Properties of the Mobile Radio Propagation Channel
Statistical Description of Multipath Fading

The basic Rayleigh / Ricean model gives the PDF of envelope

- But: how fast does the signal fade?
- How wide in bandwidth are fades?

Typical system engineering questions:

- What is an appropriate packet duration, to avoid fades?
- How much ISI will occur?
- For frequency diversity, how far should one separate carriers?
- How far should one separate antennas for diversity?
- What is good a interleaving depth?
- What bit rates work well?
- Why can't I connect an ordinary modem to a cellular phone?

The models discussed in the following sheets will provide insight in these issues
The Mobile Radio Propagation Channel

A wireless channel exhibits severe fluctuations for small displacements of the antenna or small carrier frequency offsets.

Channel Amplitude in dB versus location (= time*velocity) and frequency
## Time Dispersion vs Frequency Dispersion

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Two distinct mechanisms

1.) Time dispersion
   - Time variations of the channel are caused by motion of the antenna
   - Channel changes every half a wavelength
   - Moving antenna gives Doppler spread
   - Fast fading requires short packet durations, thus high bit rates
   - Time dispersion poses requirements on synchronization and rate of convergence of channel estimation
   - Interleaving may help to avoid burst errors

2.) Frequency dispersion
   - Delayed reflections cause intersymbol interference
   - Channel Equalization may be needed.
   - Frequency selective fading
   - Multipath delay spreads require long symbol times
   - Frequency diversity or spread spectrum may help
Time dispersion of narrowband signal (single frequency)

Transmit: \( \cos(2\pi f_c t) \)

Receive: \( I(t) \cos(2\pi f_c t) + Q(t) \sin(2\pi f_c t) = R(t) \cos(2\pi f_c t + \phi) \)

I-Q phase trajectory

- As a function of time, \( I(t) \) and \( Q(t) \) follow a random trajectory through the complex plane
- Intuitive conclusion: Deep amplitude fades coincide with large phase rotations
Doppler shift and Doppler spread

- All reflected waves arrive from a different angle
- All waves have a different Doppler shift

The Doppler shift of a particular wave is

\[ f_0 = \frac{v}{c} f_c \cos \phi \]

Maximum Doppler shift:

\[ f_D = \frac{f_c v}{c} \]

**Joint Signal Model**

- Infinite number of waves
- First find distribution of angle of arrival, then compute distribution of Doppler shifts
- Uniform distribution of angle of arrival \( \phi \): \( f_\phi(\phi) = \frac{1}{2\pi} \)
- Line spectrum goes into continuous spectrum

Calculate
Doppler Spectrum

If one transmits a sinusoid, ... what are the frequency components in the received signal?

- Power density spectrum versus received frequency
- Probability density of Doppler shift versus received frequency
- The Doppler spectrum has a characteristic U-shape.
- Note the similarity with sampling a randomly-phased sinusoid
- No components fall outside interval \([f_c - f_D, f_c + f_D]\)
- Components of + \(f_D\) or -\(f_D\) appear relatively often
- Fades are not entirely “memory-less”
Derivation of Doppler Spectrum

The power spectrum $S(f)$ is found from

$$S(f_0) = \bar{p} \int [f_{\Phi}(\phi)G(\phi) + f_{\Phi}(-\phi)G(-\phi)] \left| \frac{d\phi}{df} \right|_{f_0}$$

where

$f_{\Phi}(\phi) = 1/(2\pi)$ is the PDF of angle of incidence

$G(\phi)$ the antenna gain in direction $\phi$

$\bar{p}$ local-mean received power, and

$$f_0 = f_c \left( 1 + \frac{v}{c} \cos \phi \right)$$

One finds

$$\left| \frac{d\phi}{df} \right| = \frac{1}{\sqrt{f_D^2 - (f - f_c)^2}}$$
Vertical Dipole

A vertical dipole is omni-directional in horizontal plane

\[ G(\theta) = 1.5 \]

We assume

- Uniform angle of arrival of reflections
- No dominant wave

Received Power Spectrum

\[ S(f) = \frac{3}{2\pi} \frac{1}{\sqrt{f_D^2 - (f - f_c)^2}} \]

- Doppler spectrum is centered around \( f_c \)
- Doppler spectrum has width \( 2 f_D \)
How do systems handle Doppler Spreads?

• **Analog**
  - Carrier frequency is low enough to avoid problems (random FM)

• **GSM**
  - Channel bit rate well above Doppler spread
  - TDMA during each bit / burst transmission the channel is fairly constant.
  - Receiver training/updating during each transmission burst
  - Feedback frequency correction

• **DECT**
  - Intended to pedestrian use: only small Doppler spreads are to be anticipated for.

• **IS95**
  - Downlink: Pilot signal for synchronization and channel estimation
  - Uplink: Continuous tracking of each signal
Autocorrelation of the signal

We now know the Doppler spectrum,

but ... how fast does the channel change?

Wiener-Kinchine Theorem:

- Power density spectrum of a random signal is the Fourier Transform of its autocorrelation
- Inverse Fourier Transform of Doppler spectrum gives autocorrelation of $I(t)$, or of $Q(t)$
Derivation of Autocorrelation of $I$-$Q$ components

We find the auto-correlation

$$g(\tau) = \mathbb{E} \, I(t)I(t+\tau) = \mathbb{E} \, Q(t)Q(t+\tau)$$

$$= \int_{f_c-f_D}^{f_c+f_D} S(f) \cos 2\pi(f - f_c)\tau \, df$$

- Auto-correlation depends on $S(f)$, thus it depends on the distribution of angle of arrival
- $g(\tau = 0) = \text{local-mean power}: \ g(0) = \mathbb{E} \, I^2(t) = \bar{p}$
For uniform angle of arrival

For a U-shaped Doppler spectrum, the auto-correlation function is

\[ g(\tau) = E I(t)I(t + \tau) = \overline{p} J_0\left(2\pi f_D \tau\right) \]

where

- \( J_0 \) is zero-order Bessel function of first kind: integral over \( \{\cos(z \sin x)\} \)
- \( f_D \) is the maximum Doppler shift
- \( \tau \) is the time difference

Note that the correlation is a function of distance or time offset:

\[ f_D \tau = f_c \frac{v}{c} \tau = \frac{d}{\lambda} \]

where

- \( d \) is the antenna displacement during \( \tau \), with \( d = v \tau \)
- \( \lambda \) is the carrier wavelength (30 cm at 1 GHz)
Relation between I and Q phase

S.O. Rice: Cross-correlation

\[ h(\tau) = E I(t)Q(t + \tau) = -E Q(t)I(t + \tau) \]

\[ = \int_{f_c-f_D}^{f_c+f_D} S(f) \sin 2\pi(f - f_c)\tau \, df \]

For uniformly distributed angle of arrival \( \phi \)

- Doppler spectrum \( S(f) \) is even around \( f_c \)
- Crosscorrelation \( h(\tau) \) is zero for all \( \tau \)
- \( I(t_1) \) and \( Q(t_2) \) are independent

For any distribution of angles of arrival

- \( I(t_1) \) and \( Q(t_1) \) are independent \( \{\tau = 0: h(0) = 0\} \)
PDF of the real amplitude $R$

For the amplitude $r_1$ and $r_2$

$$f(r_1, r_2, \varphi_1, \varphi_2) = |J|f(I_1, I_2, Q_1, Q_2)$$

$$f(r_1, r_2) = \int \int \frac{r_1 r_2}{\sigma^4(1-\rho^2)} \exp \left[ -\frac{r_1^2 + r_2^2}{2\sigma^2(1-\rho^2)} \right] I_0 \left( \frac{r_1 r_2 \rho}{\sigma^2(1-\rho^2)} \right)$$

Correlation: \[ \rho = \frac{J_0^2(\omega_c \tau)}{1 + (\Delta \omega T_{rms})^2} \]

Note: get more elegant expressions for complex amplitude $H$

$$R^2 = I^2 + Q^2$$

$NB$: \[ f(r_2 | r_1) = \frac{f(r_1, r_2)}{f(r_1)} = \text{Rician}(r_2) \]
Autocorrelation of amplitude $R^2 = I^2 + Q^2$

$$E R(t) R(t + \tau) = \int \int_{0,0}^{\infty \infty} r_1 r_2 f(r_1, r_2) dr_1 dr_2$$

The solution is known as the hypergeometric function $F(a, b; c; z)$

$$E R(t) R(t + \tau) = \frac{\pi}{2} \sigma^2 F\left(-\frac{1}{2}, -\frac{1}{2}; 1; \rho^2\right)$$

or, in good approximation, ..

$$E R(t) R(t + \tau) = \frac{\pi}{2} \frac{-p}{4} \left[ I + \frac{I(t) I(t + \tau)^2}{\left(\frac{E I(t) I(t + \tau)}{p^2}\right)^2}\right]$$
Autocorrelation of amplitude $R^2 = I^2 + Q^2$

\[ E R(t)R(t + \tau) = \frac{\pi}{2} p \left[ 1 + \frac{1}{4} \left( \frac{E I(t)I(t+\tau)}{p^2} \right)^2 \right] \]

Result for Autocovariance of Amplitude

- Remove mean-value and normalize
- Autocovariance

\[ C = J_0^2 \left( 2\pi f_D \tau \right) \]
Amplitude \( r(t_0) \) and Derivative \( r'(t_0) \) are uncorrelated

\[
E_r(t) r'(t) = \frac{d}{d\tau} E_r(t) r(t + \tau) = \frac{d J_0^2(2\pi f_D \tau)}{d\tau}
\]

\[
E_h(t) h'(t) = \frac{d}{d\tau} E_h(t) h(t + \tau) = \frac{d J_0(2\pi f_D \tau)}{d\tau}
\]

Correlation is 0 for \( \tau = 0 \)
Simulation of multipath channels

Jakes' Simulator for narrowband channel

generate the two bandpass “noise” components by adding many sinusoidal signals. Their frequencies are non-uniformly distributed to approximate the typical U-shaped Doppler spectrum.

$N$ Frequency components are taken at

$$f_i = f_m \cos \frac{2\pi i}{2(2N+1)}$$

with $i = 1, 2, \ldots, N$

All amplitudes are taken equal to unity. One component at the maximum Doppler shift is also added, but at amplitude of $1/\sqrt{2}$, i.e., at about 0.707. Jakes suggests to use 8 sinusoidal signals.
How to handle fast multipath fading?

**Analog**
User must speak slowly

**GSM**
- Error correction and interleaving to avoid burst errors
- Error detection and speech decoding
- Fade margins in cell planning

**DECT**
Diversity reception at base station

**IS95**
- Wideband transmission averages channel behaviour
  This avoids burst errors and deep fades
Frequency Dispersion
Frequency Dispersion

• *Frequency dispersion* is caused by the delay spread of the channel

• *Frequency dispersion* has no relation to the velocity of the antenna
Frequency Dispersion: Delay Profile

Typical sample of impulse response $h(t)$

If we transmit a pulse $\delta(t)$ we receive $h(t)$

**Delay profile**: PDF of received power: "average $h^2(t)$"

Local-mean power in delay bin $\Delta\tau$ is $\overline{p} \cdot f_\tau(\tau)\Delta\tau$
RMS Delay Spread and Maximum delay spread

Definitions

$n$-th moment of delay spread

\[ \mu_n = \int_0^\infty \tau^n f_\tau(\tau) d\tau \]

RMS value

\[ T_{RMS} = \frac{1}{\mu_0^2} \sqrt{\mu_0 \mu_2 - \mu_1^2} \]
Typical Values of Delay Spreads

**Macrocells** \( T_{\text{RMS}} < 8 \mu\text{sec} \)
- GSM (256 kbit/s) uses an equalizer
- IS-54 (48 kbit/s): no equalizer
- In mountainous regions delays of 8 \( \mu\text{sec} \) and more occur
  GSM has some problems in Switzerland

**Microcells** \( T_{\text{RMS}} < 2 \mu\text{sec} \)
- Low antennas (below tops of buildings)
Typical values of delay spread

**Picocells 1 .. 2 GHz:**

\[ T_{\text{RMS}} < 50 \text{ nsec} - 300 \text{ nsec} \]

- Home: 50 nsec
- Shopping mall: 100 - 200 nsec
- Railway station: 200 - 450 nsec
- Office block: 100 - 400 nsec

- Indoor: often 50 nsec is assumed
- DECT (1 Mbit/s) works well up to 90 nsec

Outdoors, DECT has problem if range > 200 .. 500
Typical Delay Profiles

1) Exponential

2) Uniform Delay Profile
   • Experienced on some indoor channels
   • Often approximated by N-Ray Channel

3) Bad Urban

For a bandlimited signal, one may sample the delay profile. This gives a tapped delay line model for the channel, with constant delay times.
COST 207 Typical Urban Reception (TU6)

COST 207 describes typical channel characteristics for over transmit bandwidths of 10 to 20 MHz around 900MHz. TU-6 models the terrestrial propagation in an urban area. It uses 6 resolvable paths.

COST 207 profiles were adapted to mobile DVB-T reception in the E.U. Motivate project.

<table>
<thead>
<tr>
<th>Tap number</th>
<th>Delay (us)</th>
<th>Power (dB)</th>
<th>Fading model</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>-3</td>
<td>Rayleigh</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0</td>
<td>Rayleigh</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>-2</td>
<td>Rayleigh</td>
</tr>
<tr>
<td>4</td>
<td>1.6</td>
<td>-6</td>
<td>Rayleigh</td>
</tr>
<tr>
<td>5</td>
<td>2.3</td>
<td>-8</td>
<td>Rayleigh</td>
</tr>
<tr>
<td>6</td>
<td>5.0</td>
<td>-10</td>
<td>Rayleigh</td>
</tr>
</tbody>
</table>
## COST 207 A sample fixed channel

<table>
<thead>
<tr>
<th>Tap number</th>
<th>Delay (μs)</th>
<th>Amplitude r</th>
<th>Level (dB)</th>
<th>Phase (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.050</td>
<td>0.36</td>
<td>-8.88</td>
<td>-2.875</td>
</tr>
<tr>
<td>2</td>
<td>0.479</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.621</td>
<td>0.787</td>
<td>-2.09</td>
<td>2.182</td>
</tr>
<tr>
<td>4</td>
<td>1.907</td>
<td>0.587</td>
<td>-4.63</td>
<td>-0.460</td>
</tr>
<tr>
<td>5</td>
<td>2.764</td>
<td>0.482</td>
<td>-6.34</td>
<td>-2.616</td>
</tr>
<tr>
<td>6</td>
<td>3.193</td>
<td>0.451</td>
<td>-6.92</td>
<td>2.863</td>
</tr>
</tbody>
</table>

How do systems handle delay spreads?

**Analog**
- Narrowband transmission

**GSM**
- Adaptive channel equalization
- Channel estimation training sequence

**DECT**
- Use the handset only in small cells with small delay spreads
- Diversity and channel selection can help a little bit
  - “pick a channel where late reflections are in a fade”

**IS95**
- Rake receiver separately recovers signals over paths with excessive delays

**DTV and Digital Audio Broadcasting**
- OFDM multi-carrier modulation
  - The radio channel is split into many narrowband (ISI-free) subchannels
Correlation of Fading vs. Frequency Separation

- When do we experience frequency-selective fading?
- How to choose a good bit rate?
- Where is frequency diversity effective?

In the next slides, we will ...
- give a model for $I$ and $Q$, for two sinusoids with time and frequency offset,
- derive the covariance matrix for $I$ and $Q$,
- derive the correlation of envelope $R$,
- give the result for the autocovariance of $R$, and
- define the coherence bandwidth.
Inphase and Quadrature-Phase Components

Consider two (random) sinusoidal signals

• Sample 1 at frequency $f_1$ at time $t_1$
• Sample 2 at frequency $f_2 = f_1 + \Delta f$ at time $t_2$

Effect of displacement on each phasor:

Spatial or temporal displacement:
• Phase difference due to Doppler
• Spectral displacement
• Phase difference due to excess delay

Mathematical Treatment:

• $I(t)$ and $Q(t)$ are jointly Gaussian random processes
• $(I_1, Q_1, I_2, Q_2)$ is a jointly Gaussian random vector
Consider complex $H_1 = I_1 + jQ_1$, $H_2 = I_2 + jQ_2$ as a jointly Gaussian random vector
Multipath Channel

- Transmit signal \( s(t) \)
  \[ s(t) = a_n \exp\{j\omega_c t\} \]

- Received signal \( r(t) = a_n g_n(t) + n(t), \)

TO DO make math consistent

\[
 r(t) = \sum_{i=0}^{I_w-1} a_n D_i \exp\{j(\omega_c + n\omega_s)(t - T_i)\} + n(t)
\]

**Channel model:**
\( I_w \) reflected waves have the following properties:
- \( D_i \) is the amplitude
- \( T_i \) is the delay

**Signal parameters:**
- \( a_n \) is the data
- \( \omega_c \) is the carrier frequency
Doppler Multipath Channel

Correlation between $p$-th and $q$-th derivative

$$E\left[ \frac{dg(t) \, dg^*(t)}{d^p t \, dt} \right] = E\left[ \sum_{i=0}^{I_w-1} \sum_{k=0}^{I_w-1} D_i D_k \exp\{j \omega_c (T_i - T_k)\} \right]$$

TO DO make math consistent
Doppler Multipath Channel

\[ \lim_{I_w \to \infty} E \sum_{i \in A_{\tau \theta}} D_i D_i^* = \frac{1}{2\pi T_{rms}} \exp \left( -\frac{\tau}{T_{rms}} \right) d\tau d\theta \]

\[ A_{\tau \theta} = \{ i : \tau < T_i < \tau + dt \} \cap \{ i : 2\pi f_{\Delta} \cos \theta < \omega_i < 2\pi f_{\Delta} \cos(\theta + d\theta) \} \]

TO DO make math consistent

\[ E g_n^{(p)} g_m^{* (q)} = \frac{\omega_c^{p+q} v_s^{p+q}}{c^{p+q}} \frac{(-1)^q j^{p+q}}{2\pi T_{rms}} \int_0^{\infty} \int_0^{2\pi} \cos^{p+q} \theta d\theta d\tau \]

\[ = \int_0^{\infty} \exp \left( -\frac{\tau}{T_{rms}} \right) d\tau \]
Random Complex-Gaussian Amplitude

Special case

\[ E(v_n v_m) = \frac{1}{1 + j(n-m)T_{\text{rms}} \omega_s} \]

TO DO make math consistent

• This defines the covariance matrix of subcarrier amplitudes at different frequencies

• This is used in OFDM for channel estimation
Coherence Bandwidth

**Definition:** Coherence. Bandwidth is the frequency separation for which the correlation coefficient is down from 1 to 0.5. Thus \( 1 = 2\pi(f_1 - f_2) T_{RMS} \)

- so Coherence Bandwidth \( BW = 1 (2\pi T_{RMS}) \)
- We derived this for an exponential delay profile

**Another rule of thumb:**
- ISI affects BER if \( T_b > 0.1 T_{RMS} \)

**Conclusion:**
- Either keep transmission bandwidth much smaller than the coherence bandwidth of the channel, or
- use signal processing to overcome ISI, e.g.
  - Equalization
  - DS-CDMA with rake
  - OFDM
Frequency and Time Dispersion

Model: Each wave has its own angle and excess delay

- Antenna motion changes phase
- Changing carrier frequency changes phase

The scattering environment is defined by

- Angles of arrival
- Excess delays in each path
- Power of each path
Scatter Function of a Multipath Mobile Channel

Gives power as function of

- Doppler Shift (derived from angle of arrival $\phi$)
- Excess Delay

Example of a scatter plot

Horizontal axes:
- x-axis: Excess delay time
- y-axis: Doppler shift

Vertical axis:
- z-axis: received power
Freq. and time selective channels

\[ C_{R_1,R_2} \approx \rho^2 = \frac{J_0^2(2\pi \frac{\nu}{\lambda} \tau)}{1 + 4 \pi^2 (f_1 - f_2)^2 T_{RMS}^2} \]

Special cases

- Zero displacement / motion: \( \tau = 0 \)
- Zero frequency separation: \( \Delta f = 0 \)
Effects of fading on modulated radio signals
Effects of Multipath (I)

Narrowband

Wideband

OFDM

Time

Frequency

JPL
Effects of Multipath (II)

DS-CDMA

Frequency Hopping

MC-CDMA

JPL
Time Dispersion Revisited

The duration of fades
In the next slides we study the temporal behavior of fades.

Outline:

• # of level crossings per second
  • Model for level crossings
  • Derivation
    • Model for $I$, $Q$ and derivatives
    • Model for amplitude and derivative
  • Discussion of result
• Average non-fade duration
• Effective throughput and optimum packet length
• Average fade-duration
Two state model

Very simple mode: channel has two-states
- Good state: Signal above “threshold”, BER is virtually zero
- Poor state: “Signal outage”, BER is 1/2, receiver falls out of sync, etc

Markov model approach:
- Memory-less transitions
- Exponential distribution of sojourn time

This model may be sufficiently realistic for many investigations
Level crossings per second

- Av. number of crossings per sec = [av. interfade time]^{-1}

\[ N = \frac{dr}{dt} f_R(r) \]

Number of level crossing per sec is proportional to
- speed \( r' \) of crossing \( R \) (derivative \( r' = \frac{dr}{dt} \))
- probability of \( r \) being in \([R, R + dR]\). This Prob = \( f_R(r) \ dr \)
Derivation of Level Crossings per Second

\[ N^+ = \int_0^\infty r' f(r', R) \, dr' \]

- Random process \( r' \) is derivative of the envelope \( r \) w.r.t. time
- Note: here we need the joint PDF; not the conditional PDF \( f(r' \mid r=R) \)
- We derive \( f(r' \mid r=R) \) from \( f(r', r) \) and \( f(i_1, i_2, q_1, q_2) \)
In-phase, quadrature and the derivatives are Jointly Gaussian with covariance matrix:

\[
\Gamma_{I,Q,i,Q} = \begin{bmatrix}
\bar{p} & 0 & 0 & b_1 \\
0 & \bar{p} & -b_1 & 0 \\
0 & -b_1 & b_2 & 0 \\
b_1 & 0 & 0 & b_2
\end{bmatrix}
\]

Here \(b_n\) is \(n\)-th moment of Doppler spectral power density

\[
b_n = (2\pi)^n \int_{f_D}^{f_c+f_D} S(f) (f - f_c)^n \, df
\]

For uniform angles of arrival

\[
b_n = \begin{cases} 
\bar{p} (2\pi f_D)^n \frac{(n-1)!!}{n!} & \text{for } n \text{ even} \\
0 & \text{for } n \text{ odd}
\end{cases}
\]

where \((n-1)!! = 1 \times 3 \times 5 \ldots \times (n-1)\)
Joint PDF of R, R', \( \Phi \), \( \Phi' \)

.... After some algebra ...

\[
f(r, \hat{r}, \phi, \hat{\phi}) = \frac{r^2}{4 \pi^2 p b_2} \exp\left\{-\frac{1}{2} \left( \frac{r^2}{p} + \frac{r'^2}{b_2} + \frac{r^2 \phi'^2}{b_2} \right) \right\}
\]

So

- \( r \) and \( r' \) are independent
- phase \( \phi \) is uniform
- Fast phase change only occur when amplitude is small

Averaging over \( \phi \) and \( \phi' \) gives

- \( r \) is Rayleigh
- \( r^\wedge \) is zero mean Gaussian with variance \( b_2 \)
We insert this pdf in

\[ N^+ = \int_0^\infty r' f(r', R = R_0) \, dr' \]

We find

\[ N^+ = \frac{\sqrt{2\pi} f_D}{\sqrt{\eta}} e^{-\frac{1}{\eta}} \]

where

\[ \eta \text{ is the fade margin } \eta = 2 p / (R_0^2) \]

Level crossing rate has a maximum for thresholds \( R_0 \) close to the mean value of amplitude. Similarly for Ricean fading

\[ N^+ = \sqrt{\pi p} f_D f_R(R_0) \]
Average Fade / Nonfade Duration

- Fade durations are relevant to choose packet duration
- Duration depend on
  - Fade margin $\gamma = 2 \frac{p}{R_0^2}$
  - Doppler spread

Average fade duration $T_F$  
$N^- T_F = \Pr(R \leq R_0)$

Average nonfade duration $T_{NF}$  
$N^+ T_{NF} = \Pr(R \geq R_0)$
Average nonfade duration

\[ T_{NF} = \frac{1 - \text{Pr(outage)}}{\text{LevelCrossRate}} = \frac{\sqrt{\eta}}{\sqrt{2\pi} f_D} \]

- Average nonfade duration is inversely proportional to Doppler spread
- Average nonfade duration is proportional to fade margin
How to handle long fades when the user is stationary?

Analog
- Disconnect user

GSM
- Slow frequency hopping
- Handover, if appropriate
- Power control

DECT
- Diversity at base station
- Best channel selection by handset

IS95
- Wide band transmission avoids most deep fades (at least in macro-cells)
- Power control

Wireless LANs
- Frequency Hopping, Antenna Diversity
Optimal Packet length

We want to optimize

\[
\text{Effective throughput} = \frac{\#\text{User bits}}{\#\text{Packet bits}} \times \text{Prob}(\text{success})
\]

This requires a trade off between

- Short packets:
  much overhead (headers, sync. words etc).

- Long packets:
  may experience fade before end of packet.
Optimal Packet length

- At 1200 bit/s, throughput is virtually zero: Almost all packets run into a fade. Packets are too long.

- At high bit rates, many packets can be exchanged during non-fade periods. Intuition:
  Packet duration $< <$ Av. nonfade duration
Derivation of Optimal Packet length

- Assume exponential, memoryless nonfade durations
  (This is an approximation: In reality many nonfade periods have duration of \( \frac{\lambda}{2} \), due to U-shaped Doppler spectrum)

- Successful reception if
  1) Above threshold at start of packet
  2) No fade starts before packet ends

Formula:

\[
P_{\text{succ}} = \exp \left( -\frac{T_L}{T_{NF}} \right) = \exp \left( -\frac{1}{\eta} - \frac{\sqrt{2\pi} f_D T_L}{\sqrt{\eta}} \right)
\]

with \( T_L \) packet duration

Large fade margin: second term dominates:
- performance improves only slowly with increasing \( \eta \)
- Outage probability is too optimistic
Average fade duration

\[ T_F = \frac{\sqrt{\eta}}{\sqrt{2\pi} f_D} \left[ \exp\left(\frac{1}{\eta}\right) - 1 \right] \]

- AFD is inversely proportional to Doppler spread
- Fade durations rapidly reduce with increasing margin, but time between fades increases much slower
- Experiments: For large fade margins: exponentially distributed fade durations
- Relevant to find length of error bursts and design of interleaving

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Conclusion

• The multipath channel is characterized by two effects: Time and Frequency Dispersion

• Time Dispersion effects are proportional to speed and carrier frequency

• System designer need to anticipate for channel anomalies
### Statistical properties of a Rayleigh signal

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Probability Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I(t_1)$ and $Q(t_1)$</td>
<td>i.i.d. Gaussian, Zero mean</td>
</tr>
<tr>
<td>$I(t_1)$ and $Q(t_2)$</td>
<td>Correlated jointly Gaussian</td>
</tr>
<tr>
<td>$I(t_2)$ given $I(t_1)$</td>
<td>Gaussian</td>
</tr>
<tr>
<td>Amplitude $r$</td>
<td>Rayleigh</td>
</tr>
<tr>
<td>$r(\tau_2)$ given $r(\tau_1)$</td>
<td>Ricean</td>
</tr>
<tr>
<td>$r'(\tau_1)$ derivative $r$</td>
<td>Gaussian, Independent of amplitude</td>
</tr>
<tr>
<td>Phase</td>
<td>Uniform</td>
</tr>
<tr>
<td>Derivative of Phase</td>
<td>Gaussian, Dependent on amplitude</td>
</tr>
<tr>
<td>Power</td>
<td>Exponential</td>
</tr>
</tbody>
</table>

The nice thing about jointly Gaussian r.v.s is that the covariance matrix fully describes the behavior.
Taylor Expansion of Amplitude

\[ r(t) = a_n \sum_{i=0}^{I_w-1} D_i \exp \{ j(\omega_c)(t - T_i) + j\omega_i t \} + n(t) \]

- Rewrite the Channel Model as follows

\[ r(t) = a_n g_n(t) \exp \{ j\omega_c t \} + n(t) \]

- Taylor expansion of the amplitude

\[ g_n(t) = g_n^{(0)} + g_n^{(1)}(t - \Delta t) + g_n^{(2)}(t - \Delta t)^2/2 + \ldots \]

\( g_n^{(q)} \): the \( q \)-th derivative of amplitude with respect to time, at instant \( t = \Delta t \).

\( g_n^{(p)} \) is a complex Gaussian random variable

- To address frequency separation \( n\omega_s \) (later):

\[ g_n^{(q)} = \sum_{i=0}^{I_w-1} (j\omega_i)^q D_i \exp \{ -j(\omega_c + n\omega_s T_i + j\omega_i \Delta t) \} \]
Doppler Multipath Channel

Simplified baseband model for narrowband linear modulation

Received signal \( r(t) = a_n g(t) + n(t) \), with time-varying channel amplitude \( g(t) \)

\[
g^{(q)}(t) = \sum_{i=0}^{I_w-1} \left( j \omega_i \right)^q D_i \exp\{-j(\omega_c + n \omega_s)T_i + j \omega_i t\}
\]

Channel model:

\( I_w \) reflected waves, each has the following properties:

- \( D_i \) is the amplitude
- \( \omega_i \) is the Doppler shift
- \( T_i \) is the path delay

Signal parameters:

- linear modulation
- \( a_n \) is the user data
- \( \omega_c \) is the carrier frequency

Note \( g(t) \) is not an impulse response;
It is the channel amplification and phase

- \( n(t) \) is the noise
Doppler Multipath Channel

$p$-th Derivative $g^p(t)$ of the channel w.r.t. to time $t$

$$\frac{d^p g(t)}{dt^p} = j^p \sum_{i=0}^{I_w-1} D_i \omega_i^p \exp\{j \omega_c T_i + j \omega_i t\}$$

All derivatives are gaussian rv’s if $I_w \to \infty$ and $D_i$ i.i.d.

The covariance matrix fully describes the statistical properties
Doppler Multipath Channel

Correlation between $p$-th and $q$-th derivative

\[
E \left[ \frac{dg^{(p)}(t)}{d^p t} \frac{dg^{*(q)}(t)}{d t} \right] = \left[ -1^q j^{p+q} \sum_{i=0}^{I_w-1} \sum_{k=0}^{I_w-1} \omega_i^p \omega_k^q D_i D_k \exp\{ j \omega_c (T_i - T_k) + j (\omega_i - \omega_k) t \} \right]
\]
Doppler Multipath Channel

\[ \lim_{I_w \to \infty} E \sum_{i \in A_{\tau \theta}} D_i D_i^* = \frac{1}{2\pi T_{rms}} \exp \left( - \frac{\tau}{T_{rms}} \right) d\tau d\theta \]

\[ A_{\tau \theta} = \left\{ i : \tau < T_i < \tau + dt \right\} \cap \left\{ i : 2\pi f_\Delta \cos \theta < \omega_i < 2\pi f_\Delta \cos(\theta + d\theta) \right\} \]

\[ E g_n^{(p)} g_m^{* (q)} = \frac{\omega_c^{p+q} v_s^{p+q}}{c^{p+q}} \frac{(-1)^q j^{p+q}}{2\pi T_{rms}} \int_0^{2\pi} \int_0^{\cos^{p+q} \theta} \theta d\theta \]

\[ \int_0^\infty \exp \left( - \frac{\tau}{T_{rms}} \right) d\tau \]

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Random Complex-Gaussian Amplitude

It can be shown that for $p + q$ is even

$$E v_n^{(p)} v_m^{*(q)} = \left(2\pi f_D\right)^{p+q} \frac{(p + q - 1)!!}{(p + q)!!} \frac{(-1)^q j^{p+q}}{1 + j(n - m)T_{rms} \omega_s}$$

and 0 for $p + q$ is odd.

- This defines the covariance matrix of subcarrier amplitudes and derivatives,
- OFDM: allows system modeling and simulation between the input of the transmit I-FFT and output of the receive FFT.