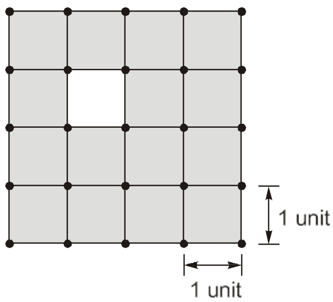
Q.4 The figure shows a grid formed by a collection of unit squares. The unshaded unit square in the grid represents a hole.



What is the maximum number of squares without a "hole in the interior" that can be formed within the 4 × 4 grid using the unit squares as building blocks?

(a) 15 (b) 20
(c) 21 (d) 26

Ans. (b)



Number of squares without hole

$$4 \times 4 - 1 = 16 - 1 = 15$$

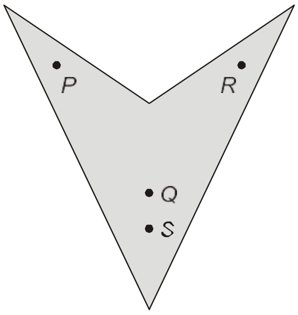
These are (1×1) squares

Now 2×2 squares = 5

Total number of squares = 15 + 5 = 20

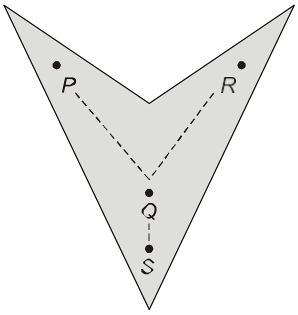
End of Solution

Q.5 An art gallery engages a security guard to ensure that the items displayed are protected. The diagram below represents the plan of the gallery where the boundary walls are opaque. The location the security guard posted is identified such that all the inner space (shaded region in the plan) of the gallery is within the line of sight of the security guard. If the security guard does not move around the posted location and has a 360° view, which one of the following correctly represents the set of ALL possible locations among the locations P, Q, R and S, where the security guard can be posted to watch over the entire inner space of the gallery.



(a) *P* and *Q*
(b) *Q*
(c) *Q* and *S*
(d) *R* and *S*

Ans. (c)



Security guard posting at *Q* and *S* can watch entire space of gallery.

End of Solution

Q.6 Mosquitoes pose a threat to human health. Controlling mosquitoes using chemicals may have undesired consequences. In Florida, authorities have used genetically modified mosquitoes to control the overall mosquito population. It remains to be seen if this novel approach has unforeseen consequences.

Which one of the following is the correct logical inference based on the information in the above passage?

(a) Using chemicals to kill mosquitoes is better than using genetically modified mosquitoes because genetic engineering is dangerous.

(b) Using genetically modified mosquitoes is better than using chemicals to kill mosquitoes because they do not have any side effects.

(c) Both using genetically modified mosquitoes and chemicals have undesired consequences and can be dangerous.

(d) Using chemicals to kill mosquitoes may have undesired consequences but it is not clear if using genetically modified mosquitoes has any negative consequence.

Ans. **(d)**

End of Solution

Q.7 Consider the following inequalities.

(i) $2x - 1 > 7$

(ii) $2x - 9 < 1$

Which one of the following expressions below satisfies the above two inequalities?

(a) $x \leq -4$ (b) $-4 < x \leq 4$

(c) $4 < x < 5$ (d) $x \geq 5$

Ans. **(c)**

$2x - 1 > 7$	$2x - 9 < 1$
$2x > 8$	$2x < 10$
$x > 4$	$x < 5$

$\therefore \quad 4 < x < 5.$

End of Solution

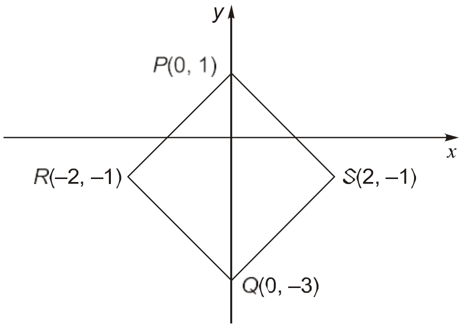
Q.8 Four points $P(0, 1)$, $Q(0, -3)$, $R(-2, -1)$, and $S(2, -1)$ represent the vertices of a quadrilateral. What is the area enclosed by the quadrilateral?

(a) 4 (b) $4\sqrt{2}$

(c) 8 (d) $8\sqrt{2}$

Ans. (c)

According to data quadrilateral *PSQR*



$$PS = \sqrt{(2-0)^2 + (-1-1)^2} = \sqrt{8}$$

$$QS = \sqrt{(2-0)^2 + (-1+3)^2} = \sqrt{8}$$

$$QR = \sqrt{(0+2)^2 + (-3+1)^2} = \sqrt{8}$$

$$PR = \sqrt{(-2-0)^2 + (1+1)^2} = \sqrt{8}$$

∴ All sides are equal

Diagonals, $$PQ = \sqrt{(0-0)^2 + (1+3)^2} = 4$$

$$PQ = \sqrt{(2+2)^2 + (-1+1)^2} = 4$$

Diagonals are equal

∴ It is a square

Area of square = $(\sqrt{8}) \times (\sqrt{8}) = 8$ square units

End of Solution

Q.9 In a class of five students P, Q, R, S and T, only one student is known to have copied in the exam. The disciplinary committee has investigated the situation and recorded the statements from the students as given below.

Statement of P: R has copied in the exam.

Statement of Q: S has copied in the exam.

Statement of R: P did not copy in the exam.

Statement of S: Only one of us is telling the truth.

Statement of T: R is telling the truth.

The investigating team had authentic information that S never lies.

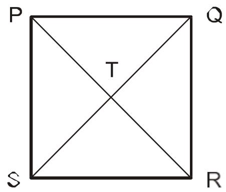
Based on the information given above, the person who has copied in the exam is

(a) R
(b) P
(c) Q
(d) T

Ans. (b)

End of Solution

Q.10 Consider the following square with the four corners and the center marked as P, Q, R, S and T respectively.



Let X, Y and Z represent the following operations:

X: rotation of the square by 180 degree with respect to the S-Q axis.

Y: rotation of the square by 180 degree with respect to the P-R axis.

Z: rotation of the square by 90 degree clockwise with respect to the axis perpendicular, going into the screen and passing through the point T.

Consider the following three distinct sequences of operation (which are applied in the left to right order).

(1) XYZZ

(2) XY

(3) ZZZZ

Which one of the following statements is correct as per the information provided above?

(a) The sequence of operations (1) and (2) are equivalent
(b) The sequence of operations (1) and (3) are equivalent
(c) The sequence of operations (2) and (3) are equivalent
(d) The sequence of operations (1), (2) and (3) are equivalent

Ans. (b)

End of Solution

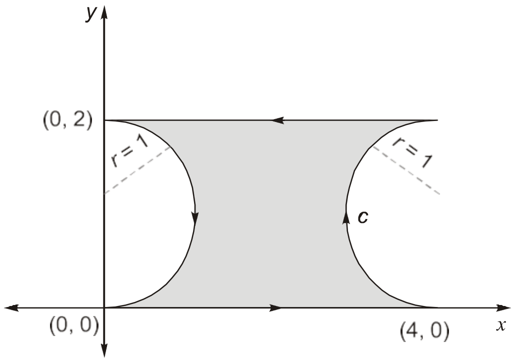
■■■■

SECTION - B TECHNICAL

Q.11 Consider the two-dimensional vector field $\vec{F}(x, y) = x\vec{i} + y\vec{j}$, where $\vec{i}$ and $\hat{j}$ denote the unit vectors along the x-axis and the y-axis, respectively. A contour C in the x-y plane, as shown in the figure, is composed of two horizontal lines connected at the two ends by two semicircular arcs of unit radius. The contour is traversed in the counter-clockwise sense. The value of the closed path integral

$$\oint_C \vec{F}(x, y) \cdot (dx\vec{i} + dy\vec{j})$$

is _______.



(a) 0
(b) 1
(c) $8 + 2\pi$
(d) -1

Ans. **(a)**

$$\oint_C \vec{F}(x,y) \cdot (dx\mathrm{i} + dy\mathrm{j})$$

$$= \oint_C (x\vec{i} + y\vec{j}) \cdot (dx\vec{i} + dy\vec{j}) = \oint_C x\,dx + y\,dy$$

By Green's theorem

$$= \iint_R (0 - 0)\,dx\,dy$$

$$= 0$$

End of Solution

Q.12 Consider a system of linear equations $Ax = b$, where

$$A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix}, \; b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

This system of equations admits _______.

(a) a unique solution for x
(b) infinitely many solutions for x
(c) no solutions for x
(d) exactly two solutions for x

Ans. (c)

$$Ax = B$$

$$x - \sqrt{2}y + 3z = 1 \quad \text{(Inconsistent)}$$

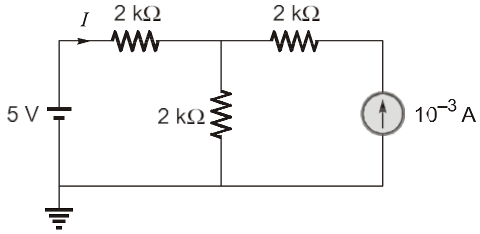
$$-x + \sqrt{2}y - 3z = 3 \Rightarrow x - \sqrt{2}y + 3z = -3 \neq 1 \quad \text{(Inconsistent)}$$

$\therefore$ Both lines are parallel to each other.

$\therefore$ It has no solution.

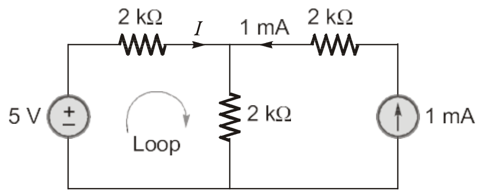
End of Solution

Q.13 The current I in the circuit shown is ________.



(a) 1.25×10^{-3} A
(b) 0.75×10^{-3} A
(c) -0.5×10^{-3} A
(d) 1.16×10^{-3} A

Ans. (b)



Applying KVL in the loop,

$$5 = 2kI + 2k(I + 10^{-3})$$

$$5 = I(4k) + 2 \times 10^{3} \times 1 \times 10^{-3}$$

$$I(4\ k) = 5 - 2$$

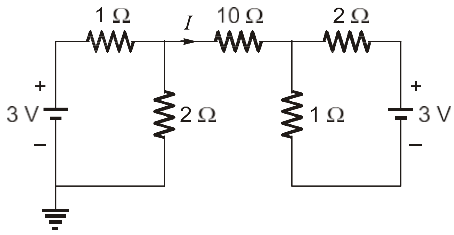
$$I = \frac{3}{4k}$$

$$I = 0.75 \times 10^{-3}\ \text{A}$$

$$I = 0.75\ \text{mA}$$

End of Solution

Q.14 Consider the circuit shown in the figure. The current I flowing through the 10 Ω resistor is ________.



(a) 1 A
(b) 0 A
(c) 0.1 A
(d) –0.1 A

Ans. **(b)**

As there is no return path for current,

$\therefore \quad I = 0$

End of Solution

Q.15 The Fourier transform $X(j\omega)$ of the signal

$$x(t) = \frac{t}{(1+t^2)^2}$$

is _______.

(a) $\frac{\pi}{2j}\omega e^{-|\omega|}$
(b) $\frac{\pi}{2}\omega e^{-|\omega|}$
(c) $\frac{\pi}{2j} e^{-|\omega|}$
(d) $\frac{\pi}{2} e^{-|\omega|}$

Ans. **(a)**

Consider, $x(t) = e^{-|t|}$

by taking Fourier transform,

$$e^{-|t|} \xleftrightarrow{F.T} \frac{2}{1+\omega^2}$$

Apply differentiation in frequency,

$$t x(t) \xleftrightarrow{F.T} j\frac{d}{d\omega}X(\omega)$$

$$te^{-|t|} \xleftrightarrow{F.T} j\left[\frac{d}{d\omega}\left(\frac{2}{1+\omega^2}\right)\right]$$

$$te^{-|t|} \xleftrightarrow{F.T} j\left[\frac{[0-2(2\omega)]}{(1+\omega^2)^2}\right]$$

$$te^{-|t|} \xleftrightarrow{F.T} \frac{-4j\omega}{(1+\omega^2)^2}$$

Apply duality property,

$$\frac{-4jt}{(1+t^2)^2} \xleftarrow{F.T} 2\pi(-\omega)e^{-|-\omega|}$$

$$\frac{t}{(1+t^2)^2} \xleftrightarrow{F.T} \frac{-2\pi\omega\, e^{-|\omega|}}{-4j}$$

$$\frac{t}{(1+t^2)^2} \xleftrightarrow{F.T} \frac{\pi}{2j}\omega e^{-|\omega|}$$

End of Solution

Q.16 Consider a long rectangular bar of direct bandgap p-type semiconductor. The equilibrium hole density is 10^{17} cm^{-3} and the intrinsic carrier concentration is 10^{10} cm^{-3}. Electron and hole diffusion lengths are 2 μm and 1 μm, respectively.

The left side of the bar (x = 0) is uniformly illuminated with a laser having photon energy greater than the bandgap of the semiconductor. Excess electron-hole pairs are generated ONLY at x = 0 because of the laser. The steady state electron density at x = 0 is 10^{14} cm^{-3} due to laser illumination. Under these conditions and ignoring electric field, the closest approximation (among the given options) of the steady state electron density at x = 2 μm, is ___________ .

(a) 0.37×10^{14} cm^{-3}
(b) 0.63×10^{13} cm^{-3}
(c) 3.7×10^{14} cm^{-3}
(d) 10^{3} cm^{-3}

Ans. (a)

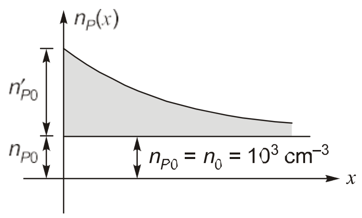
Given,

$$P = 10^{17} \text{ cm}^{-3} = P_0$$

$$n_0 = \frac{n_i^2}{P_0} = \frac{10^{20}}{10^{17}} = 10^3 \text{cm}^{-3}$$

$$L_n = 2\ \mu\text{m}$$

$$n'_{P0} = 10^{14} \text{ cm}^{-3}$$



Excess electron concentration at any distance x is

$$\delta n_P(x) = n'_{P0}e^{-x/L_n}$$
$$= 10^{14}\, e^{-2/2}$$
$$= 10^{14}\, e^{-1}$$
$$= 3.67 \times 10^{13} \text{ cm}^{-3}$$
$$= 0.367 \times 10^{14} \text{ cm}^{-3}$$

End of Solution

Q.17 In a non-degenerate bulk semiconductor with electron density $n = 10^{16}\ \text{cm}^{-3}$, the value of $E_C - E_{Fn} = 200$ meV, where E_C and E_{Fn} denote the bottom of the conduction band energy and electron Fermi level energy, respectively. Assume thermal voltage as 26 meV and the intrinsic carrier concentration is $10^{10}\ \text{cm}^{-3}$. For $n = 0.5 \times 10^{16}\ \text{cm}^{-3}$, the closest approximation of the value of $(E_C - E_{Fn})$, among the given options, is ________.

(a) 226 meV
(b) 174 meV
(c) 218 meV
(d) 182 meV

Ans. **(c)**

Given,
$$(E_C - E_{Fn})_1 = kT\ln\left(\frac{N_C}{N_{D1}}\right) = 200\text{ meV} \quad ...(1)$$

$$(E_C - E_{Fn})_2 = kT\ln\left(\frac{N_C}{N_{D2}}\right) \quad ...(2)$$

$$N_{D1} = 10^{16}\ \text{cm}^{-3}$$
$$N_{D2} = 0.5 \times 10^{16}\ \text{cm}^{-3}$$

$$(E_C - E_{Fn})_1 - (E_C - E_{Fn})_2 = kT\ln\left(\frac{N_C}{N_{D1}}\right) - kT\ln\left(\frac{N_C}{N_{D2}}\right)$$

$$200\text{ meV} - (E_C - E_{Fn})_2 = kT\ln\left(\frac{N_{D2}}{N_{D1}}\right)$$

$$= 0.026\ln\left(\frac{10^{16} \times 0.5}{10^{16}}\right)$$

$$= 0.026\ln(0.5)$$
$$= -0.01802\text{ volt}$$
$$= -18.02\text{ meV}$$

$$(E_C - E_{Fn})_2 = 200\text{ meV} + 18.02\text{ meV}$$
$$= 218.02\text{ meV}$$

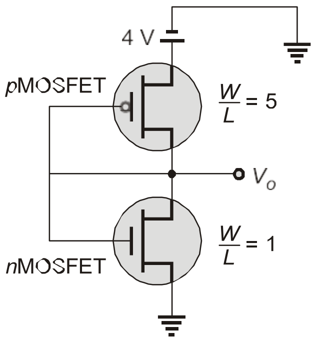
End of Solution

Q.18 Consider the CMOS circuit shown in the figure (substrates are connected to their respective sources). The gate width (W) to gate length (L) ratios $\left(\frac{W}{L}\right)$ of the transistors are as shown. Both the transistors have the same gate oxide capacitance per unit area.

For the pMOSFET, the threshold voltage is –1 V and the mobility of holes is $40\ \frac{\text{cm}^2}{\text{V.s}}$.

For the nMOSFET, the threshold voltage is 1 V and the mobility of electrons is $300\ \frac{\text{cm}^2}{\text{V.s}}$.

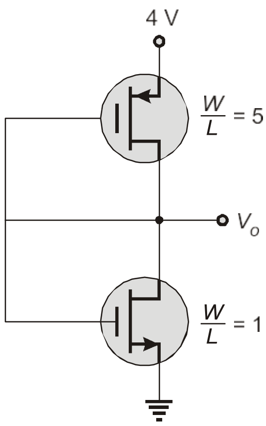
The steady state output voltage V_o is ________.



(a) equal to 0 V
(b) more than 2 V
(c) less than 2 V
(d) equal to 2 V

Ans. **(c)**

Both MOSFETs are in saturation because drain is shorted to Gate.



$$I_{DSN} = I_{SDP}$$

$$\frac{\mu_n C_{ox}}{2} \times \left(\frac{W}{L}\right)_N (V_{GSN} - V_{TN})^2 = \frac{\mu_p C_{ox}}{2}\left(\frac{W}{L}\right)_P (V_{SGP} - |V_{TP}|)^2$$

$$300 \times 1\ (V_0 - 1)^2 = 40 \times 5(4 - V_0 - 1)^2$$

$$3\left(V_0^2 + 1 - 2V_0\right) = 2\left(9 + V_0^2 - 6V_0\right)$$

$$\Rightarrow \qquad V_0^2 + 6V_0 - 15 = 0$$

$$V_0 = \frac{-6 \pm \sqrt{36 + 4 \times 15}}{2} = \frac{-6 \pm \sqrt{96}}{2}$$

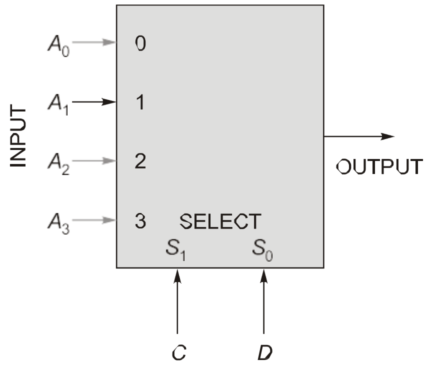
$$V_0 = 1.898\text{ V}, -7.89\text{ V}$$

V_0 cannot be negative because V_0 should lie between 0 and 4 V.

$$\therefore \qquad V_0 = 1.898\text{ V}$$

End of Solution

Q.19 Consider the 2-bit multiplexer (MUX) shown in the figure. For OUTPUT to be the XOR of C and D, the values for A_0, A_1, A_2 and A_3 are ________.



(a) $A_0 = 0$, $A_1 = 0$, $A_2 = 1$, $A_3 = 1$ (b) $A_0 = 1$, $A_1 = 0$, $A_2 = 1$, $A_3 = 0$
(c) $A_0 = 0$, $A_1 = 1$, $A_2 = 1$, $A_3 = 0$ (d) $A_0 = 1$, $A_1 = 1$, $A_2 = 0$, $A_3 = 0$

Ans. (c)

The output of MUX, F is

$$F = \bar{S}_1\bar{S}_0 I_0 + \bar{S}_1 S_0 I_1 + S_1\bar{S}_0 I_2 + S_1 S_0 I_3$$

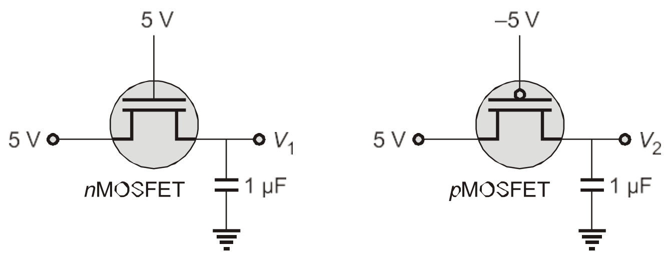
$$F = \bar{C}\bar{D}A_0 + \bar{C}DA_1 + C\bar{D}A_2 + CDA_3$$

But output, $$F = C \oplus D = \bar{C}D + C\bar{D}$$

$\therefore$ Inputs of MUX are, $A_0 = 0$, $A_1 = 1$, $A_2 = 1$, $A_3 = 0$

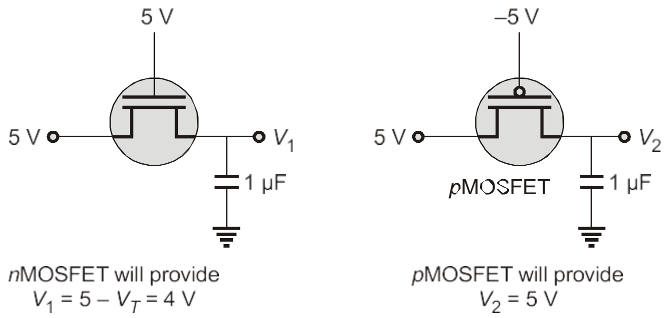
End of Solution

Q.20 The ideal long channel *n*MOSFET and *p*MOSFET devices shown in the circuits have threshold voltages of 1 V and –1 V, respectively. The MOSFET substrates are connected to their respective sources. Ignore leakage currents and assume that the capacitors are initially discharged. For the applied voltages as shown, the steady state voltages are ……………………………………….



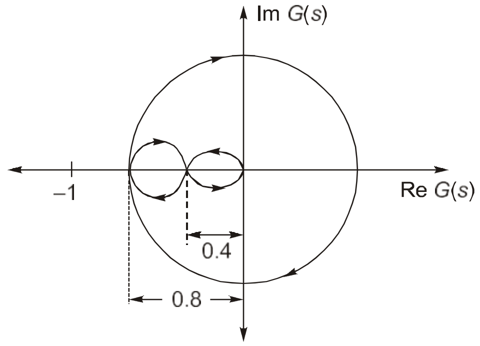
(a) $V_1 = 5$ V, $V_2 = 5$ V (b) $V_1 = 5$ V, $V_2 = 4$ V
(c) $V_1 = 4$ V, $V_2 = 5$ V (d) $V_1 = 4$ V, $V_2 = -5$ V

Ans. (c)



End of Solution

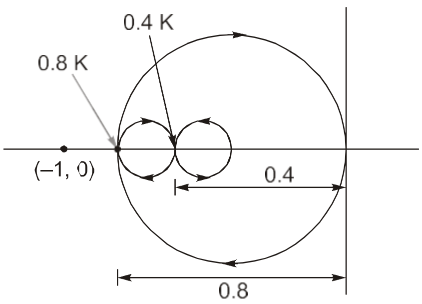
Q.21 Consider a closed-loop control system with unity negative feedback and $KG(s)$ in the forward path, where the gain $K = 2$. The complete Nyquist plot of the transfer function $G(s)$ is shown in the figure. Note that the Nyquist contour has been chosen to have the clockwise sense. Assume $G(s)$ has no poles on the closed right-half of the complex plane. The number of poles of the closed-loop transfer function in the closed right-half of the complex plane is ___________.



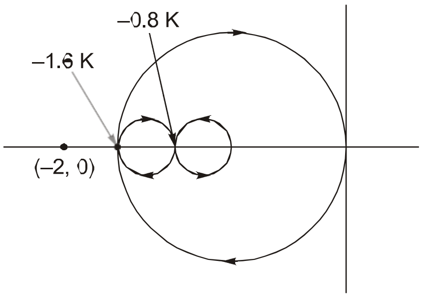
(a) 0
(b) 1
(c) 2
(d) 3

Ans. (c)

For, $K = 1$



For K = 2, the plot will be



N = No. of encirclement about (–1, 0) in anticlockwise.
P = Total number of open loop poles, in R.H.S.

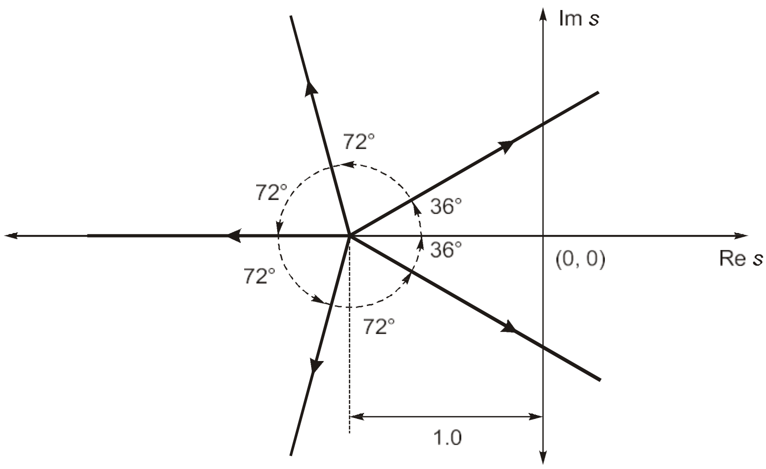
$$Z = P - N$$
$$N = -2,\ P = 0$$
$$Z = 0 - (-2) = 2$$
$$Z = 2$$

Two poles lies in right side.

End of Solution

Q.22 The root-locus plot of a closed-loop system with unity negative feedback and transfer function $KG(s)$ in the forward path is shown in the figure. Note that K is varied from 0 to ∞.

Select the transfer function $G(s)$ that results in the root-locus plot of the closed-loop system as shown in the figure.



(a) $G(s) = \dfrac{1}{(s+1)^5}$

(b) $G(s) = \dfrac{1}{s^5 + 1}$

(c) $G(s) = \dfrac{s-1}{(s+1)^6}$

(d) $G(s) = \dfrac{s+1}{s^6 + 1}$

Ans. (a)

There are 5 root locus branches from the same point, so there are 5 real multiple poles.

So, correct option is $\frac{1}{(s+1)^5}$.

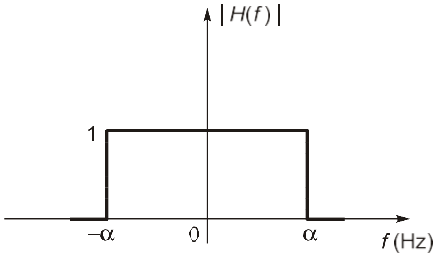
End of Solution

Q.23 The frequency response $H(f)$ of a linear time-invariant system has magnitude as shown in the figure.

Statement I: The system is necessarily a pure delay system for inputs which are bandlimited to $-\alpha \le f \le \alpha$.

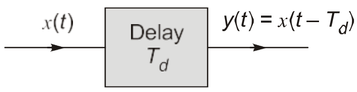
Statement II: For any wide-sense stationary input process with power spectral density $S_X(f)$, the output power spectral density $S_Y(f)$ obeys $S_Y(f) = S_X(f)$ for $-\alpha \le f \le \alpha$.

Which one of the following combinations is true?



(a) Statement I is correct, Statement II is correct
(b) Statement I is correct, Statement II is incorrect
(c) Statement I is incorrect, Statement II is correct
(d) Statement I is incorrect, Statement II is incorrect

Ans. (a)



$$Y(f) = X(f) \cdot e^{-j2\pi f T_d}$$

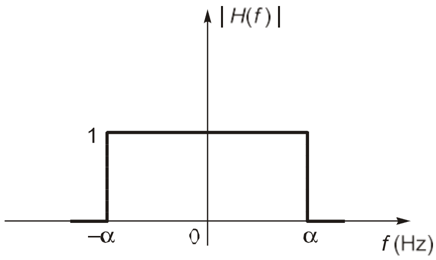
$$H(f) = \frac{Y(f)}{X(f)} = e^{-j2\pi f T_d}$$

$$|H(f)| = 1$$

$$\angle H(f) = -2\pi f T_d$$

Given that input is bandlimited to $-\alpha \le f \le \alpha$.

Magnitude response of the system given



Based on the magnitude response given ; statement (I) is correct.

Note : Phase response of the system is not given in the question without knowing phase response of the system we can't comment about exact nature of the system.

$$(PSD)_{o/p} = (PSD)_{i/p} \cdot |H(f)|^2$$

$$S_Y(f) = S_X(f) \quad ; \quad -\alpha \le f \le \alpha$$

Statement (II) is correct.

Hence option (a) is correct.

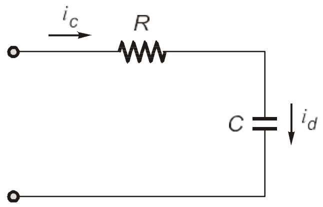
End of Solution

Q.24 In a circuit, there is a series connection of an ideal resistor and an ideal capacitor. The conduction current (in Amperes) through the resistor is $2\sin(t + \pi/2)$. The displacement current (in Amperes) through the capacitor is ________.

(a) $2\sin(t)$ (b) $2\sin(t + \pi)$

(c) $2\sin(t + \pi/2)$ (d) 0

Ans. (c)



In series connection, current through each elements remain same. Hence, $i_c = i_d$.

So, $$i_d = 2\sin\left(t + \frac{\pi}{2}\right)$$

End of Solution

Q.25 Consider the following partial differential equation (PDE)

$$a\frac{\partial^2 f(x, y)}{\partial x^2} + b\frac{\partial^2 f(x, y)}{\partial y^2} = f(x, y),$$

where a and b are distinct positive real numbers. Select the combination(s) of values of the real parameters ξ and η such that $f(x, y) = e^{(\xi x + \eta y)}$ is a solution of the given PDE.

(a) $\xi = \frac{1}{\sqrt{2a}}, \eta \frac{1}{\sqrt{2b}}$ (b) $\xi = \frac{1}{\sqrt{a}}, \eta = 0$

(c) $\xi = 0, \eta = 0$ (d) $\xi = \frac{1}{\sqrt{a}}, \ \eta = \frac{1}{\sqrt{b}}$

Ans. (a, b)

$$f(x, y) = e^{\xi x + \eta y}$$

Differentiating $f(x, y)$ two times w.r.t. x.

$$\frac{\partial^2 f(x, y)}{\partial x^2} = \xi^2 e^{\xi x + \eta y}$$

Differentiating $f(x, y)$ two times w.r.t. y,

$$\frac{\partial^2 f(x,y)}{\partial y^2} = \eta^2 e^{\xi x+\eta y}$$

$$\frac{a\partial^2 f(x,y)}{\partial x^2} + \frac{b\partial^2 f(x,y)}{\partial y^2} = \left(a\xi^2 + b\eta^2\right) f(x,y)$$

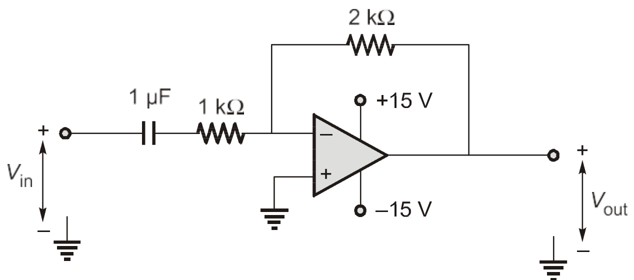
$$a\xi^2 + b\eta^2 = 1$$

For $$\xi = \frac{1}{\sqrt{a}}, \eta = 0$$

$$a \times \left(\frac{1}{\sqrt{a}}\right)^2 + b \times 0 = \frac{a}{a} = 1$$

End of Solution

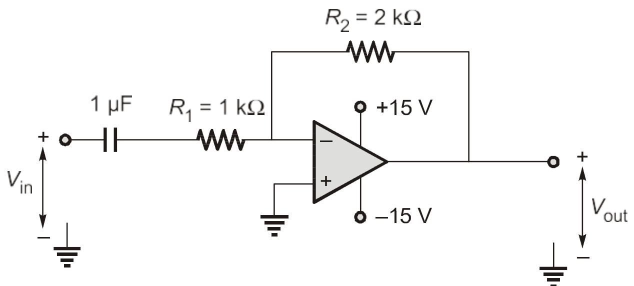
Q.26 An ideal OPAMP circuit with a sinusoidal input is shown in the figure. The 3 dB frequency is the frequency at which the magnitude of the voltage gain decreases by 3 dB from the maximum value. Which of the options is/are correct?



(a) The circuit is a low pass filter.
(b) The circuit is a high pass filter.
(c) The 3 dB frequency is 1000 rad/s.
(d) The 3 dB frequency is $\frac{1000}{3}$ rad/s.

Ans. **(b, c)**

Given circuit is a high pass filter.



$$\omega_c = \frac{1}{R_1 C_1} = \frac{1}{10^3 \times 10^{-6}} = 1000 \text{ rad/sec}$$

Hence, options (b) and (c) is correct.

End of Solution

Q.27 Select the Boolean function(s) equivalent to $x + yz$, where x, y, and z are Boolean variables, and + denotes logical OR operation.

(a) $x + z + xy$
(b) $(x + y)(x + z)$
(c) $x + xy + yz$
(d) $x + xz + xy$

Ans. **(b, c)**

Given : $f = x + yz$

(a) $x + xy + xz = x(1 + y) + xz = x + xz = x$

(b) $(x + y)(x + z) = x + yz$

(c) $x + xy + yz = x(1 + y) + yz = x + yz$

(d) $xy + yz = y(x + z)$

So, option (b, c) are correct.

End of Solution

Q.28 Select the correct statement(s) regarding CMOS implementation of NOT gates.

(a) Noise Margin High (NM_H) is always equal to the Noise Margin Low (NM_L), irrespective of the sizing of transistors.

(b) Dynamic power consumption during switching is zero.

(c) For a logical high input under steady state, the nMOSFET is in the linear regime of operation.

(d) Mobility of electrons never influences the switching speed of the NOT gate.

Ans. **(c)**

(a)
$$NM_L = V_{IL} - V_{OL}$$
$$NM_H = V_{OH} - V_{IH}$$

when, $V_{TN} = |V_{TP}|$

and $K_n = K_p$

$$V_{IT} = \frac{V_{DD}}{2} \text{ and } \boxed{NM_L = NM_H}$$

when $\uparrow \frac{K_p}{K_n} > 1, V_{IL}\uparrow, V_{IH}\uparrow$

$NM_H \downarrow$ and $NM_L \uparrow$

When, $\downarrow \frac{K_p}{K_n} < 1, V_{IL}\downarrow, V_{IH}\uparrow$

$NM_H \uparrow$ and $NM_L \uparrow$

i.e. NM_L and NM_H depends on transistor sizing and they are equal for certain condition only.

(b) Dynamic power consumption during switching is non-zero due to capacitive loading of next stage.

(c) For $V_{DD} - |V_{TP}| \le V_{in} \le V_{DD} \Rightarrow$ PMOS $\rightarrow$ Cut-off
(logic high input) NMOS $\rightarrow$ Linear

(d) Switching speed depends on charging and discharging of load capacitor for pull up and pull down of output voltage respectively.

Fast charging and discharging of capacitor depends on mobility of charge carrier.

$$\tau_{\text{charging}} = R_{\text{PMOS}}\, C_L \text{ (to bring o/p at logic high)}$$

$$\tau_{\text{discharging}} = R_{\text{NMOS}}\, C_L \text{ (to bring o/p at logic low)}$$

and also propagation delay,

$$\tau_p = \frac{\tau_{\text{PLH}} + \tau_{\text{PHL}}}{2}$$

where,

$$T_{\text{PLH}} = \frac{C_L V_{DD}}{\mu_p C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{Tp})^2}$$

$$T_{\text{PHL}} = \frac{C_L V_{DD}}{\mu_p C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2}$$

Using average charge model.

Hence, (c) is only correct statement.

End of Solution

Q.29 Let $H(X)$ denote the entropy of a discrete random variable X taking K possible distinct real values. Which of the following statements is/are necessarily true?

(a) $H(X) \leq \log_2 K$ bits

(b) $H(X) \leq H(2X)$

(c) $H(X) \leq H(X^2)$

(d) $H(X) \leq H(2^X)$

Ans. **(a, b, d)**

Given that X is a discrete random variable taking K possible distinct real values. If 'K' symbols having equal probability then entropy will be maximum $H(X)_{max} = \log_2 K$.

If symbols having different probabilities then $H(X) < \log_2 K$

So, that, $H(X) \leq \log_2 K$

Option (a) is correct.

Given option (b), $H(X) \leq H(2X)$

Let,

$X \in \{x_i\}$	-1	0	1
$P_X(x_i)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$H(X) = \sum_i P_X(x_i) \log_2 \frac{1}{P_X(x_i)}$$

$$= \frac{1}{4}\log_2 4 + \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4$$

$$H(X) = 1.5 \frac{\text{bits}}{\text{symbol}}$$

Let, $Y = 2X$

X	Y	$P(Y)$
–1	–2	1/4
0	0	1/2
1	2	1/4

$Y \in \{y_i\}$	–2	0	2
$P_Y(y_i)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$H(2X) = H(Y) = \sum_i P_Y(y_i)\log_2 \frac{1}{P_Y(y_i)} = 1.5\,\frac{\text{bits}}{\text{symbol}}$$

For $Y = 2X$, distant X values results in distinct 'Y' values so that $H(X) = H(Y)$.

So, option (b) is true i.e., $H(X) = H(2X)$

Given option (c), $H(X) \le H(X^2)$;

Let $Y = X^2$,

X	Y	$P(Y)$
–1	1	1/4
0	0	1/2
1	1	1/4

$Y \in (y_i)$	0	1
$P(Y = y_i)$	$\frac{1}{2}$	$\frac{1}{2}$

$$H(X^2) = H(Y) = \sum_i P_Y(y_i)\log_2 \frac{1}{P_Y(y_i)}$$

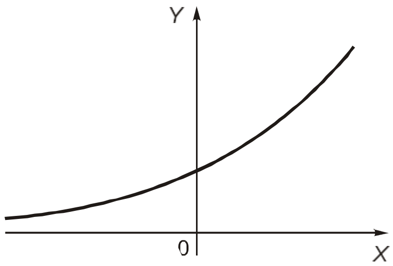
$$= \frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2 = 1\,\frac{\text{bit}}{\text{symbol}}$$

whereas, $H(X) = 1.5\,\frac{\text{bits}}{\text{symbol}}$

Option (c) is incorrect.

Given option (d) $H(X) \le H(2^X)$

Let $Y = 2^X$



Here distinct 'X' values results in distinct 'Y' values.

So that, $H(X) = H(Y)$

i.e. $H(X) = H(2^X)$

Option (d) is true.

End of Solution

Q.30 Consider the following wave equation,

$$\frac{\partial^2 f(x,t)}{\partial t^2} = 10000 \frac{\partial^2 f(x,t)}{\partial x^2}$$

Which of the given options is/are solution(s) to the given wave equation?

(a) $f(x,t) = e^{-(x-100t)^2} + e^{-(x+100t)^2}$

(b) $f(x,t) = e^{-(x-100t)} + 0.5e^{-(x+1000t)}$

(c) $f(x,t) = e^{-(x-100t)} + \sin(x+100t)$

(d) $f(x,t) = e^{j100\pi(-100x+t)} + e^{j100\pi(100x+t)}$

Ans. **(a, c)**

Given wave equation,

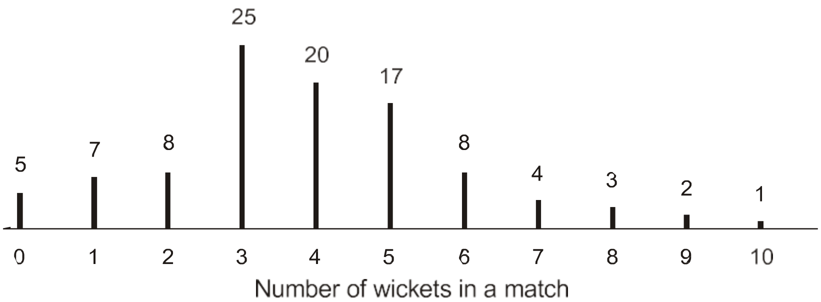
$$\frac{\partial^2 f(x,t)}{\partial t^2} = c^2 \frac{d^2 f}{dt^2} \; ; \; c = 100$$

Solution is given as $F = f(x \pm ct)$.

Hence, (a) and (c) satisfies the above solution.

End of Solution

Q.31 The bar graph shows the frequency of the number of wickets taken in a match by a bowler in her career. For example, in 17 of her matches, the bowler has taken 5 wickets each. The median number of wickets taken by the bowler in a match is ________ (rounded off to one decimal place).



Ans. **(4)**

$$\Sigma f = 5 + 7 + 8 + 25 + 20 + 17 + 8 + 4 + 3 + 2 + 1$$
$$= 100$$

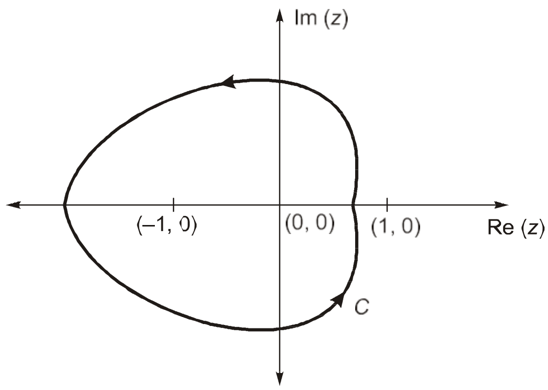
Median = Average of 50^{th} and 51^{st} matches
$= 4$

End of Solution

Q.32 A simple closed path C in the complex plane is shown in the figure. If

$$\oint_C \frac{2^z}{z^2 - 1} dz = -i\pi A,$$

where $i = \sqrt{-1}$, then the value of A is ______ (rounded off to two decimal places).



Ans. **(0.5)**

Poles are given as,

$$z^2 - 1 = 0$$
$$z = \pm 1$$

$$\therefore \quad \oint_C f(z) \cdot dz = 2\pi i [\text{Residue at } -1]$$

$$= 2\pi i \frac{2^{-1}}{(-1-1)} = \frac{-2\pi i}{2 \times 2}$$

$$= -\frac{\pi i}{2}$$

Comparing with $-\pi i A$

$$A = \frac{1}{2}$$

End of Solution

Q.33 Let $x_1(t) = e^{-t} u(t)$ and $x_2(t) = u(t) - u(t-2)$, where $u(\cdot)$ denotes the unit step function. If $y(t)$ denotes the convolution of $x_1(t)$ and $x_2(t)$, then $\lim_{t \to \infty} y(t) =$ ______ (rounded off to one decimal place).

Ans. **(0)**

Given that,

$$x_1(t) = e^{-2t} u(t)$$
$$x_2(t) = u(t) - u(t-2)$$
$$y(t) = x_1(t) * x_2(t)$$

By applying Laplace transform

$$Y(s) = X_1(s) \cdot X_2(s) = \frac{1}{(s+1)} \frac{(1-e^{-2s})}{s}$$

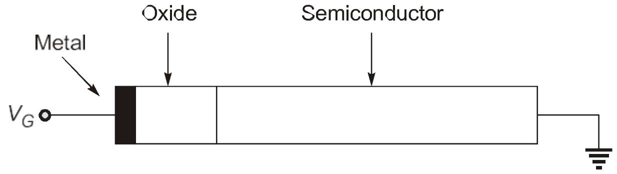
By applying final value theorem,

$$y(t)\big|_{t=\infty} = \lim_{s\to 0} sY(s) = \lim_{s\to 0}\left(\frac{1-e^{-2s}}{s+1}\right)$$

$$= 0$$

End of Solution

Q.34 An ideal MOS capacitor (p-type semiconductor) is shown in the figure. The MOS capacitor is under strong inversion with $V_G = 2$ V. The corresponding inversion charge density (Q_{IN}) is 2.2 μC/cm². Assume oxide capacitance per unit area as C_{ox} = 1.7 μF/cm². For $V_G = 4$ V, the value of Q_{IN} is ______ μC/cm² (rounded off to one decimal place).



Ans. **(5.6)**

$$Q_{in} = C_{ox}(V_{GS} - V_T) \quad \text{or} \quad C_{ox}(V_G - V_T)$$

$$\frac{Q_{in}}{C_{ox}} = V_G - V_T$$

$$V_T = V_G - \frac{Q_{in}}{C_{ox}} = 2 - \frac{2.2}{1.7} = 2 - 1.294$$

$$V_T = 0.706 \text{ volt}$$

Now, Q_{in} at $V_G = 4$ V

$$Q_{in} = C_{ox}(V_G - V_T)$$

$$= 1.7 \times 10^{-6} \times (4 - 0.706)$$

$$Q_{in} = 5.5998\ \mu\text{C/cm}^2$$

$$\simeq 5.6\ \mu\text{C/cm}^2$$

End of Solution

Q.35 A symbol stream contains alternate QPSK and 16-QAM symbols. If symbols from this stream are transmitted at the rate of 1 mega-symbols per second, the raw (uncoded) data rate is _______ mega-bits per second (rounded off to one decimal place).

Ans. **(3)**

Given that symbol stream contains alternate QPSK and 16-QAM symbols.
Symbol rate given as 1 mega symbols per second.

$$\text{Bit rate} = \text{Symbol rate} \times \log_2 M$$

For QPSK,

$$\text{Bit rate} = 1\text{M}\frac{\text{symbol}}{\text{sec}} \times \log_2 4\frac{\text{bits}}{\text{symbol}}$$

$$\text{Bit rate} = 2 \text{ Mbps}$$

For 16-QAM,

$$\text{Bit rate} = 1\text{M}\frac{\text{symbol}}{\text{sec}} \times \log_2 16\frac{\text{bits}}{\text{symbol}}$$

Bit rate = 4 Mbps

Since, alternate QPSK and 16-QAM used, data rate of uncoded data is

$$= \frac{4\,\text{Mbps} + 2\,\text{Mbps}}{2} = 3\,\text{Mbps}.$$

End of Solution

Q.36 The function $f(x) = 8\log_e x - x^2 + 3$ attains its minimum over the interval $[1, e]$ at x = _________.

(Here $\log_e x$ is the natural logarithm of x.)

(a) 2
(b) 1
(c) e
(d) $\frac{1+e}{2}$

Ans. **(b)**

$$f(x) = 8\ln(x) - x^2 + 3$$

Stationary points,

$$f'(x) = \frac{8}{x} - 2x = 0$$

$$2x^2 = 8$$

$$x = \pm 2$$

$$x = 2 \in [1, e]$$

$$f''(x) = -\frac{8}{x^2} - 2\Big|_{\text{at } x=2} = -\frac{8}{4} - 2 = -6$$

$$f''(2) < 0$$

$\therefore$ $f(x)$ is maximum.

$\therefore$ Minimum of $f(x)$ in $[1, e]$

$$= \min\{f(1), f(e)\}$$

$$= \min\{2, f(e)\}$$

$$f(e) = 8\ln e - e^2 + 3$$

$$= 8 + 3 - e^2$$

$$= 11 - 7.38$$

$$f(e) = 3.61$$

$\therefore$ Minimum value of $f(x)$ occurs at $x = 1$.

End of Solution

Q.37 Let α, β be two non-zero real numbers and v_1, v_2 be two non-zero real vectors of size 3×1. Suppose that v_1 and v_2 satisfy $v_1^T v_2 = 0, v_1^T v_1 = 1$, and $v_2^T v_2 = 1$. Let A be the 3×3 matrix given by :

$$A = \alpha v_1 v_1^T + \beta v_2 v_2^T$$

The eigen values of A are _____.

(a) $0, \alpha, \beta$
(b) $0, \alpha + \beta, \alpha - \beta$
(c) $0, \frac{\alpha+\beta}{2}, \sqrt{\alpha\beta}$
(d) $0, 0, \sqrt{\alpha^2 + \beta^2}$

Ans. (a)

Given, $A = \alpha v_1 v_1^T + \beta v_2 v_2^T$

Post multiply by v_1 on both sides,

We get, $Av_1 = (\alpha v_1 v_1^T + \beta v_2 v_2^T) v_1$

$$Av_1 = \alpha v_1 v_1^T v_1 + \beta v_2 v_2^T v_1$$

$$A_{v1} = \alpha v_1$$

$\Rightarrow$ α is an eigen value of A.

Post multiply by v_2 on both sides,

We get $Av_2 = \alpha v_1 v_1^T v_2 + \beta v_2 v_2^T v_2$

$$Av_2 = \beta v_2$$

$\Rightarrow$ β is an eigen value of A.

$v_1 v_1^T$ and $v_2 v_2^T$ both are singular matrices.

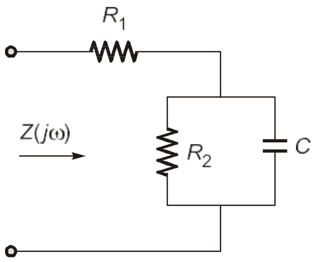
$\therefore$ A is also a singular matrix

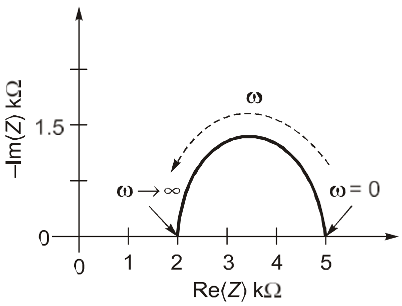
$\Rightarrow$ $|A| = 0 \Rightarrow \lambda_A = 0$

Hence, $\lambda_A = 0, \alpha, \beta$

End of Solution

Q.38 For the circuit shown, the locus of the impedance $Z(j\omega)$ is plotted as ω increases from zero to infinity. The values of R_1 and R_2 are:

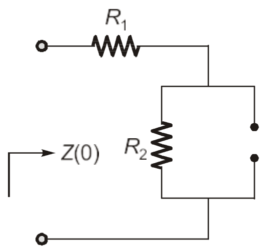




(a) $R_1 = 2$ kΩ, $R_2 = 3$ kΩ
(b) $R_1 = 5$ kΩ, $R_2 = 2$ kΩ
(c) $R_1 = 5$ kΩ, $R_2 = 2.5$ kΩ
(d) $R_1 = 2$ kΩ, $R_2 = 5$ kΩ

Ans. (a)

At $\omega = 0$ rad/s, $X_C = \dfrac{1}{\omega C} = \infty$



$$\text{Impedance, } Z(0) = R_1 + R_2$$

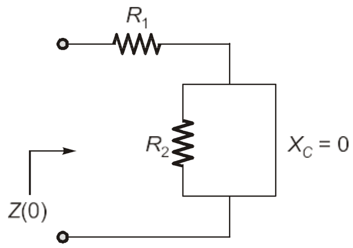
With the help of graph at $\omega = 0$.

$$Z(0) = 5 \text{ k}\Omega$$

$$R_1 + R_2 = 5 \text{ k}\Omega$$

At $\omega = \infty$ rad/s, $X_C = \dfrac{1}{\omega C}$

$$X_C = 0$$



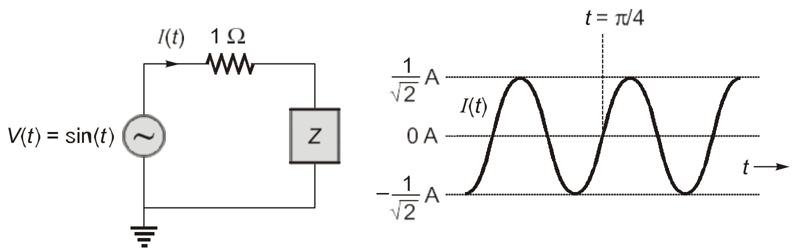
$$\text{Impedance, } Z(\infty) = R_1 = 2 \text{ k}\Omega$$

$$R_1 + R_2 = 5 \text{ k}\Omega$$

$$R_2 = 3 \text{ k}\Omega$$

End of Solution

Q.39 Consider the circuit shown in the figure with input $V(t)$ in volts. The sinusoidal steady state current $I(t)$ flowing through the circuit is shown graphically (where t is in seconds). The circuit element Z can be ________.



(a) a capacitor of 1 F
(b) an inductor of 1 H
(c) a capacitor of $\sqrt{3}$ F
(d) an inductor of $\sqrt{3}$ H

Ans. **(b)**

As the current $i(t)$ is lagging, element Z is inductor,

$$I = \frac{V}{Z_0}$$

$$I = \frac{\sin t}{Z_0}$$

Maximum value of current $i(t) = \dfrac{1}{\sqrt{2}}$

$\therefore$
$$|Z_0| = \sqrt{2}$$

$$Z_0 = R + jX_L = 1 + j\omega L$$

$$\sqrt{1+\omega^2 L^2} = \sqrt{2}$$

$$1 + \omega^2 L^2 = 2$$

given, $\omega = 1$ rad/sec

$$L^2 = 1$$

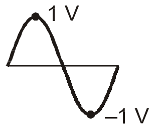
$$L = 1 \text{ H}$$

End of Solution

Q.40 Consider an ideal long channel *n*MOSFET (enhancement-mode) with gate length 10 μm and width 100 μm. The product of electron mobility (μ_n) and oxide capacitance per unit area (C_{ox}) is $\mu_n C_{ox}$ = 1 mA/V^2. The threshold voltage of the transistor is 1 V. For a gate-to-source voltage V_{GS} = [2 – sin(2t)] V and drain-to source voltage V_{DS} = 1 V (substrate connected to the source), the maximum value of the drain-to-source current is ________.

(a) 40 mA (b) 20 mA
(c) 15 mA (d) 5 mA

Ans. (c)



$$V_{GS} = [2 - \sin 2t] \text{ Volt}$$

$$V_{GS\,min} = 2 - 1 = 1 \text{ V}$$

$$V_{GS\,max} = 2 - (-1) = 3 \text{ V}$$

$$V_{DS} = 1 \text{ V}$$

$$V_{DS} < V_{GS} - V_t \quad \text{(Linear)}$$

$$I_D = \mu_n C_{ox} \frac{W}{L}\left\{(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right\}$$

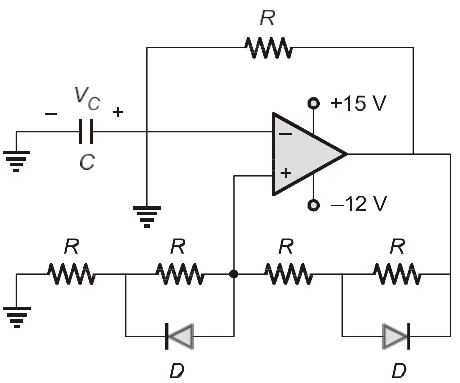
$$I_{D\,max} = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS\,max} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

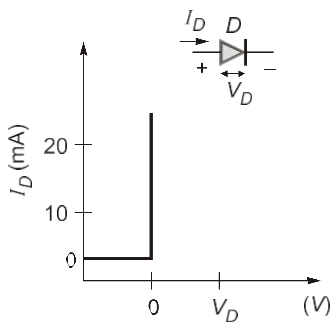
$$= 1\text{mA/V}^2 \times 10\left[(3-1)1 - \frac{1}{2} \times 1\right]$$

$$= 10[1.5] = 15 \text{ mA}$$

End of Solution

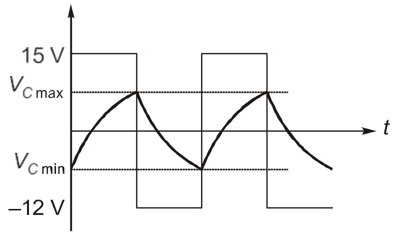
Q.41 For the following circuit with an ideal OPAMP, the difference between the maximum and the minimum values of the capacitor voltage (V_c) is __________.



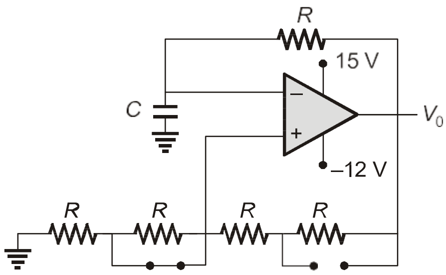


(a) 15 V (b) 27 V
(c) 13 V (d) 14 V

Ans. (c)



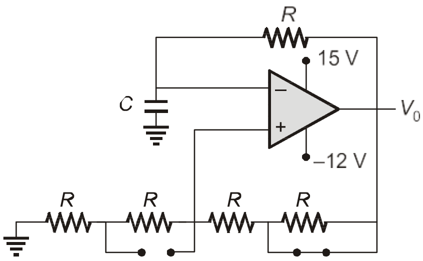
If V_0 = +15 V
D_1 is ON
D_2 is OFF



Capacitor charge upto V_{UT},

$$V_{C\,max} = V_{UT} = \frac{15 \times R}{R + 2R} = 5 \text{ V}$$

If V_0 = –12 V
D_1 is OFF
D_2 is ON
Capacitor discharges upto V_{LT}.

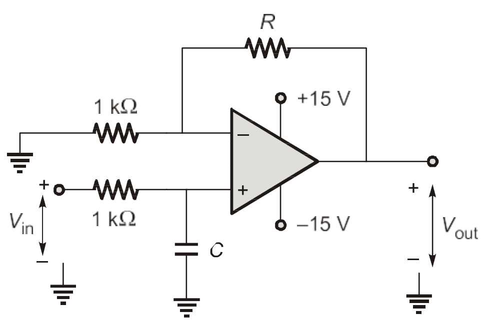


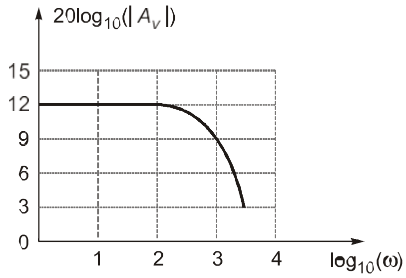
$$V_{C\,min} = V_{LT} = \frac{-12 \times 2R}{2R + R} = -8 \text{ V}$$

$$V_{C\,max} - V_{C\,min} = 5 - (-8) = 13 \text{ V}$$

End of Solution

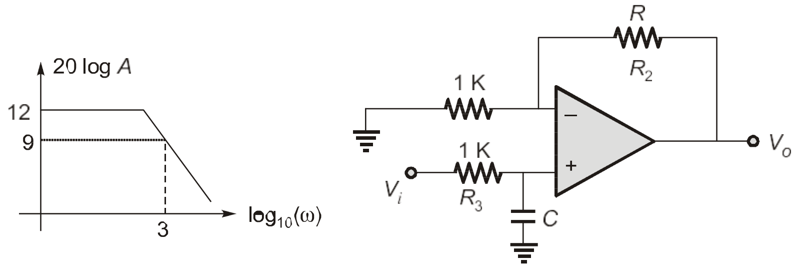
Q.42 A circuit with an ideal OPAMP is shown. The Bode plot for the magnitude (in dB) of the gain transfer function ($A_V(j\omega) = V_{out}(j\omega)/V_{in}(j\omega)$) of the circuit is also provided (here, ω is the angular frequency in rad/s). The values of R and C are ________.





(a) R = 3 kΩ, C = 1 μF
(b) R = 1 kΩ, C = 3 μF
(c) R = 4 kΩ, C = 1 μF
(d) R = 3 kΩ, C = 2 μF

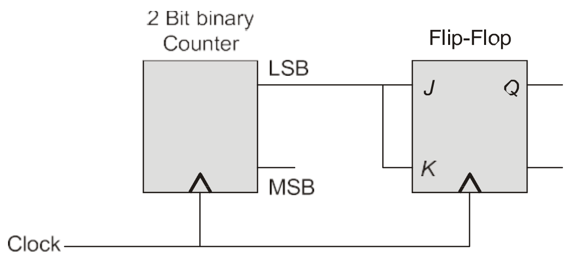
Ans. (a)



$$\text{Maximum gain} = 12 \text{ dB}$$
$$20 \times \log A_{max} = 12$$
$$A_{max} = 4$$
$$1 + \frac{R_2}{R_1} = 4 \Rightarrow R_2 = 3R_1$$
$$\Rightarrow \quad R = 3 \times 1 = 3 \text{ k}\Omega$$
$$\log_{10} \omega_c = 3 \Rightarrow \omega_c = 1000 \text{ rad/sec}$$
$$\omega_c = \frac{1}{R_3 C} \Rightarrow C = \frac{1}{R_3 \times \omega_C}$$
$$C = \frac{1}{1000 \times 1000} = 1 \mu\text{F}$$

End of Solution

Q.43 For the circuit shown, the clock frequency is f_o and the duty cycle is 25%. For the signal at the Q output of the Flip-Flop, _______.

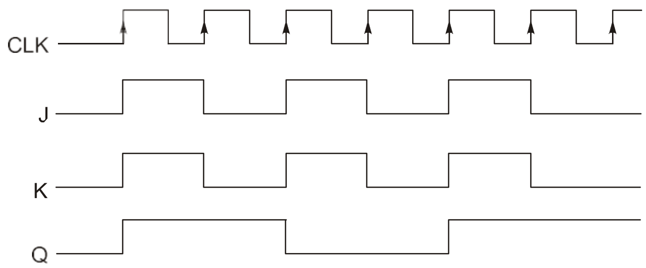


(a) frequency is $f_0/4$ and duty cycle is 50%
(b) frequency is $f_0/4$ and duty cycle is 25%
(c) frequency is $f_0/2$ and duty cycle is 50%
(d) frequency is f_0 and duty cycle is 25%

Ans. (a)

2 bit counter

MSB	$LSB(J, K)$
0	0
0	1
1	0
1	1



$\therefore$ Duty cycle = 50%

$$\text{Outeput frequency} = \frac{f_o}{4}$$

End of Solution

Q.44 Consider an even polynomial $p(s)$ given by

$$p(s) = s^4 + 5s^2 + 4 + K$$

where K is an unknown real parameter. The complete range of K for which $p(s)$ has all its roots on the imaginary axis is ________.

(a) $-4 \le K \le \frac{9}{4}$

(b) $-3 \le K \le \frac{9}{2}$

(c) $-6 \le K \le \frac{5}{4}$

(d) $-5 \le K \le 0$

Ans. **(a)**

Given: $p(s) = s^4 + 5s^2 + (4 + K)$

Routh's Table:

s^4	1	5	$(4+K)$
s^3	0	0	
s^2			
s^1			
s^0			

As all row of s^3 are zero,

$$A(s) = s^4 + 5s^2 + (4 + K)$$

$$\frac{dA(s)}{ds} = 4s^3 + 10s + 0$$

s^4	1	5	$(4+K)$
s^3	4	10	0
s^2	$\frac{10}{4} - \frac{5}{2}$	$(4+K)$	
s^1	$\frac{25-4(4+K)}{5/2}$		
s^0	$4+K$		

1st column of R-H table must be all positive.

i.e., $$\frac{25-4(4+K)}{5/2} > 0 \Rightarrow K < \frac{9}{4}$$

$$4 + K > 0 \Rightarrow K > -4$$

Range of K: $$-4 < K < \frac{9}{4}$$

End of Solution

Q.45 Consider the following series:

$$\sum_{n=1}^{\infty} \frac{n^d}{c^n}$$

For which of the following combinations of c, d values does this series converge?

(a) $c = 1, d = -1$
(b) $c = 2, d = 1$
(c) $c = 0.5, d = -10$
(d) $c = 1, d = -2$

Ans. **(b, d)**

$c = 2, d = 1$

$$\Sigma u_n = \Sigma \frac{n}{2^n}$$

Ratio test : $$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{n+1}{2^{n+1}} \times \frac{2^n}{n} = \frac{1}{2}$$

$$\frac{1}{2} < 2$$

$\therefore$ By ratio test, Σu_n is convergent.

(a)

$c = 1, d = -1$

$$\Sigma u_n = \Sigma \frac{1}{n} \text{ is divergent by } P\text{-test}$$

(c)

$c = 0.5, d = -10$

$$\Sigma u_n = \Sigma \frac{n^{-10}}{(0.5)^n}$$

Ratio test : $$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{(n+1)^{-10}}{(0.5)^{n+1}} \times \frac{(0.5)^n}{n^{10}}$$

$$= \frac{1}{0.5} = 2$$

$$2 > 1$$

Σu_n is divergent.

(d)

$c = 1, d = -2$

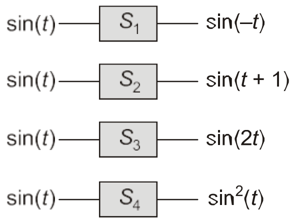
$$\Sigma u_n = \Sigma \frac{n^{-2}}{(1)^n} = \Sigma \frac{1}{n^2}$$

Σu_n is convergent by P-test.

End of Solution

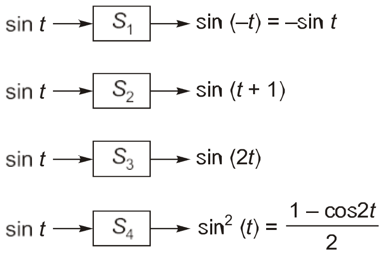
Q.46 The outputs of four systems (S_1, S_2, S_3, and S_4) corresponding to the input signal $\sin(t)$, for all time t, are shown in the figure.

Based on the given information, which of the four systems is/are definitely NOT LTI (linear and time-invariant)?



(a) S_1 (b) S_2
(c) S_3 (d) S_4

Ans. **(c, d)**



Since, LTI system does not change the frequency of sinusoidal input. So S_3 and S_4 are definitely not LTI as input and output sinusoidal frequencies are different.

End of Solution

Q.47 Select the CORRECT statement(s) regarding semiconductor devices.

(a) Electrons and holes are of equal density in an intrinsic semiconductor at equilibrium.
(b) Collector region is generally more heavily doped than Base region in a BJT.
(c) Total current is spatially constant in a two terminal electronic device in dark under steady state condition.
(d) Mobility of electrons always increases with temperature in Silicon beyond 300 K.

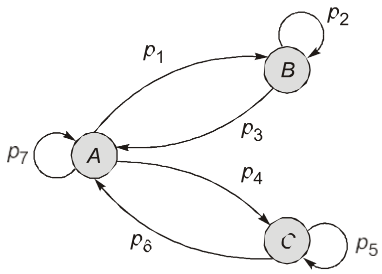
Ans. **(a, c)**

- In intrinsic semiconductor at equilibrium $n = p = n_i$
- Collector region is generally lightly doped than base region in a BJT.
- Total current is spatially constant in a two terminal electronic device, however individual currents vary spatially under dark and steady state condition.
- Beyond 300K, mobility of electron decreases with increases in temperature.

Hence, statement(s) : (a) and (c) are correct.

End of Solution

Q.48 A state transition diagram with states A, B, and C, and transition probabilities p_1, p_2, ..., , p_7 is shown in the figure (e.g., p_1 denotes the probability of transition from state A to B). For this state diagram, select the statement(s) which is/are universally true.



(a) $p_2 + p_3 = p_5 + p_6$
(b) $p_1 + p_3 = p_4 + p_6$
(c) $p_1 + p_4 + p_7 = 1$
(d) $p_2 + p_5 + p_7 = 1$

Ans. **(a, c)**

$$\left.\begin{aligned} p_2 + p_3 &= 1 \\ p_5 + p_6 &= 1 \end{aligned}\right\} \quad p_2 + p_3 = p_5 + p_6$$

Option (a) is correct.

$$p_1 + p_4 + p_7 = 1$$

Option (c) is correct.

End of Solution

Q.49 Consider a Boolean gate (D) where the output Y is related to the inputs A and B as, $Y = A + \bar{B}$, where + denotes logical OR operation. The Boolean inputs '0' and '1' are also available separately. Using instances of only D gates and inputs '0' and '1', __________ (select the correct option(s)).

(a) NAND logic can be implemented
(b) OR logic cannot be implemented
(c) NOR logic can be implemented
(d) AND logic cannot be implemented

Ans. **(a, c)**

$$F(A, B) = A + \bar{B}$$

As 0 and 1 are available.

$$F(0, B) = A + \bar{B} = 0 + \bar{B}$$
$$= \bar{B} \text{ (NOT)}$$
$$F(A + \bar{B}) = A + \bar{\bar{B}} = A + B$$
$$F(A + \bar{B}) = A + B \text{ (OR)}$$

With the combination of OR and NOT, NOR gate can be implemented.
Since NOR gate is universal logic gate, so all the functions can be implemented.
So, correct option is (a, c).

End of Solution

Q.50 Two linear time-invariant systems with transfer functions

$$G_1(s) = \frac{10}{s^2+s+1} \text{ and } G_2(s) = \frac{10}{s^2+s\sqrt{10}+10}$$

have unit step responses $y_1(t)$ and $y_2(t)$, respectively. Which of the following statements is/are true?

(a) $y_1(t)$ and $y_2(t)$ have the same percentage peak overshoot.
(b) $y_1(t)$ and $y_2(t)$ have the same steady-state value.
(c) $y_1(t)$ and $y_2(t)$ have the same damped frequency of oscillation.
(d) $y_1(t)$ and $y_2(t)$ have the same 2% settling time.

Ans. (a)

For system $G_1(s) = \frac{10}{s^2+s+1}$

Characteristics equation,

$$s^2 + s + 1 = 0$$

The standard characteristics equation is

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

On comparing, $\omega_n = 1, \ \xi = \frac{1}{2} = 0.5$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 1\sqrt{1-(0.5)^2} = 0.866$$

Settling time, $t_s = \frac{4}{\xi\omega_n} = 8$ sec

Steady-state error,

$$e_{ss} = \lim_{s\to 0} \frac{s\left(\frac{1}{s}\right)\cdot 10}{s^2+s+1} = 10$$

$$e_{ss} = 10$$

For system, $G_2(s) = \frac{10}{s^2+\sqrt{10}s+10}$

Characteristics equation,

$$s^2 + \sqrt{10}s + 10 = 0$$

Standard characteristics equation,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

On compairing, $\omega_n^2 = 10 \ \Rightarrow \ \omega_n = \sqrt{10}$

$$2\xi\omega_n = \sqrt{10} \ \Rightarrow \ \xi = 0.5$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = \sqrt{10}\sqrt{1-(0.5)^2} = 2.739$$

Settling time, $\quad t_s = \dfrac{4}{\xi\omega_n} = \dfrac{4}{0.5\sqrt{10}} = \dfrac{8}{\sqrt{10}} = 2.535$

Steady state error, $\quad e_{ss} = \lim\limits_{s\to 0} \dfrac{s\left(\dfrac{1}{s}\right)\cdot 10}{s^2 + \sqrt{10}s + 10} = 1$

$e_{ss} = 1$

Since 'ξ' value for both the system is same. So percentage peak overshoot for both system is same.

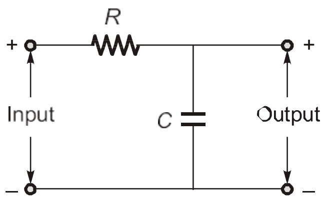
End of Solution

Q.51 Consider an FM broadcast that employs the pre-emphasis filter with frequency response

$$H_{pe}(\omega) = 1 + \frac{j\omega}{\omega_0},$$

where $\omega_0 = 10^4$ rad/sec.

For the network shown in the figure to act as a corresponding de-emphasis filter, the appropriate pair(s) of (R, C) values is/are ________.



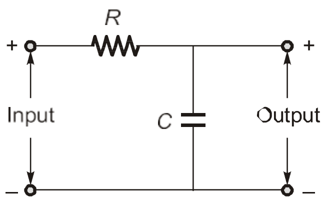
(a) R = 1 kΩ, C = 0.1 μF
(b) R = 2 kΩ, C = 1 μF
(c) R = 1 kΩ, C = 2 μF
(d) R = 2 kΩ, C = 0.5 μF

Ans. **(a)**

Given frequency response of pre-emphasis filter,

$$H_{Pe}(\omega) = 1 + \frac{j\omega}{\omega_o} \quad \text{where, } \omega_o = 10^4 \frac{\text{rad}}{\text{sec}}$$

Deemphasis filter given as



$$H_{De}(\omega) = \frac{1}{1 + j\omega RC} \longrightarrow |H_{De}(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

Relationship between pre-emphasis and De-emphasis systems is

$$|H_{Pe}(\omega)| = \frac{1}{|H_{De}(\omega)|}$$

$$|H_{Pe}(\omega)|^2 = \frac{1}{|H_{De}(\omega)|^2}$$

$$1+\left(\frac{\omega}{\omega_o}\right)^2 = 1 + (\omega RC)^2$$

$$\frac{1}{\omega_o} = RC \quad \Rightarrow RC = 10^{-4}$$

Option (a) only satisfies the required condition.

End of Solution

Q.52 A waveguide consists of two infinite parallel plates (perfect conductors) at a separation of 10^{-4} cm, with air as the dielectric. Assume the speed of light in air to be 3×10^8 m/s. The frequency/frequencies of TM waves which can propagate in this waveguide is/are ________.

(a) 6×10^{15} Hz (b) 0.5×10^{12} Hz
(c) 8×10^{14} Hz (d) 1×10^{13} Hz

Ans. **(a, c)**

The TM mode that propagates at the lowest cut-off frequency is TM_1 mode. Hence,

$$f_c|_{TM_1} = \frac{mc}{2a} \;; \quad m = 1$$

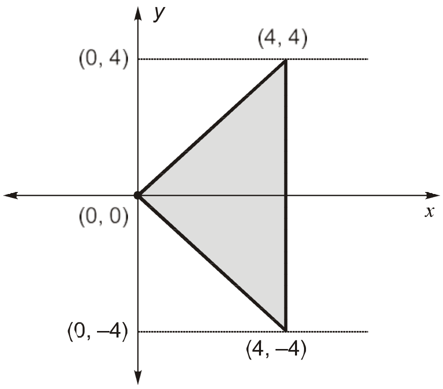
$$\therefore \quad f_c = \frac{3\times10^{10}}{2\times10^{-4}} = 1.5\times10^{14}$$

End of Solution

Q.53 The value of the integral

$$\iint_D 3(x^2 + y^2)\, dx\, dy,$$

where D is the shaded triangular region shown in the diagram, is _____ (rounded off to the nearest integer).



Ans. (512)

$$I = \int_0^4 \int_{-x}^{x} (3x^2 + 3y^2)\,dy\,dx$$

$$= \int_0^4 \left(3x^2 y + \frac{3y^3}{3}\right)_{-x}^{x} dx = \int_0^4 \left(3x^2(2x) + 2x^3\right) dx$$

$$= \int_0^4 (6x^3 + 2x^3)\,dx = \int_0^4 8x^3 dx$$

$$= \frac{8}{4}\left(x^4\right)_0^4 = 2 \times 4^4 = 512$$

End of Solution

Q.54 A linear 2-port network is shown in Fig. (a). An ideal DC voltage source of 10 V is connected across Port 1. A variable resistance R is connected across Port 2. As R is varied, the measured voltage and current at Port 2 is shown in Fig. (b) as a V_2 versus $-I_2$ plot. Note that for V_2 = 5 V, I_2 = 0 mA, and for V_2 = 4 V, I_2 = −4 mA.
When the variable resistance R at Port 2 is replaced by the load shown in Fig. (c), the current I_2 is _______ mA (rounded off to one decimal place).

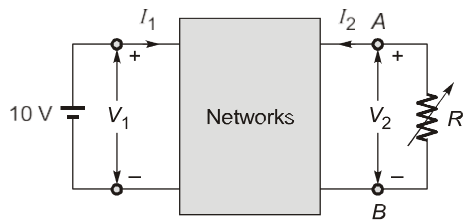


Fig. (a)

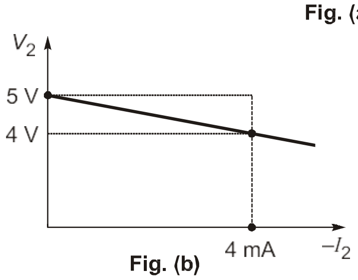


Fig. (b)

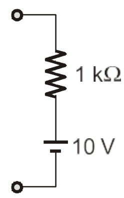
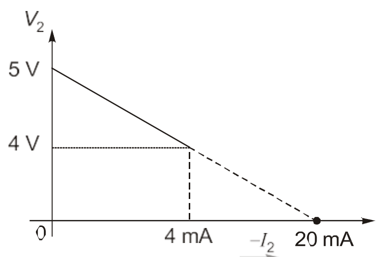


Fig. (c)

Ans. (4)

For I_2 = 0, $V_2 = V_{OC}$ = 5 V

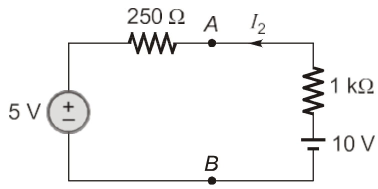
For Thevenin's resistance R_{th},



For $-I_2 = 20$ mA, $V_2 = 0$

$$I_{SC} = -I_2$$
$$I_{SC} = 20 \text{ mA}$$
$$R_{th} = \frac{V_{OC}}{I_{SC}} = \frac{5}{20} \times 10^3 = 250\ \Omega$$

Network is replaced by Thevenin's equivalent,



$$I_2 = \frac{10-5}{1.25 \times 10^3} = \frac{5}{1.25} \times 10^{-3} \text{A} = 4 \text{ mA}$$

End of Solution

Q.55 For a vector $\bar{x} = [x[0], x[1], \ldots, x[7]]$, the 8-point discrete Fourier transform (DFT) is denoted by $\bar{X} = \text{DFT}(\bar{x}) = [X[0], X[1], \ldots, X[7]]$, where

$$X[k] = \sum_{n=0}^{7} x[n] \exp\left(-j\frac{2\pi}{8}nk\right).$$

Here, $j = \sqrt{-1}$. If $\bar{x} = [1, 0, 0, 0, 2, 0, 0, 0]$ and $\bar{y} = \text{DFT}\,(\text{DFT}(\bar{x}))$, then the value of $y[0]$ is _____ (rounded off to one decimal place).

Ans. **(8)**

$$y(n) = \text{DFT}\,[\text{DFT}\,(x(n))]$$

Using Duality property

$$x(n) \xrightarrow{DFT} \xrightarrow{DFT} Nx(-k) = y(n)$$

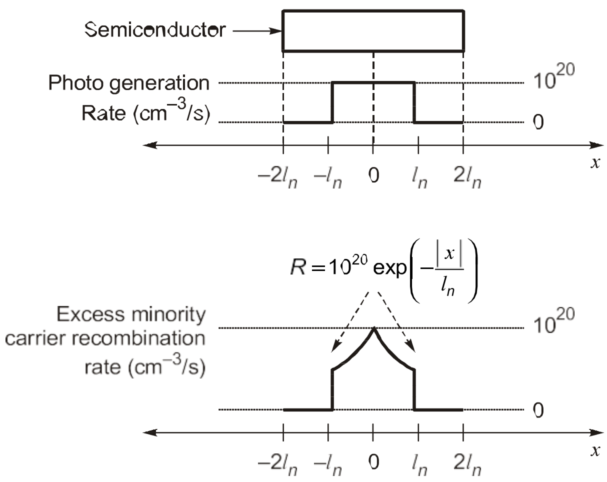
$$y(n) = Nx(-k) = Nx(N-k)$$
$$y(n) = 8\,(1, 0, 0, 0, 2, 0, 0, 0)$$

At $x = 0$

$$y(0) = 8 \times 1 = 8$$

End of Solution

Q.56 A p-type semiconductor with zero electric field is under illumination (low level injection) in steady state condition. Excess minority carrier density is zero at $x = \pm 2l_n$, where $l_n = 10^{-4}$ cm is the diffusion length of electrons. Assume electronic charge, $q = -1.6 \times 10^{-19}$ C. The profiles of photo-generation rate of carriers and the recombination rate of excess minority carriers (R) are shown. Under these conditions, the magnitude of the current density due to the photo-generated electrons at $x = +2l_n$ is _________ mA/cm^2 (rounded off to two decimal places).



Ans. (0.59)

$$\delta n(x) = R\tau_n = 10^{20} e^{-|x|/l_n} \tau_n$$

$\therefore$ $\delta n(\text{ln}) = 1020\ e^{-1}\ \tau_n$ ...(i)

$l_n \le x \le 2l_n$

Continuity equation in steady state,

$$D_n \frac{\partial^2 \delta n}{\delta x^2} + G - R = 0$$

Since, $\left.\begin{aligned} G &= 0 \\ R &= 0 \end{aligned}\right\} l_n \le x \le 2l_n$

$\therefore$ $$D_n \frac{\partial^2 \delta n}{\partial x^2} = 0$$

Whose solution is, $\delta n(x) = Ax + B$

Since at $x = 2l_n$: $\delta n(2l_n) = 0$ (given)

$$0 = A(2l_n) + B$$

$$A = -\frac{B}{2l_n}$$

$\therefore$ $$\delta n(x) = -\frac{B}{2l_n}x + B = B\left(1 - \frac{x}{2l_n}\right)$$ ...(ii)

$\therefore$ At $x = l_n$: equation (i) = equation (ii)

$$10^{20}\ e^{-1}\ \tau_n = B\left(1 - \frac{l_n}{2l_n}\right)$$

$\therefore$ $$B = 2 \times 10^{20}\ e^{-1}\ \tau_n$$

$\therefore$ $$\delta n(x) = 2 \times 10^{20} e^{-1}\ \tau_n \left(1 - \frac{x}{2l_n}\right)$$

$$l_n \le x \le 2l_n$$

∴ Electron diffusion current density :

$$|J_n|_{diff} = qD_n \frac{dn}{dx} = qD_n \times 2 \times 10^{20} \times e^{-1} \times \tau_n \left(0 - \frac{1}{2l_n}\right)$$

$$= \frac{1.6 \times 10^{-19} \times l_n^2 \times 2 \times 10^{20} \times e^{-1}}{2l_n}$$

$$= 1.6 \times 10^{-19} \times l_n \times 10^{20} \times e^{-1}$$

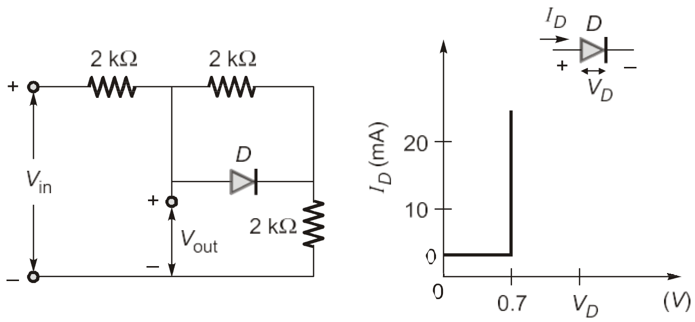
$$= 1.6 \times 10^{1} \times 1 \times 10^{-4} \times e^{-1} \qquad (l_n = 10^{-4} \text{ cm})$$

$$= 0.588 \text{ mA/cm}^2$$

$$= 0.59$$

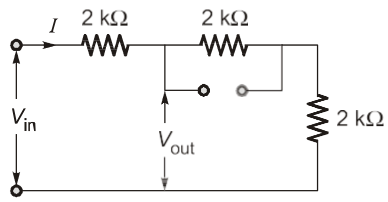
End of Solution

Q.57 A circuit and the characteristics of the diode (D) in it are shown. The ratio of the minimum to the maximum small signal voltage gain $\frac{\partial V_{out}}{\partial V_{in}}$ is ________ (rounded off to two decimal places).



Ans. **(0.75)**

When diode is OFF:

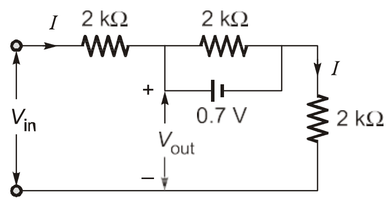


$$V_o = 4I$$

$$V_{in} = 6I$$

$$A_v = \frac{V_o}{V_{in}} = \frac{4}{6} = \frac{2}{3}$$

When diode is ON, voltage drop across diode is 0.7 V.



$$V_o = 0.7 + (2\ \text{k}\Omega)I$$
$$V_{in} = 0.7 + (4\ \text{k}\Omega)I$$
$$I = \frac{V_o - 0.7}{2\ \text{k}\Omega}$$
$$V_{in} = 0.7 + (4\ \text{k}\Omega)\cdot\frac{(V_o - 0.7)}{(2\ \text{k}\Omega)} = 0.7 + 2V_o - 1.4$$
$$V_o = \frac{V_{in} + 0.7}{2}$$
$$A'_v = \frac{\partial V_o}{\partial V_{in}} = \frac{1}{2}$$

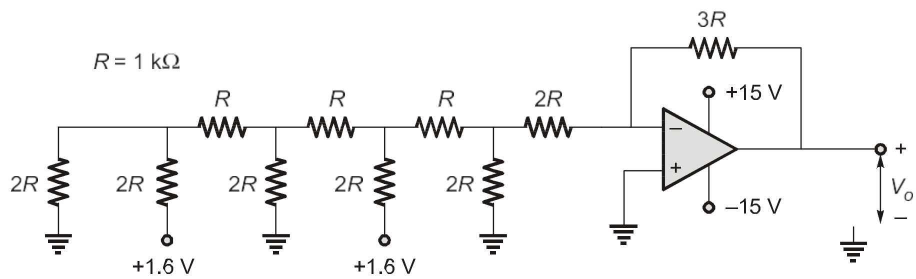
Maximum voltage gain, $A_v = \frac{2}{3}$

Minimum voltage gain, $A'_v = \frac{1}{2}$

$$\frac{(\partial V_{out}/\partial V_{in})_{min}}{(\partial V_{out}/\partial V_{in})_{max}} = \frac{A'_v}{A_v} = \frac{1}{2}\times\frac{3}{2} = \frac{3}{4} = 0.75$$

End of Solution

Q.58 Consider the circuit shown with an ideal OPAMP. The output voltage V_0 is ________V (rounded off to two decimal places).



Ans. (–0.5)

Analog output V_o = –Resolution × Gain × Decimal equivalent of binary data

$$\text{Resolution} = \frac{v_r}{2^n} = \frac{1.6}{2^4} = 0.1$$

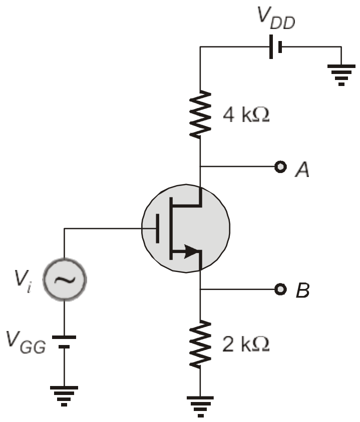
Decimal equivalent = 5

Gain = 1

$$V_o = -(0.1)\,(5)\,(1) = -0.5\ \text{V}$$

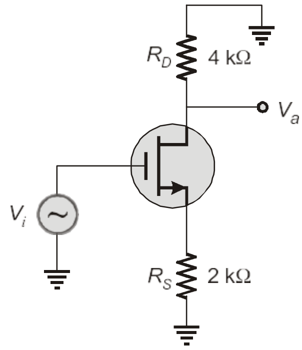
End of Solution

Q.59 Consider the circuit shown with an ideal long channel *n*MOSFET (enhancement mode, substrate is connected to the source). The transistor is appropriately biased in the saturation region with V_{GG} and V_{DD} such that it acts as a linear amplifier. v_i is the small-signal ac input voltage. v_A and v_B represent the small-signal voltages at the nodes *A* and *B*, respectively. The value of $\frac{v_A}{v_B}$ is ________ (rounded off to one decimal place).



Ans. (–2)

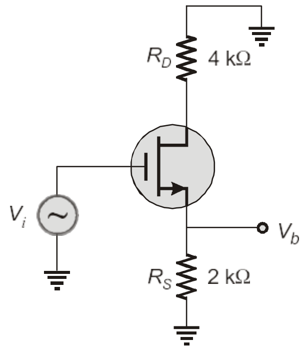
Consider V_a as an output then the small signal model is



CS in bypass amplifer

$$\text{Voltage gain } \frac{V_a}{V_{in}} = \frac{-g_m R_D}{1+g_m R_s} \qquad \text{...(1)}$$

Consider V_b as an output, then the small signal model,



CD amplifer

$$\text{Voltage gain } \frac{V_b}{V_{in}} = \frac{g_m R_s}{1+g_m R_s} \qquad \text{...(2)}$$

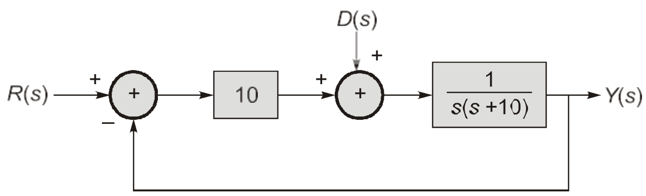
On dividing equation (1) and (2)

$$\frac{V_a / V_{in}}{V_b / V_{in}} = \frac{-R_D}{R_s}$$

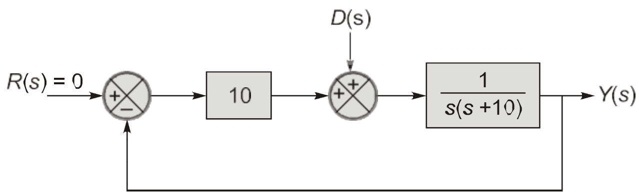
$$\frac{V_a}{V_b} = \frac{-4k}{2k} = -2$$

End of Solution

Q.60 The block diagram of a closed-loop control system is shown in the figure. $R(s)$, $Y(s)$, and $D(s)$ are the Laplace transforms of the time-domain signals $r(t)$, $y(t)$, and $d(t)$, respectively. Let the error signal be defined as $e(t) = r(t) - y(t)$. Assuming the reference input $r(t) = 0$ for all t, the steady-state error $e(\infty)$, due to a unit step disturbance $d(t)$, is ________ (rounded off to two decimal places).



Ans. (–0.1)



$$G_1(s) = 10, \quad G_2(s) = \frac{1}{s(s+10)}$$

$$\frac{E(s)}{D(s)} = \frac{-G_2(s)}{1+G_1(s)G_2(s)}$$

$$= \frac{\frac{-1}{s(s+10)}}{1+\left(10 \times \frac{1}{s(s+10)}\right)}$$

$$= \frac{-1}{s^2+10s+10}$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

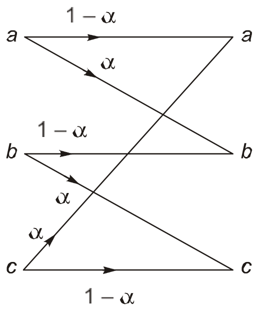
$$e_{ss} = \lim_{s\to 0} \frac{s(1/s)(-1)}{s^2+10s+10}$$

$$e_{ss} = \frac{-1}{10}$$

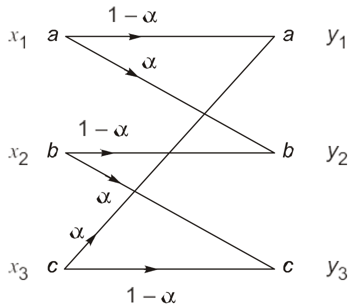
$$e_{ss} = -0.1$$

End of Solution

Q.61 The transition diagram of a discrete memoryless channel with three input symbols and three output symbols is shown in the figure. The transition probabilities are as marked. The parameter α lies in the interval [0.25, 1]. The value of .. for which the capacity of this channel is maximized, is ________ (rounded off to two decimal places).



Ans. (1)



Channel capacity, $C_s = \text{Max}[I(X\,;\,Y)]$

$$I(X\,;\,Y)] = H(Y) - H\left(\frac{Y}{X}\right)$$

where, $$H\left(\frac{Y}{X}\right) = -\sum_{i=1}^{3}\sum_{j=1}^{3} P(x_i, y_j)\log_2 P\left(\frac{y_j}{x_i}\right)$$

$$\left[P\left(\frac{Y}{X}\right)\right] = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array}\begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 1-\alpha & \alpha & 0 \\ 0 & 1-\alpha & \alpha \\ \alpha & 0 & 1-\alpha \end{bmatrix} \end{array}$$

For simplication convenience, let $[P(X)] = [1\ \ 0\ \ 0]$

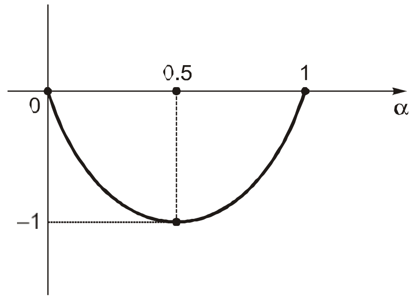
$$[P(X, Y)] = [P(X)]_d \cdot \left[P\left(\frac{Y}{X}\right)\right]$$

$$[P(X, Y)] = \begin{bmatrix} 1-\alpha & \alpha & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$H\left(\frac{Y}{X}\right) = -\{(1-\alpha)\log_2(1-\alpha) + \alpha\log_2\alpha\}$$

$$I(X\,;\,Y) = H(Y) + (1-\alpha)\log_2(1-\alpha) + \log_2\alpha$$
$$C_s = \text{Max}\{(X\,;\,Y)\}$$
$$= \text{Max}\{H(Y)\} + (1-\alpha)\log_2(1-\alpha) + \alpha\log_2\alpha$$
$$C_s = \log_2 3 + \underline{(1-\alpha)\log_2(1-\alpha) + \alpha\log_2\alpha}$$

$\alpha\log_2\alpha + (1-\alpha)\log(1-\alpha)$



C_s will be maximum at $\alpha = 0$ and 1.
Given $\alpha \in [0.25, 1]$
So, that $\alpha = 1$ will be the correct answer.

End of Solution

Q.62 Consider communication over a memoryless binary symmetric channel using a (7, 4) Hamming code. Each transmitted bit is received correctly with probability $(1-\epsilon)$, and flipped with probability ϵ. For each codeword transmission, the receiver performs minimum Hamming distance decoding, and correctly decodes the message bits if and only if the channel introduces at most one bit error.

For $\epsilon = 0.1$, the probability that a transmitted codeword is decoded correctly is ________ (rounded off to two decimal places).

Ans. **(0.85)**

Given (7, 4) Hamming code.
Number of bits in the transmitted codeword = 7.
Given is binary symmetric channel $\rightarrow P(0/1) = P(1/0)$
$$P(0/1) = P(1/0) = \epsilon = 0.1$$
Probability of correct decoding of codeword (P_c) = Probability of atmost one bit error
$$P_c = \text{No error (or) 1 bit error}$$
When 'n' bits transmitted, probability of getting error in 'r' bits is ${}^nC_r\, P^r\,(1-P)^{n-r}$
Where 'p' is bit error probability
$$P(1/0) = P(0/1) = 0.1$$
$$P_c = {}^7C_0(0.1)^0\,(1-0.1)^{7-0} + {}^7C_1(0.1)^1\,(1-0.1)^{7-1}$$
$$= (0.9)7 + 7 \times 0.1 \times (0.9)^6$$
$$P_c = 0.8503$$

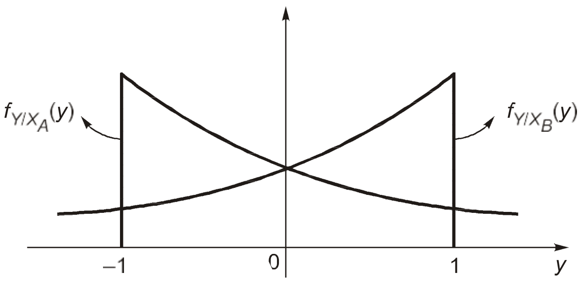
End of Solution

Q.63 Consider a channel over which either symbol x_A or symbol x_B is transmitted. Let the output of the channel Y be the input to a maximum likelihood (ML) detector at the receiver. The conditional probability density functions for y given x_A and x_B are:

$$f_{Y|x_A}(y) = e^{-(y+1)}\,u(y+1),$$
$$f_{Y|x_B}(y) = e^{(y-1)}\,(1-u(y-1)),$$

where $u(\cdot)$ is the standard unit step function. The probability of symbol error for this system is ________ (rounded off to two decimal places).

Ans. (0.23)



ML decoding $\rightarrow$ $f_{Y/X_A}(y) \underset{X_B}{\overset{X_A}{\gtrless}} f_{Y/X_B}(y)$

i.e. $f_{Y/X_A}(y) > f_{Y/X_B}(y) \rightarrow$ Decision favour of X_A

i.e. $f_{Y/X_A}(y) < f_{Y/X_B}(y) \rightarrow$ Decision favour of X_B

For $-1 < y < 0$ and $1 < y < \infty \rightarrow f_{Y/X_A}(y) > f_{Y/X_B}(y)$

For above internal decision in favour of X_A.

For $-\infty < y < -1$ and $0 < y < 1 \rightarrow f_{Y/X_B}(y) > f_{Y/X_A}(y)$

For above internal decision in favour of X_B.

$$P_e = P(X_A)\cdot P_{eX_A} + P(X_B)\cdot P_{eX_B}$$

$P_{eX_A} \rightarrow$ Probability of error X_A transmitted

$P_{eX_B} \rightarrow$ Probability of error X_B transmitted

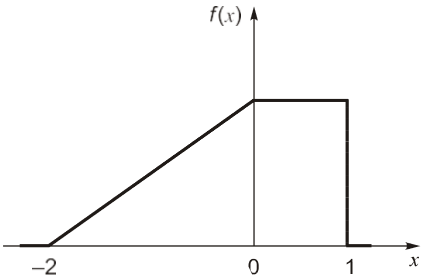
$$P_{eX_A} = \int_0^1 f_{Y/X_A}(y)\,dy = \int_0^1 e^{-(y+1)}\,u(y+1)\,dy$$
$$= \int_0^1 e^{-(y+1)}dy = e^{-1} - e^{-2} = 0.23$$

$$P_{eX_B} = \int_{-1}^0 f_{Y/X_B}(y)dy = \int_{-1}^0 e^{(y-1)}[1-u(y-1)]\,dy$$
$$= \int_{-1}^0 e^{y-1}dy = e^{-1} - e^{-2} = 0.23$$

$$P_e = P(X_A)\times 0.23 + P(X_B)\times 0.23 = 0.23[P(X_A) + P(X_B)]$$
$$P_e = 0.23$$

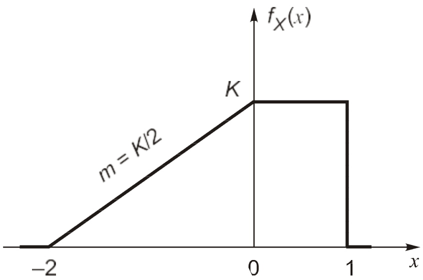
End of Solution

Q.64 Consider a real valued source whose samples are independent and identically distributed random variables with the probability density function, $f(x)$, as shown in the figure.



Consider a 1 bit quantizer that maps positive samples to value α and others to value β. If α^* and β^* are the respective choices for α and β that minimize the mean square quantization error, then $(\alpha^* - \beta^*)$ = ________ (rounded off to two decimal places).

Ans. **(1.167)**



$$\frac{1}{2}\times K\times 2+1\times K = 1 \quad \Rightarrow K = \frac{1}{2}=0.5$$

$$-2 \le x \le 0 \rightarrow f_X(x) = mx + C$$

$$f_X(x) = 0.25x + C$$

when $x = -2 \rightarrow f_X(x) = 0$

$$0 = 0.25 \times -2 + C \Rightarrow C = 0.5$$

$$f_X(x) = \frac{1}{4}x+\frac{1}{2} = -2\le x\le 0$$

$$f_X(x) = 0.5 \quad ; \quad 0 \le x \le 1$$

Quantizer output,

$$x_q = \alpha \quad ; \quad \text{for } 0 \le x \le 1$$

$$x_q = \beta \quad ; \quad \text{for } -2 \le x \le 0$$

Mean square quantization error,

$$\text{MSQ}[Q_e] = E[Q_e^2]$$

$$Q_e = (\text{Sampled value}) - (\text{quantized value})$$

$$= X - x_q$$

$$\text{MSQ}[Q_e] = \text{E}[(X - x_q)^2]$$

Quantization noise power, $N_o = \text{MSQ}[Q_e] = \int (X - x_q)^2 f_X(x)dx$

For $\quad -2 \le x \le 0 \rightarrow N_Q = \int_{-2}^{0} (x-\beta)^2 \times \left(\frac{1}{4}x+\frac{1}{2}\right)dx$

$$= \int_{-2}^{0} (x^2 + \beta^2 - 2x\beta)\left(\frac{x}{4} + \frac{1}{2}\right) dx$$

$$N_Q = \frac{\beta^2}{2} + \frac{2}{3}\beta - \frac{1}{3}$$

To find β value for which N_Q will be minimum $\rightarrow$

$$\frac{d_{NQ}}{d_\beta} = 0 \quad \Rightarrow \quad \frac{1}{2} \times 2\beta + \frac{2}{3} = 0$$

$$\beta = \frac{-2}{3}$$

For $0 \le x \le 1 \rightarrow$

$$N_Q = \int_0^1 (x - \alpha)^2 \times \frac{1}{2} dx$$

$$N_Q = \frac{1}{6}\left[(1-\alpha)^3 + \alpha^3\right]$$

To find 'α' value for which N_Q is minimum

$$\frac{d_{NQ}}{d\alpha} = 0 \quad \Rightarrow \quad \frac{1}{6}\left[3(1-\alpha)^2(-1) + 3\alpha^2\right] = 0$$

$$\alpha = \frac{1}{2}$$

Chosen α^* and β^* for which mean square quantization error (N_Q) is minimum will be $\frac{1}{2}$ and $\frac{-2}{3}$ respectively.

$$\alpha^* - \beta^* = \frac{1}{2} + \frac{2}{3} = \frac{7}{6} = 1.167$$

Q.65 In an electrostatic field, the electric displacement density vector, $\vec{D}$, is given by

$$\vec{D}(x, y, z) = (x^3\vec{i} + y^3\vec{j} + xy^2\vec{k}) \text{ C/m}^2,$$

where $\vec{i}, \vec{j}, \vec{k}$ are the unit vectors along x-axis, y-axis, and z-axis, respectively. Consider a cubical region R centered at the origin with each side of length 1 m, and vertices at ($\pm$0.5 m, $\pm$0.5 m, $\pm$0.5 m). The electric charge enclosed within R is ________ C (rounded off to two decimal places).

Ans. (0.5)

$$Q_{enclosed} = \int_v \rho_v dv = \int (\nabla \cdot D) dv$$

$$\nabla \cdot D = \frac{\partial}{\partial x}(x^3) + \frac{\partial}{\partial y}(y^3) + \frac{\partial}{\partial z}(xy^2)$$

$$\nabla \cdot D = (3x^2 + 3y^2) = 3(x^2 + y^2)$$

$$dv = dx dy dz$$

$$Q_{enc} = \int_V 3(x^2 + y^2)dxdydz$$

$$= 3\int_{x=-0.5}^{0.5}\int_{y=-0.5}^{0.5}\int_{z=-0.5}^{0.5}(x^2 + y^2)dx\,dy\,dz$$

$$= 3\left[\left\{\frac{x^3}{3}\right\}_{-0.5}^{0.5}\{y\}_{-0.5}^{0.5}\{z\}_{-0.5}^{0.5} + \{x\}_{-0.5}^{0.5}\left\{\frac{y^3}{3}\right\}_{-0.5}^{0.5}\{z\}_{-0.5}^{0.5}\right]$$

$$= \left((0.5)^3 - (-0.5)^3\right)(0.5 + 0.5)(0.5 + 0.5) \times 2$$

$$Q_{enc} = 0.5\text{ C}$$

End of Solution

■■■■